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Abstract

This paper provides an approach to estimation and inference for non-linear conditional mean panel data models, in the presence of cross-sectional dependence. We modify the common correlated effects (CCE) correction of Pesaran (2006) to filter out the interactive unobserved multifactor structure. The estimation can be carried out using non-linear least squares, by augmenting the set of explanatory variables with cross-sectional averages of both linear and non-linear terms. We propose pooled and mean group estimators, derive their asymptotic distributions, and show the consistency and asymptotic normality of the coefficients of the model. The features of the proposed estimators are investigated through extensive Monte Carlo experiments. We apply our method to estimate UK banks' wholesale funding costs and explore the non-linear relationship between public debt and output growth.

Key words: Non-linear panel data model, cross-sectional dependence, common correlated effects estimator.

JEL classification: C31, C33, C51.

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1 Introduction

Panel data are increasingly used in empirical work in economics due to the richness of their structure enabling modelling of various sources of heterogeneity. A standard feature of the most panel data models is the use of severe restrictions on the cross-sectional dependence across units. In the case of fixed effects, this takes the form of independence across units. Under a random effects assumption, a time effect term severely restricts the nature of cross-sectional dependence. To alleviate this problem, a large literature has developed over the last twenty years, modelling dependence using an interactive effects structure. This takes the form of a factor model allowing, through the presence of unobserved factor variables, for rich cross-sectional covariance structures. However, their presence results in biases in estimation and inference, by introducing endogeneity between regressors and error terms.

There are two main approaches to modify estimation and inference in unobservable multifactor setting. The first explicitly models the factor structure, usually uses some form of principal component (PC) estimation, and estimates it jointly with the rest of the model. This is carefully and exhaustively analysed, in Bai (2009), while a number of other papers extend and refine it in a variety of ways (Charbonneau (2012), Fernández-Val and Weidner (2016), Moon and Weidner (2017)).

The second approach treats factors as nuisance terms. They are not of interest in themselves, rather only as vehicles for introducing a parsimonious means of modelling cross-sectional dependence. As a result, the aim is to remove their effects by proxying them using observable counterparts. These take the form of cross-sectional averages of dependent and explanatory variables. This approach, referred as common correlated effects (CCE), was introduced by Pesaran (2006), and refined in a number of papers (Chudik and Pesaran (2013), Kapetanios, Mitchell, and Shin (2014), Westerlund and Urbain (2015)).

The two approaches have been repeatedly compared and a number of conclusions have been reached. A good summary of those is provided by Westerlund and Urbain (2015). Perhaps the main finding, is that in a restricted context, it can be theoretically shown that PC is superior when the panel regression coefficient of interest is equal to zero while CCE is superior otherwise.

All the above work has focused on linear panel data models. However, the importance of various forms of nonlinearity has been noted repeatedly in the literature. As a result, a number of papers have considered corrections for cross-sectional dependence in this context. For the PC approach, work by Chen, Fernandez-Val, and Weidner (2014) has addressed estimation and inference in a very general non-linear model using an iterative estimation. This estimator is in the spirit of the general approach of Dominitz and Sherman (2005) and Pastorello, Patilea, and Renault (2003) although no recourse to these general results is made, possibly suggesting that a potentially strong condition that guarantees convergence of the iterative scheme, can be relaxed.

CCE has been extended to limited dependent variable and quantile regression models by Boneva, Linton, and Vogt (2015, 2016). This work, which is of considerable interest, depends on the validity of a rank condition, which essentially requires that the number of factors is equal or smaller to that of the available cross-sectional proxies. This condition underlies the work of Pesaran (2006) but its necessity is sidestepped by the use of alternative weak assumptions.

An alternative class of non-linear models has recently received much attention in empirical applications. The recent financial crisis has raised the need for modelling financial phenomena related to the crises, using models with non-linear terms in their conditional mean. Baum, Checherita-Westphal, and Rother (2013) explore the non-linear impact of public debt on economic growth using a dynamic threshold panel model in the Euro area. Relatedly, Delatte, Fouquau, and Portes (2014) explore the nonlinearities in the European sovereign bond markets during debt crisis by employing a smooth transition panel model (González, Teräsvirta, and van Dijk (2005)). In financial stability front, Jude (2010) relates financial development and economic growth using González, Teräsvirta, and van Dijk (2005)'s smooth transition panel model. A potential non-linear relationship between banks' solvency and their cost of funding has started to gain attention from regulatory authorities only recently, as in Dent, Hacıoğlu Hoke, and Panagiotopoulos (2017). Combining financial stability with monetary policy, Floro and van Roye (2015) recently investigate the threshold effects of financial stability on monetary rules with a similar model to that of Hansen (1999). To investigate the transmission of fiscal policy, Auerbach and Gorodnichenko (2013) adopt a smooth transition panel method as a part of their analysis to provide empirical evidence of fiscal policy spill overs from one country to another in OECD countries. More recently, Fotiou (2017) explores the potential impact of fiscal consolidations based on tax increases on debt-to-GDP ratio using a panel smooth transition VAR model.

To the best of our knowledge, no work on CCE corrections exists for such models, while the PC approach may be both too cumbersome computationally and have less good performance, if the work of Westerlund and Urbain (2015) is of relevance in this case. This paper aims to fill that gap. We consider estimation and inference of models with non-linear but continuous conditional mean. We find that the Pesaran (2006)'s standard CCE estimator does not work unless a rank condition; which is comparable to, but stricter than, that needed for linear models, holds. We proceed to modify the standard CCE estimator by including non-linear proxies and show that this modification works to a similar degree of generality to that of the standard CCE in linear settings. We provide Monte Carlo evidence to support our theoretical finding and conclude by presenting two empirical illustrations: estimating banks' wholesale funding costs and exploring the non-linear relationship between public debt and output growth.

The rest of the paper is organised as follows: Section 2 presents the setting and our main theoretical results. Section 3 presents extensions of our estimators while Sections 4 and 5 present our Monte Carlo and empirical results. The final section concludes while proofs and additional results are relegated to the Appendix.

2 Theory

In this section, we present the model and set out the proposed estimators and their properties. Let the observed data $y_{i,t}$ and $x_{i,t}$, be generated by

$$y_{i,t} = y_{i,t}\left(\boldsymbol{\gamma}, \boldsymbol{\beta}\right) = g\left(\boldsymbol{x}_{i,t}, \gamma_0\right)' \boldsymbol{\beta}_i + u_{i,t},\ i = 1, ..., N,\ t = 1, ..., T \tag{1}$$

$$\boldsymbol{X}_i = \boldsymbol{F}\boldsymbol{\Pi}_i + \boldsymbol{V}_i,\ i = 1, ..., N \tag{2}$$

where $\boldsymbol{X}_i = (\boldsymbol{x}_{i,1}, \ldots, \boldsymbol{x}_{i,T})'$, $\boldsymbol{F} = (\boldsymbol{f}_1, \ldots, \boldsymbol{f}_T)'$, $\boldsymbol{V}_i = (\boldsymbol{v}_{i,1}, \ldots, \boldsymbol{v}_{i,T})'$ and $g\left(\boldsymbol{x}_{i,t}, \boldsymbol{\gamma}\right) = \left(g_1\left(\boldsymbol{x}_{i,t}, \boldsymbol{\gamma}\right), \ldots, g_l\left(\boldsymbol{x}_{i,t}, \boldsymbol{\gamma}\right), \boldsymbol{x}'_{i,t}\right)'$, $\boldsymbol{\gamma} \in \boldsymbol{\Gamma}$, where $\boldsymbol{\Gamma}$ is a compact subset of $\mathbb{R}^s$. Function $g_p(\cdot)$ for $p = 1, \ldots, l$ is a continuous non-linear function of $\boldsymbol{x}_i$ and $\boldsymbol{\gamma}$. Define $\boldsymbol{X}_i(\boldsymbol{\gamma}) = \left(g\left(\boldsymbol{x}_{i,1}, \boldsymbol{\gamma}\right), ..., g\left(\boldsymbol{x}_{i,T}, \boldsymbol{\gamma}\right)\right)'$ and $\boldsymbol{y}_i = (y_{i,1}, \ldots, y_{i,T})'$. Further, assume that

$$u_{i,t} = \boldsymbol{f}_t \boldsymbol{\delta}_i + \epsilon_{i,t}. \tag{3}$$

where $\boldsymbol{f}_t$ and $\boldsymbol{x}_{i,t}$ are $m \times 1$ and $k \times 1$ vectors respectively. This is a general model which allows for both linear and non-linear relations between $y_{i,t}$ and $\boldsymbol{x}_{i,t}$. As is standard in this literature, we allow for coefficient heterogeneity by setting $\boldsymbol{\beta}_i = \boldsymbol{\beta} + \boldsymbol{v}_{\boldsymbol{\beta},\boldsymbol{i}}$. We keep $\boldsymbol{\gamma}$ homogeneous as a baseline case although later on we discuss case that relax this assumption. We make the following assumptions.

Assumption 1 *The $m_f \times 1$ vector of common factors, $\boldsymbol{f}_t$, is covariance stationary with absolutely summable autocovariances, distributed independently of $\epsilon_{i,t}$ and $\boldsymbol{v}_{i',t'}$ for all i, i', t and t'.*

Assumption 2 *The individual-specific errors $\varepsilon_{i,t}$ and $\boldsymbol{v}_{i',t'}$ are distributed independently for all i, i', t and t'. They follow linear, stationary processes with absolutely summable autocovariances.*

Assumption 3 *The unobserved factor loadings are independently and identically distributed across i, and are independent of the individual-specific errors $\epsilon_{i,t}$ and $\boldsymbol{v}_{i,t}$ and the common factors $\boldsymbol{f}_t$ for all i and t. They have finite means and finite variances. In particular, we have*

$$\boldsymbol{\delta}_i = \boldsymbol{\delta} + \boldsymbol{v}_{\delta,i}$$

where $\boldsymbol{v}_{\delta,i} \sim iid\left(0, \boldsymbol{\Omega}_{\boldsymbol{\delta}}\right)$.

Assumption 4 *$\boldsymbol{\beta}_i$ follow the random coefficient specification,*

$$\boldsymbol{\beta}_i = \boldsymbol{\beta}_0 + \boldsymbol{v}_{\boldsymbol{\beta},\boldsymbol{i}} \text{ with } \boldsymbol{v}_{\boldsymbol{\beta},\boldsymbol{i}} \sim iid\left(\mathbf{0}, \boldsymbol{\Omega}_{\boldsymbol{\beta}}\right) \tag{4}$$

where $\|\boldsymbol{\beta}\| < K$ and $\boldsymbol{v}_{\boldsymbol{\beta},\boldsymbol{i}}$ are distributed independently of all other stochastic quantities.

Assumption 5 *(i) $g\left(\boldsymbol{x}_{i,t}, \gamma\right)$ is continuous and has bounded first derivative in $\boldsymbol{\gamma} \in \boldsymbol{\Gamma}$, where $\boldsymbol{\Gamma}$ is a compact subset of $\mathtt{R}^s$ and $\boldsymbol{\gamma}_0$ is an interior point of $\gneqq$. (ii) $\sup_x \left|g\left(\boldsymbol{x}, \boldsymbol{\gamma}_1\right) - g\left(\boldsymbol{x}, \boldsymbol{\gamma}_2\right)\right| = O\left(f(n)\right)$, if and only if $\left|\boldsymbol{\gamma}_1 - \boldsymbol{\gamma}_2\right| = O\left(f(n)\right)$ for any $f(n)$, such that $\lim_{n\to\infty} f(n) = 0$.*

Estimation of this model can proceed in a variety of related ways. We proceed by using non-linear least squares, by concentrating out $\boldsymbol{\beta}$. In particular, define the quadratic loss function given by

$$L(\boldsymbol{\gamma}, \boldsymbol{\gamma_0}) = \left(\sum_{i=1}^{N} \sum_{t=1}^{T} \left(y_{i,t} - g\left(\boldsymbol{x}_{i,t}, \gamma\right)' \boldsymbol{\beta}\left(\boldsymbol{\gamma}, \gamma_0\right)\right)^2 \right).$$

where

$$\boldsymbol{\beta}\left(\boldsymbol{\gamma}, \boldsymbol{\gamma}_0\right) \equiv \boldsymbol{\beta}\left(\boldsymbol{\gamma}, \boldsymbol{\gamma}_0, \boldsymbol{\beta}_0\right) = \left(\sum_{i=1}^{N} \boldsymbol{X}_i(\gamma)' \boldsymbol{M}_Z \boldsymbol{X}_i(\gamma) \right)^{-1} \left(\sum_{i=1}^{N} \boldsymbol{X}_i(\gamma)' \boldsymbol{M}_Z \boldsymbol{y}_i\left(\boldsymbol{\gamma}_0, \boldsymbol{\beta}_0\right) \right)$$

and $\boldsymbol{M}_Z = \boldsymbol{I} - \boldsymbol{Z}\left(\boldsymbol{Z}'\boldsymbol{Z}\right)^{-1}\boldsymbol{Z}'$ where $\boldsymbol{Z}$ is a regressor matrix containing proxies that account for the presence of the unboserved factor $\boldsymbol{F}$ in $u_{i,t}$ and $x_{i,t}$. Obviously the presence of this factor causes the usual bias, in unadjusted estimators, that is addressed by Pesaran (2006) in the case of linear panel models. The need for proxies arises since $\boldsymbol{f}_t$ is unobserved. Note that $E\left[\sum_{i=1}^{N}\sum_{t=1}^{T}\left(y_{i,t}-g\left(\boldsymbol{x}_{i,t},\boldsymbol{\gamma}\right)'\boldsymbol{\beta}\right)^2\right]$ is at a minimum when $\boldsymbol{\beta}=\boldsymbol{\beta}_0$ and $\boldsymbol{\gamma}=\boldsymbol{\gamma}_0$. Therefore, we are looking for proxies such that $p\lim_{N,T\to\infty}\boldsymbol{\beta}\left(\boldsymbol{\gamma}_0,\boldsymbol{\gamma}_0\right)=\boldsymbol{\beta}_0$.

One naive solution is to use $\boldsymbol{Z}=\bar{\boldsymbol{Z}}=(\bar{\boldsymbol{y}},\bar{\boldsymbol{X}})$, where $\bar{\boldsymbol{X}}=(\bar{\boldsymbol{x}}_1,...,\bar{\boldsymbol{x}}_T)'$, $\bar{\boldsymbol{x}}_t=\frac{1}{N}\sum_{i=1}^{N}\boldsymbol{x}_{i,t}$, $\bar{\boldsymbol{y}}=(\bar{y}_1,...,\bar{y}_T)'$, $\bar{y}_t=\frac{1}{N}\sum_{i=1}^{N}y_{i,t}$. In general, in what follows overbars denote averaging across i. These are the proxies proposed by Pesaran (2006) in the case of linear models. It is easy to see that if $\boldsymbol{F}\subset\boldsymbol{F}\bar{\boldsymbol{\Pi}}$, then this is a valid set of proxies. Note that in this instance $\bar{y}_t$ is not needed. However, if this rank condition does not hold the approach fails. The issue arises because $\bar{\boldsymbol{y}}$ contains $\bar{\boldsymbol{X}}\left(\boldsymbol{\gamma}\right)=\frac{1}{N}\sum_{i=1}^{N}\boldsymbol{X}_i(\boldsymbol{\gamma})$ which is not, anymore, a linear function of the factors. The problem is resolved if we augment $\bar{\boldsymbol{Z}}$ with $\bar{\boldsymbol{X}}\left(\boldsymbol{\gamma}\right)$, resulting in $\bar{\boldsymbol{Z}}_\gamma=(\bar{\boldsymbol{Z}},\bar{\boldsymbol{X}}\left(\boldsymbol{\gamma}\right))$. Of course, the true value of $\boldsymbol{\gamma}$ is not known. However, all calculations are carried out assuming a candidate value for $\boldsymbol{\gamma}$ which essentially solves this issue.

We wish to explore the theoretical properties of the estimators of $\boldsymbol{\gamma}_0$ and $\boldsymbol{\beta}_0$ based on the above discussion. They are given by

$$\hat{\boldsymbol{\gamma}}_P=\arg\min_{\boldsymbol{\gamma}}L(\boldsymbol{\gamma},\boldsymbol{\gamma}_0),$$

$$\hat{\boldsymbol{\beta}}_P=\boldsymbol{\beta}_P\left(\hat{\boldsymbol{\gamma}}_P\right)$$

and

$$\hat{\boldsymbol{\beta}}_{MG}\left(\hat{\boldsymbol{\gamma}}_P\right)=\frac{1}{N}\sum_{i=1}^{N}\hat{\boldsymbol{\beta}}_i\left(\hat{\boldsymbol{\gamma}}_P\right),$$

where

$$\boldsymbol{\beta}_i\left(\boldsymbol{\gamma}\right)\equiv\boldsymbol{\beta}_i\left(\boldsymbol{\gamma},\boldsymbol{\gamma}_0\right)=\left(\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{X}_i(\boldsymbol{\gamma})\right)^{-1}\left(\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{y}_i\left(\boldsymbol{\gamma}_0,\boldsymbol{\beta}_0\right)\right),$$

These are a pooled type estimator for $\boldsymbol{\gamma}_0$ and pooled and mean group estimators for $\boldsymbol{\beta}_0$. There are many other variants possible. For example, a mean group estimator of $\boldsymbol{\gamma}_0$, of the form

$$\hat{\boldsymbol{\gamma}}_{MG}=\frac{1}{N}\sum_{i=1}^{N}\hat{\boldsymbol{\gamma}}_i,$$

where

$$\hat{\boldsymbol{\gamma}}_i=\arg\min_{\boldsymbol{\gamma}}\sum_{t=1}^{T}\left(y_{i,t}-g\left(\boldsymbol{x}_{i,t},\boldsymbol{\gamma}\right)'\boldsymbol{\beta}_i\left(\boldsymbol{\gamma},\boldsymbol{\gamma}_0\right)\right)^2,$$

can be considered. However, numerical optimisations might be computationally cumbersome for an estimator that involves N. Further possibilities are considered in the next section. Concerning $\hat{\boldsymbol{\gamma}}_P$, we have the following Theorem.

Theorem 1 *Consider the model given by (1)-(3) and let Assumptions 1-5 hold. Then,*

$$\hat{\boldsymbol{\gamma}}_P - \boldsymbol{\gamma}_{\mathbf{0}} = O\left(\frac{1}{\sqrt{NT}}\right) + O_p\left(\frac{1}{N}\right). \tag{5}$$

The fact that $\boldsymbol{\gamma}_{\mathbf{0}}$ is homogeneous implies that the approximation error associated with the use of $\bar{\boldsymbol{Z}}_\gamma$ to proxy $\boldsymbol{F}$, is not negligible. While one could use an extension of Theorem 4 of Pesaran (2006) to the non-linear case, to derive a normal limit for $\sqrt{NT}\,(\hat{\boldsymbol{\gamma}}_P - \boldsymbol{\gamma}_{\mathbf{0}})$ when $T/N \to 0$, such a result requires also a rank condition of the form

$$Rank(\bar{\boldsymbol{C}}) = m \leq k+1$$

where

$$\bar{\boldsymbol{C}} = \frac{1}{N}\sum_{i=1}^{N} \boldsymbol{C}_i, \quad \boldsymbol{C}_i = (\boldsymbol{\delta}_i, \boldsymbol{\Pi}_i)\begin{pmatrix} 1 & 0 \\ \boldsymbol{\beta}_i & \boldsymbol{I} \end{pmatrix}.$$

Such a condition is not guaranteed to hold and so we prefer not to base a result on it. Then, the following result holds for $\hat{\boldsymbol{\beta}}_P$ and $\hat{\boldsymbol{\beta}}_{MG}$

Theorem 2 *Consider the model given by (1)-(3) and let Assumptions 1-5 hold. Then,*

$$\boldsymbol{\beta}_{MG}(\hat{\boldsymbol{\gamma}}_P) - \boldsymbol{\beta}_{MG}(\boldsymbol{\gamma}_0) = o_p\left(N^{-1/2}\right), \quad \boldsymbol{\beta}_{P}(\hat{\boldsymbol{\gamma}}_P) - \boldsymbol{\beta}_{P}(\boldsymbol{\gamma}_0) = o_p\left(N^{-1/2}\right),$$

Further,

$$\sqrt{N}\,(\boldsymbol{\beta}_{MG}(\hat{\boldsymbol{\gamma}}_P) - \boldsymbol{\beta}_{\mathbf{0}}) \to^d N(0, \boldsymbol{V}_{MG}) \tag{6}$$

where $\boldsymbol{V}_{MG}$ can be consistently estimated by

$$\hat{\boldsymbol{V}}_{MG} = \frac{1}{N}\sum_{i=1}^{N}\left(\hat{\boldsymbol{\beta}}_i(\hat{\boldsymbol{\gamma}}_P) - \boldsymbol{\beta}_{MG}(\hat{\boldsymbol{\gamma}}_P)\right)\left(\hat{\boldsymbol{\beta}}_i(\hat{\boldsymbol{\gamma}}_P) - \boldsymbol{\beta}_{MG}(\hat{\boldsymbol{\gamma}}_P)\right)'$$

Further,

$$\sqrt{N}\,(\boldsymbol{\beta}_{P}(\hat{\boldsymbol{\gamma}}_P) - \boldsymbol{\beta}_{\mathbf{0}}) \to^d N(0, \boldsymbol{\Psi}^{-1}\boldsymbol{R}\boldsymbol{\Psi}^{-1}) \tag{7}$$

where $\boldsymbol{\Psi}^{-1}\boldsymbol{R}\boldsymbol{\Psi}^{-1}$ can be obtained by using

$$\hat{\boldsymbol{R}} = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{\boldsymbol{X}_i'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{X}_i}{T}\right)\left(\hat{\boldsymbol{\beta}}_i(\hat{\boldsymbol{\gamma}}_P) - \boldsymbol{\beta}_{MG}(\hat{\boldsymbol{\gamma}}_P)\right)\left(\hat{\boldsymbol{\beta}}_i(\hat{\boldsymbol{\gamma}}_P) - \boldsymbol{\beta}_{MG}(\hat{\boldsymbol{\gamma}}_P)\right)'\left(\frac{\boldsymbol{X}_i'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{X}_i}{T}\right) \tag{8}$$

and

$$\hat{\boldsymbol{\Psi}} = \frac{1}{N}\sum_{i=1}^{N}\frac{\boldsymbol{X}_i(\hat{\boldsymbol{\gamma}}_P)'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{X}_i(\hat{\boldsymbol{\gamma}}_P)}{T},$$

3 Extensions

Following the presentation of our main findings, in this section we consider a number of extensions. We start by considering a heterogeneous setting for $\boldsymbol{\gamma}$. Therefore, we let

$$\boldsymbol{\gamma}_i = \boldsymbol{\gamma}_0 + \boldsymbol{\upsilon}_{\gamma,i} \tag{9}$$

This raises a rather non-standard question as to whether pooled estimation is consistent for $\boldsymbol{\gamma}_0$. Assume that the model is given by

$$y_{i,t}\left(\boldsymbol{\gamma},\boldsymbol{\beta}\right) = g\left(\boldsymbol{x}_{i,t},\boldsymbol{\gamma}_i\right)'\boldsymbol{\beta}_i + u_{i,t},\ i = 1,...,N,\ t = 1,...,T.$$

We need to explore the minimiser of $L(\boldsymbol{\gamma},\boldsymbol{\gamma_0})$, where we recall that

$$L(\boldsymbol{\gamma},\boldsymbol{\gamma_0}) = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(y_{i,t} - g\left(\boldsymbol{x}_{i,t},\boldsymbol{\gamma}\right)'\boldsymbol{\beta}\left(\boldsymbol{\gamma},\boldsymbol{\gamma}_i\right)\right)^2$$

$$\boldsymbol{\beta}\left(\boldsymbol{\gamma},\boldsymbol{\gamma}_i\right) \equiv \boldsymbol{\beta}\left(\boldsymbol{\gamma},\boldsymbol{\gamma}_i,\boldsymbol{\beta}_0\right) = \left(\sum_{i=1}^{N}\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{X}_i(\boldsymbol{\gamma})\right)^{-1}\left(\sum_{i=1}^{N}\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{y}_i\left(\boldsymbol{\gamma}_i,\boldsymbol{\beta}_0\right)\right).$$

We consider the scalar case for simplicity (both for g and γ_i) while noting that the vector case can be handled similarly at a cost of further notational complexity. The following Lemma states a condition under which estimation is consistent.

Lemma 3 *Consider the model given by (1)-(3) with (9) and let Assumptions 1-5 hold. Then, Theorem 1 holds if* $A_1\left(\gamma_0,\gamma_0\right)A_1\left(\gamma_0,\gamma_0\right)' < A_1\left(\gamma,\gamma_0\right)A_1\left(\gamma,\gamma_0\right)'$ *for all* $\gamma \neq \gamma_0$, *where*

$$A_1\left(\gamma,\gamma_0\right) = p\lim_{N,T\to\infty}\left(\sum_{i=1}^{N}\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{X}_i(\boldsymbol{\gamma})\right)^{-1}\sum_{i=1}^{N}\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}F_i\left(\gamma,\gamma_0\right),$$

$F_i\left(\gamma,\gamma_0\right) = \left(E\left[F\left(\boldsymbol{x}_{i,1},\upsilon_{\gamma,i},\gamma,\gamma_0\right)\right],...,E\left[F\left(\boldsymbol{x}_{i,T},\upsilon_{\gamma,i},\gamma,\gamma_0\right)\right]\right)'$ *and* $E\left[F\left(\boldsymbol{x}_{i,t},\upsilon_{\gamma,i},\gamma,\gamma_0\right)\right] = \sum_{j=1}^{\infty}\left(j!\right)^{-1}g^{(j)}\left(\boldsymbol{x}_{i,t},\gamma\right)E\left\{\left[\upsilon_{\gamma,i}+\left(\gamma_0-\gamma\right)\right]^j\right\}$.

It is clear that we cannot guarantee consistency for $\boldsymbol{\gamma}_0$ without further conditions. The problem arises because we cannot guarantee that $\left|E\left\{\left[\upsilon_{\gamma,i}+\left(\gamma_0-\gamma\right)\right]^j\right\}\right| > \left|E\left[\left(\upsilon_{\gamma,i}\right)^j\right]\right|$. However, if we are willing to impose further assumptions such as, e.g., symmetry for $\upsilon_{\gamma,i}$ then the required inequality, and therefore, consistency follows.

Next, we propose an iterative estimator that should work for heterogeneous $\boldsymbol{\gamma}_i$, irrespective of whether the condition of Lemma 3 holds. We start from some initial $\hat{\boldsymbol{\gamma}}_i$, denoted by $\hat{\boldsymbol{\gamma}}_i^{(1)}$. This could be based on $\boldsymbol{\beta}_i(\boldsymbol{\gamma}, \boldsymbol{\gamma}_0)$, using $\boldsymbol{M}_{\bar{Z}}$ or some other proxies. Then, construct $\bar{\boldsymbol{X}}^{(1)}(\boldsymbol{\gamma}) = \frac{1}{N}\sum_{i=1}^N \boldsymbol{X}_i(\hat{\boldsymbol{\gamma}}_i^{(1)})$ and $\boldsymbol{M}_{\bar{Z}_\gamma}^{(1)} = \boldsymbol{I} - \bar{\boldsymbol{X}}^{(1)}(\boldsymbol{\gamma})\left(\bar{\boldsymbol{X}}^{(1)}(\boldsymbol{\gamma})'\bar{\boldsymbol{X}}^{(1)}(\boldsymbol{\gamma})\right)^{-1}\bar{\boldsymbol{X}}^{(1)}(\boldsymbol{\gamma})'$. Then, use the recursions below, starting with $j = 2$

$$\hat{\boldsymbol{\gamma}}_i^{(j)} = \arg\min_{\boldsymbol{\gamma}} \sum_{t=1}^T \left(y_{i,t} - g(\boldsymbol{x}_{i,t}, \boldsymbol{\gamma})' \boldsymbol{\beta}_i^{(j)}(\boldsymbol{\gamma})\right)^2,$$

$$\boldsymbol{\beta}_i^{(j)}(\boldsymbol{\gamma}) = \left(\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}^{(j-1)}\boldsymbol{X}_i(\boldsymbol{\gamma})\right)^{-1}\left(\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}^{(j-1)}\boldsymbol{y}_i\right),$$

till convergence (i.e. till $\frac{1}{N}\sum_{i=1}^N \left\|\hat{\boldsymbol{\gamma}}_i^{(j)} - \hat{\boldsymbol{\gamma}}_i^{(j-1)}\right\| < \epsilon$, for some small ϵ. Denote the final j by M). Then, get MG estimates for $\boldsymbol{\gamma}_0$ and $\boldsymbol{\beta}_0$, given by

$$\hat{\boldsymbol{\gamma}}_{MG} \equiv \hat{\boldsymbol{\gamma}}_{MG}^{(M)} = \frac{1}{N}\sum_{i=1}^N \hat{\boldsymbol{\gamma}}_i^{(M)} \tag{10}$$

and

$$\hat{\boldsymbol{\beta}}_{MG} \equiv \hat{\boldsymbol{\beta}}_{MG}^{(M)} = \frac{1}{N}\sum_{i=1}^N \hat{\boldsymbol{\beta}}_i^{(M)}. \tag{11}$$

The theoretical properties of such an estimator could be explored using the work of Dominitz and Sherman (2005) and Pastorello, Patilea, and Renault (2003).

An alternative MG estimator counterpart to $\hat{\boldsymbol{\gamma}}_P$ and $\hat{\boldsymbol{\beta}}_P$, under a homogeneity assumption for $\boldsymbol{\gamma}$, is given by

$$\tilde{\boldsymbol{\gamma}}_{MG} = \arg\min_{\boldsymbol{\gamma}} \sum_{i=1}^N \sum_{t=1}^T \left(y_{i,t} - g(\boldsymbol{x}_{i,t}, \boldsymbol{\gamma})' \tilde{\boldsymbol{\beta}}_{MG}(\boldsymbol{\gamma})\right)^2 \tag{12}$$

$$\tilde{\boldsymbol{\beta}}_{MG}(\boldsymbol{\gamma}) = \frac{1}{N}\sum_{i=1}^N \boldsymbol{\beta}_i(\boldsymbol{\gamma}),$$

$$\boldsymbol{\beta}_i(\boldsymbol{\gamma}) = \left(\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{X}_i(\boldsymbol{\gamma})\right)^{-1}\left(\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{y}_i\right).$$

$$\tilde{\boldsymbol{\beta}}_{MG} = \tilde{\boldsymbol{\beta}}_{MG}(\tilde{\boldsymbol{\gamma}}_{MG})$$

Finally, a further estimator proposal is to base non-linear least squares estimation on the objective function given by

$$\tilde{L}(\boldsymbol{\gamma}, \boldsymbol{\gamma_0}) = \sum_{i=1}^N [\boldsymbol{y}_i - \boldsymbol{X}_i(\boldsymbol{\gamma})\boldsymbol{\beta}(\boldsymbol{\gamma}, \boldsymbol{\gamma}_0)]' \boldsymbol{M}_{\bar{Z}_\gamma} [\boldsymbol{y}_i - \boldsymbol{X}_i(\boldsymbol{\gamma})\boldsymbol{\beta}(\boldsymbol{\gamma}, \boldsymbol{\gamma}_0)] \tag{13}$$

rather than $L(\boldsymbol{\gamma}, \boldsymbol{\gamma_0})$.

Another extension is to use an indicator function as $g(\cdot)$ and so have a threshold panel model. In this case, the parameters of the transition function can be estimated by grid search as in Hansen (1999). Given the estimated threshold parameters, the estimation of the coefficients can be carried out by using ordinary least squares.

Last but not least, the proposed method can be adapted to accommodate lagged dependent variables and endogenous variables in the spirit of Caner and Hansen (2004), Harding and Lamarche (2011) and Chudik and Pesaran (2013).

4 Monte Carlo Analysis

This section presents the design of the Monte Carlo exercise and provides the results for different experiments. Results of an additional exercise are given in Appendix C.

Consider the panel data model where y_{it} is generated by the following data generating process (DGP),

$$y_{it} = \beta_{i1} x_{it} + \beta_{i2} x_{it} g(q_{it} : \gamma, c) + \phi_{y_{i_1}} f_{1t} + \phi_{y_{i_2}} f_{2t} + e_{it} \tag{14}$$

and

$$\begin{aligned} x_{it} &= \phi_{i1} f_{1t} + \phi_{i3} f_{3t} + v_{it} \\ g(q_{it} : \gamma, c) &= 1/(1 + \exp(-\gamma(q_{it} - c)) \end{aligned} \tag{15}$$

where x_{it} is the observable regressors on the ith cross-sectional dimension at time t for $i = 1, \ldots, N$, $t = 1, \ldots, T$; f_{jt} is the unobserved factors, $e_{it} \sim \text{IIDN}(0, 1)$ and $q_{it} = x_{it}$ for $i = 1, \ldots, N$. We address the non-linear term $x_{it} g(q_{it} : \gamma, c)$ as w_{it}.

The function $g(\cdot)$ is a logistic function used in González, Teräsvirta, and van Dijk (2005) and previously by Teräsvirta (1994). The slope parameter γ and the location parameter c are estimated endogenously. Function $g(\cdot)$ is bounded between 0 and 1. As $\gamma \to \infty$, $g(\cdot)$ becomes an indicator function so the smooth transition model reduces to a threshold model with two regimes as in Hansen (1999), i.e. higher slope parameter leads to faster transition. When $\gamma \to 0$, the model reduces to a linear panel regression with fixed effects. We take the parameters of $g(\cdot)$ constant throughout the simulations, and over time and cross sections, i.e. $\gamma = 1$ and $c = 0$. We discuss the implications of heterogeneous parameters in Appendix B.

The nature of the DGP carries the main features of Pesaran (2006) with minor differences, such as lack of observed common factors and serial correlation, and different means and variances for the distributions of some parameters.

The common factors and the individual specific errors of x_{it} are generated as independent stationary AR(1) processes with zero means and the corresponding variances below.

$$\begin{aligned} f_{mt} &= \rho_{f_m} f_{mt-1} + \nu_{f_{mt}} \quad \text{for} \quad m = 1,2,3, \\ \nu_{f_{mt}} &\sim \text{IIDN}(0, 1-\rho_{f_m}^2), \\ \rho_{f_m} &= 0.5, \\ f_{m1} &= 0 \quad \text{for} \quad m = 1,2,3. \end{aligned}$$

$$\begin{aligned} v_{it} &= \rho_{v_i} v_{it-1} + \nu_{v_{it}} \\ \nu_{v_{it}} &\sim \text{IIDN}(0, 1-\rho_{v_i}^2), \quad \text{and} \quad v_{i1} = 0 \\ \rho_{v_i} &\sim \text{IIDU}[0.05, 0.95]. \end{aligned}$$

Parameter ρ_{v_i} is fixed over replications as in Pesaran (2006). The parameters of the unobserved common factors in the x_{it} equation are generated independently across replications as

$$\boldsymbol{\Phi}_i' = \begin{pmatrix} \phi_{i1} & 0 & \phi_{i3} \end{pmatrix} \sim \begin{pmatrix} N(0.5, 0.5) & 0 & N(0.5, 0.5) \end{pmatrix}$$

For the parameters of the unobserved common factors in the y_{it} equation, we consider two different sets of parameters.

Experiment A:

$$\begin{aligned} \phi_{y_{i_1}} &\sim \text{IIDN}(1, 0.2) \\ \phi_{y_{i_2 A}} &\sim \text{IIDN}(1, 0.2) \\ \phi_{y_{i_3}} &= 0. \end{aligned}$$

This would lead to

$$E(\tilde{\boldsymbol{\Phi}}_{\mathbf{iA}}) = E(E(\phi_{y_{i_A}}), E(\boldsymbol{\Phi}_i)) = \begin{pmatrix} 1 & 0.5 & 0.5 \\ 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \end{pmatrix}$$

where the rank condition is satisfied.

Experiment B:

$$\begin{aligned} \phi_{y_{i_1}} &\sim \text{IIDN}(1, 0.2) \\ \phi_{y_{i_2 B}} &\sim \text{IIDN}(0, 1) \\ \phi_{y_{i_3}} &= 0. \end{aligned}$$

This would lead to

$$E(\tilde{\mathbf{\Phi}}_{\mathbf{i_B}}) = E(E(\phi_{y_{i_B}}), E(\mathbf{\Phi}_i)) = \begin{pmatrix} 1 & 0.5 & 0.5 \\ 0 & 0 & 0 \\ 0 & 0.5 & 0.5 \end{pmatrix}$$

where the rank condition is not satisfied.

For both experiments A and B, we conduct two other experiments,

Experiment 1: Homogeneous slopes with $\beta_{ik} = 1$ for $i = 1, \ldots, N$ and $k = 1, 2$.

Experiment 2: Heterogeneous slopes with $\beta_{ik} = 1 + \nu_{ik}$ where $\nu_{ik} \sim \text{IIDN}(0, 0.04)$ for $i = 1, \ldots, N$ and $k = 1, 2$.

For all experiments, we explore two options for the correction of cross-sectional dependence,

Option 1: correction with $\bar{y}$, $\bar{x}$, and $\bar{w}$,

Option 2: correction with $\bar{y}$, $\bar{x}$.

We repeat the simulations for each pair of $N, T = 20, 50, 100, 200, 400$ with 2000 replications. Note that the sample we use for options 1 and 2 is the same within a replication for each pair of N and T. For option 1, we report the average bias in β_1 and β_2 alongside their RMSE, empirical size and power. We report both coefficients' empirical size under true γ and c and also the average bias in γ and c and their RMSE. Under option 2, we report the bias in γ, β_1 and β_2 . These results are presented in Tables 1 to 4.

Under option 1, the bias and the RMSE substantially drop as N and T increase and this continues to hold in rank deficient cases in Tables 3 and 4 irrespective of the experiment. In general, the bias and RMSE of both coefficients seem to be smaller for homogeneous coefficients with some exceptions for small samples. In terms of bias, MG estimator depicts larger bias than the pooled estimator for small samples, regardless of the rank condition and the heterogeneity of slopes. However, the difference between pooled and MG estimators becomes negligible as N and T increase. As for RMSE, regardless of the rank condition and the heterogeneity of slopes, the difference between two estimators is quite minor for large samples. For smaller samples, pooled estimator dominates MG even if pooled estimator shows substantial bias. Overall, in all cases pooled estimator appears to be superior.

Under option 2, presented at the right bottom panel of each table, both coefficients are considerably biased. Surprisingly for small samples, for instance for $N = T = 20$, both pooled and MG estimators lead to extreme values such that the bias of both coefficients become very large. Even when T increases for $N = 20$, for instance in Table 1, the extreme values continues to appear. In all cases presented, our observations support the theoretical finding that option 2 does not correct cross-sectional dependence in non-linear models and introduces additional bias. This additional bias is due to the non-linear factor structure in $\bar{y}$. When $\bar{w}$ is not added to the regression, it simply cannot filter out this nonlinearity. This leads to substantial bias in the coefficients.

The bias of the parameters γ and c decrease steadily as sample size increases for all cases. In the full rank case, given in Tables 1 and 2, the bias and the RMSE of γ tend to be smaller compared to the rank deficient case, given in Tables 3 and 4. Curiously, the bias of c is smaller in rank deficient case although its RMSE is larger. Under option 2, γ displays substantial bias although c's bias is quite small in all cases.

For all experiments, we report the empirical size of a two-sided test of both coefficients under the null $\beta_i = 1$ and the power of both coefficients under the null $\beta_i = 0.95$. We also report the empirical size of both coefficients under the true value of the non-linear parameters under the same null hypotheses. For the variance of estimators, we follow Pesaran (2006) and use the heterogeneous versions given in (6) and (7) for MG and pooled estimators respectively.

The size of the tests is always oversized for both estimators and both coefficients in all cases when we estimate the transition function parameters. The comparison of size with the estimated vs true transition function parameters explains this suboptimal behavior. Under the true parameters, the empirical size of the tests for both estimators and both coefficients is very close to the nominal size of 5% with an exception that for smaller samples, it is slightly oversized.

The power of the tests is presented in the last column of each table. The homogeneity of slopes improves the power in general. In each case, pooled estimator appears to be slightly superior than MG estimator. For β_1, the power of the test is always very close to the desired nominal level of 95% for all cases for large N and T. For the rank deficient case, the power of the tests for β_2 decreases both for pooled and MG estimators.

Table 1: Experiment 1A, Homogeneous $\boldsymbol{\beta}$, Full Rank

β_1, correction with $\bar{y}$, $\bar{x}$, $\bar{w}$

	Bias (x100)					RMSE (x100)					Size (5% level, $H_0 : \beta_1 = 1.00$)					Size under true γ and c					Power (5% level, $H_0 : \beta_1 = 0.95$)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
Pooled																									
20	−1.27	−0.93	−0.75	−0.29	−0.08	44.63	21.62	11.68	5.23	4.99	23.65	19.35	16.30	16.60	12.25	6.95	7.70	7.85	9.20	8.10	27.90	23.40	22.85	35.00	31.00
50	−0.67	0.25	−0.32	0.02	0.18	32.91	7.94	4.34	3.86	3.68	25.35	20.45	15.05	12.35	8.30	5.45	5.80	6.70	6.50	6.05	35.25	35.85	41.25	42.55	38.70
100	−0.18	−0.20	−0.13	0.00	0.12	32.19	4.67	2.98	2.38	2.73	30.50	20.10	14.05	12.30	8.05	4.45	5.35	5.70	6.35	5.10	41.05	48.80	58.20	71.40	55.35
200	−0.75	−0.15	0.07	0.12	0.16	23.39	5.23	2.34	1.99	1.71	34.30	21.25	16.75	12.40	8.40	4.25	5.45	5.20	6.00	6.00	53.90	58.30	78.90	84.45	89.10
400	−0.86	−0.11	0.22	−0.09	0.07	9.63	2.81	3.07	1.22	0.99	38.25	24.80	17.75	13.15	9.05	5.40	4.95	5.30	4.65	5.40	65.75	81.70	86.80	99.20	99.95
MG																									
20	−8.00	−2.20	−1.02	−0.32	−0.13	76.92	28.12	13.44	5.29	4.89	24.40	19.20	15.90	17.05	12.30	6.65	7.60	6.65	7.60	6.95	26.05	22.95	20.85	35.95	31.40
50	−3.81	0.10	−0.41	0.00	0.16	59.11	9.12	4.66	4.01	3.83	24.65	21.55	17.25	13.65	9.35	5.30	6.40	5.85	5.50	5.90	32.10	36.60	42.40	43.60	38.15
100	−5.84	−0.36	−0.16	−0.03	0.16	54.82	5.50	3.21	2.43	2.87	28.25	23.05	18.00	15.65	8.70	4.90	4.60	4.90	6.50	5.75	35.70	46.55	58.65	73.85	53.85
200	−2.63	−0.30	0.05	0.11	0.18	33.16	6.56	2.57	2.08	1.74	34.20	24.10	20.55	13.65	9.50	4.40	4.70	4.60	5.40	5.85	46.00	54.70	76.00	82.90	90.55
400	−1.40	−0.21	0.21	−0.10	0.07	12.11	3.35	3.23	1.25	0.95	37.05	27.40	20.50	16.80	11.05	4.75	4.75	6.05	4.55	4.85	56.50	75.45	82.90	99.10	99.90

β_2, correction with $\bar{y}$, $\bar{x}$, $\bar{w}$

	Bias (x100)					RMSE (x100)					Size (5% level, $H_0 : \beta_2 = 1.00$)					Size under true γ and c					Power (5% level, $H_0 : \beta_2 = 0.95$)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
Pooled																									
20	0.81	2.08	1.70	0.63	0.28	69.76	38.80	14.85	6.55	4.87	19.10	19.60	20.95	19.75	20.55	6.95	7.35	7.50	6.55	7.80	18.65	20.70	24.10	32.45	41.80
50	1.86	0.29	0.70	0.23	0.18	48.68	13.28	6.18	4.59	3.17	23.40	22.90	20.10	19.80	17.15	5.20	6.35	6.20	7.45	4.40	23.80	26.15	36.55	42.95	60.05
100	1.14	0.79	0.56	0.15	0.08	49.85	7.53	4.30	2.88	2.32	32.50	22.95	19.25	18.65	17.10	4.45	4.95	4.20	5.95	6.40	31.45	34.40	48.25	68.55	80.00
200	1.88	0.74	0.28	0.05	0.03	34.66	8.43	3.33	2.31	1.54	39.25	28.15	21.85	19.25	18.50	4.95	5.45	5.05	5.95	6.15	37.20	42.30	65.50	83.10	97.30
400	1.74	0.59	0.01	0.13	0.04	21.64	4.60	5.29	1.57	1.04	40.05	28.95	26.55	22.45	20.25	5.05	5.10	4.90	5.75	4.65	44.70	62.55	76.20	98.65	100.00
MG																									
20	15.90	4.97	2.20	0.69	0.37	126.66	50.39	17.77	7.43	5.42	17.35	20.45	20.30	19.95	20.55	5.80	6.20	5.90	6.20	5.75	18.45	20.90	23.90	30.50	39.05
50	8.17	0.72	0.95	0.37	0.27	98.91	15.86	7.33	5.37	3.65	21.70	24.40	23.20	21.70	17.60	5.10	6.15	5.50	6.95	5.05	21.75	25.40	34.25	40.60	53.65
100	17.29	1.17	0.63	0.19	0.09	111.63	9.46	5.07	3.36	2.73	28.05	25.00	20.80	20.30	18.95	4.80	4.80	3.85	5.25	5.55	27.80	33.65	44.85	62.50	71.15
200	6.43	1.13	0.36	0.12	0.07	57.22	10.98	3.97	2.72	1.78	34.65	28.75	24.85	21.75	20.35	4.45	5.15	4.80	5.40	6.20	33.75	38.75	57.95	75.90	94.55
400	2.84	0.87	0.10	0.16	0.06	27.99	5.74	5.64	1.85	1.20	37.95	31.05	28.05	25.40	22.95	4.40	4.75	4.70	5.70	4.25	39.05	55.85	67.40	96.75	99.90

	correction with $\bar{y}$, $\bar{x}$, $\bar{w}$										correction with $\bar{y}$, $\bar{x}$														
	γ Bias (x100)					γ RMSE (x100)					γ Bias (x100)					β_1 Bias Pooled (x100)					β_2 Bias Pooled (x100)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
20	72.46	45.10	18.11	4.65	1.90	176.72	126.34	71.49	22.49	13.96	74.66	42.77	−6.71	−64.71	−57.91	−229.57	−285.16	23.32	22.55	19.72	1581	261.41	−47.49	−46.87	−40.79
50	58.21	16.91	2.84	1.65	0.72	151.50	62.14	20.20	13.13	8.78	33.88	−24.57	−64.56	−55.82	−47.45	19.87	20.66	21.87	17.45	17.40	−29.77	−41.94	−44.14	−35.46	−35.81
100	64.29	4.47	0.86	0.52	0.33	159.06	29.35	12.84	8.81	6.28	67.64	−56.72	−65.06	−60.93	−46.28	8.99	20.25	17.33	11.06	17.62	−28.20	−39.45	−34.86	−22.45	−36.52
200	36.98	3.42	0.27	0.44	0.12	111.62	20.39	10.01	6.98	4.42	37.11	−44.38	−60.22	−51.86	−50.64	15.99	15.91	13.35	12.87	14.35	−32.79	−31.95	−26.64	−26.51	−29.64
400	4.60	0.51	0.85	−0.19	0.01	34.00	13.57	10.89	4.85	3.20	−40.46	−61.35	−46.99	−59.84	−58.02	10.99	14.86	14.46	5.65	6.67	−22.72	−29.13	−29.74	−11.44	−13.60
	c Bias (x100)					c RMSE (x100)					c Bias (x100)					β_1 Bias MG (x100)					β_2 Bias MG (x100)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
20	−5.75	2.30	2.01	0.35	1.18	145.10	85.17	44.26	21.55	15.68	−4.74	2.55	3.61	−9.17	−3.52	−88031	−5363	29.86	25.70	24.17	802470	5465	−60.44	−53.21	−49.63
50	2.75	3.70	0.32	1.72	2.25	111.22	37.33	20.11	14.35	10.68	0.27	1.70	-2.12	1.37	0.49	1362	25.27	25.21	22.71	24.35	12397	−51.12	−50.73	−45.83	−49.54
100	3.20	1.72	0.86	1.02	2.39	107.72	24.05	14.23	9.53	8.25	−2.42	5.02	−1.07	−0.19	1.23	1918	24.20	20.95	15.03	25.04	−21898	−47.32	−42.10	−30.41	−51.18
200	2.51	2.60	1.96	2.13	2.14	78.98	22.60	10.83	8.39	5.84	1.48	3.70	2.37	0.92	0.61	−380.55	22.29	17.86	19.02	20.65	359.24	−44.63	−35.66	−38.70	−42.11
400	−0.25	1.70	2.30	0.07	1.12	43.35	13.30	10.37	4.73	3.56	−3.69	3.78	1.86	−0.37	−0.30	15.92	19.30	21.83	9.59	10.72	−184.71	−38.07	−44.45	−19.30	−21.71

Table 2: Experiment 2A, Heterogenous $\boldsymbol{\beta}$, Full Rank

β_1, correction with $\bar{y}$, $\bar{x}$, $\bar{w}$

(N,T)	Bias (x100)					RMSE (x100)					Size (5% level, $H_0 : \beta_1 = 1.00$)					Size under true γ and c					Power (5% level, $H_0 : \beta_1 = 0.95$)				
	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
Pooled																									
20	0.95	−0.56	−0.50	−0.26	−0.18	40.48	13.03	10.78	5.41	4.71	22.90	17.40	14.40	11.85	10.50	7.80	8.35	8.20	8.15	8.50	26.75	22.70	18.75	19.65	20.55
50	−0.60	−0.45	−0.09	0.08	0.18	26.18	8.16	4.74	5.19	5.14	23.15	16.65	11.45	9.80	7.95	4.95	6.10	6.60	6.60	7.00	29.75	27.25	26.75	22.05	18.65
100	−0.97	−0.33	−0.15	0.04	0.11	25.74	4.71	3.33	2.91	2.26	26.15	15.50	10.60	8.65	7.50	5.95	6.60	5.10	5.80	6.70	37.25	35.50	36.90	39.75	45.65
200	−0.52	−0.10	0.00	0.06	0.13	11.66	4.32	2.47	2.01	1.74	27.90	17.05	9.70	8.05	6.20	5.15	5.50	4.85	5.25	4.40	46.80	46.05	57.65	60.95	66.00
400	−0.72	−0.01	−0.10	0.19	0.17	25.40	3.26	1.66	1.86	1.27	38.15	20.20	11.05	8.85	7.25	5.10	4.55	4.85	6.55	5.15	55.30	65.80	82.50	76.75	91.00
MG																									
20	−4.44	−0.91	−0.63	−0.29	−0.13	70.41	14.18	11.19	5.16	4.14	21.60	18.10	14.80	11.90	10.45	5.50	7.85	7.25	7.40	7.85	24.00	22.55	18.95	20.25	20.10
50	−4.41	−0.77	−0.07	0.02	0.20	48.41	8.95	4.92	5.11	5.15	24.20	19.40	13.70	10.30	7.00	5.30	5.30	6.10	5.40	5.55	30.05	29.35	29.60	22.50	18.05
100	−3.32	−0.44	−0.23	0.03	0.12	44.90	5.32	3.35	2.71	1.94	28.75	19.05	12.05	9.15	7.90	5.25	5.75	4.35	4.80	5.50	35.10	38.30	38.55	44.55	50.90
200	−1.29	−0.24	0.01	0.08	0.14	14.55	5.02	2.63	1.90	1.54	31.05	21.40	13.45	9.15	6.85	4.75	4.60	4.70	5.20	4.55	43.25	46.55	60.55	66.70	75.15
400	−3.28	−0.07	−0.12	0.20	0.19	46.07	3.82	1.73	1.82	1.13	37.85	24.60	13.90	9.05	6.95	5.20	4.95	4.75	5.65	5.40	48.85	66.00	84.00	79.95	95.15

β_2, correction with $\bar{y}$, $\bar{x}$, $\bar{w}$

(N,T)	Bias (x100)					RMSE (x100)					Size (5% level, $H_0 : \beta_2 = 1.00$)					Size under true γ and c					Power (5% level, $H_0 : \beta_2 = 0.95$)				
	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
Pooled																									
20	−0.12	1.59	1.32	0.58	0.39	64.44	21.41	15.43	7.16	5.35	17.90	19.05	17.65	15.05	13.75	6.10	8.20	9.65	9.15	9.65	18.30	19.75	20.70	21.50	23.75
50	1.73	0.88	0.50	0.53	0.06	43.16	13.57	6.95	5.89	4.32	22.40	18.95	15.65	13.65	10.55	5.20	5.75	6.30	7.00	7.00	22.80	21.95	22.85	26.55	24.90
100	2.12	0.82	0.64	0.24	0.13	38.17	7.55	4.88	3.39	2.26	29.05	18.00	15.25	11.70	8.85	4.90	5.40	5.60	6.30	5.35	29.15	27.55	34.80	38.70	48.95
200	1.89	0.66	0.38	0.18	0.00	19.75	6.77	3.62	2.41	1.71	30.35	22.95	16.05	10.85	8.95	5.40	5.65	4.70	5.15	5.90	33.35	35.90	49.55	61.10	70.30
400	1.83	0.43	0.36	−0.01	−0.03	38.06	5.14	2.48	2.01	1.28	42.15	25.40	16.20	12.15	7.55	4.50	5.60	5.15	4.85	4.70	41.75	49.75	73.90	76.35	92.95
MG																									
20	11.10	2.41	1.58	0.71	0.40	110.81	23.70	17.30	7.39	5.12	15.85	19.60	18.00	15.70	13.05	5.35	6.75	7.75	7.50	7.20	16.75	19.30	21.00	22.35	24.50
50	8.07	1.44	0.61	0.68	0.13	64.37	15.08	7.62	6.24	4.24	21.60	19.10	18.35	16.65	11.50	4.80	5.00	5.75	5.95	5.10	22.15	23.15	25.00	28.40	27.90
100	8.13	1.08	0.90	0.28	0.11	66.33	9.02	5.36	3.57	2.22	29.10	22.40	18.35	14.85	10.95	5.25	6.30	5.15	6.05	5.30	29.05	29.90	38.20	42.10	53.10
200	3.97	1.01	0.46	0.19	0.04	26.61	8.41	4.12	2.58	1.68	32.60	27.30	20.50	15.65	11.65	4.55	4.65	5.35	4.85	6.05	33.05	37.40	50.25	63.85	74.75
400	7.20	0.63	0.40	0.02	−0.01	72.21	6.34	2.82	2.21	1.26	37.85	30.00	21.20	16.95	10.50	4.40	5.80	5.25	4.55	4.60	38.80	46.60	73.75	77.60	94.75

	correction with $\bar{y}$, $\bar{x}$, $\bar{w}$										correction with $\bar{y}$, $\bar{x}$														
	γ Bias (x100)					γ RMSE (x100)					γ Bias (x100)					β_1 Bias Pooled (x100)					β_2 Bias Pooled (x100)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
20	71.56	37.29	17.55	3.26	1.48	175.23	114.27	68.03	21.34	13.44	53.43	−7.45	−13.79	−62.49	−64.80	32.5	22.08	23.81	22.64	20.11	104.5	−44.29	−48.45	−45.92	−40.60
50	52.63	11.65	3.58	1.77	1.28	142.22	51.82	21.26	13.77	9.48	31.41	−37.03	−55.66	−40.20	−35.82	17.83	17.08	19.38	20.14	22.02	1909.75	−35.34	−39.09	−41.25	−45.90
100	44.01	4.10	0.59	0.53	0.23	126.94	34.86	13.36	8.75	5.63	36.60	−56.56	−58.28	−51.21	−54.56	17.26	15.50	14.65	13.58	10.10	−33.91	−31.43	−29.44	−27.83	−20.21
200	17.24	3.33	0.45	0.14	0.17	72.82	20.81	9.90	6.45	4.16	−15.94	−42.02	−54.71	−50.64	−50.03	12.56	14.19	10.73	12.56	13.52	−24.03	−28.80	−21.54	−25.82	−27.82
400	39.33	1.28	−0.35	0.61	0.21	116.42	14.31	7.22	5.73	3.14	41.10	−47.31	−57.69	−41.95	−50.09	16.62	12.57	6.67	18.58	13.48	−34.66	−25.31	−13.60	−38.48	−27.73
	c Bias (x100)					c RMSE (x100)					c Bias (x100)					β_1 Bias MG (x100)					β_2 Bias MG (x100)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
20	7.47	2.38	2.26	0.47	0.89	141.5	59.82	41.44	21.33	15.02	4.37	3.15	2.64	−2.65	−0.42	14088	25.52	29.73	26.22	23.56	70564	−51.14	−60.28	−52.90	−47.42
50	1.86	0.64	1.81	2.07	3.19	100.16	35.18	19.66	16.39	12.76	2.49	−3.08	1.97	-0.29	1.18	115.70	21.56	23.71	27.69	30.64	53995	−44.35	−47.69	−56.03	−62.83
100	2.31	0.66	1.18	2.04	1.62	90.10	22.69	14.52	10.25	6.79	4.99	−1.33	0.89	0.60	2.22	-41.84	19.76	18.96	18.92	14.42	−6307	−39.89	−37.95	−38.48	−28.87
200	3.76	2.66	2.10	2.00	1.94	51.96	20.74	11.04	7.75	5.58	6.52	3.12	3.30	0.42	0.13	18.23	20.98	15.81	18.12	18.97	−36.00	−42.41	−31.71	−36.89	−38.65
400	2.28	2.14	0.59	2.59	1.89	78.85	15.10	7.03	7.37	4.28	−1.76	3.87	−1.41	−0.35	−0.07	333.36	19.18	11.05	26.30	18.89	−289.94	−38.53	−22.37	−53.75	−38.47

Table 3: Experiment 1B, Homogeneous $\boldsymbol{\beta}$, Rank Deficient

β_1, correction with $\bar{y}$, $\bar{x}$, $\bar{w}$

(N,T)	Bias (x100)					RMSE (x100)					Size (5% level, $H_0 : \beta_1 = 1.00$)					Size under true γ and c					Power (5% level, $H_0 : \beta_1 = 0.95$)				
	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
Pooled																									
20	−3.79	0.39	0.39	−0.14	−0.07	60.71	24.44	16.79	9.09	5.27	21.35	23.75	20.30	17.65	19.65	6.90	6.85	7.65	7.60	8.30	23.45	29.80	29.10	32.35	42.75
50	2.10	1.30	−0.26	0.12	0.25	35.56	23.09	11.85	8.30	3.51	30.70	30.30	23.00	22.75	17.50	5.40	6.30	4.85	6.00	5.55	37.10	36.85	39.15	55.25	61.90
100	0.51	−0.59	−0.51	−0.01	0.15	29.85	17.81	8.59	3.38	2.19	35.25	31.65	27.35	25.50	22.05	5.40	5.55	6.25	4.95	4.15	45.30	46.95	57.20	72.95	88.60
200	0.95	−0.26	−0.09	0.25	0.20	25.12	18.10	8.93	2.81	1.60	41.30	37.90	35.80	30.10	26.60	5.35	5.80	5.05	5.45	5.00	58.00	64.25	71.95	87.45	97.50
400	3.96	−0.77	−0.05	0.28	0.25	31.14	13.77	8.07	8.84	1.46	56.20	47.15	44.50	40.90	31.40	5.10	5.35	5.15	5.35	4.40	68.70	73.85	85.15	88.60	99.00
MG																									
20	−15.89	−0.75	0.17	−0.31	−0.15	126.68	30.57	20.04	9.95	5.53	18.85	23.00	20.40	17.65	19.65	6.40	6.40	6.90	6.65	7.05	20.45	29.35	28.45	28.65	40.50
50	−5.51	−1.22	−0.54	0.08	0.24	80.84	38.29	13.79	9.33	4.00	28.95	28.20	23.20	25.05	18.55	5.40	6.85	5.10	4.35	5.50	35.00	34.30	36.65	53.95	54.45
100	−6.56	−1.65	−0.68	−0.05	0.16	65.52	21.79	10.12	3.86	2.46	33.65	32.30	29.45	28.55	25.90	4.85	5.20	5.35	4.95	4.90	41.80	43.40	53.65	69.65	85.75
200	−2.79	−0.71	−0.18	0.30	0.24	56.89	22.27	10.41	3.29	1.83	39.40	40.25	37.05	31.75	29.75	4.50	5.40	5.50	5.15	5.35	51.10	62.60	69.20	83.80	96.20
400	−4.54	−1.48	-0.15	0.26	0.28	85.43	17.98	9.35	10.29	1.71	51.40	48.05	47.50	38.40	34.00	4.85	4.80	5.85	4.75	4.90	59.40	70.40	81.85	81.50	98.55

β_2, correction with $\bar{y}$, $\bar{x}$, $\bar{w}$

(N,T)	Bias (x100)					RMSE (x100)					Size (5% level, $H_0 : \beta_2 = 1.00$)					Size under true γ and c					Power (5% level, $H_0 : \beta_2 = 0.95$)				
	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
Pooled																									
20	5.27	0.45	0.51	1.06	0.46	87.50	45.24	29.14	14.50	7.12	16.35	21.45	21.35	21.60	24.25	7.15	7.50	7.35	7.75	8.65	16.00	23.00	24.90	27.70	37.20
50	−3.43	−0.13	1.16	0.32	0.00	57.10	42.77	20.45	13.97	5.06	24.20	31.15	26.00	27.65	25.20	5.05	5.90	7.05	5.95	5.55	23.10	31.65	28.90	40.35	45.30
100	−1.55	0.98	0.84	0.21	−0.10	51.16	33.28	12.99	5.62	3.38	32.70	34.20	30.65	31.10	27.30	4.55	4.95	4.80	5.50	4.65	30.30	35.95	37.35	51.25	69.65
200	−1.06	1.02	0.55	−0.38	−0.31	39.63	35.51	17.13	4.74	2.66	38.05	39.95	39.50	36.20	31.10	4.95	5.40	5.45	5.20	5.90	33.45	42.50	48.15	60.25	82.60
400	−8.22	2.08	−0.06	−0.36	−0.39	48.48	24.06	20.37	12.76	2.51	57.75	49.45	48.15	44.40	37.80	5.25	5.25	5.00	4.95	5.30	51.60	51.60	59.75	69.25	90.15
MG																									
20	38.66	3.80	1.46	1.44	0.67	217.11	66.14	34.47	16.30	8.17	15.90	20.85	21.10	22.60	26.45	6.85	5.05	6.50	6.15	7.50	15.55	21.55	24.45	25.20	35.55
50	15.41	5.72	2.69	0.41	0.11	136.07	74.67	36.40	15.78	5.99	20.95	31.00	27.80	29.85	27.40	5.00	5.60	6.30	4.85	4.75	20.55	31.30	28.65	38.45	41.55
100	13.64	2.73	1.16	0.26	−0.11	124.21	55.21	15.30	6.78	4.06	29.30	35.00	33.30	33.65	30.40	4.60	4.60	4.60	5.10	5.00	28.00	36.15	35.95	48.40	62.00
200	10.87	1.95	0.87	−0.42	−0.38	109.37	51.59	20.68	5.74	3.17	35.75	43.80	42.40	38.40	34.10	4.90	5.40	4.65	5.35	5.75	32.85	41.50	44.55	55.00	76.80
400	13.18	3.58	0.09	−0.33	−0.46	167.78	30.83	24.04	14.88	3.04	49.75	50.85	50.85	45.10	39.70	5.10	5.00	5.20	4.50	4.95	46.00	49.85	57.10	63.70	86.00

	correction with $\bar{y}$, $\bar{x}$, $\bar{w}$										correction with $\bar{y}$, $\bar{x}$														
(N,T)	γ Bias (x100)					γ RMSE (x100)					γ Bias (x100)					β_1 Bias Pooled (x100)					β_2 Bias Pooled (x100)				
	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
20	100.45	59.43	32.07	13.42	5.70	209.4	152.42	106.99	55.88	31.99	106.08	17.65	−3.94	−26.62	−48.52	−9148	21.40	22.34	20.69	18.10	13939	−42.19	−46.36	−42.97	−37.24
50	89.93	60.66	19.62	5.81	4.39	190.4	152.57	75.28	28.78	19.99	73.33	59.69	−4.90	−48.82	−39.76	−1.72	19.82	19.70	13.41	16.34	−12.60	−41.16	−41.73	−27.11	−34.60
100	84.79	25.64	9.73	5.11	2.86	182.7	96.15	39.53	23.17	14.10	65.62	1.14	−38.56	−49.31	−49.52	−3.57	17.36	14.85	8.88	8.53	31.41	−36.56	−31.52	−17.70	−17.13
200	62.74	11.30	7.11	6.29	2.92	162.4	56.05	31.63	21.63	11.95	27.68	−38.39	−38.95	−43.15	−49.17	13.24	13.51	14.50	13.59	9.86	−26.01	−28.09	−31.13	−28.94	−20.45
400	115.17	14.18	6.99	5.52	2.99	217.23	61.22	29.01	23.60	12.50	105.60	−17.06	−42.87	−28.22	−44.17	2.31	15.75	12.09	19.70	14.05	9.89	−33.04	−25.69	−42.69	−29.76
(N,T)	c Bias (x100)					c RMSE (x100)					c Bias (x100)					β_1 Bias MG (x100)					β_2 Bias MG (x100)				
	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
20	0.59	5.06	5.11	3.90	3.27	176.30	99.50	62.56	38.48	22.97	−10.08	6.38	−1.85	0.24	0.48	124120	14.90	24.89	23.95	20.34	−574020	−38.34	−51.52	−49.43	−41.64
50	3.95	9.07	3.51	2.26	2.63	142.80	110.29	54.75	24.12	16.44	5.17	7.45	0.23	0.18	-0.40	−27362	25.26	23.93	15.28	19.83	239120	−52.41	−50.12	−30.90	−41.53
100	−3.22	−1.20	0.15	0.85	1.07	131.41	72.59	34.04	15.44	9.93	−3.32	−0.60	−0.15	1.66	1.35	-5192	21.36	18.05	10.30	9.93	−38396	−44.57	−37.98	−20.55	−19.98
200	2.52	1.35	1.66	0.70	0.74	111.75	66.84	38.05	14.18	7.18	2.06	−2.30	−1.11	−0.72	0.89	18467	16.28	18.27	16.73	11.91	−30720	−33.69	−38.67	−35.22	−24.62
400	0.95	3.27	−0.47	0.97	0.46	134.58	68.07	38.29	26.61	7.15	3.99	4.38	0.04	0.19	0.52	−81.20	20.22	15.40	24.78	17.17	−29401	−41.96	−32.39	−52.92	−36.10

Table 4: Experiment 2B, Heterogenous $\boldsymbol{\beta}$, Rank Deficient

β_1, correction with $\bar{y}$, $\bar{x}$, $\bar{w}$

	Bias (x100)					RMSE (x100)					Size (5% level, $H_0:\beta_1=1.00$)					Size under true γ and c					Power (5% level, $H_0:\beta_1=0.95$)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
Pooled																									
20	−0.39	−1.54	−0.13	−0.17	−0.07	51.19	27.36	16.07	10.86	5.54	18.85	21.65	19.15	15.55	13.45	7.95	8.40	8.80	8.80	9.55	22.15	25.55	22.80	20.90	23.10
50	1.02	−0.36	−0.42	−0.15	0.25	30.88	30.55	11.11	6.02	3.83	25.80	25.30	17.20	14.10	9.85	4.90	5.70	5.70	6.55	5.55	31.15	31.85	27.40	33.00	30.30
100	1.12	0.09	−0.58	0.05	0.07	29.90	19.15	11.91	4.84	2.30	31.50	26.25	19.40	15.25	10.70	5.75	5.20	5.95	5.10	5.65	39.60	40.10	42.65	50.25	54.20
200	0.85	−0.73	−0.20	0.25	0.27	27.19	12.66	8.58	2.81	2.69	38.95	30.35	26.10	19.15	12.05	5.85	4.70	5.50	6.00	5.15	52.85	56.45	58.45	66.15	79.85
400	4.67	−0.79	−0.18	0.11	0.27	32.90	16.42	5.68	3.43	1.53	50.95	41.75	31.05	24.15	14.35	5.75	5.15	5.20	4.40	5.20	64.15	67.90	76.30	72.50	93.20
MG																									
20	−14.10	−2.69	−0.35	−0.31	−0.06	111.25	34.74	16.84	11.78	5.48	17.30	22.10	19.10	15.05	13.60	5.40	7.25	6.90	7.80	8.00	19.55	26.20	22.55	21.60	24.20
50	−6.26	−3.90	−0.76	−0.22	0.22	69.55	54.18	12.85	6.80	3.86	25.50	25.50	19.70	16.20	11.00	4.85	4.85	4.30	6.05	5.90	29.75	30.35	29.10	34.95	32.55
100	−4.64	−1.10	−0.86	0.03	0.07	70.72	28.37	16.17	5.43	2.48	32.10	29.85	22.95	18.50	12.45	5.40	5.05	5.35	5.65	6.00	38.10	41.25	45.45	51.45	54.75
200	−3.82	−1.63	−0.32	0.29	0.32	59.90	23.05	9.73	3.10	2.79	38.35	34.20	30.75	23.65	14.25	5.10	4.80	5.15	5.55	5.10	50.10	56.45	60.05	68.30	80.85
400	−5.23	−2.35	−0.29	0.09	0.30	82.09	28.28	6.77	4.06	1.67	48.50	45.30	35.75	31.00	18.75	5.35	4.50	5.15	4.55	5.55	57.40	67.80	76.40	74.55	94.10

β_2, correction with $\bar{y}$, $\bar{x}$, $\bar{w}$

	Bias (x100)					RMSE (x100)					Size (5% level, $H_0:\beta_2=1.00$)					Size under true γ and c					Power (5% level, $H_0:\beta_2=0.95$)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
Pooled																									
20	4.44	2.47	0.66	0.96	0.51	94.04	43.85	26.38	17.41	7.27	15.45	21.30	18.55	20.50	14.80	7.50	7.60	8.75	8.50	8.40	15.40	21.75	21.55	23.40	22.45
50	−0.87	0.16	1.20	0.47	−0.12	51.66	46.99	20.27	9.57	5.31	21.10	29.45	23.05	19.30	17.60	5.30	6.35	5.20	7.85	7.25	20.20	30.55	25.00	26.90	28.55
100	−0.59	0.56	1.04	−0.04	−0.03	53.33	30.68	15.97	9.19	3.63	31.15	31.25	26.50	21.35	16.40	4.90	5.50	5.60	5.05	5.65	28.80	32.55	31.70	38.50	43.05
200	−2.35	1.77	0.29	−0.32	−0.41	42.46	25.23	18.03	4.48	5.04	37.25	35.95	33.25	25.10	16.85	5.00	6.40	5.35	4.90	4.55	33.45	37.05	40.85	46.85	60.10
400	−9.20	1.92	0.30	−0.09	−0.47	52.19	26.41	10.76	5.62	2.50	56.45	46.95	38.75	34.50	23.15	5.45	5.50	5.20	4.70	4.85	51.85	48.30	53.50	59.40	76.25
MG																									
20	38.86	5.54	1.24	1.16	0.57	206.82	63.89	28.80	19.31	7.81	13.25	20.35	20.00	21.30	16.45	5.50	6.65	6.30	7.50	7.30	13.25	20.60	21.50	24.95	23.00
50	16.06	6.71	2.04	0.66	0.01	118.74	77.58	24.34	11.03	5.79	18.10	30.85	25.75	21.90	19.95	5.30	4.55	4.70	7.00	6.15	18.65	30.90	27.00	29.40	31.40
100	14.94	2.62	1.63	-0.01	-0.02	119.74	40.68	20.98	10.44	4.07	27.30	33.55	30.70	26.70	20.15	4.65	5.80	5.35	5.20	5.25	25.90	34.20	33.85	39.75	45.85
200	6.05	3.31	0.50	−0.42	−0.47	93.78	35.99	20.38	5.29	5.24	34.45	40.20	37.70	30.75	23.00	4.85	5.25	4.75	5.30	4.85	31.70	38.70	42.45	47.70	61.00
400	12.03	5.52	0.49	−0.10	−0.57	166.43	51.71	12.79	7.18	2.90	48.65	50.85	43.75	41.50	29.50	4.95	4.75	4.90	4.60	5.20	44.80	48.75	53.15	58.45	75.35

	correction with $\bar{y}$, $\bar{x}$, $\bar{w}$										correction with $\bar{y}$, $\bar{x}$														
	γ Bias (x100)					γ RMSE (x100)					γ Bias (x100)					β_1 Bias Pooled (x100)					β_2 Bias Pooled (x100)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
20	116.40	51.83	32.23	13.69	5.53	222.0	142.81	103.96	59.75	26.75	117.75	20.20	−2.51	−29.10	−45.49	−1298.51	19.3	21.80	20.24	17.80	8298	−40.96	−45.09	−42.14	−36.53
50	87.55	56.66	17.07	5.22	3.93	184.5	152.02	62.96	26.61	18.03	63.08	45.36	−11.92	−46.70	−37.58	−37.01	17.75	19.04	11.94	17.09	289.16	−38.98	−40.65	−24.67	−36.41
100	79.27	28.67	9.34	5.42	2.24	178.3	97.62	42.75	22.69	12.99	64.88	6.77	−37.13	−46.99	−47.37	11.67	18.56	14.74	9.63	9.59	−65.08	−37.95	−31.32	−19.52	−19.56
200	65.07	14.26	6.79	5.45	2.49	156.9	62.55	30.69	20.11	12.04	28.07	−37.23	−36.37	−40.70	−46.65	8.20	13.49	14.86	14.60	10.91	−20.38	−27.58	−31.71	−30.79	−22.73
400	122.80	12.69	6.37	5.52	3.21	227.00	55.58	26.27	22.37	11.16	112.59	−19.84	−41.25	−27.71	−41.59	18.09	15.47	12.84	19.76	14.92	−30.31	−32.38	−27.23	−43.22	−31.74

	c Bias (x100)					c RMSE (x100)					c Bias (x100)					β_1 Bias MG (x100)					β_2 Bias MG (x100)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
20	7.00	−2.07	3.52	3.78	2.27	163.73	106.06	58.45	36.96	22.04	−6.58	−10.57	0.01	−0.16	−1.35	−220700	21.22	24.79	23.44	19.95	577040	−45.75	−50.79	−48.40	−40.81
50	3.93	−1.22	2.96	1.22	2.57	132.01	109.63	52.17	21.91	15.78	1.79	−3.75	0.85	−1.65	−0.53	−17833	23.44	23.39	13.67	20.52	27017	−47.91	−48.97	−28.14	−43.15
100	1.92	3.61	1.13	0.33	0.52	133.82	76.04	40.56	14.24	9.68	0.93	5.99	−1.16	−0.88	−0.12	−7867	23.00	18.04	11.02	11.08	−45141	−46.89	−37.76	−22.34	−22.50
200	−3.55	1.42	-0.63	0.95	0.67	108.54	58.30	30.70	13.96	7.42	−7.52	0.40	−1.01	0.04	−0.08	−4311	16.51	18.61	17.81	12.99	4471	−33.59	−39.23	−37.28	−26.89
400	4.59	1.49	−0.13	0.67	0.24	128.18	69.75	24.34	20.45	6.60	5.15	3.55	−0.49	0.08	−0.36	−13142	19.96	16.09	24.59	18.01	78242	−41.39	−33.78	−53.13	−37.98

5 Empirical Applications

In this section, we present two empirical applications with our proposed correction of cross-sectional dependence in non-linear panel models. The first depicts the nonlinearities on marginal funding costs of UK banks induced by their solvency level. The second investigates the nonlinearities caused by different levels of public indebtedness and its impact on GDP growth.

5.1 The non-linear effect of solvency on wholesale funding costs at UK banks

When banks need to raise large amounts of funding in addition to retail deposits, they turn to wholesale funding markets. In times of stress, banks bid to raise additional funds to finance their activities while uncertainty over bank solvency leads market participants charge higher interest rates to make these funds available. Historically, there have been cases where banks were shut out from wholesale funding markets due to loss of credibility and concerns over their ability to repay their debt (Shin (2009)).

Understanding how wholesale funding costs work and why they change over time is important. There has been an increasing attention to non-linear estimation techniques to model banks' funding costs (Aymanns, Caceres, Daniel, and Schumacher (2016)). In simple terms, we expect troubled banks to face higher costs to borrow. This relationship, however, is unlikely to be linear. When a bank draws closer to its default threshold, the cost of funding it faces is expected to increase at an increasing rate (Korsgaard (2017)). Similarly, beyond a certain point, an improvement in the solvency position of a healthy bank has little impact on the interest rate it must pay. Therefore we expect to see a (negative) non-linear relationship between the wholesale funding costs of banks and their solvency levels.

We apply our technique to this issue by following Dent, Hacıoğlu Hoke, and Panagiotopoulos (2017). Solvency measures, such as leverage ratio, are a good measure to explore banks' health and therefore to identify the stressful periods they go through. Following a similar approach to Dent, Hacıoğlu Hoke, and Panagiotopoulos (2017), we utilize market-based data approach to proxy for bank solvency. We use market-based capital ratio (MBLR) which is defined as $\left(\frac{\text{MarketValue of Equity}}{\text{Book Value of Assets}}\right)$, as the solvency measure.

For measuring the wholesale funding costs that banks face, market participants use banks' Credit Default Swap (CDS) premia as a proxy, Beau, Hill, Hussain, and Nixon

(2014). CDS premia is the insurance premia that investors are willing to pay to be compensated if the debt issuer, such as the bank that borrows money, defaults. CDS premia tend to go up for all financial institutions during banking or financial crises due to market-wide factors. However, banks' own characteristics are important determinants of incremental increase in their funding costs over the market-wide increase.

We work with Dent, Hacıoğlu Hoke, and Panagiotopoulos (2017)'s small panel dataset, containing the four largest banks in the UK: Barclays plc, HSBC Holdings plc, Lloyds Banking Group plc and Royal Bank of Scotland plc. The UK banking sector is highly concentrated limiting the number of cross sections which motivates the use of higher frequency data, such as market data, to increase the time dimension of this panel data set. The panel data set is in weekly frequency, aggregated from daily to eliminate outliers and noise, and run from January 2007 to December 2016. Details on the data set is given in Appendix D.1.

We see merit in employing such an exercise even though the cross-sectional dimension of the dataset is small because the results provide us a reasonable interpretation of solvency and whole funding cost interactions. However, before we explain the details of this empirical model, we provide Monte Carlo simulation results for a short experiment designed to show how our method performs under a very small number of cross sections. This is to show that this empirical illustration leads to sensible results although the banking data set we use has data for only four banks. The data generating process and the parameters are the same as (14) in Section 4. We choose $N = 4$ and $T = 521$ to create a similar data set. We run the simulation for the homogeneous full rank case with 1000 replications. In Table 5, we report the bias and RMSE of β_1, β_2, and γ and c, under correction with $\bar{x}$, $\bar{y}$ and $\bar{w}$, correction with only $\bar{x}$, $\bar{y}$, and under no correction.

Table 5: Simulation Results of Homogenous Full Rank Case for $N = 4$ and $T = 521$

		β_1		β_2		γ	c
		Pooled	MG	Pooled	MG		
Correction with $\bar{x}$, $\bar{y}$ and $\bar{w}$	Bias (x100)	−0.21	−0.22	0.10	0.01	19.69	0.57
	RMSE (x100)	10.53	10.09	12.09	12.44	91.80	33.54
Correction with $\bar{x}$, $\bar{y}$	Bias (x100)	32.45	36.15	−66.33	−73.55	−49.13	0.05
	RMSE (x100)	51.61	52.40	101.62	103.23	105.44	170.65
No correction	Bias (x100)	19.52	17.03	−0.26	0.77	31.27	−3.85
	RMSE (x100)	29.99	29.66	35.94	38.92	103.18	58.62

The table shows that correction with only $\bar{x}$, $\bar{y}$ and no correction both create larger bias and RMSE for both coefficients than correction with $\bar{x}$, $\bar{y}$ and $\bar{w}$. As mentioned in Section 4 that correction with only $\bar{x}$, $\bar{y}$ leads to substantially larger bias and RMSE. We observe that this hold true even compared to no correction.

It is worth to mention that β_2 shows relatively small bias under no correction however the bias is still larger than the correction with $\bar{x}$, $\bar{y}$ and $\bar{w}$. Compared to the bias and RMSE of both coefficients in $N = 20$, $T = 400$ case of Table 1, β_1 shows a slightly inferior performance as the number of cross sections is small. On the other hand, an increase in T seems to favor less bias in β_2. Overall, the simulation results address the importance of correcting cross-sectional dependence.

Having shown the simulation results, we specify the main model as follows:

$$\begin{aligned} \Delta y_{it} &= \alpha_i + \beta_1' \Delta x_{it} + \beta_2' \Delta x_{it} g(q_{it}; \gamma, c) + e_{it} \\ g(q_{it}; \gamma, c) &= \left(1 + \exp\left(-\gamma\left(q_{it} - c\right)\right)\right)^{-1}, \end{aligned} \tag{16}$$

where $y_{it} = \{\text{CDS premia}_{it}\}$, $x_{it} = \{\text{MBLR}_{it}\}$. Bank fixed effects are indicated by α_i. The subscript i identifies the individuals, in our case UK banks, i.e. $i = 1, \ldots, N$ where $N = 4$. The subscript t identifies time. All the variables are in first differences to ensure stationarity. We take $q_{it} = \text{MBLR}$ so that the transition of β_2 is governed by the level of MBLR.

The main feature of this model is the non-linear relationship between ΔCDS premia and ΔMBLR. The change in CDS premia is nonlinear with respect to the level of MBLR. Namely, our assumption is that wholesale funding costs of UK banks change with respect to the level of their solvency. Therefore, for each solvency level, we expect a different coefficient on ΔMBLR, which is bounded between β_1 and $\beta_1 + \beta_2$, as a result of a straightforward interpretation of the transition function $g(q_{it}; \gamma, c)$.

To estimate (16), we need to eliminate the fixed effects. We follow González, Teräsvirta, and van Dijk (2005) and rewrite the model as in (17),

$$\Delta y_{it} = \alpha_i + \beta_1' \Delta x_{it} + \beta_2' w_{it} + e_{it}, \tag{17}$$

where $w_{it} = \Delta x_{it} g(q_{it}; \gamma, c)$ and subtract individual means of the variables to reach the equation (18) below,

$$\Delta \tilde{y}_{it} = \beta_1' \Delta \tilde{x}_{it} + \beta_2' \tilde{w}_{it} + \tilde{e}_{it}, \tag{18}$$

where $\tilde{y}_{it} = y_{it} - \bar{y}_i$, $\tilde{x}_{it} = x_{it} - \bar{x}_i$, $\tilde{w}_{it} = w_{it} - \bar{w}_i$, $\tilde{e}_{it} = e_{it} - \bar{e}_i$. The individual mean of the non-linear term is defined as $\bar{w}_i \equiv T^{-1} \sum_{t=1}^{T} \Delta x_{it} g(q_{it}; \gamma, c)$ and is calculated conditional on the estimated parameters, γ and c, during non-linear least squares estimation.

First we estimate two model specifications. Model I only includes MBLR as a dependent variable as defined in (16). Model II includes the first differences of risk free rate and VFTSE to proxy for the macroeconomic environment, i.e. $\Delta Z_t = \{\Delta \text{Risk Free Rate}_t, \Delta \text{VFTSE}_t\}$. Similar variables are used in empirical work, Annaert, Ceuster, Roy, and Vespro (2013). The model is linear with respect to these exogeneous variables. Additionally, we estimate two variants of each model. We estimate Model I and II first by *not* correcting them for cross-sectional dependence. The estimated coefficients are given in the second and third column of Table 6 for both pooled and MG estimators for Model I, and in the sixth and seventh columns for Model II. The second variant corrects for cross-sectional dependence. The results are presented under the columns of correction for both model specification in Table 6.

Table 6: Estimation results for no correction and correction for specification I and II

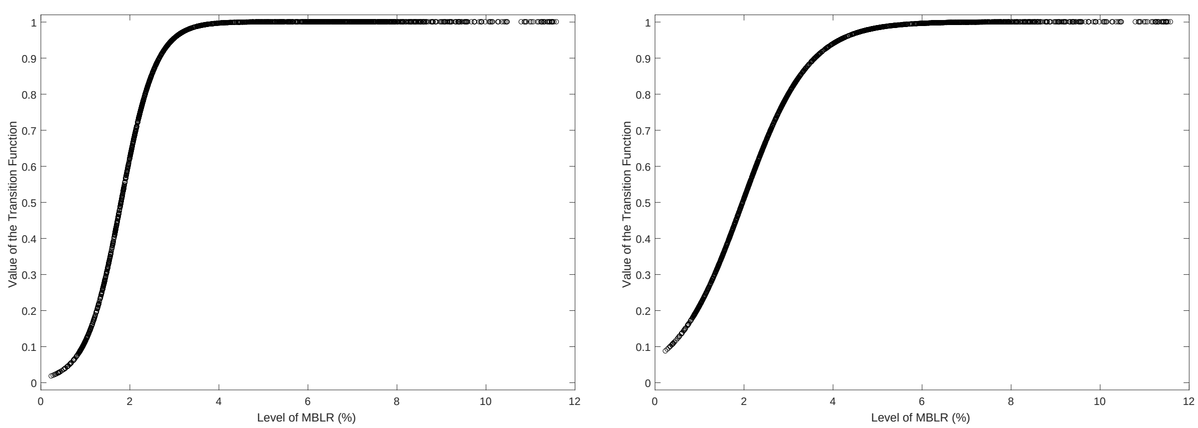
	I				II			
	No correction		Correction		No correction		Correction	
	Pooled	MG	Pooled	MG	Pooled	MG	Pooled	MG
β_1 (ΔMBLR)	-19.57	-24.81	-7.97	-8.55	-11.70	-15.40	-6.07	-6.31
β_2 (ΔMBLR*$g(q;\gamma,c)$)	12.35	16.14	4.64	4.26	11.31	11.22	4.67	4.16
ΔRisk Free Rate					-40.12	-39.02	-29.50	-28.43
ΔVFTSE					0.64	0.59	0.73	0.70

Three important observations are in order. First, the coefficient of the non-linear term, β_2, is smaller than β_1 in magnitude. Therefore, even when $g(q;\gamma,c) = 1$ for the maximum level of MBLR, the negative relationship of ΔMBLR and CDS premia is preserved both for no correction and correction of cross-sectional dependence. Second, coefficients β_1 and β_2 are smaller in magnitude under correction compared to the coefficients of no correction. For instance, for pooled estimators of Model I, the coefficient of MBLR is bounded between -19.57 and -7.22 for no correction, and -7.07 and -3.33 for correction. Therefore, if the first and second points are taken together, the magnitude of the relationship decreases under correction although the direction of the relationship always holds. Third, in Model II, the sign and the magnitude of coefficients of the exogenous variables are in line with what structural models for bank default and empirical work on the determinants of CDS premia suggest (Bongaerts, De Jong, and Driessen (2011), Longstaff and Schwatrz (1995), Annaert, Ceuster, Roy, and Vespro (2013)) although direct comparison of the coefficients under correction is not possible.

We estimate the parameters of the transition function by grid search. These parameters are used to evaluate the transition function at each MBLR level in the sample. The transition functions of Model I both for no correction and correction are presented in panels (a) and (b) of Figure 1.

Figure 1: Transition Functions of Model I

(a) No Correction (b) Correction

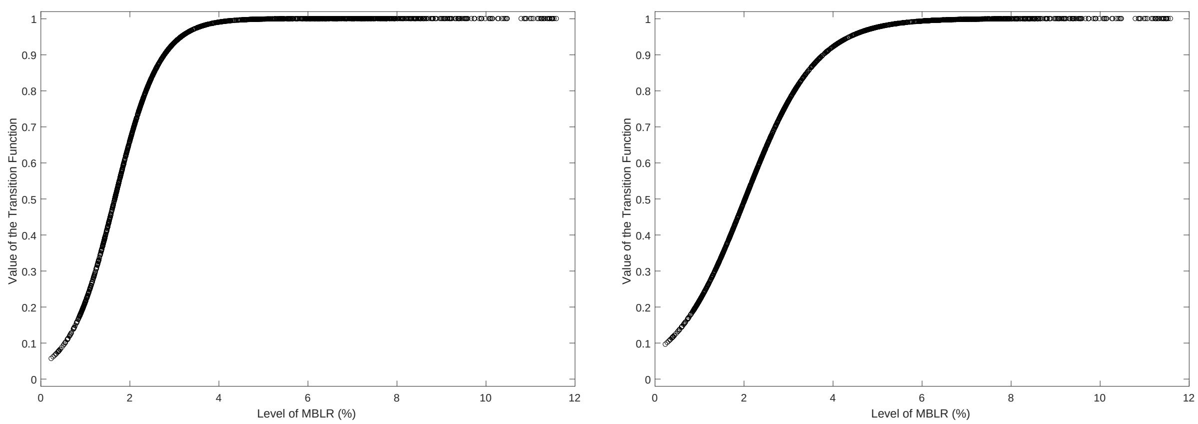


In Model I, when we do not correct for cross-sectional dependence, Figure 1a, the parameters of the transition function are estimated as $\gamma = 255$ and $c = 1.8\%$. That gives rise to a steeper slope compared to the slope of the transition function under correction with $\gamma = 135$ and $c = 2\%$ in Figure 1b. Model II's takeaway is very similar to that of Model I's because the model is linear with respect to the control variables. Inclusion of these variables change the outcome of the grid search however the qualitative results still hold. for no correction of Model II, Figure 2a, the parameters are estimated as $\gamma = 197$ and $c = 1.7\%$. For correction in panel (b) of Figure 2b, the parameters are $\gamma = 125$ and $c = 2\%$.

Our takeaway from this empirical exercise is two-fold. First, the net impact of ΔMBLR on CDS premia is always negative, as expected, for all cases regardless of the value of the transition function. Second, correcting for cross-sectional dependence leads to smaller coefficients of all variables, except for ΔVFTSE, in absolute terms. This is due to the fact that the cross-sectional averages we add to the system filter out the impact of common factors.

Figure 2: Transition Functions of Model II

(a) No Correction (b) Correction



5.2 Non-linear effect of debt on GDP growth

The impact of public indebtedness on economy has been under the radar of empirical studies (Reinhart, Reinhart, and Rogoff (2012), Baum, Checherita-Westphal, and Rother (2013)). Although indebtedness is fairly good in short run to stimulate the economy, it reduces the economic growth in long run. Especially after the recent financial crisis and sovereign debt crisis, the need to investigate potential nonlinearities in the relationship of debt-to-GDP ratio and GDP growth has elevated. Recently Chudik, Mohaddes, Pesaran, and Raissi (2017) explore the impact of public debt on countries' GDP growth. They investigate a potential debt threshold above which the GDP growth sharply falls due to increasing debt. This debt threshold is important to detect the short run and long run impact of public debt on output growth. However, they are not able to find a common threshold level of debt-to-GDP ratio after correcting cross-sectional dependence. That said, they find statistically significant threshold effects for countries with rising debt levels.

For our purposes, we adopt a simplification of Chudik, Mohaddes, Pesaran, and Raissi (2017)'s empirical illustration. We do not aim to replicate their results or claim to find a common threshold. Instead we use their methodology to illustrate the importance of correcting for cross-sectional dependence in non-linear models. We use the same data set Chudik, Mohaddes, Pesaran, and Raissi (2017) use with minor modifications to eliminate missing observations. We use the data for 36 countries data over 1971-2010 period which provides us 40 years of annual observations. The data is explained in more detail in

Appendix D.2. For simplicity, we do not distinguish between advanced and developing countries. Moreover, we do not use any lagged regressors in our model.

The model is very similar to the model in Section 5.1 and specified as follows:

$$
\begin{aligned}
\Delta y_{it} &= \alpha_i + \beta_1' \Delta x_{it} + \beta_2' \Delta x_{it} g(q_{it}; \gamma, c) + \delta' \Delta z_{it} + e_{it} \\
g(q_{it}; \gamma, c) &= \left(1 + \exp\left(-\gamma\left(q_{it} - c\right)\right)\right)^{-1},
\end{aligned}
\tag{19}
$$

where α_i is country-specific fixed effects, $y_{it} = log(\text{GDP growth}_{it})$, $x_{it} = log(\text{debt-to-GDP ratio}_{it})$ and $z_{it} = log(\text{CPI}_{it})$. The subscript i identifies the countries, i.e. $i = 1, \ldots, N$ where $N = 36$. The subscript t identifies time. All the variables are in first differences. We take $q_{it} = log(\text{debt-to-GDP ratio})$ so that the transition of β_2 is governed by the *level* of *log* debt-to-GDP ratio.

As in the previous exercise, we estimate two variants of this model after eliminating the fixed effects. First, we estimate the model first by *not* correcting for cross-sectional dependence. The estimated coefficients are given in the second and third columns of Table 7 both for pooled and MG estimators. As for the second variant, we correct for cross-sectional dependence. The results are presented in the last two columns of Table 7.

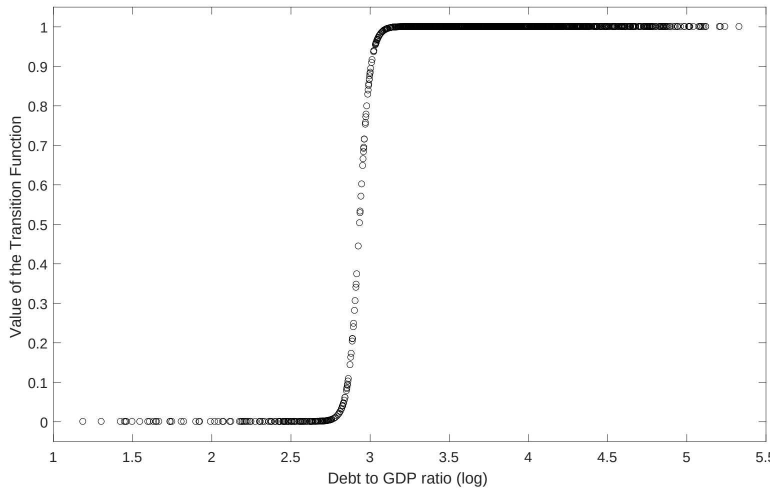
Table 7: Estimation results for no correction and correction

	No Correction		Correction	
	Pooled	MG	Pooled	MG
β_1	-0.0699	-0.1024	-0.0593	-0.0909
β_2	0.0001	0.0003	0.0004	0.0004
δ	-0.0229	0.0020	-0.0264	-0.0651

There are three observations that are worth discussing. First, regardless of the value of the transition function, the relationship between debt-to-GDP ratio and GDP growth is negative. Second, the coefficient of CPI is positive in the MG estimation of no correction case. That is not consistent with the rest of our results. Third, the magnitude of β_1 and δ are quite different between no correction and correction cases. The main conclusion of this exercise is that we need to employ the correction for cross-sectional dependence, as outlined by Chudik, Mohaddes, Pesaran, and Raissi (2017). Although direct comparison is not possible, our results are qualitatively in line with those of theirs.

We estimate the parameters of the transition function, γ and c, by grid search as the numerical optimization leads to unstable parameters. The grid search provides the same parameters for no correction and correction, $\gamma = 24$ and $c = 2.81$. Therefore, there is a single transition function which is given in Figure 3.

Figure 3: Transition Function



6 Conclusions

Increasing use of non-linear panel data estimation techniques gives rise to questions regarding how to tackle with cross-sectional dependence. Underlying unobserved factor structure, if not accounted for, causes estimation bias. In this paper, we provide an approach to estimate non-linear panel models which are subject to cross-sectional dependence. The basic idea is to augment Pesaran (2006)'s linear correction by using not only the cross-sectional averages of dependent and independent variables but also the cross-sectional averages of the non-linear term. This augmentation requires computing the estimation by non-linear least squares. We propose pooled and MG estimators and derive their asymptotic distributions. We show the consistency and asymptotic normality of the coefficients of the model. We elaborate on an extension with regard to the heterogeneity of the parameters in the non-linear term.

We illustrate the performance of both estimators by the Monte Carlo experiments. Simulation results show that the larger sample properties for pooled and MG estimators are indeed desirable. The method is first applied to an estimation of wholesale funding costs of UK banks. The estimates are in line with the theory of the determinants of funding costs. Moreover, we provide a second application where we explore the non-linear effect that public debt has on output growth.

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Appendix

A Proofs

A.1 Proof of Theorem 1

Recall that

$$\boldsymbol{\beta}(\boldsymbol{\gamma}) \equiv \boldsymbol{\beta}(\boldsymbol{\gamma},\boldsymbol{\gamma}_0) \equiv \boldsymbol{\beta}(\boldsymbol{\gamma},\boldsymbol{\gamma}_0,\boldsymbol{\beta}_0) = \left(\sum_{i=1}^{N} \boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{X}_i(\boldsymbol{\gamma})\right)^{-1}\left(\sum_{i=1}^{N} \boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{y}_i(\boldsymbol{\gamma}_0,\boldsymbol{\beta}_0)\right),$$

and

$$\boldsymbol{M}_Z = \boldsymbol{I} - \boldsymbol{Z}\left(\boldsymbol{Z}'\boldsymbol{Z}\right)^{-1}\boldsymbol{Z}',$$

for some matrix $\boldsymbol{Z}$. We note that $EL(\boldsymbol{\gamma}_0,\boldsymbol{\gamma}_0\ ) \leq EL(\boldsymbol{\gamma},\boldsymbol{\gamma}_{\mathbf{0}}), \forall\boldsymbol{\gamma}\in\boldsymbol{\Gamma}$. Then, for consistency and at rate $f(N,T)$, it is sufficient show that

$$\sup_{\boldsymbol{\gamma}_1,\boldsymbol{\gamma}_2\in\boldsymbol{\Gamma}} \|\boldsymbol{\beta}(\boldsymbol{\gamma}_1,\boldsymbol{\gamma}_2) - \boldsymbol{\beta}_0^*\| = O_p\left(f(N,T)\right). \tag{A.1}$$

where $p\lim_{N,T\to\infty}\boldsymbol{\beta}(\boldsymbol{\gamma}_1,\boldsymbol{\gamma}_2) = \boldsymbol{\beta}^*_{\boldsymbol{\gamma}_1,\boldsymbol{\gamma}_2} \equiv \boldsymbol{\beta}_0^*$. This follows by (4.1.6) of Theorem 4.1.1 of Amemiya (1985) and Assumption 5 (ii). We show the simpler result

$$\sup_{\boldsymbol{\gamma}\in\boldsymbol{\Gamma}} \|\boldsymbol{\beta}(\boldsymbol{\gamma},\boldsymbol{\gamma}) - \boldsymbol{\beta}_0\| = O_p\left(f(N,T)\right),$$

since then $p\lim_{N,T\to\infty}\boldsymbol{\beta}(\boldsymbol{\gamma},\boldsymbol{\gamma}) = \boldsymbol{\beta}^*_{\boldsymbol{\gamma},\boldsymbol{\gamma}} = \boldsymbol{\beta}_0$. The general result follows similarly. We have

$$\boldsymbol{\beta}(\boldsymbol{\gamma},\boldsymbol{\gamma}) = \boldsymbol{\beta}_0 + \left(\sum_{i=1}^{N} \boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{X}_i(\boldsymbol{\gamma})\right)^{-1}\left(\sum_{i=1}^{N} \boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{X}_i(\boldsymbol{\gamma})\boldsymbol{\upsilon}_{\beta,i}\right) + \left(\sum_{i=1}^{N} \boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{X}_i(\boldsymbol{\gamma})\right)^{-1}\left(\sum_{i=1}^{N} \boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{u}_i\right),$$

By independence of $\boldsymbol{\upsilon}_{\beta,i}$, from all other stochastic quantities, $\left(\sum_{i=1}^{N} \boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{X}_i(\boldsymbol{\gamma})\right)^{-1}\left(\sum_{i=1}^{N} \boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{X}_i(\boldsymbol{\gamma})\boldsymbol{\upsilon}_{\beta,i}\right) = O_p\left(N^{-1/2}\right)$.
We consider $\left(\sum_{i=1}^{N} \boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{X}_i(\boldsymbol{\gamma})\right)^{-1}\left(\sum_{i=1}^{N} \boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{u}_i\right)$. By (3), we substitute

$\boldsymbol{u}_i = \boldsymbol{F}\boldsymbol{\delta}_i + \boldsymbol{\epsilon_i}$

$$\begin{aligned}
&\left(\sum_{i=1}^{N} \boldsymbol{X}_i(\boldsymbol{\gamma})' \boldsymbol{M}_{\bar{Z}_\gamma} \boldsymbol{X}_i(\boldsymbol{\gamma})\right)^{-1} \left(\sum_{i=1}^{N} \boldsymbol{X}_i(\boldsymbol{\gamma})' \boldsymbol{M}_{\bar{Z}_\gamma} \boldsymbol{u}_i\right) \\
&= \left(\sum_{i=1}^{N} \boldsymbol{X}_i(\boldsymbol{\gamma})' \boldsymbol{M}_{\bar{Z}_\gamma} \boldsymbol{X}_i(\boldsymbol{\gamma})\right)^{-1} \sum_{i=1}^{N} \boldsymbol{X}_i(\boldsymbol{\gamma})' \boldsymbol{M}_{\bar{Z}_\gamma} \boldsymbol{F}\boldsymbol{\delta}_i + \\
&\left(\sum_{i=1}^{N} \boldsymbol{X}_i(\boldsymbol{\gamma})' \boldsymbol{M}_{\bar{Z}_\gamma} \boldsymbol{X}_i(\boldsymbol{\gamma})\right)^{-1} \sum_{i=1}^{N} \boldsymbol{X}_i(\boldsymbol{\gamma})' \boldsymbol{M}_{\bar{Z}_\gamma} \boldsymbol{\epsilon_i}.
\end{aligned}$$

The only term of concern is $\left(\sum_{i=1}^{N} \boldsymbol{X}_i(\boldsymbol{\gamma})' \boldsymbol{M}_{\bar{Z}_\gamma} \boldsymbol{X}_i(\boldsymbol{\gamma})\right)^{-1} \sum_{i=1}^{N} \boldsymbol{X}_i(\boldsymbol{\gamma})' \boldsymbol{M}_{\bar{Z}_\gamma} \boldsymbol{F}\boldsymbol{\delta}_i$.

We have,

$$\begin{aligned}
\bar{\boldsymbol{Z}}_\gamma &= \left(\bar{\boldsymbol{X}}_\gamma, \boldsymbol{F}\right) \begin{pmatrix} \boldsymbol{\beta}_0 & 0 & I \\ \bar{\boldsymbol{\delta}} & \bar{\boldsymbol{\Pi}} & 0 \end{pmatrix} + \left(\bar{\boldsymbol{\epsilon}} + \frac{\boldsymbol{1}}{\boldsymbol{N}} \sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} \boldsymbol{X_i}(\boldsymbol{\gamma}) \boldsymbol{v}_{\boldsymbol{\beta},\boldsymbol{i}}, \bar{\boldsymbol{V}}, \boldsymbol{0}\right) \\
&= \left(\bar{\boldsymbol{X}}_\gamma \boldsymbol{\beta}_0 + \boldsymbol{F}\bar{\boldsymbol{\delta}}, \boldsymbol{F}\bar{\boldsymbol{\Pi}}, \bar{\boldsymbol{X}}_\gamma\right) + \left(\bar{\boldsymbol{\epsilon}} + \frac{\boldsymbol{1}}{\boldsymbol{N}} \sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} \boldsymbol{X_i}(\boldsymbol{\gamma}) \boldsymbol{v}_{\boldsymbol{\beta},\boldsymbol{i}}, \bar{\boldsymbol{V}}, \boldsymbol{0}\right) \\
&= \boldsymbol{G}_\gamma \bar{\boldsymbol{P}}^* + \bar{\boldsymbol{U}}^{**} \\
&= \bar{\boldsymbol{Q}}_\gamma + \bar{\boldsymbol{U}}^{**}.
\end{aligned}$$

We know that $\boldsymbol{X}_i(\boldsymbol{\gamma})' \boldsymbol{M}_{\bar{Q}_\gamma} \bar{\boldsymbol{Q}}_\gamma = \boldsymbol{0}$ and so by Lemmas 2 and 3 of Pesaran (2006) and (A.4)-(A.5), below, it follows

$$\boldsymbol{X}_i(\boldsymbol{\gamma})' \boldsymbol{M}_{\bar{Z}_\gamma} \bar{\boldsymbol{Q}}_\gamma = O_p\left(\frac{1}{N}\right) + O\left(\frac{1}{\sqrt{NT}}\right) \tag{A.2}$$

which immediately implies

$$\boldsymbol{X}_i(\boldsymbol{\gamma})' \boldsymbol{M}_{\bar{Z}_\gamma} \bar{\boldsymbol{X}}_\gamma \boldsymbol{\beta}_0 = O_p\left(\frac{1}{N}\right) + O\left(\frac{1}{\sqrt{NT}}\right)$$

and

$$\boldsymbol{X}_i(\boldsymbol{\gamma})' \boldsymbol{M}_{\bar{Z}_\gamma} \left(\bar{\boldsymbol{X}}_\gamma \boldsymbol{\beta}_0 + \boldsymbol{F}\bar{\boldsymbol{\delta}}\right) = O_p\left(\frac{1}{N}\right) + O\left(\frac{1}{\sqrt{NT}}\right)$$

and, therefore

$$\boldsymbol{X}_i(\boldsymbol{\gamma})' \boldsymbol{M}_{\bar{Z}_\gamma} \boldsymbol{F}\bar{\boldsymbol{\delta}} = O_p\left(\frac{1}{N}\right) + O\left(\frac{1}{\sqrt{NT}}\right). \tag{A.3}$$

We need to obtain a result for $\left(\sum_{i=1}^{N}\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{X}_i(\boldsymbol{\gamma})\right)^{-1}\sum_{i=1}^{N}\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{F}\boldsymbol{\delta}_i$. We have

$$\begin{aligned}\left(\sum_{i=1}^{N}\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{X}_i(\boldsymbol{\gamma})\right)^{-1}\sum_{i=1}^{N}\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{F}\boldsymbol{\delta}_i &= \left(\sum_{i=1}^{N}\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{X}_i(\boldsymbol{\gamma})\right)^{-1}\sum_{i=1}^{N}\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{F}\bar{\boldsymbol{\delta}} \\ &+ \left(\sum_{i=1}^{N}\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{X}_i(\boldsymbol{\gamma})\right)^{-1}\times \\ &\sum_{i=1}^{N}\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{F}\left(\boldsymbol{\upsilon}_{\delta,i}-\bar{\boldsymbol{\upsilon}}_\delta\right) \\ &= O_p\left(\frac{1}{N}\right)+O\left(\frac{1}{\sqrt{NT}}\right)+O_p\left(\frac{1}{\sqrt{N}}\right)\end{aligned}$$

by (A.3) and the fact that $(\boldsymbol{\upsilon}_{\delta,i}-\bar{\boldsymbol{\upsilon}}_\delta)$ is zero mean and independent of $\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{Z}_\gamma}\boldsymbol{F}$.

It remains to derive the order of $\frac{\boldsymbol{X}_i(\boldsymbol{\gamma})'\bar{\boldsymbol{V}}}{T}$ uniformly over $\gamma \in \boldsymbol{\Gamma}$. The rest of the results needed for (A.2) are given in Lemmas 2 and 3 of Pesaran (2006). We have

$$\frac{\boldsymbol{X}_i(\boldsymbol{\gamma})'\bar{\boldsymbol{V}}}{T}=\frac{1}{N}\sum_{j=1,j\neq i}^{N}\frac{\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{V}_j}{T}+\frac{1}{N}\frac{\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{V}_i}{T}$$

We examine each term. Denote $\bar{\boldsymbol{V}}_{-i}=\frac{1}{N}\sum_{j=1,j\neq i}^{N}\boldsymbol{V}_j=(\bar{\boldsymbol{v}}_{-i,1},...,\bar{\boldsymbol{v}}_{-i,T})'$. By assumption $\bar{\boldsymbol{v}}_{-i,t}$ and $g(\boldsymbol{x}_{i,t},\boldsymbol{\gamma})$ are independently distributed processes and so

$$\sup_\gamma Var\left(\frac{\sum_{t=1}^{T}\bar{\boldsymbol{v}}_{-i,t}g_j(\boldsymbol{x}_{i,t},\boldsymbol{\gamma})}{T}\right)=\frac{\sum_{t=1}^{T}\sum_{t'=1}^{T}\sup_\gamma E\left(g_j(\boldsymbol{x}_{i,t},\boldsymbol{\gamma})\,g_j(\boldsymbol{x}_{i,t'},\boldsymbol{\gamma})\right)E\left(\bar{\boldsymbol{v}}_{-i,t}\bar{\boldsymbol{v}}'_{-i,t'}\right)}{T^2},$$

where $E\left(\bar{\boldsymbol{v}}_{-i,t}\bar{\boldsymbol{v}}'_{-i,t'}\right)=O\left(\frac{1}{N}\right)$. Hence,

$$\begin{aligned}\sup_\gamma Var\left(\frac{\sum_{t=1}^{T}\bar{\boldsymbol{v}}_{-i,t}g_j(\boldsymbol{x}_{i,t},\boldsymbol{\gamma})}{T}\right)&=O\left(\frac{1}{N}\right)\left\{\frac{\sum_{t=1}^{T}\sum_{t'=1}^{T}\sup_\gamma E\left(g_j(\boldsymbol{x}_{i,t},\boldsymbol{\gamma})\,g_j(\boldsymbol{x}_{i,t'},\boldsymbol{\gamma})\right)}{T^2}\right\} \\ &=O\left(\frac{1}{N}\right)\left\{\frac{\sum_{t=1}^{T}\sum_{t'=1}^{T}\sup_\gamma C_{g,j,\gamma}(|t-t'|)}{T^2}\right\}.\end{aligned}$$

By assumptions 1, 2 and 5,

$$\sup_\gamma\sum_{t'=1}^{T}C_{g,j,\gamma}(|t-t'|)=O(1).$$

Therefore,

$$\sup_\gamma Var\left(\frac{\sum_{t=1}^{T}\bar{\boldsymbol{v}}_{-i,t}g_j(\boldsymbol{x}_{i,t},\boldsymbol{\gamma})}{T}\right)=O\left(\frac{1}{NT}\right)$$

and

$$\sup_{\gamma} \frac{1}{N} \sum_{j=1, j\neq i}^{N} \frac{\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{V}_j}{T} = O_p\left(\frac{1}{\sqrt{NT}}\right). \tag{A.4}$$

Further, noting that by assumption

$$\sup_{\gamma} E\left(\boldsymbol{v}_{i,t} g_j\left(\boldsymbol{x}_{i,t}, \boldsymbol{\gamma}\right)\right) < \infty$$

immediately implies that

$$\sup_{\gamma} Var\left(\frac{\sum_{t=1}^{T} \boldsymbol{v}_{i,t} g_j\left(\boldsymbol{x}_{i,t}, \boldsymbol{\gamma}\right)}{T}\right) = O(1)$$

which results in

$$\sup_{\gamma} Var\left(\frac{\sum_{t=1}^{T} \boldsymbol{v}_{i,t} g_j\left(\boldsymbol{x}_{i,t}, \boldsymbol{\gamma}\right)}{NT}\right) = O(N^{-2})$$

and

$$\sup_{\gamma} \frac{1}{N} \frac{\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{V}_i}{T} = O_p\left(\frac{1}{N}\right) \tag{A.5}$$

proving the result.

From all the above, it follows that

$$\hat{\boldsymbol{\gamma}} - \boldsymbol{\gamma_0} = O\left(\frac{1}{\sqrt{NT}}\right) + O_p\left(\frac{1}{N}\right). \tag{A.6}$$

A.2 Proof of Theorem 2

We need to show that

$$\sqrt{N}\left(\boldsymbol{\beta}\left(\hat{\boldsymbol{\gamma}}\right) - \boldsymbol{\beta}\left(\boldsymbol{\gamma}_0\right)\right) = o_p\left(1\right) \tag{A.7}$$

which immediately implies that $\boldsymbol{\gamma}_0$ can be assumed known in deriving the inferential properties of $\boldsymbol{\beta}\left(\hat{\boldsymbol{\gamma}}\right)$. We have, using a Taylor expansion, that

$$\boldsymbol{\beta}\left(\hat{\boldsymbol{\gamma}}, \boldsymbol{\gamma}_0\right) = \boldsymbol{\beta}\left(\boldsymbol{\gamma}_0, \boldsymbol{\gamma}_0\right) + \left.\frac{\partial \boldsymbol{\beta}\left(\boldsymbol{\gamma}, \boldsymbol{\gamma}_0\right)}{\partial \boldsymbol{\gamma}}\right|_{\boldsymbol{\gamma}=\boldsymbol{\gamma}_0} \left(\hat{\boldsymbol{\gamma}} - \boldsymbol{\gamma}_0\right) + o_p\left(\hat{\boldsymbol{\gamma}} - \boldsymbol{\gamma}_0\right)$$

and since $\left.\frac{\partial \boldsymbol{\beta}(\boldsymbol{\gamma}, \boldsymbol{\gamma}_0)}{\partial \boldsymbol{\gamma}}\right|_{\boldsymbol{\gamma}=\boldsymbol{\gamma}_0}$ is bounded by Assumption 5(i) and (A.6) holds, (A.7) follows. The rest of the Theorem follows immediately by Theorems 1 and 2 of Pesaran (2006).

A.3 Proof of Lemma 3

We have

$$g\left(\boldsymbol{x}_{i,t},\gamma_i\right)=g\left(\boldsymbol{x}_{i,t},\gamma\right)+\sum_{j=1}^{\infty}\left(j!\right)^{-1}g^{(j)}\left(\boldsymbol{x}_{i,t},\gamma\right)\left[\upsilon_{\gamma,i}+\left(\gamma_0-\gamma\right)\right]^j=g\left(\boldsymbol{x}_{i,t},\gamma\right)+F\left(\boldsymbol{x}_{i,t},\upsilon_{\gamma,i},\gamma,\gamma_0\right).$$

Note that

$$E\left[F\left(\boldsymbol{x}_{i,t},\upsilon_{\gamma,i},\gamma,\gamma_0\right)\right]=\sum_{j=1}^{\infty}\left(j!\right)^{-1}g^{(j)}\left(\boldsymbol{x}_{i,t},\gamma\right)E\left\{\left[\upsilon_{\gamma,i}+\left(\gamma_0-\gamma\right)\right]^j\right\}$$

Recall that $F\left(\boldsymbol{x}_i,\upsilon_{\gamma,i},\gamma,\gamma_0\right)=\left(F\left(\boldsymbol{x}_{i,1},\upsilon_{\gamma,i},\gamma,\gamma_0\right),...,F\left(\boldsymbol{x}_{i,t},\upsilon_{\gamma,i},\gamma,\gamma_0\right)\right)'$ and $F_i\left(\gamma,\gamma_0\right)=\left(E\left[F\left(\boldsymbol{x}_{i,1},\upsilon_{\gamma,i},\gamma,\gamma_0\right)\right],...,E\left[F\left(\boldsymbol{x}_{i,T},\upsilon_{\gamma,i},\gamma,\gamma_0\right)\right]\right)'$. Note that if $\gamma_0=\gamma$ and $\upsilon_{\gamma,i}$ is zero mean and has a symmetric distribution then $E\left\{\left[\upsilon_{\gamma,i}+\left(\gamma_0-\gamma\right)\right]^j\right\}=0$, for all odd j. Then, noting that $\sim$ denotes asymptotic equivalence ie $A\sim B$ is defined as $A=B+o_p(B)$, we have

$$\begin{aligned}
\beta\left(\gamma_0,\gamma_i\right)=&\left(\sum_{i=1}^{N}\boldsymbol{X}_i(\gamma_0)'\boldsymbol{M}_{\bar{z}_\gamma}\boldsymbol{X}_i(\gamma_0)\right)^{-1}\left(\sum_{i=1}^{N}\boldsymbol{X}_i(\gamma_0)'\boldsymbol{M}_{\bar{z}_\gamma}\boldsymbol{y}_i\left(\gamma_i,\beta_0+\boldsymbol{\upsilon}_{\beta,i}\right)\right)\sim\\
&\beta_0+\beta_0\left(\sum_{i=1}^{N}\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{z}_\gamma}\boldsymbol{X}_i(\boldsymbol{\gamma})\right)^{-1}\sum_{i=1}^{N}\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{z}_\gamma}F_i\left(\gamma,\gamma_0\right)+\\
&+\beta_0\left(\sum_{i=1}^{N}\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{z}_\gamma}\boldsymbol{X}_i(\boldsymbol{\gamma})\right)^{-1}\sum_{i=1}^{N}\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{z}_\gamma}\left[F\left(\boldsymbol{x}_i,\upsilon_{\gamma,i},\gamma,\gamma_0\right)-F_i\left(\gamma,\gamma_0\right)\right]\\
&+\left(\sum_{i=1}^{N}\boldsymbol{X}_i(\gamma_0)'\boldsymbol{M}_{\bar{z}_\gamma}\boldsymbol{X}_i(\gamma_0)\right)^{-1}\left(\sum_{i=1}^{N}\boldsymbol{X}_i(\gamma_0)'\boldsymbol{M}_{\bar{z}_\gamma}g\left(\boldsymbol{x}_{i,t},\boldsymbol{\gamma}_i\right)'\boldsymbol{\upsilon}_{\beta,i}\right)\\
&+\left(\sum_{i=1}^{N}\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{z}_\gamma}\boldsymbol{X}_i(\boldsymbol{\gamma})\right)^{-1}\sum_{i=1}^{N}\boldsymbol{X}_i(\boldsymbol{\gamma})'\boldsymbol{M}_{\bar{z}_\gamma}\epsilon_{i,t}\\
&=\beta_0+\beta_0A_{1,NT}\left(\gamma,\gamma_0\right)+A_{2,NT}+A_{3,NT}+A_{4,NT}
\end{aligned}$$

Only $A_{1,NT}\left(\gamma,\gamma_0\right)$ is not asymptotically negligible. Then,

$$L(\gamma_0,\gamma_0)\sim\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\epsilon_{i,t}+g\left(\boldsymbol{x}_{i,t},\boldsymbol{\gamma}_i\right)'\left(\left[\upsilon_{\gamma,i}+\beta_0A_1\left(\gamma_0,\gamma_0\right)\right]\right)\right)^2$$

and

$$L(\gamma,\gamma_0)\sim\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\epsilon_{i,t}+g\left(\boldsymbol{x}_{i,t},\boldsymbol{\gamma}_i\right)'\left(\left[\upsilon_{\gamma,i}+\beta_0A_1\left(\gamma,\gamma_0\right)\right]\right)\right)^2$$

Noting that since $\epsilon_{i,t}$, $g\left(\boldsymbol{x}_{i,t},\gamma_i\right)$ and $\upsilon_{\gamma,i}$ and pairwise independent, consistency follows if $A_1\left(\gamma_0,\gamma_0\right)A_1\left(\gamma_0,\gamma_0\right)'<A_1\left(\gamma,\gamma_0\right)A_1\left(\gamma,\gamma_0\right)'$ for all $\gamma\neq\gamma_0$.

B Extensions

Here we provide the simulation results of the same set up in Section 4 for Homogeneous $\boldsymbol{\beta}$ case and Full Rank specification with *heterogeneous parameters*, γ_i and c_i. The DGP is the same as in (14) and (15). All parameters are drawn from the same distributions as specified in Section 4 except for $\gamma_i \sim N(1, 0.5)$ and $c_i \sim N(0, 0.5)$. Essentially, the results in Table A1 are the heterogeneous non-linear parameter counterparts of the results in Table 1. All the qualitative results drawn in section 4 hold for this experiment.

Table A1: Experiment 1A, Homogeneous $\boldsymbol{\beta}$, Full Rank with Heterogeneous non-linear parameters

β_1, correction with $\bar{y}$, $\bar{x}$, $\bar{w}$

	Bias (x100)					RMSE (x100)					Size (5% level, $H_0: \beta_1 = 1.00$)					Size under true γ and c					Power (5% level, $H_0: \beta_1 = 0.95$)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
Pooled																									
20	−1.77	−1.00	−0.37	−0.25	−0.07	28.06	13.31	6.24	4.41	3.90	20.45	15.40	13.65	11.75	10.60	7.65	7.75	8.40	8.70	8.20	23.95	21.40	23.95	32.65	38.75
50	−1.35	−0.50	−0.24	−0.14	0.02	21.64	4.97	4.21	2.93	3.52	22.50	13.70	10.90	9.85	7.80	6.10	5.45	5.30	6.70	6.45	29.40	32.10	39.80	49.75	37.65
100	−1.13	−0.25	−0.04	0.07	0.09	12.53	3.58	2.48	2.01	1.79	23.30	13.10	10.20	8.05	7.50	6.00	6.05	5.35	5.70	5.60	31.90	46.65	65.10	78.45	84.40
200	−0.49	0.06	−0.01	0.06	0.08	8.20	4.49	1.81	1.91	1.66	24.20	11.75	10.35	8.35	6.45	5.90	4.85	4.85	6.50	5.50	47.15	57.55	87.60	82.40	88.35
400	−0.78	−0.04	0.20	0.09	0.10	5.48	1.74	1.67	1.19	0.94	23.35	13.00	9.05	7.80	7.65	4.50	4.75	5.25	5.40	4.75	62.25	90.30	91.35	99.05	100.00
MG																									
20	−8.81	−1.54	−0.57	−0.32	−0.13	65.28	15.32	6.55	4.27	3.60	21.00	16.55	13.95	11.40	10.45	6.80	7.40	7.55	6.75	6.85	23.15	21.05	24.40	32.95	39.95
50	−3.71	−0.75	−0.32	−0.19	0.04	33.90	5.57	4.29	2.78	3.50	22.80	14.80	12.75	10.05	6.60	5.85	5.20	5.05	6.25	5.50	26.95	30.55	41.00	54.65	36.15
100	−3.05	−0.38	−0.06	0.03	0.10	20.22	4.01	2.53	1.95	1.67	23.25	15.55	11.10	9.75	7.70	5.70	5.35	6.05	5.50	5.95	30.05	43.65	66.45	83.00	89.40
200	−1.21	−0.05	−0.06	0.06	0.10	10.40	5.15	1.87	1.94	1.64	24.75	13.20	12.25	8.15	6.20	4.65	5.15	4.85	5.80	4.85	39.55	52.35	87.15	81.60	89.65
400	−1.37	−0.09	0.20	0.09	0.10	8.47	1.95	1.79	1.20	0.88	24.00	14.70	9.50	8.20	7.75	5.30	4.35	4.55	5.15	4.70	52.50	87.20	90.15	99.00	100.00

β_2, correction with $\bar{y}$, $\bar{x}$, $\bar{w}$

	Bias (x100)					RMSE (x100)					Size (5% level, $H_0: \beta_2 = 1.00$)					Size under true γ and c					Power (5% level, $H_0: \beta_2 = 0.95$)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
Pooled																									
20	7.91	2.62	0.97	0.67	0.29	58.12	23.84	8.59	5.42	3.64	16.25	16.00	15.60	14.30	13.45	7.65	7.60	8.20	7.30	9.00	18.00	21.35	23.90	33.65	49.50
50	4.28	1.60	0.58	0.33	0.09	33.76	7.94	6.60	3.27	2.59	18.75	13.45	13.65	12.35	10.35	5.80	6.05	6.05	6.50	5.25	21.40	27.15	37.20	53.10	65.55
100	5.01	0.81	0.31	0.12	0.06	33.48	5.61	3.47	2.30	1.62	23.10	15.60	12.65	12.65	11.75	5.45	6.20	4.80	5.20	5.15	27.80	33.80	51.85	76.10	94.35
200	2.48	0.24	0.19	0.05	0.05	18.09	8.05	2.48	1.87	1.29	24.85	16.50	13.85	11.55	11.50	5.40	4.85	5.20	5.90	5.45	34.55	40.80	74.55	88.85	98.75
400	2.93	0.38	-0.04	0.05	0.03	17.13	3.08	2.20	1.29	0.84	26.05	17.20	15.00	12.25	11.80	4.05	5.30	4.90	4.50	4.75	46.15	71.90	81.95	99.20	100.00
MG																									
20	24.53	4.17	1.45	0.87	0.34	139.78	28.27	9.95	6.12	4.09	16.00	17.30	15.75	15.05	15.05	6.40	6.20	7.15	6.40	7.95	17.95	21.15	23.85	32.15	44.65
50	13.97	2.26	0.78	0.39	0.16	71.81	9.87	7.25	3.74	2.87	18.50	16.00	15.55	13.80	11.20	5.90	5.15	5.25	5.80	4.95	21.15	26.10	33.80	48.20	58.90
100	14.08	1.14	0.38	0.18	0.08	82.16	6.93	4.06	2.63	1.82	23.35	16.80	15.05	14.10	13.35	5.45	5.70	5.05	4.80	4.80	27.40	31.85	46.10	70.20	90.85
200	5.19	0.60	0.31	0.10	0.10	31.61	9.70	2.96	2.16	1.47	24.00	19.70	15.60	12.95	12.95	4.50	5.45	5.60	5.00	4.85	29.50	36.55	67.10	83.40	97.10
400	4.95	0.56	0.02	0.08	0.06	30.49	3.87	2.64	1.51	0.94	26.05	19.45	16.50	14.85	14.05	4.55	4.55	3.70	4.60	4.45	36.95	61.75	73.30	97.90	100.00

	correction with $\bar{y}$, $\bar{x}$, $\bar{w}$										correction with $\bar{y}$, $\bar{x}$														
	γ Bias (x100)					γ RMSE (x100)					γ Bias (x100)					β_1 Bias Pooled (x100)					β_2 Bias Pooled (x100)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
20	64.78	32.72	19.31	6.57	3.11	174.55	119.79	74.75	38.35	21.30	83.70	35.36	17.93	−8.91	−10.67	−775.41	18.37	17.57	15.98	16.41	18261	−32.55	−31.47	−25.63	−27.57
50	47.88	11.92	4.18	1.76	0.98	148.41	63.83	35.68	17.59	12.76	59.56	13.18	−6.21	−13.34	−10.28	15.12	15.71	14.95	14.80	21.65	−24.49	−23.21	−23.35	−23.65	−43.14
100	38.70	5.10	1.61	0.47	0.33	134.23	38.04	19.02	12.95	8.52	52.14	0.05	−8.78	−18.01	−10.66	−47.54	14.94	15.02	15.31	16.58	41.13	−22.57	−23.63	−25.26	−29.02
200	19.53	5.67	0.63	0.70	0.16	87.23	31.81	13.71	9.30	6.30	23.51	12.44	−11.05	−2.73	−6.23	14.92	18.54	15.45	20.56	21.05	-20.28	−33.72	−25.39	−40.10	−41.43
400	5.53	0.16	1.74	0.28	-0.02	50.59	17.26	12.14	6.96	4.59	6.16	−9.81	0.45	−10.13	−10.27	13.81	14.65	21.00	19.11	17.22	−16.00	−21.26	−40.43	−36.15	−30.60
	c Bias (x100)					c RMSE (x100)					c Bias (x100)					β_1 Bias MG (x100)					β_2 Bias MG (x100)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
20	12.07	4.79	1.21	1.14	0.64	129.65	53.09	28.79	17.71	11.72	37.51	48.52	49.62	61.29	58.16	−12701	21.21	20.41	18.11	18.49	809770	−38.92	−38.29	−30.64	−32.70
50	5.93	2.86	1.13	0.49	1.92	86.96	27.68	16.35	10.70	8.54	46.69	66.52	62.74	60.27	41.51	−57.80	18.10	17.25	16.90	25.93	−79.78	−29.15	−29.04	−28.86	−53.38
100	5.99	2.06	0.42	0.49	1.24	74.83	18.88	11.19	7.62	5.54	41.19	64.70	62.14	56.17	54.08	−424.04	17.77	17.22	17.47	18.91	26851	−29.81	−29.17	−30.60	−34.64
200	5.97	2.20	1.11	1.77	1.92	47.45	17.36	8.20	6.66	4.88	65.71	52.97	59.09	47.43	45.62	18.27	22.78	17.84	24.45	25.02	−45.74	−44.85	−31.36	−49.75	−51.10
400	4.15	1.40	1.89	1.50	1.28	43.62	10.08	8.33	4.90	3.18	73.90	70.13	47.71	48.32	53.98	15.66	17.36	25.41	22.57	19.69	−31.16	−28.31	−51.51	−44.65	−36.65

C Additional Monte Carlo Analysis

In this section, we present the results of an additional Monte Carlo experiment. Except for the DGP, the design of the simulations and the presentation of the results are the same as in Section 4.

Let the DGP be ,

$$\begin{aligned} y_{it} &= \beta_{i1} x_{it} + \beta_{i2} g(q_{it} : \gamma, c) x_{it} + u_{it} \\ g_{it} &= 1/(1 + \exp(-\gamma(q_{it} - c)), \\ x_{it} &= \sum_{m=1}^{M} \lambda_{1_m} f_{m_t} + \varepsilon_{it} \\ u_{it} &= \sum_{m=1}^{M} \lambda_{2_m} f_{m_t} + \upsilon_{it} \end{aligned} \tag{C.1}$$

where x_{it} is the observable regressors on the ith cross-sectional dimension at time t for $i = 1, \ldots, N$, $t = 1, \ldots, T$; f_{mt} is the unobserved factors for $m = 1, \ldots, M$ with the total number of factors indicated by M. The factors, their coefficients and the idiosyncratic components are generated as

$$\begin{aligned} f_{m_t} &\sim \text{IIDN}(0, 1) \\ \lambda_{l_m} &\sim \text{IIDN}(1, 1) \\ \varepsilon_{it} &\sim \text{IIDN}(0, 1) \\ \upsilon_{it} &\sim \text{IIDN}(0, 1). \end{aligned}$$

We address the non-linear term $x_{it} g(q_{it} : \gamma, c)$ as w_{it}. The model in (C.1) is a panel smooth transition model, as in González, Teräsvirta, and van Dijk (2005), with additional cross-sectional dependence through the unobserved factors, $\boldsymbol{f}$.

The rank condition here has a different structure than the simulations in Section 4. The DGP in (C.1) constitutes only one explanatory variable. When the number of factors is more than the number of explanatory variables, i.e. in our case when $M > 1$, the rank condition is not satisfied. To test how the rank condition affects the performance of the model, we conduct different experiments and take $M = 1$ where the rank condition is satisfied, and $M = 2$ and $M = 3$ where the rank condition is not satisfied.

The results are presented in Tables A2 to A7 for both experiments for $M = 1$ (1 factor), $M = 2$ (2 factors) and $M = 3$ (3 factors) respectively. Under option 1, the bias and RMSE of both pooled and MG estimators of β_1 and β_2 tend to be large for small samples but drop substantially with N and T. The results of the case where $M = 1$ are presented in Tables A2 and A3. Heterogeneity of the coefficients has a minimal impact on the bias and RMSE of the coefficients. As the number of factors increases, Tables A4 to A7, the increase in the bias very small but the increase in the RMSE is more substantial. However, as the sample size increases, the RMSE still tends to drop considerably. Rank deficiency, when $M > 1$, increases both the bias and RMSE of both coefficients for both pooled and MG estimators in both experiments. Additionally, under option 2, results presented on the bottom right of each table, both coefficients are substantially biased with large RMSEs for both pooled and MG estimators.

The bias and RMSE of γ and c tend to be larger than the bias of the coefficients in general. This is due to the numerical optimization we employ to estimate these non-linear parameters. Heterogeneity of the coefficients improve the bias and RMSE of the parameters especially in the cases of $M = 1$ and $M = 2$. Moreover, γ is substantially biased under option 2. Curiously, c has smaller bias under option 2.

The Monte Carlo evidence seem to favor the pooled estimator over MG in terms of leading to smaller bias and RMSE. However there are exceptions. For instance, in Table A2, the MG estimator consistently leads to larger bias and RMSE for both coefficients. The same holds in Table A3 with some exceptions. For $N = T = 200$ in Table A4, the MG estimator leads to smaller bias and RMSE for β_1 however this changes in favor of the pooled estimator for β_2 in the same case and for $N = T = 400$. Heterogeneity of the coefficients does not seem to have a structured effect on the asymptotic efficiency. In Table A5, the MG estimator leads to smaller bias and RMSE in β_1 for $N = T = 400$ but not for β_2. Rank deficiency has a minimal effect on the bias and RMSE of both coefficients.

The empirical size of both coefficients is oversized in case of homogeneous coefficients when $M = 1$, Table A2. As the number of the factors increases, there is considerable gain in the empirical size in all homogeneous cases. In the heterogeneous coefficients case, the empirical size is oversized mainly for small samples. As the sample size increases, the empirical size becomes very close to the nominal size of 5%. Heterogeneity of the coefficients seem to improve the size in all cases. Moreover, in all cases and even for small samples, the empirical size is very close to the desired level under true transition function parameters.

The power of the tests are reported in the last column of each table. In the case of $M = 1$, Tables A2 and A3, both the pooled and MG estimators display the desired power, both for homogeneous and heterogeneous coefficients. As the number of the factors increases, i.e. as the system becomes rank deficient, the power of β_1 drops substantially both for the pooled and MG estimators even for large samples. Surprisingly, the power of β_2 continues to be quite high for large samples.

Table A2: Experiment 1, Homogeneous $\boldsymbol{\beta}$, $M = 1$ (1 Factor)

β_1, correction with $\bar{y}$, $\bar{x}$, $\bar{w}$

	Bias (x100)					RMSE (x100)					Size (5% level, $H_0 : \beta_1 = 1.00$)					Size under true γ and c					Power (5% level, $H_0 : \beta_1 = 0.95$)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
Pooled																									
20	−2.33	−1.84	−0.88	−0.73	−0.53	29.10	14.56	6.20	4.17	3.61	22.40	22.45	21.50	20.00	19.90	7.40	7.25	8.20	7.40	8.85	26.20	29.00	33.45	43.85	60.25
50	−2.73	−0.83	−0.32	−0.27	−0.17	26.98	6.66	3.76	2.49	1.72	30.20	21.75	20.75	18.55	17.10	6.15	6.45	6.60	6.60	6.45	36.35	37.80	53.05	71.60	92.05
100	−1.46	−0.36	−0.28	−0.14	−0.10	23.87	4.40	2.56	1.72	1.16	33.80	25.60	18.55	18.20	15.55	4.70	6.20	6.30	5.25	6.25	45.05	52.95	72.80	92.00	99.70
200	−1.29	−0.32	−0.05	−0.07	−0.07	17.11	3.11	1.85	1.21	0.80	41.65	24.80	20.00	17.50	14.85	5.30	5.40	5.70	5.70	4.90	56.10	69.85	90.95	99.50	100.00
400	−1.35	−0.24	0.02	−0.02	−0.04	12.29	2.40	1.35	0.86	0.59	43.00	30.15	21.90	17.75	17.80	4.80	5.05	5.00	5.05	5.10	67.10	84.20	98.95	100.00	100.00
MG																									
20	−7.67	−2.53	−1.04	−0.83	−0.58	54.08	16.72	6.78	4.51	3.78	21.75	23.05	20.65	19.90	19.60	6.10	5.75	6.75	6.40	7.90	23.65	26.65	31.85	40.45	57.85
50	−5.77	−1.13	−0.40	−0.32	−0.19	40.32	7.67	4.14	2.72	1.87	28.50	22.40	20.15	18.40	18.00	5.75	5.80	5.90	6.35	6.45	33.30	35.95	47.55	67.80	89.10
100	−3.23	−0.51	−0.34	−0.14	−0.11	31.17	5.13	2.85	1.87	1.26	31.70	25.60	18.80	18.90	16.90	4.75	5.00	5.20	5.20	5.95	41.20	50.05	67.25	89.20	99.05
200	−2.22	−0.41	−0.06	−0.09	−0.07	21.41	3.68	2.08	1.32	0.88	39.80	26.15	22.45	18.50	16.60	6.00	5.35	5.75	4.90	4.60	50.25	64.00	86.65	98.60	100.00
400	−2.08	−0.34	0.00	−0.02	−0.05	15.16	2.84	1.52	0.95	0.65	43.70	32.70	23.75	19.00	19.30	4.60	5.40	4.65	4.55	4.95	59.55	76.95	97.90	100.00	100.00

β_2, correction with $\bar{y}$, $\bar{x}$, $\bar{w}$

	Bias (x100)					RMSE (x100)					Size (5% level, $H_0 : \beta_2 = 1.00$)					Size under true γ and c					Power (5% level, $H_0 : \beta_2 = 0.95$)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
Pooled																									
20	3.11	2.63	1.01	0.65	0.20	51.02	24.65	9.20	6.18	6.05	17.65	19.20	18.25	15.95	18.20	6.85	6.05	7.50	6.70	8.00	18.95	22.20	25.45	31.60	43.75
50	3.88	1.13	0.19	0.10	0.16	45.79	10.64	5.66	3.74	2.60	25.75	19.25	17.50	16.40	16.35	4.90	6.55	6.60	6.15	6.00	27.90	28.40	31.60	47.40	71.80
100	2.79	0.82	0.27	0.14	0.05	40.24	6.65	3.97	2.61	1.69	32.35	22.70	18.10	15.75	12.00	5.65	6.25	6.05	5.90	4.65	34.05	36.15	47.95	71.60	92.70
200	3.15	0.61	0.07	0.06	0.01	30.45	4.98	2.83	1.88	1.28	38.80	25.00	17.40	15.85	15.80	5.60	5.20	5.80	5.50	5.55	42.80	49.85	67.80	90.85	99.50
400	2.76	0.36	0.00	0.02	0.04	21.29	3.84	2.16	1.35	0.92	47.15	30.40	21.85	17.30	16.40	5.20	4.30	5.35	4.75	5.35	48.95	65.10	87.00	99.05	100.00
MG																									
20	11.85	3.96	1.34	0.85	0.28	89.04	28.65	10.52	6.92	6.43	17.05	18.75	17.75	17.00	19.10	6.15	5.30	7.00	6.30	7.40	18.00	22.95	24.15	29.10	40.45
50	10.32	1.70	0.33	0.19	0.19	73.80	12.69	6.54	4.28	2.94	23.05	20.70	18.90	18.05	17.55	5.00	5.90	5.95	5.95	6.10	24.65	26.40	29.85	42.55	65.20
100	6.08	1.09	0.39	0.15	0.06	50.35	8.24	4.64	2.99	1.93	30.75	23.65	18.50	17.30	14.25	5.20	4.80	6.05	5.65	4.85	32.50	33.30	44.65	64.65	88.65
200	5.11	0.79	0.10	0.11	0.03	37.74	6.14	3.36	2.17	1.46	38.45	26.70	20.65	17.45	16.95	5.55	5.65	5.55	5.50	5.80	39.40	43.80	60.55	85.40	98.30
400	4.31	0.54	0.02	0.03	0.05	26.86	4.79	2.56	1.56	1.05	46.45	34.15	24.60	19.65	19.00	4.90	4.65	4.35	4.45	5.00	46.05	58.25	79.50	97.60	99.95

	correction with $\bar{y}$, $\bar{x}$, $\bar{w}$										correction with $\bar{y}$, $\bar{x}$														
	γ Bias (x100)					γ RMSE (x100)					γ Bias (x100)					β_1 Bias Pooled (x100)					β_2 Bias Pooled (x100)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
20	62.47	22.70	9.82	3.10	0.90	163.35	89.16	48.39	21.21	12.94	52.08	−1.39	−24.05	−30.76	−30.50	8.65	10.55	10.46	10.28	10.21	−27.84	−22.79	−21.85	−21.51	−21.41
50	40.35	9.76	3.07	0.86	0.36	129.59	51.38	17.94	10.81	7.28	15.56	−20.18	−21.42	−22.12	−22.20	4.32	4.86	5.64	5.83	6.07	−9.77	−10.35	−11.64	−11.98	−12.22
100	27.83	4.21	1.21	0.45	0.37	103.36	30.37	11.91	7.56	4.91	−2.06	−14.81	−14.99	−14.74	−14.54	1.16	3.02	3.23	3.51	3.61	−2.01	−5.90	−6.69	−7.07	−7.28
200	10.95	1.69	0.77	0.15	0.20	67.00	16.70	9.07	5.51	3.66	−6.02	−9.13	−8.87	−8.91	−8.82	0.83	1.65	1.94	1.95	1.98	−1.04	−3.30	−3.83	−3.90	−4.00
400	9.10	1.14	0.61	0.12	0.01	57.18	14.28	7.25	4.24	2.68	−4.62	−5.25	−5.03	−5.01	−5.00	0.50	0.87	1.05	1.04	1.04	−1.00	−1.78	−2.04	−2.07	−2.07
	c Bias (x100)					c RMSE (x100)					c Bias (x100)					β_1 Bias MG (x100)					β_2 Bias MG (x100)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
20	−8.23	−5.77	−3.41	−4.64	−4.90	127.66	56.69	31.30	21.16	15.23	−5.27	−4.88	0.57	−0.33	−0.70	−5806	9.12	9.31	8.95	8.97	−84273	−20.31	−19.58	−18.86	−18.98
50	−5.86	−3.00	−1.79	−1.92	−1.61	110.43	30.61	18.50	12.55	8.53	−2.20	−2.05	0.36	0.22	0.54	1.15	2.94	3.97	4.19	4.44	−1.60	−6.67	−8.41	−8.81	−9.05
100	−0.24	0.29	−1.37	−0.66	−0.97	76.63	21.89	13.25	8.87	6.17	1.43	1.15	−0.51	0.25	−0.01	−1.60	1.76	2.02	2.37	2.46	3.28	−3.46	−4.32	−4.85	−5.05
200	2.76	−0.10	−0.10	−0.49	−0.61	57.82	15.33	9.64	6.27	4.26	2.89	0.22	0.35	0.01	−0.01	−0.33	0.97	1.29	1.28	1.33	1.31	−1.94	−2.55	−2.59	−2.71
400	0.31	−0.31	−0.06	−0.18	−0.23	40.13	12.00	7.04	4.60	3.08	−0.06	0.15	0.17	0.03	0.05	−0.06	0.50	0.70	0.70	0.69	0.18	−1.02	−1.34	−1.40	−1.38

Table A3: Experiment 2, Heterogenous $\boldsymbol{\beta}$, $M = 1$ (1 Factor)

β_1, correction with $\bar{y}$, $\bar{x}$, $\bar{w}$

	Bias (x100)					RMSE (x100)					Size (5% level, $H_0 : \beta_1 = 1.00$)					Size under true γ and c					Power (5% level, $H_0 : \beta_1 = 0.95$)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
Pooled																									
20	−3.11	−1.73	−1.20	−0.73	−0.53	35.12	12.88	6.55	4.34	3.10	22.55	16.35	15.40	12.00	11.10	8.35	8.45	8.70	8.00	8.60	23.80	20.25	21.40	21.55	21.55
50	−1.08	−0.90	−0.37	−0.35	−0.24	22.63	6.74	3.72	2.60	1.85	24.15	16.80	11.70	10.20	8.60	5.85	6.55	6.40	7.30	6.80	30.10	27.25	29.15	32.40	37.85
100	−0.78	−0.35	−0.25	−0.12	−0.13	20.88	4.20	2.68	1.77	1.28	28.50	16.60	11.45	9.15	7.20	5.80	5.15	5.50	6.15	5.75	36.95	38.75	45.20	54.30	61.10
200	−1.21	−0.26	−0.04	−0.04	−0.04	16.04	3.00	1.91	1.27	0.89	30.95	15.90	11.60	8.95	7.70	4.95	4.80	5.65	5.15	6.15	47.75	53.80	71.65	80.30	86.25
400	−1.28	−0.15	−0.06	0.00	−0.03	9.28	2.38	1.43	0.92	0.63	39.30	19.45	12.20	8.35	5.95	5.45	4.85	6.10	4.60	3.90	60.45	77.20	90.30	97.65	99.20
MG																									
20	−8.23	−2.59	−1.45	−0.86	−0.58	62.23	14.99	7.19	4.55	3.13	21.00	17.25	15.20	12.25	11.20	7.75	7.30	7.00	6.85	7.70	22.50	20.65	21.00	21.05	21.50
50	−2.56	−1.16	−0.36	−0.37	−0.28	28.88	7.91	4.01	2.72	1.85	23.90	18.15	11.90	10.45	8.40	5.80	7.05	5.55	6.70	7.00	28.65	28.85	29.65	32.55	36.50
100	−2.14	−0.61	−0.33	−0.13	−0.13	25.41	4.85	2.91	1.84	1.28	29.00	18.65	12.25	9.65	7.25	5.15	5.25	6.10	6.20	5.10	37.00	37.85	44.65	54.35	61.35
200	−1.96	−0.35	−0.08	−0.07	−0.05	19.80	3.48	2.10	1.33	0.89	33.65	19.60	13.40	9.65	8.05	4.65	5.65	5.25	4.65	6.15	46.60	51.95	69.55	79.40	86.90
400	−2.04	−0.22	−0.09	−0.01	−0.03	11.74	2.79	1.56	0.97	0.63	42.40	22.10	14.80	9.60	6.40	5.20	4.50	5.45	5.35	3.80	56.40	72.40	87.70	97.05	99.20

β_2, correction with $\bar{y}$, $\bar{x}$, $\bar{w}$

	Bias (x100)					RMSE (x100)					Size (5% level, $H_0 : \beta_2 = 1.00$)					Size under true γ and c					Power (5% level, $H_0 : \beta_2 = 0.95$)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
Pooled																									
20	4.33	2.65	1.22	0.52	0.08	56.49	21.53	10.09	6.52	4.59	18.60	16.00	15.20	13.35	12.60	7.25	7.15	8.50	8.40	8.60	19.50	18.85	22.00	20.45	21.35
50	2.01	1.15	0.27	0.22	0.03	39.61	10.27	5.80	4.00	2.91	23.50	15.85	12.85	10.95	9.25	6.15	6.45	6.70	5.75	6.70	25.60	21.55	22.00	26.65	31.30
100	0.98	0.68	0.21	0.05	0.03	37.44	6.66	4.18	2.76	2.03	27.25	16.60	13.15	9.90	8.00	5.70	5.35	5.85	6.05	5.90	30.60	28.75	33.85	41.75	50.70
200	2.87	0.33	−0.01	−0.02	−0.03	30.30	4.81	2.95	1.98	1.40	32.70	18.20	13.20	8.45	7.65	5.45	5.20	4.50	5.25	5.80	37.95	38.60	49.80	64.35	77.30
400	2.44	0.20	0.11	−0.02	0.01	17.05	3.89	2.29	1.47	1.00	43.35	23.70	16.05	10.90	7.95	6.15	4.80	5.10	4.90	5.30	45.35	54.85	73.95	87.55	96.00
MG																									
20	13.58	4.26	1.67	0.77	0.18	85.99	24.93	11.10	6.93	4.51	17.35	17.10	15.95	13.45	12.25	6.15	6.35	7.25	8.20	7.90	19.30	20.35	21.00	20.90	20.80
50	5.31	1.57	0.28	0.23	0.11	49.27	12.29	6.46	4.18	2.84	22.45	19.40	14.20	11.15	9.70	5.45	5.65	5.85	5.70	6.55	23.55	23.70	24.25	28.20	32.55
100	3.64	1.15	0.35	0.07	0.04	45.08	8.05	4.65	2.90	2.03	28.15	21.10	15.35	11.45	8.85	5.95	5.00	5.00	5.65	5.85	29.40	30.05	35.60	43.15	54.45
200	4.55	0.51	0.09	0.02	0.00	37.54	5.84	3.36	2.10	1.41	36.00	22.25	16.70	11.20	8.65	4.95	5.50	6.00	5.55	5.60	37.00	37.75	48.35	64.95	80.35
400	3.92	0.33	0.15	0.02	0.03	21.55	4.79	2.59	1.58	0.99	46.45	30.05	19.70	13.20	8.50	5.60	5.95	4.85	4.45	4.90	46.10	51.30	72.10	88.05	97.20

	correction with $\bar{y}$, $\bar{x}$, $\bar{w}$										correction with $\bar{y}$, $\bar{x}$														
	γ Bias (x100)					γ RMSE (x100)					γ Bias (x100)					β_1 Bias Pooled (x100)					β_2 Bias Pooled (x100)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
20	60.38	23.37	8.82	2.55	1.38	164.77	86.12	46.13	18.04	11.37	48.30	−3.80	−24.70	−28.83	−28.82	0.14	11.15	10.49	10.20	10.56	−13.23	−23.00	−22.56	−21.75	−22.45
50	41.50	10.18	3.44	1.19	0.44	130.67	45.02	17.71	10.55	6.94	18.86	−18.49	−20.59	−21.07	−21.20	5.07	5.24	5.89	6.10	6.38	−9.87	−10.86	−12.29	−12.71	−13.24
100	25.05	3.51	1.37	0.52	0.30	97.72	26.14	12.06	7.29	4.89	−2.46	−14.23	−14.46	−14.19	−14.10	1.82	3.30	3.55	3.76	3.79	−3.95	−6.58	−7.32	−7.63	−7.72
200	12.82	1.93	0.90	0.41	0.17	70.71	16.26	8.85	5.44	3.44	−6.91	−8.90	−8.48	−8.52	−8.47	1.10	1.79	2.07	2.10	2.16	−2.05	−3.70	−4.17	−4.24	−4.35
400	9.65	1.95	0.19	0.23	0.00	52.68	14.12	6.90	4.21	2.58	−4.04	−4.71	−4.99	−4.80	−4.81	0.71	0.96	1.07	1.13	1.12	−1.36	−1.98	−2.12	−2.25	−2.25
	c Bias (x100)					c RMSE (x100)					c Bias (x100)					β_1 Bias MG (x100)					β_2 Bias MG (x100)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
20	−9.13	−4.16	−5.50	−4.57	−4.97	132.15	59.23	32.52	20.86	15.40	−7.90	0.89	−2.24	−0.64	−0.65	−8083	9.48	8.81	8.54	9.02	8400	−19.47	−19.19	−18.26	−19.23
50	−1.80	−3.15	−2.06	−2.14	−2.36	83.52	31.16	18.63	12.33	8.73	0.84	−0.18	−0.22	−0.39	−0.42	29.43	3.33	4.19	4.29	4.49	−31.29	−7.09	−8.84	−9.13	−9.48
100	−2.37	0.01	−1.55	−0.94	−1.21	68.36	21.10	13.27	8.68	6.07	−1.22	1.05	−0.51	0.11	−0.10	−0.23	1.83	2.19	2.49	2.53	−0.48	−3.69	−4.65	−5.10	−5.23
200	0.73	−0.92	−0.49	−0.59	−0.57	51.49	15.39	9.51	6.21	4.24	−0.02	−0.12	0.14	−0.03	−0.01	0.09	1.03	1.32	1.36	1.42	0.05	−2.19	−2.64	−2.77	−2.88
400	−0.44	−0.29	−0.29	−0.21	−0.22	34.97	11.56	6.84	4.35	2.98	0.32	−0.01	0.02	0.05	0.08	0.11	0.55	0.68	0.74	0.74	−0.15	−1.17	−1.33	−1.44	−1.48

Table A4: Experiment 1, Homogeneous $\boldsymbol{\beta}$, $M = 2$ (2 Factors)

β_1, correction with $\bar{y}$, $\bar{x}$, $\bar{w}$

	Bias (x100)					RMSE (x100)					Size (5% level, $H_0 : \beta_1 = 1.00$)					Size under true γ and c					Power (5% level, $H_0 : \beta_1 = 0.95$)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
Pooled																									
20	−0.17	−0.52	−0.09	−0.43	0.17	30.77	17.95	11.52	9.86	9.56	19.00	12.60	11.10	9.60	10.75	8.65	7.80	8.75	8.15	10.25	20.65	15.60	16.95	14.80	18.05
50	−0.78	−0.37	−0.22	0.09	0.36	30.80	12.09	7.53	6.12	5.94	20.80	13.10	10.20	7.90	8.15	5.60	5.90	6.40	6.20	7.50	25.30	21.80	20.70	22.65	22.45
100	0.20	−0.02	0.10	0.25	0.20	22.34	9.86	5.23	4.48	4.24	22.65	13.90	12.20	8.05	6.85	5.10	4.95	5.55	6.00	4.85	34.25	31.25	33.75	34.10	34.15
200	−0.31	−0.08	−0.14	0.18	0.31	16.83	6.24	4.03	3.29	3.02	29.40	17.30	12.90	9.45	6.70	5.80	4.95	6.05	5.00	4.65	47.50	46.40	46.80	51.05	53.20
400	0.45	−0.12	−0.11	0.16	0.15	17.22	4.91	2.91	2.41	2.23	34.45	23.85	15.45	10.65	8.30	5.40	4.85	4.85	5.30	4.45	60.20	61.05	67.15	71.30	73.40
MG																									
20	−4.60	−1.41	−0.58	−0.50	0.20	61.06	20.18	11.11	8.52	7.78	18.15	13.20	10.90	9.15	8.95	6.70	6.45	7.35	6.50	7.25	19.60	16.45	16.55	15.90	18.05
50	−5.02	−1.08	−0.36	0.02	0.32	57.49	14.71	7.21	5.36	4.93	23.50	16.55	12.55	8.20	8.10	5.35	5.95	6.05	5.35	6.55	27.30	24.80	24.60	27.05	28.90
100	−1.67	−0.44	−0.12	0.18	0.17	31.74	11.87	5.42	4.01	3.61	25.65	18.40	14.60	10.25	8.55	4.00	5.95	4.85	5.60	4.30	35.75	36.95	41.80	42.70	43.10
200	−1.99	−0.43	−0.31	0.12	0.26	26.55	8.90	4.23	3.10	2.62	33.55	24.00	18.15	13.75	9.80	4.35	4.65	5.65	4.95	4.90	47.20	52.90	54.30	60.65	66.05
400	−0.64	−0.46	−0.20	0.11	0.14	22.16	5.87	3.26	2.34	2.03	39.45	31.75	22.65	16.00	12.25	5.50	4.95	4.75	4.50	4.50	59.45	67.70	75.15	81.70	83.35

β_2, correction with $\bar{y}$, $\bar{x}$, $\bar{w}$

	Bias (x100)					RMSE (x100)					Size (5% level, $H_0 : \beta_2 = 1.00$)					Size under true γ and c					Power (5% level, $H_0 : \beta_2 = 0.95$)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
Pooled																									
20	1.04	2.09	1.19	0.60	0.27	55.15	27.20	11.87	5.10	3.46	16.70	16.05	11.75	11.35	10.95	6.15	7.20	6.70	7.35	6.15	19.20	23.10	23.10	32.30	49.85
50	1.12	1.05	0.85	0.30	0.02	48.86	19.13	8.19	3.46	2.36	21.80	16.40	15.60	12.40	13.50	4.85	5.70	6.40	5.35	5.80	25.70	28.65	39.50	53.60	73.20
100	0.61	0.77	0.62	0.18	0.06	40.36	14.99	4.75	2.66	1.80	23.05	18.40	17.85	15.90	14.50	4.50	6.20	5.45	4.95	5.00	27.90	36.90	52.85	73.70	93.30
200	1.22	0.83	0.37	0.11	−0.01	27.56	9.05	3.91	2.11	1.43	27.05	24.90	22.55	18.65	19.90	5.30	5.15	5.65	4.70	5.40	36.45	50.40	71.15	90.90	97.95
400	−0.46	0.63	0.33	−0.04	0.01	26.90	8.83	3.15	1.68	1.19	36.50	27.50	25.40	23.35	25.70	4.85	4.30	5.35	5.30	5.30	45.55	68.45	89.30	97.40	99.85
MG																									
20	10.40	4.27	2.15	1.08	0.45	97.45	34.69	14.59	6.42	4.01	16.95	18.90	14.45	15.25	13.10	5.70	6.45	5.90	7.20	6.35	19.25	23.65	23.55	32.60	47.80
50	8.14	2.50	1.46	0.55	0.16	80.08	23.85	10.31	4.49	2.86	23.70	21.65	18.85	17.30	17.00	5.55	4.85	5.20	5.60	5.05	26.10	28.05	36.50	49.45	69.05
100	4.69	1.71	1.09	0.40	0.15	55.33	18.19	6.61	3.55	2.34	25.50	23.25	22.45	23.45	21.90	4.20	4.95	4.85	5.65	4.75	28.55	34.00	47.75	66.80	87.70
200	4.77	1.57	0.76	0.26	0.07	42.81	13.70	5.55	2.98	1.92	31.85	31.70	30.80	27.60	29.15	5.35	4.75	4.65	5.55	5.55	34.85	44.60	63.90	84.35	95.60
400	1.76	1.41	0.58	0.06	0.03	34.49	12.04	4.68	2.31	1.63	40.50	34.75	32.85	32.45	34.90	4.70	4.05	4.95	5.05	4.90	42.50	61.35	81.75	94.20	99.00

	correction with $\bar{y}$, $\bar{x}$, $\bar{w}$										correction with $\bar{y}$, $\bar{x}$														
	γ Bias (x100)					γ RMSE (x100)					γ Bias (x100)					β_1 Bias Pooled (x100)					β_2 Bias Pooled (x100)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
20	44.14	19.89	8.09	2.75	0.90	143.23	92.70	50.23	23.47	14.43	42.56	6.57	−10.41	−15.42	−17.13	12.90	12.98	12.49	11.95	12.46	−26.94	−25.86	−25.12	−25.13	−25.43
50	31.09	10.10	4.54	2.38	1.14	126.40	64.43	33.20	19.48	12.10	27.89	0.23	−4.57	−5.44	−5.88	7.83	7.26	7.15	7.62	7.79	−15.83	−14.66	−14.90	−15.34	−15.69
100	29.78	8.61	2.91	1.35	0.78	108.49	46.97	28.21	15.99	10.69	21.38	1.77	−0.61	−1.56	−1.36	4.57	4.61	5.04	4.77	4.73	−9.04	−9.17	−9.71	−9.40	−9.54
200	20.04	8.33	2.48	1.51	0.70	80.56	42.56	21.88	14.27	9.42	9.83	1.98	0.69	1.15	0.92	2.91	2.91	2.56	2.97	2.99	−5.50	−5.59	−5.47	−5.79	−5.74
400	20.19	3.94	1.94	1.34	1.00	78.56	29.14	18.29	11.81	8.64	5.51	2.09	1.38	1.24	1.22	1.69	1.60	1.54	1.67	1.62	−3.38	−3.26	−3.19	−3.27	−3.21
	c Bias (x100)					c RMSE (x100)					c Bias (x100)					β_1 Bias MG (x100)					β_2 Bias MG (x100)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
20	6.35	4.64	7.07	5.85	5.79	149.50	83.04	47.34	33.26	26.73	−0.88	−0.06	1.13	0.49	−0.31	35599	12.75	11.97	11.61	12.31	−35593	−25.28	−24.28	−24.25	−25.00
50	0.58	3.39	5.95	4.57	5.85	135.37	62.34	38.03	26.75	22.16	−2.62	−0.34	0.07	0.19	−0.19	6.58	6.66	6.61	7.05	7.25	−12.95	−13.43	−13.63	−14.21	−14.66
100	6.08	5.62	4.93	4.83	4.58	111.68	50.79	33.30	23.51	19.82	−1.17	0.93	0.11	0.25	0.07	3.75	4.24	4.55	4.40	4.32	−7.34	−8.38	−8.89	−8.59	−8.75
200	3.87	4.09	3.07	2.86	3.52	89.32	45.78	29.12	21.68	16.44	1.81	0.34	0.28	−0.26	0.04	2.30	2.63	2.33	2.70	2.70	−4.28	−5.06	−4.96	−5.25	−5.23
400	4.07	3.26	1.81	2.04	2.31	76.44	42.61	25.54	18.38	14.25	−0.06	0.10	0.06	−0.24	−0.07	1.35	1.41	1.47	1.49	1.48	−2.72	−2.91	−2.97	−2.98	−2.95

Table A5: Experiment 2, Heterogenous $\boldsymbol{\beta}$, $M = 2$ (2 Factors)

	β_1, correction with $\bar{y}$, $\bar{x}$, $\bar{w}$																								
	Bias (x100)					RMSE (x100)					Size (5% level, $H_0 : \beta_1 = 1.00$)					Size under true γ and c					Power (5% level, $H_0 : \beta_1 = 0.95$)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
Pooled																									
20	−0.25	−0.54	−0.39	0.08	0.11	39.71	15.85	12.87	10.44	9.81	18.75	12.50	10.60	10.25	9.70	7.70	9.15	8.90	9.15	9.35	19.75	15.00	13.55	13.85	15.40
50	−0.29	−0.72	−0.04	−0.08	0.24	26.83	15.22	8.28	6.54	6.51	17.75	11.35	9.10	8.70	7.35	6.05	6.70	6.15	7.85	6.20	22.85	18.20	17.35	17.95	17.00
100	0.22	−0.18	0.07	0.31	0.06	22.92	9.27	5.18	5.27	4.36	21.10	12.00	9.90	8.20	6.55	5.65	5.45	5.70	6.05	5.45	29.80	24.80	24.85	27.85	26.85
200	0.62	0.04	0.13	0.08	0.19	21.16	7.54	3.87	3.39	3.25	25.35	14.75	10.30	7.70	6.20	5.35	5.45	5.00	5.90	5.05	41.70	39.30	40.70	38.70	42.60
400	−0.62	−0.11	−0.10	0.00	0.17	12.94	7.14	2.94	2.49	2.28	32.30	19.45	12.35	8.25	6.95	5.35	4.95	5.20	4.80	4.45	56.25	56.70	58.40	60.55	63.60
MG																									
20	−2.70	−1.31	−0.98	−0.04	0.01	58.82	18.59	14.63	8.66	7.78	19.55	13.00	10.20	8.95	8.05	6.35	6.85	7.75	7.15	7.75	20.20	15.70	14.00	13.25	14.35
50	−3.33	−1.56	−0.33	−0.10	0.19	45.12	18.00	8.35	5.55	5.37	20.50	13.95	10.55	8.50	6.80	5.95	4.85	5.15	5.95	5.70	24.15	19.95	19.85	19.95	20.20
100	−2.50	−0.67	−0.09	0.27	0.11	41.36	11.01	4.95	4.74	3.61	26.15	16.45	11.80	10.05	6.95	4.90	5.35	5.45	6.00	5.05	31.80	30.20	31.65	33.80	33.80
200	−0.55	−0.30	0.01	0.07	0.15	29.74	8.93	3.93	3.05	2.72	32.00	21.50	14.65	10.55	8.60	5.25	4.75	4.85	5.05	4.90	45.20	46.55	48.40	50.00	52.85
400	−2.05	−0.51	−0.23	−0.04	0.16	19.67	9.00	3.23	2.41	1.95	38.85	27.30	17.90	13.00	9.10	4.15	4.80	5.05	5.20	4.85	56.80	64.25	66.70	71.00	76.30

	β_2, correction with $\bar{y}$, $\bar{x}$, $\bar{w}$																								
	Bias (x100)					RMSE (x100)					Size (5% level, $H_0 : \beta_2 = 1.00$)					Size under true γ and c					Power (5% level, $H_0 : \beta_2 = 0.95$)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
Pooled																									
20	1.70	2.22	1.17	0.49	0.18	60.52	22.59	10.79	7.60	4.52	16.65	14.00	10.60	10.35	10.80	6.30	8.55	7.95	8.40	9.45	17.90	18.95	16.10	20.20	23.30
50	3.26	1.57	0.80	0.33	-0.14	47.76	22.06	9.65	3.96	5.26	19.00	14.00	11.50	9.15	8.60	5.55	6.60	6.25	6.80	5.90	23.00	22.65	24.75	28.50	31.75
100	0.23	1.00	0.40	-0.05	0.14	38.06	14.96	4.57	5.49	2.19	22.80	15.95	13.30	9.70	8.70	5.05	5.75	5.20	5.70	5.85	26.80	29.40	34.60	41.60	49.95
200	−0.40	0.44	0.27	0.15	−0.03	33.75	12.60	3.69	2.38	1.62	25.65	20.20	15.85	11.50	7.65	5.25	5.55	6.15	4.65	4.65	31.80	40.10	50.20	63.25	71.50
400	1.52	0.57	0.27	0.15	0.02	24.07	10.08	3.26	1.96	1.29	34.10	23.00	17.35	14.30	9.65	5.20	5.95	4.60	5.50	4.55	42.30	58.10	72.90	86.65	93.00
MG																									
20	7.97	4.29	2.22	0.68	0.31	91.72	30.02	14.80	7.84	4.34	17.15	16.15	13.50	11.20	11.30	5.05	6.70	7.70	6.80	8.15	20.00	21.30	19.15	19.80	24.25
50	10.65	3.36	1.45	0.71	-0.03	87.35	27.17	11.49	4.60	5.24	21.85	19.40	15.45	11.10	10.15	5.10	5.60	5.85	5.15	5.80	24.40	27.60	29.00	33.20	35.75
100	5.19	2.26	0.79	0.15	0.21	60.97	19.04	5.85	5.76	2.35	24.90	23.50	20.25	15.00	12.70	5.65	4.85	6.75	5.30	5.35	27.95	35.50	37.40	45.65	57.75
200	2.71	1.33	0.58	0.31	0.07	47.17	15.90	5.09	2.94	1.94	31.90	28.95	23.20	18.75	13.40	5.80	5.05	5.25	3.95	4.50	34.35	42.75	53.15	66.95	76.60
400	5.52	1.40	0.51	0.27	0.08	50.66	12.93	4.67	2.60	1.54	40.40	33.45	27.25	23.35	17.80	4.70	5.10	5.80	5.20	5.15	41.70	56.25	70.45	85.35	94.75

	correction with $\bar{y}$, $\bar{x}$, $\bar{w}$										correction with $\bar{y}$, $\bar{x}$														
	γ Bias (x100)					γ RMSE (x100)					γ Bias (x100)					β_1 Bias Pooled (x100)					β_2 Bias Pooled (x100)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
20	40.84	21.11	9.36	3.53	1.13	142.10	92.12	49.34	27.45	14.68	37.28	6.73	−11.33	−15.20	−16.96	2.86	12.83	12.60	12.66	12.53	−16.67	−25.83	−25.59	−25.66	−25.76
50	29.53	9.90	3.25	1.37	0.98	119.85	63.63	29.70	17.97	12.82	28.11	−1.39	−5.43	−4.88	−5.11	7.61	7.30	7.60	7.67	7.99	−15.26	−14.99	−15.27	−16.01	−16.34
100	31.35	6.42	3.76	2.28	0.66	117.86	46.97	26.61	16.64	10.71	18.12	1.06	−0.17	−0.28	−0.21	4.44	4.71	4.98	5.02	4.95	−9.18	−9.60	−9.97	−10.09	−10.22
200	19.56	6.86	2.32	0.99	0.72	84.63	39.16	20.94	13.82	9.41	10.73	2.19	0.98	1.18	1.54	3.02	2.88	2.88	2.87	3.05	−5.93	−5.85	−5.81	−5.93	−6.10
400	20.09	3.04	2.25	0.57	0.29	78.94	26.80	18.18	11.98	7.64	6.41	1.76	1.91	1.02	1.02	1.55	1.66	1.58	1.62	1.67	−3.36	−3.30	−3.40	−3.28	−3.29
	c Bias (x100)					c RMSE (x100)					c Bias (x100)					β_1 Bias MG (x100)					β_2 Bias MG (x100)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
20	7.66	5.89	3.44	5.43	4.50	146.92	75.68	50.61	34.49	25.38	−3.24	−0.80	−0.16	0.14	−0.10	−32227	12.24	11.79	12.12	11.79	32222	−24.28	−24.13	−24.69	−24.38
50	11.39	3.33	4.90	5.16	4.15	135.57	63.36	37.19	26.19	21.71	1.17	−1.59	−0.13	−0.40	−0.40	6.36	6.27	6.90	6.90	7.13	−28.55	−12.92	−13.86	−14.23	−14.65
100	3.90	6.66	4.33	5.00	4.20	107.49	50.16	31.85	23.26	18.32	−1.93	0.65	0.10	0.34	0.24	3.03	4.15	4.40	4.56	4.52	−6.70	−8.42	−8.77	−9.06	−9.16
200	6.66	5.06	4.07	3.00	3.01	95.73	45.50	29.58	21.12	16.12	1.78	0.06	0.27	−0.34	−0.11	2.54	2.48	2.62	2.64	2.70	−4.73	−5.03	−5.19	−5.31	−5.45
400	1.74	3.16	1.45	1.65	2.37	82.86	42.37	24.78	18.48	13.88	−1.23	0.14	−0.33	0.03	0.19	1.20	1.41	1.38	1.47	1.53	−2.52	−2.78	−3.04	−2.94	−2.94

Table A6: Experiment 1, Homogeneous $\boldsymbol{\beta}$, $M = 3$ (3 Factors)

β_1, correction with $\bar{y}$, $\bar{x}$, $\bar{w}$

	Bias (x100)					RMSE (x100)					Size (5% level, $H_0 : \beta_1 = 1.00$)					Size under true γ and c					Power (5% level, $H_0 : \beta_1 = 0.95$)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
Pooled																									
20	0.49	0.11	−0.48	−0.22	0.27	33.11	15.81	11.77	10.63	10.55	16.30	11.60	9.00	8.45	8.60	8.10	8.90	8.65	8.25	8.50	17.65	13.90	12.45	12.40	12.90
50	−0.05	−0.03	0.13	0.15	0.09	23.61	9.59	7.49	6.89	6.66	15.10	9.05	7.20	8.25	7.60	6.20	6.60	6.45	8.20	7.10	20.05	15.55	15.45	14.80	14.95
100	−0.67	−0.17	0.05	0.13	0.18	15.97	7.00	4.92	4.90	4.74	16.00	9.75	5.85	6.85	6.35	5.35	5.85	4.90	6.20	6.20	23.85	21.25	20.75	22.10	21.85
200	−0.16	0.10	0.04	0.06	0.27	7.95	4.75	3.59	3.45	3.28	14.95	8.60	6.95	6.50	5.00	5.05	5.50	4.75	5.95	4.75	31.60	32.50	35.25	34.95	38.50
400	−0.21	0.17	0.09	-0.02	0.16	10.94	4.51	2.69	2.46	2.40	19.05	9.90	8.15	6.10	6.45	5.40	4.95	5.60	5.20	5.70	48.10	53.00	56.30	56.85	60.50
MG																									
20	−2.36	−0.62	−0.63	−0.17	0.23	49.42	16.90	11.01	9.40	9.29	16.90	11.00	8.30	6.50	7.20	6.40	6.90	7.50	5.80	7.35	18.55	13.35	11.15	10.15	11.95
50	−1.93	−0.55	−0.16	0.17	0.12	32.11	10.24	6.94	6.06	5.85	16.65	9.30	8.35	7.00	6.50	5.20	5.15	6.50	5.70	5.85	19.35	14.95	15.55	16.60	16.55
100	−2.99	−0.53	−0.01	0.08	0.15	34.14	7.29	4.65	4.40	4.22	19.25	11.35	6.55	6.55	6.60	4.55	5.30	5.20	5.60	5.95	26.70	24.50	24.80	25.75	25.65
200	−1.00	−0.08	−0.06	0.13	0.27	10.86	5.04	3.53	3.18	2.97	20.90	11.70	7.85	7.55	5.80	5.35	5.05	4.55	5.90	4.65	34.55	37.35	40.90	43.45	46.60
400	−1.10	0.16	0.07	−0.02	0.15	20.21	4.74	2.59	2.24	2.11	24.90	12.45	10.45	7.60	6.30	5.55	5.15	5.45	4.95	4.80	48.65	57.55	64.40	66.30	71.70

β_2, correction with $\bar{y}$, $\bar{x}$, $\bar{w}$

	Bias (x100)					RMSE (x100)					Size (5% level, $H_0 : \beta_2 = 1.00$)					Size under true γ and c					Power (5% level, $H_0 : \beta_2 = 0.95$)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
Pooled																									
20	−0.16	0.93	1.23	0.42	0.24	50.67	21.40	9.87	4.46	3.16	16.10	12.15	9.45	8.85	8.95	6.85	7.40	6.90	7.40	7.80	18.60	19.55	23.70	32.55	50.00
50	1.07	0.89	0.38	0.14	0.05	38.62	11.94	5.97	2.83	1.93	17.30	11.70	8.50	7.90	7.55	5.75	5.55	5.45	6.20	6.45	21.30	25.05	34.05	54.35	80.95
100	1.11	0.46	0.15	0.04	0.00	25.77	8.87	2.91	1.97	1.38	16.40	11.30	8.20	7.85	7.65	5.50	5.65	5.70	5.80	5.65	24.00	35.05	53.75	79.05	95.85
200	0.25	0.12	0.07	−0.05	−0.04	12.11	5.84	2.24	1.40	0.99	16.30	9.85	9.00	7.60	7.80	4.60	4.70	5.05	3.90	5.40	28.95	49.60	78.25	96.00	99.90
400	0.23	−0.32	−0.01	−0.04	−0.02	15.80	7.08	1.58	1.06	0.73	20.30	12.70	9.90	9.45	8.90	5.15	5.55	4.90	4.95	5.95	42.20	72.70	94.30	99.70	100.00
MG																									
20	8.10	2.47	1.83	0.62	0.32	77.58	26.20	12.44	4.82	3.35	18.50	13.75	9.40	9.45	8.75	6.15	5.90	5.65	6.55	6.55	21.45	20.75	22.70	31.90	48.55
50	5.57	1.90	0.87	0.21	0.07	53.01	14.54	6.69	3.18	2.18	18.15	14.25	11.55	10.35	9.20	4.70	5.95	6.05	5.25	5.70	22.20	26.75	36.40	51.45	75.30
100	5.28	1.01	0.36	0.11	0.03	44.14	10.29	3.56	2.35	1.58	19.30	14.55	11.35	10.95	10.40	4.15	4.70	4.40	4.90	5.90	25.40	32.15	49.05	73.60	94.65
200	1.98	0.52	0.26	-0.03	0.00	17.44	6.76	2.99	1.74	1.17	21.75	14.00	14.55	12.65	11.30	4.90	4.35	4.90	4.50	5.10	27.90	44.90	71.95	92.15	99.75
400	1.86	−0.23	0.03	-0.03	0.00	25.37	7.62	1.98	1.33	0.88	25.65	17.15	13.90	13.25	13.55	5.30	5.10	5.35	4.80	5.55	36.20	62.90	90.55	99.05	100.00

	correction with $\bar{y}$, $\bar{x}$, $\bar{w}$										correction with $\bar{y}$, $\bar{x}$														
	γ Bias (x100)					γ RMSE (x100)					γ Bias (x100)					β_1 Bias Pooled (x100)					β_2 Bias Pooled (x100)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
20	35.34	22.73	11.46	2.20	1.09	145.87	101.65	61.60	23.32	14.45	38.47	12.72	−1.57	−10.48	−12.37	14.05	13.07	12.91	13.10	13.43	−28.07	−25.67	−26.25	−26.80	−26.64
50	21.96	14.20	3.18	2.02	0.94	108.49	69.04	27.70	16.82	10.51	29.12	10.74	1.27	−0.29	−0.11	7.34	7.88	8.03	8.19	8.08	−14.86	−15.33	−15.80	−16.16	−16.42
100	22.88	6.19	2.54	1.15	0.71	98.37	45.97	20.90	12.88	8.94	23.34	6.73	3.61	2.91	2.77	4.53	4.74	4.85	5.04	5.01	−9.67	−9.70	−9.88	−10.09	−10.01
200	20.33	5.03	1.84	1.33	0.52	77.25	33.41	15.83	10.40	6.71	19.28	3.55	3.29	3.49	3.60	3.00	2.89	2.87	2.95	3.26	−5.97	−5.58	−5.83	−5.96	−6.06
400	14.78	4.13	1.64	0.86	0.27	64.72	21.77	11.78	7.91	5.20	6.03	2.51	2.24	2.07	2.07	1.57	1.52	1.54	1.55	1.69	−3.18	−3.16	−3.08	−3.22	−3.26

	c Bias (x100)					c RMSE (x100)					c Bias (x100)					β_1 Bias MG (x100)					β_2 Bias MG (x100)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
20	13.53	7.61	5.96	5.41	5.15	156.66	77.00	48.21	31.12	25.18	9.92	1.52	0.41	0.38	0.62	6546	12.76	12.67	12.92	13.22	−6365	−25.15	−25.68	−26.27	−26.31
50	9.62	4.36	4.78	4.03	5.29	121.79	53.71	33.97	24.81	19.51	−0.45	−0.13	−0.18	0.04	0.10	6.00	7.31	7.43	7.82	7.75	−11.84	−14.42	−14.85	−15.44	−15.82
100	1.32	3.25	3.99	3.15	3.93	96.58	43.98	29.74	21.08	16.77	−4.23	−0.54	0.27	−0.56	0.03	3.52	4.33	4.61	4.73	4.75	−7.64	−9.06	−9.39	−9.60	−9.57
200	2.00	2.54	2.18	2.42	2.05	70.91	38.72	25.60	17.86	14.28	1.64	−0.53	−0.29	−0.06	0.13	2.55	2.63	2.59	2.83	3.05	−4.88	−5.11	−5.42	−5.65	−5.67
400	0.26	1.54	1.90	0.96	2.06	68.54	31.60	22.40	15.41	11.29	−0.25	0.10	−0.06	−0.06	−0.01	1.19	1.42	1.44	1.44	1.58	−2.41	−2.93	−2.93	−3.05	−3.07

Table A7: Experiment 2, Heterogenous $\boldsymbol{\beta}$, $M = 3$ (3 Factors)

β_1, correction with $\bar{y}$, $\bar{x}$, $\bar{w}$

	Bias (x100)					RMSE (x100)					Size (5% level, $H_0:\beta_1=1.00$)					Size under true γ and c					Power (5% level, $H_0:\beta_1=0.95$)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
Pooled																									
20	−0.46	−0.25	−0.18	0.23	−0.15	29.04	16.57	12.15	11.17	11.06	14.55	11.75	10.25	9.40	10.25	7.65	8.65	9.00	9.00	10.15	16.75	12.10	12.50	12.80	12.30
50	0.99	−0.16	−0.17	0.17	0.44	23.96	10.18	7.20	7.04	6.83	14.10	8.60	7.10	6.95	6.10	6.35	6.60	6.35	6.20	5.75	18.45	13.25	12.15	13.20	14.05
100	0.23	−0.38	0.02	0.13	0.21	15.77	6.29	5.19	5.21	4.88	14.20	7.45	6.50	7.35	5.65	5.60	4.70	5.25	6.35	5.55	22.25	16.90	18.05	18.50	18.55
200	−0.34	0.20	0.07	0.11	0.15	12.90	5.72	3.72	3.55	3.49	15.35	8.70	6.50	6.10	5.95	7.20	5.55	5.95	5.40	5.65	27.80	30.55	27.30	28.80	30.95
400	0.33	0.33	0.05	0.13	0.14	9.62	5.05	3.58	3.29	2.50	18.35	7.70	7.35	5.30	6.10	5.00	4.35	5.35	4.60	5.75	44.45	47.65	46.20	48.85	50.55
MG																									
20	−4.01	−1.12	−0.29	0.05	−0.09	50.92	17.34	10.85	9.65	9.28	15.75	10.75	8.70	8.35	8.25	5.65	7.15	7.05	8.20	7.75	15.80	11.85	11.75	11.05	10.95
50	−0.66	−0.56	−0.40	0.14	0.33	32.05	10.33	6.44	6.09	5.77	16.20	9.65	6.75	6.15	5.80	5.60	5.65	5.80	5.75	5.45	19.70	13.95	13.45	14.90	14.30
100	−1.06	−0.56	0.05	0.04	0.22	21.15	6.53	4.62	4.51	4.12	16.05	9.40	6.75	6.90	5.50	4.50	5.35	5.05	6.10	4.70	23.40	21.00	21.30	21.15	22.95
200	−1.31	0.06	0.03	0.07	0.16	18.54	6.09	3.39	3.11	3.00	20.25	11.35	7.25	6.80	5.95	5.15	5.15	4.60	4.80	5.85	32.30	34.75	34.70	37.10	38.80
400	−0.26	0.20	0.05	0.16	0.11	12.64	5.32	3.50	3.10	2.15	23.45	11.40	9.55	5.90	6.10	4.75	4.45	5.10	4.50	5.60	46.30	53.75	54.80	60.00	61.25

β_2, correction with $\bar{y}$, $\bar{x}$, $\bar{w}$

	Bias (x100)					RMSE (x100)					Size (5% level, $H_0:\beta_2=1.00$)					Size under true γ and c					Power (5% level, $H_0:\beta_2=0.95$)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
Pooled																									
20	3.30	1.00	1.08	0.21	0.15	48.43	22.55	8.64	5.08	3.96	15.40	11.70	9.70	9.25	8.50	7.50	7.45	8.30	8.40	8.10	17.75	17.10	19.25	18.80	21.00
50	−0.57	0.51	0.37	0.04	0.04	40.78	14.23	4.49	3.34	2.48	16.05	9.60	7.80	6.40	6.20	6.25	6.20	6.30	5.60	6.10	19.55	20.55	23.10	25.95	30.50
100	0.26	0.63	0.21	0.06	−0.03	30.31	5.34	3.27	2.36	1.84	16.50	9.00	7.15	6.20	6.45	5.40	5.75	5.65	4.90	5.55	23.10	25.55	33.40	41.20	48.95
200	0.95	0.00	0.10	0.02	−0.01	19.58	7.99	2.39	1.66	1.29	17.40	10.65	8.15	6.00	5.55	4.85	5.85	6.15	4.65	5.60	28.00	38.70	53.50	66.50	76.55
400	−0.26	−0.31	−0.05	−0.10	−0.02	16.10	8.53	4.81	4.58	0.90	18.30	9.20	7.95	5.95	5.90	4.80	5.15	5.00	4.55	5.65	35.70	57.95	78.80	91.20	96.00
MG																									
20	12.05	2.66	1.52	0.56	0.22	87.34	27.06	8.87	4.82	3.29	17.50	12.15	9.55	8.60	7.90	5.30	6.65	7.05	7.60	7.45	19.35	17.85	18.35	17.70	20.30
50	2.85	1.47	0.69	0.25	0.07	57.75	15.59	5.13	3.32	2.16	18.40	13.50	9.70	7.50	7.30	5.60	5.70	5.60	4.75	5.95	22.25	23.10	27.05	29.30	34.50
100	3.01	1.25	0.37	0.12	0.01	42.69	6.83	3.67	2.54	1.60	19.50	11.70	9.90	7.85	7.10	4.95	5.20	5.05	5.10	5.65	24.05	29.40	37.85	45.95	56.25
200	3.30	0.32	0.23	0.08	-0.02	30.62	9.14	2.84	1.75	1.18	22.35	14.05	11.40	9.25	7.30	4.55	6.60	5.05	4.50	5.25	30.05	39.95	55.80	72.15	82.50
400	1.01	−0.11	0.03	−0.11	0.02	20.38	9.07	4.96	4.61	0.90	22.85	16.30	12.25	8.95	7.65	4.65	5.95	5.30	5.50	5.10	33.65	57.45	79.75	93.10	97.85

	correction with $\bar{y}$, $\bar{x}$, $\bar{w}$										correction with $\bar{y}$, $\bar{x}$														
	γ Bias (x100)					γ RMSE (x100)					γ Bias (x100)					β_1 Bias Pooled (x100)					β_2 Bias Pooled (x100)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
20	29.99	18.83	10.01	3.30	1.81	127.08	91.65	62.61	29.17	21.09	34.55	12.43	−1.11	−8.19	−10.53	15.11	12.89	13.59	13.70	13.52	−29.38	−26.67	−27.08	−27.46	−27.92
50	26.23	10.79	4.37	2.31	1.11	109.33	64.46	28.29	18.79	10.98	33.15	11.12	3.43	0.25	−0.24	8.31	8.01	8.10	8.42	8.56	−16.67	−16.45	−16.79	−16.92	−16.83
100	23.52	4.90	3.42	1.47	1.05	98.27	37.27	22.10	13.17	8.87	24.34	6.21	3.72	3.90	4.17	4.88	4.83	5.06	5.19	5.36	−9.80	−10.04	−10.18	−10.37	−10.66
200	17.34	4.47	1.73	0.84	0.79	75.60	27.12	16.32	10.55	7.43	12.31	4.50	3.52	3.19	3.56	2.81	3.09	3.01	2.97	3.07	−5.67	−5.97	−6.00	−6.00	−6.16
400	15.71	3.09	1.45	0.84	0.26	62.69	20.80	13.58	9.45	5.42	7.33	3.21	2.39	2.57	2.16	1.65	1.85	1.63	1.74	1.70	−3.17	−3.49	−3.29	−3.42	−3.28

	c Bias (x100)					c RMSE (x100)					c Bias (x100)					β_1 Bias MG (x100)					β_2 Bias MG (x100)				
(N,T)	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400	20	50	100	200	400
20	5.15	6.39	4.52	4.71	5.32	157.60	76.01	42.56	30.71	23.29	1.64	0.75	−0.32	−1.09	0.84	−1233	11.96	13.20	12.86	13.01	1218	−24.89	−25.89	−26.00	−26.67
50	8.20	4.52	4.84	4.57	4.85	114.56	52.56	31.92	23.71	19.26	0.43	−1.09	0.04	0.32	0.27	6.72	7.17	7.36	7.82	7.96	−8.68	−14.92	−15.49	−15.63	−15.75
100	4.01	3.10	2.32	3.95	3.88	97.81	43.76	27.24	20.97	16.29	1.61	−0.38	−0.19	0.22	0.07	4.10	4.38	4.76	4.67	4.94	−8.27	−9.01	−9.39	−9.54	−9.77
200	1.26	3.20	2.67	2.83	2.51	82.14	38.47	25.14	18.09	13.95	−1.30	0.38	0.04	−0.15	−0.10	2.28	2.87	2.84	2.71	2.87	−4.57	−5.49	−5.59	−5.49	−5.67
400	4.71	1.39	1.54	1.70	1.55	67.40	32.91	22.24	15.60	11.35	1.23	−0.27	−0.29	−0.05	−0.10	1.24	1.63	1.49	1.61	1.54	−2.36	−3.11	−3.01	−3.16	−3.00

D Data Appendix

This section provides some details on the data sets we use in Section 5. We provide the basic explanation and refer the interested readers to the main papers cited below, respectively for the first and second applications.

D.1 Explanation of the data used in Section 5.1

We use CDS premia to proxy for banks Euro wholesale cost of funding (Beau, Hill, Hussain, and Nixon (2014)). The daily five-year senior euro CDS premia data are acquired from Bloomberg. To construct the MBLR series, we obtain the market capitalisation data from Datastream. The book value of assets are taken from banks' published results. The risk free rate is essentially the daily yield on the five year treasury data from Bloomberg. VFTSE index is from Bloomberg. Our data set is a panel of the four largest banks in the UK: Barclays plc, HSBC Holdings plc, Lloyds Banking Group plc and Royal Bank of Scotland plc. It is in weekly frequency, aggregated from daily to eliminate outliers and noise, and run from January 2007 to December 2016 with the total of 522 observations. For more detail, refer Dent, Hacıoğlu Hoke, and Panagiotopoulos (2017).

D.2 Explanation of the data used in Section 5.2

The data set and the data explanation are provided by Kamiar Mohaddes in his website, `http://www.econ.cam.ac.uk/people/cto/km418/research.html`. The original data set contains annual data from 1965 to 2010 on the log of CPI, log of GDP, and log of gross government debt/GDP for the following 40 countries: Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, China, Ecuador, Egypt, Finland, France, Germany, India, Indonesia, Iran, Italy, Japan, Korea, Malaysia, Mexico, Morocco, Netherlands, New Zealand, Nigeria, Norway, Peru, Phillippines, Singapore, South Africa, Spain, Sweden, Switzerland, Syria, Thailand, Tunisia, Turkey, United Kingdom, United States, Venezuela. The construction of data and the underlying sources are described in the Data Appendix of Chudik, Mohaddes, Pesaran, and Raissi (2017). To eliminate missing observations, we start the data from 1971 and omit China, Iran, Korea and Nigeria from the sample.