



BANK OF ENGLAND

# Staff Working Paper No. 692

## Do macro shocks matter for equities?

Will Dison and Konstantinos Theodoridis

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## Do macro shocks matter for equities?

Will Dison<sup>(1)</sup> and Konstantinos Theodoridis<sup>(2)</sup>

### Abstract

We investigate the role of macroeconomic shocks in driving equity price dynamics, focusing in particular on the United Kingdom as a small open economy. Using a vector error correction model estimated on 34 macroeconomic and financial time series, we show that shocks to demand, supply, monetary policy and total factor productivity account for a significant proportion of the variation in both UK and US equity prices. In contrast to some of the earlier literature, we find that shocks to total factor productivity play a particularly important role in explaining equity price movements, particularly at longer horizons. Reflecting the international nature of the FTSE All-Share, we find that most of the variation in UK equity prices is accounted for by foreign shocks, even for relatively UK-focused sectors.

**Key words:** Asset prices, stock markets, open economy macroeconomics, small open economies, international financial markets, financial forecasting.

**JEL classification:** E44, F41, G15, G17.

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(1) Bank of England. Email: [will.dison@bankofengland.co.uk](mailto:will.dison@bankofengland.co.uk)

(2) Bank of England. Email: [konstantinos.theodoridis@bankofengland.co.uk](mailto:konstantinos.theodoridis@bankofengland.co.uk)

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Publications and Design Team, Bank of England, Threadneedle Street, London, EC2R 8AH  
Telephone +44 (0)20 7601 4030 email [publications@bankofengland.co.uk](mailto:publications@bankofengland.co.uk)

# 1 Introduction

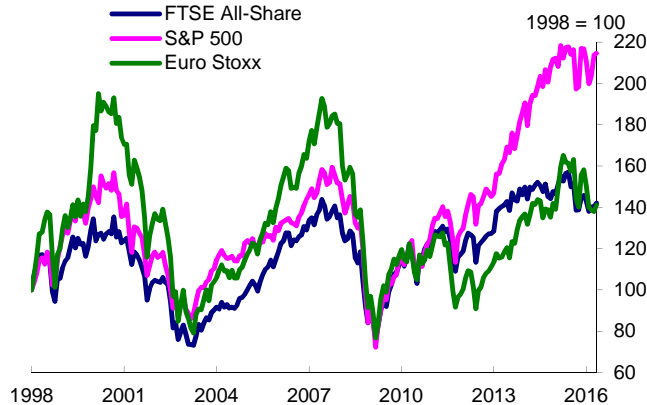
The macro-finance literature has often found it challenging to establish tight connections between asset prices and macroeconomic fundamentals (Cochrane, 2005). In this paper we apply a 34-variable cross-country vector error correction model (VECM) to show that seven ‘traditional’ structural macroeconomic shocks account for a significant proportion of the variation in UK and US equity prices. In contrast to Lettau and Ludvigson (2013) and Greenwald et al. (2014) (GLL hereafter) we find that shocks to total factor productivity (TFP) are an important driver of equity price movements. Alongside US equity prices, we focus in particular on UK equity prices, which we model in a small open economy framework. We show that foreign shocks account for most of the variation in the FTSE All-Share, consistent with the international nature of the index. But foreign shocks also play a key role in driving equity prices even for relatively domestically focused sectors of the FTSE.

The novelty of our approach is that it combines three key features that have not previously been brought together in a single model. First, we use a VECM to model equity prices in levels. Second, we apply well-motivated restrictions to identify macroeconomic shocks. And third, we apply a cross-country approach to assess the relative role of foreign and domestic factors. While a number of papers in the literature apply some of these approaches to modelling equity prices, none apply all three. For example, Kim (2003) uses a VECM to study the dynamic interaction between US equity prices and macroeconomic variables, but only considers US equity prices and does not investigate the role of structural macroeconomic shocks. Rapach (2001) uses a structural VAR to study the impact of macroeconomic shocks on US equity prices, but models equity prices in first differences and restricts his analysis to the US. And Eun and Shim (1989) use a VAR to study the linkages between equity prices in 9 countries, but their analysis does not address connections to the macroeconomy.

Modelling equity prices in levels allows us to capture long-run interactions with the macroeconomy that are lost when series are transformed to be stationary. GLL have a similar aim to us, of understanding the structural drivers of equity prices. But their analysis focuses on the stationary fluctuations of equity prices around a deterministic trend. By using a VECM to model equity prices in levels, our approach sheds light on the role of non-stationary TFP shocks in driving the stochastic trend growth in equity prices, something on which GLL’s analysis is silent.

Employing a cross-country model enables us to quantify the relative role of foreign and domestic shocks in driving UK equity prices. Much of the equity pricing literature focuses on the US, where international factors play a smaller role. But foreign shocks are likely to be particularly important for UK equity prices, given the nature of the UK as a small open economy and the international character of the FTSE All-Share. Figure 1 illustrates the strong co-movement of the FTSE All-Share with other international benchmark equity indices (Hamao et al., 1990; Longin and Solnik, 1995; Ramchand and Susmel, 1998). In part, this co-movement reflects the significant international exposure of FTSE All-Share firms, which generate around 60% of their revenues overseas and have around 50% of their assets located outside of the

Figure 1: International benchmark equity indices.



Source: Thomson Reuters Datastream.

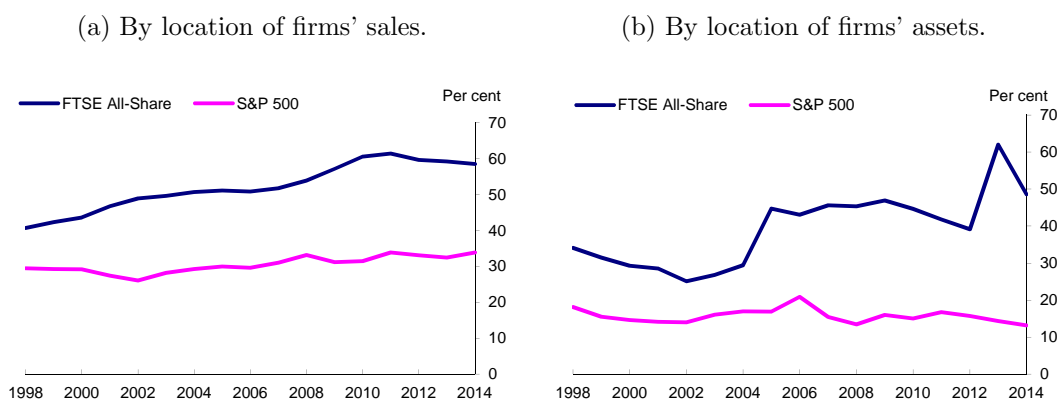
UK (Figure 2). On these measures, FTSE firms are much more internationally exposed than the firms making up the S&P 500. International comovement in equity prices also reflects comovement in equity risk premia (Campbell and Hamao, 1992; Ammer and Mei, 1996). Risk premia comovement is driven by a number of factors, including the correlation of economic risks and uncertainty across countries (Gourio et al., 2013; Cesa-Bianchi et al., 2017; Nakamura et al., 2017) and the international nature of equity ownership. For example, over half of UK-listed shares are owned by overseas investors (Figure 3).

While the FTSE All-Share is very internationally exposed at the aggregate level, this masks significant variation across sectors. Figure 4 shows that FTSE manufacturing and resource firms, for example, earn a large proportion of their revenues overseas, while the FTSE construction sector is much more domestically focused. In order to capture this heterogeneity, we include UK sectoral equity indices in our model, allowing us to investigate how the relative role of foreign and domestic factors varies across sectors.

We identify seven ‘traditional’ macroeconomic shocks in our model. The finance literature often considers the role of discount rate news and cash flow news in driving equity prices (Campbell and Shiller, 1988; Campbell, 1991; Campbell and Vuolteenaho, 2004). But we want to assess the role of deeper, more fundamental, factors. We identify a permanent TFP shock, distinguished by the persistence of its impact on the system, and six transient shocks: three US shocks, to demand, supply and monetary policy; and three domestic UK shocks, again to demand, supply and monetary policy. The UK and US shocks are distinguished using zero restrictions to impose small open economy restrictions that UK domestic shocks have no impact on foreign variables at any horizon. The transient demand, supply and monetary policy shocks are identified using sign restrictions.

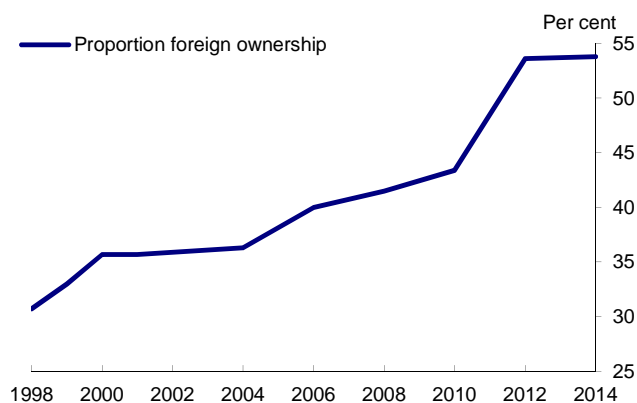
We highlight three key findings from our results. First, we successfully link equity prices to macroeconomic fundamentals. The seven structural macroeconomic shocks we identify account for around 25-30% of the variation in UK and US equity prices at short horizons, rising to

Figure 2: International exposure of equity indices.



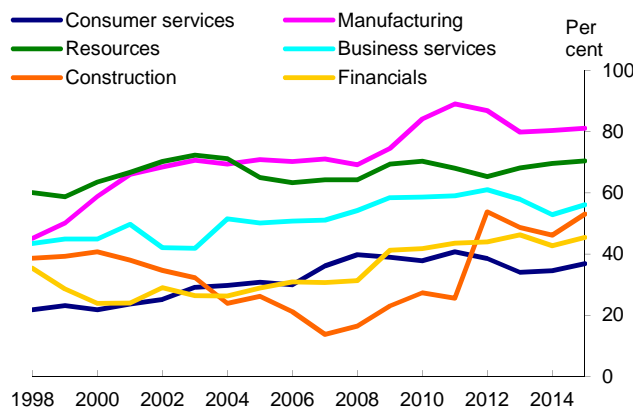
The blue line in the left-hand chart shows the percentage of FTSE All-Share firms' sales made outside of the UK. The pink line in the left-hand chart shows the percentage of S&P 500 firms' sales made outside of the US. Data in the left-hand chart cover firms making up 91% of the market capitalisation of the FTSE All-Share and 92% of the market capitalisation of the the S&P 500. The blue line in the right-hand shows the percentage of FTSE All-Share firms' assets located outside of the UK. The pink line in the right-hand shows the percentage of S&P 500 firms' assets located outside of the US. Data in the right-hand cover firms making up 77% of the market capitalisation of the FTSE All-Share and 81% of the market capitalisation of the the S&P 500. Sources: Worldscope and Thomson Reuters Datastream.

Figure 3: Foreign ownership of UK equities.



The chart shows the proportion of UK-listed equities owned by the rest of the world, by value. Source: Office for National Statistics.

Figure 4: International exposure of FTSE All-Share sectors.



The chart shows the proportion of sales made outside of the UK by firms in each sector of the FTSE All-Share. Sectors as defined in Table 4. Sources: Worldscope and Thomson Reuters Datastream.

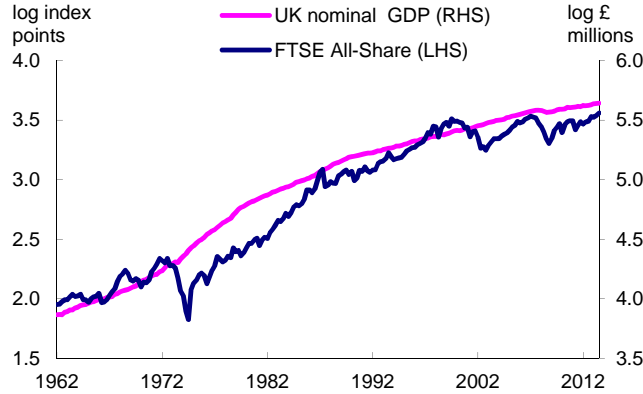
around 40% at the 5-year horizon. This is a significant proportion, given that the macro-finance literature has often found it challenging to establish tight connections between equity prices and macroeconomic variables (Cochrane, 2005).

Second, we show that permanent TFP shocks play a key role in driving equity price movements. GLL had previously found that stationary TFP shocks account for only a small proportion of the variation in equity prices around their deterministic trend. By modelling equity prices in levels, our approach sheds light on the role of non-stationary TFP shocks in driving the stochastic trend growth in equity prices, something not addressed in GLL's work. We find that permanent TFP shocks account for around 20-25% of the variation in UK and US equity prices at the 5-year horizon.

Third, we find that most of the variation in UK equity prices — some 75-80% at short horizons, rising to over 90% at the 5-year horizon — is accounted for by foreign shocks. In part this reflects the significant international exposure of FTSE firms. But even for relatively domestically focused sectors of the FTSE, the foreign shocks still play a significant role. This result is consistent with the view that foreign shocks affect domestic equity prices in part through their impact on equity risk premia, mediated by globally interconnected capital markets and the international nature of equity ownership.

This paper is structured as follows. Section 2 describes the structure of our model and explains how we use theory to pin down the long-run cointegration structure. Section 3 gives a brief overview of the data. (Appendix A describes the data in more detail.) The estimation and identification of the model are described in Sections 4 and 5 respectively. Section 6 presents our results, and Section 7 concludes.

Figure 5: Equity prices and output share a common stochastic trend.



Source: Thomson Reuters Datastream.

## 2 Model

We model equity prices using a VECM. A vector autoregression (VAR) framework is a natural approach to modelling the dynamic interactions between equity prices and macroeconomic variables. A number of authors show that equity prices help forecast macroeconomic variables, including output and inflation (Schwert, 1990; Stock and Watson, 2003; Andersson et al., 2011; Croux and Reusens, 2013). Conversely, a range of papers find that macroeconomic variables have predictive power for future equity returns (Cochrane, 1991; Lettau and Ludvigson, 2001; Piazzesi et al., 2007). Using a VECM allows us to model variables in levels and so capture long-run relationships between them. For example, Cheung and Ng (1998) show that equity prices and output are cointegrated. Intuitively, these variables share a long-run stochastic growth trend driven by increases in the productive capacity of the economy (Figure 5).

Our model takes the form

$$\begin{bmatrix} \Delta \mathbf{y}_t^* \\ \mathbf{x}_t^* \\ \Delta \mathbf{y}_t \\ \mathbf{x}_t \end{bmatrix} = \mathbf{C} + \mathbf{A} \left( \mathbf{B}' \begin{bmatrix} \mathbf{y}_{t-1}^* \\ \mathbf{y}_{t-1} \end{bmatrix} - \mathbf{c} - \phi t \right) + \mathbf{\Gamma} \begin{bmatrix} \Delta \mathbf{y}_{t-1}^* \\ \mathbf{x}_{t-1}^* \\ \Delta \mathbf{y}_{t-1} \\ \mathbf{x}_{t-1} \end{bmatrix} + \mathbf{u}_t \quad (1)$$

where  $\mathbf{z}'_t = [\Delta \mathbf{y}_t^{*'} \ \mathbf{x}_t^{*'} \ \Delta \mathbf{y}_t' \ \mathbf{x}_t']$  is an  $n$ -dimensional vector of time- $t$  observations. Stationary variables are denoted  $\mathbf{x}$  and non-stationary variables  $\mathbf{y}$ . Foreign (non-UK) variables are denoted with an asterisk.  $\mathbf{u}_t$  is an  $n$ -dimensional vector of reduced-form residuals which we assume are Gaussian white noise with  $n \times n$  covariance matrix  $\mathbf{\Sigma}_u$ :

$$\mathbf{u}_t \sim_{iid} N(\mathbf{0}, \mathbf{\Sigma}_u).$$

To fix notation, let  $n_S$  and  $n_N$  be the number of stationary and non-stationary variables re-

spectively, so  $n_S + n_N = n$ , and let  $n_D$  and  $n_F$  be the number of domestic and foreign variables respectively, so  $n_D + n_F = n$ .

The matrices  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{\Gamma}$  control the dynamics of the model. The  $n \times n$  matrix  $\mathbf{\Gamma}$  determines the short-run autoregressive behaviour of the system. The columns of the  $n_N \times r$  matrix  $\mathbf{B}$  are the  $r$  cointegration relations, which describe linear combinations  $[\mathbf{y}_t^{*'} \mathbf{y}_t'] \mathbf{B}$  of the non-stationary variables that are trend-stationary, ie stationary once a linear deterministic trend  $\mathbf{c} - \phi t$  has been removed, where  $\mathbf{c}$  and  $\phi$  are  $r$ -dimensional vectors of constants. The  $n \times r$  matrix  $\mathbf{A}$  contains the adjustment coefficients that determine how the systems responds to deviations from the cointegration relations.  $\mathbf{C}$  is an  $n$ -dimensional vector of constants.

A novel feature of our model is the inclusion of both stationary and non-stationary variables. To the best of our knowledge, this is the first example in the literature of a VECM that also includes stationary variables. This modelling choice is necessary as many macroeconomic and financial variables that help explain equity prices (for example government bond yields, central bank policy rates, and unemployment) are stationary.<sup>1</sup> The inclusion of stationary variables, while improving the fit and explanatory power of the model, does, however, make identifying the estimated model significantly more challenging.

In order to ensure that the variables  $\mathbf{x}_t$  and  $\mathbf{x}_t^*$  are stationary in the model, we impose the restriction that the error-correction innovations act only on the non-stationary variables. This is achieved by setting to zero the rows of  $\mathbf{A}$  corresponding to the stationary variables:

$$\mathbf{A} = \begin{bmatrix} * & * \\ 0 & 0 \\ * & * \\ 0 & 0 \end{bmatrix}.$$

For future reference, define  $\mathbf{I}_S$  and  $\mathbf{I}_N$  to be the vectors containing the indices in  $\mathbf{z}_t$  of the stationary and non-stationary variables respectively. Then the restrictions on  $\mathbf{A}$  are given by

$$\mathbf{A}_{ij} = 0 \text{ if } i \in \mathbf{I}_S. \tag{2}$$

There are a number of possible approaches to specifying the cointegration rank  $r$  and the cointegration relations  $\mathbf{B}$  in a VECM. One approach would be to estimate these from the data (Johansen and Juselius, 1990). Instead we use theory to pin down  $r$  and  $\mathbf{B}$ , in the manner of King et al. (1991). This approach makes it easier to provide a structural interpretation of the stochastic trend. A further advantage, given the large number of explanatory variables in the model, is that once  $\mathbf{B}$  is fixed the model becomes a normal linear regression model, making estimation significantly less computationally demanding.

We impose the cointegration relations to be consistent with a wide class of structural models, including both real-business cycle (RBC) models and New Keynesian models. RBC models were first introduced by Kydland and Prescott (1982). King and Rebelo (1999) exposit the basic RBC

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<sup>1</sup>While it is often empirically difficult to reject the hypothesis that bond yields contain a unit root, it is standard practice in the finance literature to model yields as being stationary (Sarno et al., 2007).



model and discuss the RBC literature. On the New Keynesian side, our model is consistent with models described in Christiano et al. (2005) and Justiniano et al. (2010). It is also consistent with the Bank of England’s central forecasting model COMPASS, as described in Burgess et al. (2013).

In common with all these models, our model features a single common stochastic trend that can be identified with the log level of total factor productivity (TFP). We assume a cointegration structure in which the difference between each pair of non-stationary variables is stationary.<sup>2</sup> The duality between the VECM model (1) and its common trends representation (Juselius, 2006) implies that the cointegration rank is equal to the number of non-stationary variables minus the number of common stochastic trends. In our case, this gives  $r = n_N - 1$ . For definiteness we take the cointegration matrix  $\mathbf{B}$  to be

$$\mathbf{B} = \begin{bmatrix} -\mathbf{1}_{1 \times r} \\ \mathbf{I}_{r \times r} \end{bmatrix}$$

where  $\mathbf{1}_{1 \times r}$  is the length- $r$  row vector all of whose entries are 1 and  $\mathbf{I}_{r \times r}$  is the  $r \times r$  identity matrix. This choice implies that each cointegration relation is of the form  $v_1 - v_2$  for some non-stationary variables  $v_1$  and  $v_2$  with  $v_1$  foreign.

We impose the small open economy restriction that foreign variables do not react to domestic variables. This assumption is standard in both international DSGE models such as COMPASS (Burgess et al., 2013) and international VAR models (for example Cushman and Zha, 1997; Kim, 2001; Mumtaz and Theodoridis, 2012).

The small open economy restrictions amount to zero restrictions on  $\mathbf{A}$  and  $\mathbf{\Gamma}$ . The restrictions on  $\mathbf{\Gamma}$  are easily seen to be of the form

$$\mathbf{\Gamma} = \begin{bmatrix} * & * & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & * \\ * & * & * & * \end{bmatrix}.$$

For future reference, let  $\mathbf{I}_D$  be the length- $n_D$  vector containing the indices in  $\mathbf{z}_t$  of the domestic variables, and let  $\mathbf{I}_F$  be the length- $n_F$  vector containing the indices in  $\mathbf{z}_t$  of the foreign variables. Define  $SOE_\Gamma = \mathbf{I}_F \times \mathbf{I}_D$ . Then the small open economy restrictions on  $\mathbf{\Gamma}$  are given by

$$\mathbf{\Gamma}_{ij} = 0 \text{ if } (i, j) \in SOE_\Gamma. \quad (3)$$

The small open economy restrictions on  $\mathbf{A}$  are slightly harder to describe as they amount

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<sup>2</sup>King et al. (1988) show that in a standard RBC model, the non-stationary variables can be decomposed into the sum of  $\log z_t$  plus a stationary component. King et al. (1991) drew attention to the fact that the stationarity of the ratios between output, consumption and investment implies the existence of cointegration relations between these variables.

to restrictions on the product  $\mathbf{\Pi} = \mathbf{A}\mathbf{B}'$ . It will turn out that  $\mathbf{A}$  should have the form

$$\mathbf{A} = \begin{bmatrix} * & 0 \\ * & 0 \\ * & * \\ * & * \end{bmatrix}.$$

To see this, note that  $\mathbf{A}_{ij}$  is the adjustment of the  $i^{\text{th}}$  variable in  $\mathbf{z}_t$  to deviations from the  $j^{\text{th}}$  cointegrating residual  $v_1 - v_2$ . The choice of  $\mathbf{B}$  ensures that  $v_1$  is always a foreign variable, so the restriction on  $\mathbf{A}$  will be to set  $\mathbf{A}_{ij}$  to zero when  $i \in \mathbf{I}_F$  and  $v_2$  is a domestic variable. To specify this restriction on  $j$ , define  $\mathbf{R}_D$  to be the vector containing the indices of the columns of  $\mathbf{B}$  describing cointegration relations  $v_1 - v_2$  for which  $v_2$  is a domestic variable. Then the small open economy restrictions on  $\mathbf{A}$  are given by

$$\mathbf{A}_{ij} = 0 \text{ if } (i, j) \in \mathbf{SOE}_A \tag{4}$$

where  $\mathbf{SOE}_A = \mathbf{I}_F \times \mathbf{R}_D$ .

### 3 Data

Our data are monthly observations of 34 macroeconomic and financial variables spanning December 1998 to November 2013. Each series relates to either the UK, US, euro area, or aggregate world. We label UK variables as domestic and all other variables as foreign. For the UK, US and euro area, the data include series capturing activity (the unemployment rate and a purchasing manager index), non-financial prices (CPI inflation), monetary policy (the central bank policy rate) and financial prices (a benchmark aggregate equity index, the 10-year government bond yield, and a measure of equity implied volatility). As our study focusses on the UK, we include additional UK series. These include further measures of activity (GDP and its demand components) and a wider set of financial prices (including a measure of the sterling exchange rate and equity indices representing each of the output sectors of the economy). The world variables are oil price inflation and PPP-weighted world GDP. All non-stationary variables (equity indices, GDP, and the output components) are measured in real log terms. Appendix A describes each series in more detail, including whether we classify it as stationary or non-stationary.

One aim of our work is to understand how the role of different macroeconomic shocks in driving equity price movements varies across sectors. For example, the variation in firms' international exposure across sectors of the FTSE All-Share (Figure 4) suggests that the relative role of foreign and domestic shocks might vary across sectors too. We therefore include in our model equity indices representing six sectors of the UK economy: consumer services, business services, manufacturing, construction, finance, and resources. The composition of these sectors is chosen to correspond to the UK Office for National Statistics definitions of the output sectors. We construct these sectoral equity indices ourselves on a market-capitalisation-weighted basis.

Each stock in the FTSE All-Share is included in at most one sector, and collectively the six sectors account for over 95% of the total market capitalisation of the index. More details on the construction of these sectoral equity indices are given in Appendix B.

## 4 Estimation

We estimate our model using Bayesian techniques. In principle, given a sufficiently long sample, the model could be estimated by OLS-based methods. But the model is heavily over-parameterised, with several thousand parameters to be estimated from observations spanning 180 time periods, and OLS estimation would likely result in over fitting and poor out-of-sample forecasting performance. Litterman (1986) showed that the forecasting performance of small-scale VAR models can be improved by using Bayesian methods to incorporate prior beliefs. Litterman proposed what are usually called the ‘Minnesota priors’. Banbura et al. (2010), building on work by Kadiyala and Karlsson (1997), showed that a generalisation of the Minnesota priors enables the effective estimation of much larger VAR models, provided the priors are taken to be progressively tighter as the number of variables in the system is increased. Our approach broadly follows Banbura et al., although extensions and modifications are needed to deal with the cointegration structure, the small open economy restrictions, and the inclusion of stationary variables.

As a preliminary step, we first determine  $\mathbf{B}$ ,  $\mathbf{c}$ , and  $\phi$ , after which the model (1) reduces to a normal linear regression model. The matrix  $\mathbf{B}$  is chosen as described in Section 2, and we estimate  $\mathbf{c}$  and  $\phi$  by OLS to deterministically detrend the cointegrating residuals  $(\mathbf{y}_t^{*'} \mathbf{y}_t) \mathbf{B}$ .

The key step in estimating the resulting model is the choice of priors for the remaining parameters:  $\mathbf{A}$ ,  $\mathbf{C}$ ,  $\mathbf{\Gamma}$  and  $\mathbf{\Sigma}_u$ . Once the priors have been set, estimation proceeds by a standard Gibbs sampling approach. At a high level, our priors are that  $\mathbf{A}$ ,  $\mathbf{C}$ ,  $\mathbf{\Gamma}$  and  $\mathbf{\Sigma}_u$  are independent, with  $\mathbf{A}$ ,  $\mathbf{C}$ , and  $\mathbf{\Gamma}$  each following a multivariate normal distribution, and  $\mathbf{\Sigma}_u$  following an inverse Wishart distribution. Given the properties of these distributions, to give a complete description of the priors it suffices to set the prior means and covariances of the matrices  $\mathbf{A}$ ,  $\mathbf{C}$ , and  $\mathbf{\Gamma}$ , the prior mean of  $\mathbf{\Sigma}_u$ , and the degrees of freedom parameter for the inverse Wishart distribution.

We assume prior independence between  $(\mathbf{A}, \mathbf{C}, \mathbf{\Gamma})$  and  $\mathbf{\Sigma}_u$  as the structure of our model requires our priors to follow an independent, rather than conditional, normal-inverse-Wishart distribution. Posterior inference in Bayesian modelling is typically easier when a natural conjugate prior is used, which for a normal VAR model would be a conditional normal-inverse-Wishart distribution. But this prior distribution is incompatible with our model structure, as it imposes strong conditions on the prior covariance structure. In particular, it implies that the prior covariance matrices of the coefficients in each pair of VAR equations are proportional to one another (Koop, 2009). In our model, this restriction would be incompatible with the small open economy restrictions and with the Minnesota-type priors on  $\mathbf{\Gamma}$ . The independent normal-inverse-Wishart offers the freedom to impose these conditions. For this prior distribution, however, the posterior does not have a convenient analytical form, so we have to apply a simulation approach to posterior inference.

We set the prior means and variances of the elements of  $\mathbf{\Gamma}$  to be

$$\mathbb{E}(\mathbf{\Gamma}) = \text{diag}(\delta_1, \dots, \delta_n)$$

and

$$\mathbb{V}(\mathbf{\Gamma}_{ij}) = \begin{cases} \lambda_{\Gamma}^2 & i = j, \\ \lambda_{\Gamma}^2 \frac{\sigma_i^2}{\sigma_j^2} & i \neq j \text{ and } (i, j) \notin SOE_{\Gamma}, \\ \epsilon & (i, j) \in SOE_{\Gamma}, \end{cases} \quad (5)$$

and assume that distinct coefficients of  $\mathbf{\Gamma}$  are a priori independent.

The hyperparameter  $\lambda_{\Gamma}$  controls the degree of shrinkage, and hence the relative weight placed on data versus priors in the posterior distribution. We set  $\lambda_{\Gamma} = 0.5$ . This value is sufficiently small to ensure that the priors are tight enough to avoid over fitting, while still allowing the data to influence the parameter estimates (Banbura et al., 2010). This degree of tightness is reasonably standard for a model of this size.

Three basic principles of the original Minnesota priors are incorporated in the priors for  $\mathbf{\Gamma}$ . First, the variation in a variable should be largely explained by its own lags, rather than the lags of other variables. This notion is captured by the zero off-diagonal entries of  $\mathbb{E}(\mathbf{\Gamma})$ .

Second, the diagonal entries of  $\mathbb{E}(\mathbf{\Gamma})$  should reflect prior beliefs about the persistence and autoregressive properties of the variables. In the original Minnesota priors, all the variables are non-stationary and the priors are centred around a random walk. In our setup, the original Minnesota priors would correspond to setting each  $\delta_i = 0$ . But we need to adapt this approach to account for the fact that our data include stationary, but persistent, variables. In principle the priors should be set without reference to the data, but it is standard practice to set the prior mean of  $\Sigma_u$  with reference to the estimated variability of the data, and we follow a similar approach here. Let  $z_t^i$  be the  $i^{\text{th}}$  element of  $\mathbf{z}'_t = [\Delta \mathbf{y}'_t \ \mathbf{x}'_t \ \Delta \mathbf{y}'_t \ \mathbf{x}'_t]$  and apply OLS to estimate the univariate autoregressions

$$z_t^i = \theta_i z_{t-1}^i + \eta_t^i. \quad (6)$$

It would be most theoretically consistent to set each  $\delta_i = \hat{\theta}_i$ , but stability of our model requires that  $|\delta_i| < 1$  for each  $i$ . We therefore set

$$\delta_i = \begin{cases} \hat{\theta}_i & |\hat{\theta}_i| < 1, \\ 0.99 & |\hat{\theta}_i| \geq 1. \end{cases}$$

Third, the prior variances of the elements of  $\mathbf{\Gamma}$  should adjust for the relative scale and variability of the data. This is the purpose of the factors  $\frac{\sigma_i^2}{\sigma_j^2}$  in (5), where  $\sigma_i$  is the standard deviation of the residuals  $\eta_t^i$  from the  $i^{\text{th}}$  autoregression described in (6).

A complication of the estimation procedure is the need to impose the small open economy restrictions (3) on  $\mathbf{\Gamma}$ . We do this by setting the prior variance of the relevant entries of  $\mathbf{\Gamma}$  to  $\epsilon = 10^{-14}$ , a very small number.

Following Banbura et al. (2010), we set the priors on  $\mathbf{C}$  to be loose, reflecting a lack of

strong prior beliefs about the means of the stationary variables and the mean growth rates of the non-stationary variables. The priors are centred around zero,

$$\mathbb{E}(\mathbf{C}_i) = 0,$$

and the prior variances are given by

$$\mathbb{V}(\mathbf{C}_i) = \lambda_C^2 \sigma_i^2.$$

We enforce looseness by setting the hyperparameter  $\lambda_C$  to a large number, specifically  $10^3$ . Distinct coefficients of  $\mathbf{C}$  are assumed to be a priori independent.

Our priors for  $\mathbf{A}$  are based on the cointegrated VAR priors developed, in various forms, by Villani and Warne (Villani and Warne, 2003; Villani, 2005; Warne, 2006; Villani, 2009). These priors are centred around zero,

$$\mathbb{E}(\mathbf{A}_{ij}) = 0,$$

and the prior variances are given by

$$\mathbb{V}(\mathbf{A}_{ij}) = \begin{cases} \lambda_A \sigma_i^2 \left[ (\mathbf{B}' \boldsymbol{\Omega}_N^{-1} \mathbf{B})^{-1} \right]_{jj} & i \in \mathbf{I}_N \text{ and } (i, j) \notin \text{SOE}_A \\ \epsilon & i \in \mathbf{I}_S \text{ or } (i, j) \in \text{SOE}_A \end{cases}$$

Distinct elements of  $\mathbf{A}$  are assumed to be a priori independent.

To understand the motivation for this prior, note that if we were in the special case where all of our variables were non-stationary and domestic, then we could use the Villani-Warne priors in unmodified form. These would be given by

$$\mathbb{V}(\text{vec}(\mathbf{A}) \mid \boldsymbol{\Sigma}_u) = \lambda_A \text{diag}(\sigma_1^2, \dots, \sigma_n^2) \otimes (\mathbf{B}' \boldsymbol{\Sigma}_u^{-1} \mathbf{B})^{-1}.$$

Here  $\lambda_A$  is a hyperparameter that controls the degree of shrinkage, which we set to 0.1, and the  $\text{vec}$  operator stacks the columns of a  $p \times q$  matrix into a  $pq$ -dimensional vector. For the intuition behind this prior see Villani and Warne (2003) and Villani (2005).

Our prior for  $\mathbf{A}$  incorporates three modifications relative to the baseline Villani-Warne prior. First, we set the prior covariances of distinct elements of  $\mathbf{A}$  to zero, to align the treatment of the prior covariances of  $\mathbf{A}$  with our treatment of the prior covariances of  $\boldsymbol{\Gamma}$ . Second, we need to extend the Villani-Warne priors to account for the fact that our model contains stationary, as well as non-stationary, variables. In particular, we want to impose the restrictions (2) that the stationary variables should not respond to deviations from the cointegration relations. We achieve this restriction by setting the prior variances of elements of  $\mathbf{A}$  corresponding to stationary variables to  $\epsilon$ , and the prior variance of elements of  $\mathbf{A}$  corresponding to non-stationary variables to the prior variances given by the Villani-Warne priors. This requires the introduction of the notation

$$\boldsymbol{\Omega}_N = \text{diag}\{\sigma_i^2 : i \in \mathbf{I}_N\}$$

for the diagonal matrix consisting of the residual variances  $\sigma_i^2$  of the non-stationary variables. And third, we wish to impose the small open economy restrictions (4) on  $\mathbf{A}$ . As described in Section 2, this amounts to setting to zero the elements of the block of  $\mathbf{A}$  with  $(i, j) \in SOEA$ . This restriction is imposed by setting the prior variances of the elements of  $\mathbf{A}$  in this block to  $\epsilon$ .

Following Kadiyala and Karlsson (1997), we set the prior mean of the covariance matrix  $\mathbf{\Sigma}_u$  to be

$$\mathbb{E}(\mathbf{\Sigma}_u) = \text{diag}(\sigma_1^2, \dots, \sigma_n^2).$$

We set the degrees of freedom parameter of the inverse Wishart distribution to  $n + T + 1$ , where  $T$  is the number of observations in the data.

Once the priors have been established, our approach to posterior inference is standard. We use a Gibbs sampler (described in eg Gelfand and Smith, 1990) to draw from the posterior distribution, as for the independent Normal-Wishart prior the posterior does not have a convenient analytical form. The full posterior conditional distributions are well known for this prior (see eg Koop, 2009). We iterate the sampler to obtain 1000 parameter draws from the posterior distribution.

## 5 Identification

We identify seven structural macroeconomic shocks: a TFP shock, which has a permanent impact on the system, and six transient shocks. Three of the transient shocks are domestic: a UK monetary policy shock, a UK demand shock, and a UK stationary supply shock. The remaining three transient shocks originate in the US: a US monetary policy shock, a US demand shock, and a US stationary supply shock.

We identify the structural shocks by applying three types of restrictions. First, only the TFP shock should have a long-run impact on the system. Second, the small open economy restrictions imply that UK shocks should have no impact on foreign variables (in either the short or long run). And third, the signs of the impacts of the transient shocks on certain variables should conform with a priori assumptions about the effects of these shocks. These sign restrictions, which are assumed to hold for three periods, are set out in Table 1.

We now set out more precisely the conditions that the identification must satisfy. Let  $\mathbf{X}$  be the  $n \times n$  matrix that achieves the identification and let  $\boldsymbol{\epsilon}_t = \mathbf{X}^{-1}\mathbf{u}_t$  be the identified structural shocks. The aim is to find an identification matrix  $\mathbf{X}$  with respect to which the structural shocks can be partitioned as

$$\boldsymbol{\epsilon}_t = \begin{bmatrix} \boldsymbol{\epsilon}_t^{P*} \\ \boldsymbol{\epsilon}_t^{T*} \\ \boldsymbol{\epsilon}_t^T \end{bmatrix}$$

where (informally)  $\boldsymbol{\epsilon}_t^{P*}$  is a univariate series of permanent shocks,  $\boldsymbol{\epsilon}_t^{T*}$  is a series of foreign transient shocks, and  $\boldsymbol{\epsilon}_t^T$  is a series of domestic transient shocks. As the permanent shocks  $\boldsymbol{\epsilon}_t^{P*}$  will impact on both foreign and domestic variables, we treat them as foreign so as to maintain

Table 1: Sign and zero restrictions identifying the transient shocks

	US				UK			
	$u_t$	$\pi_t$	$r_t$	$p_t$	$y_t$	$\pi_t$	$r_t$	$p_t$
US monetary policy	+	-	+	-				
US demand	-	+	+	+				
US supply	+	+	+	-				
UK monetary policy	0	0	0	0	-	-	+	-
UK demand	0	0	0	0	+	+	+	+
UK supply	0	0	0	0	-	+	+	-

$u_t$  = unemployment,  $\pi_t$  = inflation,  $r_t$  = policy interest rate,  $y_t$  = real GDP,  $p_t$  = real aggregate equity prices (FTSE All-Share for UK, S&P 500 for US). All restrictions are imposed for 3 periods.

the small open economy conditions. With this interpretation, there are  $n_F$  foreign shocks, in positions indexed by  $\mathbf{I}_F$ , and  $n_D$  domestic shocks, in positions indexed by  $\mathbf{I}_D$ . Without loss of generality, we assume that the US monetary policy shock, the US demand shock, and the US stationary supply shock are the first three elements of  $\boldsymbol{\epsilon}_t^{T*}$ . Similarly, we assume that the UK monetary policy shock, the UK demand shock, and the UK stationary supply shock are the first three elements of  $\boldsymbol{\epsilon}_t^T$ .

Our identification restrictions are restrictions on the dynamic responses of the variables to the structural shocks. In order to describe the restrictions precisely, we introduce some notation for impulse response functions. Let  $\mathbf{IRF}^t$  be the horizon- $t$  impulse response matrix for the variables in levels, i.e. the  $n \times n$  matrix whose  $ij^{\text{th}}$  entry is the impact after  $t$  periods of the  $j^{\text{th}}$  structural shock on the  $i^{\text{th}}$  variable in  $[\mathbf{y}_t^{*'} \mathbf{x}_t^{*'} \mathbf{y}_t' \mathbf{x}_t']$ . The small open economy restrictions and sign restrictions amount to restrictions on the coefficients of the matrices  $\mathbf{IRF}^t$ .

In order to discuss permanent and transitory effects, we introduce notation for the long-run impact of the structural shocks. Let  $\mathbf{IRF}^\infty$  be the long-run impulse response matrix defined by

$$\mathbf{IRF}_{ij}^\infty = \lim_{t \rightarrow \infty} \mathbf{IRF}_{ij}^t.$$

The structure of the model has various implications for the form of  $\mathbf{IRF}^\infty$ . The rows of  $\mathbf{IRF}^\infty$  corresponding to the stationary variables are all zero, as in the long run the impact of any shock on a stationary variable will die out. Our choice of ordering of the structural shocks, such that  $\boldsymbol{\epsilon}_t$  is partitioned into permanent and transitory blocks, implies that the first column of  $\mathbf{IRF}^\infty$  is non-zero and the remaining columns of  $\mathbf{IRF}^\infty$  are all zero. And the form of the cointegration relations implies that the long-run impact of the permanent shocks will be constant across the non-stationary variables. We normalise the permanent shocks by imposing that the long-run response to a positive TFP shock is positive.

We can now specify precisely the conditions that  $\mathbf{X}$  must satisfy to be a valid identification:

(C1)  $\boldsymbol{\Sigma}_u = \mathbf{X}\mathbf{X}'$ .



(C2)  $\mathbf{IRF}_{i1}^\infty > 0$  for each  $i \in \mathbf{I}_N$ .

(C3)  $\mathbf{IRF}_{ij}^\infty = 0$  for each  $i \geq 1$  and each  $j \geq 2$ .

(C4)  $\mathbf{IRF}_{ij}^t = 0$  for each  $i \in \mathbf{I}_F$ , each  $j \in \mathbf{I}_D$  and each  $t$ .

(C5) The relevant entries of  $\mathbf{IRF}^t$  satisfy the sign restrictions in Table 1 for  $t = 0, 1, 2$ .

Condition (C1) says that the structural shocks are orthogonal, as

$$\mathbb{E}(\boldsymbol{\epsilon}_t \boldsymbol{\epsilon}_t') = \mathbb{E}(\mathbf{X}^{-1} \mathbf{u}_t \mathbf{u}_t' \mathbf{X}'^{-1}) = \mathbf{X}^{-1} \boldsymbol{\Sigma}_u \mathbf{X}'^{-1} = \mathbf{I}.$$

Conditions (C2) and (C3) make precise the restrictions that the permanent shocks  $\boldsymbol{\epsilon}_t^{P*}$  have a permanent and positive impact on the system and the transient shocks  $\boldsymbol{\epsilon}_t^{T*}$  and  $\boldsymbol{\epsilon}_t^T$  have a transient impact on the system. Condition (C4) describes the small open economy restrictions that the domestic structural shocks should have no impact on foreign variables at any horizon. And condition (C5) is the sign restrictions.

As we use sign restrictions rather than zero restrictions to identify the shocks, there will be many matrices  $\mathbf{X}$  satisfying conditions (C1) to (C5). If we had only a single estimate of the parameters of the model, then one approach to dealing with this uncertainty would be to produce many valid draws of  $\mathbf{X}$  and use these draws to derive distributions for the objects of interest in our posterior inference. But as we have multiple draws for the estimated parameters, we instead produce a single valid draw of  $\mathbf{X}$  for each draw from the joint posterior distribution of the parameters and use these build up the required distributions.

We build the matrix  $\mathbf{X}$  in three stages, successively producing matrices  $\mathbf{G}$ ,  $\mathbf{H}$  and  $\mathbf{Q}$  such that  $\mathbf{X} = \mathbf{GHQ}$ . The matrix  $\mathbf{G}$  separates the permanent from the transitory shocks, ensuring that  $\mathbf{X}$  satisfies conditions (C2) and (C3). The matrix  $\mathbf{H}$  orthogonalises the shocks, ensuring that  $\mathbf{X}$  satisfies the condition (C1). And the matrix  $\mathbf{Q}$  ensures that the shocks satisfy the sign restrictions, i.e. that  $\mathbf{X}$  satisfies the condition (C5). All three matrices will be restricted to ensure that  $\mathbf{X}$  satisfies the small open economy conditions (C4).

We produce the matrix  $\mathbf{G}$  that achieves the permanent-transient separation using a result of Gonzalo and Ng (2001, GN hereafter). GN show that it is straightforward to write down  $\mathbf{G}$  from a VECM expressed in standard form. We convert the VECM representation (1) into this standard form in two steps. First, as GN's result applies to deterministically detrended time series, we eliminate the deterministic trend from (1) and rewrite the model as

$$\begin{bmatrix} \Delta \tilde{\mathbf{y}}_t^* \\ \tilde{\mathbf{x}}_t^* \\ \Delta \tilde{\mathbf{y}}_t \\ \tilde{\mathbf{x}}_t \end{bmatrix} = \mathbf{AB}' \begin{bmatrix} \tilde{\mathbf{y}}_{t-1}^* \\ \tilde{\mathbf{y}}_{t-1} \end{bmatrix} + \boldsymbol{\Gamma} \begin{bmatrix} \Delta \tilde{\mathbf{y}}_{t-1}^* \\ \tilde{\mathbf{x}}_{t-1}^* \\ \Delta \tilde{\mathbf{y}}_{t-1} \\ \tilde{\mathbf{x}}_{t-1} \end{bmatrix} + \mathbf{u}_t \quad (7)$$

where  $[\Delta \tilde{\mathbf{y}}_t^* \ \tilde{\mathbf{x}}_t^* \ \Delta \tilde{\mathbf{y}}_t \ \tilde{\mathbf{x}}_t]'$  is an  $n$ -dimensional vector of transformed time- $t$  observations. Second, our time- $t$  observations mix variables in levels with variables in first differences. To apply



GN's result we rewrite the model in a homogeneous form with all the variables in levels. Let  $\tilde{\mathbf{z}}_t = [\tilde{\mathbf{y}}_t^* \tilde{\mathbf{x}}_t^* \tilde{\mathbf{y}}_t' \tilde{\mathbf{x}}_t']'$  and rearrange (7) into the form

$$\Delta \tilde{\mathbf{z}}_t = \tilde{\mathbf{A}} \tilde{\mathbf{B}}' \tilde{\mathbf{z}}_t + \tilde{\mathbf{\Gamma}} \Delta \tilde{\mathbf{z}}_{t-1} + \mathbf{u}_t \quad (8)$$

where  $\tilde{\mathbf{A}}$ ,  $\tilde{\mathbf{B}}$  and  $\tilde{\mathbf{\Gamma}}$  are matrices of dimension  $n \times (n-1)$ ,  $n \times (n-1)$  and  $n \times n$  respectively. Appendix C sets out how  $\tilde{\mathbf{A}}$ ,  $\tilde{\mathbf{B}}$  and  $\tilde{\mathbf{\Gamma}}$  are related to  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{\Gamma}$ . Note that (8) has the same form as Equation 2 in GN.

We can now write down  $\mathbf{G}$  in terms of  $\tilde{\mathbf{A}}$  and  $\tilde{\mathbf{B}}$ . Let  $\tilde{\mathbf{A}}_\perp$  be an orthogonal complement to  $\tilde{\mathbf{A}}$ , ie an  $n \times 1$  matrix satisfying  $\tilde{\mathbf{A}}_\perp' \tilde{\mathbf{A}} = \mathbf{0}$ . Then  $\mathbf{G}$  is given by

$$\mathbf{G} = \begin{bmatrix} \tilde{\mathbf{A}}_\perp' \\ \tilde{\mathbf{B}}' \end{bmatrix}^{-1}.$$

If  $\mathbf{v}_t = \mathbf{G}^{-1} \mathbf{u}_t$  then Proposition 1 in GN implies that  $\mathbf{v}_t$  is partitioned into permanent and transient components. The first element of  $\mathbf{v}_t$ , given by  $\tilde{\mathbf{A}}_\perp' \mathbf{u}_t$ , is a univariate series of shocks having a permanent impact on the system. The remaining  $n-1$  elements of  $\mathbf{v}_t$ , given by  $\tilde{\mathbf{B}}' \mathbf{u}_t$ , are a vector of shocks having only a transient impact on the system. We collect these properties in the following proposition.

**Proposition 1.** *The reduced form shocks  $\mathbf{u}_t$  have a permanent impact on the system if and only if the first entry of the transformed shocks  $\mathbf{v}_t = \mathbf{G}^{-1} \mathbf{u}_t$  is non-zero.*

While the matrix  $\mathbf{G}$  separates the permanent from the transitory shocks, it does not ensure that the long-run impact of a permanent shock is positive. In order to ensure that this condition is met, we compute the long-run response to a shock  $\mathbf{u} = \mathbf{G} \mathbf{e}_1$  where  $\mathbf{e}_1$  is first column of the  $n \times n$  identity matrix. As described above, the response will be zero for stationary variables and constant across non-stationary variables. If the impact on the non-stationary variables is negative, we multiply  $\mathbf{G}$  by  $-1$ .

We produce the orthogonalisation matrix  $\mathbf{H}$  using a Cholesky decomposition. The covariance matrix  $\Sigma_v$  of the shocks  $\mathbf{v}_t$  is given by

$$\Sigma_v = \mathbb{E}(\mathbf{v}_t \mathbf{v}_t') = \mathbb{E}(\mathbf{G}^{-1} \mathbf{u}_t \mathbf{u}_t' \mathbf{G}'^{-1}) = \mathbf{G}^{-1} \Sigma_u \mathbf{G}'^{-1}.$$

Let  $\mathbf{H}$  be the Cholesky factor of  $\Sigma_v$ , i.e. the lower triangular matrix with  $\Sigma_v = \mathbf{H} \mathbf{H}'$ , and let  $\mathbf{w}_t = \mathbf{H}^{-1} \mathbf{v}_t$ . Then the covariance matrix  $\Sigma_w$  of  $\mathbf{w}_t$  is given by

$$\Sigma_w = \mathbb{E}(\mathbf{w}_t \mathbf{w}_t') = \mathbb{E}(\mathbf{H}^{-1} \mathbf{v}_t \mathbf{v}_t' \mathbf{H}'^{-1}) = \mathbf{H}^{-1} \Sigma_v \mathbf{H}'^{-1} = \mathbf{I}$$

so the shocks  $\mathbf{w}_t$  are mutually orthogonal.

We produce  $\mathbf{Q}$ , the matrix that ensures  $\mathbf{X}$  satisfies the sign restriction condition (C5), by a Monte Carlo approach of the type first proposed by Uhlig (2005). As there are infinitely many matrices  $\mathbf{X}$  satisfying (C1) to (C5), while the procedures for producing  $\mathbf{G}$  and  $\mathbf{H}$  are

deterministic, there will be many potential choices for  $\mathbf{Q}$ .<sup>3</sup> We repeatedly draw candidates for  $\mathbf{Q}$  uniformly at random until we find a  $\mathbf{Q}$  such that the identification  $\mathbf{X} = \mathbf{GHQ}$  satisfies all the required restrictions.

We restrict the space from which we draw  $\mathbf{Q}$  to ensure that  $\mathbf{Q}$  does not undo the identification conditions (C1)-(C4) that have already been achieved by  $\mathbf{G}$  and  $\mathbf{H}$ . Specifically, we draw candidates for  $\mathbf{Q}$  from the space  $\mathcal{S}_Q$  of block diagonal matrices whose submatrices are  $I_1$ ,  $\mathbf{Q}_F$  and  $\mathbf{Q}_D$  where  $I_1$  is the  $1 \times 1$  identity matrix and  $\mathbf{Q}_F$  and  $\mathbf{Q}_D$  are orthogonal matrices of dimensions  $(n_F - 1) \times (n_F - 1)$  and  $n_D \times n_D$  respectively. This choice of  $\mathcal{S}_Q$  ensures that the identification  $\mathbf{X} = \mathbf{GHQ}$  satisfies conditions (C1) to (C4) if and only if  $\mathbf{Q} \in \mathcal{S}_Q$ . Thus once a candidate  $\mathbf{Q}$  is drawn from  $\mathcal{S}_Q$ , only the sign restrictions condition (C5) needs to be checked.

**Proposition 2.**  *$\mathbf{X}$  satisfies conditions (C1) to (C4) if and only if  $\mathbf{Q} \in \mathcal{S}_Q$ .*

Both the necessity and sufficiency of  $\mathcal{S}_Q$  are needed in Proposition 2. Necessity ensures that the identification produced is valid, while sufficiency ensures that any valid identification can potentially be achieved. Without this latter condition, the distributions we produce for posterior inference would be inconsistent estimators of the true distributions.

The proof of Proposition 2 is given in Appendix D. The intuition for the form of  $\mathcal{S}_Q$  is as follows. Since  $I_1$ ,  $\mathbf{Q}_F$  and  $\mathbf{Q}_D$  are all orthogonal, any matrix  $\mathbf{Q} \in \mathcal{S}_Q$  is itself orthogonal, so will maintain the orthogonality of the identified shocks achieved by  $\mathbf{H}$ . That  $\mathbf{Q}$  is block diagonal with blocks of size 1 and  $n - 1$  ensures that the permanent-transient separation induced by  $\mathbf{G}$  is maintained. And that  $\mathbf{Q}$  is block diagonal with blocks of size  $n_F$  and  $n_D$  ensures that the small open economy restrictions are maintained.

With Proposition 2 in place we can complete the description of the identification procedure. For each parameter draw from the posterior distribution, we produce matrices  $\mathbf{G}$  and  $\mathbf{H}$  following the procedure described above. We then repeatedly draw candidates for  $\mathbf{Q}$  from  $\mathcal{S}_Q$ . For each candidate draw for  $\mathbf{Q}$  we produce impulse response functions  $\mathbf{IRF}^0$ ,  $\mathbf{IRF}^1$  and  $\mathbf{IRF}^2$  for the candidate identification  $\mathbf{X} = \mathbf{GHQ}$ . If the impulse response functions satisfy the sign restrictions in Table 1 we keep the draw for  $\mathbf{Q}$ , otherwise we discard it. We repeatedly draw candidates for  $\mathbf{Q}$  until a valid identification is found. We repeat this identification procedure for each of the 1000 parameter draws from the posterior distribution.<sup>4</sup>

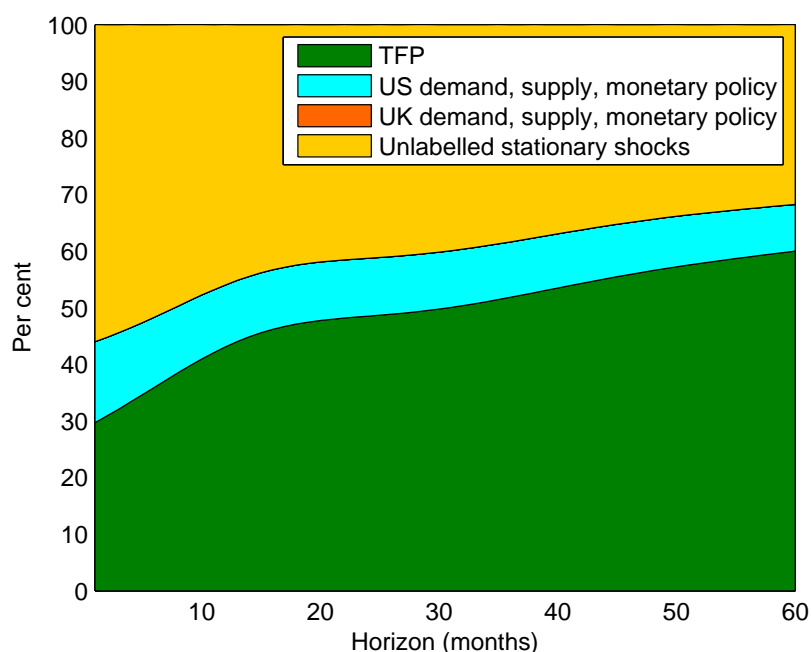
## 6 Results

Before turning to examine equity prices, we first gain some intuition into the working of our model by examining its views on the drivers of GDP.

<sup>3</sup>To be precise, the matrix  $\tilde{\mathbf{A}}'_\perp$  is uniquely defined only up to scalar multiplication.

<sup>4</sup>If for a particular parameter draw the process of repeatedly drawing candidates for  $\mathbf{Q}$  fails to produce a valid identification within a reasonable length of computational time we discard this parameter draw. This process yields 931 identified parameter draws.

Figure 6: Forecast variance decomposition for world GDP.



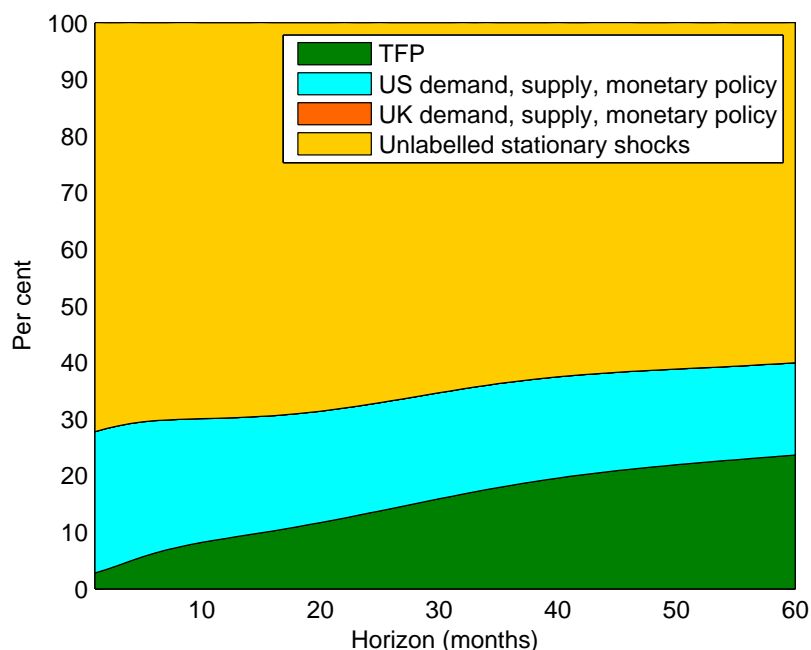
Shows the proportion of the forecast error in the level of world GDP attributable to each group of identified shocks.

## 6.1 A forecast error variance decomposition for GDP

Figure 6 shows a forecast error variance decomposition for world GDP. The shocks are aggregated into four groups: the non-stationary world TFP shock; the stationary UK demand, supply and monetary policy shocks; the stationary US demand, supply and monetary policy shocks; and the remaining 27 orthogonalised but unlabelled shocks. The small open economy conditions imply that only the foreign shocks will affect world GDP, so in particular the UK shocks account for none of the variation.

Three insights emerge from this decomposition. First, the seven identified shocks account for a significant proportion of the variation in world GDP: around 44% at the one-month horizon, rising to over 68% at the five-year horizon. This confirms that our identification scheme is successfully capturing the key sources of variability in the data. Second, the world TFP shocks are the most important of the identified shocks, consistent with the findings of the RBC literature that output variation is predominantly driven by permanent productivity shocks. And third, the unlabelled shocks still account for a significant proportion of the variation in world GDP: around 50% at the 12-month horizon. This result is perhaps not surprising as a number of important structural drivers, such as uncertainty and financial shocks, are not identified in our model. We will return below to the role of the unlabelled shocks in driving equity prices.

Figure 7: Forecast variance decomposition for the S&P 500.



Shows the proportion of the forecast error in the level of the S&P 500 attributable to each group of identified shocks.

## 6.2 The fundamental drivers of US equity prices

We turn now to discuss the role of macroeconomic shocks in driving equity prices. In order to compare our results to those of GLL, who focus on the US, we begin with the S&P 500, before turning in the next section to consider the FTSE All-Share. Figure 7 presents a forecast error variance decomposition for the S&P 500, with the shocks grouped into the same four categories used in Figure 6.

It is clear from Figure 7 that the structural macroeconomic shocks identified in the model do matter for US equity prices. At short horizons these shocks account for 28% of the variation in the S&P 500, rising to 40% at the 5-year horizon. This is a significant proportion, given that the macro-finance literature has often found it challenging to establish tight connections between equity prices and macroeconomic variables (Cochrane, 2005).

Of the identified macro shocks, the TFP shocks play an important role, accounting for 24% of the variation in the S&P 500 at the 5-year horizon. This result contrasts with the findings of GLL. Of the three stationary structural shocks they consider — to TFP, the factor share, and risk aversion — they find that the TFP shocks play only a small role at all horizons. This contrast reflects differences in modelling approaches. As GLL model the fluctuations of equity prices around a deterministic trend, they are necessarily focused on the role of stationary factors. By modelling variables in levels, our analysis can shed light on the role of non-stationary TFP growth in driving the stochastic trend growth in equity prices. The RBC literature emphasises the importance of productivity shocks in driving variation in output growth. Our model suggests

that permanent TFP shocks play a key role in driving asset price dynamics too.

Figure 7 also illustrates how the relative role of the stationary and non-stationary shocks varies across horizons. At short horizons, the transient monetary policy, demand and supply shocks are the most important of the identified shocks. At the 1-month horizon these account for around 25% of the variation in the S&P 500, compared to around 3% for the permanent TFP shock. But at longer horizons, the permanent productivity shocks become more important, accounting for around 24% of the variation after five years, while the three transient shocks account for only around 16%. The more important role for the TFP shocks at longer horizons reflects the fact that they induce a permanent shift in the level of output and wealth, and hence of equity prices. So while the impact of the transient shocks will die out over time, the impact of multiple TFP shocks is cumulative.

It is also notable that while the identified macro shocks account for a significant proportion of the variation in US equity prices, the portion left to be explained by the unlabelled shocks is even greater than for world GDP. That suggests to us that many of these unlabelled shocks may be financial, rather than macro, in nature. For example, the consensus view of the macro-finance literature is that variation in risk premia is a key driver of asset price fluctuations (Cochrane, 2011). Factors such as risk aversion, uncertainty, sentiment and liquidity are therefore likely to be important determinants of equity prices, but shocks to these factors are not identified in our model.

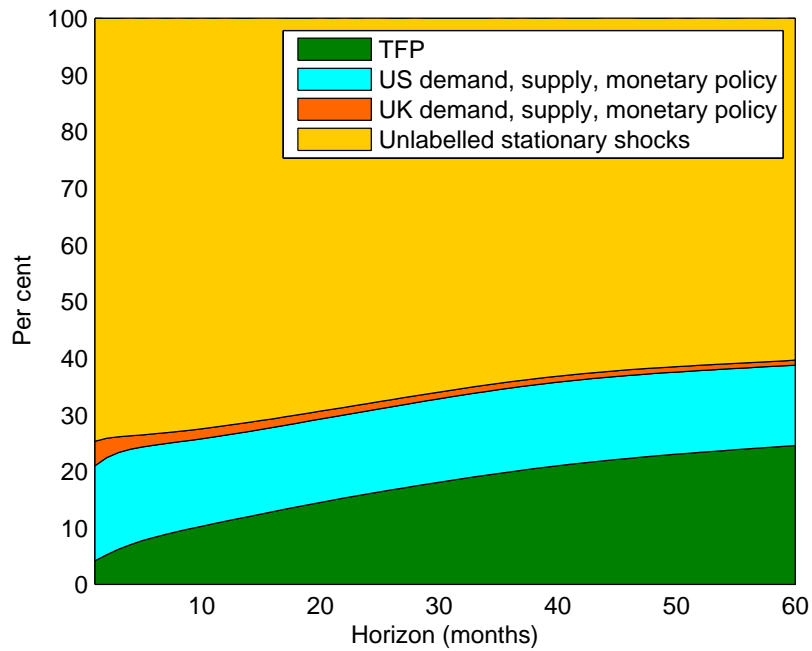
### 6.3 The fundamental drivers of UK equity prices

We now turn to discuss the role of structural macroeconomic shocks in driving UK equity prices. Figure 8 shows a forecast variance decomposition for the FTSE All-Share, with the shocks grouped into the same categories as in the previous figures.

It is clear from the figure that the key conclusions on the drivers of US equity prices apply equally to the UK. The seven structural macro shocks identified in the model account for a significant proportion of the variation in the FTSE All-Share: some 25% at short horizons, rising to 40% by the 5-year horizon. And among these macro drivers, the permanent TFP shocks play a key role, particularly at longer horizons, with the TFP shocks accounting for 25% of the variation after 5 years.

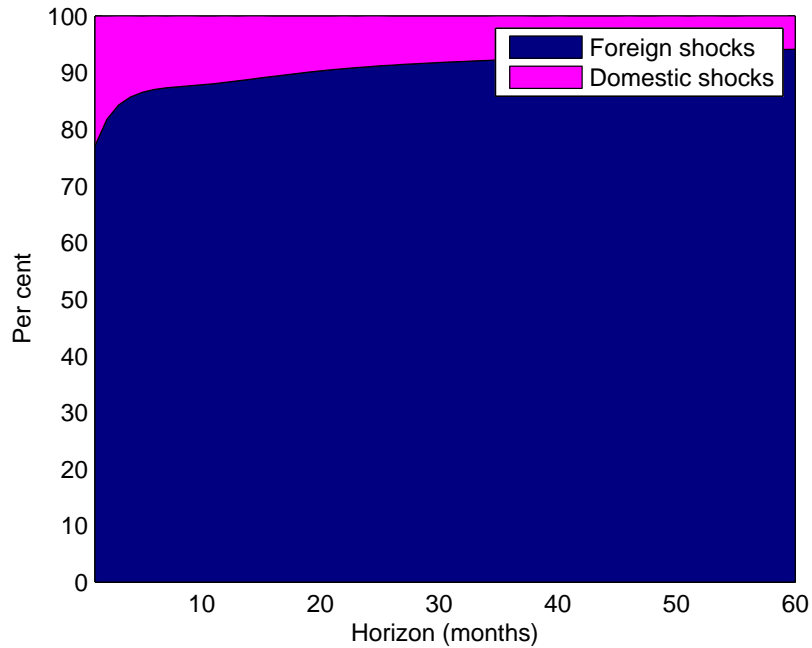
Our model also sheds light on the relative role of foreign and domestic factors in driving UK equity prices. Figure 9 shows a forecast variance decomposition for the FTSE All-Share with the shocks grouped into foreign and domestic categories. All 34 shocks are included in this decomposition, including both the labelled and unlabelled shocks. Figure 9 shows that the majority of the variation in UK equity prices is accounted for by foreign shocks, around 77% at short horizons and over 94% by the 5-year point. This key role for foreign factors is consistent with the stylised facts presented in the introduction regarding the high international exposure of FTSE firms, and the international comovement of discount rates. While it is unsurprising that foreign factors are important, our model provides a quantification of their combined influence on equity prices.

Figure 8: Forecast variance decomposition for the FTSE All-Share.



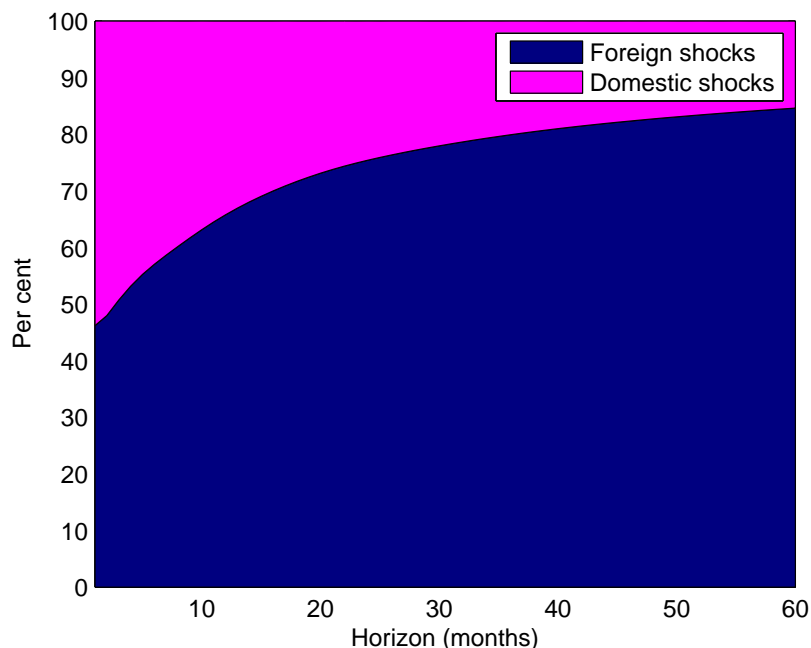
Shows the proportion of the forecast error in the level of the FTSE All-Share attributable to each group of identified shocks.

Figure 9: Forecast variance decomposition for the FTSE All-Share.



Shows the proportion of the forecast error in the level of the FTSE All-Share attributable to each group of identified shocks.

Figure 10: Forecast variance decomposition for FTSE construction equity prices.



Proportion of the forecast error in the level of the FTSE All-Share construction-sector equity index attributable to each group of identified shocks.

Our model also shows that foreign shocks are important even for relatively domestically focused sectors. For example, Figure 10 shows a forecast variance decomposition for FTSE construction-sector equity prices, with the shocks again grouped into foreign and domestic categories. Reflecting the lower international exposure of the firms in this sector (Figure 4) the domestic shocks are more important here than for the aggregate index — accounting for around half of the variation at short horizons. But, crucially, the foreign shocks still account for a significant proportion of the variation — over 84% at the 5-year horizon. This highlights the role of internationally co-moving discount rates that will influence equity prices of even relatively domestically focused firms.

## 7 Conclusion

We employed a 34-variable cross-country VECM to investigate the macroeconomic drivers of equity prices. We found three key results. First, we successfully linked equity prices to macroeconomic fundamentals, with the seven ‘traditional’ structural macroeconomic shocks identified in the model accounting for around 40% of the variation in UK and US equity prices at the 5-year horizon. Second, we showed that permanent TFP shocks play a key role in driving equity price dynamics. This compares with previous work by GLL, who found that stationary TFP shocks are relatively unimportant in explaining the fluctuation of equity prices around a deterministic trend. And third, we showed that most of the variation in UK equity prices is

accounted for by foreign shocks. In part this result reflects the high international exposure of FTSE All-Share firms. But even for relatively domestically focused sectors of the FTSE, foreign shocks still play a significant role, highlighting the importance of global factors in discount rate determination.

A promising avenue for future research would be to extend our model to incorporate financial factors. Shocks to factors such as risk aversion, uncertainty, liquidity and sentiment are not identified in our model, but our results hint that financial shocks may capture some of the variation in equity prices not accounted for by the seven macroeconomic drivers identified in our model.



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# Appendices

## A Data description

Tables 2 and 3 describe the stationary and non-stationary variables in the model. Variables are classified as either world, UK, US or euro area. All non-stationary variables are measured in real log terms.



Table 2: Stationary variables

Name	Description	Source <sup>5</sup>
<b>World</b>		
Oil price inflation	Monthly oil price inflation computed as the log difference of the average end-of-day sterling Brent crude oil price.	Bloomberg and Bank of England
<b>US</b>		
Government bond yield	Average end-of-day nominal spot 10-year zero-coupon US treasury bond yield, taken from the Bank of England's fitted yield curve.	Bloomberg
Fed funds rate	Average end-of-day federal funds target rate. Where the target rate is expressed as a range (as it has been since 16 December 2008) we use the upper bound of this range.	Bloomberg
Manufacturing PMI	Log US seasonally-adjusted monthly manufacturing purchasing managers index constructed by the Institute for Supply Management.	Datastream
S&P 500 implied volatility	Average end-of-day S&P 500 implied volatility at the 12-month horizon as implied by the Black-Scholes formula and the nearest-to-the-money index option.	Chicago Mercantile Exchange
CPI inflation	Monthly US CPI inflation, computed as the log difference of the monthly US Consumer Price Index produced by the Bureau of Labor Statistics (all urban consumers, US city average, all items, not seasonally adjusted).	Datastream
<b>Euro area</b>		
Government bond yield	Average end-of-day nominal spot 10-year zero-coupon yield, taken from the Bank of England's fitted euro-area government bond yield curve. The curve is constructed using German and French bond yields.	Bloomberg
Unemployment	Headline monthly euro-area unemployment rate, seasonally adjusted.	Datastream
Manufacturing PMI	Log euro-area seasonally-adjusted monthly manufacturing purchasing managers index constructed by Markit.	Datastream
CPI inflation	Monthly euro-area CPI inflation, computed as the log difference of the monthly euro-area harmonised Consumer Price Index, seasonally adjusted.	Datastream
<b>UK</b>		
Government bond yield	Average end-of-day nominal spot 10-year zero-coupon UK government bond yield, taken from the Bank of England's fitted yield curve.	Bloomberg
BoE Bank Rate	Average end-of-day Bank of England Bank Rate.	Bloomberg
Unemployment	Headline monthly UK Labour Force Survey unemployment rate, seasonally adjusted.	Office for National Statistics



Table 2: Stationary variables

Name	Description	Source <sup>5</sup>
Manufacturing PMI	Log UK monthly manufacturing purchasing managers index constructed by the Chartered Institute of Procurement and Supply.	Chartered Institute of Procurement and Supply
FTSE 100 implied volatility	Average end-of-day FTSE 100 implied volatility at the 12-month horizon as implied by the Black-Scholes formula and the nearest-to-the-money index option.	London International Financial Futures Exchange
Sterling exchange rate	Log sterling trade-weighted effective exchange rate index produced by Bank of England staff.	Bank of England

<sup>5</sup>With the exception of the unemployment rates, all series are subject to additional calculations, as described in the table, by Bank of England staff.



Table 3: Non-stationary variables

Name	Description	Source <sup>6</sup>
<b>World</b>		
World real GDP	Quarterly estimates of PPP-weighted world real GDP produced Bank of England staff by weighting together GDP outturns for the US, UK, euro area, Japan, Canada, Australia, New Zealand, Switzerland, Sweden, Norway, Denmark, Hong Kong, Singapore, Korea, Taiwan, Central and Eastern Europe, Middle East and North Africa, Commonwealth of Independent States, Western Hemisphere, Sub Saharan Africa, and Developing Asia. The quarterly series is converted to monthly observations by linear interpolation.	Datastream and IMF
<b>US</b>		
Real S&P 500	Average end-of-day value of the S&P 500 equity index, deflated by US CPI.	Bloomberg
<b>Euro area</b>		
Real Euro Stoxx	Average end-of-day value of the Dow Jones Euro Stoxx 300 equity index, deflated by euro-area CPI.	Bloomberg
<b>UK</b>		
Real FTSE All-Share	Average end-of-day value of the FTSE All-Share equity index, deflated by UK CPI.	Bloomberg
Consumer services real equity index	Average end-of-day value of a market-capitalisation-weighted equity index representing the UK consumer services sector, deflated by aggregate UK CPI. See Appendix B for more details on the construction of the index.	Datastream
Manufacturing real equity index	Average end-of-day value of a market-capitalisation-weighted equity index representing the UK manufacturing sector, deflated by aggregate UK CPI. See Appendix B for more details on how this index is constructed.	Datastream
Resources real equity index	Average end-of-day value of a market-capitalisation-weighted equity index representing the UK resources sector, deflated by aggregate UK CPI. See Appendix B for more details on how this index is constructed.	Datastream
Business services real equity index	Average end-of-day value of a market-capitalisation-weighted equity index representing the UK business services sector, deflated by aggregate UK CPI. See Appendix B for more details on how this index is constructed.	Datastream
Construction real equity index	Average end-of-day value of a market-capitalisation-weighted equity index representing the UK construction sector, deflated by aggregate UK CPI. See Appendix B for more details on how this index is constructed.	Datastream





Table 3: Non-stationary variables

Name	Description	Source <sup>6</sup>
Financials real equity index	Average end-of-day value of a market-capitalisation-weighted equity index representing the UK financial sector, deflated by aggregate UK CPI. See Appendix B for more details on how this index is constructed.	Datastream
Real GDP	Monthly UK real GDP estimate produced by the National Institute of Economic and Social Research.	National Institute of Economic and Social Research
Real business investment	Quarterly seasonally-adjusted real UK business investment converted to monthly observations by linear interpolation.	Office for National Statistics
Real consumption	Quarterly seasonally-adjusted real UK consumption converted to monthly observations by linear interpolation.	Office for National Statistics
Real imports	Quarterly seasonally-adjusted real UK imports of goods and services converted to monthly observations by linear interpolation.	Office for National Statistics
Real exports	Quarterly seasonally-adjusted real UK exports of goods and services converted to monthly observations by linear interpolation.	Office for National Statistics

<sup>6</sup>All series are subject to additional calculations, as described in the table, by Bank of England staff.

## B Construction of the sectoral equity indices

This section describes how we construct the six UK real equity indices representing the consumer services, business services, manufacturing, construction, finance, and resources sectors. For each sector, we first construct a nominal equity index by weighting together Datastream equity indices representing subsectors of the sector. The composition of the sectors is chosen to match, as closely as possible, the definition of the UK Office for National Statistics output sectors. Suppose a sector is composed of  $n$  subsectors, each represented by a nominal equity index  $p_i$ . Let  $p_i(t)$  be the value at time  $t$  of the equity index representing subsector  $i$ , and  $w_i(t)$  be the market capitalisation at time  $t$  of the subsector  $i$ . Then  $W(t) = \sum_{i=1}^n w_i(t)$  is the market capitalisation at time  $t$  of the whole sector. The nominal equity index  $P$  representing the whole sector is generated by the recursion relation

$$P(t) = \sum_{i=1}^n \frac{w_i(t-1)}{W(t-1)} \frac{p_i(t)}{p_i(t-1)}$$

and the initialisation  $P(1) = 100$ . Table 4 lists the Datastream codes and market capitalisations of the subsectors comprising each sector. The sectoral nominal equity indices are first constructed at daily frequency, and then converted to monthly frequency by averaging across the daily observations. Finally, the monthly sectoral nominal equity indices are converted to real indices by deflating by aggregate UK CPI inflation.

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<sup>7</sup>Market capitalisation as of 20 January 2015.

Table 4: Datastream codes for the subsectors comprising each sectoral equity index.

Subsector name	Datastream code	Market capitalisation <sup>7</sup> (£ millions)
<b>Consumer services</b>		<b>184923</b>
Retail	RTAILUK	93468
Travel and leisure	TRLESUK	91455
<b>Business services</b>		<b>305572</b>
Industrial transportation	INDTRUK	7724
Support services	SUPSVUK	98153
Media	MEDIAUK	80052
Telecommunications	TELCMUK	105481
Software and computer services	SFTCSUK	14162
<b>Manufacturing</b>		<b>608097</b>
Aerospace and defensive	AERSPUK	44354
General industrials	GNINDUK	13092
Electronic and electric equipment	ELTNCUK	10917
Industrial engineering	INDENUK	17054
Automobiles and parts	AUTMBUK	6037
Beverages	BEVESUK	108281
Food products	FDPRDUK	30724
Durable household products	DURHPUK	96
Nondurable household products	NDRHPUK	38516
Furnishings	FURNSUK	350
Leisure goods	LEISGUK	531
Personal goods	PERSGUK	46210
Tobacco	TOBACUK	95526
Medical equipment	MEDEQUK	10795
Medical supplies	MEDSPUK	1398
Pharmaceuticals	PHRMCUK	165039
Technology, hardware and equipment	TECHDUK	19177
<b>Construction</b>		<b>39398</b>
Construction and materials	CNSTMUK	17167
Home construction	HOMESUK	22231
<b>Finance</b>		<b>583232</b>
Financials	FINANUK	583232
<b>Resources</b>		<b>423408</b>
Oil and gas	OILGSUK	257168
Chemicals	CHMCLUK	15838
Industrial metals and mining	INDMTUK	2639
Mining	MNINGUK	147763

## C The parameter matrices for the model in standardised VECM form

This appendix sets out how the parameter matrices  $\tilde{\mathbf{A}}$ ,  $\tilde{\mathbf{B}}$  and  $\tilde{\mathbf{\Gamma}}$  of the model expressed in standardised VECM form (8) are related to the parameter matrices  $\mathbf{A}$ ,  $\mathbf{B}$ , and  $\mathbf{\Gamma}$  of the model in its original VECM form (1). In order to define  $\tilde{\mathbf{A}}$ ,  $\tilde{\mathbf{B}}$  and  $\tilde{\mathbf{\Gamma}}$ , we first partition  $\mathbf{A}$ ,  $\mathbf{B}$ , and  $\mathbf{\Gamma}$  into block matrix form:

$$\mathbf{A} = \begin{array}{cc} & \begin{array}{c} n_{NF-1} \quad n_{ND} \end{array} \\ \left[ \begin{array}{cc} \mathbf{A}_{y^*y^*} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{A}_{yy^*} & \mathbf{A}_{yy} \\ \mathbf{0} & \mathbf{0} \end{array} \right] & \begin{array}{c} n_{NF} \\ n_{SF} \\ n_{ND} \\ n_{SD} \end{array} \end{array}$$

$$\mathbf{B}' = \begin{array}{cc} & \begin{array}{c} n_{NF} \quad n_{ND} \end{array} \\ \left[ \begin{array}{cc} \mathbf{B}_{y^*y^*} & \mathbf{0} \\ \mathbf{B}_{yy^*} & \mathbf{B}_{yy} \end{array} \right] & \begin{array}{c} n_{NF-1} \\ n_{ND} \end{array} \end{array}$$

$$\mathbf{\Gamma} = \begin{array}{cccc} & \begin{array}{c} n_{NF} \quad n_{SF} \quad n_{ND} \quad n_{SD} \end{array} \\ \left[ \begin{array}{cccc} \mathbf{\Gamma}_{y^*y^*} & \mathbf{\Gamma}_{y^*x^*} & \mathbf{\Gamma}_{y^*y} & \mathbf{\Gamma}_{y^*x} \\ \mathbf{\Gamma}_{x^*y^*} & \mathbf{\Gamma}_{x^*x^*} & \mathbf{\Gamma}_{x^*y} & \mathbf{\Gamma}_{x^*x} \\ \mathbf{\Gamma}_{yy^*} & \mathbf{\Gamma}_{yx^*} & \mathbf{\Gamma}_{yy} & \mathbf{\Gamma}_{yx} \\ \mathbf{\Gamma}_{xy^*} & \mathbf{\Gamma}_{xx^*} & \mathbf{\Gamma}_{xy} & \mathbf{\Gamma}_{xx} \end{array} \right] & \begin{array}{c} n_{NF} \\ n_{SF} \\ n_{ND} \\ n_{SD} \end{array} \end{array}$$

The labels above and to the right of each matrix give the width of the block columns and the depth of the block rows respectively. Here  $n_{NF}$  is the number of non-stationary foreign variables,  $n_{SF}$  is the number of stationary foreign variables,  $n_{ND}$  is the number of non-stationary domestic variables, and  $n_{SD}$  is the number of stationary domestic variables, so  $n_{NF} + n_{SF} + n_{ND} + n_{SD} =$

*n.* Then  $\tilde{\mathbf{A}}$ ,  $\tilde{\mathbf{B}}$  and  $\tilde{\mathbf{\Gamma}}$  are given by:

$$\tilde{\mathbf{A}} = \begin{bmatrix} & n_{NF-1} & n_{SF} & n_{ND} & n_{SD} \\ \mathbf{A}_{y^*y^*} & \mathbf{\Gamma}_{y^*x^*} & \mathbf{0} & \mathbf{0} & \\ \mathbf{0} & \mathbf{\Gamma}_{x^*x^*} - \mathbf{I} & \mathbf{0} & \mathbf{0} & \\ \mathbf{A}_{yy^*} & \mathbf{\Gamma}_{yx^*} & \mathbf{A}_{yy} & \mathbf{\Gamma}_{yx} & \\ \mathbf{0} & \mathbf{\Gamma}_{xx^*} & \mathbf{0} & \mathbf{\Gamma}_{xx} - \mathbf{I} & \end{bmatrix} \begin{matrix} n_{NF} \\ n_{SF} \\ n_{ND} \\ n_{SD} \end{matrix}$$

$$\tilde{\mathbf{B}}' = \begin{bmatrix} & n_{NF} & n_{SF} & n_{ND} & n_{SD} \\ \mathbf{B}_{y^*y^*} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \\ \mathbf{B}_{yy^*} & \mathbf{0} & \mathbf{B}_{yy} & \mathbf{0} & \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \end{bmatrix} \begin{matrix} n_{NF-1} \\ n_{SF} \\ n_{ND} \\ n_{SD} \end{matrix}$$

$$\tilde{\mathbf{\Gamma}} = \begin{bmatrix} & n_{NF} & n_{SF} & n_{ND} & n_{SD} \\ \mathbf{\Gamma}_{y^*y^*} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \\ \mathbf{\Gamma}_{x^*y^*} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \\ \mathbf{\Gamma}_{yy^*} & \mathbf{0} & \mathbf{\Gamma}_{yy} & \mathbf{0} & \\ \mathbf{\Gamma}_{xy^*} & \mathbf{0} & \mathbf{\Gamma}_{xy} & \mathbf{0} & \end{bmatrix} \begin{matrix} n_{NF} \\ n_{SF} \\ n_{ND} \\ n_{SD} \end{matrix}$$

## D Proof of Proposition 2

**Proposition 2.**  $X$  satisfies conditions (C1) to (C4) if and only if  $Q \in \mathcal{S}_Q$ .

The proof of this proposition follows immediately from the sequence of lemmas below.

**Lemma 1.**  $X$  satisfies condition (C1) if and only if  $Q$  is orthogonal.

*Proof.*  $XX' = GHQQ'H'G'$ , so  $XX' = \Sigma_u$  if and only if  $QQ' = H^{-1}G^{-1}\Sigma_u G'^{-1}H'^{-1} = \Sigma_w = I$  since the shocks  $w_t$  are orthogonal.  $\square$

**Lemma 2.**  $X$  satisfies condition (C2) if and only if  $Q_{11} \neq 0$ .

*Proof.* Consider the impact of the first structural shock  $\epsilon_t^1 = [1 \ 0 \ \dots \ 0]'$ . Then  $w_t^1 = Q\epsilon_t^1$  has first entry equal to  $Q_{11}$  and  $v_t^1 = Hw_t^1$  has first entry equal to  $H_{11}Q_{11}$ . By Proposition 1,  $\epsilon_t^1$  has a permanent impact if and only if the first element of  $v_t^1$  is non-zero. Thus, since  $H_{11}$  is non-zero,  $X$  will satisfy condition (C2) if and only if  $Q_{11}$  is non-zero.  $\square$

**Lemma 3.**  $X$  satisfies condition (C3) if and only if  $Q_{1j} = 0$  for each  $j \geq 2$ .

*Proof.* For each  $j \geq 2$ , let  $\epsilon_t^j$  be the  $j^{\text{th}}$  structural shock, i.e. the vector  $[0 \ \dots \ 0 \ 1 \ 0 \ \dots \ 0]$  with a 1 in the  $j^{\text{th}}$  position. Then  $w_t^j = Q\epsilon_t^j$  has first entry equal to  $Q_{1j}$  and  $v_t^j = Hw_t^j$  has first entry equal to  $H_{11}Q_{1j}$ . By Proposition 1,  $\epsilon_t^j$  has a transient impact on the system if and only if the first element of  $v_t^j$  is zero. Thus condition (C3) is satisfied if and only if  $H_{11}Q_{1j} = 0$  for all  $j \geq 2$ . The result follows on noting that  $H_{11}$  is non-zero.  $\square$

For the next lemma it is useful to introduce the notation  $\mathcal{S}_{SOE}$  for the space of  $n \times n$  matrices  $M$  such that  $M_{ij} = 0$  whenever  $i \in I_F$  and  $j \in I_D$ . For example, the small open economy restrictions on  $\Gamma$  are identical to requiring  $\Gamma \in \mathcal{S}_{SOE}$ . It is easy to show that  $\mathcal{S}_{SOE}$  is closed under taking inverses and products, and that  $H$  lies in  $\mathcal{S}_{SOE}$  since it is lower triangular matrix.

**Lemma 4.**  $G \in \mathcal{S}_{SOE}$ .

*Proof.* Since  $\mathcal{S}_{SOE}$  is closed under taking inverses it suffices to show that

$$G^{-1} = \begin{bmatrix} \tilde{A}'_{\perp} \\ \tilde{B}' \end{bmatrix}$$

lies in  $\mathcal{S}_{SOE}$ . It is clear from the structure of  $\tilde{B}'$  described in Appendix C that the required entries of  $\tilde{B}'$  are zero. It thus remains to show that the final  $n_D$  entries of  $\tilde{A}'_{\perp}$  are zero. Let  $\alpha$  be an orthogonal complement to the  $n_F \times (n_F - 1)$  submatrix

$$\begin{bmatrix} A_{y^*y^*} & \Gamma_{y^*x^*} \\ \mathbf{0} & \Gamma_{x^*x^*} - I \end{bmatrix}$$

of  $\tilde{A}$ , where  $A_{y^*y^*}$ ,  $\Gamma_{y^*x^*}$  and  $\Gamma_{x^*x^*}$  are as described in Appendix C. Then it is immediate from the structure of  $\tilde{A}$  that the vector  $[\alpha' \ 0]'$  is an orthogonal complement to  $\tilde{A}$ , where  $\mathbf{0}$  is a vector



of  $n_D$  zeros. Since  $\tilde{\mathbf{A}}_{\perp}$  is uniquely defined up to scalar multiplication, any choice of  $\tilde{\mathbf{A}}_{\perp}$  will also have its final  $n_D$  entries equal to zero.  $\square$

**Lemma 5.**  $\mathbf{X}$  satisfies condition (C4) if and only if  $\mathbf{Q} \in \mathcal{S}_{SOE}$ .

*Proof.* Condition (C4) can be restated as requiring that  $\mathbf{IRF}^t \in \mathcal{S}_{SOE}$  for all  $t \geq 0$ . This condition holds if and only if  $\mathbf{X} \in \mathcal{S}_{SOE}$ , since  $\mathbf{IRF}^0 = \mathbf{X}$ , and the small open economy restrictions on  $\mathbf{A}$  and  $\mathbf{\Gamma}$  ensure that if  $\mathbf{IRF}^t \in \mathcal{S}_{SOE}$  then  $\mathbf{IRF}^{t+1} \in \mathcal{S}_{SOE}$ . But  $\mathbf{X} \in \mathcal{S}_{SOE}$  if and only if  $\mathbf{Q} = \mathbf{H}^{-1}\mathbf{G}^{-1}\mathbf{X} \in \mathcal{S}_{SOE}$  since  $\mathbf{G}, \mathbf{H} \in \mathcal{S}_{SOE}$ .  $\square$

**Lemma 6.** Suppose  $\mathbf{M}$  is a matrix with block form

$$\begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{B} & \mathbf{C} \end{bmatrix}$$

for some submatrices  $\mathbf{A}$ ,  $\mathbf{B}$ , and  $\mathbf{C}$ . If  $\mathbf{M}$  is orthogonal then  $\mathbf{B} = \mathbf{0}$ .

*Proof.* From the definition of orthogonality,

$$\mathbf{I} = \mathbf{M}\mathbf{M}' = \begin{bmatrix} \mathbf{A}\mathbf{A}' & \mathbf{A}\mathbf{B}' \\ \mathbf{B}\mathbf{A}' & \mathbf{B}\mathbf{B}' + \mathbf{C}\mathbf{C}' \end{bmatrix}.$$

Thus  $\mathbf{B}\mathbf{A}' = \mathbf{0}$  whence  $\mathbf{B} = \mathbf{0}$  since  $\mathbf{A}\mathbf{A}' = \mathbf{I}$  implies that  $\mathbf{A}'$  is non-singular.  $\square$