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Uncertain forward guidance

Alex Haberis, Richard Harrison and Matt Waldron

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Alex Haberis,[1] Richard Harrison[2] and Matt Waldron[3]

Abstract

We explore the effects of forward guidance at the zero lower bound when there is uncertainty over the lift-off date arising from: (i) the imperfect credibility of time-inconsistent forward-guidance promises; (ii) incomplete communication. We use a simple New Keynesian model to demonstrate that a forward guidance announcement to delay lift-off may be no more powerful in a more interest rate sensitive economy. We also demonstrate that attempts to delay lift-off further may fail to generate additional stimulus if the temptation to renege on the announcement is sufficiently great. In an empirical application, we consider counterfactual policy experiments based on the Federal Open Market Committee's 'threshold-based' forward guidance, in which we link the probability of lift-off to the amount by which the announced unemployment threshold is breached. We show that a more precise articulation of the lift-off conditions requires a lower unemployment threshold in order to deliver the same amount of stimulus as a less precise one.

Key words: Forward guidance, uncertainty, zero lower bound.

JEL classification: E12, E17, E20, E30, E42, E52.

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(1) Bank of England. Email: alex.haberis@bankofengland.co.uk
(2) Bank of England. Email: richard.harrison@bankofengland.co.uk
(3) Bank of England. Email: matthew.waldron@bankofengland.co.uk

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Publications and Design Team, Bank of England, Threadneedle Street, London, EC2R 8AH
Telephone +44 (0)20 7601 4030 email publications@bankofengland.co.uk

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1 Introduction

Since the financial crisis, policy rates in most advanced economies have been set at, or near, their effective lower bounds. As a consequence, central banks have turned to various unconventional policy measures, such as large scale asset purchases, as well as an increased use of public announcements about their intentions for the future path of the policy rate—a policy that has come to be known as 'forward guidance.'

From a theoretical perspective, a powerful motivation for forward guidance is to provide stimulus as a substitute for cutting the policy rate: by lowering expectations for the policy rate in the future, a central bank can stimulate activity and hence inflation today via an intertemporal-substitution channel. In this paper, we study the link between a forward-guidance announcement and the resulting change in private sector interest rate expectations—a transmission channel that has not been carefully studied in the literature to date. In particular, we argue that forward-guidance announcements do not pin down the future behavior of policy with certainty. The uncertainty associated with forward-guidance announcements, and the nature of it, is important in determining their effects on interest-rate expectations and hence the economy.

We study two different sources of uncertainty about the outlook for policy following a forward-guidance announcement. First, uncertainty stemming from the fact that forward-guidance announcements are incomplete descriptions of state-contingent policy behavior. For example, in August 2011 the FOMC issued guidance that "... economic conditions ... are *likely to* warrant exceptionally low levels for the federal funds rate *at least* through mid-2013" (emphasis added). This statement is unclear about the likelihood that the federal funds rate would stay at the lower bound until mid-2013 and, if it did, what probabilities should be assigned to it lifting-off from the zero bound at dates beyond that. The statement also implies that the probabilities are likely to depend on economic conditions. The FOMC made that dependence more explicit in their December 2012 'threshold-based' guidance, stating that "... [an] exceptionally low range for the federal funds rate will be appropriate *at least* as long as the unemployment rate remains above $6\frac{1}{2}$ percent ..." (emphasis added). While this announcement describes the conditions under which the federal funds rate could be expected to remain at the lower bound, it does not state the circumstances under which rates would lift-off. It remains, therefore, an incomplete description of how policy should be expected to behave.

The second source of uncertainty that we study arises from the fact that forward guidance promises may be imperfectly credible; a possibility that has been widely recognized by policymakers. In order for forward guidance to provide stimulus, it must embody a commitment to hold the policy rate lower for longer than had previously been expected. There is a well-known time-inconsistency problem associated with such promises. Namely, that the central bank has an incentive, once the recovery has taken hold, to renege and tighten policy earlier than originally promised. In light of that, private agents may doubt that the central bank will implement the announced plan, leading to uncertainty about future policy and an associated reduction in the effect of the forward-guidance announcement on interest-rate expectations.

To study the impact of policy uncertainty on the effectiveness of forward-guidance announcements, we use standard macroeconomic models to construct scenarios in which the economy experiences a sufficiently large recessionary shock that the policymaker becomes constrained by the lower bound. We then examine the macroeconomic effect of alternative forward-guidance announcements, motivated by a desire to stimulate the economy and improve outcomes. We model forward guidance as a promise to hold the policy rate lower for longer than would be implied by the policymaker's usual policy rule. Thus, we model forward guidance as a temporary deviation from normal policy behavior under full information in the spirit of Laséen and Svensson (2011), Campbell et al. (2012), Del Negro et al. (2015) and English et al. (2015).[1] We model policy uncertainty as a non-zero probability that the policymaker reverts to its usual policy rule. We allow for that probability to depend on the evolution of endogenous variables and, therefore, to be time-varying. We assume that there is no uncertainty about future macroeconomic shocks, which, combined with our approach to modeling policy uncertainty and a linear(ized) model of the economy, means that the problem is piecewise linear. This facilitates its application to large-scale models of the type used in policy institutions.

We study the two alternative sources of policy uncertainty described above. In the first application we consider policy uncertainty in a small textbook New Keynesian model. We focus on the macroeconomic effect of a time-inconsistent announcement by the policymaker that they intend to hold the policy rate at the lower bound for longer than agents had been expecting.

We demonstrate three key results. First, the macroeconomic effect of the lower-for-longer announcement is materially reduced for even relatively small per-period probabilities that the policymaker will renege on the promise and revert to its usual policy rule earlier than announced. Second, a promise to hold rates lower for even longer can be counterproductive if there is a sufficiently strong feedback from the incentive to renege (as measured by a welfare-consistent loss function) to the probability of reverting to the usual rule. Third, a given lower-for-longer announcement is no more powerful in a version of the model with a higher interest elasticity of demand because the credibility of the announcement is lower in that case.

In the second application, we apply our method to a large-scale model; the Federal Reserve Board's model, FRB/US. We perform counterfactual policy experiments based on the FOMC's December 2012 'threshold-based' guidance. We model the probability of lift-off from the zero bound as a function of the gap between unemployment and its threshold value, calibrated using surveys of financial-market expectations. We also consider an alternative in which the policymaker is more precise about the conditions under which lift-off will occur (a mapping that is more tightly linked to the threshold breach).

We demonstrate that increasing the precision in the announcement to steer expectations better should go alongside a *lower* unemployment threshold to achieve a comparable impact on the economy. That is because the effects of threshold-based guidance policies in this setting depend on the balance of two opposing forces. Other things equal, a lower unemployment threshold tends to reduce expected real interest rates, increasing stimulus. But there is also greater certainty that the policy rate will rise after the threshold has been breached, which tends to increase interest-rate expectations, reducing stimulus.

Our paper is related to several strands of the literature. A number of authors have examined the effects of temporary, though possibly persistent, deviations of monetary

[1] The full-information assumption—that both the private sector and the policymaker know the model of the economy—implies that our framework cannot be used to study clarification of the reaction function as a motivation for forward guidance because prior to the uncertain forward-guidance announcement there is no uncertainty about what the reaction function is. It also implies that our framework cannot be used to study the possibility that forward guidance announcements may contain information about how the central bank expects the economy to evolve (so-called 'Delphic' forward guidance) because both the private sector and policymaker have identical and correct estimates of the state of the economy.

policy from its usual behavior in a range of models. Most do so on the assumption that the policymaker has the ability to perfectly control private-sector expectations about the policy instrument (for example: Blake, 2012; Carlstrom et al., 2012; Levin et al., 2010).

Some existing research has relaxed the assumption that an announced policy deviation is fully credible (for example, Blake, 2013; Carlstrom et al., 2013; Weale, 2013). Our method is more general because we allow the probability that the policymaker reverts to its usual behavior earlier than announced to vary over time and to depend on the endogenous variables in the model.[2]

Another paper that is closely related to ours is that of Bodenstein et al. (2012), who study imperfect credibility at the zero lower bound using a 'loose commitment' framework. While less general in some ways, our framework is more general in one key respect because we allow the credibility of a forward-guidance announcement to depend on the size of the associated time-inconsistency problem, as well as on the inherent credibility (or commitment level) of the policymaker. Our results show that this mechanism can materially affect the efficacy of policy promises at the zero lower bound and, as such, is an important part of the overall transmission mechanism.

Allowing the probability of reversion to be endogenous to outcomes enables us to simulate the effects of threshold-based forward guidance. Our methodology extends the perfect-foresight approximation of expectations as in the threshold-based guidance experiments of Coenen and Warne (2014), De Graeve et al. (2014), and English et al. (2015), to allow for a non-zero probability that lift-off will not occur after the threshold conditions have been met, which is consistent with the real-world threshold-based guidance policy of the FOMC.

We also contribute to the growing literature that documents the mechanics of forward guidance in New Keynesian models and the "forward-guidance puzzle"—the pathology that forward guidance in standard macroeconomic models is implausibly powerful when compared to empirical evidence (Del Negro et al., 2015; De Graeve et al., 2014; Carlstrom et al., 2013). Much of the literature that seeks to resolve the puzzle takes interest-rate expectations as given and then focuses on reducing their effect, either by reducing the sensitivity of aggregate demand to future interest rates (Del Negro et al., 2015; McKay et al., 2015), by reducing the sensitivity of inflation to expected cost developments (Chung et al., 2014; Kiley, 2016), or by dispensing with full rationality (Gabaix, 2016). Our paper provides a different way of looking at the forward-guidance puzzle and demonstrates that it is paradoxical in an imperfect-credibility setting: the stronger the transmission channel from interest-rate expectations to activity and inflation, the lower the credibility of announcements that seek to impart stimulus by influencing interest-rate expectations, and the greater the attenuation of the macroeconomic effect.

The rest of the paper proceeds as follows. Section 2 explains our approach to incorporating uncertainty in forward-guidance announcements, the solution procedure and how that relates to existing approaches. Section 3 uses our procedure in our imperfect-credibility application. Section 4 presents counterfactual experiments based on the FOMC's December 2012 threshold-based guidance. Section 5 concludes.

[2]Our focus is on endogeneity of credibility to macroeconomic outcomes. Michelacci and Paciello (2017) examine an alternative featuring ambiguity-averse agents, whereby the credibility of announcements varies across households depending on their balance sheet positions (creditors versus debtors) in a way that matters for the aggregate effect.

2 Method

2.1 An overview of the policy experiment

The policy experiment that we analyze is one in which the central bank announces the time-path for its policy instrument that it intends to set over the next K periods, $\{b_t\}_{t=1}^K$, assuming that the private sector places a non-zero probability, $\{p_t\}_{t=1}^K$, on the policymaker deviating from that announcement in each period t and reverting to their usual policy rule (which they follow thereafter). We allow for the possibility that the probabilities that the policymaker deviates from the announcement could be endogenous to economic outcomes. After the K periods are up (and thereafter) the central bank sets the instrument according to their usual rule with certainty.

2.2 Model

Our policy experiments can be applied to linear (or linearized) rational-expectations models of the following form:

$$H^F \mathbb{E}_t x_{t+1} + H^C x_t + H^B x_{t-1} = \Psi z_t \tag{1}$$

where x is a vector of endogenous variables (both predetermined and non-predetermined), z is a vector of structural, orthogonal shocks, and the matrices H^F, H^C, H^B and Ψ are functions of the model's parameters. The majority of the equations in the system (1) describe the behavior of households and firms in the model economy. Others represent the evolution of exogenous processes. One equation that is of particular significance to our analysis is the central bank's policy rule. This describes how the policy instrument (the short-term nominal interest rate in our examples) responds systematically to a set of other macroeconomic variables (such as inflation, the output gap, and so on).

We also require that the model (1) has a unique determinate rational-expectations solution:

$$x_t = Bx_{t-1} + \sum_{s=0}^{\infty} F^s \Phi \mathbb{E}_t z_{t+s} \tag{2}$$

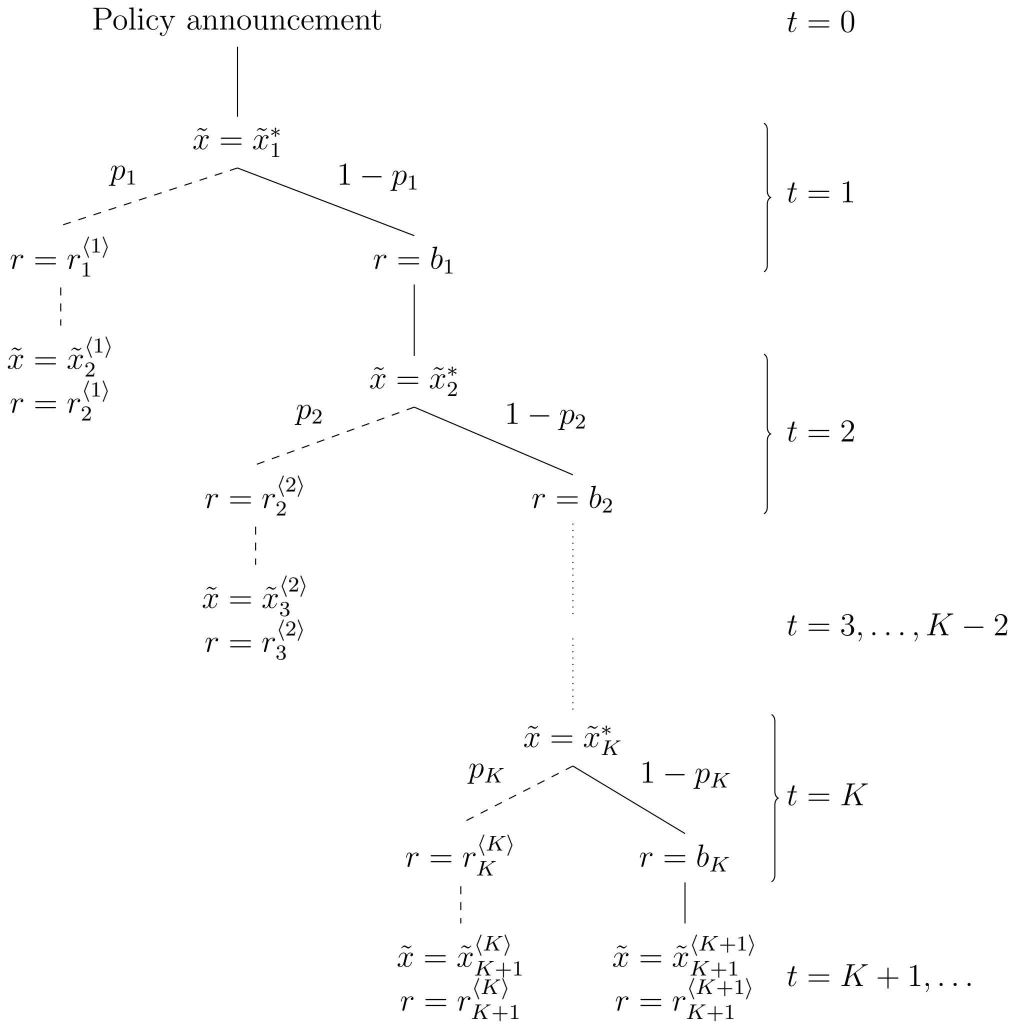
where $\Phi = \left(H^C + H^F B\right)^{-1} \Psi$, $F = -\left(H^C + H^F B\right)^{-1} H^F$, and the matrix B, which can be solved for using a variety of methods, satisfies $B = -\left(H^F B + H^C\right)^{-1} H^B$.[3]

2.3 Timing assumptions

Figure 1 details the timing assumptions in the policy experiments: r denotes the monetary-policy instrument and $\tilde{x}$ comprises all elements of x except the policy instrument, r. The diagram begins at the top, at the end of period $t = 0$, with the announcement by the policymaker that the policy rate will follow a particular path, $\{b_t\}_{t=1}^K$, for periods $t = 1, \ldots, K$. It ends at the bottom, in period $t = K$. The solid branches trace out the sequence of events that leads to the policymaker implementing the announced interest-rate path in full. The dashed branches show the K states of the world in which the policymaker reverts to their usual policy rule earlier than announced.

[3] Equation (2) incorporates anticipated or 'news' shocks. Section 3.5 provides an example where anticipated shocks are used in the construction of the policy experiment.

Figure 1: Diagrammatic representation of the policy experiment



Notes: In period 0 the policymaker announces that the policy instrument r will follow the path $\{b_t\}_{t=1}^{K}$ for K periods. In each period, $t = 1, 2, \ldots, K$, the sequence of events is such that the private sector makes decisions ($\tilde{x}_t$) based on an intra-period expectation that nominal rates will be set in line with the usual policy rule, $r_t^{\langle t \rangle}$, with probability p_t, and set in line with the announced path, b_t, with probability $1 - p_t$. The policymaker then sets the interest rate. The dotted lines show the cases in which the policymaker reverts to the rule in period t, while the solid lines show its trajectory when the policymaker implements the announced path. There are K trajectories in which the policymaker reverts to the rule earlier than announced and a single trajectory in which the announcement is fully implemented.

The nature of our policy experiment requires paying careful attention to the timing of policy and private-sector decisions. We assume that both the policymaker and private sector observe the state, including realizations of the shocks, at the start of each period, and then that private agents make their decisions *before* the policymaker. This within-period timing assumption allows us to incorporate the possibility that the policymaker may revert to its usual policy rule with some probability even in period $t = 1$. The implication is that the private sector must make its decisions based on expectations for what the policymaker will do in the current period, and, consequently, probability-weight later periods appropriately. We use an asterisk (*) to denote decisions made conditional on the policymaker not having reverted to its usual policy rule in a previous period. The vector $\tilde{x}_t^*$, therefore, denotes private-sector decisions made at the start of period t, conditional on the policymaker implementing the announced policy plan up to, and including, period $t-1$.

To see the sequence of events in more detail, consider period $t = 1$. At the start of the period, after the shocks have been observed, the private sector chooses $\tilde{x}_1^*$ on the basis of its expectations for the behavior of the policymaker in period $t = 1$ and beyond. Then, the policymaker sets the policy instrument, r_1. With probability $1 - p_1$ the policymaker implements the announced interest rate $r_1 = b_1$. This is represented by the solid branch of the $t = 1$ tree in Figure 1.

The other possible outcome in period $t = 1$ is for the policymaker to deviate from the announced rate. This happens with probability p_1 and is represented by the dashed branch of the $t = 1$ tree. In this case, the policymaker sets policy according to the rule, $r_1 = r_1^{\langle 1 \rangle}$. We use the '$\langle s \rangle$' superscript to denote outcomes in the case that the policymaker reverts to the rule in period s. Thus $\{\tilde{x}_t^{\langle s \rangle}, r_t^{\langle s \rangle}\}$ are the outcomes in period t when the policymaker has reverted to the rule in period $s \leq t$. Figure 1 shows that there are K such trajectories, corresponding to the policymaker reverting to the rule in periods $s = 1, \ldots, K$. Taken together with the case in which the policymaker implements the announced interest-rate path, there are $K + 1$ possible outcomes.

2.4 A sketch of the solution

In this section we provide an outline of our solution approach. The general method is presented in Appendix A. To aid exposition in this sketch, we make the following simplifying assumptions: no shocks arrive after period $t = 0$ ($z_{t+s} = 0 \, \forall \, t, s > 0$); no leads or lags of the policy rate appear in any model equations; the policy rule depends only on contemporaneous variables such that:

$$r_t = \Gamma \widetilde{x}_t \tag{3}$$

The primary objective is to solve for the path along which the policymaker implements the announced plan in full and returns to setting policy according to its usual rule in period $K + 1$. This is the sequence of outcomes along the solid branches of Figure 1. As explained below, the paths associated with all other outcomes can be constructed by building the appropriate 'branches' stemming from this solution.

Our algorithm exploits the structure of the policy experiment to construct a 'stacked-time' solution for the K periods simultaneously. To construct this solution, we use a representation of the model equations in which the policy instrument, r, is partitioned from the remaining endogenous variables, $\tilde{x}$, and the policy rule is removed from the model. The remaining equations of the model (1) can be written as follows (which exploits the simplifying assumptions outlined above):

$$\widetilde{H}_{\tilde{x}}^F \mathbb{E}_t^* \widetilde{x}_{t+1} + \widetilde{H}_{\tilde{x}}^C \widetilde{x}_t^* + \widetilde{H}_{\tilde{x}}^B \widetilde{x}_{t-1}^* + \widetilde{H}_r^C \mathbb{E}_t^* r_t = 0 \tag{4}$$

where the matrices with tilde ($\tilde{}$) superscripts are submatrices of the relevant matrices in (1). As explained in Section 2.3, we use asterisks to denote solutions conditional on the policymaker not having reverted to the rule in a previous period. We also apply this notation to the expectations operator, $\mathbb{E}^*$, to highlight the same conditionality. Our goal is to solve for the path $\widetilde{X}^* \equiv [\widetilde{x}_1^* \ldots \widetilde{x}_K^*]'$.

To proceed, note that the expectations terms can be defined as:

$$\mathbb{E}_t^* r_t = p_t r_t^{\langle t \rangle} + (1 - p_t)\, b_t \tag{5}$$

$$\mathbb{E}_t^* \tilde{x}_{t+1} = p_t \widetilde{x}_{t+1}^{\langle t \rangle} + (1 - p_t)\, \widetilde{x}_{t+1}^* \tag{6}$$

Equation (5) expresses the expected policy rate in period t as a probability-weighted average of the announced policy rate, b_t, and the rate consistent with the policy rule in the event that the policymaker reverts to the rule in period t, $r_t^{\langle t \rangle}$. This follows from our within-period timing assumption, which implies that private sector decisions depend on within-period expectations of the policymaker's actions. Equation (6) performs a similar operation to define the expectation of $\widetilde{x}_{t+1}$. In the event that the policymaker reverts to the rule in period t (with probability p_t), private sector decisions in period $t+1$ will be given by $\widetilde{x}_{t+1}^{\langle t \rangle}$. If the policymaker sets the planned policy rate b_t (with probability $1-p_t$), private sector decisions in period $t+1$, $\tilde{x}_{t+1}^{*}$, will be consistent with the solutions to the model equations (4) evaluated in period $t+1$.

To proceed further, we can make two additional substitutions. First, we can substitute out $r_t^{\langle t \rangle}$ in equation (5) using the policy rule (3) and the fact that the policy rate is set after private-sector decisions have been made (which means that $r_t^{\langle t \rangle} = \Gamma \widetilde{x}_t^{*}$) to give:

$$\mathbb{E}_t^{*} r_t = p_t \Gamma \widetilde{x}_t^{*} + (1 - p_t)\, b_t \tag{7}$$

Second, we can use the rational-expectations solution of the model (2), which applies in all periods after the policymaker has reverted to the rule, to substitute out $\widetilde{x}_{t+1}^{\langle t \rangle}$ in equation (6). Given our simplifying assumptions and using the same partitioning logic as above, the rational-expectations solution for $\widetilde{x}_t$ has the following form:

$$\widetilde{x}_t = B_{\tilde{x}\tilde{x}} \widetilde{x}_{t-1} \tag{8}$$

For the case in which the policymaker reverts to the rule in period t, this implies that:

$$\widetilde{x}_{t+1}^{\langle t \rangle} = B_{\tilde{x}\tilde{x}} \widetilde{x}_t^{*} \tag{9}$$

which again reflects the fact that private-sector decisions in period t, $\widetilde{x}_t^{*}$, are made before the policymaker sets the instrument.

Using equation (9) in equation (6) allows us to write the expectations of $\widetilde{x}$ in period $t+1$ as:

$$\mathbb{E}_t^{*} \widetilde{x}_{t+1} = p_t B_{\tilde{x}\tilde{x}} \widetilde{x}_t^{*} + (1 - p_t)\, \widetilde{x}_{t+1}^{*} \tag{10}$$

Substituting equations (7) and (10) into equation (4) gives:

$$\widetilde{H}_{\tilde{x}}^{F} \left[p_t B_{\tilde{x}\tilde{x}} \widetilde{x}_t^{*} + (1 - p_t)\, \widetilde{x}_{t+1}^{*} \right] + \widetilde{H}_{\tilde{x}}^{C} \widetilde{x}_t^{*} + \widetilde{H}_{\tilde{x}}^{B} \widetilde{x}_{t-1}^{*} + \widetilde{H}_{r}^{C} \left[p_t \Gamma \widetilde{x}_t^{*} + (1 - p_t)\, b_t \right] = 0$$

After collecting terms, this can be written as:

$$J_t^{F} \widetilde{x}_{t+1}^{*} + J_t^{C} \widetilde{x}_t^{*} + J_t^{B} \widetilde{x}_{t-1}^{*} = C_t \tag{11}$$

where:

$$\begin{aligned} J_t^{F} &= (1 - p_t)\, \widetilde{H}_{\tilde{x}}^{F} \\ J_t^{C} &= p_t \widetilde{H}_{\tilde{x}}^{F} B_{\tilde{x}\tilde{x}} + \widetilde{H}_{\tilde{x}}^{C} + p_t \widetilde{H}_{r}^{C} \Gamma \\ J_t^{B} &= \widetilde{H}_{\tilde{x}}^{B} \\ C_t &= -(1 - p_t)\, \widetilde{H}_{r}^{C} b_t \end{aligned}$$

The set of equations implied by equation (11) can be stacked together to give the following system of equations for the solution conditional on the policymaker implementing the policy path as announced ($\widetilde{X}^*$):

$$\mathbb{J}\widetilde{X}^* = \mathbb{C} \tag{12}$$

where:

$$\widetilde{X}^* \equiv \begin{bmatrix} \widetilde{x}_1^* \\ \vdots \\ \widetilde{x}_K^* \end{bmatrix}, \mathbb{C} \equiv \begin{bmatrix} C_1 \\ \vdots \\ C_K \end{bmatrix}$$

and

$$\mathbb{J} \equiv \begin{bmatrix} J_1^C & J_1^F & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ J_2^B & J_2^C & J_2^F & 0 & \dots & 0 & 0 & 0 & 0 \\ & & & & \vdots & & & & \\ 0 & 0 & \dots & J_t^B & J_t^C & J_t^F & 0 & \dots & 0 \\ & & & & \vdots & & & & \\ 0 & 0 & 0 & 0 & \dots & 0 & J_{K-1}^B & J_{K-1}^C & J_{K-1}^F \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & J_K^B & J_K^C \end{bmatrix}$$

For $2 \leq t \leq K-1$, each (block) row of the $\mathbb{J}$ matrix can be constructed using the matrices described above. The cases $t=1$ and $t=K$ require a slightly different treatment to reflect the inherited initial condition in period 1 and the fact that the policymaker sets policy according to the rule with certainty in period $K+1$.[4] Once $\mathbb{J}$ and $\mathbb{C}$ have been constructed appropriately, the solution to (12) is $\widetilde{X}^* = \mathbb{J}^{-1}\mathbb{C}$.

The vector $\widetilde{X}^*$ can be used to construct the trajectories consistent with the policymaker reverting to the rule in period t as follows. Up to period $t-1$ the solutions for the policy instrument and non-policy variables are $[b_1 \dots b_{t-1}]$ and $\left[\widetilde{x}_1^* \dots \widetilde{x}_{t-1}^*\right]$ respectively. Then in period t we have $r_t^{\langle t \rangle} = \Gamma \widetilde{x}_t^*$ and $\widetilde{x}_t^{\langle t \rangle} = \widetilde{x}_t^*$. From period $t+1$ onwards we use the rational expectations solution: $x_{t+i}^{\langle t \rangle} = B x_{t+i-1}^{\langle t \rangle}$.

2.5 Endogenous probabilities

In the solution sketched above, the private sector's beliefs about the policymaker reverting to the rule earlier than announced, as represented by probabilities $P \equiv [p_1 \dots p_K]$, were taken as given. In this section, we outline a method for computing a solution in which the probabilities, P, depend on outcomes. There are intuitive reasons why that might be the case, two of which are analyzed in Sections 3 and 4.

In Section 3, we consider the case in which a monetary policymaker announces that the policy rate will be held lower than implied by its usual reaction function as an attempt to stimulate the economy at the zero bound. This promise is time inconsistent as the policymaker has an incentive to deviate from the announced path as the economy improves. We link the credibility of the announcement (captured by the beliefs, P) to the policymaker's incentive to renege on it, measured as the difference between welfare when the policymaker exits early and welfare when the plan is enacted.

[4]Specifically $C_1 = -(1-p_1)\widetilde{H}_r^C b_1 - \widetilde{H}_r^B r_0 - \widetilde{H}_{\tilde{x}}^B \widetilde{x}_0$, $J_K^C = \widetilde{H}_{\tilde{x}}^F (B_{\tilde{x}\tilde{x}} + B_{\tilde{x}r}\Gamma) + \widetilde{H}_{\tilde{x}}^C + p_K \widetilde{H}_r^C \Gamma$ and $C_K = -\left[(1-p_K)\widetilde{H}_r^C + \widetilde{H}_r^B + (1-p_K)\widetilde{H}^F B_{\tilde{x}r}\right] b_K$.

In Section 4, we consider threshold-based forward guidance in which lift-off from the zero bound is tied to macroeconomic developments. The policymaker announces that lift-off will not occur *at least* until specified macroeconomic variables satisfy particular conditions. We use a mapping to link the distance of the macroeconomic variables from their threshold values to the probability of lift-off from the zero bound.

To implement endogeneity of beliefs in general, we allow for the mapping from outcomes to probabilities to be given by:

$$P = F\left(\widetilde{X}^*\left(P, \mathcal{B}, \mathcal{M}\right), P, \mathcal{M}\right) \tag{13}$$

where $\widetilde{X}^*\left(P, \mathcal{B}, \mathcal{M}\right)$ is a function defined implicitly by equation (11), $\mathcal{B} \equiv [b_1 \dots b_K]$ is the announced time path for the policy rate, and $\mathcal{M}$ is the economic model.

The arguments of the function F are sufficient to characterize a wide range of cases. In particular, the dependence on P and $\mathcal{M}$, in addition to $\widetilde{X}^*$, allows beliefs to be affected by expected outcomes, including expected developments in cases when the policymaker reverts to its usual rule earlier than announced.[5] The fact that P may depend on $\widetilde{X}^*$, which is a function of P, as well as possibly on itself indicates that we seek a fixed point of this mapping. To do so, we employ a simple function iteration approach:

1. Guess an initial vector of exit probabilities P.
2. Given the policy plan $\mathcal{B}$ and the latest guess for the exit probabilities P, solve for the equilibrium trajectories, $\widetilde{X}^*$, using the algorithm described in Section 2.4 and detailed in Appendix A.
3. Update the guess for the probabilities using $P = F\left(\widetilde{X}^*, P, \mathcal{M}\right)$.
4. If the updated guess for P is close to the previous guess, stop. Otherwise return to step 2.

2.6 Comparison with other solution approaches

Our stacked-time solution approach shares some similarities with methods used to study optimal policy subject to constraints on the policy instrument(s) in a perfect-foresight setting (for example, Jung et al., 2005; Harrison, 2012). Carlstrom et al. (2012) also use a stacked-time solution to study the properties of simulations in which the policy instrument is 'pegged' to a particular value for a finite number of periods.

In addition to the simplifications made to the model itself, another simplification in the sketch of our approach in Section 2.4 is that we ignore the effects of the zero bound on policy behavior in the event that the policymaker reverts to the rule earlier than announced. In this paper we consider experiments in which the policy rate is initially constrained by the zero bound, so such a simplification is not valid and the full solution algorithm detailed in Appendix A is required. In Appendix A.3 we explain how the

[5] As detailed in Section 2.4, expected outcomes are a weighted average of outcomes in the event that the announced plan is implemented, $\widetilde{X}^*$, and outcomes in the event that the policymaker reverts to the rule. The dependence on P reflects that the probabilities of reversion determine the weights used to compute the average. And the dependence on $\mathcal{M}$ reflects that outcomes on reversion to the rule can be computed by projecting forward outcomes along the non-reversion path using the rational-expectations solution to the model as, for example, defined in equation (9).

quadratic-programming approach developed by Holden and Paetz (2012) is used to ensure that all solution trajectories respect the zero bound on the policy rate. For linearized models, this approach gives identical results to the OccBin toolkit for solving piecewise linear models developed by Guerrieri and Iacoviello (2015). We use the Holden-Paetz approach for convenience, given that we use the Anderson and Moore (1985) algorithm to compute the rational-expectations solution of the model.[6]

Our method and those discussed above all incorporate a certainty-equivalence assumption. That is, agents' expectations are equated to the outcome of the model under the assumption that no unanticipated shocks arrive. In the presence of a zero bound on the policy instrument this assumption is typically invalid. That is because the constraint on the instrument implies different responses to equally-sized but oppositely-signed disturbances, generating a skewed distribution of outcomes.[7] Taking account of these effects requires the model to be solved using global methods (as in, for example, Adam and Billi, 2006, 2007; Bodenstein et al., 2012). However, the computational demands of these methods are such that they are only feasible for relatively small models. Our approach is designed to provide approximate solutions using medium and large-scale models of the type used by central banks and other policy institutions.

3 Imperfectly-credible forward guidance

In this section we use our algorithm to study the effects of uncertainty around a forward-guidance policy arising from the possibility that the announcement is not fully credible.

3.1 Model and baseline scenario

The model we use to analyze imperfect credibility is the standard New Keynesian model:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa y_t \tag{14}$$
$$y_t = \mathbb{E}_t y_{t+1} - \sigma \left(r_t - \mathbb{E}_t \pi_{t+1} - r_t^n \right) \tag{15}$$
$$r_t^n = \rho r_{t-1}^n + \varepsilon_t^n \tag{16}$$
$$r_t = \phi_\pi \pi_t + r_t^n \tag{17}$$

Equation (14) is a New Keynesian Phillips curve, which relates current inflation, π_t, to expected inflation and the output gap, y_t. Equation (15) is a dynamic *IS* curve, which relates the expected change in the output gap to the gap between the ex-ante real interest rate, $r_t - \mathbb{E}_t \pi_{t+1}$ (where r_t is the nominal interest rate), and the natural rate of interest, r_t^n. Equation (16) specifies an exogenous process for the natural rate of interest, including the natural-rate shock, ε_t^n. Equation (17) is the policymaker's (usual) reaction function. We adopt this specification over a more standard assumption of a Taylor rule since it is a simple way to replicate the equilibrium under optimal discretionary policy for $\phi_\pi > 1$, as noted by Galí (2009).[8] As such, it is a useful baseline for considering alternative policy

[6] The Anderson-Moore algorithm provides a representation of the solution that includes the effects of anticipated disturbances, as shown in equation (2). Laséen and Svensson (2011) note that such a representation is useful for imposing anticipated paths on the policy instrument and the Holden and Paetz (2012) approach makes use of this insight. In our experience, the Anderson-Moore solution algorithm is a particularly robust algorithm for solving large-scale linearized rational-expectations models.

[7] In particular, the mean and the mode of the distribution of outcomes differ. Certainty-equivalent approaches impose that expectations are equal to modal outcomes.

[8] In the absence of a zero bound on the policy instrument and mark-up shocks.

announcements that seek to improve outcomes via some form of commitment to deviate from this rule.

The model's parameters are given in Table 1. For the structural parameters we use the calibration in Carlstrom et al. (2012). We set ϕ_π to a value slightly above 1 to ensure a determinate rational-expectations solution. We follow Levin et al. (2010) and calibrate the persistence of the natural-rate shock to $\rho = 0.85$.

Table 1: Parameter values for small model

	Description	Value
β	Discount factor	0.99
κ	Slope of the Phillips curve	0.025
σ	Interest elasticity of demand	1
ρ	Persistence of natural rate	0.85
ϕ_π	Policy response to inflation	1.01

To shed light on the effects of forward-guidance policies we construct a baseline scenario in which a policymaker may be able to improve outcomes by following such a policy. The scenario is inspired by the Great Recession, but, for expositional purposes, is stylized. In particular, we focus on a scenario in which the natural rate, r^n, falls to minus 8% on an annualized basis in period 1. Absent the zero lower bound (ZLB), the policymaker would cut rates in line with this fall in the natural rate. In the presence of the zero bound, this is not possible. This leads to a positive real-rate gap ($r_t - \mathbb{E}_t \pi_{t+1} - r_t^n > 0$) and, hence, a large recession—as shown by the solid gray lines in Figure 2. Facing the constraint of the ZLB, the policymaker must consider other options for stimulus. One such policy is to announce a delayed lift-off from the ZLB—i.e. providing forward guidance that the policy rate will be held low until after the economy has recovered, thereby committing to an overshoot of output and inflation.

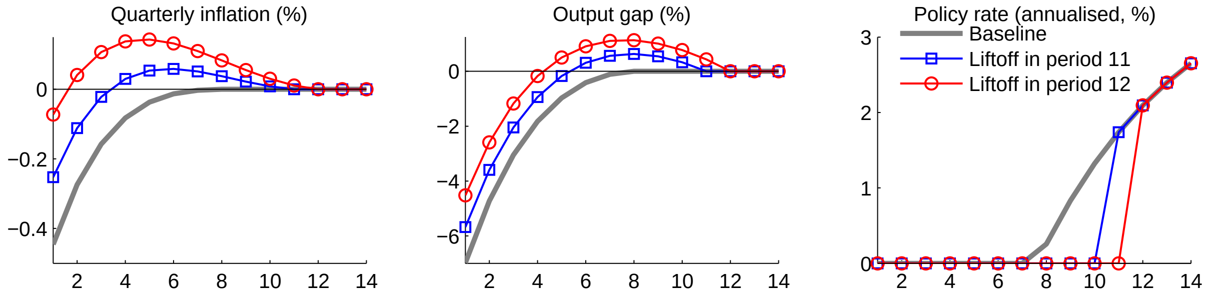
3.2 The time inconsistency of a forward-guidance promise

A forward-guidance policy, when fully credible, can improve outcomes when the policy rate is constrained by the zero bound by reducing expected future real interest rates. As a result, private agents would expect a negative real-rate gap ($r_t - \mathbb{E}_t \pi_{t+1} - r_t^n < 0$) in the future, which would offset some of the effects of the positive real-rate gap experienced during the period for which policy is constrained.

Figure 2 illustrates this in the context of our recessionary scenario. In the baseline case (solid gray lines), the policymaker sets policy according to the rule, given by equation (17), subject to the zero bound. Lift-off from the zero bound occurs in quarter 8. In each panel we also plot the results of two policy experiments in which the policymaker commits to lift-off in quarters 11 (square markers) and 12 (circle markers). After lift-off, policy reverts to the rule (17). In both cases, we assume that the policy is fully credible. From Figure 2 we observe that the immediate effects of the recessionary shock on both inflation and the output gap are smaller. As described above, this is because longer-term real interest rates are reduced by the credible promise of a subsequent overshoot in inflation.

This type of forward-guidance policy is time inconsistent. Once output and inflation have responded to the promise of a lower future policy rate, the policymaker has an incentive to renege on the announced plan and set a tighter policy to avoid the overshooting of output and inflation.

Figure 2: Responses for baseline policy and fully credible forward-guidance policies



Notes: The model starts in steady state and experiences a shock ϵ^n that pushes the natural real interest rate r^n to -8% (in annualized terms) in period 1. The solid gray lines show the responses when policy is set according to the standard policy rule, equation (17), subject to the zero bound. This implies lift-off from the zero bound in period 8. The blue lines with square markers and the red lines with circle markers show the effects of a fully credible promise to lift-off in periods 11 and 12 respectively. Policy follows the rule, equation (17), thereafter.

It is possible to illustrate this time inconsistency in the context of our scenario by comparing the benefit to the policymaker of keeping their promise to delay lift-off with the benefit from reneging on the promise (and reverting to the policy prescribed by the rule). Consistent with the literature on the objectives of monetary policy, we can assess the benefit from following a particular policy vis-$\grave{a}$-vis another by comparing the expected discounted losses associated with each policy. The policymaker's losses can be derived from a second-order approximation to the representative household's utility function (as shown by, e.g., Woodford, 2003). In the simple model we use here, this can be written as an expression involving just the output gap and inflation:

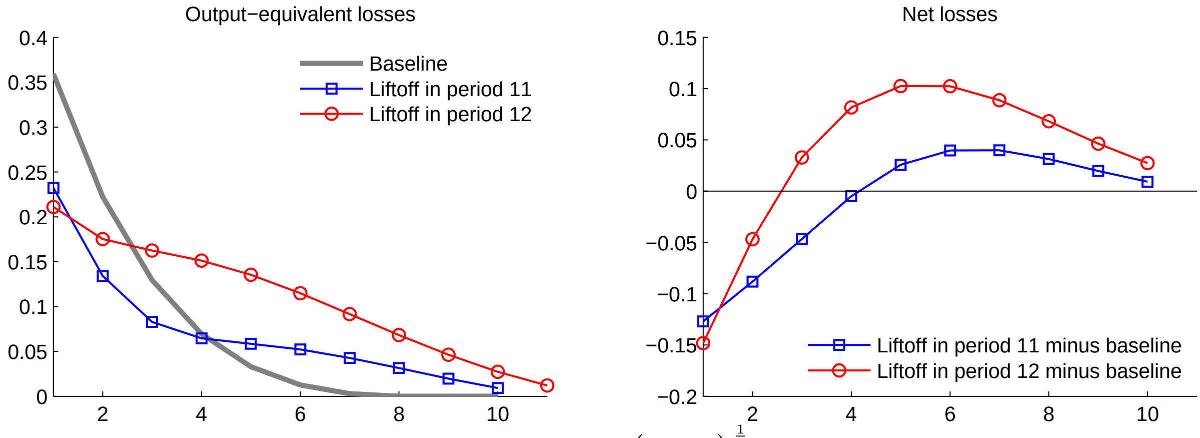
$$\mathbb{L}_t \equiv \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left[\pi_{t+i}^2 + \lambda y_{t+i}^2\right] \tag{18}$$

where $\lambda \equiv \kappa/\theta$ and $\theta > 1$ is the elasticity of demand between varieties in the Dixit-Stiglitz aggregator underlying household demand. We use a conventional value of $\theta = 10$. There is a net benefit from following through with the promise in period t when the expected losses of doing so, as re-calculated from the viewpoint of period t (so that 'bygones are bygones'), are lower than those associated with reverting to the rule.

Figure 3 shows the evolving costs and benefits of the two forward-guidance policies in terms of losses. At the time of the announcement, both forward-guidance policies reduce the size of the recession and improve welfare. As time passes, however, the benefits of the smaller recession become history, leaving only the cost of the overshoot. At this point, the policymaker would ideally like to renege on the original promise and tighten policy to bring inflation back to target. Anticipating this, it seems reasonable to suppose that the private sector might not regard the original promises as fully credible. This feature of forward-guidance promises has been noted by a number of monetary policymakers (see Nakata, 2015). For example, Carney (2012) argues that:

> Today, to achieve a better path for the economy over time, a central bank may need to commit credibly to maintaining a highly accommodative policy even after the economy and, potentially, inflation picks up. Market participants may doubt the willingness of an inflation-targeting central bank to respect this commitment if inflation goes temporarily above target. These doubts reduce the effective stimulus of the commitment and delay the recovery.

Figure 3: Output-equivalent losses from alternative policies



Notes: The left panel shows output equivalent losses, $\left(\frac{1-\beta}{\kappa}\mathbb{L}_t\right)^{\frac{1}{2}}$, with $\mathbb{L}_t$ computed using equation (18). The solid gray line shows output-equivalent losses for the baseline policy, consistent with lift-off from the zero bound in period 8. The blue line with square markers and red line with circle markers show output-equivalent losses for fully credible forward guidance to delay lift-off to periods 11 and 12 respectively. The right panel plots the net losses (measured relative to the baseline policy) for these two policies.

3.3 Forward guidance with exogenous imperfect credibility

In light of the time inconsistency of forward guidance promises, in this section we examine the effect of imperfect credibility when the degree of imperfect credibility associated with the policy—as represented by the probability that the private sector attaches to the policymaker reverting to the rule in a given period—is exogenous. This is a useful case for illustrating the basic mechanism. As we shall see, however, it may be implausible insofar as the credibility of the promise is independent of the net benefit to the policymaker from reneging on that promise.

For simplicity, we model exogenous imperfect credibility by assuming that the policymaker reverts to their usual rule each period with a constant probability. We limit our attention in this case to a promise to lift-off in period 12. In the notation of Section 2, we have $p_t = p > 0$, for $t = 1, \ldots, 12$.

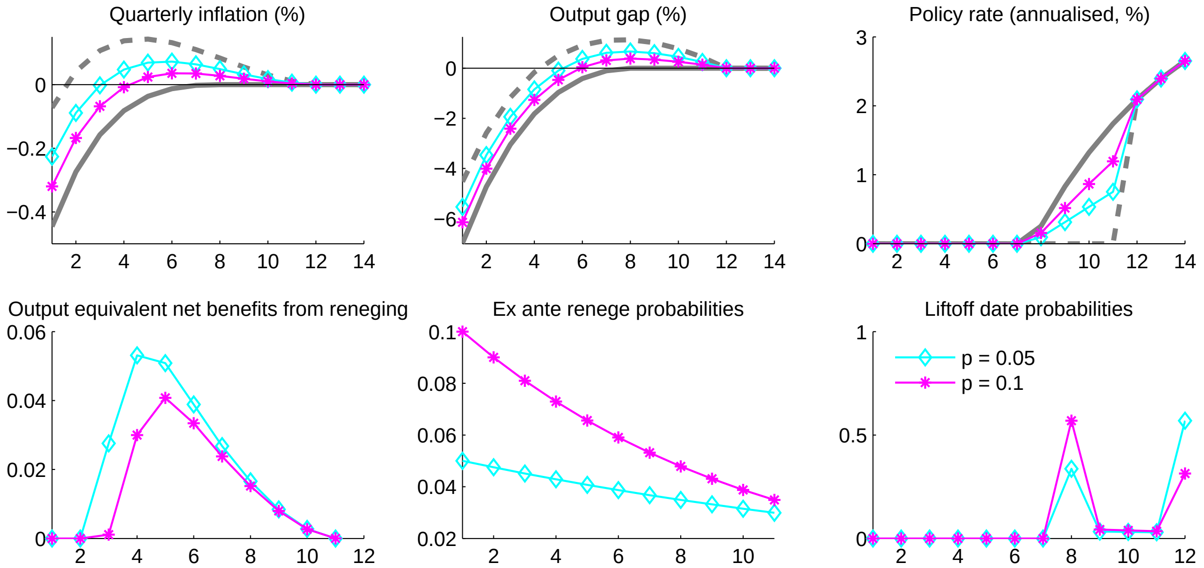
The top row of Figure 4 demonstrates that less credible forward-guidance announcements are associated with less policy stimulus. In particular, the responses of macroeconomic variables to a promise to lift-off in period 12 when credibility is low ($p = 0.1$, as shown by the magenta lines with asterisk markers) are more muted than those when credibility is higher ($p = 0.05$, the cyan lines with diamond markers).

The bottom right panel of Figure 4 shows the probabilities of lift-off from the zero bound in each period. When the policy is less credible ($p = 0.1$), the likelihood that lift-off occurs in period 12 as promised is around 30%, whereas the probability that lift-off occurs in period 8 (as in the baseline) is more than 50%.[9] When the policy is more credible, these probabilities are roughly reversed. In that case, the greater likelihood of fulfilling the forward-guidance promise to delay lift-off leads to a greater reduction in the expected path of interest rates (top right panel) and hence more stimulus.

Figure 4 also shows that the assumption of exogenous renege probabilities is somewhat implausible. The bottom left panel plots the temptation to renege from the policy

[9]The probabilities of lift-off in all periods prior to period 8 are equal to 0 despite the fact that there is a non-zero probability that the policymaker will revert to their usual rule in periods 1 to 7. This is because the policy rate is constrained at the ZLB until period 8 when policy is set according to the rule.

Figure 4: Imperfectly credible forward-guidance policies



Notes: The solid gray lines are the baseline responses to the recessionary scenario. The dashed gray lines show the effects of a fully credible policy to delay lift-off from the zero bound to period 12. The cyan line with diamond markers shows the expected effect of an imperfectly credible policy to delay lift-off to period 12, with a probability $p = 0.05$ per period that the policymaker reverts to its standard reaction function (17) each period. The magenta line with asterisk markers shows the expected effect of a policy with a reversion probability of $p = 0.1$ per period.

promise, expressed as the net benefit from reneging, $\mathcal{T}_t$. This is computed by comparing the expected loss incurred from reneging (and reverting to the usual rule, equation (17), earlier than announced), $\mathbb{L}_t^{\langle t \rangle}$, to the expected loss incurred by continuing with the plan, $\mathbb{L}_t^*$, converted into output-equivalent units following Jensen (2002):

$$\mathcal{T}_t \equiv \max\left\{0, \left(\frac{1-\beta}{\kappa}\right)^{\frac{1}{2}} \left([\mathbb{L}_t^*]^{\frac{1}{2}} - \left[\mathbb{L}_t^{\langle t \rangle}\right]^{\frac{1}{2}}\right)\right\} \tag{19}$$

The loss incurred by reverting to the usual policy rule in period t is given by:

$$\mathbb{L}_t^{\langle t \rangle} \equiv \sum_{i=0}^{H} \beta^i \left[\left(\pi_{t+i}^{\langle t \rangle}\right)^2 + \lambda \left(y_{t+i}^{\langle t \rangle}\right)^2\right]$$

for a suitably large H. The expected loss from continuing with the policy can be computed from the backward recursion:

$$\mathbb{L}_t^* = (\pi_t^*)^2 + \lambda (y_t^*)^2 + \beta \left[p_{t+1}\mathbb{L}_{t+1}^* + (1-p_{t+1})\,\mathbb{L}_{t+1}^{\langle t \rangle}\right]$$

for $t = 1, \ldots, K$, starting from $\mathbb{L}_{K+1}^* \equiv \sum_{i=0}^{H} \beta^i \left(\left(\pi_{K+1+i}^{\langle K+1 \rangle}\right)^2 + \lambda \left(y_{K+1+i}^{\langle K+1 \rangle}\right)^2\right)$, which exploits the fact that, conditional on not having already reverted to the usual policy rule, reversion happens with certainty in period $K+1$.

The bottom middle panel plots the *ex-ante* probability that the policymaker reneges in each period, t, which is given by $q_t = p\,(1-p)^{t-1}$.[10] It is decreasing in t (even though the per-period probability is constant at p), because reneging in later periods requires

[10] This reflects the constant per-period probability of reneging. More generally, $q_t \equiv p_t \prod_{j=1}^{t-1}(1-p_j)$.

the policymaker to have not previously reneged (which occurs with probability $1-p$ per period).

Taken together, the bottom left and bottom middle panels show that the *ex-ante* probability of reneging is highest in period 1 when the output-equivalent net benefit from reneging is zero. This is difficult to justify on economic grounds. Therefore, while considering the case of exogenous imperfect credibility is useful for expositional purposes, we might in general expect the credibility of a plan to be endogenously related to the temptation to renege on it.

3.4 Forward guidance with endogenous imperfect credibility

In this section we analyze the case in which the credibility of the forward-guidance announcement is endogenous. To do that, we define a mapping, F, from the temptation to renege in a given period, $\mathcal{T}_t$, to the probability that the private sector attaches to the policymaker reneging in that period, p_t.

One formalization of our approach is that in each period the policymaker draws a random cost of reneging from a distribution function, F. If the temptation to renege in that period, $\mathcal{T}_t$, exceeds the randomly drawn cost, the policymaker reneges on the policy plan. To the extent that the renege temptation varies over time, this implies that the policymaker experiences a time-varying propensity to succumb to the temptation to renege on past policy promises.[11]

For our application, we choose a Weibull distribution function for F, with parameters $\alpha_1, \alpha_2 > 0$:

$$p_t = F\left(\mathcal{T}_t\right) = 1 - \exp\left[-\left(\alpha_1^{-1}\mathcal{T}_t\right)^{\alpha_2}\right] \tag{20}$$

where the temptation to renege, $\mathcal{T}_t$ is computed using equation (19). Since the probability of observing a cost smaller than $\mathcal{T}_t$ is given by the right hand side of equation (20), this is the probability p_t that the policymaker reneges.

We use the output-equivalent losses from fully-credible policy announcements in Figure 3 to help calibrate the function F in equation (20). Because those losses are computed from simulations that assume perfect credibility, they do *not* correspond to the losses associated with a particular forward guidance promise when credibility is imperfect. However, they do provide some guidance for the calibration of the scale parameter, α_1. Specifically, we use the differences between net losses shown in the right-hand panel of Figure 3 as a very rough guide to calibrate α_1 to 0.075. The distribution function we use has the property that $F\left(\alpha_1\right) = 1 - e^{-1} \approx 0.63$, which means that it is more likely than not that a policymaker will renege on a policy plan if the benefit of doing so is 0.075. We deem this plausible as a 0.075 percent permanent output (or consumption) gain is quite large for models of this type.[12] The parameter α_2 controls the curvature of

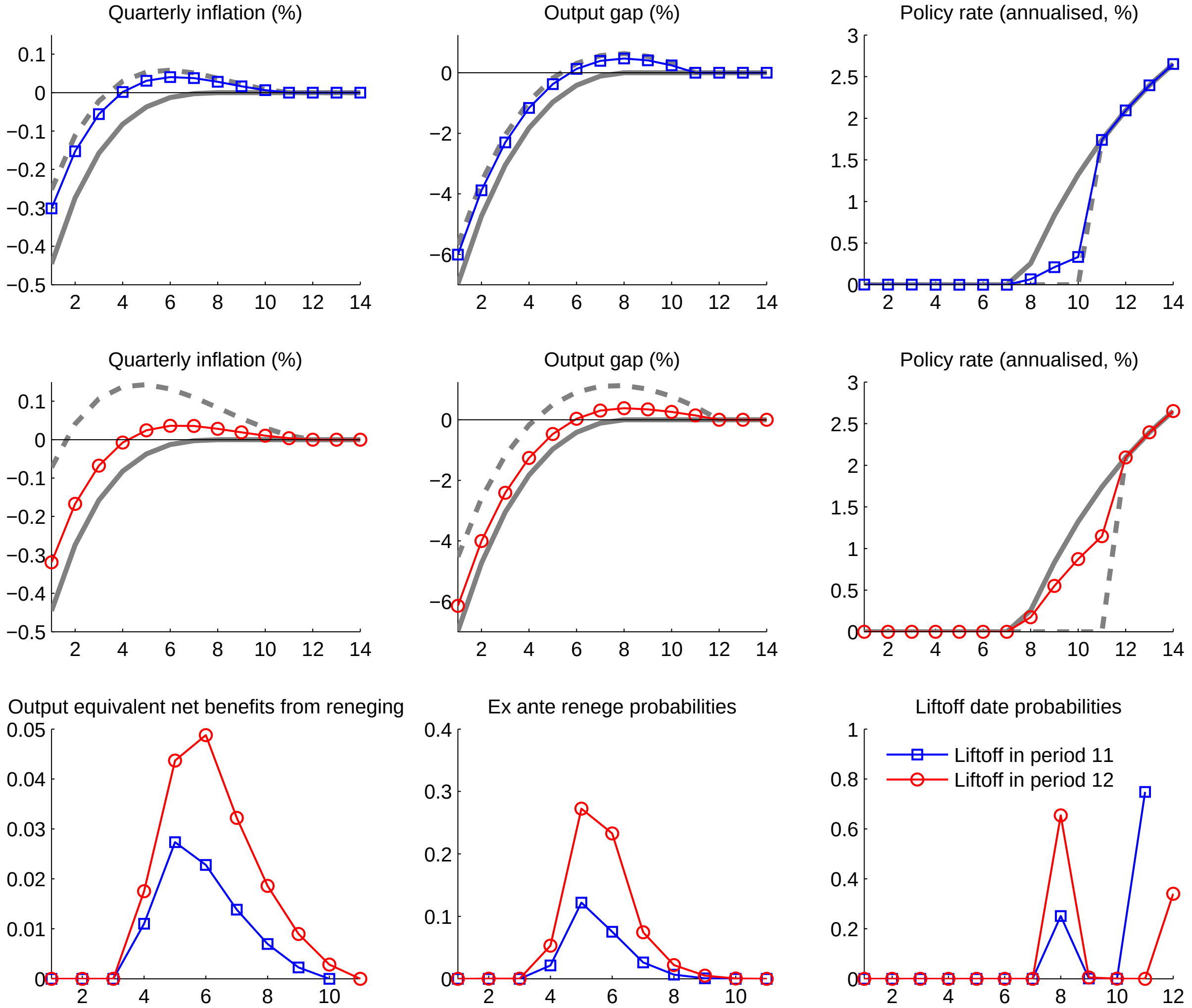
[11] An assumption that the policymaker's preferences vary over time has been employed in other models of imperfectly-credible monetary policy (see, for example, Cukierman and Meltzer, 1986) and is often motivated by the observation that policymaking-committee membership varies over time. For example, with reference to the forward guidance issued by the Bank of England's MPC in August 2013, Bean (2013) argues that: "While such a time-inconsistent [lower for longer] policy may be desirable in theory, in an individualistic committee like ours, with a regular turnover of members, it is not possible to implement a mechanism that would credibly bind future members in the manner required."

[12] Applying the approach taken by Lucas (1987), using our calibration of $\sigma = 1$ and the fact that the model is expressed in quarterly units, suggests that the welfare cost of all business cycle fluctuations is around 0.2 percent of permanent quarterly consumption. Another common experiment in the literature is to consider the welfare benefits of allowing the policymaker access to a commitment technology.

F and our baseline calibration is $\alpha_2 = 2$. We consider values of 1 and 3 in the sensitivity analysis in Appendix B.

Figure 5 shows the macroeconomic effects of the forward-guidance policies with lift-off in periods 11 and 12 with endogenous imperfect credibility under our baseline calibration of (20) ($\alpha_1 = 0.075, \alpha_2 = 2$).

Figure 5: Effects of imperfectly credible policy announcements



Notes: The solid gray lines are the baseline responses to the recessionary scenario. The top row shows policies in which lift-off is delayed until period 11: the dashed gray line is the fully-credible case; the blue line with square markers shows expected outcomes under imperfect credibility. The middle row shows policies that delay lift-off to period 12: the dashed gray line is the fully-credible case; the red line with circle markers shows expected outcomes under imperfect credibility. The imperfect credibility cases are determined by the temptation to renege as in equation (20), with $\alpha_1 = 0.075, \alpha_2 = 2$.

An imperfectly-credible plan to lift-off in period 11 has a similar expected macroeconomic effect to a fully-credible announcement of that plan, as shown in the top row. This reflects the fact that lift-off is most likely in quarter 11 (as announced), with around a

These experiments are based on a comparative statics comparison of the asymptotic properties of two economies: one in which the policymaker sets policy optimally under discretion and the other in which policy is set optimally under commitment. Using an estimated medium-scale DSGE model, Levine et al. (2008) estimate the welfare benefits of commitment to be equivalent to around 0.4 percent of permanent consumption. In this context, a 0.075 percent benefit from a short-lived policy commitment of the type we consider is relatively large.

70 percent probability attached to the policymaker implementing the announced plan in full (as seen in the bottom right panel).[13] As a result, the expected path for the policy rate is relatively close to the announced path.

By contrast, a plan to lift-off in period 12 that is imperfectly credible has a much more muted impact compared to its fully-credible counterpart (middle row). If fully credible, this policy further reduces the initial size of the recession at the cost of a larger overshoot further out, thereby creating a greater temptation to renege (bottom left panel). The greater temptation to renege results in a lower likelihood of the policymaker implementing the plan (bottom middle and right panels). Indeed, relative to the period-11 lift-off plan, the probabilities of implementing the plan in full versus lifting-off early are roughly reversed: there is less than a 30 percent chance that the policymaker implements the plan in full. Consistent with this, the expected path of the policy rate lies a long way from the full-credibility case (middle right panel) and so much less stimulus is imparted relative to that case.

These results depend on the calibration of the function that links the probability that the policymaker will renege to the benefits of doing so. How plausible is our baseline calibration? We informally assess this in two ways, neither of which map precisely into our framework, but which can, nevertheless, provide an indication of the plausibility of our calibration. First, we compare the fall in the expected path for the policy rate to the announcement effect of the FOMC's August 2011 date-based forward guidance reported by Raskin (2013). Their August 2011 meeting statement indicated that the FOMC expected "low levels for the federal funds rate at least through mid-2013." In the context of our experiment, we can interpret this as a commitment to hold rates at zero beyond the baseline lift-off date (period 8). Raskin (2013) reports that the expected federal funds rate two years ahead fell by around 0.25 percentage points on the day that the statement was released. The two experiments shown in Figure 5 generate falls in the eight-quarter-ahead expected policy rate of 0.08 and 0.19 percentage points. This suggests that actual forward-guidance announcements may have been more credible than implied by our baseline calibration.

A second way to assess our calibration is by comparison with applications of the 'loose commitment' framework. In this framework, there is a constant probability that the policymaker ignores past promises and implements a newly optimized policy plan (taking into account the constant probability of re-optimizing in each future period). This modeling approach has been used by Bodenstein et al. (2012) and Debortoli and Lakdawala (2014) to estimate the commitment levels of central banks. Bodenstein et al. (2012) study forward guidance in the US and Sweden and estimate the per-period re-optimization probability to be around 0.4 for the FOMC and 0.55 for the Riksbank. These estimates imply that the probabilities of a policymaker delivering 10 and 11 period plans without re-optimizing are less than 1 percent, significantly below the equivalent probabilities in our experiments. Debortoli and Lakdawala (2014) estimate the FOMC's degree of commitment using a medium-scale DSGE model. Their posterior mode suggests a somewhat smaller re-optimization probability of 0.19. Nevertheless, this implies the probability of delivering a 10 or 11 period plan without re-optimization is around 10 percent, again markedly lower than in our baseline calibration. Despite the differences in approach, these studies both tend to suggest that our baseline calibration may embody

[13] The majority of the remaining 30 percent of the lift-off probability is attached to period 8. This reflects the facts that: a) there is a sizable net benefit from reneging around periods 5 to 8; b) the policy rate is constrained at the ZLB until period 8 when policy is set according to the rule.

too much credibility.

Given that our baseline calibration could be regarded as embodying either too much or too little credibility, we consider two alternative calibrations in Appendix B. Even in the higher credibility variant, the feedback from the renege temptation to the probability of reneging is sufficiently strong to reduce the effectiveness of the period-12 liftoff plan substantially relative to the full-credibility case.

These results have an important practical implication for policymaking. In cases where the endogenous link from the time inconsistency of a forward guidance promise to the credibility of that promise is weak (or non-existent as in the special case where credibility is exogenously determined), the policy implication is the same as in Bodenstein et al. (2012): a low-credibility policymaker needs to promise to hold rates at the lower bound for longer than a high-credibility policymaker in order to deliver the 'optimal' amount of stimulus (conditional on the credibility level). We have shown that this result can be overturned in our framework. If the feedback from renege temptation to credibility is sufficiently strong, then promises to hold rates lower for even longer can be ineffective. The implication is that, in the absence of a commitment device, an inflation-targeting central bank is unlikely to be able to mitigate the deflationary consequences of the policy rate being constrained at the lower bound using forward guidance alone.[14] This conclusion is consistent with the behavior of central banks since the financial crisis, who have tended to use large scale asset purchases alongside forward guidance to meet their macroeconomic objectives, rather than making time-inconsistent forward-guidance promises.[15]

3.5 Imperfect credibility and the forward-guidance puzzle

It has been well documented that forward guidance is very powerful in standard New Keynesian models. Indeed, the effect is so large relative to the available empirical evidence that Del Negro et al. (2015) have coined the term "forward-guidance puzzle" to describe it. Our preceding results demonstrate that the macroeconomic effects of forward-guidance announcements can be substantially reduced when they are perceived as being imperfectly credible by the private sector. How should we interpret the forward-guidance puzzle in light of that finding?

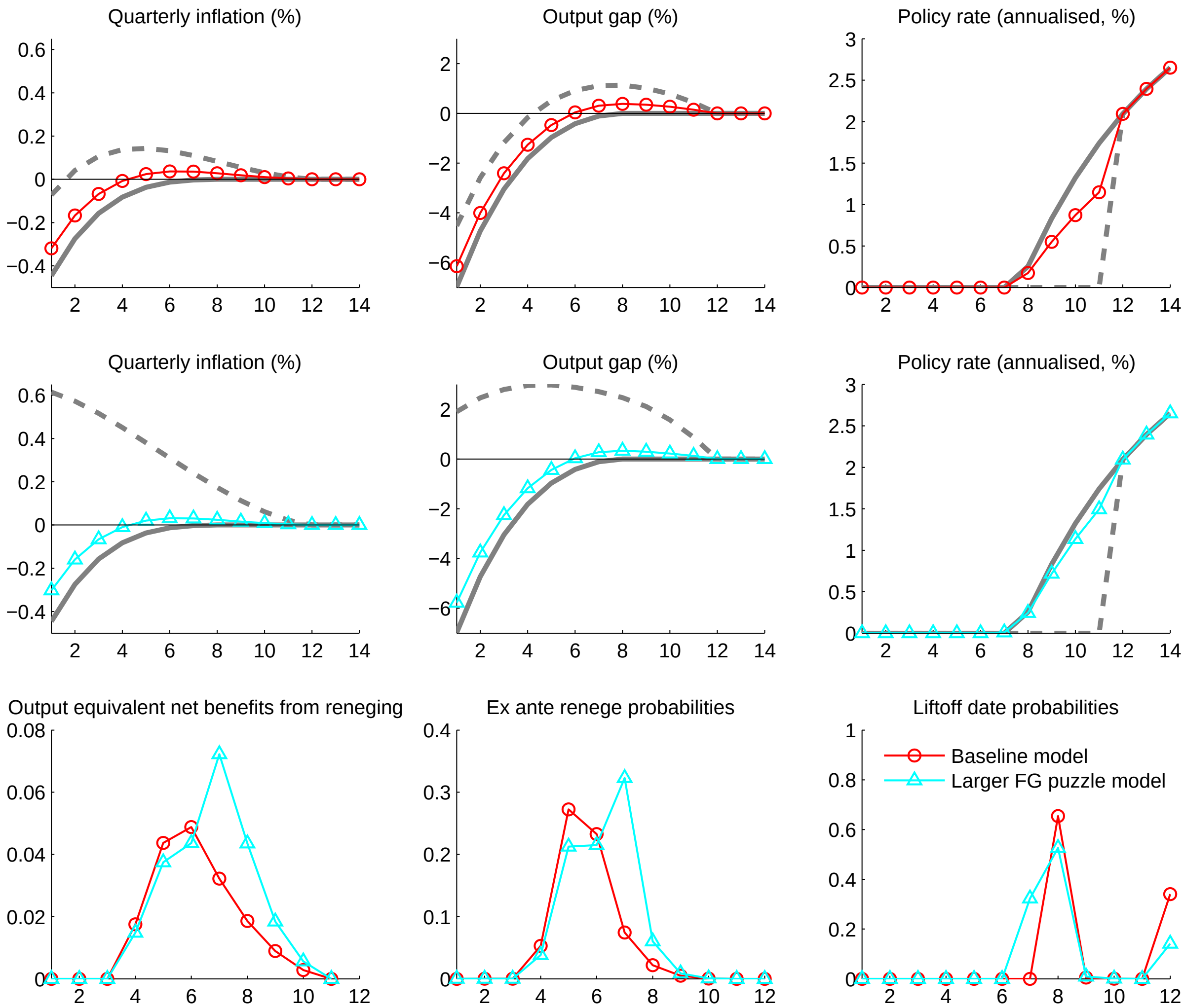
To illustrate the relationship between imperfect credibility and the forward-guidance puzzle, we re-examine the endogenous imperfect credibility experiments in Section 3.4 using a version of the model in which the interest elasticity of demand is twice as large ($\sigma = 2$, rather than our baseline assumption of $\sigma = 1$). Forward guidance has the potential to be more powerful in this alternative model because output is more responsive to changes in real interest rate expectations (which can be seen by iterating the IS curve (15) forward). To investigate how imperfect credibility affects this result, we first reconstruct the recessionary scenario described in Section 3.1 by finding a path for the natural rate (r^*) that delivers the same output gap, inflation and policy-rate paths as in the base-

[14] Eggertsson and Woodford (2003) show that optimal commitment policy at the ZLB can be approximated by the adoption of a price-level target. It follows that the most straightforward way to commit a central bank to a lower-for-longer policy would be to amend their remit accordingly.

[15] Engen et al. (2015) argue that both LSAPs and forward guidance contributed to a perception on the part of the private sector that the Federal Reserve's implicit policy rule gradually become more accommodative following the financial crisis. Note that this 'reaction function clarification' transmission channel for forward guidance is distinct from a 'lower-for-longer' forward guidance policy of the type analyzed in this paper in which the central bank promises to hold the policy rate below what their usual (and fixed) reaction function would imply.

line experiment.[16] We then compare the responses in the two alternative versions of the model to an announcement that the policymaker intends to hold the policy rate at the zero bound for an additional 4 periods (liftoff in period 12). Figure 6 shows the results.

Figure 6: Effects of imperfectly credible policy announcements: model comparison



Notes: The solid gray lines are the baseline responses to the recessionary scenarios. The top row shows policies in which lift-off is delayed until period 12 in the baseline model ($\sigma = 1$): the dashed gray line is the fully-credible case; the red line with circle markers shows expected outcomes under imperfect credibility. The middle row shows the effects of the same policy announcement in a version of the model that exhibits a larger forward guidance puzzle ($\sigma = 2$): the dashed gray line is the fully-credible case; the cyan line with triangle markers shows expected outcomes under imperfect credibility. The imperfect credibility cases are determined by the temptation to renege as in equation (20), with $\alpha_1 = 0.075, \alpha_2 = 2$.

Under perfect credibility (dashed gray lines), the forward-guidance policy has a much larger effect in the variant of the model with the higher interest elasticity of demand. Under endogenous imperfect credibility, however, the forward-guidance announcement has a very similar effect in both variants of the model. In both cases, the presence of

[16]To do this, we compute the sequence of anticipated natural rate shocks that delivers the same output gap path for the 7 periods over which the policy rate is constrained by the ZLB as in the baseline model scenario, while imposing the zero bound for the policy rate for those 7 periods. We also set the natural rate in period 8 to the same value as in the baseline scenario (which ensures that the natural rate in period 8 and thereafter is identical to the baseline scenario in which it decays in line with the auto-regressive process (16)).

imperfect credibility substantially reduces the amount of stimulus that is imparted by the policy. However, the reduction in stimulus is proportionately much larger when $\sigma = 2$ because the policymaker has a much stronger incentive to renege on the announcement in this case. As a result, there is a higher probability that rates will lift-off earlier and so the expected path of the policy rate moves by less in response to the announcement than in the baseline variant of the model.

This demonstrates that forward-guidance announcements should not necessarily be expected to have large macroeconomic effects even if demand is very sensitive to changes in interest-rate expectations. In those circumstances, forward guidance could be perceived as less credible precisely because the interest-rate expectations channel is more powerful. In that sense, the forward-guidance puzzle is an artifact of the perfect-credibility assumption, as well as of the properties of the New Keynesian model.[17]

This analysis also has implications for what we might expect to observe empirically in response to forward-guidance announcements by central banks, as studied in, for example, Campbell et al. (2012) and Del Negro et al. (2015). As these studies make clear, identifying the macroeconomic effect of any attempt by a central bank to use forward guidance to stimulate the economy is complicated by the fact that the policy and accompanying communications may reveal additional information to the private sector (e.g. the central bank's forecasts for the economy). That information can affect private-sector expectations independently of any intention by the central bank to stimulate the economy. The possibility that any forward-guidance announcement may not be perfectly credible further complicates empirical identification, particularly given that the credibility of any announcement, and hence its effectiveness, is likely to vary depending on the prevailing circumstances (including the extent to which the central bank is attempting to stimulate the economy).

4 'Threshold-based' forward guidance

'Threshold-based' forward-guidance announcements can generate policy uncertainty if they do not specify a complete strategy for the conditions under which the policy rate will exit from the lower bound. For example, in December 2012 the FOMC issued guidance that linked lift-off from the zero bound to a threshold on the unemployment rate of 6.5 percent and the prospects for inflation. The guidance that the FOMC issued was set out in its December 12 statement:

> [T]he Committee [...] currently anticipates that this exceptionally low range for the federal funds rate will be appropriate at least as long as the unemployment rate remains above $6\frac{1}{2}$ percent, inflation between one and two years ahead is projected to be no more than a half percentage point above the Committee's 2 percent longer-run goal, and longer-term inflation expectations continue to be well anchored.

While this statement makes it clear that the federal funds rate will remain low *at least until* the unemployment rate reaches the threshold (alongside the criteria on inflation and

[17] In a narrower sense, the forward guidance puzzle is still present in Figure 6. Under imperfect credibility, the responses of activity and inflation are similar in the two model variants, but those responses are consistent with a much smaller change in policy rate expectations when $\sigma = 2$. The puzzle in terms of the properties of the New Keynesian model could, therefore, more accurately be defined in terms of the macroeconomic responses to changes in private sector interest rate expectations induced by forward guidance announcements, rather than the announcements themselves.

inflation expectations), it does not spell out the conditions that would lead to rates *rising*. Therefore, even after accounting for uncertainty about the economic outlook, we would expect there to be uncertainty around the precise lift-off date attributable to uncertainty about the FOMC's intentions. This view would seem to be corroborated by Charles I. Plosser, the President of the Federal Reserve Bank of Philadelphia:[18]

> The FOMC has not been clear about the purpose of its forward guidance. Is it purely a transparency device, or is it a way to commit to a more accommodative future policy stance to add more accommodation today? This lack of clarity makes it difficult to communicate the stance of policy and the conditionality of policy on the state of the economy.

There is evidence of uncertainty around the FOMC's exit strategy in the Federal Reserve Bank of New York's Primary Dealer Survey (PDS) of financial-market participants. In the January 2013 survey, the first conducted after the threshold-based guidance was issued, respondents were asked for their *modal* estimates of the joint outcomes for the unemployment rate and the headline 12-month PCE inflation rate at the date of the first increase in the federal funds rate. For the unemployment rate, the median response was 6.5 percent, indicating that a large proportion of respondents viewed the guidance as a 'trigger'—that is, breaching the unemployment threshold would lead to an automatic and immediate increase in the policy rate. However, the 25th percentile response was 6.25 percent, suggesting that 25 percent of respondents thought it *most likely* that lift-off would occur with the unemployment rate 0.25 percentage points or more below the announced threshold. Conditional on the economic outlook, therefore, there was some remaining lift-off uncertainty associated with uncertainty about policy behavior.[19]

Typically, however, we would expect more precise communication by central banks about their intentions to improve the management of private-sector expectations and, hence, the effectiveness of monetary policy in achieving its stabilization objectives (see Woodford, 2005, for a discussion of the importance of communication for monetary policy). We can use our framework to assess the effect of alternative degrees of precision in the communication of threshold-based forward guidance. To do so, we consider the following counterfactual policy experiment. Suppose that in December 2012 the FOMC had wanted to increase stimulus relative to what was actually announced. Suppose also, that it wanted to issue a precise form of guidance, more clearly articulating the exit strategy. How would these concerns have affected the choice of its threshold for unemployment?

To conduct this experiment, we use a variant of the FRB/US model, developed by economists at the Federal Reserve Board.[20] Our motivation for using this model is that it was used for simulations supporting the FOMC's communications during the period in which forward guidance was in operation (see, for example, Yellen, 2012). In addition, the scale of the model demonstrates that our technique can be readily applied to very large models: our version of FRB/US has over 350 endogenous variables and

[18] See Plosser (2012).

[19] Note that the survey arguably understates the amount of uncertainty about the FOMC's policy intentions. Individual respondents were asked to report their modal forecast for unemployment at the lift-off date. The survey reveals some disagreement about the most likely lift-off conditions. Separately from that disagreement, individual respondents may have been uncertain about the lift-off conditions. Given the 'one-sided' nature of the announcement and that the majority of respondents viewed the threshold as being like a trigger, taking into account individuals' uncertainty (which was not recorded in the survey) would very likely give rise to a fatter tail in the distribution of lift-off probabilities.

[20] See Brayton and Tinsley (1996) for a description of the first version of the model.

more than 60 shocks. The version of FRB/US we use is the linearized model in the Macroeconomic Model Data Base (MMB) developed by Wieland et al. (2012). We use the rational-expectations variant of the model to allow for the expectational effects of forward-guidance policies. Further details on the model specification are provided in Appendix C.1.

We model threshold-based forward guidance in our framework by assuming that the probability of lift-off is endogenous and linked to the threshold variable—in this case, the unemployment rate, u_t. We implement this using the approach outlined in Section 2.5.[21] The mapping between the probability of lift-off and the unemployment rate is given by:

$$p_t = f\left(u_t - \bar{u}\right) = \begin{cases} 0 & \text{if} \quad u_t \geq \bar{u} \\ 1 - \exp\left(\alpha_1^{-1}\left(u_t - \bar{u}\right)\right) & \text{if} \quad u_t < \bar{u} \end{cases} \tag{21}$$

where $\alpha_1 > 0$ and $\bar{u}$ is the unemployment threshold. Equation (21) is a version of the Weibull distribution used in Section 3.4, with $\alpha_2 = 1$.[22]

Using a function like (21) allows there to be a positive probability that lift-off does *not* occur after the unemployment rate has fallen below the threshold value $\bar{u}$. The functional form provides flexibility over the degree of precision in the simulated forward-guidance announcement via the parameter α_1, which determines the extent to which the lift-off probability responds to threshold breaches. In the context of our counterfactual experiment, we consider two parameterizations for α_1. First, we parameterize α_1 to match the degree of precision in the FOMC's actual December 2012 announcement. To do so, we use information from the question in the January 2013 PDS on respondents' expectations for the unemployment rate at the time of the first increase in the federal funds rate. As described in Appendix C.4, this information is consistent with a calibration of $\alpha_1 \approx 0.36$. which implies that breaching the unemployment threshold by 0.25 percentage points would be associated with a lift-off probability of 50 percent. Second, we parameterize α_1 so that a breach of the threshold triggers lift-off from the zero bound with (near) certainty. Specifically we set $\alpha_1 \approx 0.0014$, consistent with a 99.9 percent probability of lift-off for a breach of the threshold by 0.01 percentage points.[23]

In constructing our counterfactual policy simulation, the first step is to build a baseline projection that is consistent with the FOMC's December 2012 projections and their announced guidance. We incorporate the latter by ensuring that the forecast path for the federal funds rate lifts-off after the unemployment projection has fallen below the 6.5 percent threshold. We view the baseline projection as a set of expectations for macroeconomic variables and monetary policy that are consistent with the actual guidance issued by the FOMC. Since the guidance announcement did not lead to noticeable changes in market-based measures of interest-rate expectations, we also interpret these projections as consistent with those of private agents, given the FOMC's actual announcement. See Appendix C.2 for evidence on the market reaction and Appendix C.3 for further details on how we build the baseline projection.

[21] This is a perfect foresight algorithm and so does not take into account that expectations should be affected by the possibility that exit may occur in any future period even if that is not the modal outcome. See Boneva et al. (2017) for a discussion of the equilibrium concept of threshold-based forward guidance in a fully stochastic setting.

[22] The sign of the argument of the exponential function is reversed because non-zero probabilities of lift-off are created by unemployment falling *below* the threshold value.

[23] Note that there is no guarantee that an equilibrium exists when the threshold is treated as a (near) trigger (though it happens to in this case). See Appendix A of Boneva et al. (2017) for a discussion.

Figure 7 plots the two counterfactual policies that we consider: the empirically-based calibration of α_1 (green dash-dotted lines), and the calibration that is consistent with a trigger-like policy (red dashed lines). We refer to the former as "open-ended" insofar as it is consistent with the *at least until* language used by the FOMC to describe its policy for holding the federal funds rate low in relation to a breach of the unemployment threshold.[24] The figure shows that for there to be broadly similar outcomes for macroeconomic variables given the different degrees of certainty in communications, the unemployment threshold for the trigger-like policy must be set at a lower level. Indeed, under the trigger-like policy, we set the threshold for unemployment at 5.75 percent, rather than 6 percent as in the open-ended case. Appendix C.5 confirms that this interpretation of these results is correct. It compares the effects of the two alternative threshold polices (5.75 percent and 6 percent) with both degrees of precision in the communication of the exit strategy ("trigger" and "open-ended"). It shows that announcing a 5.75 percent unemployment threshold delivers more stimulus if it is open-ended and that a 6 percent threshold delivers less stimulus if it is trigger-like compared to the two alternative policies described above. These results indicate that, when designing a threshold-based guidance policy, increasing the precision in communications to steer expectations more precisely should go alongside a *lower* unemployment threshold to achieve a comparable macroeconomic impact.

These findings reflect the fact that the overall effects of threshold-based guidance policies in this setting depend on the balance of two opposing forces. On the one hand, the lower threshold for the trigger-like policy, would, other things equal, lead to lower expected real rates and greater stimulus compared to the higher threshold policy. On the other hand, the greater certainty under the trigger-like policy that interest rates will rise after the threshold has been breached tends to increase longer-term interest rate expectations and reduce stimulus compared to the more open-ended policy.

These effects can be seen in the bottom right panel in Figure 7. In the case of the trigger-like guidance, the probability of lift-off in the modal lift-off quarter, 2015Q4, is around 60 percent. For the open-ended guidance, there a probability of lift-off of around 30 percent in the modal lift-off quarter (also 2015Q4). However, in this case, since the description of the exit strategy leaves more uncertainty, the private sector attaches a greater probability to the FOMC holding the federal funds rate at its existing low level for several quarters after the threshold has been breached. The effect of this is to lower the expected path of the federal funds rate, despite the higher unemployment threshold.

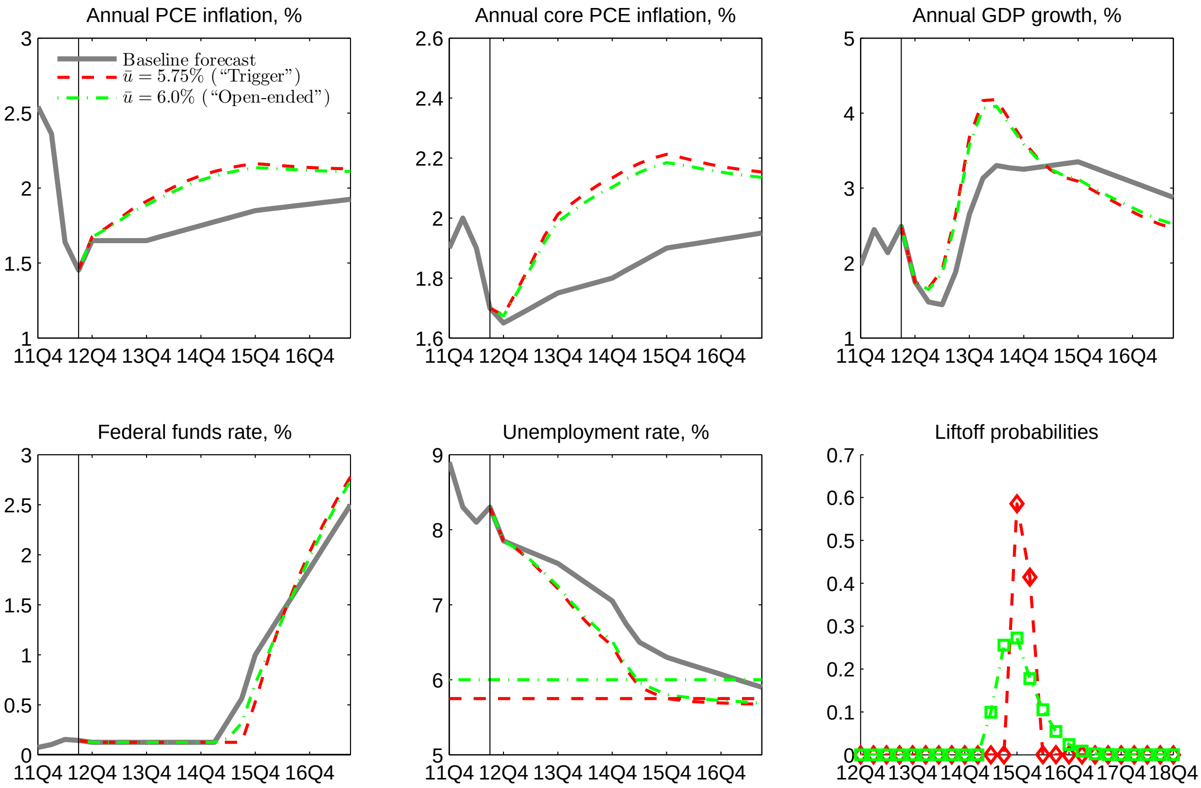
Both of the counterfactual policies that we consider deliver better stabilization outcomes than in the baseline forecast. We can assess stabilization performance with reference to a loss function that represents the FOMC's dual mandate for price stability and full employment. In particular, we use a quadratic loss function over the deviation of annual PCE inflation from a target of 2 percent, the unemployment rate from its long-run level (5.4 percent in our version of FRB/US), and the change in the federal funds rate (to reflect a concern for adjusting the instrument smoothly).[25] The improved stabilization performance under the trigger-like policy would have been worth around 0.25 percent of steady-state output relative to the baseline forecast, while the open-ended guidance policy would have improved stabilization by around 0.22 percent of steady-state output.[26]

[24]As noted above, the policy was interpreted in a reasonably 'trigger-like' way. The "at least until" language could have resulted in substantially more uncertainty than recorded in the PDS survey (notwithstanding the possibility that the mapping from disagreement to uncertainty is very weak).

[25]This specification is used by Yellen (2012).

[26]To calculate losses in terms of output equivalences, we follow the approach of Jensen (2002), as used in Section 3. To do so, we translate the unemployment gap into an output gap using the Okun's law

Figure 7: Threshold-based forward guidance using the FRB/US model



Notes: The solid gray lines show the baseline forecast derived from the FOMC's December 2012 Summary of Economic Projections. For inflation rates, growth and unemployment these forecasts are interpolated through the the midpoint of the central tendency end-year forecasts. The baseline projection for the federal funds rate is constructed using the median of the FOMC's end-year projection. The red dashed lines show the expected effects of a policy in which the FOMC has announced that the federal funds rate will rise from the zero lower bound with near certainty when unemployment falls below 5.75 percent ("trigger"). The green dash-dotted lines show the expected effects of a policy in which the federal funds rate will remain at the zero lower bound at least until the unemployment rate is below 6 percent ("open-ended"). The probability of lift-off is linked to the unemployment rate by equation (21) and the lift-off probabilities are shown in the bottom right panel.

5 Conclusion

We demonstrate that the macroeconomic effects of 'uncertain forward guidance' announcements can differ substantially from the effects of announcements in which the future path of the policy rate is known with certainty, as is typically assumed in the literature.

Our results have implications for policy design. They suggest that the calibration of threshold values and communication about the interpretation of the threshold conditions should be considered jointly in the design of threshold-based forward-guidance policies. And they highlight the importance of assessing the extent to which forward guidance policies aimed at imparting stimulus may be viewed as imperfectly credible in gauging their likely macroeconomic effects.

The possibility that forward-guidance announcements may be imperfectly credible also has implications for our understanding of the 'forward-guidance puzzle'. Forward guidance may be no more powerful in economies that are more sensitive to future expected

relationship reported for the US by Yellen (2012), $Y_t - \bar{Y} = 2.3\,(u_t - \bar{u})$.

real interest rates because the credibility of announcements in those economies is likely to be lower.

Finally, our results suggest that empirical investigations of the effects of forward guidance should attempt to control for the effects of uncertainty about future policy behavior on the expected path for the policy rate.

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A Derivation of the algorithm

This appendix details our general, perfect-foresight algorithm for imposing a path for the policy instrument under imperfect credibility in linear rational-expectations models.

A.1 Statement of the problem

The environment we consider is one in which the policymaker announces that they intend to set the policy rate to follow an arbitrary path for a finite number of periods (after which policy will be set according to the policy rule in the model). In a perfect foresight, perfect credibility setting, computing the equilibrium of the economy given known initial conditions (and a terminal condition pinned down by the policy rule in the model) is straightforward. In particular, the forward-looking linear class of models for which our algorithm applies can be written as follows:[27]

$$H^F \mathbb{E}_t x_{t+1} + H^C x_t + H^B x_{t-1} = \Psi z_t \tag{A.1}$$

where x is a vector of endogenous variables (both predetermined and non-predetermined), z is a vector of structural, orthogonal disturbances or shocks and the matrices H^F, H^C, H^B and Ψ are functions of the model's parameters. If it exists, the rational expectations (RE) solution of the model can be written as:

$$x_t = Bx_{t-1} + \sum_{i=0}^{\infty} F^i \Phi z_{t+i} \tag{A.2}$$

Given this RE solution, it is straightforward to implement an anticipated path for the policy instrument under perfect credibility using standard inversion techniques as, for example, described in Appendix C of Burgess et al. (2013) or using the algorithm outlined in Laséen and Svensson (2011). In that setting, a sequence of anticipated disturbances to the monetary policy rule (one row of the sequence of vectors $\{z_t, \ldots, z_{t+K}\}$) can be chosen to ensure that the policy instrument follows the desired trajectory for periods t to $t+K$.

The problem our algorithm solves is how to compute the equilibrium of the economy when the private sector attributes a non-zero probability to the policymaker deviating from the announced policy plan and reverting to set interest rates according to the policy rule earlier than had been announced.[28] More specifically, the imperfect credibility environment we consider is one in which the central bank makes an announcement at the end of period 0 that the interest rate will follow an arbitrary path $\{b_t\}_{t=1}^{K}$, for a finite number of periods. We suppose that the private sector attributes a probability, $\{p_t\}_{t=1}^{K}$, to the policymaker reverting back to the policy rule earlier than had been announced (with $p_{K+1} = 1$ as in the standard perfect credibility case). So, the policy regime follows a two-state stochastic process in which reversion to the rule is an absorbing state. To make the probability that the private sector attributes to reversion to the rule in period 1 relevant, we alter the standard timing assumption by assuming that in each period the

[27] Note that higher-order leads and lags can be nested in this parsimonious description using lead and lag identities.

[28] Reversion to the policy rule in the event of exiting the plan early is a logical extension of the assumption in the perfect credibility setting that policy reverts back to the rule after the plan has been implemented.

private sector makes their decisions *before* the outcome for the policy rate is revealed (see Figure 1 of Section 2).

In this environment, it is no longer possible to use the rational-expectations solution to compute the equilibrium paths of the model economy because expectations are a non-linear function of the state. The rest of this appendix details the derivation of an algorithm to solve for the equilibrium paths under perfect foresight.[29]

In what follows, we denote x_t^* as the vector of endogenous variables in any period $t = 1...K$ conditional on the policymaker having delivered the announced policy path and $x_t^{\langle i \rangle}$ as the vector of endogenous variables in any period $t = 1, \ldots, \infty$ conditional on the policymaker having already reverted to the policy rule in period $i = 1, \ldots, K+1$ (which is valid for all $t \geq i$). Note that once the policymaker has reverted to the policy rule, it is straightforward to compute any path using the rational expectations solution to the model:[30]

$$x_t^{\langle i \rangle} = Bx_{t-1}^{\langle i \rangle} + \sum_{j=0}^{\infty} F^j \Phi z_{t+j}^{\langle i \rangle} \tag{A.3}$$

Solving for the equilibrium therefore requires us to solve for the set of paths, $\{x_t^*\}_{t=1}^{K}$, and the outcomes on reversion to the rule, $\{x_i^{\langle i \rangle}\}_{i=1}^{K+1}$.

A.2 Derivation of the equilibrium paths

Without loss of generality, we proceed by partitioning the vector of endogenous variables, x, and shocks, z, as follows, where r denotes the policy rate variable, ε^r the policy shock, $\widetilde{x}$ the vector of endogenous variables excluding the policy rate and $\widetilde{z}$ the vector of shocks excluding the policy shock:[31]

$$x_t \equiv \begin{bmatrix} \widetilde{x}_t \\ r_t \end{bmatrix} \tag{A.4}$$

$$z_t \equiv \begin{bmatrix} \widetilde{z}_t \\ \varepsilon_t^r \end{bmatrix} \tag{A.5}$$

This partitioning implies conformable partitioning of the model solution matrices, so that we can write the rational-expectations solution in cases where the policymaker has already reverted to the policy rule (equation (A.3)) as:

$$\begin{aligned} \begin{bmatrix} \widetilde{x}_t^{\langle i \rangle} \\ r_t^{\langle i \rangle} \end{bmatrix} &= \begin{bmatrix} B_{\tilde{x}\tilde{x}} & B_{\tilde{x}r} \\ B_{r\tilde{x}} & B_{rr} \end{bmatrix} \begin{bmatrix} \widetilde{x}_{t-1}^{\langle i \rangle} \\ r_{t-1}^{\langle i \rangle} \end{bmatrix} \\ &+ \begin{bmatrix} S_{\tilde{x}} \sum_{j=0}^{\infty} F^j \Phi_{\tilde{z}} \widetilde{z}_{t+j} \\ S_r \sum_{j=0}^{\infty} F^j \Phi_{\tilde{z}} \widetilde{z}_{t+j} \end{bmatrix} + \begin{bmatrix} S_{\tilde{x}} \sum_{j=0}^{\infty} F^j \Phi_{\varepsilon^r} \varepsilon^{r}{}_{t+j}^{\langle i \rangle} \\ S_r \sum_{j=0}^{\infty} F^j \Phi_{\varepsilon^r} \varepsilon^{r}{}_{t+j}^{\langle i \rangle} \end{bmatrix} \end{aligned} \tag{A.6}$$

where: $B_{\tilde{x}\tilde{x}}$ is the upper left block of the reordered B matrix containing loadings on the lagged set of endogenous variables that excludes the interest rate for that same set of

[29]The non-linearity inherent in the problem implies that global solution methods would be needed to extend our algorithm to a stochastic setting. In the application of it to medium-scale DSGE models, this would present a massive computational challenge.

[30]The superscript $\langle i \rangle$ applies to the vector of shocks in the event that reversion to the rule would violate the zero or effective lower bound. This is discussed further below.

[31]This partitioning is primarily for exposition. A trivial re-ordering of variables can be undertaken to deliver the desired partitioning, though even that is not strictly necessary since it is straightforward to extract the relevant sub-matrices using the relevant indices.

variables. $B_{\tilde{x}r}$, $B_{r\tilde{x}}$ and B_{rr} are defined analogously. $S_{\tilde{x}}$ is an $(n_x-1)\times n_x$ selector matrix extracting the rows pertaining to $\widetilde{x}$ and S_r is a $1\times n_x$ matrix extracting the row relevant for the nominal interest rate. $\Phi_{\tilde{z}}$ is constructed from Φ by removing the column loading on the monetary policy shock and Φ_{ε^r} is constructed from Φ by removing all columns other than that loading on the monetary policy shock.

In this perfect foresight setting, the non-policy shocks, $\{\widetilde{z}_t\}_{t=1}^{\infty}$, are known by agents at the start of period $t=1$. The policy shocks, $\{\{\varepsilon^{r}{}_t^{\langle i\rangle}\}_{t=i}^{\infty}\}_{i=1}^{K+1}$, are included in the event that reversion to the policy rule violates the effective lower bound such that agents' perfect-foresight expectations of the interest rate fall below the effective lower bound in any period following reversion to the rule.[32] In such cases, the shocks $\{\{\varepsilon^{r}{}_t^{\langle i\rangle}\}_{t=i}^{\infty}\}_{i=1}^{K+1}$ are applied to ensure that the expected path for the policy rate respects the zero bound as described in Section A.3.

As discussed above, the objective is to compute the equilibrium in each period conditional on the policymaker not already having reverted to the rule, $\{x_t^*\}_{t=1}^{K}$, and the equilibrium in the period in which reversion does take place, $\{x_i^{\langle i\rangle}\}_{i=1}^{K+1}$. In order to compute $\{x_t^*\}_{t=1}^{K}$, the path for the endogenous variables conditional on the announced policy path being followed, we construct a "stacked-time" solution from the structural model equations with the equation governing the interest rate – the monetary policy rule – removed. As noted, it is more convenient to work with the partitioning in (A.4) and (A.5). To do so, we rearrange the structural model equations (A.1) to express them in terms of equations for the non-policy variables, $\widetilde{x}$, and non-policy shocks, $\widetilde{z}$, with the interest rate taken as exogenous for the duration of the announced policy plan. Specifically, we have:[33]

$$\widetilde{H}_{\tilde{x}}^{F}\mathbb{E}_t^*\widetilde{x}_{t+1}+\widetilde{H}_{\tilde{x}}^{C}\widetilde{x}_t^*+\widetilde{H}_{\tilde{x}}^{B}\widetilde{x}_{t-1}^*+\widetilde{H}_{r}^{F}\mathbb{E}_t^*r_{t+1}+\widetilde{H}_{r}^{C}\mathbb{E}_t^*r_t+\widetilde{H}_{r}^{B}r_{t-1}=\widetilde{\Psi}_{\tilde{z}}\widetilde{z}_t \tag{A.7}$$

where $\widetilde{x}_0^*=\widetilde{x}_0$. The matrices are constructed as follows: $\widetilde{H}_{\tilde{x}}^{C}$ is an $(n_x-1)\times(n_x-1)$ matrix constructed from H_C by removing the row associated with the policy rule and removing the column of loadings on the contemporaneous nominal interest rate (with the matrices $\widetilde{H}_{\tilde{x}}^{F}$ and $\widetilde{H}_{\tilde{x}}^{B}$ constructed analogously). $\widetilde{H}_{r}^{C}$ is an $(n_x-1)\times 1$ matrix constructed from H_C by removing the row associated with the policy rule and extracting the column of loadings on the contemporaneous nominal interest rate (with the matrices $\widetilde{H}_{r}^{F}$ and $\widetilde{H}_{r}^{B}$ constructed analogously). $\widetilde{\Psi}_{\tilde{z}}$ is constructed from Ψ by removing the row corresponding to the policy rule and the column corresponding to the policy shock.

The expectations operator, $\mathbb{E}_t^*$, denotes the expectations of the private sector conditional on information at the start of period t and conditional on the policy plan being implemented in periods $1...t-1$. Note that the interest rate within each period is unknown by the private sector when they make their decisions (hence the use of $\mathbb{E}_t^*r_t$ in equation (A.7)). That is because the private sector form their expectations and make their decisions *before* the policymaker chooses whether to continue with the plan or to revert to the policy rule. Note also that this timing assumption means that $\widetilde{x}_t^{\langle t\rangle}\equiv\widetilde{x}_t^*$.

In period $t=1$ (the first period in which the policy plan is active), equation (A.7) can be written as:

$$\widetilde{H}_{\tilde{x}}^{F}\mathbb{E}_1^*\widetilde{x}_{2}+\widetilde{H}_{\tilde{x}}^{C}\widetilde{x}_1^*+\widetilde{H}_{\tilde{x}}^{B}\widetilde{x}_{0}+\widetilde{H}_{r}^{F}\mathbb{E}_1^*r_{2}+\widetilde{H}_{r}^{C}\mathbb{E}_1^*r_1+\widetilde{H}_{r}^{B}r_{0}=\widetilde{\Psi}_{\tilde{z}}\widetilde{z}_1 \tag{A.8}$$

[32]Note that this means they are state dependent since they depend on precisely when policy reverted to the rule.

[33]This is valid for any model in which the policy shock appears only in the monetary policy rule. It would be straightforward to extend the analysis to a model in which the policy shock drove a persistence forcing process that appeared in the rule.

The stacked-time solution procedure relies on expectations being integrated out of the stacked equations.[34] We start by describing how we do that for period $t = 1$ and then generalize to all other periods. In order to compute the private sector's expectation of the outturn for the interest rate at the end of period $t = 1$, $\mathbb{E}_1^* r_1$, note that there are two possibilities: either the policymaker implements the policy plan or they do not. This means that the expectation of the interest rate is a weighted average of the planned setting for the policy instrument, b_1, and that which would prevail if the policymaker set the interest rate in line with the policy rule, where the weighting is determined by the probability that the private sector attributes to the policymaker reverting:

$$\mathbb{E}_1^* r_1 = p_1 r_1^{\langle 1 \rangle} + (1 - p_1)\, b_1 \tag{A.9}$$

If the policymaker does not enact the policy plan in period 1, then they take private sector decisions as given and set the interest rate as prescribed by the monetary policy rule:[35]

$$r_1^{\langle 1 \rangle} = \frac{\left[\begin{array}{c} \widehat{\Psi}_{\varepsilon^r} \varepsilon^{r}{}_{1}^{\langle 1 \rangle} - \widehat{H}_r^B r_0 - \widehat{H}_{\tilde{x}}^B \widetilde{x}_0 - \left(\widehat{H}_{\tilde{x}}^C + \widehat{H}_r^F B_{r\tilde{x}} + \widehat{H}_{\tilde{x}}^F B_{\tilde{x}\tilde{x}} \right) \widetilde{x}_1^* - \\ \left(\widehat{H}_{\tilde{x}}^F S_{\tilde{x}} + \widehat{H}_r^F S_r \right) \left(\Xi^{r}{}_{2}^{\langle 1 \rangle} + \widetilde{\mathbb{Z}}_2 \right) \end{array} \right]}{\widehat{H}_r^C + \widehat{H}_r^F B_{rr} + \widehat{H}_{\tilde{x}}^F B_{\tilde{x}r}} \tag{A.10}$$

where $\widehat{H}_r^B$ is the element of the monetary policy rule row of H^B that loads on to the lagged policy rate (with $\widehat{H}_r^C$ and $\widehat{H}_r^F$ defined analogously), $\widehat{H}_{\tilde{x}}^B$ is the monetary policy rule row of H^B with the element that loads off the lagged interest rate removed (with $\widehat{H}_{\tilde{x}}^C$ and $\widehat{H}_{\tilde{x}}^F$ defined analogously), $\widehat{\Psi}_{\varepsilon^r}$ is the element of the monetary policy rule row of Ψ that loads on the monetary policy shock and where:

$$\Xi^{r}{}_{2}^{\langle 1 \rangle} = \sum_{j=0}^{\infty} F^j \Phi_{\varepsilon^r} \varepsilon^{r}{}_{2+j}^{\langle 1 \rangle} \tag{A.11}$$

$$\widetilde{\mathbb{Z}}_2 = \sum_{j=0}^{\infty} F^j \Phi_{\tilde{z}} \widetilde{z}_{2+j} \tag{A.12}$$

[34] This is possible in a perfect foresight setting by integrating over the $K + 1$ alternative states of the world. As briefly discussed above, the problem would be much harder in a stochastic setting.

[35] This can be derived in the following way. Partition the structural equations to include only the row corresponding to the interest rate (monetary policy) rule (noting that the partitioning of the shocks is only valid in models in which the only shock appearing in the monetary policy rule is the policy shock):
$\widehat{H}_{\tilde{x}}^F \mathbb{E}_1^{\langle 1 \rangle} \widetilde{x}_2^{\langle 1 \rangle} + \widehat{H}_{\tilde{x}}^C \widetilde{x}_1^* + \widehat{H}_{\tilde{x}}^B \widetilde{x}_0 + \widehat{H}_r^F \mathbb{E}_1^{\langle 1 \rangle} r_2^{\langle 1 \rangle} + \widehat{H}_r^C r_1^{\langle 1 \rangle} + \widehat{H}_r^B r_0 = \widehat{\Psi}_{\varepsilon^r} \varepsilon^{r}{}_{1}^{\langle 1 \rangle}$
where $\widehat{H}^B$, $\widehat{H}^C$, $\widehat{H}^F$ and $\widehat{\Psi}$ are constructed from H^B, H^C, H^F and Ψ by extracting the rows corresponding to the policy rule and partitioning by columns (to produce $\widehat{H}_{\tilde{x}}^C$, $\widehat{H}_r^C$ and $\widehat{\Psi}_{\varepsilon^r}$ etc) in an analogous way to that described in the main text for the non-policy rule equations. The expectations can be computed using the rational expectations solution to the model:
$\mathbb{E}_1^{\langle 1 \rangle} r_2^{\langle 1 \rangle} = B_{r\tilde{x}} \widetilde{x}_1^* + B_{rr} r_1^{\langle 1 \rangle} + S_r \sum_{j=0}^{\infty} F^j \Phi_{\tilde{z}} \widetilde{z}_{2+j} + S_r \sum_{j=0}^{\infty} F^j \Phi_{\varepsilon^r} \varepsilon^{r}{}_{2+j}^{\langle 1 \rangle}$ and
$\mathbb{E}_1^{\langle 1 \rangle} \widetilde{x}_2^{\langle 1 \rangle} = B_{\tilde{x}\tilde{x}} \widetilde{x}_1^* + B_{\tilde{x}r} r_1^{\langle 1 \rangle} + S_{\tilde{x}} \sum_{j=0}^{\infty} F^j \Phi_{\tilde{z}} \widetilde{z}_{2+j} + S_{\tilde{x}} \sum_{j=0}^{\infty} F^j \Phi_{\varepsilon^r} \varepsilon^{r}{}_{2+j}^{\langle 1 \rangle}$.
These expectations can be substituted into the partitioned, structural equation for the policy rule to give:
$\widehat{H}_{\tilde{x}}^F \left(B_{\tilde{x}\tilde{x}} \widetilde{x}_1^* + B_{\tilde{x}r} r_1^{\langle 1 \rangle} + S_{\tilde{x}} \widetilde{\mathbb{Z}}_2 + S_{\tilde{x}} \Xi^{r}{}_{2}^{\langle 1 \rangle} \right) + \widehat{H}_{\tilde{x}}^C \widetilde{x}_1^* + \widehat{H}_{\tilde{x}}^B \widetilde{x}_0 + \widehat{H}_r^F \left(B_{r\tilde{x}} \widetilde{x}_1^* + B_{rr} r_1^{\langle 1 \rangle} + S_r \widetilde{\mathbb{Z}}_2 + S_r \Xi^{r}{}_{2}^{\langle 1 \rangle} \right) +$
$\widehat{H}_r^C r_1^{\langle 1 \rangle} + \widehat{H}_r^B r_0 = \widehat{\Psi}_{\varepsilon^r} \varepsilon^{r}{}_{1}^{\langle 1 \rangle}$ where:
$\Xi^{r}{}_{2}^{\langle 1 \rangle} = \sum_{j=0}^{\infty} F^j \Phi_{\varepsilon^r} \varepsilon^{r}{}_{2+j}^{\langle 1 \rangle}$ and
$\widetilde{\mathbb{Z}}_2 = \sum_{j=0}^{\infty} F^j \Phi_{\tilde{z}} \widetilde{z}_{2+j}$
And this can be rearranged to give equation (A.10).

Period 1 expectations for the vector of endogenous variables in period 2 can be defined using the same logic:

$$\mathbb{E}_1^* \widetilde{x}_2 = p_1 \widetilde{x}_2^{\langle 1 \rangle} + (1 - p_1)\, \widetilde{x}_2^* \tag{A.13}$$

where:[36]

$$\widetilde{x}_2^{\langle 1 \rangle} = B_{\tilde{x}\tilde{x}} \widetilde{x}_1^* + B_{\tilde{x}r} r_1^{\langle 1 \rangle} + S_{\tilde{x}} \left(\widetilde{\mathbb{Z}}_2 + \Xi^r{}_2^{\langle 1 \rangle} \right) \tag{A.14}$$

And, finally, period 1 expectations for the period 2 interest rate must take into account that there are two periods of uncertainty over the realisation of the interest rate (which reflects the timing assumption discussed above):

$$\mathbb{E}_1^* r_2 = p_1 r_2^{\langle 1 \rangle} + (1 - p_1)\, \mathbb{E}_2^* r_2 \tag{A.15}$$

where:

$$r_2^{\langle 1 \rangle} = B_{r\tilde{x}} \widetilde{x}_1^* + B_{rr} r_1^{\langle 1 \rangle} + S_r \left(\widetilde{\mathbb{Z}}_2 + \Xi^r{}_2^{\langle 1 \rangle} \right) \tag{A.16}$$

and:

$$\mathbb{E}_2^* r_2 = p_2 r_2^{\langle 2 \rangle} + (1 - p_2)\, b_2 \tag{A.17}$$

with $r_2^{\langle 2 \rangle}$ defined analogously to $r_1^{\langle 1 \rangle}$ from equation (A.10) as (noting that $r_1^* \equiv b_1$):

$$r_2^{\langle 2 \rangle} = \frac{\left[\begin{array}{c} \widehat{\Psi}_{\varepsilon^r} \varepsilon^r{}_2^{\langle 2 \rangle} - \widehat{H}_r^B b_1 - \widehat{H}_{\tilde{x}}^B \widetilde{x}_1^* - \left(\widehat{H}_{\tilde{x}}^C + \widehat{H}_r^F B_{r\tilde{x}} + \widehat{H}_{\tilde{x}}^F B_{\tilde{x}\tilde{x}} \right) \widetilde{x}_2^* - \\ \left(\widehat{H}_{\tilde{x}}^F S_{\tilde{x}} + \widehat{H}_r^F S_r \right) \left(\Xi^r{}_3^{\langle 2 \rangle} + \widetilde{\mathbb{Z}}_3 \right) \end{array} \right]}{\widehat{H}_r^C + \widehat{H}_r^F B_{rr} + \widehat{H}_{\tilde{x}}^F B_{\tilde{x}r}} \tag{A.18}$$

where:

$$\Xi^r{}_3^{\langle 2 \rangle} = \sum_{j=0}^{\infty} F^j \Phi_{\varepsilon^r} \varepsilon^r{}_{3+j}^{\langle 2 \rangle} \tag{A.19}$$

$$\widetilde{\mathbb{Z}}_3 = \sum_{j=0}^{\infty} F^j \Phi_{\tilde{z}} \widetilde{z}_{3+j} \tag{A.20}$$

These definitions can be plugged into the original structural equation (A.8) and the result rearranged to put 'known' terms on the right hand side (those that agents take as given) and unknown terms (those as functions of $\widetilde{x}_1^*$ and $\widetilde{x}_2^*$ on the left hand side):[37]

$$\begin{aligned}
&\left(\widetilde{H}_{\tilde{x}}^C + p_1 \left(\widetilde{H}_{\tilde{x}}^F B_{\tilde{x}\tilde{x}} + \widetilde{H}_r^F B_{r\tilde{x}} - \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{\mathbb{H}}_{\tilde{x}} \right) - (1 - p_1)\, p_2 \widehat{\mathbb{H}}_r^{-1} \widetilde{H}_r^F \widehat{H}_{\tilde{x}}^B \right) \widetilde{x}_1^* \\
&+ (1 - p_1) \left(\widetilde{H}_{\tilde{x}}^F - p_2 \widehat{\mathbb{H}}_r^{-1} \widetilde{H}_r^F \widehat{\mathbb{H}}_{\tilde{x}} \right) \widetilde{x}_2^* \\
&= \widetilde{\Psi}_{\tilde{z}} \widetilde{z}_1 + p_1 \left(\widetilde{\mathbb{H}}_r \left(\widehat{H}_{\tilde{x}}^F S_{\tilde{x}} + \widehat{H}_r^F S_r \right) - \left(\widetilde{H}_{\tilde{x}}^F S_{\tilde{x}} + \widetilde{H}_r^F S_r \right) \right) \left(\Xi^r{}_2^{\langle 1 \rangle} + \widetilde{\mathbb{Z}}_2 \right) \\
&+ (1 - p_1)\, p_2 \widetilde{H}_r^F \left(\widehat{H}_{\tilde{x}}^F S_{\tilde{x}} + \widehat{H}_r^F S_r \right) \left(\Xi^r{}_3^{\langle 2 \rangle} + \widetilde{\mathbb{Z}}_3 \right) \\
&- p_1 \widetilde{\mathbb{H}}_r \widehat{\Psi}_{\varepsilon^r} \varepsilon^r{}_1^{\langle 1 \rangle} - (1 - p_1)\, p_2 \widetilde{H}_r^F \widehat{\Psi}_{\varepsilon^r} \varepsilon^r{}_2^{\langle 2 \rangle} \\
&+ \left(p_1 \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{H}_{\tilde{x}}^B - \widetilde{H}_{\tilde{x}}^B \right) \widetilde{x}_0 + \left(p_1 \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{H}_r^B - \widetilde{H}_r^B \right) r_0 \\
&+ (1 - p_1) \left(p_2 \widehat{\mathbb{H}}_r^{-1} \widetilde{H}_r^F \widehat{H}_r^B - \widetilde{H}_r^C \right) b_1 - (1 - p_1)(1 - p_2)\, \widetilde{H}_r^F b_2
\end{aligned} \tag{A.21}$$

[36]This makes use of the timing assumption and associated definition that $\widetilde{x}_1^{\langle 1 \rangle} \equiv \widetilde{x}_1^*$.

[37]Note that the anticipated policy shocks necessary to impose the zero bound on reversion to the rule are unknown, but are treated as known within the stacked-time algorithm. See Section A.3 for a description of how we find their values.

where:

$$\widehat{\mathbb{H}}_{\tilde{x}} = \widehat{H}^{C}_{\tilde{x}} + \widehat{H}^{F}_{r} B_{r\tilde{x}} + \widehat{H}^{F}_{\tilde{x}} B_{\tilde{x}\tilde{x}} \tag{A.22}$$

$$\widehat{\mathbb{H}}_{r} = \widehat{H}^{C}_{r} + \widehat{H}^{F}_{r} B_{rr} + \widehat{H}^{F}_{\tilde{x}} B_{\tilde{x}r} \tag{A.23}$$

$$\widetilde{\mathbb{H}}_{r} = \widetilde{H}^{C}_{r} + \widetilde{H}^{F}_{r} B_{rr} + \widetilde{H}^{F}_{\tilde{x}} B_{\tilde{x}r} \tag{A.24}$$

Assuming the policymaker does not revert to the policy rule at the end of period $t = 1$, then in period $t = 2$ we have (noting that $r^*_1 = b_1$):

$$\widetilde{H}^{F}_{\tilde{x}} \mathbb{E}^*_2 \widetilde{x}_3 + \widetilde{H}^{C}_{\tilde{x}} \widetilde{x}^*_2 + \widetilde{H}^{B}_{\tilde{x}} \widetilde{x}_1 + \widetilde{H}^{F}_{r} \mathbb{E}^*_2 r_3 + \widetilde{H}^{C}_{r} \mathbb{E}^*_2 r_2 + \widetilde{H}^{B}_{r} b_1 = \widetilde{\Psi}_{\tilde{z}} \widetilde{z}_2 \tag{A.25}$$

We can use analogous arguments to substitute out expectations and rearrange to give:

$$\begin{aligned}
&\left(\widetilde{H}^{B}_{\tilde{x}} - p_2 \widehat{\mathbb{H}}^{-1}_{r} \widetilde{\mathbb{H}}_{r} \widehat{H}^{B}_{\tilde{x}}\right) \widetilde{x}^*_1 \\
&+ \left(\widetilde{H}^{C}_{\tilde{x}} + p_2 \left(\widetilde{H}^{F}_{\tilde{x}} B_{\tilde{x}\tilde{x}} + \widetilde{H}^{F}_{r} B_{r\tilde{x}} - \widehat{\mathbb{H}}^{-1}_{r} \widetilde{\mathbb{H}}_{r} \widehat{\mathbb{H}}_{\tilde{x}}\right) - (1-p_2)\, p_3 \widehat{\mathbb{H}}^{-1}_{r} \widetilde{H}^{F}_{r} \widehat{H}^{B}_{\tilde{x}}\right) \widetilde{x}^*_2 \\
&+ (1-p_2) \left(\widetilde{H}^{F}_{\tilde{x}} - p_3 \widehat{\mathbb{H}}^{-1}_{r} \widetilde{H}^{F}_{r} \widehat{\mathbb{H}}_{\tilde{x}}\right) \widetilde{x}^*_3 \\
&= \widetilde{\Psi}_{\tilde{z}} \widetilde{z}_2 + p_2 \left(\widetilde{\mathbb{H}}_{r} \left(\widehat{H}^{F}_{\tilde{x}} S_{\tilde{x}} + \widehat{H}^{F}_{r} S_{r}\right) - \left(\widetilde{H}^{F}_{\tilde{x}} S_{\tilde{x}} + \widetilde{H}^{F}_{r} S_{r}\right)\right) \left(\Xi^{r\langle 2\rangle}_{3} + \widetilde{\mathbb{Z}}_3\right) \\
&+ (1-p_2)\, p_3 \widetilde{H}^{F}_{r} \left(\widehat{H}^{F}_{\tilde{x}} S_{\tilde{x}} + \widehat{H}^{F}_{r} S_{r}\right) \left(\Xi^{r\langle 3\rangle}_{4} + \widetilde{\mathbb{Z}}_4\right) \\
&- p_2 \widetilde{\mathbb{H}}_{r} \widehat{\Psi}_{\varepsilon^r} \varepsilon^{r\langle 2\rangle}_{2} - (1-p_2)\, p_3 \widetilde{H}^{F}_{r} \widehat{\Psi}_{\varepsilon^r} \varepsilon^{r\langle 3\rangle}_{3} \\
&+ \left(p_2 \widehat{\mathbb{H}}^{-1}_{r} \widetilde{\mathbb{H}}_{r} \widehat{H}^{B}_{r} - \widetilde{H}^{B}_{r}\right) b_1 + (1-p_2) \left(p_3 \widehat{\mathbb{H}}^{-1}_{r} \widetilde{H}^{F}_{r} \widehat{H}^{B}_{r} - \widetilde{H}^{C}_{r}\right) b_2 \\
&- (1-p_2)(1-p_3)\, \widetilde{H}^{F}_{r} b_3
\end{aligned} \tag{A.26}$$

where $\Xi^{r\langle 3\rangle}_{4}$ and $\widetilde{\mathbb{Z}}_4$ are defined analogously to $\Xi^{r\langle 2\rangle}_{3}$ and $\widetilde{\mathbb{Z}}_3$.

Inspection of the expression for $t = 2$ reveals the generic form of the equations for periods $t = 2, \ldots, K-1$, which is:

$$\begin{aligned}
&\left(\widetilde{H}^{B}_{\tilde{x}} - p_t \widehat{\mathbb{H}}^{-1}_{r} \widetilde{\mathbb{H}}_{r} \widehat{H}^{B}_{\tilde{x}}\right) \widetilde{x}^*_{t-1} \\
&+ \left(\widetilde{H}^{C}_{\tilde{x}} + p_t \left(\widetilde{H}^{F}_{\tilde{x}} B_{\tilde{x}\tilde{x}} + \widetilde{H}^{F}_{r} B_{r\tilde{x}} - \widehat{\mathbb{H}}^{-1}_{r} \widetilde{\mathbb{H}}_{r} \widehat{\mathbb{H}}_{\tilde{x}}\right) - (1-p_t)\, p_{t+1} \widehat{\mathbb{H}}^{-1}_{r} \widetilde{H}^{F}_{r} \widehat{H}^{B}_{\tilde{x}}\right) \widetilde{x}^*_t \\
&+ (1-p_t) \left(\widetilde{H}^{F}_{\tilde{x}} - p_{t+1} \widehat{\mathbb{H}}^{-1}_{r} \widetilde{H}^{F}_{r} \widehat{\mathbb{H}}_{\tilde{x}}\right) \widetilde{x}^*_{t+1} \\
&= \widetilde{\Psi}_{\tilde{z}} \widetilde{z}_t + p_t \left(\widetilde{\mathbb{H}}_{r} \left(\widehat{H}^{F}_{\tilde{x}} S_{\tilde{x}} + \widehat{H}^{F}_{r} S_{r}\right) - \left(\widetilde{H}^{F}_{\tilde{x}} S_{\tilde{x}} + \widetilde{H}^{F}_{r} S_{r}\right)\right) \left(\Xi^{r\langle t\rangle}_{t+1} + \widetilde{\mathbb{Z}}_{t+1}\right) \\
&+ (1-p_t)\, p_{t+1} \widetilde{H}^{F}_{r} \left(\widehat{H}^{F}_{\tilde{x}} S_{\tilde{x}} + \widehat{H}^{F}_{r} S_{r}\right) \left(\Xi^{r\langle t+1\rangle}_{t+2} + \widetilde{\mathbb{Z}}_{t+2}\right) \\
&- p_t \widetilde{\mathbb{H}}_{r} \widehat{\Psi}_{\varepsilon^r} \varepsilon^{r\langle t\rangle}_{t} - (1-p_t)\, p_{t+1} \widetilde{H}^{F}_{r} \widehat{\Psi}_{\varepsilon^r} \varepsilon^{r\langle t+1\rangle}_{t+1} \\
&+ \left(p_t \widehat{\mathbb{H}}^{-1}_{r} \widetilde{\mathbb{H}}_{r} \widehat{H}^{B}_{r} - \widetilde{H}^{B}_{r}\right) b_{t-1} + (1-p_t) \left(p_{t+1} \widehat{\mathbb{H}}^{-1}_{r} \widetilde{H}^{F}_{r} \widehat{H}^{B}_{r} - \widetilde{H}^{C}_{r}\right) b_t \\
&- (1-p_t)(1-p_{t+1})\, \widetilde{H}^{F}_{r} b_{t+1}
\end{aligned} \tag{A.27}$$

where:

$$\Xi^{r\langle i\rangle}_{t} = \sum_{j=0}^{\infty} F^j \Phi_{\varepsilon^r} \varepsilon^{r\langle i\rangle}_{t+j} \tag{A.28}$$

$$\widetilde{\mathbb{Z}}_{t+s} = \sum_{j=0}^{\infty} F^j \Phi_{\tilde{z}} \widetilde{z}_{t+s+j} \tag{A.29}$$

If period $t = K$ is reached with the policy plan having been followed, then rates will revert to the policy rule in period $K + 1$ with certainty. The solution in this period is then given by:[38]

$$\begin{aligned}
&\left(\widetilde{H}^B_{\tilde{x}} - p_K \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{H}^B_{\tilde{x}}\right) \widetilde{x}^*_{K-1} + \left(\widetilde{H}^C_{\tilde{x}} + \widetilde{H}^F_{\tilde{x}} B_{\tilde{x}\tilde{x}} + \widetilde{H}^F_r B_{r\tilde{x}} - p_K \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{\mathbb{H}}_{\tilde{x}}\right) x^*_K \\
&= \widetilde{\Psi}_{\tilde{z}} \widetilde{z}_K + p_K \left(\widetilde{\mathbb{H}}_r \left(\widehat{H}^F_{\tilde{x}} S_{\tilde{x}} + \widehat{H}^F_r S_r\right) - \left(\widetilde{H}^F_{\tilde{x}} S_{\tilde{x}} + \widetilde{H}^F_r S_r\right)\right) \left(\Xi^{r\langle K\rangle}_{K+1} + \widetilde{\mathbb{Z}}_{K+1}\right) \\
&- p_K \widetilde{\mathbb{H}}_r \widehat{\Psi}_{\varepsilon^r} \varepsilon^{r\langle K\rangle}_K - (1 - p_K) \left(\widetilde{H}^F_{\tilde{x}} S_{\tilde{x}} + \widetilde{H}^F_r S_r\right) \left(\Xi^{r\langle K+1\rangle}_{K+1} + \widetilde{\mathbb{Z}}_{K+1}\right) \\
&+ \left(p_K \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{H}^B_r - \widetilde{H}^B_r\right) b_{K-1} - (1 - p_K) \widetilde{\mathbb{H}}_r b_K
\end{aligned} \tag{A.30}$$

unless $K = 1$, in which case we have the following:

$$\begin{aligned}
&\left(\widetilde{H}^C_{\tilde{x}} + \widetilde{H}^F_{\tilde{x}} B_{\tilde{x}\tilde{x}} + \widetilde{H}^F_r B_{r\tilde{x}} - p_1 \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{\mathbb{H}}_{\tilde{x}}\right) \widetilde{x}^*_1 \\
&= \widetilde{\Psi}_{\tilde{z}} \widetilde{z}_1 + p_1 \left(\widetilde{\mathbb{H}}_r \left(\widehat{H}^F_{\tilde{x}} S_{\tilde{x}} + \widehat{H}^F_r S_r\right) - \left(\widetilde{H}^F_{\tilde{x}} S_{\tilde{x}} + \widetilde{H}^F_r S_r\right)\right) \left(\Xi^{r\langle K\rangle}_2 + \widetilde{\mathbb{Z}}_2\right) \\
&- p_1 \widetilde{\mathbb{H}}_r \widehat{\Psi}_{\varepsilon^r} \varepsilon^{r\langle 1\rangle}_1 - (1 - p_1) \left(\widetilde{H}^F_{\tilde{x}} S_{\tilde{x}} + \widetilde{H}^F_r S_r\right) \left(\Xi^{r\langle 2\rangle}_2 + \widetilde{\mathbb{Z}}_2\right) \\
&+ \left(p_1 \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{H}^B_r - \widetilde{H}^B_r\right) r_0 + \left(p_1 \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{H}^B_{\tilde{x}} - \widetilde{H}^B_{\tilde{x}}\right) \widetilde{x}_0 - (1 - p_1) \widetilde{\mathbb{H}}_r b_1
\end{aligned} \tag{A.31}$$

To solve for the equilibrium path conditional on the announced policy plan being implemented, $\{x^*_t\}^K_{t=1}$, we stack the equations derived above to form the following system:

$$\mathbb{J} \widetilde{X}^* = \mathbb{C} \tag{A.32}$$

where:

$$\widetilde{X}^* \equiv \begin{bmatrix} \widetilde{x}^*_1 \\ \vdots \\ \widetilde{x}^*_K \end{bmatrix}$$

It is straightforward to form the matrix $\mathbb{J}$ by collecting the loadings on the $\widetilde{x}$ terms from the left hand sides of the equations outlined above as follows:

$$\mathbb{J} \equiv \begin{bmatrix}
J^C_1 & J^F_1 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\
J^B_2 & J^C_2 & J^F_2 & 0 & \dots & 0 & 0 & 0 & 0 \\
 & & & & \vdots & & & & \\
0 & 0 & \dots & J^B_t & J^C_t & J^F_t & 0 & \dots & 0 \\
 & & & & \vdots & & & & \\
0 & 0 & 0 & 0 & \dots & 0 & J^B_{K-1} & J^C_{K-1} & J^F_{K-1} \\
0 & 0 & 0 & 0 & \dots & 0 & 0 & J^B_K & J^C_K
\end{bmatrix}$$

[38] This follows from noting that:
$\mathbb{E}^*_K \widetilde{x}_{K+1} = p_K \widetilde{x}^{\langle K\rangle}_{K+1} + (1 - p_K) \widetilde{x}^{\langle K+1\rangle}_{K+1}$ where:
$\widetilde{x}^{\langle K\rangle}_{K+1} = B_{\tilde{x}\tilde{x}} \widetilde{x}^*_K + B_{\tilde{x}r} r^{\langle K\rangle}_K + S_{\tilde{x}} \left(\widetilde{\mathbb{Z}}_{K+1} + \Xi^{r\langle K\rangle}_{K+1}\right)$ and
$\widetilde{x}^{\langle K+1\rangle}_{K+1} = B_{\tilde{x}\tilde{x}} \widetilde{x}^*_K + B_{\tilde{x}r} b + S_{\tilde{x}} \sum_{j=0}^{\infty} F^j \Phi_{\tilde{z}} \widetilde{z}_{K+1+j} + S_{\tilde{x}} \sum_{j=0}^{\infty} F^j \Phi_{\varepsilon^r} \varepsilon^{r\langle K+1\rangle}_{K+1+j}$.
$\mathbb{E}^*_K r_K = p_K r^{\langle K\rangle}_K + (1 - p_K) b$ where $r^{\langle K\rangle}_K$ is defined analogously to $r^{\langle 1\rangle}_1$ and $r^{\langle 2\rangle}_2$.
$\mathbb{E}^*_K r_{K+1} = p_K r^{\langle K\rangle}_{K+1} + (1 - p_K) r^{\langle K+1\rangle}_{K+1}$ where:
$r^{\langle K+1\rangle}_{K+1} = B_{r\tilde{x}} \widetilde{x}^*_K + B_{rr} b + S_r \sum_{j=0}^{\infty} F^j \Phi_{\tilde{z}} \widetilde{z}_{K+1+j} + S_r \sum_{j=0}^{\infty} F^j \Phi_{\varepsilon^r} \varepsilon^{r\langle K+1\rangle}_{K+1+j}$.

where:

$$J_t^B = \widetilde{H}_{\tilde{x}}^B - p_t \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{H}_{\tilde{x}}^B$$
$$J_t^C = \widetilde{H}_{\tilde{x}}^C + p_t \left(\widetilde{H}_{\tilde{x}}^F B_{\tilde{x}\tilde{x}} + \widetilde{H}_r^F B_{r\tilde{x}} - \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{\mathbb{H}}_{\tilde{x}} \right) - (1-p_t)\, p_{t+1} \left(\widehat{\mathbb{H}}_r^{-1} \widetilde{H}_r^F \widehat{H}_{\tilde{x}}^B \right)$$
$$J_t^F = (1-p_t) \left(\widetilde{H}_{\tilde{x}}^F - p_{t+1} \widehat{\mathbb{H}}_r^{-1} \widetilde{H}_r^F \widehat{\mathbb{H}}_{\tilde{x}} \right)$$
$$J_K^C = \widetilde{H}_{\tilde{x}}^C + \widetilde{H}_{\tilde{x}}^F B_{\tilde{x}\tilde{x}} + \widetilde{H}_r^F B_{r\tilde{x}} - p_K \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{\mathbb{H}}_{\tilde{x}}$$

and where J_1^C, J_1^F, J_2^B, J_2^C, J_2^F, J_{K-1}^B, J_{K-1}^C, J_{K-1}^F and J_K^B can be written as specific cases of the general period t case defined above. It is also straightforward to form the vector $\mathbb{C}$ by collecting the terms from the right hand sides of the equations above:

$$\mathbb{C} \equiv \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_t \\ \vdots \\ C_{K-1} \\ C_K \end{bmatrix}$$

where:

$$\begin{aligned}
C_1 = {} & \widetilde{\Psi}_{\tilde{z}} \tilde{z}_1 + p_1 \left(\widetilde{\mathbb{H}}_r \left(\widehat{H}_{\tilde{x}}^F S_{\tilde{x}} + \widehat{H}_r^F S_r \right) - \left(\widetilde{H}_{\tilde{x}}^F S_{\tilde{x}} + \widetilde{H}_r^F S_r \right) \right) \left(\Xi^{r\langle 1 \rangle}_{\;2} + \widetilde{\mathbb{Z}}_2 \right) \\
& + (1-p_1)\, p_2 \widetilde{H}_r^F \left(\widehat{H}_{\tilde{x}}^F S_{\tilde{x}} + \widehat{H}_r^F S_r \right) \left(\Xi^{r\langle 2 \rangle}_{\;3} + \widetilde{\mathbb{Z}}_3 \right) \\
& - p_1 \widetilde{\mathbb{H}}_r \widehat{\Psi}_{\varepsilon^r} \varepsilon^{r\langle 1 \rangle}_{\;1} - (1-p_1)\, p_2 \widetilde{H}_r^F \widehat{\Psi}_{\varepsilon^r} \varepsilon^{r\langle 2 \rangle}_{\;2} \\
& + \left(p_1 \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{H}_{\tilde{x}}^B - \widetilde{H}_{\tilde{x}}^B \right) \tilde{x}_0 + \left(p_1 \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{H}_r^B - \widetilde{H}_r^B \right) r_0 \\
& + (1-p_1) \left(p_2 \widehat{\mathbb{H}}_r^{-1} \widetilde{H}_r^F \widehat{H}_r^B - \widetilde{H}_r^C \right) b_1 - (1-p_1)(1-p_2)\, \widetilde{H}_r^F b_2 && \text{(A.33)} \\
C_t = {} & \widetilde{\Psi}_{\tilde{z}} \tilde{z}_t + p_t \left(\widetilde{\mathbb{H}}_r \left(\widehat{H}_{\tilde{x}}^F S_{\tilde{x}} + \widehat{H}_r^F S_r \right) - \left(\widetilde{H}_{\tilde{x}}^F S_{\tilde{x}} + \widetilde{H}_r^F S_r \right) \right) \left(\Xi^{r\langle t \rangle}_{\;t+1} + \widetilde{\mathbb{Z}}_{t+1} \right) \\
& + (1-p_t)\, p_{t+1} \widetilde{H}_r^F \left(\widehat{H}_{\tilde{x}}^F S_{\tilde{x}} + \widehat{H}_r^F S_r \right) \left(\Xi^{r\langle t+1 \rangle}_{\;t+2} + \widetilde{\mathbb{Z}}_{t+2} \right) \\
& - p_t \widetilde{\mathbb{H}}_r \widehat{\Psi}_{\varepsilon^r} \varepsilon^{r\langle t \rangle}_{\;t} - (1-p_t)\, p_{t+1} \widetilde{H}_r^F \widehat{\Psi}_{\varepsilon^r} \varepsilon^{r\langle t+1 \rangle}_{\;t+1} \\
& + \left(p_t \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{H}_r^B - \widetilde{H}_r^B \right) b_{t-1} + (1-p_t) \left(p_{t+1} \widehat{\mathbb{H}}_r^{-1} \widetilde{H}_r^F \widehat{H}_r^B - \widetilde{H}_r^C \right) b_t \\
& - (1-p_t)(1-p_{t+1})\, \widetilde{H}_r^F b_{t+1} && \text{(A.34)} \\
C_K = {} & \widetilde{\Psi}_{\tilde{z}} \tilde{z}_K + p_K \left(\widetilde{\mathbb{H}}_r \left(\widehat{H}_{\tilde{x}}^F S_{\tilde{x}} + \widehat{H}_r^F S_r \right) - \left(\widetilde{H}_{\tilde{x}}^F S_{\tilde{x}} + \widetilde{H}_r^F S_r \right) \right) \left(\Xi^{r\langle K \rangle}_{\;K+1} + \widetilde{\mathbb{Z}}_{K+1} \right) \\
& - p_K \widetilde{\mathbb{H}}_r \widehat{\Psi}_{\varepsilon^r} \varepsilon^{r\langle K \rangle}_{\;K} - (1-p_K) \left(\widetilde{H}_{\tilde{x}}^F S_{\tilde{x}} + \widetilde{H}_r^F S_r \right) \left(\Xi^{r\langle K+1 \rangle}_{\;K+1} + \widetilde{\mathbb{Z}}_{K+1} \right) \\
& + \left(p_K \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{H}_r^B - \widetilde{H}_r^B \right) b_{K-1} - (1-p_K)\, \widetilde{\mathbb{H}}_r b_K && \text{(A.35)}
\end{aligned}$$

and where C_2 and C_{K-1} can be written as specific cases of the general period t case defined above. Finally, the special case of $K = 1$ is as follows:

$$\begin{aligned} C_1 \equiv C_K = {} & \widetilde{\Psi}_{\tilde{z}}\widetilde{z}_1 + p_1\left(\widetilde{\mathbb{H}}_r\left(\widehat{H}^F_{\tilde{x}}S_{\tilde{x}} + \widehat{H}^F_r S_r\right) - \left(\widetilde{H}^F_{\tilde{x}}S_{\tilde{x}} + \widetilde{H}^F_r S_r\right)\right)\left(\Xi^{r\langle K\rangle}_2 + \widetilde{\mathbb{Z}}_2\right) \\ & - p_1\widetilde{\mathbb{H}}_r\widehat{\Psi}_{\varepsilon^r}\varepsilon^{r\langle 1\rangle}_1 - (1-p_1)\left(\widetilde{H}^F_{\tilde{x}}S_{\tilde{x}} + \widetilde{H}^F_r S_r\right)\left(\Xi^{r\langle 2\rangle}_2 + \widetilde{\mathbb{Z}}_2\right) \\ & + \left(p_1\widehat{\mathbb{H}}^{-1}_r\widetilde{\mathbb{H}}_r\widehat{H}^B_r - \widetilde{H}^B_r\right)r_0 + \left(p_1\widehat{\mathbb{H}}^{-1}_r\widetilde{\mathbb{H}}_r\widehat{H}^B_{\tilde{x}} - \widetilde{H}^B_{\tilde{x}}\right)\widetilde{x}_0 - (1-p_1)\widetilde{\mathbb{H}}_r b_1 \end{aligned} \tag{A.36}$$

With the solution for the case in which the policymaker implements the announced plan in hand, it is straightforward to recover the solution for the $K + 1$ paths in which the policymaker could revert to the rule.[39] As noted above, the private sector's decisions are unaffected by the policymaker's decision to implement the plan or otherwise *within each period*, reflecting that the policymaker sets the policy instrument at the end of the period after private sector decisions have been made. It follows that we can define:

$$x^{\langle t\rangle}_t \equiv \begin{bmatrix} \widetilde{x}^*_t \\ r^{\langle t\rangle}_t \end{bmatrix} \tag{A.37}$$

which is valid for all periods $t = 1...K$ and in which the interest rate on reverting to the rule can be defined as follows (which is just the general case of the period $t = 1$ and $t = 2$ definitions from above):

$$r^{\langle t\rangle}_t = \frac{\left[\begin{array}{c} \widehat{\Psi}_{\varepsilon^r}\varepsilon^{r\langle t\rangle}_t - \widehat{H}^B_r b - \widehat{H}^B_{\tilde{x}}\widetilde{x}^*_{t-1} - \left(\widehat{H}^C_{\tilde{x}} + \widehat{H}^F_r B_{r\tilde{x}} + \widehat{H}^F_{\tilde{x}}B_{\tilde{x}\tilde{x}}\right)\widetilde{x}^*_t - \\ \left(\widehat{H}^F_{\tilde{x}}S_{\tilde{x}} + \widehat{H}^F_r S_r\right)\left(\Xi^{r\langle t\rangle}_{t+1} + \widetilde{\mathbb{Z}}_{t+1}\right) \end{array}\right]}{\widehat{H}^C_r + \widehat{H}^F_r B_{rr} + \widehat{H}^F_{\tilde{x}}B_{\tilde{x}r}} \tag{A.38}$$

In the case where the policymaker implements the announced plan and period $t = K + 1$ is reached without the policymaker having already reverted to the rule, then the interest rate reverts back to the rule with certainty, in which case the following definitions apply:

$$x^{\langle K+1\rangle}_{K+1} \equiv \begin{bmatrix} \widetilde{x}^{\langle K+1\rangle}_{K+1} \\ r^{\langle K+1\rangle}_{K+1} \end{bmatrix} \tag{A.39}$$

where:

$$\widetilde{x}^{\langle K+1\rangle}_{K+1} = B_{\tilde{x}\tilde{x}}\widetilde{x}^*_K + B_{\tilde{x}r}b_K + S_{\tilde{x}}\left(\widetilde{\mathbb{Z}}_{K+1} + \Xi^{r\langle K+1\rangle}_{K+1}\right) \tag{A.40}$$

$$r^{\langle K+1\rangle}_{K+1} = B_{r\tilde{x}}\widetilde{x}^*_K + B_{rr}b_K + S_r\left(\widetilde{\mathbb{Z}}_{K+1} + \Xi^{r\langle K+1\rangle}_{K+1}\right) \tag{A.41}$$

Having computed the vectors $x^{\langle i\rangle}_i$ for periods $i = 1...K + 1$, it is then straightforward to compute the paths for the endogenous variables in any future period, $x^{\langle i\rangle}_t$ for any $t > i$, using the rational expectations solution to the model in equation (A.3).

A.3 Imposing the zero bound on reversion to the rule

Note that a feature of the solution approach outlined above is that any monetary policy shocks necessary to impose a zero or effective lower bound on reversion to the rule are

[39] And it is clearly trivial to define the complete vector of endogenous variables in states where the policy plan has been followed as: $x^*_t \equiv \begin{bmatrix} \widetilde{x}^*_t \\ b_t \end{bmatrix}$.

taken as given by agents in the model. The values of these shocks depend on the state of the economy prior to reversion, which is the vector that our procedure provides a solution for. To ensure that our algorithm does not permit the policy rate to violate the lower bound on reversion, we employ the following algorithm:

1. Form the system (A.32) under the assumption that the lower bound does not bind in any period and any state in which the policymaker can revert to the rule. This means setting ${\varepsilon^{r}}_{t}^{\langle i \rangle} = 0, \forall i = 1...K+1, t \geq i$.

2. Compute the equilibrium paths in the model economy using the algorithm outlined above.

3. Check whether the assumption made in step 1 is valid by computing the expected path of the policy rate conditional on reversion to the rule in each state $i = 1, \ldots, K+1$. If none of the projections for the policy rate falls below the zero or effective bound, then stop. Otherwise, iterate as follows:

   (a) Conditional on the equilibrium path along which the policymaker does not revert to the rule early, $\{x_t^*\}_{t=1}^{K}$, compute the shocks required to enforce the lower bound ($\{\{{\varepsilon^{r}}_{t}^{\langle i \rangle}\}_{i=1}^{K+1}\}_{t=i}^{\infty}$) using the algorithm described below and use this set to update the guess for the set of anticipated policy shocks necessary to enforce the lower bound in all states of the world.[40]

   (b) Re-compute the equilibrium non-early-reversion path conditional on this updated guess using the algorithm in Section A.2.

   (c) If the distance between the previous and current guesses for the equilibrium paths are small enough and the lower bound is respected in all states and time periods, then stop. Otherwise go back to 3a.

We implement step 3a using a similar approach to that described in Holden and Paetz (2012). Specifically, for each case in which the expected path for the policy rate violates the lower bound when the policymaker reverts in period i, we find the minimal set of shocks, $\{{\varepsilon^{r}}_{t}^{\langle i \rangle}\}_{t=i}^{\infty}$, that ensures that the lower bound constraint is not violated in any period following reversion to the rule. As described in Holden and Paetz (2012), an efficient way to find that set of shocks is to write the problem as a quadratic programming problem (with the constraint that the anticipated policy shocks must be positive - i.e. the bound can only be violated from below). This allows us to use well-established and efficient algorithms to solve the problem.[41]

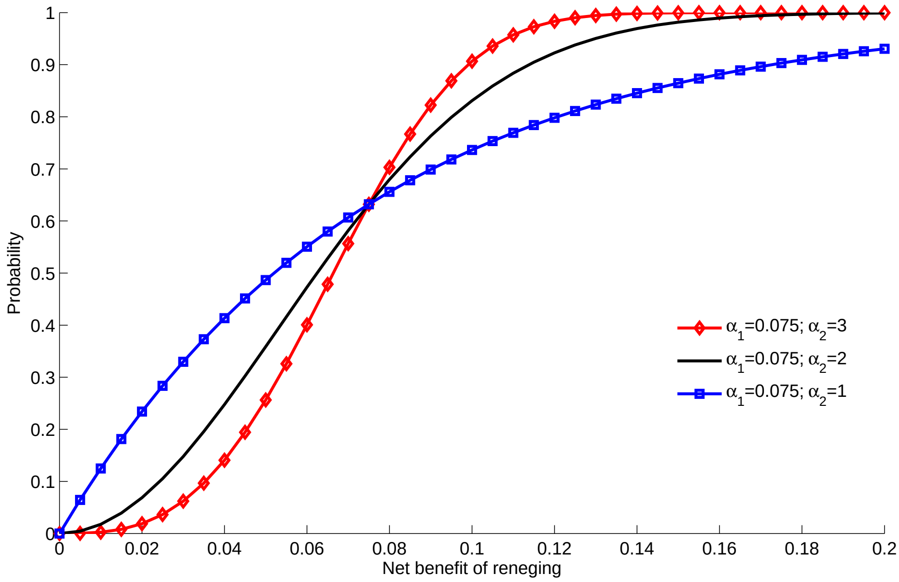
[40]We use a simple heuristic that weights the previous guesses and the new shock values equally.

[41]Note that an alternative approach that would eliminate the iterative approach described in the algorithm above would be to cast the whole problem as a quadratic programming problem and to simultaneously solve for the equilibrium paths and any anticipated policy shocks necessary to ensure that the lower bound constraint is not violated in any of the $K+1$ possible states. This is in principle a better solution to the problem (and may be faster computationally), but requires more algebra and would be more difficult to implement.

B Imperfectly-credible forward guidance sensitivity

In this Appendix, we explore the effects on the endogenous imperfect credibility simulations in Section 3.4 of alternative assumptions about the extent to which the probability of reneging is influenced by the temptation to renege. We do so using the two alternative parameterizations of the function F in equation (20), as depicted in Figure B1.

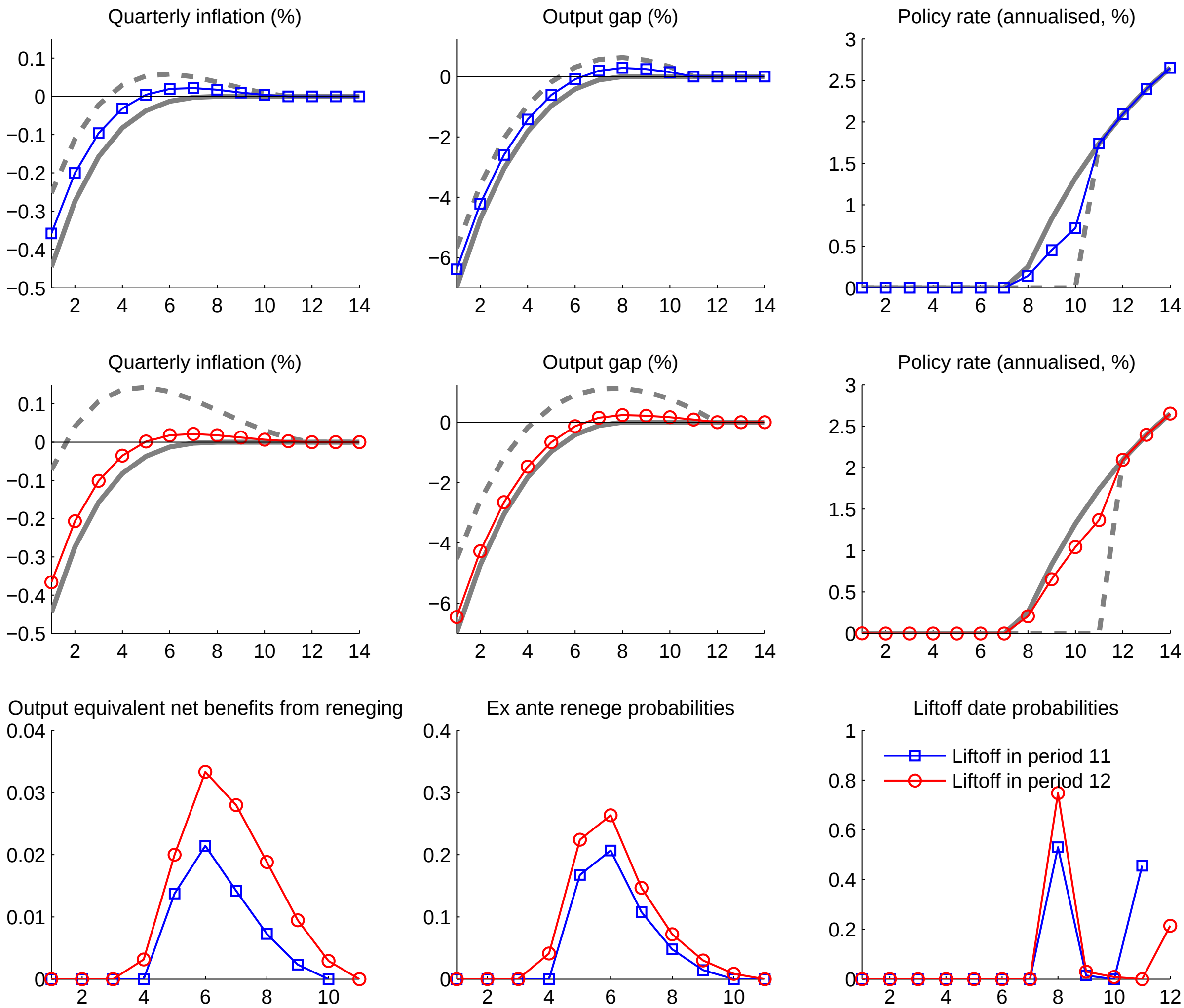
Figure B1: Mapping from net benefits to probability of reneging



Our baseline calibration assumes that $\alpha_2 = 2$, depicted by the black line in Figure B1. When $\alpha_2 = 1$ the distribution function initially rises more steeply so that, for relatively small benefits of reneging, the probability of reneging is higher. The converse is true for larger benefits of reneging, where the distribution function is flatter than the baseline calibration. When $\alpha_2 = 3$ the distribution function is more steeply S-shaped than the baseline case. For relatively small net benefits of reneging, the probability of reneging is smaller (and conversely for large net benefits). Because our experiments involve relatively small temptations to renege, we refer to the $\alpha_2 = 1$ calibration as the 'low credibility' case and $\alpha_2 = 3$ as 'high credibility'.

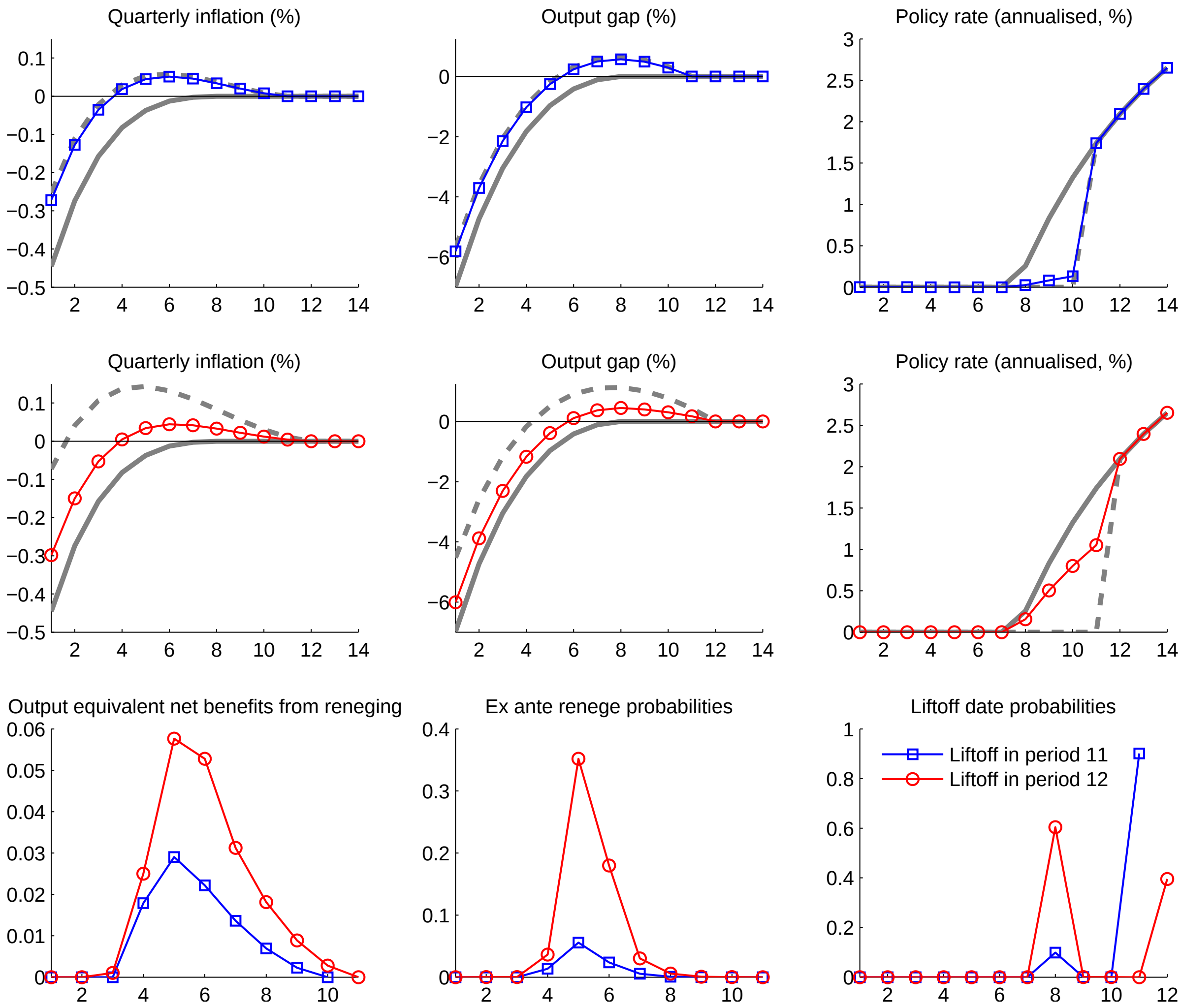
Figures B2 and B3 repeat the policy experiments of Section 3.4 for the low and high credibility cases respectively. The results are in line with intuition. In an environment of low credibility, both policy announcements are less effective than in the baseline case. Indeed, the bottom right panel of Figure B2 shows that the most likely liftoff date is period 8 in both policy experiments. The pattern of renege probabilities underlying the two simulations is very similar. Figure B3 shows that the high credibility calibration implies that a plan to liftoff in period 11 is almost fully credible (with a 90% chance of being fulfilled). Although a plan to liftoff in period 12 is more effective in the high credibility case, the most likely liftoff date remains period 8. The probability of reneging in period 5 is actually higher than the corresponding low credibility simulation, because the benefits of doing so are correspondingly higher given the greater stimulus imparted by the policy overall.

Figure B2: Effects of less credible policy announcements



Notes: The solid gray lines are the baseline responses to the recessionary scenario. The top row shows policies in which lift-off is delayed until period 11: the dashed gray line is the fully-credible case; the blue line with square markers shows expected outcomes under imperfect credibility. The middle row shows policies that delay lift-off to period 12: the dashed gray line is the fully-credible case; the red line with circle markers shows expected outcomes under imperfect credibility. The imperfect credibility cases are determined by the temptation to renege as in equation (20), with $\alpha_1 = 0.075, \alpha_2 = 1$.

Figure B3: Effects of more credible policy announcements



Notes: The solid gray lines are the baseline responses to the recessionary scenario. The top row shows policies in which lift-off is delayed until period 11: the dashed gray line is the fully-credible case; the blue line with square markers shows expected outcomes under imperfect credibility. The middle row shows policies that delay lift-off to period 12: the dashed gray line is the fully-credible case; the red line with circle markers shows expected outcomes under imperfect credibility. The imperfect credibility cases are determined by the temptation to renege as in equation (20), with $\alpha_1 = 0.075, \alpha_2 = 3$.

C Threshold-based forward guidance details

C.1 The model

In the threshold-based forward guidance application in Section 4 we use the linearized version of FRB/US supplied in the Macroeconomic Model Data Base (MMB) developed by Volcker Wieland (Wieland et al., 2012). With the minor changes we made to the model (detailed below), the model has 373 variables and 63 shocks.

The model structure is similar to the version of FRB/US documented and published by the Federal Reserve in 2014 (see, Brayton et al., 2014). However, the version made available more recently is not provided in linearized form (and so is not directly amenable to our approach without a costly linearizion process) and also includes supply side model changes that have been implemented in recent years. The version we use (dating to 2008) is likely to be closer to the version that would have been in active use at the Federal Reserve Board during the periods covered by our experiments in Section 4.

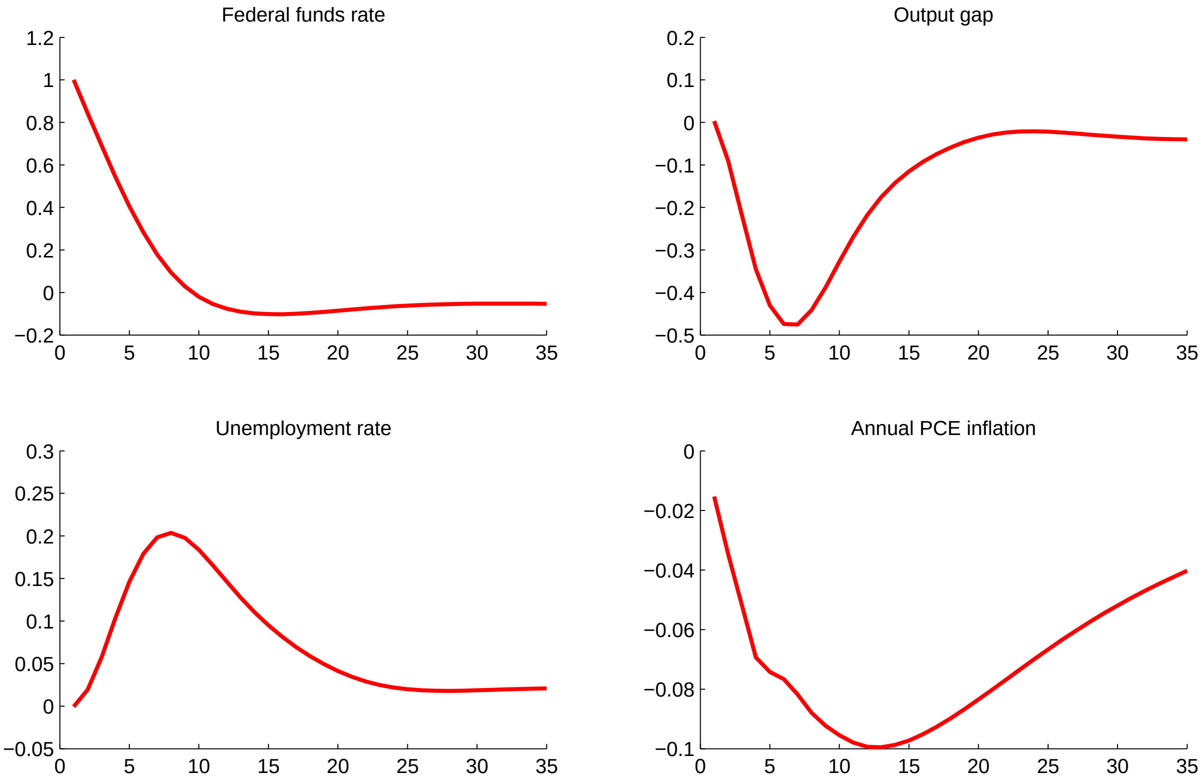
One amendment we make to this version of the model is the form and parameterization of the monetary policy rule. We use the following formulation:

$$i_t = 0.85 i_{t-1} + (1 - 0.85)\left[r_t^* + \pi_t + 0.5 \times (\pi_t - \bar{\pi}) + 1 \times x_t\right] \tag{C.1}$$

where i is the annualized effective federal funds rate, r^* is the natural real rate (exogenous), π is *annual* inflation and x is the output gap. This rule is used by Brayton et al. (2014).

Under this specification for monetary policy, the responses of key variables to an unanticipated one percentage point increase in the federal funds rate are plotted in Figure C1.

Figure C1: Responses to one percentage point shock to monetary policy rule

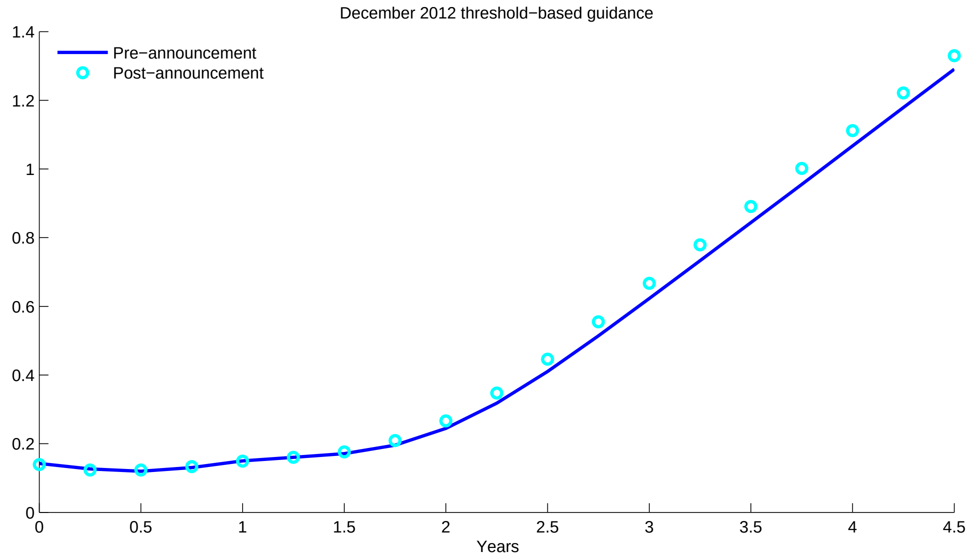


Notes: All variables are plotted as percentage point deviations from steady state.

C.2 Yield curve reaction to announcements

Figure C2 plots estimates of the US yield curve at the end of the day before and the day after the FOMC's December 2012 threshold-based forward-guidance announcement, as analyzed in Section 4.

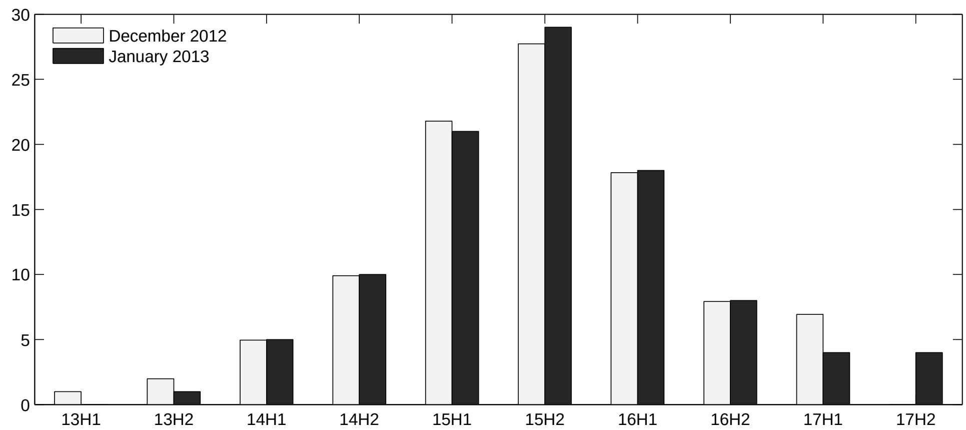
Figure C2: Estimates of market expectations of federal funds rate



Notes: Yield curves are computed from prices of government liabilities using the method of Anderson and Sleath (1999).

Figure C3 plots respondents' to the December 2012 and January 2013 Primary Dealer Surveys, conducted by the Federal Reserve Bank of New York, expectations for the first increase in the federal funds target rate.[42]

Figure C3: FRBNY Primary Dealer Survey Liftoff Date Expectations



Notes: This figure plots the percent chance primary dealers attached to the timing of the first federal funds target rate increase in the December 2012 (distributed: 29/11/2012; received by: 3/12/2012) and January 2013 Primary Dealer Surveys (distributed: 17/1/2013; received by: 22/1/2013). Source: Federal Reserve Bank of New York.

[42]The survey results are available here: `https://www.newyorkfed.org/markets/primarydealer_survey_questions.html`.

C.3 Constructing the baseline forecasts

To construct the baseline forecasts used in Section 4, we start from the observation that, as noted in Section 2.2, the rational expectations solution of the model can be written as:

$$x_t = Bx_{t-1} + \sum_{i=0}^{\infty} F^i \Phi z_{t+i}$$

The forecast is assumed to run from dates $T+1, \ldots, T+H$, with the initial condition x_T given. We also assume that the values of shocks beyond the forecast horizon are zero: $z_{T+H+j} = 0, \forall j \geq 1$. In that case we can use the rational expectations solution to write the forecast paths as:

$$\begin{bmatrix} \mathcal{I} & 0 & \ldots & 0 & 0 \\ -B & \mathcal{I} & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & \mathcal{I} & 0 \\ 0 & 0 & \ldots & -B & \mathcal{I} \end{bmatrix} \begin{bmatrix} x_{T+1} \\ x_{T+2} \\ \vdots \\ x_{T+H-1} \\ x_{T+H} \end{bmatrix} =$$

$$\begin{bmatrix} Bx_T \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \Phi & F\Phi & \ldots & F^{H-1}\Phi & F^H\Phi \\ 0 & \Phi & \ldots & F^{H-2}\Phi & F^{H-1}\Phi \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & \Phi & F\Phi \\ 0 & 0 & \ldots & 0 & \Phi \end{bmatrix} \begin{bmatrix} z_{T+1} \\ z_{T+2} \\ \vdots \\ z_{T+H-1} \\ z_{T+H} \end{bmatrix}$$

This representation implies that a sequence of anticipated shocks $\{z_{T+i}\}_{i=1}^{H}$ can be chosen to deliver a particular forecast for the endogenous variables $\{x_{T+i}\}_{i=1}^{H}$ that is fully anticipated by agents at date $T+1$. Finding the required values of the shocks can be done by matrix inversion and the method implemented by Burgess et al. (2013, Appendix C) provides a powerful and flexible toolkit for this task.

To implement the method we require assumptions about the desired forecasts to be matched (a subset of $\{x_{T+i}\}_{i=1}^{H}$) and the choice of the shocks to use to deliver those forecasts (a subset of $\{z_{T+i}\}_{i=1}^{H}$).

The variables we focus on are those for which the FOMC provides forecasts: real GDP growth, the unemployment rate, PCE inflation, core PCE inflation and the federal funds rate. The FOMC's Summary of Economic Projections (SEP) gives distributional information for the FOMC's forecasts for these variables at the end of the next three calendar years. The SEP reports ranges of forecasts rather than explicit moments. We use the midpoint of the reported 'central tendency' of forecasts. To create quarterly forecasts we linearly interpolate between the end-year forecasts. The SEP also includes information about the FOMC's view about longer-term trends. Again we take the midpoint of the central tendency as our measure of FOMC expectations. We interpolate the forecasts to reach the 'longer-term' forecasts at a horizon of five years.

The exception to this approach is the forecast for the federal funds rate, for which we use the median of the FOMC's 'dot plot' forecasts for the Fed funds rate at the end of the subsequent three calendar years and linearly interpolate between these points. We adjust the projection for the federal funds rate in 2015 to ensure that the forecast is consistent with the FOMC not lifting-off from the zero bound until the 6.5% unemployment threshold is breached.

In the context of our experiment, the shocks used to deliver the forecast are of little interest, since the properties of the experiments rely on the responses of the model to marginal changes in expected policy behavior. As long as the number of shocks is at least as large as the number of forecasts we wish to impose (and the mapping from these shocks to these variables via the Φ is non-degenerate) our results will be unaffected. We used all of the shocks in the model, excluding the oil price shock and the policy shock used to implement the experiments. This required us to add an additional shock to the monetary policy rule in order to deliver the forecast of the federal funds rate. We also added an additional shock to the mapping from aggregate demand to the unemployment rate in order to allow us to match the forecasts for both GDP growth and unemployment simultaneously.

C.4 Calibration of the liftoff probability function

To inform our calibration of the lift-off probability function, we use the January 2013 Primary Dealer Survey, the first conducted after the guidance was issued on 12 December 2012. This survey investigated respondents' views about the conditions under which the FOMC would lift off from the zero bound.

Question 12a of the survey asked respondents for their *modal* estimates of the joint outcomes for the unemployment rate and headline 12-month PCE inflation rate at the date of the first increase in the federal funds rate. For the unemployment rate, the median and 75th percentile responses were 6.5 percent indicating that the majority of respondents viewed the threshold as a trigger. However, the 25th percentile response was 6.25 percent so that (at least) 25 percent of respondents thought it *most likely* that lift-off would occur with the unemployment rate 0.25 percentage points or more below the threshold.

It is impossible to uniquely map from a survey distribution of heterogeneous beliefs about the modal rate of unemployment consistent with lift-off to uncertainty about policy as encapsulated in our f function (21). There are two reasons for that. First, the FOMC's guidance also included consideration (in the form of 'knock-outs') for the FOMC's own inflation forecast and broader inflation expectations measures. Modal beliefs about unemployment at lift-off could have differed because of alternative views about the inflation outlook (which are assumed to be identical in our setup). Second and probably more importantly, the survey measures disagreement in the most likely outcomes, not uncertainty (or disagreement about that uncertainty). Since we do not have any information about the individual respondents' uncertainty around their responses, we assume that the heterogeneity of beliefs about the modal unemployment rate at which lift-off occurs maps directly into policy uncertainty. It is likely that we understate policy uncertainty as a result of this because, although most respondents viewed the threshold as most likely being like a trigger, they may also have attached some probability to a more open-ended interpretation.

We calibrate the α_1 parameter in the mapping function (21) so that the probability that lift-off occurs when unemployment is more than 0.25 percentage points below the threshold is equal to 0.5. We choose a probability of 0.5 to match the fact that the median response in the survey was for lift-off at an unemployment rate of 6.5 percent and the 25th percentile response was 6.25% so that up to 50% of respondents thought that the most likely case was an unemployment rate of 6.25% (or less) at lift-off. The result is $\alpha_1 = \frac{-0.25}{\log 0.5} \approx 0.36$.

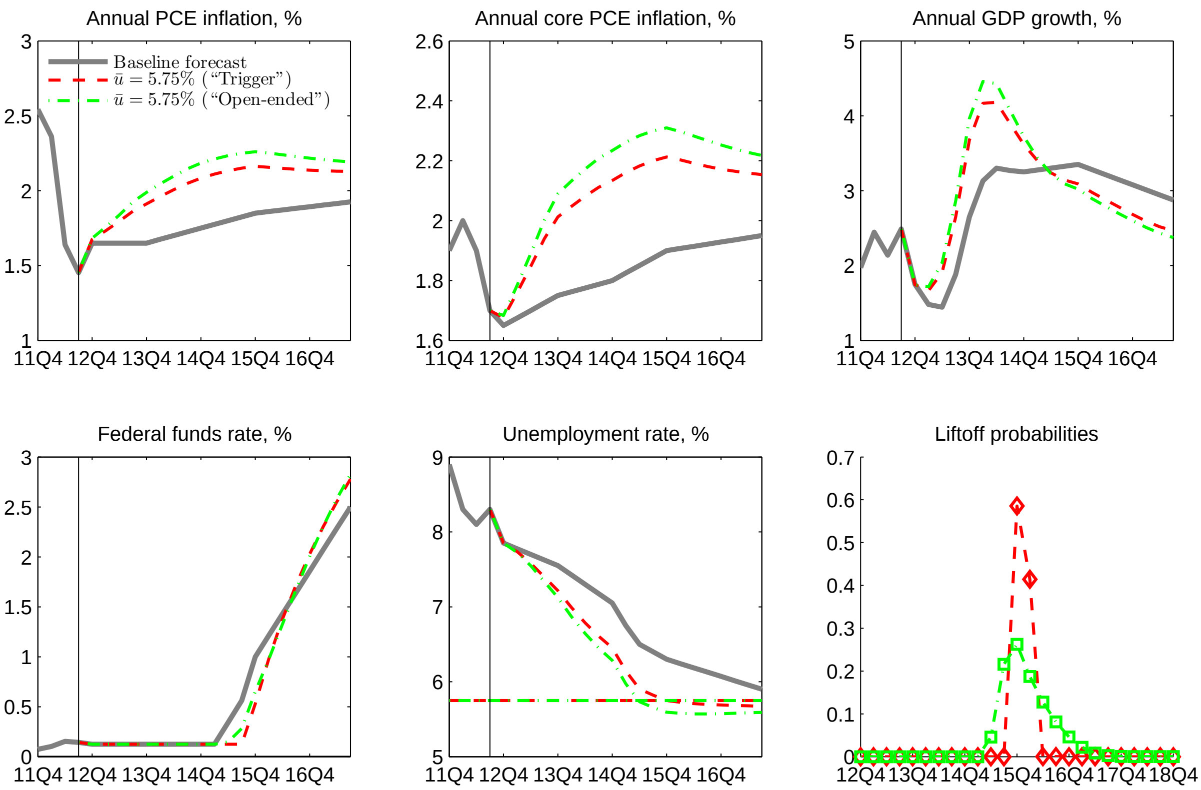
For our trigger-like calibration of equation (21) we set $\alpha_1 = \frac{-0.01}{\log 0.001} \approx 0.0014$. This implies that the probability of lift-off after breaching the threshold by 0.01 percentage points is 0.999.

C.5 Full comparison of threshold-based guidance policies

Section 4 uses a version of FRB/US to show that the macroeconomic effects of an "open-ended" threshold-based forward guidance policy with a 6 percent unemployment threshold are similar to those of a "trigger-like" policy with a 5.75 percent unemployment threshold. This appendix contains a full comparison of the two alternative thresholds under both treatments of the exit conditions. The motivation is to demonstrate that the results in Section 4 reflect the difference in policy uncertainty resulting from the difference in precision of the guidance about exit conditions, rather than being driven entirely by the different values for the unemployment thresholds.

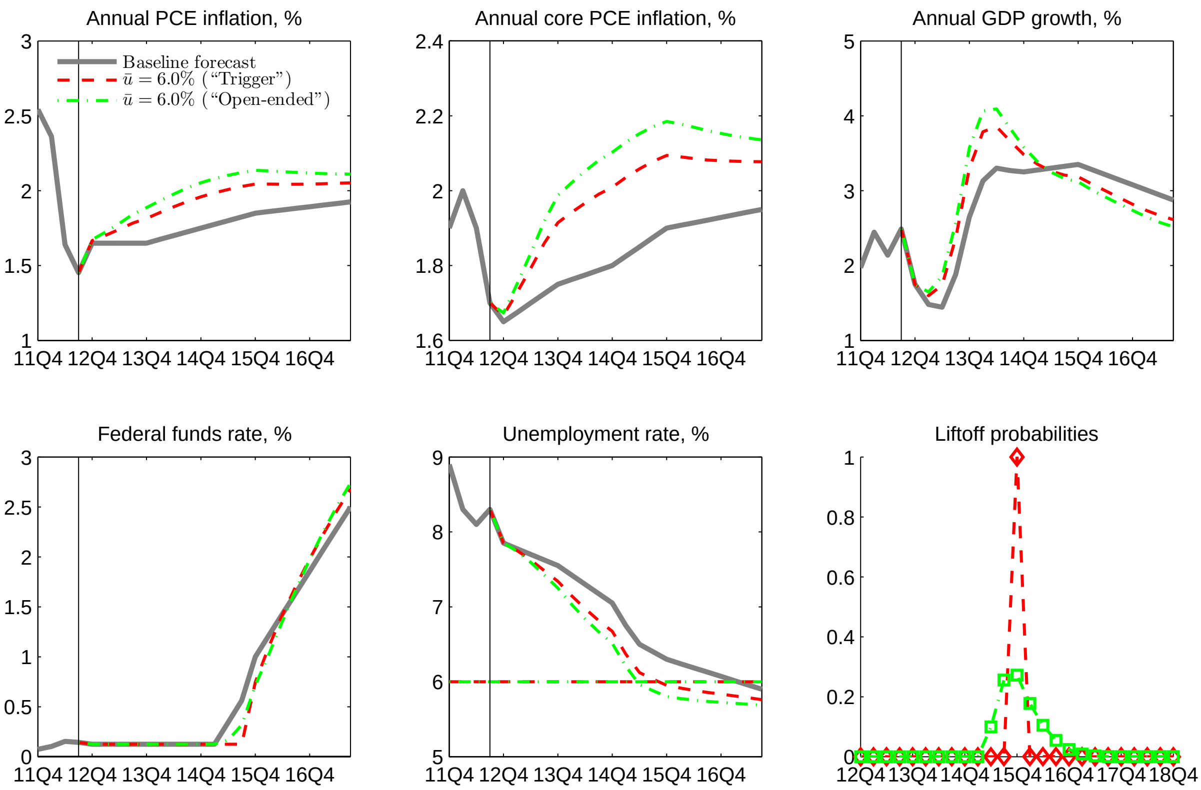
Figures C4 and C5 show the effects of the trigger-like (red dashed lines) and open-ended (green dash-dotted lines) policies for threshold values of 5.75 and 6 percent respectively. It is clear from the figures that the degree of precision in the guidance announcement has an impact on the effect of the policies, with the open-ended policies deliver greater stimulus. This reflects the greater probability attached to later lift-off dates lowering expected real interest rates and, hence, boosting demand. It is also clear from the figures that the lower threshold delivers greater stimulus than the higher threshold, conditional on the degree of precision in the guidance about exit conditions.

Figure C4: Forward guidance using FRB/US with a 5.75% unemployment threshold



Notes: The solid gray lines show the baseline forecast derived from the FOMC's December 2012 Summary of Economic Projections. For inflation rates, growth and unemployment these forecasts are interpolated through the the midpoint of the central tendency end-year forecasts. The baseline projection for the federal funds rate is constructed using the median of the FOMC's end-year projection. The red dashed lines show the expected effects of a policy in which the FOMC has announced that the federal funds rate will rise from the zero lower bound with near certainty when unemployment falls below 5.75 percent ("trigger"). The green dash-dotted lines show the expected effects of a policy with the same 5.75 percent threshold but with less certainty communicated about the exit conditions ("open-ended"). The probability of lift-off is linked to the unemployment rate by equation (21) and the lift-off probabilities are shown in the bottom right panel.

Figure C5: Forward guidance using FRB/US with a 6% unemployment threshold



Notes: The solid gray lines show the baseline forecast derived from the FOMC's December 2012 Summary of Economic Projections. For inflation rates, growth and unemployment these forecasts are interpolated through the the midpoint of the central tendency end-year forecasts. The baseline projection for the federal funds rate is constructed using the median of the FOMC's end-year projection. The red dashed lines show the expected effects of a policy in which the FOMC has announced that the federal funds rate will rise from the zero lower bound with near certainty when unemployment falls below 6 percent ("trigger"). The green dash-dotted lines show the expected effects of a policy with the same 6 percent threshold but with less certainty communicated about the exit conditions ("open-ended"). The probability of lift-off is linked to the unemployment rate by equation (21) and the lift-off probabilities are shown in the bottom right panel.