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Caterina Lepore,[1] Misa Tanaka,[2] David Humphry[3] and Kallol Sen[4]

Abstract

This paper examines how the interactions between the valuation regime and solvency requirements influence investment behaviour of long-term investors with stable liabilities, such as life insurers. Under limited liability, solvency requirements based on historical cost valuation encourage risk-shifting to the detriment of policyholders, while those based on fair value regime can induce procyclical asset sales. A hybrid valuation regime, intended to address these unfavourable outcomes, does not strictly dominate the other two regimes. But both fair value and hybrid regimes outperform the historical cost regime if the regulators can set the penalty imposed on insurers based on supervisory information about their asset quality, even if this information is imperfect.

Key words: Valuation, historical cost accounting, mark-to-market, risk-shifting, fire sales, prudential regulation, insurance.

JEL classification: M41, G28, G22.

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(1) Bank of England. Email: caterina.lepore@bankofengland.co.uk. Corresponding author.
(2) Bank of England. Email: misa.tanaka@bankofengland.co.uk
(3) Bank of England. Email: david.humphry@bankofengland.co.uk
(4) Bank of England. Email: kallol.sen@bankofengland.co.uk

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Publications and Design Team, Bank of England, Threadneedle Street, London, EC2R 8AH
Telephone +44 (0)20 7601 4030 email publications@bankofengland.co.uk

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I. Introduction

The distinct features of long-term investors – such as life insurers and pension funds – present a number of challenges in designing an appropriate valuation framework[1] for solvency regulation. Their liabilities (e.g. annuities) are typically long-term and stable in nature, and hence are not normally subject to 'runs' in the same way that bank deposits are. This feature give rise to a dilemma for designing the valuation framework for solvency regulation. On the one hand, these financial institutions may be able to conceal and accumulate losses on their assets for a long time if the valuation regime allows it. On the other hand, requiring them to value assets at market prices could make them prone to asset fire sales, when system-wide fall in asset prices brings them to the brink of breaching the market-based solvency ratio[2].

This paper examines how the valuation regimes used to define solvency requirements affect insurers' behaviour[3], using a simple theoretical model which incorporates conflicts of interest between policyholders and the management. We first demonstrate that, when the management does not internalise the losses borne by policyholders, both the historical cost (HC) and fair value (FV) regimes can lead to inefficient behaviour: whereas the HC regime allows insurers to hide losses and 'gamble for resurrection' at the expense of policyholders, the FV regime could make them prone to asset fire sales if insurers expect a heavy regulatory penalty for breaching the FV solvency constraint. We then examine whether a hybrid regime, which places only some weight on market values in calculating the solvency ratio, can overcome the dual problems of gambling for resurrection and asset fire sales, and establish that a hybrid regime cannot, by itself, solve these problems. We demonstrate, however, that both the FV and the hybrid regimes can outperform the HC regime, as long as the regulator can set the penalty for breaching the market-based solvency constraint based on supervisory information about the fundamental asset quality of the insurers – even if this information is imperfect. Our findings thus suggest that adjusting the valuation method, by itself, may not offer a solution to the problems associated with the fair value regime that were identified in the existing literature (e.g. Plantin et al. (2008), Ellul et al. (2015)).

Our analysis is particularly pertinent to the ongoing policy debate over the design of the regulatory valuation regime for life insurers. How to measure the assets and liabilities of life insurers has been a subject of debate through history. In the United Kingdom, the 1870 Life Assurance Companies Act mandated life insurers to produce periodic actuarial valuations of their assets and liabilities, but it allowed discretion in the valuation of assets, so that the use of market prices was common but not universal[4]. From the 1990s on, however, there was growing support for making greater use of market prices in valuing assets and liabilities of insurers, for two main reasons. First, there was a general movement in accounting standard-setting to enhance transparency by giving greater emphasis to the use of fair values. In the United States, the issue of the Statement of Financial Accounting Standards (SFAS) 115

[1] Whilst we use the words valuation and accounting interchangeably in the paper, we are referring throughout to valuation for solvency purposes rather than for the general purpose of financial reporting.

[2] Long-term investors with stable liabilities like insurance companies may typically be thought of as being able to behave counter-cyclically, in the absence of mark-to-market pressures, taking advantage of temporary price falls. Czech and Roberts-Sklar (2017) studied investors behaviour in the sterling corporate bond markets and found that UK insurers tended to behave counter-cyclically in the period 2011-16. Similar results have been obtained by Timmer (2017) for the insurance industry in Germany.

[3] From now on we will focus on life insurers, but the model is general enough to cover other financial institutions with long-lived risky assets and stable liabilities.

[4] We refer to Horton and Macve (1994) for a discussion on the development of life assurance accounting and regulation in the UK.

in 1993 required firms to measure securities that were not held to maturity at fair value[5] . International accounting standards moved in the same direction: the International Accounting Standards (IAS) 39 in 1998 followed by IAS 40 in 2000 increased the use of fair value accounting in financial instruments and investment properties, respectively[6]. Second, there were a number of high-profile failures of life insurers in several parts of the world from the 1990s onwards, including Executive Life Insurance Company (ELIC) in the United States, multiple life insurers in Japan, and Europavie in France, as the setting of guaranteed benefits to policyholders at high levels left them exposed to interest rate risks. Importantly, the perceived failure of life insurers to act promptly on the problem once it had been identified strengthened the case for fair value accounting in some jurisdictions, on the ground that it would have made the build-up of risks more visible and thus would have encouraged corrective actions earlier. In the United Kingdom, 'realistic' financial reporting requirements, based on market-consistency[7] and including expected future bonuses as a liability, were mandated for insurers following the near-collapse of Equitable Life Assurance Society, which was forced to close to new business in 2000.

But the flaws of the fair value (FV) regime were exposed during the 2007-9 global financial crisis. As asset prices fell across global markets, financial institutions that were forced to mark their assets to market dumped their assets to reduce leverage, thus causing asset prices to spiral down further[8]. Insurance regulators across the world responded to the financial crisis by exercising discretion.

For example, the falls in asset prices in 2007-9 could have caused certain UK insurers to breach their Individual Capital Guidance (ICG) set by the Financial Services Authority (FSA), which was responsible for financial regulations at that time. But the FSA explicitly reiterated its approach to breaches of ICG, stating that economic circumstances were "exceptional" and that "very occasionally" it was possible that a breach due to extraordinary market conditions would not lead to automatic regulatory action.[9] US agencies also took action in the wake of the 2008 crisis. In some states, statutory accounting practices set by the insurance regulator were revised in a way that allowed insurers to improve their capital positions. Both Life and Property & Casualty insurers benefited from an improvement in capital position in the years following 2008 as a result. In addition, following rating agencies' downgrade of non-agency residential mortgage-backed securities (RMBS) that would require increased capital charges for these assets, the National Association of Insurance Commissioners (NAIC) took the view that ratings had become overly pessimistic. As a result, a new methodology based on expected recovery values was introduced that had the effect of reducing capital requirements for 59% of the insurance industry's non-agency RMBS

[5]These standards form part of US Generally Accepted Accounting Principles (US GAAP). As well as being used by large non-mutual insurers in the US, the principles of US GAAP have been adopted by insurers in a number of other jurisdictions. Developments in US GAAP have in the past also influenced standard-setting by other accounting bodies.

[6]Use of international accounting standards was growing through this period, and international financial reporting standards (IFRSs) became mandatory for listed entities in the EU in 2005. As of 2017, 126 countries require the use of IFRS for entities that are either listed or otherwise publicly accountable.

[7]Market-consistent valuation is a type of economic valuation of assets and liabilities. It is based on the valuation of the future cash flows of the asset or liability allowing for the riskiness of those cash flows and the time value of money. It may use market prices, where these are available, or modelled-values that seek to approximate market prices. Adjustments may be made to observable prices in exceptional circumstances where it is deemed that they are not representative of fair value, for example where a sale is forced or similar distortions exist. We refer to `https://www.iaisweb.org/index.cfm?event=icp:getICPList&nodeId=25227&icpAction=listIcps&std_id=144&icp_id=18&showStandard=1&showGuidance=1&s=144` for more details.

[8]See, for example, Adrian and Shin (2010) for some evidence for investment banks.

[9]This is documented in a letter from Paul Sharma (Director, Wholesale & Prudential Policy, FSA) to Angela Knight (Chief Executive, British Bankers Association). See Financial Services Authority 2009, 'ICG and GENPRU 1.2.26R'. The letter is available at `http://www.fsa.gov.uk/static/pubs/other/letter_to_bba.pdf`

investments United States Government Accountability Office (2013). Regulatory changes as a result of extraordinary market conditions, whether to the valuation regime or some other part of the framework, were also made in Denmark, the Netherlands, Sweden and Switzerland (see the Bank of England and the Procyclicality Working Group (2014)).

The 2007-9 financial crisis also gave rise to the new academic literature, which questioned the desirability of fair value accounting. For example, Allen and Carletti (2008) argued that fair value accounting can lead to contagion between the banking and insurance sectors, as it induces insurance companies to sell their assets in response to a shock, and the resulting fall in asset prices can make otherwise healthy banks insolvent. Similarly, Plantin et al. (2008) developed a theoretical model to demonstrate that FV accounting could generate larger welfare losses than the historical cost (HC) accounting when applied to long-lived, illiquid and senior assets. Yet, their analysis makes a number of assumptions which may not be suitable for considering the behaviour of life insurers. For instance, they assume that the managers of financial institutions simply maximise the accounting values of their assets rather than the shareholder values, and decide on selling assets based on what they *expect* to get in the market, rather than based on observed market prices. Their framework also does not allow an analysis of how the accounting regime interacts with solvency regulation, and does not consider agency problems between shareholders and debt holders. Thus, it does not incorporate the possibility that HC accounting can generate other types of inefficiencies, such as risk-shifting, by allowing financial institutions to conceal losses.

Indeed, existing empirical studies suggest that the HC accounting has problems of its own: for example, Ellul et al. (2014) and Ellul et al. (2015) found that US insurers subject to the HC accounting were more prone to invest in risky assets and less likely to sell the downgraded assets during the global financial crisis, compared to those subject to fair value accounting. Recognition of the strengths and weaknesses of the FV versus HC regimes has also led to the exploration of hybrid valuation approaches in policy circles, including the matching adjustment and volatility adjustment[10] introduced under the Solvency II regime that codifies and harmonises insurance regulation within the European Union, and the 'market adjusted valuation' (MAV) approach under development by the International Association of Insurance Supervisors (IAIS). Thus, key outstanding questions are i) whether the hybrid approach – which combines some features of both FV and HC regimes by placing some weight on market prices in calculating the solvency ratio – dominates these regimes by reducing their inefficiencies, and ii) whether it eliminates the need for regulatory penalty to be calibrated on supervisory information, as was the case during the global financial crisis.

This paper contributes to this debate by examining how the interactions between the regulatory valuation regime and the solvency capital requirement influence the investment behaviour of 'life insurers'. Our model incorporates three key frictions. First, we assume that policyholders, due to asymmetric information, cannot monitor, and thus price in their policy premium, insurers' investment decisions. This feature gives rise to agency problems between the policyholders on the one hand, and the insurance companies' management and shareholders, on the other. Under limited liability, life insurers have the incentive to take excessive risks at the expense of uninformed policyholders by holding on to assets that

[10]The matching adjustment looks to address the balance sheet volatility that some insurers experience in the short term when using a market-based valuation approach. It is a specific adjustment to the discount rate that insurers are able to use to value certain predictable liabilities, for example, annuity payments, where asset cash flows closely match liability cash flows. The adjustment derives in part from yields on a firms own assets. The volatility adjustment is a countercyclical measure designed to operate in a similar way to the matching adjustment. The criteria for its use and the adjustment to discount rates differ to those of the matching adjustment, however, with the latter being based on prescribed, representative asset portfolios.

have suffered a deterioration in credit quality ('gambling for resurrection'). Second, we assume that market prices can deviate from fundamental asset values, both due to market illiquidity and other 'noisy' factors, such as over-optimism and -pessimism by other market participants. In our model, insurers hold common assets which are traded in illiquid markets, such that the sale of assets by one insurer imposes pecuniary externalities on other insurers by lowering the price at which others can sell their assets. We characterise 'asset fire sales' as a situation in which insurers sell their assets even when their market prices are below the levels consistent with the expected return on those assets. Finally, we assume that the regulator is imperfectly informed about the fundamental quality of the assets held by insurers.

We use the model to examine how life insurers might behave in response to a credit downgrade of their assets under the three alternative valuation regimes: historical cost (HC), fair value (FV) and a hybrid regime. Our analysis yields the following results. First, we demonstrate that the HC regime prevents asset fire sales but makes life insurers excessively prone to hold on to downgraded assets, even when the market is pricing them at levels that are higher than the expected return. While this behaviour maximises shareholders' expected returns under limited liability, this represents risk-shifting to policyholders whose expected losses increase as a result of insurers' refusal to sell their downgraded assets.

Second, we show that insurers' behaviour under a FV regime depends crucially on the regulatory penalty imposed on those that breach the solvency constraint. At one extreme, if insurers don't expect to be heavily penalised for breaching the solvency constraint, then the FV regime will generate the same 'gambling' behaviour as under the HC regime. By contrast, if insurers expect the regulator to impose a severe penalty for breaching the solvency constraint, then insurers under the FV regime will respond to negative asset price shocks with asset fire sales. Our paper is therefore closely related to the existing literature which focuses on how the valuation regimes affect agency problems (Burkhardt and Strausz (2009); Lu et al. (2011)).

Then the question is: can a market-based valuation regime solve the dual problems of risk-shifting and asset fire sales? Our paper makes two key contributions to this live policy debate. First, we demonstrate that a FV regime could eliminate both inefficiencies if the regulator can set penalties imposed on insurers that breach the solvency constraint based on supervisory information about their asset quality - even if this information is imperfect. We also demonstrate that a FV regime combined with appropriately calibrated regulatory penalty would encourage efficient investment decisions by insurers *ex-ante*, whereas the HC regime would encourage excessively risky investment decisions which increase expected losses for policyholders. These results are consistent with the empirical findings of Ellul et al. (2014) and Ellul et al. (2015) that insurers under the HC regime tended to invest in riskier asset portfolios than those under the FV regime.

Second, we demonstrate that a hybrid valuation regime, which places only some weight on the market valuation in calculating the solvency ratio, smoothes out some of the extremes of the other two regimes but does not, by itself, entirely solve the two inefficiencies. In fact, making the hybrid regime work effectively would still require regulatory penalty to be set based on supervisory information, for the simple reason that noisy asset prices can still cause insurers to breach the hybrid solvency constraint – albeit less frequently than under the FV regime – even if their asset portfolios are fundamentally sound. Plantin et al. (2008) asked: is marking-to-market a panacea or Pandora's box? Our answer is that it is unlikely that any valuation regime will offer a panacea, and so regulators should not expect adjustments in the valuation regime by themselves to solve the problems associated with the FV regime. Rather, there is a need to develop market-based solvency regimes to work well, by considering the interaction between valuation, capital requirements, and regulatory responses as whole[11].

The rest of the paper is organized as follows. Section II outlines the set-up of the model. Section III analyzes insurers' decisions on whether to sell or hold risky assets following an asset downgrade, and examines how their behaviour differs depending on the valuation method (HC, FV and hybrid) used to calculate the regulatory solvency ratio. Section IV studies how these valuation regimes affect the *ex-ante* asset allocation of forward-looking insurers that can anticipate solvency pressures. The last section concludes.

II. Model

The model consists of three time periods: $t = 0, 1, 2$. There is a continuum of *ex-ante* identical financial institutions, which we call life insurers, with a unit mass. Insurers are risk-neutral and maximise shareholders' payoff, given by the expected value of assets net of the expected payments to policyholders. However, insurers are protected by limited liability, thus they do not internalize policyholders' losses. Policyholders cannot monitor insurers' investment risk and cannot exert any discipline on them. This agency problem between policyholders and shareholders gives rise to a rationale for prudential regulation in our model[12].

At $t = 0$, each insurer has equity equal to E_0, and collects one unit of premium from policyholders in exchange for life insurance contracts[13]. More specifically, each insurance company promises to pay out to policyholders an amount D_1 at $t = 1$ and an amount D_2 at $t = 2$. Throughout the paper we assume that there is no uncertainty about D_1 and D_2. This is because certain types of insurance contracts, such as annuities and life-insurance contracts, have predictable payouts and we restrict our attention to policies which are not surrendable[14]. For example, the insurance contract can be thought as an annuity, paying policyholders that are still alive in both periods. In this case, we have that $D_1 = (1 - d)D$ and $D_2 = (1-d)^2 D$, where d is the death probability and D the annuity payout. Alternatively, a life insurance contract could promise an amount D upon the death of the insured person. In this case we would have $D_1 = dD$ and $D_2 = d(1-d)D$. In the rest of the paper we do not specify the form of insurance payments.

At $t = 0$ the insurer can invest the policy premium and its own equity, $A_0 = 1 + E_0$, in two types of securities: a short-term safe asset (e.g. government bond) yielding unitary return at $t = 1$, and a long-term risky asset (e.g. corporate bond) which matures at $t = 2$. Although very stylised, this asset allocation does capture some important features of life insurers' portfolio. In particular, the UK life sector[15] has assets of approximately £2 trillion, although approximately £1 trillion of these are assets

[11] Little academic work has been done towards this end. We refer to Douglas et al. (2017) for an analysis of life insurers' asset allocation and their propensity to act procyclically under Solvency II in a structural model accounting for different parts of the regulatory framework.

[12] This assumption is in line with other papers in the literature. For example, Plantin and Rochet (2009), present the absence of a "tough" claims-holder as a rationale for prudential regulation of insurers. See also Filipović et al. (2015), who examine the agency problem between shareholders and policyholders and risk-shifting. Other papers in the insurance literature have instead explored the implications of policyholders being informed about the risks of default and reflecting this in their demand and willingness to pay for insurance, taking an options pricing approach.
We refer for instance to Ibragimov et al. (2010).

[13] Note in our model insurance contracts' supply and demand are taken as given and we do not endogenize insurance price.

[14] It is not usually possible to surrender UK annuities. See `https://www.gov.uk/hmrc-internal-manuals/pensions-tax-manual/ptm062400` for further information on the definition of an annuity, as part of pension tax rules. Moreover, other contracts may have significant surrender penalties or tax implications that deter surrender.

[15] We refer to Bank of England, Prudential Regulation Authority (2017) for a comprehensive report on the UK insurance sector data under Solvency II over 2016.

used to back unit-linked liabilities (Figure 1), which are economically similar to mutual funds. For assets not held to back unit-linked liabilities the majority of assets are bonds, most of which are corporate bonds (Figure 2). Focusing on their bond portfolios, almost all securities are investment grade (Figure 3). However, UK life insurers have been increasing in recent years their holdings of illiquid assets, such as property-related non-linked assets and lower credit rated bonds[16]. Approximately 80% of their bonds have a maturity greater than 5 years, and 60% more than 10 years (Figure 4). Life insurers in the US holds similar assets, although they are more skewed towards the illiquid end of the market. As documented by Chodorow-Reich et al. (2016), bonds constitute $70\% - 75\%$ of their invested assets, of which more than 30% is represented by bonds to non-financial corporations[17] (Figure 5).

For simplicity, we assume that the risky asset yields $R > 1$ if there is no default and zero in case of default. Unlike the safe asset, which yields a unitary return in all states, the risky asset is subject to an adverse credit quality shock (in which case it is downgraded) at $t = 1$. If there is no adverse credit quality shock at $t = 1$, the risky asset yields R at $t = 2$ with probability one[18], while if there is a shock, it is expected to yield R only with probability $0 < z_1 < 1$. We assume that, as at $t = 0$, the probability of an adverse credit quality shock occurring at $t = 1$ is $p \in [0, 1]$, so the expected return of the risky asset is given by $[(1 - p) + pz_1]R$. Insurers can acquire both the risky and safe assets at unitary value at $t = 0$, but we assume that the risky asset yields a higher expected return, that is $[(1-p)+pz_1]R > 1$ (see Figure 6). Throughout we assume insurers have perfect information on their assets' payoff, including their credit quality following a downgrade (i.e. z_1).

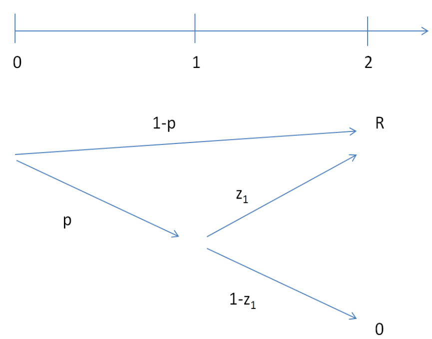


Figure 6. Payoff structure of the risky asset as at $t = 0$.

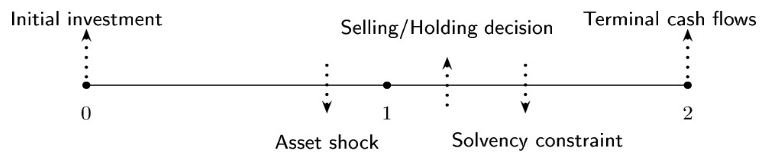
At $t = 0$, insurers need to invest enough in the safe asset to be able to pay out D_1 to policyholders at $t = 1$. Thus, in our model insurers have short-term liabilities that are perfectly matched with their short-term assets. For the moment, we take as given that every insurer invests a fraction b of its assets in the short-term safe security at $t = 0$, which is just sufficient to meet the $t = 1$ insurance payout:

$$b = \hat{b} \equiv \frac{D_1}{A_0}. \tag{II.1}$$

[16]As reported in Bank of England (2017) BBB and BB-rated holdings increased from 27% to 37% of insurers' total corporate bond holdings between 2009 Q4 and 2017 Q2.

[17]In the paper Chodorow-Reich et al. (2016) the choice of illiquid and risky assets by life insurers is explained by proposing and testing that they act as assets insulators. Because of their relatively stable liabilities, in the absence of regulatory pressures, life insurers are able to hold these assets for a long time, insulating their asset valuations from market fluctuations.

[18]Note, while this assumption has been made for simplicity, making the asset's payoff risky even in the absence of a downgrade would not change the main results of the paper.



Insurers invest the remaining fraction $1-b$ in the risky asset. Note that this is indeed the optimal asset allocation for insurers that do not anticipate the possibility of an adverse credit quality shock and any associated solvency pressures. In Section IV we will analyze the optimal initial asset allocation of forward-looking insurers and provide conditions under which this asset allocation continue to remain optimal. In order to set up the insurance company at $t = 0$, insurers need to satisfy the solvency constraint:

$$\frac{\text{Liabilities}}{\text{Assets under stress}} = \frac{D_1 + D_2}{bA_0 + c(1-b)A_0} \leq 1 \tag{II.2}$$

where $0 < c \leq 1$ represents the asset mark-down in stress, i.e. one minus the percentage change in the asset value calculated following a one-in-200-year shock[19]. The lower c, the tighter the regulatory capital requirement on risky assets relative to safe assets. We refer to the Appendix for a formal derivation of this constraint. Note that the above constraint is satisfied as long as insurers have enough initial equity:

$$E_0 \geq \frac{cD_1 + D_2}{c} - 1. \tag{II.3}$$

At $t = 1$, insurers observe the credit quality of the risky asset and decide whether to sell it at the prevailing market price (and obtain short-term, safe assets that yield unitary return at $t = 2$), or to hold it until maturity at $t = 2$. In the latter case, insurers are able to pay out policyholders at $t = 2$ only if the risky asset does not default. We assume that in that case the payoff of the risky asset is large enough to cover the policyholders' payout entirely:

$$R \geq \frac{D_2}{A_0 - D_1} \equiv X \tag{II.4}$$

For simplicity, we assume that, if there is no adverse credit quality shock at $t = 1$, there will be no downgrade and the market price of the risky asset will be its fair value R. Therefore, if there is no adverse credit quality shock, insurers do not have incentives to sell as the market prices simply reflect the fair value of the asset. We therefore assume that insurers hold the asset till maturity and pay back their policyholders at $t = 2$ the full amount that is due[20]. Thus, in the rest of this section we will focus on the more interesting case of the adverse asset quality shock.

When the risky asset suffers an adverse credit quality shock and is downgraded at $t = 1$, insurers will choose whether to sell it and obtain the safe asset, or hold it to maturity. We assume that insurers which want to sell form a queue, and the place of a given insurer in the queue is uniformly distributed in the interval $[0, \hat{s}]$ where $\hat{s}$ is the total fraction of insurers selling. In particular, insurers sell *sequentially*, and the s-th seller decides whether to sell or hold when its turn comes, depending on the price observed

[19] Since liabilities are fixed they do not change value under stress.

[20] To be precise insurers are indifferent between selling and holding the asset when there is no downgrade. In either case their payoff is $R - D_2$, that is the asset payoff net of the policyholders payments.

in the market. This assumption contrasts with that made by Plantin et al. (2008), who assumed that financial institutions decide *simultaneously* whether to sell or not depending on the *expected* (rather than observed) price obtained by selling[21]. In essence, financial institutions in Plantin et al. (2008) decide on trading before they observe the price which they can obtain. We think that this modeling strategy is more suitable for the analysis of banks, which face the possibility of a depositor run and therefore may have to act on the asset side in order to avoid that possibility. By contrast, life insurers' liabilities tend to be stable, and hence it is more appropriate to assume that they decide whether to trade or not depending on the actual price they can obtain by selling their assets.

Following the adverse credit shock, the market for downgraded assets becomes noisy and illiquid. Therefore, the market price will not only reflect the asset fundamental value, given by $z_1 R$, but also some short-term volatility. The price that the s-th seller can obtain for selling the downgraded risky asset at $t = 1$ is given by:

$$v_1(s) = \max(z_1 R + \varepsilon - \gamma s, 0) \tag{II.5}$$

where γ is the measure of market liquidity. The higher γ, the more illiquid the market, as the marginal sales by the insurers will have a larger negative impact on the market price. The term γs captures the pecuniary externalities that asset sales by one insurer imposes on others that hold similar assets. Note that (II.5) is the standard linear price impact function, as adopted in Plantin et al. (2008), with a floor of zero to ensure that prices are non-negative. In addition to market liquidity, the price differs from the fundamental value of the asset due to market noise, denoted by ε. This captures the feature that credit and liquidity spreads do not account for the entire bond spread[22].

It has been well documented that changes in market prices, particularly for fixed income securities, generally exceed changes that could be attributed to changes in credit risk. For instance, Webber and Churm (2016) have decomposed corporate bond spreads using a structural credit risk model and have shown that compensation for bearing non-credit related illiquidity risk have been a particularly important driver, particularly during the recent financial crisis. Furthermore, even after accounting for a liquidity premium, the residual component can be quite sizable, as shown in Figure 7 for sterling investment grade corporate bonds.

The residual can be interpreted as market noise, due to market participants' aggregate perceptions and confidence about the value of the assets and the financial health of the issuer. The mispricing could also arise because of their risk-preferences or their need to trade for other reasons, for instance because subject to different regulation. Therefore, ε represent the extent to which the security is over- or undervalued by other market participants. In the rest of the paper we interpret it as market pessimism or optimism, although we do not take a stand on what drives it. In particular, a negative value $\varepsilon < 0$ indicates excessive market pessimism (assets are undervalued) whereas $\varepsilon > 0$ indicates over-optimism (assets could be overvalued, depending on the liquidity premium). We assume that the market noise is uniformly distributed $\varepsilon \sim \mathcal{U}[\varepsilon^L, \varepsilon^H]$, where $\varepsilon^L < 0$ and $\varepsilon^H < (1 - z_1)R$. This last assumption implies that the market cannot overvalue the asset so much that the market price after the downgrade is higher than the asset's return in case of no downgrade. We denote the probability density function of ε with

[21] Specifically in their model the expected return depends on the noisy private signal they receive about the fundamental, and the trading strategies of other players that they infer from that noisy signal.

[22] In the model by Plantin et al. (2008) market prices are always below the assets' fundamental value, due to illiquidity and a discount factor accounting for the fact that buyers are second-best owner. By contrast in our model market prices can also be above the assets' fundamental value, if markets are over-exuberant. Effectively, ther may be a discount or a premium relative to the fundamental value.

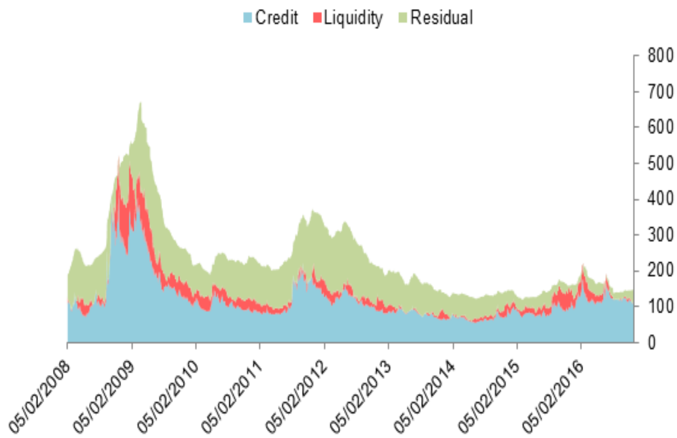


Figure 7. Estimates of the spread for credit risk, liquidity risk and noise for Sterling investment grade corporate bonds. Data source: Bank of England, ICE BofAML, and JP Morgan DataQuery.

$f(\varepsilon) = \frac{1}{\varepsilon^H - \varepsilon^L}$.

In the next section we examine insurers' decision to sell or hold the risky asset after a downgrade, under different valuation regimes. In Section IV we will then study the ex-ante asset allocation choice between a safe and risky strategy, given that insurers anticipate the possibility of a downgrade.

III. Interim asset sales

In the interim period, insurers observe whether the adverse credit quality shock has materialised. Given this information, insurers can decide whether to keep the asset until maturity or to sell it at the prevailing market price. When insurers decide to hold the asset to maturity, they can realize a strictly positive profit only if the asset does not default. Then, given limited liability, their expected profits from holding are:

$$z_1(RA_0(1-b) - D_2). \tag{III.1}$$

With probability $1 - z_1$ the asset defaults at $t = 2$ and policyholders lose entirely their payouts. By contrast, if insurers sell their assets, they can hold the proceeds in the form of safe asset which can be used to pay back policyholders, so that their payoff is given by:

$$[v_1(s)A_0(1-b) - D_2]^+. \tag{III.2}$$

In the absence of regulation, the lowest payoff that the sellers can end up with is zero, because the insurance company is wound down if it cannot meet its obligations towards its policyholders.

Under limited liability, insurers have incentives to 'risk-shift' to policyholders by taking excessive risks in the interim to benefit shareholders at the expense of policyholders. In our model, insurers can risk-shift by refusing to sell downgraded assets, rather than taking on risky projects as in the traditional models by Jensen and Meckling (1976). The risk-shifting problem we study is similar in spirit to the one analyzed in Diamond and Rajan (2011), in which a bank's manager avoids selling illiquid assets, even though such sales could save the bank. The intuition is that, as in our setting, by selling assets the manager would raise cash, which makes the bank's debt safer, but would also sacrifice the returns in case the asset recovers.

However, while Diamond and Rajan (2011) focus on liquidity shocks caused by depositors' withdrawals, we are interested in the impact of an adverse credit quality shock. This is because we consider insurers' liabilities to be more stable; and, therefore, illiquidity represents only a second-order issue relative to credit risk.

In order to reduce the problem of risk-shifting, the regulator requires insurers that decide to hold the risky assets to meet a solvency constraint in the interim period. Specifically, the $t = 1$ solvency constraint is:

$$\frac{D_2}{cV(A_0(1-b))} \leq 1 \tag{III.3}$$

that is the insurers' liabilities, i.e. policyholders' payout D_2 due at $t = 2$, must be lower than the value under stress of the remaining assets invested in the risky security, denoted by $V(A_0(1-b))$. The asset value will depend on the valuation regime adopted. In what follows, we consider the solvency regulation under three different valuation regimes: i) historical cost (HC), ii) fair value (FV), and iii) market-adjusted valuation (MAV).

We assume that the regulator imposes a penalty on those insurers that breach the solvency constraint at $t = 1$. We assume that the penalty is proportional to the size of their assets, $\rho * A_0(1-b)$, where $\rho > 0$. More concretely, ρ can be interpreted as the cost of the mandatory recapitalization or the cost of restrictions on new business ordered by the regulator. This can include the cost due to dilution of claims by the existing shareholders and the length of time given to recapitalize, or the effect on the value of the franchise arising from the restrictions on new business. In general, we think of ρ as a measure of the regulator's tolerance for breaches of the solvency requirement. In what follows, we examine how the regulatory parameters ρ and c affect the behaviour of insurers under different valuation regimes.

A. The socially optimal solution

Before analysing different valuation regimes, we first examine the socially optimal equilibrium which maximises the total expected payoffs of shareholders and policyholders in the insurance industry[23]. The analysis in this section provides a benchmark against which welfare implications of different valuation regimes can be assessed.

The expected return of shareholders at $t = 1$ if they hold the asset until maturity is given by (III.1). In this case, policyholders receive their final payouts only in case of no default, hence their expected payoff is $z_1 D_2$. By contrast, when insurers sell the assets, their shareholders' payoff is given by (III.2) and their policyholders receive:

$$\min(D_2, (1-b)A_0 v_1(s)), \tag{III.4}$$

that is, the policyholders receive entire amount due at $t = 2$ only if the sales proceeds exceed D_2. As all returns from assets accrue to policyholders and shareholders, the social welfare – given by the sum of payoffs of policyholders and shareholders – is simply captured by the expected value of insurers' assets, which is equal to $A_0(1-b)z_1 R$ when the assets are held to maturity, and is equal to their market value $A_0(1-b)v_1(s)$ when sold. Thus, a social planner will allow insurers to hold the asset to maturity, given

[23] Note that insurers' investment behaviour, such as fire sales, can be associated with negative externalities on other sectors. This is for instance accounted for by Allen and Carletti (2008), where a shock to the insurance sector propagates to the banking sector under fair value accounting. In this paper we do not model such externalities and focus our attention on the insurance sector only.

that a fraction s sold, only if the expected return on that asset is higher than the market value:

$$z_1 R \geq v_1(s), \tag{III.5}$$

Proposition III.1. ***(Socially Optimal Equilibrium)***

The socially optimal level of sales is given by:

$$s^* = \begin{cases} 0 & \text{if } \varepsilon < 0 \\ \frac{\varepsilon}{\gamma} & \text{if } 0 \leq \varepsilon \leq \gamma \\ 1 & \text{if } \varepsilon > \gamma \end{cases} \tag{III.6}$$

and the market price that prevails after insurers sell is given by:

$$v_1^* = \begin{cases} \max(z_1 R + \varepsilon, 0) & \text{if } \varepsilon < 0 \\ z_1 R & \text{if } 0 \leq \varepsilon \leq \gamma \\ z_1 R + \varepsilon - \gamma & \text{if } \varepsilon > \gamma \end{cases} \tag{III.7}$$

Proof. Given the planner's decision rule (III.5) and the asset price (II.5), the socially optimal choice is for no insurer to sell ($s^* = 0$) if $\varepsilon < 0$, as the market is under-valuing the asset in this case, and thus insurers can sell the asset only below its fundamental value. All insurers should sell ($s^* = 1$) if $\varepsilon > \gamma$, i.e. the market is overly optimistic and relatively liquid, such that all the insurers can liquidate their long-term assets at a profit. Finally, only a fraction π^s should sell if $0 \leq \varepsilon \leq \gamma$, that is the market is over-valuing the asset but is relatively illiquid, where in equilibrium:

$$\pi^s = \frac{\varepsilon}{\gamma}. \tag{III.8}$$

□

Thus, the socially optimal choice is for insurers to behave counter-cyclically, selling when the market is over-exuberant and relatively liquid while holding when market prices are depressed. By doing so insurers would act as market stabilizers, dampening inflated prices during market exuberance and protecting asset valuations from market fluctuations during downturns. Importantly, the socially optimal choice is independent of credit risk z_1. This implies that insurers should sell only when the market is underestimating the credit risk, regardless of the level of credit risk itself. Note that the socially optimal rule is not equivalent to the one that minimizes policyholders losses[24]. While the socially optimal rule never induces sales when they are detrimental for policyholders, it does lead to greater holdings of the risky asset relative to the behaviour that would minimize policyholders' losses.

B. Historical cost valuation regime

When the regulator adopts historical cost valuation in specifying the solvency constraint, insurers' assets at $t = 1$ are valued at their book value $A_0(1 - b)$ as long as they choose to hold on to them. It follows

[24]In our model the socially optimal equilibrium is characterised by maximizing the payments to policyholders and the returns to shareholders. Note that this can differ from the objective of a microprudential insurance regulator, which may only have the objective to protect payments to policyholders.

that, the regulator expects those insurers that decide to hold the long-term assets to maturity to meet the following constraint at $t = 1$:

$$X \leq c \tag{III.9}$$

Note that, any insurer that has satisfied the $t = 0$ constraint (II.2) automatically satisfies the $t = 1$ solvency constraint (III.9). This is because, when assets are measured at their book value, the heightened credit risk due to the downgrade is not reflected in their solvency positions. Therefore, under the historical cost regime, the solvency constraint is never binding in the interim period, and the insurers' decision problem is the same as under no regulation. That is, insurers will prefer to hold the asset, given that a fraction s sold, if:

$$\underbrace{z_1(R - X)}_{\text{Payoff when holding}} \quad \geq \quad \underbrace{[v_1(s) - X]^+}_{\text{Payoff when selling}} \tag{III.10}$$

Proposition III.2. ***(Historical Cost Equilibrium)***

Under the historical cost valuation regime the equilibrium fraction of sales is:

$$s^{HC} = \begin{cases} 0 \ \textit{if} \ \varepsilon < X(1 - z_1) \\ \pi^{HC} = \frac{1}{\gamma}[\varepsilon - X(1 - z_1)] \ \textit{if} \ X(1 - z_1) \leq \varepsilon \leq \gamma + X(1 - z_1) \\ 1 \ \textit{if} \ \varepsilon > \gamma + X(1 - z_1) \end{cases} \tag{III.11}$$

and the market price that prevails after insurers sell is given by:

$$v_1^{HC} = \begin{cases} \max(z_1 R + \varepsilon, 0) \ \textit{if} \ \ \varepsilon < X(1 - z_1) \\ z_1 R + \varepsilon - \gamma\pi^{HC} = z_1 R + X(1 - z_1) \ \textit{if} \ X(1 - z_1) \leq \varepsilon \leq \gamma + X(1 - z_1) \\ z_1 R + \varepsilon - \gamma \ \textit{if} \ \varepsilon > \gamma + X(1 - z_1) \end{cases} \tag{III.12}$$

Historical cost equilibrium results in excessive holding (insufficient sales) if $0 < \varepsilon < \gamma + X(1 - z_1)$.

Proof. Given the insurers' decision rule under the HC regime (III.10) and asset price (II.5), s-th insurer will hold as long as $\varepsilon \leq \gamma s + X(1 - z_1)$. Thus, no insurer sells if $\varepsilon < X(1 - z_1)$; $\pi^{HC} = \frac{1}{\gamma}[\varepsilon - X(1 - z_1)]$ sells if $X(1 - z_1) \leq \varepsilon \leq \gamma + X(1 - z_1)$; and all insurers sell if $\varepsilon > \gamma + X(1 - z_1)$: this yields (III.11). Plugging in (III.11) into (II.5), we obtain (III.12). Comparing (III.11) with (III.6), it is clear that $s^{HC} < s^*$ if $0 < \varepsilon < \gamma + X(1 - z_1)$. □

Note that under the historical cost valuation regime, insurers will sell their assets in the interim only when market conditions are relatively favourable and they can achieve a price above the fundamental value. Further, for given market conditions, insurers with high liabilities-to-assets ratios are the ones that prefer to excessively hold on to the risky assets rather than liquidating. This is in line with the standard risk-shifting motives: due to the implicit protection provided by limited liability, highly leveraged insurers prefer to hold on to the risky asset and bet on its recovery, rather than liquidating and securing cash to pay the policyholders.

Proposition (III.2) shows that solvency requirements based on historical cost valuation lead to sub-optimal level of asset sales by insurers after assets are downgraded. This in turn leads to overinflated asset prices when the market is over-optimistic. Note that holding the asset after the downgrade is particularly detrimental when credit risk is high, while it is less inefficient when credit risk remains low. In other words, the range of inefficient excessive holding is reduced the higher z_1 (i.e. lower credit risk).

Proposition III.3. *The historical cost regime achieves an outcome close to the socially optimal solution when the credit risk is low (z_1 is close to 1), while it deviates from the socially optimal solution when the credit risk is high (z_1 is close to 0) by generating incentives for excessive holding.*

Proof. From (III.11), we see that as $z_1 \to 1$, the solutions approach the socially optimal values given by (III.6), while it departs more sharply from the socially optimal solution as $z_1 \to 0$. □

C. Fair value regime

An alternative to historical cost valuation is to measure assets at their fair value, i.e. their market price at the interim date. When the fair value of assets is used to specify the regulatory solvency constraint, risky assets are valued at the prevailing market value even if the insurer decides to hold on to the assets. Thus, the $t = 1$ FV solvency constraint for those insurers who choose to hold the long-term assets to maturity, given that a fraction s sells, is given by:

$$\frac{X}{v_1(s)} \leq c \tag{III.13}$$

We assume that the insurer faces a regulatory penalty proportional to its assets, $\rho * A_0(1 - b)$, if it decides not to sell the assets and breaches this regulatory constraint as prices fall. By contrast, insurers that decide to sell can remain in operation if the sales proceeds exceed the amount due to policyholders at $t = 2$, while they are closed down otherwise. We assume that the regulator sets c and ρ at $t = 0$, but has imperfect supervisory information about the asset quality of the insurers. Specifically, we assume that, at $t = 0$, the regulator knows that z_1 is bounded, i.e. $z_1 \in [\underline{z}_1, \bar{z}_1]$. At $t = 1$, the regulator observes whether the downgrade has taken place or not, but does not receive more precise information about z_1 when the downgrade happens.

For a given c, we define the threshold $\bar{s}^{FV}$, such that if the aggregate sales exceeds $\bar{s}^{FV}(c)$, then all the remaining insurers that do not sell will breach the solvency constraint (III.13):

$$\bar{s}^{FV} = \frac{c(z_1 R + \varepsilon) - X}{c\gamma}. \tag{III.14}$$

Therefore, under the FV regime, insurers will prefer to keep holding the risky asset, given a fraction s sells, only if its expected payoff accounting for the penalty from breaching the solvency constraint (III.14) is higher than the proceeds from selling:

$$\underbrace{z_1(R - X) - \rho * I(s > \bar{s}^{FV})}_{\text{Payoff when holding}} \geq \underbrace{[v_1(s) - X]^+}_{\text{Payoff when selling}} \tag{III.15}$$

where $I(s > \bar{s}^{FV})$ is an indicator function which takes the value one if $s > \bar{s}^{FV}$ and zero otherwise. Unlike the historical cost regime, the FV solvency requirement can become binding in the interim period if the fraction of sales is large enough. This is because in illiquid markets prices are sensitive to aggregate sales, which are therefore reflected in the insurers' solvency adequacy under fair value. As a result, insurers' behaviour under the fair value regime depends crucially on the specifications of the regulatory parameters,

c and ρ. In particular, we define:

$$\varepsilon^{FV} \equiv \frac{X}{c} - z_1 R,$$

and

$$c^{FV} \equiv \frac{X}{z_1 R + (1 - z_1) X}. \tag{III.16}$$

Then the following proposition characterises the equilibrium under the fair value regime.

Proposition III.4. ***(Fair value equilibrium)***

Under the fair value regime the equilibrium fraction of sales, denoted by s^{FV}, is:

- *the same as under the historical cost regime ($s^{FV} = s^{HC}$), where s^{HC} is given by (III.11), when the FV constraint (III.13) does not bind for $s = s^{HC}$. Specifically, this is the case if $c > c^{FV}$ and $\varepsilon > \varepsilon^{FV}$. Under these assumptions, the resulting market price after the interim sales is given by (III.12).*
- *otherwise, if $\varepsilon < \varepsilon^{FV}$ or $c \leq c^{FV}$ the constraint (III.13) binds on the solution (III.11). Then in equilibrium*

  i) everyone sells ($s^{FV} = 1$) when the penalty is large $\rho > z_1(R - X)$. In this case the equilibrium market price is $v_1(1) = max(z_1 R + \varepsilon - \gamma, 0)$.

  ii) when the penalty is sufficiently low ($\rho \leq z_1(R - X)$), the fraction of sales is given by:

$$s^{FV} = \begin{cases} 0 \ \textit{if} \ \varepsilon < X(1-z_1) - \rho \\ \pi^{FV} \equiv \frac{1}{\gamma}[\varepsilon - X(1-z_1) + \rho] \ \textit{if} \ X(1-z_1) - \rho \leq \varepsilon \leq \gamma + X(1-z_1) - \rho \\ 1 \ \textit{if} \ \varepsilon > \gamma + X(1-z_1) - \rho \end{cases} \tag{III.17}$$

and the equilibrium prices are:

$$v_1^{FV} = \begin{cases} \max(z_1 R + \varepsilon, 0) \ \textit{if} \ \ \varepsilon < X(1-z_1) - \rho \\ z_1 R + X(1-z_1) - \rho \ \textit{if} \ X(1-z_1) - \rho \leq \varepsilon \leq \gamma + X(1-z_1) - \rho \\ z_1 R + \varepsilon - \gamma \ \textit{if} \ \varepsilon > \gamma + X(1-z_1) - \rho \end{cases} \tag{III.18}$$

Proof. See Annex. □

Observe that FV generally leads to at least as much, or more sales than the HC regime, thus eliminating some of the excessive holding generated by HC. However, this can come at the cost of generating excessive sales when the regulatory parameters are not set appropriately. More specifically, we now examine how the equilibrium under the fair value regime compares with the socially optimal equilibrium depending on the regulatory parameters[25] c and ρ.

Proposition III.5. ***(Welfare properties of the fair value equilibrium)***

Comparing the equilibrium under fair value to the socially optimal equilibrium, three possible cases can arise:

[25]Note that the equilibrium changes also relative to ϵ being lower or higher than ε^{FV}. However, since the market noise is not under the control of the regulator we ignore it in this discussion.

1. *Insurers with positive going concern values (i.e. $z_1 R \geq X$) implement the socially optimal solution under the fair value regime for all ε, if the regulatory parameters are set appropriately. That is $c \leq c^{FV}$ and the penalty ρ for breaching the solvency constraint is set equal to:*

$$\rho = X(1 - z_1). \tag{III.19}$$

2. *The fair value regime generates excessive asset sales by the insurers if:*
   - *i)* $c \leq c^{FV}$ *and* $\rho > z_1(R - X)$, *for* $\varepsilon < \gamma$.
   - *ii)* $c \leq c^{FV}$ *and* $X(1 - z_1) < \rho < z_1(R - X)$, *for* $X(1 - z_1) - \rho \leq \varepsilon < \gamma$ *and* $z_1 R \geq X$;

3. *The fair value regime generates excessive holding if:*
   - *i)* $c \leq c^{FV}$, *and* $\rho < \min(X(1-z_1), z_1(R-X))$ *in the parameter range* $0 < \varepsilon < \gamma + X(1-z_1) - \rho$. *Nonetheless, FV improves on the equilibrium under HC by reducing excessive holding.*
   - *ii)* $c > c^{FV}$, *the equilibrium under FV coincides with the one under HC, and represents excessive holding when* $\varepsilon > \varepsilon^{FV}$ *and* $0 \leq \varepsilon \leq \gamma + X(1 - z_1)$.

Proof. See Annex. □

Note that, in general there is a trade-off between two possible inefficient behaviours: risk-shifting through excessive holding, and fire sales due excessive selling. Specifically, when the penalty from breaching the solvency constraint is moderate, FV generates excessive holding similarly to HC, while when the penalty is relatively large, FV reduces excessive holding but at the cost of generating excessive sales.

However, the above results suggest that the regulator can induce all solvent insurers to behave in a socially optimal way if it has perfect information about insurers' asset quality z_1 and can set regulatory parameters such that $c \leq c^{FV}$ and $\rho = X(1 - z_1)$. Thus, the first best outcome can be achieved if the regulator can commit to penalise the insurers that have breached the solvency constraint depending on the fundamental asset quality. In particular, (III.19) implies that the regulator should treat those insurers with relatively high asset quality (high z_1) more leniently than those with low asset quality (low z_1).

More importantly, it is also possible to show that, even when the regulator is imperfectly informed, the regulatory parameters can be set in such a way to ensure that the FV regime outperforms the HC regime.

Proposition III.6. ***(FV can achieve higher social welfare than HC)***

Suppose that the regulator cannot perfectly observe z_1 but knows at $t = 0$ that the credit quality is range-bound, such that $z_1 \in [\underline{z}_1, \bar{z}_1]$. Then it is possible to calibrate regulatory parameters c and ρ to ensure that the FV regime dominates the HC regime. Specifically, FV always dominates HC if the regulator sets $c = \frac{X}{\bar{z}_1 R + (1-\bar{z}_1)X}$ *and* $\rho = \min(X(1 - \bar{z}_1),\, \underline{z}_1(R - X))$.

Proof. See Annex. □

The above proposition suggests that the regulator can eliminate the inefficient excessive sales that can arise under FV, as long as it can set the regulatory penalty ρ based on its supervisory information about the fundamental state of insurers' balance sheets. It is not necessary for the regulator to observe the fundamental quality of insurers' assets, z_1, precisely. Instead, if the regulator knows that z_1 is range bound, it is possible to calibrate the regulatory parameters to ensure that the FV regime dominates the HC regime. In particular, the penalty should be set as the minimum between the lowest expected policyholders' losses from the insurers' assets defaulting, and the lowest shareholders' expected payoff from holding the assets.

D. *Policyholders' losses*

So far we have compared the equilibria under the two valuation regimes to the one that is socially optimal, and in terms of asset sales and market prices. However, important real effects determined by insurers' behaviour are the losses for their policyholders. It is not clear a priori which regime performs better in this respect. While in general the risk-shifting behaviour generated under historical cost is detrimental for policyholders, when insurers are forced to sell at depressed prices they could impose losses on policyholders as well.

In particular, when insurers *hold* the asset, policyholders lose the entire insurance payout due at $t = 2$ if the asset defaults. Hence, the total expected amount of losses, as at $t = 1$, is:

$$L^{Hold} = (1 - z_1)D_2$$

Note that when the asset does not default, policyholders get paid the entire benefit and do not incur any losses, since by assumption (II.4) $R > X$. By contrast, when a fraction s of insurers *sell* the asset, policyholders incur losses if the sales proceeds are not enough to cover the entire policy payout. In this case, the expected policyholders' losses are:

$$L^{Sell}(s) = \max\{D_2 - A_0(1 - b)(z_1 R + \varepsilon - \gamma\frac{s}{2}), 0\}.$$

It is important to note that, unless all insurers are forced to sell, market prices remain high enough such that policyholders do not incur any losses. This is because in our model insurers sell sequentially and they can observe the market price. Hence, they would never sell at a loss, unless forced by regulatory solvency pressure. However, when the penalty is large enough to force everyone to sell, excessive sales can generate strictly positive losses for policyholders. Specifically, we have that

$$L^{Sell}(1) = D_2 - A_0(1 - b)(z_1 R + \varepsilon - \frac{\gamma}{2}) > 0,$$

when $\varepsilon < X - z_1 R + \frac{\gamma}{2}$, i.e. the market is relatively pessimistic and thus under-valuing the asset.

It follows that the expected losses at t=1 under HC are:

$$L_1^{HC} = \begin{cases} L^{Hold}, \text{if } \varepsilon < X(1 - z_1) \\ (1 - \pi^{HC})L^{Hold}, \text{if } X(1 - z_1) \leq \varepsilon \leq \gamma + X(1 - z_1) \\ L^{Sell}(1) = 0, \text{if } \varepsilon > \gamma + X(1 - z_1) \end{cases} \tag{III.20}$$

which coincides with the expected losses under FV when $c > c^{FV}$ and $\varepsilon > \varepsilon^{FV}$. By contrast, when $c < c^{FV}$ or $\varepsilon < \varepsilon^{FV}$, and $\rho > z_1(R - X)$, then the expected policyholder losses under FV are equal to:

$$L^{FV} = L^{Sell}(1) = \max(D_2 - A_0(1 - b)(z_1 R + \varepsilon - \frac{\gamma}{2}), 0).$$

But if $c < c^{FV}$ or $\varepsilon < \varepsilon^{FV}$, and $\rho < z_1(R - X)$, then the expected policyholder losses under FV are given by:

$$L^{FV} = \begin{cases} L^{Hold}, \text{if } \varepsilon < X(1-z_1) - \rho \\ (1-\pi^{FV})L^{Hold}, \text{if } X(1-z_1) - \rho \leq \varepsilon \leq \gamma + X(1-z_1) - \rho \\ L^{Sell}(1) = 0, \text{if } \varepsilon > \gamma + X(1-z_1) - \rho \end{cases} \tag{III.21}$$

Comparing the expected losses when the two regimes differ, we have the following result.

Proposition III.7. *When the regulator sets regulatory parameters c and ρ as in Proposition (III.6), then the policyholders losses under the FV regime are less than, or equal to, those under the HC regime.*

Proof. See Annex. □

Therefore, we can confirm that, in general, the FV regime can reduce losses to policyholders relative to the HC regime, as long as the regulatory parameters are appropriately calibrated. This is because insurers are prone to hold the risky assets even when they should be selling, to the detriment to the policyholders who may end up suffering a larger loss when the insurer ultimately defaults on its annuity obligation.

E. Market Adjusted Valuation

From the previous sections we can conclude that both fair value and historical cost can generate inefficiencies. On the one hand, historical cost accounting enables insurers to hide the deterioration in the fundamental values of the assets they hold, and thus discourages them from selling the non-performing assets even when the market is offering relatively favorable prices. The reluctance of insurers to sell assets in turn leads to overinflated asset prices and larger policyholders losses. On the other hand, when insurers are subject to a solvency requirement which is based on fair value accounting, excessive asset sales and depressed asset prices are possible when the regulatory penalty for breaching the solvency constraint is set too severely.

Recognizing these problems associated with pure historical cost or fair value valuation regimes, policymakers have and are currently considering 'hybrid' approaches, which combine elements of the two regimes. In the European Union, the Solvency II provisions on the matching adjustment and volatility adjustment are intended to mitigate the effect of short-term price movements on assets held longer term to meet liabilities to policyholders. For instance, Bank of England (2015) explain the role of the matching adjustment in the following way: "annuity writers often invest in long-term, illiquid bond assets that pay cash flows equivalent to the amount owed to policyholders. In this scenario, the annuity writers have matched their cash in-flows with their payments to policyholders and are only exposed to the risk of the bond issuer defaulting. The matching adjustment (MA) looks to address the balance sheet volatility that some insurers experience in the short term when using a market-consistent approach. It is a specific adjustment to the discount rate that insurers will be able to use to value certain predictable liabilities, for example, annuity payments. When a higher discount rate is used to calculate the present value of a firms expected future claims, the present value falls. By adjusting the liability side of the balance sheet, this retains the market-consistent valuation of assets, while reducing the impact of asset-price fluctuations on the balance sheet."

In the international setting, IAIS consulted on a market adjusted valuation approach in 2016, as one of the valuation approaches explored in its development of an international Insurance Capital Standard

(ICS)[26]. According the IAIS, ICS Principle 7 requires a valuation approach that prompts supervisory attention when appropriate. Such supervisory attention should not over-emphasise volatility that does not affect the solvency of an IAIG (Internationally Active Insurance Group). Prudentially sound behaviour by IAIGs is promoted where the ICS does not encourage IAIGs to take actions in a stress event that exacerbate the impact of that event (for example fire sales of assets) or to focus on short term goals to the detriment of appropriate long term objectives. Stability in valuation is important in that context[27].

Our setting is a fairly general application of adjustments to fair value, but does not exactly replicate the features of any specific regime, current or in development. Hence, in the following analysis, we refer to market adjusted valuation as a general adjustment to the fair value approach, rather than being specific to any existing regulatory regime. Broadly speaking, the market adjusted valuation (MAV) aims to reduce the impact of 'noise' in market prices, i.e. price movements that are not reflective of changes in default risk of the asset, on regulatory solvency constraints by adjusting the discount rate on insurers' liabilities commensurately. In principle, changes in asset prices not attributed to credit risk should not impact insurers' solvency, so long as the insurer does not need to sell the asset. Discounting liabilities' at a higher rate when asset values fall, causing liabilities to fall similarly, would alleviate (at least in part) solvency pressure.

Hence, the key question for MAV is how to design the discount factor on insurers' liabilities, in order to adjust for the fluctuations in market prices which affect the asset side of insurers' balance sheets. As we have observed previously, the FV regime can achieve the optimal outcome as long as the regulator can distinguish the fluctuations in fundamental credit risk from noise, and set the penalty for breaching the solvency constraint appropriately. Thus, the relevant question is whether a MAV-based solvency regime dominates a FV-based solvency regime when the regulator is not fully informed about whether the fluctuations in asset prices reflect noise or changes in fundamentals. This section examines how such a regime might perform relative to FV and HC regimes.

One approach is for MAV to discount liabilities at a rate which is proportional to the asset's total spread. The proportion, which we denote by $\theta \in [0, 1]$ is a constant set by the regulator. The deducted proportion of the spread should capture the asset credit risk, and the remaining proportion the price movements that are not reflective of changes in default risk. In this way the discount factor would alleviate solvency pressures due to market volatility and noise while keeping the pressure due to an increase in credit risk. Such an approach would better reflect the long-term nature of insurance liabilities, that allows the insurers to earn part of that spread risk-free.

In our stylized framework, we define the liabilities discount rate as:

$$\delta \equiv 1 + \theta\alpha \tag{III.22}$$

that is the risk-free rate, normalized to one, plus a proportion of the total asset spread[28] α:

$$v_1(s) = \frac{R}{1+\alpha} \Leftrightarrow \alpha \equiv \frac{R - v_1(s)}{v_1(s)}.$$

[26]IAIS (2016) Risk-based Global Insurance Capital Standard ICS Consultation Document, https://www.iaisweb.org/page/supervisory-material/insurance-capital-standard

[27]Page 29 IAIS (2016) ibid.

[28]Note that since $\varepsilon \leq (1 - z_1)R$, we have that $v_1(0) \leq R$, hence the spread is always positive.

It follows that the solvency constraint under MAV can be written as:

$$\frac{X}{\delta v_1(s)} \leq c. \tag{III.23}$$

As we would expect, since $\delta \geq 1$, the MAV constraint (III.23) relaxes the FV constraint (III.13) for any given c, except in the special case when there is no discount of liabilities, i.e. $\delta = 1$ and the MAV constraint coincides with the FV constraint. Substituting (III.22) in (III.23), we can rewrite the MAV solvency constraint as follows:

$$\frac{X}{\theta R + (1-\theta) v_1(s)} \leq c \tag{III.24}$$

Thus, under MAV, the regulator is basically evaluating assets as a weighted average of their market price $v_1(s)$ and their final return in the absence of default R, where the weight on the final return is given by θ. In what follows, we do not make any restrictions on the value of θ, but we will show with some numerical examples how the regime performs for different choices of this parameter. Define the threshold $\bar{s}^{MAV}$ such that, for any given c, if the aggregate sales exceeds $\bar{s}^{MAV}$, then all the remaining insurers that do not sell will breach the solvency constraint (III.24):

$$\bar{s}^{MAV} = \frac{(1-\theta)(z_1 R + \varepsilon) + \theta R - \frac{X}{c}}{\gamma(1-\theta)}. \tag{III.25}$$

Note that $\bar{s}^{MAV}(c) < 0$ if $\varepsilon < \varepsilon^{MAV} \equiv \frac{1}{(1-\theta)}\left[\frac{X}{c} - (1-\theta) z_1 R - \theta R\right]$. Under this condition, when the market is sufficiently pessimistic, the MAV constraint binds for all insurers even when no one sells. It follows that, as under the FV regime, under MAV insurers will prefer holding the assets, conditional on a fraction s selling, if:

$$\underbrace{z_1(R-X) - \rho * I(s > \bar{s}^{MAV})}_{\text{Payoff when holding}} \geq \underbrace{[v_1(s) - X]^+}_{\text{Payoff when selling}}.$$

Define:

$$c^{MAV} \equiv \frac{X}{\theta R + (1-\theta)\left[z_1 R + (1-z_1) X\right]} \tag{III.26}$$

then the equilibrium under MAV can be described as follows.

Proposition III.8. ***(Market Adjusted Valuation Equilibrium)***

Under market adjusted valuation the equilibrium fraction of sales, denoted by s^{MAV}, is:

- *the same as under the historical cost regime ($s^{MAV} = s^{HC}$), where s^{HC} is given by (III.11), when the MAV constraint (III.23) does not bind for $s = s^{HC}$. This occurs if $c > c^{MAV}$ and $\varepsilon > \varepsilon^{MAV}$.*
- *otherwise, when $\varepsilon < \varepsilon^{MAV}$ or $c < c^{MAV}$ the constraint (III.23) binds on the HC solution. Then the equilibrium solution is the same as under the fair value regime:*
  - *i) when $\rho > z_1(R-X)$ everyone sells ($s^{MAV} = 1$) and the market price is $v_1^{MAV} = \max(z_1 R + \varepsilon - \gamma, 0)$.*
  - *ii) when $\rho \leq z_1(R-X)$, the equilibrium fractions of sales and market prices are given respectively by (III.17) and (III.18).*

Proof. See Annex. □

Note that analogously to the FV regime, MAV can replicate the social planner's solution by setting the regulatory parameters appropriately for those insurers with positive going concern values ($z_1 R > X$): $c \leq c^{MAV}$ and $\rho = X(1 - z_1)$. Apart from this particular case, when the regulatory parameters are set too loose or too tight, MAV generates inefficient behaviours as under HC or FV respectively. Specifically, three different cases can arise.

First, the *three regimes generates the same equilibrium* when $c > c^{FV}$ and $\varepsilon > \varepsilon^{FV}$[29]. This is because the FV and MAV constraints do not bind in this case, and thus all the regimes generate excessive holding (for $0 < \varepsilon < \gamma + X(1-z_1)$). Second, *MAV coincides with HC but deviates from the FV equilibrium* when the solvency constraint is set loose enough, i.e. $c^{MAV} < c < c^{FV}$ and $\varepsilon^{MAV} < \varepsilon < \varepsilon^{FV}$. Then MAV generates excessive holding (for $0 < \varepsilon < \gamma + X(1 - z_1)$) and is dominated by FV. FV dominates MAV (and HC) as long as the penalty is set low enough $\rho \leq \min(X(1 - z_1), z_1(R - X))$, as FV reduces excessive holding. However, if the penalty from breaching the requirement ρ is set too high ($\rho > \min(X(1-z_1), z_1(R-X))$), it is not clear whether MAV (and HC) outperforms or underperforms FV: whereas FV generates excessive sales, HC generates excessive holding. MAV eliminates the problem of excessive sales generated by FV but only by re-introducing the problem of excessive holding under HC. Proposition III.3 implies that MAV (and HC) would be expected to outperform FV if credit risk is relatively low, and there is limited incentives for insurers to risk-shift to policyholders.

Finally, *MAV and FV coincide, but both deviate from the HC equilibrium* if the solvency ratio constraints are set relatively tightly, i.e. $c < c^{FV}$ (or $\varepsilon < \varepsilon^{FV}$), and $c < c^{MAV}$ (or $\varepsilon < \varepsilon^{MAV}$). In this case MAV and FV outperform HC if the penalty is not too high ($\rho < \min(X(1 - z_1), z_1(R - X))$) since they reduce excessive holding[30]. By contrast, both regimes generate excessive sales if $\rho > X(1 - z_1)$, in which case no regime strictly dominates the others, as shown in the figure below.

Taken together, we can state the following proposition for MAV, which is analogous to the Proposition (III.6) above for the FV regime:

Proposition III.9. *Suppose that the regulator cannot perfectly observe z_1 but knows at $t = 0$ that the credit quality is range-bound, such that $z_1 \in [\underline{z}_1, \bar{z}_1]$. Then it is possible to calibrate regulatory parameters c and ρ to ensure that the MAV regime dominates the HC regime. Specifically, MAV always dominates HC if the regulator sets $c = \frac{X}{\theta R + (1-\theta)[\bar{z}_1 R + (1-\bar{z}_1)X]}$ and $\rho = \min(X(1 - \bar{z}_1),\ \underline{z}_1(R - X))$.*

Proof. This is obvious given the proof of Proposition (III.6), and Proposition (III.8). □

In conclusion, MAV simply switches between FV and HC depending on whether the solvency constraint becomes binding, and hence does not simultaneously outperform both. The above proposition implies that, even with imperfect supervisory information about asset quality, the regulator can set parameters in order to ensure that the MAV regime dominates the HC regime. Note that, as we have discussed in Section C, this is also the case under FV under analogous conditions.

By contrast, when the regulator cannot calibrate the regulatory parameters appropriately, both MAV and FV generates inefficiencies. The MAV regime can reduce the excessive sales generated under FV by

[29] Note that $c^{FV} > c^{MAV}$ and $\varepsilon^{FV} > \varepsilon^{MAV}$ (this is true because $c > \frac{X}{R}$ given that insurers satisfy $t = 0$ solvency constraint (II.2)). Therefore $c > c^{FV}$ and $\varepsilon > \varepsilon^{FV}$ imply that $c > c^{MAV}$ and $\varepsilon > \varepsilon^{MAV}$.

[30] MAV and FV still generate excessive holding but only for $0 < \varepsilon < \gamma + X(1 - z_1) - \rho$.

switching to HC, but only at the cost of re-introducing risk-shifting[31]. Thus, it is not possible to calibrate a MAV regime that strictly dominates the FV regime.

This underscores the point that a hybrid (or MAV) regime, by itself, does not necessarily dominate the FV regime: adjusting the valuation regime, by itself, does not solve the problem associated with the FV regime. Instead, under both FV and MAV regimes, regulator needs to be able to appropriately calibrate both the solvency constraint and the penalty based on supervisory information about the asset quality of the insurers in order to ensure that they outperform the HC regime.

F. Numerical examples

We now present some numerical examples, to illustrate the main results of the model. In particular, we provide a comparison of the three different valuation regimes with the socially optimal solution, in terms of the real effects they generate on asset prices and policyholders' losses.

In the baseline simulation, we set the key parameters as follows: $R = 1.01$, $z_1 = 0.9$, $\theta = 0.5$, $D_1 = 0.50$, $D_2 = 0.49$, and $\rho = 0.01$. The calibration of the underwriting parameter is based on mortality rate of 1% over one year for a man aged 60, using data from the UK's Office for National Statistics. Assuming that the ex-ante probability of downgrade is $p = 1\%$, the ex-ante default probability on assets held by insurers is given by $p * (1 - z_1) = 0.1\%$. This value is consistent with the estimates of compensation investors require for expected and unexpected defaults that are obtained using data from the decomposition of sterling investment-grade corporate bonds[32]. Capital charges for bonds are not straightforward to derive, as they depend on factors such as credit quality and duration, contributing to spread risk, and changes in interest rates (although this is not present in our model). Informed by data from the European Insurance and Occupational Pensions Authority (EIOPA)[33], we set our mark-down of the value of bonds c as approximately 80% in the baseline analysis, but we also explore the implications of changing this parameter. The market depth γ has been obtained using liquidity compensation data from JP Morgan DataQuery. Specifically, we have decomposed the residuals from the sterling investment-grade corporate bond spreads after accounting for credit risk compensation, into compensation for liquidity and an unexplained, or a "noise" component. Informed by these data, we set $\gamma = 5\%$ which implies that, when all insurers sell an asset, the price of that asset will fall by 5.5% relative to its fundamental value ($\frac{\gamma}{z_1 R} = 0.055$).

Further, we assume that insurers start $t = 0$ with capital to investable asset ratio (E_0/A_0) of 35%. Thus, we set $E_0 = 0.54$ given that the insurers collect one unit of premium, implying that $A_0 = E_0 + 1 = 1.54$. This choice was informed by Solvency II regulatory returns gathered by the UK's Prudential Regulation Authority[34]. The median value of the ratio of assets to liabilities for large companies that specialise in underwriting products with guarantees, consistent with our calibration, is approximately 1.35. Note that given these values, the penalty ρ is lower than $X(1 - z_1) = 0.047$, for the baseline calibration.

[31] Note that, when credit risk is low, holding good quality assets becomes socially optimal. Hence MAV would dominate FV under these conditions. However, when credit risk is high, both FV and HC generate inefficient investment behaviours. Thus, even a state-dependent MAV (e.g. with θ set as a function of z_1) would not be able to strictly dominate the two regimes under all circumstances.

[32] These estimates are regularly produced by the Bank of England, see for instance Bank of England (2016). The methodology is based on Webber and Churm (2016).

[33] European Insurance and Occupational Pensions Authority (2017)

[34] See Chart 9 of Bank of England, Prudential Regulation Authority (2017).

Under this calibration, the three regimes coincide and generate too little sales relative to the socially optimal sales at $t = 1$ when the market is moderately optimistic (i.e. ε is moderately positive) (see Figure 8). As a result, as shown in Figure 9, when the market is moderately optimistic, the asset price is overinflated relative to the fundamental value. Note that, even under the socially optimal decision rule, over- and under-valuation of assets can occur in the presence of market noise. The expected policyholder losses generated under the three regimes are higher than the social optimum, as insurers are reluctant to sell downgraded assets, even when prices are favourable (see Figure 10).

Figures 11 and 12 show the equilibrium sales and the resulting market prices under an alternative scenario, in which the asset mark-down c is lowered to 0.49, such that $c^{MAV} < c < c^{FV}$, while maintaining the baseline calibrations for other parameters. A lower c could be justified if the regulator expects larger stressed losses and thus requires insurers to have more capital. In this case, FV outperforms MAV which coincides with HC, by encouraging more sales when the market price is favourable and thus reducing the inefficient holding of downgraded assets. As a result, policyholder losses are lower under FV than under MAV and HC, as shown in Figure 13.

Finally, we consider the case when MAV and FV coincide, and both generate excessive sales: to do this, we set $c = 0.47 < c^{MAV}$ and $\rho = 0.1 > X(1 - z_1)$, while maintaining baseline calibrations for other parameters. Figures 14 and 15 show that, under both FV and MAV, insurers dumping into the market lower prices when the market is already depressed. This occurs because the penalty for breaching the solvency constraint is set at a high level. Interestingly, policyholder losses (see Figure 16) are lower than under the socially optimal case, implying that the cost of inefficient sales are borne entirely by insurers' shareholders as long as the insurers remain solvent. This example highlights the potentially conflicting objectives that a regulator can face between protecting policyholders and preventing asset price volatility which could destabilise the financial system, that is between micro- and macroprudential regulation.

IV. Ex-ante asset choice

Thus far, we have taken insurers' asset portfolio as given[35], in order to focus our analysis on the impact of solvency requirements on *ex-post* insurers' investment behaviour. Starting from the same asset allocation allowed us to obtain a cleaner comparison between the different valuation regimes. However, valuation rules, through their interaction with solvency requirements, can generate important incentives ex-ante when insurers decide how to allocate their funding among different asset classes. In particular, forward-looking insurers who anticipate future solvency pressures might adopt more prudent investment strategies, relative to those who are less constrained by regulation.

Thus, in this section, we consider how expectations about future downgrades and associated solvency pressures influence insurers' *ex-ante* asset allocation when different valuation regimes are in place[36]. As before at $t = 0$ insurers must hold enough safe assets to meet the $t = 1$ insurance payout. This implies that the minimum fraction needed to be invested in safe assets is

$$b \geq \hat{b} \equiv \frac{D_1}{A_0}. \tag{IV.1}$$

[35] Specifically we have assumed that the fraction invested in the risk-free asset is (II.1) and the remaining $1 - b$ is invested in the risky asset. Note that this asset allocation is the optimal asset allocation of an insurer who cannot anticipate at t=0 the asset downgrade at t=1.

[36] Since MAV generates the same investment behaviour as HC or FV, we only focus on these two regimes in this section, as the results for MAV follows analogously.

We now examine how the insurers will allocate the remaining assets by optimizing between the risk-free and the risky asset. More specifically, at $t = 0$, insurers have a choice of investing the funds remaining after meeting the requirement (IV.1) either in the safe asset, which yields a return normalised to 1, or in the risky asset which is subject to a possibility of a credit downgrade at $t = 1$. As at $t = 0$, the probability of a downgrade is p, so the expected return of the risky asset is $[(1-p)+pz_1]R > 1$. To ensure tractability we only allow for two different strategies: either insurers invest everything in the safe asset, i.e. $b = 1$, or only the minimum to meet the constraint (IV.1), i.e. $b = \hat{b}$. In the latter case the asset allocation remains (II.1) as considered in the previous sections. We thus perform a comparison between the safest and the riskiest portfolios possible, which allow us to obtain clear predictions on the asset allocation chosen under different valuation regimes. As we will show, insurers will choose the riskiest portfolios, for different values of the probability of downgrade p under different valuation regimes.

Note that, if insurers invest all their funds in the safe asset, they will never breach the solvency constraint at $t = 1$, provided that their initial equity is large enough to satisfy the solvency constraint at $t = 0$, as specified in (II.3)[37]. By contrast, if they invest in the risky asset, they need to consider the possibility of breaching the constraint in the interim period due to a fall in asset prices, if their assets are measured by their market values. Thus, insurers will compare the expected return from the safe asset with the one from the risky asset, taking into account the expected market value in the interim period and the possibility of breaching the solvency constraint. In particular, at $t = 0$ insurers do not know their place in the queue of sellers, therefore they have to base their decision on expected prices:

$$v_0(s) = E_0[v_1(s)] = E_0[z_1 R + \varepsilon - \gamma s] = z_1 R + E_0[\varepsilon] - \gamma \frac{s}{2},$$

where $v_1(s)$ is the market price at $t = 1$. The above expression uses the fact that, at $t = 0$, the place in the queue of a given insurer that sell at $t = 1$ is uniformly distributed in $[0, s]$, if s is the total fraction of insurers that sell at $t = 1$. In the following, we restrict the parameter ε^H to be larger than $\gamma + X(1 - z_1)$, in order to cover all possible cases at $t = 1$.

First, we characterize the asset allocation that maximizes social welfare, given by insurers' and policyholders' expected value. Under the safe strategy $b = 1$, the social welfare is simply equal to the insurers' total initial assets value A_0, while under the risky strategy $b = \hat{b}$ the social welfare depends on the probability p of a downgrade. In particular, it is given by

$$A_0 \hat{b} + (1-p) A_0 (1-\hat{b}) R + p A_0 (1-\hat{b}) w^s$$

where w^s is the expected social welfare given a downgrade and given that the socially optimal rule (III.6) is followed at $t = 1$:

$$w^s = \int_{\varepsilon_L}^{0} z_1 R f(\varepsilon)\, d\varepsilon + \int_0^{\gamma} \left[\frac{\epsilon}{\gamma} \left(z_1 R + \varepsilon - \frac{\gamma}{2} \frac{\varepsilon}{\gamma} \right) + (1 - \frac{\varepsilon}{\gamma}) z_1 R \right] f(\varepsilon)\, d\varepsilon$$

[37] Note that when insurers invest all their funds in the risk-free asset, the initial constraint should be

$$E_0 \geq D_1 + D_2 - 1.$$

This is a looser requirement relative to (II.3), implying that less initial equity is required to set up the company. In the case the insurance contract is priced at fair value, i.e. $D_1 + D_2 = 1$, this constraint is never binding. However, we do not take into account the cost of equity in the asset allocation problem and just take the initial equity as given.

$$+\int_{\gamma}^{\varepsilon_H}\left[z_1 R+\varepsilon-\frac{\gamma}{2}\right] f(\varepsilon)\, d\varepsilon$$

Thus, it is socially optimal to invest in risky asset as long as

$$\hat{b}+(1-p)(1-\hat{b})R+p(1-\hat{b})w^s>1 \tag{IV.2}$$

Proposition IV.1. ***(Socially optimal ex-ante asset allocation)***
The risky strategy $\hat{b}$ is socially optimal if and only if $R+p(w^s-R)>1$. This implies that:

- *If $w^s>R$, the risky strategy $\hat{b}$ is always socially optimal, for any probability of downgrade, p.*
- *If $w^s<R$, the risky strategy $\hat{b}$ is socially optimal only if*

$$p<p^* \equiv \frac{R-1}{R-w^s}.$$

Note the socially optimal asset allocation differs from the one minimizing policyholders' losses, which would make insurers invest everything in the risk-free asset for any value of p. This is because by doing so policyholders would not incur any losses, while by investing a fraction $\hat{b}$ in the risky asset they would incur losses if the asset defaults.

Let us now analyze the insurers' asset allocation under different valuation regimes. If insurers choose the safe investment strategy $b=1$, their expected payoff is independent of the valuation regime, and given by their initial assets net of the payments to policyholders:

$$A_0-D_1-D_2=A_0(1-\hat{b})(1-X).$$

For ease of comparison with the risky strategy $\hat{b}$, we have expressed this payoff in terms of the interim liabilities-to-assets ratio under the risky strategy $X=\frac{D_2}{A_0(1-\hat{b})}$. When insurers choose the risky strategy they become exposed to the possibility of a downgrade at $t=1$. In this case, the valuation rule used to define the solvency requirement after the downgrade plays a key role.

In particular, *under the historical cost regime*, when the risky asset is downgraded, which happens with probability p, insurers will only sell the asset if its market price is high enough, as described by (III.11). By contrast, with probability $1-p$ the asset will not be downgraded and all insurers will hold it to maturity obtaining a certain return R. Thus, the expected payoff from the risky strategy under HC is given by:

$$(1-p)(RA_0(1-\hat{b})-D_2)+pA_0(1-\hat{b})w^{HC}=A_0(1-\hat{b})[(1-p)(R-X)+pw^{HC}]$$

where w^{HC} is the insurers' expected payoff in the case of a downgrade, given that they will follow the optimal rule (III.11) at t=1:

$$\begin{gathered}w^{HC}=\int_{\varepsilon_L}^{(1-z_1)X} z_1(R-X) f(\varepsilon)\, d\varepsilon \\ +\int_{(1-z_1)X}^{\gamma+X(1-z_1)}\left\{\pi^{HC}\left[z_1 R+\varepsilon-\gamma\left(\frac{\pi^{HC}}{2}\right)-X\right]+(1-\pi^{HC}) z_1(R-X)\right\} f(\varepsilon)\, d\varepsilon \\ +\int_{\gamma+X(1-z_1)}^{\varepsilon^H}[z_1 R+\varepsilon-\tfrac{\gamma}{2}-X] f(\varepsilon)\, d\varepsilon\end{gathered}$$

Proposition IV.2. ***(Historical cost regime ex-ante asset allocation)***

Under the historical cost regime insurers will choose the risky strategy $\hat{b}$ if

$$(R - X) + p(w^{HC} - (R - X)) > 1 - X. \tag{IV.3}$$

This implies that

- *If $w^{HC} > R - X$ insurers will always choose the risky strategy, for any value of p.*
- *If $w^{HC} < R - X$ insurers will choose the risky strategy only if*

$$p < p^{HC} = \frac{R-1}{R - X - w^{HC}}.$$

By contrast, *under fair value*, insurers' decision will also be driven by expectations of future solvency pressures, and thus will be highly dependent on the regulatory parameters. In order to focus our attention on the interesting case where FV and HC differ, in the following we assume that $c \leq c^{FV}$ and analyze the *ex-ante* asset allocation under the fair value regime for different values of the penalty ρ.

In particular, at $t = 0$ the expected payoff for insurers given a downgrade is given by:

$$\begin{aligned}
w^{FV} &= \int_{\varepsilon_L}^{(1-z_1)X-\rho} z_1(R-X) f(\varepsilon)\, d\varepsilon \\
&+ \int_{(1-z_1)X-\rho}^{\gamma+(1-z_1)X-\rho} \left\{\pi^{FV}\left[z_1 R + \varepsilon - \gamma\left(\frac{\pi^{FV}}{2}\right) - X\right] + (1-\pi^{FV})\left[z_1(R-X) - \rho\right]\right\} f(\varepsilon)\, d\varepsilon \\
&+ \int_{\gamma+(1-z_1)X-\rho}^{\varepsilon^H} \left[z_1 R + \varepsilon - \frac{\gamma}{2} - X\right] f(\varepsilon)\, d\varepsilon
\end{aligned}$$

if the penalty is small enough $\rho \leq z_1(R-X)$, such that they will follow the optimal rule (III.17) at $t = 1$, and by:

$$w^{FV} = z_1(R-X) + \int_{\varepsilon^L}^{\varepsilon^H} (\varepsilon - \frac{\gamma}{2} - X(1-z_1)) f(\varepsilon)\, d\varepsilon$$

when the penalty is large $\rho > z_1(R-X)$ and everyone sells at $t = 1$ in equilibrium. Then, under the fair value regime, insurers' expected payoff under the risky strategy $b = \hat{b}$ can be written as:

$$(1-p)(RA_0(1-\hat{b}) - D_2) + pA_0(1-\hat{b})w^{FV} = A_0(1-\hat{b})[(1-p)(R-X) + pw^{FV}]$$

Proposition IV.3. ***(Fair value regime ex-ante asset allocation)***

Under the fair value regime insurers will choose the risky strategy $\hat{b}$ if and only if:

$$(R - X) + p(w^{FV} - (R - X)) > 1 - X. \tag{IV.4}$$

This implies that

- *If $w^{FV} > R - X$ insurers will always choose the risky strategy for any value of p.*

- *If $w^{FV} < R - X$ insurers will choose the risky strategy only if*

$$p < p^{FV} = \frac{R-1}{R-X-w^{FV}}.$$

We now want to compare the asset allocations under HC and FV and to the socially optimal asset allocation. Note that while the safe strategy is always optimal from the policyholders' prospective, the risky strategy generates higher returns for shareholders. Thus, if credit risk is not elevated, the risky strategy can be socially optimal. However, when insurers are not restricted by regulation, since they maximize shareholders' value and are protected by limited liability, they have incentives to undertake excessive risks. In particular, they will prefer to invest in the risky strategy even for higher levels of credit risk than socially optimal, at the expense of policyholders. This is exactly what happens under the historical cost regime. Insurers anticipate that there will be no solvency pressure due to future credit downgrades, since assets are measured at their book value, and will therefore undertake the risky strategy for higher levels of p. Hence, we can conclude that historical cost leads to excessive risks both *ex post*, by 'gambling for resurrection' through excessive holding of downgraded assets, and *ex ante* by inducing riskier asset allocations.

By contrast, under the fair value regime, insurers' regulatory solvency will depend on the asset credit risk. Thus insurers, anticipating the possibility of a downgrade and the associated solvency pressures, will undertake a more prudent asset allocation than under the historical cost regime, as established formally in the following proposition.

Proposition IV.4. *The HC regime leads to riskier asset allocation than the FV regime:*

$$w^{FV} < w^{HC}.$$

Proof. See Annex. □

Note that this result implies that insurers under FV will choose to invest in securities with a lower probability of downgrade relative to insurers subject to the HC valuation regime. Furthermore, when the regulator sets the regulatory parameters appropriately, we can establish the following result.

Proposition IV.5. *If the regulator sets the FV regime regulatory parameter c and ρ as described in Proposition (III.6), then:*

$$w^s < w^{FV} + X < w^{HC} + X,$$

i.e. both HC and FV regimes lead to riskier ex ante asset allocation than would be socially optimal, but FV improves on HC.

Thus, when the regulatory parameters are appropriately calibrated, the FV regime induces insurers to have a more prudent asset allocation than the HC regime, even though both regimes lead to a riskier asset allocation than socially optimal. Note that this result further implies that, under the conditions of Proposition (III.6), the ex-ante expected losses for policyholders under FV are lower than under HC (although still larger than what socially optimal). This is because insurers would invest in riskier assets and would prefer holding on to them even after a downgrade, unless market conditions are particularly favourable. It follows that the FV regime can be calibrated to dominate the HC regime both from an *ex* *post* and an *ex ante* perspective. To do so, as discussed in the previous section, the regulator does not need perfect information on insurers' asset quality. Note that Proposition (III.9) implies that the same statement holds true for the MAV regime, as long as it is appropriately calibrated.

V. Empirical predictions

Our model has produced several empirical predictions, which are in line with empirical evidence on investment behaviour of the insurance industry.

First, we find that insurers subject to mark-to-market valuation and strict regulatory constraints are more prone to fire sales of downgraded assets during downturns. This prediction has been confirmed empirically by Ellul et al. (2014), who have studied the interaction between accounting and the regulatory framework for US insurers, and found that those subject to mark-to-market accounting were more likely to sell downgraded assets being hit by large price declines. Further, Merrill et al. (2012) have shown that insurance companies that became more capital-constrained and were forced to recognized fair value losses, sold comparable residential mortgage-backed securities (RMBS) at much lower prices than other insurance companies during the crisis.

Second, we find that insurers under the fair value regime adopt more prudent asset allocations. This is in line with the findings of Bank of England and the Procyclicality Working Group (2014), which provide some evidence that insurers and pension funds subject to prudential regulation based on market values have moved away from equities toward long-duration bonds. This remains true for more recent data for the UK insurance sector under Solvency II, as reported in Bank of England, Prudential Regulation Authority (2017). For US insurers Ellul et al. (2014) find that property and casualty ($P\&C$) insurers, which are subject to fair value accounting, adopted more prudent portfolio strategies relative to life insurers that were subject to historical cost accounting. Specifically, they find that their portfolio allocations were safer both across asset classes and for the choice of securities in each class. Our model is able to generate both predictions. In our framework, insurers under the fair value regime are more likely to invest everything in the safe asset, and when they invest in risky assets they choose securities with a lower probability of downgrade, relative to those under the historical cost regime.

While a significant amount of work has been done to analyze the effects of valuation rules during crisis times, less attention has been paid on their effects on portfolio choices in normal times before shocks materialize. Our model shows that an appropriately calibrated market-based valuation regime leads to more prudent asset allocation *ex ante* and more efficient asset sales *ex post*, i.e. following a negative credit quality shock. By contrast, the HC regime encourages excessively risky investment *ex ante* and insufficient asset sales *ex post*.

VI. Conclusions

What is the appropriate valuation regime for solvency regulation for financial institutions such as life insurers with long-term, annuity-like liabilities, held by dispersed and 'uninformed' policyholders? Our analysis suggests that, for such institutions, the historical cost regime is undesirable because it could encourage insurers to hide the deterioration in asset quality and avoid offloading assets when they should, ultimately magnifying losses for policyholders. The ability to hide losses *ex post* makes them also more likely to invest in riskier assets *ex ante* under the historical cost regime than under market-based valuation regimes. By contrast, the two market-based regimes – the fair value regime and the hybrid valuation regimes, which only places some weight on market prices in calculating the solvency ratio – could encourage asset fire sales following a negative shock to asset prices, if the regulatory penalty for breaching the solvency constraint is too severe. But when the regulatory penalty is set appropriately, insurers are likely to sell downgraded assets when they are trading above the fundamental values, to the benefit of policyholders. In general, we expect the presence of market discipline to also encourage insurers to invest in less risky assets *ex ante* under the market-based regimes than under the historical cost regimes. Our analytical results are consistent with Ellul et al. (2014) empirical findings that US insurers that were subject to historical cost accounting were less likely to sell in a falling market, but more likely to hold riskier portfolios, relative to those insurers that were subject to the fair value accounting regime.

We have shown that, if the regulator can calibrate the solvency constraint and the penalty for breaching the constraint based on supervisory information, then both fair value and hybrid regimes can be made to outperform the historical cost regime, even if that supervisory information is imprecise. Such appropriately calibrated market-based regimes yield higher total social surplus (shareholder surplus plus policyholder surplus), minimise policyholder losses, and lead to less risky and welfare enhancing investment allocations, relative to the historical cost regime. However, a hybrid regime does not in general outperform the fair value regime.

Our results therefore suggest that a market-based solvency regime – i.e. the fair value regime or the hybrid regime – could be made effective as long as regulators have good, if not perfect, supervisory information about insurers' balance sheets and can use that information to prevent asset fire sales during exceptional periods of market turmoil. In particular, when the regulator judges that an insurer has breached its solvency requirement despite having a fundamentally strong balance sheet, then treating it leniently, or suspending penalty, could not only be reasonable, but in fact be necessary to stop asset fire sales by insurers in times of market turbulence. Removing scope for such discretion, which was deployed during the 2007-9 Global Financial Crisis, would not be desirable in any market-based valuation regime when market prices do not always reflect fundamental, long-term asset values.

Finally, our results suggests that adjusting the valuation regime, by itself, is unlikely to offer a solution to the dual problem of risk shifting and asset fire sales. The valuation regime determines the circumstance under which an insurer breaches the solvency constraint, but it does not determine what happens when the breach occurs. It is important to recognise that the regulator's response to a breach is a critical part of making any market-based valuation regime work, as it determines how insurers behave when they breach or expect to breach their solvency constraints. Our analysis therefore suggests that the quest for a 'Goldilocks' valuation regime may turn out to be elusive, and that a more holistic approach – which includes appropriate calibration of regulatory penalty for insurers that have breached the solvency requirement – would be needed in order to make market-based valuation regimes work.

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A. Figures

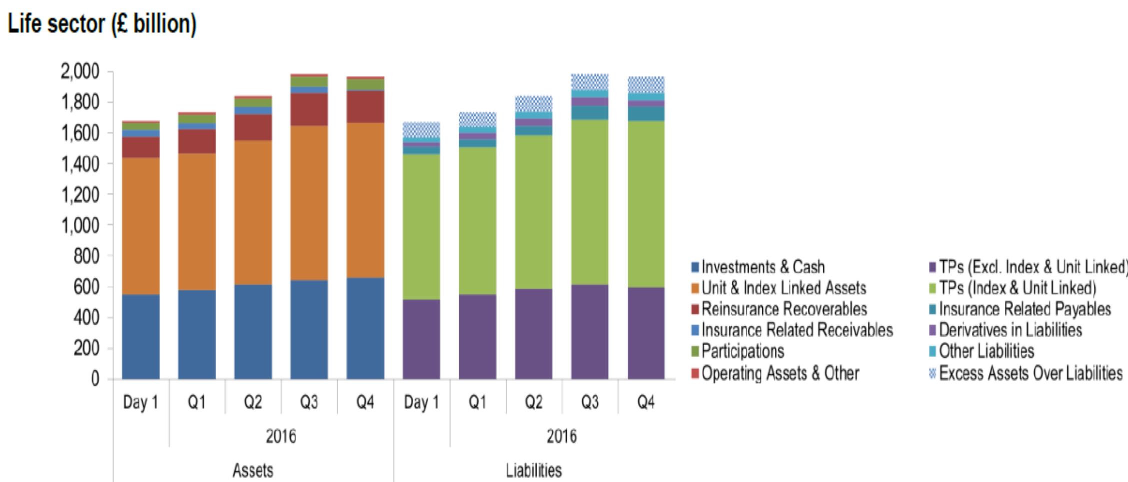

Figure 1. Breakdown of movements in the balance sheet of UK life insurers from day one to 2016 Q4. Data source: Bank of England.

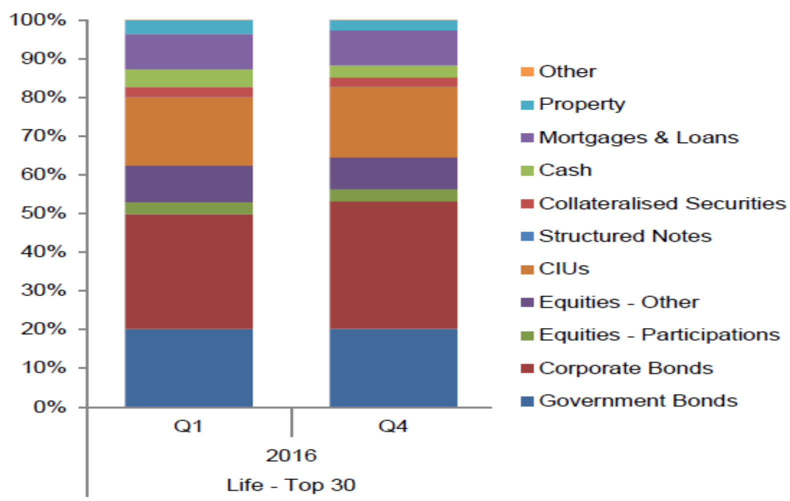

Figure 2. Proportion of investments of UK life insurers other than assets held for index-linked and unit-linked contracts by investment categories as at 2016 Q1 compared to 2016 Q4. Data source: Bank of England.

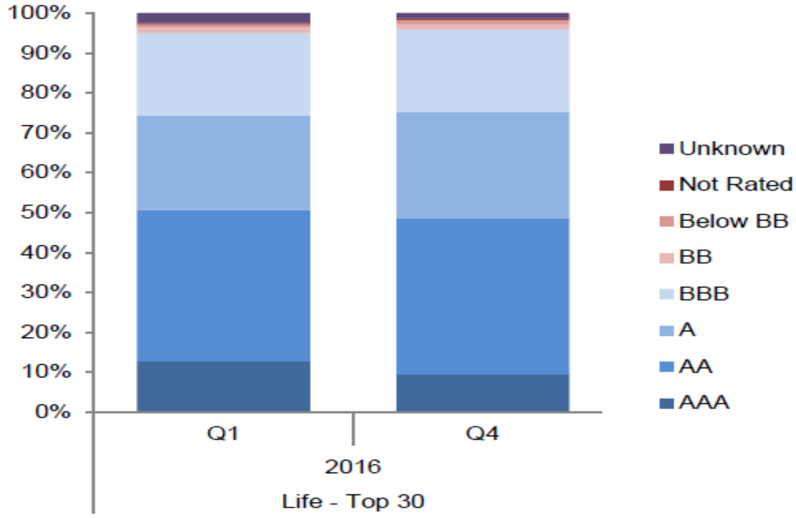

Figure 3.
Proportion of bonds of UK life insurers (government and corporate bonds only, excludes index-linked and unit-linked) with each credit rating as at 2016 Q1 compared to 2016 Q4. Data source: Bank of England.

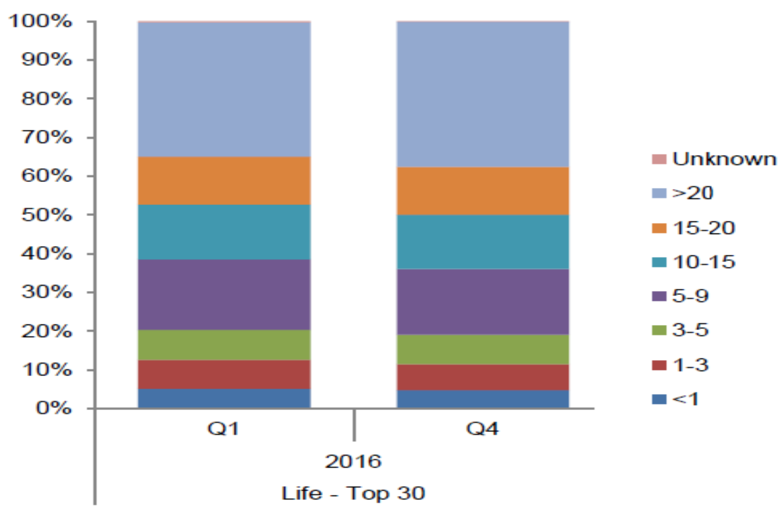

Figure 4.
Proportion of bonds of UK life insurers (government and corporate bonds only, excludes index-linked and unit-linked) with each maturity bucket as at 2016 Q1 compared to 2016 Q4. Data source: Bank of England.

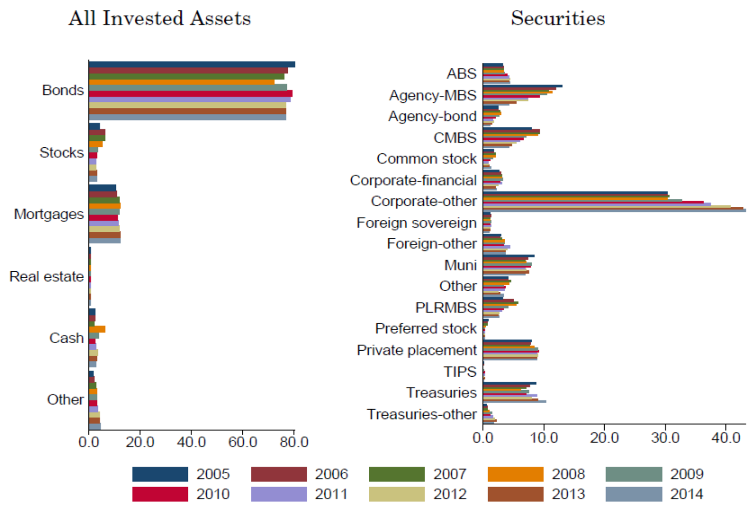

Figure 5. US life insurers asset allocation by year and category. Data source: SNL Financial, Chodorow-Reich et al. (2016).

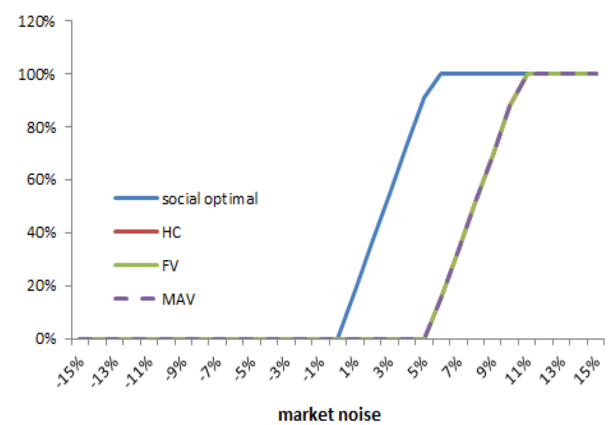

Figure 8. Equilibrium sales under the baseline calibration

The figure plots the equilibrium fraction of sales s^{HC}, given by (III.11), as a function of market noise expressed as a percentage of the fundamental value of the asset.

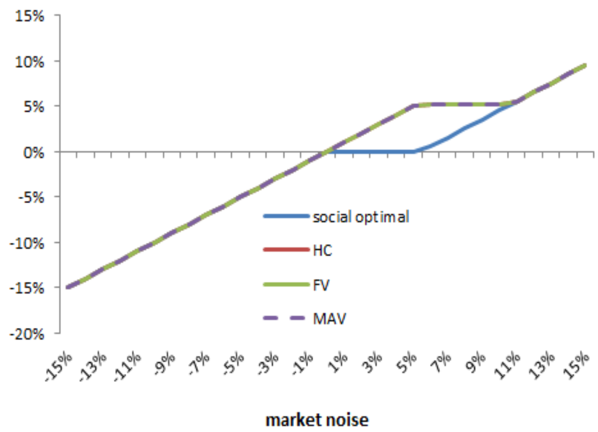

Figure 9. Market prices under the baseline calibration
The figure plots the market prices that prevails after insurers sell, v_1^{HC} given by (III.12), as a function of market noise expressed as a percentage of the fundamental value of the asset.

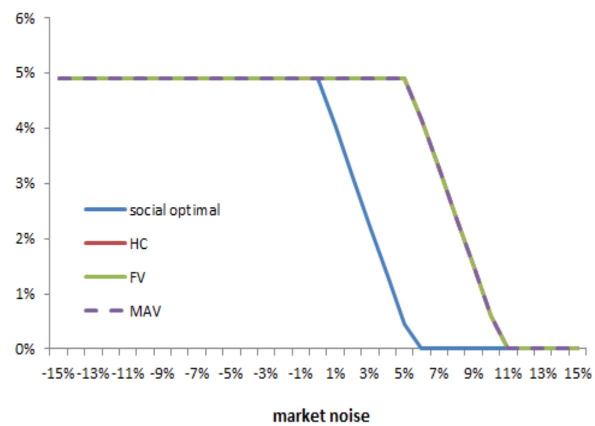

Figure 10. Policyholders losses under the baseline calibration
The figure plots the expected policyholders losses, when the three regimes coincide, as a function of market noise expressed as a percentage of the fundamental value of the asset.

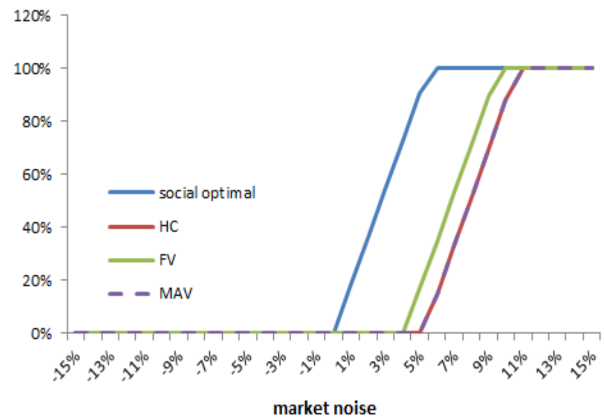

Figure 11. Equilibrium sales when $c^{MAV} < c < c^{FV}$ and $\rho < \min(X(1-z_1), z_1(R-X))$
The figure plots the equilibrium fraction of sales, when $c^{MAV} < c < c^{FV}$ and $\rho < \min(X(1-z_1), z_1(R-X))$, as a function of market noise expressed as a percentage of the fundamental value of the asset.

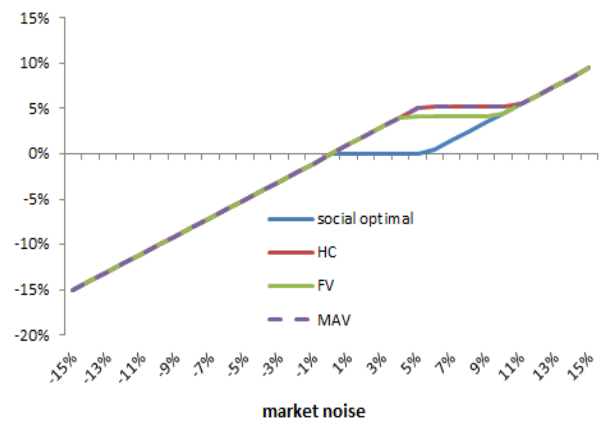

Figure 12. Market prices when $c^{MAV} < c < c^{FV}$ and $\rho < \min(X(1-z_1), z_1(R-X))$
The figure plots the market prices that prevails after insurers sell, when $c^{MAV} < c < c^{FV}$ and $\rho < \min(X(1-z_1), z_1(R-X))$, as a function of market noise expressed as a percentage of the fundamental value of the asset.

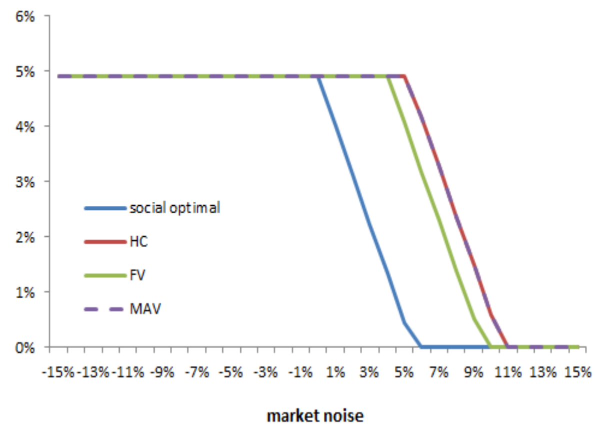

Figure 13. Policyholders losses when $c^{MAV} < c < c^{FV}$ and $\rho < \min(X(1-z_1), z_1(R-X))$
The figure plots the expected policyholders losses, when $c^{MAV} < c < c^{FV}$ and $\rho < \min(X(1-z_1), z_1(R-X))$, as a function of market noise expressed as a percentage of the fundamental value of the asset.

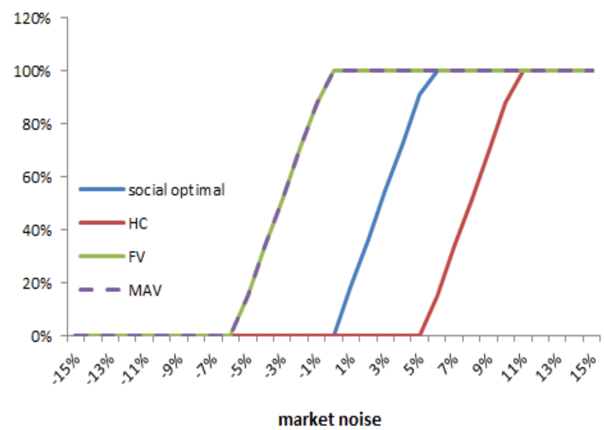

Figure 14. Equilibrium sales when $c < c^{MAV}$ and $\rho > \min(X(1-z_1), z_1(R-X))$
The figure plots the equilibrium fraction of sales, when $c < c^{MAV}$ and $\rho > \min(X(1-z_1), z_1(R-X))$, as a function of market noise expressed as a percentage of the fundamental value of the asset.

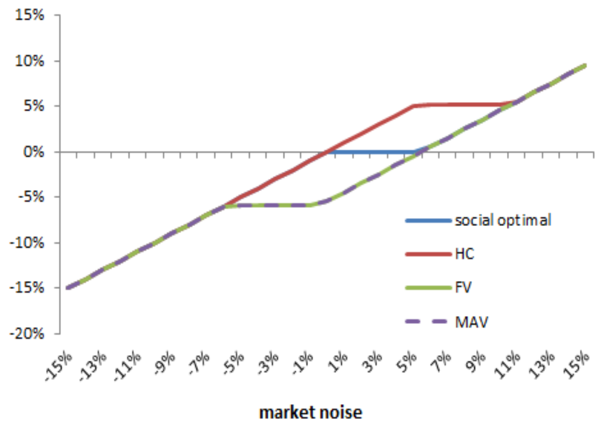

Figure 15. Market prices when $c < c^{MAV}$ and $\rho > \min(X(1-z_1), z_1(R-X))$
The figure plots the market prices that prevails after insurers sell, when $c < c^{MAV}$ and $\rho > \min(X(1 - z_1), z_1(R - X))$, as a function of market noise expressed as a percentage of the fundamental value of the asset.

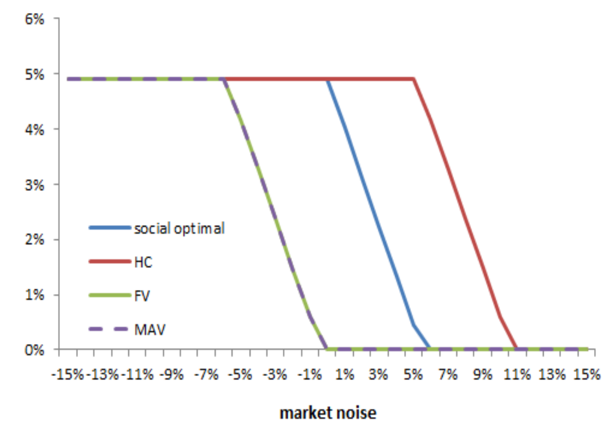

Figure 16. Policyholders losses when $c < c^{MAV}$ and $\rho > \min(X(1-z_1), z_1(R-X))$
The figure plots the expected policyholders losses, when $c < c^{MAV}$ and $\rho > \min(X(1 - z_1), z_1(R - X))$, as a function of market noise expressed as a percentage of the fundamental value of the asset.

B. Proofs

A. *Derivation of the solvency constraint*

Below, we explain why the solvency requirement at $t = 0$ can be expressed in the form of (II.2). Define insurers' expected liabilities at time $t = 0$ as $L_0 = D_1 + D_2$. In practice insurance companies must have enough equity to cover:

$$E_0 \geq \Delta A_0 + \Delta L_0, \tag{B.1}$$

where ΔA_0 and ΔL_0 represent the change in assets and liabilities following a one-in-200-year shock. In our model we assume liabilities and interest rates to be constant, therefore it follows that $\Delta L_0 = 0$. On the other hand, assets are risky and therefore subject to changes in value in periods of stress. In particular we can write the asset value under stress as:

$$A_0^{stressed} = A_0 b + A_0(1-b)c$$

where $1-c$ is the percentage decline in the value of the risky asset that the regulator expects under stress. Note that, consistent with the rest of the model, we consider the risk-free asset constant and not affected by the shock. We then have that $\Delta A_0 = A_0(1-b)(1-c)$ and we can rewrite (B.1) as:

$$\begin{aligned}
&A_0 - (D_1 + D_2) \geq A_0(1 - b - (1-b)c) \\
&1 - \frac{D_1 + D_2}{A_0} \geq 1 - b - (1-b)c \\
&\frac{D_1 + D_2}{A_0(b + (1-b)c)} \leq 1,
\end{aligned}$$

which is exactly (II.2). Analogously, we can derive the solvency constraint at $t = 1$ (III.3), starting from the following constraint:

$$V(A_1) - D_2 \geq \Delta V(A_1) + \Delta D_2 = \Delta V(A_1)$$

where we are still assuming no variation for the value of liabilities $\Delta D_2 = 0$. We indicate with $V(A_1)$ the value of the assets at time $t = 1$, where $A_1 = A_0(1-b)$, which will depend on the valuation regime adopted. Using that $\Delta V(A_1) = V(A_1)(1-c)$ we can re-write the above constraint as:

$$\frac{D_2}{cV(A_0(1-b))} \leq 1$$

which is exactly (III.3).

B. *Proof of Proposition (III.4): Fair Value Equilibrium*

Under the fair value regime the expected payoff of the insurer that hold the assets, given that a fraction s sells, is:

$$z_1(R^H - X) - \rho * I(s > \bar{s}^{FV}) \tag{B.2}$$

where $\bar{s}^{FV}(c)$ is given by (III.14). Note that there are two extreme cases:

1. $\bar{s}^{FV} < 0$, which is equivalent to $c < \frac{X}{z_1 R+\varepsilon}$ or $\varepsilon < \frac{X}{c} - z_1 R$. In this case the constraint is always binding, even when there are no sales. This is the case when the market is very pessimistic, the fundamental value is very low, or the constraint is set very tight.

2. $\bar{s}^{FV} > 1$, which is equivalent to $c > \frac{X}{z_1 R+\varepsilon-\gamma}$. Then the constraint is never binding, not even when everyone sells. This is because the market price is always high enough (because either the market is very optimistic, highly liquid or the fundamental value is very high), such that even if everyone sells the constraint does not bind.

Define

$$c^{FV} \equiv \frac{X}{z_1 R + (1 - z_1)X}. \tag{B.3}$$

Then for $c \geq c^{FV}$ if a fraction $\pi^{HC} = \frac{1}{\gamma}[\varepsilon - X(1 - z_1)]$ sells the remaining fraction $1 - \pi^{HC}$ of insurers holding the asset satisfy the constraint. On the other hand when $c < c^{FV}$, the constraint is no longer satisfied and π^{HC} cannot be an equilibrium. In the rest of the proof we then analyse these two cases separately.

Case 1: $c \geq c^{FV}$

i) When $\varepsilon < X(1 - z_1)$ no insurer sells under the historical cost regime and this remains the solution also under the fair regime, i.e. $s^{FV} = s^{HC} = 0$, as long as $\bar{s}^{FV} > 0$, that is $\varepsilon > \frac{X}{c} - z_1 R$. No insurer sells for this range of parameters because the market is unfavourable and the constraint is not breached as long as nobody sells. Note that the value of the penalty ρ is irrelevant in this case as nobody breaches the constraint.

ii) When $\varepsilon < X(1 - z_1)$ and $\varepsilon < \frac{X}{c} - z_1 R$, $s^{FV} = 0$ is still a solution if and only if the payoff from holding is greater than the payoff from selling, i.e.:

$$z_1(R - X) - \rho > [z_1 R + \varepsilon - X]^+$$

This is because $\bar{s}^{FV} \leq 0$ hence the solvency constraint is violated even if no insurer sells. Then $s = 0$ is a solution if the penalty is low enough $\rho \leq z_1(R - X)$ so that the payoff from holding remains positive and the market price is low due to over- relatively pessimistic $\varepsilon < X(1 - z_1) - \rho$. When $\rho \leq z_1(R - X)$ and $\varepsilon > X(1 - z_1) - \rho$ the aggregate sales is determined by the indifference condition of the last seller given that, by holding, it will breach the FV constraint. Thus, the equilibrium fraction of sellers π^{FV} is such that $z_1(R - X) - \rho = v_1(\pi^{FV}) - X$, from which we obtain:

$$\pi^{FV} = \frac{1}{\gamma}[\varepsilon - X(1 - z_1) + \rho] = \pi^{HC} + \frac{\rho}{\gamma}$$

Thus, if $c \geq c^{FV}$ and $\rho \leq z_1(R - X)$, but both $\varepsilon < X(1 - z_1)$ and $\varepsilon < \frac{X}{c} - z_1 R$, the equilibrium fraction of sales is given by (III.17).

While if $c \geq c^{FV}$, $\varepsilon < X(1 - z_1)$ and $\varepsilon < \frac{X}{c} - z_1 R$, but $\rho > z_1(R - X)$ everyone sells and the solution is $s^{FV} = 1$. This is because the penalty is so high that the payoff from holding becomes negative.

iii) When $X(1-z_1) \le \varepsilon \le \gamma + X(1-z_1)$ the equilibrium fraction of sales under the historical cost regime is π^{HC}. Because $c > c^{FV}$ we have that $\bar{s}^{FV} > \pi^{HC}$. Thus, this is still the equilibrium under the fair value regime since no insurer breaches the solvency constraint when π^{HC} sells. In fact there is no incentive for more than π^{HC} insurers to sell, given that their selling decision is determined by the condition (III.10). Thus, $s^{FV} = s^{HC} = \pi^{HC}$ in the parameter range $X(1-z_1) \le \varepsilon \le \gamma + X(1-z_1)$, regardless of the value of ρ.

iv) When $\varepsilon > \gamma + X(1-z_1)$ all insurers sell under the historical cost regime i.e. $s^{HC} = 1$. This is still a feasible solution, since $c \ge c^{FV}$ implies that $\bar{s}^{FV} > 1$ in this parameter range. Therefore the constraint is satisfied even when everyone sells and there is no incentive to deviate from the solution $s^{FV} = s^{HC} = 1$ regardless of the value of ρ.

To summarise, when $c \ge c^{FV}$ the equilibrium sales under fair value coincides with the one under historical cost $s^{FV} = s^{HC}$ when $\varepsilon > \frac{X}{c} - z_1 R$. Instead when $\varepsilon < \frac{X}{c} - z_1 R$ the equilibrium is $s^{FV} = 1$ if $\rho > z_1(R-X)$ and (III.17) if $\rho \le z_1(R-X)$.

Case 2: $c < c^{FV}$

Consider now the case where the regulator sets the solvency requirement, such that $c < c^{FV}$. Under this constraint, if a fraction π^{HC} sells, then the remaining insurers will breach the solvency constraint regardless of whether they sell or hold the assets. In this case, (III.10) no longer pins down the indifference condition between selling and holding, and thus π^{HC} is no longer a feasible equilibrium.

i) When $\varepsilon < X(1-z_1)$ the solution is the same as in case 1 for $c \ge c^{FV}$. Thus $s^{FV} = s^{HC} = 0$, as long as $\bar{s}^{FV} > 0 \Leftrightarrow \varepsilon > \frac{X}{c} - z_1 R$.

ii) When $\varepsilon < X(1-z_1)$ and $\varepsilon < \frac{X}{c} - z_1 R$, then the solution is given by (III.17) when $\rho \le z_1(R-X)$, and $s^{FV} = 1$ when $\rho > z_1(R-X)$.

iii) When $X(1-z_1) \le \varepsilon \le \gamma + X(1-z_1)$, $c < c^{FV}$ implies that $\bar{s}^{FV} < \pi^{HC}$. Thus, insurers that continue to hold the assets breach the solvency constraint when a fraction π^{HC} sells. In this case, all insurers sell and $s^{FV} = 1$ if the penalty is larger than $\rho > z_1(R-X)$, while $s^{FV} = \min(\pi^{FV}, 1)$ if $\rho \le z_1(R-X)$. We verify that $\pi^{FV} > \bar{s}^{FV}$ that is $c < \frac{X}{z_1 R + (1-z_1)X - \rho}$, which always holds when $c < c^{FV}$. We also verify that the payoff from selling is non-negative, that is $z_1 R + \varepsilon - \gamma \pi^{FV} > 0$, which is always true when $\rho \le z_1(R-X)$.

iv) When $\varepsilon > \gamma + X(1-z_1)$ everyone sells $s^{FV} = 1$ and remain solvent as long as $\frac{X}{z_1 R + \varepsilon - \gamma} < c$. On the other hand if $c < \frac{X}{z_1 R + \varepsilon - \gamma}$ when all insurers sell the solvency constraint is breached. Then if $\rho \le z_1(R-X)$, the payoff from holding would still be positive and $s = 1$ arises when the market is relatively optimistic such that all insurers can realise a higher payoff by selling rather than holding. That is the case when $z_1(R-X) - \rho < v_1(1) - X$, which can be re-organised as:

$$\varepsilon > \gamma + X\,(1-z_1) - \rho$$

Finally, if $\rho > z_1(R-X)$ the payoff from holding becomes negative and everyone sells for any $\varepsilon > \gamma + X(1-z_1)$.

To summarise, if $c < c^{FV}$ and $\rho \ge z_1(R-X)$, $s^{FV} = 1$, while if $\rho \le z_1(R-X)$, the solution is given by (III.17).

C. Proof of Proposition (III.5): Welfare properties of the fair value equilibrium.

We prove the results for the three different cases separately.

1. When insurers have positive concern values, $z_1 R \geq X$, it follows that $X(1 - z_1) \leq z_1(R - X)$. Comparing (III.17) with (III.6) when $c \leq c^{FV}$, we obtain that the equilibrium sales under the FV regime coincides with the socially optimal level of sales $s^{FV} = s^*$, for any level of market noise ε, if the regulator sets the penalty from breaching the constrain equal to $\rho = X(1 - z_1)$. Note that, however, this is no longer true when insurers concern values are negative, $z_1 R < X$, as it implies that $X(1 - z_1) > z_1(R - X)$. It follows that setting $\rho = X(1 - z_1)$ implies that $\rho > z_1(R - X)$. Under this condition, all insurers would sell their assets under the fair value regime, which is not always socially optimal.

2. We now analyse the case when FV generates excessive sales of the risky asset. If $c \leq c^{FV}$ everyone sells their risky assets ($s^{FV} = 1$) for any value of ε under the FV regime when the penalty is large $\rho > z_1(R - X)$. By contrast, it socially optimal for everyone to sell only when $\varepsilon > \gamma$. Therefore, under the conditions of case 2i), FV generates excessive sales for $\varepsilon < \gamma$. When instead the penalty is $X(1-z_1) < \rho < z_1(R-X)$, as in case 2ii), then the equilibrium sales under FV are given by (III.17). Note that this condition on the penalty can only be satisfied if insurers have positive concern values, $z_1 R \geq X$. Comparing (III.17) to the socially optimal sales, it follows that FV generates excessive sales for $X(1 - z_1) - \rho \leq \varepsilon \leq \gamma$.

3. We now analyse the case when FV generates excessive holding of the risky asset. Specifically, this is the case when $c \leq c^{FV}$ and $\rho < \min(X(1 - z_1), z_1(R - X))$ (case 3i), which imply that the equilibrium sales under FV are given by (III.17). Note that, because $\rho < X(1 - z_1)$, we have that $X(1-z_1)-\rho > 0$. This in turn implies that $\pi^{FV} < \frac{\varepsilon}{\gamma}$. Hence $s^{FV} > s^*$ for $0 < \varepsilon < \gamma + X(1-z_1) - \rho$. Finally, when $c > c^{FV}$ and $\varepsilon > \varepsilon^{FV}$, under case 3ii), the equilibrium under FV coincides with the one under HC. Thus the result follows from proofs of Propositions (III.2) and (III.4).

D. Proof of Proposition (III.6): FV can achieve higher social welfare than HC.

In order to ensure that the FV constraint binds following a negative credit quality shock, the regulator needs to ensure that $c \leq c^{FV}$ for all $z_1 \in [\underline{z}_1, \bar{z}_1]$. This can be achieved by setting $c = \frac{X}{\bar{z}_1 R + (1-\bar{z}_1)X}$.

In addition, in order to ensure that FV dominates HC, the regulator needs to meet the following two conditions:

i) Negative concern values insurers (with $z_1 R < X$) don't engage in fire sales, and have lower levels of excessive asset holding relative to HC, by setting $\rho \leq z_1(R - X)$ for this group.

ii) Positive concern values insurers (with $z_1 R > X$) don't engage in fire sales, and have lower levels of excessive holding relative to the HC regime by ensuring that $\rho \leq X(1 - z_1)$ for this group (see case 3i) in Proposition III.5).

Even when the regulator only knows that $z_1 \in [\underline{z}_1, \bar{z}_1]$ at $t = 0$, and cannot verify solvent versus insolvent insurers at $t = 1$, FV achieves superior outcomes relative to HC if the regulatory penalty ρ is set as follows:

a) If $X(1-\bar{z}_1) < \underline{z}_1(R-X)$, then set $\rho = X(1-\bar{z}_1)$ will ensure that the FV outperforms HC. This ensures that $\rho \leq z_1(R-X)$ for all $z_1 \in [\underline{z}_1, \bar{z}_1]$, so the condition i) for insolvent insurers is satisfied; and $\rho \leq X(1-z_1)$ for all $z_1 \in [\underline{z}_1, \bar{z}_1]$.

b) If $X(1-\bar{z}_1) \geq \underline{z}_1(R-X)$, then the regulator sets $\rho = \underline{z}_1(R-X)$. This ensures that insolvent insurers don't engage in fire sales and reduce excessive holdings relative to HC, because $\rho \leq z_1(R-X)$ for all $z_1 \in [\underline{z}_1, \bar{z}_1]$; and solvent insurers don't engage in fire sales and reduce excessive holding relative to HC, as $\rho \leq X(1-z_1)$ for all $z_1 \in [\underline{z}_1, \bar{z}_1]$ (see 3i) in Proposition III.5 above).

Thus, even with imperfect supervisory information about asset quality, the regulator can set parameters in order to ensure that the FV regime dominates the HC regime.

E. Proof of Proposition (III.7): Policyholders losses

Note that, if the regulator sets the penalty as described in Proposition (III.6), then it is always the case that $\rho \leq X(1-z_1)$ for all $z_1 \in [\underline{z}_1, \bar{z}_1]$. Thus, we first analyse the expected losses when $\rho \leq z_1(R-X)$. In this case the losses under FV are given by (III.21). Then for $\varepsilon < X(1-z_1) - \rho$ the losses are L^{Hold} under both HC and FV, and for $\varepsilon > \gamma + X(1-z_1)$ are zero. Hence the losses under HC and FV differs only when $X(1-z_1) - \rho < \varepsilon < \gamma + X(1-z_1)$. In particular, if $\gamma > \rho$ we have that:

i) for $X(1-z_1) - \rho < \varepsilon < X(1-z_1)$, $L^{FV} = (1-\pi^{FV})L^{Hold} < L^{HC} = L^{Hold}$

ii) for $X(1-z_1) < \varepsilon < \gamma - \rho + X(1-z_1)$, $L^{FV} = (1-\pi^{FV})L^{Hold} < L^{HC} = (1-\pi^{HC})L^{Hold}$

iii) for $\gamma - \rho + X(1-z_1) < \varepsilon < \gamma + X(1-z_1)$ then $L^{FV} = L^{Sell}(1) = 0 < L^{HC} = (1-\pi^{HC})L^{Hold}$

Note that in this last case we have that $L^{Sell}(1) = 0$ since $\varepsilon > \gamma - \rho + X(1-z_1)$ implies that $\varepsilon > X - z_1 R + \frac{\gamma}{2}$, using that $\rho < z_1(R-X)$. If instead $\gamma < \rho$ we have that:

i) for $X(1-z_1) - \rho < \varepsilon < \gamma - \rho + X(1-z_1)$, $L^{FV} = (1-\pi^{FV})L^{Hold} < L^{HC} = L^{Hold}$

ii) for $\gamma - \rho + X(1-z_1) < \varepsilon < X(1-z_1)$, $L^{FV} = L^{Sell}(1) = 0 < L^{HC} = L^{Hold}$

iii) for $X(1-z_1) < \varepsilon < \gamma + X(1-z_1)$, $L^{FV} = L^{Sell}(1) = 0 < L^{HC} = (1-\pi^{HC})L^{Hold}$

Note that in the last two cases, we have that $L^{Sell}(1) = 0$ because $\varepsilon > \gamma - \rho + X(1-z_1)$ implies that $\varepsilon > X - z_1 R + \frac{\gamma}{2}$, using that $\rho < z_1(R-X)$. This proves that policyholder losses under the FV regime are always lower than or equal to the losses under the HC regime.

F. Proof of Proposition (III.8): Market Adjusted Valuation Equilibrium

Under the market adjusted valuation the expected payoff of the insurer that hold the assets, given that a fraction s sells, is:

$$z_1(R-X) - \rho * I(s > \bar{s}^{MAV})$$

where $\bar{s}^{MAV}$ is defined in (III.25) such that once $\bar{s}^{MAV}$ or more insurers sell their assets, the remaining insurers that hold the asset will breach the solvency constraint c. Note that:

1. $\bar{s}^{MAV} < 0$ if and only if $\varepsilon < \varepsilon^{MAV} \equiv \frac{1}{1-\theta}[\frac{X}{c} - (1-\theta)z_1 R - \theta R]$. In this case the constraint is always binding even when there are no sales.
2. $\bar{s}^{MAV} > 1$ if and only if $\varepsilon > \varepsilon^{MAV} + \frac{1}{1-\theta}\gamma$ or equivalently, when $\epsilon > \gamma - z_1 R$, if $c > \frac{X}{\theta R + (1-\theta)(z_1 R + \varepsilon - \gamma))}$. In this case the constraint is never binding, not even when everyone sells.

Define

$$c^{MAV} \equiv \frac{X}{\theta R + (1-\theta)\left[z_1 R + (1-z_1)X\right]}$$

Then for $c < c^{MAV}$ if a fraction π^{HC} sells the constraints binds on the remaining fraction $1 - \pi^{HC}$ who hold. The rest of the proof will follow the same lines of the proof of the equilibrium under the fair value regime.

Case 1: $c \geq c^{MAV}$

i) When $\varepsilon < X(1-z_1)$, no insurers sell under the historical cost regime. This remains the solution also under MAV, i.e. $s^{MAV} = s^{HC} = 0$, as long as $\bar{s}^{MAV} > 0 \Leftrightarrow \varepsilon > \varepsilon^{MAV}$.

ii) When $\varepsilon < X(1-z_1)$ and $\varepsilon \leq \varepsilon^{MAV}$, we have that $\bar{s}^{MAV} \leq 0$ so that the solvency constraint is violated even if no insurer sells. In this case, $s^{MAV} = 0$ is still a solution if and only if the payoff from holding is greater than the payoff from selling, i.e.:

$$z_1(R-X) - \rho > [z_1 R + \varepsilon - X]^+$$

Then, analogously to Case 1i) of proposition (III.4), if the penalty is low enough $\rho < z_1(R-X)$ everyone holds ($s^{MAV} = 0$) if the market is relatively pessimistic $\varepsilon < X(1-z_1) - \rho$. While when $\rho < z_1(R-X)$ and $\varepsilon > X(1-z_1) - \rho$ the aggregate sales is determined by the indifference condition of the last seller given that, by holding, it will breach the MAV constraint, and the solution in this case is given by π^{FV}. Thus in this case the equilibrium fraction of sales is given by (III.17) if $\rho < z_1(R-X)$. On the other hand everyone sells and $s^{MAV} = 1$ when the penalty is large $\rho > z_1(R-X)$.

iii) When $X(1-z_1) \leq \varepsilon \leq \gamma + X(1-z_1)$ the equilibrium fraction of sales is π^{HC} as under the historical cost regime. This is because, since $c \geq c^{MAV}$, the remaining fraction $1-\pi^{HC}$ that holds the assets does not breach the constraint.

iv) When $\varepsilon > \gamma + X(1-z_1)$ everyone sells under the historical cost regime $s^{HC} = 1$, and this is still a feasible solution when $c > c^{MAV}$, since the MAV constraint is satisfied in this parameter range.

Case 2: $c < c^{MAV}$

Suppose now that the regulator sets the fair value solvency requirement, such that $c \leq c^{MAV}$. Under this regulatory solvency constraint, if π^{HC} sells, then the remaining insurers will breach the solvency constraint. In this case, (III.10) no longer pins down the indifference condition between selling and holding, and thus π^{HC} is no longer a feasible equilibrium.

i) When $\varepsilon < X(1-z_1)$ everyone holds and the solution is $s^{MAV} = s^{HC} = 0$, as long as $\bar{s}^{MAV} > 0$ $\Leftrightarrow \varepsilon > \varepsilon^{MAV}$.

ii) When both $\varepsilon < X(1-z_1)$ and $\varepsilon < \varepsilon^{MAV}$, then analogously to the fair value regime, the solution is (III.17) when $\rho \leq z_1(R-X)$, and $s^{MAV} = 1$ when $\rho > z_1(R-X)$.

iii) When $X(1-z_1) \leq \varepsilon \leq \gamma + X(1-z_1)$, $c \leq c^{MAV}$ implies that $\bar{s}^{MAV} \leq \pi^{HC}$. Thus, insurers that continue to hold the assets breaches the MAV solvency constraint when π^{HC} sells. In this case, all insurers sell and $s^{MAV} = 1$ if $\rho > z_1(R-X)$, while $s^{MAV} = \pi^{FV}$ if $\rho \leq z_1(R-X)$. We verify that $\pi^{FV} > \bar{s}^{MAV}$ as long as

$$c < \frac{X}{\theta R + (1-\theta)\left[z_1 R + (1-z_1)X - \rho\right]},$$

which always holds for $c < c^{MAV}$. We also verify that the payoff from selling is non-negative, i.e. $v_1(\pi^{FV}) > 0$, as long as $\rho \leq z_1(R-X)$.

iv) When $\varepsilon > \gamma + X(1-z_1)$ everyone sells $s^{MAV} = 1$ as long as

$$\frac{X}{\theta R + (1-\theta)(z_1 R + \varepsilon - \gamma)} < c \leq c^{MAV}.$$

On the other hand if $c < \frac{X}{\theta R+(1-\theta)(z_1 R+\varepsilon-\gamma)}$ and $\rho \leq z_1(R-X)$, $s = 1$ arises when the market is very optimistic such that all insurers can realise a higher payoff by selling than holding and breaching the constraint: $z_1(R-X) - \rho < v_1(1) - X$, which can be re-organised as:

$$\varepsilon > \gamma + X\left(1-z_1\right) - \rho$$

Finally, if $\rho > z_1(R-X)$ then $s^{MAV} = 1$

To summarise, if $c < c^{MAV}$ and $\rho > z_1(R-X)$, $s^{MAV} = 1$. If $c < c^{MAV}$ and$\rho \leq z_1(R-X)$, the solution is given by (III.17).

G. *Proof of Proposition (IV.4): Ex-ante asset allocation*

The proof consists in showing that $w^{HC} > w^{FV}$. That is insurers' expected payoff from the risky strategy, given a downgrade, is higher under historical cost than under fair value. It follows that if $w^{HC} < R - X$ then $p^{HC} > p^{FV}$. Thus insurers under HC choose to invest in the risky asset for an higher probability of a downgrade than under FV. Therefore we can conclude that the ex-ante asset allocation is safer under fair value. Note that we can rewrite w^{HC} as:

$$\begin{aligned}
w^{HC} \quad &= z_1(R-X) + \int_{(1-z_1)X}^{\gamma+X(1-z_1)} \left\{\pi^{HC}\left[\varepsilon - \gamma\left(\frac{\pi^{HC}}{2}\right) - X(1-z_1)\right]\right\} f\left(\varepsilon\right) d\varepsilon \\
&+ \int_{\gamma+X(1-z_1)}^{\varepsilon^H} [\varepsilon - \frac{\gamma}{2} - X(1-z_1)] f\left(\varepsilon\right) d\varepsilon \\
&= z_1(R-X) + \int_{(1-z_1)X}^{\gamma+X(1-z_1)} \left\{\pi^{HC}\frac{1}{2}(\varepsilon - X(1-z_1))\right\} f\left(\varepsilon\right) d\varepsilon \\
&+ \int_{\gamma+X(1-z_1)}^{\varepsilon^H} [\varepsilon - \frac{\gamma}{2} - X(1-z_1)] f\left(\varepsilon\right) d\varepsilon.
\end{aligned}$$

When $\rho \leq z_1(R-X)$, w^{FV} is given by:

$$\begin{aligned}
w^{FV} \quad &= z_1(R-X) + \int_{(1-z_1)X-\rho}^{\gamma+(1-z_1)X-\rho} \left\{ \pi^{FV} \left[\varepsilon - \gamma \left(\frac{\pi^{FV}}{2} \right) - X(1-z_1) \right] \right\} f(\varepsilon)\, d\varepsilon \\
&-\rho \int_{(1-z_1)X-\rho}^{\gamma-(1-z_1)X-\rho} (1-\pi^{FV}) f(\varepsilon)\, d\varepsilon + \int_{\gamma+(1-z_1)X-\rho}^{\varepsilon^H} \left[\varepsilon - \frac{\gamma}{2} - X(1-z_1) \right] f(\varepsilon)\, d\varepsilon \\
&= z_1(R-X) + \int_{(1-z_1)X-\rho}^{\gamma+(1-z_1)X-\rho} \left\{ \pi^{FV} \frac{1}{2} (\varepsilon - \rho - X(1-z_1)) \right\} f(\varepsilon)\, d\varepsilon \\
&-\rho \int_{(1-z_1)X-\rho}^{\gamma-(1-z_1)X-\rho} (1-\pi^{FV}) f(\varepsilon)\, d\varepsilon + \int_{\gamma+(1-z_1)X-\rho}^{\varepsilon^H} \left[\varepsilon - \frac{\gamma}{2} - X(1-z_1) \right] f(\varepsilon)\, d\varepsilon.
\end{aligned}$$

Therefore we can write the difference between the two expected payoffs as follows:

$$\begin{aligned}
w^{HC} - w^{FV} \quad &= \int_{X(1-z_1)}^{\gamma+X(1-z_1)} \left\{ \pi^{HC} \frac{1}{2} (\varepsilon - X(1-z_1)) \right\} f(\varepsilon) + \int_{\gamma+X(1-z_1)}^{\varepsilon^H} [\varepsilon - \frac{\gamma}{2} - X(1-z_1)] f(\varepsilon)\, d\varepsilon \\
&- \int_{(1-z_1)X-\rho}^{\gamma+(1-z_1)X-\rho} \left\{ \pi^{FV} \frac{1}{2} (\varepsilon - \rho - X(1-z_1)) \right\} f(\varepsilon)\, d\varepsilon \\
&+\rho \int_{(1-z_1)X-\rho}^{\gamma-(1-z_1)X-\rho} (1-\pi^{FV}) f(\varepsilon)\, d\varepsilon - \int_{\gamma+(1-z_1)X-\rho}^{\varepsilon^H} \left[\varepsilon - \frac{\gamma}{2} - X(1-z_1) \right] f(\varepsilon)\, d\varepsilon.
\end{aligned}$$

Note that the second and fourth terms simplify to:

$$\int_{\gamma+X(1-z_1)}^{\varepsilon^H} [\varepsilon - \frac{\gamma}{2} - X(1-z_1)] f(\varepsilon)\, d\varepsilon - \int_{\gamma+(1-z_1)X-\rho}^{\varepsilon^H} \left[\varepsilon - \frac{\gamma}{2} - X(1-z_1) \right] f(\varepsilon)\, d\varepsilon = \frac{1}{(\varepsilon^H - \varepsilon^L)} \frac{\rho}{2} [\rho - \gamma].$$

Furthermore, the first term equals to:

$$\int_{X(1-z_1)}^{\gamma+X(1-z_1)} \left\{ \pi^{HC} \frac{1}{2} (\varepsilon - X(1-z_1)) \right\} f(\varepsilon) = \frac{1}{(\varepsilon^H - \varepsilon^L)} \frac{\gamma^2}{6},$$

and the remaining terms can be expressed as:

$$- \int_{(1-z_1)X-\rho}^{\gamma+(1-z_1)X-\rho} (\pi^{FV} \frac{1}{2} (\varepsilon - \rho - X(1-z_1)) + \rho \pi^{FV}) f(\varepsilon)\, d\varepsilon \quad = \quad - \frac{1}{(\varepsilon^H - \varepsilon^L)} \frac{1}{6} \gamma^2.$$

Then the above expression for the difference between the two expected payoffs can be rewritten as

$$w^{HC} - w^{FV} \quad = \quad \frac{\rho}{(\varepsilon^H - \varepsilon^L)} \left(\frac{\rho + \gamma}{2} \right),$$

which is always positive. Therefore, when $\rho \leq z_1(R-X)$, we have that:

$$w^{HC} > w^{FV}. \tag{B.4}$$

We will now show that the same is true also when $\rho > z_1(R-X)$. In this case the expected payoff under FV is given by:

$$w^{FV} = z_1(R-X) + \int_{\varepsilon^L}^{\varepsilon^H} (\varepsilon - \frac{\gamma}{2} - X(1-z_1)) f(\varepsilon)\, d\varepsilon. \tag{B.5}$$

Note that we can rewrite the payoff under HC as:

$$w^{HC} = z_1(R-X) + \frac{1}{(\varepsilon^H - \varepsilon^L)}\frac{\gamma^2}{6} + \int_{\gamma+X(1-z_1)}^{\varepsilon^H} (\varepsilon - \frac{\gamma}{2} - X(1-z_1)) f(\varepsilon)\, d\varepsilon.$$

Then

$$w^{HC} - w^{FV} = \frac{1}{(\varepsilon^H - \varepsilon^L)}\frac{\gamma^2}{6} + \frac{1}{(\varepsilon^H - \varepsilon^L)}(\frac{\gamma}{2}X(1-z_1) - \frac{\gamma}{2}\varepsilon^H + \frac{(X(1-z_1))^2}{2} - \varepsilon^L X(1-z_1) + \frac{(\varepsilon^H)^2}{2})$$

which is always strictly positive since $\varepsilon^L < 0$. Therefore we have proved that also in this case $w^{HC} > w^{FV}$, from which it follows that when $w^{HC} < R - X$ we have:

$$p^{HC} > p^{FV}. \tag{B.6}$$

This proves that HC leads always to a riskier asset allocation than FV.

H. Proof of Proposition (IV.5): Ex-ante asset allocation, comparison with the socially optimal strategy

First, we want show the following property: $w^{HC} + X > w^s$. This implies that under the historical cost regime insurers adopt riskier investment strategies than what socially optimal. Note that we can write

$$w^{HC} + X = z_1 R + X(1-z_1) + \int_{(1-z_1)X}^{\gamma+X(1-z_1)} \left\{\pi^{HC}\frac{1}{2}(\varepsilon - X(1-z_1))\right\} f(\varepsilon)\, d\varepsilon$$
$$+ \int_{\gamma+X(1-z_1)}^{\varepsilon^H} [\varepsilon - \frac{\gamma}{2} - X(1-z_1)] f(\varepsilon)\, d\varepsilon$$

and

$$w^s = z_1 R + \int_0^{\gamma} \frac{\varepsilon^2}{2\gamma} f(\varepsilon)\, d\varepsilon + \int_{\gamma}^{\varepsilon_H} (\varepsilon - \frac{\gamma}{2}) f(\varepsilon)\, d\varepsilon$$

Then, it is possible to show that:

$$w^{HC} + X - w^s = X(1-z_1)\Big[1 - \int_{X(1-z_1)}^{\gamma+X(1-z_1)} \frac{\pi^{HC}}{2} f(\varepsilon)\, d\varepsilon - \int_{\gamma+X(1-z_1)}^{\varepsilon^H} f(\varepsilon)\, d\varepsilon - \int_{X(1-z_1)}^{\gamma+X(1-z_1)} \frac{\varepsilon}{2\gamma} f(\varepsilon)\, d\varepsilon\Big]$$

$$+\int_{X(1-z_1)}^{\gamma+X(1-z_1)}\frac{\epsilon^2}{2\gamma}f(\varepsilon)\,d\varepsilon-\int_0^{\gamma}\frac{\epsilon^2}{2\gamma}f(\varepsilon)\,d\varepsilon+\int_{\gamma+X(1-z_1)}^{\varepsilon^H}(\varepsilon-\frac{\gamma}{2})f(\varepsilon)\,d\varepsilon-\int_{\gamma}^{\varepsilon^H}(\varepsilon-\frac{\gamma}{2})f(\varepsilon)\,d\varepsilon$$

$$= X(1-z_1)\left[1+\frac{1}{\varepsilon^H-\varepsilon^L}\left(\frac{X(1-z_1)}{2}-\varepsilon^H+\gamma+X(1-z_1)\right)\right]$$

$$+\frac{1}{(\varepsilon^H-\varepsilon^L)2\gamma}\left[\frac{3\gamma^2X(1-z_1)+3\gamma(X(1-z_1))^2}{3}\right]+\frac{1}{2(\varepsilon^H-\varepsilon^L)}\left[\gamma X(1-z_1)-(\gamma+X(1-z_1))^2+\gamma^2\right]$$

$$= X(1-z_1)\left(\frac{\frac{3}{2}X(1-z_1)+\gamma-\varepsilon^L}{\varepsilon^H-\varepsilon^L}\right)$$

which is always positive, since ε^L is negative. Therefore we have shown that

$$w^{HC}+X>w^s, \tag{B.7}$$

which implies that, if $w^{HC}<R-X$ (and hence $w^s<R$) we have:

$$p^{HC}>p^*. \tag{B.8}$$

That is insurers invest in riskier assets under HC than socially optimal.

Now we want to understand when the optimal asset allocation under FV is safer than socially optimal, that is $w^s>w^{FV}+X$. We first note that, when the regulatory parameters are set as in Proposition (III.6), it follows that $\rho\le z_1(R-X)$ for all $z_1\in[\underline{z}_1,\bar{z}_1]$. Then, we have that w^{FV} is given by:

$$w^{FV}=z_1(R-X)+\int_{(1-z_1)X-\rho}^{\gamma+(1-z_1)X-\rho}\left\{\pi^{FV}\frac{1}{2}(\varepsilon-\rho-X(1-z_1))\right\}f(\varepsilon)\,d\varepsilon$$

$$-\rho\int_{(1-z_1)X-\rho}^{\gamma-(1-z_1)X-\rho}(1-\pi^{FV})f(\varepsilon)\,d\varepsilon+\int_{\gamma+(1-z_1)X-\rho}^{\varepsilon^H}\left[\varepsilon-\frac{\gamma}{2}-X(1-z_1)\right]f(\varepsilon)\,d\varepsilon$$

It follows that:

$$w^s-w^{FV}-X=-X(1-z_1)+\int_0^{\gamma}\frac{\varepsilon^2}{2\gamma}f(\varepsilon)\,d\varepsilon+\int_{\gamma}^{\varepsilon^H}(\varepsilon-\frac{\gamma}{2})f(\varepsilon)\,d\varepsilon-\int_{\gamma+X(1-z_1)-\rho}^{\varepsilon^H}(\varepsilon-\frac{\gamma}{2})f(\varepsilon)\,d\varepsilon$$

$$+\int_{X(1-z_1)-\rho}^{\gamma-\rho+X(1-z_1)}\frac{\pi^{FV}}{2}(X(1-z_1)-\rho-\varepsilon)f(\varepsilon)\,d\varepsilon+\rho\int_{(1-z_1)X-\rho}^{\gamma+(1+z_1)X-\rho}f(\varepsilon)\,d\varepsilon+\int_{\gamma-\rho+X(1-z_1)}^{\varepsilon^H}X(1-z_1)f(\varepsilon)\,d\varepsilon$$

Note that the third and fourth terms can be simplified to:

$$\int_{\gamma}^{\varepsilon^H}(\varepsilon-\frac{\gamma}{2})f(\varepsilon)\,d\varepsilon-\int_{\gamma+X(1-z_1)-\rho}^{\varepsilon^H}(\varepsilon-\frac{\gamma}{2})f(\varepsilon)\,d\varepsilon=\frac{1}{2(\varepsilon^H-\varepsilon^L)}((1-z_1)X-\rho)((1-z_1)X-\rho+\gamma),$$

and the fifth term is equal to:

$$\int_{X(1-z_1)-\rho}^{\gamma-\rho+X(1-z_1)}\frac{\pi^{FV}}{2}(X(1-z_1)-\rho-\varepsilon)f(\varepsilon)\,d\varepsilon=-\frac{1}{2\gamma}\int_{X(1-z_1)-\rho}^{\gamma-\rho+X(1-z_1)}(\rho+\varepsilon-X(1-z_1))^2f(\varepsilon)\,d\varepsilon$$

$$= -\frac{1}{(\varepsilon^H-\varepsilon^L)}\frac{1}{6}\gamma^2.$$

Then we can simplify the above expression to:

$$
\begin{aligned}
w^s - w^{FV} - X &= -X(1-z_1) + \frac{\gamma^2}{6(\varepsilon^H - \varepsilon^L)} + \frac{1}{2(\varepsilon^H - \varepsilon^L)}((1-z_1)X - \rho)((1-z_1)X - \rho + \gamma) \\
= & \frac{1}{(\varepsilon^H - \varepsilon^L)} X(1-z_1)\left(\varepsilon^L - \frac{1}{2}X(1-z_1) - \frac{\gamma}{2}\right) + \frac{1}{(\varepsilon^H - \varepsilon^L)}\frac{\rho}{2}(\rho+\gamma).
\end{aligned}
$$

This expression is positive if

$$\rho(\rho+\gamma) > X\,(1-z_1)\,[X(1-z_1) + \gamma - 2\varepsilon^L].$$

Note that this condition is never satisfied when $\rho \leq X(1-z_1)$. Thus, we have shown that when $\rho \leq X(1-z_1)$ the following result holds:

$$w^{FV} + X > w^s, \tag{B.9}$$

which implies that if $w^{FV} < R - X$ we have

$$p^{FV} \geq p^*. \tag{B.10}$$

This means that, under the conditions of Propostion (III.6), insurers subject to the FV regime invest in riskier assets relative to what is socially optimal. Thus, (B.8), (B.6) and (B.10) jointly prove that:

$$w^s < w^{FV} + X < w^{HC} + X$$

when regulatory parameters are set as in Proposition (III.6). It follows that, if $w^{HC} < R - X$, then:

$$p^* \leq p^{FV} < p^{HC}.$$

Thus although both under the HC and FV regime the insurers ex-ante asset allocations is riskier than socially optimal, the FV regime improves on the HC regime.