BANK OF ENGLAND

Staff Working Paper No. 707

Bank liquidity and the cost of debt

Sam Miller and Rhiannon Sowerbutts

October 2018

This is an updated version of the Staff Working Paper originally published on 19 January 2018

Staff Working Papers describe research in progress by the author(s) and are published to elicit comments and to further debate. Any views expressed are solely those of the author(s) and so cannot be taken to represent those of the Bank of England or to state Bank of England policy. This paper should therefore not be reported as representing the views of the Bank of England or members of the Monetary Policy Committee, Financial Policy Committee or Prudential Regulation Committee.

BANK OF ENGLAND

Staff Working Paper No. 707

Bank liquidity and the cost of debt

Sam Miller[1] and Rhiannon Sowerbutts[2]

Abstract

Since the 2007–09 crisis, tougher bank liquidity regulation has been imposed which aims to ensure banks can survive a severe funding stress. Critics of this regulation suggest that it raises the cost of maturity transformation and reduces productive lending. In this paper we build a bank run model with a unique equilibrium where solvent banks can fail due to illiquidity. We endogenise banks' funding costs as a function of their liquidity and show how they are negatively related, therefore offsetting some of the costs from higher liquidity requirements. We find evidence for this relationship using post-crisis data for US banks, implying that liquidity requirements may be less costly than previously thought.

Key words: Bank runs, global games, liquidity.

JEL classification: G21, G28.

(1) Bank of England. Email: sam.miller@bankofengland.co.uk
(2) Bank of England. Email: rhiannon.sowerbutts@bankofengland.co.uk

The views expressed in this paper are those of the authors, and not necessarily those of the Bank of England, the Monetary Policy Committee, Financial Policy Committee or Prudential Regulation Committee. We are grateful to Bill Francis, Antoine Lallour, Pete Zimmerman and participants at the Bank of England's internal seminar series and the 2nd Annual University of Bordeaux Conference on Quantitative Finance for helpful comments. All remaining errors are solely the authors.

The Bank's working paper series can be found at www.bankofengland.co.uk/working-paper/Working-papers

Publications and Design Team, Bank of England, Threadneedle Street, London, EC2R 8AH
Telephone +44 (0)20 7601 4030 email publications@bankofengland.co.uk

© Bank of England 2018
ISSN 1749-9135 (on-line)

1 Introduction

Global bank liquidity requirements are a fairly recent development.[1] These requirements are seen as beneficial for making banks runs less likely. However critics, such as IIF (2011), argue that they have had a negative economic impact by raising the cost of maturity transformation and reducing lending. This is plausible as more liquid assets usually yield less and longer term funding is more costly. However there is no academic consensus yet whether this is true in practice: some papers (such as Covas and Driscoll (2014)) find negative economic consequences, whereas others (such as Banerjee and Mio (2015) and Bonner and Eijffinger (2016)) do not.

We think the debate so far has missed a key channel: the offsetting impact on funding costs. Investors should recognise that banks with more liquidity are less likely to fail from a run, so the risk premia on their funding should be lower. In turn this offsets some of the cost to banks from forcing them to hold lower yield assets. This is the first paper that explicitly focusses on a funding cost offset for liquidity requirements.

To illustrate this mechanism, we build a 3 period theoretical model. A representative bank takes demand deposits from a continuum of investors in order to fund a risky, illiquid project. In the intermediate period, investors each receive a private signal about the bank's solvency and decide whether to withdraw their funding. We setup the problem as a Global Game (see Morris and Shin (2000)) and solve for the unique equilibrium where solvent banks can suffer runs. In this equilibrium, banks that hold more liquidity are less likely to fail.[2]

Our main theoretical contribution is to endogenise the bank's funding costs by explicitly modelling the investors' behaviour. In our model, investors have a safe outside option and the bank must offer a higher deposit rate to attract them. We find that investors recognise more liquid banks will be safer and therefore require a lower risk premium. In doing so we show that treating funding costs as exogenous, as in previous cost-benefit analyses of liquidity requirements such as LEI (2010), will overestimate the cost of liquidity requirements.

Previous Global Games papers have assumed funding costs are exogenous. The literature began with Morris and Shin (2000), who showed that multiple equilibria could be reduced to a unique equilibrium if depositors did not have identical information sets. Much of their analysis focussed on the information structure necessary to generate a unique equilibrium. Rochet and Vives (2004) analysed the impact of various bank fundamentals, such as solvency and liquidity, on failure probabilities. We perform similar analysis in

[1] Basel's introduction of the Liquidity Coverage Ratio (LCR) and Net Stable Funding Ratio (NSFR) in 2010 actually marked the first global liquidity requirements.

[2] In the classic bank run setup, such as Diamond and Dybvig (1983) and Bryant (1980), investors run based on a sunspot rather than expectations over the bank's fundamentals. There are multiple equilibria: one where the investors run and one where they do not. Changing the bank's fundamentals does not alter the probability of survival.

the presence of endogenised funding costs. Goldstein and Pauzner (2005) made a crucial contribution by analysing the optimal deposit contract under Global Games, but with more realistic payoffs for depositors.[3]

We also solve for the bank's profit-maximising cash choice and find it is never enough to prevent socially wasteful runs from co-ordination failure. Our results are consistent with Ahnert (2016), who was the first to explicitly model the bank's optimisation problem with Global Games and is therefore the closest paper to ours. They found that the bank's privately optimal cash choice would be socially inefficient due to a fire sale externality, even when the bank manager's incentives are aligned with depositors. We show this result holds even after endogenising funding costs, although our results are instead driven by a quasi-guarantee on deposits that limits investors' ability to fully price in liquidity risk.

We then empirically test our model's prediction that banks with more liquidity have lower funding costs. Using post-crisis data for large US banks, we find a significant negative association between asset liquidity and credit-default swap (CDS) spreads. This indicate that investors may be conscious of liquidity risk and are pricing it into firms' funding costs, implying liquidity requirements are less costly than previously thought. Our central estimate is that a 10% rise in liquid assets is associated with a 2.4% fall in CDS spreads, so this effect could be economically significant. However our empirical work is only an initial exploration - more work is needed to fully identify whether the relationship is causal.

Our empirical work mostly builds on the literature analysing the economic costs of liquidity regulation. Both Boissay and Collard (2016) and Roger and Vlcek (2011) find that liquidity requirements may be less costly than the yield penalty implies because raising liquidity relaxes risk-weighted capital requirements. We reach the same conclusion, that the yield spread between non-liquid and liquid assets overstates the cost, but through a different channel - investors recognising the bank should be safer reduces their funding costs.

This is consistent with empirical studies finding limited economic costs from liquidity requirements. Banerjee and Mio (2015) examine the liquidity requirements applied by the UK's Financial Services Authority in the aftermath of the crisis. They find that affected banks reduced intra-financial lending, but did not reduce lending to the non-financial sector. Bruno, Onali, and Schaeck (2016) analyse market reactions to announcements about liquidity regulation, which were made as the Basel framework was negotiated, and find negative abnormal returns. However these are mainly driven by announcements which also tighten capital regulation. The authors interpret this as markets likely not considering liquidity regulation binding.

[3] An issue with Rochet and Vives (2004) was their assumption, for tractability, that depositors were fund managers whose payoffs were determined solely by whether they made the correct decision whether to run - they did not care about monetary payoffs. Goldstein and Pauzner (2005) therefore made a key contribution by showing that the unique equilibrium still exists even with more realistic monetary payoffs However their results are less tractable, so our setup is closer to Rochet and Vives (2004).

We also contribute to the literature on bank funding costs and market discipline. [4] Our econometric approach is similar to recent studies (see Aymanns, Caceres, Daniel, and Schumacher (2016) and Dent, Hacioglu Hoke, and Panagiotopoulos (2017)) that find evidence of a relationship between bank funding costs and leverage in the post-crisis period. Our setup is also comparable to papers (see Miles, Yang, and Marcheggiano (2013), Yang and Tsatsaronis (2012) and Hanson, Kashyap, and Stein (2011)) finding evidence thatModigliani and Miller (1958) theorem holds partially for banks, therefore reducing the cost of capital requirements. Although our paper focusses on liquidity, the policy implication is similar - higher optimal liquidity requirements.

2 The model

Consider a three-period economy with time periods t={0,1,2} in which there are two types of agent. The first is a representative bank whose size is normalised to 1. The liability side of the bank's balance sheet is fixed with uninsured short-term debt (D) and equity (E). The bank optimises over its assets consisting of cash (c) and loans (L). The return on cash is 1 and the return on loans, R, is realised in period 2 with density f(R) and distribution F(R), which is common knowledge.

The second type of agent is a continuum of investors of size 1, providing D units of funding to the bank in period 0. The investors have preferences $u()$, with $u' > 0$, and outside option utility of $U > 1$. The bank offers investors a contract in period 0 that follows Table 1. If the investors withdraw in period 1, they receive 1 with certainty. If they wait until period 2, they receive r_D (which is endogenous) if the bank succeeds, but 0 if it fails.[5] Endogenising r_D, by ensuring it is high enough to satisfy the investor participation constraint, is our main theoretical contribution.

Table 1: Payoffs for investors

Action	Bank fails	Bank Survives
Withdraw in period 1	1	1
Don't withdraw	0	r_D

The bank's profit is any surplus left after repaying investors at time 2. In period 0 the risk-neutral bank chooses cash and r_D to maximise expected profit, subject to the investors' participation constraint of $u(invest) \geq U$.

[4]Market discipline refers whether wholesale investors price in bank risk. Noss and Sowerbutts (2012) and Sironi (2003) note that Too Big To Fail guarantees can blunt market discipline distort bank funding costs. This is a significant empirical challenge to overcome, with methods summarised by Kroszner (2016), and is one reason why we focus on the post-crisis period.

[5]Given that there is no consumption timing issue in this model then the optimal contract is strictly to issue all stable funding. In this model, as in Rochet and Vives (2004) and Ahnert (2016), we abstract from the reason why the bank must issue uninsured short-term debt rather than long-term debt. There are numerous reasons in the literature on the disciplining role of short-term debt, such as the need for liquidity and payment services.

In period 1 a fraction of investors $w \in [0, 1]$ decide to withdraw based on a private signal over the risky asset's return $x_i = R + e_i$, where e_i is independently and identically distributed $N(0, \sigma^2)$. The purpose of early withdrawals is to allow investors (who get better information in period 1) to close a bad project down early, because project failures in period 2 are very costly. The bank can pay withdrawing investors using cash or via interest-free secured borrowing from the central bank. We assume that there are no other available forms of funding.[6]

Withdrawing investors receive a fixed payoff even if the bank fails from illiquidity. This is a simplification in order to make the modelling more tractable, and has been used in previous papers such as Ahnert (2016) and Rochet and Vives (2004). Similarly, the payoff is fixed at 1 as a normalisation. Previous papers, such as Goldstein and Pauzner (2005), have analysed the trade-off between providing liquidity for depositors and preventing runs as part of an optimal contract. Our paper is more narrowly focussed on the relationship between bank liquidity and funding costs, so we do not address the optimal contract. We more fully discuss our modelling choices in Appendix A.

The central bank runs a committed facility and lend at a haircut $(1-\theta)$, where $\theta \in (0, 1)$. The bank can therefore borrow up to $\theta R(1-c)$ and will receive more central bank funding if its assets are high quality. For tractability we assume the central bank does not charge interest and knows R with a high degree of precision, due to its supervision of banks, but cannot reveal it to the market.

The central bank lends with a haircut despite knowing R with near-certainty. In practice central banks charge large haircuts in their liquidity facilities to protect themselves against possible falls in the value of collateral. Furthermore, the return R may only be realised if the bank itself is the owner of the project. If the bank failed, the assets would either have to be sold or the central bank would have to manage them with worse technology.

We solve the model first by solving for the investor's strategy in the middle period, given the signal that they receive and the bank's choice of cash. Given this strategy, we learn the failure frequency of the bank and can then solve backwards for the interest rate needed for investors to participate. Finally given this interest rate we find the bank's optimal cash holdings.

2.1 Critical thresholds

We assume that the bank can use all of its cash and liquid assets to pay depositors in the event of a run. [7] The bank will fail in period 1 if withdrawals exceed available funds:

[6] We could, as Rochet and Vives (2004) add a repo market, but we wanted to reflect the increased commitment to public liquidity support in the post-crisis era.

[7] We acknowledge the criticisms of Goodhart (2008) and Diamond and Rajan (2005) about the usability of liquid asset buffers but assume they do not apply in our model.

$$wD > \theta R(1-c)+c. \tag{1}$$

Withdrawing investors receive 1 and other investors receive 0. If the bank survives then it repays the central bank in period 2 along with r_D to the remaining investors. The firm will fail from insolvency in period 2 if its remaining deposit liabilites exceeds its assets i.e.

$$R < \frac{(1-w)r_d D + wD - c}{1-c} = R_s \tag{2}$$

Note that the total value of its assets is invariant to period 1 withdrawals, because the central bank does not charge interest. Therefore runs will not harm the bank's solvency position.[8] To isolate the impact of cash choice on solvency risk, assume that no depositors withdraw [9], i.e. $w = 0$. Evaluating the partial derivative:

$$\frac{\partial R_s}{\partial c} = \frac{Dr_D - 1}{(1-c)^2} \tag{3}$$

Holding cash can both reduce or increase the bank's solvency risk, depending on D and r_D. The bank earns a negative interest margin from holding cash to pay depositors of $Dr_D - 1$, because they need to pay depositors Dr_D and cash yields 1. Equation 3 shows that if $Dr_D - 1 > 0$ then holding more cash will make the bank less likely to be solvent (higher R_s), because they do not have enough equity to absorb the loss from using cash to pay their interest-bearing liabilities. In the limit $c \to 1$ the bank will always be insolvent, because their assets yield 1 with certainty which is not enough to pay depositors.

However if $Dr_D - 1 < 0$ then holding more cash makes the bank more likely to be solvent (lower R_s), because they have enough equity to absorb the negative margin from holding cash. In the limit $c \to 1$ the bank is always solvent, because their assets yield 1 with certainty which is enough to pay depositors.

An implication for banks with no equity is, given that $r_D > 1$, raising cash will always increase their solvency risk. We explore this dynamic further in section 2.4.

[8] This assumption was for simplicity but will hold so long as the rate charged by the central bank is no greater than r_D, because repaying the central bank will be cheaper than repaying depositors.

[9] Solvency can actually improve if some depositors withdraw early, because they receive 1 in period 1 but would have received r_D if they had waited for period 2. This reduces the value of the bank's liabilities.

2.2 Investors' withdrawing decisions

As stated before, the bank fails from illiquidity at $t = 1$ if:

$$wD > \theta R(1-c) + c$$

Therefore, investors will decide to wait until the end of the contract if:

$$\Delta u \equiv u(wait, w, x) - u(run, w, x) \geq 0 \tag{4}$$

where $u(a, w, x$ is the investor's payoff when the investor takes action $a \in \{wait, run\}$, w is the proportion of other investors that run and x is the signal they receive. Therefore we have that:

$$\Delta u = \begin{cases} r_D - 1 \text{ if } wD < \theta R(1-c) + c \textit{ and } R(1-c) + c \geq r_D(1-w)D + wD \\ -1 \text{otherwise} \end{cases} \tag{5}$$

The first condition in equation (5) is that the bank is not illiquid in period 1. The second is that it is solvent in the final period. We will show that a solvent bank can fail due to illiquidity but an insolvent bank will never survive period 1, so only the first condition is relevant in equilibrium.

2.3 Period 1 equilibrium run decision

We use techniques from the Global Games literature to solve for the period 1 equilibrium investor strategy. We will show that there is a unique equilibrium asset return, R^*, under which the bank fails and a unique equilibrium failure frequency, $F(R^*)$. The equilibrium is fully determined by:

- Exogenous parameters - $F(R), U, \theta$ and D.
- The bank's period 0 choices of c and r_D, which are taken as given in period 1.

This differs from the classic Diamond and Dybvig (1983) setup, where there are multiple equilibria and we cannot determine ex ante which will occur.

Proposition 1: There exists a threshold strategy equilibrium where all investors stay if they receive a signal above some value x, and run if they receive a signal below x:

$$s_i(x_i) = \begin{cases} \text{stay if } x_i \geq x \\ \text{run if } x_i < x \end{cases} \tag{6}$$

At this equilibrium:

$$R^* = \frac{1}{\theta(1-c)} \left(\frac{D}{r_D} - c \right) \tag{7}$$

Proof: See Appendix B.1. An interesting property of the threshold equilibrium is that it exists for low values of σ, which is counter-intuitive. Suppose σ is just above zero. An investor could receive some x_i just below R^*, but well above the insolvency threshold. They also know that other investors will have received a signal well above the insolvency threshold, given that σ is near zero, but in equilibrium they all run anyway. We unpack the intuition further in Appendix B.2.

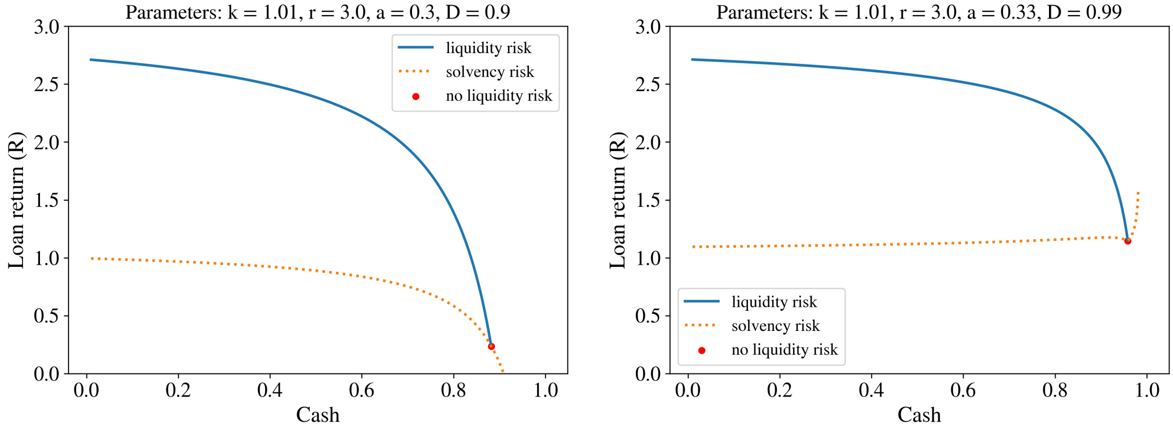
Proposition 2: The threshold strategy equilibrium given by R^* is the only strategy surviving iterated deletion of dominated strategies. It is therefore the unique equilibrium for the investor decision in period 1. Proof: See Appendix B.2. The uniqueness property is crucial for analysing comparative statics - we can see that the equilibrium run threshold is:

- Falling in c - banks with more cash can survive more withdrawals at any given R.
- Falling in θ - higher θ means the bank can raise more liquidity from the central bank at any given R.
- Falling in r_D - greater payoff from rolling over means investors are less incentivised to run after a given signal.
- Increasing in D - banks that have more flighty funding are more vulnerable to runs.

Proposition 3: If the firm holds sufficient cash $c \geq \hat{c}$ then there will be no liquidity risk in the model. Only insolvent firms will fail in period 1. Proof: see Appendix B.3. The bank could choose to eliminate its liquidity risk but will never do so in equilibrium. We discuss this further in section 4.5.

Figure 1 shows how the failure point depends on cash choice. The return at which the bank is insolvent is always below the return at which the bank is potentially illiquid i.e. solvent firms sometimes fail due to illiquidity and insolvent firms will always be illiquid. The left diagram shows that, for a firm with high equity, failure always becomes less likely as cash increases. Raising cash reduces both liquidity risk and solvency risk, so raising cash improves its chance of survival even after the point where there is no liquidity risk .

Figure 1: Failure point vs. cash choice



(a) High-equity firm

(b) Low-equity firm

However for a poorly capitalised firm, shown in the right diagram, cash makes insolvency more likely after liquidity risk is eliminated, as discussed in section 2.1.

Knowing that the unique period 1 equilibrium is failure for R below R^*, we can now solve backwards for the period 0 equilibrium. We do this by first solving for the r_D that investors demand for a given amount of cash held by the firm - the participation constraint. Then the firm optimises its cash choice to maximise expected profits, subject to the participation constraint.

2.3.1 Period 0 equilibrium - parameter restrictions

From Proposition 3, the firm can choose in period 0 to eliminate its liquidity risk by holding $c = \hat{c}$, or it can hold $c \in [0, \hat{c})$ and suffer some runs while solvent. Below we derive their optimal choice of $\{c, r_D\}$.

For tractability we assume that R is distributed uniformly on $[0, \bar{R}]$ and investors are risk neutral. This yields some parameter restrictions:

1. E(R) $= \frac{1}{2}\bar{R} > U$.
2. $\theta\bar{R} \geq D$

The first restriction is from the bank's need to make positive expected profits - it is sufficient to assume that the expected loan return exceeds the expected payout to investors. The second restriction is from the Global Games framework. There needs to be a state of the world for which it is strictly dominant not to run, because the bank has access to sufficient liquidity to survive a run from all investors.

2.4 Period 0 equilibrium - deposit rate and cash choice

2.4.1 Deposit rate

We have shown that there is a unique equilibrium run decision which depends on c, so we are now able to solve for the deposit rate. Investors have a safe outside option, $U > 1$, and they require their expected return from investing in the bank to exceed U. The depositors' participation constraint strictly binds because the bank is profit-maximising.

Equation 7 pins down the highest value of R for which the firm will fail, R^*, for a given c and r_D. Depositors will run in response to signals below R^* and receive 1. They will stay given signals above R^* and receive r_D.[10]

The participation constraint is therefore:

$$P(R < R^*) * 1 + P(R \geq R^*) * r_D = U \tag{8}$$

We substitute the in for the probabilities using equation 7:

$$\frac{1}{\bar{R}}(\frac{1}{\theta(1-c)}(\frac{D}{r_D} - c)) + r_D(1 - \frac{1}{\bar{R}}(\frac{1}{\theta(1-c)}(\frac{D}{r_D} - c))) - U = 0 \tag{9}$$

Applying the quadratic formula, we solve for the equilibrium deposit rate:

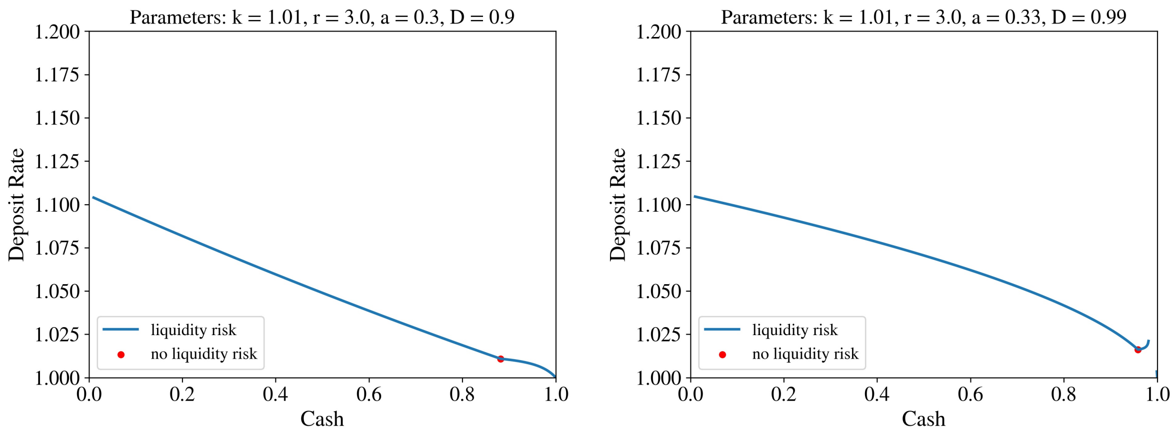
$$r_D^* = \frac{D + c + U\theta\bar{R}(1-c) + \sqrt{(D + c + U\theta(1-c))^2 - 4D(\theta\bar{R}(1-c) + c)}}{2(\theta\bar{R}(1-c) + c)} \tag{10}$$

The equilibrium deposit rate is falling in the firm's cash choice, up until the point that the firm is run-proof. Whether it falls after that will depend on how much equity the firm has. Figure 2 shows the relationship graphically for both a firm with high equity and a firm with low equity.

For a firm with high enough equity the deposit rate falls for all levels of cash. For a firm with low equity the deposit rate falls until the point at which it could survive all

[10] We note that there must be some reason for investors to have a debt contract which involves demandable debt in the first period. In this model it is efficient to liquidate the bank in the first period if signals about R are low enough, so we can justify the existence of demandable debt.

Figure 2: Deposit rate vs. cash choice



(a) High-equity firm

(b) Low-equity firm

'liquidity' based runs. It then increases after this point because the low return on holding cash means that the bank effectively becomes less solvent, as discussed in section 2.1.

However the bank never chooses more cash than needed to survive liquidity-based runs, as we discuss in Section 3.5. Therefore the equilibrium deposit rate is always falling in cash choice. This is a key result of the paper *that when a bank holds more cash, the probability of a run falls and so funding costs fall too.*

2.4.2 Cash choice

The bank maximises profits by choosing cash, subject to the participation constraint from equation 10. As the equilibrium deposit rate (r_D^*) is a function of cash, it becomes the only choice variable.

$$c^* = argmax_c(\frac{1}{\bar{R}} \int_{R^*}^{\bar{R}} R(1-c) + c - Dr_D^* dR) \tag{11}$$

We know that $c^* \in [0, \hat{c}]$ where $\hat{c}$ is the cash choice that eliminates liquidity risk. If there is an interior solution, then it will satisfy the following first-order condition:

$$-(\bar{R} - R^*)(\frac{1}{2}(\bar{R} + R^*) - 1) - \frac{dR^*}{dc}(R^*(1-c) + c^* - Dr_D^*) - \frac{Dr_D{}^*}{dc}(\bar{R} - R^*) = 0 \tag{12}$$

The first term is the cost of insuring against runs, given by the expected return on foregone loans. The second term is the positive effect from less frequent runs. The final term is the *funding cost offset* from investors knowing the bank is less risky. The optimal cash choice will trade off these 3 effects.

Both interior and exterior ($c = 0$) solutions are numerically possible. Figure 4 shows an example of each. Cash choice will be higher when:

- R is low, because the opportunity cost of holding cash is low.
- U is low, because this will relax the participation constraint and reduce r_D. Furthermore, if r_D is low then deposits become more "flighty" because the relative pay-off from running is higher.
- D is low, so that the firm has more "skin in the game" and is able to absorb the loss of $r_D - 1$ from holding cash.

Proposition 4: If the bank has no equity then it will never choose to hold $c > 0$*, regardless of the other parameters.* Proof: See Appendix B. This result is driven by two dynamics:

- The bank has no "skin in the game" and therefore little incentive to insure against illiquidity.
- Holding cash would make the bank less solvent because they have no equity to absorb the margin of $-(r_D - 1)$ from holding cash.

2.5 The funding cost offset and the cost of liquidity regulation

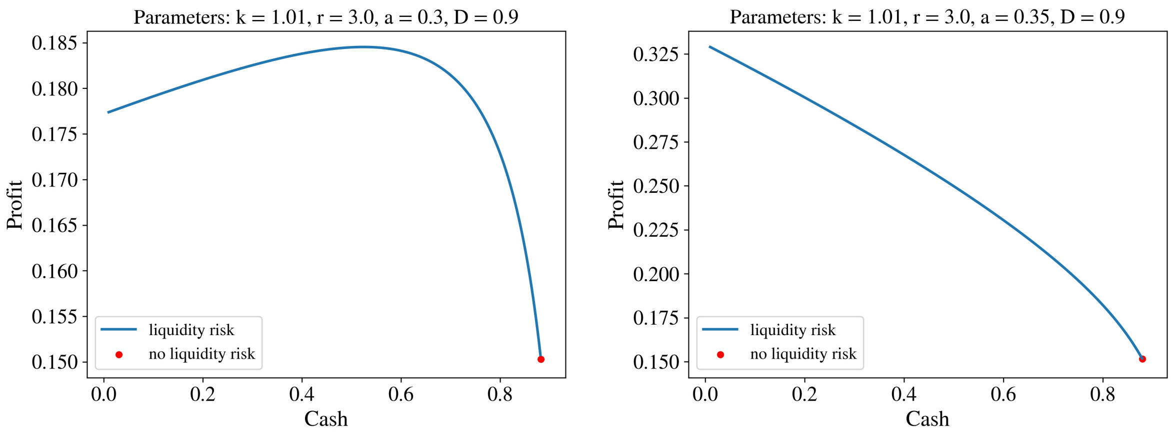
If the firm chooses $c = \hat{c}$ there will only be solvency risk, and investors will only run if the firm is insolvent. Let R_s denote the loan return for which the firm is just solvent.

$$(1 - \hat{c})R_s + c^* = Dr_D \tag{13}$$

$$R_s = R^* = \frac{Dr_D - \hat{c}}{1 - \hat{c}} \tag{14}$$

This outcome is never supported in equilibrium. The bank has no incentive to prevent runs for states s.t. $R \leq R_s$, because their equity will be wiped out in period 2. They will be indifferent over survival and failure, because their payoff will be zero in either case. In equilibrium banks will therefore always choose some liquidity risk.

Figure 3: Profits vs. cash choice



(a) Cash > 0, low E(R)

(b) Cash = 0, high E(R)

However, bank runs may be socially costly. There are many ways we could justify this, but it is not the focus of the paper. Instead we assume that under some circumstances a social planner would want the bank to hold $c = \hat{c}$ such that they only fail when insolvent.

By definition this will reduce the bank's profits. However this reduction will be somewhat offset by the falling deposit rate. Ignoring this offset would lead us to overestimate the negative impact of liquidity requirements on firm profitability. Let profits under the assumption of an exogenous deposit rate be "naive profits". We define the offset below and show it graphically in Figure 4.

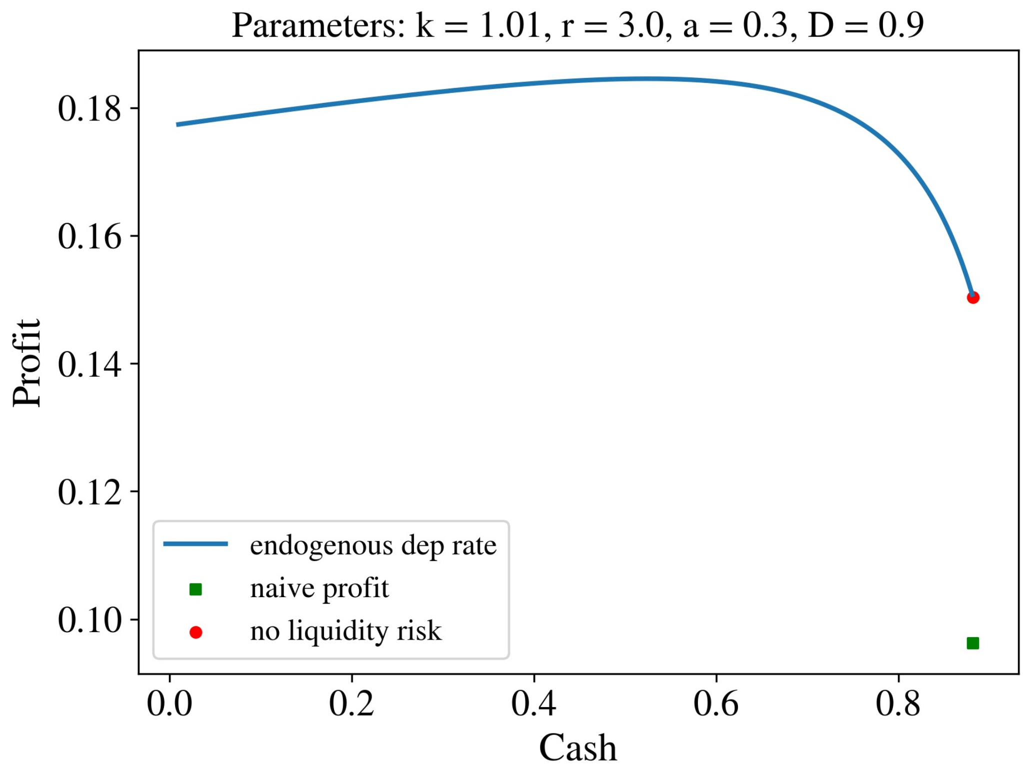
$$\text{offset} = \text{profit}_{|c=c^*} - \text{"naive" profit}_{|c=\hat{c}} \tag{15}$$

The offset will be larger when the opportunity cost of holding cash is low, and when the deposit rate is more elastic with respect to cash.

3 Empirics

We want to test our model's prediction that bank funding costs are negatively related to their asset liquidity.

Figure 4: Funding cost offset



3.1 Empirical specification

We use post-crisis data for a sample of large US bank holding companies, who have both publically available balance sheet data and funding cost measures. Our empirical specification is:

$$\text{cost of funding}_{it} = \beta_1 \text{LAR}_{it} + \beta_2 \text{STD}_{it} + \beta_3 \text{LEV}_{it} + VIX_t + UST_t + \alpha_i + \epsilon_{it} \tag{16}$$

- cost of funding_{it} is the 5 year senior credit default swap (CDS) spread for firm i in period t. Although CDS spreads are not actually a funding cost, they are a good proxy(Beau, Hill, Hussain, and Nixon (2014)). We use them instead of direct measures of funding cost, such as secondary market bond yields, because the CDS market is very deep and liquid, whereas any given funding instrument may have periods of non-trading.
- LAR is the ratio of liquid assets to total assets. β_1 is our main coefficient of interest, because it measures the association between a firm's asset-side liquidity position and cost of funding. Our model predicts a negative relationship.

- STD is the ratio of short term deposits to total assets, which is a measure of funding risk. β_2 is therefore a secondary coefficient of interest - our model predicts a positive coefficient. Funding fragility is also an important control because a firm may have more liquid assets because of an increased reliance on short term funding.

- LEV is the ratio of equity to total assets - their leverage ratio. Augustin, Subrahmanyam, Tang, and Wang (2014) shows these are correlated to firms' CDS spreads. Leverage could also be correlated with liquidity, as a more prudent firm may have both higher capital and higher liquidity.

- VIX_t is a control for S&P 500 volatility in period t, which has been found to be a significant driver of CDS spreads in previous studies (Fama and French (1989)). The VIX also serves as proxy for investor sentiment.

- UST_t is a control for the average yield on 5 year US treasuries in period t, which we use as a proxy for the risk-free rate. Risk free rates should be negatively related to CDS yields for two reasons. Firstly Longstaff and Schwartz (1995) finds that higher risk-free rates are generally associated with better macroeconomic conditions. Second Annaert, Ceuster, Roy, and Vespro (2009) argues that higher risk-free rates reduce default probabilities.

- α_i are firm-level fixed effects. These control for time invariant firm-level unobservables, such as business model, which may affect CDS spreads.

We run our specification in logs because we expect there to be diminishing returns from holding more liquidity. The coefficients therefore estimate the percentage change in CDS spreads associated with a marginal percentage change in each variable. Also logs are invariant to whether total assets are on the numerator or denominator, whereas for levels this would matter. For example if our independent variable were $\frac{\text{liquid assets}}{\text{total assets}}$, the specification would be linear in liquid assets and non-linear in total assets. But if our independent variable were $\frac{\text{total assets}}{\text{liquid assets}}$ then it would be non-linear in liquid assets and linear in total assets.

3.2 Data

Bloomberg is the source of our CDS data. These are available daily for 6 of the largest US firms: Bank of America, Citigroup, Goldman Sachs, JPMorgan Chase, Morgan Stanley and Wells Fargo. Bloomberg is also the source of the VIX and US treasury yield data.

We use the Federal Reserve's Financial Reports (form FRY-9C) to obtain data for the balance sheet variables:

- Liquid assets are the sum of cash, withdrawable reserves and US treasury securities. Our liquid asset measure is quite narrow - it excludes demand deposits at other banks and non-US government securities.

- Leverage is the ratio of Core Equity Tier 1 capital to total assets. Again this is a relatively narrow definition as it excludes other forms of loss-absorbing capital.
- Short term debt is time deposits with remaining maturity of less than a year. This covers both retail and wholesale deposits.

The FRY-9C Reports are publically available (from the Chicago Fed website), so it is plausible that investors may use them when analysing banks. They date back to 1986 at quarterly frequency for bank holding companies. However Goldman Sachs and Morgan Stanley were purely investment firms, not banks, until Q4 2008. Therefore their first Y-9C submission is Q4 2008 and the time period for our sample is Q4 2008 - Q1 2017. We have a balanced panel of 198 firm-quarter observations.

The balance sheet variables are reported for quarter-end dates, and we aggregate the daily CDS data to match the period following that reporting date. For example: the Y-9C report for Q1 2009 would refer to the firm's balance sheet on 31st March 2009, which we would match to the average of CDS spreads from 1st April to 30th June 2009. Therefore, our balance sheet variables are "lagged" by a quarter. We think this helps deal with reverse causality issues, such as rising CDS spreads triggering a run and causing a firm's liquidity to fall.

Figure 5 shows how our aggregated variables have evolved over time. Annex C provides a fuller breakdown of descriptive statistics and how these variables evolve over time by firm.

CDS spreads spiked during the financial crisis, when investors suddenly became aware of risks that had built up in the banking system. There was another spike in 2012 during the Eurozone crisis, as banks were very exposed to distressed Eurozone sovereign debt. Since then spreads have been more stable, although higher than the pre-crisis period. This could reflect permanently higher awareness of financial risks, or a better resolution regime reducing the likelihood of future bailouts.

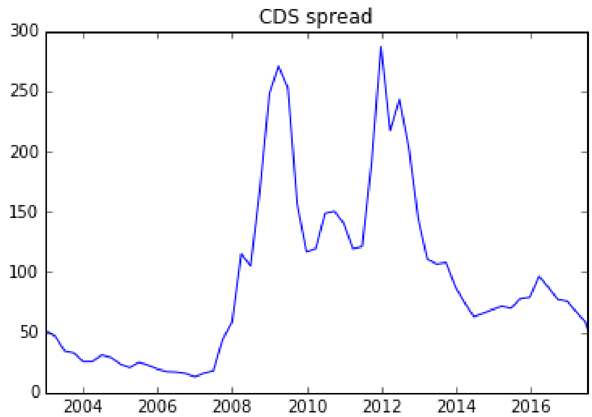
Liquidity positions were less robust pre-crisis. There was high reliance on short-term funding and banks held few liquid assets. Investors were generally confident that financial markets had become so efficient that a solvent firm could always find liquidity. Therefore there was little belief in liquidity risk and we would not expect a funding cost offset pre-crisis. After the crisis, banks built up their liquid assets and reduced short-term funding. They have continued to build liquidity, likely in response to more stringent regulation.[11]

Capital positions were also worse pre-crisis, and firms took significant losses during the crisis. However, firms were forced to re-capitalise quickly, some in response to the Troubled Asset Relief Program (TARP), and have continued to build capital since then.

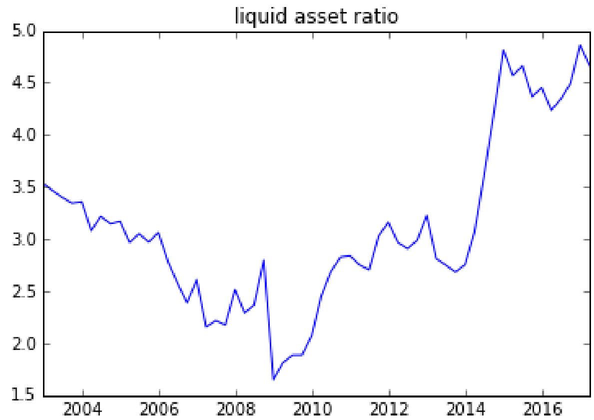
Figure 6 shows the within-firm correlations between liquid assets and CDS spreads. These

[11] The US implemented the LCR at the end of 2014.

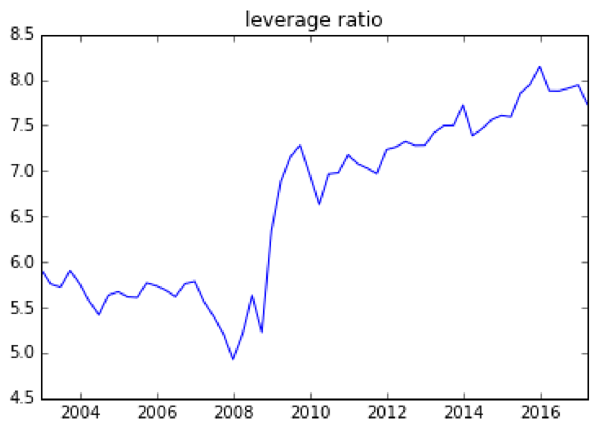
Figure 5: Variables over time



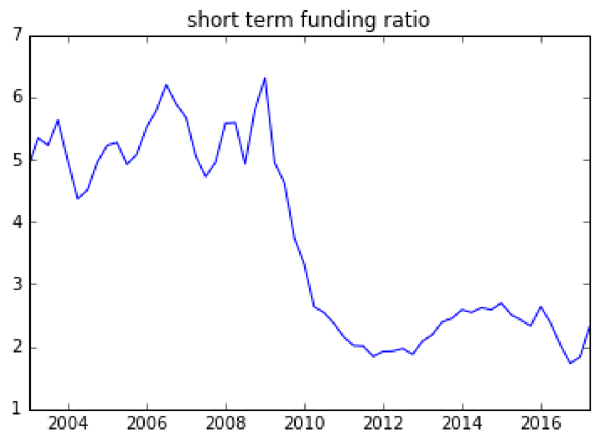
(a) CDS spreads



(b) Liquid asset ratio

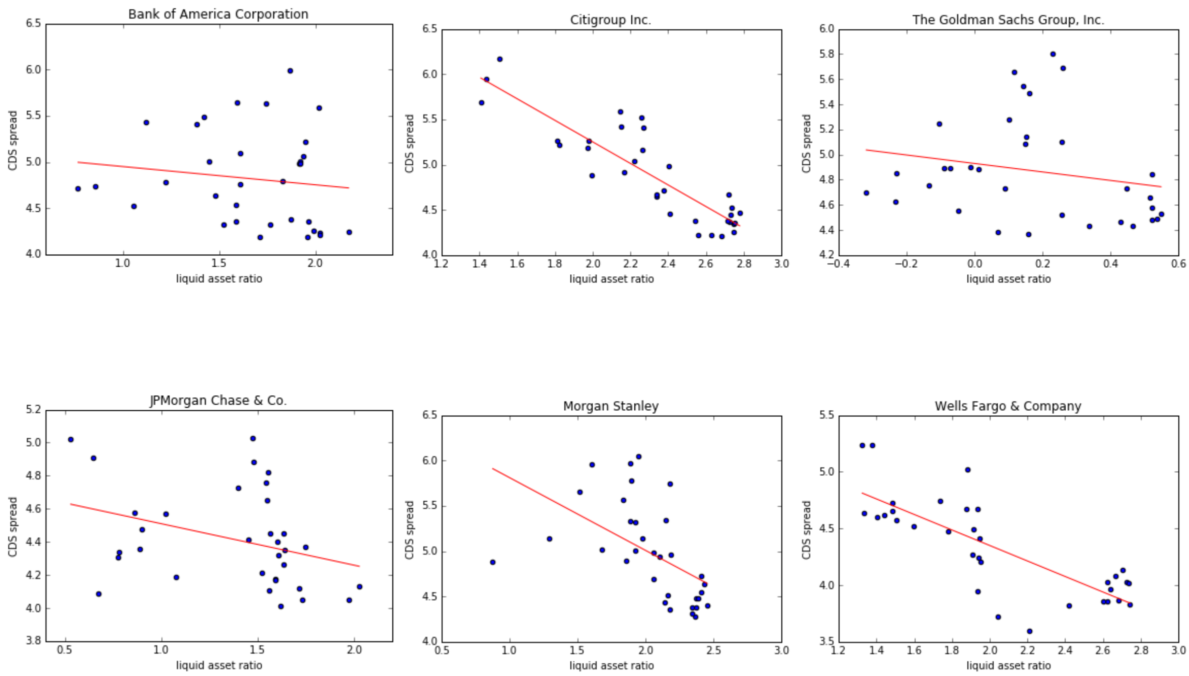


(c) Leverage



(d) Short term debt ratio

Figure 6: Within-firm correlations (variables in logs)



are generally negative, although the strength of the relationship varies.

3.3 Initial results

Table 2 presents the regression results as we build up the specification. Column 1 includes only the liquid asset ratio and firm fixed effects. There is a significant negative association between liquidity and CDS spreads. Column 2 adds controls for the firm's leverage and reliance on short term funding. The estimated size of the association between liquidity and funding costs falls, but it gains significance. Column 3 adds firm-invariant controls for stock market volatility and the risk-free rate. The magnitude of the liquidity coefficient falls, but it remains highly significant.

The coefficient in column 3 implies that a 1% rise in a firm's liquid asset ratio is associated with a -0.24% decline in their CDS spreads. Note that this is *not* a percentage point change i.e. a firm that raised its liquid asset ratio from 10% to 11% would have raised their ratio by 10%, but only 1 percentage points. For example, a bank starting with CDS spreads of 100 bps would see a fall to 97.6bps. We also find evidence of a negative association between leverage and CDS spreads, consistent with previous research, such as Augustin et al. (2014) on leverage and funding costs.

We do not find evidence that funding fragility (the variable STD) is associated with funding costs in any of our specifications. This may be due to collinearity with leverage

Table 2: Regression results

VARIABLES	(1) FE only	(2) FE + BS Variables	(3) FE + BS Variables + Controls
liq asset ratio	-0.465**	-0.389***	-0.243***
	(-3.086)	(-4.251)	(-4.276)
leverage ratio		-1.813***	-1.115***
		(-4.947)	(-6.007)
ST debt ratio		0.0398	0.0130
		(0.915)	(0.609)
Constant	5.178***	8.704***	6.921***
	(34.47)	(11.80)	(14.15)
Observations	198	198	198
R-squared	0.181	0.301	0.706
Number of firmid	6	6	6
Firm FE	YES	YES	YES
Controls	NO	NO	YES

Robust t-statistics in parentheses
*** p<0.01, ** p<0.05, * p<0.1

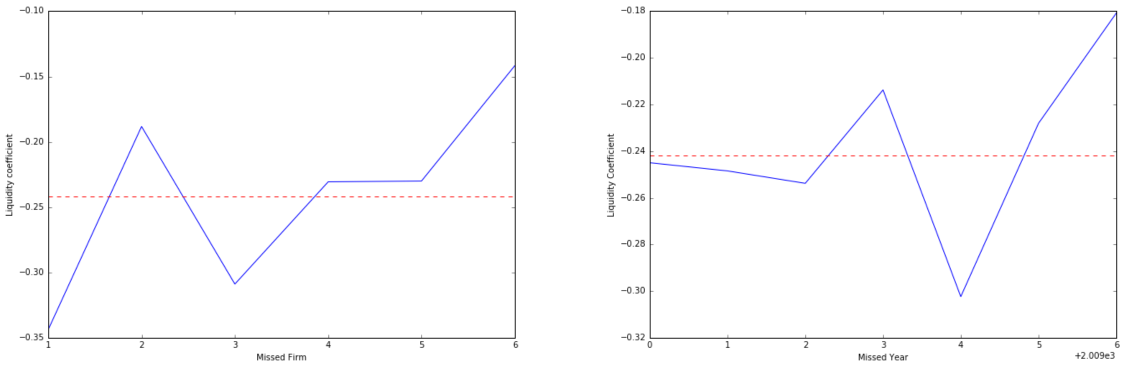
- debt funding is negatively correlated with equity funding. Alteratively, investors are perhaps less informed on funding risks than liquidity risks. The funding data is not very granular as the only maturity buckets we have are greater than / less than a year. In reality, runs can be much faster than this, so the funding data may not be a good proxy for funding risk.

3.4 Robustness to outliers

We perform two outlier robustness checks of our specification. The first is to drop out each firm individually and re-estimate without that firm. The second test is similar: we re-estimate without each of the years in our sample. If the association between liquidity and funding costs is relatively stable then we can conclude that our results are not being driven by any given firm or year.

Figure 7 shows the results of these checks. The liquid asset ratio coefficient remains fairly stable in both cases. Dropping out each year yields a range of coefficients from -0.18 to -0.30. Therefore we can conclude that our results are not being driven by a single firm or year.

Figure 7: Robustness checks



(a) Dropping each firm out

(b) Dropping each year out

3.5 Robustness to specification changes

We also test the robustness of our results to changes in specification. If changes to the specification were to dramatically change our results, the underlying relationship may not be robust.

Table 3: Robustness - different specifications

VARIABLES	(1) Broader Liquidity	(2) Lag 1	(3) Lag 2	(4) Lag 3	(5) Linear spec
liq asset ratio	-0.300*	-0.205**	-0.234**	-0.176**	-7.587*
	(-2.124)	(-3.988)	(-3.311)	(-4.029)	(-2.151)
leverage ratio	-0.928**	-1.034***	-0.983**	-1.395***	-19.06**
	(-3.182)	(-5.228)	(-3.793)	(-7.347)	(-3.773)
ST debt ratio	0.000187	0.00337	0.00910	0.00539	-0.897
	(0.00709)	(0.171)	(0.416)	(0.203)	(-0.235)
R-squared	0.706	0.698	0.717	0.740	0.614
Firm FE	YES	YES	YES	YES	YES
CONTROLS	YES	YES	YES	YES	YES

Robust t-statistics in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3 presents the results. The liquidity coefficient varies but is significant to at least the 10% level in all specifications. Column 1 broadens the liquid asset measure to include other government securities, such as local governments, and interest-bearing demand deposits at other banks. The significance of the relationship falls but the point estimate is fairly similar to our baseline result of -0.24. Columns 2-4 deepen the lag of the independent variables by 1, 2 and 3 periods, respectively. The coefficient is slightly smaller but still significant at the 5% level. Finally column 5 re-estimates with a linear specification, rather than logs, therefore we cannot compare coefficient size. Reduced significance and R^2 suggest this is a poorer fit, but the association is still significant.

4 Conclusion

In this paper, we have built a model of bank runs with a unique equilibrium where solvent banks may fail due to illiquidity. We go beyond the existing literature by endogenising the firm's funding costs to take account of this risk. While forcing the bank to hold more liquidity may impose some cost, we have shown that it is somewhat offset by reducing the bank's funding costs.

We test our model's prediction that banks with stronger liquidity positions have lower funding costs. Using post-crisis data for US banks, we find evidence of such an association. Our baseline estimate suggests doubling a bank's liquid asset ratio would be associated with a 24.4% decline in their CDS spreads. However we find no evidence for a relationship between the proportion of short-term debt a bank holds and its CDS spread.

Our results show that liquidity requirements, which force banks to hold particular levels of liquid assets relative to their liabilities, may be less costly than previously thought, at least in terms of bank's profitability and the cost of financial intermediation. This has a clear policy implication: any analysis of optimal liquidity requirements should account for the beneficial effect on funding costs.

References

Ahnert, T., December 2016. Rollover Risk, Liquidity and Macroprudential Regulation. Journal of Money, Credit and Banking 48, 1753–1785.

Annaert, J., Ceuster, M. D., Roy, P. V., Vespro, C., June 2009. What determines euro area bank CDS spreads ? Financial Stability Review 7 (1), 153–169.
URL https://ideas.repec.org/a/nbb/fsrart/v7y2009i1p153-169.html

Augustin, P., Subrahmanyam, M. G., Tang, D. Y., Wang, S. Q., 2014. Credit default swaps: A survey. Foundations and Trends(R) in Finance 9 (1-2), 1–196.
URL http://EconPapers.repec.org/RePEc:now:fntfin:0500000040

Aymanns, C., Caceres, C., Daniel, C., Schumacher, L. B., Mar. 2016. Bank Solvency and Funding Cost. IMF Working Papers 16/64, International Monetary Fund.
URL https://ideas.repec.org/p/imf/imfwpa/16-64.html

Banerjee, R., Mio, H., 2015. The impact of liquidity regulation on banks. Bank of England working papers 536, Bank of England.
URL https://EconPapers.repec.org/RePEc:boe:boeewp:0536

Beau, E., Hill, J., Hussain, T., Nixon, D., 2014. Bank funding costs: what are they, what determines them and why do they matter? Bank of England Quarterly Bulletin 54 (4), 370–384.
URL http://EconPapers.repec.org/RePEc:boe:qbullt:0156

Boissay, F., Collard, F., Dec. 2016. Macroeconomics of bank capital and liquidity regulations. BIS Working Papers 596, Bank for International Settlements.
URL http://www.bankofengland.co.uk/publications/Documents/quarterlybulletin/2014/qb14q4prereleasebankfundingcosts.pdfl

Bonner, C., Eijffinger, S. C. W., 2016. The Impact of Liquidity Regulation on Bank Intermediation. Review of Finance 20 (5), 1945–1979.
URL https://ideas.repec.org/a/oup/revfin/v20y2016i5p1945-1979..html

Bruno, B., Onali, E., Schaeck, K., 2016. Market reaction to bank liquidity regulation. Journal of Financial and Quantitative Analysis.

Bryant, J., 1980. A model of reserves, bank runs, and deposit insurance. Journal of Banking and Finance 4 (4), 335–344.
URL https://EconPapers.repec.org/RePEc:eee:jbfina:v:4:y:1980:i:4:p:335-344

Covas, F., Driscoll, J. C., Sep. 2014. Bank Liquidity and Capital Regulation in General Equilibrium. Finance and Economics Discussion Series 2014-85, Board of Governors of the Federal Reserve System (U.S.).
URL https://ideas.repec.org/p/fip/fedgfe/2014-85.html

Dent, K., Hacioglu Hoke, S., Panagiotopoulos, A., Oct. 2017. Solvency and wholesale funding cost interactions at UK banks. Bank of England working papers 681, Bank of England.
URL https://ideas.repec.org/p/boe/boeewp/0681.html

Diamond, D. W., Dybvig, P. H., June 1983. Bank Runs, Deposit Insurance, and Liquidity. Journal of Political Economy 91 (3), 401–419.
URL https://ideas.repec.org/a/ucp/jpolec/v91y1983i3p401-19.html

Diamond, D. W., Rajan, R. G., 04 2005. Liquidity Shortages and Banking Crises. Journal of Finance 60 (2), 615–647.
URL https://ideas.repec.org/a/bla/jfinan/v60y2005i2p615-647.html

Fama, F. E., French, K. R., 1989. Business conditions and expected returns on stocks and bonds. Journal of Financial Economics 25 (1), 23–49.

Goldstein, I., Pauzner, A., 06 2005. Demand-Deposit Contracts and the Probability of Bank Runs. Journal of Finance 60 (3), 1293–1327.
URL https://ideas.repec.org/a/bla/jfinan/v60y2005i3p1293-1327.html

Goodhart, C., February 2008. Liquidity risk management. Financial Stability Review (11), 39–44.
URL https://ideas.repec.org/a/bfr/fisrev/2008116.html

Hanson, S. G., Kashyap, A. K., Stein, J. C., Winter 2011. A Macroprudential Approach to Financial Regulation. Journal of Economic Perspectives 25 (1), 3–28.
URL https://ideas.repec.org/a/aea/jecper/v25y2011i1p3-28.html

IIF, 2011. The cumulative impact on the global economy of changes in the financial regulatory framework. Iif report, Institute of International Finance.
URL https://www.iif.com/system/files/nciseptember2011.pdf

Kroszner, R., 2016. A review of bank funding cost differentials. Journal of Financial Services Research 49 (2), 151–174.
URL https://EconPapers.repec.org/RePEc:kap:jfsres:v:49:y:2016:i:2:d:10.1007_s10693-016-0247-0

LEI, t. B., Aug 2010. An assessment of the long-term economic impact of stronger capital and liquidity requirements . Tech. rep., Basel Committee on Banking Supervision.
URL https://www.bis.org/publ/bcbs173.pdf

Longstaff, F., Schwartz, E. S., 1995. A simple approach to valuing risky fixed and floating rate debt. Journal of Finance 50 (3), 789–819.
URL http://EconPapers.repec.org/RePEc:bla:jfinan:v:50:y:1995:i:3:p:789-819

Miles, D., Yang, J., Marcheggiano, G., 03 2013. Optimal Bank Capital. Economic Journal 123 (567), 1–37.
URL https://ideas.repec.org/a/ecj/econjl/v123y2013i567p1-37.html

Modigliani, F., Miller, M. H., 1958. The cost of capital, corporation finance and the theory of investment. The American Economic Review 48 (3), 261–297.
URL http://www.jstor.org/stable/1809766

Morris, S., Shin, H. S., Sep. 2000. Global Games: Theory and Applications. Cowles Foundation Discussion Papers 1275R, Cowles Foundation for Research in Economics, Yale University.
URL https://ideas.repec.org/p/cwl/cwldpp/1275r.html

Noss, J., Sowerbutts, R., May 2012. Financial Stability Paper No 15: The implicit subsidy of banks. Bank of England Financial Stability Papers 15, Bank of England.
URL https://ideas.repec.org/p/boe/finsta/0015.html

Rochet, J.-C., Vives, X., December 2004. Coordination Failures and the Lender of Last Resort: Was Bagehot Right After All? Journal of the European Economic Association 2 (6), 1116–1147.
URL https://ideas.repec.org/a/tpr/jeurec/v2y2004i6p1116-1147.html

Roger, S., Vlcek, J., May 2011. Macroeconomic Costs of Higher Bank Capital and Liquidity Requirements. IMF Working Papers 11/103, International Monetary Fund.
URL https://ideas.repec.org/p/imf/imfwpa/11-103.html

Sironi, A., 2003. Testing for market discipline in the european banking industry: Evidence from subordinated debt issues. Journal of Money, Credit and Banking 35 (3), 443–72.
URL https://EconPapers.repec.org/RePEc:mcb:jmoncb:v:35:y:2003:i:3:p:443-72

Yang, J., Tsatsaronis, K., 2012. Bank stock returns, leverage and the business cycle. BIS Quarterly Review.
URL https://ideas.repec.org/a/bis/bisqtr/1203g.html

A Discussion of Modelling Choices

Our model is based off Rochet and Vives (2004). They design a bank run model where the contract looks similar to ours: investors get a fixed payoff if they run, regardless of whether the bank has enough liquidity to pay depositors when it fails. They justify this by making their investors fund managers whose utility depends only on whether they made the right decision to roll-over funding, rather than the monetary value of their decision. The fixed pay-offs are necessary for investors to have global strategic complementarities, which is required for the standard Global Games proof.

We cannot re-use their exact assumptions in our model because we want our investors to care about their monetary payoff, which is the bank's funding cost. To endogenise funding costs, and show they are related to liquidity risk, our investors require a higher expected return in successful states for banks which fail more often. Therefore we fix the investor payoffs to preserve global strategic complementarities and make investor utility depend on their monetary payoff. The investors' participation constraint pins down their incentive to provide funding to the bank.

The fixed non-zero payoff for running (even when the bank fails) in period 1 violates our model's resource constraint. There are some states of the world where the asset return is so low (or zero) that a failing bank would clearly not have the liquidity to repay a full run, yet the investors receive D in total anyway. We accept this leads to our model not being fully consistent, but have made this choice for tractability. The alternative would be to use the techniques in Goldstein and Pauzner (2005). They have investor payoffs that entirely depend on the bank's available liquidity in period 1 i.e. a failing bank will not fully repay its depositors. They show that the unique threshold equilibrium still exists, as in Rochet and Vives (2004), despite the lack of global strategic complementarities. However their equilibrium is defined implicitly so it is less tractable. Our focus is on funding costs, which are defined in period 0, so we value tractability very highly. Our focus is not on proving the equilibrium exists under certain assumptions, nor that the possibility of runs destroys optimal risk-sharing, which other papers have already proved.

Ahnert (2016) uses a similar contract to us for the intermediate period payoffs, although his final period return is an equity contract rather than a debt contract. Equity contracts introduce risk and profit sharing between the bank and investors, rather than a fixed pay-off in the event of success. In practice, equity has an infinite maturity, rather than being redeemable short-term. Equity is therefore less appropriate for us - our focus is funding costs so we require external investors for whom a debt contract is the more natural assumption.

We have also chosen to make only the final period payoff a choice variable for the bank, rather than the intermediate payoff too. This is again due to our focus on building a simple model which can be empirically tested with a link between liquidity risk and funding costs. There are previous papers, such as Goldstein and Pauzner (2005), that look at the trade-off between providing liquidity insurance for depositors (by raising the intermediate period payoff for running) and increasing liquidity risk (higher intermediate period payoff makes runs more attractive). Again this is not the focus of our paper and would significantly complicate the model, by adding another choice variable, which cannot be taken to the data. We have therefore decided to fix our intermediate period payoff at 1, which is a normalisation.

B Proofs

We will first show that a threshold strategy equilibrium exists, where all investors stay if their signal exceeds a certain value but run if the signal drops below that value. We then show that this threshold strategy equilibrium is the unique equilibrium surviving iterated deletion of dominated strategies. Finally we show that banks can hold enough cash to become "run-proof", such that they will only fail due to insolvency.

B.1 Proof of existence of a threshold strategy equilibrium

Proposition 1: There exists a threshold strategy equilibrium where all investors stay if they receive a signal above some value x, and run if they receive a signal below x:

We begin by assuming that each investor is using a threshold strategy around some signal x.

$$s_i(x_i) = \begin{cases} \text{stay if } x_i \geq x \\ \text{run if } x_i < x \end{cases} \tag{17}$$

The key question for investors in the intermediate period: given that my signal x_i is the threshold signal, what is the probability that enough investors run for the bank to become illiquid? Remember that if $w > \frac{\theta R(1-c)+c}{D} = z$ investors run then the bank will become illiquid. Investors want to know $Pr(w > z|x_i)$ i.e. the bank fails.

Given that x is the threshold signal, the bank will fail if fewer than z investors get a signal of at least x. We want to know the maximum value of R that would cause less than z investors to get a signal of at least x. Given that we have a continuum of investors and $x_i = R + e_i$, the fraction of investors with a signal higher than x will be $1 - \Phi(\frac{x-R}{\sigma})$.

$$1 - \Phi(\frac{x-R}{\sigma}) \leq z \tag{18}$$

$$R \leq x - \sigma\Phi^{-1}(1-z) = R^* \tag{19}$$

Therefore the bank will fail due to illiquidity if $R < R^*$, given that depositors withdraw when they receive a signal lower than x. To find the probability that the bank will fail, given I have received the threshold signal x, we simply need to find the probability that R is below R^*.

$$P(R < R^*|x_i = x) = \Phi(\frac{R^* - x}{\sigma}) \tag{20}$$

$$= 1 - \Phi(\frac{x - R^*}{\sigma}) \tag{21}$$

$$= 1 - \Phi(\frac{x - (x - \sigma\Phi^{-1}(1-z)}{\sigma}) = z \tag{22}$$

So if I receive the threshold signal then I believe the bank fails with probability z, where z is the critical number of people needed to run i.e. $Pr(w \geq z) = z$. This is the $U[0,1]$ distribution.

Now that we know beliefs over the distribution of investors that will run is $U[0,1]$, we can determine the threshold equilibrium loan return R^* that will cause the bank to fail. At equilibrium, investors must be indifferent staying (LHS) and running (RHS).

$$\int_{W=0}^{\frac{\theta R^*(1-c)+c}{D}} r_D dl + \int_{W=\frac{\theta R^*(1-c)+c}{D}}^{1} 0 dl \tag{23}$$

$$= \int_{W=0}^{1} 1 dl R^* = \frac{1}{\theta(1-c)}(\frac{D}{r_D} - c) \tag{24}$$

So a threshold equilibrium is that depositors will only stay if they receive $x_i \geq \frac{1}{\theta(1-c)}(\frac{1}{r_D} - c)$. We know this is a stable equilibrium because the expected staying payoff (LHS) is strictly increasing in R^*, so investors receiving a signal above R^* would strictly prefer to stay and those receiving a signal below R^* would strictly prefer to run.

B.2 Proof of Uniqueness

Proposition 2: The threshold strategy equilibrium given by R^* is the only strategy surviving iterated deletion of dominated strategies. It is therefore the unique equilibrium for the investor decision in period 1.

The key to understanding the threshold equilibrium is iterated deletion of dominated strategies. Let's continue with the assumption that $\sigma \to 0$, such that investors disregard their prior in forming an expectation over R. Let Pr(bank fails) = P. Investors will run if $P > (1-P)r_D$ i.e. $P > 1 - \frac{1}{r_D} = \gamma$.

Let $\underline{R}_0$ be the lowest return at which the firm is solvent i.e. $\underline{R}_0(1-c)+c = Dr_D$. The probability that the firm is insolvent, given signal x, is $Pr(R < \underline{R}_0|x) = \Phi(\frac{\underline{R}_0 - x}{\sigma})$. Therefore it is strictly dominant to run if investors observe any signal x such that $\Phi(\frac{\underline{R}_0 - x}{\sigma}) > \gamma$. Denote $\underline{x}_0$ the highest signal for which it is strictly dominant to run s.t. $\underline{x}_0 = \underline{R}_0 - \sigma\Phi^{-}1(\gamma)$. We can delete all strategies that rollover at $x < \underline{x}_0$.

Let $\underline{R}_1 > \underline{R}_0$ be the highest return for which a firm will fail from illiquidity given that investors with a signal below $\underline{x}_0$ run, because the proportion of investors that run will exceed available liquidity i.e. $\Phi(\frac{\underline{x}_0 - \underline{R}_1}{\sigma}) > \theta\underline{R}_1(1-c)+c$. The probability that the firm fails due to illiquidity because $R \leq \underline{R}_1$, given signal x, is at least $\Phi(\frac{\underline{R}_1 - x}{\sigma})$. Given the previous round of deletion, it is now strictly dominant to run if investors recevie signal x such that $\Phi(\frac{\underline{R}_1 - x}{\sigma}) > \gamma$. Therefore investors will run below any signal $\underline{x}_1 = \underline{R}_0 - \sigma\Phi^{-}1(\gamma) > \underline{x}_0$ and we can delete all strategies that rollover with signal $x < \underline{x}_1$.

We can iterate this deletion until we reach some pair $\underline{x}_k = \underline{R}_k - \sigma\Phi^{-}1(\gamma)$ s.t. $\nexists \underline{R}_{k+1} > \underline{R}_k$ where $\Phi(\frac{\underline{x}_k - \underline{R}_{k+1}}{\sigma}) > \theta\underline{R}_{k+1}(1-c)+c$. In English, the firm will hold enough liquidity to survive a run at any return $R_{k+1} > \underline{R}_k$, where the proportion of runners is given by the threshold signal $\underline{x}_k$ from the previous round of deletion. We have therefore deleted all strategies that involve rolling over with signals $x < \underline{x}_k$.

Now denote $\bar{R}_0$ as the lowest loan return that the bank is immune to runs i.e. $\theta\bar{R}_0(1-c)+c = 1$. The probability of this, given signal x, is $Pr(R > \bar{R}_0|X) = 1 - \Phi(\frac{\bar{R}_0 - x}{\sigma})$. Therefore $Pr(fail) \leq \Phi(\frac{\bar{R}_0 - x}{\sigma})$. Denote $\bar{x}_0$ as the lowest signal for which it is strictly dominant to roll over because $\Phi(\frac{\bar{R}_0 - \bar{x}_0}{\sigma}) = \gamma \therefore \bar{x}_0 = \bar{R}_0 - \sigma\Phi^{-}1(\gamma)$. We can delete all strategies that run with signals $x \geq \bar{x}_0$, because any investor expect the bank to survive often enough even if all other investors run.

Let $\bar{R}_1 < \bar{R}_0$ be the smallest return for which a firm cannot fail due to illiquidity given that investors with a signal above $\underline{x}_0$ roll over i.e. $\Phi(\frac{\bar{x}_0 - \bar{R}_1}{\sigma}) < \theta\bar{R}_1(1-c)+c$. The probability that a firm cannot fail due to illiquidity because $R \geq \bar{R}_1$, given signal x, is at least $1 - \Phi(\frac{\bar{R}_1 - x}{\sigma})$. Denote $\bar{x}_1$ as the lowest signal for which it is dominant to roll over because $\Phi(\frac{\bar{R}_1 - \bar{x}_1}{\sigma}) = \gamma \therefore \bar{x}_1 = \bar{R}_1 - \sigma\Phi^{-}1(\gamma) < \bar{x}_0$. There we can delete all strategies that run with signals $x \geq \bar{x}_1$.

We can iterate this deletion until we reach some pair $\bar{x}_k = \bar{R}_k - \sigma\Phi^{-}1(\gamma)$ s.t. $\nexists \bar{R}_{k+1} < \bar{R}_k$ where $\Phi(\frac{\bar{x}_k - \bar{R}_{k+1}}{\sigma}) < \theta\bar{R}_{k+1}(1-c)+c$. In other words, we eventually we reach some $\bar{R}_{k+1}$ s.t. it is no longer strictly dominant to roll over, where the proportion of investors definitely rolling over is given by the threshold signal $\bar{x}_k$ from the previous round of deletion. We have therefore deleted all strategies that involve running with signals $x > \bar{x}_k$.

Given continuity of the distributions and payoff functions, the limits of these two sequences will converge i.e. $\underline{x}_k = \bar{x}_k = x^*$ and $\underline{R}_k = \bar{R}_k = R^*$. Therefore there will be a unique equilibrium where investors roll over if they observe signals above x^*, and run if they receive signals below. Moreover $\lim_{\sigma \to 0} x^* = R^*$. Our model satisfies the general conditions laid out in Morris and Shin (2000) for existence of a unique equilibrium.

B.3 Proof of existence of cash choice that eliminates liquidity risk

Proposition 3: If a firm holds sufficient liquidity $\hat{c}$ s.t. $\theta\underline{R}_0(1-\hat{c})+\hat{c} \geq D - \frac{1}{r_D} = \gamma$, there will be no liquidity risk as investors will only run if they observe $x < \underline{x}_0$.

We prove this by contradiction. Recall that the point at which it is strictly dominant to run, due to solvency concerns, is $\underline{x}_0 = \underline{R}_0 - \sigma\Phi^{-}1(\gamma)$. Suppose $c \geq \hat{c}$ and $R^* > \underline{R}_0$ i.e. there is some liquidity risk because solvent banks can fail. There must exist at least 1 possible value R_1 s.t.$\underline{R}_0 < \underline{R}_1 < R^*$ where it is still strictly dominant to run. At signal $\underline{R}^1$, the firm has liquidity of $\theta\underline{R}^1(1-\hat{c})+\hat{c} > \gamma$ therefore it can no longer be strictly dominant to run, so $\nexists\, R^* > R_0$. There is no liquidity risk if $c \geq \hat{c}$ - only insolvent banks will fail.

B.4 Proof of unique solution for deposit rate

We show that we can rule out the lower root of the quadratic solution for the deposit rate, given by:

$$r_D^* = \frac{D+c+U\theta\bar{R}(1-c) \pm \sqrt{(D+c+U\theta(1-c))^2 - 4D(\theta\bar{R}(1-c)+c)}}{2(\theta\bar{R}(1-c)+c)} \tag{25}$$

We know that $r_D^* \geq U$ is a necessary condition to satisfy the participation constraint. The lower root is bounded above by:

$$B = \frac{1+c+U\theta\bar{R}(1-c)}{2(\theta\bar{R}(1-c)+c)} \tag{26}$$

The derivative wrt. c:

$$\frac{\delta B}{\delta c} = \frac{1}{4(\theta\bar{R}(1-c)+c)^2}(1-U\theta\bar{R})(\theta\bar{R}(1-c)+c) - 2(1+c+U\theta\bar{R}(1-c))(1-\theta\bar{R}) \tag{27}$$

$$= \frac{1}{4(\theta\bar{R}(1-c)+c)^2}(\theta\bar{R}(2-U)-1) \tag{28}$$

This could be positive or negative, depending on the values of the exogenous parameters. However whether it is positive or negative is independent of c i.e. we know it is montonically increasing or decreasing in c. It is therefore sufficient to show that that $B_{c\in\{0,1\}} < U$. If this condition holds at the extremes then it will hold at all intermediate points due to montonicity of the bound with respect to c.

$$B_{c=0} = \frac{1}{2\theta\bar{R}}(1 + U\theta\bar{R}) = \frac{1}{2\theta\bar{R}} + \frac{U}{2} < U \tag{29}$$

$$B_{c=1} = \frac{2}{2} < U \tag{30}$$

Therefore the lower root of r_D^* is always less than U, so we can rule it out.

B.5 Proof that interior solutions exist only if the bank has equity

We show that the bank will never choose $c > 0$ in equilibrium unless they have some equity. It's sufficient to show that $\frac{\delta\pi}{\delta c}|(c = 0, D = 1) < 0$.

$$\frac{\delta\pi}{\delta c}_{|c=0} = -(\bar{R} - R^*)(\frac{1}{2}(\bar{R} + R^*) - 1) - \frac{dR^*}{dc}(R^* - Dr_D) - \frac{Dr_D}{dc}(\bar{R} - R^*) \tag{31}$$

We evaluate each of these terms individually at $c = 0$. The firm is most likely to hold cash if it can raise less liquidity from its loans e.g. $\theta\bar{R} = D$. Evaluating $r_D|c = 0$ and its derivative:

$$r_{D|c=0,\theta\bar{R}=D} = \frac{1}{2}(1 + U + \sqrt{(U+3)(U-1)}) \tag{32}$$

$$\frac{\delta r_D}{\delta c}_{|c=0,\theta\bar{R}=D} = \frac{1}{2D}[1 - UD - 2(D(U+2) - 1)\sqrt{\frac{U-1}{U+3}} - (1-D)(1 + U + \sqrt{(U+3)(U-1)})] \tag{33}$$

Evaluate the failure point R^* and its derivative:

$$R^*_{|c=0,\theta\bar{R}=D} = \frac{\bar{R}}{r_D} \tag{34}$$

$$\frac{\delta R^*}{\delta c}_{|c=0,\theta\bar{R}=D} = -\bar{R}[\frac{1}{2}(1 + U + \sqrt{(U+3)(U-1)}) - D] \tag{35}$$

We can make one final simplification by evaluating at $lim_{U\to1}$, because lower reserve utilities reduce the loss from holding cash.

$$\frac{\delta\pi}{\delta c}_{|c=0,\theta\bar{R}=D,U\to1} = \bar{R}(1-D)(\bar{R} - D) > 0 \text{ iff } D < 1 \tag{36}$$

If the bank has no equity ($D = 1$), it will never hold any cash because the profit function is downward sloping at $c = 0$, even with the parameter choices that most incentivise holding cash. The intuition is that without equity, the bank has no "skin in the game" and therefore little incentive to insure against runs.

However if the bank has equity then there will be parameters for which this derivative is positive, therefore the bank may hold some cash.

C Descriptive Statistics

Table 4: Summary statistics

VARIABLES	(1) N	(2) mean	(3) sd	(4) min	(5) max
cds	210	125.2	80.00	36.49	477.9
5y UST yield	210	1.532	0.485	0.662	2.453
VIX	210	18.88	7.084	10.92	44.84
capital ratio	198	7.451	0.870	5.965	10.04
liq asset ratio	198	3.407	2.229	0.682	8.099
ST debt ratio	198	2.469	1.274	0.0149	7.076

Figure 8: CDS spread by firm

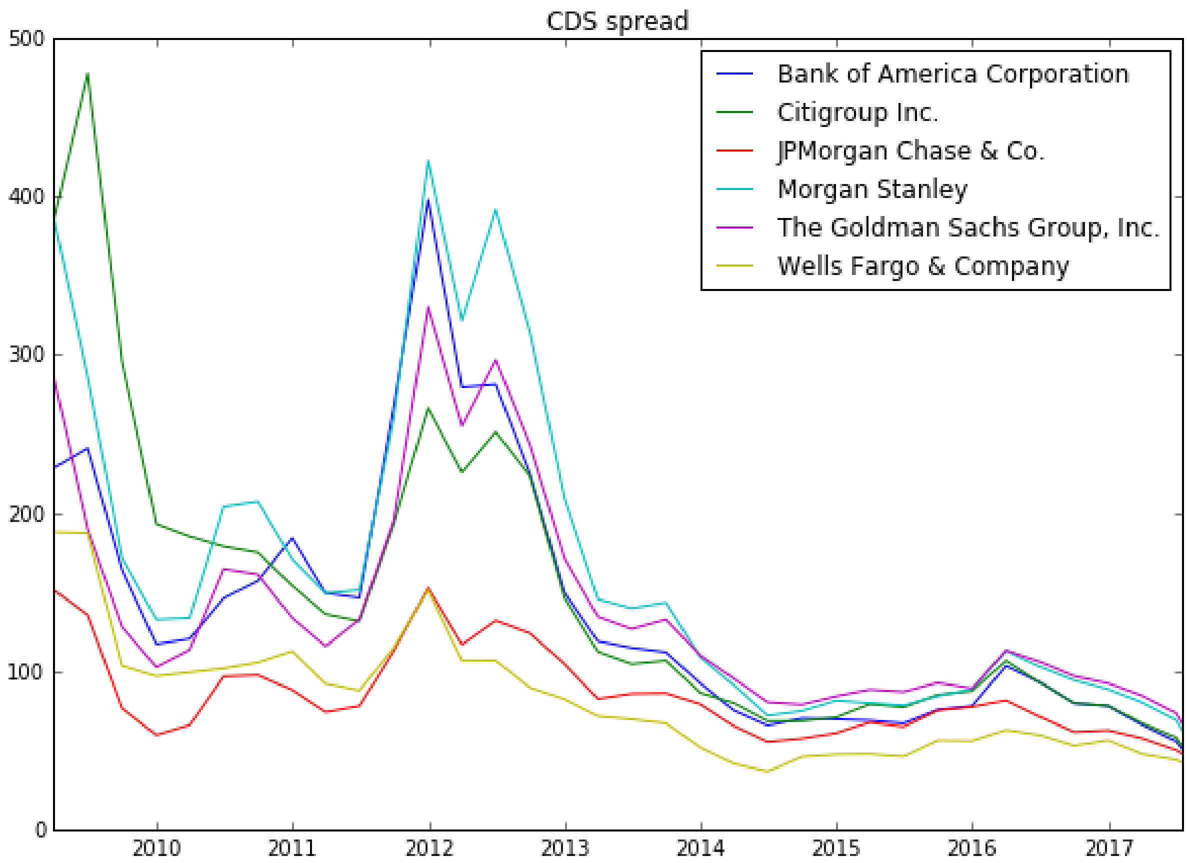


Figure 9: Liquid asset ratio by firm

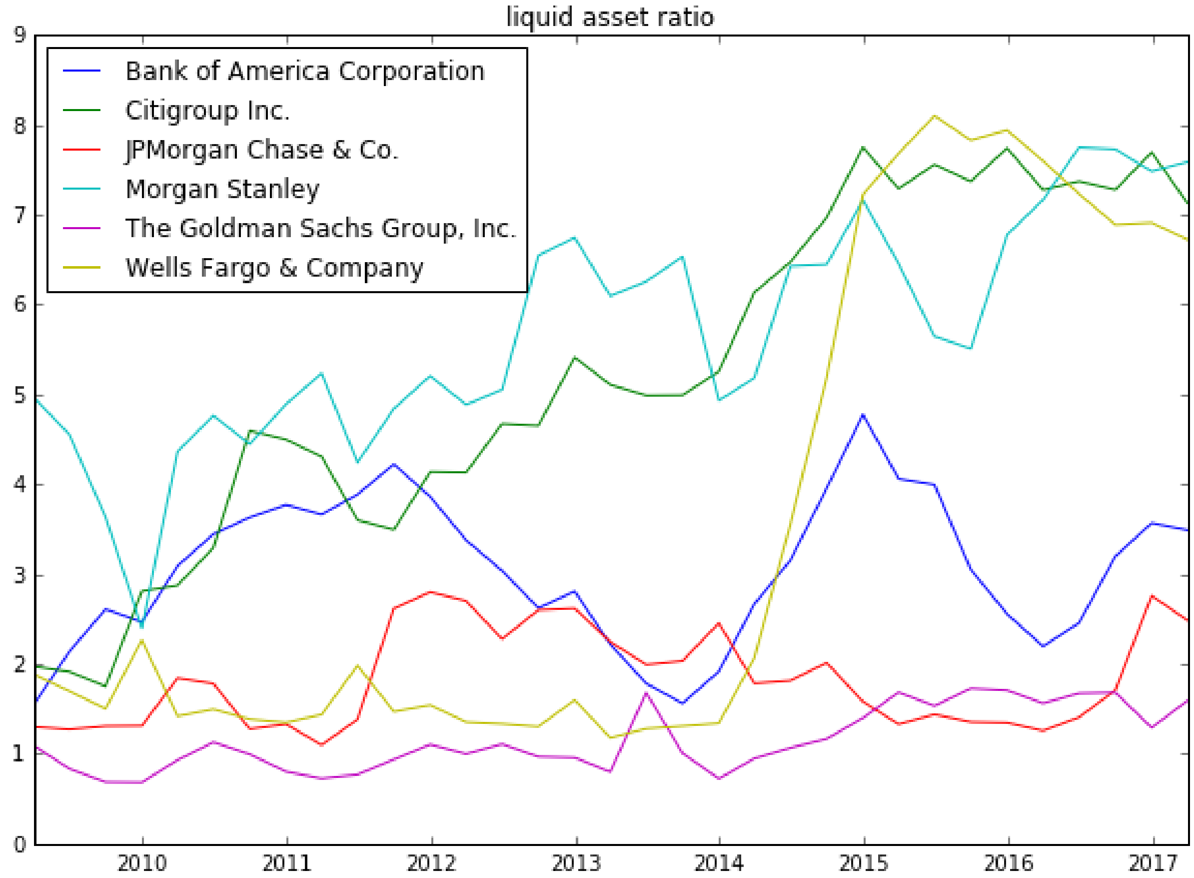


Figure 10: Leverage ratio by firm

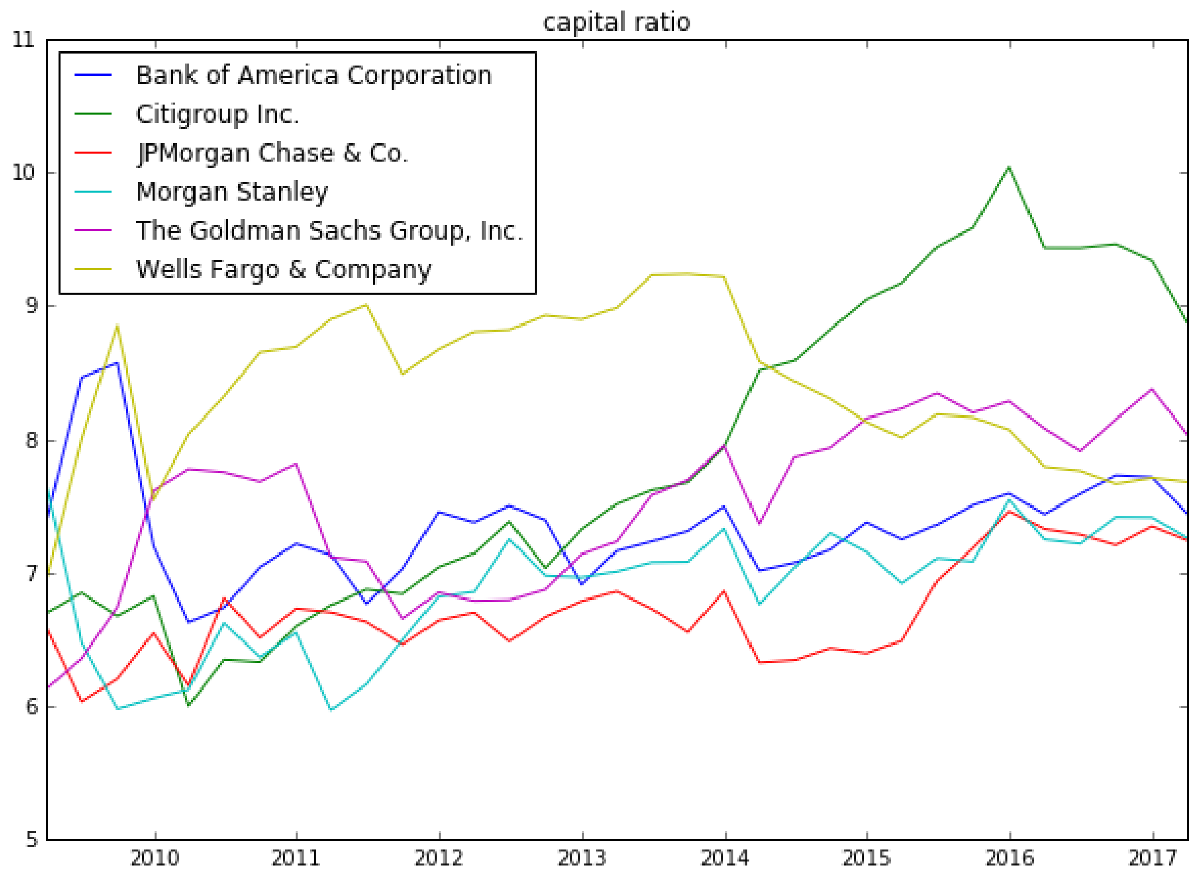


Figure 11: Short term funding ratio by firm