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Bank runs, prudential tools and social welfare in a global game general equilibrium model

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Abstract

I develop a general equilibrium model that features endogenous bank runs in a global game framework. A bank run probability — systemic risk — is increasing in bank leverage and decreasing in bank liquid asset holdings. Bank risk shifting and pecuniary externalities induce excessive leverage and insufficient liquidity, resulting in elevated systemic risk from a social welfare viewpoint. Addressing the inefficiencies requires prudential tools on both leverage and liquidity. Imposing one tool only causes risk migration: banks respond by taking more risk in another area. I extend the model and study risk migration in other fields including sectoral lending, concentration risk and shadow banking.

Key words: Bank runs, global games, capital and liquidity requirements, risk migration.

JEL classification: E44, G01, G21, G28.

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1 Introduction

Ten years have passed since the global financial crisis. In 2019 the new regulatory framework, Basel III, to prevent the recurrence of such a crisis, will be fully implemented. The recovery phase of the banking system is over and we are moving toward the evaluation phase of the financial regulatory reform.

A key to the evaluation phase is to understand how and to what extent new regulations enhance financial system resilience, and to assess their social benefits and costs. Doing so is challenging, however, due to the still incomplete financial intermediation theory (Financial Stability Board, 2017). What we need is a model that helps us understand more about how the financial system responds to new regulations. In light of the objective of Basel III – building a more resilient financial system to systemic risk – and given its multiple-tool approach, three ingredients are essential for such a model: (i) a systemic event that triggers a financial crisis, (ii) financial system resilience to such an event, and (iii) externalities that warrant the implementation of multiple tools.

In this paper, I develop a simple model that features the three essential ingredients. Specifically, I embed a bank run global game model studied by Rochet and Vives (2004) into a two-period general equilibrium model in the spirit of Christiano and Ikeda (2013, 2016). The model features bank runs as a systemic event, reflecting the historical fact that most of the financial crises have involved bank runs (Gorton, 2012 and Reinhart and Rogoff, 2009). The probability of no bank run – financial system resilience – is endogenously determined as a function of banking system fundamentals such as bank leverage and bank liquid asset holdings.

Using the model, I study its implications for social welfare and policy. The findings are triplet. First, in the laissez-faire economy, bank leverage is excessive and bank liquid asset holdings are insufficient, resulting in elevated systemic risk from a social welfare perspective. Second, bank risk shifting and pecuniary externalities are the culprit for such inefficiencies. Third, addressing the inefficiencies requires prudential tools on both leverage and liquidity. With one instrument only, another risk area, either leverage or liquidity, is always at an inefficient level. I show these three results analytically. In addition, I numerically illustrate risk migration: in response to a tightening in requirements in one area, risk can migrate from the targeted area to another, attenuating the intended effects of the requirements. Due to bank risk migration, social welfare may deteriorate if the coverage of prudential tools lacks comprehensiveness.

The model consists of three types of agents: households, banks and fund managers. Households and banks receive an endowment – household income and bank capital, respectively – in the beginning of the first period. Households allocate the income into consumption and bank deposits. Banks offer a deposit contract such that banks pay a promised interest rate as long as they do not default. The contract allows early withdrawals of funds. Banks invest the sum of deposits and bank capital in a portfolio of risky lending and safe liquidity. If the asset return is low enough, the banks, unable to pay the interest rate, default and the depositors incur a loss. Households can avoid such a loss if they successfully withdraw deposits early. Households are assumed to delegate their early withdrawal decision to fund managers, because fund managers have information advantages about the bank asset return. But early withdrawals are costly for banks. They have to sell some assets at a fire sale price if their liquidity holdings are not enough to cover withdrawals. This costly liquidation causes illiquidity-driven-bank-defaults if a large number of fund managers withdraw funds. As shown by Rochet and Vives (2004) this structure gives rise to a global game in which a bank run occurs if the bank asset return is lower than a certain threshold. Both households and banks take into account the bank run probability in choosing how much to lend and borrow, respectively. In the second period, banks distribute the profits to their owners, households, and the households consume everything at hand.

A unique feature of this model is that, unlike various financial frictions models such as Kiyotaki and Moore (1997), Bernanke et al. (1999), Jermann and Quadrini (2012), Gertler and Kiyotaki (2015) and Christiano and Ikeda (2016), bank leverage is pinned down without any binding constraints. Without bank runs, banks would increase leverage as long as the expected bank asset return is greater than the interest rate. With bank runs, however, higher leverage increases bank-run-led default probability, which in turn dampens the expected profits. Thus, market discipline, if not perfect, restrains bank behaviour. Balancing an asset return and a default probability, bank leverage has an interior solution.

To derive welfare implications, I set up a second best problem in which a benevolent regulator chooses leverage and liquidity subject to bank run risk. The first best outcome should involve no bank run because bank runs are driven by coordination failures. However, prohibiting early withdrawals is neither possible in the model nor practical in reality. Instead, by restricting leverage and liquidity, the regulator can affect financial system resilience to runs. In this model, bank liquidity holdings, required by regulators or not, are usable liquidity and hence the model abstracts away from Goodhart (2008)'s concern on liquidity usability.

The analytical characterization of the competitive equilibrium and the regulator's problem reveals two sources of externalities that cause excessive leverage and insufficient liquidity. Namely, bank risk shifting and pecuniary externalities, as I stated previously. A question is: what generates these externalities? First, bank risk shifting arises from banks' limited commitment and households' limited enforcement. Specifically, banks cannot commit to their actions that they take after receiving deposits and that households cannot enforce banks to take specific actions. This lack of commitment and enforcement gives rise to a credit market in which only a deposit interest rate works as a market signal. For households, banks offering the same interest rate look perfectly identical ex ante. Households take into account the riskiness of banks and provide funds, rationally expecting that banks will behave to maximize profits given the interest rate. Thus, the deposits are fairly priced. But each bank chooses inefficiently high leverage, knowing that doing so would not affect the interest rate they face individually. In aggregate, the banks end up with taking excessive risk in leverage and liquidity areas, a phenomenon known as risk shifting in the spirit of Jensen and Meckling (1976).

Second, the model has pecuniary externalities that work through the interest rate as in Christiano and Ikeda (2016). A bank run depends on the size of bank liabilities, specifically its important determinant, the interest rate, which, in turn, is affected by leverage in a general equilibrium. Banks ignore the effect because they behave, taking the interest rate as given in a competitive equilibrium.

The two sources of externalities affect leverage and liquidity differently. Bank risk shifting affects both leverage and liquidity, but pecuniary externalities affect leverage only. This is because a liquidity choice – how much liquidity a bank holds relative to deposits – is about the composition of assets and does not affect the total amount of borrowing. Therefore, bank risk shifting is essential for obtaining the result of insufficient liquidity and considering a liquidity tool.

The excessive leverage and insufficient liquidity warrant prudential instruments on leverage and liquidity to limit systemic risk. Doing so, however, involves a trade-off by restricting financial intermediation from households to the real sector. A leverage tool restricts banks' capacity to borrow and a liquidity tool limits liquidity transformation – the amount of deposits that are transformed into lending. Prudential policy has to balance between stabilizing the financial system – decreasing a crisis probability – and promoting the real economy by maintaining the functioning of financial intermediation.

The model highlights a general equilibrium effect on jointly optimal leverage and liquidity requirements. A numerical example illustrates that a leverage restriction should be tightened more than a liquidity requirement relative to the competitive equilibrium allocation if the supply curve of funds, which is derived from the household problem, is relatively steep. As the supply curve becomes steeper, restricting leverage lowers the interest rate more, yielding an additional benefit of reducing the crisis probability. This result suggests that jointly optimal requirements can differ significantly depending on the supply side of funds, e.g. a small open economy or a closed economy.

The model is so stylised that it has rich applications for banks' behaviour and other prudential instruments including bank/sector specific capital requirements and caps on concentration risk. Yet, risk migration between two risk areas continues to be at the heart of the applications. In the model with heterogeneous banks, the two risk areas are leverage of regulated banks and leverage of unregulated 'shadow' banks. This model also allows us to study bank-specific capital requirements and sectoral capital requirements if both types of banks are regulated. In the model with a bank portfolio choice, the two risk areas are leverage and a portfolio choice. Because of risk shifting motives, banks prefer a riskier portfolio than perfectly diversified one. But, unlike Kareken and Wallace (1978), the banks do not necessarily choose the riskiest portfolio because doing so makes the default probability too high, lowering the banks' expected profits. This model allows us to study concentration risk in specific lending.

As a further application, the paper considers a role of deposit insurance. Deposit insurance has been regarded as an institutional milestone for addressing bank runs by retail depositors. In the model, however, bank runs can occur as long as deposit insurance is imperfect, which is the case for large depositors and non-banks in practice. The paper shows that imperfect deposit insurance makes excessive leverage even excessive and further increases systemic risk.

Related literature

This paper contributes to the emerging literature on the interaction of multiple prudential tools. As I have emphasized, this paper features endogenous bank-run-led crises in a general equilibrium model. In this regard, the most closely related paper is Kashyap et al. (2017), who *numerically* study a general equilibrium version of Diamond and Dybvig (1983) in a global game framework and argue that no single regulation is sufficient to implement the social optimum. In contrast, this paper, building on a simpler global game bank run model à la Rochet and Vives (2004) and a simpler general equilibrium model à la Christiano and Ikeda (2013, 2016), derives *analytical* results on the sources of inefficiencies and the role of multiple prudential tools.[1]

In a global game framework, Vives (2014), using the model of Rochet and Vives (2004), argues that regulations should focus on the balance sheet composition of financial intermediaries. Ahnert (2016), extending the model of Morris and Shin (2000), studies intermediaries' choice of liquidity and capital separately and argues that prudential regulation should target liquidity rather than capital under fire sale externalities.[2] This paper models both the size and composition of a bank balance sheet endogenously and studies multiple prudential tools jointly.

In a dynamic general equilibrium framework, De Nicolò et al. (2012), Covas and Driscoll (2014), Van den Heuvel (2016) and Boissay and Collard (2016) study capital and liquidity regulations. This paper's model is static but features endogenous financial crises, which allows us to study a link between a crisis probability and bank fundamentals, especially capital/leverage and liquidity. In a static setting but without endogenous bank runs, Goodhart et al. (2012a, 2012b) also study the role of multiple regulatory tools.

This paper shares similar policy implications with Kara and Ozsoy (2016). They study a model with fire sale externalities and analytically show that both capital and liquidity requirements are essential to achieve constrained efficiency. With only one tool imposed on one risk area, risk migrates to and increases in other areas. Focusing on similar externalities, Walther (2015) also studies the role of capital and liquidity requirements.

This paper is also related to the huge literature on capital requirements and the emerging literature on liquidity requirements. Recent surveys on the literature on capital requirements include Rochet (2014), Martynova (2015), and Dagher et al. (2016). Gorton and Winton (2003) provides a comprehensive review of the literature. Regarding liquidity requirements, as put by Allen and Gale (2017), 'the literature on liquidity regulation is still at an early stage.' Diamond and Kshayp (2016) and Allen and Gale (2017) review the early-stage literature.

Broadly this paper can be seen as an initial step toward the recent developments in

[1] Because of its simplicity, the global game of Rochet and Vives (2004) is incorporated into the Bank of Canada's stress-test model for the banking sector (Fique 2017). Another important global game bank run model is Goldstein and Pauzner (2005). Yet, this paper adopts Rochet and Vives (2004) for analytical tractability in the general equilibrium framework.

[2] Konig (2015) and Morris and Shin (2016) focus on the role of liquidity and liquidity tools only in a global game framework. Bebchuk and Goldstein (2011) study alternative government responses to an endogenous credit market freeze similar to a bank run considered in this paper.

macroeconomic models that incorporate bank runs: Ennis and Keister (2003) and Martin, et al. (2014), both of which build on the idea of Diamond and Dybvig (1983); Gertler and Kiyotaki (2015) and Gertler et al. (2016, 2017), all of which study runs as self-fulfilling rollover crises, following Calvo (1988) and Cole and Kehoe (2011); Angeloni and Faia (2013) who extend Diamond and Rajan (2000, 2001) bank run models.

The rest of the paper is organized as follows. Section 2 presents a benchmark model in which banks choose leverage only. Section 3 conducts welfare analysis and clarifies the source of inefficiencies. Section 4 extends the model to incorporate bank liquidity and studies the role of and the interaction of leverage and liquidity requirements. Section 5 presents further extensions of the benchmark model to study bank/sector specific capital requirements, risk weights, risk concentration, deposit insurance and shadow banks. Section 6 concludes by summarising the paper's theoretical predictions for bank behaviour.

2 Model with Leverage

In this section, I present a model in which banks choose leverage only. This is the simplest general equilibrium model that features bank runs in a global game framework in this paper. It serves as a benchmark model for extensions that incorporate liquidity and others, to be studied in Section 4 and 5, respectively. In the following I first describe the environment of the model and the behaviour of agents. Then I define an equilibrium and conduct a comparative statics analysis. The derivation of non-trivial equations and the proof of all propositions are provided in the appendix.

2.1 Environment

The model has two periods, $t = 1, 2$. There is one type of goods, which can be used for consumption or investment. The economy is inhabited by three types of agents: households, fund managers and banks. Each type consists of a continuum of agents with measure unity. Banks are owned by households. In period $t = 1$, households and banks receive an endowment, y and n units of the goods, respectively. Households consume and save in banks for next period consumption. Banks invest the sum of bank capital n and deposits in a risky project. Fund managers, as delegates of households, decide whether to withdraw funds early or not. Banks default if they cannot pay the promised interest rate. In period

$t = 2$, banks pay the interest rate if they can, and transfer their profits to households, who consume everything at hand.

2.2 Households

For each household preferences are given by the quasi-linear utility,

$$u(c_1) + \mathbb{E}(c_2),$$

where c_t is consumption in period t, $\mathbb{E}(\cdot)$ is an expectation operator, and $u(\cdot)$ is a strictly increasing, strictly concave and twice differentiable function and satisfies $\lim_{c_1 \to 0} u'(c_1) = \infty$. In period $t = 1$ households consume c_1 and make a bank deposit of d, subject to the flow budget constraint, $c_1 + d \leq y$. A contract between households and banks is a deposit contract. Specifically, banks pay an interest rate of vR, where R is the promised non-state-contingent interest rate and v is the recovery rate which takes 1 if banks do not default and $v < 1$ if they default. The recovery rate v is given by the ratio of the banks' liquidation value to the debt obligation value of Rd. Households are assumed to delegate the management of deposits to fund managers who have information advantages. Fund managers can withdraw funds early at a right timing as will be explained in Section 2.3. Households diversify the management of their funds in banks over a continuum of fund managers, so that the realization of v is the same for all households, which allows the model to keep the representative agent framework. In period $t = 2$, households consume c_2, subject to $c_2 \leq vRd + \pi$, where π is bank profits. Both R and v are endogenously determined.

A key assumption is that households cannot enforce banks to choose certain actions after banks take in deposits and that banks cannot commit to any such actions. Under this assumption, knowing that banks make a choice for their own interest, the households rationally form a bank default probability P – systemic risk – and the recovery rate v. Then, solving the household problem yields the upward-sloping supply curve of deposits:

$$R = \frac{u'(y - d)}{1 - P + \mathbb{E}(v|\text{default})P}, \tag{1}$$

where $\mathbb{E}(\cdot|\text{default})$ is an expectation operator conditional on the event of bank defaults. In equilibrium, there is no idiosyncratic bank default; there is, if any, only system-wide bank default i.e. all banks default at the same time, as will be shown in Section 2.4. The supply curve (1) implies that households are willing to supply funds d at the interest rate R given the systemic risk P.

2.3 Fund managers

Fund managers have information advantages over households about a stochastic bank asset return R^k on a risky project. In the beginning of period $t = 2$, just after R^k is realized, but before it is known, fund manager $i \in (0, 1)$ receives a private noisy signal s_i about R^k, which follows a normal distribution:

$$s_i = R^k + \epsilon_i, \quad \text{with} \quad \epsilon_i \sim N\left(0, \sigma_\epsilon^2\right).$$

Parameter σ_ϵ captures the degree of the noise of the information. While s_i itself is private information, the distribution is public information.

A role of fund managers is to make a decision of withdrawing funds early or not. If a fund manager on behalf of a household withdraws early and the bank is solvent at this stage, the fund manager secures R per unit of funds and the household receives R per unit of deposits. But if a fund manager does not withdraw and the bank defaults later, the household receives an interest rate strictly less than R. Only fund managers can provide this professional service of early withdrawals with a right timing.

For analytical tractability in the general equilibrium setting, following Rochet and Vives (2004), I assume that fund managers and households adopt a behavioural rule of this type: fund manager i withdraws early if and only if the perceived probability of bank default, P_i, exceeds some threshold $\gamma \in (0, 1)$:

$$P_i > \gamma. \tag{2}$$

This rule is followed, for example, by an exogenous payoff for fund managers such that they are rewarded if they make the 'right decision' about costly withdrawals. If a net benefit of withdrawing over non-withdrawing is given by $\Gamma_0 > 0$ when the bank defaults and $-\Gamma_1 < 0$ when the bank survives, maximising the expected payoff yields the behavioural threshold, given by $\gamma = \Gamma_1/(\Gamma_0 + \Gamma_1)$. In this case, the payoffs Γ_0 and Γ_1, irrespective of goods or non-goods such as efforts or reputations, are assumed to be infinitesimally small, so that these values can be ignored in the general equilibrium consideration.

As shown by Rochet and Vives (2004), in this environment fund managers employ a threshold strategy such that they withdraw if and only if $s_i < \bar{s}$. The threshold $\bar{s}$ is determined jointly with banks' problem described below.

The behavioural rule (2) is surely the source of inefficiencies that leads to a coordination failure in the form of bank runs. But in this paper, I regard it as an inevitable nature of the financial system, and focus on the resilience of the financial system that is vulnerable to runs.

2.4 Banks

In period $t = 1$, banks offer a deposit contract to households and take in a deposit of d. Banks then combine their net worth n and the deposits d and invest in a risky project with a stochastic return R^k, which follows a normal distribution:

$$R^k \sim N\left(\mu, \sigma_k^2\right).$$

I focus on a case in which the return of the risky project is high enough to satisfy $\mu > R$ in equilibrium, so that banks always take in deposits and invest in a risky project. Also, the standard deviation σ_k is assumed to be such that the probability of the gross return R^k falling below zero is essentially zero.[3] Although there is no firm, this modelling is equivalent to the presence of firms with such a linear technology and with no frictions between banks and firms. Hence, the banks' investment in a risk project should be interpreted as financial intermediation from households to firms.

In the beginning of period $t = 2$, R^k is realized. But, before the return $R^k(n + d)$ is observed, some fund managers may withdraw their funds from banks. In response, the banks have to sell some assets. This early liquidation is costly: early liquidation of one unit of bank asset generates only a fraction $1/(1 + \lambda)$ of R^k, where $\lambda > 0$. The underlying assumption is that in response to early withdrawal requests banks raise funds by selling illiquid assets, which have been invested in the risky project, to households who have a linear but inferior technology than bankers. The technology transforms one unit of bank assets into $1/(1 + \lambda)R^k$ units of goods. With perfect competition and no friction between households and banks, the fire sale price of the early liquidated asset is $1/(1+\lambda)R^k$, where λ captures the degree of the discounting of the fire sale, or put simply the cost of early liquidation.

Let x denote the number of fund managers who withdraw funds. Then, to cover the

[3] For example, the probability of R^k falling below zero for the normal distribution with the mean return of $\mu = 1.035$ and the standard deviation of $\sigma_k = 0.025$ is smaller than $1e$-300 percent.

early withdrawal of xRd, banks have to liquidate $(1+\lambda)xRd/R^k$ units of bank assets.[4] After the liquidation, the banks have $R^k(n+d)-(1+\lambda)xRd$ in hand. If this amount is less than the promised payment under the deposit contract, $(1-x)Rd$, the banks go bankrupt. That is, the banks default if and only if

$$R^k < R\left(1-\frac{1}{L}\right)(1+\lambda x)\,, \tag{3}$$

where $L \equiv (n+d)/n$ is bank leverage.

Under the fund managers' withdrawal strategy i.e. fund manager i withdraws if and only if $s_i < \bar{s}$, the number of fund managers who withdraw is given by $x(R^k,\bar{s}) = \Pr(s_i < \bar{s}) = \Pr(\epsilon_i < \bar{s} - R^k) = \Phi((\bar{s}-R^k)/\sigma_\epsilon)$, where $\Phi(\cdot)$ is a standard normal distribution function. Condition (3) implies that the probability of bank default perceived by fund manager i is given by

$$P_i = \Pr\left(R^k < R\left(1-\frac{1}{L}\right)\left[1+\lambda x(R^k,\bar{s})\right] | s_i\right). \tag{4}$$

Conditions (2)-(4) imply that the equilibrium threshold $\bar{s}^*$ is a solution to the following set of equations:

$$\Pr\left(R^k < R^{k*}|\bar{s}^*\right) = \gamma, \tag{5}$$

$$R^{k*} = R\left(1-\frac{1}{L}\right)\left[1+\lambda x(R^{k*},\bar{s}^*)\right]. \tag{6}$$

These two equations have a unique solution for $\bar{s}^*$ and R^{k*} if the standard deviation of the signal σ_ϵ is small relative to that of bank asset return σ_k as shown by Rochet and Vives (2004). Hereafter this condition is imposed on the model.

Both the thresholds $\bar{s}^*$ and R^{k*} depend on the interest rate R and leverage L. In particular, an increase in leverage raises $\bar{s}^*$ and R^{k*} so that more fund managers withdraw funds and the probability of bank default increases. Banks take into account this effect in choosing leverage.

Banks are subject to a technical restriction such that leverage should not be too high: $L \leq L_{\max}$. This restriction differs from a prudential tool introduced later. With a high enough $L_{\max}$, the restriction is not binding in equilibrium, but it plays a role of excluding an

[4] To see this, let z denote the quantity of bank assets to be liquidated. Then, z should satisfy $1/(1+\lambda)R^k z = xRd$, which leads to $z = (1+\lambda)xRd/R^k$.

uninteresting profitable deviation of $L=\infty$ as will be discussed shortly. One interpretation of this restriction is a physical limit $L_{\max}=(y-1)/n$ at which households lend all their funds to banks.

Banks are protected by limited liability. In addition, they cannot commit to their choice of leverage in advance. This lack of commitment and the households' inability to enforce banks to take certain actions imply that banks cannot write a deposit contract that depends on leverage and equivalently the probability of bank default. Hence, the problem of banks is to choose leverage L to maximize the expected profits $\mathbb{E}(\pi)$, taking the interest rate R as given,

$$\max_{\{L\}}\int_{R^{k*}(L)}^{\infty}\left\{R^kL-R\left[1+\lambda x\left(R^k,\bar{s}^*(L)\right)\right](L-1)\right\}ndF(R^k), \tag{7}$$

subject to $L\leq L_{\max}$, where $F(\cdot)$ is a normal distribution function with mean μ and standard deviation σ_k, and $\bar{s}^*(L)$ and $R^{k*}(L)$ are solutions for $\bar{s}^*$ and R^{k*} as a function of L, respectively. In problem (7) the banks ignore the potential feedback effect of leverage on the interest rate: if the banks chose lower leverage, they would become safer and the interest rate they face would be reduced accordingly. This potential deviation to be safer is not credible, however, due to the limited commitment by banks and the limited enforcement by households. If banks offered a lower interest rate, they would not be able to attract any depositors. This induces banks to maximise the expected profits given the interest rate, giving rise to risk shifting in the spirit of Jensen and Meckling (1976).

Given that the technical restriction $L\leq L_{\max}$ is non-binding, the first-order condition is:

$$0=\int_{R^{k*}}^{\infty}(R^k-R)dF(R^k)-R\lambda\left(L-1\right)\int_{R^{k*}}^{\infty}\frac{\partial x}{\partial\bar{s}^*}\frac{\partial\bar{s}^*}{\partial L}dF\left(R^k\right)-R\lambda\int_{R^{k*}}^{\infty}xdF\left(R^k\right). \tag{8}$$

The first term in the right-hand-side of equation (8) is the expected marginal return by increasing leverage and the remaining two terms in the right-hand-side of (8) comprise the expected marginal costs. The second term is the expected marginal liquidation cost. An increase in L raises threshold $\bar{s}^*$ and increases the number of fund managers who withdraw, which leads to an increase in the liquidation cost. The third term is the expected liquidation cost. Equation (8) and assumption $\mu>R$ implies that $\partial\mathbb{E}(\pi)/\partial L|_{L=1}>0$, so that a unique solution to (8) satisfies the second-order condition as well.[5]

[5] At $L=1$, $R^{k*}=0$ and there is essentially no bank run. Hence, given that the probability of the gross return R^k falling below zero is essentially zero, the final term in the right-hand-side of equation (8) is essentially zero.

Is there no profitable deviation from the solution to (8)? This is where the technical restriction, $L \leq L_{\max}$, comes into play. For the sake of exposition and analytical tractability, consider a limit equilibrium in which the fund managers' noisy signal becomes perfectly accurate, i.e. $\sigma_\epsilon \to 0$. In the limit equilibrium, the thresholds are given by:

$$\bar{s}^* = R^{k*} = R\left(1 - \frac{1}{L}\right)\left[1 + \lambda\left(1 - \gamma\right)\right], \tag{9}$$

and the optimality condition of the banks' problem (8) is reduced to:

$$0 = \int_{R^{k*}}^{\infty} (R^k - R)dF(R^k) - \lambda\left(1 - \gamma\right) f\left(R^{k*}\right)\left[1 + \lambda\left(1 - \gamma\right)\right] R^2 \frac{L-1}{L^2}. \tag{10}$$

Equation (9) implies that $\lim_{L\to\infty} R^{k*} = R\left[1 + \lambda(1-\gamma)\right]$, so that even with infinite leverage the default probability is strictly less than unity: $\lim_{L\to\infty} F\left(R^{k*}\right) < 1$. This and condition (10) suggest $\partial\mathbb{E}(\pi)/\partial L > 0$ for a large value of L. Were it not for $L \leq L_{\max}$, there would be a profitable deviation by choosing $L = \infty$. This issue has to do with the fact that the domain of the distribution for R^k is unbounded above. Should it exist the upper bound, which is lower than $\mu < R\left[1 + \lambda\left(1 - \gamma\right)\right]$, for example, as in a uniform distribution, there would be no need for such a technical restriction.[6]

The banks' problem implies that all banks choose the same level of leverage and default, if any, at the same time. Hence, the bank default probability – systemic risk – is given by:

$$P = \Pr(R^k < R^{k*}) = F\left(R^{k*}\right). \tag{11}$$

If banks default, the banks are liquidated and their value is distributed among creditors. Consequently, the recovery rate v in equation (1) is given by:

$$v = \min\left\{1, \max\left\{\frac{R^k}{R}\frac{L}{L-1} - \lambda x(R^k, \bar{s}^*), \frac{1}{1+\lambda}\frac{R^k}{R}\frac{L}{L-1}\right\}\right\}. \tag{12}$$

If the banks have survived, the recovery rate is 1. If they have defaulted but have not sold all the bank assets, the recovery rate is given by the first term in the max operator in (12). The second term in the max operator corresponds to the recovery rate when the banks have sold all the assets and have defaulted. The banks sell all the assets if the return on

[6] A uniform distribution has also a lower bound, which implies that bank run probability can fall to zero if leverage is sufficiently low. But, with a normal distribution bank run probability is always positive. This is a main reason why this paper assumes a normal distribution.

bank assets is lower than $\underline{R}^k$, which is defined by:

$$\underline{R}^k = R\left(1 - \frac{1}{L}\right)(1+\lambda)x(\underline{R}^k, \bar{s}^*).$$

The threshold $\underline{R}^k$ is clearly lower than the default threshold R^{k*}.

2.5 Equilibrium

A competitive equilibrium for this economy consists of the interest rate R and leverage L that satisfy the supply curve for funds (1), the demand curve for funds (8) and the market clearing condition, $d = (L-1)n$, where R^{k*}, $\bar{s}^*$, P and v in these curves are given by (5), (6), (11) and (12), respectively. With the solution of R and L at hand, household consumption series c_1 and c_2 are obtained from the household budget constraints.

A unique feature of the bank problem that leads to the demand curve (8) is that bank leverage L is uniquely determined as an interior solution without any financial frictions that directly constrain leverage. Many papers have such frictions, which include banks' moral hazard of defaulting i.e. running away with borrowings (Gertler and Kiyotaki, 2015, Jermann and Quadrini, 2012), banks' hidden effort as moral hazard (Christiano and Ikeda, 2016), asymmetric information and costly state verification (Bernanke et al., 1999), and limited pledgeability (Kiyotaki and Moore, 1997). In this model, however, it is bank run risk and the resulting market discipline that help pin down bank leverage. This effect is captured by the second and third terms of the right-hand-side of equation (8) (equation (10) in the limit case). Too high leverage makes banks' liability vulnerable to runs, increases the bank run probability, raises expected liquidation costs and lowers profits. Because of such an effect banks refrain from choosing too high leverage and as a result bank leverage has an interior solution.

2.6 Comparative Statics

The competitive equilibrium for this economy depends on parameters such as μ, γ, λ, y and n. The following proposition summarizes how the supply curve (1) and the demand curve (10) shift in response to changes in these parameter values.

Proposition 1 (Comparative statics)**.** *Consider the credit market described by the supply curve (1) and the demand curve (10). Consider a limit case where $\sigma_\epsilon \to 0$. Assume* *that bank default probability is not too high,* $P \leq 0.5$, *and the leverage is not too low,* $L > 5/3 > \left(1 - \frac{0.4}{1+\lambda(1-\gamma)}\right)^{-1}$. *Then, the following results hold.*

(i) An increase in the mean return of bank assets μ *shifts the demand curve outward.*

(ii) An increase in the liquidation cost λ *(or a decrease in the threshold probability* γ*) shifts the demand curve inward.*

(iii) An increase in the household endowment y *shifts the supply curve outward.*

(iv) An increase in the bank capital n *shifts the supply curve inward.*

The comparative statics analysis supports a view that credit booms tend to be associated with vulnerability to financial crises (Schularick and Taylor, 2012). In the model a typical credit boom would feature increases in the mean return of bank assets μ, the household endowment y and the bank capital n. On the demand side, the demand curve for funds shifts outward as a result of an increase in the bank asset return. In addition, a perception of low liquidation costs (low λ) and low threshold probability by fund managers (low γ) could shift the demand curve outward further. This works to increase both leverage and the interest rate. On the supply side, if the effect of y dominates the effect of n, the supply curve shift outward as well. This works to increase leverage but to lower the interest rate. In total, these developments lead to an increase in leverage, and if the interest rate does not fall as the demand effect dominates the supply effect, the crisis probability surely rises. A credit boom builds up financial system vulnerability that triggers a banking crisis.

3 Welfare Analysis

In this section, I conduct a welfare analysis of the benchmark model presented in the previous section. The results are twofold. First, leverage is excessive in a competitive equilibrium relative to that chosen by a benevolent regulator, so that restraining leverage can improve welfare. Second, the source of the inefficiencies is bank risk shifting and pecuniary externalities.

3.1 Regulator's Problem

What is an optimal allocation for this economy? The first best should involve no bank run. But, in this paper, a bank run is regarded as an inevitable feature of the financial system.

Hence, I take a regulator perspective and set up a benevolent regulator's problem in which the regulator chooses leverage L to maximize social welfare subject to bank run risk and the supply curve for funds (1). In other words, in place of banks the regulator chooses leverage, but unlike banks the regulator maximizes social welfare and takes into account the general equilibrium effect of the choice of leverage on the interest rate. The social welfare, SW, is given by the expected households' utility, $SW = u(c_1) + \mathbb{E}(c_2)$, because banks are owned by the households.

The regulator's problem is explicitly written as:

$$\max_{\{L\}} \; u\left(y - (L-1)\,n\right) + \mathbb{E}(R^k)Ln$$
$$-\lambda \left\{ \int_{\underline{R}^k}^{\infty} \left[x(R^k, \bar{s}^*(L, R(L)))\right] R(L)(L-1)n dF(R^k) + \int_{-\infty}^{\underline{R}^k} \frac{R^k L}{1+\lambda} dF(R^k) \right\} n. \quad (13)$$

subject to $L \leq L_{\max}$, where $R(L)$ is implicitly given by the supply curve (1) and threshold $\bar{s}^*$ is written as a function of R as well as L to take into account the effect of R on the threshold. The regulator balances the expected benefit of financial intermediation, which is given by the first row of the regulator's objective (13), and the expected loss due to the fire sale of bank assets, which is given by the second row of the regulator's objective (13). The loss is governed by the parameter, $\lambda > 0$, that captures the cost of early liquidation.

The first-order condition of the regulator's problem is given by:

$$0 = -\,R\left[1 - P + \mathbb{E}(v|\text{default})P\right] + \mathbb{E}(R^k) - \lambda R \int_{\underline{R}^k}^{\infty} x dF - \lambda \int_{-\infty}^{\underline{R}^k} \frac{R^k}{1+\lambda} dF$$
$$-\lambda R(L-1)R \int_{\underline{R}^k}^{\infty} \frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L} dF - \lambda(L-1)\int_{\underline{R}^k}^{\infty} \left(\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} R + x\right)\frac{\partial R}{\partial L} dF, \quad (14)$$

where the supply curve (1) was used to substitute out for $u'(y - (L-1)n)$. Condition (14) distinguishes itself from the banks' optimality condition (8) in two respects. First, while the regulator takes into account all possible states including bank run states, the banks focus only on non-default states due to limited liability. It should be noted, however, that as will be shown later limited liability per se is not the source of inefficiencies. Second, the regulator internalizes the impact of leverage L on the interest rate R, which is captured by the final term in (14), while the banks do not as they take R as given.

3.2 Roles of leverage restrictions

Now we are in a position to study whether the competitive equilibrium has excessive leverage. If the slope of the social welfare evaluated at the competitive equilibrium allocation, $\partial SW/\partial L|_{\text{CE}}$, is negative, the leverage is excessive so that restricting it can improve welfare. Because the competitive equilibrium solves the banks' optimal condition, it has to be $\partial \mathbb{E}(\pi)/\partial L|_{\text{CE}} = 0$. Then, $\partial SW/\partial L|_{\text{CE}}$ is written and expanded as:

$$\begin{aligned}
\left.\frac{\partial SW}{\partial L}\right|_{\text{CE}} =& \left.\frac{\partial SW}{\partial L}\right|_{\text{CE}} - \left.\frac{\partial \mathbb{E}(\pi)}{\partial L}\right|_{\text{CE}} \\
=& -\frac{1}{L-1}\left[\int_{\underline{R}^k}^{R^{k*}} R^k dF + \int_{-\infty}^{\underline{R}^k} \frac{R^k}{1+\lambda} dF\right] \\
& - \lambda(L-1)\left[\int_{\underline{R}^k}^{R^{k*}} R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L} dF + \int_{\underline{R}^k}^{\infty}\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right)\frac{\partial R}{\partial L} dF\right], \qquad (15)
\end{aligned}$$

where $\partial x/\partial \bar{s}^* = \Phi'((\bar{s}^* - R^k)/\sigma_\epsilon)(1/\sigma_\epsilon) > 0$.[7] The first term on the right-hand-side of (15) is negative under the assumption that the probability of R^k falling below zero is essentially zero. As shown in the appendix, an increase in leverage L raises the threshold $\bar{s}^*$ and an increase in the interest rate R raises the threshold: $\partial \bar{s}^*/\partial L > 0$ and $\partial \bar{s}^*/\partial R > 0$. In addition, under a plausible condition the supply curve (1) is upward-sloping, i.e. $\partial R/\partial L > 0$. In this case, equation (15) implies $\partial SW/\partial L|_{\text{CE}} < 0$, which leads to the following proposition.

Proposition 2 (Excessive leverage)**.** *Assume that the supply curve (1) is upward sloping. Then, in a competitive equilibrium, bank leverage is excessive from a social welfare viewpoint. Restricting leverage can improve social welfare.*

A corollary of Proposition 2 is that the probability of bank runs – the systemic risk – is too high in a competitive equilibrium. The excessive leverage implies the high threshold R^{k*}, which in turn implies the elevated systemic risk.

The excessive leverage and the resulting elevated systemic risk in the competitive equilibrium provides a rational for policymakers to introduce prudential policy to improve social welfare. The second best allocation, which solves the benevolent regulator's problem, can be achieved, for example, by imposing a leverage restriction $\bar{L}$ on banks such that $L \leq \bar{L} = L^*$, where L^* is a solution to the regulator's problem, (14). Similarly, the second

[7] Bank capital n is abstracted away from these conditions because they are all proportional to n. The same applies hereafter in calculating the slope of the social welfare.

best can be achieved by imposing restrictions on a capital ratio, $n/(n+d)$, such that it is no less than $1/L^*$.

3.3 Sources of inefficiencies

What is the source of inefficiencies that give rise to excessive leverage? Equation (15) is suggestive, but it is not entirely clear about what causes the excessive leverage. To address this question, I consider the same problem but without bank risk shifting. In this economy, the household optimality condition (1) stays the same, but what changes is the banks' behaviour. Now the banks can commit to their choice of leverage so that they can provide a deposit contract that specifies leverage L as well as the interest rate R. The banks choose a pair of leverage and the interest rate, $\{L, R\}$, to maximize the same expected profits (7) subject to the technical constraint $L \leq L_{\max}$ and the households' participation constraint:

$$R[1-F(R^{k*})] + \int_{\underline{R}^k}^{R^{k*}} \left[R^k \frac{L}{L-1} - R\lambda x(R^k, \bar{s}^*)\right] dF + \int_{-\infty}^{\underline{R}^k} \frac{R^k}{1+\lambda} \frac{L}{L-1} dF \geq R^e, \quad (16)$$

for some return R^e, where R^{k*}, $\bar{s}^*$ and $\underline{R}^k$ are all a function of L and R. The left-hand-side of (16) corresponds to the expected return received by households, $R[1-P+\mathbb{E}(v|\text{default})]$. As long as condition (16) holds, which promises the expected return of R^e, households are willing to supply funds irrespective of a pair of L and R. In equilibrium, the constraint (16) is binding and $R^e = u'(y-(L-1)n)$.

The binding constraint (16) disciplines the banks' behaviour as an increase in leverage and a resulting increase in bank riskiness raises the interest rate. Indeed, the binding constraint (16) implicitly defines R as a function of L, which is denoted as $R = R_B(L)$, where $\partial R_B/\partial L > 0$.[8] Because of this feedback effect, the banks choose lower leverage than that in the benchmark model presented in Section 2. The optimality condition of the banks' problem is delegated to the appendix.

Leverage is still excessive in a competitive equilibrium even in the economy without bank risk shifting, but the degree of excessiveness is mitigated. Let CE' denote such a competitive equilibrium. The slope of the social welfare evaluated at the competitive

[8] A condition for $\partial R_B/\partial L > 0$ is the same as that for $\partial R/\partial L > 0$, which is assumed to hold. A relationship between $\partial R/\partial L$ and $\partial R_B/\partial L$ is such that $\partial R/\partial L - \partial R_B/\partial L \propto -u''(c_1) > 0$. Hence, the slope of $R(L)$ is sleeper than that of $R_B(L)$.

equilibrium is given by

$$\left.\frac{\partial SW}{\partial L}\right|_{\mathrm{CE}'} = \lambda(L-1)\left[\int_{\underline{R}^k}^{\infty}\left(\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R}R + x\right)dF\right]u''(c_1) \in \left(\left.\frac{\partial SW}{\partial L}\frac{1}{n}\right|_{\mathrm{CE}}, 0\right). \tag{17}$$

This equation shows that the only source of inefficiencies is the pecuniary externalities that work through the interest rate R, which is captured by the second derivative of the utility function, $u''(c_1)$. This result is summarized in the following proposition.

Proposition 3 (Excessive leverage in the model without bank risk shifting)**.** *Consider the benchmark model without bank risk shifting in which the supply curve (1) is upward sloping. In a competitive equilibrium, bank leverage is excessive because only of the pecuniary externalities that work through the interest rate.*

Propositions 2 and 3 reveal that the source of inefficiencies in the benchmark model is twofold: bank risk shifting and pecuniary externalities. Regarding bank risk shifting, in the benchmark model, banks compete for attracting deposits by using the interest rate only. Even if a bank attempts to become safer by lowering leverage, the bank would not offer a lower interest rate because they would lose depositors. Hence, such an attempt cannot be a profitable deviation from the equilibrium. Instead, in the model without bank risk shifting, the market works through bank riskiness as well as the interest rate. In this case, banks take into account the effect of leverage on the interest rate, and consequently the leverage becomes lower than in the benchmark model. Regarding pecuniary externalities, an increase in bank leverage raises the interest rate through a general equilibrium effect and increases the costs associated with bank asset fire sales, $\lambda Rx(L-1)$. This effect is ignored by banks which take the interest rates, R or R^e, as given in the economy with or without bank risk shifting, respectively.

4 Leverage and Liquidity

In this section, I extend the benchmark model presented in Section 2 to incorporate liquid assets in a bank balance sheet. This section first presents the extended model and shows analytical results on a competitive equilibrium, social welfare, the source of inefficiencies and prudential tools on leverage and liquidity. It then proceeds to numerical analyses on the role of and interaction between the two tools regarding social welfare and systemic risk.

4.1 Model with Leverage and Liquidity

In this model, a bank balance sheet consists of safe liquidity as well as risky lending. Specifically, banks have an access to a safe technology with gross return unity. Assets invested in a safe technology are called liquidity, which can be drawn at any time without any cost.

In period $t = 1$, banks allocate the sum of their net worth n and the deposit d to liquidity M and lending $n + d - M$. In response to fund managers' early withdrawal claim of xRd, banks use liquidity first because doing so is not costly, and they sell the assets invested in a risky project to households at a fire sale price if the amount of liquidity is not enough to cover the amount of the claim: $xRd > M$. In this case, the banks sell $(1+\lambda)(xRd-M)/R^k$ units of the bank lending. If the banks' revenue, $R^k(n+d-M)-(1+\lambda)(xRd-M)$, cannot cover the promised payment to the depositors who have not withdrawn early, $(1-x)Rd$, the banks go bankrupt. Instead, if the banks can cover the early withdrawal request by using liquidity, i.e. $xRd < M$, they do not liquidate any risky assets and they are subject to only a fundamental default. Hence, banks default if and only if

$$R^k < \frac{R-m}{\frac{L}{L-1}-m}\left(1+\lambda\frac{\max\{xR-m,0\}}{R-m}\right), \tag{18}$$

where $m \equiv M/d$ is a liquidity-deposit ratio (hereafter a liquidity ratio or liquidity for short) and $L \equiv (n+d)/n$ is leverage. This condition is reduced to condition (3) in the case of bank leverage only, i.e. $m = 0$. Condition (18) implies that thresholds $\bar{s}^*$ and R^{k*} are determined by equation (5) and

$$R^{k*} = \frac{R-m}{\frac{L}{L-1}-m}\left(1+\lambda\frac{x(R^{k*},\bar{s}^*)R-m}{R-m}\right), \tag{19}$$

where $x(R^{k*},\bar{s}^*) = \Phi((\bar{s}^*-R^{k*})/\sigma_\epsilon)$. At the thresholds of R^{k*} and $\bar{s}^*$, the amount of early withdrawals exceeds the bank liquidity, i.e. $x(R^{k*},\bar{s}^*)R-m > 0$. Otherwise, the banks would not default for R^k close to but smaller than R^{k*}. Equation (19) is the extension of equation (6) to incorporate a bank liquidity choice.

The banks' problem is to maximize the expected profits $\mathbb{E}(\pi)$ by choosing leverage and liquidity,

$$\max_{\{L,m\}} \int_{R^{k*}(L,m)}^{\infty} \Big\{ R^k L - (R^k-1)(L-1)m \\ - \left[R + \lambda \max\left\{ x\left(R^k, \bar{s}^*(L,m)\right) R - m, 0 \right\} \right] (L-1) \Big\} n dF(R^k),$$

subject to $L \leq L_{\max}$ and $0 \leq m \leq L/(L-1)$, where the thresholds $\bar{s}^*(L,m)$ and $R^{k*}(L,m)$ are a solution to equations (5) and (19), written as a function of L and m. A marginal increase in the liquidity ratio m is associated with the opportunity cost of $(R^k-1)(L-1)$, but it reduces the likelihood of fire sales and its cost $\lambda \max\{xR-m,0\}(L-1)$. High enough liquidity, e.g. $m = L/(L-1)$, insulates banks from bank runs and makes them perfectly bank-run-proof, but $R > 1$ is assumed so that such a choice cannot be a solution to the problem.[9]

For solving the banks' problem, let $\bar{R}^k$ define a threshold for R^k such that bank liquidity just covers the amount of early withdrawal, i.e. $x(\bar{R}^k, \bar{s}^*)R = m$. Solving for $\bar{R}^k$ yields:

$$\bar{R}^k = \bar{s}^* - \sigma_\epsilon \Phi^{-1}\left(\frac{m}{R}\right).$$

Now the first-order conditions of the banks' problem, which characterize an interior solution for leverage L and liquidity m, are given by:

$$0 = \int_{R^{k*}}^{\infty} [R^k - (R^k-1)m - R] dF(R^k) \\ - \int_{R^{k*}}^{\bar{R}^k} \left[\lambda(Rx - m) + (L-1)\lambda R \frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L} \right] dF(R^k), \tag{20}$$

$$0 = -\int_{R^{k*}}^{\infty} (R^k-1) dF(R^k) + \lambda \int_{R^{k*}}^{\bar{R}^k} \left(1 - R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial m}\right) dF(R^k). \tag{21}$$

Equation (20) corresponds to $0 = \partial \mathbb{E}(\pi)/\partial L$, which is reduced to the optimal condition in the benchmark model (10) when $m = 0$. Equation (20) implies that the fire sale cost due to a marginal increase in leverage appears only when liquidity cannot cover the amount of early withdrawals, i.e. when $R^k < \bar{R}^k$. Similar to the benchmark model, the marginal

[9]If leverage is too low, the gross interest rate can fall below unity, violating the assumption of $R > 1$. One way to address this problem is to assume that the gross return of liquidity is lower than unity. Another way is to assume that the gross return of liquidity depends on R and is given by $R - \xi$, where $\xi > 0$ is a liquidity premium. In this case the presence of R would become another source of pecuniary externalities. To make the model as simple as possible, I restrict my attention to a case in which $R > 1$.

impact of raising the threshold $\bar{s}^*$ on x, the number of fund managers who withdraw funds early, is positive and the marginal impact of leverage L on the threshold $\bar{s}^*$ is positive: $\partial x/\partial \bar{s}^* > 0$ and $\partial \bar{s}^*/\partial L > 0$.

Equation (21) corresponds to $0 = \partial \mathbb{E}(\pi)/\partial m$. The first term in the right-hand-side of equation (21) is the opportunity cost of holing liquidity, i.e. the net expected return on the risky project the banks would have earned if they had not held liquidity but invested in the risky project. The second term in the right-hand-side of equation (21) is the marginal benefit of holding liquidity by lowering the number of fund managers who withdraw early, x. As shown in the appendix, an increase in liquidity lowers the threshold $\bar{s}^*$, i.e. $\partial \bar{s}^*/\partial m < 0$ if and only if the interest rate is not high enough to satisfy:

$$R < \frac{1+\lambda}{1+\lambda x}\frac{L}{L-1}. \tag{22}$$

Under condition (22) an increase in liquidity m reduces the thresholds $\bar{s}^*$ and R^{k*} and lowers the bank run probability $F(R^{k*})$ and thereby increases the resilience of the financial system. Instead, if condition (22) does not hold, the interest cost of bank liabilities is so high that a decrease in the expected revenue due to an increase in liquidity holding causes the banks to be more vulnerable to runs, raising the threshold R^{k*} and the bank run probability $F(R^{k*})$. Hereafter condition (22) is imposed on this model.

Given a unique solution to the first-order condition with respect to liquidity, (21), the banks' liquidity holding is positive if and only if $\partial \mathbb{E}(\pi)/\partial m|_{m=0} > 0$ i.e.

$$\int_{R^{k*}}^{\infty} \left[-(R^k - 1) + \lambda \left(1 - R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial m} \right) \right] dF(R^k) > 0.$$

Hence, given a unique solution to (21), the sufficient condition for $m > 0$ is:

$$\mathbb{E}(R^k | \text{no default}) < 1 + \lambda. \tag{23}$$

That is, banks hold liquidity if the expected return of the risky loan conditional on no default is not so high, satisfying condition (23). In other words, the banks hold low return safe assets when the opportunity cost of doing so is not high.

It is worth noting that the sufficient condition for positive liquidity (23) does not apply to a limit equilibrium in which $\sigma_\epsilon \to 0$. In this case $\bar{R}^k \to R^{k*}$ for $m > 0$ and as a result the second term of the right-hand-side of condition (21) vanishes. Hence, in the limit equilibrium, banks do not hold liquidity. This is intuitive. In the limit equilibrium, when $R^k \neq R^{k*}$, it is either all fund managers withdraw or no one withdraws. Because a marginal increase in liquidity is not enough to prevent banks from defaulting due to runs by all fund managers, it generates no marginal benefit. However, if there is a region of fire sales with no default, i.e. $\bar{R}^k - R^{k*} > 0$, as in the case of $\sigma_\epsilon > 0$, building additional liquidity yields the benefits of reducing the costs of fire sales. Hence, the noisy information, $\sigma_\epsilon > 0$, is essential for analysing positive bank liquidity holdings in this model.

The supply side of funds – the household problem – is the same as in the benchmark model except for the recovery rate v. Assuming that banks can satisfy early withdrawal requests, a fraction x of fund managers who withdraw early receive R per unit of deposit. When banks default, a remaining fraction, $1 - x$, of fund managers divide banks' return $[R^k(n+d-M) - \lambda(xRd - M)]$ equally and receive $[R^k(n+d-M) - \lambda(xRd - M)]/[(1-x)d]$ per unit of deposit. Because households diversify over fund managers, households receive a weighted sum of the returns when banks default: $R^k(L/(L-1) - m) + m - \lambda(Rx - m)$. This recovery rate assumes that banks have not sold all the risky assets. The recovery rate when the banks have sold all the risky assets is given by $(R^k/(1+\lambda))(L/(L-1) - m) + m$. Consequently, the recovery rate is given by:

$$v = \min\left\{1, \max\left\{R^k\left(\frac{L}{L-1} - m\right) + m - \lambda(Rx - m), \frac{R^k}{1+\lambda}\left(\frac{L}{L-1} - m\right) + m\right\}\right\}. \tag{24}$$

This expression also applies to a case when banks default because they cannot satisfy the request of early withdrawals. The recovery rate is increasing in liquidity m when banks do not sell all the assets and $R^k < 1 + \lambda$, which holds under the assumption of (23). As in the benchmark model presented in Section 2, it is useful to define a threshold $\underline{R}^k$ under which banks sell all the risky assets:

$$\underline{R}^k = (1+\lambda)\frac{Rx(\underline{R}^k, \bar{s}^*) - m}{\frac{L}{L-1} - m}.$$

From equation (19) it is clear that $\underline{R}^k < R^{k*}$.

A competitive equilibrium for this economy consists of the interest rate R, leverage L and liquidity m that satisfy the supply curve for funds (1), the demand curve for funds (20), the optimality condition for liquidity (21) and the market clearing condition, $d = (L-1)n$, where R^{k*}, $\bar{s}^*$, P and v in these equations are given by (5), (19), (11) and (24), respectively.

4.2 Roles of Liquidity and Leverage Requirements

Is liquidity insufficient in a competitive equilibrium from a social welfare viewpoint? Does leverage continue to be excessive in the extended model? To address these questions, as in Section 3 I set up a benevolent regulator's problem, where the regulator chooses leverage L and liquidity m to maximize social welfare:

$$\begin{aligned}\max_{\{L,m\}} \quad u(y-(L-1)n) + \Bigg\{ & \int_{\bar{R}^k}^{\infty} \left[R^k L - (R^k-1)(L-1)m\right] dF \\ & + \int_{\underline{R}^k}^{\bar{R}^k} \left[R^k L - (R^k-1)(L-1)m - \lambda(xR-m)(L-1)\right] dF \\ & + \int_{-\infty}^{\underline{R}^k} \left[\frac{R^k}{1+\lambda}L - \left(\frac{R^k}{1+\lambda}-1\right)(L-1)m\right] \Bigg\} n,\end{aligned}$$

subject to $L \leq L_{\max}$ and $0 \leq m \leq L/(L-1)$, where $R = R(L,m)$ is given by the supply curve (1) and $\bar{s}^* = \bar{s}^*(L,m,R)$ is given by a solution to equations (5) and (19). The interest rate depends on liquidity in addition to leverage because the interest rate depends on the recovery rate, which is affected by liquidity.

The first-order condition of the regulator's problem with respect to liquidity yields:

$$\begin{aligned}0 = & -\int_{\underline{R}^k}^{\infty} (R^k-1)dF - \int_{-\infty}^{\underline{R}^k} \left(\frac{R^k}{1+\lambda}-1\right) dF \\ & + \lambda \int_{\underline{R}^k}^{\bar{R}^k} \left[1 - R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial m} - \left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right)\frac{\partial R}{\partial m}\right] dF. \qquad (25)\end{aligned}$$

In contrast to equation (21) that characterizes the banks' privately optimal choice of liquidity, equation (25) takes into account the opportunity cost of holding liquidity and the benefits of mitigating fire sales in default states. Furthermore, it considers the effect of liquidity on the interest rate. As shown in the appendix, if condition (23) holds and the supply curve (1) is upward sloping, an increase in liquidity lowers the interest rate, $\partial R/\partial m < 0$, by decreasing the default probability and increasing the recovery rate.[10]

The slope of the social welfare with respect to liquidity, evaluated at the level of liquidity

[10] This negative relationship is consistent with the empirical finding by Miller and Sowerbutts (2018) for the major US banks.

$m = m^*$ implied by the privately optimal choice (21), is given by:

$$\begin{aligned}
\left.\frac{\partial SW}{\partial m}\right|_{m=m^*} =& \left.\frac{\partial SW}{\partial m}\right|_{m=m^*} - \left.\frac{\partial \mathbb{E}(\pi)}{\partial m}\right|_{m=m^*} \\
=& \int_{\underline{R}^k}^{R^{k*}} (1-R^k)dF + \int_{-\infty}^{\underline{R}^k} \left(1 - \frac{R^k}{1+\lambda}\right) dF \\
&+ \lambda \int_{-\infty}^{R^{k*}} \left(1 - R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial m}\right) dF - \lambda \int_{-\infty}^{\bar{R}^k} \left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right)\frac{\partial R}{\partial m} dF.
\end{aligned}$$

The slope of the social welfare evaluated at m^* consists of four terms. The sign of the last two terms is positive if the supply curve (1) is upward sloping and conditions (22) and (23) hold. Hence, if the first two terms are positive as well, $\partial SW/\partial m|_{m=m^*} > 0$ follows. This leads to the following proposition.

Proposition 4 (Insufficient liquidity)**.** *Assume that the supply curve (1) is upward sloping and conditions (22) and (23) hold. Assume further that the threshold R^{k*} is low enough to satisfy $\int_{-\infty}^{R^{k*}} (1-R^k)dF > 0$. Then, for given leverage, banks' liquid asset holdings are insufficient from a social welfare viewpoint: raising liquidity can improve social welfare.*

Proposition 4 does not require that leverage is at the competitive equilibrium level. Indeed, Proposition 4 holds for an arbitrary level of L. Hence, Proposition 4 implies that bank liquidity is insufficient not only in a competitive equilibrium but also in an equilibrium with $m > 0$ where leverage is restrained e.g. by a prudential tool on leverage. This result suggests that there is room for imposing a liquidity tool to improve social welfare even if a leverage restriction is already put in place.

Turning to welfare implications for leverage in this model, the first-order condition of the regulator's problem with respect to leverage is given by:

$$\begin{aligned}
0 = \frac{\partial SW}{\partial L} =& -R\left[1 - P + P\mathbb{E}(v|\text{default})\right] + \int_{\underline{R}^k}^{\infty} \left[R^k - (R^k-1)m\right] dF \\
&- \lambda \int_{\underline{R}^k}^{\bar{R}^k} \left[(xR - m) + R(L-1)\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L} + (L-1)\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right)\frac{\partial R}{\partial L}\right] dF \\
&+ \int_{-\infty}^{\underline{R}^k} \left[\frac{R^k}{1+\lambda} - \left(\frac{R^k}{1+\lambda} - 1\right) m\right] dF.
\end{aligned}$$

Similar to the choice of liquidity, the slope of the social welfare with respect to leverage, evaluated at the banks' privately optimal choice $L = L^*$ implied by condition (20), is given by:

$$\begin{aligned}\left.\frac{\partial SW}{\partial L}\right|_{L=L^*} =& \left.\frac{\partial SW}{\partial L}\right|_{L=L^*} - \left.\frac{\partial \mathbb{E}(\pi)}{\partial L}\right|_{L=L^*} \\ &= -\frac{1}{L-1}\left[\int_{\underline{R}^k}^{R^{k*}} R^k dF + \int_{-\infty}^{\underline{R}^k} \frac{R^k}{1+\lambda} dF\right] \\ &\quad -\lambda(L-1)\left[\int_{\underline{R}^k}^{R^{k*}} R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L} dF + \int_{\underline{R}^k}^{\bar{R}^k}\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right)\frac{\partial R}{\partial L} dF\right].\end{aligned}$$

The sign of $\partial SW/\partial L|_{L=L^*}$ is negative if the supply curve of funds (1) is upward sloping, $\partial R/\partial L > 0$. This leads to the following proposition.

Proposition 5 (Excessive leverage)**.** *Assume that the supply curve (1) is upward sloping. Then, for given liquidity, bank leverage is excessive from a social welfare viewpoint: restricting leverage can improve social welfare.*

Similar to Proposition 4 that shows insufficient liquidity, Proposition 5 holds for any level of bank liquidity. Even if liquidity is at some regulated level, banks choose excessive leverage relative to the constrained optimal level. Hence, Propositions 4 and 5 warrant imposing both leverage and liquidity requirements.

The source of inefficiencies that give rise to excessive leverage and insufficient liquidity is the same as those in the benchmark model, i.e. bank risk shifting and pecuniary externalities. However, the choice of liquidity is free from the pecuniary externalities as the composition of bank assets does not affect the marginal utility in period 1. Hence, without risk shifting the banks' liquidity choice would coincide with the solution to the regulator's problem. This result is formalized in the following proposition.

Proposition 6 (Optimal liquidity but excessive leverage without bank risk shifting)**.** *Consider a version of the extended model in which banks have no risk shifting motives. Suppose that the supply curve (1) is upward sloping and conditions (22) and (23) hold. In a competitive equilibrium, given leverage, liquidity is at the level that would be chosen by the benevolent regulator. But, given liquidity, leverage is excessive because of the pecuniary externalities that work through the interest rate.*

Proposition 6 highlights that in this model externalities arising from bank risk shifting are essential for obtaining the result of insufficient liquidity in a competitive equilibrium.

4.3 Parameterization

The previous section analytically showed that leverage is excessive *given* liquidity and liquidity is insufficient *given* leverage in a competitive equilibrium. However, it is not clear whether the results hold *jointly*. In addition, how banks respond to changes in leverage and liquidity requirements and changes in key parameter values are yet to be known. Addressing these questions requires numerical analyses. To this end, this section parameterizes the extended model presented in Section 4.1.

Parameter values are set so that the extended model generates key endogenous variables similar to those observed for major US banks. Yet, it should be noted that the model aims to capture a financial system as a whole which issues short-term liabilities vulnerable to runs. After all, the model is so stylized that numerical analyses are intended to show qualitative implications rather than quantitative ones.

Parameters σ_ϵ, γ and λ and y are set so that the model hits the following target values in a competitive equilibrium: the leverage of $L = 15$, the liquidity ratio of $m = 0.05$, the crisis probability of $P = 0.05$ and the borrowing interest rate of $R = 1.02$. For the six largest US banks, over the period of 2008–2017 the leverage, measured by the ratio of total assets to Core Equity Tier 1 capital, is 13.4 on average and the liquidity ratio, measured by the ratio of liquid assets to total liabilities, is 0.037 on average (Miller and Sowerbutts, 2018).[11] The target values for L and m are not far from these observations. The target value of $P = 0.05$ is consistent with the historical fact that suggests that in any given country, banking crises occur on average once in every 20 to 25 years, i.e. the average annual crissi probability of 4–5 percent (Basel Committee on Banking Supervision 2010 (BCBS hereafter)). The bank capital n is set so that the consumption in period 1 is close to the consumption in period 2. The resulting parameter values are $\sigma_\epsilon = 8.68/10000$, $\gamma = 0.66$, $\lambda = 0.17$, $y = 1.63$ and $n = 0.055$.

The mean return on bank lending is set to $\mu = 1.035$ so that the after-taxed return on equity at the mean return when there is no fire sales is about 15 percent, which is higher than those observed in the post-crisis period of 2008–17, but it is in line with the pre-crisis period of 2000–07.[12] The standard deviation of the return on bank lending is set

[11] The liquid assets are the sum of cash, withdrawable reserves and US treasury securities. The ratio of liquid assets to total assets, reported by Miller and Sowerbutts (2018), is transformed into the ratio of liquid assets to total liabilities using leverage. The data are available from 2008 because two banks in the sample were purely investment banks until 2008 and their data source – the Federal Reserve's Financial Reports (form FRY-9C) – was not available before 2008 for these two banks.

[12] The after-taxed return on equity at the mean return when there is no fire sales is given by $(1-\tau)[\bar{R}^k L -$

to $\sigma_k = 0.025$ so that there exists an equilibrium that satisfy the target values discussed above.[13] Admittedly the return is highly volatile, but such volatility is required for the equilibrium to have the target level of a 5 percent crisis probability.

Finally, the functional form of the period-1 utility is assumed to be $u(c_1) = (c_1^{1-\alpha})/(1-\alpha)$ and two values $\alpha = 0.01$ and 0.1 are considered. A smaller value of α means that the utility function becomes close to be linear and the degree of the pecuniary externalities identified in the model gets smaller. Although the two values are small relative to an often-assumed case of log utility ($\alpha = 1$), the model considers quasi-linear utility and these values are enough to show contrasting implications for prudential tools, highlighting a general equilibrium effect through the curvature of $u(\cdot)$.

4.4 Leverage and Liquidity Requirements

To understand the joint impact of leverage and liquidity requirements, first, I consider cases of one tool only for leverage and liquidity, respectively, which is followed by an analysis on the joint effects of the two tools.

4.4.1 Leverage restriction only

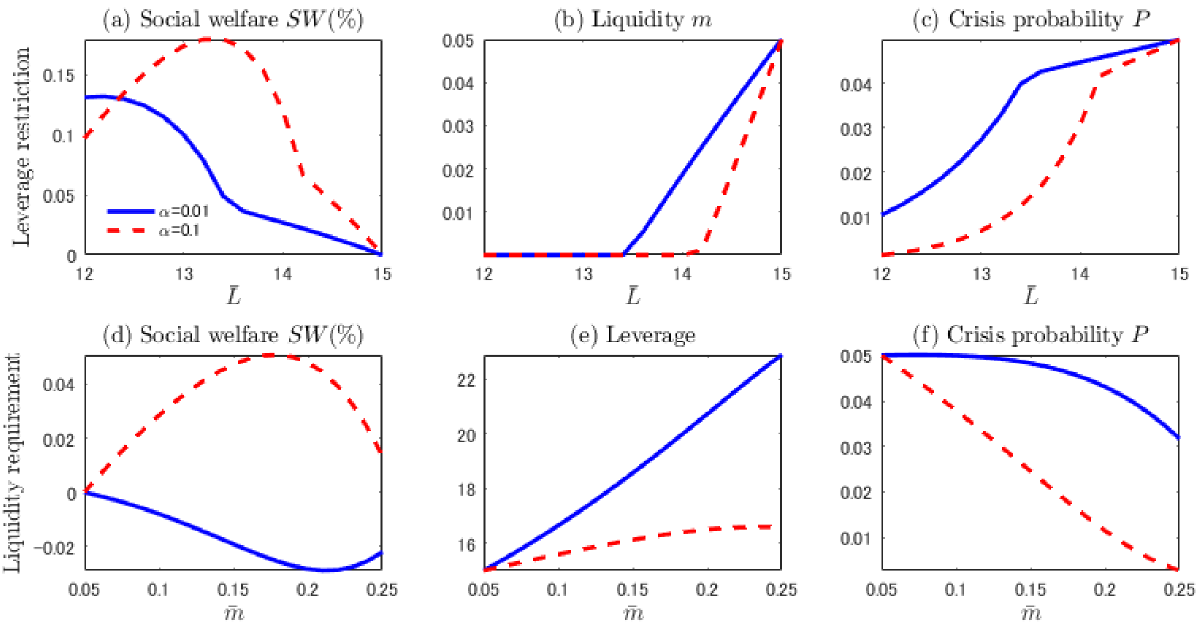
Consider the extended model presented in Section 4.1 in which only a restriction on leverage is put in place. This situation is reminiscent of the periods under the Basel I and II in which liquidity requirements were absent. Panels (a)-(c) of Figure 1 show the impacts of the leverage restriction, $L \leq \bar{L}$, on social welfare, liquidity and the crisis probability, respectively, for the economies with $\alpha = 0.01$ (blue solid line) and 0.1 (red dashed line). Without any restriction the leverage is $L = 15$. As the leverage restriction is tightened from $L = 15$ to lower values, the social welfare is improved (Panel (a)) and the crisis probability is reduced (Panel (c)). However, the banks respond by reducing liquidity holdings (Panel (b)). Hence, imposing a leverage tool only induces banks to migrate risk from leverage to liquidity.

The speed of a decrease in liquidity is faster for the economy with $\alpha = 0.1$ than that with $\alpha = 0.01$. This is because a tightening in leverage limits the amount of deposits and lowers the interest rate R, which further reduces the crisis probability. The decreased crisis

$(\mu - 1)(L - 1)m - R(L - 1) - 1]$, where τ is the tax rate. In the calculation, the tax rate is assumed to be 30 percent.

[13] If σ_k is set too low, there is no parameter value for e.g. $\gamma \in (0, 1)$ that supports the equilibrium with the target values.

Figure 1: Impacts of leverage (upper panels) and liquidity (lower panels) requirements



Note: In Panels (a) and (d), social welfare is measured as a percentage deviation from the level of social welfare at the competitive equilibrium without any restrictions.

probability allows the banks to take more risk in another area, i.e. liquidity, leading to a decrease in liquidity. The impact on liquidity is stronger for the economy with a greater general equilibrium effect of leverage on the interest rate, which is governed by parameter α, the curvature of the period-1 utility function.

Another consequence of the general equilibrium effect is the optimal level of leverage that maximises the social welfare. The economy with $\alpha = 0.01$ – a relatively small general equilibrium effect – calls for a tighter leverage restriction around $\bar{L} = 12$ than the economy with $\alpha = 0.1$ where such an optimal leverage restriction is above $\bar{L} = 13$. In the latter case, tightening leverage reduces the crisis probability more through its impact of lowering the interest rate, and hence the optimal leverage restriction becomes milder.

4.4.2 Liquidity requirement only

Next, consider a situation in which only a liquidity requirement, $m \geq \bar{m}$, is put in place. Panels (d)-(f) of Figure 1 show the impacts of the liquidity tool on social welfare, leverage and the crisis probability, respectively, for the economies with $\alpha = 0.01$ and $\alpha = 0.1$. As the liquidity requirement is tightened, the crisis probability is reduced for both economies (Panel (f)). However, while the social welfare is improved for the economy with $\alpha = 0.1$, it is deteriorated for the economy with $\alpha = 0.01$ (Panel (d)). This difference is driven by the divergent responses of leverage (Panel (e)). For the economy with the lower curvature of the utility function, the effect of increasing leverage on the interest rate is smaller, so that the banks respond by increasing leverage to a tightened liquidity requirement much more than in the economy with the higher curvature of the utility function. This negative effect dominates the benefit of increasing bank liquidity holdings, and as a result, imposing the liquidity requirement worsens welfare rather than improves it. This numerical example is still consistent with Proposition 4, which states that imposing a liquidity requirement can improve welfare *given* leverage. In this example, doing so worsens welfare, because leverage is not fixed; the banks respond by increasing leverage. This risk migration is a culprit of the welfare deterioration as a result of imposing the liquidity requirement only for the economy with $\alpha = 0.01$.

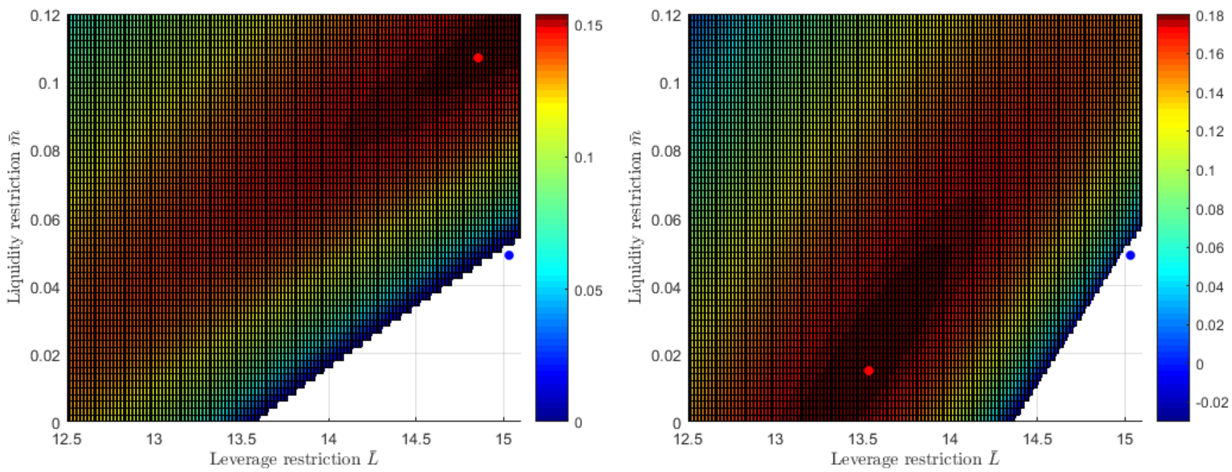
4.4.3 Coordination of leverage and liquidity tools

The previous analysis on one tool only highlights need for joint restrictions on leverage and liquidity to address risk migration from one area to another. Then, what is an optimal policy coordination between leverage and liquidity tools? How does the optimal coordination differ from the cases of one tool only?

Figure 2 addresses these questions by plotting social welfare as a function of the two requirements for the model with $\alpha = 0.01$ (Panel (a)) and that with $\alpha = 0.1$ (Panel (b)). Let subscript $_{BR}$ and $_{CE}$ denote a solution to the benevolent regulator's problem and the competitive equilibrium, respectively. First, the optimal coordination $\{L_{BR}, m_{BR}\}$ depends crucially on the curvature of the period-1 utility function, i.e. the general equilibrium effect of leverage on the interest rate. Relative to the competitive equilibrium, the solution to the regulator's problem features tightened leverage and tightened liquidity, i.e. $L_{BR} < L_{CE}$ and $m_{BR} > m_{CE}$, in the case of $\alpha = 0.01$ (Panel (a)). But the solution features tightened leverage and loosened liquidity, i.e. $L_{BR} < L_{CE}$ and $m_{BR} < m_{CE}$, in the case of $\alpha = 0.1$ (Panel (b)). In this case, the general equilibrium effect of the leverage restriction on the crisis probability, through its effect on the interest rate, is so great that lowering leverage is more effective than increasing liquidity to address the inefficiencies. It is worth noting that even though the optimal level of liquidity is lower than that in the competitive equilibrium, the liquidity requirement is still binding. Without the requirement, the banks would choose a lower level of liquidity as shown in Figure 1(b). In the case of $\alpha = 0.01$, the

Figure 2: Impacts of leverage and liquidity requirements on social welfare

(a) $\alpha = 0.01$ (b) $\alpha = 0.1$



Note: Social welfare is measured as a percentage deviation from that of the competitive equilibrium. A red circle corresponds to a solution to the constrained planner problem and a blue circle corresponds to the competitive equilibrium.

general equilibrium effect is small and hence tightening both leverage and liquidity becomes optimal.

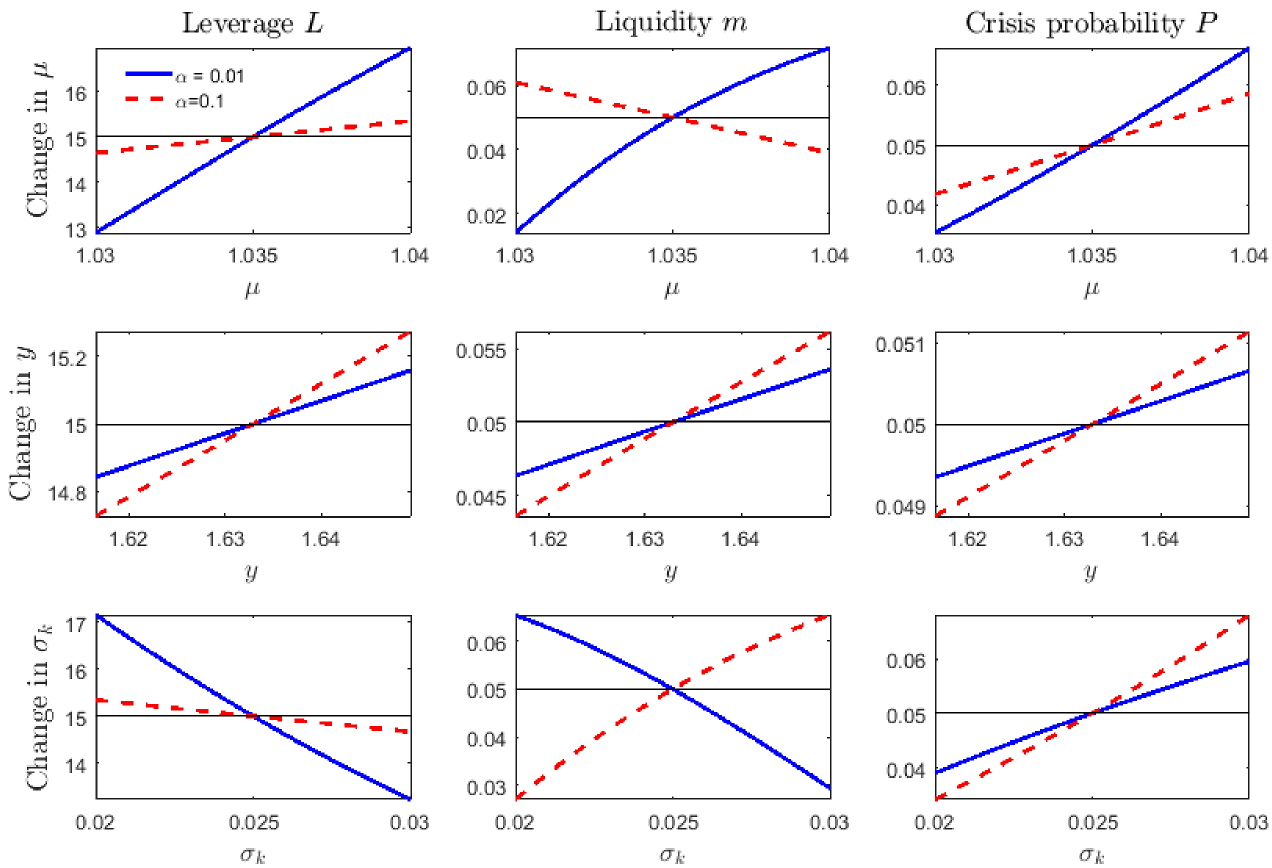
Next, relative to the cases of a leverage tool only, the optimal coordination between leverage and liquidity requirements calls for milder requirements on leverage. On the one hand, in the case of the leverage tool only, the optimal level of leverage that achieves the highest possible welfare is 12 and 13.2 for $\alpha = 0.01$ and 0.1, respectively. On the other hand, the optimal coordination requires the leverage of 14.9 and 13.5, respectively. Hence, with a liquidity requirement put in place, a less strict leverage restriction is called for to achieve the highest possible social welfare than in the case of a leverage tool only.

A similar implication holds for liquidity in the case of $\alpha = 0.1$: a liquidity tool only requires the liquidity ratio of around 0.18, while the optimal coordination calls for the liquidity ratio of only 0.016. However, this result does not hold for $\alpha = 0.01$ because tightening a liquidity requirement worsens welfare as discussed in Section 4.4.2.

4.5 Comparative Statics Analysis

Having studied the welfare implications of the model, in this section, I study how the economy without any restriction and the economy under jointly optimal leverage and liquidity requirements respond to changes in key parameter values.

Figure 3: Comparative statics of the competitive equilibrium



4.5.1 Comparative statics: competitive equilibrium

Figure 3 plots how leverage, liquidity and the crisis probability react in response to changes in the mean return on bank assets μ, the household income y and the standard deviation of the bank asset return σ_k for the cases of $\alpha = 0.01$ and 0.1, respectively

Figure 3 reveals three findings. First, similar to Proposition 1 for the baseline model with a bank leverage choice only, both leverage and the crisis probability increase as the mean return μ and the household income y increase. This result holds for both cases of $\alpha = 0.01$ and 0.1.

Second, in response to an increase in the standard deviation – uncertainty – of the bank asset return σ_k, banks lower leverage but the crisis probability increases for both cases of $\alpha = 0.01$ and 0.1. Although leverage is an important determinant of the crisis probability, the crisis probability increases when banks are deleveraging.

Third, in response to changes in the mean bank asset return μ and the uncertainty of the bank asset return σ_k, in the case of $\alpha = 0.01$, leverage and liquidity move in the opposite direction in terms of contributions to a crisis probability. But in the case of $\alpha = 0.1$ leverage and liquidity move in the same direction, both of which contributes to increasing or decreasing the crisis probability. Specifically, when the mean return rises, the banks respond by increasing leverage and thereby contribute to raising a crisis probability in both cases of $\alpha = 0.01$ and 0.1, but they behave differently in a liquidity choice: they increase liquidity, which restrains a crisis probability, in the case of $\alpha = 0.01$ while they decrease liquidity, which raises a crisis probability, in the case of $\alpha = 0.1$.

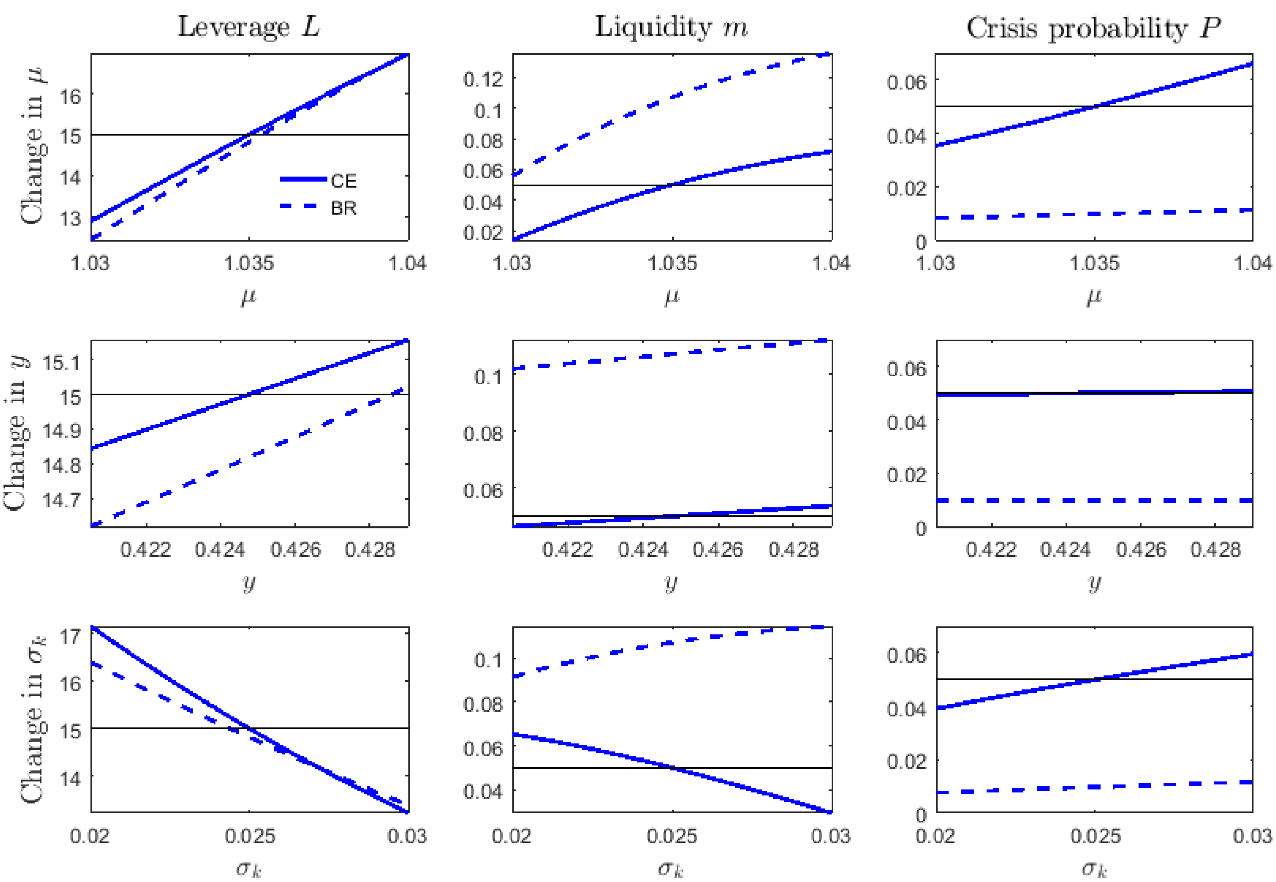
This difference has to do with the general equilibrium effect of leverage on the interest rate and on the crisis probability. When the curvature of the period-1 utility is relatively flat, $\alpha = 0.01$, a higher leverage is less associated with a rise in the interest rate than the case of $\alpha = 0.1$. Hence, the banks find it profitable to increase leverage and limit the associated increase in the crisis probability by increasing liquidity holdings. If, instead, the general equilibrium effect is relatively strong, $\alpha = 0.1$, the banks use leverage to restrain the crisis probability and use liquidity to take more risk. In response to an increase in the average return μ the banks slightly increase leverage but at a slower pace than in the economy with $\alpha = 0.01$ and reduce liquidity to raise the asset return. A similar mechanism works in the case of a change in the uncertainty of the bank asset return.

4.5.2 Comparative statics: constrained optimal allocation

How does the constrained optimal allocation – a solution to the benevolent regulator's problem – react in response to changes in key parameter values? Figure 4 plots the constrained optimal allocation for leverage, liquidity and the crisis probability in the case of $\alpha = 0.01$ in response to changes in the mean return on bank assets μ, the household income y and the uncertainty of bank asset returns σ_k. The case of $\alpha = 0.1$ is omitted as its implications are similar.

Figure 4 reveals two findings. First, the constrained optimal levels of leverage and liquidity change in response to changes in the parameter values. In most cases the constrained optimal levels change in parallel with changes in the competitive equilibrium allocation. For example, both the constrained optimal level and the competitive equilibrium level of leverage increase as the mean return on bank assets rises. However, this is not always a case: the two levels can move in the opposite direction. For example, in response to an increase in the uncertainty of the bank asset return the constrained optimal level of liquidity increases while its counterpart in the competitive equilibrium decreases (bottom

Figure 4: Comparative statics of the constrained optimal allocation



Note: CE (blue solid line) and BR (blue dashed line) represent the competitive equilibrium and the solution to the benevolent regulator's problem, respectively.

medium panel of Figure 4). These observations suggest that the optimal prudential policy, which aims to achieve the constrained optimal allocation, differs in a non-trivial manner depending on parameter values that characterize the banking system and the economy.

Second, the constrained optimal level of the crisis probability is relatively stable around 1 percent, irrespective of changes in the parameter values. This makes a contrast with the volatile crisis probability in the competitive equilibrium. The stable crisis probability implies that the degree of the crisis probability curbed by the optimal prudential policy – a difference between P_{CE} and P_{BR} – becomes greater as P_{CE} increases. This is evident in response to increases in the mean return on bank assets (top right panel of Figure 4) and the uncertainty of bank asset returns (bottom right panel of Figure 4). The stable crisis probability in the constrained optimal allocation implies that if the crisis (default) probability were observable, instead of imposing multiple tools, setting a target level for the crisis probability and letting banks to behave freely as long as the probability is no higher than the target level might be a robust way to improve welfare in various economies with a different banking system.

5 Extensions

The benchmark model presented in Section 2 is so stylised that it can be extended in various ways. In this section, I provide some extensions that are used to discuss bank/sector specific capital requirements, risk weights and deposit insurance. In addition, the extensions bring some implications for shadow banking and concentration risk. Unless mentioned otherwise, the same parameter values set in Section 4.3 are used in this section. Main implications are unaffected by the discussed values of the curvature of the utility function, and hence $\alpha = 0.1$ is used in this section.

5.1 Model with Heterogeneous Banks

5.1.1 Overview of the model

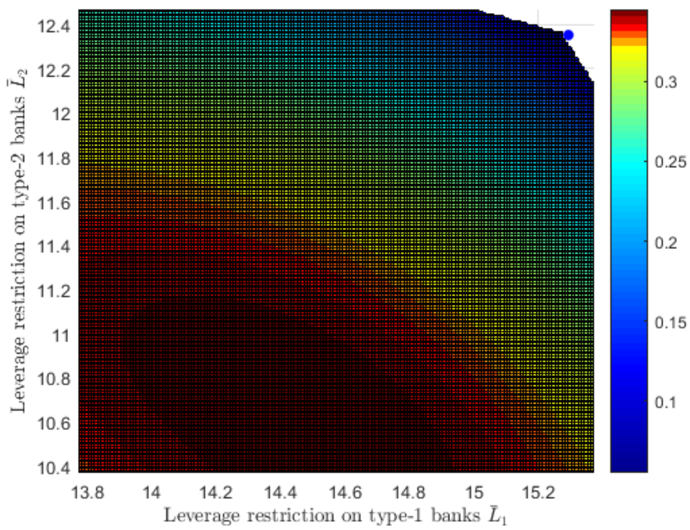
I extend the benchmark model to incorporate two types of banks, indexed by $j \in \{1, 2\}$. For simplicity, the two types of banks differ only in the riskiness of lending. The type-j banks specialise in lending to sector j and cannot lend to the other sector. Lending to sector j yields the same expected return μ, but the riskiness differs between the two sectors: $R_j^k \sim N(\mu, \sigma_j^2)$ with $\sigma_1 \neq \sigma_2$. The remaining part of the model is essentially the same as in the benchmark model.

The equilibrium for this economy is characterized by the following four equations with four unknowns $\{R_j, L_j\}_{j=1}^2$: for $j = 1, 2$,

$$
\begin{aligned}
R_j =& \frac{u'(y - (L_1 - 1)n - (L_2 - 1)n)}{1 - P_j + \mathbb{E}(v_j|\text{default})P_j}, \\
0 =& \int_{R_j^{k*}}^{\infty} (R^k - R) dF_j - R_j \lambda (L_j - 1) \int_{R_j^{k*}}^{\infty} \frac{\partial x_j}{\partial \bar{s}_j^*} \frac{\partial \bar{s}_j^*}{\partial L_j} dF_j - R_j \lambda \int_{R_j^{k*}}^{\infty} x_j dF_j,
\end{aligned}
$$

where $P_j = F_j(R_j^{k*})$ is the default probability for the type-j banks, $F_j(\cdot)$ is the cumulative normal distribution function with mean μ and standard deviation σ_j. The thresholds R_j^{k*} and $\bar{s}_j^*$ are given by equations (5) and (6) and the recovery rate v_j is given by (12) with a modification to add subscript j.

Figure 5: Impacts of bank-specific leverage restrictions on social welfare



Note: Social welfare is measured as a percentage deviation from that of the competitive equilibrium. A blue circle at the upper right corner indicates the competitive equilibrium.

For a numerical illustration, the type-2 banks are assumed to be riskier than the type-1 banks. Specifically, the standard deviation of the type-2 bank asset return is 1.5 times as high as that of the type-1 banks. The bank net worth is set to a half of the level set in Section 4.3 for each type of banks so that the aggregate bank net worth remains the same.

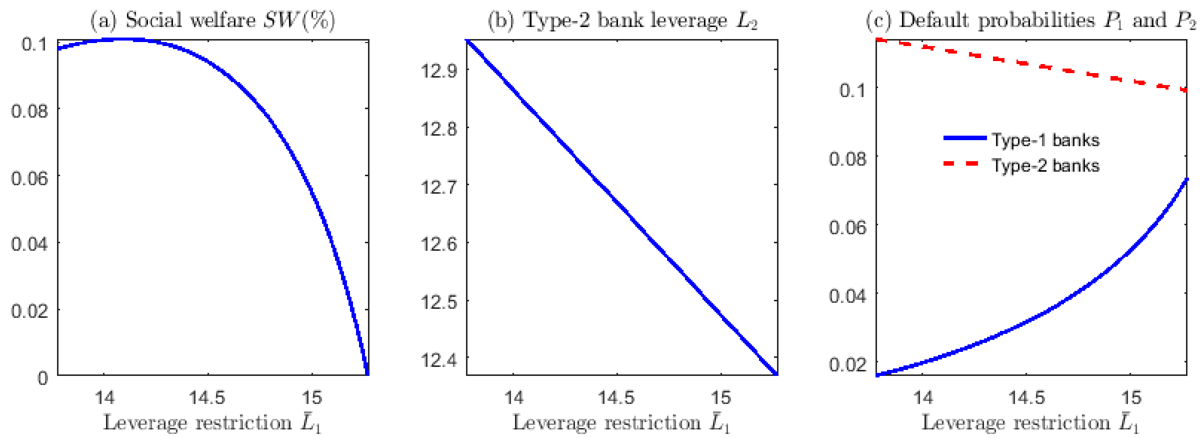
In a competitive equilibrium, the type-2 banks have a lower leverage but a higher default probability than do the type-1 banks, reflecting the higher riskiness of the bank asset return. The leverage and the default probability are $L_1 = 15.3$ and $P_1 = 0.074$ for the type-1 banks and $L_2 = 12.4$ and $P_2 = 0.099$ for the type-2 banks. Hence, in this model, low leverage reflects the riskiness of the banks and does not necessarily signals the safety of the banks.

5.1.2 Heterogeneous capital requirements and risk weights

A heterogeneity in bank riskiness calls for bank-specific leverage/capital requirement. Figure 5 shows the joint effects of bank-specific leverage restrictions on social welfare. Limiting leverage for both types of banks improves social welfare and the optimum is attained around $\bar{L}_1 = \bar{L}_1^* \equiv 14.5$ and $\bar{L}_2 = \bar{L}_2^* \equiv 10.6$. Reflecting the heterogeneous riskiness of bank assets, the leverage restriction imposed on banks differs between the two types of banks.

A single capital/leverage restriction can achieve the same outcome if it is complemented

Figure 6: Impacts of a leverage restriction on the type-1 banks only



Note: Social welfare is measured as a percentage deviation from that of the competitive equilibrium.

by risk weights. This is so-called risk-weighted-based capital requirement. A risk weight is normalised at 100 percent for the type-1 bank loans and 100ω percent for the type-2 bank loans and a risk-weighted-based capital requirement is normalised at $1/\bar{L}_1^*$. By construction, the capital ratio (or leverage) is restrained at the optimal level for the type-1 banks. To achieve the optimal level for the type-2 banks i.e. $n/(n+d_2) = 1/\bar{L}_2^*$, the risk weight ω has to be such that $n/(n+\omega d_2) = 1/\bar{L}_1^*$. Solving the equations for ω yields

$$\omega = \omega^* \equiv \frac{\bar{L}_1^* - 1}{\bar{L}_2^* - 1} > 1.$$

The optimal risk weight for the type-2 bank loans is more than 100 percent, reflecting their high riskiness.

5.1.3 Shadow banks

Shadow banks, by definition, lie outside the reach of banking regulations. In the model with heterogeneous banks, the type-2 banks, which specialise in riskier loans, can be interpreted as shadow banks if they are free from regulations, while the type-1 banks, which specialise in less risky loans, can be seen as traditional banks if they are regulated.

With restrictions imposed only on the traditional banks, the traditional banks become safer, but the shadow banks become riskier. Figure 6 plots the impacts of a leverage restriction on the type-1 banks only on social welfare, the type-2 bank leverage and the default probabilities. As the leverage restriction is tightened, the type-2 banks react by increasing leverage and as a result their default probability rises. The social welfare is improved for somewhat, but its highest achievable level of around 0.1 percent is far below the optimum of above 0.3 percent when both types of banks are regulated.

5.2 Model with a Portfolio Choice

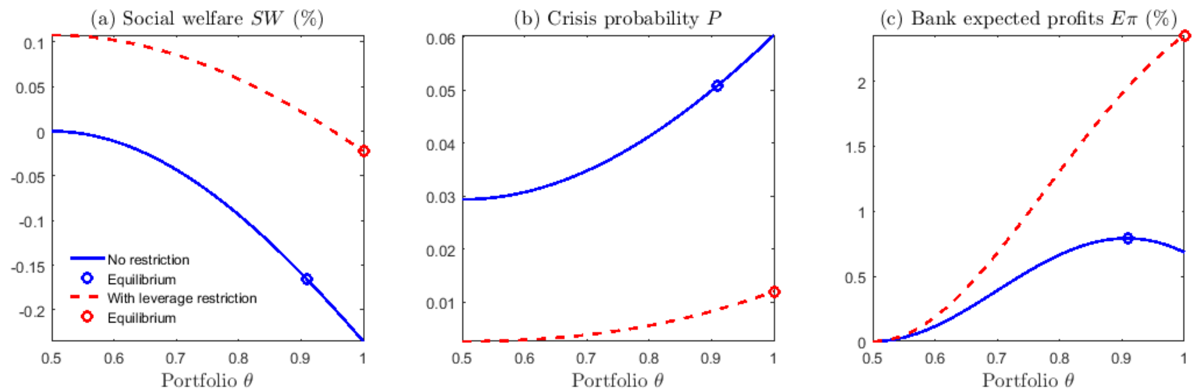
Banks may choose a less-diversified and riskier portfolio than socially desirable one when they have risk shifting motives. To formalise this idea, I extend the baseline model presented in Section 2 to incorporate a portfolio of loans. Specifically, banks make loans to two sectors, indexed by $j \in \{1, 2\}$. The returns of the two sectors follow a joint normal distribution, $\boldsymbol{R}^k \sim N(\boldsymbol{\mu}, \Sigma)$, where $\boldsymbol{R}^k \equiv [R_1^k, R_2^k]'$ is a vector of returns of the two sectors. In addition to leverage banks choose a portfolio of loans, $\boldsymbol{\theta} \equiv [\theta, 1-\theta]'$, where $\theta \in [0, 1]$ is a fraction of total loans invested in sector $j = 1$. The return of the bank asset portfolio is then given by $R^k(\theta) \equiv \boldsymbol{\theta}'\boldsymbol{R}^k$, which follows $N(\mu(\theta), \sigma_k(\theta)^2)$, where $\mu(\theta) \equiv \boldsymbol{\theta}'\boldsymbol{\mu}$ is the mean return and $\sigma_k(\theta) \equiv (\boldsymbol{\theta}'\Sigma\boldsymbol{\theta})^{\frac{1}{2}}$ is the standard deviation of the portfolio. Each fund manager i observes a bank portfolio as well as leverage and receives independent signals $s_{ij} = R_j^k + \epsilon_{ij}$ with $\epsilon_{ij} \sim N(0, \sigma_{\epsilon_j}^2)$ for $j = 1, 2$. Given a bank portfolio, this extended model works essentially the same way as in the benchmark model. Fund manager i withdraws deposits early if and only if $\boldsymbol{\theta}'\boldsymbol{s}_i$ is less than the threshold $\bar{s}^*(L, \theta)$, where $\boldsymbol{s}_i \equiv [s_{i1}, s_{i2}]'$ is a vector of noisy signals. A difference is that now the threshold depends on bank asset portfolio θ as well as leverage L.

To illustrate concentration risk, I assume that the two sectors are identical.[14] The only difference from the benchmark model is that banks can reduce their loan risk by diversifying over loans to the two sectors. Specifically, banks are able to minimize the risk of their loan portfolio by setting $\theta = 0.5$. Not surprisingly, the smallest portfolio risk achieves the highest social welfare, as shown by the solid line in Figure 7(a). However, banks do not choose such a portfolio but instead pick the riskier and more concentrated portfolio of around $\theta = 0.9$ to maximise the profits (Figure 7(c)). As a result, the crisis probability rises to 5 percent from 3 percent, a level which would be realised if the banks chose the perfectly diversified portfolio (Figure 7(b)).

The model and its numerical example highlights need for addressing concentration risk with a unique prudential instrument. Imposing a leverage restriction can improve welfare, but as in the model with liquidity and leverage and the model with heterogeneous banks, doing so causes risk to migrate to a non-regulated area, which is a portfolio choice in this

[14]BCBS (2014) points out the concentration risk as potential risk.

Figure 7: Bank portfolio choice and concentration risk



Note: Social welfare is measured as a percentage deviation from that in the economy with no restriction and with $\theta = 0.5$. Bank expected profits are measured as a percentage deviation from those with $\theta = 0.5$ for each curve.

model. For example, if a regulator imposes the leverage restriction, $\bar{L} = L_{CE} - 1$, that is tighter by 1 than what banks would choose without any restriction, the banks respond by concentrating completely in sector-1 lending, i.e., by setting $\theta = 1$, as shown in the red dashed line in Figure 7(c). As a result, the crisis probability becomes materially higher and the social welfare gets significantly lower than what would be achievable if the banks chose the perfectly diversified portfolio of $\theta = 0.5$. Hence, a prudential tool that limits exposure to single type of lending – a cap on concentration risk – is required to address the risk migration from the leverage area to the portfolio area.

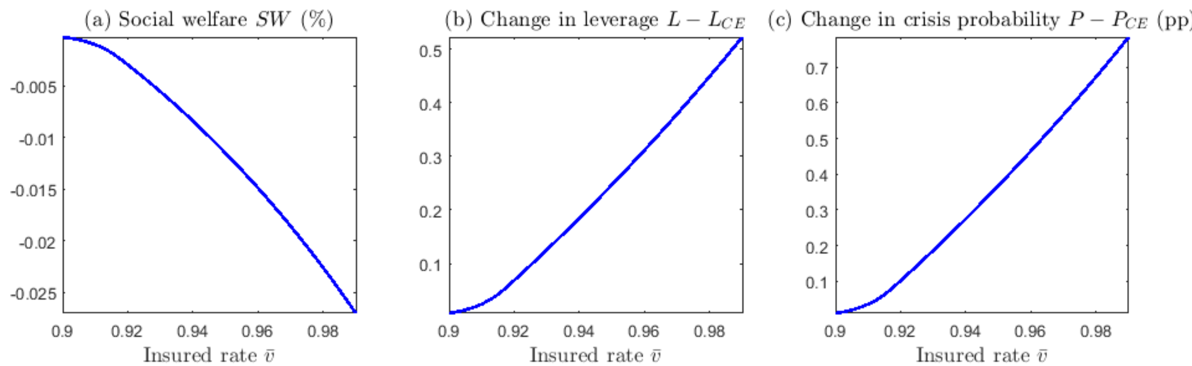
5.3 Model with Deposit Insurance

Perfect deposit insurance, which ensures the recovery rate of unity, $v = 1$, will eliminate bank runs in theory, but such an insurance is hardly put in place in practice. Typically, the coverage of bank deposit insurance is limited and there is no insurance for money-like short-term debt. In short, deposit insurance is imperfect in practice.

Imperfect deposit insurance falls short of eliminating bank runs. As long as households have a chance of losing some deposits and fund managers follow the behavioural rule (2), bank runs can still occur. A key modelling assumption is that the fund managers' incentive to run, summarized by parameter γ in (2), is unaffected by the presence of deposit insurance.

To explore the impact of deposit insurance on financial stability and social welfare, the benchmark model presented in Section 2 is extended to incorporate imperfect deposit insurance that protect households from incurring losses more than $100(1-\bar{v})$ percent of the

Figure 8: Impacts of imperfect deposit insurance



Note: Social welfare is measured as a percentage deviation from that of the competitive equilibrium with no deposit insurance. L_{CE} and P_{CE} denote the leverage and the crisis probability in the competitive equilibrium with no deposit insurance.

promised interest rate R. Hence, $\bar{v}$ forms the floor of the recovery rate. The government finances $(\bar{v} - v)R$ per unit of funds by imposing lump-sum taxes on households in period $t = 2$. Then, the supply curve for funds (1) is modified to:

$$R = \frac{u'(y - (L-1)n)}{1 - P + \mathbb{E}(\max\{v, \bar{v}\}|\text{default})P}. \tag{26}$$

Equation (26) implies that an increase in the insurance rate $\bar{v}$ shifts the supply curve outward and makes excessive leverage even more excessive and worsens the crisis probability.

Figure 8 confirms this prediction. As the coverage rate of the deposit insurance rises, the leverage becomes more excessive (Panel (b)), the crisis probability increases further (Panel (c)), and as a result, the social welfare deteriorates (Panel (a)).

6 Conclusion

This paper has developed a model of endogenous bank runs in a global game general equilibrium framework. The benchmark model presented in Section 2 has highlighted banks' risk shifting and pecuniary externalities as the source of inefficiencies that give rise to an elevated financial crisis probability. The paper has extended the benchmark model and studied the role of multiple prudential tools for addressing the inefficiencies: leverage and liquidity tools in Section 4; bank/sector specific capital requirements in Section 5.1; a leverage restriction and a cap on concentration risk in Section 5.2. These tools are closely related to and motivated by the actual regulations and reforms implemented after the global financial crisis (BCBS 2011, 2013, 2014). The benchmark model, upon which the extended models are built and used to study these tools, hence provides a basic framework for studying banking crises, banks' behaviour and prudential policy tools.

The models studied in the paper offer several empirical predictions. Their common theme is that risk can migrate from one area to others. And this is a main reason why multiple restrictions are required to address the issue. In the case of capital/leverage and liquidity discussed in Section 4, a tightening in a leverage restriction causes banks to reduce the holdings of liquid assets. In the case of traditional and shadow banks discussed in Section 5.1, tightening a leverage restriction on traditional banks induces shadow banks to grow and make them riskier. In the case of capital requirements and caps on concentration risk discussed in Section 5.2, restricting leverage induces banks to choose a riskier asset portfolio by increasing exposure to one sector.

The paper has highlighted risk migration between two different risk spaces, e.g. capital/leverage and liquidity, for simplicity and clarity. In practice there would be risk migration among more than two areas, e.g. capital/leverage, liquidity and portfolios, under the name of 'balance sheet optimisation.' The paper abstracts away from a heterogeneity in bank liabilities, but this can be another area of risk migration. Analysing risk migration in all possible areas would be extremely difficult, if not impossible. Yet, the models presented in this paper have allowed us to disentangle the impacts of one or two prudential tools on two risk spaces, a crisis probability and social welfare. In the case of leverage and liquidity tools, the model has also shed light on the general equilibrium effect through the interest rate on the constrained optimal allocation.

The models presented in this paper have considered various prudential tools on risk spaces, but they still lack an important dimension: time. Adding a time dimension is essential for considering time-varying tools, e.g. countercyclical capital buffers, and also for highlighting other potential sources of externalities. Having kept this potential extension in mind, I have constructed the benchmark model so that it would be easily incorporated into a dynamic general equilibrium model. I plan to tackle on this in a future work.

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Appendix

Derivation of equation (9). As shown in Section 2 the threshold R^{k*} is a solution to equations (5) and (6). These equations are written explicitly as:

$$\Phi\left(\sqrt{\frac{1}{\sigma_k^2}+\frac{1}{\sigma_\epsilon^2}}R^{k*}-\frac{\frac{1}{\sigma_k^2}\mu+\frac{1}{\sigma_\epsilon^2}\bar{s}^*}{\sqrt{\frac{1}{\sigma_k^2}+\frac{1}{\sigma_\epsilon^2}}}\right)=\gamma, \tag{27}$$

$$R^{k*}=R\left(1-\frac{1}{L}\right)\left[1+\lambda\Phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)\right], \tag{28}$$

where $\Phi(\cdot)$ is the standard normal distribution function. Equation (27) implies that $\lim_{\sigma_\epsilon\to 0}\Phi((R^{k*}-\bar{s}^*)/\sigma_\epsilon)=\gamma$. Therefore, $\lim_{\sigma_\epsilon\to 0}\Phi((\bar{s}^*-R^{k*})/\sigma_\epsilon)=1-\gamma$. Substituting this result into equation (28) yields (9).

Derivation of equation (10). Equation (10) is the limiting case of equation (8) where $\sigma_\epsilon\to 0$. First, consider the derivation of $\partial\bar{s}^*(L)/\partial L$ in equation (8). Totally differentiating equations (27) and (28) yields:

$$dR^{k*}=\frac{1}{\frac{\sigma_\epsilon^2}{\sigma_k^2}+1}d\bar{s}^*,$$

$$dR^{k*}=\frac{R}{L^2}\left[1+\lambda\Phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)\right]dL+R\left(1-\frac{1}{L}\right)\lambda\phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)\frac{1}{\sigma_\epsilon}(d\bar{s}^*-dR^{k*})$$

Combining these equations yields:

$$\frac{d\bar{s}^*}{dL}=\frac{\left(\sigma_k^2+\sigma_\epsilon^2\right)\frac{R}{L^2}\left[1+\lambda\Phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)\right]}{\sigma_k^2-\left(1-\frac{1}{L}\right)\lambda\phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)\sigma_\epsilon},$$

where $\phi(\cdot)$ is the standard normal pdf. Note that $\lim_{\sigma_\epsilon\to 0}\phi((\bar{s}^*-R^{k*})/\sigma_\epsilon)=\phi(\lim_{\sigma_\epsilon\to 0}(\bar{s}^*-R^{k*})/\sigma_\epsilon)=\phi(\Phi^{-1}(1-\gamma))$. Then, in the limit, $d\bar{s}^*/dL$ is given by:

$$\lim_{\sigma_\epsilon\to 0}\frac{d\bar{s}^*}{dL}=\frac{R}{L^2}\left[1+\lambda(1-\gamma)\right].$$

Next, consider $\int_{R^{k*}}^{\infty}[\partial x(R^k,\bar{s}^*)/\partial\bar{s}^*]dF(R^k)$ in equation (8), where $F(\cdot)$ is the normal distribution function with mean μ and variance σ_k^2. It is explicitly written as:

$$\begin{aligned}\int_{R^{k*}}^{\infty}\frac{\partial x(R^k,\bar{s}^*)}{\partial\bar{s}^*}dF(R^k)&=\int_{R^{k*}}^{\infty}\phi\left(\frac{\bar{s}^*-R^k}{\sigma_\epsilon}\right)\frac{1}{\sigma_\epsilon}dF(R^k)\\&=\int_{R^{k*}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\bar{s}^*-R^k}{\sigma_\epsilon}\right)^2}\frac{1}{\sigma_\epsilon}\frac{1}{\sqrt{2\pi}\sigma_k}e^{-\frac{1}{2}\left(\frac{R^k-\mu}{\sigma_k}\right)^2}dR^k.\end{aligned}$$

The terms in the power of e are arranged as:

$$
\begin{aligned}
& -\frac{1}{2}\left(\frac{\bar{s}^* - R^k}{\sigma_\epsilon}\right)^2 - \frac{1}{2}\left(\frac{R^k - \mu}{\sigma_k}\right)^2 \\
=& -\frac{1}{2}\left[\frac{\bar{s}^{*2} - 2\bar{s}^* R^k + R^{k2}}{\sigma_\epsilon^2} + \frac{R^{k2} - 2R^k\mu + \mu^2}{\sigma_k^2}\right] \\
=& -\frac{1}{2}\left[\left(\frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma_k^2}\right) R^{k2} - 2\left(\frac{\bar{s}^*}{\sigma_\epsilon^2} + \frac{\mu}{\sigma_k^2}\right) R^k + \frac{\bar{s}^*}{\sigma_\epsilon^2} + \frac{\mu^2}{\sigma_k^2}\right] \\
=& -\frac{1}{2}\left(\frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma_k^2}\right)\left[R^{k2} - 2\frac{\frac{\bar{s}^*}{\sigma_\epsilon^2} + \frac{\mu}{\sigma_k^2}}{\frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma_k^2}} R^k + \frac{\frac{\bar{s}^*}{\sigma_\epsilon^2} + \frac{\mu^2}{\sigma_k^2}}{\frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma_k^2}}\right] \\
=& -\frac{1}{2}\left(\frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma_k^2}\right)\left[\left(R^k - \frac{\frac{\bar{s}^*}{\sigma_\epsilon^2} + \frac{\mu}{\sigma_k^2}}{\frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma_k^2}}\right)^2 - \left(\frac{\frac{\bar{s}^*}{\sigma_\epsilon^2} + \frac{\mu}{\sigma_k^2}}{\frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma_k^2}}\right)^2 + \frac{\frac{\bar{s}^*}{\sigma_\epsilon^2} + \frac{\mu^2}{\sigma_k^2}}{\frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma_k^2}}\right] \\
=& -\frac{1}{2}\left(\frac{R^k - \frac{\frac{\bar{s}^*}{\sigma_\epsilon^2} + \frac{\mu}{\sigma_k^2}}{\frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma_k^2}}}{\sqrt{\frac{\sigma_\epsilon^2\sigma_k^2}{\sigma_\epsilon^2 + \sigma_k^2}}}\right)^2 + \frac{1}{2}\left[\frac{\left(\frac{\bar{s}^*}{\sigma_\epsilon^2} + \frac{\mu}{\sigma_k^2}\right)^2}{\frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma_k^2}} - \frac{\bar{s}^*}{\sigma_\epsilon^2} - \frac{\mu^2}{\sigma_k^2}\right].
\end{aligned}
$$

Then, $\int_{R^{k*}}^{\infty}[\partial x(R^k, \bar{s}^*)/\partial \bar{s}^*]dF(R^k)$ is written as:

$$
\int_{R^{k*}}^{\infty} \frac{\partial x(R^k, \bar{s}^*)}{\partial \bar{s}^*} dF(R^k) = \left(\int_{z^*}^{\infty} \phi(z)dz\right) \frac{1}{\sqrt{2\pi}} \sqrt{\frac{1}{\sigma_\epsilon^2 + \sigma_k^2}} \exp\left\{\frac{1}{2}\left[\frac{\left(\frac{\bar{s}^*}{\sigma_\epsilon^2} + \frac{\mu}{\sigma_k^2}\right)^2}{\frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma_k^2}} - \frac{\bar{s}^*}{\sigma_\epsilon^2} - \frac{\mu^2}{\sigma_k^2}\right]\right\},
$$

where

$$
z^* = \frac{R^{k*} - \frac{\frac{\bar{s}^*}{\sigma_\epsilon^2} + \frac{\mu}{\sigma_k^2}}{\frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma_k^2}}}{\sqrt{\frac{\sigma_\epsilon^2\sigma_k^2}{\sigma_\epsilon^2 + \sigma_k^2}}}
$$

Note that $\lim_{\sigma_\epsilon \to 0} = \Phi^{-1}(\gamma)$ and

$$
\lim_{\sigma_\epsilon \to 0} \frac{1}{2}\left[\frac{\left(\frac{\bar{s}^*}{\sigma_\epsilon^2} + \frac{\mu}{\sigma_k^2}\right)^2}{\frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma_k^2}} - \frac{\bar{s}^*}{\sigma_\epsilon^2} - \frac{\mu^2}{\sigma_k^2}\right] = -\frac{1}{2}\left(\frac{\bar{s}^* - \mu}{\sigma_k}\right)^2.
$$

Therefore, the limit of $\int_{R^{k*}}^{\infty}[\partial x(R^k, \bar{s}^*)/\partial \bar{s}^*]dF(R^k)$ is given by:

$$
\lim_{\sigma_\epsilon \to 0} \int_{R^{k*}}^{\infty} \frac{\partial x(R^k, \bar{s}^*)}{\partial \bar{s}^*} dF(R^k) = (1-\gamma) f(\bar{s}^*),
$$

where $f(\cdot)$ is the pdf of the normal distribution with mean μ and variance σ_k^2.

Finally, the term, $\int_{R^{k*}}^{\infty} x(R^k, \bar{s}^*(L))dF(R^k)$, in equation (8) goes to zero as $\sigma_\epsilon \to 0$. Therefore, in the limit of $\sigma_\epsilon \to 0$, equation (8) is reduced to equation (10).

Proof of Proposition 1.

(i) The first-order condition of the banks' problem in the limit equilibrium (10) is written as $0 = \partial\mathbb{E}(\pi)/\partial L$, where

$$\frac{\partial\mathbb{E}(\pi)}{\partial L} = \int_{\frac{R^{k*}-\mu}{\sigma_k}}^{\infty} (\mu + \sigma_k z)\, d\Phi(z)$$
$$-\left\{\left[1-\Phi\left(\frac{R^{k*}-\mu}{\sigma_k}\right)\right] R + \lambda(1-\gamma)\left[1+\lambda(1-\gamma)\right]\phi\left(\frac{R^{k*}-\mu}{\sigma_k}\right) R^2\frac{L-1}{L^2}\right\}.$$

A marginal change in this derivative with respect to a marginal increase in μ is given by:

$$\begin{aligned}\frac{\partial^2\mathbb{E}(\pi)}{\partial L\partial\mu} &= 1-\Phi\left(\frac{R^{k*}-\mu}{\sigma_k}\right) + \left[R^{k*} - R\phi\left(\frac{R^{k*}-\mu}{\sigma_k}\right)\right]\frac{1}{\sigma_k} \\ &+ \frac{\lambda(1-\gamma)\left[1+\lambda(1-\gamma)\right]}{\sigma_k}\phi'\left(\frac{R^{k*}-\mu}{\sigma_k}\right) R^2\frac{L-1}{L^2}.\end{aligned}$$

Because $\max_z \phi(z) < 0.4$, the assumptions of this proposition imply $R^{k*} > R\phi\left(\frac{R^{k*}-\mu}{\sigma_k}\right)$ and $\phi'(\cdot) > 0$, and thereby the sign of the above derivative is positive: $\partial^2\mathbb{E}(\pi)/(\partial L\partial\mu) > 0$. Given that the solution L is an optimal solution, the $\partial\mathbb{E}(\pi)/\partial L$ curve is downward sloping. Then, $\partial^2\mathbb{E}(\pi)/(\partial L\partial\mu) > 0$ implies that the $\partial\mathbb{E}(\pi)/\partial L$ curve shifts upward, implying that the optimal L increases. Hence, the demand curve shifts outward.

(ii) A marginal change in $\partial\mathbb{E}(\pi)/\partial L$ with respect to a marginal increase in λ is given by:

$$\begin{aligned}\frac{\partial^2\mathbb{E}(\pi)}{\partial L\partial\lambda} &= -\left[R^{k*} - R\phi\left(\frac{R^{k*}-\mu}{\sigma_k}\right)\right]\frac{1}{\sigma_k}\frac{\partial R^{k*}}{\partial\lambda} \\ &\quad -\frac{\lambda(1-\gamma)\left[1+\lambda(1-\gamma)\right]}{\sigma_k}\phi'\left(\frac{R^{k*}-\bar{R}^k}{\sigma_k}\right) R^2\frac{L-1}{L^2}\frac{\partial R^{k*}}{\partial\lambda} \\ &\quad -(1-\gamma)[1+2\lambda(1-\gamma)]\phi\left(\frac{R^{k*}-\mu}{\sigma_k}\right) R^2\frac{L-1}{L^2},\end{aligned}$$

where $\partial R^{k*}/\partial\lambda = R(1-1/L)(1-\gamma) > 0$. Hence, $\partial^2\mathbb{E}(\pi)/(\partial L\partial\lambda) < 0$, which implies that an increase in λ shifts the demand curve inward. Similarly, a marginal change in $\partial\mathbb{E}(\pi)/\partial L$ with respect to a marginal increase in γ is given by:

$$\begin{aligned}\frac{\partial^2\mathbb{E}(\pi)}{\partial L\partial\gamma} &= -\left[R^{k*} - R\phi\left(\frac{R^{k*}-\mu}{\sigma_k}\right)\right]\frac{1}{\sigma_k}\frac{\partial R^{k*}}{\partial\gamma} \\ &\quad -\frac{\lambda(1-\gamma)\left[1+\lambda(1-\gamma)\right]}{\sigma_k}\phi'\left(\frac{R^{k*}-\mu}{\sigma_k}\right) R^2\frac{L-1}{L^2}\frac{\partial R^{k*}}{\partial\gamma} \\ &\quad +\lambda[1+2\lambda(1-\gamma)]\phi\left(\frac{R^{k*}-\mu}{\sigma_k}\right) R^2\frac{L-1}{L^2}\end{aligned}$$

where $\partial R^{k*}/\partial\gamma = -R(1-1/L)\lambda < 0$. Hence, $\partial^2\mathbb{E}(\pi)/(\partial L\partial\gamma) > 0$, which implies that a decrease in γ shifts the demand curve inward.

(iii) The supply curve (1) is written as:

$$R = \frac{u'\left(y - (L-1)\,n\right)}{1 - P + \mathbb{E}(v|\text{default})P}.$$

From this it is clear that an increase in y shifts the supply curve outward.

(iv) Similarly, the supply curve implies that an increase n shifts the curve inward.

Derivation of equation (15). The first-order condition of the regulator's problem is $\partial\text{SW}/\partial L = 0$, where

$$\begin{aligned}\frac{\partial \text{SW}}{\partial L} =& -R\left[1 - P + \mathbb{E}(v|\text{default})P\right] + \mathbb{E}(R^k) - \lambda R\int_{\underline{R}^k}^{\infty} x dF - \lambda \int_{-\infty}^{\underline{R}^k} \frac{R^k}{1+\lambda} dF \\ & -\lambda(L-1)\int_{\underline{R}^k}^{\infty} R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L} dF - \lambda(L-1)\int_{\underline{R}^k}^{\infty} R\left(\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right)\frac{\partial R}{\partial L} dF,\end{aligned}$$

The first-order condition of the bank's problem is $\partial\mathbb{E}(\pi)/\partial L = 0$, where

$$\frac{\partial \mathbb{E}(\pi)}{\partial L} = \int_{R^{k*}}^{\infty} (R^k - R) dF - \lambda\,(L-1)\int_{R^{k*}}^{\infty} R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L} dF - R\lambda\int_{R^{k*}}^{\infty} x dF.$$

Then, $\partial\text{SW}/\partial L$ evaluated at the competitive equilibrium is given by:

$$\begin{aligned}\left.\frac{\partial \text{SW}}{\partial L}\right|_{\text{CE}} =& \left.\frac{\partial \text{SW}}{\partial L}\right|_{\text{CE}} - \left.\frac{\partial \mathbb{E}(\pi)}{\partial L}\right|_{\text{CE}} \\ =& \int_{\underline{R}^k}^{R^{k*}} R^k dF + \frac{1}{1+\lambda}\int_{-\infty}^{\underline{R}^k} R^k dF - R\mathbb{E}(v|\text{default})P - \lambda R\int_{\underline{R}^k}^{R^{k*}} x dF \\ & -\lambda(L-1)\left[\int_{\underline{R}^k}^{R^*} R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L} dF + \int_{\underline{R}^k}^{\infty}\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right)\frac{\partial R}{\partial L} dF\right]\end{aligned}$$

Because the recovery rate v is given by equation (12), $R\mathbb{E}(v|\text{default})P$ is given by:

$$R\mathbb{E}(v|\text{default})P = \int_{\underline{R}^k}^{R^{k*}}\left(R^k\frac{L}{L-1} - \lambda R x\right) dF + \frac{1}{1+\lambda}\int_{-\infty}^{\underline{R}^k} R^k\frac{L}{L-1} dF.$$

Then, the first-order condition of the regulator's problem, evaluated at the competitive equilibrium, is written as:

$$\begin{aligned}\left.\frac{\partial \text{SW}}{\partial L}\right|_{\text{CE}} =& -\frac{1}{L-1}\left[\int_{\underline{R}^k}^{R^{k*}} R^k dF + \frac{1}{1+\lambda}\int_{-\infty}^{\underline{R}^k} R^k dF\right] \\ & -\lambda(L-1)\left[\int_{\underline{R}^k}^{R^*} R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L} dF + \int_{\underline{R}^k}^{\infty}\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right)\frac{\partial R}{\partial L} dF\right].\end{aligned}$$

This completes the derivation of (15).

Derivation of $\partial\bar{s}^*/\partial L$ and $\partial\bar{s}^*/\partial R$ in the benchmark model. Totally differentiating equations (27) and (28) with respect to R, $\bar{s}^*$ and R^{k*} yields:

$$
\begin{aligned}
dR^{k*} &= \frac{\sigma_k^2}{\sigma_\epsilon^2 + \sigma_k^2} d\bar{s}^*, \\
dR^{k*} &= \left(1 - \frac{1}{L}\right)(1+\lambda x)dR + R\left(1 - \frac{1}{L}\right)\phi\left(\frac{\bar{s}^* - R^{k*}}{\sigma_\epsilon}\right)\frac{1}{\sigma_\epsilon}(d\bar{s}^* - dR^{k*}).
\end{aligned}
$$

Also, totally differentiating equation (28) with respect to L, $\bar{s}^*$ and R^{k*} yields:

$$
dR^{k*} = \frac{R}{L^2}(1+\lambda x)dL + R\left(1 - \frac{1}{L}\right)\phi\left(\frac{\bar{s}^* - R^{k*}}{\sigma_\epsilon}\right)\frac{1}{\sigma_\epsilon}(d\bar{s}^* - dR^{k*}).
$$

Rearranging these equations leads to:

$$
\begin{aligned}
\frac{\partial \bar{s}^*}{\partial R} &= \frac{\left(1 + \frac{\sigma_\epsilon^2}{\sigma_k^2}\right)\left(1 - \frac{1}{L}\right)(1+\lambda x)}{1 - \frac{\sigma_\epsilon}{\sigma_k^2} R\left(1 - \frac{1}{L}\right)\phi\left(\frac{\bar{s}^* - R^{k*}}{\sigma_\epsilon}\right)} > 0, \\
\frac{\partial \bar{s}^*}{\partial L} &= \frac{\left(1 + \frac{\sigma_\epsilon^2}{\sigma_k^2}\right)\frac{R}{L^2}(1+\lambda x)}{1 - \frac{\sigma_\epsilon}{\sigma_k^2} R\left(1 - \frac{1}{L}\right)\phi\left(\frac{\bar{s}^* - R^{k*}}{\sigma_\epsilon}\right)} > 0.
\end{aligned}
$$

The sign of these derivatives is positive because the denominator, which is identical between the two, is positive for the threshold $\bar{s}^*$ to uniquely exist, which is assumed to hold.

The slope of the supply curve (1). The supply curve (1) is written in terms of leverage as:

$$
R(1-P) + \int_{\underline{R}^k}^{R^{k*}} \left(R^k \frac{L}{L-1} - R\lambda x\right) dF + \frac{1}{1+\lambda}\int_{-\infty}^{\underline{R}^k} R^k \frac{L}{L-1} dF = u'(y - (L-1)n),
$$

Totally differentiating the equation with respect to R and L yields:

$$
\begin{aligned}
&\left\{1 - P - \lambda \int_{\underline{R}^k}^{R^{k*}} \left[R \frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right] dF\right\} dR \\
=& \left\{\int_{\underline{R}^k}^{R^{k*}} \left[\frac{R^k}{(L-1)^2} + R\lambda \frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L}\right] dF + \frac{1}{1+\lambda}\int_{-\infty}^{R^{k*}} \frac{R^k}{(L-1)^2} dF - u''(c_1)n\right\} dL.
\end{aligned}
$$

Then, the slope of the supply curve is given by:

$$
\frac{dR}{dL} = \frac{\int_{\underline{R}^k}^{R^{k*}} \left[\frac{R^k}{(L-1)^2} + R\lambda \frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L}\right] dF + \frac{1}{1+\lambda}\int_{-\infty}^{R^{k*}} \frac{R^k}{(L-1)^2} dF - u''(c_1)n}{1 - P - \lambda \int_{\underline{R}^k}^{R^{k*}} \left[R \frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right] dF}. \tag{29}
$$

The numerator of (29) is positive. Hence, the slope of the supply curve is positive if and only if the denominator is positive.

Banks' problem without bank risk shifting motives. Banks choose leverage L to maximize

$$
\int_{R^{k*}}^{\infty} \left\{R^k L - R\left[1 + \lambda x(R^k, \bar{s}^*(L,R))\right](L-1)\right\} n dF(R^k),
$$

subject to the technical constraint $L \le L_{\max}$ and the households' participation constraint (16), which is rewritten here for convenience:

$$R[1 - F(R^{k*})] + \int_{\underline{R}^k}^{R^{k*}} \left[R^k \frac{L}{L-1} - R\lambda x(R^k, \bar{s}^*) \right] dF + \int_{-\infty}^{\underline{R}^k} \frac{R^k}{1+\lambda} \frac{L}{L-1} dF \ge R^e.$$

The technical constraint is non-binding as in the benchmark model. It is obvious that the households' participation constraint is binding. The binding constraint implicitly defines the interest rate as a function of leverage: $R = R_B(L)$. The slope of this curve is derived in a similar manner as in the supply curve and is given by:

$$\frac{dR_B}{dL} = \frac{\int_{\underline{R}^k}^{R^{k*}} \left[\frac{R^k}{(L-1)^2} + R\lambda \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial L} \right] dF + \frac{1}{1+\lambda} \int_{-\infty}^{R^{k*}} \frac{R^k}{(L-1)^2} dF}{1 - P - \lambda \int_{\underline{R}^k}^{R^{k*}} \left[R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial R} + x \right] dF} \tag{30}$$

Compared to the slope of the supply curve, (29), the only difference in the slope of R_B is the absence of $-u''(c_1)n$ in the numerator.

Substituting $R = R_B(L)$ into the banks' objective function, the first-order condition with respect to L is written as:

$$0 = \frac{\partial \mathbb{E}(\pi)}{\partial L} = \int_{R^{k*}}^{\infty} R^k dF - (1-P)R - \lambda R (L-1) \int_{R^{k*}}^{\infty} \frac{\partial x}{\partial \bar{s}^*} \left(\frac{\partial \bar{s}^*}{\partial L} + \frac{\partial \bar{s}^*}{\partial R} \frac{\partial R_B}{\partial L} \right) dF$$
$$-\lambda R \int_{R^{k*}}^{\infty} x dF - \frac{\partial R_B}{\partial L} (L-1) \int_{R^{k*}}^{\infty} (1 + \lambda x) dF.$$

Derivation of equation (17). The slope of the social welfare function is given by equation (14), which is rewritten here for convenience:

$$\frac{\partial SW}{\partial L} = -R\left[1 - P + \mathbb{E}(v|\text{default})P\right] + \mathbb{E}(R^k) - \lambda R \int_{\underline{R}^k}^{\infty} x dF - \lambda \int_{-\infty}^{\underline{R}^k} \frac{R^k}{1+\lambda} dF$$
$$-\lambda R(L-1)R \int_{\underline{R}^k}^{\infty} \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial L} dF - \lambda(L-1) \int_{\underline{R}^k}^{\infty} \left(\frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial R} R + x \right) \frac{\partial R}{\partial L} dF,$$

where constant proportional term n is omitted for simplifying notations. Let CE' denote the competitive equilibrium without bank risk shifting motives. Then, the slope of the social welfare evaluated at this competitive equilibrium is given by:

$$
\begin{aligned}
\left.\frac{\partial SW}{\partial L}\right|_{\mathrm{CE}'} =& \left.\frac{\partial SW}{\partial L}\right|_{\mathrm{CE}'} - \left.\frac{\partial \mathbb{E}(\pi)}{\partial L}\right|_{\mathrm{CE}'} \\
=& -\frac{1}{L-1}\left[\int_{\underline{R}^k}^{R^{k*}} R^k dF + \frac{1}{1+\lambda}\int_{-\infty}^{\underline{R}^k} R^k dF\right] \\
& -\lambda(L-1)\left[\int_{\underline{R}^k}^{R^*} R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L}dF + \int_{\underline{R}^k}^{\infty}\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right)\frac{\partial R}{\partial L}dF\right] \\
& + \frac{\partial R_B}{\partial L}\left[\int_{R^{k*}}^{\infty}\left(\lambda\frac{\partial \bar{s}^*}{\partial R} + 1 + \lambda x + \lambda R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R}\right)dF\right](L-1) \\
=& -\int_{\underline{R}^k}^{R^{k*}}\left[\frac{R^k}{L-1} + R\lambda(L-1)\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L}\right]dF + \frac{1}{1+\lambda}\int_{-\infty}^{R^{k*}}\frac{R^k}{L-1}dF \\
& + \frac{\partial R_B}{\partial L}\left[1 - P - \lambda\int_{\underline{R}^k}^{R^{k*}}\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right)dF\right](L-1) \\
& -\lambda(L-1)\Delta R\int_{\underline{R}^k}^{R^{k*}}\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right)dF,
\end{aligned}
$$

where $\Delta R \equiv \partial R/\partial L - \partial R_B/\partial L \propto -u'' > 0$. Substituting $\partial R_B/\partial L$ out using (30) yields:

$$
\left.\frac{\partial SW}{\partial L}\right|_{\mathrm{CE}'} = \lambda(L-1)\left[\int_{\underline{R}^k}^{R^{k*}}\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right)dF\right]u''(c_1) < 0.
$$

Derivation of $\partial\bar{s}^*/\partial L$, $\partial\bar{s}^*/\partial m$, $\partial\bar{s}^*/\partial R$ and condition (22) in Section 4. In the model with leverage and liquidity, the thresholds $\bar{s}^*$ and R^{k*} are characterized by equations (5) and (19). Equation (19) is written as:

$$
R^{k*} = \frac{R-m}{\frac{L}{L-1} - m}\left[1 + \lambda\frac{\Phi\left(\frac{\bar{s}^* - R^{k*}}{\sigma_\epsilon}\right)R - m}{R-m}\right]. \tag{31}
$$

Totally differentiating equations (5) with respect to R^{k*} and $\bar{s}^*$ yields:

$$
dR^{k*} = \frac{1}{\frac{\sigma_\epsilon^2}{\sigma_k^2} + 1}d\bar{s}^*.
$$

Totally differentiating equation (31) with respect to R^{k*}, $\bar{s}^*$ and L yields:

$$
dR^{k*} = \frac{1}{[L - m(L-1)]^2}\left(1 + \lambda\frac{xR - m}{R-m}\right)dL + \frac{\lambda R x'}{\frac{L}{L-1} - m}\frac{d\bar{s}^* - dR^{k*}}{\sigma_\epsilon},
$$

where $x' \equiv \phi((\bar{s}^* - R^{k*})/\sigma_\epsilon)$. Then, $d\bar{s}^*/dL$ is given by:

$$
\frac{d\bar{s}^*}{dL} = \frac{\frac{\sigma_\epsilon^2/\sigma_k^2 + 1}{[L - m(L-1)]^2}\left(1 + \lambda\frac{xR-m}{R-m}\right)}{1 - \frac{\sigma_\epsilon}{\sigma_k^2}\frac{\lambda R x'}{\frac{L}{L-1} - m}} > 0.
$$

Note that the denominator is positive for the model to have a unique solution for $\bar{s}^*$ and R^{k*}. Next, totally differentiating equation (31) with respect to R^{k*}, $\bar{s}^*$ and m yields:

$$dR^{k*} = \frac{-(1+\lambda)\frac{L}{L-1} + (1+\lambda x)R}{[L/(L-1) - m]^2} dm + \frac{\lambda R x'}{\frac{L}{L-1} - m} \frac{d\bar{s}^* - dR^{k*}}{\sigma_\epsilon},$$

Then, $d\bar{s}^*/dm$ is given by:

$$\frac{d\bar{s}^*}{dm} = \frac{\frac{\sigma_\epsilon^2/\sigma_k^2+1}{[L/(L-1)-m]^2}\left[-(1+\lambda)\frac{L}{L-1} + (1+\lambda x)R\right]}{1 - \frac{\sigma_\epsilon}{\sigma_k^2}\frac{\lambda R x'}{\frac{L}{L-1}-m}}.$$

Hence, $d\bar{s}^*/dm < 0$ if the interest rate is low enough to satisfy condition (22):

$$R < \frac{1+\lambda}{1+\lambda x}\frac{L}{L-1}.$$

Finally, totally differentiating equation (31) with respect to R^{k*}, $\bar{s}^*$ and R yields:

$$dR^{k*} = \frac{1}{\frac{L}{L-1} - m}(1+\lambda x)dR + \frac{\lambda R x'}{\frac{L}{L-1} - m}\left(d\bar{s}^* - dR^{k*}\right).$$

Then, $d\bar{s}^*/dR$ is given by:

$$\frac{d\bar{s}^*}{dR} = \frac{\frac{\sigma_\epsilon^2/\sigma_k^2+1}{L/(L-1)-m}(1+\lambda x)}{1 - \frac{\sigma_\epsilon}{\sigma_k^2}\frac{\lambda R x'}{\frac{L}{L-1}-m}} > 0.$$

Derivation of $\partial R/\partial L$ and $\partial R/\partial m$ in Section 4. Using the recovery rate in the model with liquidity, the supply curve of funds (1) is written as:

$$\begin{aligned} u'(y-(L-1)n) = R(1-P) + &\int_{\underline{R}^k}^{R^{k*}} \left[R^k\left(\frac{L}{L-1} - m\right) + m - \lambda(Rx - m)\right] dF \\ &+ \int_{-\infty}^{\underline{R}^k} \left[\frac{R^k}{1+\lambda}\left(\frac{L}{L-1} - m\right) + m\right] dF. \end{aligned}$$

Totally differentiating this equation with respect to L and R yields:

$$\begin{aligned} -u''(c_1)ndL = &\left[1 - P - \lambda\left(\int_{\underline{R}^k}^{R^{k*}} R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial R} + x\right) dF\right] dR \\ &- \left[\int_{\underline{R}^k}^{R^{k*}} \left(\frac{R^k}{(L-1)^2} + \lambda R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L}\right) dF + \frac{1}{1+\lambda}\int_{-\infty}^{\underline{R}^k} \frac{R^k}{(L-1)^2} dF\right] dL. \end{aligned}$$

Similarly, totally differentiating it with respect to R and m yields:

$$0 = \left[1 - P - \lambda \left(\int_{\underline{R}^k}^{R^{k*}} R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial R} + x \right) dF \right] dR$$
$$+ \left[\int_{\underline{R}^k}^{R^{k*}} \left(-R^k + 1 + \lambda - \lambda R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial m} \right) dF + \int_{-\infty}^{\underline{R}^k} \left(-\frac{R^k}{1+\lambda} + 1 \right) dF \right] dm.$$

Hence, $\partial R / \partial L$ and $\partial R / dm$ are given by:

$$\frac{\partial R}{\partial L} = \frac{\int_{\underline{R}^k}^{R^{k*}} \left(\frac{R^k}{(L-1)^2} + \lambda R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial L} \right) dF + \frac{1}{1+\lambda} \int_{-\infty}^{\underline{R}^k} \frac{R^k}{(L-1)^2} dF - u''(c_1) n}{1 - P - \lambda \int_{\underline{R}^k}^{R^{k*}} \left(R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial R} + x \right) dF},$$
$$\frac{\partial R}{\partial m} = \frac{\int_{\underline{R}^k}^{R^{k*}} \left[\lambda R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial m} - (1 + \lambda - R^k) \right] dF - \int_{-\infty}^{\underline{R}^k} \left(1 - \frac{R^k}{1+\lambda} \right) dF}{1 - P - \lambda \int_{\underline{R}^k}^{R^{k*}} \left(R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial R} + x \right) dF}$$

The numerator of the equation for $\partial R / \partial L$ is positive. Hence, the slope of the supply curve is positive, i.e. $\partial R / \partial L > 0$, if and only if

$$1 - P - \lambda \int_{-\infty}^{R^{k*}} \left[x + R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial R} \right] dF(R^k) > 0.$$

The numerator of the equation for $\partial R / \partial m$ is negative under the assumptions of (22) and (23). If the slope of the supply curve is positive, the slope of the interest rate curve with respect to liquidity is negative, i.e. $\partial R / \partial m < 0$.

Proof of Proposition 5. As provided in Section 4, the first-order condition of the regulator's problem with respect to leverage is given by:

$$0 = \frac{\partial SW}{\partial L} = - R \left[1 - P + P \mathbb{E}(v | \text{default}) \right] + \int_{\underline{R}^k}^{\infty} \left[R^k - (R^k - 1) m \right] dF$$
$$- \lambda \int_{\underline{R}^k}^{\bar{R}^k} \left[(xR - m) + R(L-1) \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial L} + (L-1) \left(R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial R} + x \right) \frac{\partial R}{\partial L} \right] dF$$
$$+ \int_{-\infty}^{\underline{R}^k} \left[\frac{R^k}{1+\lambda} - \left(\frac{R^k}{1+\lambda} - 1 \right) m \right] dF.$$

where $RP\mathbb{E}(v | \text{default})$ is given by:

$$RP\mathbb{E}(v | \text{default}) = \int_{\underline{R}^k}^{R^{k*}} \left[R^k \left(\frac{L}{L-1} - m \right) + m - \lambda (Rx - m) \right] dF$$
$$+ \int_{-\infty}^{\underline{R}^k} \left[\frac{1}{1+\lambda} R^k \left(\frac{L}{L-1} - m \right) + m \right] dF.$$

On the other hand, as provided in the main text, the first-order condition of the banks' problem with respect to leverage is given by:

$$0 = \frac{\partial \mathbb{E}(\pi)}{\partial L} = \int_{R^{k*}}^{\infty} [R^k - (R^k - 1)m - R] dF - \lambda \int_{R^{k*}}^{\bar{R}^k} \left[(xR - m) + R(L-1) \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial L} \right] dF.$$

Hence, the slope of the social welfare, evaluated at the banks' privately optimal choice of leverage $L = L^*$, is given by:

$$\begin{aligned} \left. \frac{\partial SW}{\partial L} \right|_{L=L^*} =& - \frac{1}{L-1} \left[\int_{\underline{R}^k}^{R^{k*}} R^k dF + \int_{-\infty}^{\underline{R}^k} \frac{R^k}{1+\lambda} dF \right] \\ & - \lambda(L-1) \left[\int_{\underline{R}^k}^{R^{k*}} R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial L} dF + \int_{\underline{R}^k}^{\bar{R}^k} \left(R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial R} + x \right) \frac{\partial R}{\partial L} dF \right]. \end{aligned}$$

Hence, under the assumption of the upward-sloping supply curve, $\partial R / \partial L > 0$, the sign of $\partial \mathrm{SW} / \partial L|_{L=L^*}$ is negative. This completes the proof of Proposition 5.

Proof of Proposition 6. When banks do not have risk shifting motives, they maximize the profits subject to the households' participation constraint, $R[1 - P + \mathbb{E}(v|\text{default})] \geq R^e$ for some R^e, and the technical constraint $L \leq L_{\max}$. Given R^e, the households' participation constraint implicitly defines the interest rate as a function of leverage and liquidity, $R = R_B(L, m)$. In particular, the derivatives with respect to L and m respectively are given by:

$$\begin{aligned} \frac{\partial R_B}{\partial L} =& \frac{\int_{\underline{R}^k}^{R^{k*}} \left(\frac{R^k}{(L-1)^2} + \lambda R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial L} \right) dF + \frac{1}{1+\lambda} \int_{-\infty}^{\underline{R}^k} \frac{R^k}{(L-1)^2} dF}{1 - P - \lambda \int_{\underline{R}^k}^{R^{k*}} \left(R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial R} + x \right) dF}, \\ \frac{\partial R_B}{\partial m} =& \frac{\partial R}{\partial m} = \frac{\int_{\underline{R}^k}^{R^{k*}} \left[\lambda R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial m} - (1 + \lambda - R^k) \right] dF - \int_{-\infty}^{\underline{R}^k} \left(1 - \frac{R^k}{1+\lambda} \right) dF}{1 - P - \lambda \int_{\underline{R}^k}^{R^{k*}} \left(R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial R} + x \right) dF} \end{aligned}$$

Taking into account $R = R_B(L, m)$, the first-order condition of the banks' problem with respect to m is given by:

$$\begin{aligned} 0 = & - \int_{R^{k*}}^{\infty} (R^k - 1) dF(R^k) + \lambda \int_{R^{k*}}^{\bar{R}^k} \left(1 - R \frac{\partial x}{\partial \bar{s}^*} \frac{\partial \bar{s}^*}{\partial m} \right) dF(R^k) \\ & - (1 - P) \frac{\partial R_B}{\partial m} - \lambda \frac{\partial R_B}{\partial m} \int_{R^{k*}}^{\bar{R}^k} \left(x + R \frac{\partial x}{\partial \bar{s}^*} \frac{\bar{s}^*}{\partial R} \right) dF(R^k). \end{aligned}$$

The last two terms in the right-hand-side of the equation correspond to those related to the effect of liquidity on the interest rate. Because the sign of these terms are positive, banks which have no risk shifting motives have higher liquidity holdings than otherwise would be the case. Evaluating the first-order condition of the regulator's problem with respect to liquidity at the competitive equilibrium level of liquidity $m = m^*$ yields:

$$\begin{aligned}
\left.\frac{\partial SW}{\partial m}\right|_{m=m^*} &= \frac{\partial R_B}{\partial m}\left[1 - P - \lambda\int_{\underline{R}^k}^{R^{k*}}\left(R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial m} + x\right)dF\right] \\
&- \left\{\int_{\underline{R}^k}^{R^{k*}}\left[\lambda R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial m} - (1 + \lambda - R^k)\right]dF - \int_{-\infty}^{\underline{R}^k}\left(1 - \frac{R^k}{1+\lambda}\right)dF\right\} \\
&= 0.
\end{aligned}$$

The final equality was derived by using the expression for $\partial R_B/\partial m$. The first-order condition of the regulator's problem, evaluated at the competitive equilibrium level of leverage, can be derived similarly to the benchmark model. This completes the proof of Proposition 6.

Calibration: the extended model with bank leverage and liquidity. The unit of time is annual. The calibration strategy is to set target values for endogenous variables L, m, R and P and pin down parameter values for σ_ϵ, γ, λ and y jointly. The four parameters, σ_ϵ, γ, λ and y, are set as follows. The probability of bank default is given by $P = \Phi((R^{k*} - \mu)/\sigma_k)$, so that the threshold R^{k*} is given by $R^{k*} = \mu + \sigma_k\Phi^{-1}(P)$. Condition (5) is arranged as:

$$\frac{\bar{s}^* - R^{k*}}{\sigma_\epsilon} = \frac{\sigma_\epsilon}{\sigma_k^2}(R^{k*} - \mu) - \sqrt{1 + \frac{\sigma_\epsilon^2}{\sigma_k^2}}\Phi^{-1}(\gamma).$$

Also, condition (19) is arranged as:

$$\lambda = \left[R^{k*}\frac{\frac{L}{L-1} - m}{R - m} - 1\right]\frac{R - m}{\Phi\left(\frac{\bar{s}^* - R^{k*}}{\sigma_\epsilon}\right)R - m} \equiv \lambda(\sigma_\epsilon, \gamma),$$

where the equation for $(\bar{s}^* - R^{k*})/\sigma_\epsilon$ was used in deriving the final equivalence. The first-order conditions (20) and (21) are written as:

$$\begin{aligned}
0 &= \int_{R^{k*}}^{\infty}[R^k - (R^k - 1)m - R]dF(R^k) - \int_{R^{k*}}^{\bar{R}^k}\left[\lambda(Rx - m) + (L-1)\lambda R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial L}\right]dF(R^k), \\
0 &= -\int_{R^{k*}}^{\infty}(R^k - 1)dF(R^k) + \lambda\int_{R^{k*}}^{\bar{R}^k}\left(1 - R\frac{\partial x}{\partial \bar{s}^*}\frac{\partial \bar{s}^*}{\partial m}\right)dF(R^k),
\end{aligned}$$

where

$$\begin{aligned}
\mu =& \bar{s}^* - \sigma_\epsilon \Phi^{-1}\left(\frac{m}{R}\right), \\
\frac{\partial x}{\partial \bar{s}^*} =& \phi\left(\frac{\bar{s}^* - R^k}{\sigma_\epsilon}\right)\frac{1}{\sigma_\epsilon}, \\
\frac{\partial \bar{s}^*}{\partial L} =& \frac{\frac{\sigma_\epsilon^2/\sigma_k^2+1}{[L-m(L-1)]^2}\left(1+\lambda\frac{xR-m}{R-m}\right)}{1-\frac{\sigma_\epsilon}{\sigma_k^2}\frac{\lambda R}{\frac{L}{L-1}-m}\phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)}, \\
\frac{\partial \bar{s}^*}{\partial m} =& \frac{\frac{\sigma_\epsilon^2/\sigma_k^2+1}{[L/(L-1)-m]^2}\left[-(1+\lambda)\frac{L}{L-1}+(1+\lambda x)R\right]}{1-\frac{\sigma_\epsilon}{\sigma_k^2}\frac{\lambda R}{\frac{L}{L-1}-m}\phi\left(\frac{\bar{s}^*-R^{k*}}{\sigma_\epsilon}\right)}.
\end{aligned}$$

These two equations are solved for σ_ϵ and γ. In solving the simultaneous equations, σ_ϵ and γ have to satisfy conditions (22) and (23). Also, these parameters have to be such that the denominator of $\partial\bar{s}^*/\partial L$ is positive. With σ_ϵ and γ at hand, parameter λ is determined. Finally, y is set to satisfy equation (1), i.e.

$$y = (L-1)\,n + \frac{1}{[R\,(1-P+\mathbb{E}(v|\text{default})P)]^\alpha},$$

where $\mathbb{E}(v|\text{default})P$ is given by:

$$R\mathbb{E}(v|\text{default})P = \int_{\underline{R}^k}^{R^{k*}}\left[R^k\left(\frac{L}{L-1}-m\right)+m-\lambda(Rx-m)\right]dF + \int_{-\infty}^{\underline{R}^k}\left[\frac{1}{1+\lambda}R^k\left(\frac{L}{L-1}-m\right)+m\right]dF.$$