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Abstract

No-arbitrage dynamic term structure models (DTSMs) have regularly been used to estimate interest rate expectations and term premia, but are beset by an identification problem that results in inaccurate estimates. I propose the augmentation of DTSMs with overnight indexed swap (OIS) rates to better estimate interest rate expectations and term premia along the whole term structure at daily frequencies. I illustrate this with a Gaussian affine DTSM augmented with 3 to 24-month OIS rates, which provide accurate information about interest rate expectations. The OIS-augmented model generates estimates of US interest rate expectations that closely correspond to those implied by federal funds futures rates and survey expectations out to a 10-year horizon, accurately depict their daily frequency evolution, and are more stable across samples. Against these metrics, interest rate expectation estimates, and therefore term premia, from OIS-augmented models are superior to estimates from existing Gaussian affine DTSMs.

Key words: Dynamic term structure model, monetary policy expectations, overnight indexed swaps, term premia, term structure of interest rates.

JEL classification: C32, C58, E43, E47, G12.

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1 Introduction

Financial market participants, researchers and policymakers closely monitor the daily frequency evolution of interest rate expectations and term premia. Dynamic term structure models (DTSMs) have increasingly been used to estimate these two components of nominal government bond yields (e.g. Christensen and Rudebusch, 2012). By imposing no-arbitrage, these models provide estimates of interest rate expectations that extend to horizons in excess of what can be accurately imputed from financial market prices directly (Lloyd, 2018). However, these models suffer from an identification problem that results in estimates of interest rate expectations that are inaccurate in two senses. First, estimates of interest rate expectations are spuriously stable and, therefore, the majority of variation in yields is attributed to term premia. In practice, model-implied interest rate expectations differ from survey- and market-implied measures of interest rate expectations on dates and at horizons where such a comparison is possible. Second, interest rate expectation estimates vary in real time. That is, the interest rate expectation estimate on a given date will vary as the sample length changes.

Central to the identification problem is an informational insufficiency. Bond yield data is the sole input to a DTSM unaugmented with other information. These yields provide information of direct relevance to the estimation of the fitted bond yields. Absent additional information, estimates of interest rate expectations are poorly identified as they must also be derived from information contained within the actual bond yields. To do this, maximum likelihood or ordinary least squares estimates of, *inter alia*, the persistence of the (pricing factors derived from the) actual yields must be attained. However, as a symptom of the identification problem, a 'finite sample' bias will arise in these persistence parameters when there is insufficient information and a limited number of interest rate cycles in the observed yield data.[1] Finite sample bias will result in persistence parameters that are spuriously estimated to be less persistent than they really are and estimates of interest rate expectations that are too stable.[2] Because bond yields are highly persistent, the finite sample bias can be severe. Moreover, the severity of the bias is increasing in the persistence of actual yield data. For daily frequency yields, which display greater persistence than lower frequency data, the problem is particularly pertinent.

In this paper, I propose augmenting DTSMs with overnight indexed swap (OIS) rates as an additional input to improve the identification of interest rate expectations and term premia from yields. OIS contracts are over-the-counter traded interest rate derivatives in which two counterparties exchange fixed and floating interest rate payments. A counterparty will enter into an OIS if they expect the payments they swap to exceed those they take on. In deep and liquid markets, OIS rates should reflect investors' expectations of future short-term interest rates. Lloyd (2018) demonstrates that 1 to 24-month OIS rates tend to accurately reflect

[1] Kim and Orphanides (2012, p. 242) state that "in a term structure sample spanning 5 to 15 years, one may not observe a sufficient number of mean reversions."

[2] This 'finite sample' bias is well documented for ordinary least squares estimation of a univariate autoregressive process, where estimates of the autoregressive parameter will be biased downwards, implying less persistence that the true process. Within Gaussian affine DTSMs, the finite sample bias is a multivariate generalisation of this. Further discussion of this is in section 3.2.

interest rate expectations in the US, Eurozone and Japan.[3] Here, I show that, by providing additional daily frequency information for the identification of interest rate expectations, OIS-augmentation can tackle the informational insufficiency at the heart of the DTSM identification problem, with sizeable gains at daily frequencies.

To present the OIS-augmented DTSM, I derive expressions for OIS pricing factor loadings that account for the payoff structure of OIS contracts in a Gaussian affine framework. I estimate the OIS-augmented model using maximum likelihood via the Kalman filter with 3 to 24-month OIS rates and 3-month to 10-year US Treasury yields. The model provides estimates of interest rate expectations and term premia out to, at least, a 10-year horizon. The OIS pricing expressions are derived under the assumption that OIS rates *on average* reflect interest rate expectations, consistent with the empirical testing and results in Lloyd (2018). The Kalman filter maximum likelihood setup is well-suited to account for this.

This is not the first paper to propose a solution to the DTSM identification problem. Kim and Orphanides (2012) suggest augmenting DTSMs with survey expectations of future short-term interest rates for the same purpose. They document that, between 1990 and 2003, a Gaussian affine survey-augmented model produces sensible estimates of interest rate expectations. Guimarães (2014) shows that, relative to an unaugmented Gaussian affine DTSM, the survey-augmented model provides estimates of interest rate expectations that better correspond with survey expectations of future interest rates and delivers gains in the precision of interest rate expectation estimates. However, I show that estimated interest rate expectations from the OIS-augmented model more closely match market- and survey-based measures of interest expectations than estimates from a survey-augmented model for the 2002-2016 period.

Bauer et al. (2012) propose an alternative solution, focused on directly resolving the finite sample bias via bias-correction. They claim that their bias-correct estimates of interest rate expectations "are more plausible from a macro-finance perspective" (p. 454) than those from an unaugmented Gaussian affine DTSM. However, as Wright (2014) states, the fact bias-correction has notable effects on DTSM-estimated interest rate expectations is merely a *symptom* of the identification problem. Bias-correction does not directly address the identification problem at the heart of DTSM estimation: the informational insufficiency. Moreover, Wright (2014) argues that the bias-corrected estimates of interest rate expectations are "far too volatile" (p. 339). I find that interest rate expectation estimates from the OIS-augmented model more closely match other measures of interest expectations than bias-corrected estimates for the 2002-2016 period.

OIS-augmentation is closest in philosophy to survey-augmentation. Both use additional information to better identify interest rate expectations. However, OIS-augmentation differs in a number of important respects, helping to explain its superior performance. Primarily, although surveys help to address the informational insufficiency problem, they are ill-equipped for estimation of daily frequency expectations. Survey expectations of interest rates are only available at quarterly or monthly frequencies, at best. Thus, surveys are unlikely to provide sufficient information to accurately identify the daily frequency evolution of interest rate expectations.

[3] In the UK, 1 to 18-month OIS rates tend to accurately reflect interest rate expectations.

OIS rates offer significant advantages over survey expectations for DTSM estimation at daily frequencies. Most importantly, OIS rates are available at daily frequencies, so provide information at the same frequency at which interest rate expectations are estimated. Secondly, OIS contracts are traded instruments, so may better reflect financial market participants' expectations. Third, the information in survey expectations is limited in comparison to OIS rates. Survey expectations typically provide information about expected interest rates for a short time period in the future.[4] In contrast, there is a term structure of OIS rates containing information about interest rate expectations from now until a specified future date. The horizon of these OIS contracts corresponds exactly to the horizon of nominal government bond yields.

Away from the DTSM-literature, OIS rates are increasingly being used to infer investors' expectations of interest rates (e.g. Woodford, 2012). Lloyd (2018) formally studies the empirical performance of OIS rates as financial market-based measures of interest rate expectations from a global perspective.[5] The conclusions of Lloyd (2018)—that 1 to 24-month OIS contracts in the US, Eurozone and Japan (1 to 18-month in the UK) tend to accurately reflect interest rate expectations—imply that the OIS-augmented DTSM is applicable in other advanced economies.

In this paper, I document that an OIS-augmented Gaussian affine DTSM accurately captures investors' expectations of short-term interest rates out to a 10-year horizon. The in-sample estimates of interest rate expectations align closely with federal funds futures rates and survey expectations of interest rates at horizons and on dates where such a comparison is possible. Against these metrics, the OIS-augmented model is superior to three other Gaussian affine DTSM classes: (i) the unaugmented model, which only uses bond yield data to estimate both actual yields and interest rates expectations; (ii) the bias-corrected model (Bauer et al., 2012); and (iii) the survey-augmented model.[6] The OIS-augmented model is also best able to capture qualitative daily frequency movements in interest rate expectations implied by other financial market instruments. Moreover, unlike the other models, the interest rate expectation estimates from the OIS-augmented model obey the zero lower bound for the US, despite the fact that additional restrictions are not imposed to achieve this. This represents an important contribution in the light of recent computationally burdensome proposals for term structure modelling at the zero lower bound (e.g. Christensen and Rudebusch, 2013). In addition, estimates of interest rate expectations and term premia from the OIS-augmented model are more stable across sample periods—a desirable model feature for real-time policy analysis.

The remainder of this paper is structured as follows. Section 2 introduces OIS contracts. Section 3 describes the DTSM identification problem with reference to the unaugmented Gaussian affine model. Section 4 presents the OIS-augmented model. Section 5 documents the data and estimation methodology. Section 6 presents results. Section 7 concludes.

[4]For example, the US *Survey of Professional Forecasters* provides expectations of the average 3-month T-Bill rate during the current quarter, and the first, second, third and fourth quarters ahead.

[5]Joyce, Relleen, and Sorensen (2008) evaluate the performance of interest rate swaps for the UK only.

[6]For the most direct comparison with the OIS-augmented model, I estimate the survey-augmented model using the algorithm of Guimarães (2014) which uses the same Joslin et al. (2011) identification restrictions as the OIS-augmented model, as opposed to the Kim and Wright (2005) survey-augmented model that applies the Kim and Orphanides (2012) identification algorithm, first proposed in Kim and Orphanides (2005). Lloyd (2017b) shows that the OIS-augmented model outperforms estimates from Kim and Wright (2005).

2 Overnight Indexed Swaps

An overnight indexed swap (OIS) is an over-the-counter traded interest rate derivative with two participating agents who agree to exchange fixed and floating interest rate payments over a *notional* principal for the life of the contract. The floating leg of the contract is constructed by calculating the accrued interest payments from a strategy of investing the notional principal in the overnight reference rate—the effective federal funds rate in the US—and repeating this on an overnight basis, investing principal plus interest each time. The 'OIS rate' represents the fixed leg of the contract. For vanilla US OIS contracts with a maturity of one year or less, money is only exchanged at the contract's conclusion. Upon settlement, only the net cash flow is exchanged between the participants.[7] That is, if the accrued fixed interest rate payment exceeds the floating interest payment, the agent who took on the former payments must pay the other at settlement. Importantly, there is no exchange of principal at any time for OIS contracts of all maturities.

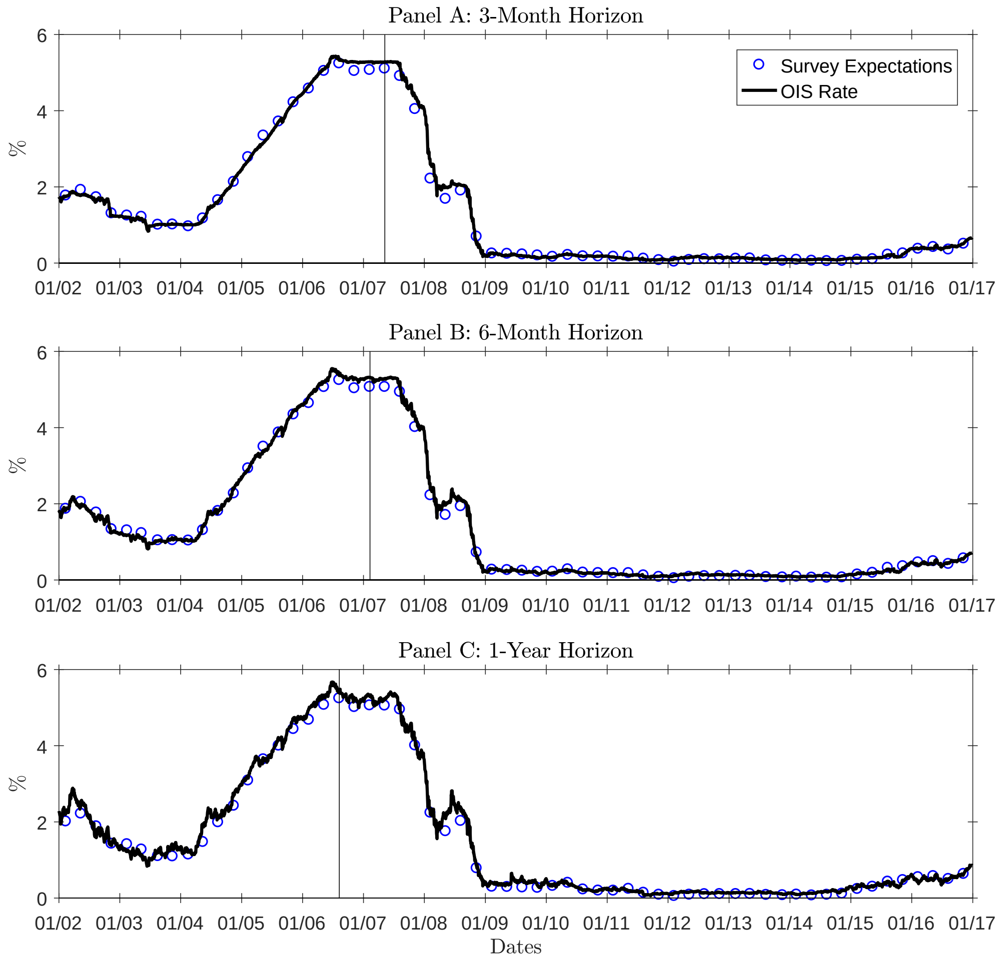
Given the features of an OIS contract, OIS rate changes can reasonably be associated with changes in investors' expectations of future overnight interest rates over the horizon of the contract (Michaud and Upper, 2008). Short-horizon OIS contracts should contain only very small excess returns, although longer-horizon OIS rates may contain term premia. Notably, because OIS contracts do not involve any exchange of principal, their associated counterparty risk is small. Because many OIS trades are collateralised, credit risk is also minimised (Tabb and Grundfest, 2013, pp. 244-245). Unlike many LIBOR-based instruments, OIS contracts have increased in popularity amongst investors following the 2007-2008 financial crisis (Cheng, Dorji, and Lantz, 2010).

The key assumption underlying the OIS-augmented decomposition presented in section 4 is that the OIS tenors used are accurate measures of interest rate expectations *on average*. Consistent with this, Lloyd (2018) assesses the properties of OIS excess returns and finds that, when accounting for the 2007-2008 money market turmoil and the US monetary policy of 2008 that was unanticipated *ex ante*, the average *ex post* excess returns on 1 to 24-month US OIS are statistically insignificant. Lloyd (2018) reaches similar conclusions for UK, euro area and Japanese OIS rates, supporting the global applicability of the proposal in this paper. Motivated by these results, only OIS rates with horizons of two years or less are used to augment DTSMs to inform estimation of expectations and term premia along the whole term structure.

To further illustrate that short-horizon OIS rates provide accurate information about expectations of future short-term interest rates, figure 1 plots daily 3, 6 and 12-month OIS rates between January 2002 and December 2016 against quarterly frequency survey expectations of the future short-term nominal interest rate over the corresponding horizon on survey submission dates.[8] Visual inspection further supports the key assumption underlying the OIS-augmented decomposition: short-horizon OIS rates co-move closely with corresponding-horizon survey expectations over the whole period. But, importantly, OIS rates are available at daily frequencies.

[7] For OIS contracts with maturity in excess of one year, net cash flows are exchanged at the end of every year.
[8] Appendix B describes how comparable-horizon survey expectation approximations are constructed.

Figure 1: US OIS Rates and Corresponding-Horizon Survey Expectations



Note: Daily OIS rates and quarterly survey expectations; January 2002 to December 2016. The survey expectation, at each horizon, is attained by constructing the geometric weighted average of the median response of forecasters relating to their expectation of the average 3-month T-Bill rate over the relevant periods (see appendix B). Survey expectations are plotted on the forecast submission deadline date for each quarter. See appendix A for detailed data source information. Vertical lines in each panel are plotted 3, 6 and 12 months prior to August 9, 2007 respectively, the date BNP Paribas froze funds citing US sub-prime mortgage sector problems.

3 Term Structure Model

This section presents a discrete-time Gaussian affine DTSM that is commonplace in the literature (e.g. Ang and Piazzesi, 2003) and describes the identification problem in unaugmented models with reference to model parameters. Since this paper is focused on the identification of interest rate expectations and term premia at daily frequencies, t is a *daily* time index.[9]

[9]The model can be estimated at lower frequencies, with the label for t changing correspondingly.

3.1 Unaugmented Model Specification

The discrete-time model has three key foundations. First, a $K \times 1$ vector of pricing factors $\mathbf{x}_t$ follows a first-order vector autoregressive process under the actual probability measure $\mathbb{P}$:

$$\mathbf{x}_{t+1} = \boldsymbol{\mu} + \boldsymbol{\Phi}\mathbf{x}_t + \boldsymbol{\Sigma}\boldsymbol{\varepsilon}_{t+1} \tag{1}$$

where $\boldsymbol{\varepsilon}_{t+1}$ is a stochastic disturbance with conditional distribution $\boldsymbol{\varepsilon}_{t+1}|\mathbf{x}_t \sim \mathcal{N}(\mathbf{0}_K, \mathbf{I}_K)$; $\mathbf{0}_K$ is a $K \times 1$ vector of zeros; and $\mathbf{I}_K$ is a $K \times K$ identity matrix. $\boldsymbol{\mu}$ is a $K \times 1$ vector and $\boldsymbol{\Phi}$ is a $K \times K$ matrix of parameters. $\boldsymbol{\Sigma}$ is a $K \times K$ lower triangular matrix.

Second, the one-period nominal interest rate i_t is an affine function of pricing factors:

$$i_t = \delta_0 + \boldsymbol{\delta}_1'\mathbf{x}_t \tag{2}$$

where δ_0 is a scalar and $\boldsymbol{\delta}_1$ is a $K \times 1$ vector of parameters.

Third, no-arbitrage is imposed. The pricing kernel M_{t+1} that prices all assets when there is no-arbitrage is of the following form:

$$M_{t+1} = \exp\left(-i_t - \frac{1}{2}\boldsymbol{\lambda}_t'\boldsymbol{\lambda}_t - \boldsymbol{\lambda}_t'\boldsymbol{\varepsilon}_{t+1}\right) \tag{3}$$

where $\boldsymbol{\lambda}_t$ represents a $K \times 1$ vector of time-varying market prices of risk, which are affine in the pricing factors, following Duffee (2002):

$$\boldsymbol{\lambda}_t = \boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1\mathbf{x}_t \tag{4}$$

where $\boldsymbol{\lambda}_0$ is a $K \times 1$ vector and $\boldsymbol{\Lambda}_1$ is a $K \times K$ matrix of parameters.

The assumption of no-arbitrage guarantees the existence of a risk-adjusted probability measure $\mathbb{Q}$, under which the bonds are priced (Harrison and Kreps, 1979).[10] Given the form of the market prices of risk in (4), the pricing factors $\mathbf{x_t}$ also follow a first-order vector autoregressive process under the risk-adjusted probability measure $\mathbb{Q}$:

$$\mathbf{x}_{t+1} = \boldsymbol{\mu}^{\mathbb{Q}} + \boldsymbol{\Phi}^{\mathbb{Q}}\mathbf{x}_t + \boldsymbol{\Sigma}\boldsymbol{\varepsilon}_{t+1}^{\mathbb{Q}} \tag{5}$$

where:[11]

$$\boldsymbol{\mu}^{\mathbb{Q}} = \boldsymbol{\mu} - \boldsymbol{\Sigma}\boldsymbol{\lambda}_0, \qquad \boldsymbol{\Phi}^{\mathbb{Q}} = \boldsymbol{\Phi} - \boldsymbol{\Sigma}\boldsymbol{\Lambda}_1.$$

and $\boldsymbol{\varepsilon}_{t+1}^{\mathbb{Q}}$ is a stochastic disturbance with the conditional distribution $\boldsymbol{\varepsilon}_{t+1}^{\mathbb{Q}}|\mathbf{x}_t \sim \mathcal{N}(\mathbf{0}_K, \mathbf{I}_K)$.

[10] The risk-adjusted probability measure $\mathbb{Q}$ is defined such that the price V_t of any asset that does not pay any dividends at time $t+1$ satisfies $V_t = \mathbb{E}_t^{\mathbb{Q}}[\exp(-i_t)V_{t+1}]$, where the expectation $\mathbb{E}_t^{\mathbb{Q}}$ is taken under the risk-adjusted probability measure $\mathbb{Q}$.

[11] See appendix C.2 for a formal derivation of these expressions.

Bond Pricing Since M_{t+1} is the nominal pricing kernel that prices all nominal assets in the economy, the gross one-period return R_{t+1} on any nominal asset must satisfy:

$$\mathbb{E}_t\left[M_{t+1}R_{t+1}\right] = 1 \tag{6}$$

Let $P_{t,n}$ denote the price of an n-day zero-coupon bond at time t. Then, using $R_{t+1} = P_{t+1,n-1}/P_{t,n}$, (6) implies that the bond price is recursively defined:

$$P_{t,n} = \mathbb{E}_t\left[M_{t+1}P_{t+1,n-1}\right] \tag{7}$$

Alternatively, with no-arbitrage, the price of an n-period zero-coupon bond must also satisfy the following relation under the risk-adjusted probability measure $\mathbb{Q}$:

$$P_{t,n} = \mathbb{E}_t^{\mathbb{Q}}\left[\exp(-i_t)P_{t+1,n-1}\right] \tag{8}$$

By combining the dynamics of the pricing factors (5) and the short-term interest rate (2) with (8), the bond prices can be shown to be exponentially affine function in the pricing factors:

$$P_{t,n} = \exp\left(\mathcal{A}_n + \mathcal{B}_n\mathbf{x}_t\right) \tag{9}$$

where the scalar $\mathcal{A}_n \equiv \mathcal{A}_n\left(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}, \boldsymbol{\Sigma}; \mathcal{A}_{n-1}, \mathcal{B}_{n-1}\right)$ and $\mathcal{B}_n \equiv \mathcal{B}_n\left(\boldsymbol{\delta}_1, \boldsymbol{\Phi}^{\mathbb{Q}}; \mathcal{B}_{n-1}\right)$, a $1 \times K$ vector, are recursively defined loadings:[12]

$$\begin{aligned}
\mathcal{A}_n &= -\delta_0 + \mathcal{A}_{n-1} + \frac{1}{2}\mathcal{B}_{n-1}\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\mathcal{B}_{n-1}' + \mathcal{B}_{n-1}\boldsymbol{\mu}^{\mathbb{Q}} \\
\mathcal{B}_n &= -\boldsymbol{\delta}_1' + \mathcal{B}_{n-1}\boldsymbol{\Phi}^{\mathbb{Q}}
\end{aligned}$$

with initial values $\mathcal{A}_0 = 0$ and $\mathcal{B}_0 = \mathbf{0}_K'$ ensuring that the price of a 'zero-period' bond is one.

The continuously compounded yield on an n-day zero-coupon bond at time t, $y_{t,n} = -\frac{1}{n}\ln\left(P_{t,n}\right)$, is given by:

$$y_{t,n} = A_n + B_n\mathbf{x}_t \tag{10}$$

where $A_n \equiv -\frac{1}{n}\mathcal{A}_n\left(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}, \boldsymbol{\Sigma}; \mathcal{A}_{n-1}, \mathcal{B}_{n-1}\right)$ and $B_n \equiv -\frac{1}{n}\mathcal{B}_n\left(\boldsymbol{\delta}_1, \boldsymbol{\Phi}^{\mathbb{Q}}; \mathcal{B}_{n-1}\right)$.

The risk-neutral yield on an n-day bond reflects the expectation of the average short-term interest rate over the n-day life of the bond, corresponding to the yields that would prevail if investors were risk-neutral.[13] That is, the yields that would arise under the expectations hypothesis of the yield curve. The risk-neutral yields can be calculated using:

$$\widetilde{y}_{t,n} = \widetilde{A}_n + \widetilde{B}_n\mathbf{x}_t \tag{11}$$

[12]See appendix C.1 for a formal derivation of these expressions.

[13]There is a small difference between risk-neutral yields and expected yields due to a convexity effect. In the homoskedastic model considered here, these effects are constant for each maturity and, in practice, small, corresponding to the $\frac{1}{2}\mathcal{B}_{n-1}\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\mathcal{B}_{n-1}'$ term in the recursive expression for $\mathcal{B}_n$ above.

where $\widetilde{A}_n \equiv -\frac{1}{n}\mathcal{A}_n\left(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}, \boldsymbol{\Phi}, \boldsymbol{\Sigma}; \mathcal{A}_{n-1}, \mathcal{B}_{n-1}\right)$ and $\widetilde{B}_n \equiv -\frac{1}{n}\mathcal{B}_n\left(\boldsymbol{\delta}_1, \boldsymbol{\Phi}; \mathcal{B}_{n-1}\right)$.[14] Note that, the risk-neutral yields are attained, *inter alia*, using parameters specific to the actual probability measure $\mathbb{P}$, $\{\boldsymbol{\mu}, \boldsymbol{\Phi}\}$. But, because no-arbitrage is assumed, the bonds are priced under the risk-adjusted measure $\mathbb{Q}$, so the fitted yields are attained, *inter alia*, by using parameters specific to the risk-adjusted probability measure $\mathbb{Q}$, $\left\{\boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}\right\}$.

The n-day term premium is the difference between (10) and (11):

$$tp_{t,n} = y_{t,n} - \widetilde{y}_{t,n} \tag{12}$$

3.2 Unaugmented DTSMs and the Identification Problem

Numerous studies have documented problems with separately identifying expectations of future short-term interest rates (risk-neutral yields) from term premia (e.g. Bauer et al., 2012; Kim and Orphanides, 2012; Guimarães, 2014). The underlying source of difficulty is an informational insufficiency, which gives rise to finite sample bias.

The unaugmented Gaussian affine model uses zero-coupon bond yield data as its sole input. This data provides a complete set of information about the dynamic evolution of the cross-section of yields — the yield curve. This provides sufficient information to accurately identify the risk-adjusted $\mathbb{Q}$ dynamics — specifically, the parameters $\left\{\boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}\right\}$ in (5) — which (10) shows are of direct relevance to estimating *actual* yields. However, if there is no additional information and the sample of yields contains too few interest rate cycles,[15] this data is not sufficient for the identification of the actual $\mathbb{P}$ dynamics — specifically, the parameters $\{\boldsymbol{\mu}, \boldsymbol{\Phi}\}$ in (1) — which (11) illustrates are of relevance to the estimation of *risk-neutral* yields.[16] Estimates of $\boldsymbol{\Phi}$ for the autoregressive process in (1) will suffer from finite sample bias. In particular, the persistent yields will have persistent pricing factors, so maximum likelihood or ordinary least squares estimates of the persistence parameters of the vector autoregressive process in (1) $\boldsymbol{\Phi}$ will be biased downwards.[17] That is, the estimated $\widehat{\boldsymbol{\Phi}}$ will understate the true persistence of the pricing factors, implying a spuriously fast mean reversion of future short-term interest rates. Because, in the model, agents form expectations of future short-term interest rates based on estimates of pricing factor mean reversion in $\widehat{\boldsymbol{\Phi}}$, their estimates of the future short-term interest rate path will mean revert spuriously quickly too. Consequently, the estimated risk-neutral yields, which summarise the average of the expected path of future short-term interest rates, will vary little and will not accurately reflect the evolution of interest rate expectations.

The magnitude of the finite sample bias is increasing in the persistence of the data. For daily frequency yield data, which is highly persistent, the bias will be more severe. This not only motivates the augmentation of DTSMs with additional data, but motivates the use of additional *daily frequency* data, namely OIS rates.

[14]See appendix C.3 for a formal derivation of these expressions.

[15]Kim and Orphanides (2012, p. 242) state that 5 to 15-year samples may contain too few interest rate cycles.

[16]Note that because $\boldsymbol{\mu} = \boldsymbol{\mu}^{\mathbb{Q}} + \boldsymbol{\Sigma}\boldsymbol{\lambda}_0$ and $\boldsymbol{\Phi} = \boldsymbol{\Phi}^{\mathbb{Q}} + \boldsymbol{\Sigma}\boldsymbol{\Lambda}_1$, estimates of the time-varying market prices of risk, $\boldsymbol{\lambda}_0$ and $\boldsymbol{\Lambda}_1$, are required to estimate $\{\boldsymbol{\mu}, \boldsymbol{\Phi}\}$ and the risk-neutral yields.

[17]This is a multivariate generalisation of the downward bias in the estimation of autoregressive parameters by ordinary least squares in the univariate case.

4 The OIS-Augmented Model

I estimate the OIS-augmented model using Kalman filter-based maximum likelihood. The Kalman filtering approach is particularly convenient for the augmentation of Gaussian affine DTSMs, as it can handle mixed-frequency data. Specifically, for OIS-augmentation, this allows model estimation for periods extending beyond that for which OIS rates are available.[18]

To implement the Kalman filter-based estimation, I use (1), the vector autoregression for the latent pricing factors under the actual probability measure $\mathbb{P}$, as the transition equation.

The observation equation depends on whether OIS rates are observed on day t. On days when the OIS rates are *not* observed (i.e. days prior to January 2002), the observation equation is formed by stacking the N yield maturities in (10) to form:

$$\mathbf{y}_t = \mathbf{A} + \mathbf{B}\mathbf{x}_t + \mathbf{\Sigma}_Y \mathbf{u}_t \tag{13}$$

where: $\mathbf{y}_t = [y_{t,n_1}, ..., y_{t,n_N}]'$ is the $N \times 1$ vector of bond yields; $\mathbf{A} = [A_{n_1}, ..., A_{n_N}]'$ is an $N \times 1$ vector and $\mathbf{B} = \left[B'_{n_1}, ..., B'_{n_N}\right]'$ is an $N \times K$ matrix of bond-specific loadings; $A_{n_\iota} = -\frac{1}{n_\iota}\mathcal{A}_{n_\iota}\left(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}^{\mathbb{Q}}, \mathbf{\Phi}^{\mathbb{Q}}, \mathbf{\Sigma}; \mathcal{A}_{n_{\iota-1}}, \mathcal{B}_{n_{\iota-1}}\right)$ and $B_{n_\iota} = -\frac{1}{n_\iota}\mathcal{B}_{n_\iota}\left(\boldsymbol{\delta}_1, \mathbf{\Phi}^{\mathbb{Q}}; \mathcal{B}_{n_{\iota-1}}\right)$ are the bond-specific loadings; and $\iota = 1, 2..., N$ such that n_ι denotes the maturity of bond ι in days. The $N \times 1$ vector $\mathbf{u}_t \sim \mathcal{N}\left(\mathbf{0}_N, \mathbf{I}_N\right)$ denotes the yield measurement error, where $\mathbf{0}_N$ is an N-vector of zeros and $\mathbf{I}_N$ is an $N \times N$ identity matrix. Here, for sake of exposition, I impose a homoskedastic form for the yield measurement error, such that $\mathbf{\Sigma}_Y$ is an $N \times N$ diagonal matrix with common diagonal element σ_e, the standard deviation of the yield measurement error. The homoskedastic error is characterised by a single parameter σ_e, maintaining computational feasibility for an already high-dimensional maximum likelihood routine.

On days when OIS rates are observed, the Kalman filter observation equation is augmented with OIS rates. The following proposition illustrates that OIS rates can (approximately) be written as an affine function of the pricing factors with loadings A_j^{ois} and B_j^{ois} for J different OIS maturities, where $j = j_1, j_2, ..., j_J$ denote the J OIS horizons in days. The loadings are calculated by assuming that the OIS tenors included in the model reflect interest rate expectations *on average* with some measurement error, an assumption discussed in Lloyd (2018) and section 2. Moreover, the loadings explicitly account for the geometric payoff structure of an OIS contract. This is an important technical difference between OIS and survey-augmented models.

Proposition The j-day OIS rate on date t $i_{t,t+j}^{ois}$, where $j = j_1, j_2, ...j_J$, can be (approximately) written as an affine function of the pricing factors $\mathbf{x}_t$:

$$i_{t,t+j}^{ois} = A_j^{ois} + B_j^{ois}\mathbf{x}_t \tag{14}$$

[18] This paper uses daily US OIS rates from 2002—the first date for which these rates are consistently available at all the relevant tenors on Bloomberg—to directly isolate the effect of OIS rates. However, given the Kalman filter method, the model can be estimated over longer periods.

where $A_j^{ois} \equiv \frac{1}{j}\mathcal{A}_j^{ois}\left(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}, \boldsymbol{\Phi}, \boldsymbol{\Sigma}; \mathcal{A}_{j-1}^{ois}, \mathcal{B}_{j-1}^{ois}\right)$ and $B_j^{ois} \equiv \frac{1}{j}\mathcal{B}_j^{ois}\left(\boldsymbol{\delta}_1, \boldsymbol{\Phi}; \mathcal{B}_{j-1}^{ois}\right)$ are:

$$\begin{aligned} \mathcal{A}_j^{ois} &= \delta_0 + \boldsymbol{\delta}_1'\boldsymbol{\mu} + \mathcal{A}_{j-1}^{ois} + \mathcal{B}_{j-1}^{ois}\boldsymbol{\mu} \\ \mathcal{B}_j^{ois} &= \boldsymbol{\delta}_1'\boldsymbol{\Phi} + \mathcal{B}_{j-1}^{ois}\boldsymbol{\Phi} \end{aligned}$$

where $\mathcal{A}_0^{ois} = 0$ and $\mathcal{B}_0^{ois} = \mathbf{0}_K'$, where $\mathbf{0}_K$ is a $K \times 1$ vector of zeros.

Proof: See appendix D.

Given this, the Kalman filter observation equation on the days OIS rates are observed is:

$$\begin{bmatrix} \mathbf{y}_t \\ \mathbf{i}_t^{ois} \end{bmatrix} = \begin{bmatrix} \mathbf{A} \\ \mathbf{A}^{ois} \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{B}^{ois} \end{bmatrix}\mathbf{x}_t + \begin{bmatrix} \boldsymbol{\Sigma}_Y & \mathbf{0}_{N\times J} \\ \mathbf{0}_{J\times N} & \boldsymbol{\Sigma}_O \end{bmatrix}\begin{bmatrix} \mathbf{u}_t \\ \mathbf{u}_t^{ois} \end{bmatrix} \tag{15}$$

where, in addition to the definitions of $\mathbf{y}_t$, $\mathbf{A}$, $\mathbf{B}$, $\boldsymbol{\Sigma}_Y$ and $\mathbf{u}_t$ above, $\mathbf{i}_t^{ois} = \left[i_{t,j_1}^{ois}, ..., i_{t,j_J}^{ois}\right]'$ is the $J \times 1$ vector of OIS rates; $\mathbf{A}^{ois} = \left[A_{j_1}^{ois}, ..., A_{j_J}^{ois}\right]'$ is a $J \times 1$ vector and $\mathbf{B}^{ois} = \left[B_{j_1}^{ois\prime}, ..., B_{j_J}^{ois\prime}\right]'$ is a $J \times K$ matrix of OIS-specific loadings; $\mathbf{0}_{N\times J}$ and $\mathbf{0}_{J\times N}$ denote $N \times J$ and $J \times N$ matrices of zeros respectively; and $\mathbf{u}_t^{ois} \sim \mathcal{N}(\mathbf{0}_J, \mathbf{I}_J)$ denotes the OIS measurement error, where $\mathbf{0}_J$ is an J-vector of zeros and $\mathbf{I}_J$ is an $J \times J$ identity matrix. The inclusion of the measurement error permits non-zero OIS forecast errors, imposing that the forecast error is zero *on average*. I compared two parameterisations of $\boldsymbol{\Sigma}_O$; a homoskedastic model, with common diagonal elements in $\boldsymbol{\Sigma}_O$, and a heteroskedastic model, with distinct diagonal elements. A likelihood ratio test of the two did not reject the null hypothesis that all diagonal elements are equal, so I impose a homoskedastic form for the OIS measurement error such that $\boldsymbol{\Sigma}_O$ has common diagonal element σ_o, the standard deviation of the OIS measurement error, and zero elsewhere. The homoskedastic OIS measurement errors also provide computational benefits, as there are fewer parameters to estimate than if a more general covariance structure was permitted.[19]

5 Methodology

To compare the OIS-augmented model with the existing literature, I estimate the following Gaussian affine DTSMs: (i) an unaugmented OLS/ML model, estimated using the Joslin et al. (2011) identification scheme, where K portfolios of yields are observed without error and are measured with the first K principal components of bond yields; (ii) the Bauer et al. (2012) bias-corrected model; (iii) a survey-augmented model, using survey expectations of future interest rates for the subsequent four quarters as an additional input, estimated with the Kalman filter using the algorithm of Guimarães (2014) (see appendix E);[20] and (iv) the OIS-augmented model.

[19] Kim and Orphanides (2012) and Guimarães (2014) impose homoskedasticity on the survey measurement errors in their Kalman filter setup too.

[20] For direct comparison to the OIS-augmented model, I estimate the survey-augmented model with the Guimarães (2014) algorithm, which uses the same Joslin et al. (2011) identification scheme. Kim and Orphanides (2012) implement a different identification scheme in the estimation of their survey-augmented model. Like Guimarães (2014), I use survey expectations from the US *Survey of Professional Forecasters* for the 3-month T-Bill rate for the remainder of the current quarter and the first, second, third and fourth quarters ahead.

5.1 Data

In all models, I use the following bond yields $\mathbf{y}_t$: 3 and 6 months, 1 year, 18 months, 2 years, 30 months, 3 years, 42 months, 4 years, 54 months, 5, 7 and 10 years.[21] For the 3 and 6-month yields, I use US T-Bill rates in accordance with much of the existing dynamic term structure literature.[22] The remaining rates are from the continuously compounded zero-coupon yields of Gürkaynak, Sack, and Wright (2007b). This data is constructed from daily-frequency fitted Nelson-Siegel-Svensson yield curves. Using the parameters of these curves, which are published along with the estimated zero-coupon yield curve, I back out the cross-section of yields for the 11 maturities from 1 to 10 years.

I use combinations of 3, 6, 12 and 24-month OIS rates in the OIS-augmented models. The choice of these maturities is motivated by evidence in section 2 and Lloyd (2018). I estimate three variants of the OIS-augmented model. The first, baseline setup, includes the 3, 6, 12 and 24-month OIS rates (4-OIS-Augmented model). The second and third models include the 3, 6 and 12-month (3-OIS-Augmented model) *and* 3 and 6-month (2-OIS-Augmented model) tenors respectively. Of the three OIS-augmented models, I find that the 4-OIS-Augmented model provides risk-neutral yields that best fit the evolution of interest rate expectations. The 2-OIS-Augmented model performs least well and may do so because the 3 and 6-month OIS rates that augment the model add little information on interest rate expectations over-and-above the 3 and 6-month T-Bill rates. The 3 and 4-OIS models benefit from longer-maturity OIS rates.

Since US OIS rates are consistently available from January 2002, the sample runs from January 2002 to December 2016 to isolate the effect of OIS augmentation.

In accordance with the evidence of Litterman and Scheinkman (1991)—that the first three principal components of bond yields explain well over 95% of their variation—I estimate the models with three pricing factors ($K = 3$).[23] By using the three-factor specification, for which the pricing factors have a well-understood economic meaning (the level, slope and curvature of the yield curve respectively), I am able to isolate and explain the economic mechanisms through which the OIS-augmented model provides superior estimates of expectations of future short-term interest rates *vis-à-vis* the unaugmented, bias-corrected and survey-augmented models.

5.2 Estimation

The OIS-augmented model relies on Kalman filter-based maximum likelihood estimation, for which the pricing factors $\mathbf{x}_t$ are latent. Normalisation restrictions must be imposed on the parameters to achieve identification. For this, I use to the Joslin et al. (2011) scheme, which allows "computationally efficient estimation of G[aussian affine] DTSMs" (Joslin et al., 2011, p. 928) and fosters faster convergence to the global optimum of the model's likelihood function

[21] These maturities are also used by Adrian, Crump, and Moench (2013).

[22] The T-Bill rates are converted from their discount basis to the yield basis, and are preferred to short-horizon zero-coupon yields because of fitting errors at the short-end of the yield curve.

[23] I also estimate a four-factor specification in the light of evidence by Cochrane and Piazzesi (2005, 2008) and Duffee (2011) who argue that more than three factors are necessary to explain the evolution of nominal Treasury yields. These results are shown in appendix F.2.

than other normalisation schemes (e.g. Dai and Singleton, 2000).[24] This permits a two-stage approach to estimating the OIS-augmented model.

To benefit fully from the computational efficiency of the Joslin et al. (2011) normalisation scheme, I first estimate the unaugmented model (hereafter, the OLS/ML model), presented in section 3.1, assuming that K portfolios of yields are priced without error, to attain initial values for the Kalman filter used in the second estimation stage. In particular, these K yield 'portfolios', $\mathbf{x}_t$, correspond to the first K estimated principal components of the bond yields. Under the Joslin et al. (2011) normalisation, this itself enables a two sub-stage estimation: first the $\mathbb{P}$ parameters are estimated by OLS on equation (1) using the K estimated principal components in the vector $\mathbf{x}_t$; second the $\mathbb{Q}$ parameters are estimated by maximum likelihood (see appendix E).

Having attained these OLS/ML parameter estimates, I estimate the OIS-augmented model—which assumes all yields are observed with error—using the OLS/ML parameter estimates as initial values for the Kalman filter-based maximum likelihood routine.

6 Decomposition Results

6.1 Model Fit

This sub-section discusses four aspects of model fit: estimated bond yields, OIS rates, pricing factors and parameters.

Fitted Bond Yields Importantly, OIS-augmentation does not compromise the overall model fit with respect to actual bond yields. Table 1 presents the root mean square error (RMSE) for fitted yields. The fit is strikingly similar across all six models. The average RMSE for each of the models at all thirteen maturities is no more than 11 basis points, and differences for a given maturity are negligible.

This is intuitive. I augment the model with OIS rates to provide additional information with which to better estimate parameters under the actual probability measure $\mathbb{P}$ $\{\boldsymbol{\mu}, \boldsymbol{\Phi}\}$, which directly influence estimates of the risk-neutral yields. Estimates of the fitted yield depend upon the risk-adjusted measure $\mathbb{Q}$ parameters $\{\boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}\}$, which are not directly influenced by the OIS rates in the model, and are well-identified with bond yield data that provide information on the dynamic evolution of the cross-section of yields.

Fitted OIS Rates The OIS-augmented models also provide fitted values for OIS rates. Figure 2 plots the 3, 6, 12 and 24-month OIS rates alongside fitted-OIS rates from these models. It illustrates that the OIS-augmented models provide broadly accurate estimates of OIS rates.

[24]The computational benefits of the Joslin et al. (2011) normalisation scheme arise because it only imposes restrictions on the short-term interest rate i_t and the factors $\mathbf{x}_t$ under the $\mathbb{Q}$ probability measure. Consequently, the $\mathbb{P}$ and $\mathbb{Q}$ dynamics of the model do not exhibit strong dependence. Under the Dai and Singleton (2000) scheme, restrictions on the volatility matrix $\boldsymbol{\Sigma}$, which influences both the $\mathbb{P}$ and $\mathbb{Q}$ evolution of the factors (see equations (1) and (5)), create a strong dependence between the parameters under the two probability measures, engendering greater computational complexity in the estimation.

Table 1: Model Fit: Root Mean Square Error (RMSE) of the Fitted Yields *vis-à-vis* Actual Yields

Sample: January 2002 to December 2016						
Maturity	OLS/ML	BC	Survey	2-OIS	3-OIS	4-OIS
3-Months	0.0979	0.0983	0.1028	0.1000	0.1091	0.1076
6-Months	0.0516	0.0513	0.0530	0.0524	0.0489	0.0534
1-Year	0.0714	0.0717	0.0775	0.0774	0.0777	0.0739
18-Months	0.0567	0.0564	0.0591	0.0598	0.0598	0.0605
2-Years	0.0403	0.0396	0.0395	0.0404	0.0396	0.0435
30-Months	0.0240	0.0234	0.0228	0.0237	0.0222	0.0265
3-Years	0.0161	0.0159	0.0181	0.0182	0.0177	0.0179
42-Months	0.0223	0.0223	0.0256	0.0249	0.0262	0.0237
4-Years	0.0313	0.0311	0.0339	0.0328	0.0349	0.0330
54-Months	0.0378	0.0374	0.0393	0.0380	0.0405	0.0401
5-Years	0.0410	0.0403	0.0414	0.0400	0.0425	0.0440
7-Years	0.0273	0.0263	0.0267	0.0265	0.0249	0.0333
10-Years	0.0638	0.0629	0.0609	0.0559	0.0608	0.0558
Average	0.0499	0.0497	0.0517	0.0507	0.0526	0.0525

Note: RMSE of the fitted yields from each of the six three-factor Gaussian affine DTSMs, computed by comparing the model-implied fitted yield to the actual yield on each day. All figures are expressed in annualised percentage points. The six Gaussian affine DTSMs are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (BC); (iii) the survey-augmented model (Survey); (iv) the 2-OIS-augmented model (2-OIS); (v) the 3-OIS-augmented model (3-OIS); and (vi) the 4-OIS-augmented model (4-OIS).

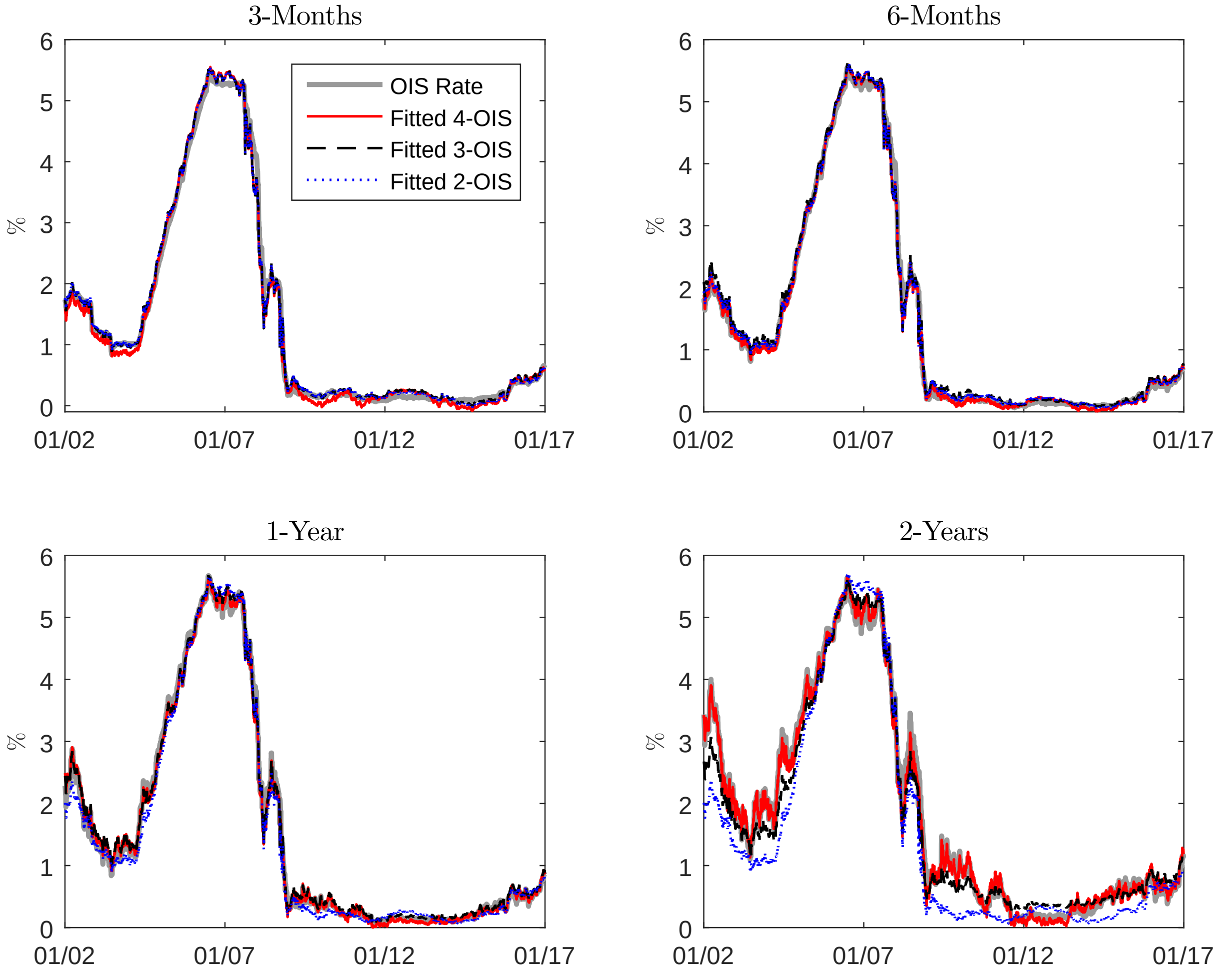
The 4-OIS-augmented model provides the best fit for the 6, 12 and 24-month OIS rates, while the 2-OIS-augmented model best fits the 3-month OIS rate. Although the differences between the OIS-augmented models at the 3-month horizon are marginal, the 4-OIS-augmented model fits the 24-month OIS rate substantially better than the 3 and 2-OIS-augmented models.[25] This is unsurprising, as this OIS tenor is observed in the 4-OIS-augmented model. The 2-OIS-augmented model fits the 1 and 2-year OIS rates least well. This is unsurprising, as it uses the fewest OIS rates as observable inputs.

Pricing Factors Of additional interest for the OIS-augmented model is whether the inclusion of OIS rates affects the model's pricing factors $\mathbf{x}_t$. To investigate this, I compare the estimated principal components of the bond yields—used as pricing factors in the OLS/ML model—to the estimated model-implied pricing factors from Kalman filter estimation of the OIS-augmented models. Figure 3 plots the time series of the first three principal components, estimated from the panel of bond yields, and the estimated pricing factors from the 4-OIS-augmented model. For all three factors, the Kalman filter-implied pricing factors are nearly identical to the estimated principal components.[26] This implies that OIS rates do not include any additional information,

[25] Table 6, in appendix F.1.1, provides detailed numerical evidence.

[26] Table 7, in appendix F.1.2, demonstrates that the summary statistics of the estimated principal components and pricing factors are very similar too.

Figure 2: Fitted OIS Rates from the OIS-Augmented Models



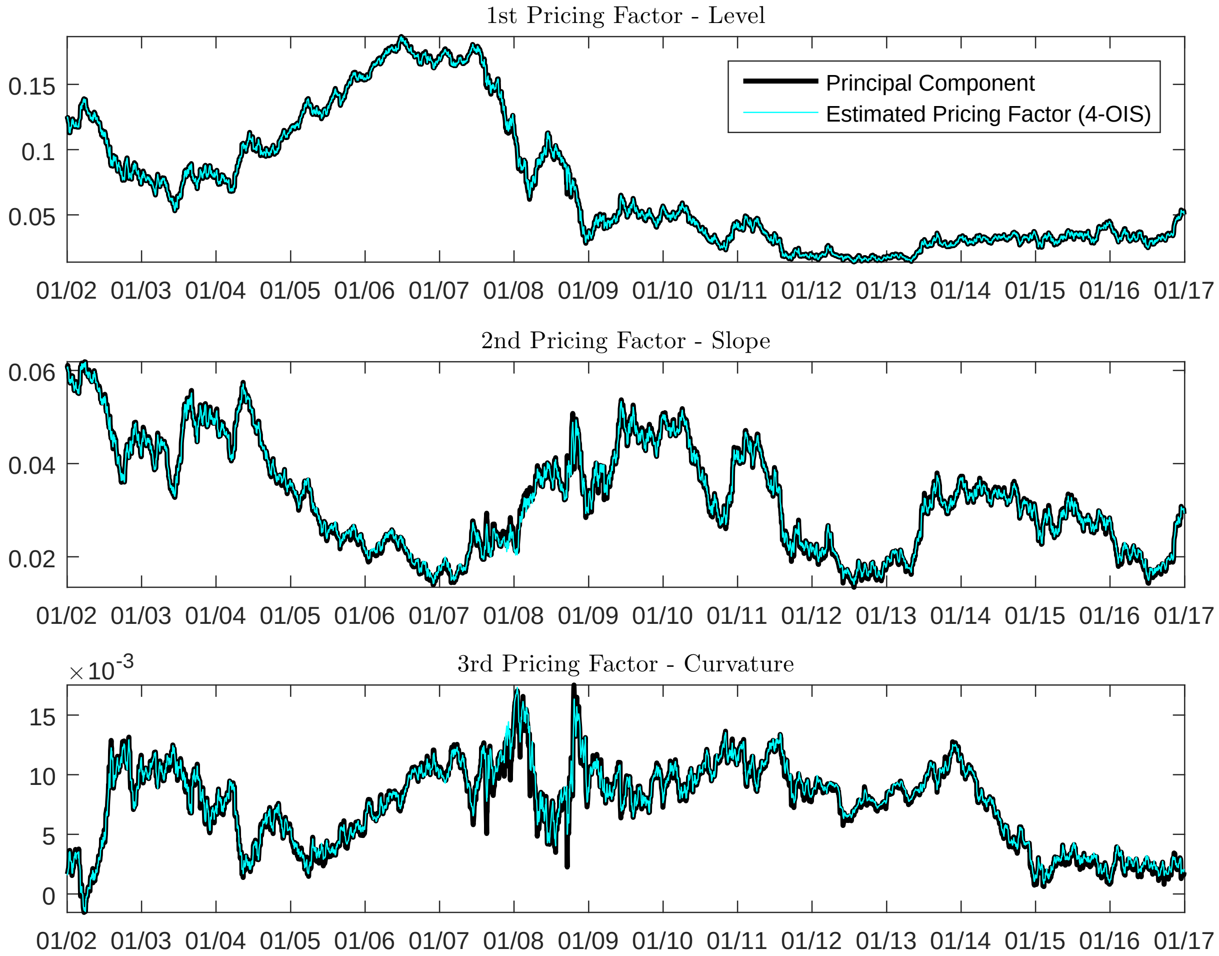
Note: Fitted and actual 3, 6, 12 and 24-month OIS rates. Fitted OIS rates are from the 4, 3 and 2-OIS-augmented Gaussian affine DTSMs. The models are estimated with three pricing factors using daily data from January 2002 to December 2016. All figures are in annualised percentage points.

over and above that in bond yields, of value in fitting the actual yields. This, again, is intuitive: OIS rates are included in the model to provide information useful for the identification of the risk-neutral yields, not the fitted yields.

Parameter Estimates Recall, from section 3.2, that informational insufficiency in DTSMs gives rise to finite sample bias. Persistent yields will have persistent pricing factors, resulting in estimates of the persistence parameters $\widehat{\mathbf{\Phi}}$ that are biased downwards. Following Bauer et al. (2012), I numerically assess the extent to which OIS-augmentation reduces finite sample bias by reporting the maximum eigenvalues of the estimated persistence parameters $\widehat{\mathbf{\Phi}}$. The higher the maximum eigenvalue, the more persistent the estimated process.

As a benchmark, the maximum absolute eigenvalue of $\widehat{\mathbf{\Phi}}$ for the unaugmented OLS/ML model is 0.9985. The maximum absolute eigenvalue of $\widehat{\mathbf{\Phi}}$ for the survey-augmented model is 0.9983. However, for the 4-OIS-augmented model, the corresponding figure is 0.9988, indicat-

Figure 3: Estimated Principal Components of the Actual Bond Yields and Estimated Pricing Factors from the 4-OIS-Augmented Model



Note: Estimated principal components from the actual bond yield data with the following maturities: 3, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 84 and 120 months. Estimated pricing factors from the three-factor 4-OIS-augmented model, implied by the Kalman filter.

ing that, in comparison to the unaugmented model, augmentation with OIS rates does serve to mitigate finite sample bias.[27] This indicates that OIS-augmentation does help to resolve the informational insufficiency in Gaussian affine DTSMs, and its associated symptoms. However, to assess this more thoroughly, a comparison of model-implied interest rate expectations is necessary. A well-identified model should accurately reflect the evolution of interest rate expectations.

6.2 Model-Implied Interest Rate Expectations

The central focus of this paper is the identification and estimation of interest rate expectations within DTSMs. Panels A and B of figure 4 plot the 2-year risk-neutral yields and term premia

[27]The corresponding statistic for the bias-corrected model, which performs bias-correction directly on the estimated $\mathbf{\Phi}$, is 1.0000 (to four decimal places). However, the ‘true’ pricing factor persistence is unknown.

from Gaussian affine model estimated between January 2002 and December 2016, respectively.

Panel A illustrates the effect of OIS-augmentation on estimates of expected future short-term interest rates. Over the 2002-2016 sample, the five models exhibit similar qualitative patterns, rising to peaks and falling to troughs at similar times. However, there are a number of notable differences between the series that reflect the benefits of OIS-augmentation.

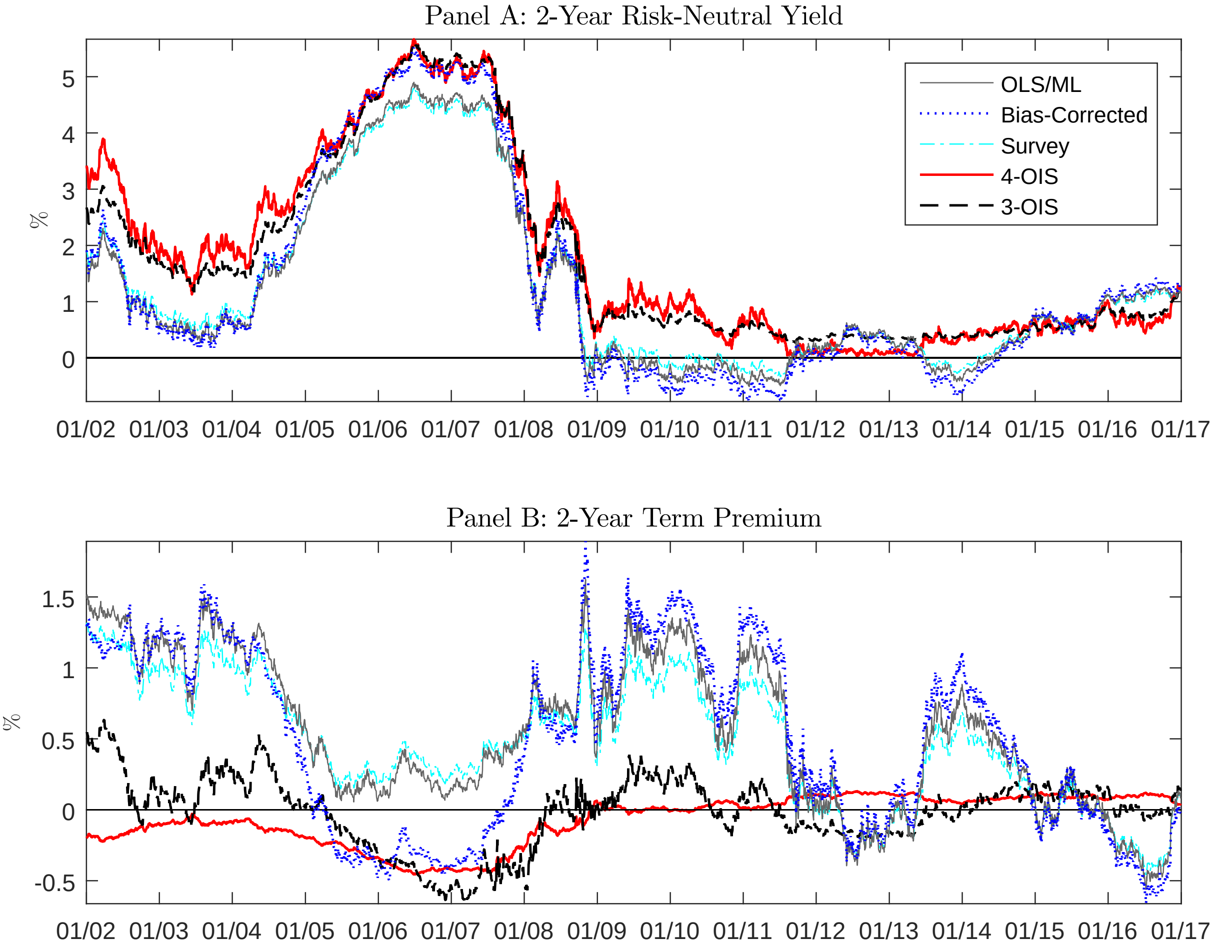
For the majority of the sample, the OIS-augmented models generate 2-year risk-neutral yields that exceed those from the OLS/ML and bias-corrected models.[28] Moreover, marked differences exist in the evolution of risk-neutral yield estimates from late-2008 onwards, with differing implications for the efficacy of monetary policy. First, from late-2008 to late-2011, the risk-neutral yields from the OLS/ML and bias-corrected models are persistently negative, implying counterfactual expectations of negative interest rates. In contrast, unlike the other models, the risk-neutral yields implied by the 3 and 4-OIS-augmented models obey a zero lower bound, with estimated interest rate expectations never falling negative, despite the fact that additional restrictions are not imposed to achieve this. This is true at all horizons, and indicates that the OIS-augmented model may be used to attain non-negative point estimates for interest rate expectations, without applying computationally burdensome proposals for term structure modelling at the lower bound (e.g. Christensen and Rudebusch, 2013).

Second, between mid-2011 and 2013, the 2-year risk neutral yields from the OLS/ML and bias-corrected models reach a peak, indicating an increase in expected future short-term interest rates at that horizon. In contrast, during the same period, the 2-year risk-neutral yield estimates from the 3-OIS-augmented model remain broadly stable, while the corresponding estimates from the 4-OIS-augmented model fall slightly. From August 2011, the Federal Reserve engaged in calendar-based forward guidance designed to influence investors' expectations of future short-term interest rates, signalling that interest rates would be kept at a low level for an extended period of time. For instance, on August 9, 2011, the Federal Open Market Committee (FOMC) stated that it expected to keep the federal funds rate near zero "at least through mid-2013." This, and other forward guidance statements, were effective at deferring investors' expectations of future rate rises between mid-2011 and 2013. Swanson and Williams (2014) show that private sector expectations of the time until a US rate rise, from Blue Chip surveys, jumped from between 2 and 5 quarters to 7 or more quarters. In this respect, the finding that expectations of future short-term interest rates increased during this period—implied by the OLS/ML and bias-corrected models—wrongly suggests that forward guidance policy was counterproductive. Unlike the OLS/ML and bias-corrected model, the OIS-augmented models imply that investors were expecting rate rises no sooner, and possibly slightly later, than they had in previous period. Subsequent quantitative analysis further demonstrates that the OIS-augmented models provide superior estimates of interest rate expectations during this period.

In figure 4, the estimated 2-year term premium from the 4-OIS-augmented model is per-

[28] Longer-horizon (i.e. 10-year) risk-neutral yields from the OIS-augmented models also exceed those from the OLS/ML and bias-corrected models. This is consistent with Meldrum and Roberts-Sklar (2015), who argue that unaugmented models provide "implausibly low estimates of long-term expected future short-term interest rates" (p. 1), "which in turn means that long-maturity term premium estimates are likely to be too high" (p. 3).

Figure 4: Estimated Yield Curve Decomposition



Note: Estimated risk-neutral yields (panel A) and term premia (panel B) from each of five Gaussian affine DTSMs, respectively. The five models are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 4-OIS-augmented model (4-OIS); and (v) the 3-OIS-augmented model (3-OIS). The models are estimated with three pricing factors, using daily data from January 2002 to December 2016. All figures are in annualised percentage points.

sistently negative from 2002 to 2008. This is a direct consequence of the accurate fitting of risk-neutral yields. However, this feature is not true for all maturities; the estimated term premia at longer-horizons are frequently and persistently positive. For instance, the 10-year term premium from the 4-OIS-augmented model peaks at 79 basis points in late-2008.

6.2.1 Risk-Neutral Yields and Federal Funds Futures

To quantitatively evaluate the estimated risk-neutral yields, I first compare them to expectations implied by 1 to 11-month federal funds futures contracts with matching horizon. Federal funds futures have long been used as measures of investors' expectations of future short-term interest rates (e.g. Gürkaynak, Sack, and Swanson, 2007a). To facilitate the comparison, I first calculate 1, 2, ..., 11-month risk-neutral yields using the estimated parameters from each model. I then

Table 2: Model-Implied Expectations: Root Mean Square Error (RMSE) of the Risk-Neutral 1-Month Forward Yields *vis-à-vis* Corresponding-Horizon Federal Funds Futures Rates

Sample: January 2002 to December 2016					
Horizon	OLS/ML	BC	Survey	3-OIS	4-OIS
0 to 1 Months	0.2230	0.2196	0.2025	**0.1649**	0.1823
1 to 2 Months	0.2134	0.2049	0.1802	**0.1201**	0.1293
2 to 3 Months	0.2307	0.2167	0.1842	0.1205	**0.0929**
3 to 4 Months	0.2678	0.2496	0.2147	0.1394	**0.0828**
4 to 5 Months	0.3099	0.2888	0.2534	0.1552	**0.0898**
5 to 6 Months	0.3573	0.3344	0.2981	0.1593	**0.1021**
6 to 7 Months	0.4103	0.3865	0.3480	0.1576	**0.1176**
7 to 8 Months	0.4648	0.4422	0.3983	0.1545	**0.1316**
8 to 9 Months	0.5243	0.5030	0.4528	0.1530	**0.1429**
9 to 10 Months	0.9773	0.9599	0.9200	0.6755	**0.6564**
10 to 11 Months	1.3392	1.3297	1.2776	0.9915	**0.9635**

Note: RMSE of the risk-neutral 1-month forward yields from each of the five Gaussian affine DTSMs in comparison to corresponding-horizon federal funds futures rates. The five models are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 3-OIS-augmented model (3-OIS); and (v) the 4-OIS-augmented model (4-OIS). The models are estimated with three pricing factors, using daily data from January 2002 to December 2016. The risk-neutral forward yields and the federal funds futures rates are compared on the final day of each calendar month. All figures are in annualised percentage points. The lowest RMSE model at each maturity has been emboldened for ease of reading.

calculate risk-neutral 1-month *forward* yields using the estimated risk-neutral yields.[29] Like federal funds futures contracts, the risk-neutral 1-month forward rates settle based on outcomes during a 1-month period in the future. However, because of the settlement structure of federal funds futures contracts, I only compare risk-neutral forward yields and federal funds futures rates on the final day of each calendar month.[30] I find that the risk-neutral forward yields from the OIS-augmented models most closely align with the expectations implied by federal funds futures rates with matching horizon, implying that they provide superior estimates of investors' expected future short-term interest rates compared to existing models.

Table 2 provides formal evidence to support this conclusion, presenting the RMSE of risk-neutral 1-month forward yields from different models and corresponding-horizon federal funds futures rates. On a RMSE basis, the OIS-augmented models unambiguously provide superior estimates of expected future short-term interest rates, as measured by federal funds futures rates, at every horizon. The 4-OIS-augmented model outperforms the 3-OIS-augmented model

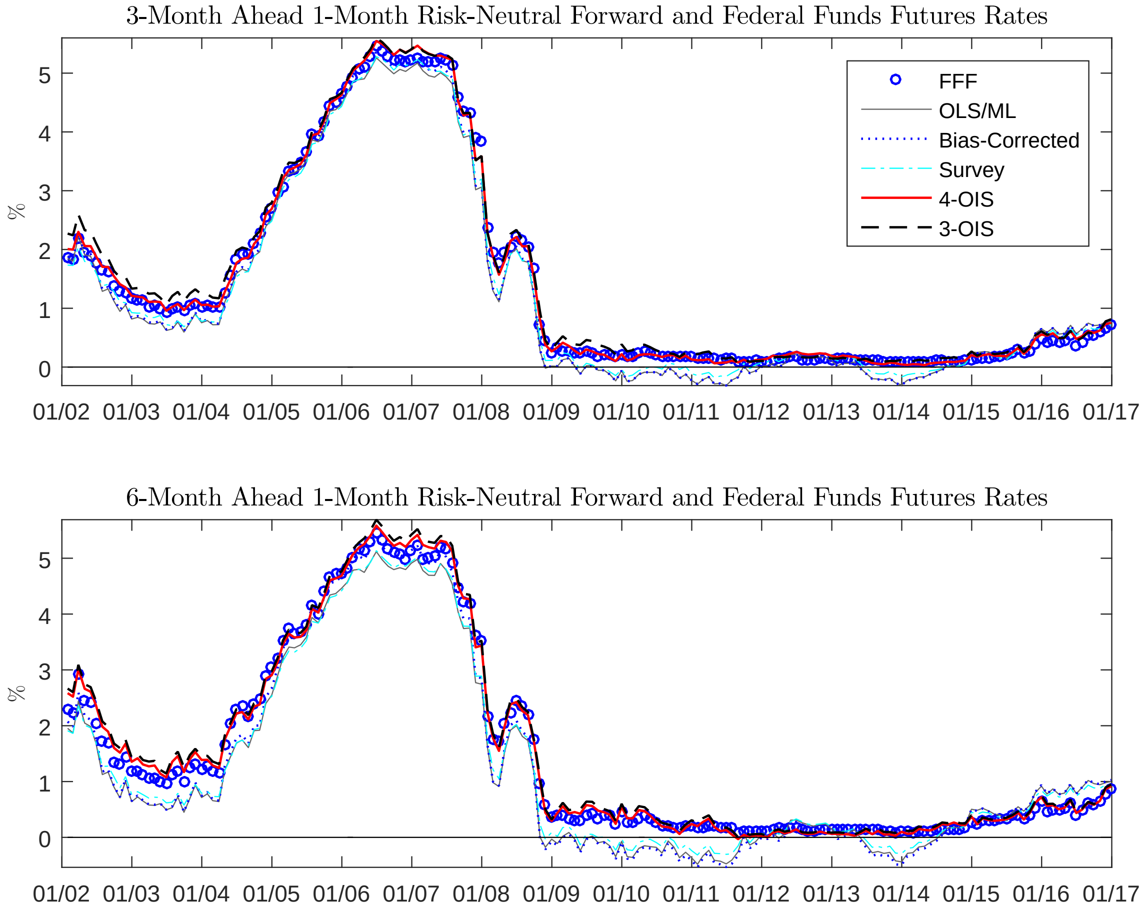
[29]To calculate the risk-neutral forward rate $\widetilde{f}_{t_1,t_2}$ from day t_1 to day t_2, I use:

$$(1+\widetilde{y}_2)^{d_2} = (1+\widetilde{y}_1)^{d_1}(1+\widetilde{f}_{t_1,t_2})^{d_2-d_1}$$

where $\widetilde{y}_1$ ($\widetilde{y}_2$) is the risk-neutral yield for the time period $(0,t_1)$ ($(0,t_2)$) and d_1 (d_2) is the length of time between time 0 and time t_1 (t_2) in years.

[30]See Lloyd (2018) for a detailed description of the settlement structure of federal funds futures contracts. The salient point is that an n-month federal funds futures contract traded on day t_j of the calendar month t has the same settlement period as an n-month contract traded on a different day t_k in the same calendar month t. For this reason, the horizon of a federal funds futures contract and the risk-neutral forward yield only align on the final calendar day of each month.

Figure 5: Estimated Risk-Neutral 1-Month Forward Yields and Comparable-Horizon Federal Funds Futures (FFF) Rates



Note: Estimated 3 month (top panel) and 6 month (bottom panel) ahead 1-month risk-neutral forward yields from each of five models. The five models are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 4-OIS-augmented model (4-OIS); and (v) the 3-OIS-augmented model (3-OIS). The models are estimated with three pricing factors, using daily data from January 2002 to December 2016. I compare the estimated risk-neutral forward yields to corresponding-horizon federal funds futures (FFF) rates, plotted on the final day of each calendar month. All figures are in annualised percentage points.

at all but two horizons. Even at extremely short horizons, the benefits of OIS-augmentation are large: the RMSE fit of the unaugmented OLS/ML and bias-corrected models at the 3 to 4 month horizon is over three times larger than that of the 4-OIS-augmented model.

Despite fitting federal funds futures-implied interest rate expectations worse than the OIS-augmented models, the survey-augmented model does perform better than the unaugmented OLS/ML and bias-corrected models. This supports the claim that, while survey-augmentation does help to reduce the informational insufficiency problem in Gaussian affine DTSMs, quarterly frequency survey expectations are not sufficient for the accurate identification of interest rate expectations at higher frequencies.

Figure 5 provides a visual comparison of the model-implied risk-neutral 1-month forward yields and federal funds futures rates at the 3 to 4 and 6 to 7 month horizons. The plot highlights two important determinants of the differences in fit highlighted by table 2.

First, in the pre-crisis period (to mid-2007), the OLS/ML and survey-augmented models generate risk-neutral forward yield estimates that persistently fall below the corresponding-horizon federal funds futures rates. The bias-corrected risk-neutral forward yields also fall below the corresponding-horizon federal funds futures rates until 2005. In contrast, the estimated risk-neutral forward yields from the OIS-augmented models align more closely with federal funds futures rates during this period, especially at the 3 to 4-month horizon.

Second, from late-2008 until late-2011, and from mid-2013 to mid-2014, the risk-neutral forward yields from the OLS/ML, bias-corrected, and survey-augmented models differ greatly from the corresponding-horizon federal funds futures rates. Moreover, these models offer counterfactual predictions for the evolution of interest rate expectations during this period. In particular, in these periods, the risk-neutral forward yields from the OLS/ML, bias-corrected and survey-augmented models are persistently negative. Moreover, from late-2011 to mid-2012, the risk-neutral forward yields from the OLS/ML and bias-corrected models rise to a peak. Not only is this contrary to the policy narrative at the time—policymakers were engaging in calendar-based forward guidance that sought to push back the date investors expected policy rates to lift-off from their lower bound—it is also counterfactual with respect to market-implied interest rate expectations. In contrast, the OIS-augmented models align closely with federal funds futures-implied expectations for most of the 2009-2016 period.

In sum, the comparison of risk-neutral forward yields and federal funds rates supports the claim that OIS-augmentation improves the identification of interest rate expectations over the whole sample.

6.2.2 Risk-Neutral Yields and Short-Horizon Survey Expectations

As further evidence in support of this claim, I compare the model-implied interest rate expectations to short-horizon survey expectations and, again, show that the OIS-augmented model performs the most accurate measures of interest rate expectations.

I use the same US SPF survey expectations data as in figure 1, and compare it to model-implied 1.5, 4.5, 7.5, 10.5 and 13.5-month risk-neutral yields. Appendix B describes the construction of corresponding-horizon survey expectations. Table 3 presents the RMSE comparison of model-implied risk-neutral yields and corresponding-horizon survey expectations on survey submission deadline dates.[31] On a RMSE basis, the OIS-augmented models unambiguously provide superior estimates of expected future short-term interest rates at each horizon.

At the 4.5, 7.5, 10.5 and 13.5-month horizons the 4-OIS-augmented model provides the superior fit of survey expectations. Strikingly, at the 10.5 and 13.5-month horizons, the RMSE fit of the OLS/ML and bias-corrected models are around three times the RMSE fit of the 4-OIS-augmented model. Although the 3-OIS-augmented model provides the lowest RMSE fit for the 1.5-month survey expectation, the RMSE fit of the 4-OIS-augmented model is only 0.85 basis points higher at this horizon. In contrast, at the 10.5-month horizon the RMSE fit of the

[31] Appendix F.1.3 presents supporting visual evidence.

Table 3: Model-Implied Expectations: Root Mean Square Error (RMSE) of the In-Sample Risk-Neutral Yields *vis-à-vis* 1.5, 4.5, 7.5, 10.5 and 13.5-Month Survey Expectations

	Sample: January 2002 to December 2016				
	RMSE vs. Survey Expectation at Different Horizons				
Model	1.5-Month	4.5-Month	7.5-Month	10.5-Month	13.5-Month
OLS/ML	0.1853	0.2005	0.2700	0.3639	0.4744
Bias-Corrected	0.1857	0.1979	0.2643	0.3570	0.4686
Survey	0.1749	0.1661	0.2216	0.3087	0.4135
3-OIS	**0.1642**	0.1437	0.1514	0.1545	0.1634
4-OIS	0.1727	**0.1354**	**0.1199**	**0.1311**	**0.1577**

Note: RMSE of the risk-neutral yields from each of the five Gaussian affine DTSMs in comparison to approximated *Survey of Professional Forecasters* survey expectations, using estimated risk-neutral yields on SPF deadline dates. The five models are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 3-OIS-augmented model (3-OIS); and (v) the 4-OIS-augmented model (4-OIS). The models are estimated with three pricing factors, using daily data from January 2002 to December 2016. The construction of the survey expectation approximations is described in appendix B. All figures are in annualised percentage points. The lowest RMSE model at each maturity has been emboldened for ease of reading.

4-OIS-augmented model is 2.34 basis points lower than the 3-OIS-augmented model.

Surprisingly, the survey-augmented model, which uses the same SPF survey expectations as an input to estimation, does not provide a superior fit for these expectations at any horizon *vis-à-vis* the OIS-augmented models. This supports the claim that quarterly frequency survey expectations are not sufficient for the accurate identification of higher frequency interest rate expectations within a Gaussian affine DTSM framework. Nevertheless, the RMSE fit of the survey-augmented model is superior to the fit of both the OLS/ML and bias-corrected models at all horizons, supporting the claim that augmentation of DTSMs with additional information can aid the identification of risk-neutral yields.

Overall, the results further support the claim that the OIS-augmentation of DTSMs improves the identification of interest rate expectations. At short horizons, OIS-augmented models provide superior estimates of investors' expectations of future short-term interest rates over the 2002-2016 sample.

6.2.3 Risk-Neutral Yields and Long-Horizon Survey Expectations

The expectational horizons considered in the previous sub-section are short-term. However, DTSMs provide estimates of risk-neutral yields for the whole term structure, at horizons further into the future. This is an important motive for using DTSMs to estimate interest rate expectations, instead of market-based financial measures which seldom provide accurate measures of investors' interest rate expectations at horizons in excess of 2 years (e.g. Lloyd, 2018).

Within a DTSM, the 10-year risk-neutral yield at date t provides an estimate for the expected average short-term interest rate for the 10-year period following date t. In general, survey data on these longer-term interest rate expectations are not readily available, making it difficult to systematically test the long-horizon interest rate expectations attained from DTSMs.

Table 4: Model-Implied Expectations: Root Mean Square Error (RMSE) of the In-Sample Risk-Neutral Yields *vis-à-vis* 10-Year Survey Expectation

Model	Sample: October 2013 to December 2016 RMSE vs. 10-Year Expectation, Survey of Primary Dealers
OLS/ML	1.5878
Bias-Corrected	1.8747
Survey	1.5309
3-OIS	0.7831
4-OIS	**0.7034**

Note: RMSE of the risk-neutral yields from each of the five Gaussian affine DTSMs in comparison to the 10-year survey expectation, using estimated risk-neutral yields on survey deadline dates. The five models are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 3-OIS-augmented model (3-OIS); and (v) the 4-OIS-augmented model (4-OIS). The models are estimated with three pricing factors, using daily data from January 2002 to December 2016. The median survey expectation is from the Survey of Primary Dealers, New York Federal Reserve. All figures are in annualised percentage points. The lowest RMSE model at each maturity has been emboldened for ease of reading.

However, in recent years, the New York Federal Reserve's *Survey of Primary Dealers* has asked respondents an increasing number of questions regarding their longer-horizon interest rate expectations.[32] Specifically, since October 2013, respondents have been asked to: "provide your estimate of the longer run target federal funds rate and your expectation for the average federal funds rate over the next 10 years".[33] The latter of these requests corresponds to the information contained within the 10-year risk-neutral yields attained from the models: the expectation of the average of the short-term interest rate over a 10-year horizon.

To quantitatively assess the model-implied longer-horizon interest rate expectations, table 4 presents a RMSE of the estimated 10-year risk-neutral yields and the median "expectations for the average federal funds rate over the next 10 years" of survey respondents on the survey deadline dates. Although the sample of long-horizon survey expectations is relatively short—including 26 surveys from October 2013 to December 2016—the results support the primary conclusion of this paper. The OIS-augmented models provide unambiguously superior estimates of future short-term interest rate expectations. The RMSE fit of the OLS/ML, bias-corrected and survey-augmented models are over double the RMSE fit of the 4-OIS-augmented model. Moreover, the RMSE fit of the 4-OIS-augmented is smaller than that of the 3-OIS-augmented model, indicating that longer-horizon OIS rates do help to improve the model's fit of interest rate expectations at longer tenors.

[32] The questions and results of these surveys are publicly available from: `www.newyorkfed.org/markets/primarydealer_survey_questions.html`.

[33] In the surveys, the question preceding this was: "provide your estimate of the most likely outcome (i.e., the mode) for the target federal funds rate or range at the end of each half-year period".

6.2.4 Daily Changes in Model-Implied Interest Rate Expectations

OIS-augmentation offers benefits for the identification and estimation of interest rate expectations from DTSMs at *daily frequencies*. As OIS rates are available at a daily frequency, the benefits are potentially sizeable. To illustrate this, I analyse the daily changes in model-implied risk-neutral yields.

The analysis of daily changes in interest rate expectations is an integral part of historical monetary policy analysis. Most recently, a number of authors have used daily changes in interest rate expectations and term premia to assess the relative efficacy of various interest rate channels of unconventional monetary policies (see Lloyd, 2017b, and the references within). For the OIS-augmented model to be well-suited to historical policy analysis of this sort, risk-neutral yields should, at the very least, qualitatively match numerical measures of investors' interest rate expectations. To test this, I compare the sign of daily changes in 3, 6 12 and 24-month risk-neutral yields to the sign of daily changes in comparable-maturity OIS rates, which I use because their horizon corresponds exactly to that of the nominal government bond yields I use. Although it may seem somewhat tautological to compare an OIS-augmented model to OIS rates, previous results indicate that this need not be the case. In table 3, the survey-augmented model does not provide the best fit for the survey-expectations which are used as an input to its estimation. To reasonably reflect investors' expectations, the sign of the daily change in the risk-neutral yield should match to the sign of the daily change in the comparable horizon OIS rate. I record the proportion of positive and negative daily changes in OIS rates that are matched in sign by the change in the corresponding-horizon risk-neutral yields. To focus on significant changes in OIS rates, I omit days on which OIS rates changed by less, in absolute value, than one standard deviation of the daily changes in the OIS rate over the whole sample (approx 2-5 basis points). The results are presented in table 5.

The results indicate that the 4-OIS-augmented model provides the best qualitative match for the sign of daily changes in 3, 6, 12 and 24-month OIS rates. For example, the 4-OIS-augmented model is the only one to match over 96% of positive daily changes in 1-year OIS rates. Moreover, at the 2-year horizon, the sign of daily changes in the risk-neutral yield from the 4-OIS-augmented model matches 97.99% (98.69%) of positive (negative) OIS rate, around 3 (5) percentage points more than the OLS/ML and bias-corrected models match.

Overall, the results in table 5 are consistent with the claim that the 4-OIS-augmented model best reflects the daily frequency evolution of short-term interest rate expectations.

6.2.5 Sub-sample Stability

In addition to reflecting the daily frequency evolution of interest rate expectations, it is desirable for estimates of fitted yields, risk-neutral yields and term premia on any given date to be stable across different sample periods for application of the model to real-time policy analysis. That is, the model should provide similar estimates of interest rate expectations on a given date regardless of the sample period used. In principle, OIS-augmentation can be helpful in this regard. By augmenting the model with additional information about interest rate expectations

Table 5: Proportion of Daily Changes in OIS Rates Matched in Sign by the Daily Changes in Estimated Risk-Neutral Yields

Sample: January 2002 to December 2016				
	Maturity			
Model	3-Months	6-Months	1-Year	2-Years
Proportion of Positive Daily Changes Matched				
OLS/ML	84.33%	93.20%	95.10%	94.27%
Bias-Corrected	83.58%	92.72%	95.10%	95.13%
Survey	82.09%	91.26%	92.81%	95.13%
3-OIS	**85.82%**	92.72%	94.44%	96.28%
4-OIS	**85.82%**	**94.17%**	**96.08%**	**97.99%**
Proportion of Negative Daily Changes Matched				
OLS/ML	87.21%	93.20%	94.93%	93.18%
Bias-Corrected	86.63%	93.13%	95.22%	93.18%
Survey	86.05%	91.85%	94.93%	96.85%
3-OIS	90.12%	**93.99%**	94.93%	98.69%
4-OIS	**90.70%**	**93.99%**	**96.42%**	**98.69%**

Note: Proportion of daily changes in 3, 6, 12 and 24-month OIS rates (in excess of one standard deviation of their daily change in absolute value) that are matched in sign by the daily change in the corresponding maturity Gaussian affine DTSM risk-neutral yield. All proportions are expressed as a percentage to two decimal places. Five models are compared: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 3-OIS-augmented model (3-OIS); and (v) the 4-OIS-augmented model (4-OIS). The models are estimated with three pricing factors, using daily data from January 2002 to December 2016. The highest percentage model at each maturity has been emboldened for ease of reading.

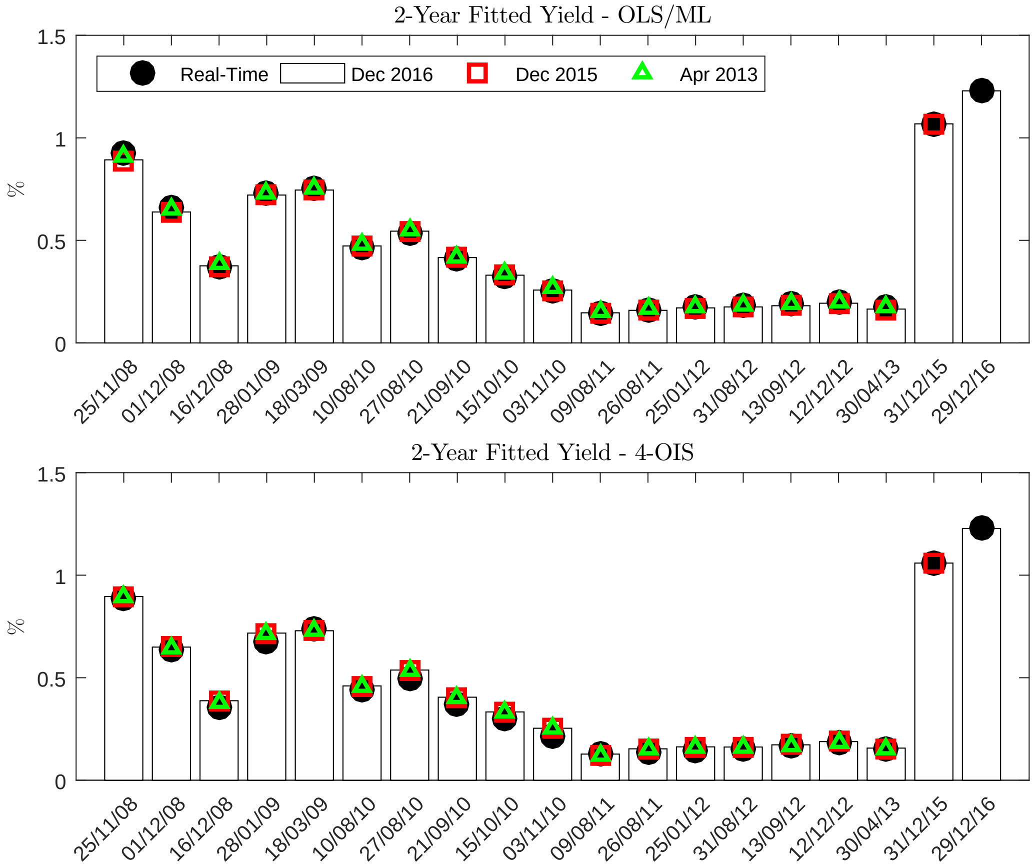
to solve the identification problem within DTSMs, the OIS-augmented model should provide estimates of interest rate expectations that vary less with respect to the sample period in comparison to unaugmented models.

To assess this, I compare real-time estimates of fitted and risk-neutral yields to estimates from the same model estimated using three different samples. To focus on monetary policy, I compare estimates on 19 different dates, the first 16 of which were major US unconventional monetary policy announcement days (see Lloyd, 2017b, table 3). The remaining 3 dates are: 30/04/2013, the end of the month prior to the May 2013 'taper tantrum'; 31/12/2015; and 31/12/2016.

I attain real-time estimates by re-estimating each model using data up to the date of interest. The start date of all real-time samples is January 2002. The three alternative samples all begin in January 2002 and end in April 2013, December 2015 and December 2016, respectively.

Figures 6 and 7 plot estimates of 2-year fitted yields and 2-year risk-neutral yields, respectively, from the unaugmented OLS/ML (top panel) and 4-OIS-augmented (bottom panel) models on the 19 different dates. The black dots represent the real-time estimates, which are most timely for monetary policy. The white bars illustrate the 2-year fitted yields and 2-year risk-neutral yields from the January 2002 to December 2016 sample. The red squares plot the same quantities from the January 2002 to December 2015 sample, while the green triangles plot

Figure 6: Estimates of the 2-Year Fitted Yield from the Unaugmented OLS/ML and 4-OIS-Augmented Models on 19 Different Dates

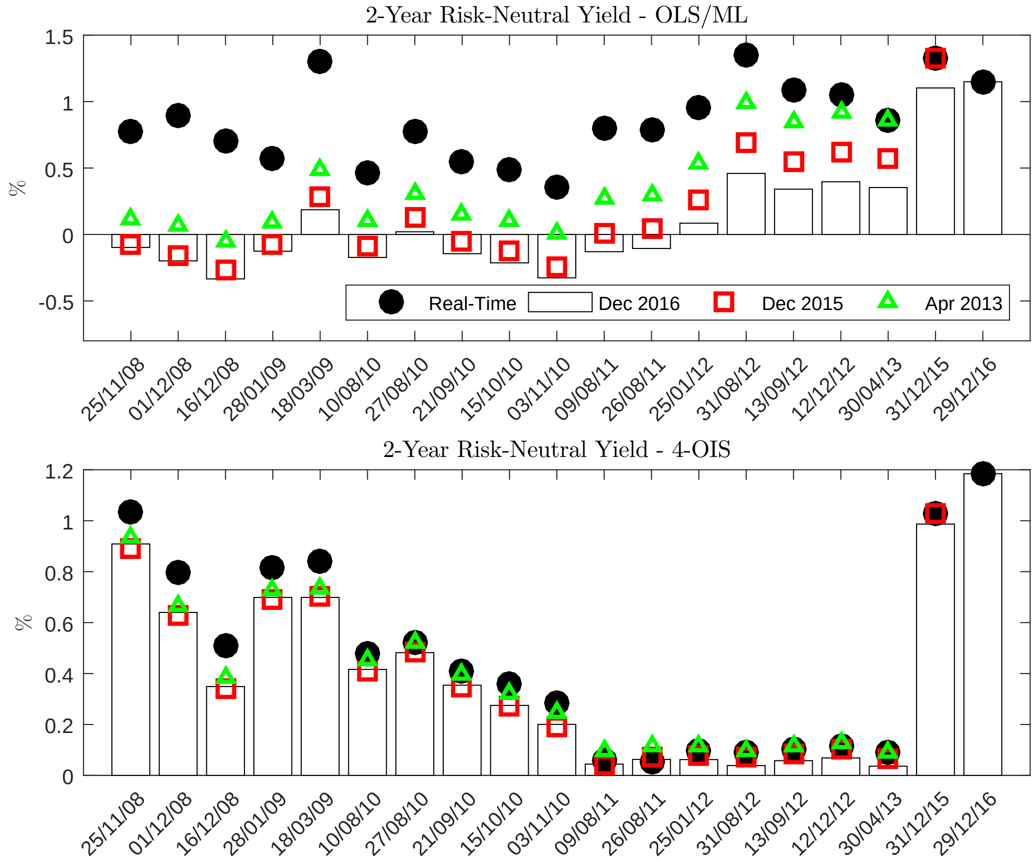


Note: Estimates of the 2-year fitted yield from the unaugmented OLS/ML (top panel) and 4-OIS-augmented (bottom panel) models on 19 dates, 16 of which are associated with US unconventional monetary policy announcements. All samples begin in January 2002. The black dots represent real-time estimates of bond yields, using data up to the event date. The white bars represent estimates using a sample that ends in December 2016, the red squares represent estimates using a sample that ends in December 2015, and the green triangles represent estimates using a sample that ends in April 2013. All models are estimated with three pricing factors, using daily data. Yields are plotted in annualised percentage points. The date format for events, on the horizontal axis, is DD/MM/YY.

the results from the January 2002 to April 2013 sample. To attain reliable inference from an event study using a DTSM, estimates of interest rate expectations and term premia on a given date from a desirable model should not vary across sample periods. For example, the estimated influence of the initial announcement of large-scale asset purchases (25/11/2008) on interest rate expectations and term premia should not change significantly as the estimation sample period is extended.

Figure 6 illustrates that real-time estimates of the fitted yield are very similar to those attained using the three longer samples. This is unsurprising, as the identification problem pertains to the risk-neutral yields. However, the top panel of figure 7 demonstrates that the identification problem does generate instability in risk-neutral yield estimates in the unaugmented OLS/ML model. Real-time estimates of the level of interest rate expectations differ

Figure 7: Estimates of the 2-Year Risk-Neutral Yield from the Unaugmented OLS/ML and 4-OIS-Augmented Models on 19 Different Dates



Note: Estimates of the 2-year risk-neutral yield from the unaugmented OLS/ML (top panel) and 4-OIS-augmented (bottom panel) models on 19 dates, 16 of which are associated with US unconventional monetary policy announcements. All samples begin in January 2002. The black dots represent real-time estimates of risk-neutral yields, using data up to the event date. The white bars represent estimates using a sample that ends in December 2016, the red squares represent estimates using a sample that ends in December 2015, and the green triangles represent estimates using a sample that ends in April 2013. All models are estimated with three pricing factors, using daily data. Yields are plotted in annualised percentage points. The date format for events, on the horizontal axis, is DD/MM/YY.

substantially from those attained from the three longer samples. Moreover, the estimates from the three longer samples substantially differ from one another. For example, on March 18, 2009, the real-time estimate of the 2-year risk-neutral yield from the OLS/ML model is 111 basis points above the estimate attained from the January 2002 to December 2016 sample, which, in turn, is 29 basis points below the estimate from the January 2002 to April 2013 sample. In contrast, the bottom panel of figure 7 illustrates that estimates of risk-neutral yields from the 4-OIS-augmented model are remarkably stable across samples. Although, there are differences between the real-time estimates and longer-sample estimates for early events that peak at 17 basis points on December 1, 2008 and December 16, 2008, this is likely to be due to parameter instability around 2008-2010. As the sample is extended to include more post-2008 data, the differences between estimates decline. On December 12, 2012, the range of estimates from the

4-OIS-augmented model is just 7 basis points; the corresponding figure for the OLS/ML model is 65 basis points. Moreover, the differences between real-time and longer-sample estimates for early events cannot be explained by small-sample issues, because shorter-sample estimates are close to real-time and longer-sample estimates on other event days. For instance, on event day 6 (August 10, 2010), the real-time estimate of the OIS-augmented 2-year yield is 48 basis points, the estimate from the 2002-2015 sample is 41 basis points, while an estimate using a 6.5-year sample from January 2004 to the event date is 45 basis points. Therefore, inference about interest rate expectations can be reliably made from the OIS-augmented model, regardless of the sample period chosen.

6.3 Explaining the Benefits of OIS-Augmentation

The preceding discussion highlights that OIS-augmented models provide more accurate and stable estimates of expected future short-term interest rates than existing models. Moreover, within the class of OIS-augmented models considered, the 4-OIS-augmented model, on balance, outperforms the 2 and 3-OIS-augmented models.

To understand the economic reasons behind the differences in risk-neutral yield estimates from different models (e.g. in figure 9), I draw on the canonical description of the first three principal components of bond yields as the level, slope and curvature of the yield curve respectively, together with the model-implied loadings on these factors.[34] Figure 8 plots these loadings for both the calculation of fitted yields $B_n \equiv -\frac{1}{n}\mathcal{B}_n\left(\boldsymbol{\delta}_1, \boldsymbol{\Phi}^{\mathbb{Q}}; \mathcal{B}_{n-1}\right)$ (top row) and the risk-neutral yields $\tilde{B}_n \equiv -\frac{1}{n}\mathcal{B}_n\left(\boldsymbol{\delta}_1, \boldsymbol{\Phi}; \mathcal{B}_{n-1}\right)$ (bottom row) for the 3-month to 10-year maturities. To refine discussion, loadings are presented for the two most inferior models (OLS/ML and bias-corrected) and the most superior (4-OIS-augmented) models. These loadings illustrate the extent to which the fitted and risk-neutral yields react to a one unit shock to a pricing factor at a given maturity, keeping all other pricing factors constant.

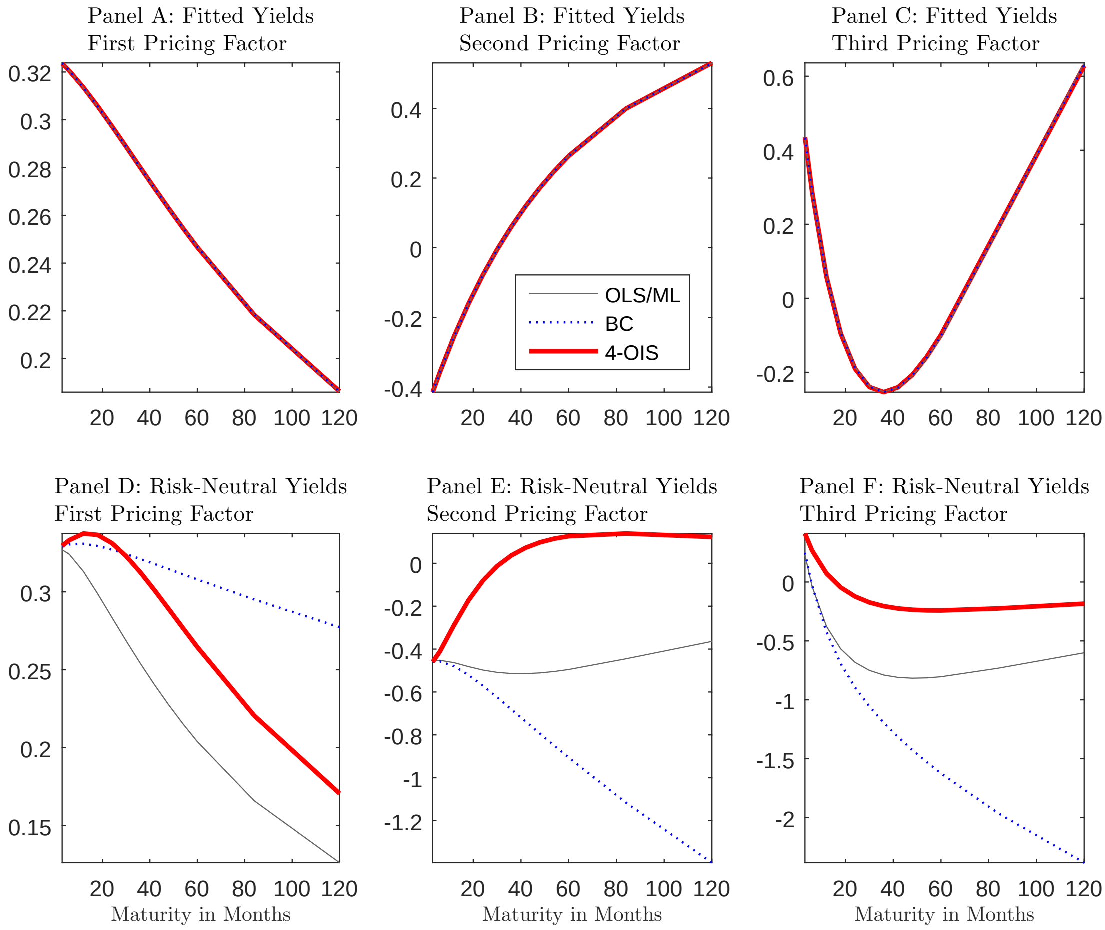
Unsurprisingly, the loadings for the fitted yields from the OLS/ML, bias-corrected and 4-OIS-augmented models are almost identical at all maturities, reinforcing the similarities in their fitted yields. The benefits of OIS-augmentation arise from the separate identification of interest rate expectations and term premia, rather than the fitting of actual yields.

However, the loadings for the risk-neutral yields differ at all horizons, helping to explain why the 4-OIS-augmented model is superior as a measure of interest rate expectations and, *inter alia*, providing economic reasons for the differences in risk-neutral yields from late-2008 onwards. In particular, the risk-neutral loadings on the slope and curvature factors from the OIS-augmented model are close to zero for most maturities. In the OIS-augmented model, shocks to the level of the yield curve, such as the reduction of interest rates to their ELB in 2008, have the strongest effect on risk-neutral yields, helping to explain why OIS-augmented risk-neutral yields remain positive during the ELB period.

From late-2011 to 2013, the risk-neutral yields from the OLS/ML and bias-corrected models

[34] Because the estimated pricing factors from the three-factor OIS-augmented models almost exactly correspond with the estimated principal components (see figure 3), this economic intuition is valid for these models.

Figure 8: Fitted and Risk-Neutral Yield Factor Loadings



Note: I plot the estimated yield loadings B_n for the fitted and risk-neutral yields, for each the three pricing factors (level, slope and curvature respectively), from the OLS/ML, bias corrected and 4-OIS-augmented models estimated with three factors from January 2002 to December 2016. These coefficients can be interpreted as the *ceteris paribus* response of the fitted and risk-neutral bond yields at a given maturity to a contemporaneous shock to the respective pricing factor. The horizontal axis labels denote the maturity, in months. The three models are denoted by: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (BC); and (iii) the 4-OIS-augmented model (4-OIS).

rise to a peak during 2012, falling back below zero for a short period from mid-2013 to mid-2014. The risk-neutral yields from the OIS-augmented models do not peak during 2012. This period was characterised by two notable phenomena. First, the target federal funds rate was at its effective lower bound. Having been set at this level in December 2008, the FOMC were signalling, through forward guidance, that it would be kept at this rate into the future. Second, the Eurozone sovereign debt crisis elevated Eurozone government bond yields, peaking in 2011-2012. This was associated with a reduction in yields on, comparatively safe, longer-term US government bonds. During the 2011-2013 period therefore, the US yield curve was characterised by a reduction in its *slope*, with no change in the *level* of short-term interest rates.

Panel E of figure 8 illustrates that a decrease in the slope of the yield curve places upward pressure on estimated risk-neutral yields in the OLS/ML and bias-corrected models at all maturities. That is, decreases in the yield curve slope, for a given level and curvature, tend to be associated with diminished term premia. However, the risk-neutral yields from the 4-OIS-augmented model react less strongly to a change in the yield curve slope, and, at longer-term horizons, a decrease in the yield curve slope will place downward pressure on estimated risk-neutral yields. This helps to explain why the risk-neutral yields from the 4-OIS-augmented do not rise to a peak in mid-2012, while those from the OLS/ML and bias-correct models do. The 4-OIS-augmented model does not exhibit the same peak, because the inclusion of OIS rates in the estimation alters the loading on that pricing factor. This constellation of factor loadings helps to attain risk-neutral yields from the 4-OIS-augmented model that align more closely with survey and market-implied expectations of future short-term interest rates.

7 Conclusion

Financial market participants and policymakers closely monitor the evolution of interest rate expectations using a wide range of financial market instruments and models. In this paper, I propose the augmentation of DTSMs with OIS rates to better estimate interest rate expectations and term premia along the whole term structure at daily frequencies.

To illustrate this proposal, I present an OIS-augmented Gaussian affine DTSM with pricing equations that explicitly account for the payoff structure of OIS contracts. Although I illustrate the OIS-augmentation procedure in a Gaussian affine framework estimated using data from January 2002 to December 2016, the proposal can be applied to a wider class of DTSMs and, thanks to the Kalman filter framework, to a longer span of data. Notwithstanding this, the improved identification of interest rate expectations, and thus term premia, within the Gaussian affine framework for the 2002-2016 sample is striking. By comparing model-implied risk-neutral yields to federal funds futures rates and survey expectations, I demonstrate that estimates of interest rate expectations from the OIS-augmented model are more accurate than those from existing unaugmented, bias-corrected and survey-augmented models at a range of horizons. In addition, estimated risk-neutral yields from the OIS-augmented model most closely match qualitative daily patterns exhibited by financial market instruments and are more stable across sub-samples, lending the proposal to real-time policy analysis that can improve our understanding of monetary policy transmission through bond yields (see Lloyd, 2017b,a, for existing policy-relevant applications of the OIS-augmented model).

To conclude, by providing daily frequency information about investors' near-term expectations of future short-term interest rates, OIS rates can be used to improve identification of daily frequency interest rate expectations and term premia at a range of horizons within DTSMs.

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Appendix

A Data Sources

- *US Treasury Bill Rates*: Federal Reserve Statistical Release H.15.
- *US Zero-Coupon Treasury Yields*: Gürkaynak, Sack, and Wright (2007b); updated data on Federal Reserve website.
- *US Federal Funds Futures Rates*: Bloomberg.
- *US OIS Rates*: Bloomberg.
- *US Survey Forecasts of the 3-month T-Bill Rate*: Survey of Professional Forecasters, Federal Reserve Bank of Philadelphia.
- *Merrill Lynch Option Volatility Estimate (MOVE)*: Bloomberg.

B Approximated Survey Forecasts

Here, I present the details underlying the survey forecast approximation presented in figure 1 and section 6.2.2 using data from the *Survey of Professional Forecasters* (SPF). The survey is published every quarter and reports forecasters' median expectations of the average 3-month T-Bill rate over a specified period: the current quarter $\bar{i}_{t|t}^{3m,sur}$; and the first $\bar{i}_{t+1|t}^{3m,sur}$, second $\bar{i}_{t+2|t}^{3m,sur}$, third $\bar{i}_{t+3|t}^{3m,sur}$ and fourth $\bar{i}_{t+4|t}^{3m,sur}$ quarters subsequent to the current one, where t denotes the current quarter. All quantities are plotted on the survey submission deadline dates.

To construct a geometric approximation for the average expectation of the 3-month T-Bill rate over the 3-months following the deadline date, I construct an equally weighted geometric average of the median expectation of the 3-month rate for the current and the subsequent quarter. An equal weighting is made possible because the survey deadline date lies approximately halfway through the 'current' quarter. I use a geometric average to facilitate direct comparison with OIS contracts, which have a geometric structure.

To achieve this, I first use the survey expectation for the average 3-month T-Bill rate over the current quarter $\bar{i}_{t|t}^{3m,sur}$ and the realised average of the 3-month T-Bill rate in the current quarter up to the SPF deadline date $\bar{i}_{t}^{3m,real}$ to approximate the survey expectations for the average 3-month T-Bill rate over the remainder of the current quarter, denoted $\bar{i}_{t_+|t}^{3m,sur}$. This is calculated from the following expression:

$$\bar{i}_{t|t}^{3m,sur} = \frac{1}{2}\bar{i}_{t}^{3m,real} + \frac{1}{2}\bar{i}_{t_+|t}^{3m,sur}$$

For figure 1, I calculate the average survey expectation of the 3-month T-Bill rate over the 3, 6 and 12 months following the SPF deadline date. To calculate the average survey expectation of the 3-month T-Bill rate over the three months from the SPF deadline date t, $i_{t|t}^{3m,sur}$, I use the approximation:

$$i_{t|t}^{3m,sur} = \left[\left(1 + \frac{\bar{i}_{t_+|t}^{3m,sur}}{100}\right)^{\frac{1}{2}} \times \left(1 + \frac{\bar{i}_{t+1|t}^{3m,sur}}{100}\right)^{\frac{1}{2}} - 1\right] \times 100$$

where $i_{t|t}^{3m,sur}$, $\bar{i}_{t_+|t}^{3m,sur}$ and $\bar{i}_{t+1|t}^{3m,sur}$ are all reported in percentage points.

The average expectation of the 3-month T-Bill rate over the six months following the deadline date t, $i_{t|t}^{6m,sur}$, is approximated using a similar geometric *weighted* average procedure: the expectation of the

3-month rate for the remainder of the current quarter and second quarter ahead are both given weights of 1/4; and the first-quarter-ahead expectation has weight 1/2. Mathematically, this is written as:

$$i^{6m,sur}_{t|t} = \left[\left(1+\frac{\bar{i}^{3m,sur}_{t+|t}}{100}\right)^{\frac{1}{4}} \times \left(1+\frac{\bar{i}^{3m,sur}_{t+1|t}}{100}\right)^{\frac{1}{2}} \times \left(1+\frac{\bar{i}^{3m,sur}_{t+2|t}}{100}\right)^{\frac{1}{4}} - 1\right] \times 100$$

The average expectation of the 3-month T-Bill rate over the year following the submission date t, $i^{1y,sur}_{t|t}$, is approximated by a geometric weighted average of the remainder of the current quarter and first, second, third and fourth quarter ahead expectations, of the form:

$$\begin{aligned} i^{1y,sur}_{t|t} = &\left[\left(1+\frac{\bar{i}^{3m,sur}_{t+|t}}{100}\right)^{\frac{1}{8}} \times \left(1+\frac{\bar{i}^{3m,sur}_{t+1|t}}{100}\right)^{\frac{1}{4}} \times \left(1+\frac{\bar{i}^{3m,sur}_{t+2|t}}{100}\right)^{\frac{1}{4}}\right. \\ &\left. \times \left(1+\frac{\bar{i}^{3m,sur}_{t+3|t}}{100}\right)^{\frac{1}{4}} \times \left(1+\frac{\bar{i}^{3m,sur}_{t+4|t}}{100}\right)^{\frac{1}{8}} - 1\right] \times 100 \end{aligned}$$

There are two minor caveats to the comparison in figure 1 which help to explain small differences between survey expectations and OIS rates. First, the expectational horizons of OIS rates and the T-Bill expectations do not exactly correspond, because the latter also reflect 3-month T-Bill rate expectations 1.5 months beyond the horizon. Second, 3-month T-Bill rates are on a discount basis, whereas OIS rates include expectations of interest rates on a yield basis.

In section 6.2.2, I calculate the average expectation of the 3-month T-Bill rate over the months following the deadline date. To do this, I construct weighted geometric averages of the median expectation of the 3-month rate for the current and subsequent quarters. Again, the weighting, which facilitates direct comparison of the survey and model-implied expectations, is possible because the survey deadline date lies approximately halfway through the 'current' quarter. Mathematically, the survey expectation of the 3-month T-Bill rate over the 1.5, 4.5, 7.5, 10.5 and 13.5 months from the deadline date t is:

$$\begin{aligned} i^{1.5m,sur}_{t|t} =& \bar{i}^{3m,sur}_{t+|t} \\ i^{4.5m,sur}_{t|t} =& \left[\left(1+\frac{\bar{i}^{3m,sur}_{t+|t}}{100}\right)^{\frac{1}{3}} \times \left(1+\frac{\bar{i}^{3m,sur}_{t+1|t}}{100}\right)^{\frac{2}{3}} - 1\right] \times 100 \\ i^{7.5m,sur}_{t|t} =& \left[\left(1+\frac{\bar{i}^{3m,sur}_{t+|t}}{100}\right)^{\frac{1}{5}} \times \left(1+\frac{\bar{i}^{3m,sur}_{t+1|t}}{100}\right)^{\frac{2}{5}} \times \left(1+\frac{\bar{i}^{3m,sur}_{t+2|t}}{100}\right)^{\frac{2}{5}} - 1\right] \times 100 \\ i^{10.5m,sur}_{t|t} =& \left[\left(1+\frac{\bar{i}^{3m,sur}_{t+|t}}{100}\right)^{\frac{1}{7}} \times \left(1+\frac{\bar{i}^{3m,sur}_{t+1|t}}{100}\right)^{\frac{2}{7}} \times \left(1+\frac{\bar{i}^{3m,sur}_{t+2|t}}{100}\right)^{\frac{2}{7}}\right. \\ &\left. \times \left(1+\frac{\bar{i}^{3m,sur}_{t+3|t}}{100}\right)^{\frac{2}{7}} - 1\right] \times 100 \\ i^{13.5m,sur}_{t|t} =& \left[\left(1+\frac{\bar{i}^{3m,sur}_{t+|t}}{100}\right)^{\frac{1}{9}} \times \left(1+\frac{\bar{i}^{3m,sur}_{t+1|t}}{100}\right)^{\frac{2}{9}} \times \left(1+\frac{\bar{i}^{3m,sur}_{t+2|t}}{100}\right)^{\frac{2}{9}}\right. \\ &\left. \times \left(1+\frac{\bar{i}^{3m,sur}_{t+3|t}}{100}\right)^{\frac{2}{9}} \times \left(1+\frac{\bar{i}^{3m,sur}_{t+4|t}}{100}\right)^{\frac{2}{9}} - 1\right] \times 100 \end{aligned}$$

C Unaugmented Gaussian Affine Model

C.1 Bond Pricing Using the Risk-Adjusted Probability Measure $\mathbb{Q}$

To guarantee the existence of a risk-adjusted probability measure $\mathbb{Q}$, under which the bonds are priced, no-arbitrage is imposed (Harrison and Kreps, 1979). The risk-adjusted probability measure $\mathbb{Q}$ is defined such that the price V_t of any asset that does not pay any dividends at time $t+1$ satisfies $V_t = \mathbb{E}_t^{\mathbb{Q}}[\exp(-i_t)V_{t+1}]$, where the expectation $\mathbb{E}_t^{\mathbb{Q}}$ is taken under the $\mathbb{Q}$ probability measure. Thus, with no-arbitrage, the price of an n-day zero-coupon bond must satisfy the following relation:

$$P_{t,n} = \mathbb{E}_t^{\mathbb{Q}}[\exp(-i_t)P_{t+1,n-1}] \tag{8}$$

Using this, it is possible to show that the nominal bond price is an exponentially affine function of the pricing factors:

$$P_{t,n} = \exp\left(\mathcal{A}_n + \mathcal{B}_n \mathbf{x}_t\right) \tag{9}$$

such that the corresponding continuously compounded yield $y_{t,n}$ is affine in the pricing factors:

$$y_{t,n} = -\frac{1}{n}\ln\left(P_{t,n}\right) = A_n + B_n \mathbf{x}_t \tag{16}$$

where $A_n \equiv -\frac{1}{n}\mathcal{A}_n\left(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}, \boldsymbol{\Sigma}; \mathcal{A}_{n-1}, \mathcal{B}_{n-1}\right)$ and $B_n \equiv -\frac{1}{n}\mathcal{B}_n\left(\boldsymbol{\delta}_1, \boldsymbol{\Phi}^{\mathbb{Q}}; \mathcal{B}_{n-1}\right)$.

To attain recursive expressions for $\mathcal{A}_n$ and $\mathcal{B}_n$:

$$\begin{aligned}
\mathcal{A}_n + \mathcal{B}_n \mathbf{x}_t &= \ln P_{t,n} \\
&= \ln \mathbb{E}_t^{\mathbb{Q}}[\exp(-i_t)P_{t+1,n-1}] \\
&= \ln \mathbb{E}_t^{\mathbb{Q}}[\exp\left(-i_t + \mathcal{A}_{n-1} + \mathcal{B}_{n-1}\mathbf{x}_{t+1}\right)] \\
&= \ln \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\delta_0 - \boldsymbol{\delta}_1'\mathbf{x}_t + \mathcal{A}_{n-1} + \mathcal{B}_{n-1}\left[\boldsymbol{\mu}^{\mathbb{Q}} + \boldsymbol{\Phi}^{\mathbb{Q}}\mathbf{x}_t + \boldsymbol{\Sigma}\boldsymbol{\varepsilon}_{t+1}^{\mathbb{Q}}\right]\right)\right] \\
&= -\left(\delta_0 + \boldsymbol{\delta}_1'\mathbf{x}_t\right) + \mathcal{A}_{n-1} + \mathcal{B}_{n-1}\left[\boldsymbol{\mu}^{\mathbb{Q}} + \boldsymbol{\Phi}^{\mathbb{Q}}\mathbf{x}_t\right] + \ln \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(\mathcal{B}_{n-1}\boldsymbol{\Sigma}\boldsymbol{\varepsilon}_{t+1}^{\mathbb{Q}}\right)\right] \\
&= -\left(\delta_0 + \boldsymbol{\delta}_1'\mathbf{x}_t\right) + \mathcal{A}_{n-1} + \mathcal{B}_{n-1}\left[\boldsymbol{\mu}^{\mathbb{Q}} + \boldsymbol{\Phi}^{\mathbb{Q}}\mathbf{x}_t\right] + \frac{1}{2}\mathcal{B}_{n-1}\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\mathcal{B}_{n-1}' \\
&= \left\{-\delta_0 + \mathcal{A}_{n-1} + \frac{1}{2}\mathcal{B}_{n-1}\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\mathcal{B}_{n-1}' + \mathcal{B}_{n-1}\boldsymbol{\mu}^{\mathbb{Q}}\right\} + \left\{-\boldsymbol{\delta}_1' + \mathcal{B}_{n-1}\boldsymbol{\Phi}^{\mathbb{Q}}\right\}\mathbf{x}_t
\end{aligned}$$

using (9) in the third line, (2) and (5) in the fourth line, and using the property of the log-normal distribution in conjunction with the fact that $\boldsymbol{\varepsilon}_{t+1}^{\mathbb{Q}}|\mathbf{x}_t \sim \mathcal{N}\left(\mathbf{0}_K, \mathbf{I}_K\right)$ to write $\ln \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(\mathcal{B}_{n-1}\boldsymbol{\Sigma}\boldsymbol{\varepsilon}_{t+1}^{\mathbb{Q}}\right)\right]$ as $\frac{1}{2}\mathcal{B}_{n-1}\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\mathcal{B}_{n-1}'$ in the sixth line.

The recursive definitions for the scalar $\mathcal{A}_n \equiv \mathcal{A}_n\left(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}, \boldsymbol{\Sigma}; \mathcal{A}_{n-1}, \mathcal{B}_{n-1}\right)$ and the $1 \times K$ vector $\mathcal{B}_n \equiv \mathcal{B}_n\left(\boldsymbol{\delta}_1, \boldsymbol{\Phi}^{\mathbb{Q}}; \mathcal{B}_{n-1}\right)$ follow from the final line by the method of undetermined coefficients:

$$\mathcal{A}_n = -\delta_0 + \mathcal{A}_{n-1} + \frac{1}{2}\mathcal{B}_{n-1}\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\mathcal{B}_{n-1}' + \mathcal{B}_{n-1}\boldsymbol{\mu}^{\mathbb{Q}} \tag{17}$$

$$\mathcal{B}_n = -\boldsymbol{\delta}_1' + \mathcal{B}_{n-1}\boldsymbol{\Phi}^{\mathbb{Q}} \tag{18}$$

with initial values $\mathcal{A}_0 = 0$ and $\mathcal{B}_0 = \mathbf{0}_K'$, where $\mathbf{0}_K$ is a $K \times 1$ vector of zeros.

C.2 Bond Pricing Using the Pricing Kernel and the Actual Probability Measure $\mathbb{P}$

Under the actual $\mathbb{P}$ probability measure, the bond price is given by equation (7):

$$P_{t,n} = \mathbb{E}_t \left[M_{t+1} P_{t+1,n-1} \right]$$

where this expectation is taken under the $\mathbb{P}$ measure.

Using this, it is also possible to show that the nominal bond price is an exponentially affine function of the pricing factors, as in (9). To attain recursive expressions for $\mathcal{A}_n$ and $\mathcal{B}_n$:

$$\begin{aligned}
\mathcal{A}_n + \mathcal{B}_n \mathbf{x}_t &= \ln P_{t,n} \\
&= \ln \mathbb{E}_t \left[M_{t+1} P_{t+1,n-1} \right] \\
&= \ln \mathbb{E}_t \left[\exp \left(-i_t - \frac{1}{2} \boldsymbol{\lambda}_t' \boldsymbol{\lambda}_t - \boldsymbol{\lambda}_t' \boldsymbol{\varepsilon}_{t+1} + \mathcal{A}_{n-1} + \mathcal{B}_{n-1} \mathbf{x}_{t+1} \right) \right] \\
&= \ln \mathbb{E}_t \left[\exp \left(-\delta_0 - \boldsymbol{\delta}_1' \mathbf{x}_t - \frac{1}{2} \left(\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1 \mathbf{x}_t \right)' \left(\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1 \mathbf{x}_t \right) \right. \right. \\
&\quad \left. \left. - \left(\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1 \mathbf{x}_t \right)' \boldsymbol{\varepsilon}_{t+1} + \mathcal{A}_{n-1} + \mathcal{B}_{n-1} \left(\boldsymbol{\mu} + \boldsymbol{\Phi} \mathbf{x}_t + \boldsymbol{\Sigma} \boldsymbol{\varepsilon}_{t+1} \right) \right) \right] \\
&= -\delta_0 - \boldsymbol{\delta}_1' \mathbf{x}_t - \frac{1}{2} \left(\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1 \mathbf{x}_t \right)' \left(\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1 \mathbf{x}_t \right) + \mathcal{A}_{n-1} + \mathcal{B}_{n-1} \left(\boldsymbol{\mu} + \boldsymbol{\Phi} \mathbf{x}_t \right) \\
&\quad + \ln \mathbb{E}_t \left[\exp \left(\left(- \left(\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1 \mathbf{x}_t \right)' + \mathcal{B}_{n-1} \boldsymbol{\Sigma} \right) \boldsymbol{\varepsilon}_{t+1} \right) \right] \\
&= -\delta_0 - \boldsymbol{\delta}_1' \mathbf{x}_t - \frac{1}{2} \left(\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1 \mathbf{x}_t \right)' \left(\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1 \mathbf{x}_t \right) + \mathcal{A}_{n-1} + \mathcal{B}_{n-1} \left(\boldsymbol{\mu} + \boldsymbol{\Phi} \mathbf{x}_t \right) \\
&\quad + \frac{1}{2} \left(- \left(\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1 \mathbf{x}_t \right)' + \mathcal{B}_{n-1} \boldsymbol{\Sigma} \right) \left(- \left(\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1 \mathbf{x}_t \right)' + \mathcal{B}_{n-1} \boldsymbol{\Sigma} \right)' \\
&= -\delta_0 - \boldsymbol{\delta}_1' \mathbf{x}_t + \mathcal{A}_{n-1} - \mathcal{B}_{n-1} \boldsymbol{\Sigma} \left(\boldsymbol{\lambda}_0 + \boldsymbol{\Lambda}_1 \mathbf{x}_t \right) \\
&\quad + \frac{1}{2} \mathcal{B}_{n-1} \boldsymbol{\Sigma} \boldsymbol{\Sigma}' \mathcal{B}_{n-1}' + \mathcal{B}_{n-1} \left(\boldsymbol{\mu} + \boldsymbol{\Phi} \mathbf{x}_t \right)
\end{aligned}$$

using (3) and (9) in the third line, and (1), (2) and (4) in the fourth line.

By the method of undetermined coefficients, the recursive definitions for the scalar $\mathcal{A}_n$ and the $1 \times K$ vector $\mathcal{B}_n$ follow from the final line:

$$\mathcal{A}_n = -\delta_0 + \mathcal{A}_{n-1} + \frac{1}{2} \mathcal{B}_{n-1} \boldsymbol{\Sigma} \boldsymbol{\Sigma}' \mathcal{B}_{n-1}' + \mathcal{B}_{n-1} \left(\boldsymbol{\mu} - \boldsymbol{\Sigma} \boldsymbol{\lambda}_0 \right) \quad (19)$$

$$\mathcal{B}_n = -\boldsymbol{\delta}_1' + \mathcal{B}_{n-1} \left(\boldsymbol{\Phi} - \boldsymbol{\Sigma} \boldsymbol{\Lambda}_1 \right) \quad (20)$$

with initial values $\mathcal{A}_0 = 0$ and $\mathcal{B}_0 = \mathbf{0}_K'$, where $\mathbf{0}_K'$ is a $K \times 1$ vector of zeros.

Comparing (17) and (18) with (19) and (20) yields the relationship between $\mathbb{P}$ and $\mathbb{Q}$ parameters:

$$\boldsymbol{\mu}^{\mathbb{Q}} = \boldsymbol{\mu} - \boldsymbol{\Sigma} \boldsymbol{\lambda}_0, \qquad \boldsymbol{\Phi}^{\mathbb{Q}} = \boldsymbol{\Phi} - \boldsymbol{\Sigma} \boldsymbol{\Lambda}_1.$$

C.3 Risk-Neutral Yields

The n-day risk-neutral yield reflects the yield that would prevail if investors were risk-neutral. That is, the risk-neutral yield corresponds to that which would arise under the actual probability measure $\mathbb{P}$.

The risk-neutral bond price $\widetilde{P}_{t,n}$ is of the form:

$$\widetilde{P}_{t,n} = \mathbb{E}_t \left[\exp(-i_t) \widetilde{P}_{t+1,n-1} \right] \quad (21)$$

and can be shown to be an exponentially affine function of the pricing factors:

$$\widetilde{P}_{t,n} = \exp\left(\mathcal{A}_n + \mathcal{B}_n \mathbf{x}_t\right) \tag{22}$$

where $\mathcal{A}_n \equiv \mathcal{A}_n\left(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}, \boldsymbol{\Phi}, \boldsymbol{\Sigma}; \mathcal{A}_{n-1}, \mathcal{B}_{n-1}\right)$ and $\mathcal{B}_n \equiv \mathcal{B}_n\left(\boldsymbol{\delta}_1, \boldsymbol{\Phi}; \mathcal{B}_{n-1}\right)$. Thus, the risk-neutral yield, $\widetilde{y}_{t,n} = -\frac{1}{n}\ln \widetilde{P}_{t,n}$, is affine in the pricing factors:

$$\widetilde{y}_{t,n} = \widetilde{A}_n + \widetilde{B}_n \mathbf{x}_t \tag{23}$$

where $\widetilde{A}_n = -\frac{1}{n}\mathcal{A}_n\left(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}, \boldsymbol{\Phi}, \boldsymbol{\Sigma}; \mathcal{A}_{n-1}, \mathcal{B}_{n-1}\right)$ and $\widetilde{B}_n = -\frac{1}{n}\mathcal{B}_n\left(\boldsymbol{\delta}_1, \boldsymbol{\Phi}; \mathcal{B}_{n-1}\right)$.

To attain the recursive expressions for $\widetilde{A}_n$ and $\widetilde{B}_n$, note that from equation (22):

$$\begin{aligned}
\widetilde{y}_{t,n} &= -\frac{1}{n}\ln \mathbb{E}_t\left[\exp\left\{-i_t + \mathcal{A}_{n-1} + \mathcal{B}_{n-1}\mathbf{x}_{t+1}\right\}\right] \\
\widetilde{A}_n + \widetilde{B}_n \mathbf{x}_t &= -\frac{1}{n}\ln \mathbb{E}_t\left[\exp\left\{-\left(\delta_0 + \boldsymbol{\delta}_1'\mathbf{x}_t\right) + \mathcal{A}_{n-1} + \mathcal{B}_{n-1}\left[\boldsymbol{\mu} + \boldsymbol{\Phi}\mathbf{x}_t + \boldsymbol{\Sigma}\boldsymbol{\varepsilon}_{t+1}\right]\right\}\right] \\
&= -\frac{1}{n}\left[\left\{-\delta_0 + \mathcal{A}_{n-1} + \frac{1}{2}\mathcal{B}_{n-1}\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\mathcal{B}_{n-1}' + \mathcal{B}_{n-1}\boldsymbol{\mu}\right\} + \left\{-\boldsymbol{\delta}_1' + \mathcal{B}_{n-1}\boldsymbol{\Phi}\right\}\mathbf{x}_t\right]
\end{aligned}$$

using (21) and (22) in the first line, and (2) and (1) in the second line. The expectation is taken under the actual probability measure $\mathbb{P}$. By the method of undetermined coefficients, it follows that:

$$\widetilde{y}_{t,n} = \widetilde{A}_n + \widetilde{B}_n \mathbf{x}_t$$

where $\widetilde{A}_n = -\frac{1}{n}\mathcal{A}_n\left(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}, \boldsymbol{\Phi}, \boldsymbol{\Sigma}; \mathcal{A}_{n-1}, \mathcal{B}_{n-1}\right)$ and $\widetilde{B}_n = -\frac{1}{n}\mathcal{B}_n\left(\boldsymbol{\delta}_1, \boldsymbol{\Phi}; \mathcal{B}_{n-1}\right)$.

D OIS-Augmented Model

To calculate the loadings for the OIS observation equation, first note that the *annualised* floating leg of a j-day OIS contract, with trade day t and starting day $t+1$, is given by:

$$i_{t,t+j}^{flt} = \left(\left[\prod_{k=1}^{j}\left(1 + \gamma_{t+k} i_{t+k}\right)\right] - 1\right) \times \frac{T_{yr}}{j}$$

where γ_{t+k} is an accrual factor, which is set to $1/T_{yr}$ for all time periods, and $T_{yr} = 252$ is the number of trading days in a year.[35] i_t is the one-period short-term floating interest rate (2) used as the reference rate for the swap — the effective federal funds rate. Rearranging this, taking logs, and using a first-order Taylor approximation around $x = 0$ such that $\ln(1+x) \approx x$, yields:

$$i_{t,t+k}^{flt} \approx \left[\sum_{k=1}^{j}\gamma_{t+k} i_{t+k}\right] \times \frac{T_{yr}}{j} \tag{24}$$

[35]For the term structure model, the accrual and annualisation factors use the convention that there are 252 business trading days in a year, as opposed to the market quoting convention of 360 days used in Lloyd (2018). Given that daily yield data is only available on 252 days per year, I adopt this convention to ensure that the *horizon* for each OIS rate corresponds to their actual maturity date and that of a corresponding maturity zero-coupon bond. This convention is also adopted for daily frequency term structure estimation by, amongst others, Bauer and Rudebusch (2014).

Therefore, under the expectations hypothesis, the OIS rate $i^{ois}_{t,t+j}$ will be:

$$i^{ois}_{t,t+j} = \mathbb{E}_t \left[\sum_{k=1}^{j} \gamma_{t+k} i_{t+k} \right] \times \frac{T_{yr}}{j} \tag{25}$$

For a one-period OIS contract, $j = 1$:

$$\begin{aligned} i^{ois}_{t,t+1} &= \mathbb{E}_t \left[\gamma \left(\delta_0 + \boldsymbol{\delta}_1' \mathbf{x}_{t+1} \right) \right] \times T_{yr} \\ &= \mathbb{E}_t \left[\gamma \left(\delta_0 + \boldsymbol{\delta}_1' \left[\boldsymbol{\mu} + \boldsymbol{\Phi} \mathbf{x}_t + \boldsymbol{\Sigma} \boldsymbol{\varepsilon}_{t+1} \right] \right) \right] \times T_{yr} \\ &= \left(\delta_0 + \boldsymbol{\delta}_1' \boldsymbol{\mu} + \boldsymbol{\delta}_1' \boldsymbol{\Phi} \mathbf{x}_t \right) \end{aligned}$$

where $\gamma \equiv 1/T_{yr}$, and noting that $\mathbb{E}_t \left[\boldsymbol{\varepsilon}_{t+1} \right] = \mathbf{0}_K$ in the second line.

For a two period OIS contract, $j = 2$, the expectations hypothesis requires that:

$$\begin{aligned} i^{ois}_{t,t+2} &= \mathbb{E}_t \left[\gamma \left(\delta_0 + \boldsymbol{\delta}_1' \boldsymbol{\mu} + \boldsymbol{\delta}_1' \boldsymbol{\Phi} \mathbf{x}_t \right) + \gamma \left(\delta_0 + \boldsymbol{\delta}_1' \boldsymbol{\mu} + \boldsymbol{\delta}_1' \boldsymbol{\Phi} \mathbf{x}_{t+1} \right) \right] \times \left(T_{yr}/2 \right) \\ &= \mathbb{E}_t \left[\gamma \left(\delta_0 + \boldsymbol{\delta}_1' \boldsymbol{\mu} + \boldsymbol{\delta}_1' \boldsymbol{\Phi} \mathbf{x}_t \right) + \gamma \left(\delta_0 + \boldsymbol{\delta}_1' \boldsymbol{\mu} + \boldsymbol{\delta}_1' \boldsymbol{\Phi} \left[\boldsymbol{\mu} + \boldsymbol{\Phi} \mathbf{x}_t + \boldsymbol{\Sigma} \boldsymbol{\varepsilon}_{t+1} \right] \right) \right] \times \left(T_{yr}/2 \right) \\ &= \frac{1}{2} \left(2\delta_0 + 2\boldsymbol{\delta}_1' \boldsymbol{\mu} + \boldsymbol{\delta}_1' \boldsymbol{\Phi} \boldsymbol{\mu} + \boldsymbol{\delta}_1' \boldsymbol{\Phi} \mathbf{x}_t + \boldsymbol{\delta}_1' \boldsymbol{\Phi}^2 \mathbf{x}_t \right) \end{aligned}$$

For a three period OIS contract, $j = 3$, the same steps as above yield the following expression:

$$i^{ois}_{t,t+3} = \frac{1}{3} \left(3\delta_0 + 3\boldsymbol{\delta}_1' \boldsymbol{\mu} + 2\boldsymbol{\delta}_1' \boldsymbol{\Phi} \boldsymbol{\mu} + \boldsymbol{\delta}_1' \boldsymbol{\Phi}^2 \boldsymbol{\mu} + \boldsymbol{\delta}_1' \boldsymbol{\Phi} \mathbf{x}_t + \boldsymbol{\delta}_1' \boldsymbol{\Phi}^2 \mathbf{x}_t + \boldsymbol{\delta}_1' \boldsymbol{\Phi}^3 \mathbf{x}_t \right)$$

The continued iteration can be summarised by the following expressions:

$$i^{ois}_{t,t+j} = A^{ois}_j + B^{ois}_j \mathbf{x}_t \tag{14}$$

where $A^{ois}_j \equiv \frac{1}{j} \mathcal{A}^{ois}_j \left(\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}, \boldsymbol{\Phi}, \boldsymbol{\Sigma}; \mathcal{A}^{ois}_{j-1}, \mathcal{B}^{ois}_{j-1} \right)$ and $B^{ois}_j = \dfrac{1}{j} \mathcal{B}^{ois}_j \left(\boldsymbol{\delta}_1, \boldsymbol{\Phi}; \mathcal{B}^{ois}_{j-1} \right)$ are recursively defined as:

$$\begin{aligned} \mathcal{A}^{ois}_j &= \delta_0 + \boldsymbol{\delta}_1' \boldsymbol{\mu} + \mathcal{A}^{ois}_{j-1} + \mathcal{B}^{ois}_{j-1} \boldsymbol{\mu} \\ \mathcal{B}^{ois}_j &= \boldsymbol{\delta}_1' \boldsymbol{\Phi} + \mathcal{B}^{ois}_{j-1} \boldsymbol{\Phi} \end{aligned}$$

where $\mathcal{A}^{ois}_0 = 0$ and $\mathcal{B}^{ois}_0 = \mathbf{0}_K'$, where $\mathbf{0}_K$ is a $K \times 1$ vector of zeros.

I attain an affine expression for OIS rates (14), with loadings that are additive and recursive, because I use a first-order Taylor approximation of $i^{flt}_{t,t+j}$ in (24). This ensures that OIS rates can be included in the Gaussian affine DTSM in a similar manner to bond yields, which are also affine in the pricing factors $\mathbf{x}_t$. In reality, because the floating leg of an OIS contract is compounded, a Jensen's inequality term would be expected in the OIS pricing expression, representing a term premium in OIS rates. The first-order Taylor approximation of $i^{ois}_{t,t+j}$ (24) prevents a Jensen's inequality term from arising, and considerably simplifies the expressions for loadings, $\mathcal{A}^{ois}_j$ and $\mathcal{B}^{ois}_j$, ensuring that they are additive and recursive. I circumvent this potential problem by only using OIS rates which I demonstrate have statistically insignificant *ex post* excess returns, such that any Jensen's inequality term should be negligible. Furthermore, I continue to admit measurement error in OIS rates in the Kalman filter setup (15) through $\mathbf{u}^{ois}_t$.

E Estimation Procedure

To identify the unaugmented model described in section 3, I use the normalisation scheme proposed by Joslin et al. (2011). The Joslin et al. (2011) normalisation fosters faster convergence to the global optimum of the model's likelihood function than other identification schemes for two reasons. First, this normalisation allows for the (near) separation of the $\mathbb{P}$ and $\mathbb{Q}$ probability measure likelihood functions, the product of which comprises the overall model likelihood function. Moreover, the Joslin et al. (2011) normalisation reduces the dimensionality of the parameter space. In the baseline, unaugmented model, the parameters governing bond pricing are:

$$\boldsymbol{\Theta} = \left\{\delta_0, \boldsymbol{\delta}_1, \boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}, \boldsymbol{\Sigma}\right\}$$

The Joslin et al. (2011) normalisation scheme uniquely maps these parameters to a smaller set:

$$\{i_{\infty}^{\mathbb{Q}}, \boldsymbol{\lambda}^{\mathbb{Q}}, \boldsymbol{\Sigma}\}$$

where: (i) $i_{\infty}^{\mathbb{Q}}$ is the risk-neutral expectation of the long-run short-term nominal interest rate; (ii) $\boldsymbol{\lambda}^{\mathbb{Q}}$ is a $K \times 1$ vector of the eigenvalues of $\boldsymbol{\Phi}^{\mathbb{Q}}$; and (iii) $\boldsymbol{\Sigma}$ is a lower triangular matrix with positive diagonal entries.

E.1 OLS/ML Estimation of the Unaugmented Model

Assuming that K portfolios of bonds are priced without error, then the Joslin et al. (2011) normalisation permits the complete separation of the $\mathbb{P}$ and $\mathbb{Q}$ likelihood functions. In this study, as in many others, I use the first K principal components of the observed bond yields as the set of K portfolios that are priced perfectly (e.g. Joslin et al., 2011). Defining these portfolios $\mathcal{P}_t \equiv W y_t = W y_t^{obs} \equiv \mathcal{P}_t^{obs}$, where W is the principal component weighting matrix and y_t^{obs} is the vector of observed yields, then Joslin et al. (2011) show that the likelihood function for the unaugmented model laid out in section 3.1 is:

$$\mathcal{L}\left(y_t^{obs}|y_{t-1}^{obs}; \boldsymbol{\Theta}\right) = \mathcal{L}\left(y_t^{obs}|\mathcal{P}_t; \boldsymbol{\lambda}^{\mathbb{Q}}, i_{\infty}^{\mathbb{Q}}, \boldsymbol{\Sigma}, \sigma_u\right) \times \mathcal{L}\left(\mathcal{P}_t|\mathcal{P}_{t-1}; \boldsymbol{\mu}, \boldsymbol{\Phi}, \boldsymbol{\Sigma}\right)$$

where σ_u is the standard deviation of the measurement error of the N observed yields.

This normalisation admits a two-stage estimation process. First, the parameters $\{\boldsymbol{\mu}, \boldsymbol{\Phi}\}$ are directly estimable by running OLS on the VAR in equation (1), where $\mathbf{x}_t \equiv \mathcal{P}_t$. Moreover, this provides initial values for the maximum likelihood estimation of the lower triangular elements of the matrix $\boldsymbol{\Sigma}$. Second, taking $\left\{\widehat{\boldsymbol{\mu}}, \widehat{\boldsymbol{\Phi}}\right\}$ as given, the parameters $\left\{i_{\infty}^{\mathbb{Q}}, \boldsymbol{\lambda}^{\mathbb{Q}}, \boldsymbol{\Sigma}, \sigma_u\right\}$ can be estimated by maximum likelihood.

E.2 Bias-Corrected Estimation

To estimate the bias-corrected decomposition, I rely entirely on the methodology of Bauer et al. (2012, Section 4). The MATLAB code for this is available here: `faculty.chicagobooth.edu/jing.wu/research/zip/brw_table1.zip`.

E.3 Survey-Augmentation

To augment the model with survey expectations of future interest rates, I employ Kalman filter-based maximum likelihood estimation. This estimation methodology, using survey expectations, draws most directly on Guimarães (2014).

Like Guimarães (2014), I use survey expectations from the *Survey of Professional Forecasters* at the Federal Reserve Bank of Philadelphia. I use forecasts for the 3 month T-Bill 1, 2, 3 and 4 quarters ahead, available at a quarterly frequency. I augment the model with the survey expectations on the survey submission deadline day.[36]

The survey-augmented Kalman filter has a similar form to the OIS-augmented setup presented in section 3. The transition equation of the Kalman filter is (1), the vector autoregression for the latent pricing factors under the actual $\mathbb{P}$ probability measure.

On days when the survey forecasts are *not* observed, the observation equation is given by (13). As with the OIS-augmented model, I maintain a homoskedastic form for the yield measurement error.

On days when the S survey forecasts, $s = s_1, s_2, ..., s_S$, are observed, the observation equation is:

$$\begin{bmatrix} \mathbf{y}_t \\ \mathbf{i}_t^{sur} \end{bmatrix} = \begin{bmatrix} \mathbf{A} \\ \mathbf{A}^{sur} \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{B}^{sur} \end{bmatrix} \mathbf{x}_t + \begin{bmatrix} \mathbf{\Sigma}_Y & \mathbf{0}_{N\times S} \\ \mathbf{0}_{S\times N} & \mathbf{\Sigma}_S \end{bmatrix} \begin{bmatrix} \mathbf{u}_t \\ \mathbf{u}_t^{sur} \end{bmatrix} \tag{26}$$

where, in addition to the definitions of $\mathbf{y}_t$, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{\Sigma}_Y$ and $\mathbf{u}_t$ above, $\mathbf{i}_t^{sur} = \left[i_{t,s_1}^{sur}, ..., i_{t,s_S}^{sur}\right]'$; $\mathbf{A}^{sur} = \left[A_{s_1}^{sur}, ..., A_{s_S}^{sur}\right]'$; $\mathbf{B}^{sur} = \left[B_{s_1}^{sur\prime}, ..., B_{s_S}^{sur\prime}\right]'$; $\mathbf{0}_{S\times N}$ and $\mathbf{0}_{N\times S}$ denote $S \times N$ and $N \times S$ matrices of zeros respectively; and $\mathbf{u}_t^{sur} \sim \mathcal{N}\left(\mathbf{0}_S, \mathbf{I}_S\right)$ denotes the survey measurement error, where $\mathbf{0}_S$ is an S-vector of zeros and $\mathbf{I}_S$ is an $S \times S$ identity matrix. As with the yield measurement error, I impose a homoskedastic form for the survey measurement error, such that $\mathbf{\Sigma}_S$ is a $S \times S$ diagonal matrix with common diagonal element σ_s, the standard deviation of the survey measurement error. Appendix C of Guimarães (2014) presents the functional forms for A_s^{sur} and B_s^{sur}, which account for the arithmetic nature of survey expectations.

As with the OIS-augmented model, I estimate the survey-augmented model by using the OLS/ML parameter estimates as initial values for the Kalman filter.

E.4 OIS-Augmentation and Kalman Filtering

When the Kalman filter is used, the assumption that K portfolios of yields are observed without error is no longer made. Instead, all yields (and portfolios thereof) can be observed with error. Consequently, the exact separation of the likelihood function described in section E.1 is no longer applicable. However, the parameter estimates attained from OLS/ML estimation of the unaugmented model do provide initial values for the Kalman filter-based optimisation routine.[37] Doing so, ensures that computational time is reasonably fast.

F Additional Decomposition Results

This appendix presents additional results from model estimation.

F.1 Three-Factor Specification

F.1.1 Model-Implied Fitted OIS Rates

Table 6 presents the RMSE for the fitted OIS rates from each of the OIS-augmented term structure models. The RMSE is presented for each maturity for the sample period January 2002 to December

[36]For survey submission dates that are not business days, I augment the model with survey data on the preceding business day.

[37]Guimarães (2014) follows similar steps to estimate a survey-augmented Gaussian affine DTSM using the Joslin et al. (2011) normalisation scheme.

2016. The results demonstrate that the 4-OIS-augmented model provides superior estimates of the 6, 12 and 24-month OIS rates, while the 2-OIS-augmented model provides marginally superior estimates of the 3-month OIS rate. At the 6 and 12-month horizons, the 3-OIS-augmented model is only marginally inferior to the 4-OIS-augmented model. However, at the 2-year horizon, the 4-OIS-augmented model provides a substantial improvement in fit *vis-à-vis* the 3 and 2-OIS-augmented models. The 2-OIS-augmented provides the highest RMSE estimates of 1 and 2-year OIS rates.

Table 6: Model Fit: Root Mean Square Error (RMSE) of Fitted OIS Rates *vis-à-vis* the Actual OIS Rates

Sample: January 2002 to December 2016			
Maturity	2-OIS	3-OIS	4-OIS
3-Months	0.1183	0.1206	0.1305
6-Months	0.0924	0.1088	0.0861
1-Year	0.1585	0.0926	0.0831
2-Year	0.5306	0.2634	0.0985

Note: RMSE of the fitted OIS rates from each of the three OIS-augmented Gaussian affine DTSMs, computed by comparing the model-implied fitted OIS rate to the actual OIS rate on each day. All figures are expressed in annualised percentage points. The three models are: (i) the 2-OIS-augmented model (2-OIS); (ii) the 3-OIS-augmented model (3-OIS); and (iii) the 4-OIS-augmented model (4-OIS).

The fact the OIS-augmented models do not fit OIS rates as well as they fit bond yields — the quantitative value of OIS-RMSE (approximately 10 basis points) is almost double that of the bond yield-RMSE (approximately 5 basis points) — is neither worrying nor surprising. The model uses thirteen bond yields as inputs to estimate the cross-section of fitted yields in every time period, whereas only four OIS rates are used to fit the cross-section of OIS rates. Moreover, adding additional OIS rates is not warranted given that they are included to improve the fit of model-implied interest rate expectations and that longer-maturity OIS rates contain significant term premia (Lloyd, 2018).

F.1.2 Estimated Pricing Factors and Principal Components

Table 7 presents summary statistics for the estimated principal components of the actual bond yields and the estimated pricing factors from the 4, 3 and 2-OIS-augmented models for the sample period January 2002 to December 2016. The results demonstrate that the principal components and estimated pricing factors evolve similarly, implying that OIS rates do not include any additional information, over and above that in bond yields, of value to the fitting of actual yields. In particular, the summary statistics of the estimated principal components and the estimated pricing factors from the 4-OIS-augmented models are similar.

Moreover, table 7 further demonstrates that the inclusion of different maturities of OIS rate in the term structure model does not appreciably alter estimates of actual bond yields. The summary statistics of the estimated pricing factors from the 4, 3 and 2-OIS-augmented models are all similar. Augmentation of the model with OIS rates only influences estimated parameters under the actual probability measure $\mathbb{P}$ and thus risk-neutral yields.

Table 7: Estimated Principal Components and Estimated Pricing Factors: Summary Statistics

Summary Statistics	1st Factor	2nd Factor	3rd Factor
	Estimated Principal Components		
Mean	0.0725	0.0326	0.0075
Variance	0.0026	0.0001	0.0000
Skewness	0.7771	0.3839	−0.2603
Kurtosis	2.2918	2.2017	2.2441
	4-OIS: Estimated Pricing Factors		
Mean	0.0725	0.0326	0.0076
Variance	0.0025	0.0001	0.0000
Skewness	0.7712	0.4043	−0.2515
Kurtosis	2.2792	2.2136	2.2732
	3-OIS: Estimated Pricing Factors		
Mean	0.0725	0.0326	0.0075
Variance	0.0025	0.0001	0.0000
Skewness	0.7756	0.3909	−0.2542
Kurtosis	2.2877	2.1886	2.2772
	2-OIS: Estimated Pricing Factors		
Mean	0.0725	0.0326	0.0075
Variance	0.0025	0.0001	0.0000
Skewness	0.7756	0.3797	−0.2812
Kurtosis	2.2893	2.1930	2.2152

Note: Summary statistics for the first three estimated principal components from actual yield data and the estimated pricing factors from the 4, 3 and 2-OIS-augmented models. All statistics are reported to four decimal places.

F.1.3 Risk-Neutral Yields and Short-Horizon Survey Expectations

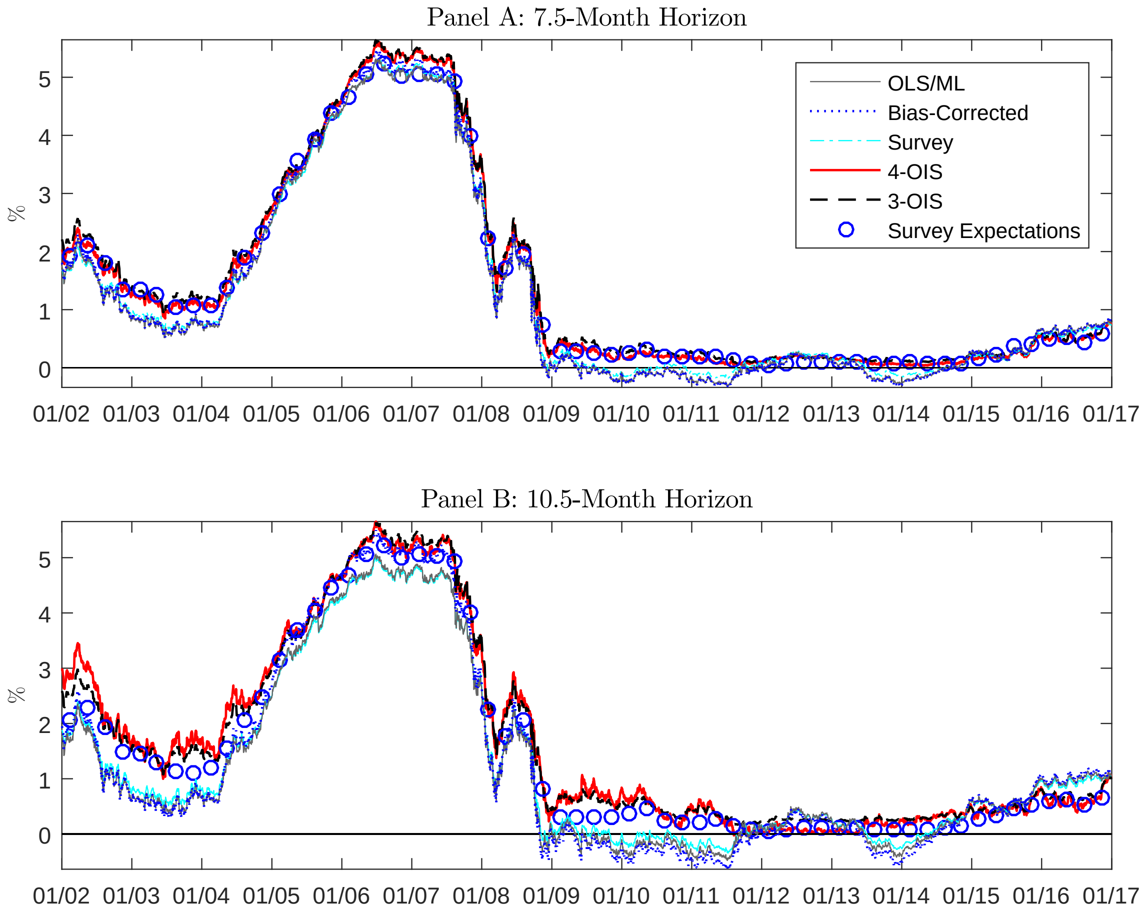
Figure 9 graphically illustrates the evolution of estimated risk-neutral yields and the approximated survey expectations at the 7.5 and 10.5-month horizons, the visual counterpart to section 6.2.2.[38] Three observations follow.

First, between 2002 and late-2004, the OLS/ML, bias-corrected and survey-augmented models generate estimated risk-neutral yields that persistently fall below the corresponding-horizon survey expectation. In contrast, the estimated risk-neutral yields from the OIS-augmented models closely co-move with the approximated survey expectations during this period, especially at the 7.5-month horizon. This corroborates with the comparison of risk-neutral forward yields and federal funds future-implied interest rate expectations in section 6.2.1.

Second, between early-2006 and mid-2007, the risk-neutral yields from the OIS-augmented models exceed interest rate expectations implied by surveys. During this short period, the 7.5-month risk-neutral yields from the OLS/ML, bias-corrected and survey-augmented models more closely align with survey expectations. Although this is not true at the 10.5-month horizon, where the OIS-augmented and bias-corrected models perform best in this period. Recall from figure 5 that the risk-neutral forward yields from the OIS-augmented models closely align with federal funds futures-implied interest rate expectations

[38]The plots for the 1.5, 4.5 and 13.5-month horizons are qualitatively similar.

Figure 9: Short-Term Interest Rate Expectations

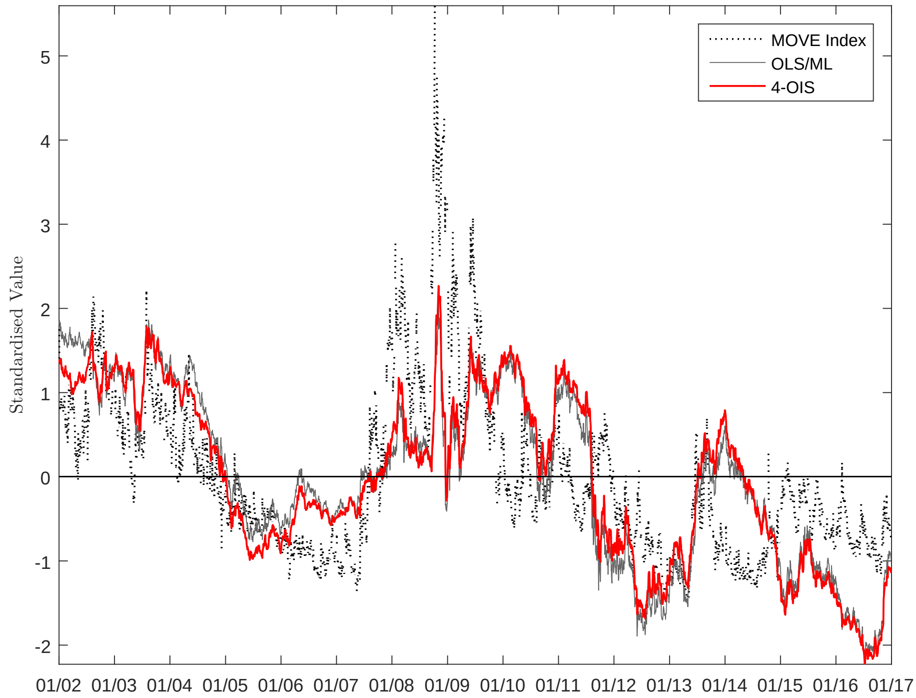


Note: Estimated 7.5-month (panel A) and 10.5-month (panel B) risk-neutral yields from each of five Gaussian affine DTSMs. The five models are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 4-OIS-augmented model (4-OIS); and (v) the 3-OIS-augmented model (3-OIS). The models are estimated with three pricing factors, using daily data from January 2002 to December 2016. I compare the estimated risk-neutral yields to approximated survey expectations of future short-term interest rates over the same horizon. The construction of the survey expectation approximations, using data from the *Survey of Professional Forecasters*, is described in appendix B. All figures are in annualised percentage points.

during this period.

Third, as in section 6.2.1, the models offer markedly different estimates of interest rate expectations from late-2008 to late-2011. During this period, the 7.5 and 10.5-month risk-neutral yields from the OLS/ML and bias-corrected models are persistently negative, implying, counter-factually, that investors expected future short-term interest rates to fall negative. From late-2011 to mid-2012, the risk-neutral yields from the OLS/ML and bias-corrected models rise to peak. Again, this is both contrary to the policy narrative and the survey expectations at the time. In contrast, the OIS-augmented models — the 4-OIS-augmented model especially — align closely with survey expectations for much of the post-2008 period.

Figure 10: Standardised 10-Year Term Premia and Merrill Lynch Option Volatility Estimate (MOVE) Index



Note: I plot the standardised one-month Merrill Lynch Option Volatility Estimate (MOVE) index against the standardised estimates of term premia from (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML), and (ii) the 4-OIS-augmented model (4-OIS). The models are estimated with three factors from January 2002 to December 2016.

F.1.4 Model-Implied Term Premia

This appendix discusses model estimates for the daily evolution of term premia. Although there is no direct metric against which to compare estimated term premia, Adrian et al. (2013) compare a standardised version of their estimated daily 10-year term premium to a standardised version of the 1-month Merrill Lynch Option Volatility Estimate (MOVE) index. This latter index is a measure of implied volatility from option contracts written on US Treasuring bonds.[39] Thus, variation in MOVE reflects changes in the risk of holding US Treasuries.

Like Adrian et al. (2013), in figure 10 I plot the standardised z-score estimates of the 10-year term premium from the OLS/ML and 4-OIS-augmented models against the standardised one-month MOVE index. The time series exhibit a strong positive correlation. The correlation coefficient between the standardised 10-year term premium estimate from the 4-OIS-augmented model and the standardised MOVE index is 0.62, marginally higher than the corresponding statistic of 0.60 for the OLS/ML model. This indicates that the estimated term premia from the 4-OIS-augmented model do reflect the risk of holding Treasury bonds.

[39]Formally, the series used here (and in Adrian et al., 2013) is defined as a yield curve weighted index of the normalised implied volatility on 1-month Treasury options. It is the weighted average of volatilities on 2, 5, 10 and 30-year bond yields.

F.2 Four-Factor Specification

In the light of evidence by Cochrane and Piazzesi (2005, 2008) and Duffee (2011), who argue that more than three factors are necessary to explain the evolution of nominal Treasury yields, I estimate a four-factor specification of the OLS/ML, bias-corrected, survey-augmented and 4-OIS-augmented Gaussian affine DTSMs. Although the four-factor model better fits actual bond yields for the 2002-2016 sample, I do not present these results in the main body of the paper because the economic meaning of the pricing factors in a three-factor model is well understood (i.e., level, slope and curvature), while the economic interpretation the fourth factor is less well understood. Moreover, the differences in risk-neutral yield estimates from the three and four-factor models are small, and there is no evidence that a single model is unambiguously preferable.

I estimate the four-factor model using the same underlying daily data as the three-factor model presented in the main body of the paper.

Fitted Yields Figure 11 demonstrates that the fitted yields from the four-factor Gaussian affine DTSMs do not differ markedly from one another. Here I plot the residual of the 2-year fitted yield from the four-factor model. The average RMSE for each of the models at all thirteen maturities is around 3 basis points, around 2 basis points smaller than from the three-factor model.

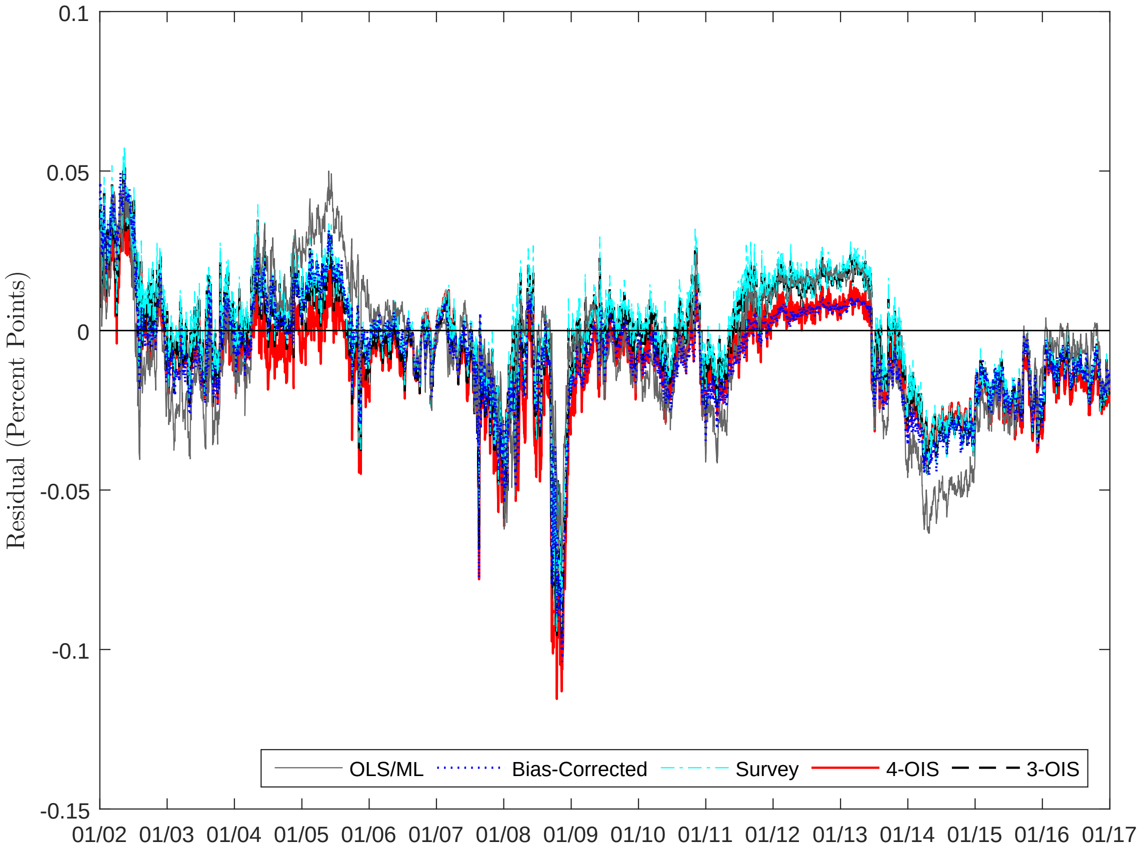
Fitted OIS Rates As with the three-factor models, the four-factor OIS-augmented models accurately fit OIS rates. Figure 12 demonstrates this, plotting the actual and fitted 3, 6, 12 and 24-month OIS rates. Table 8 presents the RMSE for the fitted OIS rates from each of the OIS-augmented models, comparing the results from the three and four-factor models. The table illustrates that the three-factor OIS-augmented models perform marginally better than their four-factor counterparts on a RMSE basis.

Pricing Factors Figure 13 plots the first four estimated principal components of the daily frequency bond yield data, and the four estimated pricing factors from the 4-OIS-augmented model. As with the three-factor model, the plot demonstrates that the inclusion of OIS rates in the estimation of Gaussian affine DTSMs does not significantly influence the bond pricing factors, as the quantities closely co-move.

Interest Rate Expectations Figure 14 plots the 3 and 6-month ahead 1-month risk-neutral forward yields from the four-factor models against comparable-horizon federal funds futures rates. The figure demonstrates that the OIS-augmented models provide the closest fit for market-based measures of interest rate expectations for the majority of the 2002-2016 sample. Moreover, the risk-neutral forward yields from the four-factor OIS-augmented models are similar to those from their three-factor variants plotted in figure 5. In contrast, the four-factor OLS/ML performs visibly worse than its three-factor counterpart.

Table 9 presents a RMSE comparison of the risk-neutral 1-month forward yields from the four-factor models. For comparison, the rightmost column of the table presents the corresponding RMSE from the three-factor 4-OIS-augmented model. The table demonstrates that, of all four-factor models, the OIS-augmented models continue provide the best estimates of interest rate expectations on a RMSE basis. However, a comparison of the four and three-factor models indicates that neither unambiguously outperforms the other. The three-factor 4-OIS-augmented model provides the lowest RMSE fit for 9 of the 11 forward yields.

Figure 11: Residual of the 2-Year Fitted Yield from Four-Factor Models



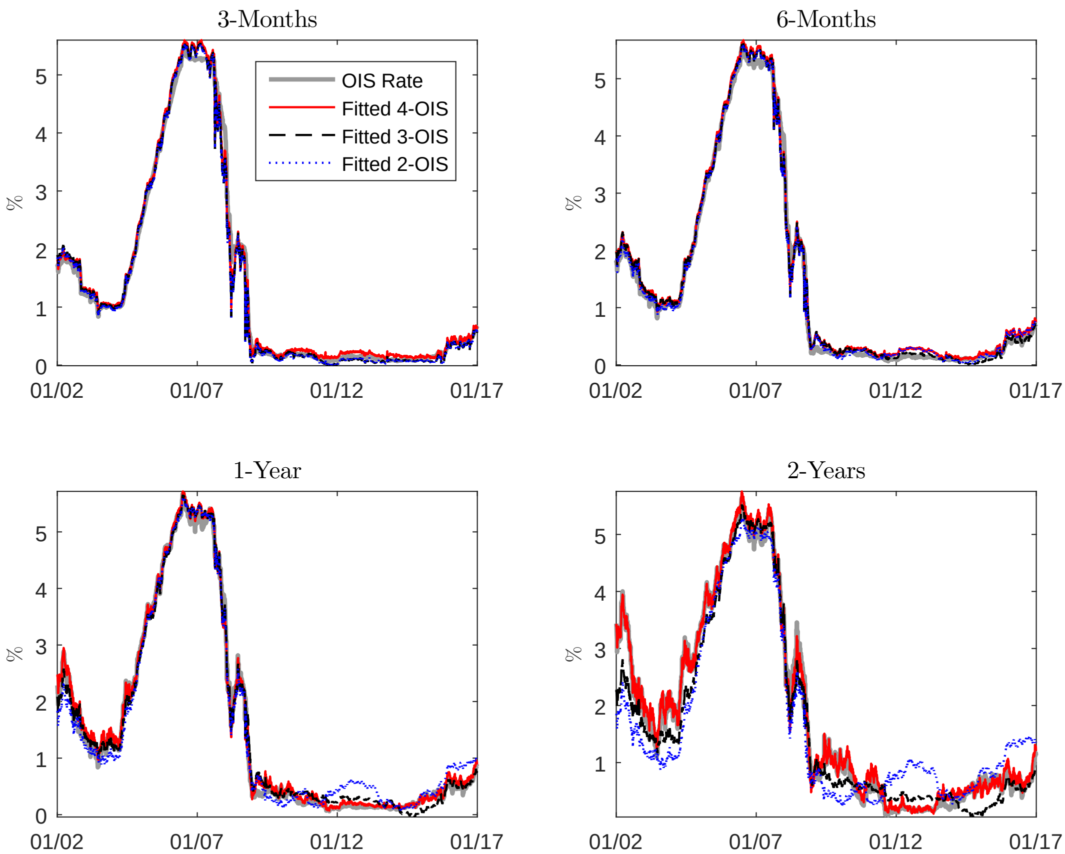
Note: Residuals of the 2-year fitted yield from five monthly frequency Gaussian affine DTSMs: (i) the unaugmented model estimates by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey; (iv) the 4-OIS-augmented model; and (v) the 3-OIS-augmented model (3-OIS). The models are estimated with four pricing factors, using daily data from January 2002 to December 2016. The residual is defined as the actual yield subtracted by the model-implied fitted yield. The residuals are presented in annualised percentage points.

Table 8: Model Fit: Root Mean Square Error (RMSE) of Fitted OIS Rates *vis-à-vis* the Actual OIS Rates for Three and Four-Factor Models

Sample: January 2002 to December 2016						
	Three-Factor Model			Four-Factor Model		
Maturity	2-OIS	3-OIS	4-OIS	2-OIS	3-OIS	4-OIS
3-Months	**0.1183**	0.1206	0.1305	0.1565	0.1561	0.1414
6-Months	0.0924	0.1088	**0.0861**	0.1136	0.1113	0.1219
1-Year	0.1585	0.0926	**0.0831**	0.2215	0.1324	0.0932
2-Year	0.5306	0.2634	**0.0985**	0.5710	0.3501	0.1091

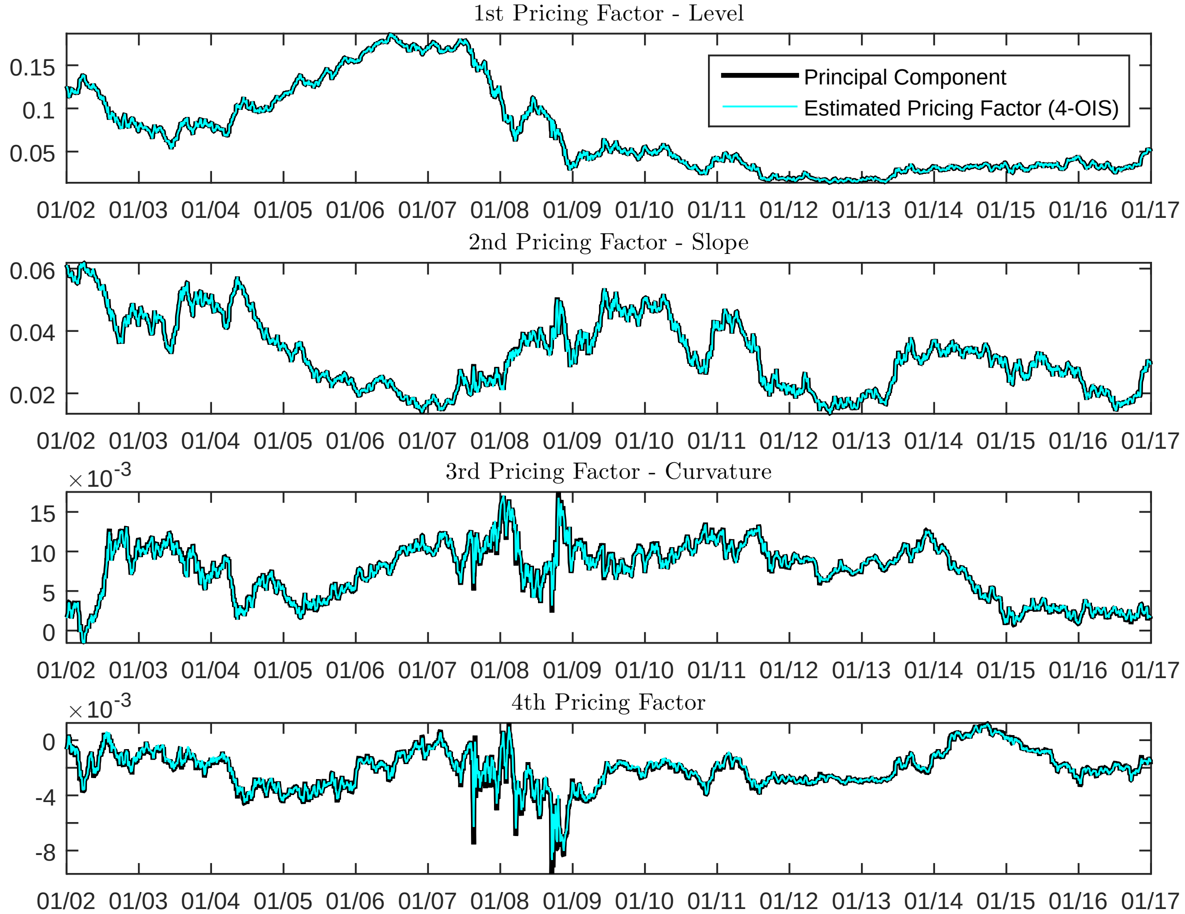
Note: RMSE of the fitted OIS rates from each of the three and four-factor OIS-augmented Gaussian affine DTSMs, computed by comparing the model-implied fitted OIS rate to the actual OIS rate on each day. All figures are expressed in annualised percentage points. The three models are: (i) the 2-OIS-augmented model (2-OIS); (ii) the 3-OIS-augmented model (3-OIS); and (iii) the 4-OIS-augmented model (4-OIS). The lowest RMSE model at each maturity has been emboldened for ease of reading.

Figure 12: Fitted OIS Rates from the Four-Factor OIS-Augmented Models



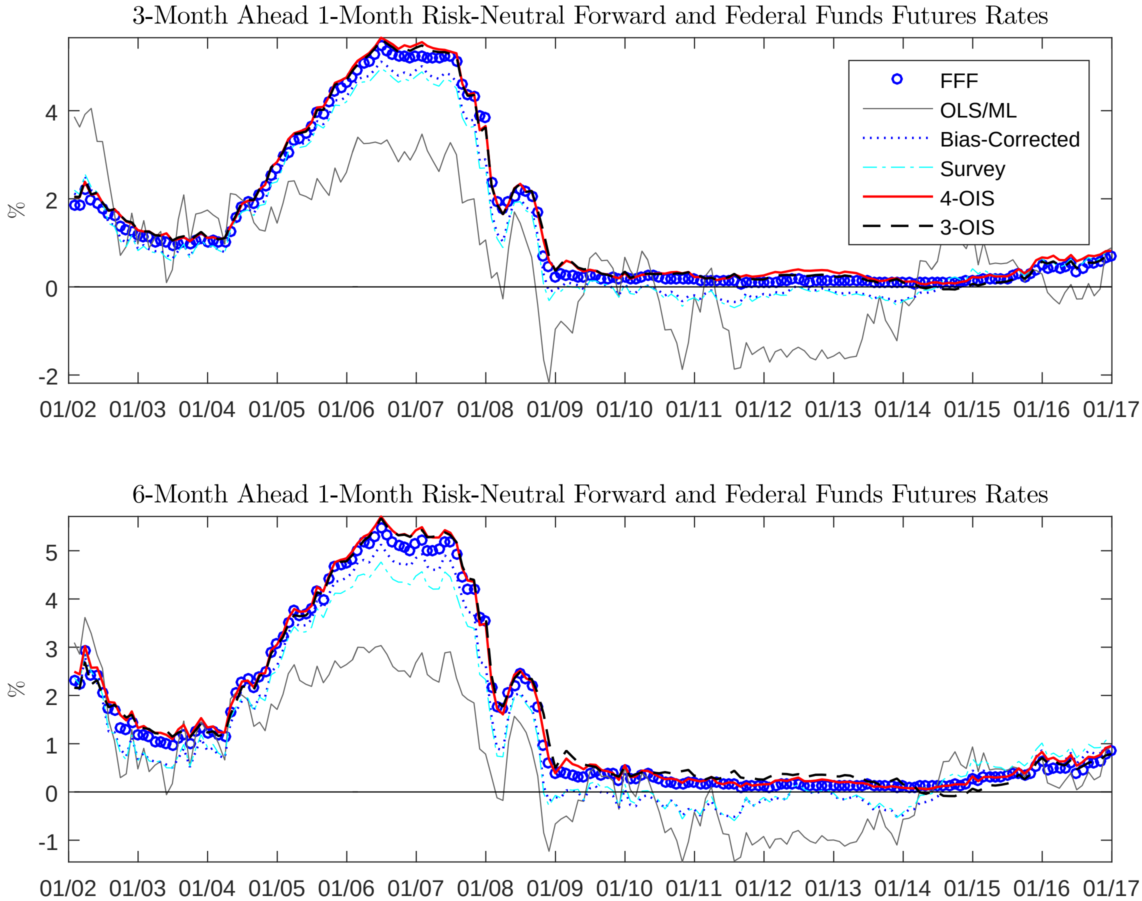
Note: Fitted and actual 3, 6, 12 and 24-month OIS rates. Fitted OIS rates are from the 4, 3 and 2-OIS-augmented Gaussian affine DTSMs. The models are estimated with four pricing factors using daily data from January 2002 to December 2016. All figures are in annualised percentage points.

Figure 13: First Four Estimated Principal Components of the Actual Bond Yields and Estimated Pricing Factors from the Four-Factor 4-OIS-Augmented Model



Note: Estimated principal components from the actual bond yield data with the following maturities: 3, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 84 and 120 months. Estimated pricing factors from the four-factor 4-OIS-augmented model implied by the Kalman filter.

Figure 14: Estimated Risk-Neutral 1-Month Forward Yields from the Four-Factor Models and Comparable-Horizon Federal Funds Futures (FFF) Rates



Note: I plot estimated 3 to 4-month ahead and 6 to 7-month ahead risk-neutral forward yields from each of five Gaussian affine DTSMs. The five models are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the survey-augmented model (Survey); (iv) the 4-OIS-augmented model (4-OIS); and (v) the 3-OIS-augmented model (3-OIS). The models are estimated with four pricing factors, using daily data from January 2002 to December 2016. I compare the estimated risk-neutral forward yields to corresponding-horizon federal funds futures (FFF) rates. All figures are in annualised percentage points.

F.3 Monthly Frequency Results

For robustness, I also estimate the models at a monthly frequency. The monthly frequency models have the same structure as described in the main body of the paper, with the time index t now representing a month, rather than a day. To estimate the model, I use bond yields and OIS rates from the final day of each calendar month. I estimate the monthly frequency models using the same 13 bond yields and 4 OIS rates for the January 2002 to December 2016. The headline conclusion is as follows: the benefits of OIS-augmentation for estimates of future short-term interest rate expectations carry over from daily frequency estimation to lower frequencies, such as the monthly frequency.

Fitted Yields Figure 15 illustrates that the fitted yields from the monthly frequency models do not differ markedly (i) from one another and (ii) in comparison to the daily frequency estimates presented in the main body of the paper. Here, I plot the residual of the 2-year fitted yield from the monthly

Table 9: Model-Implied Expectations: Root Mean Square Error (RMSE) of the Risk-Neutral 1-Month Forward Yields *vis-à-vis* Corresponding-Horizon Federal Funds Futures Rates for Four-Factor Models in Comparison to the Three-Factor 4-OIS-Augmented Model

Sample: January 2002 to December 2016						
	Four-Factor Model					Three-Factor Model
Horizon	OLS/ML	BC	Survey	3-OIS	4-OIS	4-OIS
0 to 1 Months	1.9394	0.2871	0.3196	0.2406	0.1923	**0.1823**
1 to 2 Months	1.5617	0.2647	0.3164	0.1443	0.1384	**0.1293**
2 to 3 Months	1.4007	0.2785	0.3395	0.1209	0.1438	**0.0929**
3 to 4 Months	1.3265	0.3037	0.3738	0.1302	0.1515	**0.0828**
4 to 5 Months	1.2813	0.3287	0.4065	0.1436	0.1491	**0.0898**
5 to 6 Months	1.2497	0.3573	0.4391	0.1565	0.1401	**0.1021**
6 to 7 Months	1.2324	0.3937	0.4746	0.1721	0.1320	**0.1176**
7 to 8 Months	1.2253	0.4354	0.5096	0.1924	**0.1295**	0.1316
8 to 9 Months	1.2338	0.4857	0.5502	0.2151	**0.1304**	0.1429
9 to 10 Months	1.4861	0.9388	0.9627	0.7346	0.6630	**0.6564**
10 to 11 Months	1.7709	1.3067	1.3164	1.0411	0.9703	**0.9635**

Note: RMSE of the risk-neutral 1-month forward yields from six Gaussian affine DTSMs in comparison to corresponding-horizon federal funds futures rates. The six models are: (i) the unaugmented four-factor model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected four-factor model (Bias-Corrected); (iii) the survey-augmented four-factor model (Survey); (iv) the 3-OIS-augmented four-factor model (3-OIS); (v) the 4-OIS-augmented four-factor model (4-OIS); and (vi) the 4-OIS-augmented three-factor model (4-OIS) from the main body of the paper. The models are estimated using daily data from January 2002 to December 2016. The risk-neutral forward yields and the federal funds futures rates are compared on the final day of each calendar month. All figures are in annualised percentage points. The lowest RMSE model at each maturity has been emboldened for ease of reading.

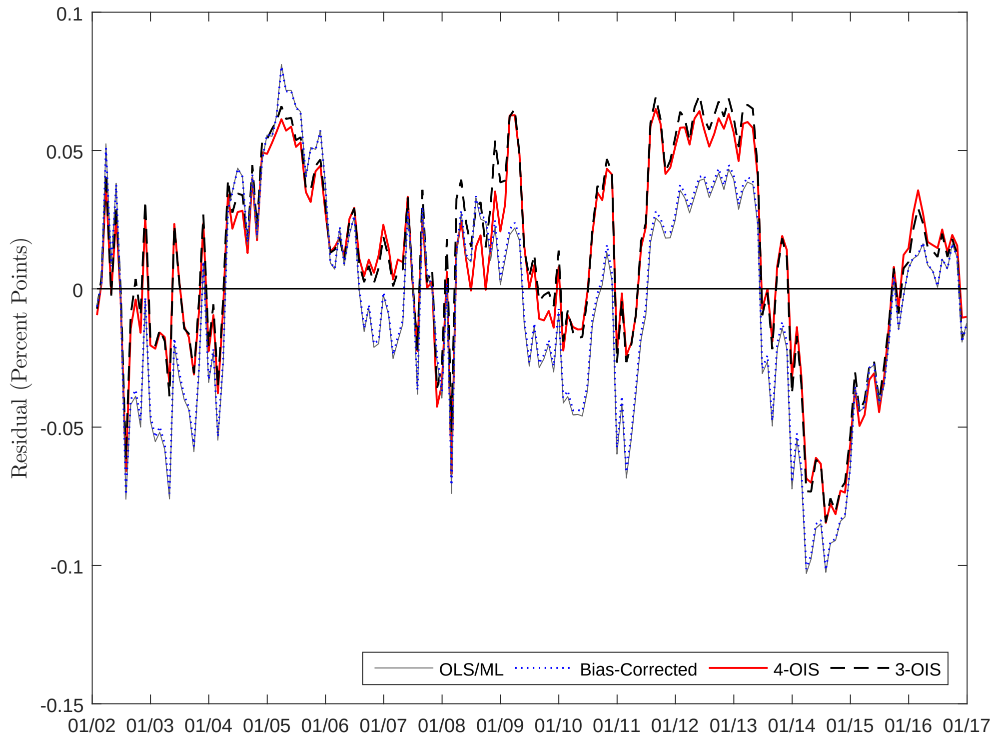
frequency models. They serve to illustrate that the models provide a similar fit for actual bond yields.

Fitted OIS Rates As with the daily frequency results, the monthly frequency OIS-augmented models accurately fit OIS rates. Figure 16 demonstrates, again, that the 4-OIS-augmented model accurately fits the 3, 6, 12 and 24-month OIS rates. Although the 4-OIS-augmented provides a visually superior fit of all four OIS rates, the 2 and 3-OIS-augmented models do provide estimates of OIS rates that fit actual OIS rates reasonably well.

Pricing Factors Figure 17 plots the estimated principal components of the monthly frequency bond yield data and the estimated pricing factors from the monthly-frequency 4-OIS-augmented model. As with the daily frequency model, the plot demonstrates that the inclusion of OIS rates in the estimation of Gaussian affine DTSMs does not significantly influence the bond pricing factors. The two quantities evolve almost identically.

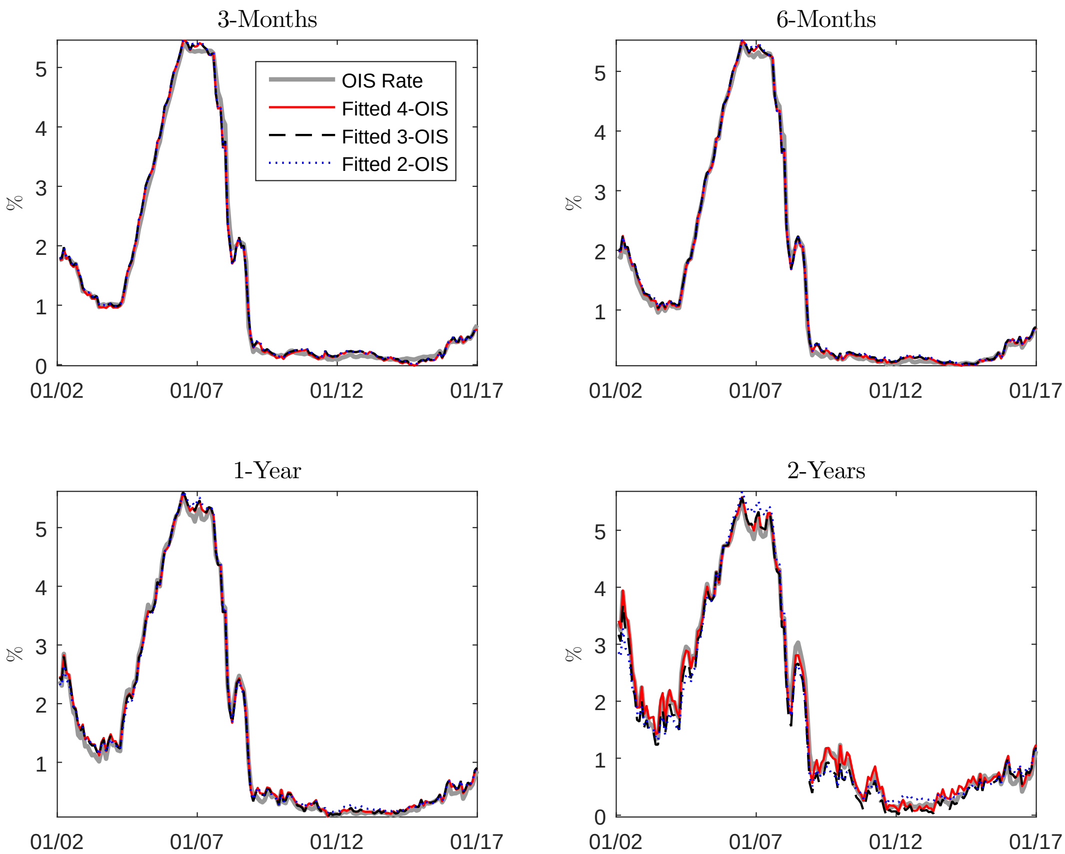
Interest Rate Expectations Finally, in figure 18, I plot the 6-month and 1-year risk-neutral yields from the monthly frequency OLS/ML, bias-corrected, 4 and 3-OIS-augmented Gaussian affine DTSMs. The figure highlights that the monthly frequency estimates for the level of interest rate expectations at a given time are similar, qualitatively and quantitatively, to estimates using daily frequency data.

Figure 15: Residual of the 2-Year Fitted Yield from Monthly Frequency Monthly



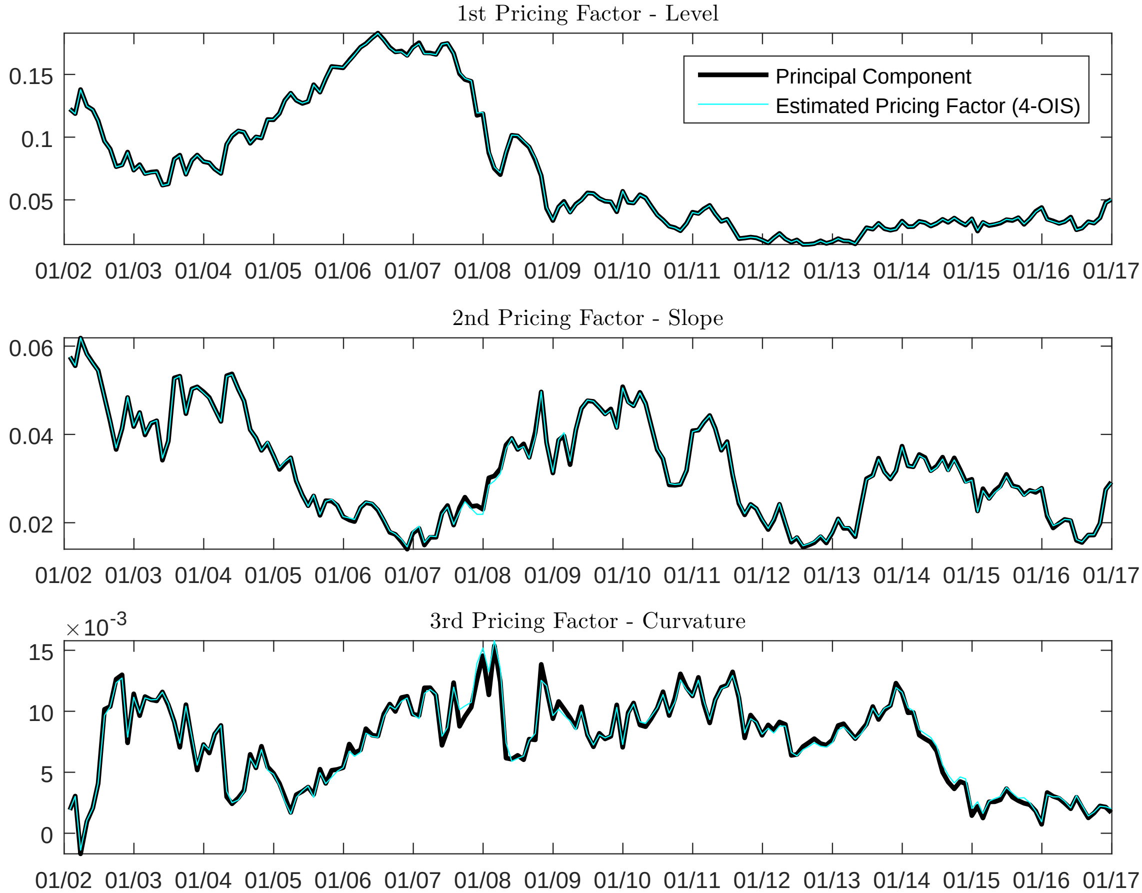
Note: Residuals of the 2-year fitted yield from four monthly frequency Gaussian affine DTSMs: (i) the unaugmented model estimates by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the 4-OIS-augmented model; and (iv) the 3-OIS-augmented model (3-OIS). The models are estimated with three pricing factors, using end of month data from January 2002 to December 2016. The residual is defined as the actual yield subtracted by the model-implied fitted yield. The residuals are presented in annualised percentage points.

Figure 16: Fitted OIS Rates from the Monthly Frequency OIS-Augmented Models



Note: Fitted and actual 3, 6, 12 and 24-month OIS rates. Fitted OIS rates are from the 4, 3 and 2-OIS-augmented Gaussian affine DTSMs. The models are estimated with three pricing factors using end of month data from January 2002 to December 2016. All figures are in annualised percentage points.

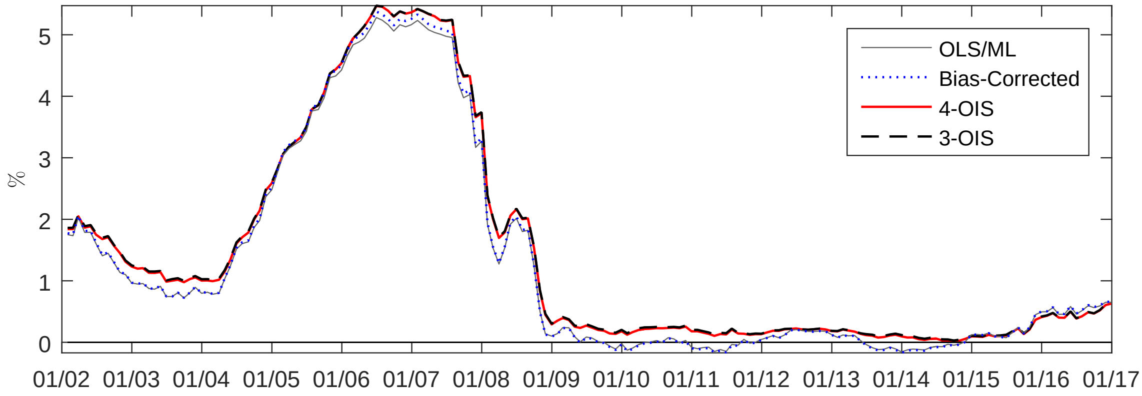
Figure 17: Estimated Principal Components of the Actual Bond Yields and Estimated Pricing Factors from the 4-OIS-Augmented Model at a Monthly Frequency



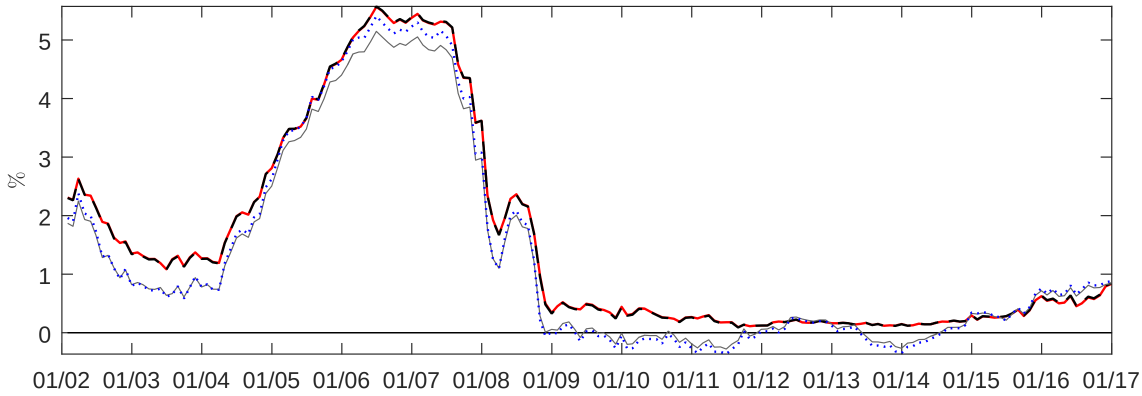
Note: Estimated principal components from the actual bond yield data with the following maturities: 3, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 84 and 120 months. Estimated pricing factors from the three-factor 4-OIS-augmented model implied by the Kalman filter.

Figure 18: Short-Term Interest Rate Expectations from the Monthly Frequency Models

Panel A: 6-Month Horizon



Panel B: 12-Month Horizon



Note: I plot estimated 6-month and 1-year risk-neutral yields from each of four Gaussian affine DTSMs in panels A and B, respectively. The four models are: (i) the unaugmented model estimated by OLS and maximum likelihood (OLS/ML); (ii) the bias-corrected model (Bias-Corrected); (iii) the 4-OIS-augmented model (4-OIS); and (iv) the 3-OIS-augmented model (3-OIS). The models are estimated with three pricing factors, using end of month data from January 2002 to December 2016. All figures are in annualised percentage points.