



BANK OF ENGLAND

Staff Working Paper No. 774

Interest rates, capital and bank risk-taking

Jonathan Acosta-Smith

December 2018

Staff Working Papers describe research in progress by the author(s) and are published to elicit comments and to further debate. Any views expressed are solely those of the author(s) and so cannot be taken to represent those of the Bank of England or to state Bank of England policy. This paper should therefore not be reported as representing the views of the Bank of England or members of the Monetary Policy Committee, Financial Policy Committee or Prudential Regulation Committee.



BANK OF ENGLAND

Staff Working Paper No. 774

Interest rates, capital and bank risk-taking

Jonathan Acosta-Smith⁽¹⁾

Abstract

Are low interest rates more likely to incentivise greater bank risk-taking? This is the question we seek to answer. Using a model in which banks raise funds from depositors to create an investment portfolio which can differ in its risk and return, we suggest so. In particular, we show that lowering the interest rate makes it more likely banks will make risky investments. This is because reducing the interest rate makes safer assets less attractive, while increasing the relative gains from gambling. We show that risk-taking is highly dependent on banks' skin-in-the-game, as banks always ignore the full extent of losses on bankruptcy. Raising the interest rate has a similar effect. It reinforces this behaviour, as by increasing the yield on the portfolio, banks have more to lose on bankruptcy.

Key words: Banking, monetary policy, risk-taking, interest rates.

JEL classification: E44, E58, G21.

(1) Bank of England. Email: jonathan.smith@bankofengland.co.uk

I would like to thank Orhan Erem Atesagaoglu, Thorsten Beck, Petra Geraats, Sean Holly, Marcus Miller and Pontus Rendahl for helpful comments and suggestions. I also thank seminar, conference and workshop participants at the University of Cambridge, Bank of England, the Scottish Economic Society 2015 Annual Conference, the 9th Biennial Conference of the Czech Economic Society, the 4th Bordeaux Workshop in International Economics and Finance, the XII Annual Seminar on Risk, Financial Stability and Banking at the Central Bank of Brazil, the Spanish Economic Association 2017 Symposium in Barcelona, the 2018 International Finance and Banking Society Conference in Porto, and the 2018 INFINITI Conference on International Finance in Poland for their attention and feedback. All remaining errors are my own. The views expressed in this paper are those of the author and do not necessarily reflect those of the Bank of England.

The Bank's working paper series can be found at www.bankofengland.co.uk/working-paper/staff-working-papers

Bank of England, Threadneedle Street, London, EC2R 8AH

Telephone +44 (0)20 3461 4030 email publications@bankofengland.co.uk

© Bank of England 2018

ISSN 1749-9135 (on-line)

1 Introduction

Taking off from the events of the financial crisis, an intense debate has emerged concerning the effects of monetary policy on bank risk-taking. Does loose monetary policy spur bank risk-taking? This lies at the heart of whether monetary policy bears some blame for the credit boom and its subsequent bust. Many have argued that the persistence of low real rates prior to the crisis led financial institutions to take excessive risk, thereby fueling asset prices and leverage (see for example, Dell’Ariccia et al. [2014] and Borio and Zhu [2012]). Proponents argue that if central banks had raised interest rates earlier and more aggressively (‘leaned against the wind’), the crisis may have been less severe (cf. Acharya and Naqvi [2012]). Yet the modern literature has emphasised price stability as the primary goal of monetary policy (the inflation targeting framework exemplifying this idea); therefore an understanding of the relationship between risk-taking and monetary policy may have serious implications for optimal policy.

The emergence of this debate led commentators to look at a new dimension of the monetary transmission mechanism, dubbed by Borio and Zhu [2012] as the ‘risk-taking channel’ of monetary policy. While the bank lending channel has been extensively covered (e.g. Bernanke and Blinder [1992]; Kashyap and Stein [2000]), the literature on the risk-taking channel is still relatively scarce, particularly in the theoretical space. As De Nicolo et al. [2010] noted “theory has had surprisingly little to offer on [this] subject. Few macroeconomic models have explicitly considered the impact of policy rates on bank risk-taking, and models of bank risk-taking have yet to incorporate the effects of monetary policy” (p. 2). For the most part, the literature has focused on the composition of credit in response to changes in the riskiness of borrowers (e.g. financial accelerator models, e.g. Gertler and Gilchrist [1994]; Bernanke et al. [1999]). Only recently, has the impact of policy rates on the risk attitudes of banks and hence the composition of the supply of credit been considered. But almost all of these considerations have been empirical (e.g. Buch et al. [2011]; Jimenez et al. [2014]). There have been hardly any theoretical papers that have explicitly attempted to model the relationship between monetary policy and the risk-taking behaviour of banks.

This paper therefore aims to add to the literature by explicitly addressing this gap. We construct a simple micro model in which there are n banks, and each of these has the opportunity to invest into a portfolio of a different riskiness and return. This endogenously determines the bank’s probability of default, and as such banks

trade-off risk for return. We find that low interest rates can spur banks to increase risk-taking and this occurs via the impact on the bank's portfolio return. Higher interest rates increase future returns. As a result, it forces them to reconsider their risk-taking - as they gamble with the potential loss of a more valuable portfolio. Indeed, higher interest rates bring about a similar effect to directly increasing bank capital, since it makes banks more cautious about losing it. Banks, therefore, may find assets they otherwise would have invested in, now too risky.

We can summarise the direction of risk-taking into two channels: (1) a risk-taking channel, and (2) a bankruptcy channel. The risk-taking channel summarises the incentives banks face in terms of increasing returns via risk. Due to limited liability, banks effectively have an option-like payoff structure. Consequently, banks may choose to invest in an asset with a lower expected return if there is a state of the world in which the payoff is large enough to offset this. This is gambling in its true sense, and the incentive exists because of the ability to enter bankruptcy in the bad state of the world. On the other hand, the bankruptcy channel works to disincentivise risk-taking. The bankruptcy channel describes the effect greater risk has on a bank's probability of default. Although taking an additional unit of risk may increase potential profitability, this additional risk may decrease a bank's probability of survival by such an amount that it no longer finds it optimal to take that risk on. The extent to which this deters risk-taking will depend on the amount of skin-in-the-game banks possess, since then bankruptcy imposes a harsher penalty. The risk-taking choice therefore, prior and post an interest rate change can be seen as the culmination of these two channels. We find that higher interest rates strengthen the bankruptcy channel as banks become more concerned about losing their higher yielding portfolio.

The rest of the paper is organised as follows. Section 2 reviews the literature; section 3 presents the model setup; section 4 solves the model; section 5 and 6 offer extensions by altering the asset structure; section 7 extends the model to a two-period analysis; and section 8 discusses some of the insights and concludes.

2 Related Literature

Following the financial crisis, there is now a growing literature studying the risk-taking channel of monetary policy. However, the main contributions have been in the empirical literature in which there seems to be a growing consensus that low prolonged interest rates increase bank risk-taking. Using a panel of countries from

the Euro Area over the pre-crisis years, Maddaloni and Peydro [2011] suggest that credit standards decline significantly following a loosening of monetary policy, with the decline exacerbated if rates are held low for a prolonged period of time. Altunbas et al. [2010] find similarly using regression quantile techniques over a large sample of listed banks operating in the European Union and the United States from 2007-2009. In addition they find that institutions with higher risk exposure had less capital, a greater reliance on short-term market funding and aggressive credit growth. See also Jimenez et al. [2014] and Ioannidou et al. [2009] for similar findings.

On the theoretical side, a few papers in the quantitative macro model literature have explicitly considered bank risk-taking. Angeloni and Faia [2009] model risk-taking on the liability side. They introduce banks, modeled as in Diamond and Rajan [2012], into a standard DSGE model and find that “bank leverage depends positively on the uncertainty of projects and on the bank’s relationship lender skills, and negatively on short-term interest rates” (p. 1). They conclude that the best policy is a combination of mildly anti-cyclical capital ratios and a monetary policy response to asset prices or leverage. Cociuba et al. [2011] model risk-taking on the asset side. Similar to this paper, they evaluate whether lower than optimal interest rates lead to excessive risk-taking by financial intermediaries. In contrast to this paper however, financial intermediaries do not know ex-ante the riskiness of the project they are investing in, but find this out only in the second stage. Interest rate policy affects risk-taking by changing the amount of safe bonds intermediaries use as collateral in the repo market. Cociuba et al. [2011] find that with properly priced collateral, lower than optimal interest rates reduce risk taking. This is contrary to the popularised view (and empirical suggestion) that low interest rates increase risk taking. However, they note that if the intermediary is able to misprice their collateral favourably (such as underestimating its credit risk as happened during the build up to the financial crisis), then lower than optimal interest rates can contribute to excessive risk-taking.

Outside the quantitative macro literature, other papers have tackled this question from a different angle. Dell’Ariccia and Marquez [2006] build a microeconomic model in which banks face adverse selection problems in selecting borrowers. They suggest a policy cut can lead to lower lending standards and a credit boom since it reduces the sustainability of the separating equilibrium in which bad borrowers are screened out. In a summary article, De Nicolo et al. [2010] suggest there are two mechanisms through which monetary policy may affect bank risk-taking: (1) portfolio effects and the search for yield, and (2) limited liability and risk-shifting. Under the first channel, a cut in interest rates leads banks to rebalance their portfolios in favour

of more risky assets. This is because the decline in yield on safe assets leads risk-neutral banks to increase their demand for risky assets until in equilibrium expected returns on both types of assets are equalised (cf. De Nicolo et al. [2010]). Following Rajan [2005] one can also identify a search for yield channel if financial institutions have long-term commitments (e.g. pension funds). A policy cut will lead these institutions to search for yield since the lower yield on their assets may no longer be sufficient to match the yield they promised on their long-term liabilities.

Under the second channel, risk-taking arises through moral hazard because of limited liability. Since the bank has a lower bound of zero on their payoffs, they can shift risk onto depositors. Dell’Ariccia et al. [2014] formally develop this channel within a static model. They find that when banks can adjust their capital structures, monetary policy easing unequivocally leads to greater leverage and higher risk; the risk-shifting effect is all dominant. However, when the capital structure is fixed, the effect depends on the degree of leverage: well-capitalised banks increase risk, while highly leveraged banks decrease it. This ties into an option value argument of bank risk-taking as presented by Agur and Demertzis [2010]. Agur and Demertzis [2010] develop a static model in which banks can choose between a good project and a bad project (which has a lower expected value and higher volatility). They suggest that due to limited liability, one can think of the bank’s decision as akin to that of an investor who owns a call option, with the bank’s cash flow as the underlying asset, and the point of default as the strike price. As known from standard option value theory, the volatility of the underlying asset is worth more when the option is less-in-the-money; since if the option is deep in-the-money, the option is all but certain to be exercised. Using this logic, they find that more efficient banks choose the good project, since “efficient banks have options that are deep in-the-money . . . [so they] care about both the upside and downside” (p. 9), while less efficient banks (because default is more likely) choose the bad project since the greater volatility is valuable to them; they do not fully internalise the credit risk.

This paper thus builds on the previous literature. We build a simple model to understand the mechanism underlying why low interest rates may induce higher bank risk-taking. We differ from the previous literature such as Dell’Ariccia et al. [2014] in that we move away from a monitoring argument such that lower monitoring is seen as greater risk-taking. Instead, we take the model back to a simple choice of assets for banks, assets which differ in their riskiness and return. We suggest that the risk-taking channel of monetary policy is different for banks than for other financial institutions such as pension funds or insurance companies. Unlike these institutions, banks are not to such an extent tied down by fixed liabilities, since they

can adjust their liabilities and increase the interest rate on their loans to compensate for deposit rate rises. The result then is driven less by a pure search for yield. Instead we show that for banks, the key mechanism runs through how interest rates affect the option-like structure of the bank's profit maximisation.

3 The Model

3.1 Agents and Market Structure

Consider an environment in which there are $n \geq 1$ risk-neutral banks. Each bank is endowed with a capital level k and a license to operate. Each bank j offers a gross interest rate $i_j \geq 1$ to depositors. There is a continuum of identical, risk-averse depositors and these depositors are negligible in size relative to banks. Depositors have two options: they can either invest in a risk-free bond yielding a gross return $r \geq 1$, or alternatively they can deposit their funds at a bank. There is full deposit insurance, therefore deposits yield the gross return i_j for sure.¹

We assume that in order to invest in the risk-free security, depositors must pay a transaction cost of $\eta \in [0, 1)$ per unit (this can be seen as a brokerage fee). Thus, the return from investing in the risk-free security for depositors is $r - \eta$, where $r > \eta$. As a result, depositors consider bank j if and only if the gross deposit rate on offer $i_j \geq r - \eta$. Since we are primarily interested in the asset decision of banks, we assume depositors supply funds infinitely elastically at the rate which leaves them indifferent between the risk-free asset and deposits. As such, banks can obtain all their financing needs at $i_j = r - \eta$ - the rate at which depositors will be indifferent between the two assets - and no bank will offer a greater deposit rate.

Each bank must determine the structure of its loan portfolio, where herein lies the potential moral hazard problem. There are two possible states of the world $\mathbf{s} = \{s_1, s_2\}$, and two assets. There exists a safe prudent asset (denoted asset 1), i.e. a risk-free bond that yields the gross return r for sure, and there exists a risky asset (denoted asset 2), which we can refer to as risky loans. Risky loans yield a higher gross return $R_2^h > r$ if state s_1 occurs, but $R_2^l = 0$ if state s_2 occurs.² State s_1 and s_2

¹For simplicity, as in Hellmann et al. [2000] and Repullo [2004], we assume the insurance premium is zero. Nevertheless, our results hold for any fixed insurance premium. Furthermore, it is worth noting that since there exists full deposit insurance, depositors never have an incentive to engage in a bank run (see Diamond and Dybvig [1983]).

²All results continue to hold if there is a small residual value in state s_2 , i.e. $R_2^l \in (0, 1)$. This however merely complicates the model with no additional insight. R_2^l is thus set to zero for

occur with probability $p \in (0, 1)$ and $(1 - p)$ respectively, and each asset pays off at the end of each period. In expected terms, although risky loans offer the potential for a superior private return $R_2^h > r$, as in Hellmann et al. [2000], it is assumed that they offer an inferior expected return relative to the bond: $pR_2^h + (1 - p)R_2^l = pR_2^h < r$.³ Hence, a mean-variance maximiser would always prefer the prudent bond to risky loans. The moral hazard arises in that if the gamble succeeds, the bank can earn a higher private return ($R_2^h > r$) without taking all of the risk.

Since changes to the risk-free rate have consequences for both the asset and liability side of the balance sheet, in order to isolate the mechanism behind switching between prudent and gambling actions, we assume that the return on the risky loan has a constant risk premium ξ over the risk-free rate, namely $R_2^h = r + \xi$. This assumption is relaxed in section 6.⁴

The asset structure of the economy is therefore one of partial segmentation. The risky asset (i.e. risky loans) is available only to banks, for example due to asymmetric information or regulation (see Challe et al. [2013]), whereas the risk-free bond is available to both. At the end of the period, the regulator inspects the balance sheets of all banks, closing those in negative equity and compensating their depositors. As noted in Hellmann et al. [2000], this setup is consistent with the regulatory structure that monitors “the risk-management system of the bank, rather than examining each individual financial transaction” (p. 152).

3.2 The problem

The objective for each bank is to maximise expected profits. Since we are interested in the asset choice of banks, we normalise the loan book to 1. Hence each bank chooses the optimal combination to hold of each asset. Denote by ω the amount invested in the safe asset. The end of period profit of a bank can be written as:⁵

parsimony.

³This assumption is relaxed in section 6 in which we allow the expected return to increase with risk. The key results are unchanged.

⁴Assuming a constant risk premium over r allows us to isolate the mechanism behind any switch between prudent and gambling actions. Clearly, if we allowed the return on risky loans to increase substantially following an interest rate rise, it would be optimal to switch into gambling, but this does not help us to understand the underlying mechanism at work. Indeed, one could argue that the return on a risk-free bond should increase by a greater amount relative to the return on riskier assets following an interest rate rise. This however would merely exacerbate our results, the qualitative results would be the same.

⁵The second state involves a maximum function due to the presence of limited liability.

$$\pi(\omega, i) = \begin{cases} \omega r + (1 - \omega)R_2^h - iD & \text{if } s = s_1 \\ \max\{\omega r - iD, 0\} & \text{if } s = s_2 \end{cases}$$

where D denotes deposits, and $D = 1 - k$, where k denotes bank capital.

Since there are two possible states of the world, the expectation will be conditional on the probability of each state occurring and the amount invested in each asset. The bank observes the values r , R_2^h , $R_2^l = 0$, and p before investment, so the only uncertainty here involves the potential state of the world. Formally, we can write each bank's problem as:⁶

$$\max_{\omega} \{E[\pi(\omega, i)]\} \tag{1}$$

subject to

$$\pi(\omega, i) = \begin{cases} \omega r + (1 - \omega)R_2^h - iD & \text{if } s = s_1 \\ \max\{\omega r - iD, 0\} & \text{if } s = s_2 \end{cases} \tag{2}$$

$$D + k = 1 \tag{3}$$

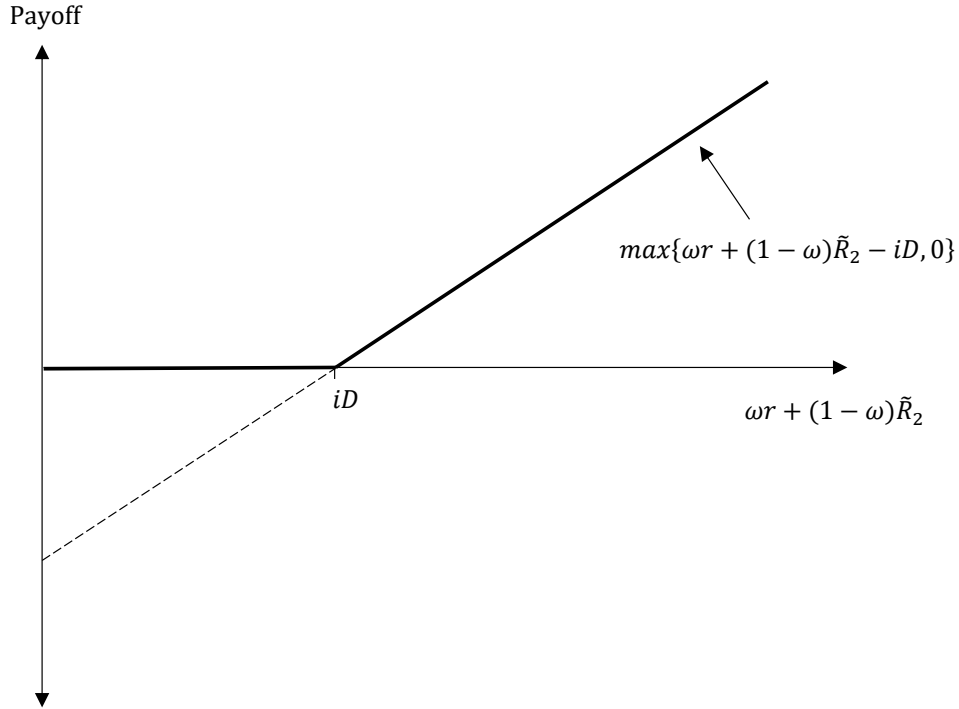
$$i \geq r - \eta \tag{4}$$

where $k \in [0, 1]$ is given. Equation 1 states that the bank's objective is to maximise expected profits. Equation 2 reiterates the state specific end of period profits. Equation 3 is the balance sheet constraint, while equation 4 is the depositors' participation constraint, since below a rate of $r - \eta$, no deposits will occur.

Recalling the discussion above, it is now possible to illustrate how one can think of the bank's payoff as having an option-like structure. The bank's end of period portfolio return will be $\max\{\omega r + (1 - \omega)\tilde{R}_2 - iD, 0\}$, where $\tilde{R}_2 = R_2^h$ if $s = s_1$, or

⁶Since all banks are identical, we drop the j subscript to denote bank j , and instead use a representative bank.

Figure 1: Payoff at the end of the period as a function of the return on assets



$\tilde{R}_2 = 0$ if $s = s_2$. It is as if the bank owns a European call option on the portfolio, with the strike price iD and maturity at the end of the period. Banks only exercise their right to buy the portfolio when its value at the maturity date, $\omega r + (1 - \omega)\tilde{R}_2$ is greater than the pre-established sale price, iD . Not to exercise the option is equivalent to going bankrupt and leaving assets and liabilities with the regulator. This is illustrated in figure 1.

4 Solution

Proposition 1 states the solution to the above problem. It illustrates that there exists two possible types of equilibria: one in which all banks invest only in risky loans (denoted the ‘gambling equilibrium’), and another in which all banks invest only in safe bonds (denoted the ‘prudent equilibrium’). We will use these terms throughout the paper, and denote the banks respectively as gambling banks and prudent banks.

Proposition 1. *There exist two types of equilibria:*

1. *A prudent equilibrium, in which the optimal portfolio satisfies $(\omega^*, 1 - \omega^*) =$*

(1, 0); and

2. A gambling equilibrium, in which the optimal portfolio satisfies $(\omega^*, 1 - \omega^*) = (0, 1)$.

The prudent equilibrium exists if $[R_2^h - (r - \eta)]p \leq \eta$. If $[R_2^h - (r - \eta)]p > \eta$, then the prudent equilibrium exists if and only if $k \geq \hat{k} \equiv \frac{[R_2^h - (r - \eta)]p - \eta}{(r - \eta)(1 - p)} \in (0, 1)$.

Proof. See appendix A.1. □

Proposition 1 illustrates that two possible types of symmetric equilibria exist and that both are corner solutions. In other words, we obtain a specialisation result as in Hellmann et al. [2000] and Repullo [2004]. Intuitively, this can be understood by considering a bank's payoff in each state of the world. Consider first a bank which holds a portfolio more heavily weighted towards risky loans such that the state s_2 payoff from their investment in the risk-free asset is insufficient to pay depositors. This means that whenever state s_2 arises, the bank enters bankruptcy. As a result, when considering payoffs from the two assets, the bank will only consider state s_1 payoffs (state s_2 payoffs are always zero). Since in state s_1 the payoffs are R_2^h and r , and $R_2^h > r$, the bank will not find it optimal to invest any amount in the prudent bond. This is true for all cases in which the bank defaults in state s_2 .

Consider now the bank shifting more and more of its portfolio into the risk-free bond. We know so long as the bank defaults in state s_2 , the bank will not wish to hold any of the prudent bond, but now consider the case in which the bank has shifted sufficiently into the bond such that it will also survive in state s_2 . In this case, the bank will consider the asset payoffs in both states of the world. In particular, since the bank is an expected profit maximiser, it will look at expected returns. We know $E[R_2] < r$, so if the bank holds a portfolio in which it is sufficiently invested in the risk-free bond so as to rule out bankruptcy in state s_2 , the bank will not find it optimal to hold any risky loans, since they offer a lower expected return. Accordingly, we can conclude that the bank will never hold a mixture of both assets; the portfolio will consist solely of either the prudent bond or risky loans.

Which equilibrium exists will depend on two conditions. First let us consider the sufficient condition. Proposition 1 states that if $[R_2^h - (r - \eta)]p \leq \eta$ banks will always invest solely into the risk-free bond. This is because when this condition holds, the risky loan is essentially a bad investment. The expected additional return $R_2^h p$ is not large enough compared to the guaranteed return of r on the risk-free

bond, and thus no bank invests into risky loans. Since banks are rational, there must exist a sufficient payoff from gambling. For example, as p approaches zero, the expected return on the risky loan approaches zero, and thus it is likely the bank will instead prefer the risk-free bond. When the risky asset offers a sufficient expected return however (i.e. $[R_2^h - (r - \eta)]p > \eta$) such that banks are enticed to consider investing, which equilibrium exists will depend on how much capital the bank has. This is because, when the risky asset is somewhat attractive, the bank will make its decision as if it has option value on the portfolio, and as will be seen, the level of capital determines the value of this option.

The bank knows that if it enters bankruptcy, it does not have to repay depositors. As a result, there exists an incentive to choose the gambling asset because it has a greater payoff in state s_1 . This option value arises from the fact that banks have limited liability. If a bank defaults in state s_2 , it will not receive any of the returns that arise in this state, but it will also not have to repay depositors. The safe asset is a better investment in expected terms (when both states of the world are considered), but conditional on default in state s_2 , the gambling asset yields a better expected return. This higher payoff in state s_1 , rather than the expected value over both states, has an influence because of the deposit-funded part of the bank's portfolio. These borrowed funds will not have to be repaid on bankruptcy, and given deposit insurance, depositors will not price risk.⁷ Since banks have limited liability, banks will weigh up, not the expected returns on each asset, but the conditional expected returns on each asset. This conditional expected return will be a function of k because firstly, the probability of failure is a function of k (the higher capital, the more likely the bank is to survive a state s_2 shock), and secondly, because on bankruptcy, losses will wipe out any bank capital, k , and banks prefer this not to occur. Hence, the higher k , the more banks will try to avoid this capital level being wiped out.

If the bank were an all-equity firm, it would never invest in the gambling asset, since it would always consider both states of the world. On the other hand, if the bank is an all-deposit firm and $[R_2^h - (r - \eta)]p > \eta$, it will always invest in the gambling asset, since given it is using entirely borrowed money (which does not have to be repaid on bankruptcy), it is optimal to gamble. The level of capital thus steers banks from one end of the spectrum to the other. The point at which the incentive to gamble starts to dominate depends on the relative size of deposits versus capital, with \hat{k} the tipping point. Above \hat{k} it is optimal to invest in the safe asset as the

⁷Since there is deposit insurance, depositors do not demand higher returns from banks for investing in riskier assets.

bank has sufficient capital (or ‘skin-in-the-game’ as it is known) to make the bank take into account both states of the world. Intuitively, since the bank cares about capital (since it cares about shareholders) and does not wish to lose its capital on bankruptcy, it will invest as if it were all-equity. Sufficient ‘skin-in-the-game’ makes banks consider enough of the distribution as to make them invest in the safe asset. This is a formalisation of the idea that bank capital or ‘skin-in-the-game’ reduces moral hazard. As Thompson [2012] states “in theory, skin in the game is a good way to prevent the bad loans that can originate” (p. 165). Indeed, forcing banks to hold more skin-in-the-game can rule out the bad equilibrium.

We can summarise this as the result of two opposing channels: a risk-taking channel and a bankruptcy channel. If banks have sufficient capital (i.e. $k > \hat{k}$), the bankruptcy channel will dominate as banks are funding a large proportion of their investment book with equity capital, and this incentivises them to consider both states of the world. The prudent equilibrium will exist in which all banks act prudently, since it is only by investing in the safe bond that banks can reduce their bankruptcy risk. Whenever $k < \hat{k}$ however, the opposite occurs. Since banks are funding little via capital, the risk-taking channel will dominate as banks take advantage of their option-like payoff structure. Banks have little capital and depositors do not price risk (due to deposit insurance), therefore banks will not fully internalise the downside risk of their portfolios; the worst that can happen is a zero payoff and the loss of the bank’s small capital buffer. When $k < \hat{k}$, the level of capital is not sufficient to entice prudent behaviour; the potential private benefit in terms of the extra return outweighs any potential loss of capital. Capital thus acts to internalise the externality, because if bank capital increases, a larger proportion of any loss is covered by bank capital. This can be seen in option terms. The pre-established sale price is iD , since this is what it costs the bank to invest in the portfolio of assets. Since $D = (1 - k)$, the option price is decreasing in k , so increasing k makes it more likely that the option will be exercised, and vice versa for large k .

This threshold value \hat{k} is a function of the risk-free rate. This is formalised in proposition 2. Proposition 2 shows that the threshold value \hat{k} is decreasing in the risk-free rate, r . Indeed, $\lim_{r \rightarrow \infty} \hat{k} = 0$, so for a given $k > 0$, for large enough r , in particular, for $r \geq \hat{r}$, the gambling equilibrium can be ruled out.

Proposition 2. *\hat{k} is decreasing in the risk-free rate, r . Indeed, for a given $k > 0$, we can rule out the gambling equilibrium for $r \geq \hat{r} \equiv \eta + \frac{p(\xi+\eta)-\eta}{(1-p)k}$.*

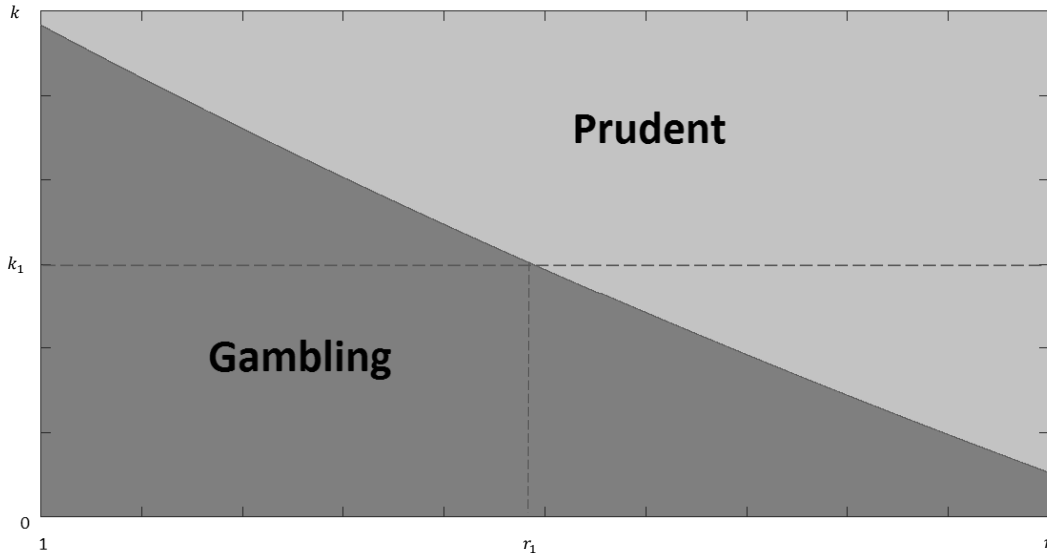
Proof. See appendix A.2. □

The reasoning behind this result lies in the trade-off banks face when determining whether to invest in the risky portfolio or the prudent portfolio. Consider the threshold bank with capital \hat{k} . The bank will be indifferent between the two portfolios, and if we consider this in option terms, the strike price on its portfolio will be $iD = i(1 - \hat{k})$. We know from above that $i = r - \eta$, thus when the interest rate, r , rises, the strike price increases making the call option less valuable. Given this option is on the gambling portfolio, this increase in r makes the gambling portfolio relatively less valuable compared to the prudent portfolio. Hence the threshold bank that was indifferent between the gambling and prudent portfolio, will instead prefer the prudent portfolio.

Intuitively, we can see this by considering the ex-post returns on the capital-funded part of the portfolio. Consider why banks invest in the prudent asset in the first place. As noted in proposition 1, when the risky asset is sufficiently attractive, banks only invest in the prudent asset if they are sufficiently concerned about losing their capital base. Raising the interest rate, is in effect another way to make banks sufficiently concerned about losing their capital, because for any given k , the return on this capital will be larger at higher interest rates. This will make banks less willing to gamble to avoid losing this now higher yielding capital. Remember, banks will invest both the deposits they raised plus their capital into the chosen portfolio. Hence, at the end of the period, on the capital-funded portion of their investment book, banks are hoping to receive the return rk or $R_2^h k$, depending on which asset they invest in. In expected terms, this translates into rk or $pR_2^h k$, which we know lies in favour of the prudent asset. Banks only choose to ignore this because they can more than compensate for this by gambling with depositors funds. As rates rise however, the return on the capital-funded portion of the investment book rises by more than the return on the deposit-funded part, because any increase in return on the deposit-funded part is offset by the increase in deposit costs. The return on the capital-funded portion thus becomes marginally more important in determining which portfolio to invest in. Furthermore, since the gambling portfolio only pays off with probability p , it will also be true that any increase in return will be subdued by p , but not so for the safe portfolio, which pays off with probability 1. At higher interest rates therefore, the capital level at which banks choose to shift into prudent action declines because the higher interest rate in effect makes all levels of capital more valuable (given this capital will now return more in the next period), and hence given that the threshold bank is now gambling with a more valuable portfolio, it will find it optimal to shift from gambling into prudent action.

Figure 2 illustrates this discussion. The figure shows when a bank will choose the

Figure 2: Prudent vs. Gambling equilibrium: relationship between k and r



prudent or gambling portfolio for a given combination of k and r . The diagonal line illustrates all the points at which the bank will be indifferent between the prudent and gambling portfolio. To the left of this line, banks will prefer to gamble, whereas to the right of the line, banks will prefer to play safe and invest in the prudent portfolio. The figure demonstrates that for a given k , there exists an r at which the gambling equilibrium can be ruled out. For example, as shown in the figure, if a bank has a capital level k_1 , the bank will not gamble if interest rates are set above r_1 . This then adds an additional tool to the regulator's arsenal. Past studies (e.g. Hellmann et al. [2000]) have shown that raising capital requirements can induce banks into prudent action. However raising capital requirements can be costly. Proposition 2 suggests that interest rates are also important in this decision; the required capital requirement is a declining function of the risk-free rate, so it can be decreased by raising the risk-free rate. Consequently, where the required capital level may be costly to enforce, the risk-free rate can be used to reduce this level to one that is more practical.

5 Risky bond portfolio

In order to explore our results further, we now alter the asset structure. Let us suppose the market is completely segmented in that only depositors have access to the risk-free bond. The bank still has access to two assets, however now the banks has access to a risky bond and a risky loan. Relative to the previous section, while

the risky loan is equivalent, the bond the bank previously had access to is no longer risk-free, it now has a probability of failure attached to it. Indeed, suppose that it has the same structure as the risky loan: it returns $R_1^h > 1$ with probability $p_1 \in (0, 1)$ and $R_1^l = 0$ otherwise. The bond is independent of the risky loan, which has an identical structure to before: it returns $R_2^h > 1$ with probability $p_2 \in (0, 1)$ and $R_2^l = 0$ otherwise. Furthermore, let us assume $p_1 > p_2$, so the bond is still safer in that it is more likely to pay off. Proceeding in a similar vein, we also assume that $R_2^h > R_1^h$, but $p_1 R_1^h > p_2 R_2^h$, so again, we can say that the riskier asset (i.e. risky loans) offers a greater ex-post return in the case of success, but it is an ex-ante inefficient investment (it has a lower expected return and higher variance). All else is left unchanged. Again, as before, since we are dealing with changes in the interest rate, for both assets we assume a constant risk premium over r (denoted by ξ_j , $j = 1, 2$). Namely, $R_1^h = r + \xi_1$, and $R_2^h = r + \xi_2$, where $\xi_2 > \xi_1$.

Propositions 3 and 4 provide the solution to the problem and illustrate the robustness of our previous results. The key results remain: banks can choose to take the maximal gamble if they have insufficient ‘skin-in-the-game’, and the interest rate can be used as a way to increase this ‘skin-in-the-game’, so that at lower interest rates banks are more likely to invest in gambling assets than at higher interest rates. Indeed, as before, proposition 4 again shows that for a given $k > 0$, there exists an \bar{r} above which the gambling equilibrium can be ruled out.

Proposition 3. *There exist two equilibria:*

1. *A prudent equilibrium in which the optimal portfolio satisfies $(\omega^*, 1 - \omega^*) = (1, 0)$; and*
2. *A gambling equilibrium in which the optimal portfolio satisfies $(\omega^*, 1 - \omega^*) = (0, 1)$.*

The prudent equilibrium exists if $p_2 R_2^h \leq p_1 R_1^h - (r - \eta)(p_1 - p_2)$. Otherwise, the prudent equilibrium exists if and only if $k \geq \bar{k}$, where $\bar{k} \equiv \frac{p_2 R_2^h - p_1 R_1^h + (r - \eta)(p_1 - p_2)}{(r - \eta)(p_1 - p_2)} \in (0, 1)$.

Proof. See appendix A.3. □

Proposition 4. *\bar{k} is decreasing in the risk-free rate, r . Also, for a given $k > 0$, we can rule out the gambling equilibrium for $r \geq \bar{r}(k)$, where $\bar{r}(k) \equiv \frac{\eta(1-k)}{k} + \frac{p_2 \xi_2 - p_1 \xi_1}{(p_1 - p_2)k}$.*

Proof. See appendix A.4. □

The intuition behind proposition 3 follows exactly the same logic as before. The bank trades off potentially greater ex-post returns from investing in a more risky loan portfolio with the cost that ex-ante the expected return on the risky loan is lower, and it will decrease the bank's probability of survival. The bank will only invest in the riskier asset if $p_2 R_2^h > p_1 R_1^h - (r - \eta)(p_1 - p_2)$, i.e. if the numerator of \bar{k} is positive. This is the sufficient condition given in proposition 3. As before, this sufficient condition essentially states that the additional return on the riskier asset must be high enough to make the gamble worth it. If the additional return is too low compared to investing prudently, or the probability of success, p_2 , is too low, it will never be optimal to invest in the riskier asset. If $p_2 R_2^h > p_1 R_1^h - (r - \eta)(p_1 - p_2)$ is satisfied however, whether the bank invests in the prudent or gambling portfolio depends on its capital level.

With little capital, the gambling option becomes more attractive, since if the bank enters bankruptcy, it has very little to lose in terms of capital. The option value inherent in the maximisation therefore drives banks to choose the risky loan portfolio. The risk-taking channel dominates since with little capital, the incentives to reduce the probability of bankruptcy are weak. As capital rises however, like before, the bank starts to act more like an all-equity bank in which it is always optimal to choose the more prudent asset portfolio. When $k \geq \bar{k}$, the bankruptcy channel begins to dominate and banks will invest prudently.

The interest rate acts to reinforce this behaviour since, as before, it increases the future value of the portfolio and particularly the value of the capital-funded part of the portfolio, thereby making the capital-funded part more important in the investment decision. Proposition 4 shows us that as in proposition 2, the threshold value \bar{k} is decreasing in the interest rate r , and for a given $k > 0$, the interest rate can be set sufficiently high to rule out the gambling equilibrium.

In optimum, the bank will increase risk up to the point at which the marginal revenue from increasing its risk level (thereby obtaining potentially greater returns from risky loans) equals the marginal cost. From an all-equity funded perspective, this trade-off is not favourable given the existence of the safer asset, and hence it is only the deposit-funded part that incentivises any investment in the risky asset. Raising the interest rate alters this trade-off because firstly, higher interest rates increase the portfolio return, thus making it more costly for banks to enter bankruptcy, as this higher return will be lost. Secondly, raising the interest rate marginally decreases the importance of the deposit-funded part in the investment decision because the increase in return on the capital-funded part of the portfolio is

greater. This is because any increase on the deposit-funded part is offset by rising deposit costs. The capital-funded part thus gains a greater impact in the marginal decision, and when considering the investment from the perspective of the capital-funded part, the prudent portfolio is the optimal investment. The threshold bank that was otherwise indifferent between the gambling and prudent portfolio, given the now more highly valued capital-funded portion of the investment book, therefore starts to prefer the prudent portfolio, since it does not need so much capital to incentivise prudent investment. Combining these results and those of the previous section, we can say that lower interest rates incentivise riskier investment.

6 Continuum of risk levels

Let us now relax the two asset case. We extend the model so as to fully endogenise the risk-taking decision of banks, and furthermore we allow the riskier asset to be mean superior. Suppose banks face a full spectrum of portfolios varying in their degree of risk. Along this spectrum of risk, banks are able to choose any risk level they desire. In particular, suppose each bank can choose a portfolio $0 \leq \sigma \leq \bar{\sigma}$, where a higher value of σ denotes a riskier portfolio, which returns $R(\sigma, r) \geq r$ with probability $p(\sigma) \in (0, 1]$, and $\alpha \in [0, 1)$ otherwise.⁸ Formally, we can write the gross return $\tilde{R}(\sigma, r)$ on a portfolio of type σ as given by:

$$\tilde{R}(\sigma, r) = \begin{cases} R(\sigma) & \text{with probability } p(\sigma) \\ \alpha & \text{with probability } [1 - p(\sigma)] \end{cases}$$

Under this extension, we impose the following assumptions:

Assumption 1

1. $R(0, r) = r$ and $p(0) = 1$ meaning if a bank chooses $\sigma = 0$, the bank is effectively choosing the risk-free asset.
2. $p_\sigma(\sigma) < 0$ meaning the probability of failure is increasing in risk-taking.⁹

⁸It is assumed that $\alpha < r - \eta$, otherwise banks can always repay depositors. The residual value is set to α to allow for the setup to also encapsulate a portfolio of risky and risk-free assets, or different risky assets.

⁹Where subscripts denote the derivative with respect to that variable.

3. $R_\sigma > 0$, $R_{\sigma\sigma} \leq 0$, $R_r > 0$ meaning the return on the portfolio in the good state increases with risk (and the risk-free rate), but at a declining rate.

Assumption 1 implies banks can trade-off additional return by increasing σ , but only at the expense of a lower payoff probability $p(\sigma)$.

We follow the same setup as before. Using the same notation, we can write the bank's maximisation problem as:

$$\begin{aligned} & \max_{\sigma} \{E[\pi(\sigma, i)]\} \\ \pi(\sigma, i) &= \begin{cases} R(\sigma, r) - iD & \text{with probability } p(\sigma) \\ \max\{\alpha - iD, 0\} & \text{with probability } [1 - p(\sigma)] \end{cases} \\ & i = r - \eta \\ & D + k = 1 \end{aligned}$$

where $k \in [0, 1]$ is given. Given the residual value, α , the bank's probability of default will depend on k . If $k \geq 1 - \frac{\alpha}{r-\eta}$, the bank will survive with probability 1, and the first order condition (FOC) that characterises the optimal risk level σ will be given by:

$$p(\sigma)R_\sigma(\sigma, r) = -p_\sigma(\sigma)[R(\sigma, r) - \alpha] \quad (5)$$

If $k < 1 - \frac{\alpha}{r-\eta}$, the bank will survive with probability $p(\sigma)$ since the residual value is not sufficient to repay depositors. The FOC that characterises the optimal risk-level σ will be given by:

$$p(\sigma)R_\sigma(\sigma, r) = -p_\sigma(\sigma)[R(\sigma, r) - (r - \eta)(1 - k)] \quad (6)$$

What is immediately apparent from the two FOCs, is that when the bank fails (because the residual value is not sufficient to repay depositors), it will take this into consideration when determining its optimal risk. This can be seen in the term $p_\sigma(\sigma)(r - \eta)(1 - k)$, which is apparent in equation 6, but not equation 5. In equation 6, banks know that by increasing risk (σ), they can decrease their probability of

success, $p(\sigma)$, and thus reduce the chance they will have to repay depositors. Also, given they enter bankruptcy in the bad state, they completely disregard the residual value α from consideration, since they will never receive this residual value. Either the portfolio pays off yielding R , or they enter bankruptcy. This is precisely the option value discussed earlier in section 3.

Considering the FOCs more carefully. The equations state that the bank will increase risk up to the point at which the marginal revenue from risk-taking equals its marginal cost. The marginal revenue is given by the left hand side (LHS), which is the same for both equations. Increasing risk (σ), increases the potential return on the portfolio $R(\sigma, r)$ since $R_\sigma(\sigma, r) > 0$. The right hand side (RHS) of the equations represent the marginal cost of taking more risk - i.e. the increased potential of losing $R(\sigma)$, since $p_\sigma(\sigma) < 0$. This cost is however slightly offset in both equations, firstly in equation 5 by the fact that in the bad state, the bank will receive the residual value α , and in equation 6, by the fact that if the bank enters bankruptcy in the bad state, it will not have to repay depositors. This ability to avoid repayment of depositors in the bad state is larger than the offset from the residual value α ,¹⁰ and thus the bank takes more risk in equation 6. The equations therefore show that when choosing the optimal level of risk, the bank will assess the increase in return it could achieve (shown on the LHS), against the cost of a lower probability of success and the potential loss of profits. The option value discussed previously is again apparent here.

Proposition 5. *In the model where banks can choose from a continuum of portfolios varying in their degree of risk and expected return.*

1. If $R_{\sigma r} \leq 0$, then:

(a) If $R_r \geq 1 - \frac{R_{\sigma r} p(\sigma)}{p_\sigma(\sigma)}$, then $\frac{d\sigma}{dr} \leq 0$, where if $R_{\sigma r} \leq \frac{p_\sigma(\sigma)}{p(\sigma)}$ this is always true.

(b) If $R_r < 1 - \frac{R_{\sigma r} p(\sigma)}{p_\sigma(\sigma)}$, then $\frac{d\sigma}{dr} \leq 0$ if and only if $k \geq \tilde{k}$

2. If $R_{\sigma r} > 0$, then:

(a) If $R_r \geq 1 - \frac{R_{\sigma r} p(\sigma)}{p_\sigma(\sigma)}$, then $\frac{d\sigma}{dr} \leq 0$

(b) If $R_r \in \left[-\frac{R_{\sigma r} p(\sigma)}{p_\sigma(\sigma)}, 1 - \frac{R_{\sigma r} p(\sigma)}{p_\sigma(\sigma)} \right)$, then $\frac{d\sigma}{dr} \leq 0$ if and only if $k \geq \tilde{k}$

(c) If $R_r < -\frac{R_{\sigma r} p(\sigma)}{p_\sigma(\sigma)}$, then $\frac{d\sigma}{dr} > 0$.

where $\tilde{k} \equiv \min \left\{ 1 - \frac{\alpha}{r-\eta}, 1 - R_r - \frac{R_{\sigma r} p(\sigma)}{p_\sigma(\sigma)} \right\} \in (0, 1]$

¹⁰Since equation 6 operates in the region where $k < 1 - \frac{\alpha}{r-\eta}$, it must be that $(r - \eta)(1 - k) > \alpha$.

Proof. See appendix A.5. □

Proposition 5 offers us an interesting insight into the actions of banks following a change in r when the choice is not so stark. While previously a shift in the optimal portfolio involved a large change - from taking the maximal gamble of only investing in risky loans to shifting the entire portfolio into safer bonds - here the choice is much finer. As a result, proposition 5 shows us that when banks can adjust risk along a continuum of levels, what is of key importance is not just the level of capital, but also the interaction between the sensitivity of the portfolio return to interest rates (R_r), and the sensitivity of the function that determines how much $R(\sigma, r)$ increases with larger σ at higher or lower interest rates ($R_{\sigma r}$). This is because as can be seen from the FOCs, this is what ultimately determines the optimal trade-off between risk and return, along with the capital level as seen in equation 6, which influences the bank's option value.

To understand this proposition, let us discuss the conditions in turn. First, consider the two conditions labelled (a) in proposition 5, and suppose for concreteness that there is an interest rate rise. Under both cases ($R_{\sigma r} \leq 0$ and $R_{\sigma r} > 0$), the conditions state that if the return on the portfolio in the good state, $R(\sigma, r)$, increases sufficiently following an interest rate rise, then banks will optimally reduce risk-taking. This supports the hypothesis that lower interest rates spur bank risk-taking in a search for yield type mechanism. When interest rates rise such that the portfolio return increases sufficiently, banks will optimally reduce their risk-taking since they no longer need to take these risks. It is as though previously the bank only took this risk in a search for yield. This mechanism is apparent here because since the portfolio return is sufficiently sensitive to interest rate rises, banks are not forced to increase risk to search for yield in an effort to meet higher deposit costs (which also rise with higher r). Instead banks can re-evaluate their optimal risk from the perspective that their portfolio is now more valuable, since it is more highly yielding, thereby strengthening the bankruptcy channel discussed before.

What is deemed sufficient will be a function of how the relationship between higher risk-taking (σ) and higher return $R(\sigma, r)$ (i.e. the payoff from higher risk-taking) changes as interest rates rise, i.e. $R_{\sigma r}$. This is what differentiates point 1 and 2 of proposition 5. If the payoff from higher risk-taking is diminishing in r or constant, i.e. $R_{\sigma r} \leq 0$, as in point 1, then the increase in portfolio return that is required to induce banks to decrease risk-taking at higher interest rates will be lower than otherwise. This is because the trade-off between higher risk-taking and return becomes less favourable (or unchanged) at higher interest rates. Since banks are now

gambling with a higher portfolio return, which may be lost on failure, the benefit of taking additional risk at these higher interest rates falls. Indeed, if $R_{\sigma r} \leq \frac{p_{\sigma}(\sigma)}{p(\sigma)} < 0$, the trade-off becomes so unfavourable that banks will always optimally reduce their risk-taking. Given the higher value portfolio, the bankruptcy channel strengthens, and it is optimal to decrease risk-taking. On the other hand, if at higher r , the payoff from risk-taking becomes more favourable, i.e. $R_{\sigma r} > 0$, as in point 2, this will strengthen the risk-taking channel. Thus, despite the higher R the bank is now gambling with (which strengthens the bankruptcy channel), given the risk-return trade-off is now more favourable, this strengthening of the risk-taking channel can outweigh the bankruptcy channel. Hence to incentivise banks not to risk-up requires the bankruptcy channel to be stronger than otherwise, and thus a larger increase in the portfolio return is required for the bankruptcy channel to dominate and for risk to decline.

Consider now condition (c) under point 2, and it is the mirror of our discussion above. The condition states that if the sensitivity of the portfolio return to an interest rate rise is too low, banks will increase risk-taking following an interest rate rise. This is precisely for the same arguments as above. If interest rates rise, deposit costs rise, and as a result, if the portfolio return does not rise sufficiently, profits will be squeezed. This weakens the bankruptcy channel, since banks have less to lose on bankruptcy, and thus in an attempt to offset this squeeze in profits, banks will take more risk. What is noticeable is that this condition only applies when $R_{\sigma r} > 0$. This is because if the payoff from risk-taking increases in the interest rate ($R_{\sigma r} > 0$), there will be a natural tendency to increase risk-taking as interest rates rise, since this strengthens the risk-taking channel: the risk-return trade-off is more favourable at higher interest rates rates. On the other hand, if the increase in portfolio return from taking on more risk declines (or stays constant) as interest rates rise ($R_{\sigma r} \leq 0$), since this weakens (or leaves unchanged) the risk-taking channel, while $R_r > 0$, thereby strengthening the bankruptcy channel, when $R_{\sigma r} \leq 0$, this condition does not apply. The condition would not be feasible as the RHS of the inequality would be negative.

The conditions labelled (a) and (c) thus take extreme points. The conditions labelled (b) can be seen as what occurs otherwise. The conditions suggest that what determines whether banks increase or decrease their risk-taking after an interest rate rise will be how much of the portfolio is funded with borrowed deposits. Following an interest rate rise, banks will decrease risk-taking if they have sufficient ‘skin-in-the-game’, k . This is because, with the portfolio return sensitivity in between the two extremes as laid out in conditions (a) and (c), the amount of capital banks hold will determine to what extent banks are hit by rising deposit costs, and thus how

much the bank is incentivised to compensate for this increase in deposit costs by increasing risk-taking.

In order to gain a fuller intuitive understanding of this, let us consider the two extremes of an all-equity funded bank, and an all-deposit bank.¹¹ An all-equity funded bank will be completely shielded from any increase in deposit rates. As a result, when interest rates rise, banks feel the full benefit of its effect on revenues and none of its disadvantages in cost. Consequently, at all levels of risk, returns will be higher. This means the bank is now gambling with a more valuable pot, hence any potential loss from gambling is now greater, and the marginal trade-off with respect to potentially losing all profit becomes less attractive. Even if the sensitivity of the portfolio return is at its lowest in this region, an all equity bank will find it optimal to decrease risk-taking, thereby making survival more probable. This is because with the portfolio now more valuable at higher interest rates, and in point 1, the risk-return trade-off less favourable at higher interest rates, the bankruptcy channel strengthens sufficiently that it is optimal to reduce risk-taking. It is as though the bank was only investing in the more risky portfolio because rates were low. At higher interest rates, the bank can reduce its risk without losing too much return and since the bankruptcy channel dominates when capital is high, this is optimal given the higher probability of success.

For an all-deposit bank, this logic is completely reversed. The bank is funding its entire portfolio with deposits, so when interest rates rise, the bank will be hit hard with rising deposit costs. This presents banks with a large incentive to offset this via risk-shifting; offsetting the cost with higher risk-taking. Given the portfolio return does not rise as much as in the conditions labelled (*a*), it may be that profits are squeezed, and this therefore gives an additional incentive to increase risk since there is less profit to be lost on bankruptcy. In addition, since the bank is entirely deposit financed, any further risk can be shifted onto deposit funds that do not have to be repaid on default. Hence, since the bank has little ‘skin-in-the-game’, the bankruptcy channel will be very weak, and this gives rise to a powerful option value on the portfolio that exacerbates the incentive to offset rising deposit costs with higher risk-taking. Indeed, for all values of R_r in this region, an all-deposit bank will increase risk-taking following an interest rate rise.

Intermediate values of k exhibit characteristics of both these scenarios, and the size of k will determine whether banks increase or decrease risk-taking, as this deter-

¹¹Importantly, as discussed in section 3, these depositors do not share in the profits of the bank. The setup thus differs from, for example mutual, or depositor-owned institutions, such as building societies in the UK.

mines whether the bankruptcy or the risk-taking channel dominates. When $k \geq \tilde{k}$, banks will decrease risk-taking following an interest rate rise as the incentives that influence an all-equity bank dominate over the incentives to risk-shift because of higher deposit rates. In other words, the bankruptcy channel dominates as banks become more concerned about losing this now higher portfolio return R . As before, \tilde{k} can be a function of $R_{\sigma r}$ and R_r for similar reasons.¹² If the portfolio return is less sensitive to interest rate changes, the threshold capital level will need to be higher to induce a reduction in risk-taking at higher r , since higher deposit rates will hit the bank more than otherwise. Equally, if the payoff from increasing risk-taking is more favourable at higher interest rates ($R_{\sigma r} > 0$), then there will be an additional incentive to increase risk-taking at higher interest rates, and thus, to outweigh this risk-taking incentive, a higher capital level must prevail so that banks are sufficiently concerned about bankruptcy.

These arguments mirror the discussions in sections 4 and 5. As discussed there, higher capital levels reduce the extent to which banks can increase risk without bearing all of the costs. With higher capital, the option value inherent in the maximisation declines, so the choice of risk moves closer to that of an all-equity bank. An interest rate rise can perform a similar function to an increase in capital itself, since when interest rates rise, the return on this capital at the end of the period will be larger, thus making banks less willing to gamble with it. This drives banks to reduce risk-taking at higher interest rates as was seen in sections 4 and 5 wherein the threshold value that determines prudent investment decreases. The additional insight that the above extension provides, is that when the decision can be a marginal change in risk-taking, if the capital level is too low, this incentive to decrease risk can be offset, and banks may marginally increase risk-taking following an interest rate rise. This is because although an increase in r makes capital more valuable (since its end of period return will be larger), given the low amount of capital, and hence large increase in deposit costs, this strengthening of the bankruptcy channel may not be sufficient to induce banks to reduce risk-taking at higher interest rates. Indeed, whenever $k < \tilde{k}$, banks will only decrease risk-taking following an interest rate rise if the portfolio return rises sufficiently to compensate and strengthen the bankruptcy channel. In other words, conditions (a) hold.

¹²This will be the case when $1 - \frac{\alpha}{r-\eta} > 1 - R_r - \frac{R_{\sigma r} p(\sigma)}{p_{\sigma}(\sigma)}$.

7 Two-period model

We now extend the model such that there exists two periods, $t = 0, 1$. This is done in order to capture the future franchise value of the bank, and how future profitability can impact risk-taking decisions today. We return to the initial asset structure of section 3 with two assets: one risk-free and the other risky. The two-period model proceeds as in section 3 with the exception that if banks survive the first period, there is a second period in which banks can earn further profit. As before, the per-period profit of a bank is given by: $\pi_t = \max \{ \omega_t r + (1 - \omega_t) R_2^h - i_t D_t, 0 \}$, $t = 0, 1$. At the end of the first period, banks can payout dividends to shareholders. Any remaining profit is carried over as capital into the following period. The extent to which banks can payout profit as dividends however is capped by a capital requirement, $k_1 \geq \underline{k}$, which banks must satisfy. All else is left unchanged.

At the beginning of the first period, banks are endowed with $k_0 \in [0, 1]$ and they observe the values r , R_2^h , $R_2^l = 0$, and $p \in (0, 1)$ that prevail throughout the two periods. Banks determine the structure of their asset portfolio at the beginning of each period. Uncertainty is revealed at the end of each period. We solve the model by backward induction, starting from the final stage.

The final stage will be identical to the static problem. Banks will have entered the period with a given $k_1 \in [0, 1]$, and they will maximise their expected profit as in section 4:

$$V = \max_{\omega_1} \{ E [\pi (\omega_1, i_1)] \} \quad (7)$$

where

$$\pi (\omega_1, i_1) = \begin{cases} \omega_1 r + (1 - \omega_1) R_2^h - i_1 D_1 & \text{if } s = s_1 \\ \max \{ \omega_1 r - i_1 D_1, 0 \} & \text{if } s = s_2 \end{cases}$$

$$D_1 + k_1 = 1$$

$$i_1 \geq r - \eta$$

As a result, the final stage solution will be identical to that of section 4, and the results of proposition 1 will continue to apply: the bank will invest either solely into

the gambling asset $(\omega_1^*, 1 - \omega_1^*) = (0, 1)$, or solely into the safe asset $(\omega_1^*, 1 - \omega_1^*) = (1, 0)$. The choice will depend firstly on whether $[R_2^h - (r - \eta)]p > \eta$, as if $[R_2^h - (r - \eta)]p \leq \eta$, the risky asset does not offer a sufficiently high expected payoff to induce investment, and no bank will invest in the risky asset (see section 4 for further details). If $[R_2^h - (r - \eta)]p > \eta$ however, the choice will depend on k_1 : the bank will invest solely into the safe asset when $k_1 \geq \hat{k}$, and solely into the risky gambling asset when $k_1 < \hat{k}$, where $\hat{k} \in (0, 1)$ is defined as in proposition 1. Given this, we can now solve the first stage.

From a first stage perspective, the addition of a future period brings two new concerns relative to the static problem. First, decisions today will have repercussions in terms of the probability of receiving any profit tomorrow. Default thus has a larger cost compared to the static model, since if banks enter bankruptcy in the first period, they will lose not only their capital, but also the future period of profit. Second, the bank must determine how much profit to reinvest next period. Foregoing dividends today for a higher capital level tomorrow can reduce the number of deposits needed tomorrow, and thus cost. The bank can therefore forego payout today for a higher payout tomorrow, and in doing so, also potentially increase their probability of survival tomorrow, as with higher capital, loss absorbency is increased. In the first stage therefore, the bank will need to determine firstly its optimal first stage asset portfolio, $(\omega_0, 1 - \omega_0)$, in the knowledge that this will influence its probability of survival, and thus also its probability of receiving any profit tomorrow. Secondly, the bank will need to determine how much it would like to payout in dividends (denoted by d) at the end of the period, in the knowledge that any remaining profit will be carried over as capital into the following period. The first stage problem can be written as follows:

$$\max_{\omega_0, k_1} \{E[d] + \beta Pr(\text{survival})V(k_1)\}$$

s.t.

$$d = \max \{\pi(\omega_0, i_0) - k_1, 0\} \tag{8}$$

$$k_1 \geq \underline{k} \tag{9}$$

$$\pi(\omega_0, i_0) = \begin{cases} \omega_0 r + (1 - \omega_0)R_2^h - i_0 D_0 & \text{if } s = s_1 \\ \max \{\omega_0 r - i_0 D_0, 0\} & \text{if } s = s_2 \end{cases}$$

$$D_0 + k_0 = 1$$

$$i_0 \geq r - \eta$$

where $\beta = \frac{1}{r}$ is the discount factor. The problem illustrates that the bank will determine its optimal investment portfolio in the knowledge that at the end of the period, the bank will pay out a proportion of any profit as dividends, with any remaining capital carried over into the following period. The problem further demonstrates how tomorrow's profit can influence decisions today, as risk-taking decisions today will impact the bank's probability of receiving any profit tomorrow, and hence tomorrow's expected profit, V , enters with a probability. Lemma 6 and proposition 7 present the solution to the first stage problem and show that firstly banks will always wish to payout the maximum they can in dividends (the capital constraint will bind), and that secondly, a familiar result arises: skin-in-the-game incentivises prudent investment in the first period.

Lemma 6. *Banks will payout the maximum dividend, thus the capital requirement will bind, $k_1^* = \underline{k}$.*

Proof. See appendix A.6. □

To understand this, consider the bank's choice when it determines dividends, and suppose there is no capital constraint. If the bank decides to payout dividends today, the bank will receive the full value, d . If on the other hand, the bank decides not to payout these dividends so that $d = 0$, but to carry these earnings over as capital tomorrow, k_1 , it will reduce the number of deposits needed tomorrow. Instead of raising unit 1 of deposits, it can raise $1 - k_1$, where k_1 is the amount of capital carried over. An additional unit of capital will thus save the bank $(r - \eta)$, i.e. the deposit rate. The bank trades off receiving the full value of 1 today, with a saving of $(r - \eta)$ tomorrow, however, since tomorrow is discounted, in today's terms, that saving is given by $\beta(r - \eta)$. This is smaller than 1, therefore the bank prefers to receive the dividend today. The cost of raising one additional unit of deposit tomorrow, $\beta(r - \eta)$, is smaller than the cost of foregoing one unit of dividend today. As a result, banks will want to payout all their profit as dividends and the capital constraint will bind.¹³

¹³There can also exist an incentive to carry over capital so that the bank has greater shock absorbercy in period two, but again banks will prefer to payout these as dividends in the first period. Lemma 6 shows that this is never optimal because the gain from changing the probability of default via higher capital is smaller than the cost of foregoing these dividends.

Banks also need to determine their optimal portfolio. This decision will be similar to the static framework, except for one additional concern: by investing in a riskier portfolio today, banks will reduce their probability of surviving to receive tomorrow's profit. Proposition 7 illustrates that a familiar result arises: when the risky asset is sufficiently attractive, banks either choose to invest solely into the safe asset, or solely into the risky asset, and this will depend on k . However, proposition 7 also adds an additional dimension that was not apparent before. If the profit in period two is sufficiently large, this can also act as skin-in-the-game so as to induce prudent action.

Proposition 7. *In the first stage, banks will invest either solely into risky loans, or solely into the safe asset. Banks will invest solely into the safe asset:*

1. *If $p[R_2^h - (r - \eta)] - \eta \leq 0$, or if $p[R_2^h - (r - \eta)] - \eta > 0$, then if tomorrow's expected profit is sufficiently large, i.e. $V \geq \hat{V} \equiv \frac{p[R_2^h - (r - \eta)] - \eta + k(1-p)}{\beta(1-p)}$.*
2. *Otherwise, banks will invest solely into the safe asset if and only if $k_0 \geq \bar{k}$, where $\bar{k} \equiv \frac{p[R_2^h - (r - \eta)] - \eta + k(1-p) - \beta(1-p)V(k)}{(r - \eta)(1-p)} \in (0, 1)$.*

Proof. See appendix A.7. □

Corollary 8. *The relationship between \bar{k} and r can be characterised as follows:*

1. *If $\bar{k} \geq \beta p \beta \{ [R_2^h - (r - \eta)] - k\eta \} \in (0, 1) \Rightarrow \frac{d\bar{k}}{dr} \leq 0$*
2. *If $\bar{k} \in [\beta \beta \eta (1 - \underline{k}), \beta p \beta \{ [R_2^h - (r - \eta)] - k\eta \})$, then $\frac{d\bar{k}}{dr} \leq 0$ if and only if $\underline{k} \geq \hat{k}$, where \hat{k} is defined as in proposition 1.*
3. *If $\bar{k} < \beta \beta \eta (1 - \underline{k}) \in (0, 1) \Rightarrow \frac{d\bar{k}}{dr} > 0$*

Proof. See appendix A.8. □

Proposition 7 carries forward our previous message: banks will act prudently if they have sufficient skin-in-the-game. Thus just as in the final period, skin-in-the-game incentivises prudent investment in the first period. Yet proposition 7 adds an additional dimension that was not apparent previously. In particular that skin-in-the-game can also take the form of future profitability. Indeed, proposition 7 shows that if $V \geq \hat{V}$, even if $k_0 = 0$ (i.e. the bank is all-deposit financed in period 1) and $p[R_2^h - (r - \eta)] > \eta$ (i.e. the risky asset is sufficiently attractive), the bank will still choose to invest in the safe prudent portfolio because it is sufficiently concerned

about losing the second period's profit that it does not gamble. Banks will act as if they had $k_0 \geq \bar{k}$ because once we extend the model to take into account a future period, banks begin to consider whether a risk today is worth potentially losing both their capital and future profit. Therefore, tomorrow's expected profit can play the pivotal role of skin-in-the-game; with profit tomorrow, banks have lower incentives to take risk today. If future profitability is lower however (if $V < \hat{V}$), then banks will only choose to invest in the prudent portfolio if their capital stock is high enough. This is because, with less concern for tomorrow, banks will essentially look at the problem from a one period perspective. Hence the same result as in proposition 1 applies. It is worth noting that again banks never wish to hold a mixture of both assets. This is for the same reasons as before, hence we will not repeat them.

Corollary 8 also shows us that as previously, the threshold value \bar{k} can be decreasing in the risk-free rate, however compared to proposition 2 this now also depends on where on the spectrum between 0 and 1, the threshold bank sits. To understand this, consider the impact of a change in interest rates on the bank's current and future return. Suppose there is an increase in the interest rate. The first impact will be a present day effect on current profitability (equivalent to that discussed in section 4). Increasing the interest rate increases the return on the portfolio, and in particular increases the value of any given capital level as it yields more. As a result, as in section 4, this incentivises banks to act more cautiously to protect their now more valuable portfolio and thereby incentivises the threshold bank to move towards the safe portfolio which has a higher probability of survival. This is an identical channel to that of proposition 2; it strengthens the bankruptcy channel.

Given there are now two periods however, there will also be a second impact from the higher interest rate tomorrow. With higher interest rates tomorrow, two opposing effects occur. First, a similar mechanism to that of the first period will occur in that the return on the portfolio tomorrow will be larger. Second, a higher interest rate means a higher discount rate, and thus in present value terms any future profit will be smaller. This means that any profit that comes tomorrow will have a lower impact on risk-taking decisions today. This discount rate effect is stronger and thus it completely offsets the effect from tomorrow's higher return. Consequently, this second impact leads to a weakening of the bankruptcy channel as tomorrow's returns become marginally less important for risk-taking decisions today.

The rationale behind corollary 8 is thus made up of the culmination of these two forces. The bankruptcy channel is strengthened by the force from today, whereas it is weakened via the more highly discounted returns tomorrow. Which effect dominates

depends on where between 0 and 1 the threshold bank sits. To understand this, consider as previously the two extremes of an all-equity financed bank and an all-deposit financed bank in the first period. An all-equity bank has a significant amount of capital. As a result, when interest rates rise the first effect on current profitability will be very strong. Capital levels become much more valuable as returns increase and thus there will be a large incentive to reduce risk-taking in an attempt to avoid losing this now more valuable capital. For an all-equity bank therefore, the first effect on current profitability will outweigh the effect from a higher discount rate. On the other hand, an all-deposit bank will see its higher return offset by rising deposit costs, and thus the first effect on current profitability will be weak. In this case therefore, the second effect - the higher discount of future profit - will drive risk-taking more than the first effect. These cases are extremes that can be seen in conditions 1 and 3 of the corollary 8 respectively. Therefore, when the threshold bank is a bank with an amount of capital $\bar{k} \geq \beta p \beta \{ [R_2^h - (r - \eta)] - \underline{k} \eta \}$, the sequence of events will be akin to that of an all-equity bank. The first effect will dominate and banks will decrease risk-taking at higher interest rates as the bankruptcy channel strengthens. On the other hand, when the threshold bank is a bank with little capital, i.e. $\bar{k} < \beta \beta \eta (1 - \underline{k})$, the sequence of events will be akin to that of an all-deposit bank. Most of the influence will come from the reduction in the present value of tomorrow's expected profit, and thus with the second effect dominating, the threshold bank will increase risk-taking by shifting to the gambling portfolio.

In the intermediary case, condition 2 shows us that the first effect will dominate (and banks will decrease risk-taking after an interest rate rise) if banks are forced to sufficiently take into consideration their period two return. This can be achieved via the capital requirement, \underline{k} , as by increasing the capital requirement, banks are forced to delay more of their payout until tomorrow. The condition states that if the capital requirement is sufficiently high (and thus a sufficient amount of the bank's payoff is pushed into the second period), i.e. $\underline{k} \geq \hat{k}$, then the bank will be forced to sufficiently take into consideration tomorrow's higher portfolio return that it will offset the higher discount rate effect enough to enable the first effect to dominate. Condition 2 shows that this occurs whenever $\underline{k} \geq \hat{k}$.

This discussion shows that there is an additional dimension that should be taken into consideration by regulators. The capital requirement which determines how much of the bank's profit it can payout as dividends, \underline{k} , will impact both risk-taking decisions today and tomorrow. First, since the last period is identical to the single period model, regulators can set the capital requirement so as to rule out the risky portfolio in the last stage, i.e. $\underline{k} \geq \hat{k}$. This however will also impact risk-taking

decisions in the first period. As discussed above, it can firstly influence the direction of risk-taking as interest rates rise or fall as shown in corollary 8, but as can be seen from the definition of \bar{k} , which is increasing in \underline{k} , the capital requirement will also influence the risk choice in period 1. This is because although it pushes the bank's payout into the future, thereby increasing the influence of any expected profit tomorrow on the bank's current risk-taking decision, by shifting today's payout to tomorrow, it reduces the pot with which the bank is gambling with (as tomorrow's returns are discounted). As a result, the bank becomes slightly less cautious as it has slightly less to lose. Hence it will require a slightly higher first period capital level to act prudently.

8 Conclusion

This paper presents an investigation into whether lowering the risk-free rate spurs banks to take on additional risk. It develops a stylised model in which banks borrow from depositors to invest into a portfolio of assets of different risk-types. Throughout the analysis, we find a consistent argument that skin-in-the-game deters risk-taking. This is because of the option-like structure of the payoff function which arises because of limited liability. The lowest possible return for banks is bounded from below by zero, therefore all their actions are influenced by this fact. Banks will not fully internalise the downside risks of their actions, banks may even choose to invest in assets that are mean-variance dominated. Skin-in-the-game aligns banks' incentives because it forces banks to internalise more of the downside risks. This skin-in-the-game can take the form of capital or future profitability, since with both forms of skin-in-the-game, the bank pays a price on bankruptcy.

The model suggests that raising the interest rate in effect reduces the option value inherent in the bank's payoff function, and thus reduces the incentive to invest in riskier assets. Intuitively, this can be seen as follows. Increasing the interest rate, increases the future return on the portfolio, therefore on bankruptcy, banks would lose a more valuable portfolio. This reduces the incentive to take risk. Furthermore, higher interest rates increase the value of any given capital level, as the return on this capital increases. This has the same effect as an outright increase in capital, which as shown, increases skin-in-the-game. The capital-funded part of the investment portfolio increases by more than the deposit-funded part since the deposit-funded part is offset by rising deposit costs. As a result, the capital-funded part becomes marginally more important in the investment decision, which incentivises banks

to act more prudently. The results therefore show that capital can be used in conjunction with the risk-free rate to induce prudent behaviour. Hence if the required level of capital to induce prudent behaviour is prohibitively high, we can incentivise prudent behaviour using a combination of capital requirements and rising interest rates.

The results can be summarised as the balance of two channels: (1) a risk-taking channel in which higher risk means higher return, and (2) a bankruptcy channel in which higher risk implies a lower probability of survival. As discussed above, skin-in-the-game is what determines the relative strengths of these channels. Hence in considering the impact of interest rates on bank risk-taking, regulators must weigh up how these channels are likely to impact risk-taking. Previous studies such as Hellmann et al. [2000] have shown that raising capital requirements can induce banks into prudent action, since it raises the relative strength of the bankruptcy channel, but our results suggest that it is not only capital that is important, future profitability can be equally important as can the interest rate. Thus actions which reduce future profitability or adjust the interest rate will have consequences for risk-taking today. The model however is highly stylised, so it would be beneficial in future research to consider further extensions, such as allowing banks to undertake their maturity transformation role,¹⁴ or incorporating supervisory action that may provide an alternative to mitigating risk.

¹⁴This would provide banks with an alternative source of risk-taking: liquidity risk.

References

- Viral V. Acharya and Hassan Naqvi. The seeds of a crisis: A theory of bank liquidity and risk-taking over the business cycle. *Journal of Financial Economics*, 106(2): 349–366, November 2012.
- Itai Agur and Maria Demertzis. Monetary policy and excessive bank risk taking. De Nederlandsche Bank Working Paper No. 271, December 2010.
- Yener Altunbas, Leonardo Gambacorta, and David Marques-Ibanez. Bank risk and monetary policy. *Journal of Financial Stability*, 6(3):121 – 129, 2010.
- Ignazio Angeloni and Ester Faia. A tale of two policies: Prudential regulation and monetary policy with fragile banks. Kiel Working Paper 1569, Kiel Institute for the World Economy, October 2009.
- Ben S. Bernanke and Alan S. Blinder. The federal funds rate and the channel of monetary transmission. *American Economic Review*, 82(4):pp. 901–921, September 1992.
- Ben S. Bernanke, Mark Gertler, and Simon Gilchrist. Chapter 21 the financial accelerator in a quantitative business cycle framework. volume 1, Part C of *Handbook of Macroeconomics*, pages 1341 – 1393. Elsevier, 1999.
- Claudio Borio and Haibin Zhu. Capital regulation, risk taking and monetary policy: a missing link in the transmission mechanism. *Journal of Financial Stability*, 8(4):236–251, 2012.
- Claudia M. Buch, Sandra Eickmeier, and Esteban Prieto. In search for yield? survey-based evidence on bank risk taking. Discussion Paper Series1: Economic Studies, 10, Deutsche Bundesbank, Research Centre, 2011.
- Edouard Challe, Benoit Mojon, and Xavier Ragot. Equilibrium risk shifting and interest rate in an opaque financial system. *European Economic Review*, 63:117–133, 2013.
- Simona Cociuba, Malik Shukayev, and Alexander Ueberfeldt. Financial intermediation, risk taking and monetary policy. Economic Policy Research Institute Working paper, 2011.
- Gianni De Nisco, Giovanni Dell’Ariccia, Luc Laeven, and Fabian Valencia. Monetary policy and bank risk taking,. IMF Staff Position Note 10/09, International Monetary Fund., 2010.

- Giovanni Dell’Ariccia and Robert Marquez. Lending booms and lending standards. *The Journal of Finance*, 61(5):2511–2546, 2006.
- Giovanni Dell’Ariccia, Luc Laeven, and Robert Marquez. Real interest rates, leverage, and bank risk-taking. *Journal of Economic Theory*, 149(1):65 – 99, 2014.
- Douglas W. Diamond and Philip H. Dybvig. Bank runs, deposit insurance, and liquidity. *Journal of Political Economy*, 91(3):401 – 419, June 1983.
- Douglas W. Diamond and Raghuram G. Rajan. Illiquid banks, financial stability, and interest rate policy. *Journal of Political Economy*, 120(3):552–591, June 2012.
- Mark L. Gertler and Simon Gilchrist. Monetary policy, business cycles and the behavior of small manufacturing firms. *Quarterly Journal of Economics*, 109(2): 309 – 340, May 1994.
- Thomas Hellmann, Kevin Murdock, and Joseph Stiglitz. Liberalization, moral hazard in banking, and prudential regulation: Are capital requirements enough? *American Economic Review*, 90(1):147 – 165, 2000.
- Vasso Ioannidou, Steven Ongena, and Jose-Luis Peydro-Alcade. Monetary policy, risk taking and pricing: evidence from a quasi-natural experiment. European Banking Center Discussion Paper No. 2009-04S., 2009.
- Gabriel Jimenez, Steven Ongena, and Jose-Luis Peydro. Hazardous times for monetary policy: What do twenty-three million bank loans say about the effects of monetary policy on credit risk-taking? *Econometrica*, 82(2):pp. 463–505, 2014.
- Anil K. Kashyap and Jeremy C. Stein. What do a million observations on banks say about the transmission of monetary policy? *American Economic Review*, 90 (3):pp. 407–428, 2000.
- Angela Maddaloni and Jose-Luis Peydro. Bank risk-taking, securitization, supervision, and low interest rates: Evidence from the euro-area and the u.s. lending standards. *Review of Financial Studies*, 24(6):2121 – 2165, 2011.
- Raghuram G. Rajan. Has financial development made the world riskier? NBER Working Papers 11728, National Bureau of Economic Research, Inc, November 2005.
- Rafael Repullo. Capital requirements, market power, and risk-taking in banking. *Journal of Financial Intermediation*, 13(2):156 – 182, 2004.

Eric Thompson. Dodd-frank and basel iii's skin in the game divergence and why it is good for the international banking system. *The Global Business Law Review*, 159(2):159 – 174, August 2012.

A Appendix

A.1 Proof of Proposition 1

The proof begins by illustrating that a bank would never wish to hold a mixture of both assets in the portfolio. The optimal choice will be either, all in the risky loan ($\omega = 0$), or all in the safe asset ($\omega = 1$).

Banks will choose ω so as to maximise $p[\omega r + (1 - \omega)R_2^h - i(1 - k)] + (1 - p)[\max\{\omega r - i(1 - k), 0\}]$. As can be seen, the choice of ω (for a given k) will determine whether the bank can survive state s_2 , so the probability of bankruptcy will be a function of ω . If $\omega r \geq i(1 - k)$, then the objective function simplifies to $\omega r + (1 - \omega)pR_2^h - i(1 - k)$. We know the bank will choose an interest rate i such that the depositors' participation constraint binds, so $i = (r - \eta)$ and the objective function is: $\omega r + (1 - \omega)pR_2^h - (r - \eta)(1 - k)$. This is clearly maximised at $\omega = 1$. So, if the bank chooses an ω such that $\omega r \geq i(1 - k)$, it must be that the maximum entails $\omega = 1$.

Now, suppose $\omega r < i(1 - k)$. As before, the depositors' participation constraint will bind in the maximum, so $i = (r - \eta)$. If this is the case, then the objective function becomes $p[\omega r + (1 - \omega)R_2^h - (r - \eta)(1 - k)]$. Given $R_2^h > r$, this is maximised at $\omega = 0$. So, if the bank chooses an ω such that $\omega r < i(1 - k)$, it must be that the maximum entails $\omega = 0$. This shows that the bank will never choose an interior solution, the maximum is a corner solution: banks will either invest all into the prudent asset $\omega^* = 1$ or all into the gambling asset $\omega^* = 0$.

The bank will prefer to invest solely into the prudent safe asset if and only if the expected profit is greater than or equal to the expected profit from gambling, namely, $r - (r - \eta)(1 - k) \geq p[R_2^h - (r - \eta)(1 - k)]$. Rearranging, this will be true if $[R_2^h - (r - \eta)]p \leq \eta$. Otherwise, this is true if and only if $k \geq \hat{k} \equiv \frac{[R_2^h - (r - \eta)]p - \eta}{(r - \eta)(1 - p)} = \frac{[\xi + \eta]p - \eta}{(r - \eta)(1 - p)}$. Lastly, rewriting \hat{k} as $\hat{k} = \frac{(R_2^h p - \eta) - (r - \eta)p}{(r - \eta) - (r - \eta)p}$, it is easily seen that since $R_2^h p < r$ and \hat{k} is defined in the region where $[R_2^h - (r - \eta)]p > \eta$, $\hat{k} \in (0, 1)$. ■

A.2 Proof of Proposition 2

Taking the derivative of \hat{k} , we find: $\frac{d\hat{k}}{dr} = -\frac{1}{(r - \eta)(1 - p)} \left\{ \frac{[R_2^h - (r - \eta)]p - \eta}{(r - \eta)} \right\}$. \hat{k} is defined in the region where $[R_2^h - (r - \eta)]p > \eta$, thus the term in curly brackets is positive. Furthermore, since $r > \eta$ and $p \in (0, 1)$, this implies that $\frac{d\hat{k}}{dr} < 0$.

Lastly, rearranging the inequality $k \geq \frac{[R_2^h - (r - \eta)]^{p - \eta}}{(r - \eta)(1 - p)}$ (given in proposition 1) in terms of r , we find the following: $r \geq \eta + \frac{p(\xi + \eta) - \eta}{(1 - p)^k} \equiv \hat{r}$. Thus for a given $k > 0$, we can rule out the gambling equilibrium for $r \geq \hat{r}$. ■

A.3 Proof of Proposition 3

The proof proceeds as in a similar manor to the proof of proposition 1. We firstly show that a bank will never choose a mixture of both assets, the optimal solution is a corner solution, and then we show which corner solution is optimal.

Given the discrete nature of the asset setup, there are four cases which can arise as regards to whether the bank will survive when a certain asset pays off. We consider each in turn. First, the bank may have a large proportion of its portfolio in asset 1, and thus it needs asset 1 to pay off for the bank to survive. If this is the case, then the expected profit function will be:

$$\begin{aligned} & \omega p_1 p_2 R_1^h + \omega p_1 (1 - p_2) R_1^h + (1 - \omega) p_1 p_2 R_2^h - p_1 p_2 (r - \eta)(1 - k) - p_1 (1 - p_2)(r - \eta)(1 - k) \\ & \Leftrightarrow \omega p_1 R_1^h + (1 - \omega) p_1 p_2 R_2^h - p_1 (r - \eta)(1 - k) \end{aligned} \quad (10)$$

Maximising this function, the optimal ω for this case is $\omega = 1$, since $p_1 R_1^h > p_1 p_2 R_2^h$.

Second, the bank may have a large proportion of its portfolio in asset 2, and thus need asset 2 to pay off for the bank to survive. If this is the case, then the expected profit function will be:

$$\begin{aligned} & \omega p_1 p_2 R_1^h + (1 - \omega) p_1 p_2 R_2^h + (1 - \omega) p_2 (1 - p_1) R_2^h - p_1 p_2 (r - \eta)(1 - k) - p_2 (1 - p_1)(r - \eta)(1 - k) \\ & \Leftrightarrow \omega p_1 p_2 R_1^h + (1 - \omega) p_2 R_2^h - p_2 (r - \eta)(1 - k) \end{aligned} \quad (11)$$

Maximising this function, the optimal ω for this case is $\omega = 0$, since $p_2 R_2^h > p_1 p_2 R_1^h$.

Third, the bank may have spread investment across both assets, and the param-

eters are such that the bank can survive if either of the two assets pay off. If this is the case, then the expected profit function will be:

$$\begin{aligned}
& \omega p_1 p_2 R_1^h + \omega p_1 (1 - p_2) R_1^h + (1 - \omega) p_1 p_2 R_2^h + (1 - \omega) p_2 (1 - p_1) R_2^h \\
& \quad - (r - \eta)(1 - k) [p_1 p_2 + p_1 (1 - p_2) + p_2 (1 - p_1)] \\
& \Leftrightarrow \omega p_1 R_1^h + (1 - \omega) p_2 R_2^h - [p_1 + p_2 (1 - p_1)] (r - \eta)(1 - k) \quad (12)
\end{aligned}$$

We show that banks will never wish to spread their investments like this, and would instead prefer the corner solution. To see this, take the first case in which banks choose $\omega = 1$. The expected profit will be given by equation 10 with $\omega = 1$, namely $p_1 R_1^h - p_1 (r - \eta)(1 - k)$. This is greater than the expected profit in equation 12 for any ω since $p_1 R_1^h > p_2 R_2^h$. Banks can increase their expected return by increasing ω to 1, plus since $p_1 < p_1 + p_2 (1 - p_1)$, banks also decrease their expected cost by decreasing their probability of survival. Holding a portfolio such that the expected return is given by equation 12 can thus never be optimal, the bank can improve its payoff by investing solely into asset 1.

Fourth, the bank may spread its investment over both assets, but given the parameter values, survival requires both assets to pay off. If this is the case, then the expected profit function will be given by:

$$\omega p_1 p_2 R_1^h + (1 - \omega) p_1 p_2 R_2^h - p_1 p_2 (r - \eta)(1 - k) \quad (13)$$

Again, we show that if this case exists, banks can increase their expected return by shifting to the corner solution, and thus a maximum solution cannot lie in this area. Take, the second case in which banks invest solely into the riskier asset ($\omega = 0$). In this second case, expected profit is given by equation 11 with $\omega = 0$, namely $p_2 R_2^h - p_2 (r - \eta)(1 - k)$. This is larger than the expected profit in equation 13 for any ω as can be seen by the following set of inequalities:

$$p_2 R_2^h - p_2 (r - \eta)(1 - k) > \omega p_1 p_2 R_1^h + (1 - \omega) p_1 p_2 R_2^h - p_1 p_2 (r - \eta)(1 - k)$$

$$\Leftrightarrow p_2 R_2^h - p_2 (r - \eta)(1 - k) + \omega p_1 p_2 R_2^h > \omega p_1 p_2 R_1^h + (1 - \omega) p_1 p_2 R_2^h - p_1 p_2 (r - \eta)(1 - k) + \omega p_1 p_2 R_2^h$$

$$\Leftrightarrow p_2 (1 - p_1) (R_2^h - (r - \eta)(1 - k)) + \omega p_1 p_2 (R_2^h - R_1^h) > 0$$

So, if this case exists, banks will never wish to spread investments across both assets, again, the bank can increase expected profit by choosing a corner solution.

The above thus illustrates that the optimal solution will be a corner solution, either $\omega = 0$ or $\omega = 1$. We now show that both corner solutions can be optimal. Banks will prefer the prudent portfolio consisting of solely the safer asset ($\omega = 1$) if and only if the expected profit from this portfolio is larger than or equal to the expected profit on the portfolio of choosing only the riskier asset ($\omega = 0$). Namely,

$$p_1 [R_1^h - (r - \eta)(1 - k)] \geq p_2 [R_2^h - (r - \eta)(1 - k)]$$

Rearranging, this simplifies to:

$$k \geq \bar{k} \equiv \frac{p_2 R_2^h - p_1 R_1^h + (r - \eta)(p_1 - p_2)}{(r - \eta)(p_1 - p_2)}$$

Since $k \geq 0$, this will always be the case if $p_2 R_2^h \leq p_1 R_1^h - (r - \eta)(p_1 - p_2)$. Otherwise, banks will prefer the prudent portfolio consisting of solely the safe asset ($\omega = 0$) if and only if $k \geq \bar{k}$. Lastly, since $p_2 R_2^h < p_1 R_1^h$, it is simple to see that $\bar{k} < 1$, and since \bar{k} is defined in the region where $p_2 R_2^h > p_1 R_1^h - (r - \eta)(p_1 - p_2)$, $\bar{k} > 0$. ■

A.4 Proof of Proposition 4

Taking the derivative of \bar{k} , we find $\frac{d\bar{k}}{dr} = -\frac{1}{(r - \eta)(p_1 - p_2)} \left\{ \frac{p_2 R_2^h - p_1 R_1^h + (r - \eta)(p_1 - p_2)}{(r - \eta)} \right\}$. Since $r > \eta$ and \bar{k} is defined in the region where $p_2 R_2^h > p_1 R_1^h - (r - \eta)(p_1 - p_2)$, the term in curly brackets is positive. Lastly, since $p_1 > p_2$, $\frac{d\bar{k}}{dr} < 0$.

Furthermore, rearranging the inequality $k \geq \frac{p_2 R_2^h - p_1 R_1^h + (r - \eta)(p_1 - p_2)}{(r - \eta)(p_1 - p_2)}$ (given in proposition 3) in terms of r , we find the following: $r \geq \frac{\eta(1 - k)}{k} + \frac{p_2 \xi_2 - p_1 \xi_1}{(p_1 - p_2)k} \equiv \bar{r}$. Thus for a given $k > 0$, we can rule out the gambling equilibrium for $r \geq \bar{r}$. ■

A.5 Proof of Proposition 5

As shown in the text, the optimal risk level when $k \geq 1 - \frac{\alpha}{r-\eta}$ will satisfy $p(\sigma)R_\sigma(\sigma, r) = -p_\sigma(\sigma)[R(\sigma, r) - \alpha]$, whereas when $k < 1 - \frac{\alpha}{r-\eta}$, the optimal risk level will satisfy: $p(\sigma)R_\sigma(\sigma, r) = -p_\sigma(\sigma)[R(\sigma, r) - (r - \eta)(1 - k)]$. The proof shows that the conditions laid out in proposition 5 apply to both these cases.

Suppose first that $k < 1 - \frac{\alpha}{r-\eta}$, then by the implicit function theorem, $\frac{d\sigma}{dr} \leq 0$ if and only if $p(\sigma)R_{\sigma r}(\sigma, r) + p_\sigma(\sigma)[R_r(\sigma, r) - (1 - k)] \leq 0$. Rearranging, this simplifies to: $R_r \geq 1 - \frac{p(\sigma)}{p_\sigma(\sigma)}R_{\sigma r} - k$. It is immediate that if $R_r \geq 1 - \frac{R_{\sigma r}p(\sigma)}{p_\sigma(\sigma)}$, this condition always holds and $\frac{d\sigma}{dr} \leq 0$. This is condition (a) in point 2 and the first half of condition (a) in point 1 of proposition 5. Taking this one stage further, it is clear that if $R_{\sigma r} \leq \frac{p_\sigma(\sigma)}{p(\sigma)} \leq 0$ then $R_r \geq 1 - \frac{R_{\sigma r}p(\sigma)}{p_\sigma(\sigma)}$ will always be true since $R_r \geq 0$. This is the second half of condition (a) in point 1 of proposition 5.

Equally, it is immediate that if $R_r < -\frac{R_{\sigma r}p(\sigma)}{p_\sigma(\sigma)}$, since $k \leq 1$, the condition $R_r \geq 1 - \frac{p(\sigma)}{p_\sigma(\sigma)}R_{\sigma r} - k$ can never hold, and thus $\frac{d\sigma}{dr} > 0$. This is point 2, condition (c) in proposition 5. However, also note that if $R_{\sigma r} \leq 0$, since $R_r \geq 0$, it would not be possible for $R_r < -\frac{R_{\sigma r}p(\sigma)}{p_\sigma(\sigma)}$. Hence if $R_{\sigma r} \leq 0$ condition (c) does not apply.

Lastly, suppose $R_r \in \left[-\frac{R_{\sigma r}p(\sigma)}{p_\sigma(\sigma)}, 1 - \frac{R_{\sigma r}p(\sigma)}{p_\sigma(\sigma)}\right)$, then the condition $R_r \geq 1 - \frac{p(\sigma)}{p_\sigma(\sigma)}R_{\sigma r} - k$ may or may not hold. Rearranging, we can show that it will hold if and only if $k \geq 1 - R_r - \frac{p(\sigma)}{p_\sigma(\sigma)}R_{\sigma r} \in (0, 1]$. Otherwise, the condition will not hold and $\frac{d\sigma}{dr} > 0$. This is condition (b) under points 1 and 2 of proposition 5, where under point 1, since $-\frac{R_{\sigma r}p(\sigma)}{p_\sigma(\sigma)} < 0$, but $R_r \geq 0$, we can simply write $R_r < 1 - \frac{R_{\sigma r}p(\sigma)}{p_\sigma(\sigma)}$.

Suppose now that $k \geq 1 - \frac{\alpha}{r-\eta}$, then by the implicit function theorem, $\frac{d\sigma}{dr} \leq 0$ if and only if $p(\sigma)R_{\sigma r}(\sigma, r) + p_\sigma(\sigma)R_r(\sigma, r) \leq 0$. Rearranging, this becomes: $R_r \geq -\frac{p(\sigma)}{p_\sigma(\sigma)}R_{\sigma r}$. Thus, when $k \geq 1 - \frac{\alpha}{r-\eta}$, $\frac{d\sigma}{dr} \leq 0$ if and only if $R_r \geq -\frac{p(\sigma)}{p_\sigma(\sigma)}R_{\sigma r}$, where if $R_{\sigma r} \leq \frac{p_\sigma(\sigma)}{p(\sigma)}$, this is always the case, otherwise, $\frac{d\sigma}{dr} > 0$.

Combining the two cases, we can state that if $R_r \geq 1 - \frac{R_{\sigma r}p(\sigma)}{p_\sigma(\sigma)} > -\frac{R_{\sigma r}p(\sigma)}{p_\sigma(\sigma)} \Rightarrow \frac{d\sigma}{dr} \leq 0$, where if $R_{\sigma r} \leq \frac{p_\sigma(\sigma)}{p(\sigma)} \leq 0$ this will always be the case. This proves conditions (a) under both point 1 and 2. On the other hand, if $R_{\sigma r} > 0$ and $R_r < -\frac{R_{\sigma r}p(\sigma)}{p_\sigma(\sigma)} \Rightarrow \frac{d\sigma}{dr} > 0$. This proves condition (c) under point 2. Lastly, if $R_r \in \left[-\frac{R_{\sigma r}p(\sigma)}{p_\sigma(\sigma)}, 1 - \frac{R_{\sigma r}p(\sigma)}{p_\sigma(\sigma)}\right)$, where this is equivalent to $R_r \in \left[0, 1 - \frac{R_{\sigma r}p(\sigma)}{p_\sigma(\sigma)}\right)$ when $-\frac{R_{\sigma r}p(\sigma)}{p_\sigma(\sigma)} < 0$, then under the second case (i.e. $k \geq 1 - \frac{\alpha}{r-\eta}$) $\Rightarrow \frac{d\sigma}{dr} \leq 0$, but under the first case (i.e. $k < 1 - \frac{\alpha}{r-\eta}$), $\frac{d\sigma}{dr} \leq 0$ if and only if $k \geq 1 - R_r(\sigma) - \frac{p(\sigma)}{p_\sigma(\sigma)}R_{\sigma r}(\sigma)$. So define \tilde{k} as $\tilde{k} \equiv \min \left\{ 1 - \frac{\alpha}{r-\eta}, 1 - R_r - \frac{R_{\sigma r}p(\sigma)}{p_\sigma(\sigma)} \right\}$, we can state that $\frac{d\sigma}{dr} \leq 0$ if and only if $k \geq \tilde{k}$. This proves conditions (b) under points 1 and 2 of proposition 5. ■

A.6 Proof of Lemma 6

There are two reasons why banks may want to carry over capital into the second period. First, banks may wish to forego dividends today to receive a higher return tomorrow. Second, banks may wish to increase their loss absorption tomorrow, so that they are more likely to survive in the second period. The proof proceeds by addressing each of these reasons in turn. We show that banks will never wish to carry any capital into the second period. Banks will carry over as little as they are allowed, namely the capital constraint will bind.

First, consider whether banks would wish to forego dividends today to receive them tomorrow. Suppose the bank is deciding between paying out dividends of d or carrying this over as retained earnings. Suppose there is no capital constraint. If the bank pays out these dividends at the end of the first period, the bank will receive d . If instead, the bank decides to forego this dividend payout and carry the payment over as capital, the bank will reduce deposit costs tomorrow since it will not need to raise as many deposits. Instead of raising deposits of 1, the bank can raise deposits of $1 - d$. The bank will thus save $id = (r - \eta)d$. Given this occurs tomorrow however, any saving will be discounted by β , thus banks will trade off a return of d today for $\beta(r - \eta)d$ tomorrow. Since $\beta = \frac{1}{r}$, $d > \beta(r - \eta)d$ and thus the bank will prefer to payout all profits as dividends in the first period.

Second, consider whether banks wish to forego dividends to increase their probability of survival in the last stage. We know that the bank will invest in the prudent portfolio in the last stage if and only if $k_1 \geq \hat{k}$, so if the capital requirement $\underline{k} \geq \hat{k}$, there will be no incentive to carry over additional capital as the bank cannot increase its probability of survival above 1. Hence, there can only exist an incentive in this regard if $\underline{k} < \hat{k}$. Furthermore, if $\underline{k} < \hat{k}$, given our argument in the previous paragraph, it can only be optimal for banks to carry over \hat{k} , as firstly any capital level above this will not influence the probability of survival, thus from above, the bank would prefer to payout this excess as dividends in the first period, and secondly, anything less will not be sufficient to affect the probability of default. The probability of default would stay the same and thus again by the previous paragraph, it would be optimal to payout this capital as dividends in the first period.

We show that even if $\underline{k} < \hat{k}$, banks will prefer to payout dividends and carry over \underline{k} than defer dividend payments and carry over \hat{k} . To see this, consider the cost of deferring these dividend payments. The bank will forego $(\hat{k} - \underline{k})$ in dividends which the bank could have paid out. In doing so however, the bank will shift

tomorrow's expected profit from $\beta p [R_2^h - (r - \eta)(1 - \underline{k})]$ to $\beta [r - (r - \eta)(1 - \hat{k})]$. In other words, tomorrow's expected profit will increase by $\beta [r - (r - \eta)(1 - \hat{k})] - \beta p [R_2^h - (r - \eta)(1 - \underline{k})]$, which the bank will prefer if and only if this is greater than the foregone dividend payment $(\hat{k} - \underline{k})$, namely $\beta [r - (r - \eta)(1 - \hat{k})] - \beta p [R_2^h - (r - \eta)(1 - \underline{k})] > (\hat{k} - \underline{k})$. Rearranging this expression, we find:

$$\beta\eta - \beta p [R_2^h - (r - \eta)] > (\hat{k} - \underline{k}) - \beta(r - \eta) (\hat{k} - p\underline{k})$$

$$\beta\eta - \beta p [R_2^h - (r - \eta)] > \hat{k} [1 - \beta(r - \eta)] - \underline{k} [1 - \beta(r - \eta)p] \quad (14)$$

Given $\hat{k} > 0$, otherwise banks will always choose the safe asset and therefore there would be no incentive to withhold dividends, the LHS of the inequality in equation 14 is negative. On the other hand, the RHS is linearly decreasing in \underline{k} . It is clear that when \underline{k} is small, the RHS will be positive and this inequality cannot hold. Hence, banks will not wish to defer these dividend payments. We show that even when \underline{k} is just above its maximum in this region (where we know from above that we only have to consider $\underline{k} < \hat{k}$), the inequality will not hold. Suppose $\underline{k} = \hat{k}$. Plugging this into the above equation, and rearranging, we find that for banks to wish to forego dividends and carry them into the following period, it must be that: $\hat{k} > \frac{p[R_2^h - (r - \eta)] - \eta}{(r - \eta)(1 - p)}$. But the RHS of this inequality equals \hat{k} by definition, so \hat{k} cannot possibly be greater than this. Therefore, given the inequality does not hold when $\underline{k} = \hat{k}$, and the RHS of equation 14 is linearly decreasing in \underline{k} , it cannot hold for any $\underline{k} < \hat{k}$.

Taking all the results together, we can state that banks will never wish to forego paying out dividends to carry them into the next period, and thus if there exists a capital requirement, it will bind. ■

A.7 Proof of Proposition 7

The proof proceeds in a similar fashion to the proof of proposition 1. In the first stage, banks will choose ω so as to maximise $p[\omega r + (1 - \omega)R_2^h - (r - \eta)(1 - k_0) - k_1] + (1 - p)[\max\{\omega r - (r - \eta)(1 - k_0) - k_1, 0\}] + Pr(\text{survival})\beta V(k_1)$.¹⁵ k_0 is given,

¹⁵As in section 3, the depositors' participation constraint must bind in optimum, thus $i_0 = r - \eta$. If $i_0 > r - \eta$, banks could offer a lower rate i_0 and still receive funding, and if $i_0 < r - \eta$, no depositor will deposit at the bank.

and from lemma 6 we know that banks will always choose $k_1 = \underline{k}$. Thus the bank's choice ω will determine whether the bank can survive state s_2 in the first period, and therefore ω will determine both the probability of obtaining the end of period dividend payout, and the probability of proceeding into the following period.

If the bank decides to choose an ω such that $\omega r \geq (r - \eta)(1 - k_0) - \underline{k}$, then the objective function simplifies to $\omega r + (1 - \omega)pR_2^h - (r - \eta)(1 - k_0) - \underline{k} + \beta V(\underline{k})$. This is maximised at $\omega = 1$, so if the bank chooses an ω such that $\omega r \geq (r - \eta)(1 - k_0) - \underline{k}$, it will always be $\omega = 1$. Now, suppose $\omega r < (r - \eta)(1 - k_0) - \underline{k}$. If this is the case, then the objective function simplifies to $p[\omega r + (1 - \omega)R_2^h - (r - \eta)(1 - k_0) - \underline{k}] + \beta pV(\underline{k})$. Given $R_2^h > r$, this is maximised at $\omega = 0$. So, if the bank chooses an ω such that $\omega r < (r - \eta)(1 - k_0) - \underline{k}$, it will always be $\omega = 0$. This shows that the bank will never choose an interior solution, the maximum is a corner solution. Banks will either invest all into the prudent asset $\omega^* = 1$ or all into the gambling asset $\omega^* = 0$.

The bank will prefer to invest solely into the safe asset if and only if the expected profit from doing so is greater than the expected profit from investing solely into the risky asset, i.e. $[r - (r - \eta)(1 - k_0) - \underline{k}] + \beta V(\underline{k}) \geq p[R_2^h - (r - \eta)(1 - k_0) - \underline{k}] + \beta pV(\underline{k})$. Rearranging, this is true if and only if $k_0 \geq \bar{k} \equiv \frac{p[R_2^h - (r - \eta)] - \eta + \underline{k}(1 - p) - \beta(1 - p)V(\underline{k})}{(r - \eta)(1 - p)}$. From this, we can derive the conditions given in proposition 7. First, consider $p[R_2^h - (r - \eta)] \leq \eta$. If this is the case, we know from proposition 1 and backward induction that in the final stage the bank will invest solely into the safe asset. Thus the numerator of \bar{k} becomes $p[R_2^h - (r - \eta)] - \eta + \underline{k}(1 - p) - \beta(1 - p)[\eta + (r - \eta)\underline{k}] = p[R_2^h - (r - \eta)] - \eta - (1 - p)\beta\eta(1 - \underline{k})$. Hence, if $p[R_2^h - (r - \eta)] \leq \eta$, the numerator will be negative and since $r > \eta$ and $p \in (0, 1)$ so that the denominator of \bar{k} is positive, banks will always invest solely in the safe asset since $k_0 \geq 0$. If $p[R_2^h - (r - \eta)] > \eta$ however, since $k_0 > 0$, if $V \geq \hat{V} \equiv \frac{p[R_2^h - (r - \eta)] - \eta + \underline{k}(1 - p)}{\beta(1 - p)}$, the numerator will again be negative and banks will invest solely in the safe asset. Otherwise, if neither of these conditions hold, banks will invest solely in the safe asset if and only if $k_0 \geq \bar{k}$. Lastly, in this region it is clear that $\bar{k} > 0$. To see that $\bar{k} < 1$, first suppose in the final stage that the bank invests solely in the safe asset. If so, then the numerator of \bar{k} can be rewritten as $p[R_2^h - (r - \eta)] - \eta + \underline{k}(1 - p) - \beta(1 - p)[\eta + (r - \eta)\underline{k}]$. It can be easily seen that this is smaller than the denominator $(r - \eta)(1 - p)$, i.e. $p[R_2^h - (r - \eta)] - \eta + \underline{k}(1 - p) - \beta(1 - p)[\eta + (r - \eta)\underline{k}] < (r - \eta)(1 - p)$. Rearranging, this becomes: $pR_2^h - r + (1 - p)\beta\eta(\underline{k} - 1) < 0$, which is always true since $\underline{k} \leq 1$ and $pR_2^h < r$. Second, suppose in the final stage that the bank invests solely in the risky asset. If so, then the numerator of \bar{k} can be written as

$p[R_2^h - (r - \eta)] - \eta + \underline{k}(1 - p) - \beta(1 - p)p[[R_2^h - (r - \eta)] + (r - \eta)\underline{k}]$. To see that this is strictly less than $(r - \eta)(1 - p)$, rearrange the inequality such that it is written as $(pR_2^h - r) - \beta(1 - p)p(R_2^h - (r - \eta)) + \underline{k}(1 - p)(1 - \beta p(r - \eta)) < 0$. This is strictly increasing in \underline{k} so if we set $\underline{k} = 1$ and the inequality holds, it will hold for all $\underline{k} \in [0, 1]$. Setting $\underline{k} = 1$, the condition simplifies to: $(R_2^h - r)[1 - \frac{(1-p)}{r}] < 0$ since $(1 - p) < 1 \leq r$. So, whether the banks invests solely into the risky asset or the safe asset in the final stage, the numerator of $\bar{k} < 1$. ■

A.8 Proof of Corollary 8

By proposition 7, \bar{k} is defined by the following equation: $\bar{k}(r - \eta)(1 - p) - p[R_2^h - (r - \eta)] + \eta - \underline{k}(1 - p) + \beta(1 - p)V(\underline{k}) = 0$. Differentiating with respect to r , we find: $\frac{d\bar{k}}{dr} = -\frac{\bar{k} - \frac{1}{r^2}V(\underline{k}) + \beta(\partial V(\underline{k})/\partial r)}{(r - \eta)}$. Given the bank behaves differently in the second period depending on whether $\underline{k} < \hat{k}$ or $\underline{k} \geq \hat{k}$, which influences the value of $V(\underline{k})$, we consider each case in turn. First, suppose $\underline{k} < \hat{k}$, then: $\frac{d\bar{k}}{dr} = -\frac{\bar{k} - \frac{1}{r^2}p[R_2^h - (r - \eta) + (r - \eta)\underline{k}] + \beta kp}{(r - \eta)}$. Rearranging this, we find:

$$\frac{d\bar{k}}{dr} = -\frac{\bar{k} - \beta p \{ \beta [R_2^h - (r - \eta)] - \underline{k}\beta\eta \}}{(r - \eta)} = -\frac{\bar{k} - \beta p \beta \{ [R_2^h - (r - \eta)] - \underline{k}\eta \}}{(r - \eta)}$$

$r > \eta$ and the term in curly brackets $\{ [R_2^h - (r - \eta)] - \underline{k}\eta \}$ is positive for all $\underline{k} \in [0, 1]$ since $R_2^h > r$. Thus, $\frac{d\bar{k}}{dr} \leq 0$ if and only if $\bar{k} \geq \beta p \beta \{ [R_2^h - (r - \eta)] - \underline{k}\eta \} \in (0, 1)$.

Suppose now, that $\underline{k} \geq \hat{k}$, then $\frac{d\bar{k}}{dr} = -\frac{\bar{k} - \frac{1}{r^2}[(r - \eta)\underline{k} + \eta] + \beta k}{(r - \eta)}$. Rearranging this, we find:

$$\frac{d\bar{k}}{dr} = -\frac{\bar{k} - \beta \{ \beta\eta - \underline{k}[1 - \beta(r - \eta)] \}}{(r - \eta)} = -\frac{\bar{k} - \beta\beta\eta(1 - \underline{k})}{(r - \eta)}$$

Again, $r > \eta$, so $\frac{d\bar{k}}{dr} \leq 0$ if and only if $\bar{k} \geq \beta\beta\eta(1 - \underline{k}) \in (0, 1)$.

Comparing the two conditions on \bar{k} , it can be seen that $\beta p \beta \{ [R_2^h - (r - \eta)] - \underline{k}\eta \} > \beta\beta\eta(1 - \underline{k})$. Rearranging, this becomes: $p[R_2^h - r] - (1 - p)\eta(1 - \underline{k}) > 0$ which is always positive. So, we can state that if $\bar{k} \geq \beta p \beta \{ [R_2^h - (r - \eta)] - \underline{k}\eta \} \Rightarrow \frac{d\bar{k}}{dr} \leq 0$. If $\bar{k} \in [\beta\beta\eta(1 - \underline{k}), \beta p \beta \{ [R_2^h - (r - \eta)] - \underline{k}\eta \})$, then $\frac{d\bar{k}}{dr} \leq 0$ if and only if $\underline{k} \geq \hat{k}$. Whereas if $\bar{k} < \beta\beta\eta(1 - \underline{k}) \Rightarrow \frac{d\bar{k}}{dr} > 0$. ■