



BANK OF ENGLAND

# Staff Working Paper No. 770

## Macroprudential capital regulation in general equilibrium

Benjamin Nelson and Gabor Pinter

December 2018

Staff Working Papers describe research in progress by the author(s) and are published to elicit comments and to further debate. Any views expressed are solely those of the author(s) and so cannot be taken to represent those of the Bank of England or to state Bank of England policy. This paper should therefore not be reported as representing the views of the Bank of England or members of the Monetary Policy Committee, Financial Policy Committee or Prudential Regulation Committee.



BANK OF ENGLAND

# Staff Working Paper No. 770

## Macroprudential capital regulation in general equilibrium

Benjamin Nelson<sup>(1)</sup> and Gabor Pinter<sup>(2)</sup>

### Abstract

We examine macroprudential bank capital policy in a macroeconomic model with a financial accelerator originating in the banking sector. Under Ramsey-optimal policy, the bank capital buffer tracks closely a model-based measure of the credit gap, defined as the gap between equilibrium credit in the economy featuring financial frictions and that in a hypothetical frictionless economy. Simple rules that vary the capital buffer in response to the credit gap perform worse than Ramsey policy, but only modestly so. When monetary policy controls inflation less aggressively, optimal macroprudential responses are smaller. Optimal macroprudential policy operates at a lower frequency than monetary policy.

**Key words:** Macroprudential policy, bank capital, monetary policy.

**JEL classification:** E5, G2.

---

(1) Rokos Capital.

(2) Bank of England. Email: [gabor.pinter@bankofengland.co.uk](mailto:gabor.pinter@bankofengland.co.uk)

This paper contains the views of the authors and not necessarily of the Bank of England. We are grateful to David Aikman and Andrew Haldane for numerous conversations on these topics over the years, and to participants at the Royal Economic Society Annual Conference 2014 for comments and suggestions. The views expressed in this paper are those of the authors and do not necessarily represent the views of the Bank of England, the Monetary Policy Committee, Financial Policy Committee or Prudential Regulation Committee.

The Bank's working paper series can be found at [www.bankofengland.co.uk/working-paper/staff-working-papers](http://www.bankofengland.co.uk/working-paper/staff-working-papers)

Bank of England, Threadneedle Street, London, EC2R 8AH  
Telephone +44 (0)20 3461 4030 email [publications@bankofengland.co.uk](mailto:publications@bankofengland.co.uk)

© Bank of England 2018  
ISSN 1749-9135 (on-line)

# 1 Introduction

The ten years since the global financial crisis of 2008 have witnessed a major overhaul of financial regulation. Bank capital standards have been raised substantially, their liquidity positions strengthened, and the complexity that characterised the trading of financial instruments within the financial system has been simplified.<sup>1</sup> Alongside these structural developments, responsibilities for macroprudential oversight of the banking sector have been established in numerous jurisdictions. Given this, attention is increasingly turning to the operation of cyclical macroprudential policy – macroprudential measures, including countercyclical bank capital requirements, aimed at stabilising the supply of financial services to the real economy over the economic cycle.

Unlike monetary policy, the ‘science’ of cyclical macroprudential policy is in its infancy.<sup>2</sup> In this paper we use a dynamic macroeconomic model featuring a banking sector to push this agenda forward. In our model, financial intermediaries - banks - channel funds from households to firms. Their ability to do this is limited, however, by the value of their net worth, which in turn is influenced by the value of the collateral assets they hold on their balance sheets. When asset values rise, so does banks’ net worth. By relaxing banks’ funding constraints, this creates an endogenous expansion in credit supply - a financial accelerator.<sup>3</sup> In this setting, macroprudential bank capital requirements that raise banks’ funding costs as collateral values rise can help ameliorate the pro-cyclicality of the banking system and stabilise the provision of credit to the real economy.

How countercyclical should macroprudential bank capital policy be? We explore this by taking household welfare as the objective of the macroprudential authority – in effect making cyclical macroprudential policy an arm of macroeconomic stabilisation policy.<sup>4</sup> We show that (Ramsey) optimal macroprudential policy under commitment is countercyclical in response to shocks to technology, the natural interest rate, and bank capital, raising capital

---

<sup>1</sup>See, *inter alia*, Carney (2014); Yellen (2017); Draghi (2017).

<sup>2</sup>The term is borrowed from Clarida, Gali, and Gertler (1999).

<sup>3</sup>See Bernanke, Gertler, and Gilchrist (1999) and Kiyotaki and Moore (1997). See also He and Krishnamurthy (2013); Brunnermeier and Sannikov (2014, 2016) that explore the global behaviour of the financial accelerator mechanism.

<sup>4</sup>See for example Haldane (2014).

requirements as output and credit rise, and relaxing requirements as they fall. This is because these macroeconomic disturbances are typically amplified by the financial accelerator mechanism arising from the financial sector. Macroprudential policy can lean against these effects.

This remains the case when nominal as well as financial frictions distort the economy. As monetary policy becomes less aggressive in stabilising inflation, however, so too does optimal macroprudential policy. This is because nominal rigidities attenuate the effects of macroeconomic shocks, dampening the effects of the financial accelerator mechanism, helping to stabilise the arguments of the macroprudential policymaker's objective function.

The optimal co-movement of monetary and macroprudential capital requirements depends on the source of the shock. Monetary and macroprudential policy sometimes moves in opposite directions, and sometimes move in the same direction. For example, positive supply shocks lower inflation, calling for a monetary loosening. However, these shocks are typically amplified by the financial accelerator, calling for a macroprudential tightening. There is no contradiction in these seemingly conflicting policy responses. Equally, because shocks arising from within the banking system itself cause inefficient fluctuations in the economy, monetary and macroprudential policies optimally co-move positively in the face of such disturbances. Negative bank capital shocks call on monetary policy to support demand and inflation and macroprudential policy to support the flow of credit to the supply side of the economy.<sup>5</sup>

A striking feature of optimal macroprudential policy under commitment is the close co-movement it delivers between the capital buffer and the model-based measure of the 'credit gap'. The practical use of the credit gap has been pioneered by e.g. [Basel Committee on Banking Supervision \(2010\)](#). We provide a theoretical definition of this gap, in the context of our model, as the gap between equilibrium credit in the economy distorted by financial frictions, and its counterpart in an economy without financial frictions.<sup>6</sup> This observation suggests simple rules based on the credit gap could provide useful practical guides to the

---

<sup>5</sup>See also, for example, [Collard, Dellas, Diba, and Loisel \(2017\)](#).

<sup>6</sup>Clearly this is distinct from the BCBS credit gap concept, which effectively de-trends the credit to GDP ratio. Our theoretical concept is closer to de-trended credit, where the trend in question is that defined by the 'efficient' path for the economy.

complex problem of Ramsey optimal policy under commitment. Both when prices are flexible and sticky, we find that the policymaker's losses associated with moving from the optimal policy to one based on a simple optimised credit gap rule are larger but only modestly so. Under our calibration, for example, losses could be between 4 and 6% larger. This finding has practical relevance because of the prevalence of measures similar to the credit gap in proposed policy guides for macroprudential policy.

We derive these results using a second-order approximation to household welfare, in common with the monetary policy literature. To do this, from a modelling perspective we effectively assume that structural reforms to financial regulation are or will be successful in eliminating the steady state distortions arising from the configuration of the financial sector. This is a strong assumption but it allows for a clean analogy to be drawn between cyclical macroprudential policy and monetary policy. Moreover, it allows us to extend the economic environment we study to include the consequences of nominal rigidities with which monetary authorities are concerned. This allows us to establish that our results regarding the cyclicity of macroprudential policy are robust to a more realistic setting in which monetary policy also has real effects around an efficient steady state.

**Related literature** Our work is closely related to the burgeoning literature examining the role of macroprudential policy in moderating macroeconomic volatility in quantitative macroeconomic models. That literature has developed both theoretical and empirical strands.<sup>7</sup>

An empirical literature examines indicators relevant for constructing macroprudential policy rules. These include [Drehmann, Borio, and Tsatsaronis \(2011\)](#), [Schularick and Taylor \(2012\)](#), [Aikman, Haldane, and Nelson \(2015\)](#) and [Giese, Andersen, Bush, Castro, Farag, and Kapadia \(2014\)](#) who examine the role of credit imbalances - in the form of deviations in the ratio of credit to GDP from a smooth trend, or rapid credit growth rates - in predicting

---

<sup>7</sup>For overviews of macroprudential policy, see eg [Morris and Shin \(2008\)](#), [Bank of England \(2009\)](#) and [Hanson, Kashyap, and Stein \(2011\)](#). For theoretical studies, see inter alia [Lorenzoni \(2008\)](#), [Jeanne and Korinek \(2010\)](#), [Bianchi \(2010\)](#), [Christiano and Ikeda \(2013\)](#), [Repullo and Suarez \(2013\)](#) and [Kashyap, Tsomocos, and Vardoulakis \(2014\)](#). [Stein \(2012\)](#) examines the role of a form of monetary policy in containing systemic risk. For an operational model, see [Clerc, Derviz, Mendicino, Moyen, Nikolov, Stracca, Suarez, and Vardoulakis \(2015\)](#); [Mendicino, Nikolov, Suarez, and Supera \(2018\)](#).

banking crises. The apparent empirical connection between credit imbalances and financial distress has led to the inclusion of these variables in proposed rule-like ‘policy guides’ for the Basel III capital buffer (eg [Basel Committee on Banking Supervision \(2010\)](#)). While our paper is unable to capture the non-linearity associated with financial crises implicit in these empirical studies, it is nonetheless complementary to the nascent empirical literature on macroprudential rules in bringing a structural interpretation to the credit gap and using this to conduct theoretically-grounded macroprudential experiments.

To do this we employ a model that embeds a financial sector closely related to [Gertler, Kiyotaki, and Queralto \(2012\)](#) which, by allowing financial intermediaries to issue both outside equity and deposit claims, has a description of bank capital structure sufficiently rich to allow us to model Basel III-type regulation relatively closely.<sup>8</sup> In providing a quantitative assessment of the role of macroprudential policy, our work complements [Gertler, Kiyotaki, and Queralto \(2012\)](#)’s study by bridging that theoretical contribution with the practical implications of the empirical literature briefly surveyed above. Unlike [Gertler, Kiyotaki, and Queralto \(2012\)](#), more recent papers such as [Angeloni and Faia \(2013\)](#) and [Angelini, Neri, and Panetta \(2014\)](#) consider the implications of banking for the conduct of monetary and macroprudential policies, a feature we also consider.<sup>9</sup>

Like the present paper, [Angelini, Neri, and Panetta \(2014\)](#) characterise the potential gains from the operation of countercyclical macroprudential policies. They employ [Gerali, Neri, Sessa, and Signoretti \(2010\)](#)’s DSGE banking model estimated on the euro area to conduct their analysis, but limit attention to productivity and financial shocks, and evaluate different monetary and macroprudential policy rules against simple ad-hoc loss functions. This is complementary to our welfare analysis which is based instead on a second-order accurate approximation of the household’s utility function. Of course, that is good insofar as our model captures the frictions relevant to welfare. As [Angelini, Neri, and Panetta \(2014\)](#) rightly note, even though recent attempts to integrate banking and financial frictions more

---

<sup>8</sup>This model enriches the capital structure of the model bank compared to earlier but related variants of this particular model of intermediation, see [Gertler, Kiyotaki, et al. \(2010\)](#), [Gertler and Karadi \(2011\)](#) and [Gertler, Karadi, et al. \(2013\)](#).

<sup>9</sup>See also [Svensson \(2017\)](#) and [Svensson \(2018\)](#) who examines the case for having monetary policy play a larger role in ‘leaning against the wind’.

fully into dynamic equilibrium macroeconomic models represents good progress, such models still, on the whole, struggle fully to articulate the causes and consequences of systemic risk, a limitation our approach also inherits.<sup>10</sup>

Unlike us, [Angelini, Neri, and Panetta \(2014\)](#) consider the implications of a non-cooperative game between monetary and macroprudential authorities, which has also received theoretical attention in [Paoli and Paustian \(2017\)](#). These are important considerations from which we abstract, effectively assuming the institutional set-up that is present in the United Kingdom holds in our model, so there is no coordination problem between monetary and macroprudential authorities. That said, we do point to cases in which monetary and macroprudential policies may *appear* to contradict one another as judged by whether the two tools co-vary positively or negatively. As we point out, this is not necessarily suboptimal, however.

The remainder of this paper proceeds as follows. Section 2 sets out the model. Section 3 studies its properties. Section 4 examines optimal macroprudential policy in this economic environment. Section 5 extends the model to include nominal rigidities and monetary policy. Section 6 concludes.

## 2 Model

The model features households who save, consume and supply labour, banks that raise funds from households and intermediate funds to final goods firms, firms that produce capital goods, and firms who produce final output. In addition, we model the behaviour of a macroprudential authority that has complete control over bank capital requirements.

### 2.1 Households

A unit mass of households consumes final goods, supplies labour to good producers, and saves via the banking system. Household  $j \in [0, 1]$  maximises:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t(j), L_t(j)), \quad (2.1)$$

---

<sup>10</sup>For a counter example, see for example [Ajello, Laubach, Lopez-Salido, and Nakata \(2018\)](#).

subject to the following budget constraint:

$$C_t(j) + D_t(j) + Q_t^e e_t(j) = W_t L_t(j) + R_{t-1} D_{t-1}(j) + R_t^e Q_{t-1}^e e_{t-1}(j) + J_t(j) - G(Q_t^e e_t(j)), \quad (2.2)$$

where  $C$  denotes consumption,  $D$  bank deposits,  $e$  bank ‘outside equity’, with price  $Q^e$ ,  $W$  the real wage,  $L$  hours,  $R$  the return on deposits,  $R^e$  the return on bank equity,  $J$  lump-sum taxes and transfers, and  $G(Q_t^e e_t(j))$  adjustment costs associated with changing the household’s bank equity portfolio. As the budget constraint makes clear, the distinction between bank deposits and bank outside equity is twofold. First, bank deposits pay a real return that is non-state contingent (risk-free), and is contracted in advance. Bank equity, by contrast, pays a state contingent return, and represents a direct claim on the cash-flows of the bank. Second, bank deposits are more liquid, in that households can adjust their quantity frictionlessly. In contrast, bank equity is costly to adjust. Imagine that households must adjust their equity portfolios via a broker, whereas they can add or remove cash from their deposit accounts essentially frictionlessly.

The first-order conditions characterising the households consumption and labour supply choices are standard, and are given by:

$$E_t \Lambda_{t+1}(j) R_t = 1, \quad (2.3)$$

$$W_t U_{ct}(j) = -U_{lt}(j), \quad (2.4)$$

where  $\Lambda_{t+1}(j) \equiv \beta U_{ct+1}(j)/U_{ct}(j)$  is the household’s stochastic discount factor,  $U_{ct}$  is the marginal utility of consumption, and  $U_{lt}$  is the marginal utility of hours worked. In the simulations below, we append a multiplicative term  $\exp(\varepsilon_t^r)$  to the consumption Euler equation – a ‘risk premium’ shock, or a shock to the natural rate of interest, where  $\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + u_t^r$ ,  $u_t^r \sim N(0, \sigma_r)$ . What is novel is the first-order condition for bank equity, which is:

$$E_t \Lambda_{t+1} R_{t+1}^e = 1 + G'(Q_t^e e_t(j)), \quad (2.5)$$



where  $G'$  represents the marginal portfolio adjustment cost. Combining this expression with the expression for household consumption results in the ‘equity supply curve’:

$$E_t \Lambda_{t+1} (R_{t+1}^e - R_t) = G'(Q_t^e e_t(j)), \quad (2.6)$$

an expression describing the cost of equity supplied to the banking system by the household sector. In general, the larger is  $G'(Q_t^e e_t(j))$ , the cost of adjusting the equity portfolio, the higher is the equity-deposits spread. We suppose that the equity adjustment cost takes a quadratic form, and that equity adjustment costs are scaled by the size of the bank asset portfolio they help to fund. Letting bank assets be  $S_t(j)$ , we assume:

$$G(Q_t^e e_t(j)) = \frac{\Psi}{2} \left( \frac{Q_t^e e_t(j)/S_t(j)}{Q^e e/S} - 1 \right)^2 \frac{Q^e e}{S} S_t(j), \quad (2.7)$$

As a result, marginal portfolio costs are:

$$G'(Q_t^e e_t(j)) = \Psi \left( \frac{Q_t^e e_t(j)/S_t(j)}{Q^e e/S} - 1 \right) = \Psi \left( \frac{\tilde{\gamma}_t(j)}{\tilde{\gamma}} - 1 \right), \quad (2.8)$$

in which:

$$\tilde{\gamma}_t(j) \equiv \frac{Q_t^e e_t(j)}{S_t(j)} \quad (2.9)$$

is the bank’s *outside equity ratio*. Using this, the equity supply curve can be written as:

$$E_t \Lambda_{t+1}(j) (R_{t+1}^e - R_t) = \Psi \left( \frac{\tilde{\gamma}_t(j)}{\tilde{\gamma}} - 1 \right), \quad (2.10)$$

such that the required return on equity over bank deposits is an increasing function of the bank’s equity ratio.

## 2.2 Banks

Next we describe the banking sector. There is a unit mass of competitive banks,  $i \in [0, 1]$ , run by bankers, owned by households, and which fund themselves with inside equity  $N$ , outside equity  $e$ , and deposits  $D$ . They hold loan portfolios given by  $S$ . The balance sheet

is then:

$$S_t(i) = D_t(i) + Q_t^e e_t(i) + N_t(i). \quad (2.11)$$

Unlike outside equity, each bank's inside equity or net worth is not traded, and is instead inherited by each bank at the start of the period. Given an endowment of net worth, the bank raises outside finance from households. (One can think of banks as raising finance from households other than the owners.) A bank's ability to raise external finance is limited, however, because it can pledge only a fraction  $1 - \theta$  of its assets as collateral. The remaining fraction  $\theta$  can be 'diverted' by the banker for private gain. If the banker chooses not to divert funds, she instead enjoys the profits from lending after paying returns to depositors and outside equity holders. The returns from these two activities are equal, and so banking is just incentive compatible, if:

$$V_t(i) \geq \theta S_t(i), \quad (2.12)$$

where the right-hand side are the fruits of asset diversion, and the left-hand side  $V_t(i)$  is the bank's franchise value. Each period, there is a probability  $1 - \sigma \in [0, 1]$  that the banker returns her net worth to the household, and a probability  $\sigma$  that the banker instead continues banking operations. Equivalently, one can think of  $1 - \sigma$  as the dividend rate. As such, the banker's franchise value is given by:

$$V_t(i) = E_t \Lambda_{t+1} ((1 - \sigma) N_{t+1}(i) + \sigma V_{t+1}(i)). \quad (2.13)$$

The bank is subject to macroprudential regulation. In particular, it must hold a capital buffer of  $\tilde{\gamma}_t(i)$  set by the macroprudential authority. In addition, we allow for the possibility of a non-zero steady state subsidy to the banking sector, which plays no role in the analysis other than to deliver an efficient steady state. For convenience, we scale this subsidy by the return on assets,  $\tilde{\tau} R_{t+1}^k S_t(i)$ . Given these, the bank's net worth then evolves according to:

$$\begin{aligned} N_{t+1}(i) &= R_{t+1}^k S_t(i) - R_t D_t(i) - R_{t+1}^e Q_t e_t(i) + \tilde{\tau} R_{t+1}^k S_t(i) \\ &= (R_{t+1}^k - R_t) S_t(i) - (R_{t+1}^e - R_t) \tilde{\gamma}_t(i) S_t(i) + R_t N_t(i) + \tilde{\tau} R_{t+1}^k S_t(i). \end{aligned} \quad (2.14)$$

where  $R_{t+1}^k$  is the return on the bank's assets. From this, one can see that to the extent that  $R_{t+1}^e - R_t > 0$ , a higher capital buffer requirement reduces the franchise value of the bank. This will tighten the borrowing constraint and contract the bank's ability to intermediate funds, i.e. shift the credit supply curve inwards. The banker's problem is to choose the size of the bank's balance sheet subject to the borrowing constraint (2.12), its franchise value (2.13), the law of motion for net worth, and the macroprudential capital buffer requirement.

**Banker equilibrium** The banker's problem can be formalised by guessing the bank's franchise value takes the form:

$$\begin{aligned} V_t(i) &= \tilde{v}_t^s S_t(i) - \tilde{v}_t^d D_t(i) - \tilde{v}_t^e Q_t e_t(i) \\ &= \tilde{v}_t^s S_t(i) - \tilde{v}_t^d ((1 - \tilde{\gamma}_t(i)) S_t(i) - N_t(i)) - \tilde{v}_t^e \tilde{\gamma}_t(i) S_t(i), \end{aligned} \quad (2.15)$$

where the  $\tilde{v}_t^j$ s are coefficients to be determined and where we used the balance sheet constraint and the definition of the capital buffer. This can be used to write the Lagrangean:

$$\mathcal{L}_t(i) = (1 + \tilde{\lambda}_t(i)) V_t(i) - \tilde{\lambda}_t(i) \theta S_t(i), \quad (2.16)$$

where  $\tilde{\lambda}_t(i)$  is the multiplier on the borrowing (or incentive compatibility) constraint (2.12). The first-order condition for total assets  $S_t(i)$  is then:

$$\tilde{v}_t^s - (1 - \tilde{\gamma}_t(i)) \tilde{v}_t^d - \tilde{\gamma}_t(i) \tilde{v}_t^e = \frac{\tilde{\lambda}_t(i)}{1 + \tilde{\lambda}_t(i)} \theta. \quad (2.17)$$

When the borrowing constraint binds in the neighbourhood of the steady state,  $\tilde{\lambda}_t(i) \neq 0$ , and:

$$\tilde{v}_t^s S_t(i) - \tilde{v}_t^d D_t(i) - \tilde{v}_t^e \tilde{\gamma}_t(i) S_t(i) = \theta S_t(i). \quad (2.18)$$

The first of these, in effect, governs the credit spread, relating it to the tightness of the bank's borrowing constraint; while the second, in effect, determines the bank's feasible leverage.

To see that, one can use the balance sheet in the second of these to write:

$$\frac{N_t(i)}{S_t(i)} = \frac{\theta - (\tilde{v}_t^s - (1 - \tilde{\gamma}_t(i))\tilde{v}_t^d - \tilde{\gamma}_t(i)\tilde{v}_t^e)}{\tilde{v}_t^d}, \quad (2.19)$$

such that for a given amount of net worth, a rise in  $\tilde{v}_t^s - (1 - \tilde{\gamma}_t(i))\tilde{v}_t^d - \tilde{\gamma}_t(i)\tilde{v}_t^e$ , a proxy for the credit spread, allows for a rise in  $S_t(i)$ , the bank's total assets.

Using the first-order condition for total assets, the maximised value of the bank's franchise can be written as:

$$\begin{aligned} V_t(i) &= (\tilde{v}_t^s - (1 - \tilde{\gamma}_t(i))\tilde{v}_t^d - \tilde{\gamma}_t(i)\tilde{v}_t^e) S_t(i) + \tilde{v}_{t+1}^d N_t(i) \\ &= (1 + \tilde{\lambda}_t(i))\tilde{v}_t^d N_t(i), \end{aligned} \quad (2.20)$$

such that it is linear in net worth. Using this in the franchise value equation gives:

$$\begin{aligned} V_t(i) &= E_t \Lambda_{t+1} ((1 - \sigma) + \sigma(1 + \tilde{\lambda}_{t+1}(i))\tilde{v}_{t+1}^d) N_{t+1}(i) \\ &= E_t \Lambda_{t+1} \Omega_{t+1} N_{t+1}(i), \end{aligned} \quad (2.21)$$

where:

$$\Omega_{t+1} \equiv (1 - \sigma) + \sigma(1 + \tilde{\lambda}_{t+1}(i))\tilde{v}_{t+1}^d. \quad (2.22)$$

Using the guess of the value function and the law of motion for net worth, we then get:

$$\tilde{v}_t^s S_t(i) - \tilde{v}_t^d D_t(i) - \tilde{v}_t^e Q_t e_t(i) = E_t \Lambda_{t+1} \Omega_{t+1} ((1 + \tilde{\tau})R_{t+1}^k S_t(i) - R_t D_t(i) - R_{t+1}^e Q_t e_t(i)), \quad (2.23)$$

such that equating coefficients yields:

$$\tilde{v}_t^s = E_t \Lambda_{t+1} \Omega_{t+1} (1 + \tilde{\tau}) R_{t+1}^k, \quad (2.24)$$

$$\tilde{v}_t^d = E_t \Lambda_{t+1} \Omega_{t+1} R_t, \quad (2.25)$$

$$\tilde{v}_t^e = E_t \Lambda_{t+1} \Omega_{t+1} R_{t+1}^e. \quad (2.26)$$

To see what these coefficients mean, substitute them into the borrowing constraint to get an expression for the credit spread:

$$E_t \tilde{\Omega}_{t+1} (R_{t+1}^k - R_t) = \theta + \tilde{\gamma}_t(i) E_t \tilde{\Omega}_{t+1} (R_{t+1}^e - R_t) - \frac{N_t(i)}{S_t(i)} E_t \tilde{\Omega}_{t+1} R_t - \tilde{\tau} E_t \tilde{\Omega}_{t+1} R_{t+1}^k \quad (2.27)$$

where  $\tilde{\Omega}_t \equiv \Lambda_t \Omega_t$ . The left-hand side is the credit spread. On the right-hand side is the asset divertibility parameter, the cost incurred by the bank of holding a capital buffer  $\tilde{\gamma}_t(i)$ , and an expression that is decreasing in the ratio of the bank's net worth to its assets – its leverage ratio. As this rises, the bank's borrowing constraint loosens, so it can expand lending, lowering the credit spread. Equally, when  $E_t \Lambda_{t+1} \Omega_{t+1} (R_{t+1}^e - R_t) > 0$ , increases in the bank's capital buffer lower the bank's franchise value and so tighten the borrowing constraint, causing a cut in credit supply and a rise in the credit spread.

### 2.2.1 Aggregation in the banking sector

In aggregate, we suppose that, in addition to the  $\sigma$  of net worth that is retained each period, there is, in addition, a capital injection of  $\xi_t$  times last period's gross banking revenues, which is exogenous and subject to random disturbances. We label these 'financial' or 'bank capital' shocks. Finally, we assume a lending subsidy equal  $\tilde{\tau}$  times gross lending revenue, financed with lump-sum (non-distortionary) taxation. The role of this subsidy is to deliver an efficient steady state and is discussed further below. As a result of these assumptions, the law of motion of bank net worth is:

$$N_{t+1} = (\sigma + \xi_t) R_{t+1}^k S_t - \sigma R_t D_t - \sigma R_{t+1}^e Q_t^e e_t + \sigma \tilde{\tau} R_{t+1}^k S_t, \quad (2.28)$$

in which we let  $\xi_t = \xi \exp(u_t^n)$ ,  $u_t^n \sim N(0, \sigma_n)$ . By symmetry among banks, the remaining equilibrium conditions are the first-order condition:

$$(1 + \tilde{\lambda}_t)(\tilde{v}_t^s - (1 - \tilde{\gamma}_t)\tilde{v}_t^d - \tilde{\gamma}_t \tilde{v}_t^e) = \tilde{\lambda}_t \theta, \quad (2.29)$$

the borrowing constraint:

$$\tilde{v}_t^s S_t - \tilde{v}_t^d D_t - \tilde{v}_t^e \tilde{\gamma}_t S_t = \theta S_t, \quad (2.30)$$

the balance sheet:

$$S_t = D_t + Q_t^e e_t + N_t, \quad (2.31)$$

the value function coefficients:

$$\tilde{v}_t^s = E_t \Lambda_{t+1} \Omega_{t+1} (1 + \tilde{\tau}) R_{t+1}^k, \quad (2.32)$$

$$\tilde{v}_t^d = E_t \Lambda_{t+1} \Omega_{t+1} R_t, \quad (2.33)$$

$$\tilde{v}_t^e = E_t \Lambda_{t+1} \Omega_{t+1} R_{t+1}^e, \quad (2.34)$$

the discount factor:

$$\Omega_{t+1} = (1 - \sigma) + \sigma(1 + \tilde{\lambda}_{t+1}) \tilde{v}_{t+1}^d, \quad (2.35)$$

and the capital buffer:

$$\tilde{\gamma}_t = \frac{Q_t^e e_t}{S_t}. \quad (2.36)$$

### 2.3 Capital goods and production

Final goods firms employ labour and capital to produce output:

$$Y_t = A_t K_{t-1}^\alpha L_t^{1-\alpha}, \quad (2.37)$$

where  $A_t \equiv \exp(\varepsilon_t^a)$  is exogenous total factor productivity, and  $\varepsilon_t^a$  follow an AR(1) stochastic process,  $\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + u_t^a$ ,  $u_t^a \sim N(0, \sigma_a)$ , and  $K$  is physical capital. The demands for labour and capital are, respectively:

$$W_t = (1 - \alpha) \frac{Y_t}{L_t}, \quad (2.38)$$

$$Z_t = \alpha \frac{Y_t}{K_{t-1}}. \quad (2.39)$$

Firms use loans from banks to rent capital from capital goods producers. By arbitrage in the market for capital goods, the return on bank loans is then the return on capital:

$$R_t^k = \frac{Z_t + (1 - \delta)Q_t}{Q_{t-1}}. \quad (2.40)$$

As such, bank loans represent claims on the capital stock, and in turn, equity claims on the banking sector also represent claims on the capital stock:

$$R_t^e = \frac{Z_t + (1 - \delta)Q_t^e}{Q_{t-1}^e}. \quad (2.41)$$

Capital is produced subject to ‘flow’ adjustment costs  $f(I_t/I_{t-1})$  by perfectly competitive producers, where  $f(1) = f'(1) = 0$ , and  $f''(1) \equiv \omega > 0$ . The resulting price of capital is standard and described by:

$$Q_t = 1 + f(I_t/I_{t-1}) + (I_t/I_{t-1})f'(I_t/I_{t-1}) - E_t \Lambda_{t+1} (I_{t+1}/I_t)^2 f'(I_{t+1}/I_t). \quad (2.42)$$

#### 2.4 Market clearing

In aggregate, the goods market must clear, accounting for the adjustment costs associated with investment and the portfolio adjustment costs borne by households:

$$Y_t = C_t + (1 + f(I_t/I_{t-1}))I_t + \frac{\Psi}{2} \left( \frac{\tilde{\gamma}_t}{\tilde{\gamma}} - 1 \right)^2 \tilde{\gamma} S_t, \quad (2.43)$$

the bank loans market must clear:

$$S_t = Q_t K_t, \quad (2.44)$$

and the labour market must clear:

$$(1 - \alpha) \frac{Y_t}{L_t} = - \frac{U_{lt}}{U_{ct}}. \quad (2.45)$$

Finally, the capital stock evolves according to:

$$K_{t+1} = (1 - \delta)K_t + I_t. \quad (2.46)$$

The exogenous disturbances in the economy are:

$$\varepsilon_t^j = \rho_j \varepsilon_{t-1}^j + u_t^j, \quad j = a, r, n, \quad (2.47)$$

i.e. to technology, the natural interest rate, and to bank capital, in which  $u_t^j \sim \mathcal{N}(0, \sigma_j^2)$ ,  $j = a, r, \theta$ , and to which we add a ‘macroprudential’ disturbance:

$$\tilde{\gamma}_t = \tilde{\gamma} \exp(\varepsilon_t^\gamma), \quad \varepsilon_t^\gamma = \rho_\gamma \varepsilon_{t-1}^\gamma + u_t^\gamma. \quad (2.48)$$

### 3 Model properties

#### 3.1 Steady state analysis

In this section, we consider some properties of the steady state. We begin with households. As usual, we assume labour supply preference parameters deliver equilibrium hours of 2/3. The real risk-free interest rate is pinned down by household time preferences:  $R = \beta^{-1}$ . Suppose the macroprudential authority sets a steady state capital buffer of  $\tilde{\gamma}^*$ . Then the steady state spread on outside equity over deposits is:

$$R^e - R = \frac{\Psi}{\beta} \left( \frac{\tilde{\gamma}^*}{\tilde{\gamma}} - 1 \right). \quad (3.1)$$

Clearly, as  $\tilde{\gamma}^* \rightarrow \tilde{\gamma}$ , this spread tends to zero, and the equity-debt spread is non-zero only away from the perfect foresight steady state. The effects of the capital requirement on the equilibrium in the real economy occur via the steady state credit spread equation, which is:

$$R^k - R = \frac{\theta}{\beta\Omega} + \tilde{\gamma}^*(R^e - R) - \frac{N}{S}R - \tilde{\tau}R^k. \quad (3.2)$$



In a partial equilibrium sense, a rise in  $\tilde{\gamma}^*$  then causes a rise in the aggregate credit spread whenever  $R^e - R > 0$  in steady state. Even when  $R^e = R$  in the steady state, however, the steady state credit spread will in general be non-zero for the simple reason that the borrowing constraint generically prevents enough saving from flowing through to investment in the economy to drive the return on capital down to the risk-free rate. One can see that, in order for the credit spread to be zero in steady state, the remaining banking variables must satisfy:

$$\theta = \Omega(\eta + \tilde{\tau}), \quad (3.3)$$

where  $\eta \equiv \frac{N}{S}$  is the steady state leverage ratio of the bank. We now explore what parameter values are needed to deliver this efficient steady state.

### 3.1.1 Efficient steady state with subsidy

In the perfect foresight steady state, the law of motion for net worth implies:

$$\xi = \frac{\frac{N}{S} + \sigma R \frac{D}{S} + \sigma R^e \frac{Q^e e}{S} - \sigma \tilde{\tau} R^k - \sigma R^k}{R^k}. \quad (3.4)$$

As  $R^k \rightarrow R$ , then:

$$\xi \rightarrow (\beta - \sigma)\eta - \sigma\tilde{\tau},$$

so that transfers have to be higher the greater is the discrepancy between the household's discount rate and the rate at which banks pay dividends, and transfers have to be lower the higher is the lending subsidy.

Similarly, the borrowing constraint gives:

$$\tilde{v}^s - \tilde{v}^d((1 - \tilde{\gamma}) - \eta) - \tilde{v}^e \tilde{\gamma} = \theta, \quad (3.5)$$

and the bank's first-order condition gives:

$$\frac{1 + \tilde{\lambda}}{\tilde{\lambda}}(\tilde{v}^s - \tilde{v}^d) = \theta, \quad (3.6)$$

so that equating the two yields an expression for the Lagrange multiplier:

$$\tilde{\lambda} = \frac{(\tilde{v}^s - \tilde{v}^d)}{(\tilde{v}^s - \tilde{v}^d((1 - \tilde{\gamma}) - \eta) - \tilde{v}^e \tilde{\gamma}) - (\tilde{v}^s - \tilde{v}^d)}. \quad (3.7)$$

Assume that the macroprudential authority sets  $\tilde{\gamma}^* = \tilde{\gamma}$ , so that  $R = R^e$ , in turn implying that  $\tilde{v}^d = \tilde{v}^e$  to get:

$$\tilde{\lambda} = \frac{\tilde{v}^s - \tilde{v}^d}{\eta \tilde{v}^d} \quad (3.8)$$

And finally use that  $\tilde{v}^s = \beta\Omega(1 + \tilde{\tau})R^k$ , and  $\tilde{v}^d = \beta\Omega R$  to get

$$\tilde{\lambda} = \frac{(R^k - R) + \tilde{\tau}R^k}{\eta R}. \quad (3.9)$$

Now note that even as  $R^k \rightarrow R$ , the Lagrange multiplier retains a positive value as long as there is a lending subsidy. In particular, as  $R^k \rightarrow R$ , then

$$\tilde{\lambda} \rightarrow \frac{\tilde{\tau}}{\eta}.$$

The intuition for this is that even though the credit spread has shrunk to zero, the lending subsidy still makes it worthwhile for the bank to push up against its borrowing constraint, because every unit of borrowing it does at cost  $R$  yields a return of  $(1 + \tilde{\tau})R > R$  – so it still has a positive Lagrange multiplier in the steady state. This allows us to study the dynamics of the economy subject to financial frictions around a steady state that is efficient, corresponding to the equilibrium of the counterpart RBC economy. Finally, using this, the discount factor can be computed from:

$$\Omega = \frac{1 - \sigma}{1 - \sigma(1 + \tilde{\lambda})}, \quad (3.10)$$

which stays in excess of unity even as  $R^k \rightarrow R$  because the lending subsidy means there is, in effect, a positive return to the bank to lending, and so in a sense the bank remains capital constrained.

If the banking subsidy is appropriately calibrated, then the return on capital converges

to the risk-free rate. In that case, the steady state becomes efficient. That is because the aggregate capital-output ratio is:

$$\frac{K}{Y} = \frac{\alpha}{R^k - (1 - \delta)}, \quad (3.11)$$

which converges to its efficient level as  $R^k \rightarrow R = \beta^{-1}$ , in which case  $\frac{K}{Y} = \frac{\alpha\beta}{1-\beta(1-\delta)}$ .

### 3.2 *The banking friction and dynamic inefficiency*

Suppose that the subsidy discussed above is in place and that in steady state the macroprudential authority sets  $\tilde{\gamma}^* = \tilde{\gamma}$ . The steady state of the model is then efficient as the correspond exactly with the simple RBC case. However, its dynamics are not. In particular, supposing the household could directly accumulate claims on the capital stock, its portfolio of equity and deposits would satisfy:

$$E_t \Lambda_{t+1} (R_{t+1}^k - R_t) = 0. \quad (3.12)$$

We can use this condition to gauge the size of the misallocation that occurs in response to shocks. Construct a parallel economy in which (3.12) holds instead of (2.27). Denote the vector of endogenous variables in this parallel economy by  $X_t^*$ . Then the ratio  $X_t/X_t^*$  captures the degree of misallocation. In particular,  $Y_t/Y_t^*$  is the ‘output gap’, and  $S_t/S_t^*$  is the credit gap, or in log-linear terms:

$$y_t^{\text{gap}} \equiv y_t - y_t^*, \quad s_t^{\text{gap}} \equiv s_t - s_t^*,$$

by analogy with the monetary policy literature, where lower-case variables denote log-deviations from steady state. The behaviour of  $X_t/X_t^*$  in response to fundamental shocks is the object of interest. Moreover, simple policy rules that vary the macroprudential instrument  $\gamma_t$  in response to such gaps are interesting from a policy perspective.

Parameter	Description	Value
	<i>Macro parameters</i>	
$\sigma_c$	Intertemporal elasticity	1.0
$\varphi$	Inverse Frisch elasticity	3.0
$\alpha$	Capital share	1/3
$\beta$	Discount rate	0.9938
$\delta$	Capital depreciation rate	0.025
$\omega$	Elasticity of investment to $Q$	1.0
	<i>Banking parameters</i>	
$\psi$	Elasticity of equity spread to capital buffer	1.0
$\eta$	Steady state $N/S$	0.05
$\gamma$	Steady state capital buffer	0.05
$\sigma$	One minus the dividend rate	0.975
$\tau$	Lending subsidy	0.0005
$\xi$	Net worth transfer	0.0004
$\theta$	Divertibility of bank assets	0.0828

Table 1: Calibration details

### 3.3 Model calibration and impulse responses

We linearise the model around the efficient perfect foresight steady state. The linearised equations appear in the appendix. In this section, we explore the dynamic properties of the model in response to technology shocks (‘supply shocks’), financial shocks, and time preference shocks (‘demand shocks’). We also examine the effects of a surprise increase in the capital buffer requirement.

Table 1 contains details of the calibration. We set the intertemporal elasticity to unity and inverse Frisch elasticity to 3, the capital share to one-third, and the discount factor to be consistent with a steady state risk-free rate of 2.5% annualised. The capital depreciation rate is 10% annualised, and the elasticity of investment to asset prices is unity. These are all relatively standard values. The remaining parameters are specific to the banking part of the model. Within this block, we set the elasticity of the equity spread to the capital buffer,  $\psi$ , to 1.0, such that changes in the equity buffer requirement go one-for-one into the equity-debt spread. Arguably this is a relatively powerful effect, and though ultimately an empirical question, probably delivers an upper bound for the effects if the capital buffer requirement on spreads. We set the overall capital ratio of the bank to 10%, split evenly between net worth and the capital buffer, which are each set to 0.05. We set the quarterly dividend

rate to 2.5%, implying  $\sigma$  of 0.975. The lending subsidy needed to deliver an efficient steady state is 5 basis points. The remaining banking parameters are pinned down by these other choices.

The shock processes governing technology, the natural interest rate and the capital buffer have AR(1) coefficients of 0.80, while the bank capital shock has no persistence. The technology and natural interest rate shocks have standard deviations of 1%, the bank capital shock 25%, and the capital buffer shock is scaled to deliver a 1ppt rise in the capital buffer whose steady state value (as noted above) is 5%.

Figures 1-3 show the impulse responses to technology, natural rate and bank capital shocks respectively. Both the technology and natural rate shocks generate expansions of output, lending, hours, and asset prices, together with reduction in credit spreads. This suggests credit supply endogenously shifts outwards following these expansionary shocks – in effect, rising asset prices raise the net worth of the intermediary sector, allowing for an expansion of credit supply and a loosening in financial conditions. One can see this via the final two panels in Figures 1 and 2. These show the output gap ( $y^{gap}$ ) and the credit gap ( $s^{gap}$ ) respectively. In each case, output and credit both rise above their frictionless RBC levels, consistent with amplification arising from the financial friction.

Figure 3 shows the effects of a 25% one-off loss of bank capital. This causes a sharp fall in output and a spike in credit spreads. Hours and investment fall and there is a reduction in lending in the economy. The falls in output and lending pass one for one into the output and credit gaps – both fall below the efficient levels implied by the RBC model, which, because of the absence of financial frictions, features no response to such financial shocks. That said, given the scale of the shock, the overall output fall is not huge. In part that is because as the bank's funding constraint tightens, investment falls but consumption rises. In effect this cushions the blow, and reflects both the structure of preferences (non-separable between consumption and labour) together with the absence of nominal rigidities.

Finally, Figure 4 shows the impact of a temporary 1ppt rise in the capital buffer requirement. This causes an immediate increase in banks' funding costs, lowering their franchise values and causing a steady fall in lending. At its peak, lending falls by around 0.8% (after

around four years), and output by around 0.35%) after around two years – suggesting quite slow and reasonably modest effects on the economy. Reduced lending and output pass directly into the output and credit gaps, both of which fall one-for-one with their respective variables. While modest, these responses suggest the scope for cyclical variation in the macroprudential tool to smooth macroeconomic outcomes. This is the topic of the next section.

## 4 Optimal macroprudential policy

We now examine the conduct of macroprudential policy in this context. We begin by defining the policymaker’s objectives. We take these to be to maximise the welfare of the representative household. To formalise this in a linear-quadratic setting, in the appendix we take a second-order approximation to the household’s utility function, which results in the following quadratic loss function:<sup>11</sup>

$$\begin{aligned} \mathcal{W} \equiv & -(\sigma_c - \zeta)\text{var}(c_t) - (1 - \zeta)\text{var}(i_t) - \frac{(1 - \alpha)(1 + \varphi)}{\zeta}\text{var}(l_t) \\ & - \omega \frac{1 - \zeta}{\zeta}\text{var}(\Delta i_t) - \frac{1}{\zeta}\Psi\tilde{\gamma}\frac{\alpha\beta}{1 - \beta(1 - \delta)}\text{var}(\gamma_t), \end{aligned} \quad (4.1)$$

where lower case variables or, in the case of the capital buffer, variables without tildes, denote log-deviations from steady state, and where  $\zeta \equiv C/Y$  is the steady state consumption-output ratio. The elements of this objective are intuitive but worth a quick discussion. The terms in the variances of consumption, investment and hours all make sense – arising as they do via the household’s consumption and labour/hours objectives. The fourth term in the variance the change in investment arises because of investment adjustment costs: observe that as the elasticity of investment to asset prices goes to zero,  $\omega \rightarrow 0$ , this term vanishes. Finally, the fifth term in the objective function captures the costs associated with moving the policy instrument. Recall that these impose portfolio adjustment costs on the household, proportional to  $\Psi\tilde{\gamma}$ , and so the policymaker naturally takes these into account when setting the macroprudential instrument. Even were perfect stabilisation of

---

<sup>11</sup>See [Edge \(2003\)](#) and [Svein and Weinke \(2009\)](#) for related derivations.

the macroeconomy possible in principle, then, it need not be the case that optimal policy achieves this, as it naturally balances these gains with a concern for smoothing the costs of adjusting policy in the first place. With this objective in hand, we compute the optimal path for  $\gamma_t$  under commitment in response to the technology, natural rate and bank capital shocks described above.

Figures 5–7 show the dynamic responses of the economy under the laissez-faire baseline, and under optimal Ramsey policy. The first point that stands out under each of these three shocks is that the macroprudential tool is generally moves in a countercyclical manner. Generally, when output and credit rise, the macroprudential capital buffer is increased in a way that moderates the boom. The effects of this policy are most obvious in the plots for the output gap and the credit gap. Under technology shocks, the output gap is all but returned to zero after four years under optimal policy and the credit gap is roughly halved over this period. A broadly similar picture emerges for the natural interest rate shock. Finally, under the bank capital shock, although the responses for output and credit are moved in the direction of their efficient responses, they are far from offset completely; optimal policy is countercyclical but insufficiently aggressive completely to eliminate the effects of the shock.

The second point is that the absolute scale of the macroprudential adjustments are quite modest given the changes in output and credit. For example, under technology shocks, the optimal response is to raise the macroprudential buffer by only 2bp for a 1% rise in output and a 0.5% rise in credit. The responses under the natural rate shock are also modest – perhaps a rise of 10bp in the capital buffer for a 1% rise in output and a 1.5% rise in credit. Of course these magnitudes are sensitive to calibration, but they are suggestive that relatively modest, slow-moving adjustments in the capital buffer are optimal under commitment policy.

Next we examine the extent to which a simple rule can replicate the outcomes delivered by the optimal commitment policy. Given the close correspondence between the Ramsey optimal path for the capital buffer and the equilibrium path for the credit gap this delivers,

	<i>Technology</i>	<i>Natural rate</i>	<i>Bank capital</i>	<i>All shocks</i>	<i>Ramsey</i>
$v_s$	2.387	2.875	0.628	2.437	
Loss	0.328	4.242	0.273	4.886	4.701

Table 2: Optimal credit gap rule and Ramsey policy compared under flexible prices. Loss is  $(1 - \beta)^{-1}W$ .

we examine a simple policy rule of the form:

$$\gamma_t = v_s(s_t - s_t^*). \quad (4.2)$$

We search for values of  $v_s$  to minimise the loss function defined by  $\mathcal{W}$  conditional on technology shocks, natural interest rates shocks, bank capital shocks and all three shocks respectively. Table 2 contains the results. The first row contains the optimal value of  $v_s$  for each of the three shocks, and the optimal value when all three shocks are present. For each of the shocks individually, and for all the shocks together, the optimal response is countercyclical. As a rule of thumb, for example, a 1% rise in the credit gap is met with a rise in the capital buffer of between 0.6 and 2.8%. As in the case of optimal Ramsey policy, these are relatively modest adjustments in the context of the model with a steady state capital buffer of 5%. Supposing, however, that a simple de-trended measure of aggregate credit gives a reasonable measure of the credit gap empirically, these magnitudes suggest the overall variation in the capital buffer may be larger. For example, the filtered credit series shown in Figure 8 has varied between around +/-20% over the postwar period. Taken at face value, this suggests changes in the capital buffer in the range +/- 0.20\*2.8 $\approx$  50%, or +/- 0.50\*0.05=2.5pp, which does not seem unreasonable from a practical perspective.

Table 2 also compares the loss arising from the simple credit gap rule, optimised for all three shocks, with the loss under optimal commitment policy. As expected, the simple rule delivers a larger loss than optimal policy. This excess is relatively modest, however – of the order 4% larger. This suggests that in the absence of the ability to set fully optimal state contingent commitment policy, the simple credit gap rule may not be an unreasonable practical option. To see the equilibrium outcomes delivered by such simple credit gap rules, the responses of the economy to the three shocks under each are shown in Figures 5–7. For



both technology and natural interest rate shocks, the responses are very close to the Ramsey optimal responses. For example, the paths for the output gap and the credit gap are almost completely aligned with the Ramsey paths under the credit gap rule. That is less obviously the case of the bank capital shock, where the simple rule loosens less aggressively in the initial phases of the shock, and tightens less aggressively further out.

## 5 Nominal rigidities

### 5.1 Model

Next we consider how robust these findings are to the presence of nominal rigidities. We do so by extending the baseline model described above to include rigidities in the prices of final goods. We do this in two, standard steps. First, we add monopolistic competition to the final goods production sector.<sup>12</sup> Second, we assume only a fixed fraction of these firms is able to re-set their prices each period. Specifically, we assume a continuum of firms,  $j \in [0, 1]$ . Each firm produces a unique variety, facing demand for its output from consumers of:

$$Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon} Y_t, \quad (5.1)$$

where  $Y_t(j)$  is firm  $j$ 's output and  $P_t(j)$  is its price. Firm  $j$  takes this demand function as given and chooses its price to maximise its profits. We let  $\theta_p$  denote the probability with which the firm cannot re-set its price. The complementary probability determines the likelihood that the firm can re-optimize.

By symmetry across firms, the demand for labour and capital now satisfy:

$$Z_t = M_t \alpha \frac{Y_t}{K_t}, \quad (5.2)$$

$$W_t = M_t (1 - \alpha) \frac{Y_t}{L_t}, \quad (5.3)$$

where  $M_t$  denotes real marginal cost. This is constant under flexible prices. Under sticky

---

<sup>12</sup>We make the standard assumption that a steady state production subsidy financed with lump-sum taxation offsets this distortion on the economy's steady state.

prices, the aggregate price level satisfies:

$$\Pi_t = \theta_p + (1 - \theta_p) \left( \frac{P_t^*}{P_{t-1}} \right)^{1-\varepsilon}, \quad (5.4)$$

where  $P_t^*$  denotes the optimal re-set price and where  $\Pi_t \equiv P_t/P_{t-1}$ . The usual derivation results in the following New Keynesian Phillips Curve:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa m_t, \quad (5.5)$$

where  $\kappa \equiv (1 - \theta_p)(1 - \beta\theta_p)(1 - \alpha)/\theta_p(1 - \alpha - \alpha\varepsilon)$ , and  $m_t$  is the log-deviation of real marginal cost.

Together with nominal frictions, we assume that monetary policy operates under a form of strict inflation targeting according to the simple Taylor rule:

$$r_t^n = \phi_\pi \pi_t, \quad (5.6)$$

where  $r_t^n$  is the nominal interest rate. We choose this rule because, in the absence of shocks that induce a trade-off for monetary policy between the output gap and inflation, a rule that responds aggressively to inflation will necessarily close the inflation-relevant output gap. (In other words, it would make little difference if we also included a measure of the inflation-relevant output gap, like real marginal cost, in the Taylor rule for monetary policy.) Finally, the nominal and real interest rates are related according to:

$$r_t^n = r_t + E_t \pi_{t+1}. \quad (5.7)$$

Taken together, these are the three additional equations we add to the RBC model described above. These bring two additional parameters to calibrate: the slope of the Phillips curve ( $\kappa$ ), which we set to 0.05, and the coefficient in the monetary policy rule, which vary below.

## 5.2 *Optimal macroprudential policy under nominal rigidities*

In this section, we re-compute optimal macroprudential policy when the economy also features distortions due to nominal rigidities. In doing so, we face a choice about how to specify the objective of the macroprudential authority. Strictly speaking, with the addition of the distortion in real allocations caused by slow price adjustment, the policy authority should include an inflation objective in its loss function to the extent that function reflects the losses borne by households. To make a comparison with our previous results possible, however, we conduct our experiments under the assumption that the macroprudential authority's loss function remains the same as the case above, and that it takes as given the monetary authority's monetary policy, which is characterised by the standard Taylor rule (5.6).

The question of interest is: how does the optimal macroprudential policy response vary with the conduct of monetary policy. To answer this question we conduct a comparative statics exercise in which we compute Ramsey optimal macroprudential policy for different degrees of inflation aversion from the monetary authority. In particular, we compute three sets of responses for  $\phi_\pi \in \{100, 5, 2.5\}$ . The first set of responses corresponds to strong inflation aversion and, in effect, under this parameterisation the monetary authority succeeds in perfectly stabilising inflation and closing the 'inflation relevant' output gap. The remaining two parameterisations gradually relax this inflation aversion towards more realistic values.

Figure 9 shows the response to technology shocks. The responses corresponding to  $\phi_\pi = 100$  are very close to the flexible price case. In this instance, the monetary authority stabilises inflation almost completely (by loosening policy) and the macroprudential authority tightens policy to moderate the acceleration in credit and the positive output gap that results. The apparently opposing actions of the two authorities make sense: monetary policy loosens to stabilise inflation, while macroprudential policy tightens to lean against the financial accelerator. As the monetary authority's inflation aversion falls, however, the output and credit gaps become less positive – and the output gap in particular is negative for a couple of quarters initially. This causes the macroprudential authority to moderate its own response: as monetary policy controls inflation less aggressively, so too does the macroprudential authority lean against the financial accelerator less strongly because nominal frictions are

being allowed to dampen real volatility in the economy.

Qualitatively, the same pattern is visible in Figure 10, which shows the responses to a shock to the natural interest rate. Here again as the monetary authority stabilises inflation less aggressively, the shock turns from being expansionary to contractionary, as interest rate cuts become insufficient to support demand and stabilise inflation. As in the case of the technology shock, this helps to temper the size of the financial accelerator, so the macroprudential policy responds less aggressively to the economy's responses as the monetary authority responds less strongly to inflation. And like technology shocks, macroprudential and monetary policy respond in opposite directions.

Figure 11 shows the responses of the economy to bank capital shocks. These shocks differ from the first two in that as the inflation aversion of the monetary authority falls, it is more likely that optimal macroprudential policy moves in tandem with the monetary policy response. For moderate to low inflation aversion, both the nominal interest rate and the capital buffer are reduced following a bank capital shock. This looser monetary policy response helps cushion the initial response of output and alleviates some of the stabilisation burden on macroprudential policy.

We close with one final remark on these experiments. It is clear from Figures 9–11 that, typically, the macroprudential response remains away from equilibrium for some time after the monetary authority has returned inflation and its policy instrument to equilibrium. This reflects the different distortions upon which the two policies operate. The financial friction in this case impinges on the dynamics of investment and so the capital stock, creating slow-moving deviations in total credit and output from their efficient levels. In contrast, the distortions associated with nominal rigidities last, in this case, for a shorter period of time. In this sense one would expect macroprudential policy to be conducted at lower frequency than monetary policy; the responses shown in Figures 9–11 bear this intuition out. In that sense, there is a correspondance between these theoretical findings and those of Aikman et al and Drehmann et al, who emphasise the longer-term nature of the credit cycle in comparison to the business cycle.

	<i>Technology</i>	<i>Natural rate</i>	<i>Bank capital</i>	<i>All shocks</i>	<i>Ramsey</i>
$v_s$	2.869	4.455	0.375	2.362	
Loss	0.192	2.279	0.244	2.786	2.623

Table 3: Optimal credit gap rule and Ramsey policy compared under nominal rigidities. Loss is  $(1 - \beta)^{-1}W$ .

### 5.3 Simple credit gap rule with nominal rigidities

This section considers how the optimal simple credit gap rule changes when nominal rigidities are present. To do this, we re-compute the value of  $v_s$  in the simple rule (4.2) in the economy characterised by nominal rigidities and a weak response of monetary policy to inflation ( $\phi_\pi = 2.5$ ). Table 3 contains the results, by analogy with Table 2. As when prices are flexible, the optimal simple credit gap rule features a counter-cyclical response to the credit gap. Compared to the case of flexible prices, the orders of magnitude of this response are broadly unchanged – with a slightly larger countercyclical response to technology and natural interest rate shocks, but a weaker one to bank capital shocks. Overall, when all shocks are present, the optimal simple rule coefficient is little changed compared to the flexible price case. And like that case, the simple credit gap rule delivers a loss that is greater than, but not much larger than, the case of optimal commitment policy. When nominal rigidities are present, the loss associated with the simple credit gap rule is some 6% larger than the optimal policy, compared to around 4% larger when the comparison is performed under flexible prices.

## 6 Concluding remarks

When the intermediary sector generates a financial accelerator in the economy, Ramsey-optimal policy calls for the macroprudential authority to raise bank capital buffers in response to shocks to technology, the natural interest rate and bank net worth that expand output and credit, and compress credit spreads. In other words, macroprudential policy is naturally countercyclical. Moreover, under such a policy, the optimal bank capital buffer tracks a model-based measure of the credit gap, which we define as the gap between equilibrium credit in the economy featuring financial frictions and that in a hypothetical

frictionless economy, by analogy with the output gap in monetary economics. Simple rules that vary the capital buffer in response to this definition of the credit gap generate worse outcomes than this, but only modestly so.

When monetary policy must also confront the consequences of nominal rigidities, optimal macroprudential policy is affected. If monetary policy controls inflation less aggressively, optimal macroprudential responses are smaller. This is because nominal rigidities that cause inflation also attenuate the economy's response to the financial accelerator, calling for a less robust response from macroprudential policy. Some shocks, like technology, call for monetary and macroprudential policies optimally to pull in opposite directions, whereas others, like bank net worth shocks, call for the two stabilisation tools optimally to co-move positively. Nonetheless, macroprudential policy typically remains away from equilibrium for longer than does monetary policy: in general, macroprudential policy operates at a lower frequency than interest rate policy.

The literature on optimal macroprudential policy is in its infancy. Further work is needed to understand how policy tools like the macroprudential bank capital buffer interact with other macroprudential measures aimed at stabilising financial accelerator mechanisms originating elsewhere in the economy, including the housing market, the shadow banking sector, and in the nature of cross-border capital flows. Moreover, richer descriptions of the causes and consequences of systemic risk, including non-linearities, are needed to push the study of macroprudential policies forward. These are clear avenues for future research.

# A Appendix (not for publication)

## A.1 Complete set of linearised equations

Household: euler

$$u_{ct+1} - u_{ct} + r_t + \varepsilon_t^r = 0$$

Household: labour supply

$$y_t - l_t = w_t = \varphi l_t - u_{ct}$$

Household: equity supply

$$r_{t+1}^e - r_t = \Psi \gamma_t$$

Firms: production:

$$y_t = \alpha k_t + (1 - \alpha) l_t + \varepsilon_t^a$$

Firms: capital demand

$$z_t = y_t - k_t$$

Firms: capital price, where  $f''(1) \equiv \omega$ :

$$i_t = \frac{1}{1 + \beta} i_{t-1} + \left(1 - \frac{1}{1 + \beta}\right) i_{t+1} + \frac{1}{1 + \beta} \frac{1}{\omega} q_t$$

Banks: return on loans:

$$r_t^s + q_{t-1} = (1 - \beta(1 - \delta)) z_t + \beta(1 - \delta) q_t$$

Banks: return on equity:

$$r_t^e + q_{t-1}^e = (1 - \beta(1 - \delta)) z_t + \beta(1 - \delta) q_t^e$$

Banks: loan market:

$$s_t = k_t + q_t$$

Banks: balance sheet, where  $(1 - \tilde{\gamma}) - \eta = \frac{D}{S}$ :

$$(1 - \tilde{\gamma})s_t - \tilde{\gamma}\gamma_t = (1 - \tilde{\gamma} - \eta) d_t + \eta n_t$$

Banks: net worth law of motion:

$$\beta \eta n_{t+1} = (\sigma + \xi)(r_{t+1}^k + s_t) - \sigma(1 - \tilde{\gamma} - \eta)(r_t + d_t) - \sigma \tilde{\gamma}(r_{t+1}^e + \gamma_t + s_t) + \sigma \tilde{\tau}(r_{t+1}^k + s_t) + u_t^n$$

Banks: first-order condition:

$$(1 + \tilde{\lambda})(\tilde{v}^s v_{st} - (1 - \tilde{\gamma})\tilde{v}^d v_t^d - \tilde{\gamma}\tilde{v}^e v_t^e) + (\tilde{v}^s - \tilde{v}^d)\tilde{\lambda}\lambda_t = \tilde{\lambda}\theta\lambda_t$$

Banks: borrowing constraint:

$$\tilde{v}^s(v_t^s + s_t) - \tilde{v}^d(1 - \tilde{\gamma} - \eta)(v_t^d + d_t) - \tilde{v}^e\tilde{\gamma}(v_t^e + \gamma_t + s_t) = \theta s_t$$

Banks: value function coefficients:

$$v_t^s = u_{ct+1} - u_{ct} + \omega_{t+1} + r_{t+1}^k$$

$$v_t^d = u_{ct+1} - u_{ct} + \omega_{t+1} + r_t$$

$$v_t^e = u_{ct+1} - u_{ct} + \omega_{t+1} + r_{t+1}^e$$

Banks: discount factor

$$\Omega\omega_t = \sigma(1 + \tilde{\lambda})\tilde{v}^d v_t^d + \sigma\tilde{\lambda}\tilde{v}^d\lambda_t$$

Marginal utility of consumption:

$$u_{ct} = -\sigma_c c_t$$

Market clearing: where  $\zeta \equiv C/Y$ :

$$y_t = \zeta c_t + (1 - \zeta)i_t$$

Market clearing: capital:

$$k_t = (1 - \delta)k_{t-1} + \delta i_t$$



Macropprudential instrument:

$$\gamma_t = \varepsilon_t^\gamma$$

Shock processes  $(a, r, n, \gamma)$ :

$$\varepsilon_t^j = \rho_j \varepsilon_{t-1}^j + u_t^j$$

together with exogenous processes  $u_t^a, u_t^r, u_t^n, u_t^\gamma$ .

## A.2 Linearising the $Q$ equation

Capital producers solve:

$$\max_{\{I_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \Lambda_{0,t} \left( Q_t I_t - \left( 1 + f\left(\frac{I_t}{I_{t-1}}\right) \right) I_t \right).$$

The first-order condition is:

$$Q_t - 1 - f\left(\frac{I_t}{I_{t-1}}\right) - \frac{\partial f_t}{\partial I_t} I_t - E_t \Lambda_{t,t+1} I_{t+1} \frac{\partial f_{t+1}}{\partial I_t} = 0,$$

where

$$\begin{aligned} \frac{\partial f_t}{\partial I_t} &= f'\left(\frac{I_t}{I_{t-1}}\right) \frac{1}{I_{t-1}} \\ \frac{\partial f_{t+1}}{\partial I_t} &= -f'\left(\frac{I_{t+1}}{I_t}\right) \frac{I_{t+1}}{I_t^2} \end{aligned}$$

So

$$Q_t = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}} f'\left(\frac{I_t}{I_{t-1}}\right) - E_t \Lambda_{t,t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 f'\left(\frac{I_{t+1}}{I_t}\right)$$

Or:

$$Q_t = 1 + f(X_t) + X_t f'(X_t) - E_t \Lambda_{t,t+1} X_{t+1}^2 f'(X_{t+1})$$

where  $X_t \equiv I_t/I_{t-1}$ . Linearising:

$$\begin{aligned} Q_t &= f'(X)(X_t - X) + f'(X)(X_t - X) + X f''(X)(X_t - X) \\ &\quad - E_t \beta X^2 f''(X)(X_{t+1} - X) - E_t \beta f'(X) 2X(X_{t+1} - X) - E_t X^2 f'(X)(\Lambda_{t+1} - \Lambda) \end{aligned}$$

First use that  $f'(X) = 0$ , so

$$Qq_t = X f''(X)(X_t - X) - E_t \beta X^2 f''(X)(X_{t+1} - X)$$

Next use that  $X = 1$ ,

$$Qq_t = f''(1)(X_t - 1) - E_t \beta f''(1)(X_{t+1} - 1)$$

Next use that  $Q = 1$ ,

$$\frac{1}{f''(1)} q_t = (X_t - 1) - E_t \beta (X_{t+1} - 1)$$

Finally write:

$$\frac{1}{f''(1)} q_t = X x_t - E_t \beta X x_{t+1}$$

where:

$$x_t = i_t - i_{t-1}$$

So:

$$\frac{1}{f''(1)} q_t = (i_t - i_{t-1}) - E_t \beta (i_{t+1} - i_t)$$

This rearranges to:

$$i_t = \frac{1}{1 + \beta} i_{t-1} + \frac{\beta}{1 + \beta} i_{t+1} + \frac{1}{1 + \beta} \frac{1}{\omega} q_t$$

where we let  $f''(1) \equiv \omega$ .

### A.3 Linearising the banking block

Gathering the nonlinear banking equations together:

$$N_{t+1} = (\sigma + \xi_t) R_{t+1}^k S_t - \sigma R_t D_t - \sigma R_{t+1}^e \tilde{\gamma}_t S_t + \sigma \tilde{\tau} R_{t+1}^k S_t.$$

$$(1 + \tilde{\lambda}_t)(\tilde{v}_t^s - (1 - \tilde{\gamma}_t)\tilde{v}_t^d - \tilde{\gamma}_t \tilde{v}_t^e) = \tilde{\lambda}_t \theta,$$

$$\tilde{v}_t^s S_t - \tilde{v}_t^d D_t - \tilde{v}_t^e \tilde{\gamma}_t S_t = \theta S_t,$$

$$S_t(1 - \tilde{\gamma}_t) = D_t + N_t,$$

$$\tilde{v}_t^s = E_t \Lambda_{t+1} \Omega_{t+1} (1 + \tilde{\tau}) R_{t+1}^k,$$

$$\tilde{v}_t^d = E_t \Lambda_{t+1} \Omega_{t+1} R_t,$$

$$\tilde{v}_t^e = E_t \Lambda_{t+1} \Omega_{t+1} R_{t+1}^e,$$

$$\Omega_{t+1} = (1 - \sigma) + \sigma(1 + \tilde{\lambda}_{t+1})\tilde{v}_{t+1}^d,$$

which linearise to:

$$Nn_{t+1} = (\sigma + \xi)R^k S(r_{t+1}^k + s_t) - \sigma RD(r_t + d_t) - \sigma R^e \tilde{\gamma} S(r_{t+1}^e + \gamma_t + s_t) + \sigma \tilde{\tau} R^k S(r_{t+1}^k + s_t) + \xi R^k S u_t^n$$

$$(1 + \tilde{\lambda})(\tilde{v}^s v_{st} - (1 - \tilde{\gamma})\tilde{v}^d v_t^d - \tilde{\gamma}\tilde{v}^e v_t^e - \tilde{\gamma}(\tilde{v}^e - \tilde{v}^d)\gamma_t) + (\tilde{v}^s - (1 - \tilde{\gamma})\tilde{v}^d - \tilde{\gamma}\tilde{v}^e)\tilde{\lambda}\lambda_t = \tilde{\lambda}\theta\lambda_t$$

$$\tilde{v}^s S(v_t^s + s_t) - \tilde{v}^d D(v_t^d + d_t) - \tilde{v}^e \tilde{\gamma} S(v_t^e + \gamma_t + s_t) = \theta S s_t$$

$$S(1 - \tilde{\gamma})s_t - \tilde{\gamma} S \gamma_t = D d_t + N n_t$$

$$v_t^s = u_{ct+1} - u_{ct} + \omega_{t+1} + r_{t+1}^k$$

$$v_t^d = u_{ct+1} - u_{ct} + \omega_{t+1} + r_t$$

$$v_t^e = u_{ct+1} - u_{ct} + \omega_{t+1} + r_{t+1}^e$$

$$\Omega \omega_t = \sigma(1 + \tilde{\lambda})\tilde{v}^d v_t^d + \sigma \tilde{\lambda} \tilde{v}^d \lambda_t$$

and further simplify to (in an efficient steady state)

$$\beta \eta n_{t+1} = (\sigma + \xi)(r_{t+1}^k + s_t) - \sigma(1 - \tilde{\gamma} - \eta)(r_t + d_t) - \sigma \tilde{\gamma}(r_{t+1}^e + \gamma_t + s_t) + \sigma \tilde{\tau}(r_{t+1}^k + s_t) + u_t^n$$

$$(1 + \tilde{\lambda})(\tilde{v}^s v_{st} - (1 - \tilde{\gamma})\tilde{v}^d v_t^d - \tilde{\gamma}\tilde{v}^e v_t^e) + (\tilde{v}^s - \tilde{v}^d)\tilde{\lambda}\lambda_t = \tilde{\lambda}\theta\lambda_t$$

$$\tilde{v}^s(v_t^s + s_t) - \tilde{v}^d(1 - \tilde{\gamma} - \eta)(v_t^d + d_t) - \tilde{v}^e \tilde{\gamma}(v_t^e + \gamma_t + s_t) = \theta s_t$$

$$(1 - \tilde{\gamma})s_t - \tilde{\gamma}\gamma_t = (1 - \tilde{\gamma} - \eta)d_t + \eta n_t$$

$$v_t^s = u_{ct+1} - u_{ct} + \omega_{t+1} + r_{t+1}^k$$

$$v_t^d = u_{ct+1} - u_{ct} + \omega_{t+1} + r_t$$

$$v_t^e = u_{ct+1} - u_{ct} + \omega_{t+1} + r_{t+1}^e$$

$$\Omega\omega_t = \sigma(1 + \tilde{\lambda})\tilde{v}^d v_t^d + \sigma\tilde{\lambda}\tilde{v}^d \lambda_t$$

where we scaled the bank capital shock.

## B Approximation of household welfare

### B.1 Basic RBC case

Begin by deriving an approximation to household welfare in the basic RBC case. Here, we wish to approximate

$$\mathcal{U} \equiv E_t \sum_{t=0}^{\infty} \beta^t U_t, \quad U_t \equiv U(C_t, L_t).$$

A second-order approximation of the period utility function is:

$$U_t \simeq U + U_c(C_t - C) + \frac{1}{2}U_{cc}(C_t - C)^2 + U_l(L_t - L) + \frac{1}{2}U_{ll}(L_t - L)^2.$$

Throughout, we make use of the fact that

$$X_t - X \simeq X(1 + x_t + x_t^2),$$

In this case, we get:

$$U_t \simeq U + U_c C(c_t + \frac{1}{2}c_t^2) + \frac{1}{2}U_{cc}C^2c_t^2 + U_l L(l_t + \frac{1}{2}l_t^2) + \frac{1}{2}U_{ll}L^2l_t^2.$$

**Consumption terms.** Turn first to the consumption terms. We want to eliminate the linear ones. To do this, start with the aggregate resource constraint. In the simple RBC case (e.g. with no adjustment costs), this is:

$$Y_t = C_t + I_t.$$

This approximates to:

$$C\left(c_t + \frac{1}{2}c_t^2\right) = Y\left(y_t + \frac{1}{2}y_t^2\right) - I\left(i_t + \frac{1}{2}i_t^2\right)$$

in which  $\frac{I}{Y} = 1 - \frac{C}{Y}$ . Using this, we can re-write the resource constraint as:

$$c_t + \frac{1}{2}c_t^2 = \frac{1}{\zeta} \left( y_t + \frac{1}{2}y_t^2 \right) - \frac{1-\zeta}{\zeta} \left( i_t + \frac{1}{2}i_t^2 \right)$$

where  $\zeta \equiv C/Y$ .

In the simple RBC case, the evolution of capital is:

$$K_{t+1} = (1 - \delta)K_t + I_t$$

Its first-order approximate dynamics are:

$$k_{t+1} = (1 - \delta)k_t + \delta i_t$$

so

$$i_t = \frac{1}{\delta} (k_{t+1} - (1 - \delta)k_t)$$

We can use this to eliminate the linear term in investment from the second-order approximate resource constraint.

**Labour terms.** From the production function, aggregate labour demand is:

$$L_t = \left( \frac{Y_t}{A_t K_t^\alpha} \right)^{\frac{1}{1-\alpha}},$$

which approximates to:

$$l_t = \frac{1}{1-\alpha} (y_t - a_t - \alpha k_t).$$

We can use this to eliminate the linear term in labour from the second-order approximate utility function.

**Welfare function.** Before using these, first write the period utility function as:

$$\frac{U_t - U}{U_c C} \simeq \left( c_t + \frac{1}{2}c_t^2 \right) + \frac{1}{2} \frac{U_{cc} C}{U_c} c_t^2 + \frac{U_l L}{U_c C} \left( l_t + \frac{1}{2}l_t^2 \right) + \frac{1}{2} \frac{U_{ll} L}{U_l} \frac{U_l L}{U_c C} l_t^2,$$

in which

$$U(C) = \frac{C^{1-\sigma}}{1-\sigma}, \quad U_c = C^{-\sigma}, \quad U_{cc} = -\sigma C^{-\sigma-1},$$

so

$$\frac{U_{cc}C}{U_c} = -\sigma,$$

and in which

$$U(L) = \frac{1}{1+\varphi}L^{1+\varphi}, \quad U_l = L^\varphi, \quad U_{ll} = \varphi L^{\varphi-1},$$

so

$$\frac{U_{ll}L}{U_l} = \varphi.$$

As such, we get:

$$\frac{U_t - U}{U_c C} \simeq c_t + \frac{1-\sigma}{2}c_t^2 + \frac{U_l L}{U_c C}(l_t + \frac{1+\varphi}{2}l_t^2).$$

In the steady state, which is efficient, we have that:

$$-\frac{U_l}{U_c} = W = (1-\alpha)\frac{Y}{L}.$$

So the period utility function is:

$$\frac{U_t - U}{U_c C} \simeq c_t + \frac{1-\sigma}{2}c_t^2 - \frac{1-\alpha}{\zeta}(l_t + \frac{1+\varphi}{2}l_t^2).$$

Then use the aggregate resource constraint to eliminate the linear term in consumption, substituting:

$$c_t = \frac{1}{\zeta} \left( y_t + \frac{1}{2}y_t^2 \right) - \frac{1-\zeta}{\zeta}(i_t + \frac{1}{2}i_t^2) - \frac{1}{2}c_t^2$$

into the period utility function, so giving:

$$\frac{U_t - U}{U_c C} \simeq \frac{1}{\zeta} \left( y_t + \frac{1}{2}y_t^2 \right) - \frac{1-\zeta}{\zeta}(i_t + \frac{1}{2}i_t^2) - \frac{1}{2}c_t^2 + \frac{1-\sigma}{2}c_t^2 - \frac{1-\alpha}{\zeta}(l_t + \frac{1+\varphi}{2}l_t^2).$$

simplifying to:

$$\frac{U_t - U}{U_c C} \simeq \frac{1}{\zeta} \frac{1}{2}y_t^2 - \frac{1-\zeta}{\zeta}i_t - \frac{1}{2} \frac{1-\zeta}{\zeta}i_t^2 - \frac{\sigma}{2}c_t^2 + \frac{1}{\zeta}\alpha k_t - \frac{1-\alpha}{\zeta} \frac{1+\varphi}{2}l_t^2.$$

And then use the law of motion for capital to eliminate the linear term in investment, so giving:

$$\begin{aligned}\frac{U_t - U}{U_c C} &\simeq -\frac{1-\zeta}{\zeta} \frac{1}{\delta} (k_{t+1} - (1-\delta)k_t) + \frac{1}{\zeta} \alpha k_t \\ &+ \frac{1}{\zeta} \frac{1}{2} y_t^2 - \frac{\sigma}{2} c_t^2 - \frac{1}{2} \frac{1-\zeta}{\zeta} i_t^2 - \frac{1-\alpha}{\zeta} \frac{1+\varphi}{2} l_t^2.\end{aligned}$$

Note that

$$\frac{1-\zeta}{\delta} = \frac{1-C/Y}{\delta}.$$

In the steady state:

$$R^s = \alpha \frac{Y}{K} + (1-\delta) = R = \frac{1}{\beta}.$$

So

$$\frac{K}{Y} = \frac{\alpha}{\frac{1}{\beta} - (1-\delta)}.$$

Note that  $I = \delta K$ , so

$$\frac{I}{Y} = \beta \frac{\alpha \delta}{1 - \beta(1-\delta)},$$

so

$$1 - \frac{C}{Y} = \frac{I}{Y} = \beta \frac{\alpha \delta}{1 - \beta(1-\delta)}.$$

Therefore the terms in capital in the period utility function,  $\hat{k}_t$ , defined below, can be simplified:

$$\begin{aligned}\hat{k}_t &\equiv -\frac{1-\zeta}{\zeta} \frac{1}{\delta} (k_t - (1-\delta)k_{t-1}) + \frac{1}{\zeta} \alpha k_t \\ &= \frac{1}{\zeta} \left( -\frac{1-\zeta}{\delta} (k_{t+1} - (1-\delta)k_t) + \alpha k_t \right) \\ &= \frac{\alpha}{\zeta} \left( -\frac{\beta}{1 - \beta(1-\delta)} (k_{t+1} - (1-\delta)k_t) + k_t \right) \\ &= \frac{\alpha}{\zeta} \left( \frac{-\beta}{1 - \beta(1-\delta)} k_{t+1} + \frac{1 - \beta(1-\delta)}{1 - \beta(1-\delta)} k_t + \frac{\beta(1-\delta)}{1 - \beta(1-\delta)} k_t \right) \\ &= \frac{\alpha}{\zeta} \frac{1}{1 - \beta(1-\delta)} (k_t - \beta k_{t+1})\end{aligned}$$

Following [Edge \(2003\)](#), note that the infinite discounted sum of  $\hat{k}_t$ , which is what's relevant for

welfare, can be written as:

$$\begin{aligned} E_0 \sum_{t=0}^{\infty} \beta^t \hat{k}_t &= \frac{\alpha}{\zeta} \frac{1}{1 - \beta(1 - \delta)} E_0 (k_0 - \beta k_1 + \beta(k_1 - \beta k_2) + \dots) \\ &= \frac{\alpha}{\zeta} \frac{1}{1 - \beta(1 - \delta)} k_0 \end{aligned}$$

As a result, this term in the initial capital stock is treated as being fixed and independent of policy.

Therefore, we arrive at:

$$\frac{U_t - U}{U_c C} \simeq \frac{1}{\zeta} \frac{1}{2} y_t^2 - \frac{\sigma}{2} c_t^2 - \frac{1}{2} \frac{1 - \zeta}{\zeta} i_t^2 - \frac{1 - \alpha}{\zeta} \frac{1 + \varphi}{2} l_t^2.$$

The household's welfare is decreasing in the variances of consumption, investment, and labour. Conditional on this, it is increasing in the variance of output. However, note that the variance of output is linked to that of consumption and investment through the resource constraint. For example, using:

$$y_t = \zeta c_t + (1 - \zeta) i_t$$

we get:

$$\begin{aligned} y_t^2 &= \zeta^2 c_t^2 + (1 - \zeta)^2 i_t^2 + \zeta(1 - \zeta) c_t i_t \\ &\simeq \zeta^2 c_t^2 + (1 - \zeta)^2 i_t^2 \end{aligned}$$

So

$$\frac{U_t - U}{U_c C} \simeq -\frac{1}{2} \left( (\sigma - \zeta) c_t^2 + (1 - \zeta) i_t^2 + \frac{(1 - \alpha)(1 + \varphi)}{\zeta} l_t^2 \right).$$

## B.2 Case with adjustment costs

When there are capital adjustment costs and costs associated with changing equity requirements, the aggregate resource constraint reads:

$$Y_t = C_t + I_t + f \left( \frac{I_t}{I_{t-1}} \right) I_t + \frac{\Psi}{2} \left( \frac{\tilde{\gamma}_t}{\tilde{\gamma}} - 1 \right)^2 \tilde{\gamma} S_t.$$



Let

$$h(\tilde{\gamma}_t) \equiv \frac{\Psi}{2} \left( \frac{\tilde{\gamma}_t}{\tilde{\gamma}} - 1 \right)^2 \tilde{\gamma} S_t$$

such that:

$$\begin{aligned} h'(\tilde{\gamma}_t) &= \Psi \left( \frac{\tilde{\gamma}_t}{\tilde{\gamma}} - 1 \right) \frac{\tilde{\gamma}}{\tilde{\gamma}} S_t = \Psi \left( \frac{\tilde{\gamma}_t}{\tilde{\gamma}} - 1 \right) S_t, & h'(\tilde{\gamma}) &= 0 \\ h''(\tilde{\gamma}_t) &= \Psi \frac{S_t}{\tilde{\gamma}}, & h''(\tilde{\gamma}) &= \Psi \frac{S}{\tilde{\gamma}} \end{aligned}$$

Then we get:

$$\begin{aligned} (Y_t - Y) &= (C_t - C) + (I_t - I) \\ &\quad + f'(X)I(X_t - X) + \frac{1}{2}f''(X)I(X_t - X)^2 \\ &\quad + h'(\tilde{\gamma})(\tilde{\gamma}_t - \tilde{\gamma}) + \frac{1}{2}h''(\tilde{\gamma})(\tilde{\gamma}_t - \tilde{\gamma})^2 \end{aligned}$$

where  $X_t \equiv I_t/I_{t-1}$ . We have  $X = 1$ , and  $f'(X) = f'(\tilde{\gamma}) = 0$ , so the first-order effects of these terms are zero:

$$(Y_t - Y) = (C_t - C) + (I_t - I) + \frac{1}{2}f''(X)I(X_t - X)^2 + \frac{1}{2}h''(\tilde{\gamma})(\tilde{\gamma}_t - \tilde{\gamma})^2$$

We have that  $f''(X) = \omega$  and  $h''(\tilde{\gamma}) = \Psi \frac{S}{\tilde{\gamma}}$  so

$$(Y_t - Y) = (C_t - C) + (I_t - I) + \frac{1}{2}\omega I(X_t - X)^2 + \frac{1}{2}\Psi \frac{S}{\tilde{\gamma}}(\tilde{\gamma}_t - \tilde{\gamma})^2$$

Focussing on the adjustment cost terms:

$$\begin{aligned} \frac{1}{2}\omega I(X_t - X)^2 + \frac{1}{2}\Psi \frac{S}{\tilde{\gamma}}(\tilde{\gamma}_t - \tilde{\gamma})^2 &= \frac{1}{2}\omega I X^2 \left( \frac{X_t - X}{X} \right)^2 + \frac{1}{2}\Psi \frac{S}{\tilde{\gamma}} \tilde{\gamma}^2 \left( \frac{\tilde{\gamma}_t - \tilde{\gamma}}{\tilde{\gamma}} \right)^2 \\ &= \frac{1}{2}\omega I x_t^2 + \frac{1}{2}\Psi \tilde{\gamma} S \gamma_t^2 \end{aligned}$$

where we used  $X = 1$ , in the last line. So:

$$(Y_t - Y) = (C_t - C) + (I_t - I) + \frac{1}{2}\omega I x_t^2 + \frac{1}{2}\Psi \tilde{\gamma} S \gamma_t^2$$

Finally, with  $x_t = i_t - i_{t-1}$ , we get:

$$(Y_t - Y) = (C_t - C) + (I_t - I) + \frac{1}{2}\omega I (i_t - i_{t-1})^2 + \frac{1}{2}\Psi\tilde{\gamma}S\gamma_t^2$$

Using  $W_t - W = W(w_t + \frac{1}{2}w_t^2)$  for some  $W_t$ , then

$$Y(y_t + \frac{1}{2}y_t^2) = C(c_t + \frac{1}{2}c_t^2) + I(i_t + \frac{1}{2}i_t^2) + \frac{1}{2}\omega I (i_t - i_{t-1})^2 + \frac{1}{2}\Psi\tilde{\gamma}S\gamma_t^2$$

or:

$$y_t + \frac{1}{2}y_t^2 = \frac{C}{Y}(c_t + \frac{1}{2}c_t^2) + \frac{I}{Y}(i_t + \frac{1}{2}i_t^2) + \frac{1}{2}\omega\frac{I}{Y}(i_t - i_{t-1})^2 + \frac{1}{2}\Psi\frac{\tilde{\gamma}S}{Y}\gamma_t^2$$

In the perfect foresight steady state, it remains the case that:

$$Y = C + I, \quad \frac{C}{Y} \equiv \zeta$$

so

$$(y_t + \frac{1}{2}y_t^2) = \zeta(c_t + \frac{1}{2}c_t^2) + (1 - \zeta)(i_t + \frac{1}{2}i_t^2) + \frac{1}{2}\omega(1 - \zeta)(i_t - i_{t-1})^2 + \frac{1}{2}\Psi\frac{\tilde{\gamma}S}{Y}\gamma_t^2$$

Also, note that

$$S = K$$

and that

$$\frac{K}{Y} = \frac{\alpha}{\frac{1}{\beta} - (1 - \delta)}$$

as above. Then

$$(y_t + \frac{1}{2}y_t^2) = \zeta(c_t + \frac{1}{2}c_t^2) + (1 - \zeta)(i_t + \frac{1}{2}i_t^2) + \frac{1}{2}\omega(1 - \zeta)(i_t - i_{t-1})^2 + \frac{1}{2}\Psi\tilde{\gamma}\frac{\alpha\beta}{1 - \beta(1 - \delta)}\gamma_t^2$$

Rearranging this for the linear consumption term therefore gives:

$$c_t = \frac{1}{\zeta}(y_t + \frac{1}{2}y_t^2) - \frac{1}{2}c_t^2 - \frac{1 - \zeta}{\zeta}(i_t + \frac{1}{2}i_t^2) - \frac{1}{2}\omega\frac{1 - \zeta}{\zeta}(i_t - i_{t-1})^2 - \frac{1}{2}\Psi\tilde{\gamma}\frac{\alpha\beta}{1 - \beta(1 - \delta)}\gamma_t^2$$

**Welfare function.** Return to the household's utility function:

$$\frac{U_t - U}{U_c C} \simeq c_t + \frac{1 - \sigma}{2}c_t^2 - \frac{1 - \alpha}{\zeta}(i_t + \frac{1 + \varphi}{2}i_t^2).$$

Using the expression above to eliminate  $c_t$  and the production function to eliminate  $l_t$ :

$$\begin{aligned} \frac{U_t - U}{U_c C} &\simeq \frac{1}{2} \frac{1}{\zeta} y_t^2 - \frac{1 - \zeta}{\zeta} i_t - \frac{1}{2} \frac{1 - \zeta}{\zeta} i_t^2 - \frac{1}{2} \omega \frac{1 - \zeta}{\zeta} (i_t - i_{t-1})^2 - \frac{1}{2} \frac{1}{\zeta} \Psi \tilde{\gamma} \frac{\alpha \beta}{1 - \beta(1 - \delta)} \gamma_t^2 \\ &\quad - \frac{\sigma}{2} c_t^2 + \frac{1}{\zeta} \alpha k_t - \frac{1 - \alpha}{\zeta} \frac{1 + \varphi}{2} l_t^2. \end{aligned}$$

and the capital law of motion to eliminate investment  $i_t$ :

$$\begin{aligned} \frac{U_t - U}{U_c C} &\simeq \frac{1}{2} \frac{1}{\zeta} y_t^2 + \hat{k}_t - \frac{1}{2} \frac{1 - \zeta}{\zeta} i_t^2 - \frac{1}{2} \omega \frac{1 - \zeta}{\zeta} (i_t - i_{t-1})^2 - \frac{1}{2} \frac{1}{\zeta} \Psi \tilde{\gamma} \frac{\alpha \beta}{1 - \beta(1 - \delta)} \gamma_t^2 \\ &\quad - \frac{\sigma}{2} c_t^2 - \frac{1 - \alpha}{\zeta} \frac{1 + \varphi}{2} l_t^2. \end{aligned}$$

where  $\hat{k}_t \equiv -\frac{1 - \zeta}{\zeta} \frac{1}{\delta} (k_{t+1} - (1 - \delta)k_t) + \frac{1}{\zeta} \alpha k_t$  as above. As above, this term can be shown to be proportional to the initial capital stock, so can be dropped from the period utility function, giving:

$$\begin{aligned} \frac{U_t - U}{U_c C} &\simeq \frac{1}{2} \frac{1}{\zeta} y_t^2 - \frac{\sigma}{2} c_t^2 - \frac{1}{2} \frac{1 - \zeta}{\zeta} i_t^2 - \frac{1 - \alpha}{\zeta} \frac{1 + \varphi}{2} l_t^2 - \frac{1}{2} \omega \frac{1 - \zeta}{\zeta} \Delta i_t^2 \\ &\quad - \frac{1}{2} \frac{1}{\zeta} \Psi \tilde{\gamma} \frac{\alpha \beta}{1 - \beta(1 - \delta)} \gamma_t^2. \end{aligned}$$

As above, we can use:

$$y_t^2 \simeq \zeta^2 c_t^2 + (1 - \zeta)^2 i_t^2$$

so

$$\begin{aligned} \frac{U_t - U}{U_c C} &\simeq -\frac{1}{2} \\ &\quad \times \left( (\sigma - \zeta) c_t^2 + (1 - \zeta) i_t^2 + \frac{(1 - \alpha)(1 + \varphi)}{\zeta} l_t^2 + \omega \frac{1 - \zeta}{\zeta} \Delta i_t^2 + \frac{1}{\zeta} \Psi \tilde{\gamma} \frac{\alpha \beta}{1 - \beta(1 - \delta)} \gamma_t^2 \right). \end{aligned}$$

The last two terms in this expression reflect the additional real rigidities introduced into the model: welfare losses now also arise due to (a) the volatility of *changes* in investment,  $\Delta i_t^2$ , and (b) volatility in capital requirements,  $\gamma_t^2$ .

Using this,

$$\begin{aligned}
& 2(1 - \beta)\mathcal{U} \\
&= 2(1 - \beta)E_t \sum_{t=0}^{\infty} \beta^t U_t \\
&\simeq -(1 - \beta)E_t \sum_{t=0}^{\infty} \beta^t \\
&\times \left( (\sigma - \zeta)c_t^2 + (1 - \zeta)i_t^2 + \frac{(1 - \alpha)(1 + \varphi)}{\zeta}l_t^2 + \omega \frac{1 - \zeta}{\zeta} \Delta i_t^2 + \frac{1}{\zeta} \Psi \tilde{\gamma} \frac{\alpha \beta}{1 - \beta(1 - \delta)} \gamma_t^2 \right) \\
&= -(\sigma - \zeta)\text{var}(c_t) - (1 - \zeta)\text{var}(i_t) - \frac{(1 - \alpha)(1 + \varphi)}{\zeta}\text{var}(l_t) - \omega \frac{1 - \zeta}{\zeta}\text{var}(\Delta i_t) \\
&\quad - \frac{1}{\zeta} \Psi \tilde{\gamma} \frac{\alpha \beta}{1 - \beta(1 - \delta)} \text{var}(\gamma_t)
\end{aligned}$$

## References

- AIKMAN, D., A. G. HALDANE, AND B. D. NELSON (2015): “Curbing the credit cycle,” *The Economic Journal*, 125(585), 1072–1109.
- AJELLO, A., T. LAUBACH, J. D. LOPEZ-SALIDO, AND T. NAKATA (2018): “Financial Stability and Optimal Interest-Rate Policy,” *International Journal of Central Banking*.
- ANGELINI, P., S. NERI, AND F. PANETTA (2014): “The interaction between capital requirements and monetary policy,” *Journal of Money, Credit and Banking*, 46(6), 1073–1112.
- ANGELONI, I., AND E. FAIA (2013): “Capital regulation and monetary policy with fragile banks,” *Journal of Monetary Economics*, 60(3), 311–324.
- BANK OF ENGLAND (2009): “The role of macroprudential policy,” *Discussion Paper*.
- BASEL COMMITTEE ON BANKING SUPERVISION (2010): “Countercyclical capital buffer proposal - consultative document,” *Bank for International Settlements*.
- BERNANKE, B. S., M. GERTLER, AND S. GILCHRIST (1999): “The financial accelerator in a quantitative business cycle framework,” *Handbook of macroeconomics*, 1, 1341–1393.
- BIANCHI, J. (2010): “Credit externalities: Macroeconomic effects and policy implications,” *The American Economic Review*, 100(2), 398–402.
- BRUNNERMEIER, M. K., AND Y. SANNIKOV (2014): “A Macroeconomic Model with a Financial Sector,” *American Economic Review*, 104(2), 379–421.
- (2016): “Macro, Money and Finance: A Continuous Time Approach,” NBER Working Papers 22343, National Bureau of Economic Research, Inc.
- CARNEY, M. J. (2014): “The future of financial reform,” *Speech at the Monetary Authority of Singapore*.
- CHRISTIANO, L., AND D. IKEDA (2013): “Leverage restrictions in a business cycle model,” *NBER Working Paper No. 18688*.

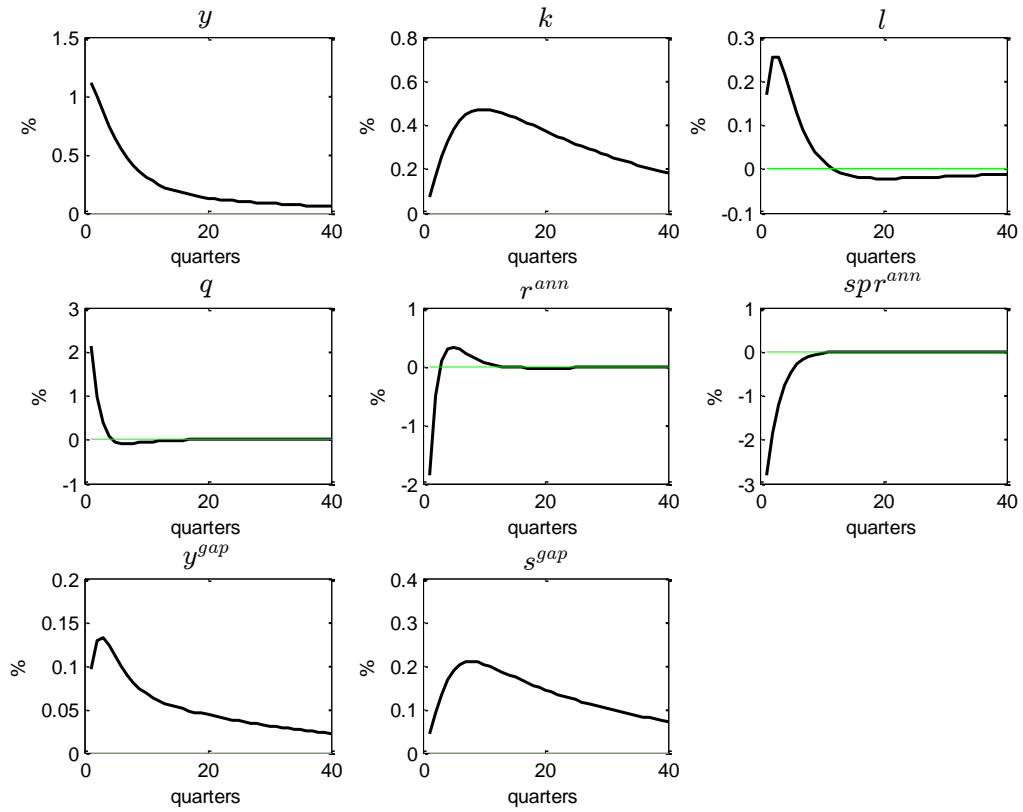
- CLARIDA, R., J. GALÍ, AND M. GERTLER (1999): “The Science of Monetary Policy: A New Keynesian Perspective,” *Journal of Economic Literature*, 37, 1661–1707.
- CLERC, L., A. DERVIZ, C. MENDICINO, S. MOYEN, K. NIKOLOV, L. STRACCA, J. SUAREZ, AND A. P. VARDOULAKIS (2015): “Capital Regulation in a Macroeconomic Model with Three Layers of Default,” *International Journal of Central Banking*, 11(3), 9–63.
- COLLARD, F., H. DELLAS, B. DIBA, AND O. LOISEL (2017): “Optimal Monetary and Prudential Policies,” *American Economic Journal: Macroeconomics*, 9(1), 40–87.
- DRAGHI, M. (2017): “The interaction between monetary policy and financial stability in the euro area,” *Speech at the First Conference on Financial Stability*.
- DREHMANN, M., C. BORIO, AND K. TSATSARONIS (2011): “Anchoring Countercyclical Capital Buffers: The role of Credit Aggregates,” *International Journal of Central Banking*, 7(4), 189–240.
- EDGE, R. M. (2003): “A utility-based welfare criterion in a model with endogenous capital accumulation,” *Finance and Economics Discussion Series (Board of Governors of the Federal Reserve System (U.S.))*, (2003-66).
- GERALI, A., S. NERI, L. SESSA, AND F. M. SIGNORETTI (2010): “Credit and Banking in a DSGE Model of the Euro Area,” *Journal of Money, Credit and Banking*, 42(s1), 107–141.
- GERTLER, M., AND P. KARADI (2011): “A model of unconventional monetary policy,” *Journal of Monetary Economics*, 58(1), 17–34.
- GERTLER, M., P. KARADI, ET AL. (2013): “Qe 1 vs. 2 vs. 3...: A framework for analyzing large-scale asset purchases as a monetary policy tool,” *International Journal of central Banking*, 9(1), 5–53.
- GERTLER, M., N. KIYOTAKI, ET AL. (2010): “Financial intermediation and credit policy in business cycle analysis,” *Handbook of Monetary Economics*, 3(3), 547–599.

- GERTLER, M., N. KIYOTAKI, AND A. QUERALTO (2012): “Financial crises, bank risk exposure and government financial policy,” *Journal of Monetary Economics*, 59, S17–S34.
- GIESE, J., H. ANDERSEN, O. BUSH, C. CASTRO, M. FARAG, AND S. KAPADIA (2014): “The Credit-To-Gdp Gap And Complementary Indicators For Macroprudential Policy: Evidence From The Uk,” *International Journal of Finance and Economics*, 19(1), 25–47.
- HALDANE, A. G. (2014): “Ambidexterity,” *Speech at the American Economic Association Annual Meeting*.
- HANSON, S. G., A. K. KASHYAP, AND J. C. STEIN (2011): “A macroprudential approach to financial regulation,” *The Journal of Economic Perspectives*, 25(1), 3–28.
- HE, Z., AND A. KRISHNAMURTHY (2013): “Intermediary Asset Pricing,” *American Economic Review*, 103(2), 732–70.
- JEANNE, O., AND A. KORINEK (2010): “Managing credit booms and busts: A Pigouvian taxation approach,” *NBER Working Paper No. 16377*.
- KASHYAP, A. K., D. P. TSOMOCOS, AND A. P. VARDOULAKIS (2014): “How does macroprudential regulation change bank credit supply?,” Working Paper 20165, National Bureau of Economic Research.
- KIYOTAKI, N., AND J. MOORE (1997): “Credit Cycles,” *The Journal of Political Economy*, 105(2), 211–248.
- LORENZONI, G. (2008): “Inefficient credit booms,” *The Review of Economic Studies*, 75(3), 809–833.
- MENDICINO, C., K. NIKOLOV, J. SUAREZ, AND D. SUPERA (2018): “Optimal Dynamic Capital Requirements,” *Journal of Money, Credit and Banking*, 50(6), 1271–1297.
- MORRIS, S., AND H. S. SHIN (2008): “Financial regulation in a system context,” *Brookings papers on economic activity*, (2008(2)), 229–274.

- PAOLI, B. D., AND M. PAUSTIAN (2017): “Coordinating Monetary and Macroprudential Policies,” *Journal of Money, Credit and Banking*, 49(2-3), 319–349.
- REPULLO, R., AND J. SUAREZ (2013): “The Procyclical Effects of Bank Capital Regulation,” *Review of Financial Studies*, 26(2), 452–490.
- SCHULARICK, M., AND A. M. TAYLOR (2012): “Credit booms gone bust: monetary policy, leverage cycles, and financial crises, 1870–2008,” *The American Economic Review*, 102(2), 1029–1061.
- STEIN, J. C. (2012): “Monetary Policy as Financial Stability Regulation,” *The Quarterly Journal of Economics*, 127(1), 57–95.
- SVEEN, T., AND L. WEINKE (2009): “Firm-specific capital and welfare,” *International Journal of Central Banking*, 5(2).
- SVENSSON, L. E. O. (2017): “Cost-benefit analysis of leaning against the wind,” *Journal of Monetary Economics*, 90, 193–213.
- (2018): “Monetary policy and macroprudential policy: Different and separate?,” *Canadian Journal of Economics*, 51(3), 802–827.
- YELLEN, J. L. (2017): “Financial Stability a Decade after the Onset of the Crisis,” *Speech at Jackson Hole*.

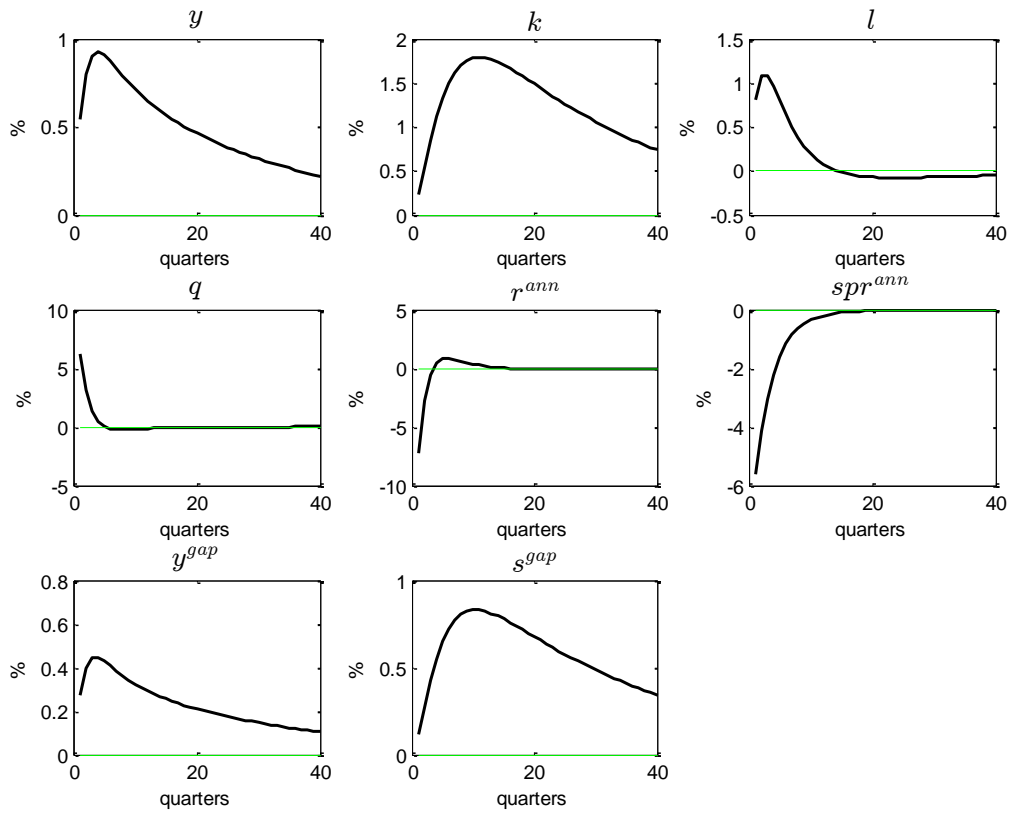


FIGURE 1: Technology shock



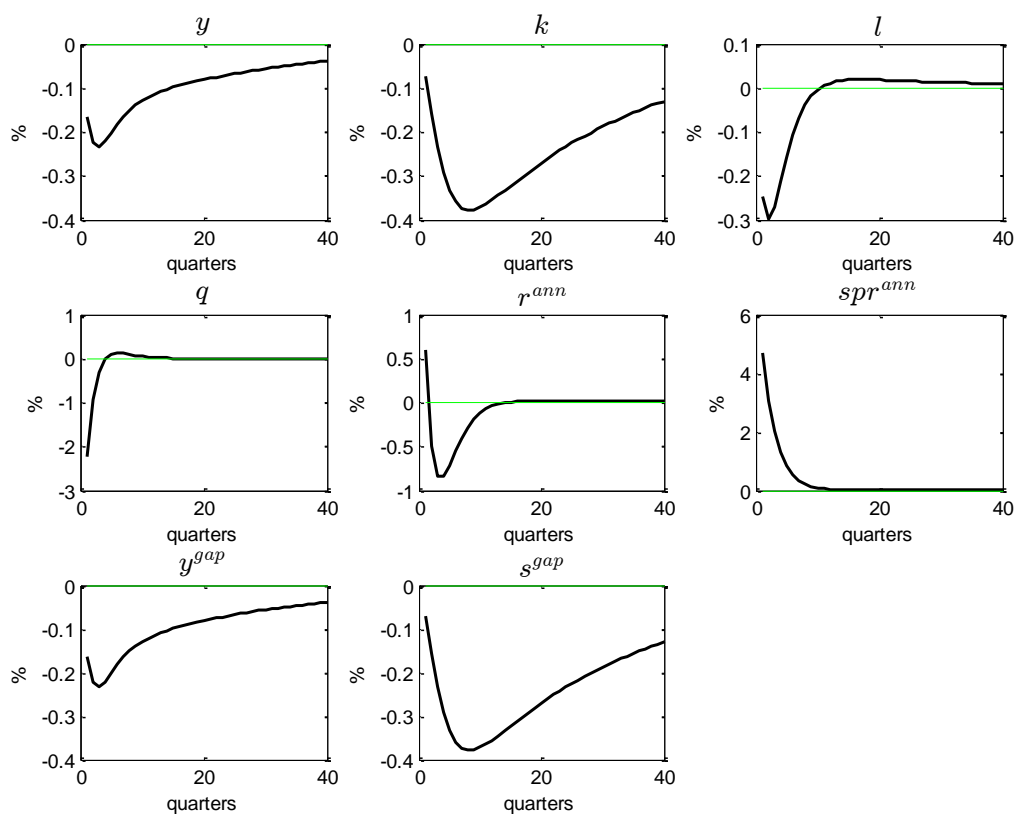
Notes:  $y$  = output,  $k$  = capital,  $l$  = hours,  $q$  = price of capital,  $r^{ann}$  = interest rate (annualised),  $spr^{ann}$  = spread (annualised),  $y^{gap}$  = output gap,  $s^{gap}$  = credit gap.

FIGURE 2: Natural interest rate shock



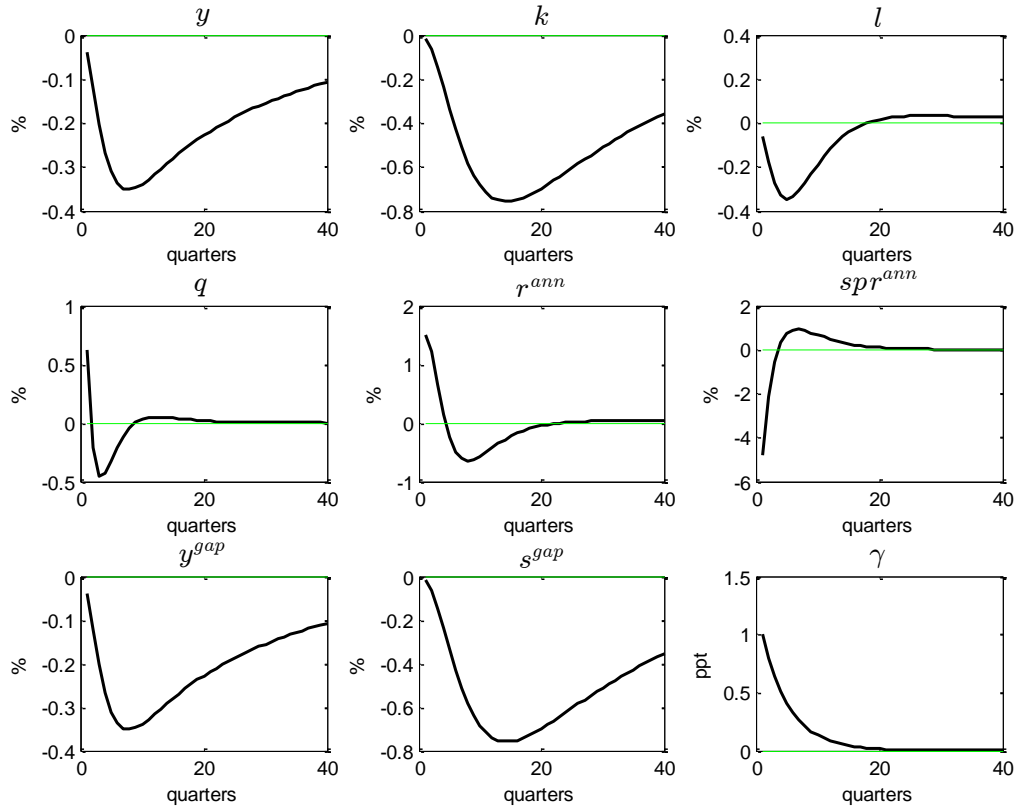
Notes:  $y$  = output,  $k$  = capital,  $l$  = hours,  $q$  = price of capital,  $r^{ann}$  = interest rate (annualised),  $spr^{ann}$  = spread (annualised),  $y^{gap}$  = output gap,  $s^{gap}$  = credit gap.

FIGURE 3: Bank capital (financial) shock



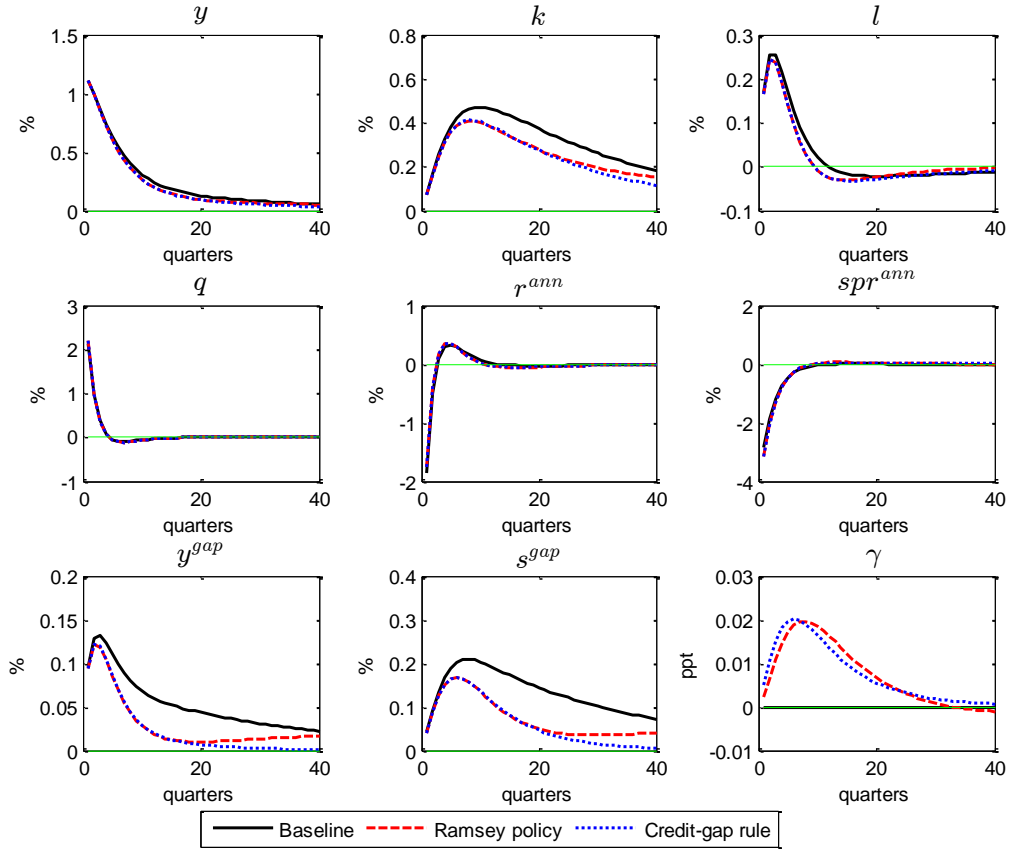
Notes:  $y$  = output,  $k$  = capital,  $l$  = hours,  $q$  = price of capital,  $r^{ann}$  = interest rate (annualised),  $spr^{ann}$  = spread (annualised),  $y^{gap}$  = output gap,  $s^{gap}$  = credit gap.

FIGURE 4: Macroprudential policy shock



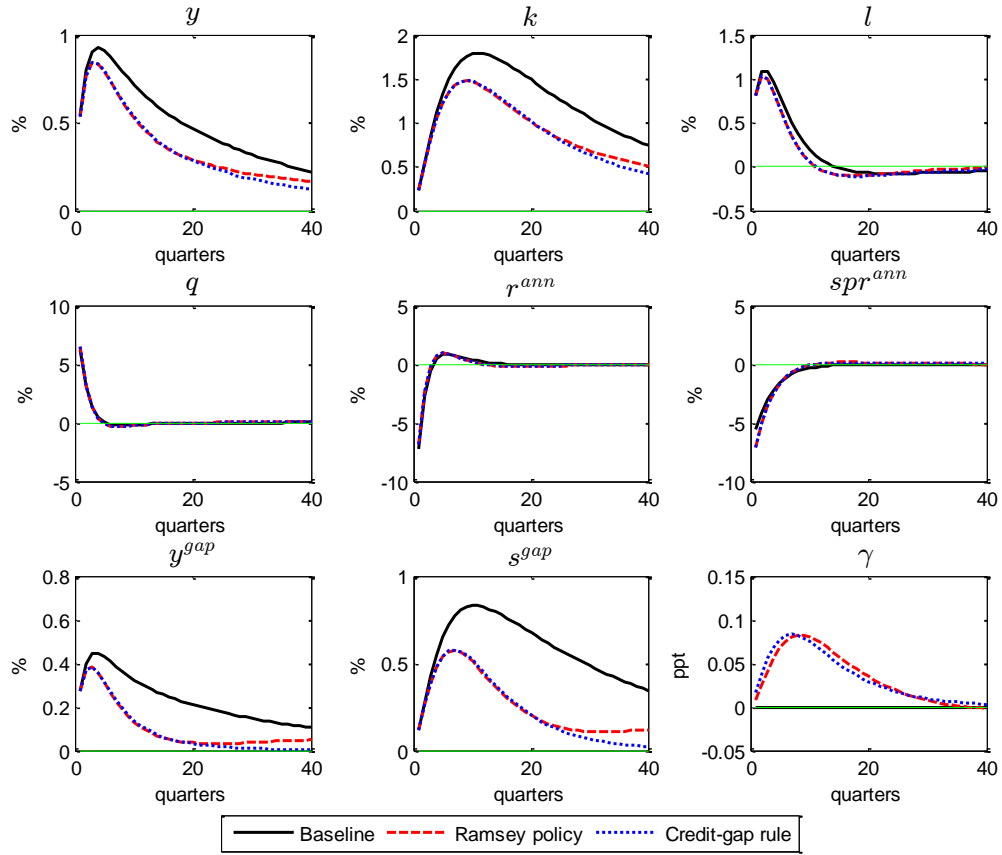
Notes:  $y$  = output,  $k$  = capital,  $l$  = hours,  $q$  = price of capital,  $r^{\text{ann}}$  = interest rate (annualised),  $\text{spr}^{\text{ann}}$  = spread (annualised),  $y^{\text{gap}}$  = output gap,  $s^{\text{gap}}$  = credit gap,  $\gamma$  = capital buffer requirement.

FIGURE 5: Technology shock under Ramsey optimal policy and optimal simple credit-gap rule



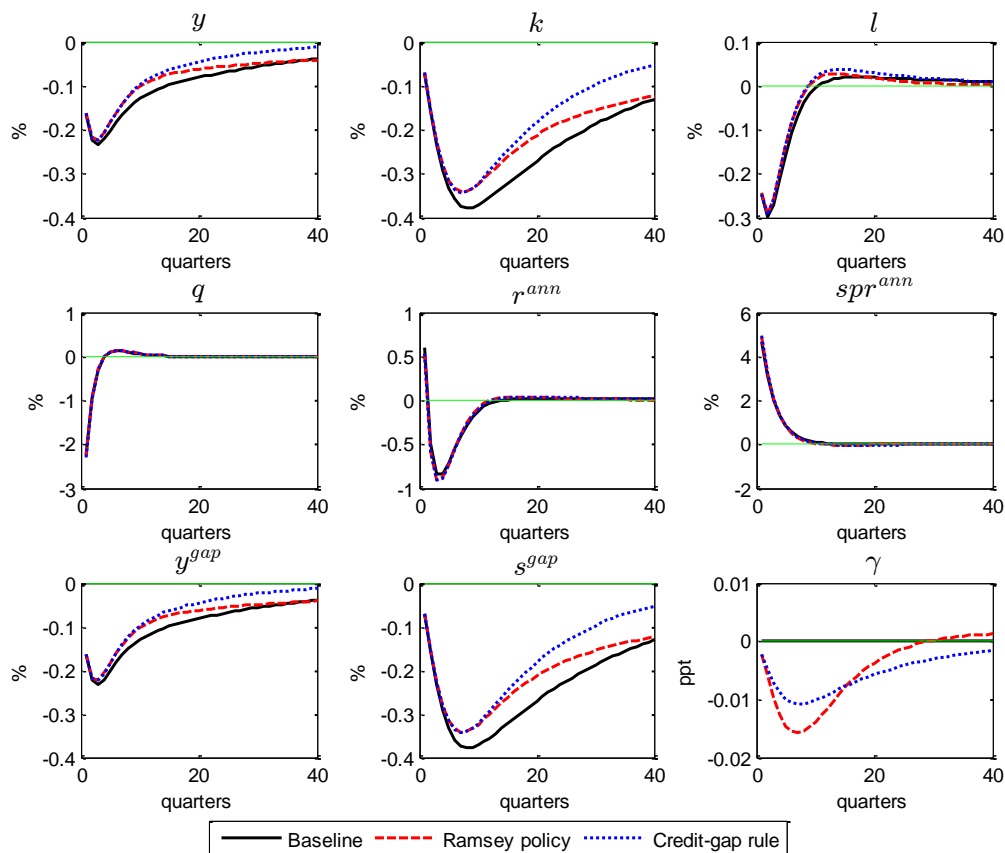
Notes:  $y$  = output,  $k$  = capital,  $l$  = hours,  $q$  = price of capital,  $r^{ann}$  = interest rate (annualised),  $spr^{ann}$  = spread (annualised),  $y^{gap}$  = output gap,  $s^{gap}$  = credit gap,  $\gamma$  = capital buffer requirement. ‘Ramsey policy’ corresponds to the case where the capital buffer  $\gamma$  is varied under the optimal commitment policy to minimise the loss  $\mathcal{W}$  defined in the text. ‘Credit-gap rule’ corresponds to the case where the capital buffer is varied according to a the rule  $\gamma_t = v_s s^{gap}$ , where  $v_s$  is computed optimally to minimise the loss  $\mathcal{W}$  defined in the text.

FIGURE 6: Natural interest rate shock under Ramsey optimal policy and optimal simple credit-gap rule



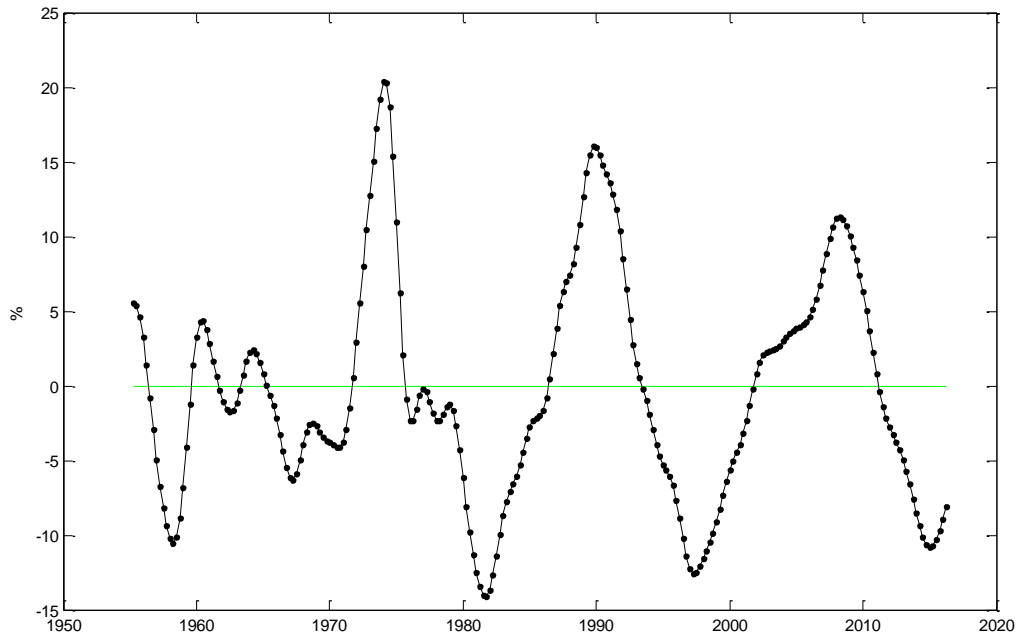
Notes:  $y$  = output,  $k$  = capital,  $l$  = hours,  $q$  = price of capital,  $r^{ann}$  = interest rate (annualised),  $spr^{ann}$  = spread (annualised),  $y^{gap}$  = output gap,  $s^{gap}$  = credit gap,  $\gamma$  = capital buffer requirement. ‘Ramsey policy’ corresponds to the case where the capital buffer  $\gamma$  is varied under the optimal commitment policy to minimise the loss  $\mathcal{W}$  defined in the text. ‘Credit-gap rule’ corresponds to the case where the capital buffer is varied according to a the rule  $\gamma_t = v_s s^{gap}$ , where  $v_s$  is computed optimally to minimise the loss  $\mathcal{W}$  defined in the text.

FIGURE 7: Bank capital shock under Ramsey optimal policy and optimal simple credit-gap rule



Notes:  $y$  = output,  $k$  = capital,  $l$  = hours,  $q$  = price of capital,  $r^{\text{ann}}$  = interest rate (annualised),  $\text{spr}^{\text{ann}}$  = spread (annualised),  $y^{\text{gap}}$  = output gap,  $s^{\text{gap}}$  = credit gap,  $\gamma$  = capital buffer requirement. ‘Ramsey policy’ corresponds to the case where the capital buffer  $\gamma$  is varied under the optimal commitment policy to minimise the loss  $\mathcal{W}$  defined in the text. ‘Credit-gap rule’ corresponds to the case where the capital buffer is varied according to a the rule  $\gamma_t = v_s s^{\text{gap}}$ , where  $v_s$  is computed optimally to minimise the loss  $\mathcal{W}$  defined in the text.

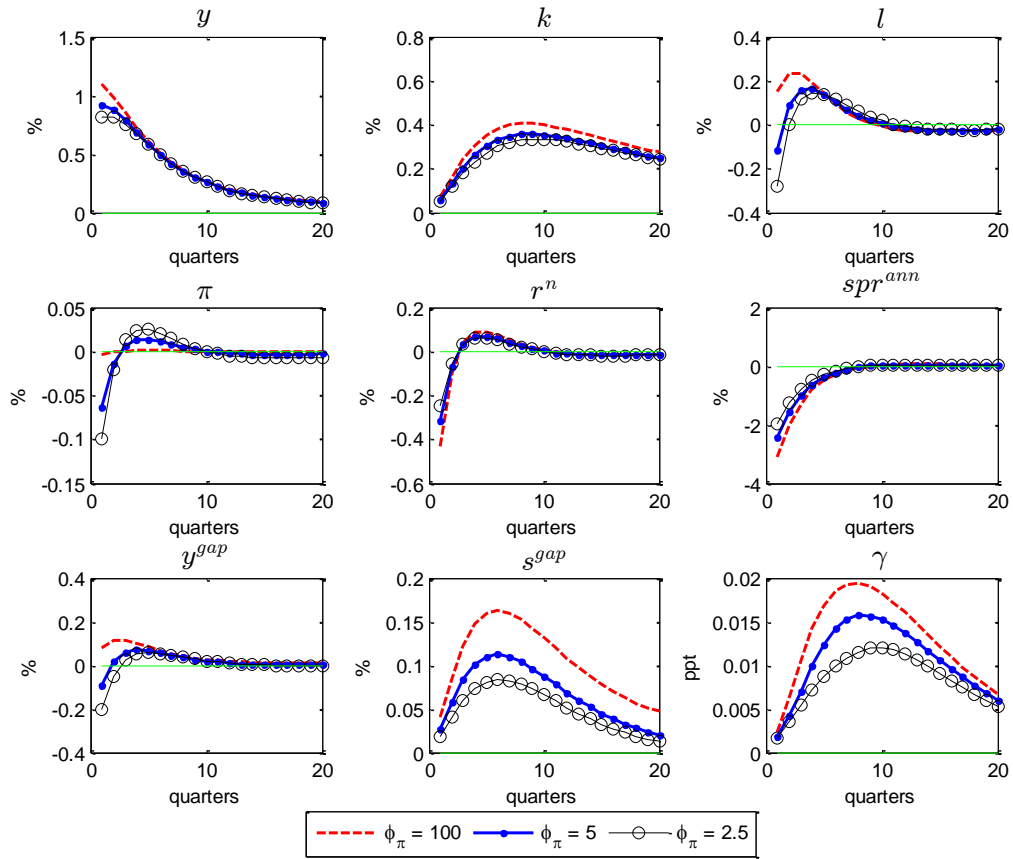
FIGURE 8: De-trended credit in the UK



Notes: Credit is defined as household liabilities plus debt liabilities of non-financial corporates. The series is deflated by the GDP deflator and filtered using a band-pass filter to isolate variation in the 2- to 20-year frequency range, thereby passing a relatively smooth trend through the series.

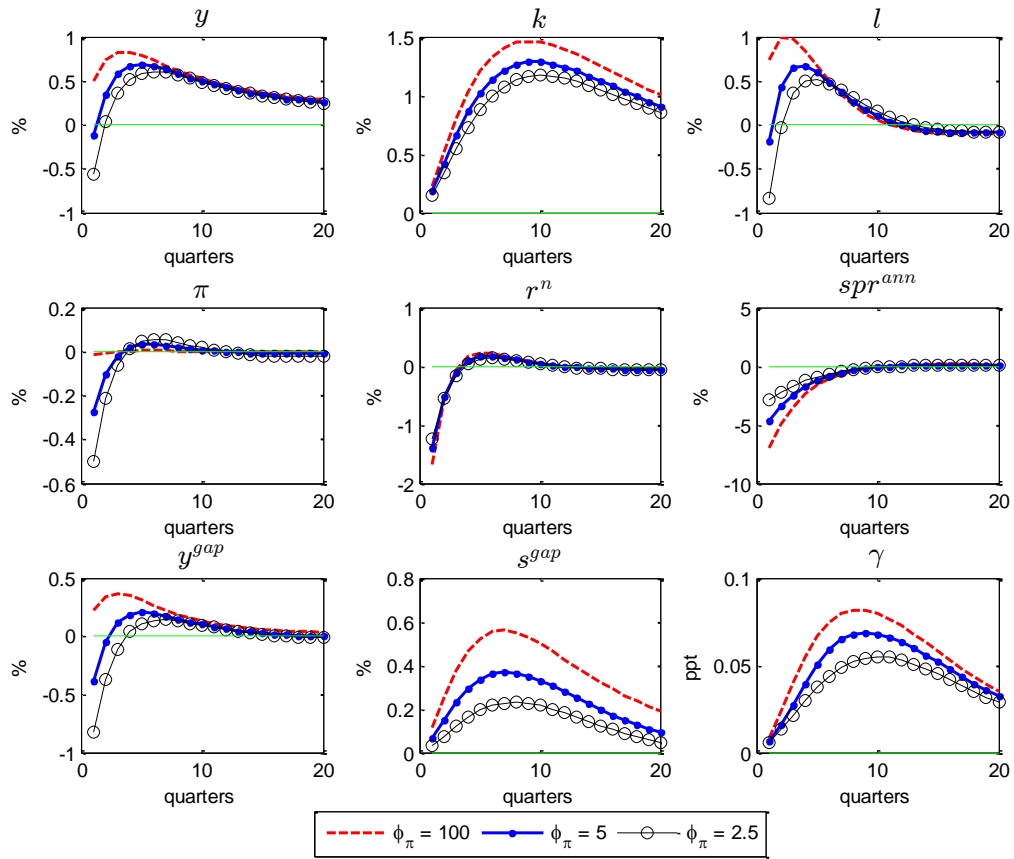


FIGURE 9: Ramsey optimal macroprudential policy response to technology shock as monetary policy response to inflation varies



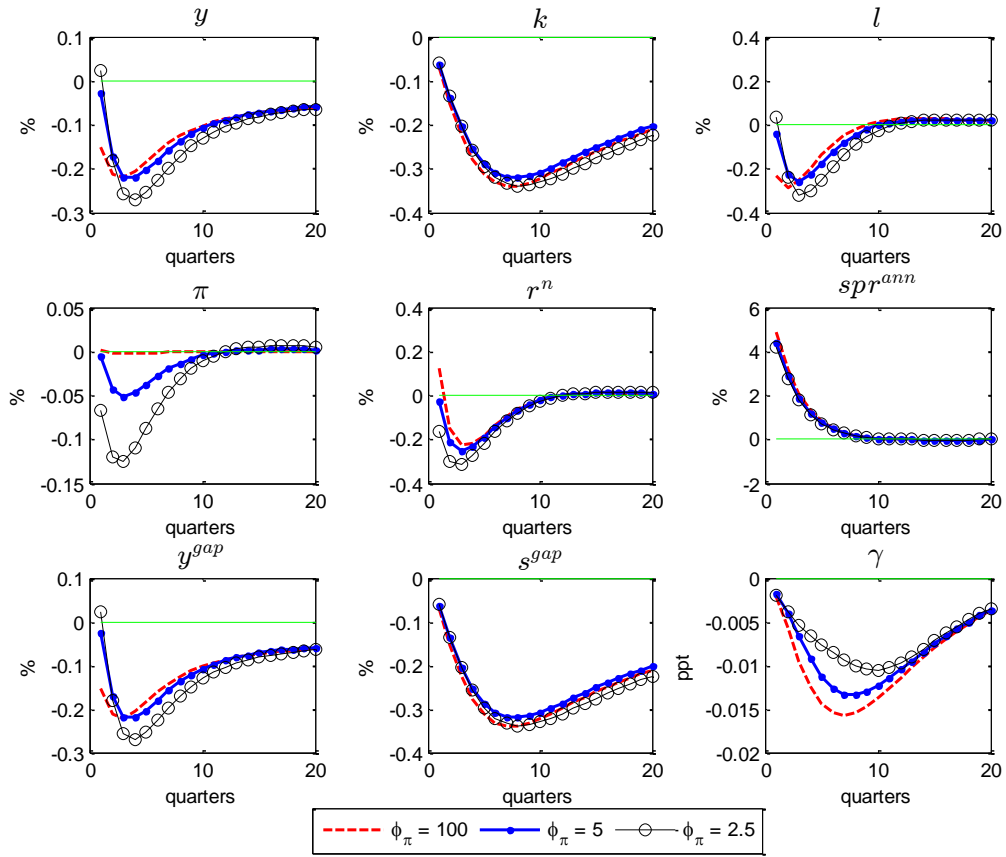
Notes:  $y$  = output,  $k$  = capital,  $l$  = hours,  $\pi$  = inflation,  $r^n$  = policy rate,  $spr^{ann}$  = spread (annualised),  $y^{gap}$  = output gap,  $s^{gap}$  = credit gap,  $\gamma$  = capital buffer requirement.  $\phi_\pi$  is the response of monetary policy to inflation in the Taylor rule.

FIGURE 10: Ramsey optimal macroprudential policy response to natural interest rate shock as monetary policy response to inflation varies



Notes:  $y$  = output,  $k$  = capital,  $l$  = hours,  $\pi$  = inflation,  $r^n$  = policy rate,  $spr^{ann}$  = spread (annualised),  $y^{gap}$  = output gap,  $s^{gap}$  = credit gap,  $\gamma$  = capital buffer requirement.  $\phi_\pi$  is the response of monetary policy to inflation in the Taylor rule.

FIGURE 11: Ramsey optimal macroprudential policy response to bank capital shock as monetary policy response to inflation varies



Notes:  $y$  = output,  $k$  = capital,  $l$  = hours,  $\pi$  = inflation,  $r^n$  = policy rate,  $spr^{ann}$  = spread (annualised),  $y^{gap}$  = output gap,  $s^{gap}$  = credit gap,  $\gamma$  = capital buffer requirement.  $\phi_\pi$  is the response of monetary policy to inflation in the Taylor rule.