



BANK OF ENGLAND

Staff Working Paper No. 766

The leverage ratio, risk-taking and bank stability

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Jonathan Acosta-Smith,⁽¹⁾ Michael Grill⁽²⁾ and Jan Hannes Lang⁽³⁾

Abstract

This paper addresses the trade-off between additional loss-absorbing capacity and potentially higher bank risk-taking associated with the introduction of the Basel III leverage ratio. This is addressed in both a theoretical and empirical setting. Using a theoretical micro model, we show that a leverage ratio requirement can incentivise banks that are bound by it to increase their risk-taking. This increase in risk-taking however, should be more than outweighed by the benefits of higher capital, thereby leading to more stable banks. These theoretical predictions are tested and confirmed in an empirical analysis on a large sample of EU banks. Our baseline empirical model suggests that a leverage ratio requirement would lead to a significant decline in the distress probability of highly leveraged banks.

Key words: Bank capital, risk-taking, leverage ratio, Basel III.

JEL classification: G01, G21, G28.

(1) Bank of England. Email: jonathan.smith@bankofengland.co.uk

(2) European Central Bank. Email: michael.grill@ecb.europa.eu

(3) European Central Bank. Email: jan-hannes.lang@ecb.europa.eu

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Bank of England, Threadneedle Street, London, EC2R 8AH

Telephone +44 (0)20 3461 4030 email publications@bankofengland.co.uk

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Non-Technical Summary

As a response to the global financial crisis, the Basel Committee on Banking Supervision (BCBS) decided to undertake a major reform to the regulatory framework of the banking system. Under the new Basel III capital framework, a non-risk based leverage ratio (LR) will be introduced alongside the risk-based capital framework. The aim is to “restrict the build-up of excessive leverage in the banking sector to avoid destabilising deleveraging processes that can damage the broader financial system and the economy” (BCBS (2014a)).¹ The LR is a non-risk based capital measure and it is defined as Tier 1 capital over a bank’s total exposure measure, which consists of on-balance sheet as well as off-balance sheet items. It has been widely expected that the LR will become a Pillar I requirement for banks under Basel III, ever since the BCBS issued a consultative document that outlined a baseline proposal for the design of the LR in December 2009.²

Nonetheless, the LR has been subject to various criticism raised by market participants and other stakeholders. The main concern relates to the risk-insensitivity of the LR: assets with the same nominal value but of different riskiness are treated equally and face the same capital requirement under the non-risk based LR.³ Given that an LR has a skewed impact, binding only for those banks with a large share of low risk-weighted assets on their balance sheets, this move away from a solely risk-based capital requirement may induce these banks to increase their risk-taking; potentially offsetting any benefits from requiring them to hold more capital. This paper addresses exactly this trade-off between additional loss-absorbing capacity and potentially higher bank risk-taking associated with an LR, in both a theoretical and empirical setting.

First, we build a simple theoretical model that is able to capture the trade-off between risk-taking and higher loss-absorption associated with an LR. The model yields two key results. First, if equity is sufficiently costly, imposing an LR indeed always incentivises banks that are bound by it to modestly increase risk-taking. This occurs because the non-risk based nature of the LR effectively reduces the marginal cost of risk-taking. Under an LR, bound banks are no longer forced to

¹The Basel III framework also includes a strengthened risk-based capital framework and two new liquidity requirements, i.e. the Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NSFR). See BCBS (2014a) for further details.

²See BCBS (2009). Also note that in the U.S., bank organizations have long been subject to an LR. However, whereas the U.S. LR is restricted to on-balance sheet items, the Basel III LR also includes off-balance sheet items in its exposure measure.

³See for example ESRB (2015).

hold additional capital when they take greater risk, and greater risk is associated with a greater expected return. If capital is expensive, under a risk-based framework this incentivises banks to reduce their risk-taking as adding capital contributes to marginal costs. Under a binding LR, this marginal cost disappears and hence banks increase their risk-taking since they can now increase risk and return without the penalty of having to hold greater capital.

Nevertheless, this increase in risk-taking is not unbounded. First, the risk-based capital framework underlies the LR, such that if the bank takes too much additional risk it will simply move back into the risk-based capital framework. Second, there exists an offsetting effect on risk-taking incentives from the fact that banks are required to hold greater capital, as this to some extent makes them more cautious (banks have more “skin in the game”). Consequently, the second key result from the model suggests that imposing an LR should be beneficial for bank stability as the additional loss-absorbing capacity of banks dominates the increase in risk-taking. In particular, the model suggests that adding an LR to the risk-based capital framework will both weakly decrease banks’ probability of distress, and if the distribution of banks is not such that the majority of banks are concentrated around the LR minimum, which is arguably the case in reality, should strictly decrease the expected loss of deposit funds.

The theoretical banking model that we develop therefore yields two testable hypotheses. First, if equity is costly, the introduction of an LR should incentivise banks for which it is a binding constraint to modestly increase risk-taking. Second, the negative impact of increased risk-taking induced by an LR should be outweighed by the beneficial impact of increased loss-absorbing capacity, resulting in more stable banks as measured by their distress probability. We take these two hypotheses and test them empirically on a large dataset of EU banks that encompasses a unique collection of bank distress events.

The empirical analysis follows in three steps. We first test whether banks with low LRs started to increase their risk-taking and capital positions after the announcement of the Basel III LR regime at the end of 2009 using a difference-in-difference type approach. We then estimate the joint effects of the LR and risk-taking on bank distress probabilities in a logit model framework, in order to quantify the risk-stability trade-off associated with an LR. Finally, we combine the first and second stage empirical results into a counterfactual simulation to test whether the negative impact of the estimated increase in risk-taking is outweighed by the benefits of increasing loss-absorbing capacity, i.e. whether an LR is beneficial for bank stability.

The empirical evidence provided in the paper lends support to both hypotheses. Our estimates suggest that banks bound by the LR increase their risk-weighted assets to total assets ratio by around 1.5 - 2.5 percentage points more than they otherwise would without an LR. Importantly, this small increase in risk-taking is more than compensated for by the substantial increase in capital positions for highly leveraged banks, which results in significantly lower estimated distress probabilities for banks bound by the LR.

The theoretical and empirical results of our paper therefore support the introduction of an LR alongside the risk-based capital framework for banks. The analysis further suggests that the LR and the risk-based capital framework reinforce each other by covering risks which the other is less able to capture, making sure banks do not operate with excessive leverage, and at the same time, have sufficient incentives for keeping risk-taking in check.

1 Introduction

Excessive leverage has been identified as one of the key drivers of the recent financial crisis and of many past crises.⁴ Moreover, in the recent crisis a significant number of banks were found to have built up excessive leverage while apparently maintaining strong risk-based capital ratios (BCBS, 2014a). As a response, the Basel Committee on Banking Supervision (BCBS) decided to introduce into the Basel III regulatory framework, a non-risk based leverage ratio (LR) alongside the risk-based capital requirement.⁵ Nevertheless, the LR has been subject to various criticism raised by market participants and other stakeholders, mainly related to its risk-insensitivity: as a non-risk based measure, assets of the same nominal value but of different riskiness are treated equally and face the same capital requirement. This has raised some concern that a move away from a solely risk-based capital framework will simply lead banks constrained by the LR to increase their risk-taking; potentially offsetting any benefits from holding higher capital.⁶

This paper addresses exactly this trade-off between additional loss-absorbing capacity and higher bank risk-taking associated with an LR. This is done in both a theoretical and empirical model. We first build a simple micro model that suggests if equity is sufficiently costly, there does exist an increased incentive to take further risk once banks become constrained by the LR. Nonetheless, our theoretical analysis suggests this increase in risk-taking should be limited and outweighed by the beneficial impact of the concurrent increase in loss-absorbing capacity arising from a higher capital requirement. The main result from the theory is therefore that banks become less likely to fail with an LR.

These theoretical results are then tested and confirmed within a three-stage empirical analysis on a large sample of EU banks for the period 2005 - 2014. First, we provide evidence of moderate increases in bank risk-taking using a difference-in-

⁴Using a historical dataset for 14 developed countries over almost 140 years, Schularick and Taylor (2012) provide ample evidence that excessive leverage contributed to recurrent episodes of financial instability.

⁵See BCBS (2009) and BCBS (2014a). The Basel III banking regulations also include a strengthened risk-based capital framework and two new liquidity requirements, i.e. the Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NSFR).

⁶For example the ex-CEO of Barclays Antony Jenkins expressed concern about the LR saying it needed “to be interpreted with care to avoid unintended consequences such as credit restriction and asset quality dilution” (see Treanor (2013)). Other examples include the Financial Supervisory Authority in Sweden (Finansinspektionen (2015)) which noted that “if non-risk-sensitive capital requirements - such as a leverage ratio requirement or standardised floor - are set at a level that makes them the binding capital restriction, Sweden may end up with a smaller, but riskier banking system. [...] A high leverage ratio requirement could consequently result in less financial stability”.

difference type approach taking the Basel III LR announcement at the end of 2009 as a treatment that only affects a subset of banks that are highly leveraged. Second, we show in a logit model framework that the marginal impact on a bank's distress probability of increasing its LR is much larger than the marginal negative impact of increased bank risk-taking, especially for highly leveraged banks. Third, we combine the first and second stage empirical results into a counterfactual simulation and find that an LR should be beneficial for financial stability by significantly reducing the distress probability of highly leveraged banks.

For our theoretical analysis, we develop a bank micro model along the lines of Dell'Ariccia et al. (2014) that is able to capture the trade-off between risk-taking and higher loss-absorption associated with an LR. In line with the Basel III regulatory framework, we consider a setting in which the risk-based capital framework is complemented with a non-risk based LR. Banks thus face the maximum of two capital charges. The LR requires banks to hold capital against its assets independent of the riskiness of its portfolio, whereas the capital requirement of the risk-based framework depends on the risk choice of the bank. Banks can choose between two types of assets: a (relatively) safe asset and a risky asset. We then introduce the key friction of our model, a correlated system-wide shock that has a small probability of occurring, but hits both the safer and the risky asset. In our setting, the risk-weighted framework is not able to perfectly cover this correlated shock, therefore providing an opportunity for the LR to improve upon a situation with only a risk-based framework. This friction relates directly to one of the Basel Committee's key reasons for the imposition of an LR: the build-up of leverage in low-risk assets and the imperfect coverage of rare shocks to these assets under the risk-based capital framework (BCBS, 2014b).

In a first step, we show that if equity is sufficiently costly, imposing an LR always incentivises banks bound by it to take more risk. This occurs because under an LR, constrained banks are no longer forced to hold additional capital when they take greater risk, and greater risk is associated with a greater expected return. If capital is expensive, under a risk-based framework banks are incentivised to reduce their risk-taking as higher risk requires higher capital and therefore contributes to marginal costs. Under a binding LR, this marginal cost disappears and hence banks increase their risk-taking since they can now increase risk and expected return without the penalty of having to hold greater capital.

Despite this however, we find that imposing an LR is beneficial as it reduces both banks' probability of failure and the expected loss of deposit funds. In other words,

the benefit of increased loss absorbing capacity brought about by the LR outweighs any negative impact from additional risk-taking in our framework. This is due to two reasons. First, there is a limit to how much additional risk a bank can take. If it takes too much additional risk, it will simply move back into the risk-based capital framework. Hence, as long as the risk-based capital requirement applies alongside the LR, it acts to constrain this risk-taking incentive. Second, there exists a skin-in-the-game effect that somewhat offsets the incentive to increase risk-taking once a bank is bound by the LR. Forcing banks to hold greater capital means they survive larger shocks. As a result, banks internalise losses which they otherwise would have ignored due to limited liability, and this decreases their incentive to take further risk.

The model therefore illustrates both how incentives adjust under a combined LR and risk-based capital framework, and how the trade-off between higher loss absorbing capacity and increased risk-taking looks once banks become constrained by the LR requirement. Our paper therefore relates to the extensive literature on bank capital and risk-taking, which has been remarkably inconclusive.⁷ Theoretical predictions have ranged from suggesting higher requirements lead to riskier asset profiles (e.g. Kahane (1977), Koehn and Santomero (1980) and Kim and Santomero (1988)) to either suggesting the effect can be ambiguous (Gennotte and Pyle (1991); Calem and Rob (1999); Blum (1999)) or lead to lower risk-taking incentives (Kealey and Furlong (1990); Flannery (1989); Hellmann et al. (2000); Repullo (2004); Repullo and Suarez (2004); Acosta Smith (2018)).

Since our paper approaches this topic from a different angle, by focussing on the effects of introducing a non-risk based LR alongside a risk-based capital framework, our paper more closely relates also to some of the more recent papers that consider a similar setting. Blum (2008) and Kiema and Jokivuolle (2014) look at the effects of imposing an LR in addition to the risk-based capital framework, but with a different focus of the analysis. Prior to Blum (2008), the literature had not considered a combined LR, risk-based capital framework, hence we are one of the first to address the benefits and costs of imposing an LR alongside the risk-based capital framework. Using an adverse selection model, Blum (2008) argues that a risk-independent capital ratio can improve bank stability through its disincentivising effect to conceal true risk-levels. Kiema and Jokivuolle (2014) consider a similar question but through a model risk perspective. They show that the introduction of an LR can induce formerly low risk banks to increase risk-taking, however in the presence of model risk,

⁷We also contribute to the more general literature on the impact of bank capital requirements on balance sheet adjustment - see e.g. Cohen and Scatigna (2016) and de Ramon et al. (2016).

which arises if some loans get incorrectly rated, an LR can improve stability due to the presence of a greater capital buffer should these mispriced loans become toxic. We move away from these papers and show that even in the absence of gaming and model risk, the LR (combined with a risk-based capital requirement) is beneficial for bank stability due to its additional loss absorbing capacity, as it helps to cover tail risks not covered in the risk-based framework.⁸ Finally, Brei and Gambacorta (2016) compare the cyclical properties of the LR and the risk-based capital ratio and find that the former is significantly more countercyclical.

Our theoretical model allows us to derive two main hypotheses, which we test empirically. To our knowledge, we are the first paper to combine a theoretical and empirical analysis of the imposition of an LR. In particular, our two hypotheses suggest that: 1) Introducing an LR incentivises those banks bound by it to increase risk-taking; 2) Forcing banks to hold greater capital via an LR is beneficial for bank stability despite the increase in risk-taking. Using a panel data set of EU banks over the period 2005-2014, we find evidence in support of both hypotheses.

First, we investigate risk-taking incentives via a difference-in-difference type approach. The announcement of the Basel III LR at the end of 2009 is taken as a treatment that only affects banks below the LR; this allows us to carve out treatment and control groups. Banks with LRs below the minimum requirement of 3% (currently being assessed by the BCBS) are the treatment group, while banks with LRs above the threshold are the control group. We use the risk-weighted asset (RWA) to total assets ratio as a proxy for risk-taking, which directly relates to our theoretical model. The results confirm our first hypothesis: an LR leads banks to increase risk-taking, but this increase is relatively contained (in the region of a 1.5 to 2.5 p.p. increase in the RWA ratio), and small relative to the required capital increase from an LR. This finding is also in line with the previous empirical literature that has suggested a positive relationship between capital requirements and bank risk-taking (see e.g. Shrieves and Dahl (1992); Aggarwal and Jacques (2001); Rime (2001); Jokipii and Milne (2011)).⁹ Yet this literature has been plagued by endogeneity issues since capital and risk are inextricably linked. Since we focus on a regime change, moving from a fully risk-based capital framework to one in which there also exists an LR, we are better able to identify any risk-taking effect from regulatory requirements without concern for reverse causality.

⁸We take as given that banks accurately measure and truthfully report their risk. The model could easily be extended to include both gaming and model risk, with results further in favour of the LR.

⁹Other studies have also suggested a negative relationship (see e.g. Jacques and Nigro (1997)).

Second, we estimate the joint effects of the LR and risk-taking on bank distress probabilities in a logit model framework using a unique dataset of EU bank distress events between 2005 - 2014. We build on the early warning literature along the lines of Betz et al. (2014) and Estrella et al. (2000) who use logit models to analyse out-of-sample forecasting properties of specific variables. Both papers emphasise the benefits of higher capital levels for financial stability, while Berger and Bouwman (2013) have shown that banks with higher capital levels are more likely to survive a financial crisis. We refine existing models to quantify the risk-stability trade-off associated with an LR, and show that the LR is a very important determinant for bank distress probabilities, both economically and statistically. Importantly, the marginal benefit of increasing a bank's LR from low levels is an order of magnitude larger than the marginal negative impact from taking on greater risk.

Third, we use the results from the first two empirical exercises to analyse whether given our estimated increase in risk-taking, bank distress probabilities would decline following the imposition of an LR. In particular, the results from the logit model are combined with the estimated increase in risk-taking from the difference-in-difference model in a counterfactual simulation. We ask whether bank distress probabilities significantly decline if an LR forces banks to increase their LRs to the minimum level, but at the same time this has the side effect of increased risk-taking (represented via higher RWA ratios). We perform the exercise with a 3%, 4% and 5% LR minimum and in all cases bank distress probabilities decline. This holds true even for our two most conservative exercises where banks are assumed to increase their risk-taking by triple the estimated amount, and by the maximum amount before moving back into the risk-based capital framework. The results therefore support the second hypothesis that banks should become more stable with the imposition of an LR despite the slight increase in bank risk-taking.

The remainder of the paper is organised as follows. Section 2 presents a brief overview of the Basel III LR framework. Section 3 develops the bank micro model and derives testable hypotheses regarding the effect of an LR on risk-taking and bank stability. Section 4 tests the hypotheses empirically, and section 5 concludes.

2 The Basel III Leverage Ratio Requirement

The build-up of excessive leverage and the subsequent deleveraging in the banking sector has been identified as one of the root causes of the financial crisis.¹⁰ The largest banks in Europe, for example, had built up significant leverage in the run-up to the crisis, with median leverage of around 33 times the level of common equity. Some banks even operated with leverage of 50 times the level of common equity.¹¹ As a response to this, the BCBS decided to undertake a major reform to the regulatory framework of the banking system. Among other measures, under the new Basel III banking regulations, was the introduction of a non-risk based LR requirement alongside the risk-based capital framework. The aim was to “restrict the build-up of excessive leverage in the banking sector to avoid destabilising deleveraging processes that can damage the broader financial system and the economy”.¹²

The LR is a non-risk based capital measure and is defined as Tier 1 capital over a bank’s total exposure measure, which consists of on-balance sheet as well as off-balance sheet assets. For on-balance sheet assets, the exposure measure generally relies on the assets’ accounting treatment, with the exception of derivatives and securities financing transactions. For these two asset classes, the differences in accounting standards across jurisdictions required the Basel committee to define a specific treatment that ensures a level playing field. For off-balance sheet assets, the credit conversion factor (CCF) from the Basel framework’s Standardised Approach for credit risk will be used for converting an off-balance sheet exposure to an on-balance sheet equivalent, subject to a floor of 10 percent.

It has been widely expected that the LR will become a binding Pillar I requirement for banks under Basel III in Europe, ever since the BCBS issued a consultative document that outlined a baseline proposal for the design of the LR in December 2009.¹³ Following further public consultations and revisions to the design, the BCBS issued the almost final LR framework in January 2014. In January 2016, the Group of Central Bank Governors and Heads of Supervision (GHOS), the Committee’s oversight body, discussed the final design and calibration of the Basel III LR. The GHOS confirmed that it should be based on a Tier 1 definition of capital and should comprise a minimum level of 3 percent. The GHOS further discussed additional LR

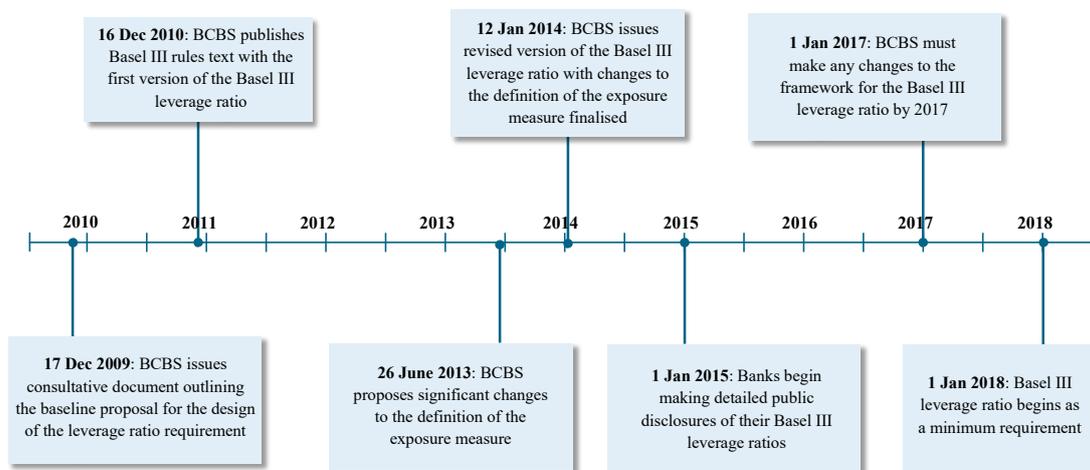
¹⁰See e.g. Eurosystem contribution to the European Commission’s call for evidence on the EU regulatory framework for financial services (ECB (2015)).

¹¹See European Systemic Risk Board (2014), “Is Europe Overbanked?” Reports of the Advisory Scientific Committee, No 4, June.

¹²See BCBS (2014a).

¹³See BCBS (2009).

Figure 1: Key dates regarding the introduction of the Basel III Leverage Ratio



requirements for global systemically important banks (G-SIBs). The Basel Committee is expected to finalise the work on its LR framework in the course of 2017.

In parallel, some jurisdictions have already provided for legislation that implements the Basel III LR as a binding Pillar I requirement for their banks. For example, both the US and Switzerland subject their global systemically important banks to an LR of 5 percent, whereas in the UK, banks are required to comply with an LR that will have a G-SIB buffer and a countercyclical buffer on top of a 3 percent minimum requirement. In Europe more generally, the European Banking Authority (EBA) recently published a report on the impact and the calibration of the LR, recommending a general minimum requirement of 3 percent and higher requirements for systemically relevant banks, in particular G-SIBs.¹⁴ Figure 1 summarises the above discussion and illustrates the key regulatory milestones related to the LR. This will be used in the empirical analysis in section 4.2 to motivate the econometric set-up to identify the impact of an LR requirement on bank risk-taking.

3 Theoretical model

The following section presents a simple microeconomic model that captures the trade-off between risk-taking incentives and higher loss-absorbing capacity associated with the introduction of an LR. It allows us to compare the outcomes for a

¹⁴See EBA (2016)

scenario where only the risk-based framework constrains banks to the outcomes of a setting where the LR is introduced in addition.

3.1 The set-up of the model environment

Consider a one-period economy with three types of agent: banks, investors and depositors. There are $n > 1$ banks, run by risk-neutral penniless bankers. The size of the bank's balance sheet is normalised to one. The bank finances itself with equity/capital k and deposits $(1 - k)$ subject to two capital requirements (a risk-based capital requirement and an LR requirement). There exists a continuum of identical, risk-neutral depositors. These depositors are negligible in size relative to banks. Depositors have two options: they either invest their endowment in bank deposits which yield a gross return of i , or alternatively deposit their endowment in a storage asset, which yields a gross return of 1. Banks are covered by limited liability, they therefore repay depositors only in the case of survival. Nevertheless, there exists full deposit insurance.¹⁵ This implies deposits are insensitive to risk-taking and will receive a deposit rate equal to the expected return on the safe asset $i = 1$.¹⁶

In addition to deposits, given bankers are wealth constrained (and must satisfy capital requirements), they can also raise funds by issuing equity. Investors are risk-neutral, they are not covered by deposit insurance, and they have an outside option yielding a gross return of $\rho > 1$ per unit of capital. As a result, banks must ensure the return they offer to shareholders is at least as large as ρ in expected terms in order to satisfy the investors' participation constraint. Similar to Dell'Ariccia et al. (2014), throughout the analysis, ρ is assumed to be constant.¹⁷

Each bank may invest its funds into two assets: a risky asset and a (relatively)

¹⁵For simplicity, as in Hellmann et al. (2000), we assume the insurance premium is zero. Nonetheless, our results hold for any fixed insurance premium

¹⁶Deposits will be insensitive to risk-taking since whether banks fail or survive, depositors will be fully compensated. As a result, as long as $i \geq 1$, depositors will prefer bank deposits (where at equality, depositors will be indifferent). Knowing this, banks will offer the lowest rate possible and thus set $i = 1$. Keeley and Furlong (1990) formally show that when there exists deposit insurance, deposit supply will not be a function of bank risk.

¹⁷Assuming a constant ρ should put the LR at a disadvantage since as will be seen in section 3.3, it is the higher cost of equity (relative to deposit costs) that drives greater risk-taking. If we follow an argument akin to Admati and Hellwig (2013), one would suggest that as capital ratios rise, the required ρ should fall, since ceteris paribus, banks are becoming safer. Given that the higher is ρ , the higher the increase in risk-taking, relaxing this assumption (which should lead to a decline in required ρ as bank LRs rise), will reduce this increase in risk-taking. Thus if the LR is beneficial under a constant ρ assumption, it is also expected to be beneficial when this assumption is relaxed; indeed the result should be strengthened.

safe asset. Denote by ω investment in the safer asset and by $(1 - \omega)$ investment in the risky asset. As in Allen and Gale (2000), there exists a convex non-pecuniary investment cost to risky investment $c(\omega)$, where $c'(\omega) < 0$ and $c''(\omega) \leq 0$, so investing in the risky asset becomes increasingly expensive. Banks face two types of capital regulation: a risk-based requirement and a non-risk-based LR requirement.

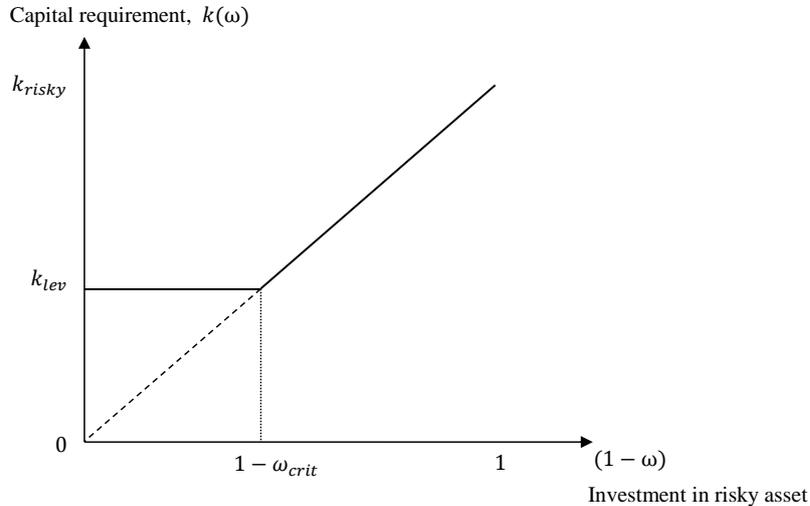
Since assets differ in their riskiness, the risk-based capital requirement is increasing in holdings of the risky asset. Specifically, as in the Basel risk-based capital framework, on each asset banks are required to hold sufficient capital such that they cover expected and unexpected losses with some probability $(1 - \alpha) \in (0, 1)$, where in the Basel requirements $\alpha = 0.001$.¹⁸ There exists a capital requirement k_{safer} on the safer asset, and k_{risky} on the risky asset, where $k_{safer} < k_{risky}$. Given asset holdings of ω , the risk-based capital requirement can be written as $k_{rw} = k(\omega) = \omega k_{safer} + (1 - \omega)k_{risky}$.

In addition, banks are subject to an LR which states that banks must hold a minimum level of capital k_{lev} independent of risk. The combined capital framework will be such that the bank must hold a capital level k greater than or equal to the higher of the two requirements, namely $k \geq \max\{k_{rw}, k_{lev}\}$. Which constraint requires the higher capital level depends on the riskiness of the bank's balance sheet. Figure 2 illustrates this. Since the risk-based requirement increases in holdings of the riskier asset, at low-risk holdings, the risk-based requirement (see the dashed diagonal line) lies below the LR. As holdings of the riskier asset increase, the requirement also increases until beyond some level, denoted $(1 - \omega_{crit})$ in Figure 2, it starts to exceed the LR. As a result, the combined capital framework exhibits a kinked structure.

There exist two possible states of nature, state 1, denoted s_1 , which can be thought of as a good state, and state 2, denoted s_2 , which can be thought of as a bad state. These states occur with probability μ and $(1 - \mu)$ respectively. Each asset's return is a function of the state of the world. The safer asset offers a gross return of $R_1 \geq 1$ if state s_1 occurs, and $(1 - \lambda_1) \in (0, 1]$ if state s_2 occurs. On the other hand, in state s_1 , the risky asset offers a gross return of $R_2^h > R_1$ with probability π and $(1 - \lambda_2) \in (0, 1)$ with probability $(1 - \pi)$, while in state s_2 , it returns $(1 - \lambda_3) \in (0, 1)$ with probability π , and 0 otherwise. The expected return on the risky asset is assumed to be greater than the expected return on the safer asset. The setup can be seen in Figure 3, where $0 \leq \lambda_1 < \lambda_2 < \lambda_3$, so it is envisioned that

¹⁸In this simplified world, the only source of provisions is the bank's own funds, hence it is analogous to bank capital.

Figure 2: Capital requirements under a combined leverage ratio requirement and risk-based framework.



Notes: The graph shows the interaction between a leverage ratio requirement (k_{lev}) and the risk-based requirement which is increasing in $(1 - \omega)$. $(1 - \omega_{crit})$ is the point at which the capital requirement under both the risk-based and the leverage ratio requirement are equalised.

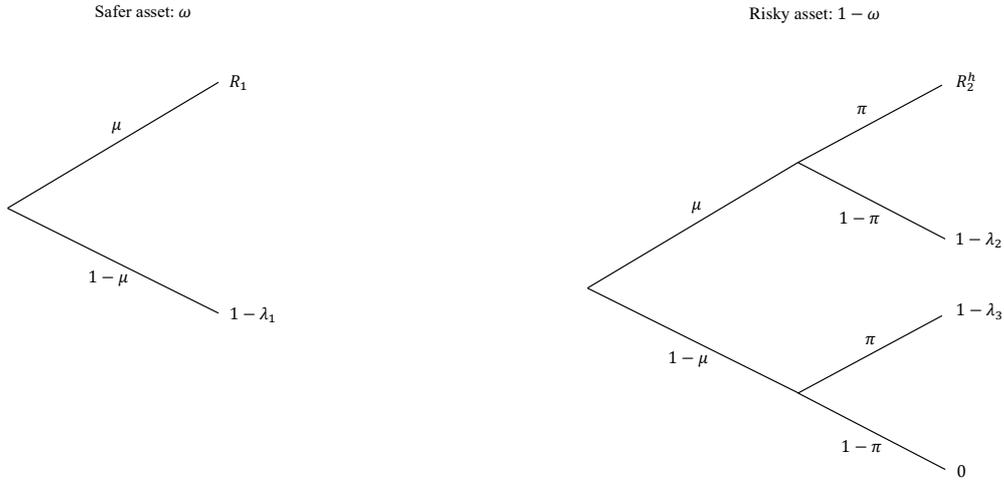
losses on the risky asset are larger in the bad state, but losses on the safer asset are smaller than for the risky asset. The risk of the bank's portfolio is thus determined by the investment proportion devoted to the risky asset relative to the safer asset.

As discussed above, under the risk-based framework, the exact capital requirement will be a function of how the probabilities μ and π relate to α . For generality, we consider all cases. For clarity, we discuss the cases separately. The baseline case $(1 - \mu) \leq \alpha$ will be discussed in this section. The alternative case, $(1 - \mu) > \alpha$, is discussed in the appendix. All results hold in both cases.¹⁹

Since $(1 - \mu) \leq \alpha$, the bank is only required to hold enough capital to survive state s_1 . So immediately, it is clear that the capital charge on the safer asset, k_{safer} , is zero. For the risky asset, since $(1 - \mu) \leq \alpha$, the bank does not need to cover losses in state s_2 . Thus the capital charge will be either λ_2 or zero. If $(1 - \mu) + \mu(1 - \pi) > \alpha$, then the bank must cover the loss in state s_1 , as otherwise the requirement is not satisfied, hence $k_{risky} = \lambda_2$. If $(1 - \mu) + \mu(1 - \pi) \leq \alpha$, then the probability of loss in state s_1 is so small that the bank does not need to hold capital against it, and so $k_{risky} = 0$. Since this case entails a zero capital requirement under both assets

¹⁹As discussed further in the appendix, the alternative case is also equivalent to the case in which $(1 - \mu) \leq \alpha$, but the risk-based capital requirement is strengthened. This has the same effect, i.e. increasing the capital charge on each asset. Hence the results presented in this section continue to hold even if the risk-based capital requirement is strengthened.

Figure 3: Payoff of the risky and safer asset



Notes: Figure shows the payoff function dependent on the state of the world for the safe and risky asset.

and hence there is no risk-based nature to it, indeed there is no capital requirement (both assets have a zero capital charge), we ignore this case for the more realistic previous case. So, if $(1 - \mu) \leq \alpha$, $k(\omega) = (1 - \omega)\lambda_2$.

The setup attempts to capture one of Basel's key reasons for the imposition of an LR: the inability of the risk-based framework to cover correlated shocks that can also impact lower risk assets. To capture this friction, the setup adds state s_2 so that the risk-based framework is able to cover some shocks, but potentially not all. We envisage state s_2 as a low probability event, but it is an event that can hit both assets. Thus it may be that the risk-weighted framework is not able to perfectly cover this correlated shock; providing an opportunity for the LR to potentially improve upon a situation with only a risk-based framework. It is worth noting that the model assumes probabilities and payoffs are known with certainty, i.e. there exists no model risk or gaming concerns. We impose this assumption to illustrate the benefits of an LR even in the absence of these concerns. Clearly if there exists model risk in addition, or if banks game their risk weights, the benefit from an LR will be further enhanced. See for example Blum (2008) and Kiema and Jokivuolle (2014).

3.2 The bank's decision problem

The objective for the bank is to maximise expected profits after paying out shareholders, and conditional on survival, while taking into account the investment cost.

In order to achieve this, each bank must determine the structure of its portfolio in terms of both its asset and liability side. Each bank must optimally choose the amount of capital and deposits to hold (subject to both a risk-adjusted capital requirement and an LR constraint), how much to pay depositors and equity holders, and their investment $(\omega, 1-\omega)$ in each asset. In order to raise funds, banks must satisfy both depositors and equity holders' participation constraints. As noted above, for depositors this implies banks must satisfy $i \geq 1$ since their outside option is to store their assets with a gross return of 1. In optimum, since banks wish to minimise costs, the bank will set $i = 1$. Investors on the other hand have an outside option ρ . Unlike depositors, they are not covered by deposit insurance, so banks must ensure they earn an expected gross return of at least their opportunity cost. Suppose $(1-\theta)$ is the share of profits given to equity holders as compensation, then it must be that the bank ensures the following participation constraint is satisfied:

$$(1 - \theta)\Pi \geq \rho k$$

where Π is expected profits, with

$$\begin{aligned} \Pi = & \mu\pi[\omega R_1 + (1 - \omega)R_2^h - id] + \mu(1 - \pi) \max\{[\omega R_1 + (1 - \omega)(1 - \lambda_2) - id], 0\} + \\ & (1 - \mu)\pi \max\{[\omega(1 - \lambda_1) + (1 - \omega)(1 - \lambda_3) - id], 0\} + (1 - \mu)(1 - \pi) \max\{[\omega(1 - \lambda_1) - id], 0\} \end{aligned}$$

As with deposits, since banks treat this like a cost, in optimum this constraint must hold with equality.

Considering the entire setup together, we can write each bank's problem formally as:

$$\max_{\omega, \theta, i, k} \{\theta\Pi - c(\omega)\}$$

subject to

$$(1 - \theta)\Pi \geq \rho k$$

$$d + k = 1$$

$$i \geq 1$$

$$k \geq \max\{k_{lev}, k(\omega)\}$$

$$\begin{aligned} \Pi = & \mu\pi[\omega R_1 + (1 - \omega)R_2^h - id] + \mu(1 - \pi) \max\{[\omega R_1 + (1 - \omega)(1 - \lambda_2) - id], 0\} + \\ & (1 - \mu)\pi \max\{[\omega(1 - \lambda_1) + (1 - \omega)(1 - \lambda_3) - id], 0\} + (1 - \mu)(1 - \pi) \max\{[\omega(1 - \lambda_1) - id], 0\} \end{aligned}$$

where d is deposits, and following Dell’Ariccia et al. (2014), we parameterise the cost function as $c(\omega) = (c/2)(1 - \omega)^2$, where it is assumed that $c > \mu[\pi R_2^h + (1 - \lambda_2)(1 - \pi) - R_1]$ so that the cost c of investing in the risky asset is larger than the increase in expected return from switching from the safer asset to the risky asset in state s_1 .²⁰

It is worth noting that the above problem illustrates how bankers and equity holders are covered by limited liability. Whenever returns are negative, payoffs become zero. Furthermore, the problem illustrates how banks can adjust their probability of survival in two ways. First, banks can choose to directly decrease risk-taking, i.e. increase ω . Second, banks can increase their probability of survival by choosing to hold more capital. Should losses then occur, the bank is able to withstand them.

3.3 Main theoretical results

3.3.1 Risk-taking under a risk-based capital requirement

Let us first analyse the solution to the model when there exists only a risk-based capital requirement. The problem will be identical except since there does not exist an LR, the capital constraint will reduce to $k \geq k(\omega)$. As outlined in the previous paragraph, if desired, banks could choose to hold enough capital such they survive all potential losses. This has two effects. First, holding additional capital is costly since equity holders require an expected return of at least $\rho > i = 1$. Second, increasing capital sufficiently will enable banks to survive further shocks in state s_2 , and this will both decrease the bank’s probability of default and generate additional return. Yet, state s_2 is a loss state; the assets yield a gross return of less than 1. As Lemma 1 shows, banks do not find it optimal to increase capital to survive these states. The cost of holding greater capital outweighs the benefit of obtaining the residual value in these states. Capital must return on average ρ to satisfy shareholders, while depositors will accept $i = 1$. Since ρ is larger, ceteris paribus, banks will prefer to fund themselves with cheaper deposits. Banks will therefore never wish to hold more than the required capital amount. Indeed, banks would prefer to be 100% deposit financed, but due to the capital requirement, banks are forced to hold at least the minimum. As a result, the capital constraint binds. Lemma 1 formalises this.

Lemma 1 *Banks always wish to hold as little capital as possible; therefore the capital requirement will bind.*

²⁰This simply ensures our results are interior solutions.

Proof. See the appendix. ■

Since the risk-weighted capital requirement binds, it will impact risk-taking decisions. Holding more of the risky asset entails holding greater capital and as we have noted, this is expensive. Hence, there exists a trade-off between holding more of the risky asset, which in expected terms yields more, and the cost of doing so. The bank will choose the point at which the marginal revenue from greater investment in the risky asset equals the marginal cost. The first order condition (FOC) depicts this:

$$\mu[\pi R_2^h + (1 - \pi)(1 - \lambda_2) - R_1] = -(\rho - \mu)k'(\omega) - c'(\omega)$$

The left hand side (LHS) of this expression shows the marginal benefit from increasing holdings of the risky asset ($1 - \omega$), while the right hand side (RHS) illustrates the marginal cost. The marginal benefit comprises the increased potential payoff the risky asset offers. By shifting funds from the safer asset to the risky asset, the bank forgoes R_1 , but gains $\pi R_2^h + (1 - \pi)(1 - \lambda_2)$ which is larger. On the other hand, the marginal cost takes into account both the cost of investing an additional unit in the risky asset, $c'(\omega) < 0$, and the fact that holding greater quantities of the risky asset requires higher capital levels (shown in the $k'(\omega) < 0$ terms) which is more expensive than deposits. Replacing one unit of deposit with one unit of capital saves $i \cdot \mu = 1 \cdot \mu$ in expected terms, but costs ρ ; hence the $(\rho - \mu)$ term; the net cost of replacing deposits with capital. In the risk-based framework there is therefore a trade-off the bank can exploit in terms of capital and risk; by choosing to hold less risk, the bank somewhat offsets the lower return by its ability to lower expensive capital. Banks trade off this potential loss of profits with the cost of risky investment, and hence choose a risk level such that the marginal benefit from increasing $(1 - \omega)$ is zero.

The condition illustrates the trade-off banks possess when risk-taking under a risk-based framework. Increasing the weight on the risky asset increases potential return, but at the same time entails costs related to investment and capital raising. A risk-weighted capital requirement thus disincentivises risk-taking, as it forces banks to hold more capital if they wish to take more risk.

3.3.2 Risk-taking with a leverage ratio requirement

Suppose that now banks are subject to an additional constraint, namely, a constraint on leverage such that $k \geq k_{lev}$ regardless of ω . Given the LR exists alongside the risk-based capital framework, any LR below the risk-weighted requirement will have no effect (since it does not bind) and the results of the previous paragraph still hold. In

order to make the LR bite, the LR must be set such that it is above the risk-weighted capital requirement of a bank. Suppose the LR is set to $k_{lev} > k(\omega_{rw}^*)$ such that it is the binding constraint, where ω_{rw}^* denotes the optimal safer asset holdings under the risk-based framework. Although banks can now potentially survive larger losses (since they hold greater capital), it may be that as a result of the LR, bound banks shift so much of their portfolio into the risky asset that even with this higher level of capital, they cannot withstand these now more probable, larger losses. Whether this increase in capital is beneficial depends on how much (if at all) the bank is incentivised to shift its portfolio into the risky asset (which is more likely to fail and its residual value is lower).

The change in risk incentives can be clearly seen by comparing the FOC with respect to ω under a risk-based framework to the FOC if the LR is binding. Suppose the LR is set just above the risk-based capital requirement, then the FOC is characterised by:

$$\mu[\pi R_2^h + (1 - \pi)(1 - \lambda_2) - R_1] = -c'(\omega)$$

As can be seen, all terms related to the risk-weighted capital requirement have disappeared due to the binding LR. Removing this dependence on risk means banks can now increase risk without having to hold additional capital. In other words, the marginal cost of risk-taking declines as there is no longer a requirement to increase expensive capital if the bank increases $(1 - \omega)$. By removing the link between capital and risk-taking, the bank will be incentivised to take more risk. This can be seen in the FOC above. The LHS of the equality (i.e. the marginal benefit) is identical to before, whereas the RHS, the marginal cost, is lower. Thus the ω that solves this equation must be lower than the ω that solves the risk-based FOC, hence implying greater risk-taking. As the LR rises however, and banks begin to hold more capital, it is possible that at the same time, depending on the LR level, the marginal benefit can also change. The marginal benefit of increasing risk can decline, since with higher capital, banks survive larger shocks, and as a result, banks are forced to internalise these returns they otherwise would have ignored - so called “skin-in-the-game”. Due to the discrete nature of the asset setup, this effect first appears when the LR is set high enough that banks also survive state s_2 when the risky asset pays

off $(1 - \lambda_3)$.²¹ The FOC becomes:

$$\mu[\pi R_2^h + (1 - \pi)(1 - \lambda_2) - R_1] - (1 - \mu)\pi(\lambda_3 - \lambda_1) = -c'(\omega)$$

Compared to the previous FOC, one can clearly see the presence of a “skin-in-the-game” effect, $(1 - \mu)\pi(\lambda_3 - \lambda_1)$, which brings down the chosen level of risk slightly. As capital holdings rise, banks survive larger and larger shocks. Since banks then attach value to these returns, this to some extent decreases the benefit of higher risk-taking as the residual value of the risky asset is lower, and hence this reduces the optimal risk level chosen. There can therefore exist two opposing effects from the imposition of an LR. The first effect (i.e. removing the link between risk and capital) - the loss of the $k'(\omega)$ terms in the FOC - incentivises greater risk-taking, whereas the second effect - the skin-in-the-game effect, as banks are forced to increase capital by more and more - incentivises less risk-taking since banks begin to internalise returns they otherwise would have ignored. Proposition 2 formalises this discussion and shows that when equity is sufficiently costly, the first effect always dominates and banks increase risk-taking with an LR.

Proposition 2

If $k_{lev} < \hat{k} \in [\lambda_1, \lambda_3]$, imposing a leverage ratio requirement will always incentivise banks to take more risk. \hat{k} is defined in the appendix.

If $k_{lev} \geq \hat{k}$, imposing a leverage ratio requirement will still always incentivise banks to take more risk if equity is sufficiently costly, i.e. $\rho > \mu + (1 - \mu)\frac{[(1 - \lambda_1) - \pi(1 - \lambda_3)]}{\lambda_2}$, where if $(1 - \lambda_1) - \pi(1 - \lambda_3) < \lambda_2$, this is always the case since $\rho > 1$.

Proof. See the appendix. ■

Proposition 2 summarises the two effects that determine whether an LR will incentivise greater risk-taking. The first condition illustrates that for lower levels of the LR, the “skin-in-the-game” effect is so small, indeed in this region it is zero due to the discrete nature of the set-up, that the only incentive driving risk-taking is the move away from linking risk to capital, which simply incentivises the bank to risk-up. As the LR rises however, as discussed before, banks will begin to survive shocks in state s_2 , and thus this “skin-in-the-game” effect will begin to appear.

²¹Whether the LR can be set at such a level that banks begin to survive state s_2 shocks depends on the extent to which banks risk-up under an LR, since if they increase risk to the maximum, this case is not possible. This will depend on the exact parameter values of the model. Nevertheless, for some parameter values, it is possible.

The second condition illustrates that as long as equity is sufficiently expensive, the move away from a risk-based requirement will always dominate the bank's decision making, and therefore banks will shift more of their portfolio into the risky asset.²² This is because, compared to the cost of equity that incentivised lower risk-taking under a risk-based capital requirement, and for which banks are now released from considering, this "skin-in-the-game" effect is small; state s_2 is a low probability state and any additional payoff is multiplied by $(1 - \mu)$ which is very small. To give an idea of the magnitude required, consider the threshold for ρ , and the reasonable parametrisation $\mu = 0.999$, $\pi = 0.8$, $\lambda_1 = 0.02$, $\lambda_2 = 0.2$ and $\lambda_3 = 0.8$, then ρ must be larger than 1.0031.

Taking proposition 2 as a whole therefore, we can conclude that if equity is sufficiently expensive, once the LR binds, risk-taking will increase because the LR in effect allows banks to engage in greater risk-shifting. Removing the binding risk-weighted capital requirement allows banks to increase risk while imposing most of that risk onto the funds raised from depositors (ultimately the responsibility of taxpayers) - since banks are not forced to raise any further capital. Since there exists full deposit insurance, depositors are not sensitive to this risk-taking; hence banks increase risk without incurring higher funding costs. With a risk-weighted capital requirement, this ability to risk-shift is somewhat offset since taking on further risk implies increasing capital, which is expensive. Once the risk-weighted capital requirement ceases to bind, banks can increase risk-taking without needing further additions of capital. This was a major inhibitor to risk-taking, hence under an LR, banks have a greater incentive to risk-shift.

Lastly, under the case in which ρ is less than the sufficient level, we cannot immediately conclude that risk-taking will therefore be lower. It may be the case, and if so, then clearly bank stability will improve as banks are more highly capitalised and take lower risk, however we cannot generalise. This is because for larger values of k , it may be that the optimal level of risk chosen by the FOC is not sufficient to satisfy the shareholders' participation constraint.²³ If so, then banks are obliged to choose a higher risk level than desired, as otherwise they are unable to raise equity, and this level can be higher than the risk-based choice.

²²If $(1 - \lambda_1) - \pi(1 - \lambda_3) < \lambda_2$ this is always the case since $\rho > 1$. This states that the capital charge on the risky asset under the risk-based framework is larger than the skin-in-the-game effect, so even if ρ is at its lowest, the move away from this capital charge will induce greater risk-taking.

²³This is possible for large k when the optimal risk choice is low and $\rho > \mu R_1 + (1 - \mu)(1 - \lambda_1)$.

3.3.3 Risk-taking vs. loss absorbing capacity

Proposition 2 showed that imposing an LR will always incentivise banks to increase risk-taking if equity is sufficiently costly. Nonetheless, this does not imply that an LR is detrimental. Quite the contrary, whether the LR improves outcomes depends on the extent of this risk-taking compared to increased loss absorbing capacity. We assess this in two important ways: first, via the impact on the bank's probability of default, and second, via the impact on the expected loss of deposit funds.²⁴ With an LR, banks may potentially survive a state s_2 shock, but in order to generate a benefit, it must be that any additional risk is outweighed by this loss-absorbing capacity. At the same time, even if the probability of default remains the same, the LR may induce a benefit via its effect on the expected loss of deposit funds - since any losses that do occur are absorbed by capital rather than deposit funds. Proposition 3 formalises this discussion.

Proposition 3 *Relative to a solely risk-based capital framework, imposing a leverage ratio requirement:*

1. *Leads to weakly lower bank failure probabilities.*
2. *If $\rho \leq \hat{\rho}$, a strictly lower expected loss of deposit funds if $k_{lev} > \underline{k}$.*
3. *If $\rho > \hat{\rho}$, a strictly lower expected loss of deposit funds if $k_{lev} \in (\underline{k}, \bar{k})$.*

where $\hat{\rho}$, \underline{k} and \bar{k} are defined in the appendix, and $\underline{k} < \bar{k}$.

Proof. See the appendix. ■

Proposition 3 illustrates that an LR can improve bank default probabilities and reduce the expected loss of deposit funds.²⁵ In other words, the increase in risk-taking identified previously is not sufficiently large to outweigh the loss-absorbing benefit. Indeed, an LR improves outcomes on both criteria for all (\underline{k}, \bar{k}) . This can be understood by considering two important points. First, the risk-based capital requirement still underlies the LR. As such, there is a limit to how much additional

²⁴In particular, the expected loss of deposit funds is defined as the expected amount of deposit funds the bank will be unable to repay on bankruptcy.

²⁵As before, to give an indication of the magnitude required of ρ , consider the reasonable parameterisation: $\mu = 0.999$, $\pi = 0.9$, $R_1 = 1.02$, $R_h^2 = 1.2$, $c = 9$ (c set following Dell'Ariccia et al. (2014)), $\lambda_1 = 0.02$, $\lambda_2 = 0.1$, $\lambda_3 = 0.8$. This gives $\hat{\rho} = 1.21$, so in reality, the third statement is unlikely to be relevant.

risk a bank can take; if it takes too much risk, it will simply move back into the risk-based framework. In terms of failure probabilities, this puts a floor on the failure probability, as if the bank takes too much risk such that it no longer covers the shocks that were required under the risk-based capital requirement, e.g. to survive state s_1 , it must be that the risk-based requirement is the higher binding requirement again. Since this acts as a backstop to risk-taking, banks are limited in the extent to which they can increase risk. Second, as we noted before, the skin-in-the-game effect somewhat offsets the incentive to increase risk-taking, and thus banks will not risk-up by vast amounts, since this to an extent subdues the risk-taking incentive. These two effects combine to prevent excessive risk-taking, thus the LR has a beneficial effect both on bank failure probabilities and on the expected loss of deposit funds, as greater losses are born by the bank's capital.

The lower bound on the expected loss of deposit funds condition is related to the amount of loss absorbing capacity available. For example, if the LR is set to an epsilon above the risk-weighted capital requirement for a bank, the LR adds barely any additional loss absorbing capacity, yet, the bank will take more risk; this therefore leads to an increase in the expected loss of deposit funds relative to the solely risk-based framework. At higher levels of capital however, the additional loss absorption is sufficient to outweigh any additional risk-taking. Since in reality it is arguably the case that banks are not all concentrated around the LR minimum, but there exists a distribution of banks with different risk-based capital requirements, we can suggest that as long as this distribution is not concentrated around the LR minimum, this lower bound should be less of a concern.

Lastly, proposition 3 shows that there can exist a potential risk when ρ is large and the LR is set very high. This however only occurs when $\rho > \hat{\rho}$, where $\hat{\rho}$ is greater than the expected return on the risky asset; so banks must be targeting very large ROEs.²⁶ This occurs because at these levels of ρ , once the LR rises beyond some point, the optimal choice of risk the bank would like to take may no longer meet the shareholders' participation constraint. As a result, banks can be forced to increase risk-taking further just to meet their required return on equity. When $\rho < \hat{\rho}$, this can also potentially occur, but the increase in risk-taking is not sufficiently fast as to outweigh the benefit from higher loss absorbing capacity.

²⁶This may cover a situation in which the expected return on risky assets has declined, but banks have not adjusted their ROE targets. When the LR is set above \bar{k} , it is not possible to rule out that the expected loss of deposit funds might be larger under an LR. Note, in this region because $\hat{\rho}$ is greater than the expected return on the risky asset, there exists a $k_{max} < 1$ above which the region $k > k_{max}$ is infeasible. In this way, proposition 3 shows that for $k \in (\bar{k}, k_{max}]$ and $\rho > \hat{\rho}$, the expected loss of deposit funds can be larger under an LR, although not necessarily.

Above $\hat{\rho}$ however, risk-taking increases so fast with increases in the LR (just to meet the shareholders' participation constraint) that at higher levels of the LR, it can lead to worse outcomes than under a solely risk-based framework. The point at which this arises will depend on the size of ρ , and as stated only occurs for large ρ . Nevertheless, at these higher levels of capital, the increase in risk-taking is not sufficiently constrained and thus an LR can lead to a higher expected loss of deposit funds. It should be noted however, that this case is somewhat a consequence of the constant ρ assumption. If one considers that ρ will decline as k rises, this forced increase in risk will either not occur, or it will be subdued. This is because if ρ declines as the LR rises, risk-taking would also decline as the target ROE falls, and hence risk-taking would not be forced to consistently rise. Indeed, if ρ falls back below the expected return on the risky asset, as would be reasonable, the upper bound would cease to exist.

Overall therefore, from proposition 3, we can suggest that the LR should improve outcomes via the dominating effect of higher loss absorbing capacity. In the next section, we will test this suggested impact on loss absorbing capacity by considering the impact of the LR on banks' distress probabilities.

4 Empirical analysis

The model presented in the previous section suggests two testable hypotheses. First, the introduction of an LR should incentivise banks for which it is a binding constraint to modestly increase risk-taking. Second, the negative impact of increased risk-taking induced by an LR constraint should be outweighed by the beneficial impact of increased loss-absorbing capacity, resulting in more stable banks. We take these two hypotheses and test them empirically on a large panel dataset of EU banks that encompasses a unique collection of bank distress events. The empirical analysis follows in three steps. We first test whether banks with low LRs started to increase their risk-taking and capital positions after the announcement of the Basel III LR at the end of 2009 using a difference-in-difference type approach. We then estimate the joint effects of the LR and risk-taking on bank distress probabilities in a logit model framework, in order to quantify the risk-stability trade-off associated with an LR. Finally, we combine the first and second stage empirical results into a counterfactual simulation to test whether the negative impact of the estimated increase in risk-taking is outweighed by the benefit of holding higher capital, i.e. whether an LR is beneficial for bank stability.

The empirical evidence provided lends support to both hypotheses. Our estimates suggest that banks bound by the LR slightly increased their risk-taking after the announcement of the Basel III LR at the end of 2009. Specifically, our point estimates suggest that bound banks increased their RWA to total assets ratios by around 1.5 - 2.5 percentage points more than they otherwise would have without an LR. Importantly, the negative effect for bank stability of this small increase in risk-taking is more than compensated for by the beneficial effect of a substantial increase in capital positions for highly leveraged banks, which results in significantly lower estimated distress probabilities for banks constrained by the LR. The remainder of this section describes the underlying dataset and detailed results of the three stages of the empirical analysis.

4.1 Dataset

The dataset consists of a large unbalanced panel of 655 EU banks covering the years 2005 - 2014, and is based on publicly available data only. There are three main building blocks of the dataset: i) a large set of bank-specific variables based on publicly available annual financial statements from SNL Financial; ii) a unique collection of bank distress events that covers bankruptcies, defaults, liquidations, state-aid cases and distressed mergers that are collected from Bankscope, Moody's, Fitch, the European Commission, Reuters and Bloomberg; iii) various country-level macro-financial variables from the ECB's Statistical Data Warehouse. The dataset builds upon and expands the dataset described in Betz et al. (2014) and Lang et al. (2015). Tables 1 and 2 display various descriptive statistics of the dataset by country. As can be seen, there is substantial variation across countries. Moreover, table 3 provides details of the distress events. In addition to direct failures (bankruptcies, liquidations and defaults on bonds), the definition of distress events also includes state interventions and distressed mergers, since in the absence of these measures, many banks would probably have failed in many cases. In total, there are 252 distress events.

4.2 Effect of a leverage ratio constraint on bank risk-taking

To identify how the risk-taking behaviour of a bank changes after the imposition of an LR, we exploit the panel structure of our dataset in combination with the timing of the Basel III LR announcement, as described in section 2. Our identification strategy builds on the programme evaluation literature by considering the

Table 1: Bank Characteristics by Country: 2005-2014 (mean values)

Country	RWA/TA	Leverage Ratio	Loans/TA	Total Assets, €bn	Pre-tax ROA	Loan-to-deposit ratio	Coverage ratio	Interest to liabilities
Austria	49.38	5.04	55.92	27.35	0.32	189.17	55.88	2.24
Belgium	35.12	5.78	47.42	105.29	0.52	106.62	66.88	3.20
Bulgaria	76.91	10.31	73.45	3.71	1.42	100.71	70.12	2.99
Cyprus	63.37	7.79	71.56	18.14	0.13	120.37	76.15	3.27
Czech Republic	50.32	7.44	60.97	13.04	1.47	107.48	46.52	1.84
Denmark	53.91	7.07	68.53	55.69	0.43	136.24	48.99	2.28
Estonia	55.20	14.57	78.61	10.78	1.87	138.45	98.65	1.24
Finland	42.56	5.03	65.09	46.92	0.72	190.37	63.74	1.84
France	43.08	6.65	58.85	97.62	0.73	208.15	77.90	2.53
Germany	47.81	5.81	46.65	41.16	0.41	115.54	60.23	2.15
Greece	65.39	6.47	74.75	35.96	-0.99	114.68	55.36	2.83
Hungary	63.96	7.28	67.56	9.65	-0.25	162.93	60.58	4.37
Ireland	47.81	7.32	50.40	51.33	-0.14	200.87	74.30	2.18
Italy	56.99	6.21	67.93	37.90	0.41	161.27	68.74	1.57
Lithuania	76.59	8.50	74.50	4.50	0.41	141.35	43.56	2.34
Luxembourg	32.09	5.98	31.99	16.68	0.59	102.23	65.91	2.27
Latvia	67.04	8.84	62.25	3.80	0.83	122.10	81.60	1.48
Malta	50.26	5.70	51.93	5.33	1.58	69.23	55.85	1.66
Netherlands	39.62	5.89	58.78	160.20	0.35	173.84	60.79	2.57
Poland	67.90	8.20	67.40	12.05	1.37	132.90	72.02	2.82
Portugal	65.12	6.64	64.93	36.82	0.11	163.70	96.37	3.69
Romania	62.64	7.90	63.75	6.63	0.60	133.62	67.17	3.80
Slovenia	73.94	7.80	68.27	6.01	-0.95	127.87	46.76	2.36
Slovakia	62.92	7.90	65.89	6.89	1.07	88.02	64.48	1.65
Spain	59.33	5.90	67.39	82.89	0.14	125.69	124.56	2.12
Sweden	27.56	6.08	69.24	103.27	0.66	198.99	77.20	2.27
United Kingdom	43.65	6.05	50.53	207.56	0.64	107.64	60.74	1.75
Total	50.18	6.31	59.66	69.98	0.48	145.06	72.32	2.25

Source: SNL Financial.

Table 2: Country characteristics: 2005-2014 (mean values)

Country	GDP growth	Inflation	Δ unemployment, y-on-y	Credit to GDP	10-yr yield	Government debt	House price growth, y-on-y	Stock market growth, y-on-y	Δ Bundspread	Private credit	Δ Banking sector securities to liabilities, y-on-y
Austria	1.57	1.95	0.03	103.52	3.68	71.44	3.68	8.06	0.05	6.31	0.18
Belgium	1.25	2.10	-0.04	82.59	3.73	100.35	5.83	3.58	0.03	13.75	-0.18
Bulgaria	3.43	5.30	-0.88	59.56	5.19	28.12	10.32	20.24	0.12	19.63	-0.10
Cyprus	1.77	2.26	0.73	244.29	5.36	64.60	5.63	15.25	0.13	18.97	-0.91
Czech Republic	2.82	2.35	-0.15	48.11	3.89	32.15	2.53	6.89	0.02	4.63	0.50
Denmark	0.63	1.87	0.05	187.78	3.52	41.07	4.23	10.54	-0.00	14.66	-0.22
Estonia	3.63	4.23	-0.26	90.32	5.51	5.55	10.70	16.56	0.46	12.95	-1.07
Finland	1.25	1.98	-0.26	80.96	3.46	42.26	3.92	7.67	0.01	9.05	-0.17
France	1.01	1.67	0.02	97.91	3.45	75.02	4.39	3.42	0.03	7.89	0.09
Germany	1.15	1.71	-0.34	98.26	3.33	68.85	1.37	9.23	0.00	1.06	-0.67
Greece	0.77	2.63	0.75	90.90	7.18	118.94	3.08	3.88	0.20	8.19	0.02
Hungary	1.02	4.62	0.33	60.48	7.49	71.27	0.57	9.95	0.05	11.39	0.07
Ireland	2.01	1.62	0.54	161.40	4.83	62.48	1.27	3.23	0.08	18.08	-0.37
Italy	-0.06	2.09	0.01	101.53	4.37	110.20	2.42	-0.64	0.11	6.20	0.41
Lithuania	3.83	2.93	-0.22	54.98	5.23	25.66	10.03	18.33	0.12	5.84	-0.37
Luxembourg	2.31	2.55	0.22	149.77	3.46	13.36	3.16	7.86	0.03	12.27	-0.24
Latvia	3.19	5.13	0.12	82.36	6.48	24.37	16.57	8.88	0.31	12.65	0.14
Malta	2.09	2.34	-0.14	119.52	4.70	66.09	4.06	12.32	-0.11	13.29	-0.05
Netherlands	1.06	1.84	0.10	169.65	3.36	55.28	1.36	3.44	0.01	8.22	0.52
Poland	3.84	3.11	-0.52	45.32	6.12	47.45	16.07	12.22	-0.11	5.63	0.23
Portugal	0.35	2.02	0.62	151.94	5.18	80.77	-0.39	1.40	0.20	11.89	0.77
Romania	3.35	8.85	-0.14	31.18	7.85	21.17	25.93	19.46	0.92	11.58	0.09
Slovenia	2.33	3.74	0.00	73.97	4.88	32.10	6.84	4.04	0.32	9.66	0.10
Slovakia	4.22	3.66	-0.46	47.45	4.06	38.92	3.63	10.14	-0.02	7.00	0.55
Spain	1.53	2.51	0.54	155.73	4.34	55.91	3.17	4.07	0.08	12.44	0.37
Sweden	1.78	1.60	0.03	124.71	3.45	41.33	6.53	7.59	-0.01	14.18	0.99
United Kingdom	1.23	2.50	0.17	155.75	3.86	57.05	2.89	3.11	0.01	9.70	0.09
Total	1.19	2.13	-0.03	113.73	3.90	69.03	3.11	5.94	0.04	6.90	-0.07

Source: Various data sources obtained from the ECB's Statistical Data Warehouse.

Table 3: Distress events by category

Distress category	Composition	Frequency
Direct failure		35
	<i>Bankruptcy / Liquidation</i>	3
	<i>Default</i>	32
Distressed merger		46
	<i>Merger with state intervention</i>	29
	<i>Merger with coverage ratio < 0</i>	20
State intervention		196
	<i>Capital injection</i>	165
	<i>Asset protection</i>	22
	<i>Asset guarantee</i>	34
Total		252

See Betz et al. (2014) for details of the definitions.

announcement of the Basel III LR at the end of 2009 as a treatment that only affects a subset of banks, i.e. only banks below the announced LR, where 3% is taken as the relevant LR threshold.²⁷ Since our dataset includes time periods where an LR was not part of the regulatory regime (only the risk-based framework was in existence), we use a difference-in-difference type analysis in which the effect of an LR on risk-taking is estimated through a treatment dummy, while controlling for a large set of bank-specific and country-level variables that capture systematic differences in bank behaviour pre- and post-treatment. Our econometric strategy therefore is to compare the periods before the existence of an LR with the periods after, and then to analyse whether banks that were affected by the imposition of an LR (i.e. those treated with an LR below 3%) increased their risk-taking behaviour.

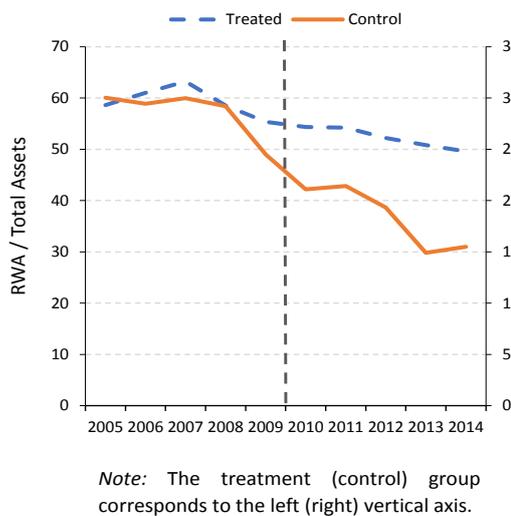
For such an analysis, it is important to investigate the comparability of the treatment and control groups. Table 4 illustrates summary statistics of bank characteristics split by group, and pre- and post-treatment eras. Figure 4 graphically demonstrates the trend for the RWA to total assets ratio, where the vertical grey line marks the LR announcement.²⁸ While the trend for both treatment and control

²⁷This classification of banks into treatment and control groups can be justified via the kinked structure of capital requirements under a combined LR and risk-based capital framework, which was illustrated in Figure 2. The LR will only bind for those banks with LRs below the minimum requirement. For all other banks, the risk-based capital framework will remain the binding constraint, so their behaviour should not be different in the pre-treatment and post-treatment periods, i.e. they can be seen as the control group.

²⁸The control group is set on a secondary axis to illustrate more clearly changes in patterns.

group banks is broadly similar before the LR announcement, afterwards there is a definite divergence, with control group banks seemingly de-risking significantly more than treated banks.

Table 5 directly tests the parallel trend assumption for changes in the RWA to total assets ratio. The table shows that before 2010, it appears that there was no significant difference between banks that had LRs below 3% and banks that had LRs above 3%, but since 2010 there was indeed a significant difference in bank behaviour. This can be seen in the indicator variables in the first and second row of Table 5. The indicator variable “Leverage Ratio \leq 3%, year $<$ 2010” is insignificant in all columns of table 5.²⁹ On the other hand, the post-treatment indicator variable “Leverage Ratio \leq 3%, year \geq 2010” is positive and highly significant in all specifications.³⁰ Columns (2) and (3) show that this result is robust to the inclusion of bank fixed effects and a lagged dependent variable respectively. Table 5 therefore illustrates that although before 2010 there was no significant difference between the treatment and control groups in terms of changes in risk-taking behaviour, significant differences arose from 2010.



	Loans to Assets	Log Total Assets	Pre-tax ROA	Loans to deposits
2005-09				
$>$ 3%	66.56	3.30	0.63	163.33
\leq 3%	41.04	4.86	0.12	189.72
2010-14				
$>$ 3%	64.28	2.50	0.33	135.09
\leq 3%	36.43	3.69	0.15	199.21

Source: SNL Financial, Bloomberg, ECB

Figure 4: Risk-weighted assets to total Assets, by group
Table 4: Summary stats split by 3% criterion (mean values)

Our identification strategy is somewhat complicated by the fact that the LR will not become a binding Pillar I regulatory requirement until 2018. Nevertheless, we rely on the assumption that banks already started to adjust their behaviour in

²⁹In particular, the variable is defined as follows: it is set equal to 1 for a given bank and year if its LR in the previous year is below 3%, but only for the years before 2010. It is set to 0 otherwise.

³⁰The variable “Leverage Ratio \leq 3%, year \geq 2010” is defined as follows: it is set equal to 1 for a given bank and year if its LR in the previous year is below 3%, but only for the years after and including 2010. It is set to 0 otherwise.

Table 5: Change in a bank’s risk-weighted assets to total assets ratio

	(1)	(2)	(3)
Leverage Ratio $\leq 3\%$, year ≥ 2010	1.314***	1.714**	0.806***
Leverage Ratio $\leq 3\%$, year < 2010	0.663	0.0870	0.706
Lagged dependent variable			0.0103
Observations	4636	4636	3704
R-squared	0.026	0.027	0.019
Bank fixed effects	No	Yes	No

Notes: The dependent variable is the first difference in a bank’s risk-weighted assets to total assets ratio. All regressions include time fixed effects. Significance is based on clustered robust standard errors.

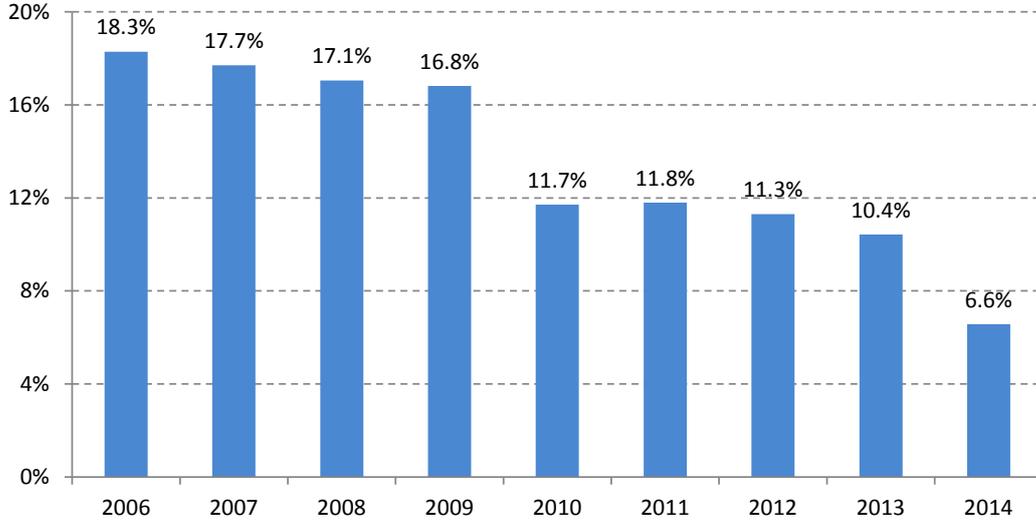
response to the Basel III LR announcement at the end of 2009. There is ample anecdotal evidence to support this assumption.³¹ Figure 5 illustrates that banks with low LRs (below 3%) started to bolster them after the announcement of the Basel III LR. While the percentage of bound versus non-bound banks remained around 17% in all years prior to 2010, there seems to have been a shift around 2010 as banks started to strengthen their LRs. There is around a 5 p.p. decline in the share of bound banks in 2010 (relative to 2009), and this share continues to decline from double digits to around 7% as of 2014. Indeed, we provide formal statistical evidence below in Table 8 that bound banks (i.e. banks with LRs below 3%) appear to have reacted to the Basel III LR announcement by bolstering their LRs.

The assumption that banks already started to react to the Basel III LR announcement at the end of 2009 is also supported by the fact that banks were required to start reporting their LRs (and its components) to supervisors from 1 January 2013 onwards. Moreover, adjusting balance sheet structures takes time, so it is reasonable to assume that banks already started to react well in advance of the LR becoming a binding regulatory requirement. Indeed, economic reasoning suggests that in order to properly identify the effect of the Basel III LR, it is necessary to take into account anticipatory effects, since by 2018 all banks will have to satisfy the LR, and thus any effects on risk-taking will probably already have occurred before that date.

Formally, our empirical strategy consists of estimating various versions of the following general panel model, where the left-hand side variable is a risk-taking proxy for bank i , located in country j , in year t :

³¹See for example <http://www.risk.net/risk-magazine/news/2388970/citi-buys-usd250bn-deutsche-bank-single-name-cds-portfolio>

Figure 5: Percentage of bound banks



$$y_{i,j,t} = \alpha + \beta T_{i,j,t} + \theta' X_{i,j,t-1} + \varphi' Y_{j,t-1} + \mu_i + \lambda_t + \epsilon_{i,j,t} \quad (1)$$

The terms μ_i and λ_t are bank and time fixed-effects respectively, $X_{i,j,t-1}$ and $Y_{j,t-1}$ are vectors of bank-specific and country-specific control variables (discussed below), and $\epsilon_{i,j,t}$ is an i.i.d error term. In the risk-taking model above, $T_{i,j,t}$ is the treatment dummy of interest. It is set equal to 1 for a given bank and year if its LR in the previous year was below the 3% minimum, but only for years following the first announcement of the Basel III LR requirement at the end of 2009. The treatment dummy is set to 0 otherwise.³² The coefficient of interest for the first stage of the empirical analysis is β , which measures how the announcement of the Basel III LR requirement has affected the risk-taking behaviour of banks.

2010 is set as the treatment start date in reference to the December 2009 BCBS consultative document that outlined the baseline proposal for the LR (see timeline presented in Figure 1). Moreover, 3% is taken as the relevant LR threshold since the BCBS has been assessing a minimum of 3%. Indeed, on 10 January 2016,

³²As will be shown, the results are also robust to instead defining treatment based on a bank's LR only at announcement date. In particular, the results are robust to instead setting the treatment dummy equal to 1 for a given bank for all periods after announcement if its LR at the announcement of the Basel III LR requirement was below 3%; setting the dummy to 0 otherwise. We do not use this definition in our baseline specification as firstly it is not clear whether a bank which increases its LR above 3% after 2010 should still be defined as a treated bank; theoretically, the incentives to risk-up will diminish. Secondly, defining treatment in this way reduces the sample size, thus we use the alternative definition to maximise the sample. Nevertheless, the results are robust to either definition.

the BCBS’s oversight body, the GHOS, confirmed that the final minimum level of the Tier 1 LR should be 3%. Since data for the Basel III definition of the LR is unavailable, the ratio of Tier 1 equity to total assets is used as our LR proxy. This variable correlates very highly with the Basel III regulatory definition of the LR, with a correlation coefficient of 0.92 for the sample where the Basel III LR data is available (from 2013-2014). Finally, as our measure of bank risk-taking, we use the ratio of RWA to total assets. While the ratio of RWA to total assets is an imperfect measure of true bank risk-taking, it is the most direct measure of risk-taking, and it is the measure that should be affected by the introduction of an LR.³³

The following bank-specific variables are used as control variables: balance sheet size (measured via the logarithm of total assets), since it may be that larger institutions behave differently than smaller institutions; the ratio of total loans to total assets, to control for the business model of a bank; the loan-to-deposit ratio, to control for liquidity; pre-tax return on assets (ROA), to control for bank profitability, since it may be that more profitable banks take less risk in a skin-in-the-game type mechanism; and the tier 1 to total asset ratio, to control for the bank’s LR.

We also include the following bank-specific dummy variables: first, a dummy variable called “Tier 1 capital ratio treatment”, which is defined in a similar way to the LR treatment dummy, but in reference to the bank’s risk-weighted capital requirements.³⁴ This is included so as to control for the concurrent strengthening of the risk-based capital framework (see BCBS (2009)), so that results captured by the LR treatment dummy are not wrongly capturing responses to changes in the risk-based capital framework.³⁵ Second, a dummy variable called “Dummy ($LR \leq 3\%$)”

³³While the RWA to total assets ratio is potentially imprecise for comparing the level of risk-taking across banks, changes in this measure for a given bank should in principle be highly correlated with actual changes in risk-taking. This should be true as long as risk-weight levels within a given bank are positively correlated with true risk. In addition, control variables for the calculation method of risk weights are included in the panel regressions, which should partly account for the fact that risk-weight levels appear to differ systematically between the standardised approach and the internal ratings based approach for determining risk-weights. The RWA to total assets ratio is the most direct measure of risk-taking, since any changes in risk for a given bank should show up immediately in the reported financial statements. This contrasts with other proxies for risk-taking such as non-performing loans or the Z-score, which should lag true risk-taking and it is not obvious at what lag changes will show up in these proxies. Lastly, although the results presented in this section take the level of the RWA to total assets ratio as the dependent variable, the results equally apply (with a similar magnitude and significance) if we instead use the change in the RWA to total assets ratio as the dependent variable.

³⁴In particular, the dummy is set equal to 1 for a given bank and year if its Tier 1 capital ratio in the previous year was below the new Basel III risk-based capital requirements (including the capital conservation buffer) of 8.5% (plus any buffer requirements for systemically important institutions), but only for years after 2009. The dummy is set to 0 otherwise.

³⁵To note that the Basel Committee has also introduced the Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NSFR) as part of the Basel III framework which may have impacted

is included in order to control for the general effect of being a highly leveraged bank with an LR below 3% in any year. In particular, for all years in the sample, the dummy is set equal to 1 for a given bank and year if that bank's LR in the previous year was below 3%. It is set to 0 otherwise. Third, dummy variables are included that capture the calculation method for risk weights that is used by each bank in a given year. In particular, there is a dummy for whether a bank is advanced IRB, foundation IRB or a mixture. This is complemented with further dummy variables controlling for the Basel regime prevailing at the time, which can be seen in the dummies Basel II, Basel II.5 and Basel III in Table 6.

Lastly, the following macro variables are controlled for: GDP growth, inflation, government debt to GDP, and the change in the unemployment rate, to control for the economic environment; the 10-year government bond yield, to control for the monetary environment, including capturing potential effects from the risk-taking channel of monetary policy; and the ratio of total bank credit to GDP, stock price growth, and house price growth since these factors may impact risk-taking incentives for banks.

Table 6 presents the baseline estimation results for the effect of the Basel III LR announcement on the risk-taking behaviour of EU banks. In line with the first hypothesis from our theoretical model of section 3, the results suggest that since the Basel III LR framework was announced at the end of 2009, EU banks with low LRs have slightly increased their risk-taking, as measured by their RWA to total assets ratio. This conclusion is robust to various specifications and estimation methods.³⁶ The estimated coefficients for the treatment effect are always positive and highly significant for all model specifications. In terms of the quantitative impact, the point estimates for the treatment effect of a 3% LR suggest that on average banks bound by it increase their RWA ratio by around 1.5 to 2.5 percentage points more than they otherwise would, which appears rather muted.

What is more, while the Basel III LR announcement seems to have incentivised slightly higher risk-taking, the concurrent strengthening of the risk-based capital

banks' behaviour as well. Unfortunately, time series data for these two ratios is not available, so we are unable to create similar dummies for these new liquidity regulations. Nevertheless, the EBA (2016) finds that there is almost no correlation between the LR and the LCR as well as the NSFR of European banks, suggesting that omitting controls for these two measures should not bias our results in one direction or the other. In addition, we include the loan-to-deposit ratio to control for the liquidity profile of banks. This is mostly insignificant - see Table 6.

³⁶Dynamic panel GMM as in Arellano and Bond (1991) is used since a lagged dependent variable is introduced in the model. In the dynamic GMM estimation, GDP growth, the change in unemployment and the Basel regime variables are considered exogenous; all other variables are instrumented using lags of the variables in question.

Table 6: Estimated effect of the Basel III leverage ratio on bank risk-taking

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Leverage ratio treatment	2.017**	1.596**	2.592**	1.750**	1.445*	2.511**	2.851**	2.575**
Tier 1 capital ratio treatment		-0.676*	-1.730***	-0.783*	-0.301	-0.709*	-0.839	-1.638*
RWA / Total assets, lag		0.527***		0.460***	0.516***	0.523***	0.434***	0.434***
RWA / Total assets, lag 2					-0.102**	-0.0766	-0.0946	-0.122
Total assets, log			-3.529**	-2.733**	-2.972**	-1.347	-2.179	-2.759
Total loans / Total assets			0.109**	0.0610	0.000692	-0.0223	0.0535	0.046
Pre-tax ROA			0.330*	0.376**	0.575**	0.656**	0.272	-0.612
Loan-to-deposit ratio			0.00773**	0.00366	0.00325	0.00185	0.00293	0.005
Leverage ratio proxy	1.06***	0.257	1.411***	0.150	0.174	0.239	0.733	0.746
Dummy (LR \leq 3)	-2.082***	-2.070***	-2.335**	-1.354*	-1.044	-0.793	-0.507	-0.388
Basel II dummy			-0.212	-0.728	-0.423	-0.823	-0.825	-0.263
Basel II.5 dummy			-0.566	-1.561	-1.103	-1.949	-1.611	-1.223
Basel III dummy			3.089	0.953	1.085	-0.162	-2.681	-4.729
Advanced IRB dummy			-0.346	0.0533	-0.236	0.231	1.693	2.302
Foundations IRB dummy			-2.154	-2.146	-2.003	-3.152*	-4.101**	-3.167
Mix IRB / SA dummy			-5.290***	-3.660***	-3.315***	-3.491***	-3.888***	-3.223**
GDP growth, y-on-y			-0.284**	-0.273**	-0.278**	-0.250*	-0.355*	-0.034
Inflation, y-on-y			0.438	0.535**	0.598***	0.553**	-0.0709	0.948**
Unempl. Rate change, y-on-y			0.0947	0.335	0.465	0.800**	0.00558	0.495
10-year Government bond yield			-0.0681	-0.130	-0.125	-0.147	-0.0106	-0.905
Total credit / GDP			-0.0211	-0.0337	-0.0463*	-0.0670**	-0.0345	-0.048
Stock price growth, y-on-y			-0.00152	0.00970	0.013	0.00841	0.0231	-0.067*
House price growth, y-on-y			-0.00286	-0.00796	-0.0116	-0.0118	0.00338	0.007
Government Debt / GDP			0.0980*	0.0594*	0.0736**	0.0679**	0.0636*	-0.016
Intercept	52.41***	28.20***	56.06***	38.46***	40.26***			
Observations	4,689	4,574	2,849	2,836	2,748	2,111	1,824	1,126
R-squared	0.171	0.390	0.261	0.407	0.410			
AR1-p						3.63e-7	2.20e-5	0.000
AR2-p						0.922	0.995	0.284
Hansen-p						0.180	0.164	0.738
Lagged dependent	No	1 Lag	No	1 Lag	2 Lags	2 Lags	2 Lags	2 Lags
Estimation method	FE	FE	FE	FE	FE	GMM	GMM	GMM
Bank & Time effects	Yes	Yes						
Sample	All EU	Western Europe	W. Europe excl. GIIPS					

Notes: The dependent variable is the risk-weighted assets to total assets ratio (expressed as a percentage). In all models, explanatory variables are lagged by one period to avoid endogeneity issues. All EU sample means estimation is based on all of the EU banks contained in the dataset. Western Europe represents the bank subsample encompassing Austria, Belgium, Germany, Denmark, Spain, Finland, France, the UK, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Sweden. The Western Europe excl. GIIPS sample represents the Western Europe sample excluding banks from Greece, Italy, Ireland, Portugal and Spain. The models in columns 1-5 are estimated with bank and time fixed-effects (FE). Columns 6-8 are estimated using dynamic GMM, where the lagged dependent variable, bank-specific variables and macro variables (excluding GDP growth and the change in unemployment) are instrumented with their own lags. GDP growth, the change in unemployment and Basel regime variables are treated as exogenous and are therefore used as IV-style instruments in columns 6-8. AR1-p, AR2-p and Hansen-p refer to the p-values of the Arellano-Bond tests for first- and second-order autocorrelation of the differenced residuals and exogeneity of the instruments using the Hansen J statistic respectively. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. Significance is based on clustered robust standard errors.

framework under Basel III seems to have had the opposite effect. This can be seen via the variable “Tier 1 capital ratio treatment”. Specifically, the range of point estimates presented in Table 6 suggests that banks with Tier 1 capital ratios below their forthcoming regulatory minimum reduced their RWA ratio by around 0.3 to 1.7 percentage points more than they otherwise would have, since the strengthening of the risk-based capital framework under Basel III was announced at the end of 2009 (although results are not as robust as the results for the LR). By controlling for the strengthening of the risk-based capital framework under Basel III, we assure that the small estimated effects on bank risk-taking from the Basel III LR announcement are not a result of the concurrent strengthening of the risk-based capital framework.

The small estimated increase in risk-taking for banks bound by the Basel III LR remains robust to various other tests, both quantitatively and in terms of statistical significance. First, columns (7) - (8) in Table 6 and column (1) in Table 7 show that the result is robust to using different bank and country samples. Second, Columns (2) - (3) of Table 7 tackle concerns related to potential misclassifications of the treatment and control groups, given the uncertainty over the final level of the LR threshold. Columns (2) and (3) show that the significant small increase in risk-taking remains when the model is re-estimated excluding all banks with LRs between 3% and 5%.³⁷

Columns (4) - (6) of Table 7 tackle the potential concern that banks with vastly different LRs are fundamentally different through a Regression Discontinuity Design (RDD).³⁸ By restricting the estimation sample to banks that are close to either side of the LR threshold it is more likely that these banks exhibit similar ex-ante behaviour. This allows us to estimate a local average treatment effect (LATE). The optimal bandwidth around the LR threshold is determined via the procedure proposed by Imbens and Kalyanaraman (2012), and then double and triple this bandwidth is tested. As can be seen from columns (4) - (6), our core result is left unchanged. The treatment dummy remains significant at all different bandwidth levels (we experiment with different bandwidth levels for robustness) and the coefficient remains within a similar region of magnitude, namely around 1 to 2 percentage points. Moreover, columns (4) - (6) suggest that as we reduce the bandwidth

³⁷This is done as it may be that banks with LRs between 3% - 5% are fuzzy in regards to whether they should be classified as treatment or control group banks. Banks with LRs just above 3% may also act to some extent. Therefore, excluding all bank observations with LRs in this range should alleviate potential misclassification problems of the treatment and control groups.

³⁸Without controls, the validity of difference-in-difference crucially relies on the identical ex-ante behaviour of banks in the control and treatment groups, so that it is only the treatment that generates differing behaviour, not differences among group participants.

Table 7: Robustness of the estimated effect on bank risk-taking

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Leverage ratio treatment, 3%	2.647**	2.552***	1.993*	0.990*	1.689***	2.193***			
Leverage ratio treatment 2, 3%							2.273**		
Leverage ratio treatment 3, 3%								3.215**	1.700*
Tier 1 capital ratio treatment	-0.901*	-0.207	-0.769	-1.53**	-1.257**	-0.951**	-1.243**	-0.914*	-0.903
Observations	1,550	2,342	1,491	760	1,344	1,950	2,111	1,738	726
AR1-p	7.90e-06		9.06e-05	0.001	6.50e-06	2.46e-07	0.000	0.000	0.006
AR2-p	0.168		0.548	0.500	0.566	0.841	0.934	0.756	0.716
Hansen-p	0.578		0.882	0.931	0.729	0.672	0.426	0.350	0.362
Lagged dependent	2 Lags	1 Lag	2 Lags	2 Lags	2 Lags	2 Lags	2 Lags	2 Lags	2 Lags
Control variables	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Estimation method	GMM	FE	GMM	GMM RDD, optimal	GMM RDD, double	GMM RDD, triple	GMM	GMM	GMM RDD, optimal
Bank sample	Euro Area	All EU $LR \notin (3, 5)$	All EU $LR \notin (3, 5)$	All EU	All EU	All EU	All EU	All EU	All EU

Notes: The dependent variable is the risk-weighted assets to total assets ratio (expressed as a percentage). The same set of control variables as in Table 6 are included in all of the regressions, including bank and time fixed-effects. All explanatory variables are lagged by one period to avoid endogeneity issues. All EU sample means estimation is based on all of the EU banks contained in the dataset. The Euro Area sample only includes banks from the 19 Euro Area countries. All EU $LR \notin (3, 5)$ excludes all observations where a given bank had a leverage ratio greater or equal than 3% and smaller or equal than 5%. RDD refers to a Regression Discontinuity Design that restricts the estimation sample to banks that are close to the leverage ratio threshold on either side. The optimal band-width is plus / minus 1.81 around the baseline 3% leverage ratio threshold. The leverage ratio treatment variables are dummy variables that indicate whether a given bank had a leverage ratio below the threshold level in the previous year, for years after 2009. The leverage ratio treatment 2 variable measures the one-sided distance from the required minimum level. Formally: treatment variable 2 = treatment dummy $\cdot (LR_{min} - LR)$. The leverage ratio treatment 3 variable is defined based on end-2009 values only. Formally: for a given bank, the treatment dummy is set to 1 for all years between 2010-2014 if the bank had a leverage ratio below 3% at end-2009, otherwise it is set to 0. It is set to 0 for all banks before 2010. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. Significance is based on clustered robust standard errors.

around the LR threshold of 3%, the estimated induced risk-taking due to the LR gets smaller.

Column (7) of Table 7 formally tests whether the risk-taking effect differs in magnitude the further a bank's LR is from the 3% threshold. Indeed, the results in column (7) suggest that the induced increase in risk-taking due to the LR is smaller, the closer a bank is to the 3% threshold.³⁹ The coefficient estimate suggests that banks with LRs of 1.5%, 2% and 2.5% adjusted their RWA ratios upward by 3.40, 2.27 and 1.14 percentage points respectively. This fits well with intuition, since the incentive for additional risk-taking should be smaller if a bank is required to increase capital only slightly, say from a 2.9% to a 3% LR, compared to if a bank is required to increase capital by a lot, for instance from a 1% to a 3% LR.

Finally, columns (8) - (9) in Table 7 show that the results are robust to instead defining the LR treatment dummy based only on a bank's LR at announcement date. In particular, the 'leverage ratio treatment 3' variable is set equal to 1 for a given bank for all periods after announcement if its LR at the announcement of the Basel III LR requirement was below 3%. It is set to 0 otherwise. Significant coefficient estimates with similar magnitudes as before are obtained. In summary, the results from the first stage empirical exercise suggest that an LR appears to incentivise additional risk-taking for banks bound by it, but this additional risk-taking appears

³⁹The 3% 'leverage ratio treatment 2' variable measures the one-sided distance from the required minimum level. Hence, if the LR of a bank was 1%, the treatment variable would be 2%. Formally: treatment variable 2 = treatment dummy $\cdot (LR_{min} - LR)$.

Table 8: Estimated effect on banks' leverage ratios

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Leverage ratio treatment, 3%	0.610***	0.831***	0.795***	0.439***	0.718***	1.081***		
Leverage ratio treatment 2, 3%							0.534***	
Leverage ratio treatment 3, 3%								0.999***
Tier 1 capital ratio treatment	0.419***	0.400***	0.354***	0.142	0.169	0.473***	0.354***	0.400***
Observations	3,064	2,631	2,393	1,021	1,807	1,826	2,437	2,631
R-squared	0.105	0.102	0.095	0.152	0.137	0.110	0.115	0.102
Control variables	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Estimation method	FE	FE	FE	FE RDD, optimal	FE RDD, double	FE	FE	FE
Bank sample	All EU	Western Europe	W. Europe excl. GIIPS	All EU	All EU	All EU $LR \notin (3, 5)$	All EU	All EU

Notes: The dependent variable is the first difference of the leverage ratio (expressed in percentage points). The same set of control variables as in the third column of Table 6 are included in all of the regressions, including bank and time fixed-effects. All explanatory variables are lagged by one period to avoid endogeneity issues. All EU sample means estimation is based on all of the EU banks contained in the dataset. Western Europe excl. GIIPS represents the bank subsample encompassing Austria, Belgium, Germany, Denmark, Finland, France, the UK, Luxembourg, the Netherlands, and Sweden. All EU $LR \notin (3, 5)$ excludes all observations where a given bank had a leverage ratio greater or equal than 3% and smaller or equal than 5%. RDD refers to a Regression Discontinuity Design that restricts the estimation sample to banks that are close to the leverage ratio threshold on either side. The bandwidth denoted optimal is plus / minus 1.81 around the baseline 3% leverage ratio threshold. The leverage ratio treatment variables are dummy variables that indicate whether a given bank had a leverage ratio below the threshold level in the previous year, for years after 2009. The leverage ratio treatment 2 variable measures the one-sided distance from the required minimum level. Formally: treatment variable 2 = treatment dummy $\cdot (LR_{min} - LR)$. The leverage ratio treatment 3 variable is defined based on end-2009 values only. Formally: for a given bank, the treatment dummy is set to 1 for all years between 2010-2014 if the bank had a leverage ratio below 3% at end-2009, otherwise it is set to 0. It is set to 0 for all banks before 2010. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. Significance is based on clustered robust standard errors.

limited, as suggested by the theoretical model of section 3.

To shed more light on banks' reactions to the Basel III LR announcement, the risk-taking regressions are also re-estimated with the change in a bank's LR as the dependent variable, to see if treated banks were increasing their LRs at the same time as taking on further risk. This indeed seems to have been the case, as can be seen from Table 8, with estimates of around a 0.44 - 1.1 percentage point greater increases in a bank's LR than would have otherwise happened. This result is again robust to different bank country and bank samples, running RDD specifications, and assuming different treatment definitions. This finding also provides further support for the assumption that banks already started to react to the Basel III LR requirement upon announcement in 2009, well before it is planned to migrate to a binding Pillar I regulatory requirement in 2018. To summarise, while treated banks may have increased their RWA to total assets ratios by around 1.5 to 2.5 p.p. more, the evidence suggests they also increased their LRs by up to 1 p.p. more over the period of consideration. This is a considerable increase in a bank's capital position relative to the estimated increase in RWA.

4.3 Trade-off between loss-absorption and risk-taking

For the second part of the empirical analysis, we use our unique dataset of EU bank distress events in a discrete choice modelling framework. We analyse the joint effects of the LR and risk-taking on bank distress probabilities. This analysis is crucial in order to quantify the net impact of the risk-stability trade-off associated with an LR. As discussed in van den Berg et al. (2008), a logit model is preferred over a probit model, because the fatter tailed error distribution matches better to the empirical frequency of bank distress events. While the early-warning literature has commonly used a pooled logit approach (see e.g. Lo Duca and Peltonen (2013)) we control for both time and country fixed-effects, since in-sample fit and unbiased coefficient estimates are more important for our analysis than optimising out-of-sample predictive performance.⁴⁰ Specifically, various versions of the following logit model are estimated, where the left-hand side variable is the binary distress indicator for bank i , located in country j , in year $t + 1$, γ_j and λ_{t+1} are country and time fixed-effects respectively, and $X_{i,j,t}$ and $Y_{j,t}$ are vectors of bank-specific and country-specific control variables respectively.⁴¹

$$P(I_{i,j,t+1} = 1) = \frac{\exp(\alpha + \theta'X_{i,j,t} + \varphi'Y_{j,t} + \gamma_j + \lambda_{t+1})}{1 + \exp(\alpha + \theta'X_{i,j,t} + \varphi'Y_{j,t} + \gamma_j + \lambda_{t+1})} \quad (2)$$

Table 9 presents the main results from our bank distress analysis, where the LR is proxied by the ratio of Tier 1 equity to total assets and risk-taking is proxied by the ratio of RWA to total assets, as in the first stage empirical exercise above. Columns (1) - (2) present the baseline estimation results excluding and including country and time fixed-effects. In line with economic intuition, the LR has a negative coefficient and risk-taking a positive coefficient. Most importantly, in comparison

⁴⁰Controlling for time and country fixed-effects should lead to better in-sample fit, as shown by Fuertes and Kalotychou (2006).

⁴¹In particular, in addition to the LR and the ratio of RWA to total assets, the following control variables are included. Bank-specific variables include: non-performing loans (NPLs) to total assets and the coverage ratio, in order to control for the bank's asset quality; pre-tax ROA, to control for profitability, since more profitable banks may have lower probabilities of distress; interest expenses to total liabilities, which allows us to control for funding costs; the loan-to-deposit ratio, to control for liquidity; and the logarithm of total assets, to control for size, since it is conceivable that larger banks have different distress probabilities than smaller banks. As in the risk-taking regressions, we also control via dummy variables for the risk-weighting method used by the bank in a given year, and what Basel regime prevailed at the time. The macro-financial control variables include: GDP growth, inflation, the government debt to GDP ratio, and the unemployment rate, to control for the economic environment; the change in the Bund-spread as a measure of country risk; the change in the banking sector's issued debt to liabilities as a measure of the banking system's funding structure; and total credit to GDP, private sector credit flow and stock price growth as further controls for the macroeconomic environment.

to risk-taking, the LR seems to be much more important for determining a bank's distress probability, both statistically and economically. For example, models (1) and (2) suggest that a 1 p.p. increase in a bank's LR is associated with around a 43-51% decline in the relative probability of distress to non-distress (the odds ratio).⁴² This is much larger than the marginal impact of taking greater risk. The coefficient estimates suggest that increasing a bank's RWA ratio by 1 p.p. is associated with an increase in its relative distress probability of only around 1-3.5%. This demonstrates the relative importance of the LR in determining bank distress probabilities.

The other models in Table 9 show that the results are robust to introducing non-linear effects in the LR and RWA ratio (columns (3) - (4)) and to considering different country and bank samples (columns (5) - (7)). Adding squared terms for both variables of interest and a cubic term for the LR indeed improves the fit of the model, as measured by the Pseudo R-squared and the Area Under the Receiver Operating Characteristics Curve (AUROC), as well as the statistical significance of the estimated effect of risk-taking on bank distress probabilities. Figure 6 illustrates graphically the estimated non-linear effects of the LR and risk-taking on bank distress probabilities obtained from model (4), which is the most complete specification. There seems to be considerable benefit for bank stability from increasing the LR from low levels, but as a bank's LR gets to around 5%, the benefits from increasing it further start to diminish slightly. Moreover, the marginal beneficial impact for bank stability of increasing the LR from low levels is much stronger than the marginal negative impact of increasing a bank's RWAs. Columns (5) - (7) confirm that this result remains robust if we restrict the estimation sample to banks from the Euro Area, Western Europe, and Western Europe excluding countries that were most affected by the European sovereign debt crisis.

4.4 Net effect of a leverage ratio constraint on bank stability

The two previous empirical exercises suggest that while banks that are constrained by an LR slightly increase risk-taking, the concurrent increase in their Tier 1 to total asset ratio appears more important for bank stability considerations. To analyse this more formally, the results from the bank distress model are combined with the estimated increase in risk-taking in a counterfactual simulation. The simulation proceeds as follows. We first take all bank-year observations in our sample where the bank had an LR below the relevant minimum, and compute the associated distress

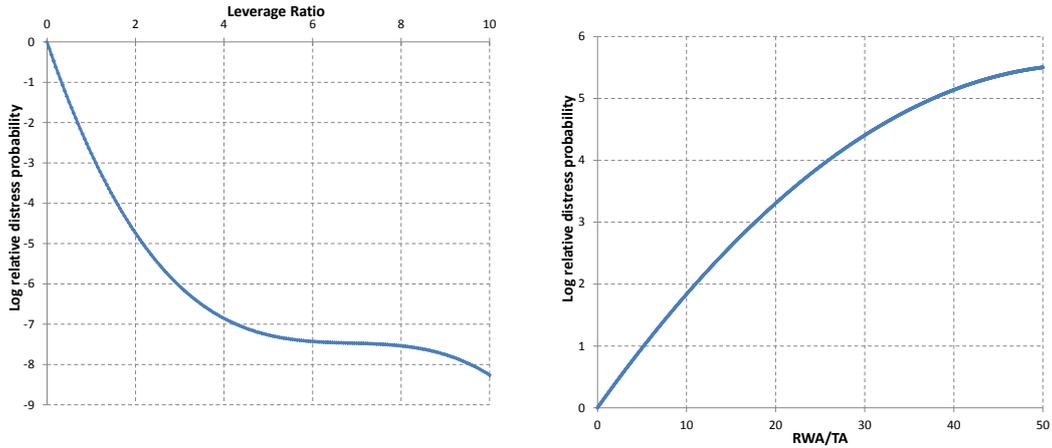
⁴²For a detailed discussion on the interpretation of logit coefficients, see Cameron and Trivedi (2005).

Table 9: Estimated effect of the leverage ratio and risk-taking on bank distress probabilities

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Leverage ratio proxy	-0.510***	-0.427***	-1.046***	-3.206***	-2.865***	-3.957***	-5.188**
Leverage ratio proxy, squared			0.054***	0.463***	0.420***	0.580***	0.465
Leverage ratio proxy, cubed				-0.023***	-0.021**	-0.028***	-0.014
RWA / Total assets	0.035***	0.011	0.166***	0.202***	0.188***	0.251***	0.406**
RWA / Total assets, squared			-0.001***	-0.002***	-0.002***	-0.002***	-0.002
NPLs / Total assets	0.072**	0.055	0.090***	0.101***	0.098***	0.097**	0.117
Coverage ratio	-0.014***	-0.011**	-0.012**	-0.012**	-0.012	-0.013**	-0.026***
Pre-tax ROA	-0.013	-0.082	-0.018	-0.001	-0.031	-0.001	-0.402
Interest expenses / Total liabilities	0.203***	0.152**	0.125**	0.140**	0.152**	0.147**	0.149
Loan-to-Deposit ratio	0.002***	0.002***	0.003***	0.003***	0.003***	0.003***	0.005***
Total assets, log	0.314***	0.345***	0.323***	0.334***	0.330***	0.341***	0.438**
Basel II dummy	0.698*	0.175	-0.001	0.018	0.171	-0.104	-1.206
Basel II.5 dummy	-1.180	-1.256	-1.492	-1.632	-1.214	-1.660	-2.539
Advanced IRB dummy	-1.967**	-1.751**	-1.593**	-1.733**	-1.844**	-1.702**	-0.496
Foundations IRB dummy	0.627	0.612	0.527	0.537	0.625	0.564	1.125*
Mix IRB / SA dummy	0.222	0.116	0.088	0.127	0.126	0.135	1.771***
Bund-spread, y-on-y change	0.284***	0.495*	0.515**	0.485*	0.553*	0.354*	1.882
Government Debt / GDP	0.009*	-0.067**	-0.070***	-0.073***	-0.096***	-0.090***	0.050
Unemployment rate	0.105***	0.218**	0.185**	0.182**	0.262***	-0.002	-0.725*
GDP growth, y-on-y	-0.009	-0.211	-0.156	-0.163	-0.209	-0.222	-0.470
Inflation, y-on-y	-0.099	-0.807**	-0.859***	-0.886***	-0.910**	-0.735***	-1.074*
Private sector credit flow	0.066***	0.096***	0.103***	0.108***	0.137***	-0.034	-0.027
Total credit / GDP	0.001	0.047**	0.057***	0.056***	0.089***	0.046*	-0.044
Bank issued debt / Liabilities, y-on-y change	-0.095	-0.200*	-0.205	-0.227*	-0.387***	-0.159	0.163
Stock price growth, y-on-y	-0.011	0.038*	0.039*	0.041**	0.044*	0.043	0.115
Intercept term	-6.157***	-26.26***	-29.96***	-26.80***	-34.80***	-22.84***	-10.76
Observations	1,661	1,661	1,661	1,661	1,234	1,334	674
Pseudo R2	0.284	0.410	0.430	0.437	0.431	0.408	0.559
AUROC	0.870	0.926	0.929	0.930	0.926	0.918	0.961
Country and time fixed-effects	No	Yes	Yes	Yes	Yes	Yes	Yes
Non-linear effects	No	No	Yes	Yes	Yes	Yes	Yes
Bank sample	All EU	All EU	All EU	All EU	Euro Area	Western Europe	W. Europe excl. GIIPS

Notes: Logit model estimates are obtained on a binary bank distress variable (See Betz et al. (2014) and Lang et al. (2015) for details on the bank distress event definitions). The numbers in the table are logit model coefficients. All right hand side variables are lagged by one year. All EU sample means estimation is based on all of the EU banks contained in the dataset. The Euro Area sample only includes banks from the 19 Euro Area countries. Western Europe represents the bank subsample encompassing Austria, Belgium, Germany, Denmark, Spain, Finland, France, the UK, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Sweden. The Western Europe excl. GIIPS sample represents the Western Europe sample excluding banks from Greece, Italy, Ireland, Portugal and Spain. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. Significance is based on clustered robust standard errors. AUROC refers to the Area Under the Receiver Operating Characteristics Curve, which is a global measure of how well the model can classify observations into distress and non-distress periods. An uninformative model has an AUROC of 0.5, while a perfect model has an AUROC of 1.

Figure 6: Non-linear effects of the Leverage Ratio and risk-taking on bank distress



Notes: The log relative distress probability is equal to the log of the probability of distress divided by the probability of non-distress. Specifically, if the probability of distress is given by p , then it is equal to $\log(p/(1-p))$. For illustrative purposes, in generating these charts, all variables except the specified variable are set to zero. Results are based on the coefficient estimates of model (4) in Table 9.

probabilities using the true data. We then compute counterfactual distress probabilities for the same set of bank-year observations, assuming that banks increase their LR up to the required minimum, but at the same time also increase their RWA ratio by the estimated amount. Finally, we look at the changes in distress probabilities across all the relevant bank-year observations in the sample to see whether bank distress probabilities decline on average and whether any decline is statistically significant. In this way, we attempt to assess quantitatively the net effect of the potential trade-off between greater loss-absorbing capacity and higher bank risk-taking associated with an LR requirement. To allow for a conservative assessment, the mid-point in the range of the estimated increase in risk-taking is assumed, i.e. a 2 p.p. increase in the RWA ratio. For robustness purposes, we also test an increase in the RWA ratio of 4 p.p., 6 p.p. and the maximum amount possible before moving back into the risk-based capital framework (denoted $\Delta(RWA/TA) = max$ in Table 10).⁴³ The simulation is performed for a 3%, 4% and 5% LR minimum.

Table 10 reports the average changes in estimated bank distress probabilities from the various simulations. The numbers can be interpreted as the average percentage change in distress probability for the relevant banks in our sample between 2005

⁴³The maximum risk possible before moving back into the risk-based capital framework is the kinked point in Figure 2 - i.e. the point at which any further increase in risk would make the risk-based capital requirement larger than the LR requirement. This critical risk level will depend on the exact LR requirement, with higher requirements increasing this critical risk level. Furthermore, since banks have differing initial risk levels, this implies that the increase in risk will differ for each bank, as it will depend on how far away its initial risk level is from this maximum.

Table 10: Simulated change in average bank distress probabilities

LR threshold:	3%	4%	5%	4%	5%	5%
Banks with an LR of:	Less than 3%			Between 3-4%		Between 4-5%
$\Delta(RWA/TA) = 2$	-0.860***	-0.986***	-0.995***	-0.694***	-0.978***	-0.709***
$\Delta(RWA/TA) = 4$	-0.792***	-0.981***	-0.994***	-0.536**	-0.969***	-0.559**
$\Delta(RWA/TA) = 6$	-0.689***	-0.973***	-0.992***	-0.293	-0.956***	-0.327
$\Delta(RWA/TA) = max$	-0.484*	-0.707*	-0.664*	-0.055	-0.462	-0.167

Notes: The numbers represent the average simulated percentage change in the distress probability for the relevant bank sample between 2005 - 2014, expressed as decimal numbers (i.e. 0.1 represents 10%). Changes in distress probabilities are derived as follows. First, distress probabilities are estimated using the underlying data. Second, each bound bank has its leverage ratio increased to the stated percentage (e.g. 3%), while at the same time increasing its risk-weighted assets ratio by the stated amount (e.g. 2 p.p.). Using this adjusted data, new distress probabilities are estimated and the percentage change is taken. The table reports average (mean) values, where the average changes are reported separately for the sample of banks with a leverage ratio less than 3%, between 3-4% and between 4-5%. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. Significance is based on bootstrapped standard errors on 10,000 replications.

and 2014. For example, if a bank had a probability of distress of 0.02, a change of -0.860 (reported in the first row for a 3% LR), would imply a fall by 86% to 0.0028. Since increasing the LR minimum increases the sample of banks below this minimum, to ensure comparability across simulations, results are reported separately for the sample of banks with an LR less than 3%, between 3-4% and between 4-5%. The results demonstrate that bank distress probabilities should significantly decline with an LR, even when taking into account much higher increases in risk-taking than were estimated in section 4.2. For example, Table 10 shows that assuming a 3% LR and an increase in the RWA ratio of 4 p.p., the average distress probability declines by 79% for the given sample of bank-years. If the increase in the RWA ratio is assumed to be 6 p.p., the average decline in distress probabilities would still be 68.9%. Even if we assume that banks increase their RWA ratio by the maximum amount possible before moving back into the risk-based capital framework (denoted $\Delta(RWA/TA) = max$ in the table), the results still indicate that bank distress probabilities should decline, although the statistical significance drops to the 10% level. The simulation results therefore lend support to the second hypothesis, namely that the beneficial impact of higher capital holdings from an LR should more than outweigh the negative impact of increased risk-taking, thus leading to more stable banks.

5 Conclusion

Theoretical considerations and empirical evidence for EU banks provided in this paper suggest that the introduction of an LR requirement into the Basel III regulatory framework should lead to more stable banks. This paper has shown that although there can indeed exist an increased incentive to take risk once banks become bound by the LR requirement, this increase should be more than outweighed by the synchronous increase in loss-absorbing capacity due to higher capital. The analysis therefore supports the introduction of an LR alongside the risk-based capital framework. The analysis further suggests that the LR and the risk-based capital framework reinforce each other by covering risks which the other is less able to capture; making sure banks do not operate with excessive leverage and at the same time, have sufficient incentives for keeping risk-taking in check.

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Appendix A: Alternative assumption $(1 - \mu) > \alpha$

This section solves the model under the alternative assumption that $(1 - \mu) > \alpha$. This impacts the model by changing the risk-based capital requirement. To see that it entails a different capital requirement, consider how this assumption maps into the requirement that on each asset banks must cover all shocks with some probability $(1 - \alpha)$. If $(1 - \mu) > \alpha$, then ensuring survival in only state s_1 is no longer sufficient; the capital charge on each asset must also ensure that some shocks in state s_2 are covered. Consider the safer asset, since $(1 - \mu) > \alpha$, the capital charge on the safer asset must ensure that banks survive an additional shock in state s_2 . But there is only one additional shock in state s_2 , and thus $k_{safer} = \lambda_1$: anything less would violate the requirement. Consider the risky asset, in state s_2 the risky asset returns $(1 - \lambda_3)$ with probability π and 0 otherwise. If the bank holds capital of λ_3 , it will fail to cover shocks with probability $(1 - \mu)(1 - \pi)$. If $(1 - \mu)(1 - \pi) \leq \alpha$, this is sufficient, and the capital charge on the risky asset will be $k_{risky} = \lambda_3$. On the other hand, if $(1 - \mu)(1 - \pi) > \alpha$, λ_3 is not sufficient to satisfy the requirement, and the capital charge on the risky asset will be $k_{risky} = 1$. This second case is less realistic since it implies a zero probability of default; the risk-based capital requirement is so high that it covers all shocks. Nevertheless, we take both cases and show that the main results found in section 3 continue to apply.⁴⁴

Since $(1 - \mu) > \alpha$, the new capital requirement will be:

$$k(\omega) = \begin{cases} \omega\lambda_1 + (1 - \omega)\lambda_3 & \text{if } (1 - \mu)(1 - \pi) \leq \alpha \\ \omega\lambda_1 + (1 - \omega)1 & \text{if } (1 - \mu)(1 - \pi) > \alpha \end{cases}$$

Aside from this strengthened capital requirement, the problem will be identical to section 3.1. We begin by showing that as before for any $\omega \in [0, 1]$, banks always wish to hold as little capital as possible, and therefore the capital requirement will bind.

First, consider $\omega = 1$. If this is the case, profits will be given by $\mu R_1 + (1 - \mu)(1 - \lambda_1) - (1 - k) - \rho k - c(1)$. Clearly since $\rho > 1$, this is maximised at $k = 0$, and hence banks will choose the minimum capital level (this is true whether $(1 - \mu)(1 - \pi)$ is

⁴⁴While this section presents results assuming $(1 - \mu) > \alpha$, the results presented here would equally apply to the case in which $(1 - \mu) \leq \alpha$ but the risk-based capital framework is strengthened - i.e. banks are forced to cover all shocks with a probability smaller than $(1 - \alpha)$. This has the same effect as altering the assumption on the probabilities, namely increasing the capital charge on each asset, and thus it is equivalent

greater than, equal to, or less than α).

Now consider $\omega \in [0, 1)$. Take the case in which $(1 - \mu)(1 - \pi) \leq \alpha$. Banks will prefer to minimise capital and make the requirement bind if and only if profits under a binding capital requirement are higher than holding excess capital. To see that this is the case, first see that when the capital requirement binds, banks only survive state s_2 if and only if the risky asset returns its residual value in this state, i.e. $(1 - \lambda_3)$. This is true iff the payoff from the safer asset in state s_2 is not sufficient, i.e.:

$$\omega(1 - \lambda_1) \leq (1 - k(\omega))$$

Rearranging, since $k(\omega) = \omega\lambda_1 + (1 - \omega)\lambda_3$, we find

$$0 \leq (1 - \omega)(1 - \lambda_3)$$

which is true since $\omega \in [0, 1)$ and $\lambda_3 \in (0, 1)$. So if the capital requirement binds, and $(1 - \mu)(1 - \pi) \leq \alpha$, the bank will survive iff the risky asset pays off its residual value, $(1 - \lambda_3)$, in state s_2 .

Compare profits under a binding capital requirement to when the bank holds excess capital. The bank will prefer the capital requirement to bind if and only if profits under a binding capital requirement are higher than holding excess capital, namely:

$$\begin{aligned} & \mu[\omega R_1 + (1 - \omega)\pi R_2^h + (1 - \omega)(1 - \lambda_2)(1 - \pi)] + (1 - \mu)[\omega(1 - \lambda_1) + (1 - \omega)(1 - \lambda_3)]\pi \\ & \quad - (1 - k(\omega))[\mu + (1 - \mu)\pi] - \rho k(\omega) - c(\omega) \\ & \quad > \\ & \mu[\omega R_1 + (1 - \omega)\pi R_2^h + (1 - \omega)(1 - \lambda_2)(1 - \pi)] + (1 - \mu)[\omega(1 - \lambda_1) + (1 - \omega)(1 - \lambda_3)]\pi \\ & \quad - (1 - k_{ex}) - \rho k_{ex} - c(\omega) \end{aligned}$$

where $k_{ex} > k(\omega)$.

Rearranging, this is true if and only if:

$$\rho > \mu + (1 - \mu) \frac{[k_{ex} - (1 - \omega)[(1 - \pi) + \lambda_3\pi] - \omega\lambda_1(1 - \pi)]}{[k_{ex} - (1 - \omega)\lambda_3]}$$

which holds since $\rho > 1$ and $\mu + (1 - \mu) \frac{[k_{ex} - (1 - \omega)[(1 - \pi) + \lambda_3\pi] - \omega\lambda_1(1 - \pi)]}{[k_{ex} - (1 - \omega)\lambda_3]} < 1$ as $\mu \in [0, 1]$ and $\frac{[k_{ex} - (1 - \omega)[(1 - \pi) + \lambda_3\pi] - \omega\lambda_1(1 - \pi)]}{[k_{ex} - (1 - \omega)\lambda_3]} < 1$ since $[k_{ex} - (1 - \omega)[(1 - \pi) + \lambda_3\pi] - \omega\lambda_1(1 - \pi)] <$

$$[k_{ex} - (1 - \omega)\lambda_3 - \omega\lambda_1(1 - \pi)] < [k_{ex} - (1 - \omega)\lambda_3].$$

Now consider $(1 - \mu)(1 - \pi) > \alpha$. Since this case implies a zero probability of default, profits will be given by

$$\begin{aligned} \mu[\omega R_1 + (1 - \omega)\pi R_2^h + (1 - \omega)(1 - \lambda_2)(1 - \pi)] + (1 - \mu)[\omega(1 - \lambda_1) + (1 - \omega)(1 - \lambda_3)\pi] \\ - (1 - k) - \rho k - c(\omega) \end{aligned}$$

where $k \geq k(\omega)$. Since $\rho > 1$, for any ω , this is maximised by minimising k , i.e. the bank will hold as little capital as possible and thus the capital requirement will bind.

Therefore, as in section 3.3, banks always wish to minimise capital, and thus the requirement will bind. Since this is the case, the risk-based capital requirement will impact risk-taking decisions. Suppose the LR does not exist, then the FOC that determines optimal bank risk-taking is given by:

$$\begin{cases} \mu[\pi R_2^h + (1 - \lambda_2)(1 - \pi) - R_1] - (1 - \mu)\pi[\lambda_3 - \lambda_1] = -k'(\omega)[\rho - (\mu + (1 - \mu)\pi)] - c'(\omega) & \text{if } (1 - \mu)(1 - \pi) \leq \alpha \\ \mu[\pi R_2^h + (1 - \lambda_2)(1 - \pi) - R_1] - (1 - \mu)[(1 - \lambda_1) - (1 - \lambda_3)\pi] = -k'(\omega)[\rho - 1] - c'(\omega) & \text{if } (1 - \mu)(1 - \pi) > \alpha \end{cases}$$

As previously, the risk-based capital requirement disincentivises risk-taking, and this can be seen in the $k'(\omega)$ terms on the RHS of both FOCs. Let us compare these to the risk level chosen under an LR. As before, we know there are two cases that can occur. Firstly, the risk level can be set by the FOC. Secondly, if this level is not sufficient to satisfy the shareholders' participation constraint at the given LR, the risk level can be set by the shareholders' participation constraint itself. By definition, the risk level set by the shareholders' participation constraint must be greater than the risk level chosen under the FOC, otherwise the original level would have satisfied the participation constraint. Hence, it is sufficient to show that the risk level chosen under the FOC is larger than the risk level under a solely risk-based framework. Suppose ω is set by the FOC therefore. If $(1 - \mu)(1 - \pi) > \alpha$, the FOC is given by:

$$\mu[\pi R_2^h + (1 - \lambda_2)(1 - \pi) - R_1] - (1 - \mu)[(1 - \lambda_1) - (1 - \lambda_3)\pi] = -c'(\omega)$$

When the LR binds, all $k'(\omega)$ terms disappear, thereby reducing the marginal cost of risk-taking (the RHS). Clearly the ω that solves this FOC is smaller than the ω that solves the FOC under a solely risk-based capital requirement; the LHS

is identical, whereas the RHS is smaller. So risk is higher under a binding LR.

Now consider the case in which $(1 - \mu)(1 - \pi) \leq \alpha$. In this case, as in our baseline scenario, the optimal ω will depend on the exact level of the LR (since this increases banks' loss absorbency), and whether given this LR, the bank can survive additional shocks in state s_2 . There are two possibilities: (1) the bank can survive state s_2 , but if and only if both assets pay off their residual value, in which case it must be that $k_{lev} < 1 - \omega(1 - \lambda_1)$; or (2) the bank can survive all shocks in state s_2 even if only the safer asset pays off its residual value $(1 - \lambda_1)$, in which case it must be that $k_{lev} \geq 1 - \omega(1 - \lambda_1)$.

For each of these possibilities, we can solve for the optimal ω that would prevail. For lower LR levels in which to survive state s_2 the bank requires both assets to pay off their residual value, the optimal risk will be characterised by:

$$\mu[\pi R_2^h + (1 - \lambda_2)(1 - \pi) - R_1] - (1 - \mu)\pi[\lambda_3 - \lambda_1] = -c'(\omega_0^*)$$

At higher LR levels, in which the bank can survive all shocks in state s_2 regardless, the optimal risk will be characterised by:⁴⁵

$$\mu[\pi R_2^h + (1 - \lambda_2)(1 - \pi) - R_1] - (1 - \mu)[(1 - \lambda_1) - (1 - \lambda_3)\pi] = -c'(\omega_0^{**})$$

Comparing these two cases, it is immediately clear that risk is lower in the second case ($\omega_0^* < \omega_0^{**}$); this is because of the skin-in-the-game effect we discussed previously. The first case always applies when $k_{lev} < \check{k} \equiv \lambda_1 + (1 - \lambda_1) \frac{\mu[\pi R_2^h + (1 - \lambda_2)(1 - \pi) - R_1] - (1 - \mu)\pi(\lambda_3 - \lambda_1) - (1 - \mu)(1 - \pi)(1 - \lambda_1)}{c}$. To see this, see that $\check{k} \in [\lambda_1, 1]$ is the point at which when the bank chooses ω_0^{**} , the bank breaks even in state s_2 when only the safer asset pays off. This means that for all $k_{lev} < \check{k}$, if the bank chooses ω_0^{**} , bankruptcy will occur in state s_2 if the risky asset does not pay off its residual value. But then this cannot be optimal. Whenever $k_{lev} < \check{k}$ therefore, ω_0^* is optimal.⁴⁶

Looking at the equation that defines ω_0^* , and comparing this to the FOC under a solely risk-based capital requirement, it is clear that whenever $k_{lev} < \check{k}$, risk is

⁴⁵As discussed previously in the proof to proposition 2, since the profit function is a negative quadratic in ω , it is relatively simple to show that the bank's optimal ω is always characterised by these two conditions. For any given k_{lev} , banks will never wish to choose an ω not characterised by the FOC, but which allows them to survive an additional shock.

⁴⁶ $\omega_0^* < \omega_0^{**}$, so it is evident that for all $k_{lev} < \check{k}$, if the bank chooses ω_0^{**} , bankruptcy will also occur in state s_2 if the risky asset does not pay off its residual value.

larger under an LR - the LHS of the equation is identical, yet the RHS is smaller since all terms relating to $k'(\omega)$ disappear. There is no skin-in-the-game effect here due to the discrete nature of the set-up. The skin-in-the-game effect appears when banks begin to survive additional shocks, and this only occurs at higher LR levels above \check{k} . As before, we can show that even when $k_{lev} \geq \check{k}$, risk can still be larger than under a solely risk-based capital requirement. Plugging in the functional forms and rearranging, risk under a binding LR will be larger than under the risk-based requirement if:

$$(\lambda_3 - \lambda_1) [\rho - [\mu + (1 - \mu)\pi]] > (1 - \mu)(1 - \pi)(1 - \lambda_1)$$

Rearranging, this becomes:

$$\rho > [\mu + (1 - \mu)\pi] + (1 - \mu)(1 - \pi) \frac{(1 - \lambda_1)}{(\lambda_3 - \lambda_1)}$$

So as before, banks will always take more risk under a binding LR if $k_{lev} < \check{k}$, or if $k_{lev} \geq \check{k}$, if ρ is sufficiently expensive (as defined above). Since we are in the case in which $(1 - \mu)(1 - \pi) \leq \alpha$, with $\alpha = 0.001$, ρ probably does not need to be very large to exceed this.

As in the baseline scenario, this does not necessarily imply that an LR is detrimental. In order to consider the consequences of imposing an LR, we again must consider this increase in risk-taking in comparison to loss absorbing capacity. We do so as before with respect to the effect on the probability of default, and the expected loss of deposit funds. Let us first consider the less realistic case in which $(1 - \mu)(1 - \pi) > \alpha$. This implied the capital charge on the safer asset was λ_1 and the capital charge on the risky asset was 1. This implies a zero probability of default and hence a zero expected loss of deposit funds. Imposing an LR will also yield a zero probability of default and a zero expected loss of deposit funds, hence it is weakly better. This case is unrealistic since it implies a zero probability of default under the risk-based framework, but illustrates even in this case that the LR does not worsen outcomes. It cannot as the risk-based framework still underlies the LR.

Consider the more realistic case now: $(1 - \mu)(1 - \pi) \leq \alpha$. Under the risk-based framework, the probability of default is $(1 - \mu)(1 - \pi)$ since it defaults only if the risky asset pays off 0 in state s_2 . We show that under an LR, the probability of default cannot be higher, and can be strictly lower. Suppose the bank takes the maximum possible risk before moving back into the risk-based framework, where this is defined when $k(\omega) = \omega_{\max}\lambda_1 + (1 - \omega_{\max})\lambda_3 = k_{lev}$, so $\omega_{\max} = \frac{k_{lev} - \lambda_3}{(\lambda_1 - \lambda_3)}$

If the bank takes this risk, the probability of default will be at least as low as under a solely risk-based framework iff it can survive when the risky asset pays off $(1 - \lambda_3)$ in state s_2 , i.e.:

$$\omega(1 - \lambda_1) + (1 - \omega)(1 - \lambda_3) \geq (1 - k_{lev})$$

Rearranging, this becomes:

$$\frac{k_{lev} - \lambda_3}{(\lambda_1 - \lambda_3)}(1 - \lambda_1) + (1 - \frac{k_{lev} - \lambda_3}{(\lambda_1 - \lambda_3)})(1 - \lambda_3) \geq (1 - k_{lev})$$

And:

$$(1 - k_{lev}) \geq (1 - k_{lev})$$

Both sides are equalised, so the bank will survive state s_2 when the risky asset pays off $(1 - \lambda_3)$, even if it takes the maximum risk.⁴⁷

Let's now consider when an LR leads to a strict decline in the probability of default. This will be true iff:

$$\omega(1 - \lambda_1) \geq (1 - k_{lev})$$

i.e.

$$k_{lev} \geq 1 - \omega_{lev}^*(1 - \lambda_1) \in (0, 1]$$

where $*$ denotes the optimal risk choice. So there can exist a region where the probability of default is strictly lower. Overall therefore, imposing an LR weakly decreases the probability of default.

Consider the expected loss of deposit funds now. Under the risk-based framework, the expected loss of deposit funds, which we denote as EL_{rw} will be:

$$EL_{rw} = (1 - \mu)(1 - \pi) [(1 - k(\omega)) - \omega_{rw}(1 - \lambda_1)]$$

Under an LR, where $k_{lev} > k(\omega)$, the expected loss of deposit funds will be:

$$max\{(1 - \mu)(1 - \pi) [(1 - k_{lev}) - \omega_{lev}(1 - \lambda_1)], 0\}$$

⁴⁷ ω_{max} as defined above only applies for $k_{lev} \leq \lambda_3$, nevertheless for levels above λ_3 , banks will still survive a λ_3 shock by definition that they hold more capital than λ_3 . If the bank chooses a higher risk level than ω_{max} , then the bank will simply move back into the risk-based framework, which by definition has a probability of default of $(1 - \mu)(1 - \pi)$ as before.

We start by showing that unlike distress probabilities, if the bank takes the maximal risk, denoted ω_{max} , the expected loss of deposit funds will be larger under an LR. To see this, first note that if banks take the maximum risk, it is not possible to survive all shocks - banks will only survive state s_2 if the risky asset pays off its residual value $(1 - \lambda_3)$. Thus the expected loss of deposit funds will be positive.

To always survive both states of the world, it must be that $\omega_{max}(1 - \lambda_1) > (1 - k_{lev})$. Taking the maximal risk implies $\omega_{lev} = \omega_{max} = \frac{k_{lev} - \lambda_3}{(\lambda_1 - \lambda_3)}$. Plugging this in and rearranging, the above expression simplifies to $k_{lev} < \lambda_1$, but this is a contradiction, it is not possible for $k_{lev} < \lambda_1$ since $k_{lev} \geq k(\omega_{rw}) = \omega_{rw}\lambda_1 + (1 - \omega_{rw})\lambda_3 \geq \lambda_1$. Therefore, if the bank takes the maximum risk, the bank can only survive state s_2 if the risky asset also pays off its residual value $(1 - \lambda_3)$ in this state. Hence, the expected loss of deposit funds will given by $(1 - \mu)(1 - \pi) [(1 - k_{lev}) - \omega_{lev}(1 - \lambda_1)] > 0$.

Given this, suppose the bank indeed takes the maximum possible risk level. Plugging this into $(1 - \mu)(1 - \pi) [(1 - k_{lev}) - \omega_{lev}(1 - \lambda_1)]$ and rearranging, we find that the expected loss of deposit funds will be lower under an LR iff:

$$k_{lev} > k(\omega) + \left[\omega_{rw} - \frac{k_{lev} - \lambda_3}{(\lambda_1 - \lambda_3)} \right] (1 - \lambda_1)$$

This simplifies to

$$k_{lev} [\lambda_3 - 1] > k(\omega) [\lambda_3 - 1]$$

but since $\lambda_3 < 1$, this is a contradiction as $k_{lev} > k(\omega)$. So if the bank takes the maximal risk, the expected loss of deposit funds will be larger under an LR.

This suggests that if the bank takes too much risk under an LR (i.e. approaches the maximal risk), the expected loss of deposit funds will be larger under an LR. As noted before, there are two cases which determine the bank's risk-taking. First, the bank's optimal risk choice can be determined by its FOC. Second, it is possible that this optimal risk-level is not sufficient to satisfy the shareholders' participation constraint and risk will be pinned down by the participation constraint. Let us take each case in turn.

Consider the first case in which the level of risk is pinned down by the FOC. Suppose the LR is set just above the risk-based capital requirement such that the expected loss of deposit funds is positive. Under this case, the expected loss of

deposit funds will be lower under an LR if:

$$(1 - \mu)(1 - \pi) [(1 - k_{lev}) - \omega_{lev} (1 - \lambda_1)] < (1 - \mu) (1 - \pi) [(1 - k(\omega)) - \omega_{rw} (1 - \lambda_1)]$$

Plugging in the optimal solution and rearranging, we find:

$$k_{lev} > k(\omega) + (\lambda_3 - \lambda_1) [\rho - [\mu + (1 - \mu)\pi]] (1 - \lambda_1)$$

k_{lev} can be set at any level greater than $k(\omega)$, so there exists a region just above $k(\omega)$ in which the expected loss of deposit funds is greater under an LR - there is risk-shifting but little loss absorption.

Consider all k_{lev} above this level then. From before, we know that as long as the bank can choose its optimal level of risk (i.e. that set by the FOC), ω_{lev}^* will either stay constant or increase in k_{lev} . Hence, if this is the case, as is clear from the expected loss of deposit funds function under an LR, as k increases, the expected loss of deposit funds under an LR will decrease and hence the expected loss of deposit funds will be strictly lower under an LR for all k_{lev} greater than this level.

However, this optimal risk level must be feasible, namely the solution must be less than the maximum possible risk level; so we must add an extra condition. As can be readily seen from the maximal possible risk level, $\omega_{\max} = \frac{\lambda_3 - k_{lev}}{(\lambda_3 - \lambda_1)}$, this function is decreasing in k_{lev} . At low k_{lev} the bank's interior solution may be larger than this maximal possible risk level, whereas at higher k_{lev} , the interior solution is possible. To be beneficial in terms of the expected loss of deposit funds therefore, we must impose that the solution be an interior one, i.e. $\omega_{lev} > \frac{\lambda_3 - k_{lev}}{\lambda_3 - \lambda_1}$ or $k_{lev} > \omega_{lev}^* \lambda_1 + (1 - \omega_{lev}^*) \lambda_3$ where ω_{lev}^* is the optimal risk choice.

Combining these two conditions, and denoting \underline{k}_1 the maximum of these conditions, we can conclude that when the optimal risk level is set by the FOC, the expected loss of deposit funds will be strictly lower under an LR if k_{lev} is set above \underline{k}_1 , i.e. $k_{lev} > \underline{k}_1 \equiv \max\{\omega_{lev}^* \lambda_1 + (1 - \omega_{lev}^*) \lambda_3, k(\omega_{rw}) + (\lambda_3 - \lambda_1) [\rho - [\mu + (1 - \mu)\pi]] (1 - \lambda_1)\}$

Let us now consider the second case in which the shareholders' participation constraint determines ω . As seen from proposition 3, when the shareholders' participation constraint determines risk-taking, risk is increasing in k_{lev} , so it may be that any benefit from an increase in k_{lev} is offset by increased holdings of the risky asset. We show that for $\rho \leq \hat{\rho}$ (defined below), ω_{lev} will not decline fast enough to lead to a detriment, yet for $\rho > \hat{\rho}$, the increase in risk-taking can outweigh increased loss absorption at high k .

To be detrimental, it must be that

$$(1 - \mu)(1 - \pi) [(1 - k_{lev}) - \omega_{lev} (1 - \lambda_1)] > EL_{rw}$$

$$\Leftrightarrow (1 - \lambda_1)(\omega_{rw} - \omega_{lev}) > (k_{lev} - k(\omega_{rw}))$$

where ω_{lev} is set by the shareholders' participation constraint, i.e.

$$\omega_{lev} = \frac{[\mu\pi R_2^h + \mu(1 - \pi)(1 - \lambda_2) + (1 - \mu)\pi(1 - \lambda_3)] - (\rho - [\mu + (1 - \mu)\pi])k - [\mu + (1 - \mu)\pi]}{[\mu\pi R_2^h + \mu(1 - \pi)(1 - \lambda_2) + (1 - \mu)\pi(1 - \lambda_3) - [\mu R_1 + (1 - \mu)\pi(1 - \lambda_1)]]}$$

Define $E_{c'}(risky) = [\mu\pi R_2^h + \mu(1 - \pi)(1 - \lambda_2) + (1 - \mu)\pi(1 - \lambda_3)]$ and $E_{c'}(safer) = [\mu R_1 + (1 - \mu)\pi(1 - \lambda_1)]$, we can rewrite this condition as:

$$k_{lev} \left[1 - \frac{(1 - \lambda_1)(\rho - (\mu + (1 - \mu)\pi))}{E_{c'}(risky) - E_{c'}(safer)} \right] < k(\omega_{rw}) \left[1 - \frac{(1 - \lambda_1)(\rho - (\mu + (1 - \mu)\pi))}{E_{c'}(risky) - E_{c'}(safer)} \right]$$

$$+ (1 - \lambda_1) \frac{[\omega_{rw}[E_{c'}(risky) - E_{c'}(safer)] - [E_{c'}(risky) - (\mu + (1 - \mu)\pi)] + (\rho - (\mu + (1 - \mu)\pi)(\omega_{rw}\lambda_1 + (1 - \omega_{rw})\lambda_3))}{E_{c'}(risky) - E_{c'}(safer)}$$

The second term on the RHS is negative, which can be seen as follows. By the shareholders' participation constraint:

$$\omega_{rw}E_{c'}(safer) + (1 - \omega_{rw})E_{c'}(risky) - (1 - k(\omega_{rw}))(\mu + (1 - \mu)\pi) \geq \rho k(\omega_{rw})$$

$$\Leftrightarrow E_{c'}(risky) - (\mu + (1 - \mu)\pi) - (\rho - (\mu + (1 - \mu)\pi))(\omega_{rw}\lambda_1 + (1 - \omega_{rw})\lambda_3) \geq \omega_{rw}[E_{c'}(risky) - E_{c'}(safer)]$$

So the second term on the RHS of the inequality must be negative. Since this is the case, we can immediately state that if $\rho \leq (\mu + (1 - \mu)\pi) + E_{c'}(risky)/(1 - \lambda_1) - E_{c'}(safer)/(1 - \lambda_1)$, as $k_{lev} \geq k(\omega)$, this inequality will never hold. Whereas, if $\rho > (\mu + (1 - \mu)\pi) + E_{c'}(risky)/(1 - \lambda_1) - E_{c'}(safer)/(1 - \lambda_1)$, we can simplify the expression to:

$$k_{lev} > \bar{k}_1 \equiv k(\omega_{rw})$$

$$+ (1 - \lambda_1) \frac{\omega[E_{c'}(risky) - E_{c'}(safer)] - [E_{c'}(risky) - (\mu + (1 - \mu)\pi)] + (\rho - (\mu + (1 - \mu)\pi)(\omega\lambda_1 + (1 - \omega)\lambda_3))}{[E_{c'}(risky) - E_{c'}(safer)] - (1 - \lambda_1)(\rho - (\mu + (1 - \mu)\pi))}$$

So there may exist a certain \bar{k}_1 above which the expected loss of deposit funds can be larger under an LR.

For this to be possible however, it must be that $\bar{k}_1 < 1$, since $k_{lev} \leq 1$. This is only true if:

$$\rho > \tilde{\rho} \equiv \frac{[E_{c'}(risky) - E_{c'}(safer)][E_{c'}(risky) - E_{c'}(safer) + (\mu + (1 - \mu)\pi)(\lambda_3 - \lambda_1)](1 - \lambda_3) + c(1 - \lambda_1)(\mu + (1 - \mu)\pi)}{c(1 - \lambda_1) + [E_{c'}(risky) - E_{c'}(safer)](1 - \lambda_3)(\lambda_3 - \lambda_1)}$$

This may or may not be larger than $(\mu + (1 - \mu)\pi) + \frac{[E_{c'}(risky) - E_{c'}(safer)]}{(1 - \lambda_1)}$, nevertheless we can immediately conclude that for any $\rho \leq \hat{\rho} \equiv \max\{(\mu + (1 - \mu)\pi) + \frac{[E_{c'}(risky) - E_{c'}(safer)]}{(1 - \lambda_1)}, \tilde{\rho}\}$, the increase in risk-taking will not be sufficient to outweigh the increased loss-absorbing capacity. Combining this result with the result when the shareholders' participation constraint does not determine ω , and we can conclude that for all $k_{lev} > \underline{k}_1$, if $\rho \leq \hat{\rho}$, the expected loss of deposit funds will be strictly lower under an LR.

Let us now show that the upper bound \overline{k}_1 is strictly greater than the lower bound \underline{k}_1 . The lower bound level on the optimal risk level was first defined at the point where $k_{lev} = k(\omega_{rw}^*) + \omega_{rw}^*(1 - \lambda_1) - \omega_{lev}^*(1 - \lambda_1)$ where $*$ denotes optimal levels. The upper bound level was defined at the point where $k_{lev} = k(\omega_{rw}^*) + \omega_{rw}^*(1 - \lambda_1) - \omega_{lev}^{pc}(1 - \lambda_1)$ where pc denotes the level determined by the shareholders' participation constraint. Since $\omega_{lev}^{pc} < \omega_{lev}^*$, it must be that the upper bound is strictly greater. The upper bound is also larger than the level required for an interior solution. We can see this by comparing the two conditions. The upper bound will be larger iff $\omega_{lev}^*\lambda_1 + (1 - \omega_{lev}^*)\lambda_3 < \omega_{rw}^*\lambda_1 + (1 - \omega_{rw}^*)\lambda_3 + (1 - \lambda_1)(\omega_{rw}^* - \omega_{lev}^{pc})$. Rearranging, we find: $(1 - \lambda_1)(\omega_{rw}^* - \omega_{lev}^{pc}) - (\omega_{rw}^* - \omega_{lev}^*)(\lambda_3 - \lambda_1) > 0$, which is true since $\omega_{lev}^{pc} < \omega_{lev}^*$ and $\lambda_3 < 1$.

We can conclude therefore that for all ρ , if $k_{lev} \in (\underline{k}_1, \overline{k}_1)$ the expected loss of deposit funds will be strictly lower under an LR.

Lastly, suppose $\rho > \hat{\rho}$, then $\overline{k}_1 < 1$. There could therefore potentially exist a region above \overline{k}_1 in which the expected loss of deposit funds is greater under an LR, i.e. the increase in loss absorption is outweighed by the increase in risk-taking. For this to be possible however, it must be that the expected loss of deposit funds does not fall to zero in this region. We show numerically that this is possible.⁴⁸ First, note that this region implies that $\rho > \hat{\rho} > E_{c'}(risky)$, so as before, for this case to exist, it must be that the bank is targeting very high ROEs (i.e. greater than the expected return on the risky asset). This can be seen as follows. For the expected loss of deposit funds to be positive, it must be that $(1 - k_{lev}) > \omega_{lev}^{pc}(1 - \lambda_1)$. Plugging

⁴⁸It is not possible to analytically prove a general statement in this region, but we can illustrate numerically that when $\rho > \hat{\rho}$, the expected loss of deposit funds can be higher under an LR when k_{lev} is set above \overline{k}_1 .

in ω_{lev}^{pc} and rearranging, we find that if $\rho > \hat{\rho}$, for this to hold it must be that:

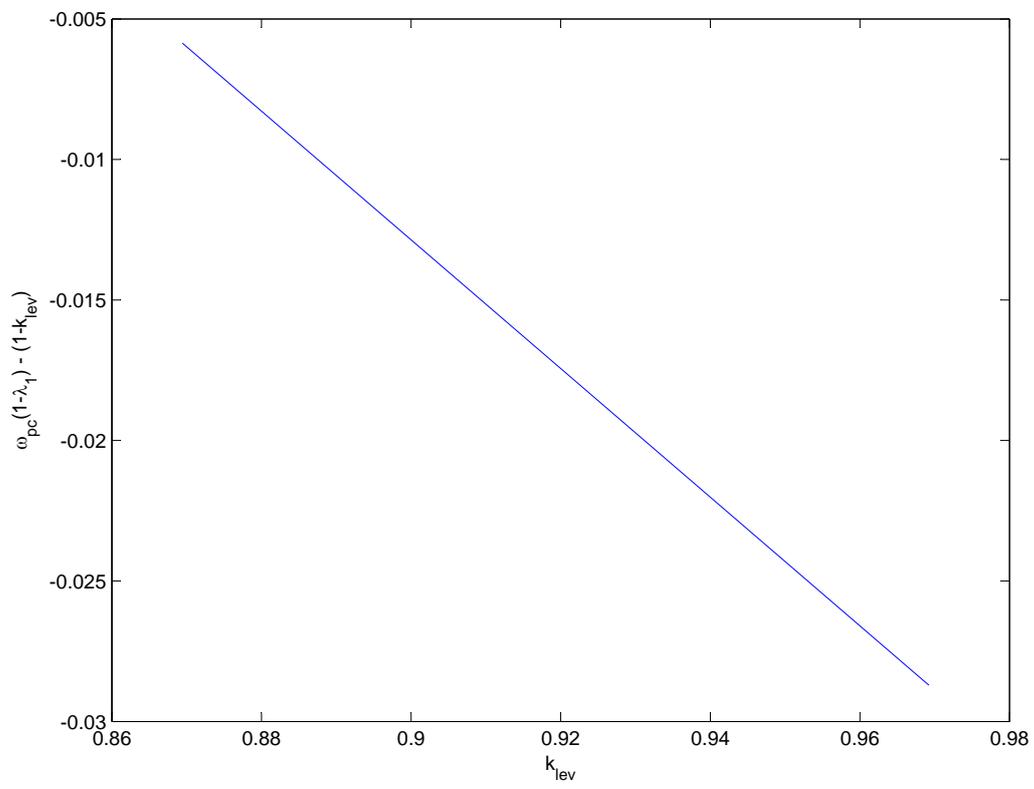
$$k_{lev} > \frac{[E_{c'}(risky) - (\mu + (1 - \mu)\pi)](1 - \lambda_1) - [E_{c'}(risky) - E_{c'}(safer)]}{[\rho - (\mu + (1 - \mu)\pi)](1 - \lambda_1) - [E_{c'}(risky) - E_{c'}(safer)]}$$

which since $k_{lev} \leq 1$, is only possible if $\rho > E_{c'}(risky)$. Since $\rho > E_{c'}(risky)$, there will exist an upper bound on the LR below 1 above which banks can no longer satisfy their shareholders' participation constraint. Therefore, since $\bar{k}_1 < 1$, it must be that risk-shifting increases at a faster rate than the benefit from loss absorption until the bank hits the corner solution of $\omega = 0$ at which point the LR cannot be set any higher since the bank would not be able to raise further equity without violating the shareholders' participation constraint. Denote this point k_{max} , which is defined at $k_{max} \equiv \frac{E_{c'}(risky) - (\mu + (1 - \mu)\pi)}{\rho - (\mu + (1 - \mu)\pi)}$. When $\rho > \hat{\rho}$, it can be that for $k > \bar{k}_1$ until k_{max} at which point the LR cannot be raised any further, the expected loss of deposit funds is larger than under a solely risk-based capital requirement. To see that this is possible, suppose the parameter values are such that $\mu = 0.99$, $\pi = 0.9$, $R_1 = 1.02$, $R_h^2 = 1.2$, $c = 9^{49}$, $\lambda_1 = 0.02$, $\lambda_2 = 0.1$, $\lambda_3 = 0.8$. This gives $\hat{\rho} = 1.1521$. Suppose $\rho = 1.155$, this gives $\bar{k}_1 = 0.8694$ and $k_{max} = 0.9692$. The expected loss of deposit funds is positive throughout this region as can be seen by plotting $\omega_{pc}(1 - \lambda_1) - (1 - k_{lev})$ for all $k \in [\bar{k}_1, k_{max}]$. This can be seen in Figure 7 which is strictly decreasing in k_{lev} . At \bar{k}_1 , this is equal to -0.0059, while at k_{max} , this is equal to -0.0287. Thus, there can exist a region where the expected loss of deposit funds is larger under an LR.

These are the same results as obtained in the main text, hence the results are robust to the alternative assumption that $(1 - \mu) > \alpha$ or strengthening the risk-based capital framework.

⁴⁹ c is set to 9 following Dell'Ariccia et al. (2014).

Figure 7



Note: The chart plots $\omega_{pc}(1 - \lambda_1) - (1 - k_{lev})$ for all $k \in [\bar{k}_1, k_{max}]$.

Appendix B: Mathematical proofs

Proof of Lemma 1

We show that for any ω , a bank will prefer to hold the minimum capital requirement. First, see that for any ω , if the capital requirement is binding, the bank will not survive any state s_2 shock, it will always enter bankruptcy in state s_2 .

In state s_2 , the safer asset returns $(1 - \lambda_1)$, while the risky asset returns a maximum of $(1 - \lambda_3)$. Suppose the bank holds the minimum capital requirement, i.e. $k(\omega) = (1 - \omega)\lambda_2$

To survive a shock in state s_2 , it must be that:

$$\omega(1 - \lambda_1) + (1 - \omega)(1 - \lambda_3) \geq (1 - k_{rw})$$

otherwise the return on the two assets is not sufficient to repay depositors, even when the risky asset pays off its highest state s_2 return. Imposing the assumption that banks hold the minimum capital requirement and rearranging, this becomes:

$$\omega\lambda_1 + (1 - \omega)(\lambda_3 - \lambda_2) \leq 0$$

which is a contradiction, since $\lambda_3 > \lambda_2$. So, for any $\omega \in [0, 1]$, this condition cannot hold. Hence if banks hold the minimum capital requirement, they can never survive state s_2 .

Given this is the case, we show that for any ω , banks will not find it optimal to hold excess capital.

The profit from holding the minimum capital requirement is:

$$\mu[\omega R_1 + (1 - \omega)\pi R_2^h + (1 - \omega)(1 - \lambda_2)(1 - \pi)] - (1 - k(\omega))\mu - \rho k(\omega) - c(\omega)$$

If the bank decides to hold excess capital, where k_{ex} denotes a capital level above the minimum, then profit will be either:

$$\begin{aligned} &\mu[\omega R_1 + (1 - \omega)\pi R_2^h + (1 - \omega)(1 - \lambda_2)(1 - \pi)] + (1 - \mu)[\omega(1 - \lambda_1) + (1 - \omega)(1 - \lambda_3)]\pi \\ &\quad - (1 - k_{ex})[\mu + (1 - \mu)\pi] - \rho k_{ex} - c(\omega) \end{aligned}$$

if the bank holds only enough excess capital to survive when the risky asset returns

$(1 - \lambda_3)$ in state s_2 , or:

$$\mu[\omega R_1 + (1 - \omega)\pi R_2^h + (1 - \omega)(1 - \lambda_2)(1 - \pi)] + (1 - \mu)[\omega(1 - \lambda_1) + (1 - \omega)(1 - \lambda_3)\pi] - (1 - k_{ex}) - \rho k_{ex} - c(\omega)$$

if the bank holds enough excess capital to survive all shocks.

We show that holding the minimum capital requirement is preferred to both these alternatives, namely:

$$\begin{aligned} & \mu[\omega R_1 + (1 - \omega)\pi R_2^h + (1 - \omega)(1 - \lambda_2)(1 - \pi)] - (1 - k(\omega))\mu - \rho k(\omega) - c(\omega) > \\ & \mu[\omega R_1 + (1 - \omega)\pi R_2^h + (1 - \omega)(1 - \lambda_2)(1 - \pi)] + (1 - \mu)[\omega(1 - \lambda_1) + (1 - \omega)(1 - \lambda_3)\pi] \\ & \quad - (1 - k_{ex}) [\mu + (1 - \mu)\pi] - \rho k_{ex} - c(\omega) \end{aligned}$$

and

$$\begin{aligned} & \mu[\omega R_1 + (1 - \omega)\pi R_2^h + (1 - \omega)(1 - \lambda_2)(1 - \pi)] - (1 - k(\omega))\mu - \rho k(\omega) - c(\omega) > \\ & \mu[\omega R_1 + (1 - \omega)\pi R_2^h + (1 - \omega)(1 - \lambda_2)(1 - \pi)] + (1 - \mu)[\omega(1 - \lambda_1) + (1 - \omega)(1 - \lambda_3)\pi] - (1 - k_{ex}) - \rho k_{ex} - c(\omega) \end{aligned}$$

Let us proceed with the first condition. Plugging in the minimum capital requirement and simplifying, we find that this is true if and only if:

$$\rho > \mu + (1 - \mu)\pi \frac{[k_{ex} - \omega\lambda_1 - (1 - \omega)\lambda_3]}{[k_{ex} - (1 - \omega)\lambda_2]}$$

which is true by definition, since $\rho > 1$, and $\mu + (1 - \mu)\pi \frac{[k_{ex} - \omega\lambda_1 - (1 - \omega)\lambda_3]}{[k_{ex} - (1 - \omega)\lambda_2]} < 1$, since $[k_{ex} - (1 - \omega)\lambda_2] > [k_{ex} - \omega\lambda_1 - (1 - \omega)\lambda_3]$ for any ω .

Performing the same exercise with the second condition, we find a similar condition stating that banks will prefer to hold the minimum capital requirement if and only if:

$$\rho > \mu + (1 - \mu) \frac{[k_{ex} - \omega\lambda_1 - (1 - \omega)[\lambda_3\pi + (1 - \pi)]]}{[k_{ex} - (1 - \omega)\lambda_2]}$$

which again is true by definition since $\rho > 1$, and $\mu + (1 - \mu) \frac{[k_{ex} - \omega\lambda_1 - (1 - \omega)[\lambda_3\pi + (1 - \pi)]]}{[k_{ex} - (1 - \omega)\lambda_2]} < 1$, since $[k_{ex} - \omega\lambda_1 - (1 - \omega)[\lambda_3\pi + (1 - \pi)]] < [k_{ex} - \omega\lambda_1 - (1 - \omega)\lambda_3] < [k_{ex} - (1 - \omega)\lambda_2]$.

Proof of Proposition 2

The proof proceeds in two stages. First, we show the optimal solution under a risk-based capital requirement. Second, we show that under a binding LR requirement, a bank's chosen risk level will always be higher than this when either the LR is set below some \hat{k} (defined below) or for a sufficiently large ρ .

We know from lemma 1 that the bank will never survive state s_2 under the risk-based framework. So, under a solely risk-based capital requirement, the bank will choose an ω that maximises:

$$\mu[\omega R_1 + (1 - \omega)\pi R_2^h + (1 - \omega)(1 - \lambda_2)(1 - \pi)] - (1 - k(\omega))\mu - \rho k(\omega) - c(\omega)$$

The optimal choice can be written as:

$$(1 - \omega_{rw}^*) = \frac{\mu [\pi R_2^h + (1 - \lambda_2)(1 - \pi) - R_1] - \lambda_2 (\rho - \mu)}{c}$$

With an LR, in contrast, the bank can potentially survive state s_2 shocks. Whether it does will depend on the exact level of the LR, and the amount of risk the bank chooses to take. As a result, the bank's optimal risk level will depend on whether the bank can only survive state s_1 , or whether it can also survive any of the shocks in state s_2 .

There are three possibilities: (1) the bank can only survive state s_1 , in which case the following must be true: $k_{lev} < \omega\lambda_1 + (1 - \omega)\lambda_3$; (2) the bank can also survive state s_2 , but only if both the safer and risky asset payoff their residual value - i.e. $\omega(1 - \lambda_1) + (1 - \omega)(1 - \lambda_3) \geq (1 - k_{lev})$, but $\omega(1 - \lambda_1) < (1 - k_{lev})$, or more succinctly, $k_{lev} \in [\omega\lambda_1 + (1 - \omega)\lambda_3, 1 - \omega(1 - \lambda_1)]$; or (3) the bank can survive all shocks in state s_2 - i.e. the bank can survive even if only the safer asset pays off its residual value $(1 - \lambda_1)$ in state s_2 , in which case the following must be true: $k_{lev} \geq 1 - \omega(1 - \lambda_1)$.

For each of these possibilities, we can solve for the optimal ω , where we use subscripts to refer to each possibility. First, if the bank can only survive state s_1 , its optimal risk will be characterised by:

$$(1 - \omega_1) = \frac{\mu[\pi R_2^h + (1 - \lambda_2)(1 - \pi) - R_1]}{c}$$

Second, if the bank can in addition survive state s_2 , but only if both assets pay

off their residual value, its optimal risk will be characterised by:

$$(1 - \omega_2) = \frac{\mu[\pi R_2^h + (1 - \lambda_2)(1 - \pi) - R_1] - (1 - \mu)\pi(\lambda_3 - \lambda_1)}{c}$$

Third, if the bank can survive all shocks in state s_2 , its optimal risk will be characterised by:

$$(1 - \omega_3) = \frac{\mu[\pi R_2^h + (1 - \lambda_2)(1 - \pi) - R_1] - (1 - \mu)\pi(\lambda_3 - \lambda_1) - (1 - \mu)(1 - \pi)(1 - \lambda_1)}{c}$$

Note that it is immediately clear that $(1 - \omega_1) > (1 - \omega_{rw}^*)$. Thus if this characterises the bank's chosen risk level, risk is clearly larger under an LR. Also note that $(1 - \omega_1) > (1 - \omega_2)$, since $\lambda_3 > \lambda_1$, and $(1 - \omega_2) > (1 - \omega_3)$.

We show that there exists a \hat{k} below which the optimal solution will be characterised by ω_1 , and in which banks only survive state s_1 . If this is the case, then clearly risk will be larger under an LR for all $k_{lev} < \hat{k}$.

Define \underline{k}_3 as the level of capital at which if the level of risk chosen by the bank is given by ω_3 , the bank will breakeven in state s_2 when only the safer asset pays off - i.e. $\underline{k}_3 \equiv 1 - \omega_3(1 - \lambda_1) \in [\lambda_1, 1]$.

Define \underline{k}_2 as the level of capital at which if the level of risk chosen by the bank is given by ω_2 , the bank will breakeven in state s_2 when both the safer and risky asset pay off their residual value - i.e. $\underline{k}_2 \equiv 1 - \omega_2(1 - \lambda_1) - (1 - \omega_2)(1 - \lambda_3) \in [\lambda_1, \lambda_3]$.

This means that if $k_{lev} < \underline{k}_3$, ω_3 cannot be the optimal solution. It is not optimal when bankruptcy occurs in state s_2 . Equally, if $k_{lev} < \underline{k}_2$, ω_2 cannot be the optimal solution. It is not optimal when bankruptcy always occurs in state s_2 , which it will if $k_{lev} < \underline{k}_2$ and the bank holds ω_2 .

Hence, if k_{lev} is less than both \underline{k}_2 and \underline{k}_3 , neither ω_2 nor ω_3 can be optimal solutions. If the bank chooses ω_3 , the bank can fail in state s_2 , but this cannot be optimal. If the bank chooses ω_2 , the bank will only survive state s_1 , but this cannot be optimal. We know from above that the optimal solution will be ω_1 , and $\omega_1 < \omega_{rw}^*$.⁵⁰

⁵⁰There is also the possibility that for the given k_{lev} , instead of choosing ω_2 or ω_3 , the bank will wish to continue reducing risk enough (i.e. shifting a sufficient amount of its portfolio into the safer asset) that it begins to survive shocks in state s_2 . For example, for a given k_{lev} , if choosing ω_2 leads only to surviving state s_1 , the bank could reduce risk further to a non-optimal level until the point at which it begins to breakeven in state s_2 when both assets pay off their residual value. This will be possible if $k_{lev} \geq \lambda_1$ since if $k_{lev} < \lambda_1$, the bank can never survive a shock in state s_2 even if $\omega = 1$. Despite this possibility, since the maximum function is a negative quadratic in ω ,

Summarising the above, we can state that if $k_{lev} < \hat{k} \equiv \min\{\underline{k}_2, \underline{k}_3\} \in [\lambda_1, \lambda_3]$, the optimal risk level will be given by ω_1 , and $(1 - \omega_1) > (1 - \omega_{rw})$ so risk is larger.⁵¹ This proves the first statement.⁵²

Suppose now that $k_{lev} \geq \hat{k}$, then it is possible that the bank may optimally choose not to hold ω_1 , but ω_2 or ω_3 .⁵³ For example, if $k_{lev} \geq \lambda_3$, the bank will always survive state s_2 if the risky asset pays off its residual value. As a result, ω_1 cannot be an optimal solution. We show that even if the bank chooses the lowest risk level, i.e. ω_3 , if ρ is sufficiently large, the risk choice under an LR is still larger than the risk-based choice.

Comparing the risk-based choice to ω_3 , and rearranging, we find that risk will always be larger under an LR if $\rho > \mu + (1 - \mu) \frac{[(1 - \lambda_1) - \pi(1 - \lambda_3)]}{\lambda_2}$. This will always be true if $(1 - \lambda_1) - \pi(1 - \lambda_3) < \lambda_2$ since $\rho \geq 1$. This proves the second statement.

Proof of Proposition 3

The proof proceeds in two steps. First, we look at failure probabilities, then we consider the expected loss of deposit funds.

and the required ω for the bank to be able to survive any additional shocks is by definition not the optimal risk level in that region (i.e. it is not ω_2 nor ω_3), it is relatively straightforward to show that the bank will not find it optimal to choose this risk level.

⁵¹To see that the bank will only ever survive state s_1 when it chooses ω_1 and $k_{lev} < \hat{k}$, such that ω_1 is optimal, first define \underline{k}_1 as the level of capital at which given ω_1 , the bank will breakeven in state s_2 when both the safer and risky asset pay off their residual value: i.e. $\underline{k}_1 \equiv 1 - \omega_1(1 - \lambda_1) - (1 - \omega_1)(1 - \lambda_3) = \omega_1\lambda_1 + (1 - \omega_1)\lambda_3 \in [\lambda_1, \lambda_3]$. From above, we know that ω_1 can only be an optimal solution if $k_{lev} < \underline{k}_1$, as if $k_{lev} > \underline{k}_1$, the bank will survive shocks in state s_2 . \hat{k} is the smaller of \underline{k}_2 or \underline{k}_3 (which is smaller will depend on the exact parameter values). Suppose $\underline{k}_2 < \underline{k}_3$, then $\hat{k} = \underline{k}_2$. Comparing \underline{k}_1 to \underline{k}_2 , it is clear that $\underline{k}_1 > \underline{k}_2$, thus it must be that for all $k_{lev} < \hat{k}$, if the bank chooses ω_1 , it will only ever survive state s_1 . Now suppose $\underline{k}_2 > \underline{k}_3$, then $\hat{k} = \underline{k}_3$. Since $\underline{k}_1 > \underline{k}_2$ and $\underline{k}_2 > \underline{k}_3$, it must be that $\underline{k}_1 > \underline{k}_3$ and hence as before for all $k_{lev} < \hat{k}$, if the bank chooses ω_1 , it will only ever survive state s_1 .

⁵²As discussed in the text, for large ρ , there can exist a point at which the shareholders' participation constraint forces banks to take further risk. This will be the point at which the optimal risk set by the FOC is too low, since it does not satisfy the shareholders' participation constraint. By definition this risk level is larger than the optimal risk the bank would otherwise choose. As a result, it is sufficient to show that if the optimal level of risk is higher than the risk-based choice, then this level of risk will also be. Throughout the proof therefore, we have supposed that ρ is low enough that even at $k_{lev} = 1$, banks can choose their optimal risk level, and they will still satisfy the shareholders' participation constraint. This puts a lower bound on the bank's chosen level of risk. Thus if risk is larger at this lower bound, it is necessarily larger when the shareholders' participation constraint forces greater risk-taking.

⁵³Again, since the maximum function is a negative quadratic in ω , for any given k_{lev} , it is relatively straightforward to show that it is never optimal to keep increasing the weight on the safer asset such that the bank survives an additional shock, but by choosing a non-optimal ω for the region. The optimal solution will always be given by one of the ω 's above.

Under the risk-based framework, by definition, the probability of default is $(1 - \mu)$. When the LR binds, the bank will have more capital, but at the same time, it will take more risk. This level of risk however is capped at the maximum possible level of risk before the bank moves back into the risk-based framework. We show that even if the bank takes this level of risk, default probabilities will not rise, and for some LR levels, default probabilities will decline relative to the risk-based probability.⁵⁴

The maximum risk level occurs at the point where the risk-based capital requirement equals the LR, i.e. $k(\omega) = (1 - \omega_{\max})\lambda_2 = k_{lev}$. In other words, the maximum the bank can increase risk to is $(1 - \omega_{\max}) = \frac{k_{lev}}{\lambda_2}$. Suppose the bank increases risk to the maximum, so $(1 - \omega_{lev}) = (1 - \omega_{\max}) = \frac{k_{lev}}{\lambda_2}$.

We show that even at this level, the bank will survive the shock in state s_1 and thus its probability of default will not be less than $(1 - \mu)$. This is true if and only if $\omega R_1 + (1 - \omega)(1 - \lambda_2) \geq (1 - k_{lev})$

Plugging the maximum risk level into the above:

$$\left(1 - \frac{k_{lev}}{\lambda_2}\right) R_1 + \frac{k_{lev}}{\lambda_2}(1 - \lambda_2) \geq (1 - k_{lev})$$

And rearranging:

$$(R_1 - 1) \left(1 - \frac{k_{lev}}{\lambda_2}\right) \geq 0$$

which is true for all $k_{lev} \leq \lambda_2$.

So, for all $k_{lev} \leq \lambda_2$, the bank can take the maximum risk and it will still survive the state s_1 shock. This is because, with the risk-based framework underlying the LR, it cannot be that the LR allows failure in this state, otherwise the risk-based capital requirement would have been higher. If $k_{lev} > \lambda_2$, the bank can still never enter bankruptcy in state s_1 . To see this, denote $k_{lev} = \lambda_2 + \varepsilon$ as any LR above λ_2 , where $\varepsilon \in [0, 1 - \lambda_2]$. For any $\omega \in [0, 1]$ and $\varepsilon \in [0, 1 - \lambda_2]$, $\omega R_1 + (1 - \omega)(1 - \lambda_2) > (1 - k_{lev}) = (1 - \lambda_2 - \varepsilon)$. So for any k_{lev} , the probability of default will not fall below $(1 - \mu)$.

We now show that the probability of default can be strictly lower under an LR. The probability of default will be strictly lower under an LR if the bank can survive

⁵⁴In addition, if the bank moves back into the risk-based framework by exceeding this level of risk, its probability of default would simply revert to $(1 - \mu)$, as the risk-based requirement kicks in again. Nevertheless, because the expected profit function is a negative quadratic in ω , it is relatively easy to show that banks will never wish to exceed the maximum possible level of risk and thereby move back into the risk-based framework (which would involve an increase in capital).

a shock in state s_2 . Suppose the parameters are such that the optimal solution lies below the maximum risk level discussed above. A bank will survive a λ_3 shock in state s_2 iff:

$$\omega(1 - \lambda_1) + (1 - \omega)(1 - \lambda_3) \geq (1 - k_{lev})$$

Plugging in the optimal ω , where $*$ denotes optimal values, and rearranging,

$$k_{lev} \geq 1 - \omega_{lev}^*(1 - \lambda_1) - (1 - \omega_{lev}^*)(1 - \lambda_3) \in [\lambda_1, \lambda_3]$$

So for k_{lev} greater than this, the probability of default can be strictly lower. This proves the first statement.

Now, consider the expected loss of deposit funds. Under a risk-based capital framework, the expected loss of deposit funds will be:

$$EL_{rw} \equiv (1 - \mu) [(1 - k(\omega)) - \omega_{rw}(1 - \lambda_1) - (1 - \omega_{rw})(1 - \lambda_3) \pi]$$

Under an LR, the expected loss of deposit funds can be one of three possibilities. First,

$$(1 - \mu) [(1 - k_{lev}) - \omega_{lev}(1 - \lambda_1) - (1 - \omega_{lev})(1 - \lambda_3) \pi] \quad (3)$$

if at the level the LR is set and the risk banks take, banks can only survive state s_1 shocks. Second,

$$(1 - \pi)(1 - \mu)(1 - \pi) [(1 - k_{lev}) - \omega_{lev}(1 - \lambda_1)] \quad (4)$$

if at the level the LR is set and risk level taken, banks can survive all state s_1 shocks and also survive state s_2 with probability π (i.e. when the risky asset pays off its higher value in that state $(1 - \lambda_3)$). Third, it will be 0 if at the level the LR is set and chosen risk level, banks can survive all states of the world, since their probability of default will be zero.

We begin by showing that if the bank takes the maximum level of risk, the expected loss of deposit funds can be larger under an LR. First see that if the bank takes the maximum risk, where the maximum risk the bank can take is $(1 - \omega_{max}) = \frac{k_{lev}}{\lambda_2}$, it will never survive a state s_2 shock. Banks never survive state s_2 when they take the maximum risk iff:

$$\omega_{max}(1 - \lambda_1) + (1 - \omega_{max})(1 - \lambda_3) < (1 - k_{lev})$$

Plugging in what we know to be ω_{max} and rearranging:

$$k_{lev} [\lambda_2 + (1 - \lambda_3) - (1 - \lambda_1)] < \lambda_1$$

If $[\lambda_2 + (1 - \lambda_3) - (1 - \lambda_1)] < 0$, then this clearly holds since $\lambda_1 > 0$. Suppose $[\lambda_2 + (1 - \lambda_3) - (1 - \lambda_1)] > 0$, then the LHS is maximised at $k_{lev} = 1$. Imposing this, the expression simplifies to:

$$\lambda_2 < \lambda_3$$

which is true by definition. Given this holds for $k_{lev} = 1$, the maximum of the function, it must be true for all $k_{lev} < 1$. So, if banks take the maximal risk, they can only survive state s_1 .⁵⁵

Given this, let us now show that in the case where banks take the maximum risk, the expected loss of deposit funds can be larger under an LR. We prove by contradiction. Suppose this is not the case and the expected loss of deposit funds is lower under an LR when banks maximise their risk-taking, then it must be that:⁵⁶

$$[(1 - \lambda_1) - (1 - \lambda_3) \pi] \left(\omega_{rw} - 1 + \frac{k_{lev}}{\lambda_2} \right) < k_{lev} - k(\omega)$$

Rearranging,

$$[(1 - \lambda_1) - (1 - \lambda_3) \pi - \lambda_2] k_{lev} < k(\omega) [(1 - \lambda_1) - (1 - \lambda_3) \pi - \lambda_2]$$

If $[(1 - \lambda_1) - (1 - \lambda_3) \pi - \lambda_2] > 0$,

$$k_{lev} < k(\omega)$$

which is a contradiction. So if the bank takes the maximal risk, the expected loss of deposit funds can be larger under an LR.⁵⁷

Let us now prove the second statement of proposition 3. As discussed before, there are two cases which determine optimal risk-taking. First, the optimal risk

⁵⁵Since $\omega_{max}(1 - \lambda_1) + (1 - \omega_{max})(1 - \lambda_3) < (1 - k_{lev})$, then it must also be that $\omega_{max}(1 - \lambda_1) < (1 - k_{lev})$, which confirms that banks can never survive state s_2 if they take the maximum risk.

⁵⁶Obtained by rearranging the inequality that equation 3 is less than EL_{rw} , since when the bank takes maximum risk, as shown above, it can never survive state s_2 .

⁵⁷For the alternative assumption, i.e. $[(1 - \lambda_1) - (1 - \lambda_3) \pi - \lambda_2] < 0$, clearly the expected loss of deposit funds would be lower since $k_{lev} > k(\omega)$, hence banks could increase risk all the way to the maximum and the expected loss of deposit funds would still be lower under an LR. Since this is the case, we do not need to consider this alternative assumption; the LR would clearly yield a better outcome.

level can be pinned down by the FOC. Second, if this is insufficient to satisfy the shareholders' participation constraint, the shareholders' participation constraint can pin down the risk level. Let us take each case in turn. Suppose the LR is set just above the risk-based capital requirement and the FOC determines ω so that the bank can survive only state s_1 .⁵⁸ Then the expected loss of deposit funds will be lower under an LR if:

$$\begin{aligned} & (1 - \mu) [(1 - k_{lev}) - \omega_{lev} (1 - \lambda_1) - (1 - \omega_{lev}) (1 - \lambda_3) \pi] \\ & < (1 - \mu) [(1 - k(\omega)) - \omega_{rw} (1 - \lambda_1) - (1 - \omega_{rw}) (1 - \lambda_3) \pi] \end{aligned}$$

Plugging in the optimal values:

$$k_{lev} > k(\omega_{rw}) + [(1 - \lambda_1) - (1 - \lambda_3) \pi] \frac{\lambda_2 (\rho - \mu)}{c}$$

Since k_{lev} can be any value above $k(\omega_{rw})$, there exists a region just above $k(\omega_{rw})$ in which the expected loss of deposit funds can be higher. To be beneficial in terms of the expected loss of deposit funds therefore, the LR should be set above this level. Consider all k_{lev} above this level. Since the solution ω_{lev}^* set by the FOC is either constant or increasing in k_{lev} ,⁵⁹ for any k_{lev} above this level, so long as the solution is set by the FOC, the expected loss of deposit funds will decrease in k_{lev} . Hence if the expected loss of deposit funds is lower under an LR at this k_{lev} , it will be lower under an LR for all k_{lev} larger than this. This implies that as long as k_{lev} is greater than this defined level and risk is determined by the bank's FOC (not the shareholders' participation constraint), the expected loss of deposit funds must be lower under an LR. However, for this to be the case, the FOC must be feasible: namely the solution must be less than the maximum possible risk level. So we must impose this additional condition. For the expected loss of deposit funds to be lower under an LR therefore, we require an interior solution and for the LR to be set higher than the level above. This means $k_{lev} > \underline{k} \equiv \max\{(1 - \omega_{lev}^*) \lambda_2, k(\omega_{rw}^*) + [(1 - \lambda_1) - (1 - \lambda_3) \pi] \frac{\lambda_2 (\rho - \mu)}{c}\}$. Summarising then, if the risk level is set by the FOC, for all $k_{lev} > \underline{k}$, the expected loss of deposit funds will be strictly lower under an LR.

⁵⁸We show when the expected loss of deposit funds under an LR is lower under these conditions, since as we know from proposition 2, if the LR is set higher and the FOC is set such that banks can also survive some shocks in state s_2 , their optimal risk choice will be lower (i.e. they will hold more of the safer asset), and their probability of default will be lower. By definition therefore, their expected loss of deposit funds must be lower than when the bank has less capital and it chooses a higher weight on the risky asset (as we are assuming here). As a result, if the expected loss of deposit funds under an LR is lower under these conditions, it will also be lower under the alternative conditions.

⁵⁹From the proof to proposition 2, we know that $\omega_1 > \omega_2 > \omega_3$ as defined there.

Now consider the second case in which the shareholders' participation constraint determines ω . The shareholders' participation constraint will bite when:

$$\omega_{lev}^* E(safer|survival) + (1 - \omega_{lev}^*) E(risky|survival) - (1 - k) \Pr(survival) < \rho k$$

where $E(safer|survival)$ and $E(risky|survival)$ denote the conditional expected returns on the safer and risky asset respectively, and $\Pr(survival)$ denotes the probability of survival for the bank.

Rearranging, this implies that :

$$\omega = \frac{E(risky|survival) - \rho k - (1 - k) \Pr(survival)}{E(risky|survival) - E(safer|survival)}$$

We show that if $\rho \leq \hat{\rho}$ (defined below), even if the shareholders' participation constraint determines risk-taking, for all $k_{lev} > \underline{k}$, the expected loss of deposit funds will be lower.

As discussed before, due to the discrete nature of the problem, at different levels of the LR, the expected loss of deposit funds can jump. At first, banks may only survive state s_1 , but as the LR rises (e.g. if $k_{lev} \geq \lambda_3$), banks may then be able to survive state s_2 with probability π , etc. Let us therefore consider each case in turn.

Consider the first case/region in which banks can only survive state s_1 . To be detrimental, it must be that:

$$(1-\mu) \left[(1 - k_{lev}) - \frac{\mu(R_2^h \pi + (1-\pi)(1-\lambda_2)) - \rho k - (1-k)\mu}{\mu(R_2^h \pi + (1-\pi)(1-\lambda_2)) - \mu R_1} (1 - \lambda_1) - (1 - \frac{\mu(R_2^h \pi + (1-\pi)(1-\lambda_2)) - \rho k - (1-k)\mu}{\mu(R_2^h \pi + (1-\pi)(1-\lambda_2)) - \mu R_1}) (1 - \lambda_3) \pi \right] > (1 - \mu) [(1 - k(\omega)) - \omega_{rw} (1 - \lambda_1) - (1 - \omega_{rw}) (1 - \lambda_3) \pi] \quad (5)$$

where we have plugged in what we know to be ω_{lev} given the shareholders' participation constraint bites.

Rearranging, we find that if $\rho > \mu + \frac{\mu[R_2^h \pi + (1-\pi)(1-\lambda_2) - R_1]}{(1-\lambda_1) - (1-\lambda_3)\pi}$, we can write a condition on k_{lev} such that above a certain level, the expected loss of deposit funds is larger

under an LR. Namely, to be detrimental, it must be that:⁶⁰

$$k_{lev} > \tilde{k}_0 \equiv$$

$$\frac{[E_c(risky) - E_c(safer)]\{[EL_{rw}/(1 - \mu)] + (1 - \lambda_3)\pi - 1\} + [E_c(risky) - \mu][(1 - \lambda_1) - (1 - \lambda_3)\pi]}{(\rho - \mu)[(1 - \lambda_1) - (1 - \lambda_3)\pi] - [E_c(risky) - E_c(safer)]}$$

where for compactness $E_c(risky) \equiv \mu\pi R_2^h + \mu(1 - \pi)(1 - \lambda_2)$ and $E_c(safer) \equiv \mu R_1$.

Considering this expression in more detail. We know the denominator is positive by definition that $\rho > \mu + \frac{\mu[R_2^h\pi + (1 - \pi)(1 - \lambda_2) - R_1]}{(1 - \lambda_1) - (1 - \lambda_3)\pi}$. Equally, we can see that the numerator is positive. This can be seen as follows:

$$\begin{aligned} & [E_c(risky) - E_c(safer)] [EL_{rw}/(1 - \mu) + (1 - \lambda_3)\pi - 1] + [E_c(risky) - \mu][(1 - \lambda_1) - (1 - \lambda_3)\pi] \\ & > [E_c(risky) - E_c(safer)] [EL_{rw}/(1 - \mu) + (1 - \lambda_3)\pi - 1 + [(1 - \lambda_1) - (1 - \lambda_3)\pi]] \\ & = [E_c(risky) - E_c(safer)](1 - \omega_{rw})[(1 - \lambda_1) - \lambda_2 - \pi(1 - \lambda_3)] > 0 \end{aligned}$$

Thus \tilde{k}_0 is positive. But for the condition to ever bite, it must also be that \tilde{k}_0 is less than 1, since $k_{lev} \leq 1$. Rearranging, we find that this is true iff:

$$\rho > E_c(risky) + \frac{EL_{rw}/(1 - \mu) + \pi(1 - \lambda_3)}{(1 - \lambda_1) - \pi(1 - \lambda_3)} [E_c(risky) - E_c(safer)]$$

Plugging in what we know to be EL_{rw} , we can state that $\tilde{k}_0 < 1$ iff:

$$\begin{aligned} \rho > \tilde{\rho}_0 \equiv & \frac{cE_c(risky)[(1 - \lambda_1) - \pi(1 - \lambda_3)] + \pi(1 - \lambda_3)c[E_c(risky) - E_c(safer)]}{c[(1 - \lambda_1) - \pi(1 - \lambda_3)] + \lambda_2[(1 - \lambda_1) - \lambda_2 - (1 - \lambda_3)\pi][E_c(risky) - E_c(safer)]} \\ & + [E_c(risky) - E_c(safer)] \frac{c\lambda_1 + [(1 - \lambda_1) - \lambda_2 - (1 - \lambda_3)\pi][\mu(\pi R_2^h + (1 - \lambda_2)\pi - R_1) + \mu\lambda_2]}{c[(1 - \lambda_1) - \pi(1 - \lambda_3)] + \lambda_2[(1 - \lambda_1) - \lambda_2 - (1 - \lambda_3)\pi][E_c(risky) - E_c(safer)]} \end{aligned}$$

So, in this region if $\rho \leq \tilde{\rho}_0$, even if the shareholders' participation constraint determines ω , risk-taking will not increase fast enough to lead to a detriment.

Let us now consider the second region wherein banks can survive state s_2 with probability π . Finding a similar condition, the expected loss of deposit funds will be larger under an LR when:

$$(1 - \mu)[(1 - k(\omega)) - \omega_{rw}(1 - \lambda_1) - (1 - \omega_{rw})(1 - \lambda_3)\pi] = EL_{rw}$$

⁶⁰If ρ is lower than the given level, the condition becomes $k_{lev} < \tilde{k}_0$. But since ρ is below this given level, this means that the denominator of \tilde{k}_0 is negative, while the numerator is positive. Hence $\tilde{k}_0 < 0$. Since $k_{lev} \geq 0$ by definition, the expected loss of deposit funds must therefore always be lower under an LR. So for the expected loss of deposit funds to be larger under an LR in this case, it must be that $\rho > \mu + \frac{\mu[R_2^h\pi + (1 - \pi)(1 - \lambda_2) - R_1]}{(1 - \lambda_1) - (1 - \lambda_3)\pi}$.

$$< (1 - \mu)(1 - \pi) [(1 - k_{lev}) - \omega_{lev}(1 - \lambda_1)]$$

Define $E_{c'}(risky) = \mu\pi R_2^h + \mu(1 - \pi)(1 - \lambda_2) + (1 - \mu)\pi(1 - \lambda_3)$ and $E_{c'}(safer) = \mu R_1 + (1 - \mu)\pi(1 - \lambda_1)$, then if $\rho > \frac{\mu[\pi R_2^h + (1 - \pi)(1 - \lambda_2)] + (1 - \mu)(1 - \lambda_3)\pi - [\mu R_1 + (1 - \mu)(1 - \lambda_1)\pi]}{1 - \lambda_1} + \mu + (1 - \mu)\pi$, this simplifies to:⁶¹

$$k_{lev} > \tilde{k}_1 \equiv \frac{[E_{c'}(risky) - E_{c'}(safer)][EL_{rw}/(1 - \mu)(1 - \pi) - 1] + [E_{c'}(risky) - \mu](1 - \lambda_1)}{(\rho - \mu)(1 - \lambda_1) - [E_{c'}(risky) - E_{c'}(safer)]}$$

Again, the RHS must be less than 1 since $k_{lev} \leq 1$. This is true iff:

$$\rho > E_{c'}(risky) + \frac{EL_{rw}/(1 - \mu)(1 - \pi)}{(1 - \lambda_1)} [E_{c'}(risky) - E_{c'}(safer)]$$

Plugging in what we know to be EL_{rw} , we can state that $\tilde{k}_1 < 1$ iff:

$$\rho > \tilde{\rho}_1 \equiv \frac{E_{c'}(risky)(1 - \pi)(1 - \lambda_1)c}{(1 - \lambda_1)(1 - \pi)c + [E_{c'}(risky) - E_{c'}(safer)]\lambda_2[(1 - \lambda_1) - \lambda_2 - (1 - \lambda_3)\pi]}$$

$$[E_{c'}(risky) - E_{c'}(safer)] \frac{\lambda_1 c + [(1 - \lambda_1) - \lambda_2 - (1 - \lambda_3)\pi][\mu(\pi R_2^h + (1 - \lambda_2)\pi - R_1) + \mu\lambda_2]}{(1 - \lambda_1)(1 - \pi)c + [E_{c'}(risky) - E_{c'}(safer)]\lambda_2[(1 - \lambda_1) - \lambda_2 - (1 - \lambda_3)\pi]}$$

So again, if $\rho \leq \tilde{\rho}_1$, even if the shareholders' participation constraint determines risk-taking, the expected loss of deposit funds will lower under an LR. If attainable, the third region in which the probability of default falls to zero (and thus so also the expected loss of deposit funds), clearly is lower than under a solely risk-based capital framework, and so, we can conclude that if $\rho < \hat{\rho} \equiv \min\{\tilde{\rho}_0, \tilde{\rho}_1\}$,⁶² even if the shareholders' participation constraint determines ω , risk-taking will not increase fast enough to lead to a detriment. Combining our previous result on the lower bound, we can state that if $\rho < \hat{\rho}$, the expected loss of deposit funds under an LR will be strictly lower for all $k_{lev} > \underline{k}$. This proves the second statement.

If $\rho > \hat{\rho}$ however, the above suggests that the expected loss of deposit funds could potentially be higher under an LR. Let us therefore now prove the third statement and show that this will not be the case for all $k \in (\underline{k}, \bar{k})$, where $\bar{k} \equiv \min\{\tilde{k}_0, \tilde{k}_1\}$. Let us first show that the upper bound levels discussed above (i.e.

⁶¹As before, if ρ is less than this, since the denominator of \tilde{k}_1 would be negative, while the numerator is positive: $[E_{c'}(risky) - E_{c'}(safer)][EL_{rw}/(1 - \mu)(1 - \pi) - 1] + [E_{c'}(risky) - \mu](1 - \lambda_1) > [E_{c'}(risky) - E_{c'}(safer)][EL_{rw}/(1 - \mu) + (1 - \lambda_3)\pi - 1] + [E_{c'}(risky) - \mu][(1 - \lambda_1) - (1 - \lambda_3)\pi] > 0$, then the condition simplifies to $k < \tilde{k}_1$ where $\tilde{k}_1 < 0$. But since $k_{lev} \geq 0$ by definition, this means that in this case, the expected loss of deposit funds will always be smaller under an LR when ρ is less than this level.

⁶²It is not possible to determine which is smaller, this will depend on the exact parameter values.

\tilde{k}_0 and \tilde{k}_1) are strictly greater than the lower bound level (\underline{k}) derived earlier. The lower bound level, \underline{k} , was first defined at the point where $k_{lev} = k(\omega_{rw}^*) + \omega_{rw}^*(1 - \lambda_1) - \omega_{lev}^*(1 - \lambda_1) - (\omega_{rw}^* - \omega_{lev}^*)(1 - \lambda_3)\pi$ where $*$ denotes optimal levels. The first upper bound level, \tilde{k}_0 , is defined at the point where $k_{lev} = k(\omega_{rw}^*) + \omega_{rw}^*(1 - \lambda_1) - \omega_{lev}^{pc}(1 - \lambda_1) - (\omega_{rw}^* - \omega_{lev}^{pc})(1 - \lambda_3)\pi$ where pc denotes the level determined by the shareholders' participation constraint. Since $\omega_{lev}^{pc} < \omega_{lev}^*$, it must be that this upper bound is strictly greater. Equally, \tilde{k}_0 is strictly greater than the k required for an interior solution, i.e. $k = (1 - \omega_{lev}^*)\lambda_2$. This will be true if and only if: $k(\omega_{rw}^*) + \omega_{rw}^*(1 - \lambda_1) - \omega_{lev}^{pc}(1 - \lambda_1) - (\omega_{rw}^* - \omega_{lev}^{pc})(1 - \lambda_3)\pi > (1 - \omega_{lev}^*)\lambda_2$. Rearranging, we find $(\omega_{lev}^* - \omega_{rw}^*)\lambda_2 + (\omega_{rw}^* - \omega_{lev}^{pc})[(1 - \lambda_1) - (1 - \lambda_3)\pi] > 0$, which must be true since $\omega_{rw}^* > \omega_{lev}^* > \omega_{lev}^{pc}$ and $[(1 - \lambda_1) - (1 - \lambda_3)\pi] > \lambda_2$. Hence $\underline{k} < \tilde{k}_0$.

Consider the second upper bound now, \tilde{k}_1 . This is defined at the point where

$$1 + k \frac{[\rho - (\mu + (1 - \mu)\pi)](1 - \lambda_1) - 1}{[E_{c'}(risky) - E_{c'}(safer)]} - \frac{E_{c'}(risky) - (\mu + (1 - \mu)\pi)}{E_{c'}(risky) - E_{c'}(safer)} = \frac{EL_{rw}}{(1 - \mu)(1 - \pi)}$$

Since $\rho > \tilde{\rho}_1$,⁶³ the LHS is increasing in k . Hence, the k that solves this equation (i.e. \tilde{k}_1), ceteris paribus, must be larger than the k that solves

$$1 + k \frac{(\rho - (\mu + (1 - \mu)\pi))(1 - \lambda_1) - 1}{[E_{c'}(risky) - E_{c'}(safer)]} - \frac{E_{c'}(risky) - (\mu + (1 - \mu)\pi)}{E_{c'}(risky) - E_{c'}(safer)} = \frac{EL_{rw}}{(1 - \mu)}$$

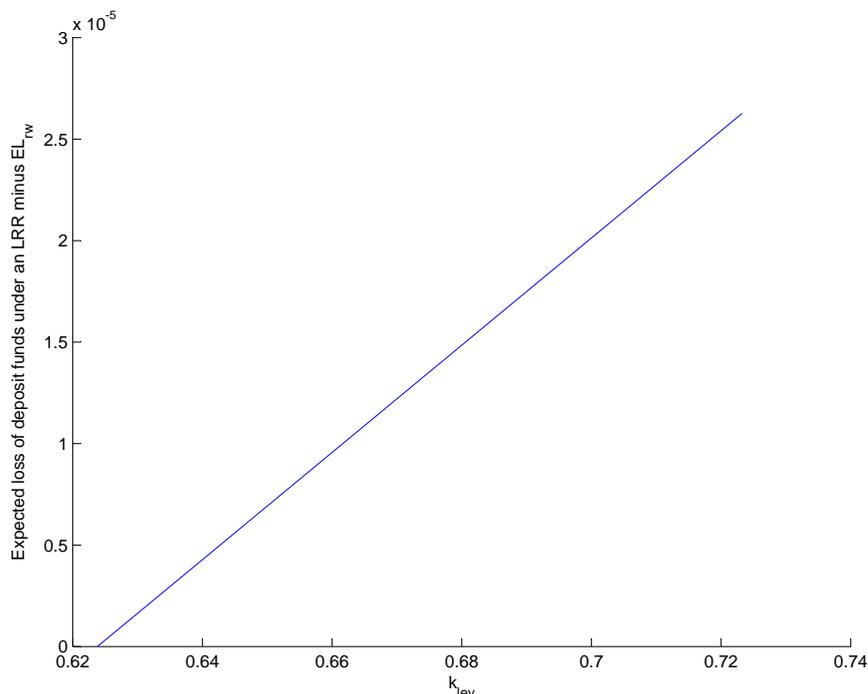
since this is identical except the RHS is smaller. Let us take this smaller k . We show that this is larger than \underline{k} and thus, it must also be that \tilde{k}_1 is too. This lower k is defined at $[1 - k_{lev} - \omega_{lev}^{pc}(1 - \lambda_1)] = EL_{rw}/(1 - \mu) = (1 - k(\omega_{rw}^*) - \omega_{rw}^*(1 - \lambda_1) - (1 - \omega_{rw}^*)(1 - \lambda_3)\pi$. Again, \underline{k} is defined first at the point where $k_{lev} = k(\omega_{rw}^*) + \omega_{rw}^*(1 - \lambda_1) - \omega_{lev}^*(1 - \lambda_1) - (\omega_{rw}^* - \omega_{lev}^*)(1 - \lambda_3)\pi$. Since $\omega_{lev}^{pc} < \omega_{lev}^*$, it must be that this upper bound is strictly greater. Equally, this k is larger than that required for an interior solution, i.e. $k > (1 - \omega_{rw}^*)\lambda_2$. This is true if: $(1 - \omega_{rw}^*)\lambda_2 + \omega_{rw}^*(1 - \lambda_1) - \omega_{lev}^{pc}(1 - \lambda_1) + (1 - \omega_{rw}^*)(1 - \lambda_3)\pi > (1 - \omega_{lev}^*)\lambda_2$ which we can rearrange as $-(\omega_{rw}^* - \omega_{lev}^*)\lambda_2 + (1 - \lambda_1)(\omega_{rw}^* - \omega_{lev}^{pc}) + (1 - \omega_{rw}^*)(1 - \lambda_3)\pi > 0$. This is true since $\omega_{lev}^* > \omega_{lev}^{pc}$ and $(1 - \lambda_1) > \lambda_2$. Hence $\underline{k} < \tilde{k}_1$.

Combining the results from above, we can conclude therefore that for all ρ , if $k_{lev} \in (\underline{k}, \bar{k})$, the expected loss of deposit funds will be strictly lower under an LR. This proves the third statement.

Lastly, we show that for $\rho > \hat{\rho}$ and $k_{lev} > \bar{k}$ it is possible that for all k_{lev} above this level, the expected loss of deposit funds can be higher than EL_{rw} . It is not

⁶³Since from above, we need not consider this region if $\rho < \tilde{\rho}_1$.

Figure 8



Note: The graph shows the difference between the expected loss of deposit funds under an LR minus the expected loss of deposit funds under a solely risk-based capital framework from \bar{k} to k_{max} .

possible to analytically prove a general statement in this region, so we illustrate numerically that the statement is possible.⁶⁴ Suppose the parameters are such that $\mu = 0.999$, $\pi = 0.9$, $R_1 = 1.02$, $R_h^2 = 1.2$, $c = 9$,⁶⁵ $\lambda_1 = 0.02$, $\lambda_2 = 0.1$, $\lambda_3 = 0.8$. This gives $\tilde{\rho}_0 = 1.2026$ and $\tilde{\rho}_1 = 1.2596$. Suppose $\rho = 1.22$, this gives $\tilde{k}_0 = 0.6238$ and $\tilde{k}_1 = 1.5116$. So, for these parameter values, $\tilde{\rho}_0 < \tilde{\rho}_1$, and $\tilde{k}_0 < \tilde{k}_1$, hence $\hat{\rho} = \tilde{\rho}_0$ and $\bar{k} = \tilde{k}_0$. We plot the expected loss of deposit funds under an LR minus EL_{rw} from \bar{k} to k_{max} , where $k_{max} \equiv \frac{E_c(risky) - \mu}{\rho - \mu}$ is the point at which for all $k > k_{max}$, even if the bank chooses $\omega = 0$, it will not satisfy the shareholders' participation constraint. Thus any $k > k_{max}$ is infeasible. Figure 8 illustrates that for these parameter values (in which $\rho > \hat{\rho}$), for all levels of the LR above \bar{k} until we hit the point k_{max} (at which point the LR cannot be set any higher), the expected loss of deposit funds under an LR will be larger.

⁶⁴Note that from the definition of $\hat{\rho}$, $\hat{\rho}$ is strictly greater than the conditional expected return on the risky asset. As such, in this region, there will exist an upper bound below 1 above which the LR cannot be set because the bank could not raise equity beyond this point without violating the shareholders' participation constraint.

⁶⁵ c is set to 9 following Dell'Ariccia et al. (2014).