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Riccardo M Masolo[(1)] and Pablo E Winant[(2)]

Abstract

Since the Great Recession policy rates have been extremely low, but neither absolutely constant, nor exactly set to zero. We thus augment a standard zero lower bound model to study the effects of a stochastic lower bound (SLB) on policy rates. We find that a less predictable SLB helps keep inflation closer to target by lowering expectations of future values of the SLB when interest rate cuts are not an option.

Key words: Zero lower bound, DSGE.

JEL classification: E31, E52.

(1) Bank of England and Centre for Macroeconomics. Email: riccardo.masolo@bankofengland.co.uk
(2) Bank of England. Email: pablo.winant@bankofengland.co.uk

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Publications and Design Team, Bank of England, Threadneedle Street, London, EC2R 8AH
Telephone +44 (0)20 7601 4030 email publications@bankofengland.co.uk

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1 Introduction

Since the Great Recession, policy rates have been extremely low and less responsive to inflation than standard policy rules would have predicted (Taylor (1993)), in a number of advanced economies. That suggests they were constrained. As Figure 1 shows, however, rates were neither necessarily zero, nor absolutely constant, as implied by ZLB models.[1]

Consequently, we modify a simple New-Keynesian model (Yun (2005)) with a zero-lower bound (Eggertsson and Woodford (2003)) to incorporate a *Stochastic Lower Bound* (SLB), i.e. time variation in the lower bound. In this paper we focus on a particular aspect of this model, which highlights the importance of the properties of the SLB. We take the ergodic distribution of the lower bound as given and focus on the effects that the *conditional* probability of the SLB switching values has on inflation. We show that the frequency with which its values switch matters for inflation. A frequently-switching SLB makes the current value of the lower bound a poor predictor of future values: the SLB is more unpredictable. This turns out to help keep inflation close to target during downturns, as it lowers future rate expectations when rate cuts are not an option.

Recent papers (Bianchi and Melosi (2017), Binning and Maih (2016), Zhutova (2017)) add a small shock to their models' ZLB to match observed variations in the *effective level* of the Fed Funds Rates within the Federal Funds Target Range.[2] Ghironi and Ozhan (2018) study changes in the volatility of policy rate shocks. Rognlie (2015) studies the effects of negative rates but not directly the effects of changes in the lower bound and its predictability. We focus on variations of the SLB, rather than trying to model the discrepancies between target and effective rates, model discrete jumps in the SLB which capture policy variations, and describe the effects that more or less frequent SLB switches have on inflation.

In our model we have two sources of variation (both Markov-switching processes). A standard demand shock, $d_t \in \{d^L, d^H\}$, where $d_t = d^H$ represents "normal" times and $d_t = d^L$, periods of low inflation and policy rates. The SLB is modeled as $s_t \in \{s^L, s^H\}$, so that our log-linear policy rule can be written as $i_t = \phi\pi_t$, if $\phi\pi_t > s_t$, and $i_t = s_t$, otherwise. We maintain symmetry in the SLB, i.e. $P\{s_{t+1} = s_t | s_t\} = \vartheta \in [.5, 1)$, $s_t \in \{s^L, s^H\}$.[3] This assumption allows us to easily vary the conditional properties of the SLB, by modifying the value for ϑ, without affecting the ergodic mean and variance of the SLB. This is important because the effects on inflation of changing the average level of the lower bound are obvious. So we want to restrict ourselves to studying the effects of changes in the frequency with which the SLB switches value while keeping the ergodic mean and variance of the SLB unchanged. In other words, ϑ allows us to control the SLB's predictability, i.e. how good a signal for s_{t+1} the current s_t is, without affecting the unconditional mean and variance of the SLB.

2 An Analytical Case

We first illustrate our result in a log-linear version of our model (which we describe in Appendix A) by making two assumptions we will relax later:

i. $\{d_t\}_{t=0}^{\infty}$ is known: $d_t = d^L$, $t = 0, 1$, and $d_t = d^H = 0$, $t \geq 2$, along the lines of Christiano, Eichenbaum, and Rebelo (2012),

ii. d^L is such that $s_t = s^H$ makes the SLB binding but $s_t = s^L$ does not.

[1] Altavilla, Boucinha, and Peydró (2017) discusses determinants of the lower bound on rates explaining why it can vary over time and across countries.

[2] As Kulish, Morley, and Robinson (2014) explain, this is primarily an estimation related problem. We are not concerned with estimation here, nor we are trying to model why effective rates can differ from target rates or target ranges. Rather we want to understand the effects of discrete changes in the target rate itself.

[3] By restricting $\vartheta \in [.5, 1)$ we rule out negatively autocorrelated SLBs on grounds of plausibility, though our analysis does not depend on this.

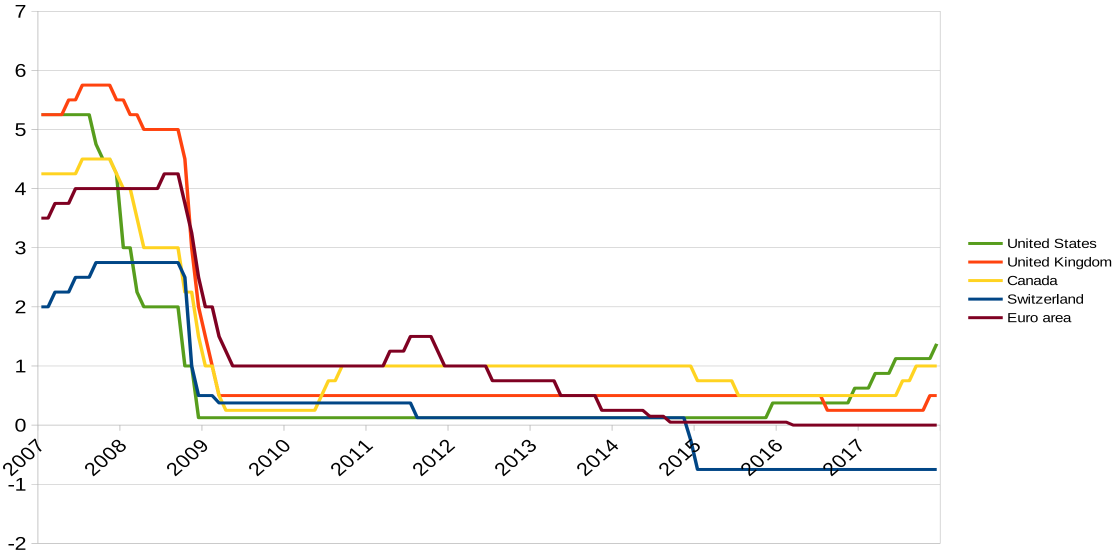

Figure 1: Monetary Policy Rates in some advanced economies. Source: BIS policy rate statistics.

Then the following system characterizes the economy:

$$y_t = \mathbb{E}_t y_{t+1} - (i_t - \mathbb{E}_t \pi_{t+1}) + d_t, \tag{1}$$

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t \pi_{t+1}, \tag{2}$$

$$i_t = \begin{cases} \phi \pi_t & \text{if } \phi \pi_t > s_t \in \{s^L, s^H\} \\ s_t & \text{if } \phi \pi_t < s_t \in \{s^L, s^H\}, \end{cases} \tag{3}$$

where y_t is output, π_t inflation and greek letters positive coefficients. Given i., $\pi_t = 0,\ t \geq 2$ and $\pi_1 < 0$, but π_1 is independent of ϑ. What matters is how the predictability of the SLB affects the ex-ante expectation of inflation in period 0, $\mathbb{E}\pi_0$.

Proposition 1. *$\mathbb{E}\pi_0$ is inversely related to ϑ, and so closest to target for $\vartheta = .5$.*

Proof. Appendix B. □

To appreciate the result, it is enough to understand how inflation at time 0 varies with ϑ:

$$\frac{\partial \pi_0}{\partial \vartheta} = \begin{cases} \frac{\kappa}{1+\kappa\phi}(1+\kappa+\beta)\left(i_1(s^H) - i_1(s^L)\right) > 0 & s_0 = s^L \\ -\kappa(1+\kappa+\beta)\left(i_1(s^H) - i_1(s^L)\right) < 0 & s_0 = s^H. \end{cases} \tag{4}$$

Conditional on $s_0 = s^L$, $\vartheta \to 1$ maximizes inflation in period 0. By a symmetric argument, when $s_0 = s^H$, the optimal value of ϑ is .5. Basically, π_0 is higher the more likely $s_{t+1} = s^L$.

Before knowing the realization for s_0, however, a lower ϑ is preferable since, when $s_0 = s^L$, inflation responds less (in absolute value) to changes in ϑ by a factor $\frac{1}{1+\kappa\phi}$.

When $s_0 = s^L$ deflationary pressures generated by an increase in $P\{s_{t+1} = s^H | s_t = s^L\}$ can be countered by a rate cut. When $s_0 = s^H$, the policy rate is independent of inflation instead, and inflation can only increase if $P\{s_{t+1} = s^L | s_t = s^H\}$ increases.

In the next section, we show numerically how this carries over to a global solution of our nonlinear model and is quantitatively relevant.

3 Global Solution

We confine a complete calibration of the model to Appendix A. It is worth noting, though, that we allow for interest-rate smoothing. This is realistic and also beneficial relative to a policy

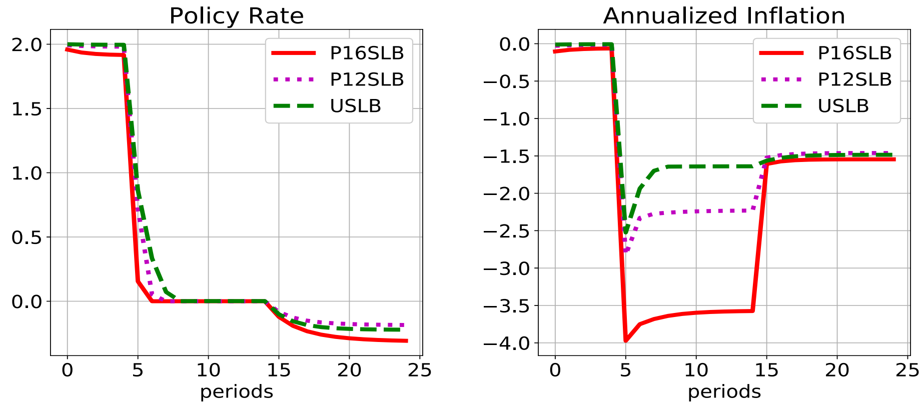

Figure 2: Policy rate (left) and inflation (right) simulated profiles.

rule like that in equation (3), in the presence of a lower bound (Nakov (2008)). We set $s^H - s^L =$ 50bps, which seems reasonable given the range of rates presented in Figure 1. We assume d^H to be very persistent ($P\left\{d_{t+1} = d^H | d_t = d^H\right\} = .99$), and we set $P\left\{d_{t+1} = d^L | d_t = d^L\right\} = .67$.

We illustrate our result with a simulation: the economy starts off in "normal times": $d_t = d^H$, $s_t = s^H$, $t = 0, ..., 4$, then switches to what corresponds to a standard ZLB scenario: $d_t = d^L$, $s_t = s^H$, $t = 5, ..., 9$; it then moves to a low-demand, low-SLB scenario: $d_t = d^L$, $s_t = s^L$, $t = 10, ..., 14$.

Figure 2 reports the simulated profiles for the policy rate and inflation. We consider three values for ϑ:

- **P16SLB**. $\vartheta = .9375$: s_t switches value on average every 16 quarters.
- **P12SLB**. $\vartheta = .9167$: s_t expected to switch every 12 quarters.
- **USLB**. $\vartheta = .5$: unpredictable SLB.

A comparison between P12SLB and USLB confirms the findings of our analytical exercise. Inflation is closer to target when the bound is unpredictable and $s_t = s^H$, but not when $s_t = s^L$. The difference, though, is more than .5 percent in the first case and less than .1 in the second, so, clearly, the USLB setup is to be preferred, as $s_t = s^H$ and $s_t = s^L$ are ex-ante equally likely.[4]

The P16SLB scenario adds another dimension to our discussion. In the (d^L, s^H) state, inflation is more than 3.5 percent below target. This causes inflation to be lower than under USLB in *all states*. Under P16SLB, the (d^L, s^H) is so bad that, even when $s_t = s^L$ inflation will be lower: despite $P\left\{s_{t+1} = s^L | s_t = s^H\right\}$ being smaller under P16SLB than under USLB, the drop in inflation, reflected in inflation expectations, would be so severe that it causes inflation to be lower under P16SLB.

We also perform a 200000-period stochastic simulation. Inflation volatility under P16SLB is higher by 47 percent relative to USLB over the entire simulation and by 133 percent if we only consider periods in which $d_t = d^L$. Consumption volatility is higher by a factor of 24 and 83 percent respectively.

These numbers should be taken with a grain of salt given the simplicity of our model and shock structure. Yet, we find it striking how a seemingly minor detail like the frequency with with the SLB changes value, in a very simple and commonly used model, can produce such large quantitative effects.

[4] We also experimented with calibrations in which $s_t = s^L$ binds for the lower-ϑ case. Results are robust to that.

4 Conclusion

Expectations are notoriously important for inflation determination. All the more so, when policy rates hit a lower bound. The success of lower-for-longer policies rests precisely on how they affect expectations.

We find that, taking the ergodic level and variance of the lower bound as given, more unpredictability in the future level of the bound helps mitigate the deflationary effects of negative demand shocks.

We show this in detail in an analytical example and in a non-linear New-Keynesian DSGE which demonstrates that these effects are quantitatively significant for a standard calibration.

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A Model

A.1 Setup

The representative household maximizes utility from consumption and leisure:

$$\max_{C_t,N_t} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(\prod_{s=0}^{j} e^{-d_{t+s-1}} \right) \left(\log\left(C_{t+j}\right) - \frac{N_{t+j}^{1+\psi}}{1+\psi} \right), \tag{A.5}$$

where $d_t \in \{d^H, d^L\}$ is a binary Markov-switching intertemporal preference shock.[5] The maximization is subject to the following flow budget constraint:

$$P_t C_t + B_t = R_{t-1} B_{t-1} + W_t N_t + T_t, \tag{A.6}$$

where P_t is the final-good price, W_t the nominal wage, B_t the holdings of zero-supply one-period bonds yielding a nominal return R_t. T_t includes profits from the diversified portfolio of firms households own and the lump sum tax used to finance the production subsidy.

The household's behavior is thus characterized by the intertemporal and intratemporal Euler equations:

$$\frac{1}{C_t} = \frac{\beta e^{-d_t} R_t}{\mathbb{E}_t [C_{t+1} \Pi_{t+1}]}, \tag{A.7}$$

$$\chi N_t^{\psi} C_t = \frac{W_t}{P_t}. \tag{A.8}$$

There is a continuum of intermediate goods $i \in [0,1]$, produced by monopolistically competitive firms according to:

$$Y_t(i) = N_t(i), \tag{A.9}$$

They face the following demand function:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t, \tag{A.10}$$

and are subject to pricing frictions à la Calvo (1983). They maximize:

$$\max_{P_t^*(i)} \sum_{s=0}^{\infty} \theta^j \beta^j \mathbb{E}_t \left[\left(\prod_{s=0}^{j} e^{-d_{t+s-1}} \right) \frac{C_{t+j}^{-1}}{P_{t+j}} \left[P_t^* Y_{t+j}(i) - (1-\tau) W_{t+j} Y_{t+j}(i) \right] \right], \tag{A.11}$$

where $(1-\tau)W_{t+j}$ is the nominal marginal cost, net of the subsidy $\tau = \frac{1}{\epsilon}$, so as to eliminate the monopolistic competition distortion in steady state. Their first-order condition can be expressed as:

$$\frac{P_t^*(i)}{P_t} = \frac{\Upsilon_t}{\Gamma_t} \tag{A.12}$$

$$\Upsilon_t = \frac{W_t}{P_t} + \beta\theta e^{-d_t} \mathbb{E}_t \Pi_{t+1}^{\epsilon} \Upsilon_{t+1} \tag{A.13}$$

$$\Gamma_t = 1 + \beta\theta e^{-d_t} \mathbb{E}_t \Pi_{t+1}^{\epsilon-1} \Gamma_{t+1}, \tag{A.14}$$

which pin down inflation when combined with the following condition:

$$\frac{P_t^*(i)}{P_t} = \left(\frac{1-\theta \Pi_t^{\epsilon-1}}{1-\theta} \right)^{\frac{1}{1-\epsilon}}, \tag{A.15}$$

[5] Our formulation of the discount factor process is identical to that in Boneva, Braun, and Waki (2016), except for notational convention. We prefer to denote with d_t the variable known at time t, while in their model d_{t+1} is assumed to be revealed at time t. Also, we add a negative sign because we want to interpret high d_t as high demand, which corresponds to more impatient households. Ultimately, what matters is that the shock that shows up in the Euler equation, $-d_t$ in our case and d_{t+1} in theirs, is known and works as a demand shock that can drive the economy to the lower bound.

implied by the law of motion of the aggregate price index. The final good Y_t is produced by perfectly competitive producers using a continuum of intermediate goods $Y_t(i)$ and a standard CES production function

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}. \tag{A.16}$$

The nominal policy rate is set according to:

$$R_t = \max\left\{R_{t-1}^{\rho}\left(\frac{1}{\beta}\Pi_t^{\phi}\right)^{1-\rho}, e^{s_t}\right\}, \tag{A.17}$$

where $s_t \in \{s^H, s^L\}$ is a Markov-switching process for which we maintain that $0 > s^H > s^L$ (the bound is not binding in the non-stochastic steady state) and $P\{s_{t+1} = s_t | s_t\} = \vartheta$, $s_t \in \{s^H, s^L\}$, which implies that the two states are equally likely unconditionally: $P\{s_0 = s^L\} = P\{s_0 = s^H\} = \frac{1}{2}$.

We assume that the government runs a balanced budget and finances the production subsidy with a lump-sum tax. Out of notational convenience, we include the firms' aggregate profits in the lump-sum transfer:

$$\begin{aligned} T_t &= P_t\left[-\tau\frac{W_t}{P_t}N_t + Y_t\left(1-(1-\tau)\frac{W_t}{P_t}\Delta_t\right)\right] \\ &= P_t Y_t\left(1-\frac{W_t}{P_t}\Delta_t\right), \end{aligned}$$

where Δ_t is the price dispersion term, defined by the market-clearing conditions we turn to now.

Market clearing in the goods markets requires that $Y_t(i) = C_t(i)$ for all $i \in [0,1]$. For the labor market to clear, the following has to hold:

$$N_t = \int_0^1 N_t(i)di = \int_0^1 Y_t(i)di = Y_t\int_0^1\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} di. \tag{A.18}$$

We define $\Delta_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} di$, the price dispersion term (Yun, 2005) which, given the law of motion for prices, is characterized by the following law of motion:

$$\Delta_t = \theta\Pi_t^{\epsilon}\Delta_{t-1} + (1-\theta)\left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta}\right)^{\frac{\epsilon}{\epsilon-1}}. \tag{A.19}$$

A.2 Calibration

The calibration is, for the most part, standard and summarized in Table 1.

The discount factor, Frisch elasticity, demand elasticity, degree of price stickiness and inflation-response coefficient, are commonly accepted in the literature.

The interest-rate smoothing coefficient is also calibrated to a standard value but it is worth noting that in ZLB models a degree of interest-rate smoothing is known to reduce the most negative effects of the low demand in that it acts as a proxy for systematic lower-for-longer policies. We will thus use a interest-rate smoothing coefficient of .7 in the global solution, while we do away with smoothing in the analytical example to make the solution as simple and transparent as possible.

We assume the inflation target to be zero but the model can be readily re-written assuming positive inflation target and indexation to steady state inflation.

We calibrate the distance between the non-stochastic steady state policy rate and the SLB to 2 and 2.5 percent depending on s_t. We think this is realistic given current scenarios of ultra low real rates and levels of the SLB that can be positive.

Indeed, while in a model in which d_t followed a continuous distribution, this distance would affect the frequency of lower-bound episodes, in this model, the distance between the

Table 1: Calibration

β	.995	Discount factor
ψ	1	Inverse Frisch Elasticity
ϵ	11	Goods Demand Elasticity
θ	.75	Probability of not adjusting price
ϕ	1.5	Coefficient on inflation in the policy rule
ρ	.7	Interest-rate smoothing coefficient
s^H	0	High level of the SLB
s^L	-.50/400	Low level of the SLB (-50bps in annualized rate space)
d^H	.00025	High demand
d^L	-.01	Low demand

SLB and the non-stochastic steady state affects the size of the d^L shock required to bring the economy to the lower bound.

We calibrate the distance between the two levels of the SLB we consider to 50 basis points ($s^H - s^L = 50bps$). It is a coarse approximation, and probably an underestimation, of the degree of variation in rates given the policy rate series presented in Figure 1. It is enough, though, to illustrate our point and to produce quantitatively large variations in inflation.

The demand process d_t is calibrated with d^H, a very persistent ($P\left\{d_{t+1} = d^H | d_t = d^H\right\} =$ .99) and frequent state, which makes it very close to the ergodic steady state. We think of it as "normal times". d^L is a level of demand low enough to drive the economy to the SLB.

We calibrate $P\left\{d_{t+1} = d^L | d_t = d^L\right\} = .67$, which implies a moderate degree of persistence of the d^L state. Davig and Leeper (2007) and, more specifically, Richter and Throckmorton (2015) discuss in detail how a persistent lower-bound state significantly shrinks the determinacy region: when rates are constrained the *Taylor principle* is temporarily not satisfied. We could try alternative calibrations consistent with longer lower-demand spells, e.g. the analysis in Richter and Throckmorton (2015) suggests that increasing price stickiness and decreasing interest-rate smoothing is bound to increase the average duration of the low-demand state consistent with a convergent solution. However, we prefer to use a standard calibration for the structural parameters and show how the effects of changes in the persistence of the SLB are large even under the assumption that the low-demand state is relatively short-lived.

A.3 Log-Linear Version

When we consider the log-linear version of our model we assume away the interest-rate smoothing term and assume that $d^H = 0$ is an absorbing state. In this case, $\Pi = 1$, $\Delta = 1$ in steady state and the log-linear version of the model can be easily derived, with one detail worth noting. The log-linear Phillips Curve reads:

$$\pi_t = \frac{(1-\theta)\left(1-\beta\theta\right)}{\theta} mc_t + \beta \mathbb{E}_t \pi_{t+1}, \tag{A.20}$$

which can be re-expressed as a function of output (or the output gap, as the natural rate of output is constant in this economy) by noting that the marginal cost equals the real wage. The log-linear version of equation (A.8), combined with the production function in equation A.9 and the market clearing condition in the goods market, implies:

$$w_t = \psi n_t + c_t = (1+\psi)c_t = (1+\psi)y_t. \tag{A.21}$$

Defining $\kappa = \frac{(1-\theta)(1-\beta\theta)(1+\psi)}{\theta}$ delivers the equation in the main text.

B Proof of Proposition

The solution is trivial once the absorbing state for demand is attained. For $t \geq 2$:

$$d_t = d^H = 0,\ s_t < 0,\ y_t = \pi_t = i_t = 0. \tag{A.22}$$

In $t = 1$:

$$\begin{aligned} d_1 &= d^L < 0, && \text{(A.23)} \\ y_1 &= -i_1 + d^L, && \text{(A.24)} \\ \pi_1 &= -\kappa i_1 + \kappa d^L. && \text{(A.25)} \end{aligned}$$

Then we have two cases:

$$\pi_1 = \begin{cases} \frac{\kappa}{1+\kappa\phi} d^L & s_1 = s^L \\ \kappa\left(d^L - s^H\right) & s_1 = s^H. \end{cases} \tag{A.26}$$

Plus the restriction $0 > s^H > \frac{\kappa\phi}{1+\kappa\phi} d^L > s^L$.
For output:

$$y_1 = \frac{\pi_1}{\kappa}. \tag{A.27}$$

In $t = 0$:

$$\begin{aligned} y_0 &= \mathbb{E}_0 y_1 - (i_0 - \mathbb{E}_0 \pi_1) + d^L \\ &= \left(1 + \frac{1}{\kappa}\right)\mathbb{E}_0 \pi_1 - i_0 + d^L \end{aligned} \tag{A.28}$$

Plugging this into the Phillips Curve:

$$\begin{aligned} \pi_0 &= \kappa\left(\left(1+\frac{1}{\kappa}\right)\mathbb{E}_0\pi_1 - i_0 + d^L\right) + \beta\mathbb{E}_t\pi_1 \\ &= \kappa\left(d^L - i_0\right) + \left(1+\kappa+\beta\right)\mathbb{E}_t\pi_1 \\ &= \kappa\left(d^L - i_0\right) + \left(1+\kappa+\beta\right)\left(p(s_0)\left(\frac{\kappa}{1+\kappa\phi}d^L\right) + (1-p(s_0))\kappa\left(d^L - s^H\right)\right) \\ &= \kappa\left(d^L - i_0\right) + \left(1+\kappa+\beta\right)\left(p(s_0)\left(1-\frac{\kappa\phi}{1+\kappa\phi}\right)\kappa d^L + (1-p(s_0))\kappa\left(d^L - s^H\right)\right) \\ &= \kappa d^L\left(1+\left(1+\kappa+\beta\right)\right) - \kappa i_0 + \left(1+\kappa+\beta\right)\left(-p(s_0)\frac{\kappa\phi}{1+\kappa\phi}\kappa d^L - (1-p(s_0))\kappa s^H\right) \\ &= \kappa d^L\left(1+\left(1+\kappa+\beta\right)\left(1-p(s_0)\frac{\kappa\phi}{1+\kappa\phi}\right)\right) - \kappa i_0 - \left(1+\kappa+\beta\right)(1-p(s_0))\kappa s^H. \end{aligned} \tag{A.29}$$

Where we define $p(s_0) = P\left\{s_1 = s^L | s_0\right\}$.
We again have two cases, depending on the value of i_0:

$$\pi_0 = \begin{cases} \kappa d^L\left(1+\left(1+\kappa+\beta\right)\left(1-\vartheta\frac{\kappa\phi}{1+\kappa\phi}\right)\right) - \kappa\phi\pi_0 - \left(1+\kappa+\beta\right)(1-\vartheta)\kappa s^H & s_0 = s^L \\ \kappa d^L\left(1+\left(1+\kappa+\beta\right)\left(1-(1-\vartheta)\frac{\kappa\phi}{1+\kappa\phi}\right)\right) - \kappa s^H - \left(1+\kappa+\beta\right)\vartheta\kappa s^H & s_0 = s^H \end{cases} \tag{A.30}$$

$$\pi_0 = \begin{cases} \frac{\kappa}{1+\kappa\phi}\left(d^L\left(1+\left(1+\kappa+\beta\right)\left(1-\vartheta\frac{\kappa\phi}{1+\kappa\phi}\right)\right) - \left(1+\kappa+\beta\right)(1-\vartheta)s^H\right) & s_0 = s^L \\ \kappa\left(d^L\left(1+\left(1+\kappa+\beta\right)\left(1-(1-\vartheta)\frac{\kappa\phi}{1+\kappa\phi}\right)\right) - \left(1+\left(1+\kappa+\beta\right)\vartheta\right)s^H\right) & s_0 = s^H, \end{cases} \tag{A.31}$$

where we use the assumption that the lower bound is binding when $s_t = s^H$ but not when $s_t = s^L$ and, in equation (A.31), we solve out for inflation.

Notice that inflation is a linear function of ϑ. So:

$$\frac{\partial\pi_0}{\partial\vartheta} = \begin{cases} \frac{\kappa}{1+\kappa\phi}\left(1+\kappa+\beta\right)\left(s^H - \frac{\kappa\phi}{1+\kappa\phi}d^L\right) & s_0 = s^L \\ -\kappa\left(1+\kappa+\beta\right)\left(s^H - \frac{\kappa\phi}{1+\kappa\phi}d^L\right) & s_0 = s^H \end{cases} \tag{A.32}$$

Given our maintained assumption $s^H - \frac{\kappa\phi}{1+\kappa\phi} d^L > s^H - s^H = 0$. But also, given the value for π_1 computed above, it is the case that $i_1(s^H) = s^H,\ i_1(s^L) = \frac{\kappa\phi}{1+\kappa\phi} d^L$.
Then, if we define $\Psi = \kappa\left(1+\kappa+\beta\right)\left(s^H - \frac{\kappa\phi}{1+\kappa\phi} d^L\right) > 0$, we can rewrite:

$$\frac{\partial \pi_0}{\partial \vartheta} = \begin{cases} \frac{\Psi}{1+\kappa\phi} & s_0 = s^L \\ -\Psi & s_0 = s^H. \end{cases} \tag{A.33}$$

So, finally, the ex-ante expected value for inflation in period 0:

$$\mathbb{E}\pi_0 = \frac{1}{2}\pi_0\left(s_0 = s^L\right) + \frac{1}{2}\pi_0\left(s_0 = s^H\right) \tag{A.34}$$

Given linearity:

$$\frac{\partial \mathbb{E}\pi_0}{\partial \vartheta} = \frac{1}{2}\frac{\Psi}{1+\kappa\phi} - \frac{1}{2}\Psi = \frac{1}{2}\frac{-\kappa\phi}{1+\kappa\phi}\Psi < 0. \quad \square \tag{A.35}$$