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Abstract

Measures of economic uncertainty are countercyclical, but economic theory does not provide definite guidance on the direction of causation between uncertainty and the business cycle. This paper takes a common-factor approach to the analysis of the interaction between uncertainty and economic activity in a multi-country model without a priori restricting the direction of causality at the level of individual countries. Motivated by the observation that cross-country correlations of volatility series are much higher than cross-country correlations of GDP growth series, we set up a multi-country version of the Lucas tree model with time-varying volatility consistent with this stylized fact and use it to identify two common factors, a real and a financial one. We then quantify the absolute and the relative importance of the common shocks as well as country-specific volatility and GDP growth shocks. The paper highlights three main empirical findings. First, it is shown that most of the unconditional correlation between volatility and growth can be accounted for by shocks to the real common factor, which is extracted from world growth in our empirical model and linked to the risk-free rate in the theoretical model and in the data. Second, the share of volatility forecast error variance explained by the real common shock and by country-specific growth shocks amounts to less than 5%. Third, common financial shocks explain about 10% of the growth forecast error variance, but when such shocks occur, their negative impact on growth is large and persistent. In contrast, country-specific volatility shocks account for less than 1%–2% of the forecast error variance decomposition of country-specific growth rates.

Key words: Uncertainty, business cycle, common factors, real and financial global shocks, multi-country, identification, realized volatility.

JEL classification: E44, F44, G15.

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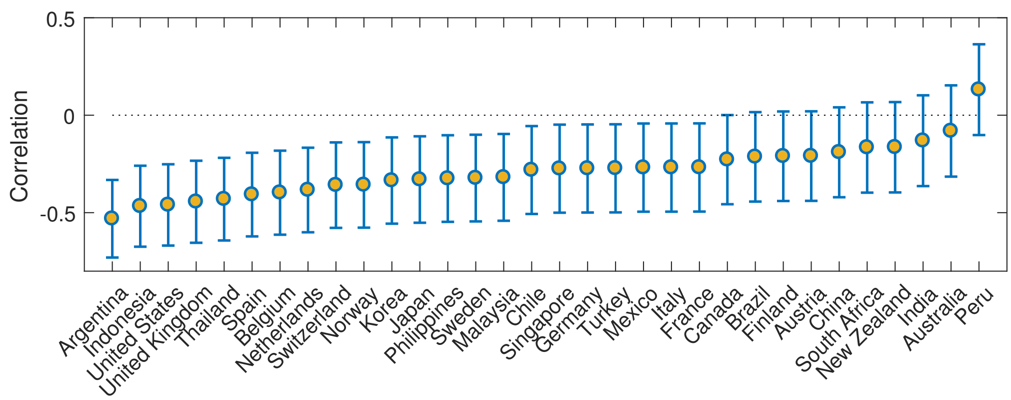
1 Introduction

It is well-established that empirical measures of uncertainty behave countercyclically in the US and most other countries.[1] This negative correlation can be seen in Figure 1 which shows the country-specific contemporaneous correlations between stock market realized volatility and real GDP growth for all 32 countries in our panel together with their 95-percent error band. On average, this correlation is about -0.3, ranging from slightly more than -0.5 for Argentina to just above zero for Peru. With the exception of Austria, China, Indonesia, Peru, and South Africa, these correlations are statistically significant.

Figure 1 Country-specific Correlations Between Volatility and Growth



Note. Correlations between (log) realized stock market volatility and real GDP growth. The dots represent the country-specific contemporaneous correlations, and the lines represent 95% confidence intervals. See equation (59) in Section 6 for a definition of realized volatility at quarterly frequency and Section 7 for a description of the data. Sample period: 1993:Q1-2011:Q2.

Interpreting correlations in economic terms is always difficult because causation can run in both directions. From a theoretical standpoint, uncertainty can cause economic activity to slowdown and even contract through a variety of mechanisms, both on the household side via precautionary savings (Kimball, 1990) and on the firm side via investment delays or other frictions (see for instance Bernanke (1983), Dixit and Pindyck (1994) and, more recently, Bloom (2009), Christiano et al. (2014), Gilchrist et al. (2013), Arellano et al. (2012), Leduc and Liu

[1]For the evidence on the United States see, for example, Schwert (1989a) and Schwert (1989b) using the volatility of aggregate stock market returns; Campbell et al. (2001), Bloom et al. (2007), and Gilchrist et al. (2013) using the volatility of firm-level stock returns; Bloom et al. (2012) and Bachmann and Bayer (2013) using the volatility of plant, firm, industry and aggregate output and productivity; Bachmann et al. (2013) using the behavior of expectations' disagreement. For the evidence on other countries see Baker and Bloom (2013), Carriere-Swallow and Cespedes (2013), and Nakamura et al. (2017) among others.

(2016).[2] But it is also possible that uncertainty responds to fluctuations in economic activity or other unobserved effects. Indeed, the theoretical literature highlights mechanisms through which spikes in uncertainty may be the result of adverse economic conditions. Examples based on information and financial frictions include Van Nieuwerburgh and Veldkamp (2006), Fostel and Geanakoplos (2012), Kozlowski et al. (2015), and Ilut et al. (2017).[3] Theory, therefore, does not provide a definite guidance on how to interpret the countercyclical nature of empirical measures of uncertainty.

In this paper we take a common factor approach to modeling the two-way relationship between volatility and growth in a multi-country framework. In addition to documenting that they are highly correlated within countries, we show that volatility and growth are also highly correlated across countries, but this cross-country correlation is much stronger for volatility than for GDP growth. We exploit this stylized fact to identify two common factors, a real and a financial one. The real factor is identified as common to both volatility and growth and is shown to be sufficient to explain the cross-country correlations of growth series. The financial factor is identified as common only to volatility, after controlling for the real common factor, and is shown to be necessary to capture the remaining cross-country correlations of the volatility series. We then show that the real common factor, which is extracted from world growth in our empirical model and associated with a proxy for the world risk-free rate in the theoretical model and the data, accounts for most of the country-specific unconditional correlation between volatility and growth documented above. We also find that the portion of country-specific volatility driven by common or country-specific growth shocks is small, while shocks to the common financial factor explain a significant share of country-specific growth rates.

For each country in our sample, Figure 2 plots the average pair-wise correlation of volatility and output growth series, together with the average across all countries.[4] It can be seen that the average pair-wise correlation across all countries for the volatility series is more than twice the average for the growth series, at 0.58 and 0.27, respectively (the two dotted lines). This is evidence that, indeed, the volatility is much more correlated across countries than growth.[5]

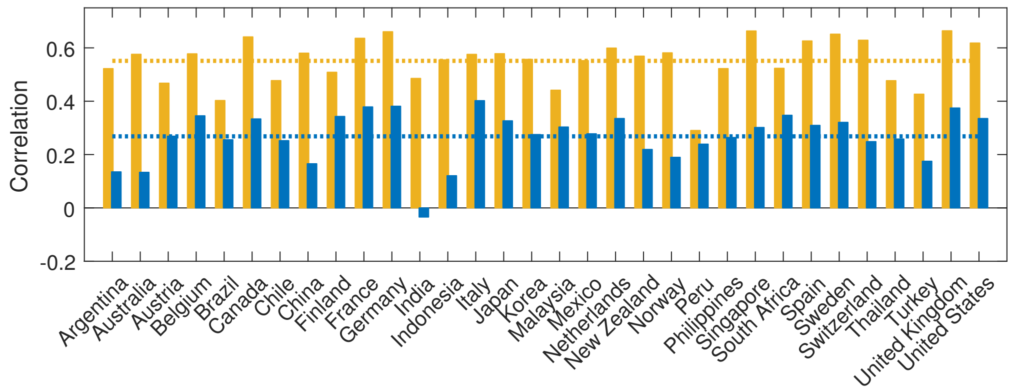
[2] Pricing frictions and the zero lower bound on nominal interest rates can amplify the impact of a volatility shock–see for instance Basu and Bundick (2017).

[3] Theoretically, the impact of uncertainty on activity could even be positive. See, for example, Mirman (1971), Oi (1961), Hartman (1976) and Abel (1983).

[4] The average pair-wise correlation of a variable x for country i (i.e., each bar in Figure 2) is defined as the average bilateral correlation of x_{it} with x_{jt} for all $j \neq i$. See equation (60) in Section 7 for a formal definition.

[5] We note here that these patterns of cross-country correlations are consistent with those documented by Tesar (1995), Colacito and Croce (2011), and Lewis and Liu (2015) for consumption growth and equity returns,

Figure 2 Average Pair-wise Correlations of Volatility and Growth



Note. For each country, the light (yellow) and the dark (blue) bar show the average pair-wise correlation with the remaining countries in the sample for volatility and GDP growth series, respectively. The dotted lines correspond to the overall average across all countries, equal to 0.55 and 0.27 for volatility and GDP growth, respectively. The average pair-wise correlation of a variable x_{it} in country i is the average of the contemporaneous correlation between x_{it} and x_{jt} for all $j \neq i$. See equation (59) in Section 6 for a definition of the realized volatility measure and Section 7 for a description of the data. Sample period: 1993:Q1-2011:Q2.

The empirical evidence in Figure 2 motivates us to adopt a common factor approach in a multi-country framework to the analysis of volatility and output growth. We proceed in two steps. We first develop a multi-country version of the Lucas (1978) tree model in which country-specific output growth (or the dividend growth process) is determined by a common component with time-varying volatility, interpreted as a global technology factor, and an all-encompassing country-specific business cycle component. In this set up, we show that country-specific equity returns and their realized volatility are driven by two common shocks, the first being the global technology factor shock and the second the shock to its conditional volatility. In effect, we develop a consumption-based international asset pricing model in which at least two risk factors are needed to explain the cross-country comovement of equity returns and their realized volatility, even though only one factor is sufficient to explain cross country comovement of output growth, a prediction that is consistent with the cross-country correlation structure of the data observed in Figure 2.

In view of these theoretical derivations, we next set up a multi-country econometric framework where we allow output growth and volatility to be driven by two common factor shocks and two country-specific shocks. To identify the two common factors in the model, we exploit the insights from the stylized facts above and the theoretical model. Specifically, we assume that

respectively. The novelty of our approach is to focus on the volatility and growth dimension.

the first factor, which is common to both country-specific volatility and growth, is sufficient to model the cross-country correlation of output growth, but not to model the volatility correlations; and that a second factor, only common to the volatility series, is needed to model the remaining cross-country correlation of the volatility series after controlling for the first factor. To identify the two common factors, we do not need to impose any restriction on the within-country correlation between country-specific volatility and growth shocks. We then quantify the dynamic response of country-specific volatility and growth to the common factor shocks, and their importance relative to country-specific shocks with impulse responses and forecast error variance decompositions. For identification of country-specific shocks we consider alternative sets of auxiliary assumptions, including the conventional one that country-specific volatility shocks cause growth contemporaneously, finding very similar results.

To measure economic uncertainty, we build on the contributions of Andersen et al. (2001, 2003) and Barndorff-Nielsen and Shephard (2002, 2004) and compute realized equity price volatility for a given quarter by using daily returns for 32 advanced and emerging economies representing more than 90 percent of the world economy. We also consider several other proxies for uncertainty and argue that they are either not suitable for the purpose of our analysis, or not readily available for a large number of countries over a sufficiently long period needed for our analysis, or that they are closely correlated with realized volatility.

The empirical analysis yields a rich set of findings. Here we highlight three main results. First, we find that the bulk of the negative country-specific correlations between volatility and output growth observed in the data can be accounted for by the real common factor. While unconditionally volatility behaves countercyclically for all but one of the 32 countries in our sample, when we condition on shocks to the real factor, the correlations between volatility and growth innovations become statistically insignificant in all but one emerging economy, and quantitatively much smaller in all countries. This result suggests that part of the explanatory power attributed to uncertainty shocks in empirical studies of individual countries, considered in isolation from the rest of the world economy, might be due to omitted common factors. In line with the insight of the theoretical model, we find that shocks to the real common factor, correlate closely with a proxy for the world risk-free rate.

Second, the paper shows that the time-variation of country-specific volatility is explained largely by shocks to the financial factor (with a share of forecast error variance being larger than

60%) and innovations to country-specific volatility series themselves (with a share of forecast error variance of about 35 percent). Shocks to the real common factor and to country-specific growth innovations jointly explain less than 5 percent of volatility forecast error variance.

Third, we find that shocks to the common financial factor explain about 10% of the forecast error variance of country-specific output growth, and they have strong and persistent contractionary effects. In contrast, country-specific volatility shocks explain only $1-2$ percent of country-specific forecast error growth variance. These results illustrate the importance of distinguishing between common and country-specific volatility shocks. In our empirical model, the forecast error variance of output growth is explained mainly by innovations to country-specific growth rates themselves (with a share of at least 60% percent) and the real common shock (with another 25% percent of the total).

The rest of the paper is organized as follows. Section 2 relates our paper to the existing literature. Section 3 sets out the theoretical multi-country model and derives country-specific equity returns and realized volatilities, showing how they relate to a world growth factor. Section 4 considers the identification of the real and financial factors in a static version of our econometric multi-country model. Section 5 extends the analysis to a dynamic setting showing that allowing for heterogeneous dynamics is important in an applied context. Section 6 considers the use of realized volatility as a proxy for uncertainty in a multi-country setting. Section 7 reports key stylized facts of the data. Section 8 reports the estimated common shocks and cross-country correlations of the volatility and growth innovations. Section 9 reports the most important empirical results of the paper that compare unconditional correlations between volatility and growth to the one obtained conditional on our real common shocks. Section 10 reports forecast error variance decompositions and discusses the alternative set of auxiliary assumptions made to identify country-specific shocks. Section 11 presents impulse responses, and Section 12 concludes. Some of the technical proofs and details of the data and their sources are provided in the Appendix. Derivation of impulse responses and variance decompositions, together with additional empirical results as well as selected country-specific results are reported in a separate online supplement to the paper.

Notations: Let $\mathbf{w}=(w_1,w_2,...,w_n)'$ and $\mathbf{A}=(a_{ij})$ be an $n\times 1$ vector and an $n\times n$ matrix, respectively, and denote the largest eigenvalue of $\mathbf{A}$, by $\varrho_{\max}(\mathbf{A})$. Then, $\|\mathbf{w}\|=\left(\Sigma_{i=1}^{n}w_i^2\right)^{1/2}$ and $\|\mathbf{A}\|=\left[\varrho_{\max}(\mathbf{A}'\mathbf{A})\right]^{1/2}$ are the Euclidean (L_2) norm of $\mathbf{w}$, and the spectral norm of $\mathbf{A}$, respectively. $\boldsymbol{\tau}_T = (1, 1, ..., 1)'$ is a $T \times 1$ vector of ones. If $\{y_n\}_{n=1}^{\infty}$ is any real sequence and $\{x_n\}_{n=1}^{\infty}$ is a sequences of positive real numbers, then $y_n = O(x_n)$, if there exists a positive finite constant C_0 such that $|y_n|/x_n \leq C_0$ for all n. $y_n = o(x_n)$ if $f_n/g_n \to 0$ as $n \to \infty$. If $\{y_n\}_{n=1}^{\infty}$ and $\{x_n\}_{n=1}^{\infty}$ are both positive sequences of real numbers, then $y_n = \mathcal{O}(x_n)$ if there exists $N_0 \geq 1$ and positive finite constants C_0 and C_1, such that $\inf_{n \geq N_0}(y_n/x_n) \geq C_0$, and $\sup_{n \geq N_0}(y_n/x_n) \leq C_1$. By "granular" we mean "asymptotically small" in the sense of Chudik and Pesaran (2013).

2 Relation to the Existing Literature

Our paper is closely related to three strands of empirical literature on volatility and growth.[6] A first strand acknowledges that uncertainty has endogenous components and could be driven by the business cycle. See, for instance, Ludvigson et al. (2015) and Berger et al. (2017). The key difference of our work relative to these contributions is that we take a common factor approach to modeling the two-way relationship between volatility and growth in a multi-country framework.[7] The restrictions that we impose to identify the common factors apply to a cross-section of countries, as opposed to a single country considered in isolation from the rest of the world, or the global economy analyzed as a single closed economy. Furthermore, these restrictions are consistent with both the stylized facts of the data and standard asset pricing theory. The identification problem that we pose cannot be addressed in a single country set up. Interestingly, despite the different approaches taken to proxy for uncertainty and to separate endogenous responses to the business cycle from exogenous changes in uncertainty, we reach very similar conclusions.

A second strand of the literature has an international focus as in our paper. For instance, Carriere-Swallow and Cespedes (2013) estimate a battery of 40 small open economy VARs for advanced and emerging economies in which the US VIX index is assumed to be exogenous, and identification is achieved imposing country-by-country restrictions. Baker and Bloom (2013) study an unbalanced panel of 60 countries, documenting the counter-cyclicality of different

[6]The literature is voluminous. See Bloom (2014) for a recent survey. Here we focus only on studies directly related to our paper.

[7]Interestingly, Berger and Vavra (2018)) also show that the open economy environment can be used to provide identification in interpreting counter cyclicality in the dispersion of economic variables. Their work, however, focuses on distinguishing between greater volatility of shocks over time from greater agents response to shocks of a constant size.

proxies for uncertainty, such as stock market volatility, sovereign bond yields volatility, exchange rate volatility and GDP forecast disagreement, and use measures of disaster risk as instruments without quantifying the importance of activity measures for uncertainty. Hirata et al. (2012) estimate a factor-augmented VAR (FAVAR), with factors computed based on data for 18 advanced economies, and use a recursive identification scheme in which the volatility variable is ordered first in the VAR. Carriero et al. (2017) estimate a large Bayesian VAR with exogenously driven stochastic volatility to quantify the impact of macroeconomic uncertainty on OECD economies. Hirata et al. (2012), Carriere-Swallow and Cespedes (2013), Carriero et al. (2017) therefore, restrict the direction of economic causation from the outset of the analysis assuming that the uncertainty proxy used is exogenous. In addition, in our framework, countries interact with each other not only via the common factors, but also via the covariance matrix of the country-specific volatility and growth innovations. In contrast, in the above studies, economies can interact only via common factors or variables like the VIX index, but do not interact with each other via other spillover channels.

Our paper also relates to contributions in the finance literature. The closest analogous to the framework we propose are mean-variance frontier models—discussed, for example, by Black (1976) and French et al. (1987). In those models, however, the focus is on the causal relation between the stock market return and its volatility, via leverage effect or other channels. We model the contemporaneous relation between country-specific GDP growth and stock market volatility. We argue that one can think of GDP as the 'dividend' or the 'cash flow' associated with the country stock market index. In this sense, the novelty of our modeling approach is to work in the dividend-volatility (or cash flow-volatility) space rather than return-volatility space. Indeed, our identification strategy exploits the fact that country-specific dividend growth processes (equated with country GDP growth rates) are less correlated across countries as compared to the cross-market correlation of equity volatilities. While other papers have highlighted similar patterns of cross-country correlations for equity returns and consumption growth (e.g., Tesar (1995), Colacito and Croce (2011), Lewis and Liu (2015), as far as we are aware of, this is the first paper that highlights this property of uncertainty proxies. The novelty of our approach is to focus on the volatility and growth dimension.

3 Equity Returns and Volatility in a Multi-country Business Cycle Model

In this section we set up a multi-country theoretical model of equity market volatility and the business cycle. The model allows us to identify and interpret two common factors in the data, a real and a financial one, which can characterize the cross-country correlation structure of the data consistent with the stylized facts reported in Figure 2. The model is a multi-country version of the Lucas (1978) tree model with global technology shocks that have time-varying volatility. In this framework, there is a link between changes in volatility and business cycle fluctuations via the common (global) risk-free rate that we are going to show to be quantitatively important for interpreting the countercyclical nature of volatility.

Specifically, consider a world consisting of N economies (countries) indexed by $i = 1, 2, ...N$, of similar but not necessarily identical relative sizes, $w_{it} = O(N^{-1})$, where $\Sigma_{i=1}^{N} w_{it} = 1$. We shall also assume that these economies have the same preferences, but are exposed differently to a world growth factor assumed to be exogenously given. This world growth factor is largely, but not exclusively, driven by technology shocks. Each economy i is inhabited by an infinitely-lived representative agent endowed with a stochastic stream of a single homogeneous good $Y_{i,t+s}$, $s = 0, 1, 2, ...$, viewed as the economy's measure of real output or GDP. It is assumed that the country's output growth rate, $\Delta y_{it} = \ln(Y_{it}/Y_{i,t-1})$ fluctuates around a deterministic steady state, $\mathfrak{g}_i$, driven by both the common world growth factor, f_t, and country-specific growth shocks, ε_{it}:

$$\Delta y_{it} = \mathfrak{g}_i + \gamma_i f_t + \varepsilon_{it}, \tag{1}$$

where ε_{it} denotes a stationary process that includes all country-specific forces driving the country's business cycles, including country-specific technology shocks as well as other shocks, possibly including also uncertainty shocks.[8] Despite its simplicity, the assumed country-specific growth process (1) is consistent with multi-country versions of the international real business cycle models of Backus et al. (1992) and Baxter and Crucini (1995), and it is at the core of typical new open-economy DSGE models.[9]

To obtain a closed form solution, we assume that $\varepsilon_{it} \sim IIDN(0, \sigma_{\varepsilon i}^2)$, and that f_t follows a

[8]For a discussion of uncertainty shocks interpreted as demand shocks see Leduc and Liu (2016) and Basu and Bundick (2017)

[9]See Section A.1 of the paper Appendix for the derivation of this specification.

stationary first order auto-regressive process with conditionally heteroskedastic innovations:

$$f_t = \phi_f f_{t-1} + \nu_t, \tag{2}$$

where $|\phi_f| < 1$, $\nu_t \sim IIDN(0, \sigma_\nu^2)$, and

$$Var_{t-1}(\nu_t) = E_{t-1}(\nu_t^2) = a_f + b_f \nu_{t-1}^2, \tag{3}$$

with $a_f > 0$, $0 < b_f < 1$. $E_{t-1}(.) = E(.\,|\mathfrak{I}_{t-1})$ and $Var_{t-1}(.) = Var(.\,|\mathfrak{I}_{t-1})$ denoting conditional expectations and variance operators with respect to the non-decreasing information set, $\mathfrak{I}_{t-1}$. Note here that ν_t is conditionally heteroskedastic, but unconditionally homoskedastic with $Var(\nu_t) = \sigma_\nu^2 = a_f/(1 - b_f) > 0$.[10]

To simplify the exposition, we also assume that ε_{it} for $i = 1, 2, ..., N$ are serially uncorrelated and independently distributed across i, as well as uncorrelated with f_t. However, richer time series dynamics, as well as *weak* forms of cross-country dependence of country specific shocks, could be allowed for. What is crucial for the model derivations is that ε_{it} contains only idiosyncratic (in this case country-specific) risk. Indeed, empirically, we will model the dynamics of country-specific equity market volatility and the business cycle jointly as factor augmented vector autoregressive processes, weakly correlated across countries.

The representative agent of country i can trade freely a globally available risk-free bond and N risky equity claims defined on the country-specific entire endowment streams, $Y_{i,t+s}$, for $s = 0, 1, 2, ...\infty$. International asset markets are complete in the Arrow-Debreau sense so that country-specific consumption growth is equalized across countries, and one can use the world endowment growth in the stochastic discount factor of country i's representative agent.[11]

The representative agent in country i has constant relative risk aversion (CRRA) period utility function and maximizes lifetime utility,

$$E_t \left[\sum_{s=t}^{\infty} \beta^s \left(\frac{C_{is}^{1-\varrho}}{1-\varrho} \right) \right], \tag{4}$$

[10]For further clarity, we will refer to f_t as the "growth factor", and to ν_t as the "innovation" or "shock" to the growth factor.

[11]In this set up, one could prove that asset market are complete in Arrow-Debreau rather than assuming it if we were to restrict the specification of the stochastic processes for ε_{it} and f_t such that the number of uncertain states of the world is less than $N + 1$. See for instance Chapter 5 of Obstfeld and Rogoff (1996) and Aiyagari (1993)).

where $\varrho > 0$ is the coefficient of relative risk aversion and β is the subjective discount factor, both common across countries. The period budget constraint is:

$$C_{it} + B_{i,t+1} + \sum_{j=1}^{N} \theta_{i,t+1}^{(j)} P_{jt} = (1 + r_t^f) B_{it} + \sum_{j=1}^{N} \theta_{it}^{(j)} \left(Y_{jt} + P_{jt} \right), \tag{5}$$

where C_{it} is consumption of country i during period t, B_{it} is the risk-free bond held by country i at the start of period t, with real gross return $1 + r_t^f$. $\theta_{it}^{(j)}$ is the share of country j^{th} income stream held by the representative agent of country i at the start of period t, with ex-dividend market value P_{jt}, subject to the adding-up constraints $\sum_{i=1}^{N} \theta_{it}^{(j)} = 1$, for $j = 1, 2, ...N$.[12] Substituting for C_{it} from (5) in (4), the first order conditions for choosing the bond holding $B_{i,t+1}$, and the N equity holdings, $\theta_{i,t+1}^{(j)}$, are:

$$1 + r_{t+1}^f = \frac{1}{E_t \left[\beta \left(\frac{C_{i,t+1}}{C_{it}} \right)^{-\varrho} \right]}, \text{ for } i = 1, 2, ..., N, \tag{6}$$

and

$$P_{jt} = E_t \left\{ \left[\beta \left(\frac{C_{i,t+1}}{C_{it}} \right)^{-\varrho} \right] \left(P_{j,t+1} + Y_{j,t+1} \right) \right\}, \text{ for } i, j = 1, 2, ..., N. \tag{7}$$

Since by assumption the equity markets are complete, the stochastic discount factor for country i is given by

$$E_t \left[\left(\frac{C_{i,t+1}}{C_{it}} \right)^{-\varrho} \right] = E_t \left[\left(\frac{Y_{w,t+1}}{Y_{wt}} \right)^{-\varrho} \right] = E_t \left[\exp \left(-\varrho \Delta \ln Y_{w,t+1} \right) \right], \tag{8}$$

where $Y_{w,t+1}$ is the world output, defined by $Y_{w,t+1} = \sum_{i=1}^{N} Y_{i,t+1}$. Thus, the above first-order conditions, (6) and (7), can be written as

$$E_t \left[\beta \exp \left(-\varrho \Delta \ln Y_{w,t+1} \right) \right] = \frac{1}{1 + r_{t+1}^f}, \tag{9}$$

and

$$E_t \left\{ R_{i,t+1} \left[\beta \left(\frac{C_{i,t+1}}{C_{it}} \right)^{-\varrho} \right] \right\} = 1, \text{ for } i = 1, 2, ..., N, \tag{10}$$

where $R_{i,t+1}$ is the gross return on country i^{th} endowment defined by $R_{i,t+1} = \left(P_{i,t+1} + Y_{i,t+1} \right) / P_{it}$.

[12] Note that the risk-free rate, r_{t+1}^f, is known at the start of period t, and hence it is included in the information set $\mathfrak{I}_t$.

3.1 Derivation of the Risk-free Rate

We now use the above first order conditions to relate the world growth factor, f_t, to the asset returns. We begin with the risk-free rate and using (1), we note that

$$\Delta \ln Y_{w,t+1} = \ln(1 + \mathfrak{g}_{w,t+1}),$$

where $\mathfrak{g}_{w,t+1} = \left(\sum_{i=1}^{N} Y_{i,t+1} / \sum_{i=1}^{N} Y_{it}\right) - 1$ is the world output growth rate, which can also be written equivalently as

$$\mathfrak{g}_{w,t+1} = \frac{\sum_{i=1}^{N}(Y_{i,t+1} - Y_{it})}{\sum_{i=1}^{N} Y_{it}} = \frac{\sum_{i=1}^{N} Y_{it}\mathfrak{g}_{i,t+1}}{\sum_{i=1}^{N} Y_{it}} = \sum_{i=1}^{N} w_{it}\mathfrak{g}_{i,t+1},$$

where $\mathfrak{g}_{i,t+1} = (Y_{i,t+1}/Y_{it}) - 1$, for $i = 1, 2, ..., N$ are country-specific growth rates, and $w_{it} = Y_{it}/\sum_{j=1}^{N} Y_{jt}$ is the size of country i in the world economy at time t. Also since $\mathfrak{g}_{i,t+1}$ and $\mathfrak{g}_{w,t+1}$ are small they can be well approximated by

$$\begin{aligned} \mathfrak{g}_{w,t+1} &\approx \ln(1 + \mathfrak{g}_{w,t+1}) = \Delta \ln(Y_{w,t+1}) \\ \mathfrak{g}_{i,t+1} &\approx \ln(1 + \mathfrak{g}_{i,t+1}) = \Delta \ln(Y_{i,t+1}) = \Delta y_{i,t+1}, \end{aligned}$$

which yields

$$\mathfrak{g}_{w,t+1} \approx \Delta \ln(Y_{w,t+1}) \approx \sum_{i=1}^{N} w_{it}\Delta y_{i,t+1}.$$

Using this result in (8) and then in (9) now yields

$$1 + r_{t+1}^{f} \approx \frac{1}{E_t\left[\beta \exp\left(-\varrho \sum_{i=1}^{N} w_{it}\Delta y_{i,t+1}\right)\right]}. \tag{11}$$

Finally, using the country-specific output growth equations (1) we also have

$$\sum_{i=1}^{N} w_{it}\Delta y_{i,t+1} = \sum_{i=1}^{N} w_{it}(\mathfrak{g}_i + \gamma_i f_{t+1} + \varepsilon_{i,t+1}) = \mathfrak{g}_{wt} + \gamma_{wt} f_{t+1} + \varepsilon_{w,t+1}, \tag{12}$$

where $\mathfrak{g}_{wt} = \sum_{i=1}^{N} w_{it}\mathfrak{g}_i$, $\gamma_{wt} = \sum_{i=1}^{N} w_{it}\gamma_i$, and $\varepsilon_{w,t+1} = \sum_{i=1}^{N} w_{it}\varepsilon_{i,t+1}$. Note that $\mathfrak{g}_{wt}$ and γ_{wt} are included in the information set $\mathfrak{I}_t$. Under the assumptions that f_{t+1} and $\varepsilon_{i,t+1}$ for

$i = 1, 2, ..., N$ are Gaussian, then conditional on $\Im_t$, $\Delta y_{w,t+1}$ is also Gaussian and we have:

$$E_t\left[\exp\left(-\varrho\sum_{i=1}^{N} w_{it}\Delta y_{i,t+1}\right)\right] = e^{-\varrho\mathfrak{g}_{wt}}E_t\left(e^{-\varrho\gamma_{wt}f_{t+1}-\varrho\varepsilon_{w,t+1}}\right)$$
$$= e^{-\varrho\mathfrak{g}_{wt}-\varrho\gamma_{wt}E_t(f_{t+1})+\frac{1}{2}\left[\varrho^2\gamma_{wt}^2 Var_t(f_{t+1})+\varrho^2 Var_t(\varepsilon_{w,t+1})\right]}.$$

Setting $\beta = 1/(1+r)$ and using the above result in (11) we obtain

$$\ln\left(\frac{1+r_{t+1}^f}{1+r}\right) = \varrho\mathfrak{g}_{wt} + \varrho\gamma_{wt}E_t\left(f_{t+1}\right) - \frac{\varrho^2}{2}\left[\gamma_{wt}^2 Var_t(f_{t+1}) + Var_t\left(\varepsilon_{w,t+1}\right)\right]. \tag{13}$$

But under (2) and (3),

$$E_t\left(f_{t+1}\right) = \phi_f f_t, \text{ and } Var_t(f_{t+1}) = a_f + b_f\nu_t^2.$$

Furthermore, since by assumption the idiosyncratic shocks, ε_{it}, are cross-sectionally independent and $w_{it} = O(N^{-1})$, we also have $Var_t\left(\varepsilon_{w,t+1}\right) = O(N^{-1})$. Therefore, overall we have:

$$r_{t+1}^f \approx \left(r + \varrho\mathfrak{g}_{wt} - \frac{1}{2}\varrho^2\gamma_{wt}^2 a_f\right) + \left(\gamma_{wt}\varrho\phi_f\right)f_t - \frac{1}{2}\left(\varrho^2\gamma_{wt}^2 b_f\right)\nu_t^2 + O\left(N^{-1}\right). \tag{14}$$

This expression shows how the global risk-free rate responds to changes in the composition of world output growth, $\mathfrak{g}_{wt}$, the expected change in the level of the global growth factor, $\left(\gamma_{wt}\varrho\phi_f\right)f_t$, and the expected volatility of the global factor, $\frac{1}{2}\left(\varrho^2\gamma_{wt}^2 b_f\right)\nu_t^2$. An expected increase in level of growth factor increases the risk-free rate, whilst a rise in the expected volatility of the global factor reduces it.

We now show the expression for the risk-free rate, given by (14), can be used to relate equity return volatility and output growth, but to simplify the exposition we abstract from time variations in the weights and set $w_{it} = w_i$. So in what follows we use the following simplified version of (14):

$$r_{t+1}^f \approx \left(r + \varrho\mathfrak{g} - \frac{1}{2}\varrho^2\gamma^2 a_f\right) + \left(\gamma\varrho\phi_f\right)f_t - \frac{1}{2}\left(\varrho^2\gamma^2 b_f\right)\nu_t^2 + O\left(N^{-1}\right), \tag{15}$$

where $\gamma = \gamma_w = \sum_{i=1}^{N} w_i\gamma_i$, and $\mathfrak{g} = \mathfrak{g}_w = \sum_{i=1}^{N} w_i\mathfrak{g}_i$.

3.2 Country Equity Returns and their Realized Volatility

Consider now the first order conditions for the equity returns given by (10), which are non-linear in current and expected future output growth. To obtain an analytical solution, we make use of the approximate present-value relation for stock market returns derived by Campbell and Shiller (1988) (CS, henceforth), and note that in our set up $D_{it} = Y_{it}$. Let $\kappa_{it} = P_{it}/(P_{it} + Y_{it})$ and, following CS, assume that κ_{it} is approximately constant over time and set it to κ_i with $0 < \kappa_i < 1$. Then using result (2') of CS we have

$$r_{i,t+1} = \Delta y_{i,t+1} + \delta_{it} - \kappa_i \delta_{i,t+1}, \tag{16}$$

where $r_{i,t+1} = \ln(R_{i,t+1}) = \ln(P_{i,t+1} + Y_{i,t+1}) - \ln(P_{i,t})$ is the realized gross log-return on country i^{th} equity, $y_{it} = \ln(Y_{it})$, and $\delta_{it} = \ln(Y_{i,t}/P_{it})$.[13] Further, CS show that irrespective of the asset pricing model considered, under rational expectations and assuming that the transversality condition ruling out rational bubbles holds, using result (6) of CS, we also have

$$\delta_{it} = \sum_{j=0}^{\infty} \kappa_i^j \left[E_t\left(r^f_{t+j+1}\right) - E_t\left(\Delta y_{i,t+j+1}\right)\right], \tag{17}$$

where r^f_{t+1} is the (world) risk-free rate as given by (15). Using (1) and (15), therefore, we have

$$E_t\left(\Delta y_{i,t+j+1}\right) = \mathfrak{g}_i + \gamma_i E_t\left(f_{t+j+1}\right), \tag{18}$$

and

$$E_t\left(r^f_{t+j+1}\right) \approx \left(r + \varrho\mathfrak{g} - \frac{1}{2}\varrho^2\gamma^2 a_f\right) + \left(\gamma\varrho\phi_f\right) E_t\left(f_{t+j}\right) - \frac{1}{2}\left(\varrho^2\gamma^2 b_f\right) E_t\left(\nu^2_{t+j}\right) + O\left(N^{-1}\right). \tag{19}$$

Also using (2) and (3) it follows that

$$E_t\left(f_{t+j}\right) = \phi_f^j\ f_t, \text{ and } E_t\left(\nu^2_{t+j}\right) = \frac{\left(1 - b_f^j\right) a_f}{1 - b_f} + b_f^j \nu_t^2. \tag{20}$$

Substituting the above results in (17) and then in (16), after some algebra and lagging by

[13] In their derivations CS use b_t, d_t and r_t, for our $r_{i,t+1}, d_{i,t+1}$ and r^f_{t+1}, respectively. See equations (1) and (5) and the related discussion in CS.

one period we obtain[14]

$$r_{it} = a_r + \gamma\varrho\phi_f f_{t-1} - \frac{1}{2}\varrho^2\gamma^2 b_f \nu_{t-1}^2 + a_{i0}\nu_t + b_{i0}\chi_t + \varepsilon_{it} + O\left(N^{-1}\right), \tag{21}$$

where

$$a_r = r + \varrho\mathfrak{g} \text{ , } \nu_t = f_t - \phi_f f_{t-1} \text{ and } \chi_t = \nu_t^2 - a_f - b_f\nu_{t-1}^2, \tag{22}$$

and

$$a_{i0} = \frac{\gamma_i - \kappa_i\gamma\varrho\phi_f}{1-\kappa_i\phi_f}, \text{ and } b_{i0} = \frac{1}{2}\left(\frac{\kappa_i\varrho^2\gamma^2 b_f}{1-\kappa_i b_f}\right). \tag{23}$$

The above return equation has a number of interesting features. First, the returns are explicitly related to the innovations in the underlying world growth factor, f_t, and its volatility. Second, the factor loadings in (21) vary across countries partly reflecting the different responsiveness of their growth process to f_t, as well as the relative importance of D_{it} in $P_{it} + D_{it}$, as captured by parameter κ_i. This heterogeneity is present even though the risk preference parameter, ϱ, is assumed to be identical across countries. Third, and crucially for our empirical analysis, while only one *common* shock is sufficient to explain cross country differences in output growth, at least two *common* shocks, ν_t and χ_t, are required to explain the cross country differences of equity returns.[15] The innovations ν_t and χ_t, can be viewed as first and second order moment shocks, respectively. The conditional covariance of these two shocks is given by $Cov_{t-1}(\nu_t, \chi_t) = E_{t-1}(\nu_t\chi_t) = E_{t-1}\left(\nu_t^3\right)$, which measures the conditional asymmetry of the technology shock in our model.[16]

In our empirical application, we consider the relationship between output growth and realized volatility of equity returns across countries, computed from squares of daily returns within a quarter to match the available data on output growth–see Section 6 below. To link the above theoretical results to our empirical analysis, denote output growth and equity returns for a given day τ within a quarter t with $\Delta y_{it}(\tau)$, and $r_{it}(\tau)$, respectively, for $\tau = 1, 2, ..., D$, where D is the number of trading days within a quarter (which we assume to be fixed across t for convenience). In this set up, the underlying daily growth factor and country-specific shocks are given by $f_t(\tau)$ and $\varepsilon_{it}(\tau)$. So, in terms of daily changes, the theoretical output growth and equity return

[14]Details of the derivations can be found in the Appendix, sub-section A.2. See equation (A.3).

[15]Note that since $E_{t-1}(\chi_t) = 0$, then χ_t can be viewed as a shock since it is serially uncorrelated and has a zero mean.

[16]Note that since $E_t\left(\zeta_{t+1}^3\right)$ is a conditional measure it need not be equal to zero, even if ζ_t is normally distributed.

equations can be written as

$$\Delta y_{it}(\tau) = \mathfrak{g}_i(\tau) + \gamma_i f_t(\tau) + \varepsilon_{it}(\tau), \tag{24}$$

and

$$r_{it}(\tau) = a_r(\tau) + b_r f_{t-1}(\tau) + c_r \nu_{t-1}^2(\tau) + a_{i0}\nu_t(\tau) + b_{i0}\chi_t(\tau) + \varepsilon_{it}(\tau) + O\left(N^{-1}\right), \tag{25}$$

where $b_r = \gamma\varrho\phi_f$, and $c_r = -\frac{1}{2}\varrho^2\gamma^2 b_f$. Using the above daily models of the output growth and the equity return, the associated quarterly output growth rates and realized equity return volatilities (respectively) are

$$\begin{aligned} \Delta y_{it} &= \sum_{\tau=1}^{D}\mathfrak{g}_i(\tau) + \gamma_i\sum_{\tau=1}^{D} f_t(\tau) + \sum_{\tau=1}^{D}\varepsilon_{it}(\tau) \\ &= \mathfrak{g}_i + \gamma_i f_t + \varepsilon_{it}, \end{aligned} \tag{26}$$

and

$$\begin{aligned} \sigma_{it}^2 &= \sum_{\tau=1}^{D}\left[r_{it}(\tau) - a_r(\tau)\right]^2 \\ &= b_r^2\sum_{\tau=1}^{D} f_{t-1}^2(\tau) + c_r^2\sum_{\tau=1}^{D}\nu_{t-1}^4(\tau) + a_{i0}^2\sum_{\tau=1}^{D}\nu_t^2(\tau) + b_{i0}^2\sum_{\tau=1}^{D}\chi_t^2(\tau) + \sum_{\tau=1}^{D}\varepsilon_{it}^2(\tau) \\ &\quad +2b_r\sum_{\tau=1}^{D} f_{t-1}(\tau)\left[c_r\nu_{t-1}^2(\tau) + a_{io}\nu_t(\tau) + b_{i0}\chi_t(\tau) + \varepsilon_{it}(\tau)\right] \\ &\quad +2c_r\sum_{\tau=1}^{D}\nu_{t-1}^2(\tau)\left[a_{i0}\nu_t(\tau) + b_{i0}\chi_t(\tau) + \varepsilon_{it}(\tau)\right] \\ &\quad +2a_{i0}\sum_{\tau=1}^{D}\nu_t(\tau)\left[b_{i0}\chi_t(\tau) + \varepsilon_{it}(\tau)\right] \\ &\quad +2b_{i0}\sum_{\tau=1}^{D}\chi_t(\tau)\varepsilon_{it}(\tau) + O\left(N^{-1}\right). \end{aligned} \tag{27}$$

It is clear that while individual country returns depend linearly on the first and second order moment innovations, ν_t and χ_t, realized volatility depends on non-linear functions of these innovations and their cross products, and their impacts cannot be identified separately. The presence of χ_t, however, induces *strong* cross sectional dependence (in the sense to be made precise in the following section) in country-specific realized volatilities even if the effects of the growth innovation, ν_t, on r_{it} and σ_{it}^2 are eliminated. In the next section, we will use the model to interpret the difference in the degree of cross sectional dependence of the country output growth rates and realized volatilities documented above, after controlling for the effects of the common growth factor shock, ν_t. All other common terms in (27) will be combined in a single common (or global) financial shock that could also include any additional factors that may influence realized equity market volatilities in the data, such as market imperfections, bubble effects, or time-varying risk preferences.

4 A Static Multi-Country Econometric Framework

Guided by the insights of our model, we now set up a multi-country econometric framework to investigate empirically the relation between realized volatility and quarterly GDP growth. To simplify the exposition, we begin with a static specification, omitting dynamics and deterministic components. We assume that two common shocks and two country-specific shocks drive country specific volatility and growth. Consistent with the theoretical equations (1) and (27), we posit the following unobservable common factor representation:

$$\Delta y_{it} = \gamma_i f_t + \varepsilon_{it}, \tag{28}$$

$$v_{it} = \lambda_i f_t + \theta_i g_t + \eta_{it}, \tag{29}$$

for $i = 1, 2, ..., N$; $t = 1, 2, ..., T$, where as before Δy_{it} is real GDP growth (also referred to 'output growth' or 'growth' for brevity) and $v_{it} = \ln(\sigma_{it})$ is the log of realized stock market volatility for country i during quarter t. In line with our theoretical derivations, f_t can be viewed as the common world growth or technology factor which affects all country growth rates and equity price volatilities contemporaneously. For brevity we shall refer to f_t as the 'real' factor. Consistent with equation (27) of the theoretical model above, the second factor, g_t, is introduced only in the volatility equations and captures all common components not accounted for by f_t. These possibly include also non-fundamental aspects of financial markets, such as over-reactions to news due to excessive optimism/pessimism about future global output growth prospects, ruled out by the no-bubble condition in the derivation of (27). However, note that we do not require the country-specific shocks ε_{it} and η_{it} to be orthogonal, nor are we imposing any ordering of the output growth and volatility variables on the individual country models.

While one could consider theoretical models for which the above triangular factor representation might not hold, as we shall see, there is strong empirical evidence in support of this specification. Indeed, one could start the empirical analysis directly from (28) and (29) as an econometric characterization of the stylized facts of the data described in the introduction. But for the purpose of interpreting the factors, a structural model of the type developed in Section 3 is required, thus providing a set of theoretical assumptions consistent with the properties of the data and the econometric model specified in (28) and (29).

4.1 Factor Identification

In the multi-country econometric specification (28)-(29), there are two identification problems to be solved: identification of the common factors and that of the country specific shocks. Let us focus first on the identification of the factors. We will discuss identification of the country specific shocks in the context of our empirical application, in which we use conventional methods for that purpose.

The main idea of this paper is to achieve identification of f_t and g_t and their loadings, λ_i, γ_i, and θ_i by placing restrictions on the *cross-country* correlations of ε_{it} and η_{it}, while leaving their *within-country* correlation unrestricted. This is a problem that, obviously, cannot be addressed in a single-country framework, or in a model of the world economy viewed as a single entity. In a single country model, the factors in (28)-(29) cannot be identified even if it is assumed that the idiosyncratic shocks are uncorrelated, or by adding more country-specific variables to the model. It is only by adopting a multi-country perspective that we can pose this factor identification problem and solve it. As we will see in the application, the two common factors will also turn out to be important for the interpretation of the relationship between volatility and growth at the level of individual countries. As an example, consider equations (28) and (29) with $N = 1$, and take $i = 1$ to refer to the U.S. economy. In this case, it is readily seen that the triangular factor representation does not impose any restriction on the observed covariance matrix of $(\Delta y_{US,t}, v_{US,t})$. In other words, our identification strategy for the common factors crucially depends on the differential patterns of cross country correlations of output growth and volatility when N is sufficiently large, and does not impose any restrictions on the country specific shocks.

To illustrate the strategy, denote global GDP growth and global volatility by $\Delta \bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$, respectively, and suppose that they are measured by the weighted cross-section averages of country-specific volatility and growth measures namely,

$$\Delta \bar{y}_{\omega,t} = \sum_{i=1}^{N} w_i \Delta y_{it}, \tag{30}$$

$$\bar{v}_{\omega,t} = \sum_{i=1}^{N} \mathring{w}_i v_{it}, \tag{31}$$

where $\{w_i\}$ and $\{\mathring{w}_i\}$ are two sets of aggregation weights, which can be the same. We make the following assumptions on the common factors, f_t and g_t, and their loadings, λ_i, γ_i, and θ_i, the weights, $\mathring{w}_i$ and w_i, and the country-specific innovations, ε_{it} and η_{it}:

Assumption 1 *(Common factors and their loadings) The common unobservable factors f_t and g_t have zero means and finite variances, normalized to one. The factor loadings, λ_i, γ_i, and θ_i, are distributed independently across i and from the common factors f_t and g_t for all i and t, with non-zero means λ, γ, and θ ($\lambda \neq 0$, $\gamma \neq 0$, and $\theta \neq 0$), and satisfy the following conditions, for a finite N and as $N \to \infty$:*

$$N^{-1}\sum_{i=1}^{N}\lambda_i^2 = \mathcal{O}(1), \quad N^{-1}\sum_{i=1}^{N}\gamma_i^2 = \mathcal{O}(1), \quad \text{and} \quad N^{-1}\sum_{i=1}^{N}\theta_i^2 = \mathcal{O}(1), \tag{32}$$

$$\lambda = \sum_{i=1}^{N}\mathring{w}_i\lambda_i \neq 0, \quad \gamma = \sum_{i=1}^{N} w_i\gamma_i \neq 0 \quad \text{and} \quad \theta = \sum_{i=1}^{N} w_i\theta_i \neq 0. \tag{33}$$

Assumption 2 *(Aggregation weights) Let $\mathbf{w} = (w_1, w_2, ..., w_N)'$ and $\mathring{\mathbf{w}} = (\mathring{w}_1, \mathring{w}_2, ..., \mathring{w}_N)'$ be the $N \times 1$ vectors of non-stochastic weights with $w_i, \mathring{w}_i > 0$, $\sum_{i=1}^{N} w_i = 1$ and $\sum_{i=1}^{N}\mathring{w}_i = 1$, such that the following "granularity" conditions are met:*

$$||\mathbf{w}|| = O(N^{-1}), \quad \frac{w_i}{||\mathbf{w}||} = O(N^{-1/2}), \tag{34}$$

and

$$||\mathring{\mathbf{w}}|| = O(N^{-1}), \quad \frac{\mathring{w}_i}{||\mathring{\mathbf{w}}||} = O(N^{-1/2}), \tag{35}$$

for all i.[17]

[17] In practice the weights, w_i and $\mathring{w}_i$ need not to be fixed and could be time-varying but predetermined. The volatility weights, $\mathring{\mathbf{w}}$, can also be allowed to have non-granular components.

Assumption 3 *(Cross-country correlations) The country-specific innovations, η_{it} and ε_{it}, have zero means and finite variances, and are serially uncorrelated, but can be correlated with each other both within and between countries. Furthermore, denoting the covariance matrices of the $N \times 1$ innovation vectors $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \varepsilon_{2t}, ..., \varepsilon_{Nt})'$ and $\eta_t = (\eta_{1t}, \eta_{2t}, ..., \eta_{Nt})'$ by $\boldsymbol{\Sigma}_{\varepsilon\varepsilon} = Var(\boldsymbol{\varepsilon}_t)$ and $\boldsymbol{\Sigma}_{\eta\eta} = Var(\eta_t)$, respectively, it is assumed that:*

$$\varrho_{\max}(\boldsymbol{\Sigma}_{\varepsilon\varepsilon}) = O(1), \tag{36}$$

$$\varrho_{\max}(\boldsymbol{\Sigma}_{\eta\eta}) = O(1). \tag{37}$$

Assumption 1 is standard in the factor literature (see, for instance, Assumption B in Bai and Ng (2002)). It ensures that f_t is a strong (or pervasive) factor for both volatility and growth so that it can be estimated consistently either using principal components or by cross-section averages of country-specific observations (see Chudik et al., 2011).

Assumption 2 requires that individual countries' contribution to world growth or world volatility is of order $1/N$. This is consistent with the notion that, since the 1990s, when our sample period starts, world growth and world capital markets have become progressively more diversified and integrated as a result of the globalization process.

The first part of Assumption 3 is also standard and leaves the causal relation between the idiosyncratic components, ε_{it} and η_{it}, unrestricted. In our model, the correlation between ε_{it} and η_{it} captures any contemporaneous causal relation between volatility and growth at the country level, conditional on f_t, on which we do not impose any restrictions for the purpose of identifying f_t.

Proposition 1 *(Identification of the real factor) Under Assumptions 1-3, f_t can be identified (up to a scalar constant) by $\bar{y}_{\omega,t} = \sum_{i=1}^{N} w_i \Delta y_{it}$, for N sufficiently large.*

Proof. Consider the model (29)-(28) for $i = 1, 2, ..., N$. Under Assumptions 1-3, and using the definitions in (31)-(30), the following model for the global variables obtains:

$$\Delta\bar{y}_{\omega,t} = \gamma f_t + \bar{\varepsilon}_{\omega,t}, \tag{38}$$

$$\bar{v}_{\omega,t} = \lambda f_t + \theta g_t + \bar{\eta}_{\omega,t}, \tag{39}$$

where $\bar{\varepsilon}_{\omega,t} = \mathring{\mathbf{w}}'\boldsymbol{\varepsilon}_t$, and $\bar{\eta}_{\omega,t} = \mathbf{w}'\eta_t$. Furthermore:

$$Var\left(\bar{\varepsilon}_{\omega,t}\right) = \mathbf{w}'\boldsymbol{\Sigma}_{\varepsilon\varepsilon}\mathbf{w} \leq \left(\mathbf{w}'\mathbf{w}\right)\varrho_{\max}\left(\boldsymbol{\Sigma}_{\varepsilon\varepsilon}\right). \tag{40}$$

Thus, under Assumption 3, we have:

$$Var\left(\bar{\varepsilon}_{\omega,t}\right) = O\left(\mathbf{w}'\mathbf{w}\right) = O\left(N^{-1}\right), \tag{41}$$

and hence:

$$\bar{\varepsilon}_{\omega,t} = O_p\left(N^{-1/2}\right). \tag{42}$$

Using this in (38), since under Assumption 1, $\gamma \neq 0$, we have:

$$f_t = \gamma^{-1}\Delta\bar{y}_{\omega,t} + O_p\left(N^{-1/2}\right), \tag{43}$$

which allows us to recover f_t form $\Delta\bar{y}_{\omega,t}$ up to the scalar $1/\gamma$. ■

This is a key result in our paper and several remarks are in order:

Remark 1 *(Estimation of f_t) As f_t is pervasive or strong, we can estimate it with either as the first principal component of the observations $\{\Delta y_{it}$, for $i = 1, 2, ..., N;\ t = 1, 2, ..., T\}$ or by cross-section averages of Δy_{it}, obtaining asymptotically equivalent results.*

Indeed, Figure S.2 in the online supplement shows that the first (static) principal component and the cross section average of Δy_{it} provide estimates of f_t that are very close, with a correlation of 0.9. In the present context, the use of the cross-section-average (CSA) estimator of f_t has two advantages. First, it can be directly interpreted as global GDP growth. Second, under Assumptions 1 and 3 the CSA estimator of f_t is consistent so long as N is large, whilst the principal component estimator requires both N and T to be large (See section 19.5.1 of Pesaran (2015)).

Remark 2 *(Principal component on the panel of volatility series) The cross-section average or the principal component of the panel of volatilities series v_{it} does not identify f_t.*

Remark 3 *(Principal component on the combined panel of growth and volatility series) For the* *same reason, applying principal component analysis to the panel of volatility and growth series does not identify f_t, either.*

Indeed, Figure S.3 in the online supplement shows that the first principal component of the combined panel of volatilities and growth series does not coincides with f_t estimated as the cross-section averages of Δy_{it}, and its correlation with $\Delta \bar{y}_{\omega,t}$ is -0.43. The first principal component extracted from the panel of v_{it} and Δy_{it} captures a linear combination of f_t and any additional common factors that exclusively affect the volatility series.

4.2 Identifying the Financial Factor

The main empirical result of the paper does not require explicit identification of the second strong factor g_t assumed to be exclusive to the volatility series, v_{it}. But doing so permits exploring other properties of the data that underpin the second and the third main empirical results of the paper summarized above. Under the assumptions we made, as the next proposition shows, g_t can be identified from the data as a liner combination of $\Delta \bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$, up to an orthonormal transformation.

Proposition 2 *(Identification of the financial factor) Under the assumptions made, the factor g_t can be identified up to a liner transformation as $N \to \infty$ and is given by*

$$g_t = \theta^{-1} \left(\bar{v}_{\omega,t} - \frac{\lambda}{\gamma} \Delta \bar{y}_{\omega,t} \right) + O_p \left(N^{-1/2} \right). \tag{44}$$

This result follows from substituting (43) and (29) into (39) and applying the same reasoning as in the proof of Proposition 1.

As we noted earlier, we label g_t a 'financial' factor to highlight its role in capturing the effect of χ_t and all other common effects in equation (27) once we account for the common real factor, f_t, as well as any bubble component, financial friction, or time-varying risk preference component that might be present in the volatility data. In effect, our identification assumptions distinguish between a first, level factor, f_t, common to both the growth and volatility series, and all other effects common only to the volatility series, lumped together in g_t. Moreover, we will see below in Proposition 3 that a consistent estimator of g_t can be obtained as the residual of a regression of $\bar{v}_{\omega,t}$ on $\Delta \bar{y}_{\omega,t}$.

4.3 Consistent Estimation of Orthogonalized Real and Financial Factors

For consistent estimation of the real and financial factors we note that under Assumptions (1)-(3), from equation (43) and (44), we have:

$$
\begin{aligned}
f_t &= \alpha_f \Delta \bar{y}_{\omega,t} + O_p(N^{-1/2}) \qquad (45)\\
g_t &= \alpha_{1g} \bar{v}_{\omega,t} - \alpha_{2g} \Delta \bar{y}_{\omega,t} + O_p(N^{-1/2}), \qquad (46)
\end{aligned}
$$

for $t = 1, 2, ...T$, where $\alpha_f = \gamma^{-1}$, $\alpha_{1g} = \theta^{-1}$, and $\alpha_{2g} = \lambda/\gamma\theta$. For N sufficiently large, f_t and g_t can be consistently estimated point-wise (at each t) by a linear combination of $\Delta \bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$, without requiring T to be large. Given the recursive structure of the relation between (f_t, g_t) and ($\Delta \bar{y}_{\omega,t}$, $\bar{v}_{\omega,t}$), f_t can be estimated up to a scalar constant by average world GDP growth. In contrast, g_t is identified as a linear combination of $\Delta \bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$ that is unique only up to an orthonormal transformation. It is evident from (45) and (46) that g_t and f_t are correlated. The next proposition illustrates that we can proxy g_t by setting it equal to the residual of a regression of $\bar{v}_{\omega,t}$ on $\Delta \bar{y}_{\omega,t}$ for all t, and thus making it orthogonal to f_t, without requiring additional economic restrictions.

Proposition 3 *(Consistent estimation of orthonormalized factors in the static case) Let $\hat{\zeta}_t$ and $\hat{\xi}_t$ be consistent, orthonormalized estimators of f_t and g_t, respectively, where f_t and g_t are defined by (43) and (44). Then, $\hat{\zeta}_t$ can be obtained by re-scaling $\Delta \bar{y}_{\omega,t}$ so that its variance is 1, while $\hat{\xi}_t$ can be obtained as the standardized residual of a least squares regression of $\bar{v}_{\omega,t}$ on $\Delta \bar{y}_{\omega,t}$.*

Proof. Consider equation (45) and (46) and set the coefficients $\alpha_g = (\alpha_{1g}, \alpha_{2g})'$, such that $T^{-1} \sum_{t=1}^{T} \hat{\zeta}_t \hat{\xi}_t = 0$. This yields:

$$
\frac{\hat{\alpha}_{2g}}{\hat{\alpha}_{1g}} = \frac{\sum_{t=1}^{T} \Delta \bar{y}_{\omega,t} \bar{v}_{\omega,t}}{\sum_{t=1}^{T} \Delta \bar{y}_{\omega,t}^2},
$$

which is the OLS estimate of the coefficient on $\Delta \bar{y}_{\omega,t}$ in a regression of $\bar{v}_t$ on $\Delta \bar{y}_{\omega,t}$. Next, set α_f and α_{1g} so that ζ_t and ξ_t have unit in-sample standard deviations. Thus:

$$
\hat{\alpha}_f^2 = \left(\frac{1}{T^{-1} \sum_{t=1}^{T} \Delta \bar{y}_{\omega,t}^2} \right)
$$

and:

$$
1 = \hat{\alpha}_{1g}^2 \left(\frac{\sum_{t=1}^{T} \bar{v}_{\omega,t}^2}{T} \right) + \hat{\alpha}_{2g}^2 \left(\frac{\sum_{t=1}^{T} \Delta \bar{y}_{\omega,t}^2}{T} \right) - 2\hat{\alpha}_{1g}\hat{\alpha}_{2g} \left(\frac{\sum_{t=1}^{T} \bar{v}_{\omega,t} \Delta \bar{y}_{\omega,t}}{T} \right).
$$

Hence, we also have:

$$\hat{\alpha}_{1g}^2 = \frac{\left(\frac{\sum_{t=1}^{T}\Delta\bar{y}_{\omega,t}^2}{T}\right)}{\left(\frac{\sum_{t=1}^{T}\Delta\bar{y}_{\omega,t}^2}{T}\right)\left(\frac{\sum_{t=1}^{T}\bar{v}_{\omega,t}^2}{T}\right) - \left(\frac{\sum_{t=1}^{T}\bar{v}_{\omega,t}\Delta\bar{y}_{\omega,t}}{T}\right)^2}.$$

Finally, use $\Delta\bar{y}_{\omega,t} - \Delta\bar{y}_{\omega}$ and $\bar{v}_{\omega,t} - \bar{v}_{\omega}$, where $\Delta\bar{y}_{\omega} = T^{-1}\sum_{t=1}^{T}\Delta\bar{y}_{\omega,t}$ and $\bar{v}_{\omega} = T^{-1}\sum_{t=1}^{T}\bar{v}_{\omega,t}$ in the above formulae to ensure that ζ_t and ξ_t have zero means.[18] ■

Proposition 3 accomplishes two objectives. It derives an observable proxy for g_t and it makes sure that the resultant estimator is orthogonal to the proxy for f_t, which will turn out to be useful when we estimate error variance decompositions and impulse responses. This is achieved simply by choosing coefficients for the linear combination of $\bar{v}_{\omega,t}$ on $\Delta\bar{y}_{\omega,t}$ such that the observable common factors have zero-means, unit variances and are orthogonal to each other.[19]

5 A Dynamic Multi-Country Heterogeneous Model

Whilst the static model considered so far is helpful for illustrative purposes, in empirical applications it is important to take dynamics, possibly differing across countries, into account. As we shall see, allowing for dynamics that differ across countries, while requiring additional regularity conditions and derivations, does not alter our main conclusions regarding identification.

Consider the following first-order dynamic version of the static model (28) and (29):

$$v_{it} = a_{iv} + \phi_{i,11}v_{i,t-1} + \phi_{i,12}\Delta y_{i,t-1} + \lambda_i f_t + \theta_i g_t + \eta_{it}, \quad (47)$$

$$\Delta y_{it} = a_{iy} + \phi_{i,21}v_{i,t-1} + \phi_{i,22}\Delta y_{i,t-1} + \gamma_i f_t + \varepsilon_{it}. \quad (48)$$

In matrix notation we have:

$$\mathbf{z}_{it} = \mathbf{a}_i + \mathbf{\Phi}_i\mathbf{z}_{i,t-1} + \mathbf{\Gamma}_i\mathbf{f}_t + \boldsymbol{\vartheta}_{it}, \quad \text{for } i = 1, 2, ..., N; \ t = 1, 2, ..., T, \quad (49)$$

[18] These mean corrections will be applied automatically if intercepts are included in the country-specific models (28) and (29).

[19] In the present static set up, ζ_t corresponds to the growth factor shock ν_t introduced in Section 3; but ξ_t need not correspond directly to the growth factor volatility innovations, χ_t, also defined in Section 3. This is because, as we noted above, realized volatilities could also be affected by other factors besides growth factor volatility innovations.

where $\mathbf{z}_{it} = (v_{it}, \Delta y_{it})'$ and:

$$\mathbf{a}_i = \begin{pmatrix} a_{iv} \\ a_{iy} \end{pmatrix}, \ \boldsymbol{\Phi}_i = \begin{pmatrix} \phi_{i,11} & \phi_{i,12} \\ \phi_{i,21} & \phi_{i,22} \end{pmatrix}, \ \boldsymbol{\Gamma}_i = \begin{pmatrix} \lambda_i & \theta_i \\ \gamma_i & 0 \end{pmatrix}, \ \mathbf{f}_t = \begin{pmatrix} f_t \\ g_t \end{pmatrix}, \ \boldsymbol{\vartheta}_{it} = \begin{pmatrix} \eta_{it} \\ \varepsilon_{it} \end{pmatrix}.$$

To accommodate the dynamic nature of the model, we now make the following additional assumptions:

Assumption 4 *(Innovations) The country-specific shocks, $\boldsymbol{\vartheta}_{it}$, are serially uncorrelated (over t), and cross-sectionally weakly correlated (over i), with zero means, positive definite covariance matrices, $\boldsymbol{\Omega}_i$, for $i = 1, 2, ..., N$.*

Assumption 5 *(Common factors) The 2×1 vector of unobserved common factors, $\mathbf{f}_t = (f_t, g_t)'$, is covariance stationary with absolute summable autocovariances, and fourth order moments, distributed independently of the country-specific shocks, $\boldsymbol{\vartheta}_{it'}$, for all i, t and t'.*

Assumption 6 *(Factor loadings) The factor loadings λ_i, θ_i, and γ_i (i.e., the non-zero elements of $\boldsymbol{\Gamma}_i$) are independently distributed across i, and of the common factors, $\mathbf{f}_t$, for all i and t, with non-zero means λ, θ, and γ, and second-order moments. Furthermore:*

$$\boldsymbol{\Gamma} = \mathbb{E}\left(\boldsymbol{\Gamma}_i\right) = \begin{pmatrix} \lambda & \theta \\ \gamma & 0 \end{pmatrix}. \tag{50}$$

Assumption 7 *(Coefficients) The constants $\mathbf{a}_i$ are bounded, $\boldsymbol{\Phi}_i$ and $\boldsymbol{\Gamma}_i$ are independently distributed for all i, the support of $\varrho\left(\boldsymbol{\Phi}_i\right)$ lies strictly inside the unit circle, for $i = 1, 2, ..., N$, and the inverse of the polynomial $\boldsymbol{\Lambda}(L) = \sum_{\ell=0}^{\infty} \boldsymbol{\Lambda}_\ell L^\ell$, where $\boldsymbol{\Lambda}_\ell = \mathbb{E}\left(\boldsymbol{\Phi}_i^\ell\right)$ exists and has exponentially decaying coefficients, namely $\left\|\boldsymbol{\Lambda}_\ell\right\| \leq K\rho^\ell$, where K is a fixed constant and $0 < \rho < 1$.*

These assumptions complement, extend and generalize those made earlier for the static case and allow us to derive consistent estimates of unobservable factors f_t and g_t in a heterogeneous factor-augmented VAR, as summarized in the proposition below. The important additional condition is to control the effects of aggregation of dynamics across the units by requiring that $\Lambda_\ell = E\left(\boldsymbol{\Phi}_i^\ell\right)$ exists and has exponentially decaying coefficients. But it is easily seen that this latter condition holds only if it is further assumed that $\sup_i E\left\|\boldsymbol{\Phi}_i\right\| < \rho < 1$. It is also worth noting that $\boldsymbol{\Gamma}$ defined by (50) is invertible since $\gamma\theta \neq 0$ under Assumption 6.

Proposition 4 *(Consistent estimation of unobservable factors in a dynamic heterogeneous multi-country model) Consider the factor-augmented bivariate VAR models for country $i = 1, 2, ..., N$ given by (49),and suppose that Assumptions 4-7 hold. Then:*

$$f_t = b_f + \gamma^{-1}\Delta\bar{y}_{\omega,t} + \sum_{\ell=1}^{\infty} \mathbf{c}'_{1,\ell}\bar{\mathbf{z}}_{\omega,t-\ell} + O_p\left(N^{-1/2}\right), \tag{51}$$

$$g_t = b_g + \theta^{-1}\left(\bar{v}_{\omega,t} - \frac{\lambda}{\gamma}\Delta\bar{y}_{\omega,t}\right) + \sum_{\ell=1}^{\infty} \mathbf{c}'_{2,\ell}\bar{\mathbf{z}}_{\omega,t-\ell} + O_p\left(N^{-1/2}\right), \tag{52}$$

where b_f and b_g are fixed constants, $\bar{\mathbf{z}}_{\omega,t} = (\bar{v}_{\omega,t}, \Delta\bar{y}_{\omega,t})$, $\{w_i, \text{ for } i = 1, 2, ..., N\}$ are fixed weights that satisfy the granularity Assumption 2, and $\mathbf{c}'_{1,\ell}$ and $\mathbf{c}'_{2,\ell}$ are the first and the second rows of $\mathbf{C}_\ell = \mathbf{\Gamma}^{-1}\mathbf{B}_\ell$, where $\mathbf{\Gamma} = \mathbb{E}(\mathbf{\Gamma}_i)$, $\mathbf{B}_\ell$ is defined by $\mathbf{\Lambda}^{-1}(L) = \mathbf{B}_0 + \mathbf{B}_1 L + \mathbf{B}_2 L^2 + ...$, $\mathbf{\Lambda}(L) = \sum_{\ell=0}^{\infty}\mathbf{\Lambda}_\ell L^\ell$, and $\mathbf{\Lambda}_\ell = \mathbb{E}\left(\mathbf{\Phi}_i^\ell\right)$, for all i.

Proof. See Appendix A.3. ■

Notice here that, as shown in Pesaran and Chudik (2014) and Chudik and Pesaran (2015), if slope heterogeneity is not extreme (i.e., if the coefficient matrices $\mathbf{\Phi}_i$ do not differ too much across i) and $\mathbf{C}_\ell$ decays exponentially in ℓ, the infinite order distributed lag functions in $\bar{\mathbf{z}}_{\omega,t}$ can be truncated. In practice, Pesaran and Chudik (2014) and Chudik and Pesaran (2015) recommend a lag length ℓ equal to $T^{1/3}$, where T is the time dimension of the panel. Notice also that f_t and g_t are unobservable, while for estimation purposes we need observable factors. However, as f_t is identified up a scalar, while g_t is identified up to a linear combination of $\bar{v}_{\omega,t}$ and $\Delta\bar{y}_{\omega,t}$, we can continue to proceed similarly to the the case of the static model, as the next proposition illustrates.

Proposition 5 *(Consistent estimation of the orthonormalized factors in the dynamic case) Consider a p^{th} order truncated approximation of the unobservable factors in equation (51) and (52) above, and note that in matrix notations we have:*

$$\mathbf{f} = \Delta\bar{\mathbf{y}}_\omega + \bar{\mathbf{Z}}_\omega\mathbf{C}_1 + O_p\left(N^{-1/2}\right), \tag{53}$$

$$\mathbf{g} = \bar{\mathbf{v}}_\omega - \lambda\Delta\bar{\mathbf{y}}_\omega + \bar{\mathbf{Z}}_\omega\mathbf{C}_2 + O_p\left(N^{-1/2}\right), \tag{54}$$

where $\mathbf{f} = (f_1, f_2, ..., f_T)'$, $\mathbf{g} = (g_1, g_2, ..., g_T)'$, $\bar{\mathbf{Z}}_\omega = (\tau_T, \bar{\mathbf{z}}_{\omega,-1}, \bar{\mathbf{z}}_{\omega,-2}, ..., \bar{\mathbf{z}}_{\omega,-p})$, $\bar{\mathbf{z}}_{\omega,-l} = (\Delta\bar{\mathbf{y}}_{\omega,-l}\bar{v}_{\omega,-l})$, $\Delta\bar{\mathbf{y}}_{\omega,-l} = (\Delta\bar{y}_{\omega,1-l}, \Delta\bar{y}_{\omega,2-l}, ..., \Delta\bar{y}_{\omega,T-l})'$, $\Delta\bar{\mathbf{y}}_\omega = \Delta\bar{\mathbf{y}}_{\omega,0}$, $\bar{\mathbf{v}}_{\omega,-l} = (\bar{v}_{\omega,1-l}, \bar{v}_{\omega,2-l}, ..., \bar{v}_{\omega,T-l})'$,

$\bar{\mathbf{v}}_\omega = \bar{\mathbf{v}}_{\omega,0}$, *and* p *denotes a suitable number of lags (or truncation order).*[20] *Consistent estimators of the common shocks, denoted* ζ *and* ξ, *can be obtained as residuals from the following OLS regressions:*

$$\Delta\bar{\mathbf{y}}_\omega = \bar{\mathbf{Z}}_\omega\hat{\mathbf{C}}_1 + \hat{\zeta}, \tag{55}$$

$$\bar{\mathbf{v}}_\omega = \hat{\lambda}\hat{\zeta} + \bar{\mathbf{Z}}_\omega\hat{\mathbf{C}}_2 + \hat{\xi}. \tag{56}$$

where $\hat{\mathbf{C}}_1$ *is the OLS estimator of the regression coefficients in the regression of* $\Delta\bar{\mathbf{y}}_\omega$ *on* $\bar{\mathbf{Z}}_\omega$, *and* $\hat{\lambda}$ *and* $\hat{\mathbf{C}}_2$ *are OLS estimators of the regression coefficients in the regression of* $\bar{\mathbf{v}}_\omega$ *on* $\hat{\zeta}$ *and* $\bar{\mathbf{Z}}_\omega$.

Proof. See Appendix A.4. ■

Remark 4 *Since* $\hat{\zeta}_t$ *and* $\hat{\xi}_t$ *are the residuals from regressions of* $\Delta\bar{y}_{\omega,t}$ *and* $\bar{v}_{\omega,t}$ *on an intercept and the lagged values* $\bar{z}_{\omega,t-1}, ..., \bar{z}_{\omega,t-p}$, *it follows that* $\hat{\zeta}_t$ *and* $\hat{\xi}_t$ *will have zero (in-sample) means and, for a sufficiently large value of* p, *will be serially uncorrelated. Therefore,* $\hat{\zeta}_t$ *and* $\hat{\xi}_t$ *can be viewed as estimators of the global innovations (or shocks) to the underlying factors,* f_t *and* g_t. *Unlike the theoretical innovations* ν_t *and* χ_t, *defined by (22), which could be correlated, the estimators* $\hat{\zeta}_t$ *and* $\hat{\xi}_t$ *are orthogonalized.*

Remark 5 *In a dynamic setting, the orthogonalized components of* $\Delta\bar{y}_{\omega,t}$ *and* $\bar{v}_{\omega,t}$, *obtained by projecting* $\bar{v}_{\omega,t}$ *on* $\Delta\bar{y}_{\omega,t}$, *are not the same as our global shocks* $\hat{\zeta}_t$ *and* $\hat{\xi}_t$, *because this would ignore the contributions of* $\bar{\mathbf{z}}_{\omega,t-\ell}$ *for* $\ell = 1, 2, ..., p$ *to the estimation of* f_t *and* g_t. *As the factors depend on lagged variables, it is important to make sure that the past values of* $\bar{\mathbf{z}}_{\omega,t}$ *are filtered out.*

Given the orthogonal factor innovations, $\hat{\zeta}_t$ and $\hat{\xi}_t$, obtained from equation (55) and (56), by substituting them in (49) we can investigate their impact and relative importance for country-

[20] The inclusion of τ_T in $\bar{\mathbf{Z}}_\omega$ ensures that the filtered factors have zero in-sample means.

specific volatility and growth based on the following regressions:[21]

$$v_{it} = a_{iv} + \phi_{i,11} v_{i,t-1} + \phi_{i,12} \Delta y_{i,t-1} + \beta_{i,11} \hat{\zeta}_t + \beta_{i,12} \hat{\xi}_t + \sum_{\ell=1}^{p} \psi'_{v,i\ell} \bar{\mathbf{z}}_{\omega,t-\ell} + \eta_{it}, \quad (57)$$

$$\Delta y_{it} = a_{iy} + \phi_{i,21} v_{i,t-1} + \phi_{i,22} \Delta y_{i,t-1} + \beta_{i,21} \hat{\zeta}_t + \sum_{\ell=1}^{p} \psi'_{\Delta y,i\ell} \bar{\mathbf{z}}_{\omega,t-\ell} + \varepsilon_{it}. \quad (58)$$

These country-specific equations can be estimated consistently by least squares so long as N and T are sufficiently large. As in the static case, large N is required so that the probability order $O_p(N^{-1/2})$ in equations (53) and (54) become negligible. Large T is required to ensure that the dynamics are estimated reasonably accurately. We are now ready to present our empirical results, but before doing so we need to discuss how we measure volatility in our multi-country setting.

6 Volatility Measurement

As a proxy for uncertainty we use realized equity price volatility. Realized volatility has been used extensively in the theoretical and empirical finance literature and implicitly assumes that uncertainty and risk can be characterized in terms of probability distributions.[22] Specifically, we use a measure of quarterly realized volatility based on the summation of daily squared stock price returns. This is a natural application of within-day measures of volatility based on high frequency within-day price changes.[23]

Denote the daily equity price of country i, measured at close of day τ in quarter t as $P_{it}(\tau)$. The realized volatility for country i in quarter t is computed as:

$$\sigma_{it}^2 = \sum_{\tau=1}^{D_t} (r_{it}(\tau) - \bar{r}_{it})^2 \quad (59)$$

where $r_{it}(\tau) = \Delta \ln P_{it}(\tau)$, and $\bar{r}_{it} = D_t^{-1} \sum_{\tau=1}^{D_t} r_{it}(\tau)$ is the average daily price changes in the quarter t, and D_t is the number of trading days in quarter t, so that the realized volatility in (59) is expressed at quarterly rate. Note that, for most time periods, $D_t = 3 \times 22 = 66$, which is

[21] We describe how we can compute the relative importance of these factors for the forecast error variance decomposition of country-specific variables, and the impulse response function of the country specific variables to these shocks in the online supplement to the paper.

[22] It therefore abstracts from Knightian uncertainty, where one cannot attach probabilities to outcomes.

[23] See, for example, Andersen et al. (2001, 2003), Barndorff-Nielsen and Shephard (2002, 2004))

larger than the number of data points typically used in the construction of daily realized market volatility in the empirical finance literature.[24] Note also that, because variances have right-skewed distributions, but logarithmic variances tend to have near Gaussian distributions, in our empirical application we will be working with the logarithm of realized volatility measures, i.e. $v_{it} = \log(\sigma_{it})$.

The realized volatility of asset prices is not the only way of measuring 'risk' or 'uncertainty'. If we consider a panel of country equities (e.g., of firms or sectors *within* a country), a different measure of uncertainty can be computed as the cross-sectional dispersion of equity prices within each country. In Section S1 of the online supplement we show that, under fairly general assumptions, and for D_t relatively large (as in our sample), the cross-sectional dispersion of equity returns within country i is closely related to the realized volatility of the country equity returns. So, in our application, we will focus on the realized volatility of country equity indexes.[25]

Realized volatility and cross-sectional dispersion encompass most measures of uncertainty and risk proposed in the literature that could be used to implement our identification strategy. Schwert (1989b), Ramey and Ramey (1995), Bloom (2009), Fernandez-Villaverde et al. (2011) use aggregate time series volatility (i.e., summary measures of dispersion over time of output growth, stock market returns, or interest rates). Bloom et al. (2007) and Gilchrist et al. (2013) use dispersion measures at the firm-level stock market returns, while Bloom et al. (2012) use cross-sectional dispersion of plant, firm, and industry profits, stocks, or total factor productivity.

In the finance literature, the focus of the volatility measurement has now shifted to implied volatility measures obtained from option prices, like the US VIX Index. However, a key input for the implementation of our identification strategy is the availability of country-specific measures of uncertainty for a large number of countries over a long period of time, and implied volatility measures are not yet available for a meaningful number of countries. Moreover, Berger et al. (2017) show that, conditional on realized volatility, VIX measures of expected future uncertainty are not associated with indicators of economic activity like the unemployment rate or output growth.

The literature has also used uncertainty measures based on expectations dispersion such as,

[24]In the case of intra-day observations, for example, prices are usually sampled at 10-minutes intervals which yield around 48 intra-daily returns in an 8-hour trading day.

[25]Daily returns are computed abstracting from dividends, which are negligible by comparison to price changes at this frequency.

for instance, the one proposed by Bachmann et al. (2013) in the case of the United States and by Rossi and Sekhposyan (2015) in the international context. While the data set of Rossi and Sekhposyan (2015) covers a large number of countries, the time series dimension is unbalanced and often not long enough for our purposes. Finally, model based measures, such as those in Jurado et al. (2015) and Ludvigson et al. (2015) could in principle be computed for all countries in our sample, but the data requirements to construct such proxies for many countries over a sufficiently long time period are prohibitive.

7 Data and Selected Stylized Facts for Volatility and Growth

This section briefly describes the data set we use in the empirical analysis and reports some stylized facts based on the unconditional moments of the data. Specifically, we consider the degree of persistence in the growth and volatility series, which is relevant for our model specification, and examine the patterns of cross-country correlations that play an important role in our identification strategy.

The sources of the data and their sampling information are reported in Appendix B. To construct a balanced panel for the largest number of countries for which we have sufficiently long time series, we first collect daily stock prices for 32 advanced and emerging economies from 1979 to 2011. We then cut the beginning of the sample in 1993, as daily equity price data are not available earlier for two large emerging economies (Brazil and China) and for Peru. Better quality quarterly GDP data for China also became available from 1993. Our results seem to be robust to excluding these three countries and starting the sample in 1988. Moreover, some steps of the empirical analysis, like the estimator of factor innovations ($\hat{\zeta}_t$ and $\hat{\xi}_t$), can be implemented with the unbalanced panel from 1979 without any significant consequence for our main findings.

7.1 Persistence

A battery of summary statistics on the realized volatility series and the real GDP series (in levels) supports our model specification in terms of the log-level of realized volatility and the log-differences of real GDP. As Table S.1 in the online supplement shows, the levels of realized volatility, even though persistent, tend to be mean-reverting. Table S.2 in the supplement shows that the first order auto-correlation coefficient for realized volatility is on average about 0.6. Also standard ADF tests reject the null hypothesis of unit roots in the volatility series. In contrast, the persistence of real GDP levels is very high (on average around 0.99). Moreover, the null of a unit root for the level of log-GDP cannot be rejected by standard ADF tests for any of the 32 countries in our sample.

7.2 Cross-country Correlations

The differential pattern of cross-country correlations of the growth and volatility *innovations* is crucial for our identification strategy. Here we consider the properties of the observed *time series* as displayed in Figure 2. In order to gauge the extent to which volatility and growth series co-move across countries, we use two techniques: standard principal component analysis and pair-wise correlation analysis across countries.

In a panel of countries indexed by $i = 1, 2, ..., N$, the average pair-wise correlation of country i in the panel ($\bar{\rho}_i$) measures the average degree of co-movement of country i with all other countries j (i.e., for all $j \neq i$). The average pair-wise correlation *across* all countries, denoted by $\bar{\rho}_N$, is defined as the cross-country average of $\bar{\rho}_i$ over $i = 1, 2, ..., N$. This statistics relates to the degree of pervasiveness of the factors, as measured by the factor loadings. To see this, consider equation (28) of our model, $\Delta y_{it} = \gamma_i f_t + \varepsilon_{it}$, where $Var(f_t) = 1$, and $Var(\varepsilon_{it}) = \sigma^2_{\varepsilon_i}$. The average pair-wise correlation across all countries is given by:

$$\bar{\rho}_N = \frac{2}{N(N-1)} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \rho_{ij} = \frac{1}{N(N-1)} \left(\sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{ij} - N \right), \tag{60}$$

where

$$\rho_{ij} = \begin{cases} \frac{\tilde{\gamma}_i}{\sqrt{1+\tilde{\gamma}_i^2}} \frac{\tilde{\gamma}_j}{\sqrt{1+\tilde{\gamma}_j^2}} & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$$

and $\tilde{\gamma}_i = \gamma_i / \sigma_{\varepsilon_i}$. Substituting the above expression for ρ_{ij} in (60) we have:

$$\bar{\rho}_N = \frac{N}{N-1} \left(\frac{1}{N} \sum_{i=1}^{N} \frac{\tilde{\gamma}_i}{\sqrt{1+\tilde{\gamma}_j^2}} \right)^2 - \frac{1}{N-1}.$$

Hence

$$\bar{\rho}_N = O\left(\bar{\gamma}_N^2\right), \tag{61}$$

where $\bar{\gamma}_N = N^{-1}\sum_{i=1}^{N}\tilde{\gamma}_i$ measures the degree of pervasiveness of the factor.

The attraction of the average pair-wise correlation, $\bar{\rho}_N$, lies in the fact that it applies to multi-factor processes, and unlike factor analysis does not require the factors to be strong. In fact, the average pair-wise correlation, $\bar{\rho}_N$, tends to be a strictly positive number if Δy_{it} contains at least one strong factor, otherwise it tends to zero as $N \to \infty$. Therefore, non-zero estimates of $\bar{\rho}_N$ are suggestive of strong cross-sectional dependence.[26] For completeness, and to show that our analysis is robust to using an alternative methodology, in what follows, we also use standard principal component analysis.[27]

Country-specific average pair-wise correlations of volatility and GDP growth are reported in Figure 2. Recall that the average pair-wise correlation across all countries for the realized volatility series is 0.56. In contrast, the average pair-wise correlation across all countries for the growth series at 0.27 is much smaller. As we can see, the pair-wise correlations of volatility and growth have a similar values for different countries, but there is a clear difference between the two variables. This suggests that both variables may share at least one strong common factor, even though volatilities seem to co-move more across countries than the GDP growth rates.

Principal component analysis yields similar results. The first principal component in our panel of realized volatility series explains 65 percent of the total variation in the log-level of volatility, whilst the first principal component of the growth series accounts for only around 30 percent of total cross-country variations in these series. Thus, both in the case of the pair-wise correlation and principal component analysis, the results point to a much higher degree of cross-country co-movements for the volatility series than for the growth series. As we will see, these differences are even more pronounced in the case of the estimated innovations series obtained using equations (57) and (58).

8 Estimated Common and Country-specific Components

The preliminary analysis above is compatible with the common factor model proposed in the paper, suggesting a stronger degree of cross-country co-movements for volatility series as compared to the growth series. The summary statistics reported also support the model specification in

[26]Formal tests of cross-sectional dependence based on estimates of $\bar{\rho}_N$ are discussed in Pesaran (2015) and reported, for our panel of countries, in the next section.

[27]See also Chapter 29 in Pesaran (2015).

terms of log-level of volatility and log-difference of growth. Now we will use our multi-country factor-augmented VAR model, (57) and (58), to interpret the observed negative association between volatility and growth.

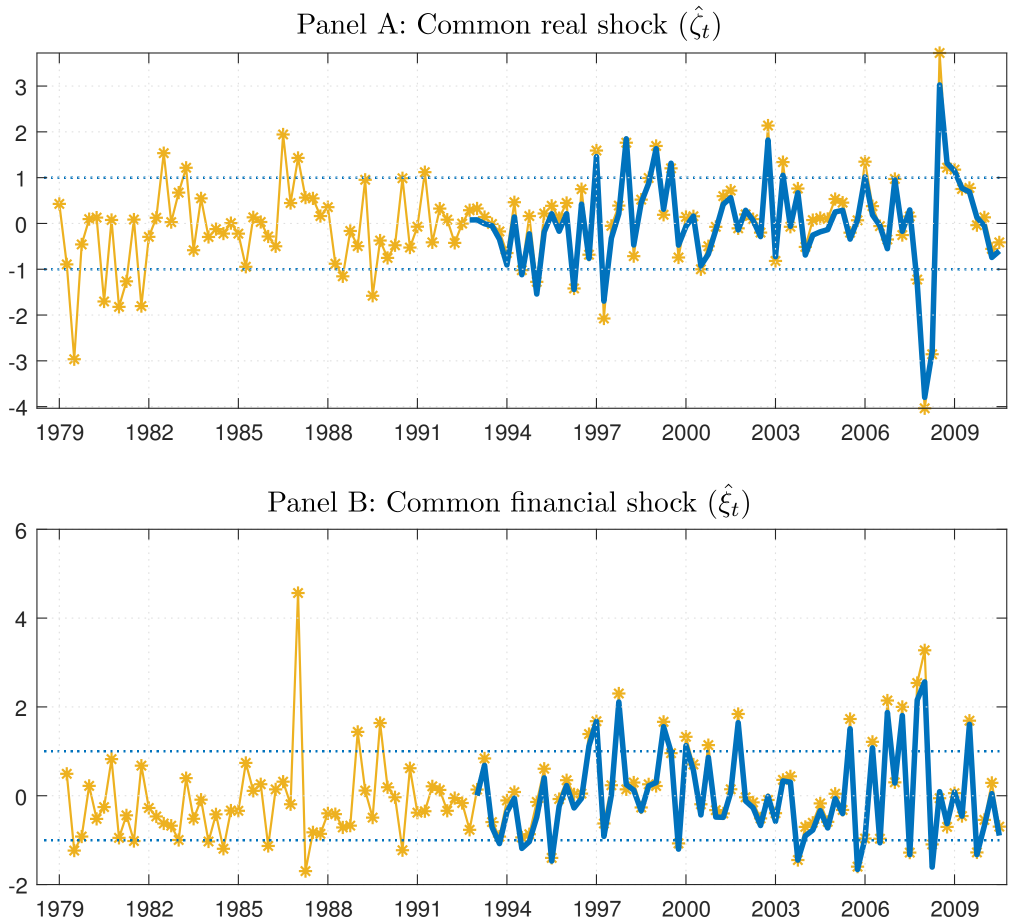
We begin by estimating the global factor innovations, ζ_t and ξ_t, using (55) and (56). We then estimate country-specific VAR models conditional on these estimated innovations and obtain the country-specific growth and volatility innovations, ε_{it} and η_{it}. The necessary computations are carried out by applying OLS to (57)-(58) for each i, separately. Finally, we compute and report *conditional* pair-wise correlations across countries for country-specific volatility and growth innovations to evaluate our identification assumptions, and within-country correlations between volatility and growth innovations to assess the model's ability to capture the countercyclical nature of realized volatility. It is important to note here that we will also estimate country-specific volatility innovations conditional only on $\hat{\zeta}_t$ in (57)-(58), rather than conditional on both $\hat{\zeta}_t$ and $\hat{\xi}_t$, and denote these estimated innovations by $\hat{u}_{it}$.

8.1 Estimated Global Real and Financial Shocks

Estimates of the global shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$, are recovered from the OLS estimation of (55) and (56). Figure 3 plots them when estimated using the unbalanced panel from 1979 (thin lines with asterisks), and when we use the balanced panel from 1993 (thick solid lines), so as to better illustrate their time profiles. The figure also reports one-standard deviation bands for the shocks. Note that the shocks are standardized and have zero means and unit in-sample variances. They are also serially uncorrelated and orthogonal to each other by construction. Interestingly, the Jarque-Bera test strongly rejects normality in the case of the real shocks, with strong evidence of left skewness and kurtosis, and marginally rejects in the case of the financial shock with only mild evidence of right skewness.

The figure shows that the largest negative realization of the real common shock was after the second oil shock in 1979, and during the fourth quarter of 2008 after the Lehman's collapse, consistent with prevailing narratives on the characterization of world recessions. Figures 4, Panel A plots the estimated real common shocks against changes in a proxy for the global risk-free rate calculated as the simple average of the country-specific estimates of the Fisherian natural rate of interest for the United States, Canada, the Euro Area, and the United Kingdon from Holston et al. (2017). Consistent with our theoretical analysis, innovations to the global real factor,

Figure 3 Estimated Common Real ($\hat{\zeta}_t$) and Financial ($\hat{\xi}_t$) Shocks

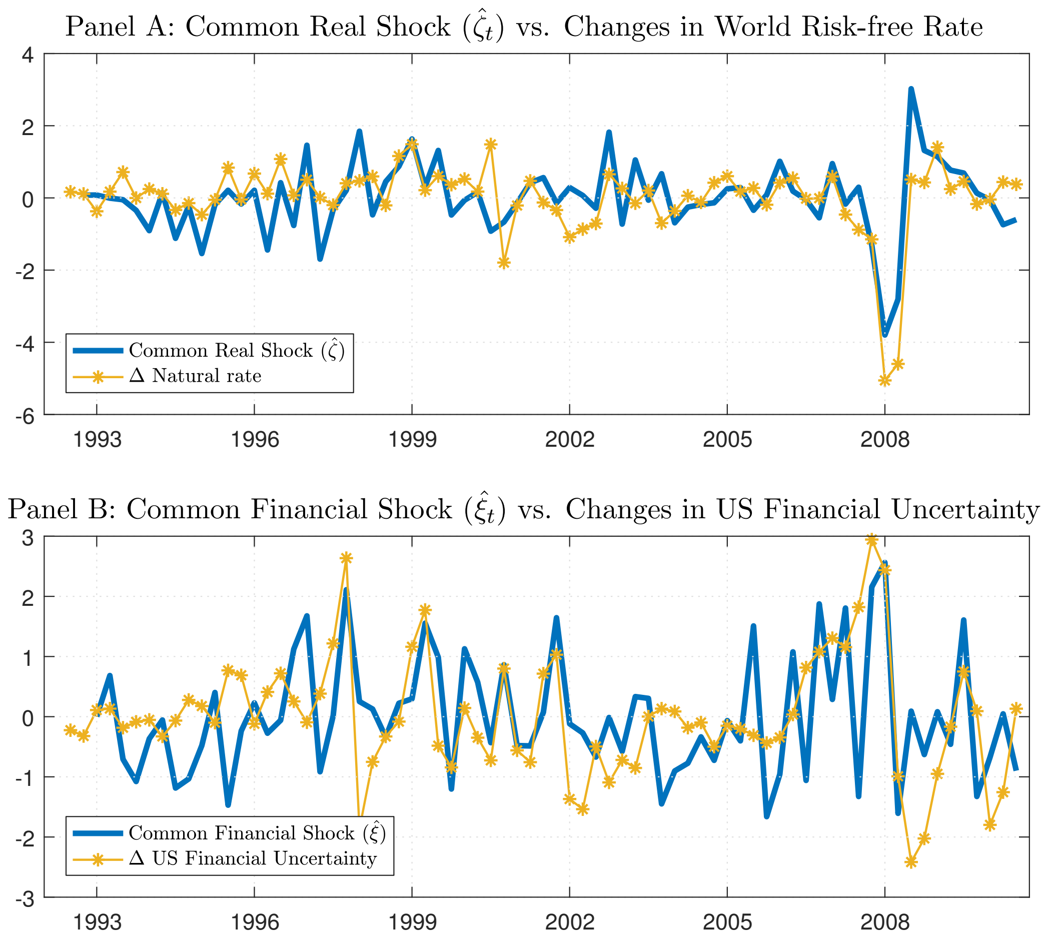


Note. The common shocks $\hat{\zeta}_t$ and $\hat{\xi}_t$ are computed using (55) and (56), with one lag of $\mathbf{z}_{it}$, using the full unbalanced sample 1979:Q2-2011:Q2 (thin lines with asterisks) and the shorter balanced sample 1993:Q1-2011:Q2 (thick solid lines). The shocks are standardized and the dotted lines are the one-standard deviation bands around the zero mean.

which are common to both country-specific volatility and output growth, are closely associated with changes in this proxy for the world risk-free rate (with a correlation coefficient of 0.65) throughout the sample period over which they overlap, except in 2002 and 2003. Note, however, that when we use only the estimate for the United States, as opposed to the average of the four rates, this correlation vanishes.

Figure 3 illustrates that the largest realizations of the common financial shock, $\hat{\xi}_t$ coincide with the 1987 stock market crash and the 2008 Lehman's collapse. However, our series of global financial shocks has a distinct information content as compared to US-specific measures of financial uncertainty. The correlation between our global financial shocks and changes in the

Figure 4 Estimated Common Shocks, the World Risk-free Rate, and US Financial Uncertainty



Note. The common shocks $\hat{\zeta}_t$ and $\hat{\xi}_t$ are the same as in Figure 3. The proxy for the risk-free rate (line with asterisks in Panel A is the simple average of the country-specific estimates of the natural rate of interest for the United States, Canada, the Euro Area, and the United Kingdon as defined and estimated by Holston et al. (2017). The US financial uncertainty measure (line with asterisks in Panel B) is taken from by Jurado et al. (2015). Both the proxy for the risk-free rate and the measure of US financial uncertainty are in first differences and standardized to have in sample zero mean and unit variance like our common shocks.

measure of US financial uncertainty of Ludvigson et al. (2015) is 0.43. The two series, differ during periods in which there is no financial stress in the United States, like 1993-1996 and 2003-2006, and they move closely together during periods in which the United States economy itself is under strain. For example, in 1998-2002 the large hedge fund Long Term Capital Management nearly collapsed after the Russian default, the dotcom bubble burst, and the Twin Towers were attached in 2011. Similarly, in 2007-2009 the United States was at the epicenter of the global financial crisis.

In the online supplement to the paper, we compare the above results with those obtained using the principal components. Specifically, we first show that, when we recover $\hat{\zeta}_t$ using principal components applied to the panel of growth rates, Δy_{it}, we obtain virtually the same results, as expected and highlighted in Remark 1 above. Second, we show that when we apply principal component analysis to the panel of volatilities, (v_{it}), or volatilities and growth rates, $(\Delta y_{it}', v_{it}')'$, we do not recover $\hat{\zeta}_t$, as also stated earlier in Remarks 2 and 3. Finally, in the online supplement, we show that one can approximately recover $\hat{\zeta}_t$ and $\hat{\xi}_t$ by applying the principal component analysis in a recursive manner, provided the recursive estimation is carried out with Δy_{it} first, followed by v_{it}, and not *vice versa*.

8.2 Cross-country Correlations of Volatility and Growth Innovations

Although the restrictions behind our identification assumptions cannot be formally tested, our multi-country approach permits us to investigate the extent to which the implications of the identified model are in line with the identification restrictions made.[28] To this end, we explore the cross-country correlations of the estimated residuals from the dynamic regressions (57) and (58), with and without conditioning on the financial shocks series, $\hat{\xi}_t$.[29] Panel A of Figure 5 plots, for each country in our sample, the average pair-wise correlation of the growth innovations, and the volatility innovations when we condition only on $\hat{\zeta}_t$ in model (57)-(58). Panel B reports the same statistics when we condition on both $\hat{\zeta}_t$ and $\hat{\xi}_t$ in model (57)-(58).[30]

Panel A of Figure 5 shows that, if we condition only on $\hat{\zeta}_t$ in (57)-(58), the volatility innovations display average pair-wise correlations comparable to those of the data reported for all countries in Figure 2. In contrast, the pair-wise correlations of the growth innovations are negligible, with an average across all countries of 0.03.[31] Panel B of Figure 5 also shows that, if we condition on both $\hat{\zeta}_t$ and $\hat{\xi}_t$, the cross-country correlations of the volatility innovations are now negligible, as in the case of the growth innovations, with an average pair-wise correlation across all countries equal to 0.02. For instance, in the specific case of the US, the average pair-wise correlation of the volatility innovations is equal to 0.6 conditioning on $\hat{\zeta}_t$ alone. But it drops

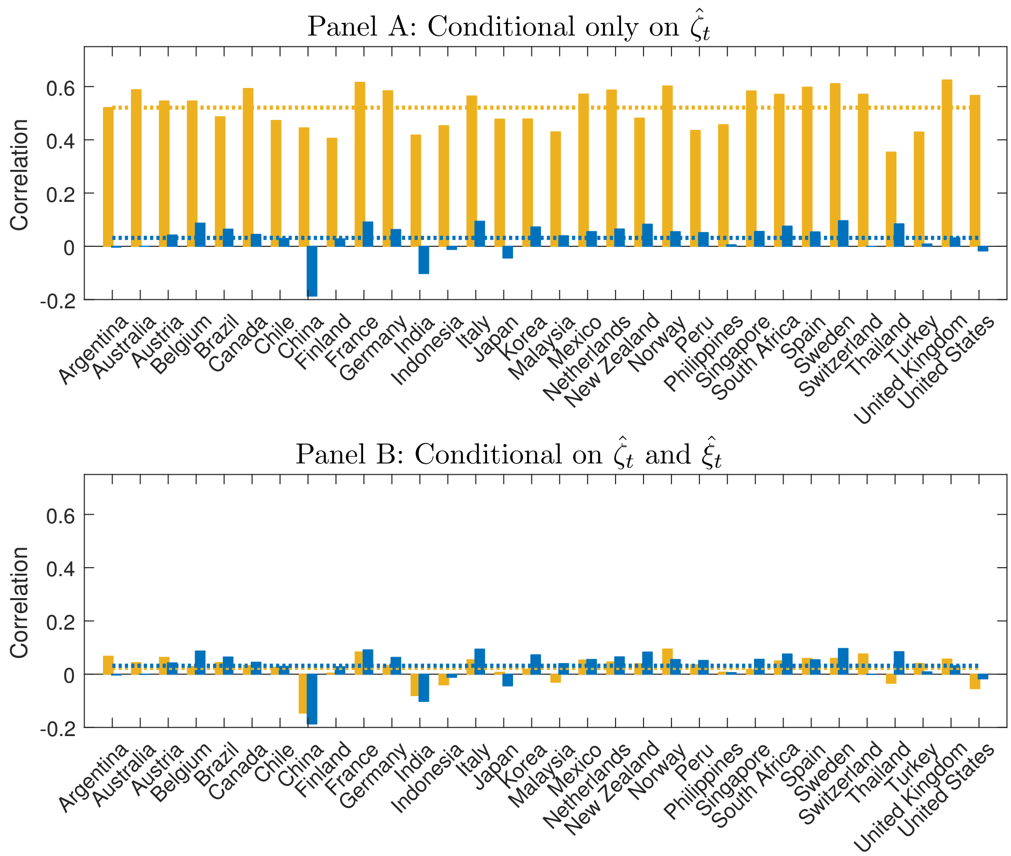
[28] Note that we can estimate ζ_t and ξ_t consistently by means of the OLS regressions (55) and (56) only under the identification assumptions made. As a result, whilst we can directly estimate pair-wise correlations of volatility and growth series, we can not examine cross-country pair-wise correlations of their innovations without imposing these identification conditions.

[29] In this case, we run OLS on (57) without conditioning on $\hat{\xi}_t$ in the regression.

[30] The same growth innovations are obtained in the two exercises.

[31] Notable exceptions are China and India.

Figure 5 CROSS-COUNTRY CORRELATION OF COUNTRY-SPECIFIC VOLATILITY AND GROWTH INNOVATIONS



NOTE. Country-specific average pair-wise correlation of volatility (yellow, lighter bars) and GDP growth (blue, darker bars) innovations conditional on $\hat{\zeta}_t$ only (Panel A) and on $\hat{\zeta}_t$ and $\hat{\xi}_t$ (Panel B). The volatility measures are based on (59). The dotted lines are the averages across all countries, equal to 0.52 and 0.03 for volatility and growth in Panel A; and equal to 0.02 and 0.03 for volatility and GDP growth in Panel B, respectively. Sample period: 1993:Q1-2011:Q2.

to -0.05 if we condition on both factor innovations. By comparison, the US average pair-wise correlation of the growth innovations is -0.02.

Figure 5, therefore, illustrates that, after conditioning on $\hat{\zeta}_t$—which is common to both growth and volatility series—not much commonality is left in the case of growth innovations, but the volatility innovations continue to share strong commonality. Moreover, after conditioning on both $\hat{\zeta}_t$ and $\hat{\xi}_t$, the volatility innovations also appear weakly correlated because of the near-zero average pair-wise correlation across all countries, thus suggesting that only two common shocks are necessary to span their correlations across-countries as we assumed in our theoretical model. It is, therefore, interesting to test whether the two sets of innovations also satisfy a formal definition of *weak* and *strong* dependence, as we assumed deriving them.

To test for weak and strong cross-section dependence, we estimate the cross-sectional dependence (CD) test statistic of Pesaran (2015) and the exponent of cross sectional dependence (α) proposed in Bailey et al. (2016). The CD statistic is normally distributed with zero-mean and unit-variance under the null of zero average pair-wise correlations. So, the critical value is around 2. When the null is rejected, Bailey et al. (2016) suggest estimating the strength of the cross-section dependence with an exponent, denoted α in the range $(1/2, 1]$, with unity giving the maximum degree of cross dependence. Any value above 1/2 and below 1, but significantly different from 1, suggests weak dependence.[32] So, in what follows, we present estimates of α for the volatility and the growth innovations, together with their confidence intervals. For comparison, we also report the same estimates for the (raw) growth (Δy_{it}) and volatility (v_{it}) series.

Table 1 Testing for the Strength of Cross-Sectional Dependence

	CD	Lower 5%	$\hat{\alpha}$	Upper 95%
Data				
v_{it}	57.00	0.96	1.00	1.05
Δy_{it}	29.64	0.83	1.00	1.17
Innovations (conditional on $\hat{\zeta}_t$)				
$\hat{u}_{it}$	57.31	0.95	1.00	1.05
$\hat{\varepsilon}_{it}$	5.07	0.75	0.80	0.86
Innovations (conditional on $\hat{\zeta}_t$ and $\hat{\xi}_t$)				
$\hat{\eta}_{it}$	2.13	0.57	0.65	0.73

Note. CD is the cross-sectional dependence test statistic of Pesaran (2015). $\hat{\alpha}$ is the estimate of the exponent of cross-sectional dependence as in Bailey et al. (2016), together with its 90-percent confidence interval ('Lower 5%' and 'Upper95%').

The results are summarized in Table 1 and are in strong accordance with the identification assumptions made. The CD test statistic for the growth series is 29.64, with the associated α exponent estimated at 1.00. The CD statistic for the volatility series is even higher at 57.00 with an estimated α of 1.00. The CD statistics and the estimates of the α exponent confirm with a high degree of confidence that both series are cross-sectionally strongly correlated, containing *at*

[32]When estimating α one also needs to take into account the sampling uncertainty, which depends on the relative magnitude of N and T, and the null of weak cross dependence, which depends on the relative rates of increase of N and T.

least one strong common factor. Conditional only on $\hat{\zeta}_t$, the CD statistic for the country-specific growth innovations ($\hat{\varepsilon}_{it}$) drops to 5.07, close to its critical value under the null of zero average pair-wise correlations, with its exponent of cross-sectional dependence estimated to be 0.80, and is significantly below 1. In sharp contrast, the CD statistic for the country-specific volatility innovations when we condition only on $\hat{\zeta}_t$ in model (57)-(58) (denoted by $\hat{u}_{it}$) remains close to that of the raw volatility series at 57.31 with an estimated α close to unity. However, when we condition on both $\hat{\zeta}_t$ and $\hat{\xi}_t$, the CD statistic for the volatility innovations ($\hat{\eta}_{it}$) also falls to 2.13, with an estimated α of 0.65 and a 95 percent confidence interval of $[0.57, 0.73]$, while the CD statistic and α are the same as before for the growth innovations ($\hat{\varepsilon}_{it}$). The battery of test statistics in Table 1, therefore, accord very well with the assumptions made that the volatility innovation share at least one more, and only one more, strong common factor than the growth innovations.

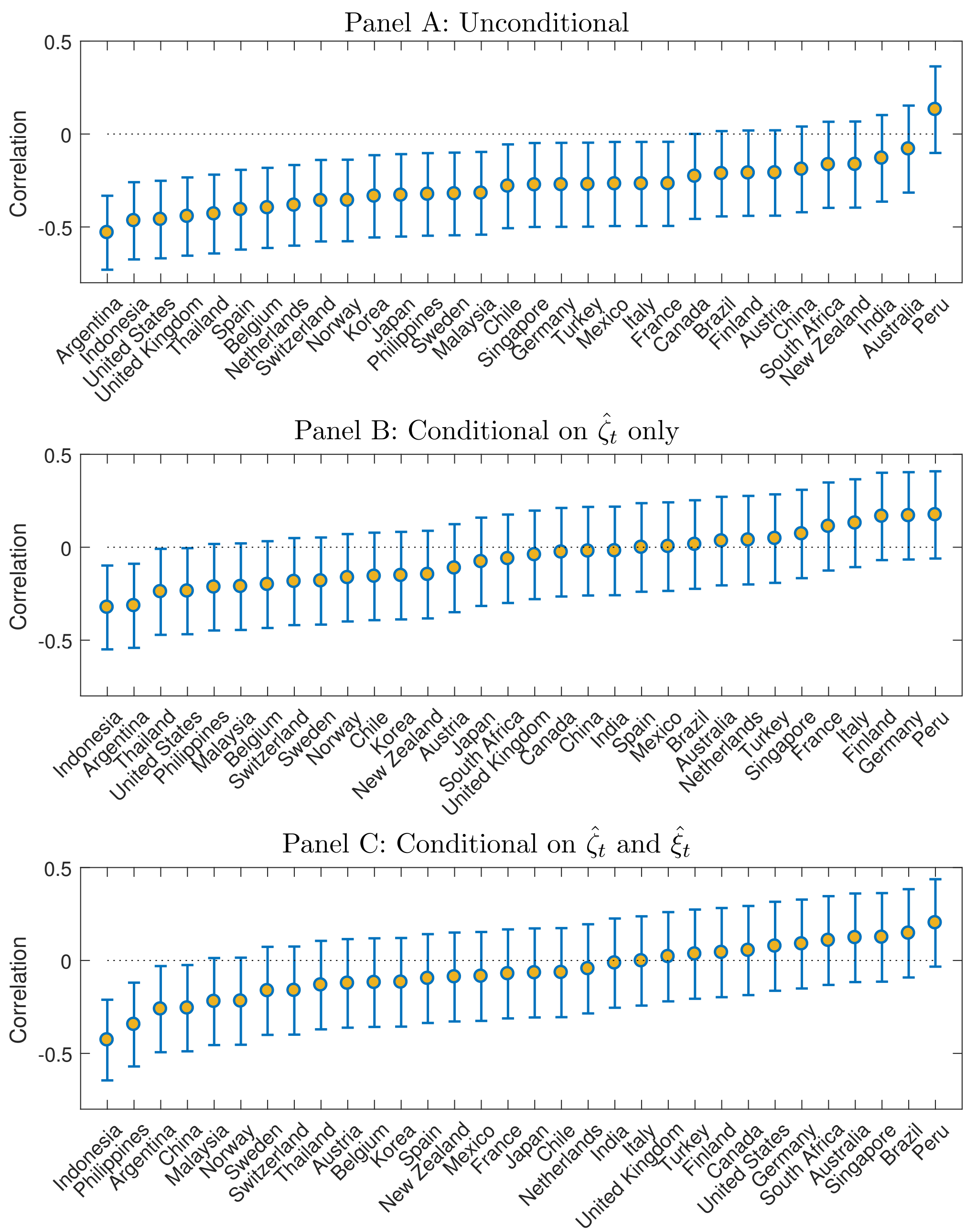
9 Country-specific Correlations Between Volatility and Growth Innovations

We are now ready to present and discuss our main empirical results. Figure 6 reports our first main empirical result. It compares unconditional and conditional contemporaneous correlations between volatility and growth, and suggests that this association is almost entirely accounted for by the global output growth innovations, $\hat{\zeta}_t$.[33] For ease of comparison, Panel A displays again the correlations of volatility and growth series reported in Figure 1. Panel B shows the correlation between volatility and growth innovations when we condition only on $\hat{\zeta}_t$ in model (57)-(58). Panel C reports the same correlation when we condition on both $\hat{\zeta}_t$ and $\hat{\xi}_t$.

When we condition only on $\hat{\zeta}_t$, the correlation between volatility and growth innovations weakens substantially for all countries and it is no longer statistically significant in all but two cases. In the case of the United States, for instance, the conditional correlation does not vanish, but drops to less than half its unconditional value and becomes borderline statistically insignificant when considered in isolation from the other correlations. But it is statistically not different from zero when estimated with the regularized covariance estimator discussed in

[33] Recall that this result does not depend on the granularity of the volatility weights in Assumption 2. Note also that this result is robust to excluding from the analysis the sample period covering the global financial crisis (i.e., ending the sample period in 2006:Q4 or 2008:Q2).

Figure 6 Country-specific Correlations Between Volatility and Growth Innovations



Note. Panel A displays the unconditional correlations plotted in Figure 1. Panel B plots the correlation between volatility and growth innovations when we condition only on $\hat{\zeta}_t$ in model (57)-(58). Panel C reports the same correlation when we condition on both $\hat{\zeta}_t$ and $\hat{\xi}_t$. The dots represent the contemporaneous correlations. The lines represent 95% confidence intervals. Sample period: 1993:Q1-2011:Q2.

Section 10.3 and reported in Table 2. Panel C of Figure 6 also shows that when we condition on both $\hat{\zeta}_t$ and $\hat{\xi}_t$ does not alter these results. This is intuitive, as $\hat{\xi}_t$ is common only to the volatility series. These results suggest that volatility and growth share an important common component at quarterly frequency and that conditioning on $\hat{\zeta}_t$ captures most of this dependence. This also implies that some of the explanatory power attributed to uncertainty shocks in empirical studies of individual countries, considered in isolation from the rest of the world, might be due to omitted common factor from the analysis.

How can we interpret this evidence? Consistent with our theoretical model, it is possible to interpret the common growth innovations, $\hat{\zeta}_t$, as capturing variations in the global risk-free rate. Specifically, in Section 3.1 and 3.2, using a simple multi-country model, we showed that expected world growth is tied to the world risk-free rate, and changes in expected world growth can affect both country volatilities and country growth rates at the same time. Moreover, we saw in Figure 4 that our estimated real common factor innovations, $\hat{\zeta}_t$, are closely associated with a proxy of the world risk-free rate. Lower expected world growth means a lower world risk-free rate, and hence higher country risk premium and country volatility, but also lower country growth. Our real common factor, therefore, can drive both variables.

This result is consistent not only with the model we set up, but also with results available in the related literature obtained in the context of different theoretical frameworks and with different empirical methodologies. For instance, Berger et al. (2017), find that conditioning on a realized equity market volatility shock, a shock to expected future volatility has no effect on output growth in the United States. Berger et al. (2017) interpret this finding in terms of negatively skewed productivity shocks, consistent with the evidence documented in Section 8 on the skewness of the our estimated real common shocks.

Note, however, that the evidence reported above does not imply that changes in volatility over time are mostly driven by $\hat{\zeta}_t$. That is, while these shocks can account for most of the contemporaneous co-movement between country-specific volatility and growth, they do not necessarily explain a significant share of the observed time variations in global innovations to volatility. Indeed, as we will see in the next section, $\hat{\zeta}_t$ explains a relatively small share of the variation of volatility over time, with a much larger share explained by $\hat{\xi}_t$.

10 Volatility and Growth Forecast Error Variance Decompositions

Forecast error variance decompositions are routinely used to quantify the importance of a given shock for the time-variation of the endogenous variables at different time horizons, relative to other shocks in the model. Our factor augmented multi-country VAR model can be readily used to decompose the forecast error variance of country volatility and growth in terms of the common shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$, as well as the 64 vector of country-specific shocks, $\hat{\eta}_{it}$ and $\hat{\varepsilon}_{it}$ for $i = 1, 2, ..., 32$. While the global real and financial shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$, are orthogonal to the country-specific shocks and to each other by construction, the country-specific shocks $\hat{\eta}_{it}$ and $\hat{\varepsilon}_{it}$ are left unrestricted, and can be correlated, both within and between countries, even conditional on $\hat{\zeta}_t$ and $\hat{\xi}_t$. In order to compute and interpret forecast error variance decompositions, we therefore have to deal with this second identification problem.

Consider the correlation between volatility and growth innovations within each country. We saw in Figure 6 that the contemporaneous within-country correlation between $\hat{\eta}_{it}$ and $\hat{\varepsilon}_{it}$ is very small and not statistically significant in most countries, once we condition on the global shocks $\hat{\zeta}_t$ and $\hat{\xi}_t$. Nonetheless, even assuming the estimated reduced form covariance matrix were truly diagonal, this would not imply that innovations $\hat{\eta}_{it}$ and $\hat{\varepsilon}_{it}$ can be interpreted as 'structural' country-specific volatility and growth shocks. As it is well known there always exists an orthonormal transformation of $\hat{\eta}_{it}$ and $\hat{\varepsilon}_{it}$ that lead to the same forecast error variance decomposition.

It is, therefore, important that the 64×64 matrix of correlations among all 32 countries and both variables is considered in a full multi-country set up. Our results show that, conditional on both real and financial common shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$, the country-specific innovations $\hat{\varepsilon}_{it}$ and $\hat{\eta}_{it}$ are weakly correlated across countries (Figure 5). The average pair-wise correlations of volatility and growth is negligible, and even in the case of China and India they were well below 0.2. As we discussed above, weak cross-sectional dependence means that, as N grows, the overall average pair-wise correlation tends to zero. This further means that, while some pairs of correlations can be different from zero, not all pairs can be so. In practice, this means that most correlation pairs will be very small and the overall covariance matrix must be sparse.

We exploit the sparsity of the correlation matrix of country-specific shocks by making alternative assumptions regarding the causal relations between the country-specific innovations $\hat{\eta}_{it}$ and $\hat{\varepsilon}_{it}$, and show that the inference we draw is reasonably robust to different estimates of the country-specific error correlation matrix. As a first approximation, we assume that the only source of interdependence among all growth and volatility series are the global real and financial shocks $\hat{\zeta}_t$ and $\hat{\xi}_t$. This implies assuming that country-specific volatility and growth shocks have no contemporaneous impact on growth or volatility series within and across countries. Despite its apparent severity, this assumption seems justified by our empirical finding that there exist very limited conditional within-country correlations and weak cross-country correlations as summarized above.

We then check the robustness of the results from this 'benchmark' case, by comparing them with those obtained under weaker assumptions. While maintaining the assumption of zero conditional correlations across countries, we assume that country-specific volatility shocks can have a contemporaneous causal impact on growth variables but not *vice-versa*, in line with much of the existing empirical literature as reviewed in the Introduction. This is done by allowing for a block-diagonal error covariance matrix in the full multi-country model, in which the only non-zero off-diagonal elements are the estimated covariances between volatility and growth errors of each country block. These within-country blocks are factorized with a Cholesky decomposition, ordering volatility before growth.

Finally, as a third possibility we refrain altogether from interpreting country-specific volatility and growth shocks as structural, and make use of a general unrestricted error covariance matrix subject to the sparsity condition, both within and across countries and compute the generalized forecast error variance decompositions (GFEVD) of Pesaran and Shin (1998), rather than orthogonal forecast error variance decompositions that require Cholesky ordering of the shocks. However, before computing GFEVDs, we use the regularized multiple testing threshold estimator of the error covariance matrix proposed by Bailey et al. (2017) and described in more detail below, to obtain a consistent estimator of the 64×64 error covariance matrix for the full multi-country model. This regularized estimator exploits the sparsity of the underlying error covariance matrix.

In what follows we report results for these three alternative specifications of the covariance matrix of the innovations $\hat{\varepsilon}_{it}$ and $\hat{\eta}_{it}$. As we wish to quantify the relative importance of both the real and the financial common shocks, all results are based on (57)-(58) that include both $\hat{\zeta}_t$ and

$\hat{\xi}_t$. Specifically, Figure 7 reports the forecast error variance decompositions (FEVDs) obtained assuming the 64×64 error covariance matrix is diagonal; Figure 8 reports the results obtained for a block-diagonal error covariance matrix and a Cholesky decomposition within each block; and Figure 9 reports the generalized FEVDs (GFEVDs) obtained using the regularized estimator of the error covariance matrix.[34] Each figure reports the 'average' variance decomposition, weighting country-specific decompositions with PPP-GDP weights. We shall now summarize the error variance decompositions that result from these three alternative specifications.

10.1 Diagonal Covariance Matrix and Orthogonal Decomposition

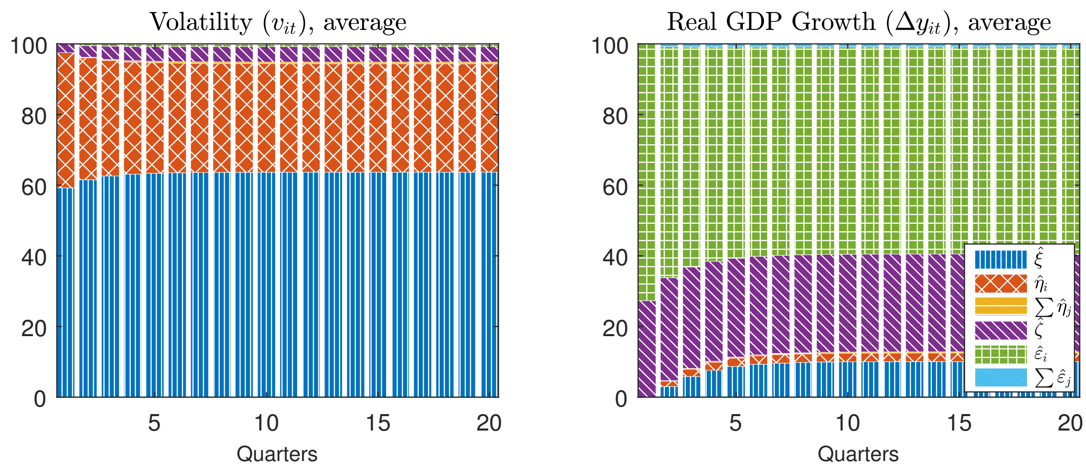
The left hand panel of Figure 7 plots the average forecast error variance decomposition of volatility across all countries in our sample under a diagonal error covariance matrix. The figure shows that country volatility is driven largely by common financial shocks (blue area with vertical lines) and country-specific volatility shocks (red area with crosses). Together, these two shocks explain about 95 percent of the total variance of realized volatility over time. These results, therefore, suggest that country-specific volatility is largely driven by global financial shocks and its own country-specific innovations. Real common shocks (purple area with diagonal lines) explain less than 5 percent of the total volatility forecast error variance. Country-specific own growth shocks, as well as all other 31 country-specific foreign growth shocks in the full model, play essentially no role.

It is worth noting that these estimated shares of the forecast error variance decomposition of country-specific realized volatility are similar to the central estimates of Ludvigson et al. (2015) for their US financial uncertainty. In that study, the share of the macroeconomic shock in the forecast error variance decomposition of the financial uncertainty measure is estimated at just above 5 percent. However, while Ludvigson et al. (2015) attribute this to the US business cycle (as proxied by a shock to US industrial production), we attribute the outcome largely to the global real shock, which can be interpreted as an international business cycle factor, as we find that country-specific growth shocks have little or no explanatory power for country-specific volatility.[35]

[34] The derivation of the FEVDs and GFEVDs is reported in section S3 of the online supplement to the paper.

[35] Results for specific countries, including the United States, are reported in the online supplement. As can be seen from Figures S.6 to S.9 also in the online supplement, countries behave pretty similarly, with some but limited heterogeneity. The results for the United States, in particular, are similar to those for the average economy reported here.

Figure 7 FORECAST ERROR VARIANCE DECOMPOSITION OF COUNTRY-SPECIFIC SHOCKS - DIAGONAL ERROR COVARIANCE MATRIX (IN PERCENT)



NOTE. Average across countries with GDP-PPP weights. $\hat{\xi}$ is common financial shock (blue area with vertical lines); $\hat{\eta}_i$ is country-specific volatility shock (red area with crosses); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area with horizontal lines); $\hat{\zeta}$ is common real shock (purple area with diagonal lines); $\hat{\varepsilon}_i$ is country-specific GDP shock (green areas with squares); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP shocks in the remaining countries (light blue areas with no pattern). Sample period: 1993:Q1-2011:Q2.

Consider now the forecast error variance decomposition of GDP growth reported on the right hand side of Figure 7. The figure shows that, on average, the forecast error variance of country specific GDP growth is driven mostly by country-specific growth shocks and global real shocks, with a combined share approaching 90 percent of the total in the long run (green areas with squares and purple area with diagonal lines, respectively). The country-specific growth shock explains more than 60 percent of the total forecast error variance in the long-run, while the real global shock on average explains around 30 percent of the total growth forecast error variance. This is in line with existing results in the international business cycle literature (see, for instance, Kose et al. (2003)).[36]

The global financial shock explains 8-10 percent of country-specific growth forecast error variance, on average, in our sample. The importance of this shock picks up gradually over the forecast horizon and stabilizes within two years. In contrast, the own country-specific volatility shock explains $1-2$ percent of the total forecast error variance of GDP growth, while the com-

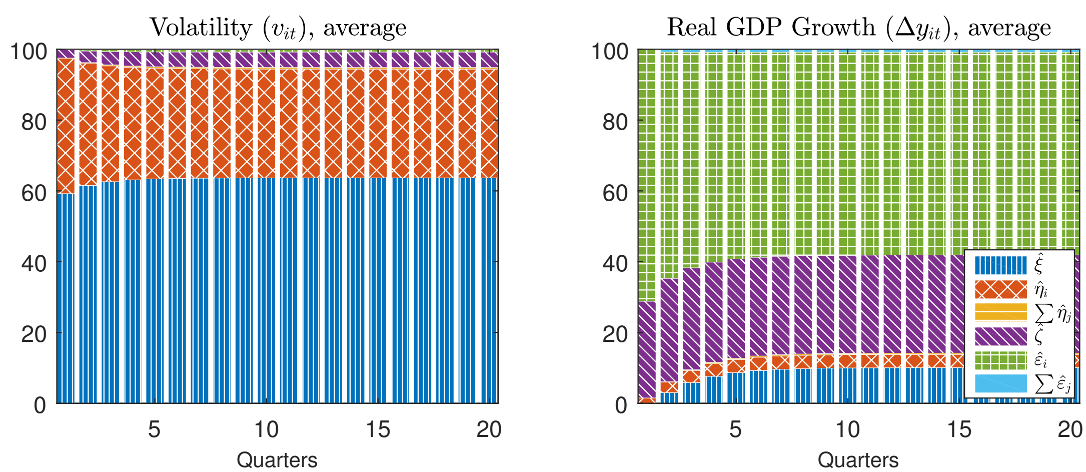
[36]Note that these results imply that countries' business cycles remain largely unexplained within our econometric model. Indeed, in the data, there are many shocks at work, and this is captured in our relatively simple empirical framework by the large share of growth forecast error variance accounted for by the own country specific growth shocks.

bination of all other 31 country-specific volatility shocks in the model explains an even smaller share of country growth variance. These results clearly illustrate the quantitative importance of distinguishing between common and country-specific volatility shocks.

10.2 Block-Diagonal Covariance Matrix and Orthogonal Decomposition

We now maintain the assumption of zero correlations of country-specific shocks (after conditioning on the common shocks) across countries, but allow for a possibly non-zero correlation between volatility and growth within each country. Specifically, we assume that, at the country level, a volatility shock can affect growth contemporaneously but not vice versa. This is the assumption typically made in the empirical and theoretical literature on volatility and the business cycle. So, here, we are 'identifying' exogenous country-specific volatility changes with a Cholesky decomposition of the within-country covariance matrix. We do so by ordering volatility first in the model (57)-(58).

Figure 8 FORECAST ERROR VARIANCE DECOMPOSITION OF COUNTRY-SPECIFIC SHOCKS - BLOCK DIAGONAL ERROR COVARIANCE MATRIX (IN PERCENT)



NOTE. Block-diagonal covariance matrix, with Cholesky decomposition of within-country covariance. Average across countries with GDP-PPP weights. $\hat{\xi}$ is common financial shock (blue area with vertical lines); $\hat{\eta}_i$ is country-specific volatility shock (red area with crosses); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area with horizontal lines); $\hat{\zeta}$ is common real shock (purple area with diagonal lines); $\hat{\varepsilon}_i$ is country-specific GDP shock (green areas with squares); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP shocks in the remaining countries (light blue areas with no pattern). Sample period: 1993:Q1-2011:Q2.

The results for this specification are given in Figure 8 and can be seen to be virtually identical to the estimates obtained for the diagonal error covariance matrix reported in Figure 7. This is perhaps not surprising given that the correlations between the country-specific innovations, once the effects of the common shocks are removed, are very small as in Figure 6.

10.3 Thresholding the Error Covariance Matrix and Generalized Decomposition

We finally allow for a fully estimated (64×64) correlation matrix, both within and across countries, and compute the GFEVDs. However, given the large size of this matrix, we regularize it by computing a threshold estimator following Bailey et al. (2017), who developed a procedure based on results from the multiple testing literature. Specifically, we first test for the statistical significance of each of the 2016 distinct off-diagonal elements of the (64×64) matrix. We then set to zero all those elements that are not statistically significant, using suitably adjusted critical values to allow for the large number of tests that are being carried out. We then finally compute the GVEDs by using the regularized estimates as derived in the online supplement to the paper.

Table 2 below lists all the non-zero correlation pairs. As can be seen, only 50 out of 2016 total off-diagonal elements are statistically different from zero. Of these, about half are positively correlated and the other half are negatively correlated, with an average value that is close to zero. Most notably, there is no surviving within-country contemporaneous correlation between volatility and growth, except for India. There are also very few significant GDP-GDP correlation pairs (i.e., $\hat{\varepsilon}_{it}$ with $\hat{\varepsilon}_{jt}$), with no obvious regional pattern of co-movements. There are a few significant pairs of volatility-volatility correlations (i.e., $\hat{\eta}_{it}$ with $\hat{\eta}_{jt}$), but involving only a handful of countries, with no evidence of a dominant role for the United States. Finally, there are a few significant GDP-volatility correlation pairs (i.e., $\hat{\varepsilon}_{jt}$ with $\hat{\eta}_{it}$) for a few countries, like Belgium, China, France, Italy and the Netherlands, again revealing no specific patterns.

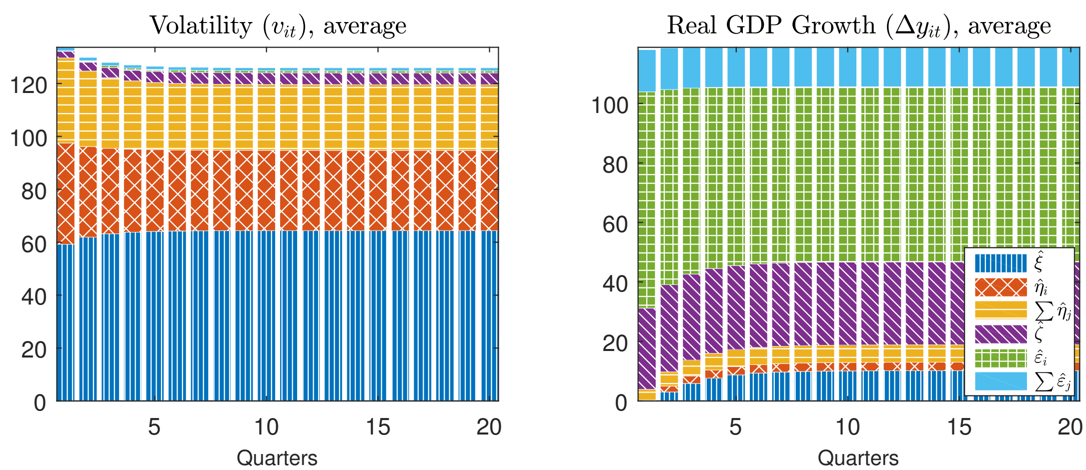
The estimated generalized forecast error variance decompositions (GFEVDs), reported in Figure 9, are consistent with those obtained assuming a diagonal or block-diagonal error covariance matrix.[37] Relative to the results with diagonal or block-diagonal covariance matrix in Figures 7 and 8, the contribution of foreign country-specific volatility (growth) shocks, $\sum \hat{\eta}_j$ ($\sum \hat{\varepsilon}_j$), to domestic volatility (growth) is now larger, but the spillover effects of foreign volatility shocks to growth (and foreign growth shocks to volatility) remain negligible. Moreover, global financial shocks and domestic country-specific volatility shocks continue to explain the bulk of

[37] Notice here that the GFEVDs need not to sum to 100 as the underlying shocks are not orthogonal.

the forecast error variance of volatility. Similarly, real global shocks and the country-specific growth shocks remain the main drivers of the forecast error variance of growth.

We interpret the above results as strong evidence of robustness of our conclusions reached by assuming a diagonal or block-diagonal covariance matrix. In particular, it remains the case that common or country-specific output growth shocks have a small quantitative importance for volatility, and home and foreign country-specific volatility shocks have little or no quantitative consequence for output growth.

Figure 9 Generalized Forecast Error Variance Decomposition of Country-specific Shocks - Estimation of Regularized Full Error Covariance Matrix (In Percent)



Note. Threshold estimator of the population covariance matrix. Average across countries with GDP-PPP weights. $\hat{\xi}$ is common financial shock (blue area with vertical lines); $\hat{\eta}_i$ is country-specific volatility shock (red area with crosses); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area with horizontal lines); $\hat{\zeta}$ is common real shock (purple area with diagonal lines); $\hat{\varepsilon}_i$ is country-specific GDP shock (green areas with squares); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP shocks in the remaining countries (light blue areas with no pattern). Sample period: 1993:Q1-2011:Q2.

Table 2 Non-zero Elements of the Regularized Error Covariance Matrix Estimate

All Significant			Between-county			Within-country
Country - Variable Pairs		Corr	$\hat{\varepsilon}_{it}$,$\hat{\varepsilon}_{jt}$	$\hat{\eta}_{it}$,$\hat{\eta}_{jt}$	$\hat{\varepsilon}_{it}$,$\hat{\eta}_{jt}$	$\hat{\varepsilon}_{it}$,$\hat{\eta}_{it}$
ARG ($\hat{\eta}_{it}$)	ZAF ($\hat{\eta}_{jt}$)	0.46				
AUT ($\hat{\varepsilon}_{it}$)	PHL ($\hat{\varepsilon}_{jt}$)	-0.43	AUT, PHL			
BEL ($\hat{\eta}_{it}$)	ITA ($\hat{\eta}_{jt}$)	0.51		BEL, ITA		
BEL ($\hat{\eta}_{it}$)	NLD ($\hat{\eta}_{jt}$)	0.60		BEL, NLD		
BEL ($\hat{\eta}_{it}$)	CHE ($\hat{\eta}_{jt}$)	0.51		BEL, CHE		
BEL ($\hat{\eta}_{it}$)	GBR ($\hat{\eta}_{jt}$)	0.54		BEL, GBR		
BEL ($\hat{\varepsilon}_{it}$)	CHN ($\hat{\varepsilon}_{jt}$)	-0.40	BEL, CHN			
BRA ($\hat{\eta}_{it}$)	MEX ($\hat{\eta}_{jt}$)	0.56		BRA, MEX		
BRA ($\hat{\varepsilon}_{it}$)	CHN ($\hat{\varepsilon}_{jt}$)	-0.44	BRA, CHN			
CAN ($\hat{\eta}_{it}$)	NOR ($\hat{\eta}_{jt}$)	0.40		CAN, NOR		
CHN ($\hat{\eta}_{it}$)	FRA ($\hat{\eta}_{jt}$)	-0.58		CHN, FRA		
CHN ($\hat{\eta}_{it}$)	ITA ($\hat{\eta}_{jt}$)	-0.42		CHN, ITA		
CHN ($\hat{\eta}_{it}$)	NLD ($\hat{\eta}_{jt}$)	-0.46		CHN, NLD		
CHN ($\hat{\eta}_{it}$)	ESP ($\hat{\eta}_{jt}$)	-0.41		CHN, ESP		
CHN ($\hat{\eta}_{it}$)	SWE ($\hat{\eta}_{jt}$)	-0.40		CHN, SWE		
CHN ($\hat{\eta}_{it}$)	CHE ($\hat{\eta}_{jt}$)	-0.45		CHN, CHE		
CHN ($\hat{\eta}_{it}$)	GBR ($\hat{\eta}_{jt}$)	-0.49		CHN, GBR		
CHN ($\hat{\eta}_{it}$)	USA ($\hat{\eta}_{jt}$)	-0.57		CHN, USA		
CHN ($\hat{\varepsilon}_{it}$)	FRA ($\hat{\varepsilon}_{jt}$)	-0.39	CHN, FRA			
CHN ($\hat{\varepsilon}_{it}$)	JPN ($\hat{\eta}_{jt}$)	0.55			CHN, JPN	
CHN ($\hat{\varepsilon}_{it}$)	USA ($\hat{\varepsilon}_{jt}$)	-0.51	CHN, USA			
FIN ($\hat{\eta}_{it}$)	KOR ($\hat{\varepsilon}_{jt}$)	-0.41			FIN, KOR	
FIN ($\hat{\eta}_{it}$)	TUR ($\hat{\varepsilon}_{jt}$)	0.41			FIN, TUR	
FRA ($\hat{\eta}_{it}$)	DEU ($\hat{\eta}_{jt}$)	0.50		FRA, DEU		
FRA ($\hat{\eta}_{it}$)	IND ($\hat{\eta}_{jt}$)	-0.46		FRA, IND		
FRA ($\hat{\eta}_{it}$)	IDN ($\hat{\eta}_{jt}$)	-0.39		FRA, IDN		
FRA ($\hat{\eta}_{it}$)	ITA ($\hat{\eta}_{jt}$)	0.46		FRA, ITA		
FRA ($\hat{\eta}_{it}$)	NLD ($\hat{\eta}_{jt}$)	0.63		FRA, NLD		
FRA ($\hat{\eta}_{it}$)	ESP ($\hat{\eta}_{jt}$)	0.61		FRA, ESP		
FRA ($\hat{\eta}_{it}$)	SWE ($\hat{\eta}_{jt}$)	0.51		FRA, SWE		
FRA ($\hat{\eta}_{it}$)	CHE ($\hat{\eta}_{jt}$)	0.55		FRA, CHE		
FRA ($\hat{\eta}_{it}$)	GBR ($\hat{\eta}_{jt}$)	0.71		FRA, GBR		
IND ($\hat{\eta}_{it}$)	NLD ($\hat{\eta}_{jt}$)	-0.39		IND, NLD		
IND ($\hat{\eta}_{it}$)	GBR ($\hat{\eta}_{jt}$)	-0.49		IND, GBR		
IND ($\hat{\eta}_{it}$)	USA ($\hat{\eta}_{jt}$)	-0.46		IND, USA		
IDN ($\hat{\eta}_{it}$)	IDN ($\hat{\varepsilon}_{jt}$)	-0.43				IDN, IDN
ITA ($\hat{\eta}_{it}$)	NLD ($\hat{\eta}_{jt}$)	0.60		ITA, NLD		
ITA ($\hat{\eta}_{it}$)	ESP ($\hat{\eta}_{jt}$)	0.61		ITA, ESP		
ITA ($\hat{\eta}_{it}$)	GBR ($\hat{\eta}_{jt}$)	0.46		ITA, GBR		
KOR ($\hat{\varepsilon}_{it}$)	MYS ($\hat{\varepsilon}_{jt}$)	0.58	KOR, MYS			
KOR ($\hat{\varepsilon}_{it}$)	THA ($\hat{\varepsilon}_{jt}$)	0.47	KOR, THA			
MYS ($\hat{\eta}_{it}$)	SWE ($\hat{\eta}_{jt}$)	-0.39		MYS, SWE		
MYS ($\hat{\varepsilon}_{it}$)	NOR ($\hat{\eta}_{jt}$)	-0.41			MYS, NOR	
NLD ($\hat{\eta}_{it}$)	ESP ($\hat{\eta}_{jt}$)	0.50		NLD, ESP		
NLD ($\hat{\eta}_{it}$)	CHE ($\hat{\eta}_{jt}$)	0.70		NLD, CHE		
NLD ($\hat{\eta}_{it}$)	GBR ($\hat{\eta}_{jt}$)	0.74		NLD, GBR		
NOR ($\hat{\varepsilon}_{it}$)	THA ($\hat{\eta}_{jt}$)	0.40			NOR, THA	
PHL ($\hat{\eta}_{it}$)	SGP ($\hat{\eta}_{jt}$)	0.44		PHL, SGP		
SGP ($\hat{\eta}_{it}$)	USA ($\hat{\eta}_{jt}$)	-0.42		SGP, USA		
CHE ($\hat{\eta}_{it}$)	GBR ($\hat{\eta}_{jt}$)	0.66		CHE, GBR		

Note. Non-zero elements of the regularized error covariance matrix estimate proposed by Bailey et al. (2017). Sample period: 1993:Q1-2011:Q2.

11 The Transmission of Global Real and Financial Shocks

The last step of our empirical analysis is the computation of impulse responses of country-specific volatility and growth to global real and financial shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$. While forecast error variance decompositions speak to the importance of a particular shock for the time-variation of the endogenous variables relative to other shocks in the model, impulse responses provide information on the size of the effects of the shocks and their transmission across variables and countries.

Figure 10 displays a weighted average of the country-specific impulse responses using PPP-GDP weights (solid line), together with two-standard deviation error bands (shaded areas). The error bands are computed based on the dispersion of the impulse responses across countries.[38] We focus on the effects of positive unit (one-standard deviation) real and financial shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$.[39]

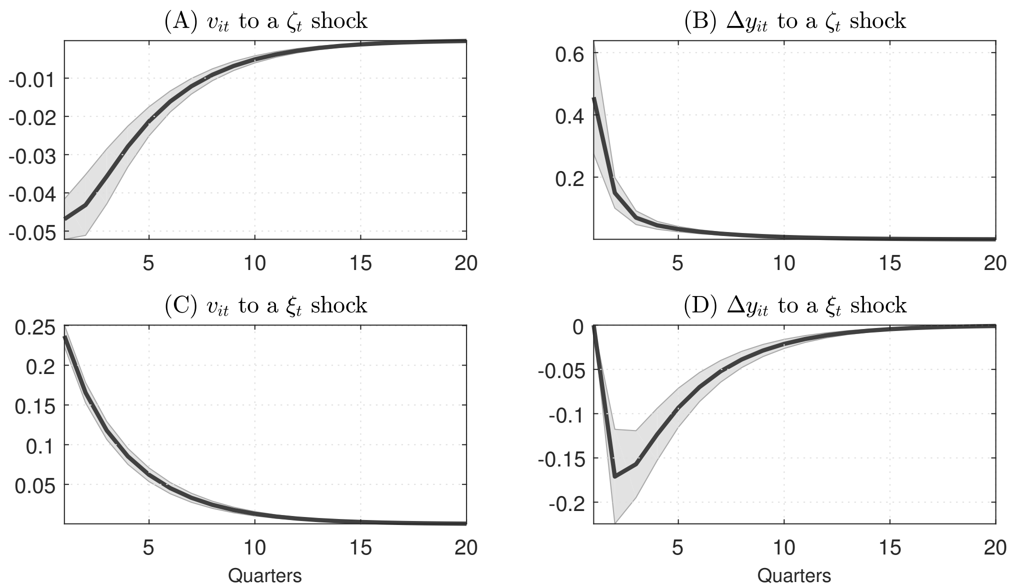
Panels (A) and (B) of Figure 10 display the average across countries of the volatility and growth responses to a real global shock. These figures show that a positive real global shock increases output growth and lowers volatility. This reflects an endogenous volatility response to the fundamental improvements in the world economy. Note that the error bands around the average responses are very tight, reflecting relatively homogeneous country responses. In fact, as can be seen from Figure S.12, provided in the online supplement, the impulse responses have a similar shape for most countries. The average impact of this shock on country volatilities is one order of magnitude smaller than its impact on country output growth, but it is quite persistent, taking more than three years for the effects of the shocks to vanish completely. Panel (B) also shows that, on average, country growth loads positively on $\hat{\zeta}_t$, with persistent effects up to 8-10 quarters, as one would expect. Country output growth increase by about half a percentage point following a one-standard error change in $\hat{\zeta}_t$. This is consistent with the existing evidence on the international business cycle, which attributes an important role to a world factor, along with regional and country-specific factors, in driving the business cycle (e.g., Kose et al. (2003)).

Panels (C) and (D) of Figure 10 report the responses of volatility and growth to a positive global financial shock, $\hat{\xi}_t$. These average responses suggest that a positive shock to $\hat{\xi}_t$ is 'bad

[38] The derivation of the average impulse response functions to common factor shocks and their error bands is provided in the online supplement S3, equations (S3.12) and (S3.13). The estimated country-specific responses to the two common shocks are reported in Figure S.12 in the online Supplement to the paper.

[39] Recall that these shocks are orthogonal to all other shocks in the model and to each other by construction.

Figure 10 Average Country Volatility and Growth Responses to Real and Financial Factor Shocks (In Percent)



Note. Average impulse responses to one-standard deviation real and financial shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$. The solid lines are the PPP-GDP weighted averages of the country-specific responses. The shaded areas are the two standard deviations confidence intervals. See equations (S3.12) and (S3.13) for the derivations and Figure S.12 for the country-specific responses. Sample period: 1993:Q1-2011:Q2.

news' for the world economy, as volatility increases and growth declines. For a one-standard deviation shock to the common financial factor, volatility increases by 25 basis points, while growth declines by about 15 basis points within two quarters after the shock.[40] Although smaller than the growth response to the real common factor shock in panel B, the average growth response to the global financial shock in panel D is of the same order of magnitude, and hence quantitatively sizable. The average responses to the common financial shock are also very persistent, but there is much more heterogeneity in the country growth responses, as can be seen from Figure S.12 provided in the online supplement. Therefore, these impulse responses suggest that, even though common financial shocks may not explain a very large share of the forecast error variance of output growth over time, they can cause large and persistent global recessions.

Impulse responses to country-specific shocks have qualitatively similar pattern of transmission but, as one would expect given the forecast error variance results, are quantitatively much

[40]Note that the delayed growth response to the global financial shock follows from our identification assumptions, but it is not imposed directly on country-specific models.

smaller than responses to common shocks.[41]

The pattern of shock transmissions in Figure 10 is consistent with country volatility increasing in response to the large declines in the world output in the second part of 2008, and the world recession being amplified by the exceptionally large common financial shock in the fourth quarter of 2008, and the first quarter of 2009. The transmission in Figure 10 can also help in explaining the seemingly puzzling coexistence of high policy volatility (as in Baker et al. (2016)) and low equity market volatility after the beginning of the Trump administration with a combination of a real and a financial shock partially offsetting each other.

12 Conclusions

Empirical measures of uncertainty behave countercyclically in most countries of the world, but economic theory suggests that causation can run both ways. In this paper, we take a common factor approach in an multi-country setting to study the interrelation between realized equity price volatility and GDP growth without imposing *a priori* restrictions on the direction of economic causation on country-specific volatility and growth shocks.

Based on the stylized facts of the data that we document in the paper and a multi-country version of the Lucas tree model with time-varying volatility, we estimate a multi-country econometric model in output growth and realized volatilities for 32 countries over the period 1993Q1-2011Q2. Common real and financial shocks are identified assuming that volatility and growth are driven by two common factors. By taking a multi-country approach, as opposed to studying a single economy in isolation, we can identify and estimate these two factors exploiting the different patterns in the correlations of volatility and output growth innovations across countries. Evidence based on the estimated innovations accords with the assumptions made to achieve factor identification.

Empirically, we report three main sets of findings. First, shocks to the real common factor, which are closely associated with changes in proxy for the world risk-free rate, account for most of the unconditional correlation between volatility and growth in all but few emerging market economies. Second, the share of forecast error variance of country-specific volatility explained by the real common factor shock and by country-specific growth shocks is less than 5 percent.

[41] These are not reported but are available from the authors on request.

Third, shocks to the financial common factor explain about 10 percent of the country-specific growth forecast error variance, while country-specific volatility shocks explain only about 1-2 percent. Moreover, when a shock to the financial common factor is realized, its negative impact on country-specific growth is large and persistent as typically estimated in the existing literature.

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Appendix: Derivations and Data Sources

A Mathematical Derivations

A.1 Country-specific Growth Process

One way to motivate the output growth specification, (1), is as follows. Assume a standard Cobb-Douglas production function in terms of output per worker and denote $(Y_{it}/L_{it}) = exp(\tilde{y}_{it})$, real GDP per capita, $A_{it} = exp(a_{it})$ the country-specific level of technology, L_{it} the labor force, and K_{it} the capital stock in country i so that we have:

$$\tilde{y}_{it} = \ln(Y_{it}/L_{it}) = a_{it} + \alpha_i \ln(K_{it}/L_{it}) = a_{it} + \alpha_i \log(k_{it})$$

for $i = 1, 2, \ldots, N$. Further assume that the processes for L_{it} and a_{it} are exogenously given by

$$\ln(L_{it}) - \ln(L_{i,t-1}) = n_i, \text{ and } a_{it} = a_{i0} + \tilde{\mathfrak{g}}_i t + \gamma_i a_t + e_{it}$$

where growth of labour force, n_i, is assumed to be fixed, a_{i0} is an initial condition, $\tilde{\mathfrak{g}}_i$ is a deterministic growth component of a_{it}, a_t is the log-level of a stochastic common technology factor, and e_{it} is the country-specific technology shock, with γ_i measuring the extent to which country i is exposed to the global technology factor a_t. A key result from the stochastic growth literature is that, for all i, $\log(k_{it})$ is ergodic and stationary, in the sense that as t tends to infinity, $\log(k_{it})$ tends to a time-invariant random variable, namely $\log(k_{it}) = \log(k_i) + \tau_{it}$, where τ_{it} is a stationary process representing all country-specific forces driving the country's business cycles, possibly reflecting the effects of aggregate demand as well as country-specific uncertainty shocks (see, for instance, Lee et al., 1997). So we have:

$$\tilde{y}_{it} = a_{i0} + \alpha_i log(k_i) + \mathfrak{g}_i t + \gamma_i a_t + e_{it} + \tau_{it}.$$

Taking first differences we obtain,

$$\Delta\tilde{y}_{it} = \tilde{\mathfrak{g}}_i + \gamma_i f_t + \varepsilon_{it}, \tag{A.1}$$

where $f_t = \Delta a_t = a_t - a_{t-1}$, and $\varepsilon_{it} = \Delta e_{it} + \Delta\tau_{it}$. In terms of log output, $y_{it} = \ln(Y_{it})$, we now obtain equation (1), with $\Delta y_{it} = y_{it} - y_{i,t-1}$, $\mathfrak{g}_i = \tilde{\mathfrak{g}}_i + n_i$.

A.2 Country-specific Equity Excess Return

We first note that using (20) in (18) and (19) we obtain

$$E_t\left(\Delta y_{i,t+j+1}\right) = \mathfrak{g}_i + \gamma_i \phi^{j+1} f_t,$$

and

$$E_t\left(r^f_{t+j+1}\right) \approx r+\varrho g+\left(\gamma\varrho\phi_f^{j+1}\right)f_t-\frac{1}{2}\left(\varrho^2\gamma^2 b_f\right)\left(\frac{\left(1-b_f^j\right)a_f}{1-b_f}+b_f^j\nu_t^2\right)+O\left(N^{-1}\right)$$

$$= r+\varrho g-\frac{1}{2}\frac{\left(1-b_f^j\right)\left(\varrho^2\gamma^2 b_f\right)a_f}{1-b_f}+\left(\gamma\varrho\phi_f^{j+1}\right)f_t-\frac{1}{2}\varrho^2\gamma^2 b_f^{j+1}\nu_t^2+O\left(N^{-1}\right).$$

Substituting the above results in (17) we have

$$\delta_{it} = \sum_{j=0}^{\infty}\kappa_i^j\left[r+\varrho g-\frac{1}{2}\frac{\left(1-b_f^j\right)\left(\varrho^2\gamma^2 b_f\right)a_f}{1-b_f}+\left(\gamma\varrho\phi_f^{j+1}\right)f_t-\frac{1}{2}\varrho^2\gamma^2 b_f^{j+1}\nu_t^2-\mathfrak{g}_i-\gamma_i\phi_f^{j+1}f_t\right]+O\left(N^{-1}\right)$$

$$= \frac{r+\varrho g-\mathfrak{g}_i-\frac{1}{2}\frac{\left(\varrho^2\gamma^2 b_f\right)a_f}{1-b_f}}{1-\kappa_i}+\frac{1}{2}\frac{\left(\varrho^2\gamma^2 b_f\right)a_f}{\left(1-b_f\right)\left(1-b_f\kappa_i\right)}+\left(\frac{\left(\gamma\varrho-\gamma_i\right)\phi_f}{1-\phi_f\kappa_i}\right)f_t-\frac{1}{2}\frac{\varrho^2\gamma^2 b_f}{\left(1-b_f\kappa_i\right)}\nu_t^2+O\left(N^{-1}\right),$$

which if used in (16), and after some algebra, yields

$$r_{i,t+1}=a_i+\varepsilon_{i,t+1}+\frac{\left(\gamma\varrho-\gamma_i\right)\phi_f}{1-\phi_f\kappa_i}f_t+\left(\frac{\gamma_i-\kappa_i\gamma\varrho\phi_f}{1-\phi_f\kappa_i}\right)f_{t+1}-\frac{1}{2}\left(\frac{\varrho^2\gamma^2 b_f}{1-b_f\kappa_i}\right)\left(\nu_t^2-\kappa_i\nu_{t+1}^2\right), \quad \text{(A.2)}$$

where

$$a_i=r+\varrho g-\frac{1}{2}\frac{\kappa_i\varrho^2\gamma^2 b_f a_f}{1-b_f\kappa_i}.$$

It is also helpful to note that (A.2) can be written equivalently as

$$r_{i,t+1} = r+\varrho g+\varepsilon_{i,t+1}+\gamma\varrho\phi_f f_t-\frac{1}{2}\varrho^2\gamma^2 b_f\nu_t^2+\left(\frac{\gamma_i-\kappa_i\gamma\varrho\phi_f}{1-\phi_f\kappa_i}\right)\left(f_{t+1}-\phi_f f_t\right) \quad \text{(A.3)}$$

$$+\frac{1}{2}\left(\frac{\varrho^2\gamma^2 b_f\kappa_i}{1-b_f\kappa_i}\right)\left(\nu_{t+1}^2-a_f-b_f\nu_t^2\right)+O\left(N^{-1}\right).$$

Subtracting r^f_{t+1} from both sides of (A.3), using the equation for the risk free rate given by (15), we also obtain the following expression for country-specific excess returns

$$r_{i,t+1}-r^f_{t+1}=\frac{1}{2}\varrho^2\gamma^2 a_f+\left(\frac{\gamma_i-\kappa_i\gamma\varrho\phi_f}{1-\phi_f\kappa_i}\right)\nu_{t+1}+\frac{1}{2}\left(\frac{\varrho^2\gamma^2 b_f\kappa_i}{1-b_f\kappa_i}\right)\left(\nu_{t+1}^2-a_f-b_f\nu_t^2\right)+\varepsilon_{i,t+1}+O\left(N^{-1}\right), \quad \text{(A.4)}$$

which yields $E_t\left(r_{i,t+1}-r^f_{t+1}\right)=\frac{1}{2}\varrho^2\gamma^2 a_f+O\left(N^{-1}\right)$. Therefore, in our multi-country model with complete markets, country-specific risk gets diversified completely, and excess return predictability only arises if N, the number of countries participating in global risk sharing, is not large enough. However, there is still a non-zero risk premium for equity holdings so long as $\varrho^2>0$. Recall that $a_f>0$, $\gamma^2>0$.

A.3 Proof of Proposition 4 (Consistent estimation of factors in a dynamic heterogeneous multi-country model)

Proof. Using the country-specific models given by (49), and solving for $\mathbf{z}_{it}$ in terms of current and past values of factors and shocks we have:

$$\mathbf{z}_{it} = \mu_i + \sum_{\ell=0}^{\infty} \mathbf{\Phi}_i^{\ell} \mathbf{\Gamma}_i \mathbf{f}_{t-\ell} + \varkappa_{it}, \tag{A.5}$$

where

$$\mu_i = (\mathbf{I}_2 - \mathbf{\Phi}_i)^{-1} \mathbf{a}_i, \;\; \varkappa_{it} = \sum_{\ell=0}^{\infty} \mathbf{\Phi}_i^{\ell} \boldsymbol{\vartheta}_{i,t-\ell}, \text{ and } \boldsymbol{\vartheta}_{it} = (\eta_{it}, \varepsilon_{it})'. \tag{A.6}$$

Assumption 7, ensures that the infinite sums are convergent. Pre-multiplying both sides of (A.5) by (w_i) and summing over i yields:

$$\bar{\mathbf{z}}_{\omega t} = \bar{\mu}_{\omega} + \sum_{\ell=0}^{\infty} \mathbf{A}_{\ell,N} \mathbf{f}_{t-\ell} + \bar{\varkappa}_{\omega t} \tag{A.7}$$

where

$$\begin{array}{l} \bar{\mathbf{z}}_{\omega t} = \sum_{i=1}^{N} w_i \mathbf{z}_{it}, \quad \bar{\mu}_{\omega} = \sum_{i=1}^{N} w_i \mu_i, \\ \mathbf{A}_{\ell,N} = \sum_{i=1}^{N} w_i \mathbf{\Phi}_i^{\ell} \mathbf{\Gamma}_i, \text{ and } \bar{\varkappa}_{\omega t} = \sum_{i=1}^{N} w_i \varkappa_{it}. \end{array} \tag{A.8}$$

Under Assumption 4, $\varkappa_{it}$ are cross-sectionally weakly correlated and the weights $\mathbf{w} = (w_1, w_2, ..., w_N)'$ are granular. By results in Pesaran and Chudik (2014), it readily follows that:

$$\bar{\varkappa}_{\omega t} = O\left(\|\mathbf{w}\|\right) = O\left(N^{-1/2}\right), \text{ for each } t. \tag{A.9}$$

Under Assumptions 6 and 7, we also have

$$\mathbb{E}\left(\mathbf{\Phi}_i^{\ell} \mathbf{\Gamma}_i\right) = \mathbb{E}\left(\mathbf{\Phi}_i^{\ell}\right) \mathbb{E}\left(\mathbf{\Gamma}_i\right) = \mathbf{\Lambda}_{\ell} \mathbf{\Gamma},$$

and since $\mathbf{\Phi}_i$ and $\mathbf{\Gamma}_i$ are distributed independently across i, using again results in Pesaran and Chudik (2014) we have:

$$\mathbf{A}_{\ell,N} - \mathbb{E}\left(\mathbf{A}_{\ell,N}\right) = \sum_{i=1}^{N} w_i \left[\mathbf{\Phi}_i^{\ell} \mathbf{\Gamma}_i - \mathbb{E}\left(\mathbf{\Phi}_i^{\ell} \mathbf{\Gamma}_i\right)\right] = O\left(\|\mathbf{w}\|\right) = O\left(N^{-1/2}\right). \tag{A.10}$$

Using (A.9) and (A.10) in (A.7) we now have:

$$\begin{aligned}\bar{\mathbf{z}}_{\omega t} &= \bar{\mu}_{\omega} + \sum_{\ell=0}^{\infty} \mathbf{\Lambda}_{\ell}\mathbf{\Gamma}\boldsymbol{f}_{t-\ell} + O_p\left(N^{-1/2}\right) \\ &= \bar{\mu}_{\omega} + \left(\sum_{\ell=0}^{\infty} \mathbf{\Lambda}_{\ell} L^{\ell}\right)\mathbf{\Gamma}\boldsymbol{f}_{t} + O_p\left(N^{-1/2}\right) \\ &= \bar{\mu}_{\omega} + \mathbf{\Lambda}\left(L\right)\mathbf{\Gamma}\mathbf{f}_{t} + O_p\left(N^{-1/2}\right).\end{aligned}$$

But under Assumptions 6 and 7, $\mathbf{\Gamma}$ and $\mathbf{\Lambda}\left(L\right)$ are both invertible and:

$$\mathbf{f}_t = \mathbf{\Gamma}^{-1}\mathbf{\Lambda}^{-1}\left(L\right)\left(\bar{\mathbf{z}}_{\omega t} - \bar{\mu}_{\omega}\right) + O_p\left(N^{-1/2}\right),$$

where:

$$\begin{aligned}\mathbf{\Gamma}^{-1} &= \begin{pmatrix} 0 & \gamma^{-1} \\ \theta^{-1} & -\frac{\lambda}{\theta\gamma}\end{pmatrix}, \\ \mathbf{\Lambda}^{-1}\left(L\right) &= \mathbf{B}_0 + \mathbf{B}_1 L + \mathbf{B}_2 L^2 + ...\end{aligned}$$

(note that $\mathbf{B}_0 = \mathbf{\Lambda}_0 = \mathbf{I}_2$). Hence,

$$\begin{aligned}\mathbf{f}_t &= \mathbf{\Gamma}^{-1}\left(\bar{\mathbf{z}}_{\omega t} - \bar{\mu}_{\omega}\right) + \left(\mathbf{C}_1 + \mathbf{C}_2 L + \mathbf{C}_3 L^2 + ...\right)\left(\bar{\mathbf{z}}_{\omega,t-1} - \bar{\mu}_{\omega}\right) + O_p\left(N^{-1/2}\right) \\ &= \mathbf{b} + \left(\sum_{\ell=0}^{\infty}\mathbf{C}_{\ell}L^{\ell}\right)\bar{\mathbf{z}}_{\omega,t} + O_p\left(N^{-1/2}\right),\end{aligned}$$

where $\mathbf{C}_{\ell} = \mathbf{\Gamma}^{-1}\mathbf{B}_{\ell}$, for $\ell = 0, 1, 2, ...$, and $\mathbf{b} = -\,\mathbf{\Gamma}^{-1}\mathbf{\Lambda}^{-1}\left(1\right)\bar{\mu}_{\omega}$. But given the lower triangular form of $\mathbf{\Gamma}^{-1}$, we have

$$f_t = \gamma^{-1}\Delta\bar{y}_{\omega,t} + \sum_{\ell=1}^{\infty}\mathbf{c}_{1,\ell}'\bar{\mathbf{z}}_{\omega,t-\ell} + O_p\left(N^{-1/2}\right), \tag{A.11}$$

$$g_t = \theta^{-1}\bar{v}_{\omega,t} - \left(\frac{\lambda}{\theta\gamma}\right)\Delta\bar{y}_{\omega,t} + \sum_{\ell=1}^{\infty}\mathbf{c}_{2,\ell}'\bar{\mathbf{z}}_{\omega,t-\ell} + O_p\left(N^{-1/2}\right), \tag{A.12}$$

where $\mathbf{c}_{1,\ell}'$ and $\mathbf{c}_{2,\ell}'$ are the first and the second rows of $\mathbf{C}_{\ell}$, respectively, and $\bar{v}_{\omega,t}$, $\Delta\bar{y}_{\omega,t}$, $\bar{\mathbf{z}}_{\omega,t}$ are defined as above.

Consider now $\mathbf{C}_{\ell}$ and note that $\left\|\mathbf{C}_{\ell}\right\| \leq \left\|\mathbf{\Gamma}^{-1}\right\|\left\|\mathbf{B}_{\ell}\right\|$, where $\left\|\mathbf{\Gamma}^{-1}\right\|$ is bounded for fixed non-zero values of γ and θ. Further, $\mathbf{B}_\ell$ is given by the following recursions

$$\begin{aligned}
\mathbf{B}_0 &= \mathbf{I}_2,\, \mathbf{B}_1 = -\boldsymbol{\Lambda}_1 \\
\mathbf{B}_2 &= -\left(\boldsymbol{\Lambda}_1\mathbf{B}_1 + \boldsymbol{\Lambda}_2\mathbf{B}_0\right), \\
&\vdots \\
\mathbf{B}_\ell &= -\left(\boldsymbol{\Lambda}_1\mathbf{B}_{\ell-1} + \boldsymbol{\Lambda}_2\mathbf{B}_{\ell-1} + ... + \boldsymbol{\Lambda}_\ell\mathbf{B}_0\right).
\end{aligned}$$

Hence, $\|\mathbf{B}_1\| \leq \|\boldsymbol{\Lambda}_1\| \|\mathbf{B}_0\|$, $\|\mathbf{B}_2\| \leq \|\boldsymbol{\Lambda}_1\| \|\mathbf{B}_1\| + \|\boldsymbol{\Lambda}_2\| \|\mathbf{B}_0\|$, and in general $\|\mathbf{B}_\ell\| \leq \|\boldsymbol{\Lambda}_1\| \|\mathbf{B}_{\ell-1}\| + \|\boldsymbol{\Lambda}_2\| \|\mathbf{B}_{\ell-1}\| + ... + \|\boldsymbol{\Lambda}_\ell\| \|\mathbf{B}_0\|$, where $\|\mathbf{B}_0\| = 1$. However,

$$\|\boldsymbol{\Lambda}_\ell\| = \left\|\mathbb{E}\left(\boldsymbol{\Phi}_i^\ell\right)\right\| \leq \mathbb{E}\left\|\boldsymbol{\Phi}_i^\ell\right\| \leq \left(\mathbb{E}\|\boldsymbol{\Phi}_i\|\right)^\ell \leq \rho^\ell.$$

Hence, $\|\mathbf{B}_1\| \leq \rho$, $\|\mathbf{B}_2\| \leq \rho^2$, and so on, and as required $\|\mathbf{C}_\ell\| \leq \left\|\boldsymbol{\Gamma}^{-1}\right\| \rho^\ell$.[42] ■

A.4 Proof of Proposition 5 (Consistent estimation of the orthonormalized factors in the dynamic case)

Proof. Consider equation (53) and (54) in the main text. Let $M_{\bar{Z}_\omega} = \mathbf{I}_T - \bar{\mathbf{Z}}_\omega\left(\bar{\mathbf{Z}}_\omega'\bar{\mathbf{Z}}_\omega\right)^{-1}\bar{\mathbf{Z}}_\omega'$, and note that:

$$\begin{aligned}
\mathbf{M}_{\bar{Z}_\omega}\mathbf{f} &= \mathbf{M}_{\bar{Z}_\omega}\Delta\bar{\mathbf{y}}_\omega \\
\mathbf{M}_{\bar{Z}_\omega}\mathbf{g} &= \mathbf{M}_{\bar{Z}_\omega}\bar{\mathbf{v}}_\omega - \lambda\mathbf{M}_{\bar{Z}_\omega}\Delta\bar{\mathbf{y}}_\omega
\end{aligned}$$

since $\mathbf{M}_{\bar{Z}_\omega}\bar{\mathbf{Z}}_\omega = \mathbf{0}$. We set the first normalized vector of innovations, denoted by $\hat{\zeta}$, to $\mathbf{M}_{\bar{Z}_\omega}\mathbf{f}$, namely $\hat{\zeta} = \mathbf{M}_{\bar{Z}_\omega}\Delta\bar{\mathbf{y}}_\omega$, and set the second factor, that we label $\hat{\xi}$, as the linear combination of $\mathbf{M}_{\bar{Z}_\omega}\mathbf{f}$ and $\mathbf{M}_{\bar{Z}_\omega}\mathbf{g}$ such that $\hat{\zeta}'\hat{\xi} = 0$. This can be achieved selecting λ so that:

$$\hat{\zeta}'\hat{\xi} = \Delta\bar{\mathbf{y}}_\omega'\mathbf{M}_{\bar{Z}_\omega}\left(\mathbf{M}_{\bar{Z}_\omega}\bar{\mathbf{v}}_\omega - \lambda\mathbf{M}_{\bar{Z}_\omega}\Delta\bar{\mathbf{y}}_\omega\right) = 0.$$

The value of λ that solves this equation is given by:

$$\hat{\lambda} = \frac{\Delta\bar{\mathbf{y}}_\omega'\mathbf{M}_{\bar{Z}_\omega}\bar{\mathbf{v}}_\omega}{\Delta\bar{\mathbf{y}}_\omega'\mathbf{M}_{\bar{Z}_\omega}\Delta\bar{\mathbf{y}}_\omega}.$$

Note that $\hat{\lambda}$ is the OLS estimator of the coefficient of the regression of $\mathbf{M}_{\bar{Z}_\omega}\bar{\mathbf{v}}_\omega$ on $\mathbf{M}_{\bar{Z}_\omega}\Delta\bar{\mathbf{y}}_\omega$. Hence, the orthogonalized factors are

$$\begin{aligned}
\hat{\zeta} &= \mathbf{M}_{\bar{Z}_\omega}\Delta\bar{\mathbf{y}}_\omega, \\
\hat{\xi} &= \mathbf{M}_{\bar{Z}_\omega}\bar{\mathbf{v}}_\omega - \hat{\lambda}\mathbf{M}_{\bar{Z}_\omega}\Delta\bar{\mathbf{y}}_\omega.
\end{aligned}$$

[42] Note that for any matrix $\mathbf{A}$, $\|\mathbf{A}^p\| \leq \|\mathbf{A}\|^p$, and for any random variable X, $\|E(X)\| \leq E\|X\|$.

In practice, this implies that $\hat{\zeta}$ can be recovered as residuals from the OLS regression of $\Delta\bar{\mathbf{y}}_\omega$ on an intercept and $\bar{\mathbf{z}}_{\omega,t-\ell}$, for $\ell = 1, 2, ..., p$:

$$\Delta\bar{\mathbf{y}}_\omega = \bar{\mathbf{Z}}_\omega \hat{\mathbf{c}}_1 + \hat{\zeta} \tag{A.13}$$

While $\hat{\xi}$ can be recovered as residuals from the OLS regression of $\bar{\mathbf{v}}_\omega$ on $\hat{\zeta}$, an intercept, and $\bar{\mathbf{z}}_{\omega,t-\ell}$, for $\ell = 1, ..., p$:

$$\bar{\mathbf{v}}_\omega = \hat{\lambda}\hat{\zeta} + \bar{\mathbf{Z}}_\omega \hat{\mathbf{c}}_2 + \hat{\xi} \tag{A.14}$$

■

B Data Sources

For equity prices we use the MSCI Index (excluding dividends) in local currency. We collected daily observations from January 1979 to June 2011, but the panel of countries is unbalanced with only 16 economies starting from the beginning of the sample. A balanced panel was also constructed with 32 countries from 1993:Q1. The data source for the daily equity price indices is Bloomberg. The countries included in the sample are the following: Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, China, Finland, France, Germany, India, Indonesia, Italy, Japan, Korea, Malaysia, Mexico, Netherlands, Norway, New Zealand, Peru, Philippines, Saudi Arabia, South Africa, Singapore, Spain, Sweden, Switzerland, Thailand, Turkey, United Kingdom, and United States.

The list of Bloomberg tickers is as follows: MSELTAG, MSDLAS, MSDLAT, MSDLBE, MSELTBR, MSDLCA, MSELTCF, MSELTCH, MSDLFI, MSDLFR, MSDLGR, MSELTIA, MSELTINF, MSDLIT, MSDLJN, MSELTKO, MXMY, MSELTMXF , MSDLNE, MSDLNO, MSDLNZ, MSELTPR, MSELTPHF, MSELTSA, MGCLSA, MSDLSG, MSDLSP, MSDLSW, MSDLSZ, MSELTTHF, MSELTTK, MSDLUK, MSDLUS.

Real GDP data come from standard sources. The data set is balanced and good quality quarterly data are also available for all countries from 1993:Q1. For more details see: https://sites.google.com/site/gvarmodelling/.

Online Supplement to

'Uncertainty and Economic Activity: A Multi-Country Perspective'

by A. Cesa-Bianchi, and M.Hashem Pesaran, and A. Rebucci

May 15, 2018

S1 Realized Volatility versus Cross-sectional Dispersion

As noted in the paper, if we consider a panel of country equities (e.g., of firms or sectors within a country), a different measure of uncertainty can be computed as the cross-sectional dispersion of equity prices. In this section we show that this concept is closely related to the realized volatility measures we consider. To illustrate the point with the data that we use in our application, we derive results at the 'country-specific versus world level' rather than 'firm-specific versus country level'.[43] Specifically, we compare the cross-sectional dispersion of equity returns across countries with the realized volatility of 'world' equity returns.

Define the daily cross-country dispersion of equity returns as:

$$\sigma_{cdt} = \sqrt{D_t^{-1} \sum_{\tau=1}^{D_t} \sum_{i=1}^{N} w_i \left(r_{it}(\tau) - \bar{r}_t(\tau) \right)^2}, \tag{S1.1}$$

and the daily realized volatility of world equity returns as:

$$\sigma_{rvt} = \sqrt{D_t^{-1} \sum_{i=1}^{N} \sum_{\tau=1}^{D_t} w_{it} \left(r_{it}(\tau) - \bar{r}_{it} \right)^2}, \tag{S1.2}$$

where $r_{it}(\tau) = \Delta \ln P_{it}(\tau)$ and $\bar{r}_{it} = D_t^{-1} \sum_{\tau=1}^{D_t} r_{it}(\tau)$ is the average daily price change over the quarter t, and D_t is the number of trading days in quarter t; and w_i is the weight attached to country i. To establish the relation between these two measures it is easier to work with their squares:

$$\begin{aligned} \sigma_{rvt}^2 &= D_t^{-1} \sum_{i=1}^{N} \sum_{\tau=1}^{D_t} w_i \left(r_{it}(\tau) - \bar{r}_{it} \right)^2, \\ \sigma_{cdt}^2 &= D_t^{-1} \sum_{\tau=1}^{D_t} \sum_{i=1}^{N} w_i \left(r_{it}(\tau) - \bar{r}_t(\tau) \right)^2. \end{aligned}$$

Note also that

$$\sigma_{rvt}^2 = D_t^{-1} \sum_{i=1}^{N} \sum_{\tau=1}^{D_t} w_i r_{it}^2(\tau) - \sum_{i=1}^{N} w_i \bar{r}_{it}^2,$$

and

$$\sigma_{cdt}^2 = D_t^{-1} \sum_{\tau=1}^{D_t} \sum_{i=1}^{N} w_i r_{it}^2(\tau) - \sum_{i=1}^{N} w_i \left(D_t^{-1} \sum_{\tau=1}^{D_t} \bar{r}_t^2(\tau) \right).$$

[43] Our analysis holds at the firm-specific versus country level as well.

Hence, since $\sum_{i=1}^{N} w_i = 1$, it follows that

$$\sigma^2_{cdt} - \sigma^2_{rvt} = \sum_{i=1}^{N} w_i \bar{r}^2_{it} - D_t^{-1} \sum_{\tau=1}^{D_t} \bar{r}^2_t(\tau),$$

where $\bar{r}_{it} = D_t^{-1} \sum_{\tau=1}^{D_t} r_{it}(\tau)$, and $\bar{r}_t(\tau) = \sum_{i=1}^{N} w_i r_{it}(\tau)$.

Suppose now that daily returns have the following single-factor structure:[44]

$$r_{it}(\tau) = \beta_i f_t(\tau) + \varepsilon_{it}(\tau),$$

where the factor is strong in the sense that (see Bailey et al., 2016)

$$\lim_{N\to\infty} \sum_{i=1}^{N} w_i \beta_i = \bar{\beta} \neq 0, \text{ and } \lim_{N\to\infty} \sum_{i=1}^{N} w_i \beta_i^2 = \sigma^2_\beta + \bar{\beta}^2 \neq 0.$$

The idiosyncratic components, $\varepsilon_{it}(\tau)$, are assumed to be independently distributed from $\beta_i f_t(\tau)$, cross-sectionally weakly correlated, and serially uncorrelated with zero means and finite variances. Also let:

$$\lim_{D_t\to\infty} D_t^{-1} \sum_{\tau=1}^{D_t} f_t^2(\tau) = h^2_{f_t}.$$

We now note that

$$\begin{aligned}
\sum_{i=1}^{N} w_i \bar{r}^2_{it} &= \left(\sum_{i=1}^{N} w_i \beta_i^2\right) \bar{f}_t^2 + \left(\sum_{i=1}^{N} w_i \bar{\varepsilon}^2_{it}\right) + 2\left(\sum_{i=1}^{N} w_i \beta_i \bar{\varepsilon}_{it}\right) \bar{f}_t \\
&= \left(\sigma^2_\beta + \bar{\beta}^2\right) \bar{f}_t^2 + O_p\left(D_t^{-1/2}\right) + O_p\left(N^{-1/2}\right),
\end{aligned}$$

where $\bar{f}_t = D_t^{-1} \sum_{\tau=1}^{D_t} f_t(\tau)$, and $\bar{\varepsilon}_{it} = D_t^{-1} \sum_{\tau=1}^{D_t} \varepsilon_{it}(\tau)$. Also

$$\begin{aligned}
D_t^{-1} \sum_{\tau=1}^{D_t} \bar{r}^2_t(\tau) &= D_t^{-1} \sum_{\tau=1}^{D_t} \left[\bar{\beta} f_t(\tau) + \bar{\varepsilon}_t(\tau)\right]^2 \\
&= \bar{\beta}^2 \left[D_t^{-1} \sum_{\tau=1}^{D_t} f_t^2(\tau)\right] + D_t^{-1} \sum_{\tau=1}^{D_t} \bar{\varepsilon}^2_t(\tau) + 2D_t^{-1} \sum_{\tau=1}^{D_t} \bar{\beta} \bar{\varepsilon}_t(\tau) f_t(\tau) \\
&= \bar{\beta}^2 h^2_{f_t} + O_p\left(N^{-1/2}\right) + O_p\left(D_t^{-1/2}\right).
\end{aligned}$$

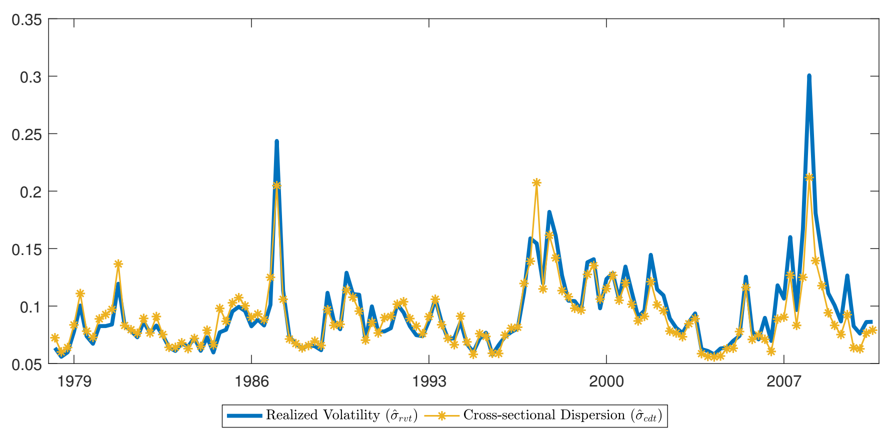
Hence

$$\begin{aligned}
\sigma^2_{cdt} - \sigma^2_{rvt} &= \left(\sigma^2_\beta + \bar{\beta}^2\right) \bar{f}_t^2 - \bar{\beta}^2 h^2_{f_t} + O_p\left(N^{-1/2}\right) + O_p\left(D_t^{-1/2}\right) \\
&= \sigma^2_\beta \bar{f}_t^2 - \bar{\beta}^2 \sigma^2_{f_t} + O_p\left(N^{-1/2}\right) + O_p\left(D_t^{-1/2}\right).
\end{aligned}$$

[44] The analysis readily extends to more general multiple factor settings.

where $\sigma^2_{f_t} = \left(h^2_{f_t} - \bar{f}^2_t\right) \geq 0$, is the variance of the common factor. This expression shows that, under fairly general assumptions (and for N and D_t sufficiently large) we would expect the cross-sectional dispersion measure to be closely related to asset-specific measures of realized volatility when the factor loadings, β_i, are not too dispersed across countries. The results also show that the relative magnitudes of the cross section dispersion and realized volatility measures depend on the relative values of $\sigma^2_\beta \bar{f}^2_t$ and $\bar{\beta}^2 \sigma^2_{f_t}$.

Figure S.1 Realized Volatility and Cross-sectional Dispersion



Note. World realized volatility of equity returns ($\hat{\sigma}_{rvt}$) and cross-sectional dispersion of equity returns across countries ($\hat{\sigma}_{cdt}$), computed as in equations (S1.2) and (S1.1), respectively. Both measures are expressed at quarterly rates and computed over the 1979:Q2-2011:Q2 period.

Figure S.1 compares world realized volatility (σ_{rvt}, light thick line) and cross-sectional dispersion (σ_{cdt}, dark thin line), computed as in equations (S1.2) and (S1.1), respectively, with equal weights. Their sample correlation over the 1979:Q1 to 2011:Q2 period is 0.92. Figure S.1 suggests that the two measures are very closely related, which is in line with the evidence provided by Bloom et al. (2012).

S2 Comparison with Principal Components

S2.1 Principal Component versus Cross-sectional Averages

In what follows we compare the cross-sectional average of the GDP growth rates ($\Delta \bar{y}_{\omega,t}$) with the first principal component of the individual output growth series $\{\Delta y_{it},\ i = 1, 2, ..., N;\ t = 1, 2, ..., T\}$, which we denote by $PC^y_{1,t}$. As can be seen from Figure S.2, the two series ($\Delta \bar{y}_{\omega,t}$, and $PC^y_{1,t}$) move very closely and have a correlation coefficient of 0.9.

S2.2 Principal Components of the Combined Panel of Output Growth Series and Realized Volatilities

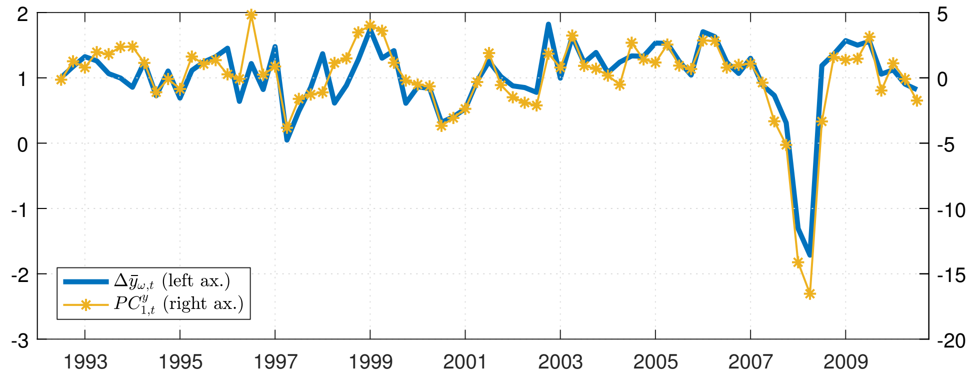
As stated in Remark 2 and 3 in the main text of the paper, applying principal component analysis to the panel of all volatility series or all volatility and growth series would not permit identifying f_t. To see this, we extract the first two principal components from the full panel of growth rates (Δy_{it}) and volatilities (v_{it}), which we label by $PC_{1,t}^{vy}$ and $PC_{2,t}^{vy}$, respectively. We then fit a VAR(1) model to these two PCs, and compute orthogonalized residuals from this VAR with a Cholesky decomposition placing the first PC, namely $PC_{1,t}^{vy}$, first (as discussed in Section 4.3). We denote these orthogonalized residuals by $\epsilon^{PC_{1,t}^{vy}}$ and $\epsilon^{PC_{2,t}^{vy}}$. Figure S.3 plots $\epsilon^{PC_{1,t}^{vy}}$ together with $\hat{\zeta}_t$ (top panel) and $\epsilon^{PC_{2,t}^{vy}}$ together with $\hat{\xi}_t$ (bottom panel), and shows that the two series behave in a very different fashion, with correlations of -0.43 and 0.43 respectively.

S2.3 Applying Principal Components Recursively

While principal components extracted from the panel of all volatility and growth series do not permit identifying f_t, here we show how principal component analysis can be used to obtain estimates of the real and financial factors reported in the paper. As discussed in the paper, we need to follow a recursive procedure where the order of the recursion plays a crucial role. We need extract the first principal component from the panel of GDP growth rates, which as before we label by $PC_{1,t}^{y}$, and as noted earlier recovers a consistent estimator of ζ_t (up to an scalar). Next, we obtain the first principal component from the combined panel of output growth rates and volatilities and label it as $PC_{1,t}^{vy}$. Then, we estimate a VAR(1) in the principal components $(PC_{1,t}^{y}, PC_{1,t}^{vy})'$ to remove any serial correlation, and orthogonalize the residuals of this VAR with a Cholesky factorization of the variance-covariance matrix of this VAR's reduced form residuals, with $PC_{1,t}^{y}$ ordered first. Denote the resultant orthogonalized residuals by $\epsilon^{PC_{1,t}^{y}}$ and $\epsilon^{PC_{1,t}^{vy}}$.

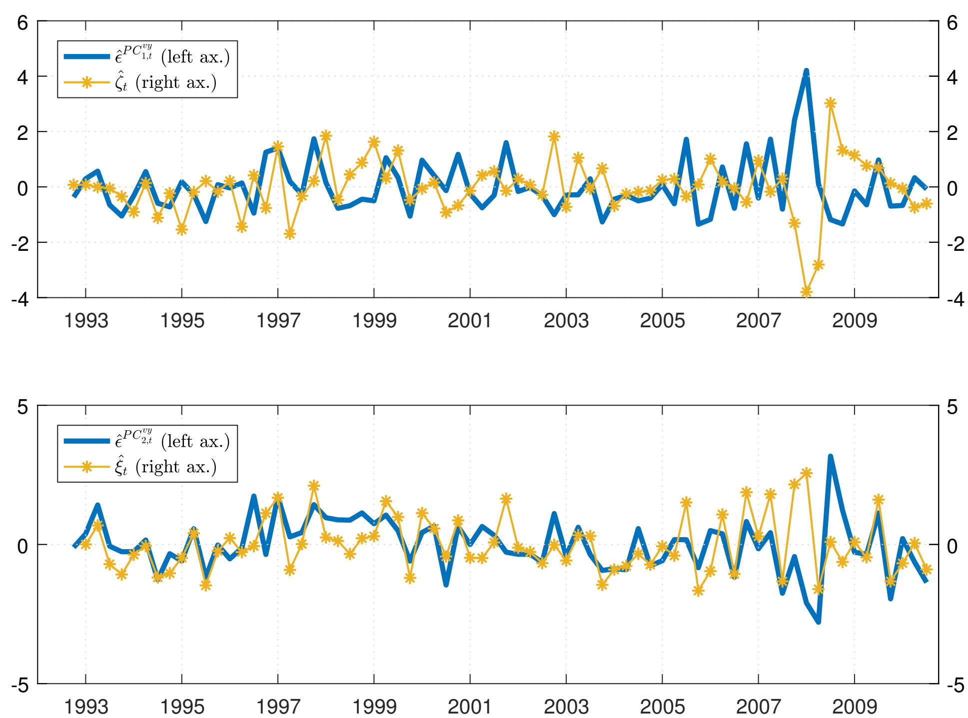
Figure S.4 plots $\epsilon^{PC_{1,t}^{y}}$ together with $\hat{\zeta}_t$ (top panel) and $\epsilon^{PC_{1,t}^{vy}}$ together with $\hat{\xi}_t$ (bottom panel), and shows that they move in tandem, with correlations of 0.82 and 0.97, respectively. Note, finally, that if we were to do a Cholesky factorization of the residuals from the same VAR but with the variables ordered in reverse, namely $(PC_{1,t}^{vy}, PC_{1,t}^{y},)'$, the resultant orthogonalized residuals will not closely follow $\hat{\zeta}_t$ and $\hat{\xi}_t$. We conclude from this exercise that principal component analysis could be used to obtain qualitatively similar results that we report in the paper if applied in the recursive manner proposed here.

Figure S.2 Estimating $\hat{\zeta}_t$: Principal Component Versus Cross-sectional Averages



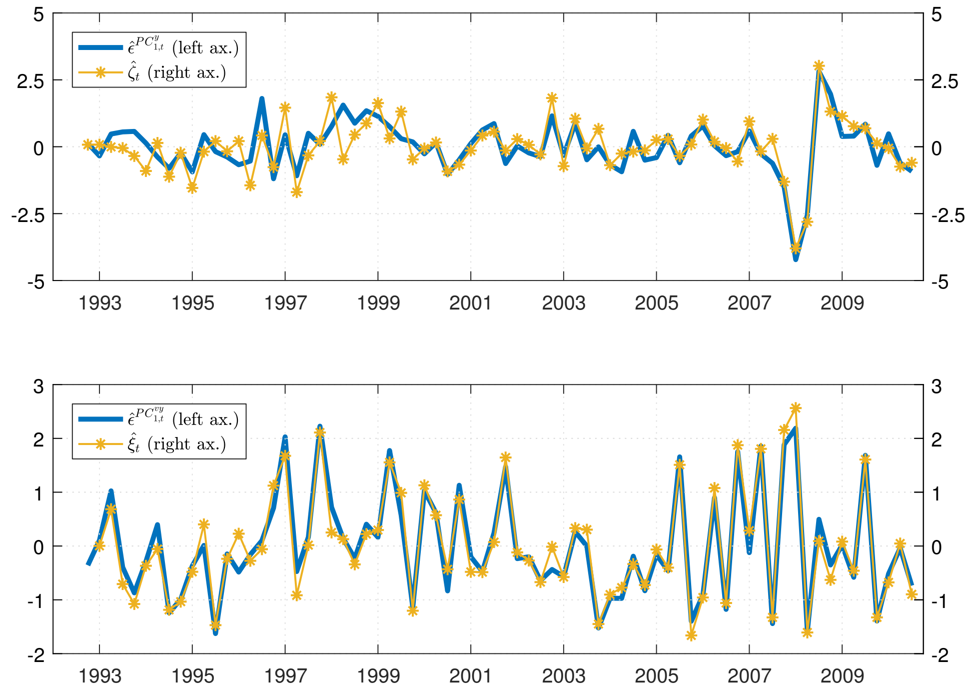
Note. Comparison of the cross-sectional average computed on the panel of GDP growth rates ($\Delta\bar{y}_{\omega,t}$) and the first principal component computed on the same data set ($PC^y_{1,t}$). Sample period: 1993:Q1-2011:Q2.

Figure S.3 Principal Component on the Full Panel of Volatility and Growth Series



Note. Comparison of $\hat{\zeta}_t$ and $\hat{\xi}_t$ with the principal components obtained from the full panel of volatilities (v_{it}) and growth rates (Δy_{it}). Sample period: 1993:Q1-2011:Q2.

Figure S.4 Principal Component of the Full Panel of Volatility and Growth Series Computed Recursively



Note. Comparison of $\hat{\zeta}_t$ and $\hat{\xi}_t$ with the principal components obtained from the full panel of volatilities (v_{it}) and growth series (Δy_{it}) following the recursive procedure described in Section S2.3. Sample period: 1993:Q1-2011:Q2.

S3 Computing Impulse Responses and Error Variance Decompositions

Consider the factor-augmented country-specific VAR models augmented with lagged cross section averages, $\bar{\mathbf{z}}_{\omega,t-\ell}$, for $\ell = 1, 2, ..., p$ as in equations (57)-(58) in the main text:

$$\mathbf{z}_{it} = \mathbf{\Phi}_i \mathbf{z}_{i,t-1} + \sum_{\ell=1}^{p} \psi_{i,\ell} \bar{\mathbf{z}}_{\omega,t-\ell} + \beta_i \upsilon_t + \boldsymbol{\vartheta}_{it}, \text{ for } i = 1, 2, ..., N, \tag{S3.1}$$

where:

$$\beta_i = \begin{pmatrix} \beta_{i,11} & \beta_{i,12} \\ \beta_{i,21} & 0 \end{pmatrix}, \ \upsilon_t = \begin{pmatrix} \zeta_t \\ \xi_t \end{pmatrix}.$$

Intercepts are omitted to simplify the exposition. Note also that $\bar{\mathbf{z}}_{\omega,t} = \sum_{i=1}^{N} w_i \Delta \mathbf{z}_{it} = \mathbf{W}\mathbf{z}_t$, where $\mathbf{z}_t = (\mathbf{z}'_{1t}, \mathbf{z}'_{2t}, ..., \mathbf{z}'_{Nt})'$, and $\mathbf{W}$ is a $2 \times 2N$ matrix of weights. Stacking the VARs in (S3.1) over i we obtain:

$$\mathbf{z}_t = \mathbf{\Phi} \mathbf{z}_{t-1} + \sum_{\ell=1}^{p} \psi_\ell \mathbf{W} \mathbf{z}_{t-\ell} + \beta \upsilon_t + \boldsymbol{\vartheta}_t, \tag{S3.2}$$

where $\boldsymbol{\vartheta}_t = (\boldsymbol{\vartheta}'_{1t}, \boldsymbol{\vartheta}'_{2t}, ..., \boldsymbol{\vartheta}'_{Nt})'$ and:

$$\boldsymbol{\Phi} = \begin{pmatrix} \boldsymbol{\Phi}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Phi}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \cdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{\Phi}_N \end{pmatrix}, \quad \psi_\ell = \begin{pmatrix} \psi_{1,\ell} \\ \psi_{2,\ell} \\ \vdots \\ \psi_{N,\ell} \end{pmatrix}, \quad \beta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_N \end{pmatrix}.$$

The high-dimensional VAR in (S3.2) can now be written as a standard FAVAR(p) model in $2N$ variables:

$$\mathbf{z}_t = (\boldsymbol{\Phi} + \psi_1 \mathbf{W}) \mathbf{z}_{t-1} + \sum_{\ell=2}^{p} \psi_\ell \mathbf{W} \mathbf{z}_{t-\ell} + \beta \upsilon_t + \boldsymbol{\vartheta}_t. \tag{S3.3}$$

For example, when $p = 1$ we have the FAVAR(1):

$$\mathbf{z}_t = (\mathbf{I}_{2N} - \boldsymbol{\Psi}_1 \mathbf{L})^{-1} (\beta \upsilon_t + \boldsymbol{\vartheta}_t),$$

where $\boldsymbol{\Psi}_1 = \boldsymbol{\Phi} + \psi_1 \mathbf{W}$ and

$$\mathbf{z}_t = (\mathbf{I}_{2N} - \boldsymbol{\Psi}_1 \mathbf{L})^{-1} \beta \upsilon_t + (\mathbf{I} - \boldsymbol{\Psi}_1 \mathbf{L})^{-1} \boldsymbol{\vartheta}_t.$$

Note that by construction υ_t and $\boldsymbol{\vartheta}_t$ are orthogonal, and for sufficiently large p, they are serially uncorrelated. Hence, the impulse response of shocks to elements of υ_t and $\boldsymbol{\vartheta}_t$ can be computed using the following moving average representation:

$$\mathbf{z}_t = \sum_{n=0}^{\infty} \mathbf{A}_n \upsilon_{t-n} + \sum_{n=0}^{\infty} \mathbf{C}_n \boldsymbol{\vartheta}_{t-n}, \tag{S3.4}$$

where $\mathbf{A}_n = \boldsymbol{\Psi}_1^n \beta$, and $\mathbf{C}_n = \boldsymbol{\Psi}_1^n$, for $n = 0, 1, 2, ...$

S3.1 Responses to Common and Country-specific Shocks

Let $\mathfrak{e}_i$ be a selection vector such that $\mathfrak{e}'_i \mathbf{z}_t$ picks the i^{th} element of $\mathbf{z}_t$. Also let $\mathbf{s}_f = (1, 0)'$ and $\mathbf{s}_g = (0, 1)'$, the vectors that select ζ_t and ξ_t from υ_t, namely:

$$\mathbf{s}'_f \upsilon_t \equiv \zeta_t, \qquad \mathbf{s}'_g \upsilon_t \equiv \xi_t. \tag{S3.5}$$

Recall now that ζ_t and ξ_t have zero means, unit variances and are orthogonal to each other. Then the impulse responses to a positive unit shock to ζ_t or ξ_t are given by:

$$IR_{i,\zeta,n} = \mathfrak{e}'_i \mathbf{A}_n \mathbf{s}_f \quad \text{and} \quad IR_{i,\xi,n} = \mathfrak{e}'_i \mathbf{A}_n \mathbf{s}_g \quad \text{for } n = 0, 1, 2, ..., \tag{S3.6}$$

where $\mathbf{A}_n$ is given by the moving average representation, (S3.4)

To derive impulse response functions for country-specific shocks, namely the j^{th} element of $\boldsymbol{\vartheta}_t$, we need to make assumptions about the correlation between volatility and growth innovations within each country and across countries. Since the elements of $\boldsymbol{\vartheta}_t$ are weakly correlated across countries, they have some, but limited correlations across countries (see Figure 5). We also documented that, conditional on the common factors ζ_t and ξ_t, the country correlation of volatility and growth innovations are statistically insignificant for all except for four countries.

As a first order approximation, therefore, we will assume that the covariance matrix of $\boldsymbol{\vartheta}_t$ in (S3.3) is diagonal. Under this assumption, the impulse response function of a positive, unit shock to the j^{th} element of $\boldsymbol{\vartheta}_t$ on the the i^{th} element of $\mathbf{z}_t$ is given by:

$$IR_{i,\vartheta_j,n} = \sqrt{\hat{\omega}_{jj}}\mathfrak{e}_i'\mathbf{C}_n\mathfrak{e}_j \ , \tag{S3.7}$$

where $\mathbf{C}_n$ is given by the moving average representation, (S3.4), $\hat{\omega}_{jj}$ is the (estimate) of the variance of the j^{th} country-specific shock and $\mathfrak{e}_j$ is a selection vector such that $\mathfrak{e}_j'\mathbf{z}_t$ picks the j^{th} element of $\mathbf{z}_t$.

The above impulse responses can be compared to the generalized impulse responses of Pesaran and Shin (1998). The latter are given by:

$$GIR_{i,\vartheta_j,n} = \frac{\mathfrak{e}_i'\mathbf{C}_n\hat{\Omega}\mathfrak{e}_j}{\sqrt{\hat{\omega}_{jj}}}, \tag{S3.8}$$

where $\hat{\Omega} = (\hat{\omega}_{ij})$ is the estimate of the covariance of $\boldsymbol{\vartheta}_t$. The generalized impulse responses allow for non-zero correlations across the idiosyncratic errors. The two sets of impulse responses coincide if the covariance matrix of $\boldsymbol{\vartheta}_t$ is diagonal.

S3.2 Forecast Error Variance Decompositions

Traditionally, the forecast error variance decomposition of a VAR model is performed on a set of orthogonalized shocks, whereby the contribution of the j^{th} orthogonalized innovation to the mean square error of the n-step ahead forecast of the model is calculated. In our empirical application this is not the case as —even if the country-specific volatility and growth innovations η_{it} and ε_{it} are weakly correlated across countries— some pairs of innovations can still display some non-zero correlation. An alternative approach is to compute Generalized Forecast Error Variance Decompositions (GVD) of Pesaran and Shin (1998). The Generalized Forecast Error Variance Decompositions consider the proportion of the variance of the n-step forecast errors of the endogenous variables that is explained by conditioning on the non-orthogonalized shocks, while explicitly allowing for the contemporaneous correlations between these shocks and the shocks to the other equations in the system.

Let $GVD_{i,\zeta,n}$ and $GVD_{i,\xi,n}$ be the share of the n-step ahead forecast error variance of the i^{th} variable in $\mathbf{z}_t$ that is accounted for by ζ_t and ξ_t, respectively, and $GVD_{i,j}$ the variance share of a generic country-specific shock, then:

$$GVD_{i,\zeta,n} = \frac{\sum\limits_{\ell=0}^{n}\left(\mathfrak{e}_i'\mathbf{A}_\ell\mathbf{s}_f\right)^2}{\sum\limits_{\ell=0}^{n}\mathfrak{e}_i'\mathbf{A}_\ell\mathbf{A}_\ell'\mathfrak{e}_i + \sum\limits_{\ell=0}^{n}\mathfrak{e}_i'\mathbf{C}_\ell\hat{\mathbf{\Omega}}\mathbf{C}_\ell'\mathfrak{e}_i}, \quad n = 1, 2, ..., H, \tag{S3.9}$$

$$GVD_{i,\xi,n} = \frac{\sum\limits_{\ell=0}^{n}\left(\mathfrak{e}_i'\mathbf{A}_\ell\mathbf{s}_g\right)^2}{\sum\limits_{\ell=0}^{n}\mathfrak{e}_i'\mathbf{A}_\ell\mathbf{A}_\ell'\mathfrak{e}_i + \sum\limits_{\ell=0}^{n}\mathfrak{e}_i'\mathbf{C}_\ell\hat{\mathbf{\Omega}}\mathbf{C}_\ell'\mathfrak{e}_i}, \quad n = 1, 2, ..., H, \tag{S3.10}$$

$$GVD_{i,j,n} = \frac{\hat{\omega}_{jj}^{-1}\sum\limits_{\ell=0}^{n}\left(\mathfrak{e}_i'\mathbf{C}_\ell\hat{\mathbf{\Omega}}\mathfrak{e}_j\right)^2}{\sum\limits_{\ell=0}^{n}\mathfrak{e}_i'\mathbf{A}_\ell\mathbf{A}_\ell'\mathfrak{e}_i + \sum\limits_{\ell=0}^{n}\mathfrak{e}_i'\mathbf{C}_\ell\hat{\mathbf{\Omega}}\mathbf{C}_\ell'\mathfrak{e}_i}, \quad j = 1, 2, ..., 2N, \quad n = 1, 2, ..., H; \tag{S3.11}$$

Note that the different assumptions we make on the covariance matrix of all country-specific shocks, $\hat{\mathbf{\Omega}}$, have implications for the error variance decompositions. Specifically, when we assume that (i) $\hat{\mathbf{\Omega}}$ is diagonal or (ii) $\hat{\mathbf{\Omega}}$ is block-diagonal with Cholesky-orthogonalized blocks, the relative importance of shocks to country volatility and growth for all countries (η_{it} and ε_{it}, for $j = 1, 2, ..., 2N$) and shocks to the two common factors ζ_t and ξ_t, is easily characterized as $VD_{i,\zeta,n} + VD_{i,\xi,n} + \sum_{j=1}^{2N} VD_{i,j,n} = 1$. That is the GVD formula coincides with the standard VD formula. In contrast, when we consider an unrestricted covariance matrix $\hat{\Omega}$, the sum of the variance shares does not necessarily add up to 1.

S3.3 Average Impulse Responses and Forecast Error Variance Decompositions

As a summary measure of the effects of shocks to the common factors we report the following average measures. Denote the impulse response (or forecast error variance decomposition) of a particular shock on the j^{th} variable in country i at horizon n by $X_{i,j,n}$. Let $w = (w_1, w_2, ..., w_N)'$ be a vector of fixed weights such that $\Sigma_{i=1}^{N} w_i = 1$. Then the average impulse response (or forecast error variance decomposition) of the shock to variable j, at horizon n, is computed as:

$$X_{\omega,j,n} = \sum_{i=1}^{N} w_i X_{i,j,n}. \tag{S3.12}$$

and its dispersion is computed by:

$$\sigma_{X_{\omega,j,n}} = \left[\sum_{i=1}^{N} w_i^2 \left(X_{i,j,n} - X_{\omega,j,n}\right)^2\right]^{1/2}, \tag{S3.13}$$

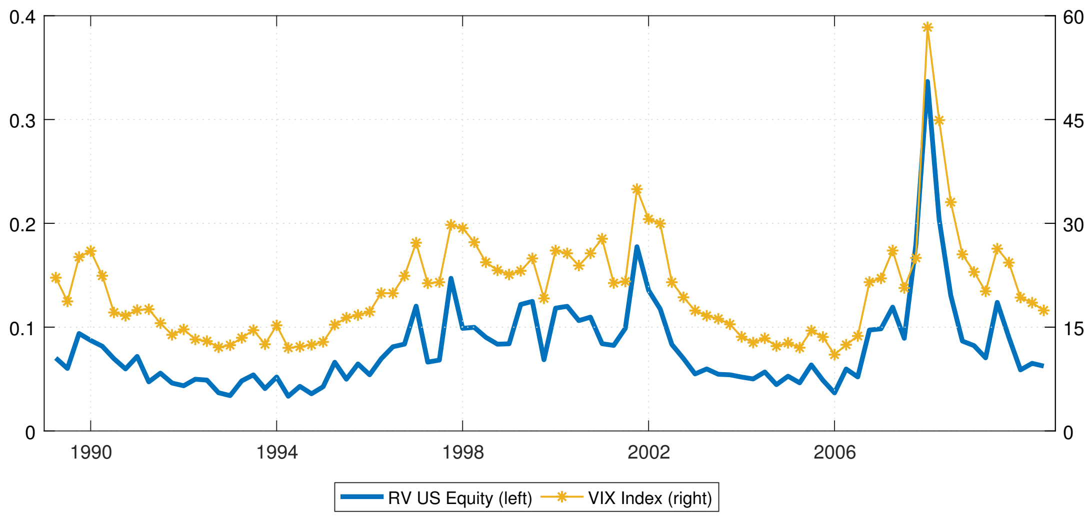
assuming country-specific impulse responses or forecast error variance decompositions are approximately uncorrelated.

S4 Country-specific Results

In this section we report selected country-specific results, including summary statistics, and the individual forecast error variance decompositions.

Figure S.5 plots the US realized volatility measure we constructed with the VIX index (during the period over which they overlap). The chart shows that the two measures co-move very closely with a correlation coefficient of around 0.9.

Figure S.5 Estimated Quarterly US Equity Realized Volatility and the Vix Index

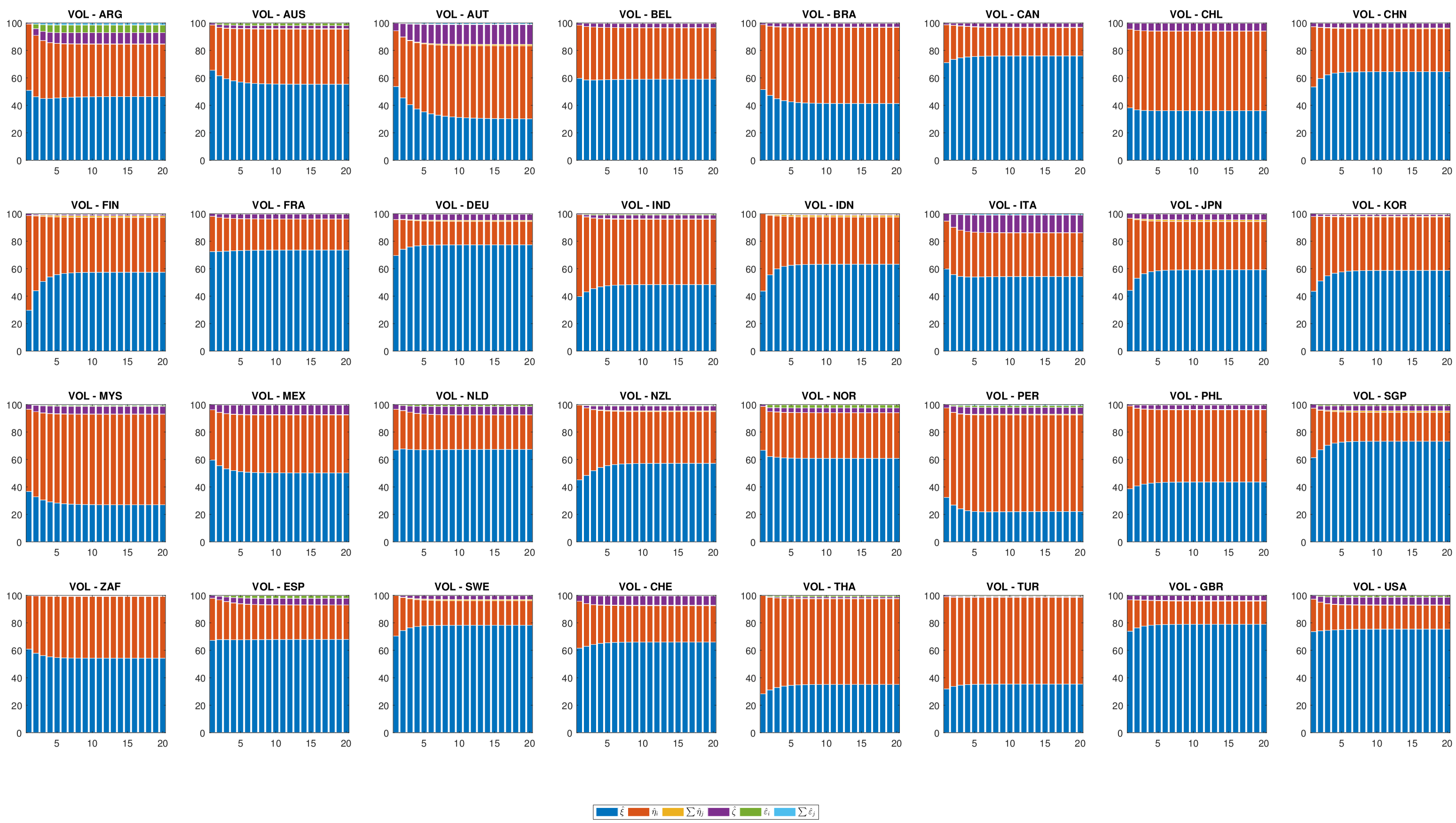


Note. 'RV US Equity' is the US realized volatility measure defined by (59). The VIX Index is the quarterly average of the daily Chicago Board Options Exchange Market Volatility Index from Bloomberg. Sample period: 1990:Q1-2011:Q2.

Figures S.6 to S.11 report forecast error variance decompositions for each country, for both volatility and growth, computed with different assumptions on the covariance matrix of the volatility and growth innovations. As can be seen the estimates are very similar across countries and for all the three schemes assumed for the error covariances.

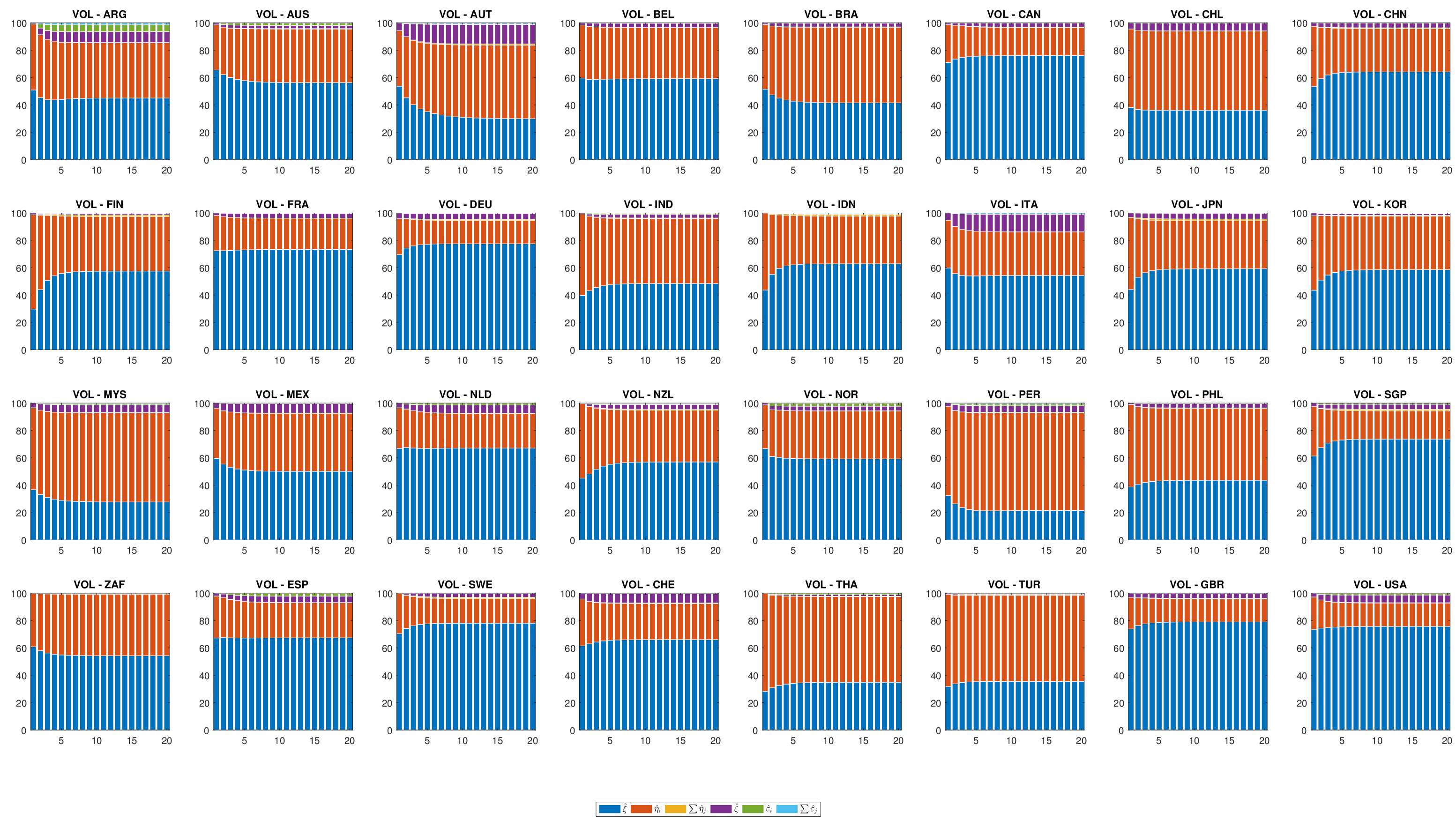
Figure S.12 plots the country-specific impulse response of volatility and growth to a positive, one-standard-deviation shock to the common shocks $\hat{\zeta}_t$ and $\hat{\xi}_t$. We can see from Figure S.12 that for most countries the impulse responses have a very similar profile.

Figure S.6 FORECAST ERROR VARIANCE DECOMPOSITION OF COUNTRY-SPECIFIC VOLATILITY SHOCKS - DIAGONAL ERROR COVARIANCE MATRIX



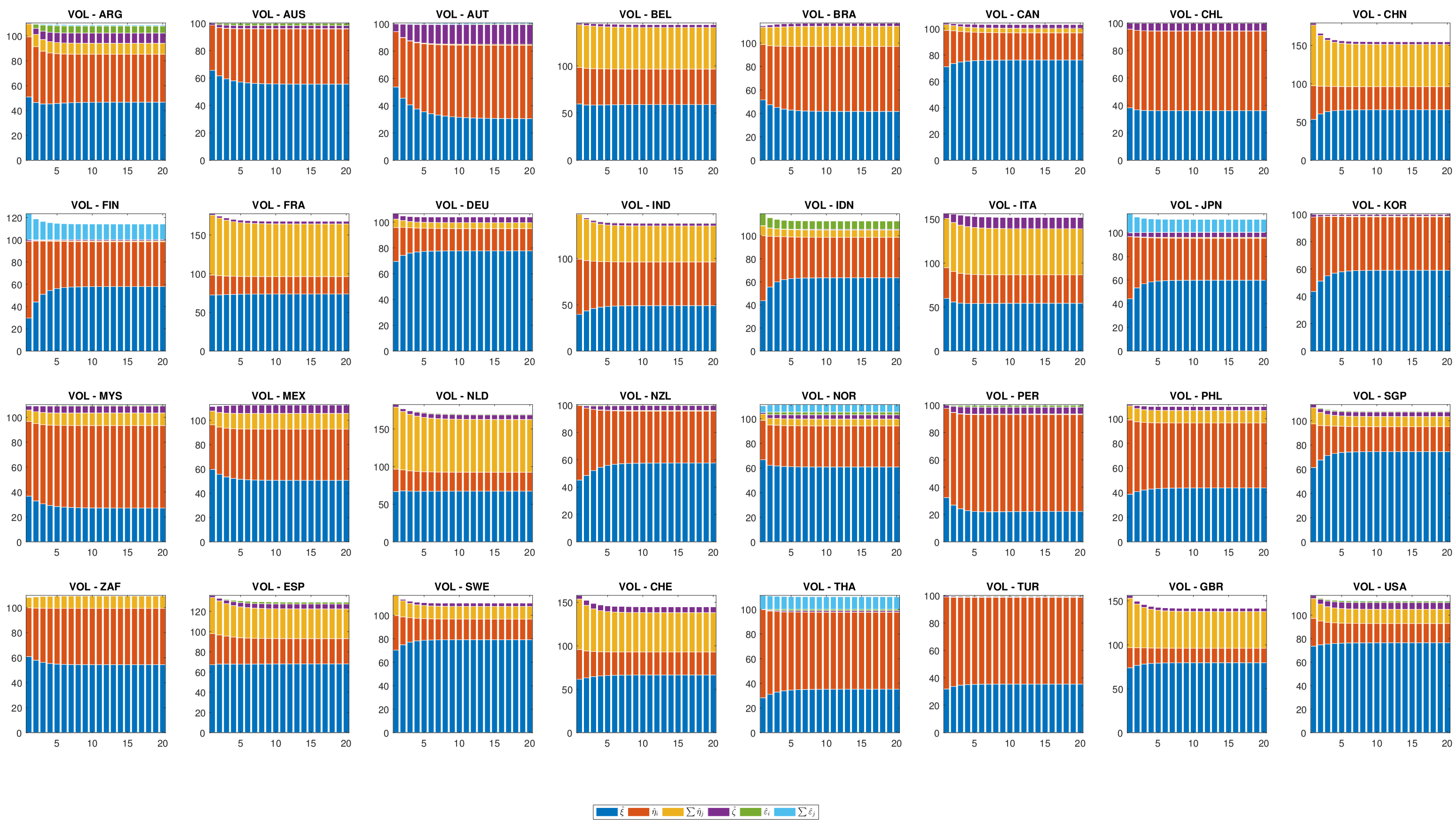
NOTE. $\hat{\xi}$ is the global financial shock; $\hat{\eta}_i$ is country-specific volatility shock; $\sum\hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries; $\hat{\zeta}$ is global real shock; $\hat{\varepsilon}_i$ is country-specific GDP shock; $\sum\hat{\varepsilon}_j$ is the sum of the contribution of the GDP shocks in the remaining countries. The horizontal axis is in quarters. Sample period: 1993:Q1-2011:Q2.

Figure S.7 Forecast Error Variance Decomposition of Country-Specific Volatility Shocks - Block Diagonal Error Covariance Matrix



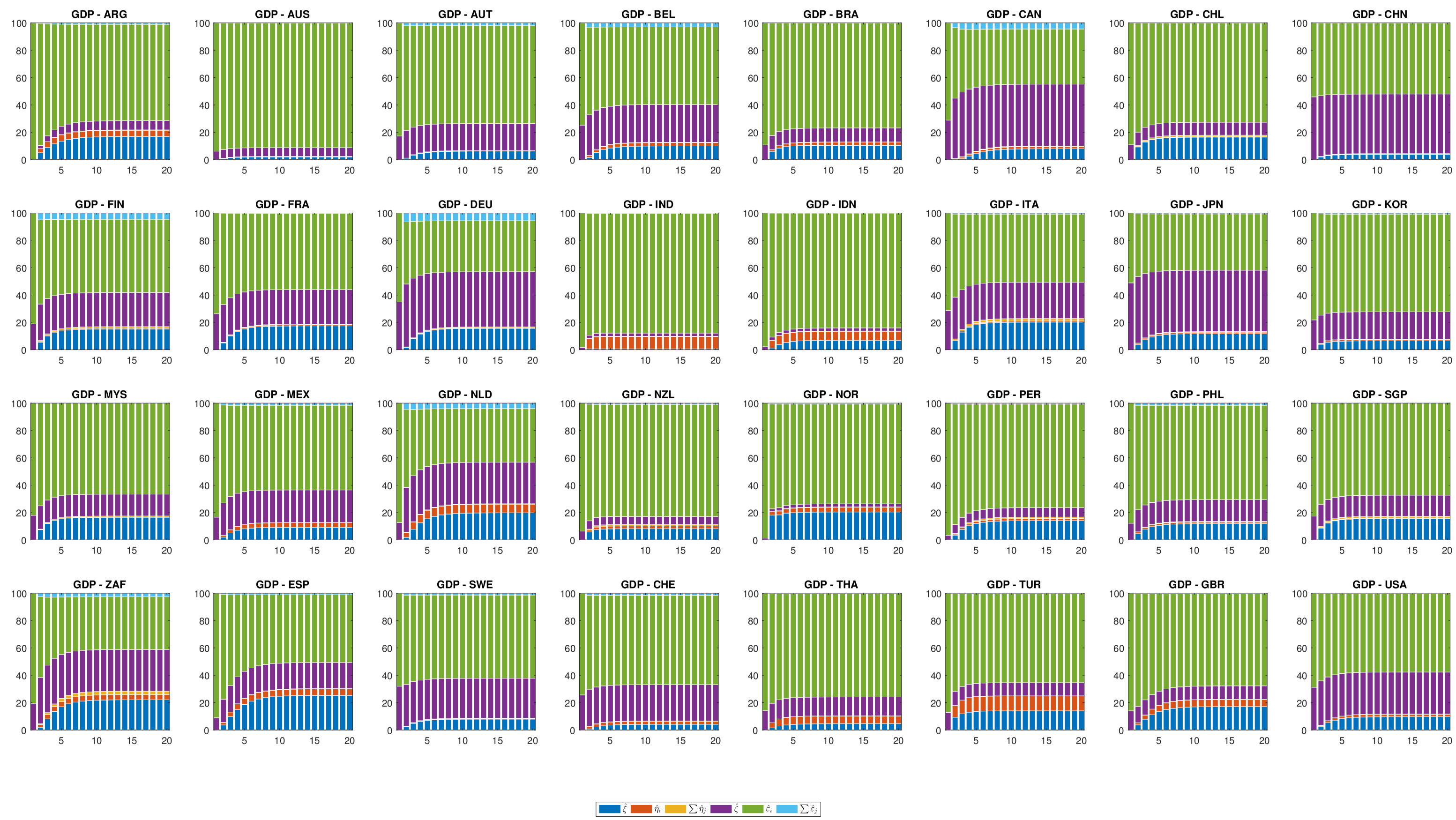
Note. $\hat{\xi}$ is the global financial shock; $\hat{\eta}_i$ is country-specific volatility shock; $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries; $\hat{\zeta}$ is global real shock; $\hat{\varepsilon}_i$ is country-specific GDP shock; $\sum \hat{\varepsilon}_j$ is the sum of the contribution of the GDP shocks in the remaining countries. The horizontal axis is in quarters. Sample period: 1993:Q1-2011:Q2.

Figure S.8 Generalized Forecast Error Variance Decomposition of Country-Specific Volatility Shocks - Regularized Estimation of Full Error Covariance Matrix



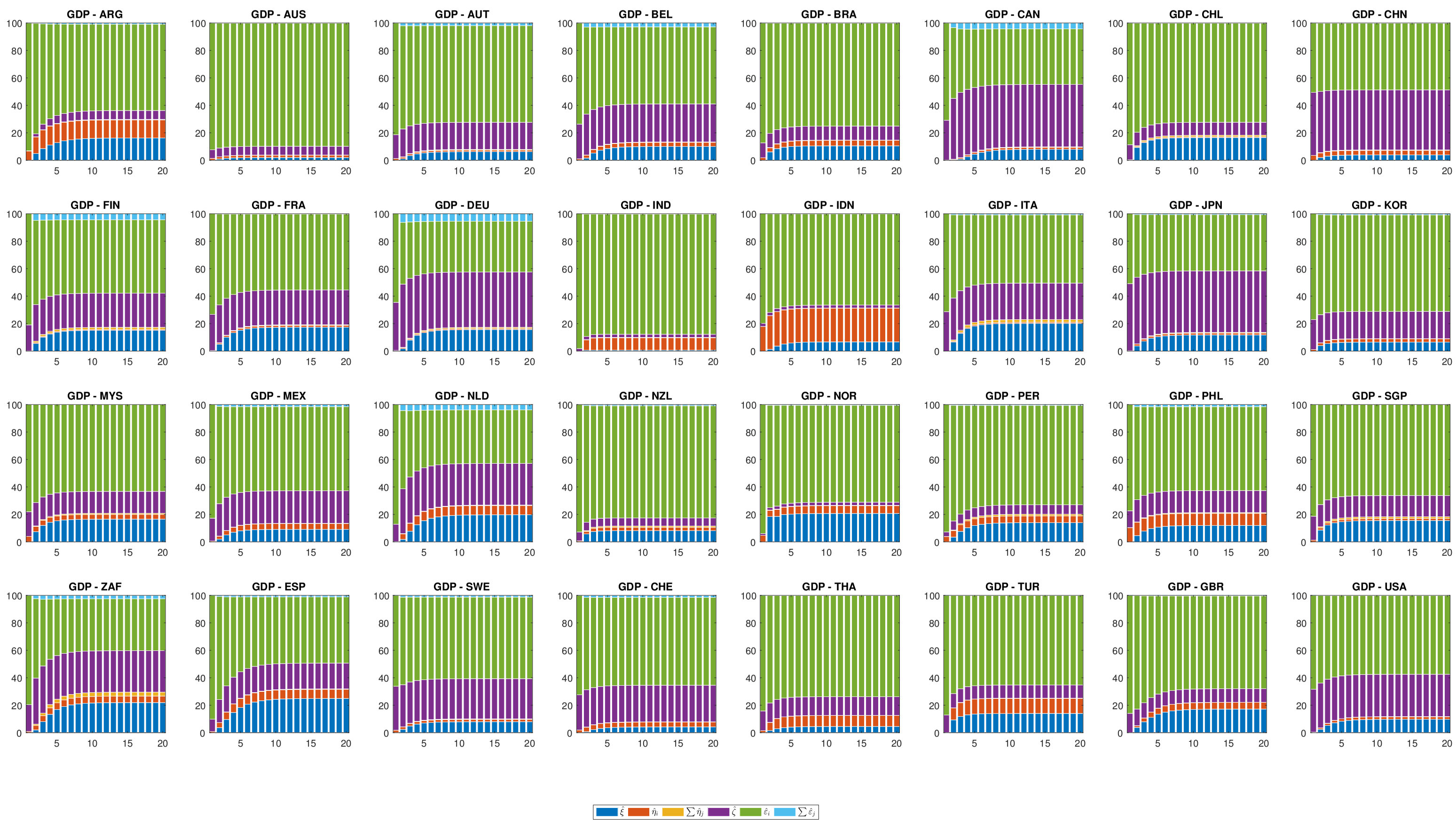
Note. $\hat{\xi}$ is the global financial shock; $\hat{\eta}_i$ is country-specific volatility shock; $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries; $\hat{\zeta}$ is global real shock; $\hat{\varepsilon}_i$ is country-specific GDP shock; $\sum \hat{\varepsilon}_j$ is the sum of the contribution of the GDP shocks in the remaining countries. The horizontal axis is in quarters. Sample period: 1993:Q1-2011:Q2.

Figure S.9 Forecast Error Variance Decomposition of Country-Specific Growth Shocks - Diagonal Error Covariance Matrix



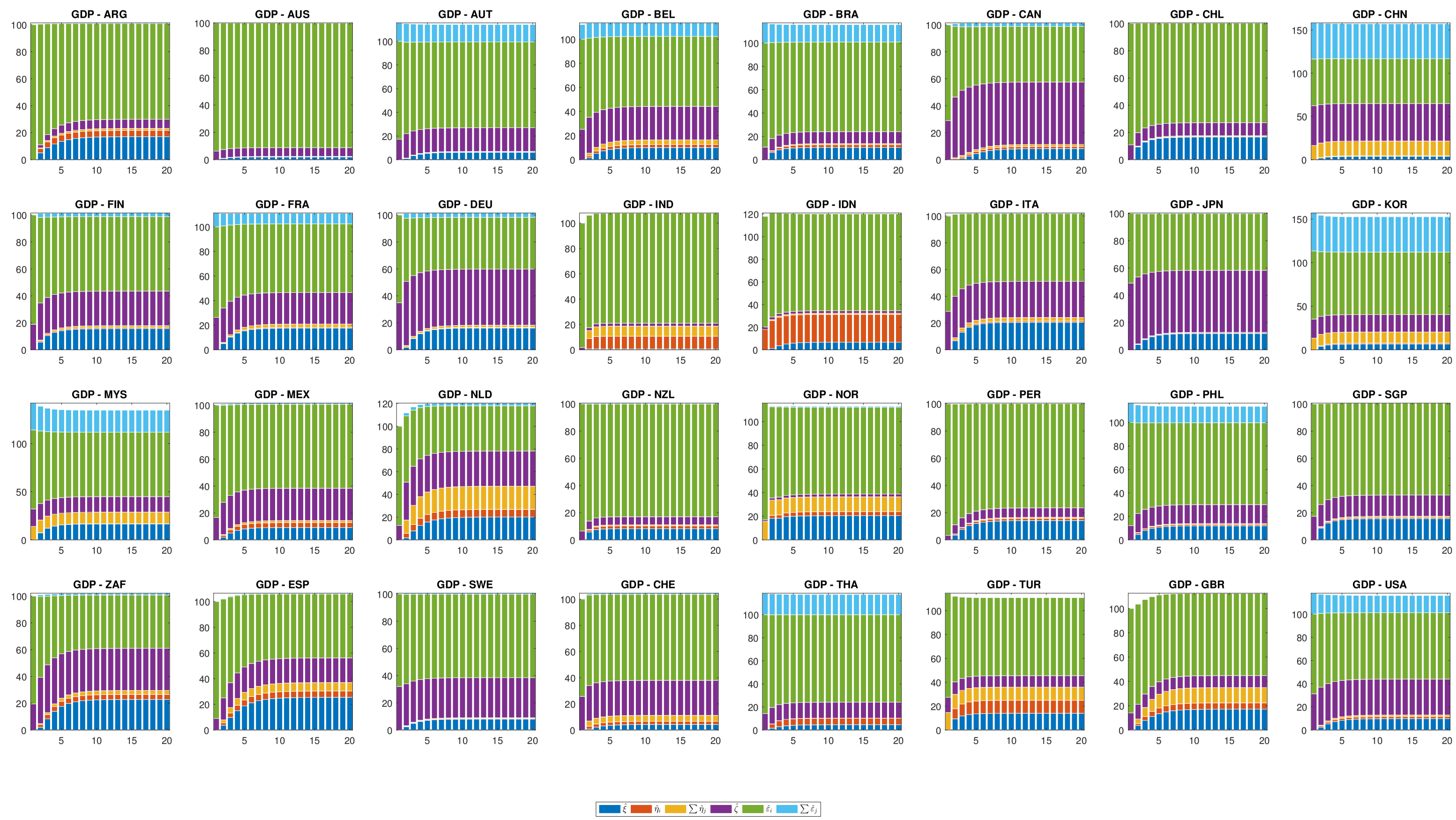
Note. $\hat{\xi}$ is the global financial shock; $\hat{\eta}_i$ is country-specific volatility shock; $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries; $\hat{\zeta}$ is global real shock; $\hat{\varepsilon}_i$ is country-specific GDP shock; $\sum \hat{\varepsilon}_j$ is the sum of the contribution of the GDP shocks in the remaining countries. The horizontal axis is in quarters. Sample period: 1993:Q1-2011:Q2.

Figure S.10 Forecast Error Variance Decomposition of Country-Specific Growth Shocks - Block Diagonal Error Covariance Matrix



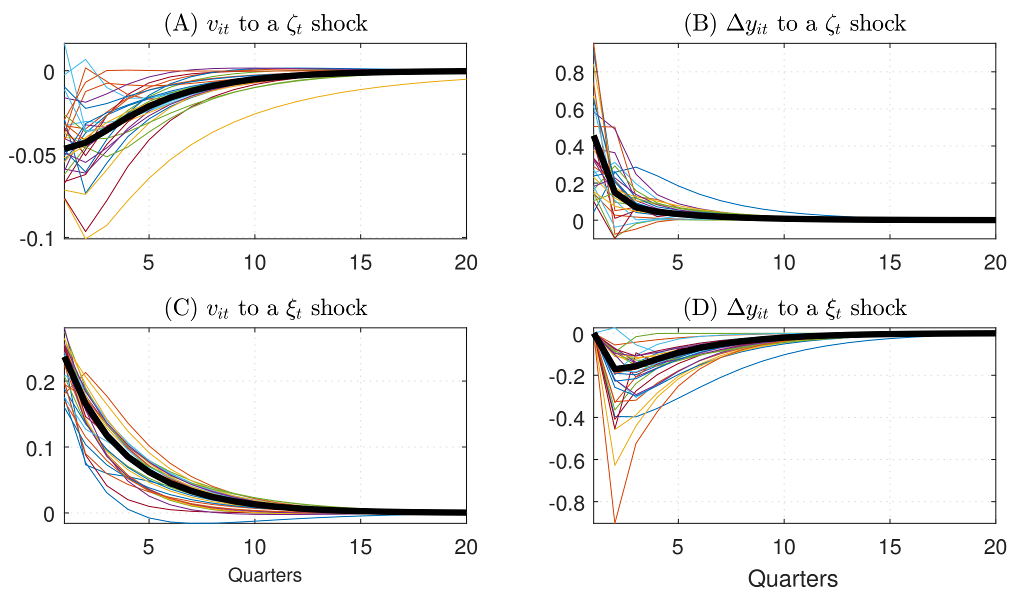
Note. $\hat{\xi}$ is the global financial shock; $\hat{\eta}_i$ is country-specific volatility shock; $\sum\hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries; $\hat{\zeta}$ is global real shock; $\hat{\varepsilon}_i$ is country-specific GDP shock; $\sum\hat{\varepsilon}_j$ is the sum of the contribution of the GDP shocks in the remaining countries. The horizontal axis is in quarters. Sample period: 1993:Q1-2011:Q2.

Figure S.11 Generalized Forecast Error Variance Decomposition of Country-Specific Growth Shocks - Regularized Estimation of Full Error Covariance Matrix



Note. $\hat{\xi}$ is the global financial shock; $\hat{\eta}_i$ is country-specific volatility shock; $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries; $\hat{\zeta}$ is global real shock; $\hat{\varepsilon}_i$ is country-specific GDP shock; $\sum \hat{\varepsilon}_j$ is the sum of the contribution of the GDP shocks in the remaining countries. The horizontal axis is in quarters. Sample period: 1993:Q1-2011:Q2.

Figure S.12 COUNTRY-SPECIFIC VOLATILITY AND GROWTH IMPULSE RESPONSES TO COMMON REAL AND FINANCIAL SHOCKS



NOTE. One standard deviation shocks to $\hat{\zeta}_t$ and $\hat{\xi}_t$. Thin lines are individual country responses. The solid lines are the PPP-GDP weighted averages, as the ones reported in the main text. Impulse responses are computed as in Appendix S3. Sample period: 1993:Q1-2011:Q2.

S5 Other results

Tables S.1, S.2, and S.3 report the summary statistics for the realized volatility series, the log-level of real GDP, and the log-difference of real GDP for each country in our sample. These results support the use of GDP growth and log-level of realized volatilities as stationary series in our empirical analysis.

Table S.1 SUMMARY STATISTICS FOR COUNTRY-SPECIFIC REALIZED VOLATILITY (LOG-LEVEL)

	ARG	AUS	AUT	BEL	BRA	CAN	CHL	CHN	FIN	FRA	DEU
Obs	94	129	129	129	86	129	94	74	94	129	129
Mean	-1.72	-2.63	-2.88	-2.79	-1.67	-2.73	-2.54	-2.01	-2.07	-2.52	-2.46
Max	0.00	-1.07	-1.17	-1.41	0.61	-1.08	-1.44	-0.88	-0.26	-1.27	-1.09
Min	-2.58	-3.40	-4.22	-3.59	-2.72	-3.44	-3.17	-2.71	-3.19	-3.23	-3.29
St. Dev.	0.51	0.35	0.63	0.45	0.65	0.44	0.36	0.42	0.54	0.38	0.34
Auto Corr.	0.63	0.51	0.78	0.61	0.84	0.70	0.47	0.64	0.78	0.53	0.54
ADF	-2.95^b	-2.97^b	-2.73^c	-3.35^b	-2.45	-2.78^c	-3.63^a	-2.57	-2.72^c	-3.25^b	-3.71^a
	IND	IDN	ITA	JPN	KOR	MYS	MEX	NLD	NZL	NOR	PER
Obs	97	94	129	129	129	125	94	129	94	125	74
Mean	-2.19	-2.12	-2.39	-2.50	-2.24	-2.51	-2.22	-2.64	-2.52	-2.31	-2.09
Max	-1.23	-0.56	-1.27	-1.07	-1.08	-0.75	-1.21	-1.21	-1.44	-0.93	-0.80
Min	-2.92	-3.19	-3.32	-3.67	-3.52	-3.64	-2.97	-3.36	-3.28	-3.05	-2.79
St. Dev.	0.39	0.50	0.41	0.44	0.44	0.50	0.35	0.43	0.37	0.36	0.40
Auto Corr.	0.52	0.55	0.60	0.64	0.72	0.61	0.55	0.67	0.65	0.54	0.62
ADF	-3.19^b	-2.09	-3.49^a	-3.85^a	-2.48	-3.18^b	-2.74^c	-2.87^c	-2.69^c	-3.2^b	-2.06
	PHL	SGP	ZAF	ESP	SWE	CHE	THA	TUR	GBR	USA	
Obs	101	129	129	129	117	129	97	94	129	129	
Mean	-2.19	-2.51	-2.21	-2.58	-2.37	-2.85	-2.11	-1.65	-2.61	-2.63	
Max	-0.82	-0.95	-0.89	-1.22	-1.21	-1.41	-1.20	-0.87	-1.23	-1.09	
Min	-3.13	-3.29	-3.27	-3.49	-3.13	-3.89	-2.85	-2.60	-3.43	-3.40	
St. Dev.	0.39	0.44	0.40	0.46	0.39	0.50	0.40	0.36	0.39	0.40	
Auto Corr.	0.50	0.55	0.45	0.67	0.59	0.64	0.53	0.61	0.55	0.68	
ADF	-4.1^a	-3.27^b	-3.72^a	-3.01^b	-3.13^b	-3.02^b	-3.31^b	-1.76	-3.24^b	-2.83^c	

NOTE. Summary statistics of the log-level of volatility (v_{it}). *ADF* is the Augmented Dickey-Fuller t-statistic computed with 4 lags and a constant, where a, b, and c denote associated p-values at 1%, 5%, and 10%.

Table S.2 Summary Statistics for Country-specific Real GDP (Log-Level)

	ARG	AUS	AUT	BEL	BRA	CAN	CHL	CHN	FIN	FRA	DEU
Obs	128	128	128	128	128	128	128	128	128	128	128
Mean	0.65	0.80	0.54	0.48	0.70	0.62	1.15	2.49	0.57	0.44	0.42
Max	5.18	2.85	3.48	2.23	5.16	2.47	8.39	5.91	4.41	1.56	2.63
Min	-6.35	-1.65	-2.61	-2.12	-7.25	-2.27	-6.72	-1.33	-6.01	-1.70	-4.16
St. Dev.	2.20	0.78	0.97	0.78	1.88	0.79	2.07	1.21	1.45	0.52	0.94
Auto Corr.	0.59	0.29	-0.03	0.26	0.21	0.55	0.27	0.27	0.07	0.40	0.13
ADF	-4.17	-4.59	-4.41	-5.00	-5.42	-4.22	-5.05	-3.63	-3.82	-4.02	-4.52

	IND	IDN	ITA	JPN	KOR	MYS	MEX	NLD	NZL	NOR	PER
Obs	128	128	128	128	128	128	128	128	128	128	128
Mean	1.53	1.24	0.36	0.45	1.45	1.42	0.65	0.50	0.52	0.67	0.77
Max	3.59	12.08	2.84	2.68	6.67	5.22	3.79	2.98	3.47	4.68	7.04
Min	-2.84	-8.17	-3.70	-4.09	-8.94	-7.10	-6.07	-2.38	-2.72	-3.48	-14.00
St. Dev.	1.06	2.19	0.73	1.02	1.82	1.61	1.58	0.78	0.96	1.27	3.11
Auto Corr.	0.27	0.02	0.36	0.29	-0.01	0.35	0.20	0.23	0.20	-0.24	0.38
ADF	-5.27	-4.80	-4.11	-4.14^{c}	-4.87^{c}	-5.26	-4.84	-3.54	-4.48	-3.56	-4.34

	PHL	SGP	ZAF	ESP	SWE	CHE	THA	TUR	GBR	USA
Obs	128	128	128	128	128	128	128	128	128	128
Mean	0.79	1.65	0.61	0.58	0.53	0.44	1.31	1.00	0.48	0.64
Max	4.63	6.77	2.50	2.49	4.41	2.50	6.06	7.03	2.18	2.23
Min	-6.88	-3.77	-2.14	-1.57	-5.12	-3.50	-5.11	-11.93	-2.40	-2.18
St. Dev.	1.55	1.91	0.86	0.56	1.32	0.83	1.65	2.68	0.76	0.76
Auto Corr.	0.12	0.23	0.58	0.80	-0.23	0.24	0.49	0.04	0.52	0.41
ADF	-3.17	-5.75	-4.33	-3.09	-4.50	-4.31	-3.11	-6.31	-4.60	-4.43

Note. Summary statistics for the log-level of real GDP (y_{it}). *ADF* is the Augmented Dickey-Fuller t-statistic computed with 4 lags and a constant, where a, b, and c denote associated p-values at 1%, 5%, and 10%.

Table S.3 SUMMARY STATISTICS FOR COUNTRY-SPECIFIC REAL GDP (LOG-DIFFERENCE)

	ARG	AUS	AUT	BEL	BRA	CAN	CHL	CHN	FIN	FRA	DEU
Obs	128	128	128	128	128	128	128	128	128	128	128
Mean	0.65	0.80	0.54	0.48	0.70	0.62	1.15	2.49	0.57	0.44	0.42
Max	5.18	2.85	3.48	2.23	5.16	2.47	8.39	5.91	4.41	1.56	2.63
Min	-6.35	-1.65	-2.61	-2.12	-7.25	-2.27	-6.72	-1.33	-6.01	-1.70	-4.16
St. Dev.	2.20	0.78	0.97	0.78	1.88	0.79	2.07	1.21	1.45	0.52	0.94
Auto Corr.	0.59	0.29	-0.03	0.26	0.21	0.55	0.27	0.27	0.07	0.40	0.13
ADF	-4.17^a	-4.59^a	-4.41^a	-5^a	-5.42^a	-4.22^a	-5.05^a	-3.63^a	-3.82^a	-4.02^a	-4.52^a

	IND	IDN	ITA	JPN	KOR	MYS	MEX	NLD	NZL	NOR	PER
Obs	128	128	128	128	128	128	128	128	128	128	128
Mean	1.53	1.24	0.36	0.45	1.45	1.42	0.65	0.50	0.52	0.67	0.77
Max	3.59	12.08	2.84	2.68	6.67	5.22	3.79	2.98	3.47	4.68	7.04
Min	-2.84	-8.17	-3.70	-4.09	-8.94	-7.10	-6.07	-2.38	-2.72	-3.48	-14.00
St. Dev.	1.06	2.19	0.73	1.02	1.82	1.61	1.58	0.78	0.96	1.27	3.11
Auto Corr.	0.27	0.02	0.36	0.29	-0.01	0.35	0.20	0.23	0.20	-0.24	0.38
ADF	-5.27^a	-4.8^a	-4.11^a	-4.14^a	-4.87^a	-5.26^a	-4.84^a	-3.54^a	-4.48^a	-3.56^a	-4.34^a

	PHL	SGP	ZAF	ESP	SWE	CHE	THA	TUR	GBR	USA
Obs	128	128	128	128	128	128	128	128	128	128
Mean	0.79	1.65	0.61	0.58	0.53	0.44	1.31	1.00	0.48	0.64
Max	4.63	6.77	2.50	2.49	4.41	2.50	6.06	7.03	2.18	2.23
Min	-6.88	-3.77	-2.14	-1.57	-5.12	-3.50	-5.11	-11.93	-2.40	-2.18
St. Dev.	1.55	1.91	0.86	0.56	1.32	0.83	1.65	2.68	0.76	0.76
Auto Corr.	0.12	0.23	0.58	0.80	-0.23	0.24	0.49	0.04	0.52	0.41
ADF	-3.17^b	-5.75^a	-4.33^a	-3.09^b	-4.5^a	-4.31^a	-3.11^b	-6.31^a	-4.6^a	-4.43^a

NOTE. Summary statistics for the log-difference of real GDP (Δy_{it}). *ADF* is the Augmented Dickey-Fuller t-statistic computed with 4 lags and a constant, where a, b, and c denote associated p-values at 1%, 5%, and 10%.