



BANK OF ENGLAND

# Staff Working Paper No. 787

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## International trade, non-trading firms and their impact on labour productivity

Stephen Millard,<sup>(1)</sup> Anamaria Nicolae<sup>(2)</sup> and Michael Nower<sup>(3)</sup>

### Abstract

In this paper we examine the impact of non-trading firms on labour productivity and its persistence in response to macroeconomic shocks, through their entry and exit into the domestic market, in a model with monopolistic competition and heterogeneous firms. We quantify the effects of various macroeconomic shocks on labour productivity and we demonstrate that non-trading domestic firms' entry and exit into the domestic market explains the persistence of labour productivity in response to transitory shocks. We also show that the model successfully replicates the sluggish recovery of labour productivity in the United Kingdom since the Great Recession.

**Key words:** International trade, heterogeneous firms, productivity, endogenous persistence.

**JEL classification:** E24, F17, J24, O40.

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(1) Bank of England, Durham University Business School and Centre for Macroeconomics.

Email: [stephen.millard@bankofengland.co.uk](mailto:stephen.millard@bankofengland.co.uk)

(2) Durham University Business School. Email: [anamaria.nicolae@durham.ac.uk](mailto:anamaria.nicolae@durham.ac.uk)

(3) Durham University Business School. Email: [michael.nower@durham.ac.uk](mailto:michael.nower@durham.ac.uk)

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Bank of England, Threadneedle Street, London, EC2R 8AH

Telephone +44 (0)20 3461 4030 email [publications@bankofengland.co.uk](mailto:publications@bankofengland.co.uk)

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# 1 Introduction

In this paper we study the role of the entry and exit of less productive non-trading domestic firms into the domestic market in driving labour productivity and its persistence, in response to shocks to macroeconomic variables. The analysis takes place in a dynamic, stochastic, general equilibrium macroeconomics model of international trade with monopolistic competition and heterogeneous firms. Recent research has thoroughly examined the effects of international trade on non-trading firms in such a framework.

For the purpose of our analysis, two important results stand out from this research. First, exports impact productivity of non-trading domestic firms thorough competition for inputs, as exporting firms demand additional labour to serve international markets, and this drives up the real wage. The increase in the real wage forces the least productive non-trading firms to exit, thus increasing average productivity. Second, imports impact labour productivity at an aggregate level thorough increased competition. Domestic firms face increased competition from importers and this forces the least productive non-trading firms to exit, thus increasing average productivity. The key papers that develop these results are: Melitz (2003), Ghironi and Melitz (2005) and Fattel Jaef and Lopez (2014), empirically supported by papers such as Amiti and Konings (2007), Yasar and Paul (2007) and Bloom *et al.* (2016). Important precursors to the analytical foundations of these results are contained in Dixit and Stiglitz (1977), Krugman (1979), Krugman (1980), Hopenhayn (1992*a*) and Hopenhayn (1992*b*).

Ghironi and Melitz (2005), Fattel Jaef and Lopez (2014) and many other important papers, make the assumption that the productivity of non-trading firms does not change as a result of the entry and exit of less productive non-trading domestic firms into the domestic market in response to macroeconomic shocks. Instead, they assume that the distribution of firm-level productivity is exogenously fixed. This assumption is appropriate when analysing the response of firms that trade internationally to macroeconomic shocks and the impact of these firms on the economy. However, this assumption may be less appealing when analysing the role of non-trading domestic firms in driving labour productivity, given that less than 1% of US firms engage in international trade (according the US Census 2016). The non-trading firms also tend to be less productive than firms that engage in international trade. A wide body of literature, such as Delgado *et al.* (2002), Andersson *et al.* (2008) and Sharma and Mishra (2015), has shown the self-selection effect, where productive firms self-select into international markets. The non-trading domestic firms are both relatively less productive and most numerous; and small macroeconomic shocks can have significant effects on their entry and exit. Unlike Ghironi and Melitz (2005) and Fattel Jaef and Lopez (2014), in our model, we allow for the entry and exit of non-trading firms into the domestic market to endogenously drive changes in the distribution of firm-level productivities.

Three recent influential papers discuss some of the issues we address in this paper. The first is Melitz (2003). He demonstrates that the distribution of non-trading firms' productivity responds to permanent shocks to barriers to trade following changes in competition for inputs. However, his analysis is conducted in a static model and it cannot therefore examine the dynamic behaviour of the distribution of non-trading firms' productivity, nor can it examine its response to temporary shocks.

The two other related key papers are by Ghironi and Melitz (2005) and Fattel Jaef and Lopez (2014). Although the analyses in Ghironi and Melitz (2005) and Fattel Jaef and Lopez (2014) are conducted in a dynamic, stochastic, general equilibrium model, the distribution of firm-level productivity

is exogenously fixed, and therefore, the impact of the entry and exit of firms into the domestic market on the productivity of domestic firms cannot be investigated. This limits their analysis of the response of productivity to macroeconomic shocks to only the extensive margin, with all changes to productivity being driven only by the entry and exit of firms. Unlike Ghironi and Melitz (2005) and Fattel Jaef and Lopez (2014) who only capture the extensive margin of production through the total number of domestic firms, we also allow for an endogenously determined productivity distribution of producing firms, capturing variations in the intensive margin of production as well. Limiting their analysis to only the extensive margins of production also limits the impact of any changes in barriers to imports, which as Bloom *et al.* (2016) show operate through both the intensive and extensive margins of production, and thus may underestimate the impact of changes in barriers to imports on productivity relative to the impact of changes in barriers to exports. According to Bloom *et al.* (2016), competition from Chinese imports alone accounts for 14% of European technological growth from 2000 to 2007 through both technological change within firms and reallocated employment between firms, as less productive firms exit the market the latter of which is ignored in Ghironi and Melitz (2005) and Fattel Jaef and Lopez (2014).

This paper is also closely related to the recent literature examining the impact of firm entry and exit on productivity in a closed economy. The main theoretical papers, starting with Hopenhayn (1992*b*) and Hopenhayn (1992*a*), are Clementi and Palazzo (2016), Woo (2016) and Lee and Mukoyama (2018), while Moreira (2017) and Sedlacek and Stern (2017) provide important empirical examinations of the impact of firm entry and exit on productivity in a closed economy. Our paper builds on this literature by extending the closed economy analysis of entry and exit dynamics to an open-economy framework in order to analyse the effect of international trade flows on the impact of firm entry and exit on productivity.

In this paper we develop a model in an open economy framework that allows us to examine the role of the entry and exit of less productive non-trading domestic firms on labour productivity, through both the extensive margin of production and, by allowing endogenous changes in the firm-level productivity distribution, through the intensive margin of production as well. We conduct our analysis in a ‘three-country’ setup to allow for a possible future analysis of changes in trade barriers which results in trade diversion, but which we do not take up here. We use our model to analyse the extent to which the response of labour productivity to macroeconomics shocks in our model could explain the behaviour of UK productivity since the Great Recession. Despite the response of labour productivity to macroeconomics shocks being a clear question of first-order policy importance, to our knowledge, it has not been addressed so far in the class of models employed here, although such models are widely employed for policy-oriented analyses.

The rest of the paper is organised as follows: In Section 2 we outline our model and discuss its features and in Section 3 we present the model calibration. In Section 4.1 we analyse the role of entry and exit of non-trading firms into the domestic market in driving labour productivity in response to shocks to macro variables. In Section 5 we use our model to analyse the behaviour of UK productivity since the Great Recession and examine the role the entry and exit of non-trading domestic firms in the UK market might have played in driving labour productivity. In Section 6 we discuss our results and offer some thought on areas for future research.

Figure 1: Timeline of a Period

	Period Starts	Production/Consumption	New Firm Entry	Period Ends	Exogenous Death Shock Hits	Next Period Starts
<b>Firms</b>	<p>Period starts with <math>N_{Dt}</math> firms</p> <p>Firms are then hit by Exogenous Shocks</p>	<p><math>N_{Dt}</math> firms with average productivity <math>\tilde{z}_{Dt}</math> make an average of <math>\tilde{d}_t</math> profits over the course of the period</p>	<p><math>N_{UEt}</math> firms pay the sunk entry cost (<math>f_E</math>) and attempt to enter the market of which <math>N_{Et} = (1 - G(E(z_{Dt+1})))N_{UEt}</math> draw a productivity above <math>z_d</math> and successfully enter the market</p>	<p>Period ends with <math>N_{Ht} = N_{Dt} + N_{Et}</math> firms</p>	<p>A proportion, <math>\delta</math>, of firms are hit by the exogenous death shock, and cease to operate</p>	<p>Next period starts with <math>(1 - \delta) N_{Ht} = N_{Dt+1}</math> Firms with average productivity <math>\tilde{z}_{Dt+1}</math></p>
<b>Households</b>	<p>Households start the period with holdings of bonds (<math>B_{t-1}</math>) and shares (<math>x_{t-1}</math>)</p>	<p>Households consume <math>C_t</math> and are paid a total wage of <math>w_t L_t</math>, the interest on their holdings of bonds <math>B_{t-1} r_{t-1}</math>, as well as a proportion of firm profits <math>x_{t-1} \tilde{d}_t</math> equal to their holdings of shares in the mutual fund (under financial autarky <math>x_{t-1} = 1</math>)</p>		<p>Households choose their holdings of Bonds <math>B_t</math> for which they will receive a known interest rate <math>r_t</math> and shares in the mutual fund <math>x_t</math> of existing firms (<math>N_{Ht}</math>) for the next period</p>		<p>Households start the next period with bond holdings <math>B_t</math> and share holdings <math>x_t</math></p>

## 2 The Model

Our basic framework builds on the model of Ghironi and Melitz (2005), the component parts of which are now familiar in the literature, therefore we develop the key equations more concisely. We develop a three-country ( $h$ ,  $i$  and  $j$ ) model with endogenous average firm-level productivity. Figure 1 presents a timeline for one period, detailing the behaviour of firms and households.

### 2.1 Households

Households are homogeneous and demand goods from both domestic and foreign producers. The representative household in country  $h$  supplies  $L_t^h$  units of labour, to only the firms in country  $h$ , at a nominal wage rate  $W_t^h$ ; the real wage rate is denoted by  $w_t^h$ . The representative household maximises their expected intertemporal utility from consumption subject to their budget constraint:

$$\max_{w.r.t.: C_t^h, B_t^h, x_t^h, C_{t+1}^h} \sum_{s=t}^{\infty} \beta^{s-t} \left( \frac{C_s^{h1-\gamma} - 1}{1-\gamma} \right),$$

$$\text{subject to: } B_t^h + \tilde{v}_t^h N_{Ht}^h x_t^h + C_t^h = (1 + r_{t-1}^h) B_{t-1}^h + (\tilde{d}_t^h + \tilde{v}_t^h) N_{Dt}^h x_{t-1}^h + w_t^h L_t^h, \quad (1)$$

where  $\beta \in (0, 1)$  is the subjective household discount factor,  $\gamma > 0$  is the inverse of the intertemporal elasticity of substitution and  $C_t$  is the consumption basket, defined over a continuum of goods  $\Omega$  in every period.  $C_t = \left( \int_{\omega \in \Omega} c_t(\omega)^{\frac{\theta-1}{\theta}} d\omega \right)^{\frac{\theta}{\theta-1}}$ , where  $\theta > 1$  is the elasticity of substitution across goods,  $B_{t-1}^h$  is the consumers holdings of bonds at the beginning of the period (chosen during the previous period), which pay a risk free rate of interest,  $r_{t-1}^h$ ;  $x_{t-1}^h$  represents the consumers holdings of shares in a mutual fund of domestic firms at the beginning of the period (chosen during the previous period);  $\tilde{v}_t^h$  and  $\tilde{d}_t^h$  are the average value and per-period profits of firms respectively;  $N_{Dt}^h$  is the number of firms at the start of a period and  $N_{Ht}^h$  is the number of firms at the end of the period. After the end of every period, an exogenously given proportion of firms  $\delta$  dies out, thus the number of firms at the start of a period,  $N_{Dt}^h$ , will be equal to the number number of firms operating in the market at the end of the previous period,  $N_{Ht-1}^h$ , adjusted to reflect the proportion of firms that die out:  $N_{Dt}^h = (1 - \delta)N_{Ht-1}^h$ . The derivations of  $\tilde{v}_t^h$ ,  $\tilde{d}_t^h$ ,  $N_{Dt}^h$  and  $N_{Ht}^h$  will be presented in the next section. Additionally, we impose financial autarky: households accumulate risk free domestic bonds and shares in only the firms in their domestic economy.

In each period, only a subset of goods,  $\Omega_t \in \Omega$ , will be available. Let  $p_t^h(\omega)$  denote the country  $h$  currency nominal price of a good  $\omega \in \Omega_t$ . The consumption based price index in country  $h$  is  $P_t^h = \left( \int_{\omega \in \Omega_t} p_t^h(\omega)^{1-\theta} d\omega \right)^{\frac{1}{1-\theta}}$  and the household demand, for each individual good  $\omega$ , is given by  $c_t^h(\omega) = \left( \frac{p_t^h(\omega)}{P_t^h} \right)^{-\theta} C_t^h$ . The representative households in countries  $i$  and  $j$  solve a similar problem.

### 2.2 Firms

There is a continuum of firms in each of the three countries,  $h$ ,  $i$  and  $j$ , each producing a different variety of good  $\omega \in \Omega$ . Firm  $\omega$  employs  $L_{Pt}(\omega)$  units of labour to produce output at time  $t$ . Their marginal cost in nominal terms will depend on: first, the country specific aggregate technology level  $Z_t$ , which evolves according to an AR(1) process with persistence  $\rho$ , common to all firms within a country; second, the firm-level productivity  $z$ , and third, the nominal wage rate  $W_t$ ; therefore,  $MC = W_t/Z_t z$ . The firm-level productivity of each firm is drawn by the firm from a distribution  $G(z)$  with support on  $[z_{min}, \infty)$ , upon market entry. To enter the market, and draw a firm-level productivity, the firm must, as in

Hopenhayn (1992b) and Melitz (2003), pay a sunk entry cost,  $f_E$ , expressed in terms of effective labour units.<sup>1</sup> In the manner of Hopenhayn (1992a) and Melitz (2003) firms also have to pay a per-period fixed cost of production,  $f_D^h$ , as well as per-period costs of entering foreign markets  $i$  and  $j$ ,  $f_{X_i}^h$  and  $f_{X_j}^h$ , respectively, all measured in terms of effective labour units. In addition, exporting firms have to pay a per-unit iceberg cost such that a firm needs to export  $\tau$  units of their good in order to sell one unit in the destination market. Finally, we assume that all three markets are monopolistically-competitive. The firms' problem is to maximise profits subject to their production function and the three consumer demand curves. Given that each firm produces a single variety of good,  $\omega$ , and that the firms optimal behaviour is determined by their firm-level productivity level  $z$ , we move from indexing by  $\omega$  to indexing by  $z$ , such that  $c_t(\omega) \equiv c_t(z)$  and  $p_t(\omega) \equiv p_t(z)$  for a firm with a given productivity  $z$ .

A firm with firm-level productivity  $z$  in country  $h$  solves the following constrained maximisation problem:

$$\text{Max: } d_t^h(z) = p_{D_t}^h(z)y_{D_t}^h(z) + p_{X_{it}}^h(z)y_{X_{it}}^h(z)\epsilon_i + p_{X_{jt}}^h(z)y_{X_{jt}}^h(z)\epsilon_j - W_t^h L_{P_t}^h(z)$$

$$-\frac{W_t^h f_D^h}{Z_t^h} - \frac{W_t^h f_{X_i}^h}{Z_t^h} - \frac{W_t^h f_{X_j}^h}{Z_t^h},$$

$$w.r.t : p_{D_t}^h(z), y_{D_t}^h(z), p_{X_{it}}^h(z), y_{X_{it}}^h(z), p_{X_{jt}}^h(z), y_{X_{jt}}^h(z), L_{P_t}^h(z),$$

$$\text{subject to : } y_{D_t}^h(z) + \tau_{ii}^h y_{X_{it}}^h(z) + \tau_{ij}^h y_{X_{jt}}^h(z) = z Z_t^h L_{P_t}^h(z),$$

$$y_{D_t}^h(z) = \left( \frac{p_{D_t}^h(z)}{P_t^h} \right)^{-\theta} C_t^h,$$

$$y_{X_{it}}^h(z) = \left( \frac{p_{X_{it}}^h(z)}{P_t^i} \right)^{-\theta} C_t^i,$$

$$y_{X_{jt}}^h(z) = \left( \frac{p_{X_{jt}}^h(z)}{P_t^j} \right)^{-\theta} C_t^j,$$

where,  $\tau_{ii}^h$  and  $\tau_{ij}^h$  are the iceberg costs of exporting from country  $h$  to countries  $i$  and  $j$  respectively at time  $t$ ; for a firm with a given firm-level productivity level  $z$  in country  $h$ ,  $d_t^h(z)$  is its total profit,  $p_{D_t}^h(z)$ ,  $p_{X_{it}}^h(z)$  and  $p_{X_{jt}}^h(z)$  are the prices of domestic goods, exports to country  $i$  and exports to country  $j$ , denominated in units of the currency of country  $h$ ,  $i$  and  $j$  respectively;  $y_{D_t}^h(z)$ ,  $y_{X_{it}}^h(z)$  and  $y_{X_{jt}}^h(z)$  are the total units of goods sold by the firm in the domestic market and countries  $i$  and  $j$  respectively, we assume that supply matches demand:  $y_t(z) = c_t(z)$ ;  $L_{P_t}^h(z)$  is the amount of labour used in production;  $C_t^h$ ,  $C_t^i$  and  $C_t^j$  are aggregate consumption in countries  $h$ ,  $i$  and  $j$  respectively;  $P_t^h$ ,  $P_t^i$  and  $P_t^j$  are the consumption-based price indices of countries  $h$ ,  $i$  and  $j$  and,  $\epsilon_i$  and  $\epsilon_j$  are the nominal exchange rates (units of  $h$  currency per unit of  $i$  and  $j$  currency) between country  $h$  and countries  $i$  and  $j$  respectively.

Solving this problem, the firm sets their output price as a mark-up over the marginal cost, where the mark-up is given by  $\theta/(\theta - 1)$ . Given this, the real prices of the firm's goods in each market are as follows: the real price of domestic goods in country  $h$  is  $\rho_{D_t}^h(z) = \frac{p_{D_t}^h(z)}{P_t^h} = \frac{\theta}{\theta-1} \frac{w_t^h}{Z_t^h}$ , the real price of goods exported to country  $i$  from country  $h$  is  $\rho_{X_{it}}^h(z) = \frac{p_{X_{it}}^h(z)}{P_t^i} = \frac{\tau_{ii}^h}{Q_t^i} \rho_{D_t}^h(z)$ , and the real price of goods exported to country  $j$  from country  $h$  is  $\rho_{X_{jt}}^h(z) = \frac{p_{X_{jt}}^h(z)}{P_t^j} = \frac{\tau_{ij}^h}{Q_t^j} \rho_{D_t}^h(z)$ , where  $Q_t^i$  is the real exchange rate between country  $h$  and country  $i$ , equal to  $\epsilon_i \frac{P_t^i}{P_t^h}$ ,  $Q_t^j$  is the real exchange rate between country  $h$

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<sup>1</sup>Effective labour units are calculated as units of labour multiplied by the technology level  $Z_t$ .

and country  $j$ , equal to  $\epsilon_j \frac{P_j^j}{P_t^h}$  and  $w_t^h = W_t^h/P_t^h$  is the real wage. Equivalent price equations hold for countries  $i$  and  $j$ .

Total firm profits are given by the sum of profit from domestic sales,  $d_{Dt}^h$ , and potential profit from exporting,  $d_{Xit}^h$  and  $d_{Xjt}^h$ , to countries  $i$  and  $j$  respectively. Given the fixed costs of domestic production and exporting, there will be some firms that do not draw high enough firm-level productivity to make a profit (or break even) in the domestic market, who then exit the market entirely, and some firms that do not export to one or the other of the two foreign markets. Therefore, there are cutoff productivity levels below which a firm will not produce for either the domestic market,  $z_{Dt}^h = \inf\{z : d_{Dt}^h \geq 0\}$  or for each of the foreign markets,  $z_{Xit}^h = \inf\{z : d_{Xit}^h \geq 0\}$  and  $z_{Xjt}^h = \inf\{z : d_{Xjt}^h \geq 0\}$  for exports to countries  $i$  and  $j$  respectively.

We assume that the lower bound of the productivity distribution  $z_{min}$  is low enough compared to the domestic cutoff level  $z_{Dt}^h$ , such that this is above  $z_{min}$ . We further assume that the domestic cutoff level,  $z_{Dt}^h$ , is low enough relative to the export cutoff levels  $z_{Xit}^h$  and  $z_{Xjt}^h$  such that both  $z_{Xit}^h$  and  $z_{Xjt}^h$  are above  $z_{Dt}^h$ . These assumptions ensure that: 1) there will be an endogenously determined subset of firms that pay the sunk entry cost  $f_E$ , but do not produce for the domestic market, and 2) there will be an endogenously determined non-traded sector - the firms with productivities between  $z_{Dt}^h$  and the lower of  $z_{Xit}^h$  and  $z_{Xjt}^h$ . The subset of firms that pay the sunk entry cost, but do not draw a productivity above the cutoff level for domestic production immediately exit the market. Therefore, if they want to enter the market again and try to draw a productivity above the cutoff level they must pay the sunk entry cost again. Firm profits are therefore:

$$d_t^h(z) = d_{Dt}^h(z) + d_{Xit}^h(z) + d_{Xjt}^h(z),$$

$$d_{Dt}^h(z) = \begin{cases} \frac{1}{\theta} (\rho_{Dt}^h(z))^{1-\theta} C_t - \frac{w_t^h f_D^h}{Z_t^h} & \text{if } z \geq z_{Dt}^h, \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

$$d_{Xit}^h(z) = \begin{cases} \frac{Q_t^i}{\theta} (\rho_{Xit}^h(z))^{1-\theta} C_t^i - \frac{w_t^h f_{X_i}^h}{Z_t^h} & \text{if } z \geq z_{Xit}^h, \\ 0 & \text{otherwise,} \end{cases} \quad (3)$$

$$d_{Xjt}^h(z) = \begin{cases} \frac{Q_t^j}{\theta} (\rho_{Xjt}^h(z))^{1-\theta} C_t^j - \frac{w_t^h f_{X_j}^h}{Z_t^h} & \text{if } z \geq z_{Xjt}^h, \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

Equivalent firm profit equations hold for countries  $i$  and  $j$ .

### 2.2.1 Firm Averages

In every period there is a number of firms,  $N_{Dt}^h$ , that produce for the domestic market, given the cutoff level of domestic production,  $z_{Dt}^h$ . A number of these firms, given by  $N_{Xit}^h$  and  $N_{Xjt}^h$ , export to countries  $i$  and  $j$  respectively. In a similar manner to Melitz (2003), we define ‘average’ productivity for all domestic firms,  $\bar{z}_D^h$ , and for firms that export to countries  $i$  and  $j$ ,  $\bar{z}_{X_i}^h$  and  $\bar{z}_{X_j}^h$ , as:

$$\bar{z}_D^h = \left[ \frac{1}{1 - G(z_{Dt}^h)} \int_{z_{Dt}^h}^{\infty} z^{\theta-1} dG(z) \right]^{\frac{1}{\theta-1}},$$



$$\tilde{z}_{Xit}^h = \left[ \frac{1}{1 - G(z_{Xit}^h)} \int_{z_{Xit}^h}^{\infty} z^{\theta-1} dG(z) \right]^{\frac{1}{\theta-1}},$$

$$\tilde{z}_{Xjt}^h = \left[ \frac{1}{1 - G(z_{Xjt}^h)} \int_{z_{Xjt}^h}^{\infty} z^{\theta-1} dG(z) \right]^{\frac{1}{\theta-1}}.$$

Melitz (2003) shows that these productivity averages contain all the information on the productivity distributions relevant for macroeconomic variables. Thus, our model is isomorphic to a model where  $N_{Dt}^h$  firms with productivity  $\tilde{z}_D^h$  produce for the domestic market, and  $N_{Xit}^h$  and  $N_{Xjt}^h$  firms with productivities  $\tilde{z}_{Xi}^h$  and  $\tilde{z}_{Xj}^h$  produce for each of the two export markets. The average price in the domestic market, will be equal to the price of the firm with average productivity,  $\tilde{p}_{Dt}^h = p_{Dt}^h(\tilde{z}_D^h)$  and the average price in each of the exporting markets will be equal to the price of the exporting firms with average productivities  $\tilde{p}_{Xit}^h = p_{Xit}^h(\tilde{z}_{Xi}^h)$  and  $\tilde{p}_{Xjt}^h = p_{Xjt}^h(\tilde{z}_{Xj}^h)$ . The nominal price index in country  $h$  reflects the nominal price of both domestic firms and imports from foreign firms. The nominal price index can therefore be written as:

$$P_t^h = [N_{Dt}^h (\tilde{p}_{Dt}^h)^{1-\theta} + N_{Xht}^i (\tilde{p}_{Xht}^i)^{1-\theta} + N_{Xht}^j (\tilde{p}_{Xht}^j)^{1-\theta}]^{\frac{1}{1-\theta}}.$$

Dividing both sides by  $P_t^h$  we obtain the following real price index:

$$N_{Dt}^h (\tilde{p}_{Dt}^h)^{1-\theta} + N_{Xht}^i (\tilde{p}_{Xht}^i)^{1-\theta} + N_{Xht}^j (\tilde{p}_{Xht}^j)^{1-\theta} = 1. \quad (5)$$

Equivalent price index equations hold for countries  $i$  and  $j$ .

Average total profits are given by the sum of average profits from domestic sales and average profits from exporting, adjusted to the proportion of firms that export to a each market:

$$\tilde{d}_t^h = \tilde{d}_{Dt}^h + (1 - G(z_{Xit}^h)) \tilde{d}_{Xit}^h + (1 - G(z_{Xjt}^h)) \tilde{d}_{Xjt}^h.$$

This equation can then be written explicitly with the ratios of exporting firms to total domestic firms:

$$\tilde{d}_t^h = \tilde{d}_{Dt}^h + \frac{N_{Xit}^h}{N_{Dt}^h} \tilde{d}_{Xit}^h + \frac{N_{Xjt}^h}{N_{Dt}^h} \tilde{d}_{Xjt}^h. \quad (6)$$

Equivalent average total profit equations hold for each of the two foreign countries,  $i$  and  $j$ .

### 2.2.2 Firm Value

All producing firms, other than the firm with productivity equal to the cutoff level ( $z = z_{Dit}$ ), make positive profits. Thus, the average profit level in country  $h$  will be positive ( $\tilde{d}_t^h > 0$ ), and the average firm will have a positive value, derived from expected future profits. After the end of a period, an exogenously determined proportion  $\delta$  of firms in each country will cease to operate. Given that these firms cease to operate after new entrants have entered the market, a proportion  $\delta$  of the successful new entrants will never operate. Since households own the firms, we can solve the households problem to calculate the average value of firms in the economy,  $\tilde{v}_t^h$ . Given that the firms are owned entirely by domestic households the value of a firm on entry will be given by the limit of the household share Euler equation: If we assume that there are no bubbles in the economy then  $\lim_{j \rightarrow \infty} \tilde{\beta} \tilde{v}_{t+j} = 0$ , where  $\tilde{\beta} = [\beta(1 - \delta)]^j E_t \left( \frac{C_{t+s}^h}{C_t^h} \right)^{-\gamma}$ , then the value of a firm will be equal to the discounted present value of its expected profit stream:

$$\tilde{v}_t^h = E_t \sum_{s=t+1}^{\infty} [\beta(1 - \delta)]^{s-1} \left( \frac{C_{t+s}^h}{C_t^h} \right)^{-\gamma} \tilde{d}_s^h.$$

Thus, as long as  $\tilde{d}_t^h$  is positive, the average firm value in country  $h$  will also be positive ( $\tilde{v}_t^h > 0$ ).

### 2.2.3 Firm Entry and Exit

In each period,  $N_{UEt}^h$  new firms will pay the sunk entry cost to commence production, and then find out their firm-level productivity,  $z$ . Upon drawing their productivity, some firms will have a productivity less than the expected cutoff level for domestic production in the following period,  $E(z_{Dt+1}^h)$ , thus a proportion of firms that pay the entry cost will not produce,  $G(E(z_{Dt+1}^h))$ , and will instead exit the market immediately. Firms will choose to enter the market until the average firm value, adjusted by the probability of entering, is equal to the initial entry cost,  $f_E^h$ , expressed in effective labour units, which leads to the free entry condition:

$$\tilde{v}_t^h (1 - G(E(z_{Dt+1}^h))) = \frac{w_t^h f_E^h}{Z_t^h}, \quad (7)$$

which, rearranged, is:

$$\tilde{v}_t^h = \frac{1}{1 - G(E(z_{Dt+1}^h))} \frac{w_t^h f_E^h}{Z_t^h}.$$

The number of firms operating at the end of the period,  $N_{Ht}^h$ , will be equal to the number of firms operating at the start of the period,  $N_{Dt}^h$ , plus the number of successful new entrants  $N_{Et}^h$ . The number of successful new entrants will be equal to the number of firms that pay the entry cost,  $N_{UEt}^h$ , adjusted by the probability of entering the market:  $N_{Et}^h = (1 - G(E(z_{Dt+1}^h)))N_{UEt}^h$ . The number of firms at the end of the period will therefore be given by:  $N_{Ht}^h = N_{Dt}^h + N_{Et}^h = N_{Dt}^h + (1 - G(E(z_{Dt+1}^h)))N_{UEt}^h$ . Given the timing of firm entry and exit we have assumed, the number of firms operating during a period will be given by:

$$N_{Dt}^h = (1 - \delta)N_{Ht-1}^h = (1 - \delta)(N_{Dt-1}^h + N_{Et-1}^h). \quad (8)$$

Note that, because the total number of firms can only change endogenously at the end of the period, the average productivity of domestic production during a period,  $\tilde{z}_{Dt}^h$ , will be predetermined during a period, and will only change in between periods, as a result of the entry and exit of less productive non-trading firms from the domestic market.

## 2.3 Parametrising Productivity

In order to solve the model we assume that the firm-level productivities,  $z$ , follow a Pareto distribution with lower bound  $z_{min}$  and shape parameter  $k$ . We assume that  $k > \theta - 1$  to ensure that the average of firm size is finite<sup>2</sup>. Thus, we have  $G(z) = 1 - (z_{min}/z)^k$ .

Average firm-level productivities are then:  $\tilde{z}_D^h = \nu z_{Dt}^h$ ,  $\tilde{z}_{Xit}^h = \nu z_{Xit}^h$ ,  $\tilde{z}_{Xjt}^h = \nu z_{Xjt}^h$ , where  $\nu = [k/(k - (\theta - 1))]^{\frac{1}{\theta-1}}$ .

The proportion of country  $h$  firms that export to each market is given by, who will have an average productivity  $z_{Dt}^h$ , predetermined at time  $t$ :

$$\frac{N_{Xit}^h}{N_{Dt}^h} = \frac{1 - G(z_{Xit}^h)}{1 - G(z_{Dt}^h)},$$

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<sup>2</sup>According to Axtell (2001),  $1/(\theta - 1)$  is around 1.06 in the US.

$$\frac{N_{Xjt}^h}{N_{Dt}^h} = \frac{1 - G(z_{Xjt}^h)}{1 - G(z_{Dt}^h)}.$$

Using  $G(z)$  and average firm-level productivities, these can then be rewritten as:

$$\frac{N_{Xit}^h}{N_{Dt}^h} = \frac{\left(\frac{z_{min}^h}{z_{Xit}^h}\right)^k}{\left(\frac{z_{min}^h}{z_{Dt}^h}\right)^k} = (z_{Dt}^h)^k (z_{Xit}^h)^{-k}, \quad (9)$$

$$\frac{N_{Xjt}^h}{N_{Dt}^h} = \frac{\left(\frac{z_{min}^h}{z_{Xjt}^h}\right)^k}{\left(\frac{z_{min}^h}{z_{Dt}^h}\right)^k} = (z_{Dt}^h)^k (z_{Xjt}^h)^{-k}. \quad (10)$$

Equivalent equations for the proportion of firms that export hold for countries  $i$  and  $j$ .

Given the parametrisation of  $G(z_{Dt}^h)$ , we rewrite the free entry condition (7) :

$$\tilde{v}_t^h = \frac{1}{1 - G(E(z_{Dt+1}^h))} \frac{w_t^h f_E^h}{Z_t^h} = \left(\frac{E(z_{Dt+1}^h)}{z_{min}^h}\right)^k \frac{w_t^h f_E^h}{Z_t^h}. \quad (11)$$

Equivalent free entry conditions hold for countries  $i$  and  $j$ .

The country  $h$  zero domestic profit cutoff condition  $d_{Dt}^h(z_{Dt}^h) = 0$ , zero export profit cutoff conditions  $d_{Xit}^h(z_{Xit}^h) = 0$  and  $d_{Xjt}^h(z_{Xjt}^h) = 0$ , and equations (2), (3) and (4) for firm profits, imply that country  $h$  average domestic profits and average export profits to each market will satisfy:

$$\tilde{d}_{Dt}^h = (\theta - 1) \left(\frac{\nu^{\theta-1}}{k}\right) \frac{w_t^h f_D^h}{Z_t^h}, \quad (12)$$

$$\tilde{d}_{Xit}^h = (\theta - 1) \left(\frac{\nu^{\theta-1}}{k}\right) \frac{w_t^h f_{Xi}^h}{Z_t^h}, \quad (13)$$

$$\tilde{d}_{Xjt}^h = (\theta - 1) \left(\frac{\nu^{\theta-1}}{k}\right) \frac{w_t^h f_{Xj}^h}{Z_t^h}. \quad (14)$$

Equivalent zero profit conditions will hold for countries  $i$  and  $j$ .

## 2.4 Market Clearing

The wage rate in each country will adjust such that the exogenously set labour supply is equal to the sum of labour used in production, labour used as a sunk cost of entry, labour used as fixed costs of domestic production, and labour used as fixed costs of exporting to each foreign market. The resultant labour market clearing condition is:

$$L_t^h = L_{Pt}^h + \left(\frac{E(z_{Dt+1}^h)}{z_{min}^h}\right)^k \frac{N_{Et}^h f_E^h}{Z_t^h} + \frac{N_{Dt}^h f_D^h}{Z_t^h} + \frac{N_{Xit}^h f_{Xit}^h}{Z_t^h} + \frac{N_{Xjt}^h f_{Xjt}^h}{Z_t^h}, \quad (15)$$

where  $L_{Pt}^h = (Y_{Dt}^h/z_{Dt}^h + Y_{Xit}^h/z_{Xit}^h + Y_{Xjt}^h/z_{Xjt}^h)/Z_t^h$  is the labour used in production, equal to the sum of labour used in domestic production,  $Y_{Dt}^h/Z_t^h z_{Dt}^h$ , and the labour used in producing for both export markets,  $Y_{Xit}^h/Z_t^h z_{Xit}^h$  and  $Y_{Xjt}^h/z_{Xjt}^h$  for exports to  $i$  and  $j$  respectively, where  $Y_{Dt}^h$ ,  $Y_{Xit}^h$  and  $Y_{Xjt}^h$  are, respectively the total number of units of output produced for the domestic market, and for exporting to countries  $i$  and  $j$ .

Given that we have assumed no government borrowing, no physical capital and financial autarky, aggregate bond holdings must equal zero at the end of the period, and the aggregate number of shares per company must equal unity. The assumption of financial autarky (value of exports=value of imports) for all countries, also yields the balanced trade equation:

$$Q_t^i N_{Xit}^h (\tilde{\rho}_{Xit}^h)^{1-\theta} C_t^i + Q_t^j N_{Xjt}^h (\tilde{\rho}_{Xjt}^h)^{1-\theta} C_t^j = N_{Xht}^i (\tilde{\rho}_{Xht}^i)^{1-\theta} C_t^h + N_{Xht}^j (\tilde{\rho}_{Xht}^j)^{1-\theta} C_t^h. \quad (16)$$

Equivalent labour market clearing and balanced trade equations will hold for countries  $i$  and  $j$ .

## 2.5 Model Summary

Table A1 in the Appendix summarizes the main equilibrium conditions of the model. The equations in this table constitute a system of 56 equations in 56 endogenous variables:  $w_t^h, w_t^i, w_t^j, \tilde{\rho}_{Dt}^h, \tilde{\rho}_{Dt}^i, \tilde{\rho}_{Dt}^j, \tilde{\rho}_{Xit}^h, \tilde{\rho}_{Xjt}^h, \tilde{\rho}_{Xht}^h, \tilde{\rho}_{Xit}^i, \tilde{\rho}_{Xjt}^i, \tilde{\rho}_{Xht}^i, \tilde{\rho}_{Xit}^j, \tilde{\rho}_{Xjt}^j, \tilde{\rho}_{Xht}^j, \tilde{d}_t^h, \tilde{d}_t^i, \tilde{d}_t^j, \tilde{d}_{Dt}^h, \tilde{d}_{Dt}^i, \tilde{d}_{Dt}^j, \tilde{d}_{Xit}^h, \tilde{d}_{Xjt}^h, \tilde{d}_{Xht}^h, \tilde{d}_{Xit}^i, \tilde{d}_{Xjt}^i, \tilde{d}_{Xht}^i, \tilde{d}_{Xit}^j, \tilde{d}_{Xjt}^j, \tilde{d}_{Xht}^j, N_{Dt}^h, N_{Dt}^i, N_{Dt}^j, N_{Et}^h, N_{Et}^i, N_{Et}^j, N_{Xit}^h, N_{Xjt}^h, N_{Xht}^h, N_{Xit}^i, N_{Xjt}^i, N_{Xht}^i, N_{Xit}^j, N_{Xjt}^j, N_{Xht}^j, \tilde{z}_{Dt}^h, \tilde{z}_{Dt}^i, \tilde{z}_{Dt}^j, \tilde{z}_{Xit}^h, \tilde{z}_{Xjt}^h, \tilde{z}_{Xht}^h, \tilde{z}_{Xit}^i, \tilde{z}_{Xjt}^i, \tilde{z}_{Xht}^i, \tilde{z}_{Xit}^j, \tilde{z}_{Xjt}^j, \tilde{z}_{Xht}^j, \tilde{v}_t^h, \tilde{v}_t^i, \tilde{v}_t^j, r_t^h, r_t^i, r_t^j, C_t^h, C_t^i, C_t^j, Q_t^i$  and  $Q_t^j$ . Of these endogenous variables, 9 are predetermined as of time  $t$ : the total numbers of firms in each country  $N_{Dt}^h, N_{Dt}^i$  and  $N_{Dt}^j$ , the average firm level productivities,  $\tilde{z}_{Dt}^h, \tilde{z}_{Dt}^i$  and  $\tilde{z}_{Dt}^j$  and the risk-free interest rates,  $r_t^h, r_t^i$  and  $r_t^j$ . Additionally, the model features 21 exogenous variables: the aggregate productivities  $Z_t^h, Z_t^i$  and  $Z_t^j$ , and the policy variables  $f_{Et}^h, f_{Et}^i, f_{Et}^j, f_{Dt}^h, f_{Dt}^i, f_{Dt}^j, f_{Xit}^h, f_{Xjt}^h, f_{Xht}^h, f_{Xit}^i, f_{Xjt}^i, f_{Xht}^i, f_{Xit}^j, f_{Xjt}^j, f_{Xht}^j, \tau_{it}^h, \tau_{jt}^h, \tau_{ht}^i, \tau_{jt}^i, \tau_{ht}^j$  and  $\tau_{it}^j$ . Changes in  $f_{Xit}^h, f_{Xjt}^h, f_{Xht}^h, f_{Xit}^i, f_{Xjt}^i, f_{Xht}^i, f_{Xit}^j, \tau_{it}^h, \tau_{jt}^h, \tau_{ht}^i, \tau_{jt}^i, \tau_{ht}^j$  and  $\tau_{it}^j$  reflect changes in trade policy. For country  $h$ , the trade policy instruments will be  $f_{Xit}^h, f_{Xjt}^h, \tau_{it}^h$  and  $\tau_{jt}^h$ , for country  $i$ , the trade policy instruments will be  $f_{Xht}^i, f_{Xjt}^i, \tau_{ht}^i$  and  $\tau_{jt}^i$ , and for country  $j$ , the trade policy instruments will be  $f_{Xht}^j, f_{Xit}^j, \tau_{ht}^j$  and  $\tau_{it}^j$ .

## 3 Calibration

The papers we address here, such as Ghironi and Melitz (2005), assume complete symmetry between the countries in their models, including in terms of country size and barriers to trade. We take a different approach. We allow for asymmetries in country size and barriers to trade, and we calibrate our model accordingly. We interpret periods as three months, which determines the discount factor  $\beta = 0.99$ , and the risk aversion parameter  $\gamma = 2$ , standard values in quarterly business cycle models. The firm exit rate,  $\delta$ , is set to 0.0235, such as to match the 9.4% UK annual firm death rate.<sup>3</sup> Following Bernard *et al.* (2003),  $\theta$  is set to 3.8<sup>4</sup> and  $k$  is set to 3.4, satisfying the condition that  $k > \theta - 1$ .

The three countries in the model are set as the UK ( $h$ ), the EU ( $i$ ) and the Rest of the World (RoW) ( $j$ ), where RoW is defined as all countries in the world that are not members of the European Union. The per unit iceberg costs  $\tau$  were calculated using data from the *World Bank Trade Costs* database, and the *ONS Pink Book*. The *World Bank Trade Costs* database provides the tariff equivalent rate,  $x$ , for trade between pairs of countries, which allows the calculation of the average tariff equivalent rate for 2005-2015 for each country pair. These tariff equivalent rates were then mapped into an iceberg cost, IC, for each country pair according to:  $IC = x/(1+x)$ .

<sup>3</sup>Firm death rate is obtained from the *ONS Business Demography Statistics* (2016).

<sup>4</sup>We note that, although the value of  $\theta$  may appear low (standard macro literature sets  $\theta = 6$  to deliver a 20% mark-up over marginal cost), the mark-up in this paper represents mark-up over average cost, including the entry cost. We have conducted sensitivity analyses on the value of  $\theta$ , and all values from 1.9 to 4.5 give similar responses to model simulations.

The iceberg costs were calculated individually for UK imports from the EU, UK imports from the RoW, UK exports to the EU and UK exports to the RoW as a weighted average on the basis of total exports/imports from each country. For example, the iceberg cost for UK exports to the EU was calculated as the sum of: the iceberg cost for UK trade with each European country multiplied by the proportion of UK exports going to each European country. Given that the UK and the EU are part of a customs union and share similar geographic characteristics, we assume that the iceberg costs of exporting and importing from the EU to the RoW are the same as the iceberg costs of exporting and importing from the UK to the RoW. The iceberg costs are therefore as follows:  $\tau_i^h = 1.316$ ,  $\tau_j^h = 1.450$ ,  $\tau_h^i = 1.326$ ,  $\tau_j^i = 1.450$ ,  $\tau_h^j = 1.459$  and  $\tau_i^j = 1.459$ .<sup>5</sup>

The fixed costs of exporting from the UK to the EU and UK to the RoW are calibrated such that the proportion of UK firms that export to the EU, and to the RoW match the proportions reported by the ONS *Annual Business Survey of Importers and Exporters* (approximately 7% and 8%, respectively). The remaining fixed exporting costs are then set in the same proportions as the iceberg costs. The fixed costs of domestic production in the EU and RoW are calibrated such that the number of firms in the EU and RoW are 9.93 and 24.2 times the number of firms in the UK, to match data from *Eurostat* and the *World Bank*. As in de Soyres (2016) we then normalize the fixed cost of domestic production in the UK so that no domestic entry threshold lies below the lower bound of the productivity distribution.

We normalize  $f_E$ ,  $z_{min}$  and  $Z$  to 1 for all three countries<sup>6</sup> as well as normalizing the labour force in the UK to 1. Finally, the Labour forces in the EU and RoW were set such that, in the calibrated model, UK GDP is equal to 1/6.58 of EU GDP and 1/24.2 of RoW GDP, in line with 2017 *World Bank* data. Table A2 in the appendix sets out the full parameter values used for the calibration.

## 4 Transmission of Macroeconomic Shocks

In this section we examine the transmission of macroeconomic shocks in our theoretical framework. First, we analyse the impact of different transitory macroeconomic shocks on productivity and its persistence. Second, we examine the relative impact of changes in barriers to imports and barriers to exports. Third, we analyse the impact of permanent macroeconomic shocks on various macroeconomic variables.

For the purpose of our analysis, to match output per worker productivity data as closely as possible, we adopt the definition for aggregate productivity specified in Kehrig (2015). Productivity is defined as total output divided by the sum of labour used in production, labour used to pay the fixed costs of domestic production and labour used to pay the fixed costs of exporting to each foreign market:

$$Z_O^h = \frac{Y_t^h}{L_{P_t}^h + F_t^h},$$

where  $Y_t^h$  is total output,  $L_{P_t}^h$  is labour used in production, as defined earlier and  $F_t^h$  is the labour used to pay the fixed costs, equal to the sum of the labour used to pay the fixed costs of domestic production

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<sup>5</sup>Although the iceberg costs appear high, particularly for UK-EU trade (given the absence of formal trade barriers) their values reflect not only the cost of formal barriers to trade, but also other costs of exporting, such as language barriers and transport costs.

<sup>6</sup>Changing the entry cost,  $f_E$ , and the fixed cost of domestic production,  $f_D$ , while maintaining the same ratios  $f_X/f_E$  and  $f_D/f_E$  does not have any effect on the firm-level productivity variables,  $z_D$  and  $z_X$ , as they are determined by the free entry condition and the zero profit conditions. Changing  $f_E$ ,  $f_D$  and  $f_X$  by the same proportion will not alter the calibrated values for firm-level productivity.

$\frac{N_{Dt}^h f_{Dt}^h}{Z_t^h}$  and the fixed costs of exporting to each foreign market  $\frac{N_{Xit}^h f_{Xit}^h}{Z_t^h}$  and  $\frac{N_{Xjt}^h f_{Xjt}^h}{Z_t^h}$  for exports to  $i$  and  $j$  respectively. Total output will be made up of output for the domestic market,  $Y_{Dt}^h$ , and output for each foreign market,  $Y_{Xit}^h$  and  $Y_{Xjt}^h$  for exports to the EU and RoW respectively. Note that, because of the variable iceberg costs of exporting, the output for exporting will be equal to the number of units sold in each foreign market, multiplied by the iceberg cost of exporting to that market. All future references to labour productivity should be taken to refer to productivity as measured above.

#### 4.1 Endogenous Productivity Persistence

In this sub-section we examine the impact of transitory shocks to aggregate technology, sunk entry costs and the fixed costs of domestic production on the persistence of labour productivity and quantify the extent of this impact for the first two shocks. Figure 2 shows the response of labour productivity in country  $h$  to three different macroeconomic shocks: a one period, 1 percentage point positive shock to aggregate technology,  $Z^h$ , shown by the dashed line; a one period, 3.56 percentage point negative shock to the sunk costs of entry,  $f_E^h$ , shown by the dotted line; and a one period, 2.88 percentage point positive shock to the fixed costs of domestic production,  $f_D^h$ , shown by the dot-dash line. The solid line shows the time path of the shocks themselves and the length of time (in quarters) after the shock is on the horizontal axis. Figure 3 shows the response of labour productivity in the closed economy version of the model, where it is assumed that countries do not trade internationally to the three types of macroeconomic shocks as in Figure 2: a one period, 1 percentage point positive shock to aggregate technology,  $Z^h$ , shown by the dashed line; a one period, 3.65 percentage point negative shock to the sunk costs of entry  $f_E^h$ , shown by the dotted line; and a one period, 1.53 percentage point positive shock to the fixed costs of domestic production,  $f_D^h$ , shown by the dot-dash line. Finally, the solid line shows the time path of the shocks themselves and the length of time (in quarters) after the shock is on the horizontal axis. In both the open and closed economy cases, the shock size is set such that the initial response of labour productivity is 1% above its steady-state level.

Figure 2: Open Economy Response of Labour Productivity to Macroeconomic Shocks

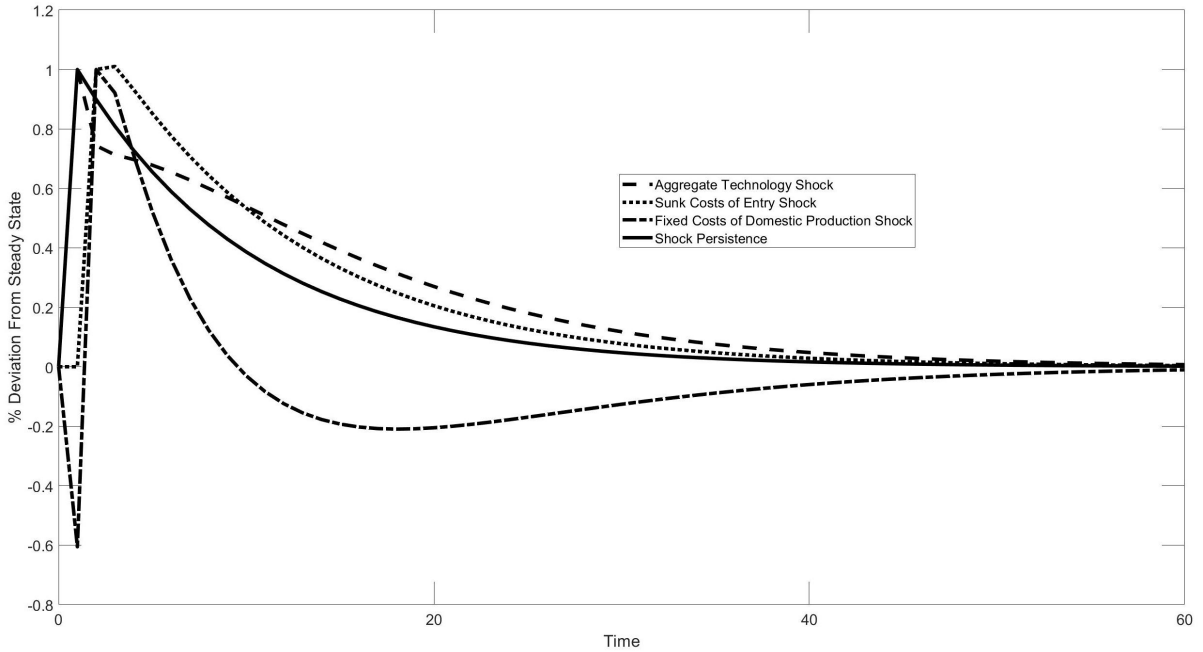
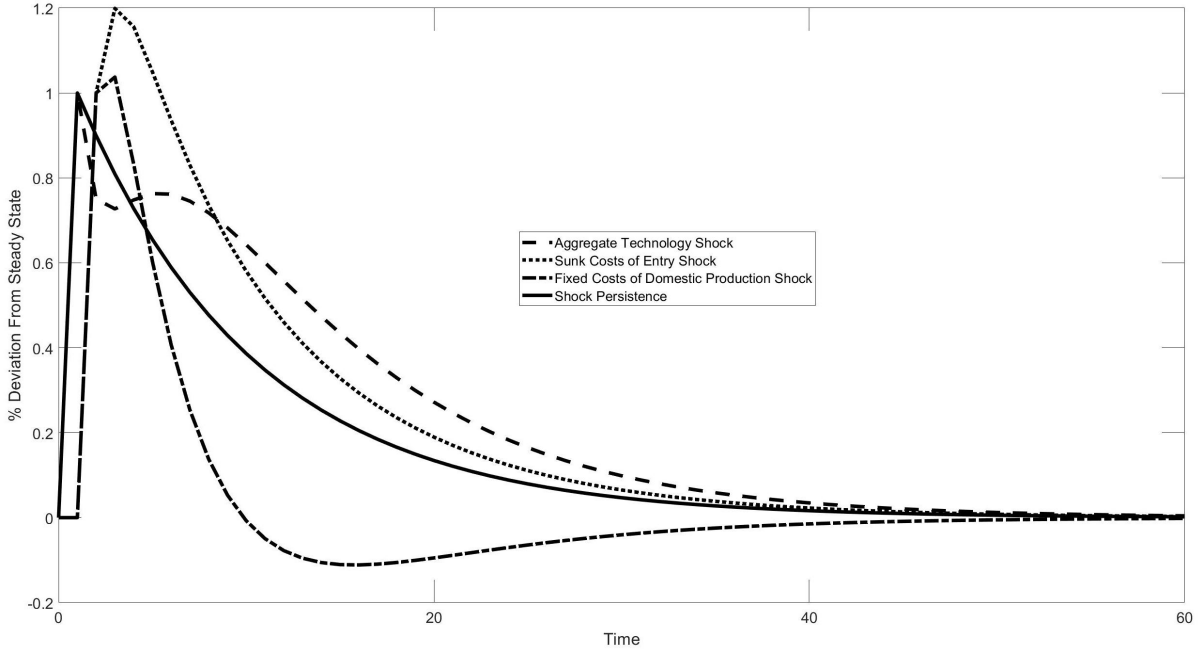


Figure 3: Closed Economy Response of Labour Productivity to Macroeconomic Shocks



We assume that these shocks hit at the beginning of the period, before production starts, and follow an AR(1) processes:

$$\hat{Z}_t^h = \rho * \hat{Z}_{t-1}^h + \epsilon_t,$$

$$\hat{f}_{Et}^h = \rho * \hat{f}_{Et-1}^h + \epsilon_t,$$

$$\hat{f}_{Dt}^h = \rho * \hat{f}_{Dt-1}^h + \epsilon_t,$$

where  $\hat{Z}_t^h$ ,  $\hat{f}_{Et}^h$  and  $\hat{f}_{Dt}^h$  are the deviations of aggregate technology, sunk costs of entry and fixed costs of domestic production respectively, from their steady-state levels, in period  $t$ ,  $\rho$  is the exogenously set persistence of shocks and  $\epsilon_t$  is the magnitude of the shock in period  $t$ . We assume that the persistence of the shocks, exogenously given, is equal to  $\rho = 0.90$ , as in Ghironi and Melitz (2005).<sup>7</sup>

If the distributions of firm productivities were fixed exogenously, then in both the open and closed economies, the persistence of labour productivity would have been solely determined by the exogenously given persistence parameter for the macroeconomic shocks,  $\rho$ , as is the case in standard RBC models. However, when the distribution of firm productivities is endogenously determined through endogenous changes in the cut-off productivity levels, an endogenous degree of persistence is now present in the response of labour productivity to each of these shocks. The extent to which this is the case, depends on the source of the macroeconomic shock, and whether the shock is occurring in an open or closed economy setting:

1. An aggregate technology shock,  $Z^h$ , in both the open and closed economy settings, induces labour productivity,  $Z_O^h$ , to return to its steady-state level at a much slower rate than aggregate technology, due to the behaviour of the average firm-level productivity,  $z_D^h$ . Firm-level productivity remains above its steady-state level for a significantly longer period of time than aggregate technology. The

<sup>7</sup>We conduct robustness checks on the value of  $\rho$ , examining the responses of labour productivity for a range of values from  $\rho = 0.85$ , as in Pancrazi and Vukotic (2011) to  $\rho = 0.994$ , as in Baxter (1995). For this range of values, the responses were similar in all cases. Results are available on request.

half life of productivity in response to the aggregate technology shock is 14 periods in the open economy and 16 periods in the closed economy, compared to a half life of 8 periods for the shock itself. Over these time periods, this is equivalent to a persistence of 0.952 for productivity in the open economy and 0.953 in the closed economy, compared to a shock persistence of 0.9. Comparing these results to Fattal Jaef and Lopez (2014), where the distribution of firm-level productivity is exogenously fixed, we see that allowing for an endogenous firm-level productivity response, through the entry and exit of less productive non-trading firms, introduces further endogenous persistence to the response of productivity. In their paper, the half life of productivity (in the open economy framework) is approximately 12 periods, with an approximate total TFP persistence of 0.95, lower than in both the open and closed economy versions of our model.

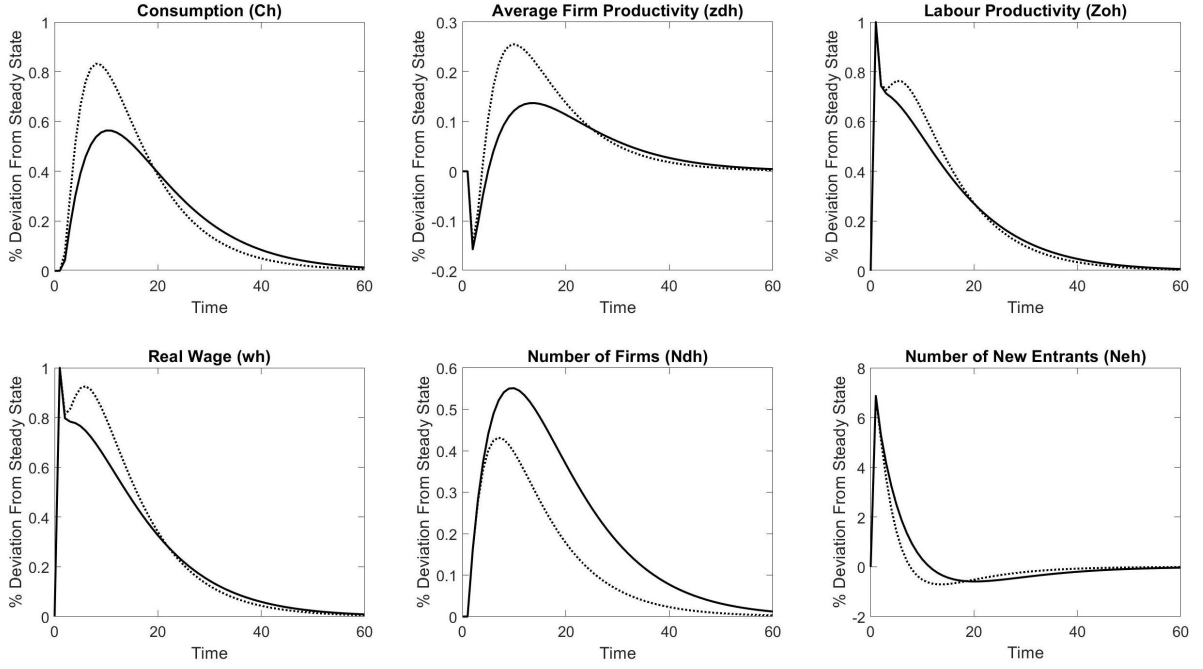
2. A shock to the sunk costs of entry,  $f_E^h$ , in both the open and closed economy settings, induces labour productivity,  $Z_O^h$ , to return to its steady-state level at a much slower rate than the sunk cost of entry, but quicker than in the case of a shock to aggregate technology, due to the behaviour of the average firm-level productivity,  $z_D^h$ . Firm-level productivity remains above its steady-state level for a significantly longer time than aggregate technology. The half life of productivity in response to the aggregate technology shock is 13 periods in the open economy and 14 periods in the closed economy, compared to a half life of 8 periods for the shock itself. Over these time periods, this is equivalent to a persistence of 0.938 for productivity in the open economy and 0.945 in the closed economy, compared to a shock persistence of 0.9.
3. In the case of a shock to the fixed costs of domestic production,  $f_D^h$ , in both the open and closed economies, the persistence of labour productivity is significantly lower than the persistence of the shock itself, driven by the rapid entry of less productive firms, when initial lower profits drive firms off the market resulting in less initial competition. In this case, labour productivity falls below its initial steady-state level, before slowly returning to it, as the shock dissipates.

The different persistence of labour productivity between the open and closed economies and across the various macroeconomic shocks studied here is driven by the responses of firms and consumers which differ in each case. Thus, in Figures 4, 5 and 6 we show the dynamic response of consumption,  $C^h$ , average productivity for domestic production,  $z_D^h$ , labour productivity,  $Z_O^h$ , the real wage,  $w^h$ , the number of firms,  $N_D^h$ , and the number of new entrants,  $N_E^h$ , to the one period transitory shocks, outlined above, to aggregate technology, the sunk costs of entry and the fixed costs of domestic production, respectively. The solid lines plot the response in the open economy, while the dotted lines plot the response in the closed economy.

Figure 4 shows that in response to a positive aggregate technology shock, in both the open and closed economy, firms with lower firm-level productivity, who would otherwise be unable to enter the market, enter the market, decreasing in the first instance average domestic firm-level productivity,  $z_D^h$ . However, as the new firms enter the market,  $N_E^h$ , driven by potentially higher profits, the relatively less productive firms are forced out of the market, due to the increased competition in the market and average domestic firm-level productivity then increases. The positive aggregate technology shock also increases the real wage,  $w^h$ , leading firms to increase their prices, thus leading to a reduction in demand. The real wage increase leads to a rise in the fixed entry cost and the fixed cost of domestic production (measured in effective labour units). These higher costs mean that when firms are hit by the exogenous death shock, new firms do not enter the market to replace them, and thus, average firm-level productivity falls back to its initial steady state, albeit at a slower rate than the aggregate technology level. However, in the open economy, the later persistence of productivity is higher than in the closed economy, driven by a



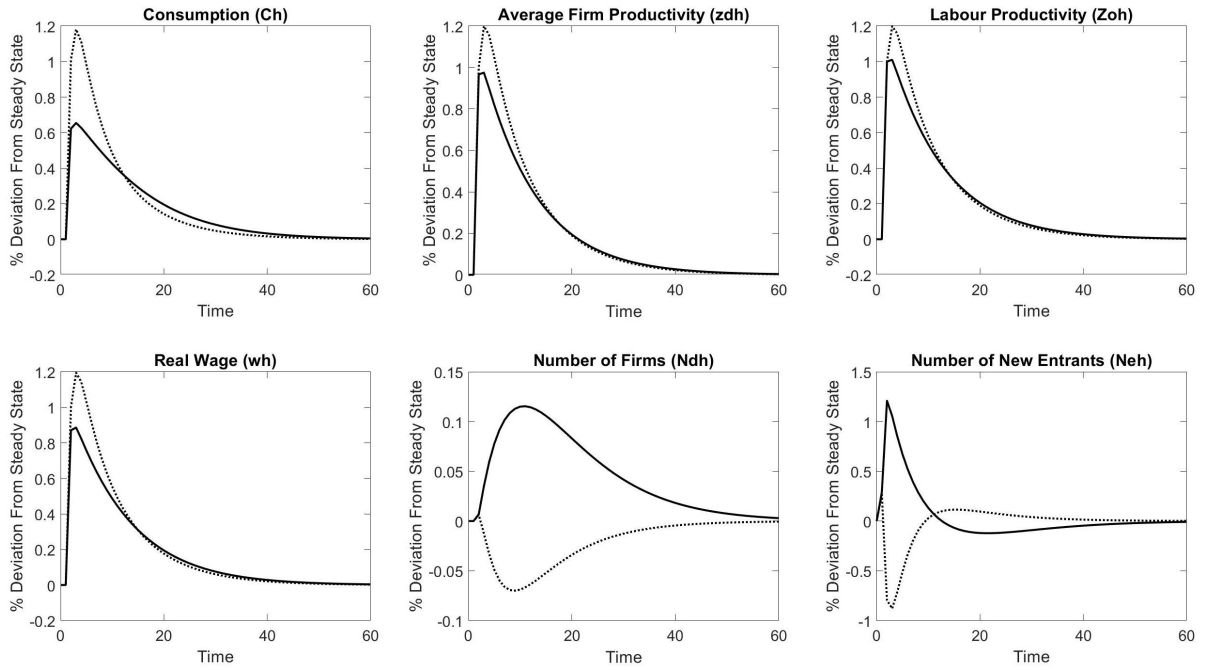
Figure 4: Response to Transitory Aggregate Technology ( $Z^h$ ) Shock



larger increase in the number of entrants and hence a larger increase in the number of domestic firms. The larger number of entrants in the open economy is caused by the increase in the competitiveness of domestic exporters in foreign markets, driven by the increase in domestic aggregate technology. The spike in technology not only increases domestic profits, but also the profits from exporting, thus more firms enter the market to take advantage of the increased total profits. The larger increase in the number of domestic firms leads to an increase in domestic wages, as competition for labour increases, causing the number of new entrants to fall. This explains the higher persistence of productivity in the open economy after around period 22. Our results are consistent with the empirical findings of Moreira (2017) and Sedlacek and Stern (2017), that the entry and exit of less productive firms into the market can drive a persistent response of labour productivity to macroeconomic shocks.

Figure 5 presents the response paths of the same macroeconomic variables as Figure 4 to a negative transitory shock to the sunk cost of entry in both the open and closed economies. Comparing to the case of the aggregate technology shocks, the response paths now appear less smooth, as a change in the sunk cost of entry predominately impacts new firms entering the market, and existing firms only indirectly, through changes in the real wage. Given its low impact on existing firms, a decrease in the sunk cost of entry leads to a lower increase in productivity, shown by the larger shock size needed for a 1% increase in productivity. A reduction in the sunk entry cost in the open economy (temporarily) allows new firms to enter the market,  $N_E^h$ , and drives up the real wage,  $w^h$ , which causes less productive firms to become unprofitable and exit the market, thus driving up both firm-level productivity,  $z_D^h$ , and labour productivity,  $Z_O^h$ . However, as the sunk cost of entry returns to its pre-shock value, the number of new entrants decreases, as firms are hit by the exogenous death shock. The real wage therefore returns to its initial steady-state level, as do productivity and consumption,  $C^h$ . The rate at which productivity and consumption return to steady state is lower than the rate at which the shock dissipates due to the increase in the number of domestic firms, which leads to increased competition in the domestic market, which, in turn, drives up productivity temporarily. It is only as the number of firms returns to its steady-state level that the average firm-level productivity also returns to steady state. In the closed

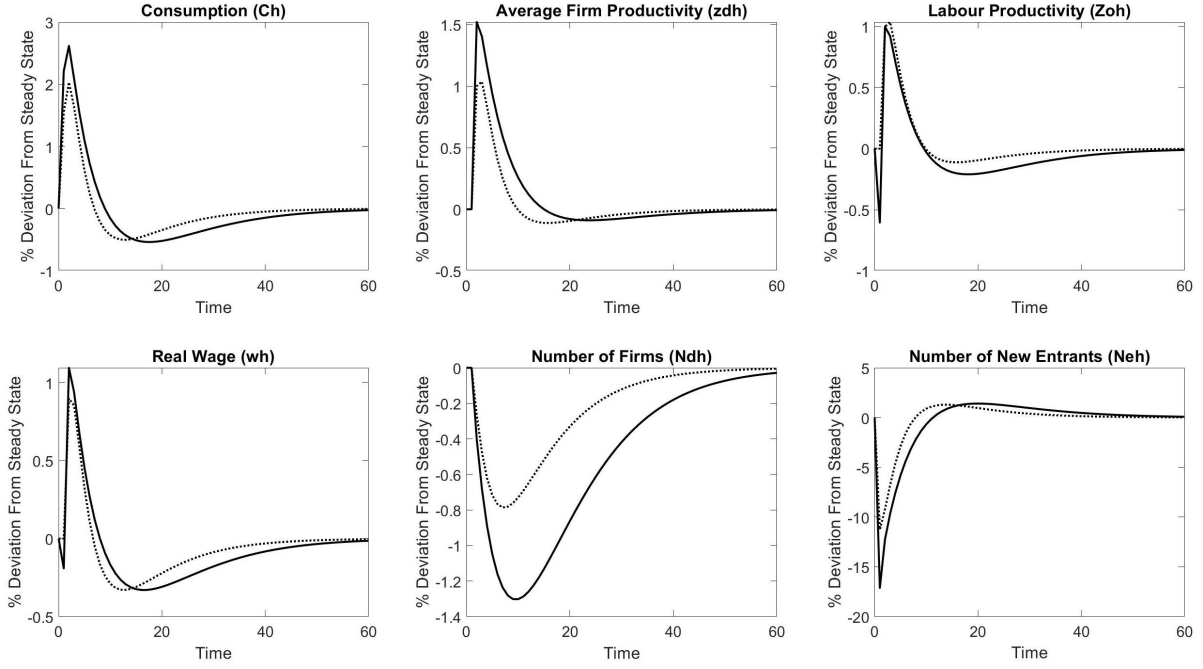
Figure 5: Response to Transitory Shock to the Sunk Cost of Entry ( $f_E^h$ )



economy the response of productivity is similar, although the driving forces are slightly different. In response to the decrease in the sunk costs of entry, more firms attempt to enter the economy, thus driving up the wage rate, as more labour is required to pay the sunk costs of entry. The increase in the wage rate drives up the cutoff productivity for domestic production, as firms now face higher fixed costs of domestic production (denominated in effective labour units) and higher wage costs. Given that the number of successful entrants into the market is dependent on the probability of drawing a productivity higher than the cutoff level, the increase in the cutoff level means that although more labour is being used to pay the sunk costs of entry, the number of successful entrants into the market decreases, as more firms now draw a productivity below the threshold level, and thus exit the market immediately. As the shock dissipates, and thus the number of firms attempting to enter the market decreases, so does the wage rate and therefore the average domestic firm productivity as well.

Figure 6 presents the response paths of the same macroeconomic variables as Figures 4 and 5 to a negative transitory shock to the fixed costs of domestic production in both the open and closed economies. When the shock to the fixed costs hits, relatively less productive firms are no longer able to make profits, and exit the market,  $N_E^h$ , thus leading to an increasing average firm productivity,  $z_D^h$ , in the period immediately after the shock. As a result of higher productivity, firms are able to pay a higher real wage,  $w^h$ , and also make higher profits. The higher real wage and higher profits are passed onto households leading to higher consumption,  $C^h$ . The increase in fixed costs also means that less productive firms are no longer able to make sufficient profits to pay back their sunk costs of entry, so they do not enter the market, thus further increasing productivity. However, as the shock dissipates, and the number of firms returns to equilibrium, the competition in the market decreases, due to a lower number of firms in the market,  $N_D^h$ . This leads to a higher number of relatively less productive firms to quickly enter the market, dragging down average firm-level productivity and causing labour productivity to dip below its original steady-state level before returning to equilibrium. Given that the competition on the domestic market is larger in the open economy due to competition from imports, the increase in the fixed costs of domestic production cause a larger number of unproductive firms to exit the market in the open economy

Figure 6: Response to Transitory Shock to the Fixed Cost of Domestic Production ( $f_D^h$ )



compared to the closed economy, resulting in a much less persistent response of productivity to such a shock in the open economy.

## 4.2 Import and Export Cost Shocks

To fully explore the impact of barriers to imports and exports in driving the entry and exit of less productive non-trading domestic firms (and thus productivity), we analyse the dynamic responses of labour productivity to transitory shocks of the same magnitude to changes in barriers to imports and barriers to exports broken down between shocks to the variable and fixed costs of trade. We assume that the shocks follow AR(1) processes similar to the previous transitory shocks. In Figure 7 we present the response of labour productivity to a one period, 1 percentage point negative shock to the iceberg costs of importing,  $\tau_h^i$ , and  $\tau_h^j$ , shown by the solid line, and a one period, 1 percentage point negative shock to the iceberg costs of exporting,  $\tau_i^h$ , and  $\tau_j^h$ , shown by the dotted line. In Figure 8 we present the response of labour productivity to a one period, 1 percentage point negative shock to the fixed costs of importing,  $f_{Xh}^i$  and  $f_{Xh}^j$ , shown by the solid line and a one period, 1 percentage point negative shock to the fixed costs of exporting,  $f_{Xi}^h$  and  $f_{Xj}^h$ , shown by the dotted line.

Figure 7 shows that a shock to the variable costs of importing induces a larger impact on labour productivity, 0.92% above the initial steady state, than a shock of the same magnitude to the variable costs of exporting, about 0.73% above the initial steady state.<sup>8</sup> Figure 8 shows a slightly different result to Figure 9. As in the case of shocks to the variable cost of trade, a shock to an importing cost, in this case the per period costs of exporting to the UK (UK imports), induces an increase in UK productivity, albeit to a more limited extent than a variable cost of importing shock, 0.08% above the initial steady state. However, a shock to the per period costs of exporting from the UK induces a reduction in UK

<sup>8</sup>We show that this result is invariant to country size. As the size of the domestic economy increases relative to the size of the other countries, the magnitude of the response of labour productivity to trade barrier shocks diminishes, however the response to import barrier changes remains higher than the response to export barrier changes.

Figure 7: Response of Labour Productivity to Transitory Shocks to the Iceberg Costs of Trade

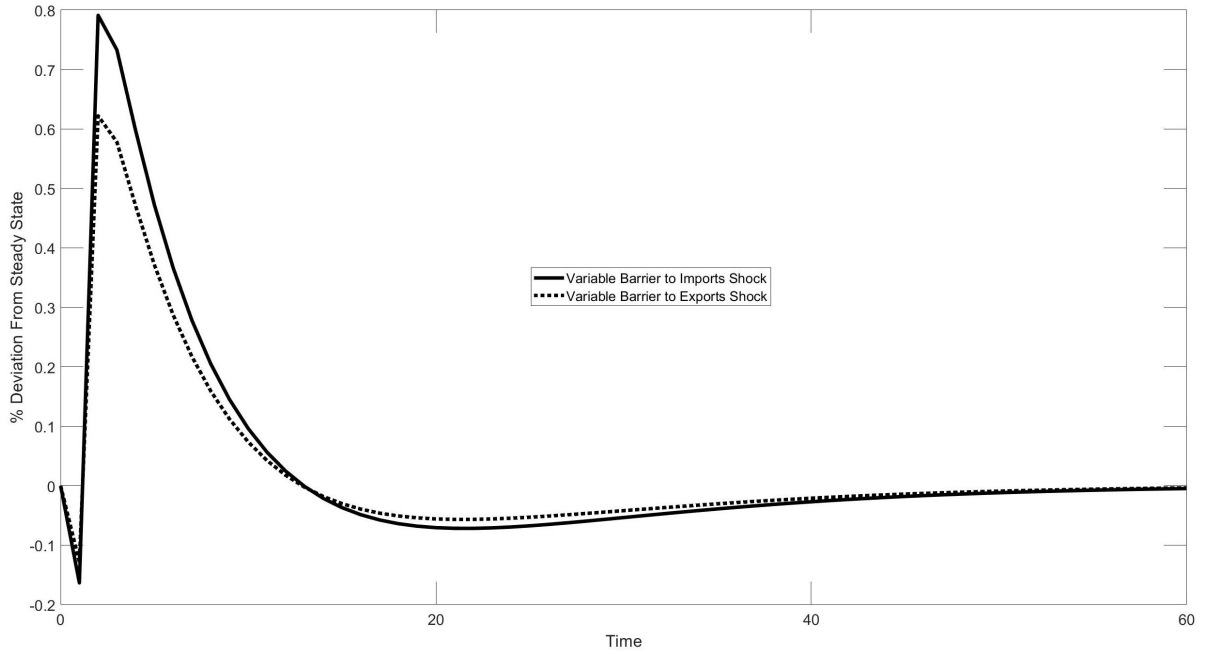
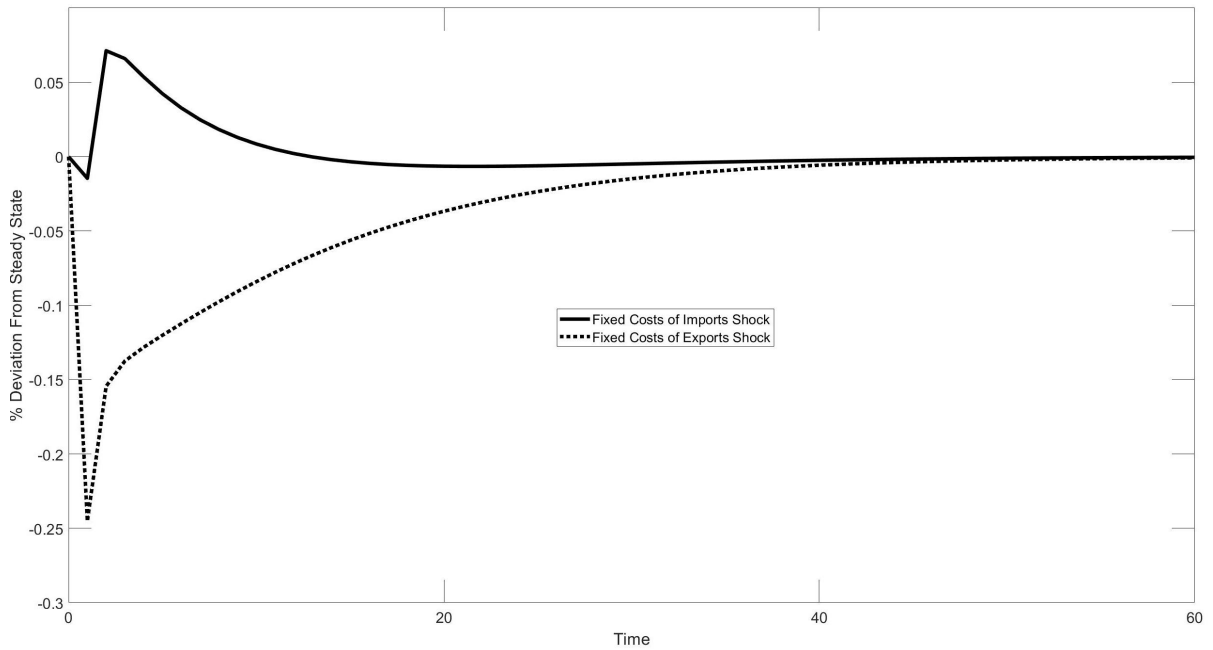


Figure 8: Response of Labour Productivity to Transitory Shocks to the Fixed Costs of Trade

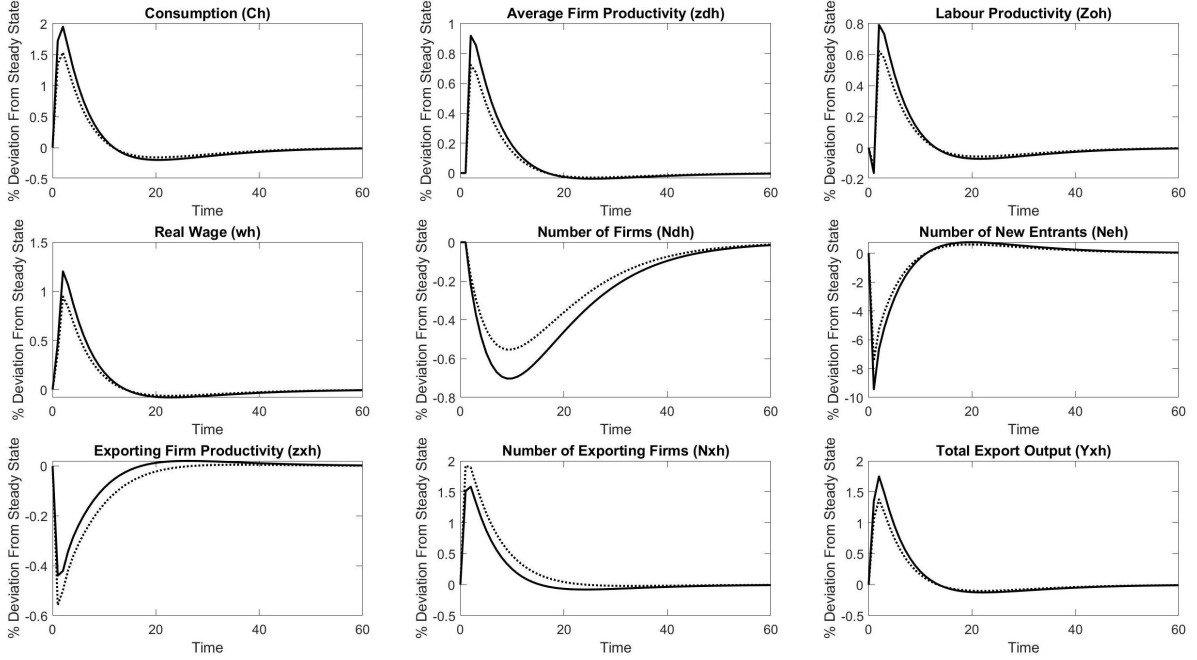


productivity of 0.17%. These results can be explained by the ways in which different types of firms are impacted by trade costs. Our result that a reduction in barriers to imports lead to an increase in productivity is consistent with the existing empirical literature, see for example Amiti and Konings (2007), Yasar and Paul (2007) and Bloom *et al.* (2016).

To further examine the driving mechanisms behind these results Figures 9 and 10 present the response path of consumption,  $C^h$ , average productivity for domestic production,  $z_D^h$ , labour productivity,

$Z_O^h$ , the real wage,  $w^h$ , the number of firms,  $N_D^h$ , the number of new entrants,  $N_E^h$ , the average firm productivity of exporters to the EU<sup>9</sup>,  $z_{X_i}^h$ , the number of exporters to the EU,  $N_{X_i}^h$ , and the total output for exporting to the EU,  $Y_{X_i}^h$ , to the one period transitory shocks presented above. The solid lines plot the response to the shocks to the barriers to imports, while the dotted lines plot the response to the shocks to the barriers to exports.

Figure 9: Breakdown of the Response to Transitory Shocks to the Iceberg Costs of Trade



As the variable cost of imports decrease, the amount of competition from imports increases too, as the domestic market becomes more attractive to foreign exporters. As competition increases, less productive non-trading firms, who are impacted by competition from imports, are no longer able to make profits and thus exit the market, driving up both firm-level productivity,  $z_D^h$ , and labour productivity,  $Z_O^h$ , significantly, as Figure 9 shows. As the less productive firms exit the market, causing decreases in the number of firms,  $N_D^h$ , the competition for inputs decreases, which allows less productive firms to re-enter the market, causing productivity to quickly drop, and even dip below its initial steady-state level, as was also seen in the case of a transitory shock to the fixed costs of domestic production, presented in section 4.1.

Changes in the variable cost of exporting meanwhile significantly impact the domestic firms that engage in international trade (export); however, these firms make up a relatively small proportion of the total number of domestic firms (less than 1% of US firms according to the 2016 Census). On the other hand, non-trading domestic firms make up the majority of firms and are only directly impacted by changes in barriers to exports through changes in the competition for inputs, which, in our model, is competition for labour<sup>10</sup>, reflected in the wage rate,  $w^h$ . The decrease in the variable cost of exporting impacts both the number of exporting firms,  $N_{X_i}^h$ , (the extensive margin of trade) and the total output for exporting,  $Y_{X_i}^h$  (intensive margin of trade), both of which increase after the variable cost of exporting

<sup>9</sup>Note that the responses of the average firm productivity, number of and output for exports to the RoW are almost identical for those for exports to the EU

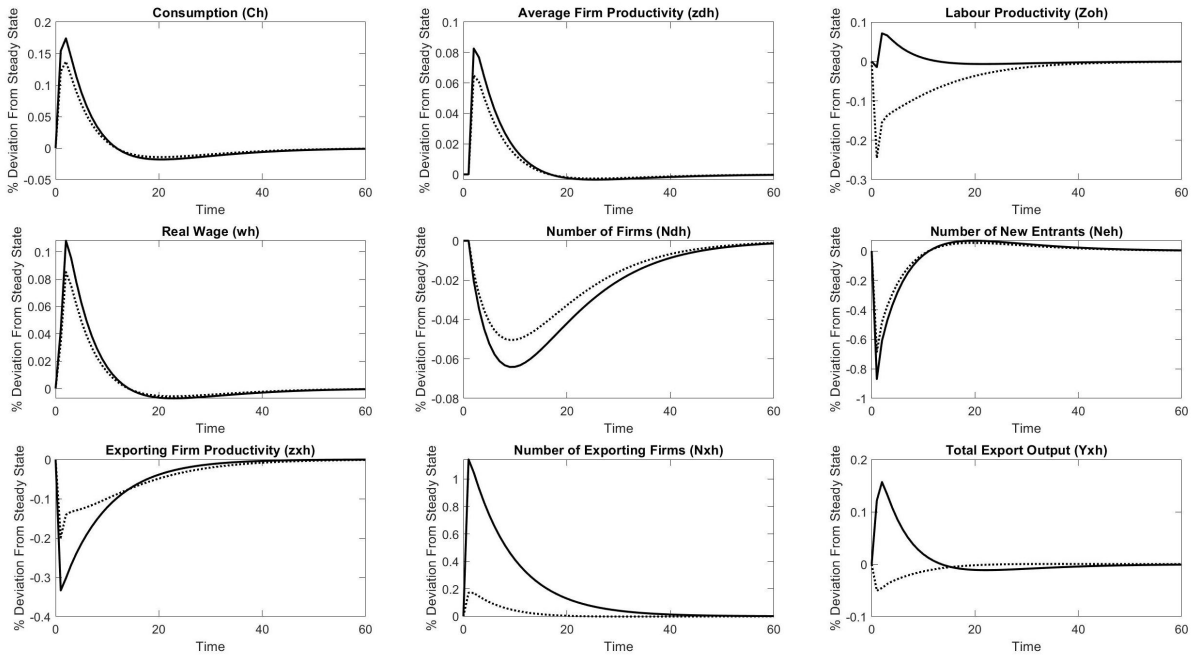
<sup>10</sup>In addition, changes in barriers to exports will indirectly change competition from imports through general equilibrium effects on foreign countries, but the impact of this change in competition is small.

decreases.

When variable costs of exporting decrease, competition for inputs and, to a lesser extent, competition from importers will change, and affect productivity in the same direction: exporting firms demand additional labour to serve international markets, and this drives up the real wage which forces the least productive non-trading firms to exit, thus increasing average productivity. Additionally, competition from importers will further increase labour productivity but only through general equilibrium effects on exporters in foreign countries. As in the case of changes in barriers to imports, as the less productive firms exit the market, the competition for inputs decreases, allowing less productive firms to re-enter the market, causing productivity to quickly decrease, and even dip below its initial steady-state level.

Given that the impact of an increase in competition from imports has a greater impact on non-trading firms than increased competition for inputs because non-trading firms are directly impacted by changes in competition from imports, but only indirectly impacted by changes in competition for inputs, the impact of changes in barriers to imports on productivity is greater than the impact of changes in barriers to exports, consistent with the empirical literature, such as Bloom *et al.* (2016).

Figure 10: Breakdown of the Response to Transitory Shocks to the Fixed Costs of Trade



As Figure 10 shows, the impact of a reduction in the fixed costs of importing causes an almost identical response to a decrease in the variable cost of importing, albeit of a smaller magnitude. In both cases, the competition from imports increase, driving less productive firms out of the market, leading to a decrease in the number of firms,  $N_D^h$ , and increasing both firm-level productivity  $z_D^h$  and labour productivity  $Z_O^h$ . As the shock dissipates, the competition from imports returns to steady state and thus the number of domestic firms also returns to equilibrium. As the number of firms returns to equilibrium, the competition for labour also intensifies, causing productivity to also return to its steady-state level.

The impact of a reduction in the fixed costs of exporting on the other hand, leads to a significantly different response of productivity. In this case, labour productivity decreases, before slowly returning to

equilibrium as the shock dissipates. As the fixed cost of exporting decreases, less productive firms are able to enter the exporting markets, as a result of the decrease in the cutoff productivity for exporting. The number of exporting firms then increases, and thus so does the number of units of labour needed to pay for the fixed costs of exporting, causing an increase in the real wage,  $w^h$ . As firms set their price equal to a fixed mark-up over marginal cost, the combination of increased wage costs and decreased average productivity of exporters causes the average price of exports to increase, decreasing foreign demand, and leading to a decrease in the number of units of output that are sold in domestic markets, even if the total value of exports increases. Thus, although the number of exporting firms,  $N_{X_i}^h$ , increases (the extensive margin of trade) the total output per firm,  $Y_{X_i}^h$ , decreases (intensive margin of trade), leading to a reduction in the total number of export units sold. Given that the labour used in production (the labour force less the labour used in the sunk cost of entry) increases as a result of the reduction in the number of new entrants, and that the number of units of output produced for export markets decreases, labour productivity, measured as output per hour falls.

### 4.3 Permanent Shocks

Next, we examine the dynamic response of consumption,  $C^h$ , average firm productivity,  $z_D^h$ , labour productivity,  $Z_O^h$ , the real wage,  $w^h$ , the number of firms,  $N_D^h$ , and the number of new entrants,  $N_E^h$ , to three separate permanent macroeconomic shocks in country  $h$ : first, a permanent 1% increase in aggregate,  $Z^h$ ; second, a permanent 1% decrease in the sunk cost of entry  $f_E^h$ ; third, a permanent 1% increase in the fixed cost in the fixed cost of domestic production,  $f_D^h$ .

Figure 11 shows the dynamic response of consumption,  $C^h$ , average firm productivity,  $z_D^h$ , labour productivity,  $Z_O^h$ , the real wage,  $w^h$ , the number of firms,  $N_D^h$ , and the number of new entrants,  $N_E^h$ , to a permanent 1% increase in aggregate technology in country  $h$ . The increase in aggregate technology allows the relatively less productive firms, previously unable to make profits, to immediately enter the market, sharply increasing the number of new entrants, and driving down average firm-level productivity and leading to an increase in the number of firms. However, the decrease in average firm-level productivity is not as large as the increase in aggregate technology (0.6% in the short run and 0.18% in the long run, compared to an 1% increase in aggregate technology), therefore labour productivity increases. The increase in labour productivity allows firms to pay a higher real wage, and make higher profits, leading to an increase in consumption. As the real wage increases, the less productive firms who initially entered the market are no longer able to pay the higher real wage and exit the market, allowing firm-level productivity to return slowly to its new, lower, steady-state level.

The endogenization of the distribution of firm productivity in our model introduces a smoothing effect to the behaviour of the endogenous variables, such as consumption and the real wage, as these variables move more gradually to their new steady-state level in our model than in the case in which the distribution of firm productivity is fixed, as in Ghironi and Melitz (2005). In our model, consumption does not jump immediately to a higher level as in Ghironi and Melitz (2005), but moves gradually to its new steady state, driven by the entry and subsequent exit of less productive firms, matching more closely the seasonally adjusted US GDP in the World Bank data. A similar behaviour is observed when examining the other endogenous variables, such as the real wage, the number of new entrants, and the number of domestic firms: the response of these variables in our model is much smoother than of those observed when the distribution of firm productivity is fixed.

Figure 11: Response to a Permanent Increase in Aggregate Technology ( $Z^h$ )

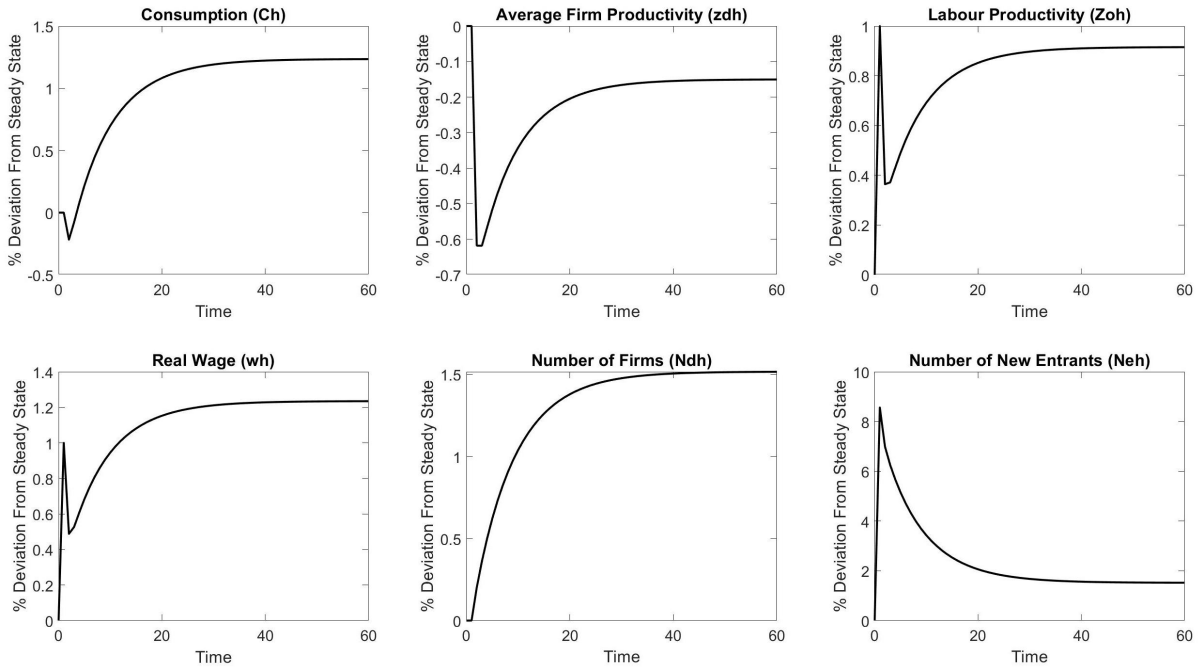


Figure 12: Response to a Permanent Decrease in Sunk Costs of Entry ( $f_E^h$ )

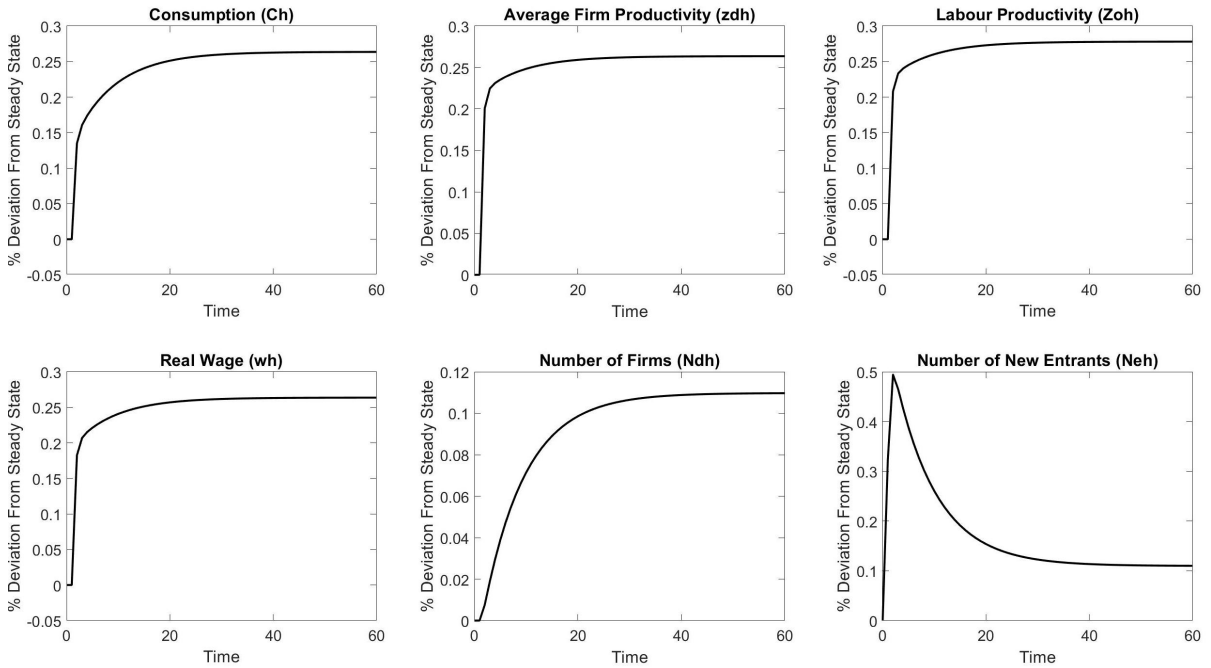


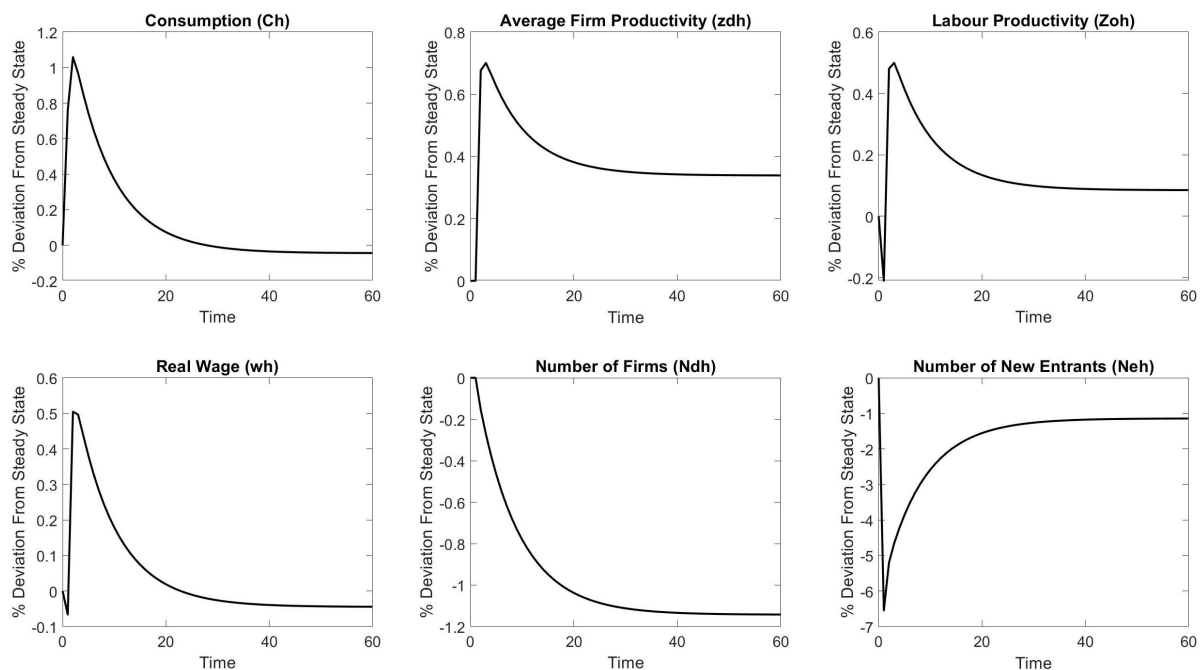
Figure 12 shows the dynamic response of consumption, average firm productivity, labour productivity, the real wage, the number of firms and the number of new entrants to a permanent 1% decrease in sunk entry costs in country  $h$ . The decrease in the sunk entry costs allows a large number of new productive firms to immediately enter the market, increasing sharply the number of new entrants,  $N_E^h$ . The increased competition for inputs from the new productive firms drives the relatively less productive firms out of the market, thus increasing average firm-level productivity,  $z_D^h$ . As in the case of the permanent increase in aggregate technology, the increase in productivity allows firms to pay a higher



real wage,  $w^h$ , and make higher profits, leading to an increase in consumption,  $C^h$ . After the initial sharp increase in labour productivity, as more productive firms slowly enter the market, the relatively less productive firms are driven out by the increased competition for inputs, and thus consumption,  $C^h$ , labour productivity,  $Z_O^h$ , and the real wage,  $w^h$ , slowly increase to their new, higher steady-state levels.

The introduction of endogenous firm-level productivity in our model has the effect of counteracting the initial consumption undershooting in Ghironi and Melitz (2005). In contrast to their model, consumption in our model raises sharply, before increasing slowly to its new higher steady-state level, driven by the sudden exit of unproductive firms from the market, giving the movement/change in the cutoff productivity level for domestic production. As in the case of the transitory shock, the decrease in sunk entry costs immediately drives up the real wage as new firms enter the market, as well as increasing average firm-level productivity, both of which increase consumption. As more firms enter the market, this effect continues, albeit at a decreasing rate, as the economy moves towards the new steady state.

Figure 13: Response to a Permanent Increase in Fixed Costs of Domestic Production ( $f_D^h$ )



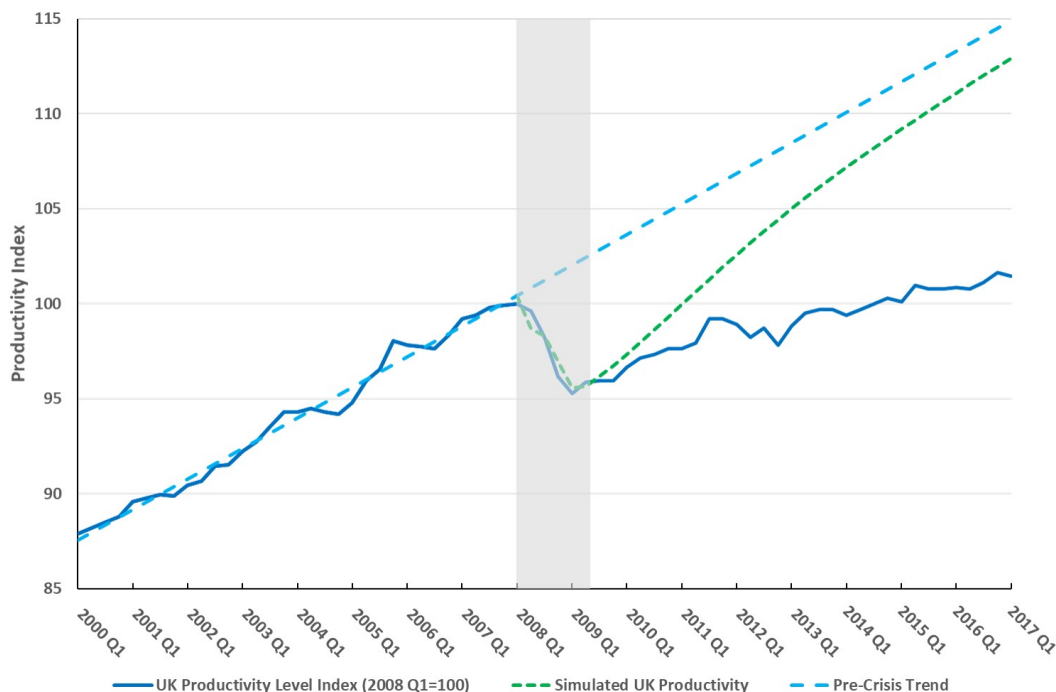
In contrast to the dynamic responses to permanent shocks to aggregate technology and the sunk costs of entry, a permanent shock to the fixed cost of domestic production can induce significant overshooting of the endogenous variables. In Figure 13 we present the dynamic response of consumption, average firm productivity, labour productivity, the real wage, the number of firms and the number of new entrants to a permanent 1% increase in the fixed cost of domestic production in country  $h$ . A permanent increase in the fixed cost of domestic production induces consumption, the real wage, labour productivity and the number of new entrants to overshoot sharply their new steady-state levels before slowly returning to their new equilibrium levels. The increase in fixed costs of domestic production drives the relatively less productive firms out of the market immediately, as they are no longer profitable, leading to an immediate increase in average firm productivity. The increase in fixed costs of domestic production also leads to a sharp fall in the number of new entrants,  $N_E^h$ , as the least productive firms, previously able to enter the market and make profits, are no longer able to enter. The exit of the least productive firms from the market decreases competition for inputs, and allows the relatively less productive firms to

slowly enter the market, driving labour productivity down to its new, higher steady-state level. Thus, productivity overshoots its new steady-state level initially, before slowly falling as the competition for inputs decreases. As with the permanent increase in aggregate technology and the permanent decrease in the sunk cost of entry, the increase in firm-level productivity allows firms to pay a higher real wage,  $w^h$ , and make higher profits, which sharply increases consumption,  $C^h$ . However, as (more) new firms enter the market, productivity moves towards its new equilibrium level, leading the real wage and consumption to fall back to their new, lower steady-state levels.

## 5 UK Productivity Puzzle

Since 2008, productivity in the UK has stagnated to such an extent that in 2017 Q4, it was 16% lower than the continuation of its pre-crisis trend would predict (measured as output per employed worker). In Figure 14, the solid (dark blue) line plots quarterly UK productivity and the dashed (light blue) line plots its pre-crisis trend.

Figure 14: UK Labour Productivity



The decline and the subsequent stagnation in UK productivity could have been driven by two factors: 1) business responses to a reduction in consumer demand, leading to a temporary drop in productivity; 2) more persistent supply-side factors, such as a reduction in investment in research and development (see for example Millard and Nicolae (2014)) or changing credit conditions (see for example Franklin *et al.* (2015)), leading to a permanent decrease in UK productivity growth.

In the first case, the reduction in consumer demand may have led to labour hoarding by businesses, as Patterson (2012) and Martin and Rowthorn (2012) argue or, according to Goodridge *et al.* (2013) and King and Millard (2014), to higher investment in intangible assets. Both labour hoarding and higher investment in intangible assets would cease once consumer demand recovered, allowing productivity to start returning to pre-crisis levels. In the case of the more persistent supply-side factors, the number of firms with relatively low productivity may have permanently increased causing a permanent drop in

average UK labour productivity. Franklin *et al.* (2015) note that after the Great Recession, in the UK, the supply of credit decreased, particularly for firms entering the market, allowing less productive firms to survive due to lower competition. The decrease in the supply of credit was also examined by Corry *et al.* (2011), who noted the lower firm creation rates in the UK after Great Recession, which they explained by the decreased availability of loans to new businesses, as a result of financial institutions increased internal capital requirements. Similar constraints on UK firm credit post Great Recession were found by Saleheen *et al.* (2017) in their survey of UK businesses, and by Chandha *et al.* (2017) in their examination of the sectoral differences of UK productivity.

In addition to examining the credit restrictions UK firms faced after the Great Recession, the recent UK Productivity Puzzle literature also examines the impact of lower debt servicing costs on UK productivity. Arrowsmith *et al.* (2013) examine the extent to which bank forbearance and low interest rates have led to increased firm survival. They argue that although the extent to which bank forbearance affected UK productivity is small, low interest rates made debt servicing costs lower which had a large effect on UK productivity. The link between the debt burden of a firm, driven by the interest burden, and firm survival in the UK was also examined by Guariglia *et al.* (2016), who concluded that there was a statistically significant link between the interest burden of a firm and their survival rate during the Financial Crisis and Great Recession. Given that the interest burden decreased post Great Recession, it can be argued that firm survival rates increased, leading to lower productivity growth.

In this paper we examine the effect of both a reduction in consumer demand and of (more) persistent supply-side factors such as a changing credit conditions, captured here by a reduction in the supply of credit for firms entering the market and by a lower debt servicing cost for existing firms (driven by lower interest payments) on labour productivity in the UK since the Financial Crisis/Great Recession, in order to gauge the extent to which they could explain the UK Productivity Puzzle.

The reduction in consumer demand, driven by the Financial Crisis and subsequently by the Great Recession, is simulated as a temporary shock to aggregate technology in all three countries, matching the magnitude of the decrease in productivity in the UK, the EU and the RoW. The reduction in the supply of credit for firms entering the market is simulated as a permanent positive shock to the sunk cost of entry. The reduction in the debt servicing cost is simulated as a permanent negative shock to the per-period fixed cost of domestic production. We run simultaneously through our model the three shocks, which are calibrated for this exercise as follows:

1. To simulate the reduction in consumer demand driven by the Financial Crisis and subsequently by the Great Recession in the UK, the EU and the RoW, per-period shocks to aggregate technology ( $Z_t^h$ ,  $Z_t^h$  and  $Z_t^h$ ) of -0.011, -0.008 and 0.00025 respectively, are run for 4 periods, to match the reductions in productivity according to the OECD data.
2. After the Great Recession, in the UK, the credit supplied by the financial sector to the domestic market decreased by 22% from 2008 to 2015.<sup>1112</sup> Approximately 25% of a new start-up cost is related to setting up a company, including the costs associated with obtaining credit. If all company's start-up costs were directly linked to the supply of credit, the increase in  $f_E^h$  would be 5.5% (22% times 25%). However, given that not all start-up costs are directly related to domestic credit, for a more accurate calibration for a more realistic simulation exercise,  $f_E^h$  is only increased

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<sup>11</sup>According to World Bank data on credit supplied by the UK financial sector to the domestic market from the start of the Great Recession to 2017.

<sup>12</sup>From 210% of GDP in 2008 to 163% of GDP in 2015.

by 4% relative to its pre-crisis level. To match the timing in the data, we assume that  $f_E^h$  increased by 0.166% in each of the first 24 periods.

3. The average interest paid by the UK companies registered with the Bureau Van Dijk Orbis database, decreased over 2008-2015 by 4%, in nominal terms. In real terms, however, the reduction was substantially larger: 20.2%. Over the same time period, the average interest paid (as a proportion of total fixed cost was) 10.5%. To simulate the impact of the reduction in the UK interest rate, and its effect on the debt servicing cost, the fixed cost of domestic production in the UK,  $f_D^h$ , is reduced by 2.1% (20.2% times 10.5%) relative to the pre-crisis level. To match the timing of the reductions in the UK interest rate,  $f_D^h$  is decreased by 0.5% in each of the first 4 periods, and by 0.1% in period 5.<sup>13</sup>

The dashed green line in Figure 14 plots the simulated UK labour productivity according to our model, when simultaneously running through the model the shocks presented above. We note that it matches the data quite well in the periods immediately following the Financial Crisis. The initial drop in the level of productivity according to the results of our simulation is 6.43% relative to pre-crisis trend, while according the data this is 6.45%. Up to 2011 Q3, simulated productivity successfully replicates the behaviour of UK productivity, when it explains about 4.3% points of the 6.4% shortfall of the UK productivity relative to a continuation of its pre-crisis trend, illustrating the role entry and exit of relatively less productive non-trading domestic firms into the domestic market can have on aggregate productivity. However, the slowdown in growth after 2011 Q3 cannot be accounted for. Our result shows that in response to a 4% increase in the sunk entry cost and a 2.1% reduction in per-period fixed cost of domestic production in the UK relative to their pre-crisis levels, UK labour productivity is expected to decrease in long run by 1.22%., while the shortfall of UK productivity from the continuation of its pre-crisis trend kept increasing to be 16% in 2017 Q4. The results of our simulation indicate that changes in credit conditions (as captured here) can explain a small but significant part of this shortfall. Our calculations show that of the 1.99% long-run decrease in UK productivity, approximately 1% is attributable to the increase in firm start-up costs, and approximately 0.22% is attributable to decreased debt servicing cost for existing firms. If the firm set-up costs were entirely depending on the supply of credit, and therefore the start-up costs increased by 5.5% after the Great Recession, then the long-run decrease in UK productivity attributable to increased start-up costs becomes larger, from 1% to 1.4%.

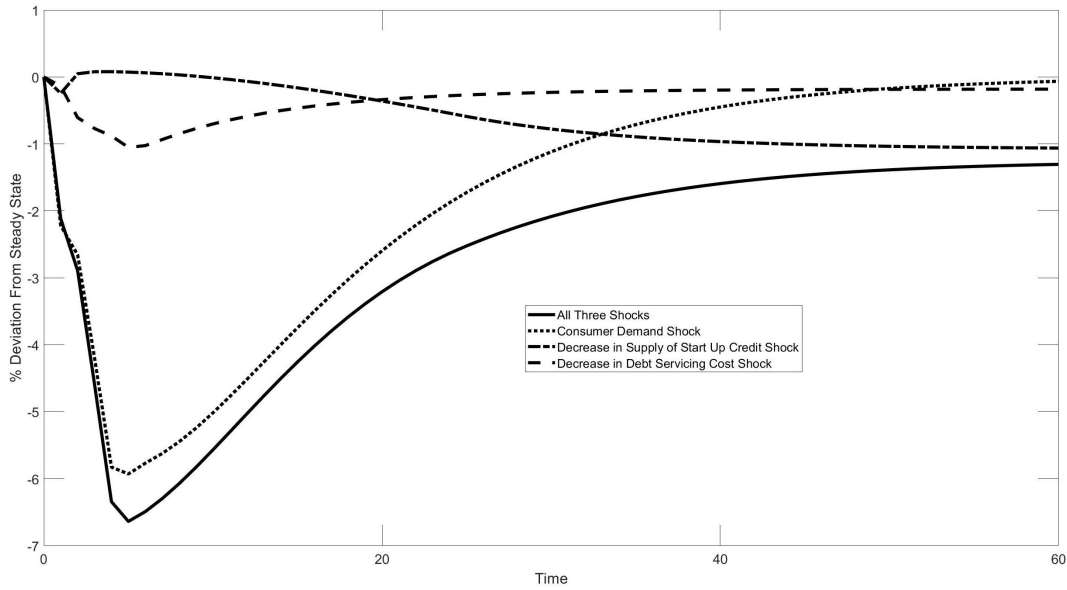
An analysis of the effect of each of the shocks on labour productivity, reveals that in response to a temporary negative consumer demand shock, labour productivity decreases to almost 5.4% below its initial steady-state level, as shown by the dotted line in Figure 15. This shows that if the reduction in UK labour productivity was solely caused by declining consumer demand, then UK productivity would have returned to its initial steady-state level within approximately 10 years with no permanent effect.

On the other hand, the response of labour productivity to a permanent positive start-up credit supply shock, shown by the dotted line, is to never recover to its initial steady-state level, reaching its new long run steady-state level which is 1% below the initial one soon after approximately 10 years. Labour productivity will first slightly increase, as a result of increased demand for labour which drives up the real wage following the increase in the sunk cost (as the fixed/sunk costs are expressed in labour unit terms), leading relatively less productive (non-trading) firms to exit the market. But as this happens, labour productivity decreases, as competition in the UK market decreases following the increase in the start-up costs, allowing unproductive firms to remain in the market, while exiting firms are not replaced at the same rate as before because of the higher start-up costs, permanently decreasing

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<sup>13</sup>the Bank of England took approximate 1 year and one quarter to reduce the interest rate to its post-crisis level.

Figure 15: The Effect of Individual Shocks on Labour Productivity



productivity.

Similarly, the response of labour productivity to a permanent negative debt servicing cost shock, as shown by the dashed line, is to permanently lower average productivity to 0.22% below the initial steady-state level. This is as the effect of lower interest payments on existing debt is to decrease the cost of staying in the market, allowing relatively less productive (non-trading) firms to stay and/or enter the market and make profits. Short term, the effect of a reduction in interest payments (through its effect on the debt servicing cost) on labour productivity is more pronounced, immediately following the lower cost of staying in the market, lowering average productivity to up to 1% below the initial steady state.

These results do not provide an explanation for the increasing shortfall of the UK productivity relative to a continuation of its pre-crisis trend, driven by lower trend productivity growth post crisis, but do provide an explanation as to why this shortfall is non-zero i.e. why the level of productivity has fallen.

We argue that changes in credit conditions in the UK following the Financial Crisis led to a higher number of relatively less productive firms entering the UK market, which - unlike in existing models - is captured and driven in our model by an endogenous average firm-level productivity, as the firm-level productivity distribution changes. However, our model shows that changes in credit conditions cannot fully explain the magnitude of the UK Productivity Puzzle. Franklin *et al.* (2015) show that 5-8% of the UK productivity decline up to 2014, could be explained by reductions in UK credit supply, while our model suggests that only 1.22% could be explained by such reductions. In Franklin *et al.* (2015) the decline of UK labour productivity is explained by the substitution of capital with more labour-intensive methods of production caused by the increased cost of capital. Given that we do not model capital in our model, the degree to which changes in credit conditions, as captured in our paper, can contribute to our understanding of the UK Productivity Puzzle is not surprising.

## 6 Conclusion

In this paper we have developed a three-country dynamic, stochastic, general equilibrium macroeconomic model of international trade with monopolistic competition and heterogeneous firms, in order to explore the role of the entry and exit into the domestic market of less productive non-trading domestic firms on labour productivity and its persistence in response to shock to macroeconomic variables. Unlike the existing literature, we allow for the distribution of firm-level productivity to be endogenously determined in our model through the entry and exit of less productive non-trading firms into the domestic market, and we run our analysis in a three-country world. Two main results stand out from our study:

First, unlike in the existing theoretical literature such as Ghironi and Melitz (2005), we show that changes in barriers to imports have a much larger effect on domestic productivity than changes in barriers to exports of an equivalent size. In this existing theoretical literature, labour productivity can only change as a result of changes in the number of domestic firms (the extensive margin of production). In our model we allow for changes in the extensive as well as intensive margins of production, through endogenous changes in both the number of domestic firms and the productivity distribution of producing firms. Our results show the key role of the entry and exit of less productive non-trading firms into the domestic market in driving productivity, a result which is consistent with empirical papers such as Yasar and Paul (2007) and Bloom *et al.* (2016).

Second, the entry and exit of less productive non-trading domestic firms into the domestic market can explain the persistence of labour productivity in response to transitory macroeconomic shocks. We show that the persistence of labour productivity in response to transitory shocks to aggregate technology and the sunk cost of entry is higher than the exogenous persistence of the two shocks, but the persistence of labour productivity in response to transitory shocks to fixed costs of domestic production is lower than the exogenous persistence of the shock, irrespective of the calibration of the persistence of the shock. Therefore, we offer a potential explanation for the wide variation in the empirically calculated figures for the persistence of productivity from 0.85 in Pancazi and Vukotic (2011) to 0.906 in Backus *et al.* (1992) and 0.994 in Baxter (1995).

We also use our model to examine the extent to which changes in credit conditions seen in the UK shortly after the Financial Crisis/Great Recession could contribute to our understanding of the UK Productivity Puzzle. We show that our model can successfully replicate the behaviour of UK productivity for the first three and a half years after the Great Recession. We conclude that changes in consumer demand and credit conditions can account for about 4.3% points of the 6.4% shortfall of the UK productivity in 2011 Q3 relative to a continuation of its pre-crisis trend, illustrating the role entry and exit of relatively less productive non-trading domestic firms into the domestic market can have on aggregate productivity. The non-trading domestic firms are both the most unproductive firms and most numerous and therefore small credit changes can have significant effects on the entry and exit of these firms, leading to large changes in aggregate productivity.

Future research might investigate the role of non-trading firms exit and entry into the domestic market in driving labour productivity and its persistence, in a context in which two current restrictive assumptions in the model are relaxed: 1) trade balances in every period; and 2) full employment. Relaxing the first assumption would allow us to account for the intertemporal consumption smoothing allowed by international trade (Obstfeld and Rogoff (1995)). Relaxing the assumption of full employment would allow us to examine the relationships between trade, firm entry and exit, job creation

and destruction, productivity and the unemployment rate.

## 7 Bibliography

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# Appendices

Table A1: Equations of the Model

Equation Name	Equation
Price Indexes	$N_{Dt}^h (\tilde{\rho}_{Dt}^h)^{1-\theta} + N_{Xht}^i (\tilde{\rho}_{Xht}^i)^{1-\theta} + N_{Xht}^j (\tilde{\rho}_{Xht}^j)^{1-\theta} = 1$ $N_{Dt}^i (\tilde{\rho}_{Dt}^i)^{1-\theta} + N_{Xit}^h (\tilde{\rho}_{Xit}^h)^{1-\theta} + N_{Xit}^j (\tilde{\rho}_{Xit}^j)^{1-\theta} = 1$ $N_{Dt}^j (\tilde{\rho}_{Dt}^j)^{1-\theta} + N_{Xjt}^h (\tilde{\rho}_{Xjt}^h)^{1-\theta} + N_{Xjt}^i (\tilde{\rho}_{Xjt}^i)^{1-\theta} = 1$
Domestic Price	$\tilde{\rho}_{Dt}^h = \frac{\theta}{\theta-1} \frac{w_t^h}{Z_t^h \tilde{z}_{Dt}^h}$ $\tilde{\rho}_{Dt}^i = \frac{\theta}{\theta-1} \frac{w_t^i}{Z_t^i \tilde{z}_{Dt}^i}$ $\tilde{\rho}_{Dt}^j = \frac{\theta}{\theta-1} \frac{w_t^j}{Z_t^j \tilde{z}_{Dt}^j}$
Export Price	$\tilde{\rho}_{Xit}^h = \frac{\theta}{\theta-1} \frac{\tau_{it}^h}{Q_t^i} \frac{w_t^h}{Z_t^h \tilde{z}_{Xit}^h}$ $\tilde{\rho}_{Xjt}^h = \frac{\theta}{\theta-1} \frac{\tau_{jt}^h}{Q_t^j} \frac{w_t^h}{Z_t^h \tilde{z}_{Xjt}^h}$ $\tilde{\rho}_{Xht}^i = \frac{\theta}{\theta-1} Q_t^i \tau_{ht}^i \frac{w_t^i}{Z_t^i \tilde{z}_{Xht}^i}$ $\tilde{\rho}_{Xjt}^i = \frac{\theta}{\theta-1} \frac{Q_t^j \tau_{jt}^i}{Q_t^i} \frac{w_t^i}{Z_t^i \tilde{z}_{Xjt}^i}$ $\tilde{\rho}_{Xht}^j = \frac{\theta}{\theta-1} Q_t^j \tau_{ht}^j \frac{w_t^j}{Z_t^j \tilde{z}_{Xht}^j}$ $\tilde{\rho}_{Xit}^j = \frac{\theta}{\theta-1} \frac{Q_t^i \tau_{it}^j}{Q_t^j} \frac{w_t^j}{Z_t^j \tilde{z}_{Xit}^j}$
Total Firm Profits	$\tilde{d}_t^h = \tilde{d}_{Dt}^h + \frac{N_{Xit}^h}{N_{Dt}^h} \tilde{d}_{Xit}^h + \frac{N_{Xjt}^h}{N_{Dt}^h} \tilde{d}_{Xjt}^h$ $\tilde{d}_t^i = \tilde{d}_{Dt}^i + \frac{N_{Xht}^i}{N_{Dt}^i} \tilde{d}_{Xht}^i + \frac{N_{Xjt}^i}{N_{Dt}^i} \tilde{d}_{Xjt}^i$ $\tilde{d}_t^j = \tilde{d}_{Dt}^j + \frac{N_{Xht}^j}{N_{Dt}^j} \tilde{d}_{Xht}^j + \frac{N_{Xit}^j}{N_{Dt}^j} \tilde{d}_{Xit}^j$
Domestic Profits	$\tilde{d}_{Dt}^h = \frac{C_t^h}{\theta} (\tilde{\rho}_{Dt}^h)^{1-\theta} - \frac{w_t^h f_D^h}{Z_t^h}$ $\tilde{d}_{Dt}^i = \frac{C_t^i}{\theta} (\tilde{\rho}_{Dt}^i)^{1-\theta} - \frac{w_t^i f_D^i}{Z_t^i}$ $\tilde{d}_{Dt}^j = \frac{C_t^j}{\theta} (\tilde{\rho}_{Dt}^j)^{1-\theta} - \frac{w_t^j f_D^j}{Z_t^j}$
Export Profits $h$	$\tilde{d}_{Xit}^h = \frac{C_t^i Q_t^i}{\theta} (\tilde{\rho}_{Xit}^h)^{1-\theta} - \frac{w_t^h f_{Xi}^h}{Z_t^h}$ $\tilde{d}_{Xjt}^h = \frac{C_t^j Q_t^j}{\theta} (\tilde{\rho}_{Xjt}^h)^{1-\theta} - \frac{w_t^h f_{Xj}^h}{Z_t^h}$ $\tilde{d}_{Xht}^i = \frac{C_t^h Q_t^h}{\theta} (\tilde{\rho}_{Xht}^i)^{1-\theta} - \frac{w_t^i f_{Xh}^i}{Z_t^i}$ $\tilde{d}_{Xjt}^i = \frac{C_t^j Q_t^j}{\theta} (\tilde{\rho}_{Xjt}^i)^{1-\theta} - \frac{w_t^i f_{Xj}^i}{Z_t^i}$ $\tilde{d}_{Xht}^j = \frac{C_t^h Q_t^h}{\theta} (\tilde{\rho}_{Xht}^j)^{1-\theta} - \frac{w_t^j f_{Xh}^j}{Z_t^j}$ $\tilde{d}_{Xit}^j = \frac{C_t^i Q_t^i}{\theta} (\tilde{\rho}_{Xit}^j)^{1-\theta} - \frac{w_t^j f_{Xi}^j}{Z_t^j}$

Equation Name	Equation
Free Entry Condition	$\tilde{v}_t^h = \left( \frac{E(z_{Dt+1}^h)}{z_{min}^h} \right)^k \frac{w_t^h f_E^h}{Z_t^h}$
	$\tilde{v}_t^i = \left( \frac{E(z_{Dt+1}^i)}{z_{min}^i} \right)^k \frac{w_t^i f_E^i}{Z_t^i}$
	$\tilde{v}_t^j = \left( \frac{E(z_{Dt+1}^j)}{z_{min}^j} \right)^k \frac{w_t^j f_E^j}{Z_t^j}$
New Firm Entry Condition	$N_{Dt}^h = (1 - \delta)(N_{Dt-1}^h + N_{Et-1}^h)$
	$N_{Dt}^i = (1 - \delta)(N_{Dt-1}^i + N_{Et-1}^i)$
	$N_{Dt}^j = (1 - \delta)(N_{Dt-1}^j + N_{Et-1}^j)$
Proportion of Firms That Export	$\frac{N_{Xit}^h}{N_{Dt}^h} = (\tilde{z}_{Dt}^h)^k (\tilde{z}_{Xit}^h)^k$
	$\frac{N_{Xjt}^h}{N_{Dt}^h} = (\tilde{z}_{Dt}^h)^k (\tilde{z}_{Xjt}^h)^k$
	$\frac{N_{Xht}^i}{N_{Dt}^i} = (\tilde{z}_{Dt}^i)^k (\tilde{z}_{Xht}^i)^k$
	$\frac{N_{Xjt}^i}{N_{Dt}^i} = (\tilde{z}_{Dt}^i)^k (\tilde{z}_{Xjt}^i)^k$
	$\frac{N_{Xht}^j}{N_{Dt}^j} = (\tilde{z}_{Dt}^j)^k (\tilde{z}_{Xht}^j)^k$
	$\frac{N_{Xit}^j}{N_{Dt}^j} = (\tilde{z}_{Dt}^j)^k (\tilde{z}_{Xit}^j)^k$
Zero Profit Condition For Domestic Firms	$\tilde{d}_{Dt}^h = (\theta - 1) \left( \frac{\nu^{\theta-1}}{k} \right) \frac{w_t^h f_D^h}{Z_t^h}$
	$\tilde{d}_{Dt}^i = (\theta - 1) \left( \frac{\nu^{\theta-1}}{k} \right) \frac{w_t^i f_D^i}{Z_t^i}$
	$\tilde{d}_{Dt}^j = (\theta - 1) \left( \frac{\nu^{\theta-1}}{k} \right) \frac{w_t^j f_D^j}{Z_t^j}$
Zero Profit Condition For Exporting Firms	$\tilde{d}_{Xit}^h = (\theta - 1) \left( \frac{\nu^{\theta-1}}{k} \right) \frac{w_t^h f_{X_i}^h}{Z_t^h}$
	$\tilde{d}_{Xjt}^h = (\theta - 1) \left( \frac{\nu^{\theta-1}}{k} \right) \frac{w_t^h f_{X_j}^h}{Z_t^h}$
	$\tilde{d}_{Xht}^i = (\theta - 1) \left( \frac{\nu^{\theta-1}}{k} \right) \frac{w_t^i f_{X_h}^i}{Z_t^i}$
	$\tilde{d}_{Xjt}^i = (\theta - 1) \left( \frac{\nu^{\theta-1}}{k} \right) \frac{w_t^i f_{X_j}^i}{Z_t^i}$
	$\tilde{d}_{Xht}^j = (\theta - 1) \left( \frac{\nu^{\theta-1}}{k} \right) \frac{w_t^j f_{X_h}^j}{Z_t^j}$
	$\tilde{d}_{Xit}^j = (\theta - 1) \left( \frac{\nu^{\theta-1}}{k} \right) \frac{w_t^j f_{X_i}^j}{Z_t^j}$
	$\tilde{d}_{Xit}^j = (\theta - 1) \left( \frac{\nu^{\theta-1}}{k} \right) \frac{w_t^j f_{X_i}^j}{Z_t^j}$
Household Bond Euler Equation	$(C_t^h)^{-\gamma} = \beta(1 + r_t^h) E_t[(C_{t+1}^h)^{-\gamma}]$
	$(C_t^i)^{-\gamma} = \beta(1 + r_t^i) E_t[(C_{t+1}^i)^{-\gamma}]$
	$(C_t^j)^{-\gamma} = \beta(1 + r_t^j) E_t[(C_{t+1}^j)^{-\gamma}]$
Household Share Euler Equation	$\tilde{v}_t^h = \beta(1 - \delta) E_t \left[ \frac{C_{t+1}^h}{C_t^h}^{-\gamma} (\tilde{v}_{t+1}^h + \tilde{d}_{t+1}^h) \right]$
	$\tilde{v}_t^i = \beta(1 - \delta) E_t \left[ \frac{C_{t+1}^i}{C_t^i}^{-\gamma} (\tilde{v}_{t+1}^i + \tilde{d}_{t+1}^i) \right]$
	$\tilde{v}_t^j = \beta(1 - \delta) E_t \left[ \frac{C_{t+1}^j}{C_t^j}^{-\gamma} (\tilde{v}_{t+1}^j + \tilde{d}_{t+1}^j) \right]$
Labour Market Clearing Equation	$L_t^h = L_{Pt}^h + \left( \frac{E(z_{Dt+1}^h)}{z_{min}^h} \right)^k \frac{N_{Et}^h f_E^h}{Z_t^h} + \frac{N_{Dt}^h f_D^h}{Z_t^h} + \frac{N_{Xit}^h f_{X_{it}}^h}{Z_t^h} + \frac{N_{Xjt}^h f_{X_{jt}}^h}{Z_t^h}$
	$L_t^i = L_{Pt}^i + \left( \frac{E(z_{Dt+1}^i)}{z_{min}^i} \right)^k \frac{N_{Et}^i f_E^i}{Z_t^i} + \frac{N_{Dt}^i f_D^i}{Z_t^i} + \frac{N_{Xht}^i f_{X_{ht}}^i}{Z_t^i} + \frac{N_{Xjt}^i f_{X_{jt}}^i}{Z_t^i}$
	$L_t^j = L_{Pt}^j + \left( \frac{E(z_{Dt+1}^j)}{z_{min}^j} \right)^k \frac{N_{Et}^j f_E^j}{Z_t^j} + \frac{N_{Dt}^j f_D^j}{Z_t^j} + \frac{N_{Xht}^j f_{X_{ht}}^j}{Z_t^j} + \frac{N_{Xit}^j f_{X_{it}}^j}{Z_t^j}$
Balanced Trade Equation	$Q_t^i N_{Xit}^h (\tilde{\rho}_{Xit}^h)^{1-\theta} C_t^i + Q_t^j N_{Xjt}^h (\tilde{\rho}_{Xjt}^h)^{1-\theta} C_t^j =$
	$1/Q_t^i N_{Xht}^i (\tilde{\rho}_{Xht}^i)^{1-\theta} C_t^h + Q_t^j / Q_t^i N_{Xjt}^i (\tilde{\rho}_{Xjt}^i)^{1-\theta} C_t^j =$

Table A2: Parameter Values Used in the Model Calibration

Parameter	Value	Description
$\beta$	0.99	Household discount factor
$\gamma$	2	Risk aversion
$\delta$	0.0235	Probability of firm death
$\theta$	3.8	Elasticity of substitution
k	3.4	Firm-level productivity dispersion
$\rho$	0.9	Aggregate persistence
$z_{min}^h$	1	Minimum firm productivity in country $h$
$z_{min}^i$	1	Minimum firm productivity in country $i$
$z_{min}^j$	1	Minimum firm productivity in country $j$
$f_E^h$	1	Firm sunk entry cost in country $h$
$f_E^i$	1	Firm sunk entry cost in country $i$
$f_E^j$	1	Firm sunk entry cost in country $j$
$f_D^h$	0.0090	Per period fixed cost of producing for the domestic market in country $h$
$f_D^i$	0.0104	Per period fixed cost of producing for the domestic market in country $i$
$f_D^j$	0.0184	Per period fixed cost of producing for the domestic market in country $j$
$\tau_i^h$	1.316	Per unit iceberg cost of exporting from country $h$ to country $i$
$\tau_j^h$	1.450	Per unit iceberg cost of exporting from country $h$ to country $j$
$\tau_h^i$	1.326	Per unit iceberg cost of exporting from country $i$ to country $h$
$\tau_j^i$	1.450	Per unit iceberg cost of exporting from country $i$ to country $j$
$\tau_h^j$	1.459	Per unit iceberg cost of exporting from country $j$ to country $h$
$\tau_i^j$	1.459	Per unit iceberg cost of exporting from country $j$ to country $i$
$f_{X_i}^h$	0.09	Per period fixed cost of producing for the country $i$ export market in country $h$
$f_{X_j}^h$	0.082	Per period fixed cost of producing for the country $j$ export market in country $h$
$f_{X_h}^i$	0.083	Per period fixed cost of producing for the country $h$ export market in country $i$
$f_{X_j}^i$	0.135	Per period fixed cost of producing for the country $j$ export market in country $i$
$f_{X_h}^j$	0.136	Per period fixed cost of producing for the country $h$ export market in country $j$
$f_{X_i}^j$	0.136	Per period fixed cost of producing for the country $i$ export market in country $j$
$L^h$	1	Size of country $h$
$L^i$	5.93	Size of country $i$
$L^j$	19.59	Size of country $j$