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Abstract

This paper presents a forward-looking approach to measure systemic solvency risk using contingent claims analysis (CCA) as a theoretical foundation for determining an institution's default risk based on the uncertainty in its asset value relative to promised debt payments over time. Default risk can be quantified as market-implied expected losses calculated from integrating equity market and balance sheet information in a structural default risk model. The expected losses of multiple banks and their non-parametric dependence structure define a multivariate distribution that generates portfolio-based estimates of the joint default risk using the aggregation technique of the Systemic CCA framework (Jobst and Gray, 2013). This market-implied valuation approach ('shadow capital adequacy') endogenises bank solvency as a probabilistic concept based on the perceived default risk (in contrast to accounting-based prudential measures of capital adequacy). The presented model adds to the literature of analytical tools for estimating market-implied systemic risk by augmenting the CCA approach with a jump diffusion process of asset changes to inform a more comprehensive and flexible assessment of common vulnerabilities to tail risks of the four largest UK commercial banks.

Key words: Systemic risk, contingent claims analysis, jump diffusion, CoVaR, systemic expected shortfall, conditional tail expectation, capital adequacy.

JEL classification: C61, C63, G01, G21, G28.

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1 Introduction

The global financial crisis underscored the importance of developing new and reliable approaches that can help quantify system-wide default risk of multiple institutions (beyond the traditional focus on firm-specific default risk) as an integral element in the design and implementation of macroprudential policy and surveillance. However, assessing the importance of systemic risk with a view towards enhancing the resilience of the financial sector is not a straightforward exercise and has many conceptual challenges. In particular, there is a danger of underestimating practical problems of applying systemic risk management approaches as complex modeling risks losing transparency and could fail to accommodate important differences in business models of financial institutions and regulatory approaches across countries.

This paper addresses some of these challenges in the specification of a forward-looking framework to measure systemic solvency risk based on market-implied expected losses. We use an advanced form of *contingent claims analysis* (CCA), which defines the probability of default based on the risk-adjusted balance sheet of firms whose assets are stochastic and may be above or below promised payments on its debt obligations. In the context of banks, this approach facilitates an integrated market-based capital assessment outside the accounting-based concept of regulatory capital adequacy and helps capture how the actual default risk impacts the capital assessment ("shadow capital adequacy"), which is consistent with Haldane (2011) who stated that "market-based metrics of bank solvency would be based around the market rather than book value of capital. The market prices of banks are known to offer useful supplementary information to that collected by supervisors when assessing bank health. And there is also evidence they can offer reliable advance warnings of bank distress."

We model the magnitude of the joint default risk of multiple financial banks falling into distress as a portfolio of individual market-implied expected losses (with individual risk parameters) to generate a distribution-based perspective of system-wide capital adequacy, which reflects the variability of both assets and liabilities at different levels of statistical confidence. Following the Systemic CCA (or SCCA) model (Jobst and Gray (2013)), we combine the market-implied expected losses and the dependence between them to estimate system-wide tail risk using multivariate generalised extreme value (GEV) analysis. This approach is applied to the four largest UK banks (Barclays, HSBC, Lloyds, and RBS) over a time horizon of more than 12 years covering the period of the global financial crisis (2008-2009) and the European debt crisis (2010-2011). However, the methodology can be replicated in any banking jurisdiction where bank equity trades in the secondary market. The four sample banks have different business models and account for about 85 percent of total banking sector assets. We find that systemic risk has greatly diminished in the wake of the global financial crisis, which implies a far lower probability of these banks experiencing extreme losses at the same time.

The paper is organised as follows. Section 2 provides a brief review of the literature on estimating systemic risk within banking systems. Section 3 describes the jump diffusion process and how the model parameters are used to calculate the implicit put option values of each bank. Section 4 describes the data and the empirical results relating to the determination of market-implied expected losses of individual banks. Section 5 discusses how these bank-specific expected losses can be used to generate a multivariate density of expected losses with extreme value dependence. Section 6 explains how the estimation results can inform the design of a market-based capital assessment to complement existing prudential approaches. Section 7 concludes.

2 Related Literature

The growing literature on systemic risk measurement has responded to greater demand for models that explore the interlinkages between banks and their implications for financial stability. During the global financial crisis, interbank linkages - both directly through money markets and indirectly through derivatives and structured finance markets - became a major source of contagion. Thus, the specification of the dependence between firms helps identify common vulnerabilities to risks based on the (assumed) collective behaviour under common distress.[1]

There are two strands of model development, which resonate with different policy objectives and corresponding risk measures: (i) a particular activity causes a firm to fail, whose importance to the system imposes marginal distress on other firms (or markets) ("contribution approach"), or (ii) a firm experiences losses from a significant exposure to a single (or multiple) large shock(s) to another firm, sector, country and/or currency (or demonstrates sufficient resilience to mitigate/absorb these shocks ("participation approach"). While the *contribution approach* presumes that individual failure causes system-wide distress, the *participation approach* implies greater nuance regarding the cause and effect of risk transmission. For instance, a highly leveraged institution's sudden disposal of large asset positions and investors reassessment of its default risk can significantly disrupt trading and increase asset and/or funding risk potentially causing significant losses for other firms with similar holdings. Due to differences in portfolio composition, market access and/or business factors some firms amplify the scale of impact, while others might demonstrate sufficient resilience to absorb these shocks.

Most institution-level measures of systemic risk have focussed on empirical, bivariate approaches with an implicit or explicit treatment of statistical dependence (using the historical dynamics of market prices) to determine the joint risk or expected losses based on the assumed directionality of systemic risk propagation:

(i) for the *contribution* approach - Conditional Value-at-Risk (CoVaR)[2] (Adrian and Brunnermeier (2016)) and related extensions, such as Component/Incremental VaR (Liao et al. (2015)), Co-Risk (Giudici and Parisi (2016)), and copula-based CoVaR (Reboredo and Ugolini (2015); Reboredo and Ugolini (2016); Karimalis and Nomikos (2018), which show the marginal contribution of firms to system-wide losses during times of individual stress, and

(ii) for the *participation* approach - Systemic Expected Shortfall (SES) (Acharya et al. (2010)) and related extensions, such as Component Expected Shortfall (CES) (Banelescu and Dumitrescu (2013)) and Systemic Capital Shortfall/SRISK (Brownlees and Engle (2011)), which show the potential losses of individual banks in response to a large drop in overall market capitalisation of the aggregate banking sector during times of stress.[3], [4]

[1] Bardoscia et al. (2017) show that solvency contagion driven by the interconnectedness between financial institutions seems to have declined in the wake of the global financial crisis.

[2] See Sheu and Cheng (2012) for a detailed discussion of the CoVaR model.

[3] The SES and CES are complementary, with the latter being focussed on the absolute rather than the marginal "contribution" of an individual firm to system-wide losses (after accounting for differences in leverage and market capitalisation) during times of general distress in the financial system (Popescu and Turcu (2014)).

[4] Karas and Szczepaniak (2019) combine both contribution and participation approaches in their specification of a composite measure (Systemic Turbulence Measures), which includes both CoVaR and MES.

However, only a few models have incorporated a multivariate[5] perspective using historically informed measures of association, such as Multiple Conditional VaR (MCoVaR) and Multiple Conditional Expected Shortfall (MCoES) (Bernardi et al. (2017)), and all of them define default based on a pre-specified threshold of negative shocks to market returns rather than a structural definition of default. [6]

There are also several studies on network analysis which model how inter-linked asset holdings generate and propagate systemic risk (Allen et al. (2010)) as an alternative to distributional approaches to model complex financial relationships and project their changes during times of stress (conditional on feedback effects) (Kanno (2016)). Haldane and Nelson (2012) argue that networks can produce non-linearity and unpredictability with attendant extreme (or fat-tailed) events. Elsinger et al. (2006) and Webber and Willison (2011) analyse systemic risk in the UK banking system with a focus on interbank exposures. But both estimate banks' marginal default probabilities using the traditional diffusion process of total assets as part of the single-firm credit risk model (Merton (1974)). As will be discussed later in the paper, this approach understates market-implied expected losses and joint default risk.[7]

We propose a forward-looking, market data-based framework for estimating systemic risk to inform a more flexible solvency assessment of banks ("shadow capital adequacy") based on the price impact of investor perception (which is expressed as expected losses) rather than accounting values (which define capital adequacy). In this framework, the general default risk of the banking sector is viewed as a portfolio of individual expected losses (with individual risk parameters) where the joint probability of all banks experiencing distress simultaneously defines the magnitude of systemic risk. Expected losses are defined by observable changes in each bank's equity and equity volatility, which determine the chance of the implied asset value (including available cash flows from operations and asset sales) falling below the amount of required funding to satisfy debt payment obligations over a specified time horizon.

We specify the statistical distribution of individual expected losses at high confidence intervals in accordance with *extreme value theory* (EVT). We then estimate multivariate dependence (bank-by-bank) through a closed-form specification, which accounts for the sensitivity of expected losses to common risk factors. The market-implied linkages affecting joint expected losses can deliver important insights about the joint tail risk of multiple entities during times of stress.

[5]This could be done via modelling multivariate (firm-by-firm) dependence through either a closed-from specification or the simulation of joint probabilities

[6]Novickyte and Dicpinigatiene (2018) as well as Hattori et al. (2014) provide a comprehensive review of the existing literature of measurement approaches for monitoring of systemic financial risk. Blancher et al. (2013) take stock of the current tools enlisted by IMF staff in the context of the bilateral and multilateral macroprudential surveillance and offers detailed and practical guidance on their use. The Global Financial Stability Report (International Monetary Fund (2009a) and International Monetary Fund (2009b)) provided a first overview of available analytical tools for the identification of systemic risk in the financial sector. The Office for Financial Research of the U.S. Treasury Department published a comprehensive survey of systemic risk analytics (Bisias et al. (2012)), which also organises various indicators according to a supervisory perspective and a research perspective.

[7]Di Cesare and Picco (2018) complement this conceptual distinction by including two taxonomies that help categorise systemic risk indicators used in official publications by international institutions based on their theoretical foundations and regulatory relevance. Indicators with features that are most relevant from a regulatory perpsective have (i) a high early warning capacity (ex-ante/near-coincident), (ii) are simple to implement, and (iii) can be updated at high frequency. Systemic risk indicators can also be categorised according to their underlying analytical techniques: (i) contingent claim analysis, (ii) probability and mathematical methods, (iii) interconnection analysis, and (iv) composite measures. See also Cerutti et al. (2011).

We identify changes in firm-specific and common factors determining individual default risk, their implications for the market-implied linkages between firms, and the resulting impact on the overall assessment of systemic risk based on three model properties:

(i) *Drawing on the market's evaluation of a firm's risk profile.* Combining market prices and balance sheet information into a measure of implied default risk links the evaluation of solvency conditions to investor perception and provides a market-based cross-validation of accounting-based measures of regulatory capital adequacy.

(ii) *Controlling for common factors affecting the firm's solvency.* Changes in market conditions (and their impact on the perceived risk profile of each bank) link the market-based assessment of each bank's risk profile with the risk characteristics of other banks that are subject to common changes in market conditions. Thus, measuring the time-varying dependence of expected losses connects banks implicitly to the markets in which they obtain equity funding (as the most junior, and, thus, most risk-sensitive claim on profits).[8]

(iii) *Quantifying the chance of simultaneous distress.* The magnitude of systemic risk is assessed based on the combined default risk of all banks during times of stress and provides a sense of relative systemic relevance of each bank. The probability of multiple firms experiencing a significant increase of expected losses simultaneously is made explicit by computing their joint probability distribution (which also accounts for the differences in the magnitude of changes in individual expected losses).

The model augments the standard option pricing formula for measuring expected losses within the multivariate set-up of the Systemic CCA framework (Jobst and Gray (2013)) with a jump diffusion component for the pricing process of the firm's asset value. This approach helps mitigate the empirical shortcomings of non-stochastic volatility in traditional single-firm credit risk models to derive more reliable estimates of expected losses. Further refinements are possible, including various simulation approaches, which might come at the expense of losing analytical tractability.[9] In the related literature, Hanson et al. (2004) use a jump diffusion model to estimate stock market returns. Synowiec (2008) uses exchange rate quotations to analyse alternative approaches for modelling asset returns using jump diffusion processes. However, there is little empirical research on incorporating jumps in the firm value process,[10] much less in estimating the expected losses of financial institutions. [11] This paper contributes to the literature of analytical tools for estimating market-implied systemic risk by using a jump diffusion process of changes in banks' asset values within a CCA model.

[8]The implied asset value of a bank (which underpins the estimation of implied expected losses) is modelled as being sensitive to the same markets as the implied asset value of every other bank but by varying degrees due to different capital structures (which impact the market's perception of their implied asset values).

[9]The ad hoc model by Dumas et al. (1998) can accommodate the implied volatility smile and is easy to implement but requires a large number of market option prices. The pricing models by Heston (1993) and Heston and Nandi (2000) allow for stochastic volatility, but the parameters driving these models can be difficult to estimate. Many other models have been proposed to incorporate stochastic volatility and interest rates. Other option pricing models include those based on copulas, Levy processes, neural networks, GARCH models, and non-parametric methods. The binomial tree proposed by Cox et al. (1979) spurred the development of lattices, which are discrete-time models that can be used to price any type of option - European or American, plain-vanilla or exotic.

[10]Law and Roache (2015) use the jump diffusion process to estimate the default risks for a large sample of Chinese firms.

[11]Note that imposing a jump diffusion process for estimating the default risk poses its own set of estimation challenges. For instance, a bank's reported total liabilities change only every quarter (in the best case), which limits the number of observations for the calibration of jump risk within a given sample.

3 Estimating Market-implied Expected losses

3.1 Payoffs to equity and debt holders

A bank's asset portfolio includes loans to non-banks, interbank loans and traded securities, which are funded by debt and equity. But the actual market value of assets is not directly observable. What is observable is the market value of equity and the face value of debt for each publicly traded bank. By viewing equity as a European call option on the bank's assets with a strike price equal to the value of debt at maturity, we can make use of this information to get an estimate of the market value of assets.

As described in Gray and Malone (2008) as well as Jobst and Gray (2013), structural credit risk models view firm's liabilities as "contingent claims" on the firm's underlying assets.[12] Firms have assets (V) that change in value over time, and a fixed amount of debt (D) that is due at some point of time in the future (T). Assets of a bank are uncertain and change due to factors such as profit flows and risk exposures. Default risk over a given horizon is driven by uncertain changes in future asset values relative to promised payments on debt.

The value of the firm is split into two - that which goes to the equity holders and that which goes to the debt holders. If, at the time (T) when the debt falls due, the assets have more than enough value to repay the liabilities (D), the excess value ($V - D$) goes to the equity holders. If they do not, the equity holders receive nothing. This means that the equity holder has a European call option on the value of the firm's assets at maturity (T), where the payoff is either zero or the value of assets less liabilities (D) whichever is greater. The strike price is the nominal value of outstanding debt D.

If the assets are not enough to repay the liabilities, then the firm defaults, and debt holders receive the recovery value of the assets. So the value of the debt is equivalent to a risk-free debt holding plus a short put option on the value of the assets. To the extent the debt may not be repaid in full, it is deemed to be risky. Debt holders offer an implicit guarantee by absorbing losses if there is a default. Thus, they receive a put option premium in the form of a credit spread above the risk-free rate in return for holding risky debt. Their payoff at maturity, B, can be described as the nominal value of outstanding debt, D minus their put option of selling the firm's assets at a strike price D, so that

$$B = \min[V, D] = D - \max[D - V, 0]. \quad (1)$$

Financial crises have caused equity holders and debt holders of a bank to become more uncertain about how the value of the bank's assets will evolve in the future. A bank's creditors participate in the downside risk but receive a maximum payoff equal to the face value of debt. Equity holders benefit from upside outcomes where default does not occur but have limited liability on the downside. So uncertainty in a bank's asset value has an asymmetric effect on the market value of debt and equity.

Having identified the nature of the payoffs to debt and equity holders, the next step would be to examine how the value of the bank's assets would evolve through time relative to a default barrier.

[12] Under this approach, shareholders hold a residual claim (or call option) on the firm's total asset value after outstanding liabilities have been paid off. Thus, bondholders effectively write a European put option to share holders and receive the option premium as compensation for holding risky debt; the put option premium can be observed as a credit spread above the risk-free rate. The spread increases with a higher probability of the implied asset value falling below the default barrier over the pre-defined horizon. This, in turn, depends on the volatility of the asset value, the duration of the debt claim, and the leverage of the firm.

Stochastic assets evolve relative to a distress barrier and determine the value of liabilities with implicit options. The probability that the assets will be below the distress barrier is the probability of default.

3.2 Asset evolution according to a diffusion process

In the Merton model each bank's assets V evolve according to a Geometric Brownian Motion (GBM) with *ex-ante* fixed coefficients $[\mu,\sigma]$:

$$\frac{dV}{V} = \mu dt + \sigma dW_t \tag{2}$$

where μ is the drift, which measures the average return. The stochastic component, σdW_t, models the random change in asset value in response to external effects. σ is equal to the standard deviation of the asset return. W_t is a standard Brownian motion that is characterised by independent identically distributed increments that are normally distributed with mean 0 and variance t.

In this framework, the payoff to debtholders B_t in present value terms is simply the sum of the safe claim payoff and a short position in a put option written on the firm's assets

$$B_t = De^{-rT} - \left(De^{-rT}N\left(-d_2\right) - V_tN(-d_1)\right). \tag{3}$$

According to the Merton (1974) model, the expected loss of the firm as given by the value of the implicit put option, is

$$P_{t_{Merton}} = De^{-rT}N(-d_2) - V_tN\left(-d_1\right), \tag{4}$$

where $d_1 = \frac{ln\frac{V_t}{D}+\left(\mu+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$ and $d_2 = d_1 - \sigma\sqrt{T}$.

The Merton (1974) model is effective in capturing default risk during periods of severe financial crises and particularly for banks that are on the brink of default. However, the emprical application of the Merton model for estimating the credit spreads on the risky debt of UK banks during non-crisis periods leads to poor results. This is consistent with Leland (2006), who finds that structural credit models tend to underestimate default probabilities over shorter time horizons, especially for investment grade-rated debt.[13] Our empirical results show that using a jump diffusion model generates more realistic estimates of default risk.

3.3 Asset evolution according to a jump diffusion process

The empirical distribution of asset returns differs in many ways from the diffusion process assumed in the Black and Scholes (1973) and Merton (1974) models. Under a standard diffusion process, such as a GBM, volatility is assumed to be constant rather than stochastic. However, if the volatility (and the correlation of assets) does not change over time, the probability of a sudden large drop in asset value is very small. And if a firm cannot default unexpectedly (without being already in financial distress), the probability of default on its short-term debt will be zero as will be its credit spreads. This is not borne out empirically as the credit spreads on even investment grade short-term corporate debt will be significantly greater than zero. Jones et al. (1984) show that credit spreads on corporate bonds are much higher than what a diffusion approach would suggest. In contrast to the Black-Scholes-Merton

[13] During the height of the crisis, several banks witnessed sharp declines in the market value of their assets over short time intervals, which are not captured by a standard diffusion process.

(BSM) framework, a reduced-form approach would treat default as a random Poisson event involving a sudden loss in market value that cannot be predicted. Despite the wide application of continuous stochastic volatility models, it has become increasingly evident that a pure diffusion process is not suitable for most asset returns. Instead, reasonable assumptions need to be made about the dynamics of the underlying pricing process, such as the incorporation of jumps.

In our paper, we add a jump process to the underlying diffusion process of the BSM framework. Jump diffusions can incorporate rare, large fluctuations in asset prices as witnessed during the global financial crisis. Moreover, jumps allow higher moment features such as skewness and leptokurtic behaviour in the distribution of asset price returns. Various specifications of jump diffusion models have been introduced to empirical finance. A short review can be found in Synowiec (2008). We apply the Merton (1976) jump diffusion model with time invariant coefficients, constant volatility and log-normally distributed jump sizes. Changes in asset prices consist of a continuous diffusion component which is modelled as a GBM, and a discontinuous jump component, which is modelled as a Poisson process. The continuous time stochastic process for the asset value V_t is given by the stochastic differential equation,

$$\frac{dV_t}{V_t} = \mu dt + \sigma dW_t + d(\sum_{q=0}^{N_t}(J_q - 1)) \tag{5}$$

where the last term models the jumps. The jumps capture the price impact of extreme events, which arrive only at discrete points in time and these arrivals are described by a Poisson process N_t, characterised by its intensity λ. A jump is modelled by a random variable J, which transforms the asset value V_t to JV_t. The difference $(J-1)$ is the relative change in price when a Poisson jump occurs. The jump size J_q is a sequence of independent identically distributed nonnegative random variables. In the absence of outside news, the asset price simply follows a GBM. In the model all sources of randomness, N_t, W_t and $J's$, are assumed to be independent. Solving the stochastic differential equation (5) gives the dynamics of the asset price:

$$V_t = V_0 \exp\{(\mu - \frac{1}{2}\sigma^2)t + \sigma W_t\} \prod_{q=0}^{N_t} J_q \tag{6}$$

where V_0 is the asset price at time zero. If we denote $Y_q = \log J_q$, we have

$$X_t = \log \frac{V_t}{V_o} = (\mu - \frac{1}{2}\sigma^2)t + \sigma W_t + \sum_{j=0}^{N_t} Y_q, \tag{7}$$

The jump size is drawn randomly from a distribution with probability density function $J(Q)$ which is independent of both the GBM and the Poisson process. The jump size J is a log-normally distributed random variable:

$$\ln J \sim N(\mu_q, \sigma_q^2). \tag{8}$$

The expected value of the jump size can be written as:

$$E[J-1] = \exp(\mu_q + \frac{\sigma^{2_q}}{2}) - 1. \tag{9}$$

Augmenting the diffusion process with jumps adds three extra parameters $(\lambda, \mu_q, \sigma_q)$ to the BSM framework which contains two parameters (μ, σ). Using the approach of Ball and Torous (1983) and discretising over $[t, t+\Delta]$, the solution takes the form

$$\Delta X_t = (\mu - \frac{1}{2}\sigma^2)\Delta + \sigma \Delta W_t + \sum_{j=0}^{\Delta N_t} Y_q, \quad (10)$$

where $\Delta W_t = W_{t+\Delta} - W_t \sim N(0, \Delta)$, where $\Delta N_t = N_{t+\Delta} - N_t$ is the number of jumps occuring in the interval $[t, t+\Delta]$.

The estimated asset values for each bank are transformed into log returns, $\Delta\left[ln\left(V_t\right)\right] = ln(V_{t+1}) - ln(V_t)$. For estimation purposes, we need the probability density function of X_t as in (7). Since the calibration is done in discrete time we work with ΔX_t as defined in (10) so that the density function now has a finite number of terms. The approximation assumes that $\lambda\triangle$ converges to zero; this type of discrete time specification is referred to as a Bernoulli diffusion model. When the time difference Δt is small, by the properties of the Poisson process N_t, we know that $P(\Delta N_t = 0) = 1 - \lambda\Delta t$, $P(\Delta N_t = 1) = \lambda\Delta t$ and $P(\Delta N_t > 1) = 0$. Therefore, the density in log returns, $f_{\Delta X_t}$ can be thought of as a Bernoulli random variable that has a mixture distribution for a small $\triangle t$. During Δt, the density in log returns is a weighted average of the diffusion density $(f_{\Delta Diff})$ and jump density $(f_{\triangle Jump})$ given by:

$$f_{\Delta X_t} = (1 - \lambda\Delta t) f_{\Delta Diff} + \lambda\Delta t (f_{\Delta Diff} * f_{\triangle Jump}). \quad (11)$$

In equation (11) the diffusion part of the process is:

$$f_{\Delta Diff} \sim N((\mu - \frac{\sigma^2}{2})\Delta t, \sigma^2 \Delta t), \quad (12)$$

and the jump part is:

$$(f_{\Delta Diff} * f_{\triangle Jump}) \sim N([(\mu - \frac{\sigma^2}{2})\Delta t + \mu_q], \sigma^2 \Delta t + \sigma_q^2) \quad (13)$$

Denoting the distribution of daily log returns in a bank's asset values as $\triangle X = \Delta \ln V_t$, the log-likelihood function over the set of parameter values $\theta = \{\mu, \sigma, \lambda, \mu_q, \sigma_q\}$, can be written as:

$$\log L(\theta \mid \Delta x_1, ...\Delta x_T) = \sum_{t=1}^{T} \log f_{\Delta x}(\Delta x_t \mid \theta) \quad (14)$$

3.4 Estimating the parameters of the jump diffusion process

The normal practice for estimating the parameters would be to maximise the log-likelihood function over a set of given parameter values $\{\mu, \sigma, \lambda, \mu_q, \sigma_q\}$. However, in this case, the standard maximum likelihood (ML) procedure is not valid as the log return in the Bernoulli diffusion model is a mixture of two normal distributions with different means and variances as given by equations (12) and (13). The likelihood function is not well-behaved, which indicates the occurrence of discontinuous jumps. Moreover, since the intensity parameter, λ, is unknown *ex ante*, it is not possible to identify from which of the two normal distributions each observation originates. This, coupled with the fact that the two normal distributions are different, the ML estimator does not exist.[14] Maximising the log-likelihood function, according to equation (14) fails to determine robust parameter estimates since the likelihood equation has a flat surface. A solution to the problem would be to devise a method that would not require all the five parameters to be optimised simultaneously.

We adopt a two-stage estimation process to calibrate the model parameters. The most intuitive way to calibrate the model would be to use a bank's values. However, these are not directly observable. If we assume that the bank's underlying market value exhibits characteristics similar to those of the observable market capitalisation, we can estimate the jump parameters $\{\lambda, \mu_q, \sigma_q\}$ directly from the available time series data. The remaining diffusion parameters $\{\mu, \sigma\}$ are then estimated using ML. This two-stage estimation process is described in the following two sections.

3.5 Detecting jumps as change points in market capitalisation returns

We estimate the jump parameters $\{\lambda, \mu_q, \sigma_q\}$ using the observable market capitalisation, of each sample bank, over an estimation window of six months from 2005 to 2017. We expect structural breaks of the time series when the mean and/or standard deviation of the series undergo significant changes. Figure 1 shows the daily log returns in the market capitalisation of a major UK bank in 2008H2 (July to December). We can see that the first break in the mean log return occurs around day number 57 (18 September 2008). However, for our estimation, we need to identify the precise time when the mean series changes abruptly, which we call "change points." A change point reflects a discontinuity or jump in the time series of log returns.

3.5.1 Model specification

Let x_i denote the log returns in the market capitalisation of a bank at time t where $1 \leq i \leq n$ and $n = 130$ days. We assume that these log returns fluctuate around some underlying signal m that could be associated with the factors driving the market capitalisation of the bank. x_i can be expressed as

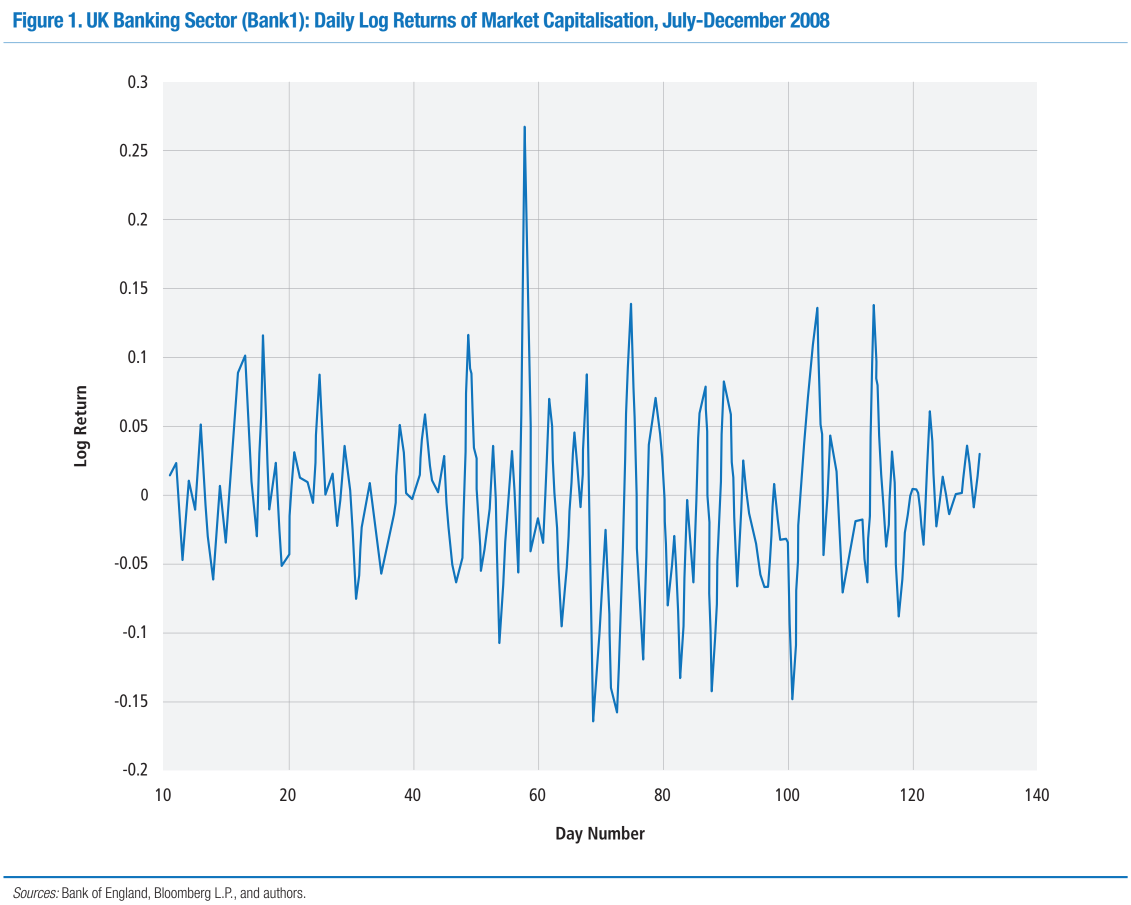
$$x_i = m(t_i) + e_i, \tag{15}$$

where e_i is a sequence of residual errors of the signal against the modelled changes. Figure 1 shows that the mean does not change during some time periods, which implies that m is piecewise constant within any of these "distinct states." The horizontal lines in Figure 2 capture those distinct states whose starting and end dates are identified by a distinct change point. The objective of this detection method is to determine these change points. For the log-return time series data $x_1, ..., x_n$ if a change point occurs at τ, then $x_{1,...,}x_\tau$ will differ from $x_{\tau+1,...,}x_n$ in some way. Following Lavielle (2005) and Killick et al. (2012), we assume that log-returns x_i follow a normal distribution, where the means m_i are piecewise constant through time. Moreover, we assume that there exists discontinuity instants $\mu_1, \mu_2, ...\mu_K$ such that

$$m(t) = \mu_k \text{ if } \tau_{k-1} < i \leq \tau_k, \tag{16}$$

[14]For further clarification see Kiefer (1978), Honore (1998), and Hamilton (1994).

Figure 1: UK Banking Sector (Bank 1): Daily Log Returns of Market Capitalisation, July-December 2008



where $k-1$ is the number of change points, which gives k homogeneous intervals where the mean of the log-returns are constant, and where $\tau_0 = 0$ and $\tau_k = n$. Thus, for any $\tau_{k-1} < i \leq \tau_k$,

$$x_i = \mu_k + e_i. \tag{17}$$

The sequence of residual errors $e_i, 1 \leq i \leq n$ is a sequence of random variables with zero mean. So x_i is a sequence of random variables with piecewise constant mean

$$E(x_i) = \mu_k \text{ if } \tau_{k-1} < i \leq \tau_k. \tag{18}$$

Assuming that the sequence of residual errors is a sequence of independent and identically distributed Gaussian variables $e_i \sim N(0, \sigma^2)$. It follows that

$$x_i \sim N(0, \sigma^2) \text{ if } \tau_{k-1} < i \leq \tau_k. \tag{19}$$

Therefore, identifying the number and sequence of jumps in the time series of log asset value returns would involve estimating (i) the number of K segments, (ii) the location of the discontinuities (τ_k, where $1 \leq k \leq K-1$) and (iii) the value of the underlying signal or mean in each segment ($\mu_k, 1 \leq k \leq K$). We derive these estimates by minimising the residual errors, which is equivalent to maximising the likelihood. Adding change points decreases the residual error but can result in overfitting. In the extreme case, every point becomes a change point, and the residual error vanishes.

The model is specified as a parametric model which depends on a vector of paramaters $\theta = (\mu_{1,...,}\mu_K, \sigma^2, \tau_1, ..., \tau_{K-1})$. Since the data from each segment represent an independent set of random variables, the overall likelihood function is a product of local likelihood functions. The overall likelihood function is denoted by $L(\theta \mid x_{1,}x_2, ..., x_n)$ and is obtained by multiplying all the local probability distributions $p(x_1, x_2, ..., x_n; \theta)$ so that

$$L(\theta \mid x_1, x_2, ..., x_n) = p(x_1, x_2, ..., x_n; \theta)$$

$$= \prod_{k=1}^{K} p(x_{\tau_{k-1}+1,...,}x_{\tau_k} : \mu_k, \sigma^2)$$

$$= \prod_{k=1}^{K} (2\pi\sigma^2)^{\frac{-(\tau_k - \tau_{k-1})}{2}} \exp\left\{ -\frac{1}{2\sigma^2} \sum_{i=\tau_{k-1}+1}^{\tau_k} (x_i - \mu_k)^2 \right\} \tag{20}$$

The ML estimation of θ can be decomposed into two steps:
(i) the means (μ_k) and change points (τ_k) are estimated by minimising

$$J(\mu_1, ..., \mu_K, \tau_1, ..., \tau_{K-1}) = \sum_{k=1}^{K} \sum_{i=\tau_{k-1}+1}^{\tau_k} (x_i - \mu_k)^2, \tag{21}$$

(ii) the variance σ^2 is estimated as the empirical variance of the estimated residuals

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{k=1}^{K}\sum_{j=\hat{\tau}_{K-1}+1}^{\hat{\tau}_k}(x_i - \hat{\mu}_k)^2. \qquad (22)$$

We will focus on the critical first step, i.e., the minimisation of $J(\mu_1, ..., \mu_K, \tau_1, ..., \tau_{K-1})$. For a given sequence of change points $\tau_1, ..., \tau_{K-1}$, J can be minimised with respect to $1; ...; K$. This is described below:

$$\hat{\mu}_k(\tau_{k-1}, \tau_k) = \overline{x}_{\tau_{k-1}+1:\hat{\tau}_k}$$

$$= \frac{1}{\tau_k - \tau_{k-1}}\sum_{i=\tau_{k-1}=1}^{\tau_k} x_i$$

minimises $\sum_{j=\tau_{k-1}+1}^{\tau_k}(y_j - \mu_k)^2$.

We insert the estimated mean values $(\mu_k(\tau_{k-1}, \tau_k))$ into J so that

$$U(\tau_1, ..., \tau_{K-1}) = J(\hat{\mu}_1(\tau_0, \tau_1), ..., \hat{\mu}_K(\tau_{K-1}, \tau_K), \tau_1, ..., \tau_{K-1})$$

$$= \sum_{k=1}^{K}\sum_{i=\tau_{k-1}+1}^{\tau_k}\left(x_i - x_{\tau_{k-1}+1:\tau_k}\right)^2.$$

The ML estimation process involves minimises

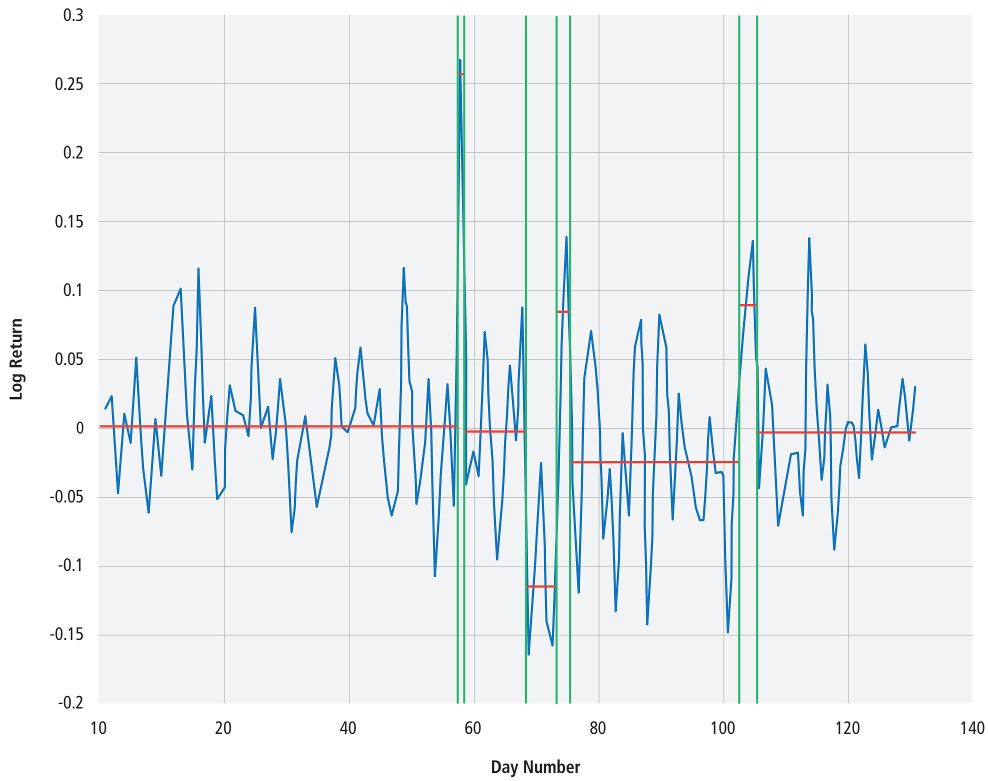
$$U(\tau_1, ..., \tau_{K-1}) = \sum_{k=1}^{K}\sum_{j=\tau_{k-1}+1}^{\tau_k}\left(x_i - \overline{x}_{\tau_{k-1+1:\tau_k}}\right)^2. \qquad (23)$$

We use a dynamic programming algorithm for solving this optimisation problem, [15] which is explained in Lavielle (2017). In our analysis, we select the optimum number of change points k (where $k = 1, 2, 3, ..., 10$) to minimise the residual returned by the model. Specifying a maximum number of k change points does not guarantee that k change points will be found. Rather, any number of change points from 1 up to k could be found.

[15] We have implemented the algorithm in MATLAB programming language by applying the *findchangepts()* function.

Figure 2: UK Banking Sector (Bank 1): Daily Log Returns of Market Capitalisation with Identified Change Points, July-December 2008

Figure 2. UK Banking Sector (Bank 1): Daily Log Returns of Market Capitalisation with Identified Change Points, July-December 2008



Sources: Bank of England, Bloomberg L.P., and authors. *Note:* number of change points = 7, total residual error = 0.31935.

As shown in Figure 2, the output from the algorithm tells us the number of times the mean of the log returns series x_i changes most significantly and also the dates at which those changes occur. In this case there were 7 such identified change points, which indicate the number of jumps in the time series of the log returns for the period under observation. The algorithm also returns the residual error of the signal against the modelled changes which, in this case, is 0.31935. Figure 2 shows the results from the algorithm for a UK bank between July and December 2008.

3.5.2 Jump diffusion parameters

Having identified the number of jumps within the time series of log returns in market capitalisation, we determine the jump rate λ, as the number of jumps divided by the total number of observations. For sample bank (Bank 1) shown in Figure 2 this was 0.0534 (7 divided by 131). The corresponding mean jump size $\hat{\mu}_q$ is -0.0456, and the jump size volatility $\hat{\sigma}_q$ is 0.2329. As expected, during the financial crisis period July-December 2008, the mean jump size of the daily log returns in market capitalisation for all the four sample banks turned out to be negative.

3.6 Calibrating the remaining parameters of the jump diffusion model

Having obtained estimates of the jump parameters $\{\lambda, \mu_q, \sigma_q\}$, we now estimate the remaining parameters μ and σ using conditional ML. We re-specify the negative of the log-likelihood for the mixed density function in equation (11) such that the parameter values, θ^*, can be written as:

$$\theta^* = \left\{\mu, \sigma, \hat{\lambda}, \hat{\mu}_q, \hat{\sigma}_q\right\}. \tag{24}$$

This indicates that the jump parameters are now held constant within the log-likelihood function for a bank's returns, and what varies is σ and μ. However, the value of the likelihood function is ultimately determined by a single unknown parameter, σ. According to the standard Merton (1974) model, the diffusion mean

$$\mu = \frac{(\log V_{t+\triangle} - \log V_t)}{T} + \frac{1}{2}\sigma^2 \tag{25}$$

is dependent on the diffusion volatility.

In order to calibrate the jump diffusion model, we choose exogenously an initial value of the unobserved volatility of asset value returns σ. A bank's actual asset value V is not observable. However, if a bank's shares are publicly traded, we can observe its market value, which is reflected in the price of equity E. So the estimation process begins by fitting the jump diffusion model to the observed time series of bank equity prices (market capitalisation series) and then producing an initial estimate for the market value over the residual maturity $T - t$ so that

$$E_t = C_t = \sum_{k=0}^{\infty} \frac{e^{-\lambda T}(\lambda T)^k}{k!}[V_t e^{\mu T + k(\mu_q + \frac{\sigma_q^2}{2})} N(d_1) - De^{-rT}.N(d_2)], \tag{26}$$

where

$$d_1 = \frac{\ln\frac{V_t}{D} + (\mu + \frac{\sigma^2}{2})T + k(\mu_q + \sigma_q^2)}{\sqrt{\sigma^2 T + k\sigma_q^2}} \tag{27}$$

$$d_2 = d_1 - \sqrt{\sigma^2 T + k\sigma_q^2}. \tag{28}$$

Expected loss is modelled as a put option where the present value of debt D represents the strike price

$$P_t = \sum_{k=0}^{\infty} \frac{e^{-\lambda T}(\lambda T)^k}{k!}[De^{-rT}.N(-d_2) - V_t e^{\mu T + k(\mu_q + \frac{\sigma_q 2}{2})} N(-d_1)]. \tag{29}$$

This assumes a more realistic evolution process of a bank's asset value than the specification of expected losses according to the traditional Merton model in equation (4). The market value of the bank (V) comprises the equity (E) and the market value of debt (B) at time t. It can be represented as:

$$V_t = E_t + B_t. \tag{30}$$

Put-call parity states that owning the asset V_t outright is equivalent to owning a portfolio comprising (i) a call option C_t at strike price D, (ii) a risk-free bond valued D at time T, and (iii) a short put option P_t with strike price D so that

$$V_t = C_t + De^{-rT} - P_t. \tag{31}$$

Rearranging we have:

$$C_t = V_t - (De^{-rT} - P_t). \tag{32}$$

In equation (32), $De^{-rT} - P_t$ is the market value of debt B_t. As an initial step, we solve for V_t using the jump diffusion parameters estimated from the observable changes in equity and equity volatility. After updating the parameter estimates based on this solution for V_t, we follow an iterative procedure until the estimates of V_t, $\mu, \sigma, \lambda, \mu_j$ and σ_j have converged. After convergence, we can plot the negative log-likelihood surface in a neighbourhood of the candidate solution point to verify that a local minimum point was identified by the ML process. Figure 3 plots such a surface for the same UK bank (shown in Figure 2) in 2008H2. The charts in Appendix A show the estimated parameter values of the diffusion volatility σ, for each of the four sample banks together with their conditional confidence intervals. For the parameter σ, the 95% confidence interval is defined as:

$$\left[\hat{\sigma} - 1.96 \times SE\left(\hat{\sigma}\right), \hat{\sigma} + 1.96 \times SE\left(\hat{\sigma}\right)\right]. \tag{33}$$

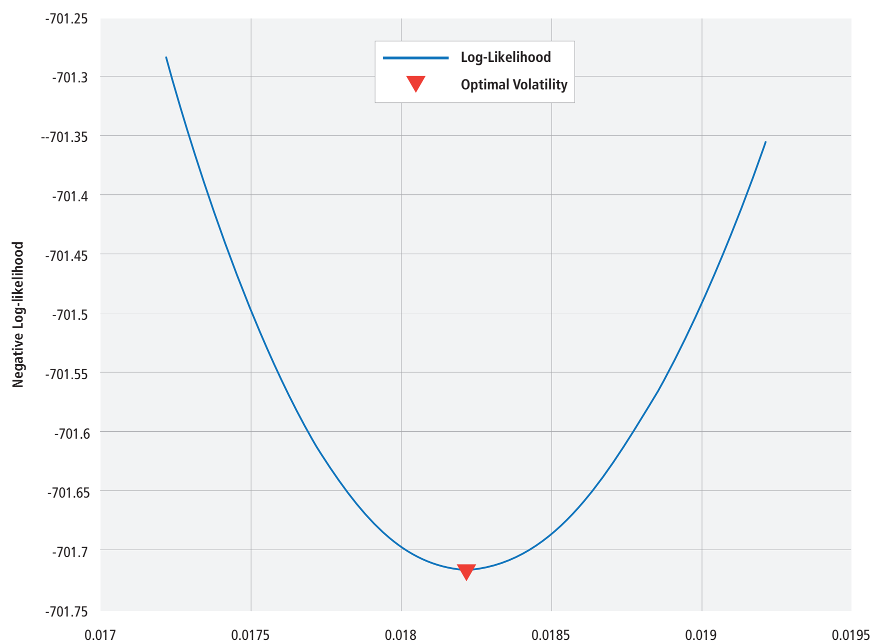
Following Zhou (1997), we can calculate the risk-neutral default probability for each individual bank by denoting the drift μ_j by r.

$$\Pr\{\frac{V}{D} \leq \varsigma\} = \sum_{k=0}^{\infty} \frac{e^{-\lambda T}(\lambda T)^k}{k!}.N(\frac{\ln(\varsigma) - \ln(\frac{V}{D}) - (r - \frac{\sigma^2}{2} - \lambda v)T - k\mu_q}{\sqrt{\sigma^2 T + k\sigma_q^2}}), \tag{34}$$

where ς is the default barrier, which is set equal to one. The time to maturity of the nominal debt

Figure 3: UK Banking Sector (Bank 1): Log-Likelihood Curve, July-December 2008

Figure 3. UK Banking Sector (Bank 1): Log-Likelihood Curve, July-December 2008



Sources: Bank of England, Bloomberg L.P., and authors.

outstanding is assumed to be one year. We define D to be the sum of a bank's total short-term liabilities and half of long-term liabilities to reflect the fact that not all of the longer maturity debt will need to be repaid within the course of a year (Crosby and Bohn (2003)). The expected loss can be decomposed into the probability of default (PD) and the loss given default (LGD),

$$P_t = \underbrace{\sum_{k=0}^{\infty} \frac{e^{-\lambda T}(\lambda T)^k}{k!} N(-d_2) \left[1 - \frac{N(-d_1)}{N(-d_2)} \frac{V_t e^{\mu T + k(\mu_q + \frac{\sigma_q^2}{2})}}{De^{-rT}}\right]}_{PD} \underbrace{De^{-rT}}_{LGD}, \tag{35}$$

and under the Merton (1974) model

$$P_{tMerton} = \underbrace{N(-d_2) \left[1 - \frac{N(-d_1)}{N(-d_2)} \frac{V_t}{De^{-rT}}\right]}_{PD} \underbrace{De^{-rT}}_{LGD}. \tag{36}$$

Equation (35) shows that the LGD is endogenously determined in this model. Banks default when they are not able to pay their debt in full. In turn, creditors that fail to receive some payments from their counterparties might not be able to pay their own creditors, potentially triggering a cascade of defaults. Creditors do not necessarily lose the full face value of their claims towards defaulted banks if the recovery rate is positive.

4 Data and Results

The model is calibrated using market information and balance sheet data of four major UK banks. Data about the amount of outstanding debt and the corresponding maturity structure are sourced from published semi-annual accounts (2005 H1 to 2017 H2), consistent with the statutory reporting cycle. We obtain daily observations on equity prices and the secondary market yields on one-year UK Treasury bonds as the risk-free interest rate. The combined panel dataset comprises observations for each bank j over a six-month period. Based on the time series of observed equity prices and the risk-free rate, the jump diffusion process given by equation (10) is used to back out a corresponding series of implied asset values over a 13-year study period from 3 January 2005 to 30 December 2017 (with a total number of 3,390 observations) for each bank over six-month rolling windows. The expected loss of a bank is the equivalent of the put option premium specified in equation (35). The expected losses of all banks peaked in 2008 and remained elevated until the end of 2009. There was another spike following the European debt crisis during the second half of 2011. Figure 4 plots the expected losses (in billions of Pound Sterling) of the four sample banks from 2005 to 2017 estimated based on the specification in equation (35) above.

Figure 4: UK Banking Sector (Largest Listed Banks): Individual Expected Losses (Put Option Values with Jump Diffusion), 2005-17 (GBP Billions)

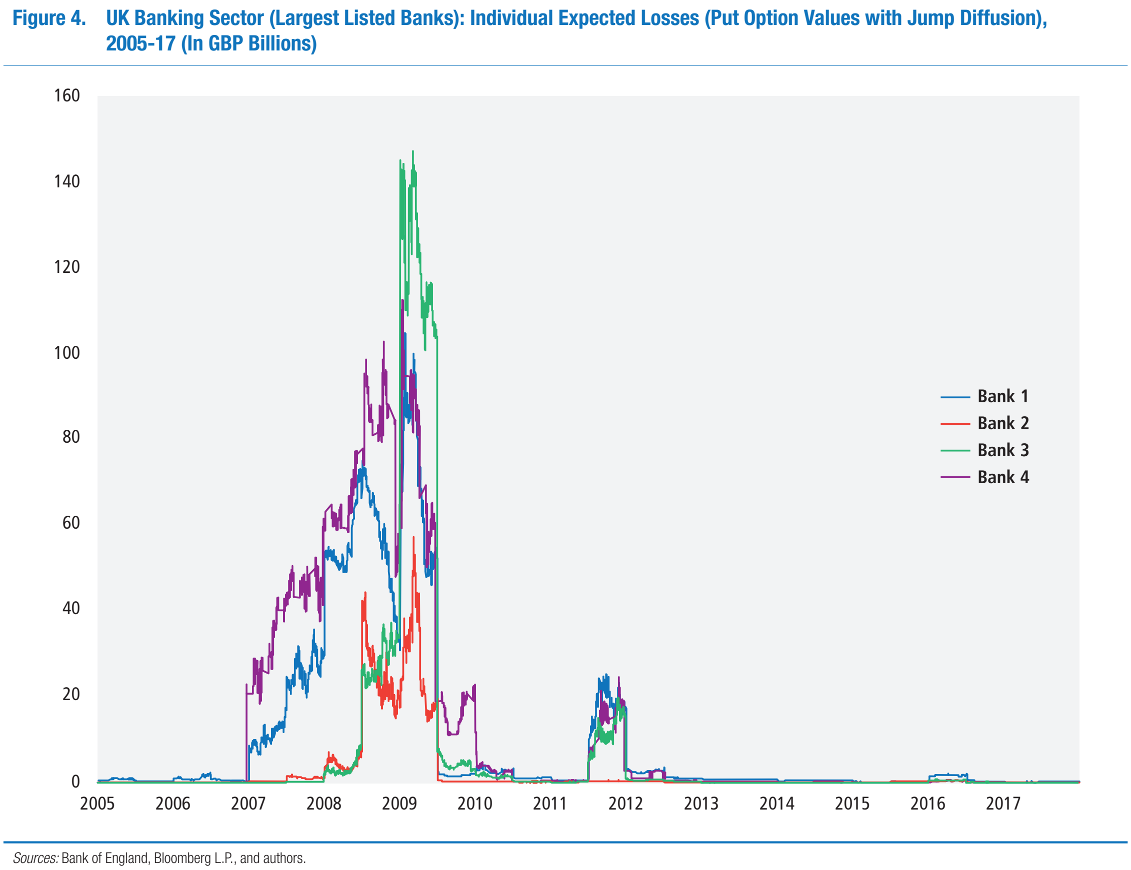
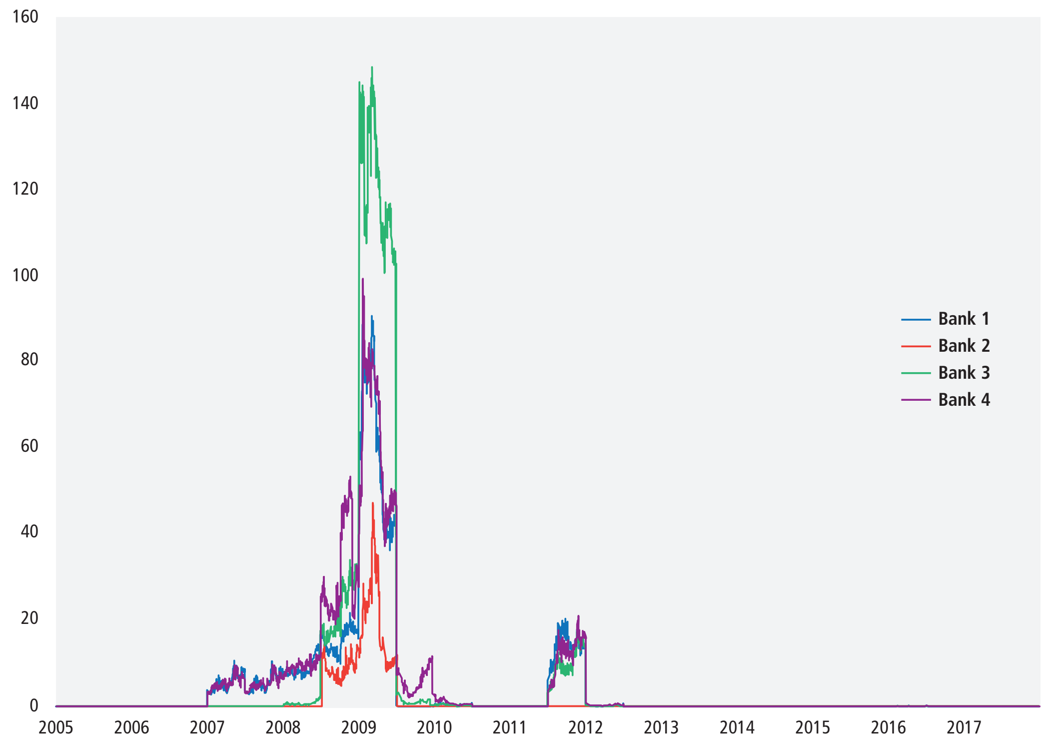


Figure 5: UK Banking Sector (Largest Listed Banks): Individual Expected Losses (Put Option Values without Jump Diffusion) 2005-17 (In GBP Billions)

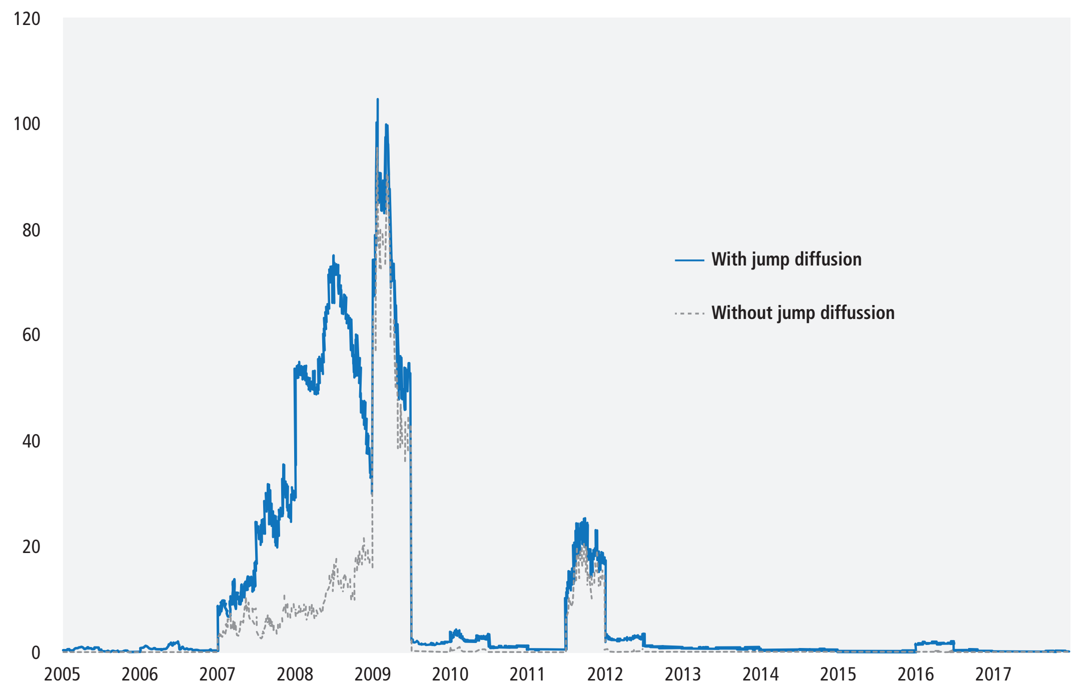
Figure 5. UK Banking Sector (Largest Listed Banks): Individual Expected Losses (Put Option Values without Jump Diffusion), 2005-17 (In GBP Billions)



Sources: Bank of England, Bloomberg L.P., and authors.

Figure 6: UK Banking Sector (Largest Listed Banks): Comparison of Expected Losses for Bank1 (with and without a Jump Diffusion of asset values) 2005-2017 (In GBP Billions)

Figure 6. UK Banking Sector (Bank 1): Individual Expected Losses (Put Option Values with and without Jump Diffusion), 2005-17 (In GBP Billions)



Sources: Bank of England, Bloomberg L.P., and authors.

Figure 5 shows the estimation results for expected losses consistent with the Merton (1974) model as specified in equation (36). While this simpler option pricing approach still captures the surge of expected losses of banks during both the global financial crisis and the European soevereign debt crisis, there is a distinct difference in the order of magnitude and timing. More importantly, sporadic stresses are not picked up under this conventional estimation of expected losses, suggesting very low credit spreads on outstanding debt outside distress periods. Figure 6 makes the comparison for the expected losses of a single bank (Bank 1), which confirms the different estimation results obtained with and without a jump diffusion of asset values underpinning the option pricing approach.

5 Estimating Joint Expected Losses

In this section, the individually estimated expected losses obtained from the previous analysis are combined to determine the magnitude of default risk on a system-wide level using their dependence structure. As part of the valuation of the different risks affecting a bank's operating performance, expected losses can be modeled by explicitly taking into account extreme events. Based on the empirical evidence presented in the previous section, the distribution of expected losses is assumed to be "fat-tailed" in keeping with *extreme value theory* (EVT). For instance, closed-form methods are frequently used to help define the limiting behaviour of extreme observations. [16]Estimating joint expected losses of multiple institutions, however, entails a non-trivial aggregation problem. Since the simple summation of individual expected losses would presuppose perfect correlation, i.e a coincidence of defaults, the correct estimation of aggregate risk requires knowledge about the dependence structure of individual balance sheets and associated expected losses. While it is necessary to move beyond "singular CCA" by accounting for the dependence structure of individual balance sheets, the estimation of systemic risk through correlation becomes exceedingly unreliable in the presence of "fat tails".

Correlation describes the complete dependence structure between two variables correctly only if the joint distribution is elliptical - an ideal assumption rarely encountered in practice. This is especially true in times of stress, when default risk is highly skewed, and higher volatility inflates conventional correlation measures automatically (as covariance increases disproportionately to the standard deviation). Alternative measures of dependence between risk factors can capture nonlinear dynamics of changes in variables far removed from the median.[17] Copula functions and other nonparametric methods provide the possibility to combine two or more distributions of variables based on a more flexible specification of their dependence structure at different levels of statistical significance. These approaches generate measures of so-called "joint asymptotic tail dependence," which define the expectation of common extreme outcomes.[18]

As part of a four-step process, we define the univariate marginal density of all firms, which are then combined with their dependence function in order to generate an aggregate measure of default risk as

[16]The generalised extreme value (GEV) distribution specifies the asymptotic tail behaviour of the order statistics of normalised maxima (or minima) drawn from a sample of dependent random variables, whereas the generalised Pareto distribution (GPD) is an exceedence function, which measures the residual risk of extremes beyond a given threshold (that is, a designated maximum (or minimum) as the conditional distribution of mean excess).

[17]For example, an expedient nonparametric method of investigating the bivariate empirical relation between two random vectors is to ascertain the incidence of shared cases of cross-classified extremes via a quantile-based statistic of independence.

[18]See also Gray and Jobst (2010) and Gray and Jobst (2011).

joint expected losses based on the the multivariate set-up underpinning the Systemic CCA framework.[19] The dependence function combines several marginal distribution functions in accordance with Sklar's theorem (Sklar (1959)) on constructing joint distributions with arbitrary marginal distributions via copula functions (which completely describes the dependence structure and contains all the information to link the marginal distributions).[20] We can then use tail-risk estimates such as *conditional tail expectation* (CTE) (which is sometimes referred to as *conditional Value-at-Risk (VaR)* or *expected shortfall*) in order to gauge systemic solvency risk in terms of stress at a statistical confidence level of choice.

5.1 Estimating the marginal distributions of individual expected losses

We first specify the statistical distribution of individual expected losses (based on the time series of put option values obtained for each bank) in accordance with EVT, which is concerned with modeling the tails of probability distributions where extreme risks are situated. Let the vector-valued series

$$X_j^n = P_1^n(t), ..., P_m^n(t) \tag{37}$$

denote a sequence of independent and identically distributed (i.i.d) random observations of expected losses (i.e., a total of n-number of daily put option values $P_m^n(t)$ up to time t), each estimated according to equation(29) above over a rolling window of τ =120 observations (covering half a year of working days) with period updating for $j \epsilon m$ banks in the sample over a 13-year study period from 3 January 2005 to 30 December 2017 (with a total number of 3,390 observations per bank). We define the cumulative distribution F of expected losses of m-banks as an ordered n-sequence of sample maxima based on

$$X = \max(P_1^n(t)..., P_m^n(t)). \tag{38}$$

Since the actual distribution is unknown, we focus on the asymptotic distribution of X, which is modelled in accordance with Fisher and Tippett (1928), which defines the attribution of a given distribution of normalised maxima (or minima) to be of extremal type (assuming that the underlying function is continuous on a closed interval). In order to obtain a non-degenerate limit for the maxima, we standardise the random variable X, by means of an affine transformation with a location parameter α_j^n and a positive scale parameter $\beta_j^n > 0$, thereby focussing on the asymptotic distribution of $(X - \alpha^n)/\beta^n$.

The probability of each ordered n-sequence of normalised sample maxima $(X - \alpha^n)/\beta^n$ converges to the non-degenerate limit distribution $H(x)$ as $n \to \infty$ and $x \in \mathbb{R}$, so that

$$F^{[\beta^n + \alpha^n]}(x) = \lim_{n\to\infty} \Pr((X - \alpha^n)/\beta^n \leq y) = [F(\beta^n y + \alpha^n)]^n \to H(x) \tag{39}$$

[19] Jobst and Gray (2013) explain how the Systemic CCA framework can also be applied in stress testing. Any capital shortfall would arise when the level of regulatory capital after accounting for joint expected losses causing capital levels to fall below the hurdle rate of minimum capital requirement.

[20] Note that the analysis of dependence in this approach is completely different from the analysis of marginal distributions, and, thus differs from the classical approach, where multivariate analysis is performed jointly for marginal distributions and their dependence structure by considering the complete variance-covariance matrix, such as the MGARCH approach.

falls within the maximum domain of attraction (MDA) of the generalised extreme value (GEV) distribution, which defines the limiting distribution of maxima of dependent random variables (Coles et al. (1999); Poon et al. (2003); Jobst (2007)). Assuming the existence of a whole sequence (α_j^n, β_j^n) of such parameters indexed by n, Gnedenko (1943) proved that the distribution of the sequence of standardised maxima converges to one of the following three non-degenerate types of distributions:

$$H_0(x) = \exp\{-\exp(-x)\}, x \in \mathbb{R} \tag{40}$$

$$H_{1,\xi}(x) = \exp(-x^{-1/\xi}) \text{ if } x \in (\mu - \sigma/\xi, \infty), \xi > 0, \text{and} \tag{41}$$

$$H_{2,\xi}(x) = \exp(-(-x)^{-1/\xi}) \text{ if } x \in (-\infty, \mu - \sigma/\xi], \xi < 0. \tag{42}$$

The parameter μ corresponds to the scale, and σ is a location parameter; the third parameter ξ is called the shape parameter or tail index and accounts for the behaviour of the tail of the distribution. The larger the tail index, the thicker the tail, i.e., the larger the weight of the tail and the slower the speed at which the tail approaches this limit. The shape parameter also indicates the number of moments of the distribution, e.g., if $\xi = 1/2$, the first moment (mean) and the second moment (variance) exist, but higher moments have an infinite value.[21] The upper tails of most (conventional) limit distributions, i.e., the ordered maxima, always (weakly converge) to this parametric specification of asymptotic behaviour, irrespective of the original distribution of observed maxima (unlike parametric VaR models).

A standard form of the cumulative GEV distribution function combines the three sub-models to

$$H_{\mu,\sigma,\xi}(x) = \left\{ \begin{array}{ll} \exp\left(-\left(1+\xi\frac{(x-\mu)}{\sigma}\right)^{-1/\xi}\right) & if \;\; 1+\frac{\xi(x-\mu)}{\sigma} \geq 0 \\ \exp\left(-\exp\left(-\frac{(x-\mu)}{\sigma}\right)\right) & if \;\; x \in \mathbb{R}, \xi = 0 \end{array} \right\}, \tag{43}$$

which, after differencing $H_{\mu,\sigma,\xi}(x)' = \frac{d}{dx} H_{\mu,\sigma,\xi}(x)$, yields the probability density function

$$h_{\mu,\sigma,\xi}(x) = \frac{1}{\sigma}(1+\xi\frac{(x-\mu)}{\sigma})^{(-1/\xi)-1} exp(-(1+\xi\frac{(x-\mu)}{\sigma})^{-1/\xi}). \tag{44}$$

Thus, the estimated j^{th} univariate marginal density function of each expected loss series (bank-by-bank) converging to GEV in the limit is defined as

$$\hat{y_j}(x) = \left(1 + \widehat{\xi}_j \frac{(x-\mu_j)}{\widehat{\sigma_j}}\right)_+^{-1/\xi} > 0 \text{ (for } j = 1, ..., m), \tag{45}$$

where $1 + \hat{\xi}_j(x-\mu_j)/\sigma_j > 0$, scale parameter $\sigma_j > 0$, location parameter μ_j, and shape parameter $\hat{\xi}_j \neq 0$. These moments are estimated concurrently by means of ML, which identifies possible limiting

[21] Any moments of order greater than $1/\xi$ are unbounded, i.e., $1/\xi$ indicates the highest bounded moment for the distribution. This is of practical importance since many results for asset pricing in finance rely on the existence of several moments. When the index is equal to zero, the distribution H corresponds to a Gumbel type. When the index is positive, it corresponds to a Fréchet distribution; when the index is negative, it corresponds to Weibull distribution. The Fréchet variant of modeling the asymptotic tail behavior has been found to be most appropriate for financial sector data exhibiting extremes.

laws of asymptotic tail behaviour, i.e., the likelihood of even larger extremes converges to zero as the level of statistical confidence approaches certainty. The ML estimator is evaluated numerically to solve for GEV parameters by using an iterative procedure (e.g., over a rolling window of τ observations with periodic updating). The ML estimator is evaluated numerically to solve for the GEV parameters by using an iterative procedure (e.g., over a rolling window of τ observations with periodic updating) to maximise the likelihood $\Pi_{i=1}^{n} h_{\mu,\sigma,\xi}(x \mid \theta)$ over all three parameters $\theta = (\mu, \sigma, \xi)$ simultaneously, using the linear combination of ratios of spacings estimator (Jobst and Gray (2013); Jobst (2013)).

5.2 Estimating the dependence structure of individual expected losses

We follow Jobst (2014) in defining a non-parametric, multivariate dependence function between the marginal distribution of expected losses by expanding the bivariate logistic method proposed by Pickands (1981) and to the multivariate case and adjusting the margins according to Hall and Tajvidi (2000) so that

$$Y(\omega) = \min\left\{1, \max\left\{n\left(\sum_{i=1}^{n} \Lambda_{j=1}^{m} \frac{y_{i,j}/\hat{y}_{\bullet j}}{\omega_j}\right), \omega, 1-\omega\right\}\right\}, \tag{46}$$

where $\hat{y}_{\bullet j} = \sum_{j=1}^{n} y_{i,j}/n$ reflects the average marginal density of all $i \in n$ put option values and $0 \leq \max(\omega_1, ..., \omega_{m-1}) \leq Y(\omega_j) \leq 1$ for all $0 \leq \omega_j \leq 1$. $Y(.)$ represents a convex function on $[0,1]$ with $Y(0) = Y(0) = 1$, i.e., the upper and lower limits of $Y(.)$ are obtained under complete dependence and mutual independence, respectively. It is estimated iteratively (over half-year rolling windows) subject to the optimisation of the $(m-1)$-dimensional unit simplex

$$S_m = \left\{(\omega_{1,...,}\omega_{m-1}) \in_{+}^{m}: \omega_j \geq 0, 1 \leq j \leq m-1; \sum_{j=1}^{m-1} \omega_j \leq 1 \text{ and } \omega_m = 1 - \sum_{j=1}^{m-1} \omega_j\right\}, \tag{47}$$

which establishes the degree of coincidence of multiple series of cross-classified random variables, similar to a χ-statistic that measures the statistical likelihood that observed values differ from their expected distribution (Jobst and Kamil (2008)). This specification stands in contrast to a general copula function that links the marginal distributions using only a single (and time-invariant) dependence parameter.

5.3 Estimating the joint distribution of expected losses

We then combine the marginal distributions of these individual expected losses with their dependence structure to generate a multivariate generalised extreme value distribution (MGEV) over the same estimation period as above. Analogous to equations (43)-(45), the resultant cumulative distribution function is specified as

$$G_{t,m}(x) = \exp\left\{-\left(\sum_{j=1} y_{t,j}\right) Y_t(\omega)\right\} \tag{48}$$

with corresponding probability density function

$$g_{t,m}(x) = \widehat{\sigma}_{t,m}^{-1} \left(\left(\sum_{j=1}^{m} y_{t,j} \right) Y_t\left(\omega\right) \right)^{\xi_m+1} \exp\left\{ - \left(\sum_{j=1}^{n} y_{t,j} \right) Y_t(\omega) \right\} \tag{49}$$

at time $t = \tau + 1$ by maximising the likelihood function $\prod_{j=1}^{n} g_{t,m}\left(x \mid \theta\right)$ over all three parameters $\theta = (\mu, \sigma, \xi)$ simultaneously. Equation (49) above represents the functional form of the joint expected losses of all firms based on the empirical observations in equation (38). Since the logarithm is a continously increasing function over the range of the likelihood, the parameter values that maximise the likelihood will also maximise the logarithm as global maximum. Thus, we can write the lognormal likelihood function as $\sum_{j=1}^{m} \ln g_j\left(x \mid \theta\right)$ so that the ML estimate of the true values θ_0 is

$$\widehat{\theta}_{MLE} = \underset{\theta \in \Theta}{\arg\max}\, \widehat{l}\left(\theta \mid x\right) \to \theta_0. \tag{50}$$

5.4 Estimating a tail risk measure of joint expected losses

Finally, we obtain the joint expected shortfall (ES) (or conditional VaR) as a CTE measure. ES defines the probability-weighted residual density (i.e., average aggregated expected losses) beyond a pre-specified statistical confidence level ("severity threshold") over a given estimation period. It is a coherent risk measure, i.e., it satisfies several axioms of convexity, positive homogeneity, and sub-additivity (Artzner et al. (1999)); this is an improvement over VaR, which, in addition to being a pure frequency measure, is incoherent (e.g. it violates the axiom of sub-additivity). Given the dependence structure defined in equation (46), and in accordance with the general definition of ES, as

$$ES_n = E\left[X \mid X > VaR_a\right],$$

which can be written as

$$\frac{1}{1 - F\left(VaR_a\right)} \int_{VaR_a}^{\infty} \left(1 - F\left(x\right)\right) dx = \frac{1}{1-a} \int_{a}^{1} VaR_a da \tag{51}$$

for a continuous random variable x, where $VaR_a = \inf\left\{x : F\left(x\right) \succeq a\right\}$ is the quantile of order $0 < a < 1$ (say, $a = 0.95$) pertaining to the distribution function F, ES in a multivariate context is defined as

$$ES_{t,\tau,m} = E\left[z_t \mid z_t \geq G_{t,\tau,m}^{-1}\left(a\right) = VaR_{t,a}\right], \tag{52}$$

where

$$\frac{1}{a} \int_{0}^{\infty} G_{t,\tau,m}^{\leftarrow}\left(z\right) dz = \frac{1}{1-a} \int_{0}^{a} G_{t,\tau,m}^{-1}\left(a\right) da,$$

$G^{\leftarrow}(a) = G^{-1}(a)$ is the quantile corresponding to probability $1-a$, with $G^{\leftarrow}(a) \equiv \inf\left(z \mid G\left(z\right) \geq a\right)$), with random variable $z \epsilon \mathbb{R}$, so that

$$VaR_{t,a} = \sup\left\{G_{t,\tau,m}^{-1}\left(a\right) \mid \Pr\left[z_t > G_{t,\tau,m}^{-1}\left(a\right)\right] \geq a\right\}, \tag{53}$$

with the point estimate of joint potential losses of m firms defined as[22]

$$G_{t,\tau,m}^{-1}(a) = \widehat{\mu}_{t,m} + \widehat{\sigma}_{t,m}/\widehat{\xi}_{t,m}\left(\left(-\frac{\ln(a)}{Y_j(\omega)}\right)^{-\widehat{\xi}_{t,m}} - 1\right). \tag{54}$$

We also apply alternative systemic risk methodologies, such as CoVaR and SES, to the expected losses generated by equation (37) to cross-validate our main results with those obtained under alternative aggregation techniques. In their original formulation, however, both approaches specify default risk as the probability of market capitalisation to drop below a pre-specified threshold based on the historical dynamics of equity price returns rather than a measure of market-implied default risk with an observable distress barrier (such as the single-firm credit risk specification of CCA). So we need to re-formulate the definition of CoVaR and SES using expected losses as a measure of market-implied default risk. In the case of the former, we also disregard controlling for the median CoVaR (which is used to derive the marginal contribution of each bank in the form of $\triangle CoVaR$), and in the case of the latter, we derive the marginal expected shortfall (MES) of each bank (without controlling for leverage and market capitalisation) to derive a system-wide measure.

For CoVaR (conditional on individual stress of any bank), we can write

$$CoVaR_{t,\tau,a} = \sup\left\{P_{t,\tau}^{-1}(a) \mid (\Pr\left[x_t > P_{t,\alpha}^{-1}(a)\right] \geq a)(1 + \prod_{j}^{m}\Pr\left[y_{t,j} > Q_{t,\tau,j}^{-1}(a)\right] \geq a)\right\}, \tag{55}$$

where $P(x) = \mathrm{X}_m^n$ and $Q(y) = \mathrm{X}_j^n$ define the distribution functions of the vector-valued series of expected losses of all sample banks (as per equation (24)) and those of each individual bank over the study period, respectively.

For SES (which is based on CTE), we have

$$SES_{t,a,m} = \sum_{j}^{m} w_{t,\tau,j}\sup\left\{Q_{t,\tau,j}^{-1}(a) \mid E\left[x_t \mid x_t \geq P_{t,\tau}^{-1}(a) = VaR_{t,a,m}\right]\right\}, \tag{56}$$

where

$$VaR_{t,a,m} = \sup\{P_{t,\tau}^{-1}(a) \mid \Pr[x_t > P_{t,\tau}^{-1}(a)] \geq a\}.$$

The marginal expected shortfall (MES) corresponds to the partial derivative of SES with respect to the weight w of bank j within the total sample, so that $MES_{t,\alpha,j} = \frac{\partial SES_{t,a,m}}{\partial w_{t,\tau,j}}$.

Since we consider all sample banks to comprise the domain of systemic risk, we simplify equation (56) above to

$$MES_{t,a,j} = \sup\left\{y_{t,j} \mid E\left[x_t \mid x_t \geq P_{t,\tau}^{-1}(a) = VaR_{t,a,m}\right]\right\}. \tag{57}$$

We keep the statistical confidence $a = 0.95$ consistent with the one used for the Systemic CCA model and maintain the same estimation period of rolling windows with daily updating. Given that

[22] This estimation has been fully implemented in the R and C# programming languages (with all relevant codes deposited as Github under https://github.com/yuxuzi/macroprudential), and is available as web-based tool at https://macroprudential.shinyapps.io/SystemCCA/. See Jobst and Liu (2017) for further information.

the two alternative methods apply different measurement concepts of tail risk (i.e., VaR vs. CTE), we also formulate CoVaR in CTE terms as "Conditional Expected Shortfall (CoES)" so that

$$CoES_{t,a,m} = E\left[x_t \mid x_t \geq P_{t,\tau,j}^{-1}(a) = VaR_{t,a,j}\right] \cap E\left[y_{t,j} \mid y_{t,j} \geq Q_{t,\tau,j}^{-1}(a) = VaR_{t,a,j}\right] \qquad (58)$$

and MES in VaR terms as "MarginalVaR (MVaR)" so that

$$\sum_{j}^{m} MVaR_{t,a,j} = \sum_{j}^{m} \sup\left\{y_{t,j} \mid P_r\left[x_t > P_{t,\tau}^{-1}(a)\right] \geq a\right\}. \qquad (59)$$

5.5 Empirical Application and Findings

Based on the specification of default risk on a system-wide level in the previous section, we estimate the multivariate probability distribution of joint expected losses for all sample banks. We combine the univariate density function of estimated individual expected losses and their non-parametric dependence structure (over a 120-day sliding window and daily updates) under the assumption of GEV-consistent asymptotic tail behaviour according to equation (48). With a fully-specified multivariate probability distribution, we can then extract point estimates of expected losses at varying levels of statistical confidence at different points in time by inverting the cumulative density function of joint expected losses (see equation 52). In order to generate a coherent risk measure of systemic risk for tail events, we focus on expected shortfall (ES) according to equation (53). Table 1 provides an overview of all estimation results at different levels of statistical confidence over the total study period (since 2005) and during episodes of significant changes in macroeconomic or market conditions. Figure 7 shows in more detail the daily time series of the sum of expected losses of each bank and the median value of the joint expected losses.

We find that the systemic risk of the largest U.K. banks has greatly diminished in the wake of the global financial crisis. During the crisis, the sharp decline in banks' asset values raised the 95% CTE of joint expected losses to levels that would have completely exhausted the total available amount of system-wide equity (GBP187.5 billion). However, since 2012, the 95% CTE dropped sharply to only GBP 38.8 billion (or slightly more than one-third of the pre-crisis value (GBP 111.5 billion), which implies a far lower probability of banks experiencing extreme losses at the same time (see Table 1).

Accounting for the occurrence of large and interrelated changes of expected losses provides an early indication of systemic risk outside known episodes of system-wide stress (such as the global financial crisis and the European debt crisis). We find long episodes of elevated joint expected losses, which are not captured by an identification strategy of systemic risk that follows a simple aggregation of individual expected losses (without considering their distributional properties and dependence accross sample banks). The pre-crisis 95% CTE of joint expected losses exceeded the sum of individual expected losses by almost a factor of three (see Table1). During times of stress (such as the global financial crisis), joint expected losses become very sensitive to extreme shocks at higher levels of statistical confidence, indicating high elasticity of systemic risk (with a certain degree of persistence). These (in-sample) results suggest some early warning capacity of our approach in identifying the stochastic relevance and persistence of changes in the way expected losses evolve over time, both individually and jointly. This is also consistent with Kerry (2019), who finds that equity market-based capital ratios

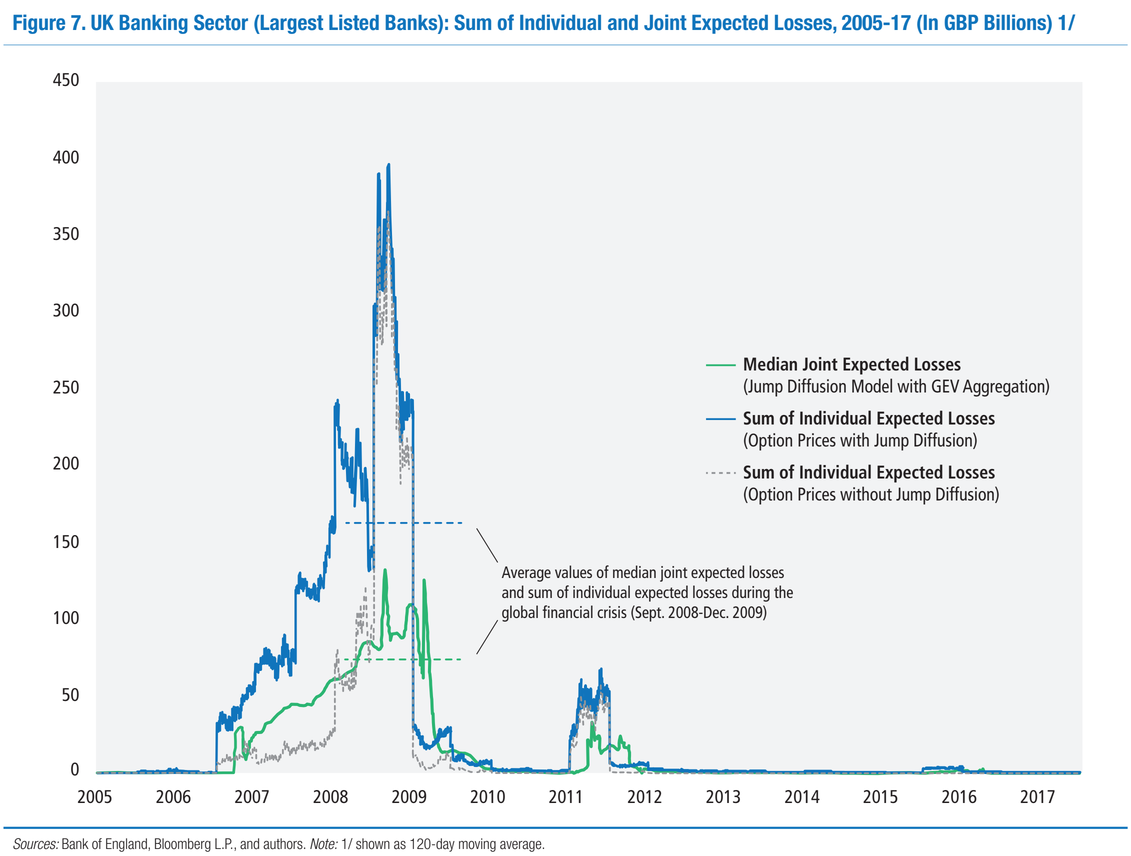
Table 1: UK Banking Sector (Largest Listed Banks): Sum of individual and joint Expected Losses at Different Levels of Statistical Confidence, 2005-2017 (In GBP Billions)

Table 1. UK Banking Sector (Largest Listed Banks): Sum of Individual and Joint Expected Losses at Different Levels of Statistical Confidence, 2005-2017 (In GBP Billions)

	Pre-Financial Crisis: June 2005– Aug. 2008	Global Financial Crisis: Aug. 2008– March 2010	Sovereign Debt Crisis: April 2010– Dec. 2011	Post-Crisis Jan. 2012– Dec. 2017	Entire Time Period
	Joint Expected Losses				
	Conditional Tail Expectation (95%)				
25th percentile	1.2	93.4	1.3	0.5	**0.9**
Median	45.1	116.0	7.2	0.9	**3.0**
75th percentile	76.3	151.4	27.9	2.9	**62.9**
Average	*111.5*	*266.2*	*106.2****	*38.8****	***119.1***
	Value-at-Risk (95%)				
25th percentile	1.0	92.1	1.3	0.5	**0.8**
Median	28.1	104.7	4.1	0.9	**2.2**
75th percentile	57.3	146.2	22.8	2.4*	**34.0**
Average	*36.2*	*187.2*	*55.3****	*27.8****	***50.9***
	Median				
25th percentile	0.4	14.4	0.7	0.3	**0.3**
Median	0.7	80.7	1.3	0.5	**0.7**
75th percentile	42.4	90.5	3.4	0.8	**14.2**
Average	*18.9*	*65.0*	*4.5*	*1.7*	***14.3***
	Sum of Individual Expected Losses				
25th percentile	0.5	18.9	1.5	0.6	**0.6**
Median	30.5	141.7	2.3	1.1	**1.5**
75th percentile	76.1	237.2	27.4	2.3*	**25.6**
Average	*49.8*	*139.2*	*14.9*	*1.6*	***33.0***

Sources: Bloomberg L.P. and authors. *Note:* The estimation window of 120 working days and daily updating for the multivariate distribution of joint expected losses means that the output time series of the estimation results starts on 21 June 2005 (based on a sample start date of 1 January 2005). The statistical significance of expected losses (for each time period) is determined via a simple two-sided t-test (based on the distribution of the sum of individual expected losses forming the expectation specific to each time period): ***=1 percent, **=5 percent, *=10 percent statistical significance.

Figure 7: UK Banking Sector (Largest Listed Banks): Sum of Individual and Joint Expected Losses, 2005-17 (In GBP Billions) 1/



would have been better at signalling bank distress in the run-up to the global financial crisis than regulatory capital ratios - particularly the Tier 1 capital ratio.

Our portfolio-based estimation approach seems to also capture considerable diversification effects due to banks' varying sensitivity to price shocks. The dependence structure of expected losses has a significant impact on the estimated magnitude of systemic risk over the study period. We find that the median of joint expected losses is less than half the sum of the individual expected losses (GBP 0.7 billion vs. GBP 1.5 billion), suggesting considerable variability in the way financial stress propagates across sample banks. The difference remains persistent but declines somewhat during stress periods (indicating high co-movement of expected losses) when the dependence of expected losses tends to increase. The parameterisation of the multivariate GEV distribution of joint expected losses according to equation (49) suggests that the average dependence value across all sample banks peaked during the global financial crisis at a value of 0.57 (up from 0.47 during the pre-crisis period) but has remained relatively stable since at 0.56 until the end of the study period in 2017 (see Table 4).

Alternative systemic risk measures deliver similar results if adapted to our panel series of expected losses but leave large gaps outside severe distress periods. The CoVaR and SES measures of joint expected losses at 95% statistical confidence converge to those generated using the Systemic CCA framework, with median values during the global financial crisis at GBP 140.8 billion and GBP 148.6 billion, respectively (see Table 2).[23]However, the bivariate specification of both approaches fails to fully capture very large changes in systemic risk over time, which does not only generate discontinuities in measuring systemic risk over time but also diminishes their early warning capacity.

Our results help address the challenges from the inherent uncertainty and time-inconsistency in measuring systemic risk - the magnitude of risk is imperfectly predictable, and its impact cannot be precisely quantified in advance, which requires flexible approaches that can capture a broad range of outcomes at varying levels of statistical confidence. However, we acknowledge the limitations of our approach (and are reflected as caveats in the interpretation of the findings). The underlying option pricing model as specified in equation (5) (given its specific distributional assumptions, the derivation of both implied assets and asset volatility and assumptions about the default barrier) could fail to capture some relevant economics underpinning default risk and, thus, could generate biased estimates of expected losses (even if we incorporate jump risk). Moreover, financial market behaviour, especially during times of stress, might defy statistical assumptions of valuation models, such as the option pricing theory and extreme value measurement. For instance, during the global financial crisis and the European sovereign debt crisis, financial market behaviour was characterised by rare and non-recurring events, and not by repeated realisations of predictable outcomes, generated by a stochastically stable process of asset price changes. Thus, sudden and unexpected realisations beyond historical precedent defy the statistical apparatus underlying conventional asset pricing theory even if we account for tail events. Also, equity prices might not only reflect fundamental values due to both shareholder dilution and trading behaviour that can obfuscate proper economic interpretation, during periods of stress, such as the global financial crisis.

[23]Note that the average values of joint expected losses under Systemic CCA are sometimes lower than those derived from conditional meaasures, such as CoVaR and SES, outside of stress periods; this can be attributed to the distributional approach of Systemic CCA, which generates continuous point estimates of joint expected losses while actual expected losses might not have breached the stochastic threshold.

Table 2: UK Banking Sector (Largest Listed Banks): Joint Expected Losses - Comparison of Systemic Risk Methodologies, 2005-2017

Table 2. UK Banking Sector (Largest Listed Banks): Joint Expected Losses—Comparison of Systemic Risk Methodologies, 2005-2017 (In GBP Billions)

	Pre-Financial Crisis: June 2005–Aug. 2008	**Global Financial Crisis:** Aug. 2008–March 2010	**Sovereign Debt Crisis:** April 2010–Dec. 2011	**Post-Crisis** Jan. 2012–Dec. 2017	**Entire Time Period**
	Systemic CCA				
	Joint Expected Losses: Conditional Tail Expectation (95%)				
Median	45.1	116.0	7.2	0.9	**3.0**
Average	*111.5*	*266.2*	*106.2*	*38.8*	***99.5***
	Joint Expected Losses: Value-at-Risk (95%)				
Median	28.1	104.7	4.1	0.9	**2.2**
Average	*36.2*	*187.2*	*55.3*	*27.8*	***54.3***
	CoVaR Methodology				
	Conditional Expected Shortfall (CoES) at 95% [1,2,3]				
Median	118.5	143.6	38.1	0.2	**27.3**
Average	*131.0*	*134.2*	*63.8*	*10.8*	*59.1*
	Conditional Value-at-Risk (CoVaR) at 95%				
Median	115.6	140.8	37.5	0.2	**26.1**
Average	*121.8*	*121.1*	*60.1*	*9.9*	***54.8***
	SES Methodology				
	Systemic Expected Shortfall (SES) at 95% [1]				
Median	119.4	148.6	39.4	0.3	**30.4**
Average	*139.0*	*142.4*	*67.1*	*11.5*	***62.7***
	Systemic Value-at-Risk (SVaR) at 95%				
Median	118.3	163.0	38.9	0.2	**30.9**
Average	*134.7*	*133.7*	*67.1*	*11.3*	***61.1***

Sources: Bloomberg L.P. and authors. *Note:* 1/ Systemic Expected Shortfall: SES aggregates (as weighted sum) the individual losses of all institutions ("marginal expected shortfall" or MES) conditional on a systemic stress event (i.e., the banking sector experiencing losses above a level consistent with its conditional tail expectation (CTE) of 95 percent, estimated over a rolling window of 120 working days) – for a consistent comparison of SES based on CCA-generated expected losses with results obtained under the CoVaR methodology, a systemic stress level of VaR at 95 percent was chosen as an alternative threshold (which turns SES into what could be termed "Systemic VaR (SVaR)"); 2/ Conditional Value-at-Risk: CoVaR measures the system-wide losses (as VaR with a statistical confidence of 95 percent) conditional on an individual stress event (i.e., a bank experiencing losses above a level consistent with a VaR of 95 percent, estimated over a rolling window of 120 working days) – for a consistent comparison of CoVaR based on CCA-generated expected losses with results obtained under the SES methodology, also CTE at 95 percent was chosen as individual stress level (which converts CoVaR into "Conditional Expected Shortfall (CoES)"); 3/ Given the concentration of the banking sector, the changes in the total expected losses of all four sample banks indicate the systemic stress level (consistent with the definition of "the market" in the CoVaR and SES methodologies); 4/ For both SES and CoVaR methodologies, the CTE at the statistical confidence level α=95 percent was approximated via VaR at 98.0 percent (based on the simplifying assumption of normally distributed expected losses, where the z-scores $\varphi(\Phi^{-1}(1-\alpha))/(1-\alpha)$ (for CTE) and $\Phi^{-1}(\alpha)$ (for VaR) are the same, where φ and Φ^{-1} are the standard normal probability distribution and quantile function).

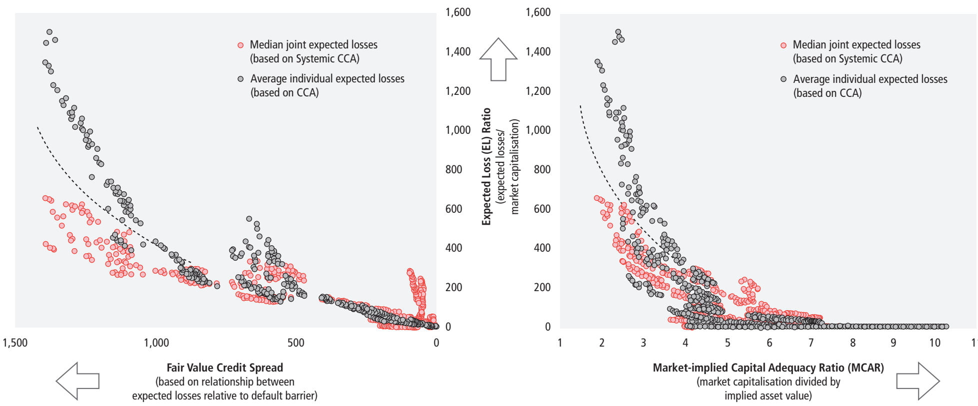
6 Integrated Market-Implied Capital Assessment

The historical estimates of individual and joint expected losses also motivate an integrated market-based capital assessment using the risk-adjusted balance sheet of each bank. We can replicate two conventional measures of solvency risk - the regulatory capital adequacy and default probability - using key variables that are fully indentified and estimated within our CCA framework (see Section 3): (i) the *market-implied capital adequacy ratio* (MCAR) between the market capitalisation and the implied asset value as well as (ii) the *fair value credit spread* (based on the ratio between expected losses and the default barrier). All variables of these two measures can be extracted from equation (29), which specifies expected losses as a put option on the bank's implied asset value subject to a pre-determined debt repayment schedule that defines the default barrier. The fair value credit spread can be obtained by rearranging the equation for a given level of expected losses. Figure 8 shows the non-linear relation between the average MCAR and the average credit spread consistent with average (individual) expected losses (that is, without considering the dependence structure between expected losses). According to equation (52) above, a lower market capitalisation implies a lower asset value to outstanding liabilities, which reduces the MCAR due to higher default risk (and increases asset volatility). Similarly, higher expected losses increases the fair value credit spread. The non-linear interaction between these variables can also be shown based on the joint expected losses and the corresponding MCAR and credit spread (consistent with their average elasticity across all sample banks). The empirical results suggest a generally lower sensitivity of both MCAR and credit spreads to changes in default risk if the joint distribution of expected issues is considered (see Figure 8).[24]

[24] Aikman et al. (2019) indirectly confirm the complementarity of accounting and market-based measures of solvency risk. They find that even in hindsight it would have been hard to diagnose fully the risks in the run-up to the crisis. This suggests that macroprudential policy frameworks should be calibrated with some built-in "slack" to account for the inherent difficulty of risk assessment, particularly in real time.

Figure 8: UK Banking Sector (Largest Listed Banks): Integrated Capital Assessment Using Contingent Claims Analysis (CCA) - Market-Implied Capital Adequacy Ratio (MCAR), Expected Loss Ratio, and Fair Value Credit Spread

Figure 8. UK Banking Sector (Largest Listed Banks): Integrated Capital Assessment Using Contingent Claims Analysis (CCA)—Market-Implied Capital Adequacy Ratio (MCAR), Expected Loss Ratio, and Fair Value Credit Spread



Sources: Bank of England, Bloomberg L.P., and authors. *Note:* The two charts show the non-linear relation between three interlinked elements of the CCA framework—the market-implied capital adequacy ratio (MCAR), the expected loss (EL), and the fair value credit spread (derived from the relation between expected losses and the default barrier). The black and red dots illustrate this integrated assessment using the average of individual (bank-by-bank) expected losses (derived from CCA) and the median value of their multivariate distribution (derived from the aggregation approach of Systemic CCA). The EL ratio scales the average expected losses and median joint expected losses to the median market capitalisation across all sample banks. Since the MCAR and the credit spread are bank-specific, we derive their values consistent with the median value of joint expected losses using (i) the elasticity of the average MCAR to the average expected loss (as a mixture of two distributions 0.5(-0.016953x + 7.413553)*0.5(-0.689ln(x) + 7.642)), and (ii) the elasticity of the average credit spread to the average expected loss (-0.107x^2+6.3676x+0.9652), where x=average expected loss in GBP billions across all sample banks over the study period.

The MCAR represents a market-based analogue to the accounting-based metrics of regulatory capital adequacy reported by the sample banks, and indicates whether the default risk implied by prudential measures is consistent with individually and jointly estimated expected losses inferred from market prices using the CCA. We compare the MCAR to the core leverage ratio to examine the difference between the economic and book value of both assets and capital (see Figure 9). The core leverage ratio is a simple indicator of solvency. [25] It relates capital resources of a bank to the size of its balance sheet exposures. But it does not take account of the riskiness of exposures like the risk-weighted capital requirements. We find that there is a persistent gap between both measures over the study period, which varies over time and declines during times of stress. Thus, the leverage ratio acts as an important guardrail against model risks embedded in risk-weighted measures of exposures and may be viewed as a stabiliser against excessive balance sheet expansion funded by debt. It is readily understood by market participants and more comparable across firms than risk-weighted measures. The average core leverage ratio of the four sample banks has roughly doubled since 2009.

Similarly, relating the MCAR to the risk-based capital requirement generates a structural model-based representation of the bank's price-to-book (PB) ratio (see Figure 10).[26] We choose the capital ratio between common equity Tier 1 capital and risk-weighted assets ("Common Equity Tier 1 (CET1) capital ratio") as a measure of capital adequacy, which is conceptually closest to the composition of MCAR. In normal times, we would expect the PB ratio to be greater than one, because it not only reflects the bank's current net asset value but also includes the present value of future profits. However, in the aftermath of the global financial crisis, the average PB ratio of the sample banks dropped well below one as the regulatory capital ratio and the MCAR have begun to diverge. This is consistent with the market taking a more pessimistic view about banks' future profitability (relative to its cost of equity) than was the case prior to the crisis, with the MCAR as a "shadow capital ratio," reflecting the actual cost of equity.[27] Forces driving prices and risk are different in a crisis than they would be out of crisis. [28] Prior to the crisis, the MCAR was considerably higher than the levels seen today despite banks holding much lower capital, which might be explained by markets having significantly under-priced risk previously.

PB ratios improved again in 2012 as equity markets recovered and banks began further strengthening capital buffers. The amount of CET1 capital that the sample banks now hold relative to their risk-weighted assets, via their minimum capital requirements and various buffers, has increased almost tenfold since the global financial crisis (Bank of England (2018)). Figure 10 shows that the sample banks' PB ratios have recovered from their intermittent trough in mid-2016 and seem to have stabilised around 0.9.

[25] This is defined here as shareholders equity divided by (accounting) balance sheet assets.

[26] A bank's PB ratio compares the market value of shareholders' equity in the bank with its accounting or book value. In theory, there are at least two reasons why companies should have a low PB ratio: investors are unsure that a company's net assets are correctly valued, or it has low or highly uncertain future profit prospects. The UK Financial Policy Committee has judged that UK banks' low PB ratios could be explained by market concerns over expected future profitability rather than by concerns about asset quality.

[27] This gap arises from differences between (i) the market valuation of banks vis-ã-vis accounting valuation of their assets and/or (ii) risk-weighted assets vis-a-vis total assets, which are influenced by (a) structural (crisis vs. post-crisis), (b) cyclical (risk aversion), and (c) technical (prudential/regulatory standards) factors.

[28] Danielsson (2002) argues that since market data is endogenous to market behaviour, statistical analysis made in times of stability does not provide much guidance in times of crisis. The distribution of risk is different during crisis than in other periods and this complicates the role of regulation.

Figure 9: UK Banking Sector (Largest Listed Banks): Accounting-Based and Market-Implied Capital Adequacy of All Sample Banks, 2005-17 (In Percent)

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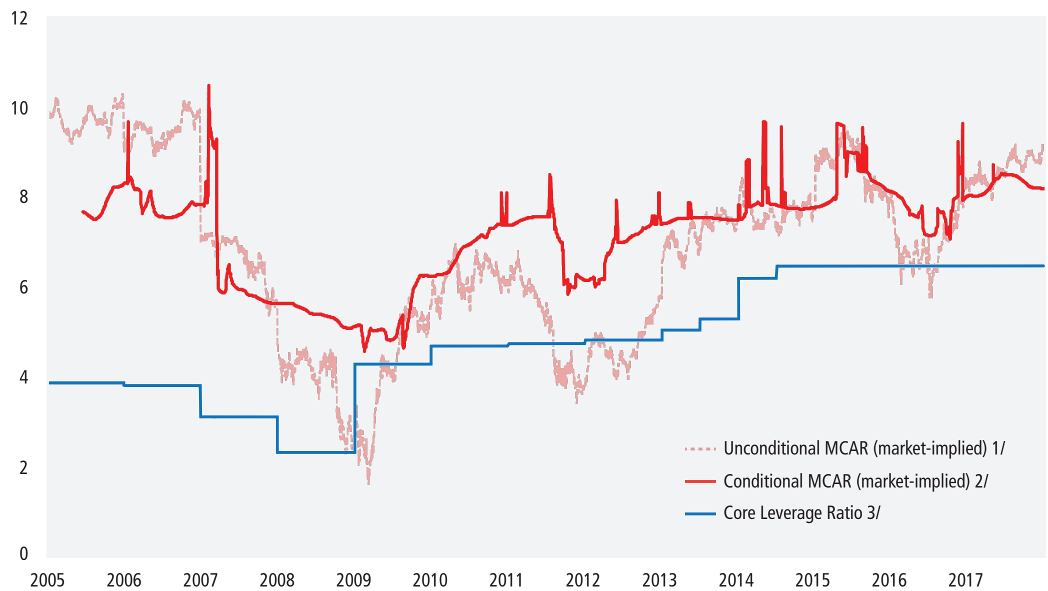
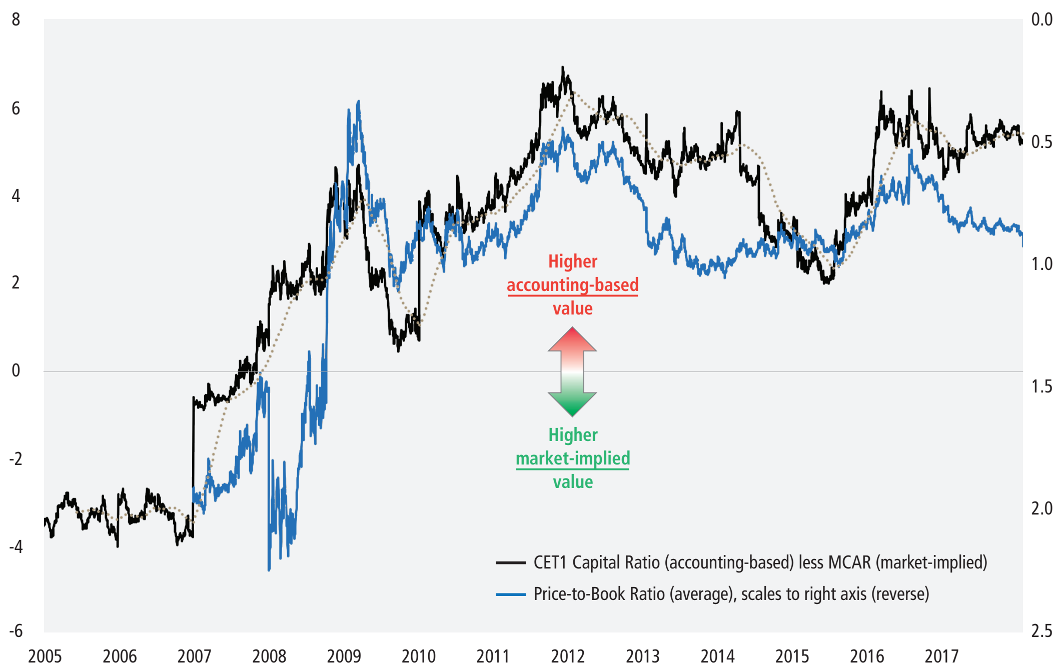


Sources: : Bank of England, Bloomberg L.P., and authors; *Note:* MCAR = market-implied capital adequacy ratio (which is defined as market capitalisation divided by option-based implied asset value); 1/ The *unconditional MCAR* is based on the average of each sample bank's individual asset value implied by expected losses; 2/ The *conditional MCAR* is based on the (theoretical) implied asset value that corresponds to the median joint expected losses consistent with the historical relationship between implied asset values and expected losses of all sample banks over the study period; 3/ The *core leverage ratio* does not control for risky assets (which are deducted in the prudential leverage ratio) and also tends to be more volatile since there is no netting of derivatives assets and liabilities (and other claims/obligations that have expanded significantly during the global financial crisis).

Figure 10: UK Banking Sector (Largest Listed Banks): Difference between Accounting-Based and Market-Implied Capital Adequacy for All Sample Banks, 2005-17 (In Percent)

Figure 10. UK Banking Sector (Largest Listed Banks): Difference between Accounting-Based and Market-Implied Capital Adequacy for All Sample Banks, 2005-17 (In Percent)



Sources: Bank of England, Bloomberg L.P., and authors. *Note:* smoothed line shows 120-working day moving average of the black line; CET1=Common Equity Tier 1; MCAR=market-implied capital adequacy ratio of market capitalisation divided by option-based implied asset value.otted line shows the 120-working day moving average. The market perception of bank solvency implies higher default risk than implied by accounting-based measures if the CET1 capital ratio exceeds MCAR.

7 Conclusion

This paper applied the Systemic CCA framework (Jobst and Gray (2013)) to model the market-implied systemic solvency risk in the U.K banking sector by controlling for common factors affecting the interlinkages between individual risk-adjusted balance sheets of the largest commercial banks. This approach explicitly acknowledges nonlinearities in measuring default risk using a GEV distribution set-up to quantify simultaneous distress. The underlying option pricing formula for measuring expected losses was augmented with a jump diffusion process of banks' asset values to mitigate the empirical shortcomings of traditional single-firm structural default models.

Accounting for the occurrence of large and interrelated changes of expected losses provides an early indication of systemic risk outside known episodes of system-wide stress (such as the global financial crisis and the European debt crisis). We find long time periods of elevated joint expected losses, which are not captured by an identification strategy of systemic risk that follows a simple aggregation of individual expected losses (without considering their distributional properties and dependence across sample banks). During times of stress (such as the global financial crisis), joint expected losses become very sensitive to extreme shocks at higher levels of statistical confidence. The cross-validation of our results with the aggregate outcomes of alternative (bivariate) systemic risk measures, such as CoVaR and SES, over the same study period suggests a greater early warning capacity of our approach.

We also exploited the integrated way of measuring expected losses to generate a market-implied measure of capital adequacy, the MCAR. This measure of "shadow capital adequacy" indicated whether estimated expected losses - individually and jointly - were consistent with the default risk implied by the accounting-based capital adequacy ratio reported by sample banks. Hence, this distribution-based perspective of market-implied solvency risk can inform a system-wide capital adequacy assessment that reflects the variability of both assets and liabilities at different levels of statistical confidence.

Market-based measures of systemic risk can effectively complement prudential reporting in informing a more comprehensive assessment of capital adequacy by considering the impact of changes in market conditions on the perceived risk profile of banks. This has become increasingly relevant due to fundamental changes in the market assessment of banks' solvency risk following the global financial crisis. Prior to the crisis markets underpriced the risk inherent in debt capital which enabled banks to fund themselves at rates that did not reflect the risk associated with such investments. Investors are now more uncertain about the value of banks' net assets and of the underlying asset risks. Low market values may also reflect weak or uncertain profits, or high equity risk premia. The contribution of each factor is not entirely independent, and will vary by bank.

Going forward, further refinements to our model are possible, including various simulation approaches, which might come at the expense of losing analytical tractability. The current specification could also be expanded in scale and scope to include different financial institutions and other types of risks. For instance, given the flexibility of this framework, the financial sector and sovereign risk analysis could be integrated with macro-financial feedback effects to design monetary and fiscal policies. Such an approach could inform the calibration of stress scenarios and sovereign balance sheets and the appropriate use of macroprudential regulation.

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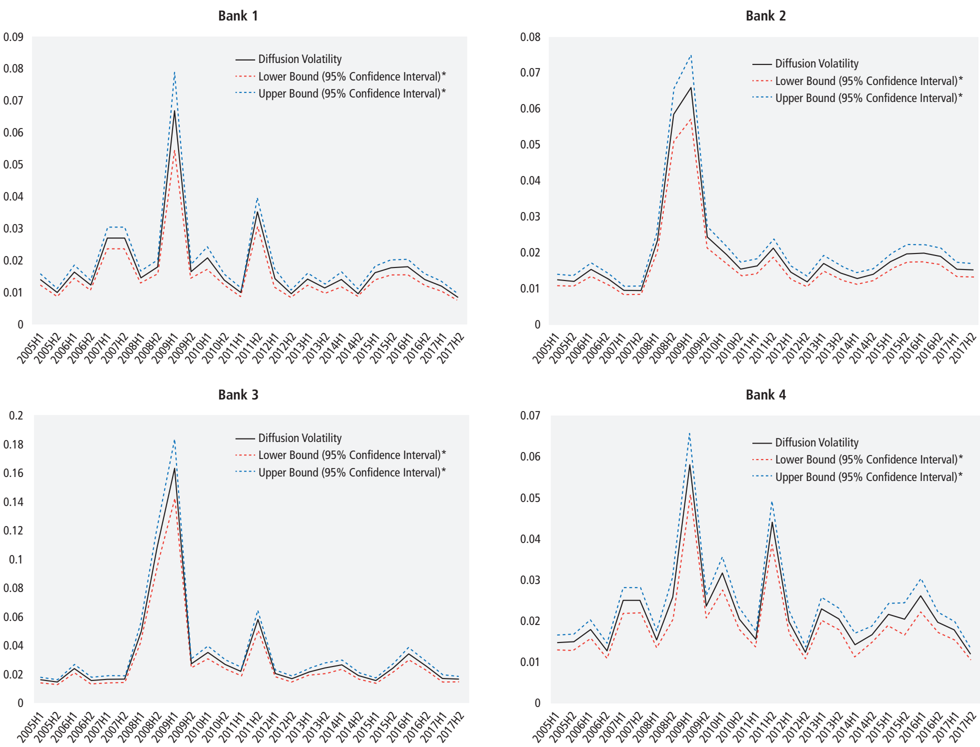
Appendix A - Parameter Variation in the Diffusion Volatility (σ)

Figure 11: Parameter Estimates: Variation of Parameter Estimates: Diffusion Volatility

Figure 11. Variation of Parameter Estimates: Diffusion Volatility



Source: authors' estimates. *Note:* */ The average standard errors over the total study period were 0.0013 (Bank 1), 0.0012 (Bank 2), 0.0021 (Bank 3), and 0.0015 (Bank 4).

Appendix B - Specification of Moments of the GEV Distribution

Since all raw moments of $G_{t,m}(\centerdot)$ are defined contingent on the tail shape, the natural estimator of is $\hat{\xi}$ derived by means of the *linear combination of ratios of spacings* (LRS) method using the linear combination

$\xi = (n/4)^{-1} \sum_{i=1}^{n/4} ((ln(\upsilon_i))/(-ln(c)))$

for n observations, where

$\hat{\upsilon}_i = \frac{x_{n(1-\alpha):n} - x_{n\alpha^c:n}}{x_{n\alpha^c:n} - x_{n\alpha:n}}$ and $c = \sqrt{\frac{ln(1-\alpha)}{ln(\alpha)}}$ for quantile $\alpha = i/n$. Since $x_{n\alpha:n} = G^{-1}(\alpha)$, the approximation $\hat{\upsilon}_i \approx \frac{G^{-1}(1-\alpha) - G^{-1}(\alpha^c)}{G^{-1}(\alpha^c) - G^{-1}(\alpha)} = c^{-1+\hat{\xi}}$ holds.

The simple statistics are defined as

mean:

$$\begin{cases} \mu + \frac{\sigma(g_1 - 1)}{\xi} & if \xi \neq 0, \xi < 1 \\ \mu + \sigma\gamma & if \xi = 0 \\ \infty & if \xi \geq 1 \end{cases}$$

variance:

$$\begin{cases} \sigma^2 \frac{g_2 - g_1^2}{\xi^2} & if \xi \neq 0, \xi < \frac{1}{2} \\ \sigma^2 \frac{\pi^2}{6} & if \xi = 0 \\ \infty & if \xi \geq \frac{1}{3} \end{cases}$$

skewness:

$$\begin{cases} \frac{g_3 - 3g_1 g_2 - g_1^3}{(g_2 - g_1^2)^{3/2}} & if 0 < \xi < \frac{1}{3} \\ -\frac{g_3 - 3g_1 g_2 - g_1^3}{(g_2 - g_1^2)^{3/2}} & if \xi < 0 \\ \frac{12\sqrt{6}\zeta 3)}{\pi^3} & if \xi = 0 \\ \infty & if \xi \geq \frac{1}{3} \end{cases}$$

and kurtosis:

$$\begin{cases} \frac{g_{4-4g_1 g_3 - 3g_2^2 - 12 g_2 g_2 g_1^2 - 6g_1^4}}{\left(g_2 - g_1^2\right)^2} & if \xi \neq 0, \xi < \frac{1}{4} \\ \frac{12}{5} & if \xi = 0 \\ \infty & if \xi \geq \frac{1}{4} \end{cases}$$

with $g_p = \Gamma(1 - p\xi)$ for $p \in [1, ..., 4]$, Euler's constant γ (Sondow (1998)), the Riemann zeta function $\zeta(\centerdot)$ (Borwein et al. (2000)), and the gamma probability density function $\Gamma(\centerdot)$.

Appendix C - Additional Figures and Tables

Table 3: UK Banking Sector (Largest Listed Banks): Individual Expected Losses (Put Option Values with and without Jump Diffusion),* 2005-2017 (In GBP Billions)

Table 3. UK Banking Sector (Largest Listed Banks): Individual Expected Losses (Put Option Values with and without Jump Diffusion), 2005-2017 (In GBP Billions)

	Pre-Financial Crisis: June 2005– Aug. 2008	Global Financial Crisis: Aug. 2008– March 2010	Sovereign Debt Crisis: April 2010– Dec. 2011	Post-Crisis: Jan. 2012– Dec. 2017	**Entire Time Period**
			Option Pricing Model (with jump diffusion)		
			Bank 1		
25th percentile	0.48	1.90	0.55	0.17	**0.36**
Median	7.77	35.27	1.15	0.42	**0.86**
75th percentile	28.96	57.94	13.36	0.86	**3.08**
Average	*18.46*	*33.60*	*6.05*	*0.70*	***10.13***
			Bank 2		
25th percentile	0.01	0.20	0.11	0.01	**0.02**
Median	0.21	15.51	0.13	0.04	**0.11**
75th percentile	1.26	22.10	0.27	0.12	**0.27**
Average	*2.61*	*13.19*	*0.18*	*0.08*	***2.40***
			Bank 3		
25th percentile	0.00	3.62	0.18	0.00	**0.01**
Median	0.03	25.13	0.27	0.03	**0.09**
75th percentile	0.21	109.14	6.59	0.15	**0.70**
Average	*1.87*	*46.31*	*3.70*	*0.14*	***6.91***
			Bank 4		
25th percentile	0.02	13.37	0.63	0.16	**0.17**
Median	21.89	51.61	0.72	0.27	**0.56**
75th percentile	47.64	82.83	7.53	0.49	**14.76**
Average	*26.87*	*46.10*	*4.99*	*0.42*	***13.58***
			Option Pricing Model (without jump diffusion)		
			Bank 1		
25th percentile	3.25	0.01	0.00	0.00	**0.00**
Median	7.62	0.03	0.12	0.00	**0.00**
75th percentile	14.20	0.14	9.13	0.00	**0.51**
Average	*14.96*	*0.13*	*4.10*	*0.01*	***4.41***
			Bank 2		
25th percentile	0.00	0.00	0.00	0.00	**0.00**
Median	0.00	0.00	0.00	0.00	**0.00**
75th percentile	7.95	0.00	0.00	0.00	**0.00**
Average	*4.47*	*0.00*	*0.00*	*0.00*	***1.14***
			Bank 3		
25th percentile	0.00	0.00	0.00	0.00	**0.00**
Median	0.33	0.12	0.05	0.00	**0.00**
75th percentile	18.25	0.41	4.79	0.00	**0.10**
Average	*22.59*	*0.27*	*2.87*	*0.00*	***6.20***
			Bank 4		
25th percentile	4.22	0.01	0.00	0.00	**0.00**
Median	7.59	0.23	0.15	0.00	**0.01**
75th percentile	24.50	1.66	5.14	0.01	**4.09**
Average	*17.99*	*1.50*	*3.61*	*0.01*	***5.29***

Source: authors' estimates. *Note:* Even though the sample starts on 1 January 2005, the first time window (June 2005-August 2008) above remains unchanged to facilitate the time-consistent comparison with the estimated joint expected losses, which are available from 21 June 2005 (based on a rolling window of 120 working days and daily updating).

Table 4: UK Banking Sector (Largest Listed Banks): Parameter Values of Fitted Multivariate Generalised Extreme Value (GEV) Distribution of Expected Losses, 2005-2017 (In GBP Billions)

Table 4. UK Banking Sector (Largest Listed Banks): Parameter Values of Fitted Multivariate Generalised Extreme Value (GEV) Distribution of Expected Losses, 2005-2017 (In GBP Billions)

	Pre-Financial Crisis: June 2005– Aug. 2008	Global Financial Crisis: Aug. 2008– March 2010	Sovereign Debt Crisis: April 2010– Dec. 2011	Post-Crisis: Jan. 2012– Dec. 2017	**Entire Time Period**
			All Banks		
			Average		
μ	6.60	29.01	1.46	0.55	**5.82**
σ	1.85	14.92	1.20	0.36	**2.70**
ξ	0.17	-0.08	0.05	-0.06	**0.01**
skewness	*0.25*	*-0.09*	*0.10*	*-0.09*	***0.02***
kurtosis	*0.57*	*0.29*	*0.54*	*0.32*	***0.41***
Υ(ω)	0.47	0.57	0.48	0.56	**0.53**
			Median		
μ	6.43	34.65	1.98	0.71	**6.63**
σ	1.41	13.68	1.31	0.42	**2.47**
ξ	0.19	-0.07	0.08	-0.09	**0.01**
skewness	*0.24*	*-0.05*	*0.14*	*-0.14*	***0.01***
kurtosis	*0.63*	*0.28*	*0.59*	*0.30*	***0.42***
Υ(ω)	0.43	0.48	0.44	0.47	**0.45**
			Individual Banks		
			Bank 1		
μ	12.45	38.10	3.15	1.52	**9.02**
σ	3.21	14.78	1.93	0.69	**3.21**
ξ	0.13	0.03	0.28	-0.15	**0.00**
skewness	*0.59*	*0.35*	*0.61*	*0.27*	***0.41***
kurtosis	*3.68*	*3.39*	*4.42*	*3.88*	***3.81***
			Bank 2		
μ	0.62	12.71	0.16	0.09	**1.83**
σ	0.36	6.94	0.04	0.03	**0.99**
ξ	0.43	0.26	-0.42	-0.04	**0.09**
skewness	*0.67*	*0.76*	*0.04*	*0.37*	***0.42***
kurtosis	*5.08*	*3.43*	*2.95*	*3.69*	***4.03***
			Bank 3		
μ	0.35	41.38	1.58	0.63	**5.78**
σ	0.23	21.63	1.15	0.47	**3.13**
ξ	0.36	-0.17	0.23	-0.07	**0.08**
skewness	*0.66*	*0.10*	*0.67*	*0.36*	***0.44***
kurtosis	*5.18*	*4.20*	*4.17*	*3.45*	***4.23***
			Bank 4		
μ	19.60	52.86	2.44	1.15	**12.47**
σ	3.61	16.32	1.66	0.64	**3.47**
ξ	0.08	-0.49	0.31	-0.17	**-0.07**
skewness	*0.34*	*-0.07*	*0.85*	*0.32*	***0.36***
kurtosis	*3.35*	*2.72*	*4.58*	*3.95*	***3.80***

Source: authors' estimates. *Note:* The table shows the average and median parameter values of the *multivariate generalised extreme value* (GEV) distribution (mean (μ), scale (σ), and shape (ξ)), which was fitted to the empirical distribution of individual expected losses with a non-parametric dependence structure (Υ(ω)).

Table 5: UK Banking Sector (Largest Listed Banks): Joint Expected Losses under Adapted CoVaR Methodology, 2005-2017 (In GBP Billions)

Table 5. UK Banking Sector (Largest Listed Banks): Joint Expected Losses under Adapted CoVaR Methodology, 2005-2017 (In GBP Billions)

	Pre-Financial Crisis: June 2005– Aug. 2008	Global Financial Crisis: Aug. 2008– March 2010	Sovereign Debt Crisis: April 2010– Dec. 2011	Post-Crisis: Jan. 2012– Dec. 2017	Entire Time Period
			All Banks, triggered by any bank		
			Conditional Expected Shortfall (CoES) at 95% 1/		
25th percentile	26.4	138.9	31.2	0.0	**0.2**
Median	118.5	143.6	38.1	0.2	**27.3**
75th percentile	195.5	173.3	117.7	23.2	**115.1**
Average	*131.0*	*134.2*	*63.8*	*10.8*	***59.1***
			Conditional Value-at-Risk (CoVaR) at 95%		
25th percentile	25.1	32.9	31.0	0.0	**0.1**
Median	115.6	140.8	37.5	0.2	**26.1**
75th percentile	123.7	166.8	111.9	22.4	**109.2**
Average	*121.8*	*121.1*	*60.1*	*9.9*	***54.8***
			All Banks, triggered by …		
			Bank 1		
			CoES at 95%		
25th percentile	25.9	140.0	64.7	0.0	**0.1**
Median	117.5	146.5	118.8	0.1	**35.1**
75th percentile	125.0	169.4	130.2	29.1	**119.6**
Average	*123.6*	*148.3*	*100.9*	*12.5*	***67.3***
			CoVaR at 95%		
25th percentile	23.7	136.0	43.0	0.0	**0.1**
Median	114.7	141.8	118.5	0.1	**26.0**
75th percentile	122.6	148.5	140.3	27.0	**116.5**
Average	*109.0*	*137.5*	*95.7*	*11.8*	***59.9***
			Bank 2		
			CoES at 95%		
25th percentile	23.2	142.2	31.9	0.0	**0.4**
Median	109.7	146.5	111.6	0.2	**31.1**
75th percentile	262.2	183.0	132.6	23.0	**127.5**
Average	*141.6*	*159.7*	*89.6*	*12.2*	***77.2***
			CoVaR at 95%		
25th percentile	20.7	139.8	31.5	0.0	**0.2**
Median	107.4	147.7	47.8	0.1	**26.0**
75th percentile	255.4	177.8	135.2	21.0	**122.5**
Average	*124.2*	*157.7*	*80.8*	*10.3*	***71.0***
			Bank 3		
			CoES at 95%		
25th percentile	69.6	140.3	30.0	0.1	**14.6**
Median	195.5	142.9	33.4	0.2	**31.4**
75th percentile	275.7	165.6	39.6	21.6	**107.6**
Average	*169.6*	*156.9*	*40.9*	*11.6*	***71.8***
			CoVaR at 95%		
25th percentile	21.7	137.4	29.9	0.1	**0.2**
Median	114.2	143.5	33.4	0.2	**31.0**
75th percentile	269.6	182.4	38.5	20.7	**43.8**
Average	*145.5*	*156.7*	*38.4*	*11.1*	***61.1***
			Bank 4		
			CoES at 95%		
25th percentile	107.3	33.1	107.1	0.1	**0.3**
Median	119.3	33.2	120.2	0.4	**25.9**
75th percentile	254.6	33.3	143.4	22.0	**119.1**
Average	*140.6*	*33.2*	*110.2*	*10.4*	***70.5***
			CoVaR at 95%		
25th percentile	23.7	31.2	44.5	0.0	**0.2**
Median	116.5	31.2	120.7	0.3	**24.8**
75th percentile	123.7	32.5	143.5	22.2	**116.6**
Average	*122.9*	*31.7*	*104.1*	*9.7*	***62.4***

Source: authors' estimates. *Note:* Conditional Value-at-Risk (CoVaR) measures the system-wide losses (as VaR at 95 percent) conditional on an individual stress event (i.e., a bank experiencing losses above a level consistent with a VaR of 95 percent, estimated over a rolling 120-day window). In applying CoVaR to CCA-generated expected losses, and for a complete comparison with the SES methodology, the individual stress level was also defined as CTE at 95 percent as a threshold (which could be termed "Conditional Expected Shortfall (CoES)"). Given the concentration of the banking sector, the changes in the total expected losses of all four sample banks indicate the systemic stress level (consistent with the definition of "the market" in the CoVaR methodology). The CTE at the statistical confidence level a=95 percent was approximated via VaR at 98.0 percent (based on the simplifying assumption of normally distributed expected losses, where the z-scores $\phi(\Phi^{-1}(1-a))/(1-a)$ (for CTE) and $\Phi^{-1}(a)$ (for VaR) are the same, where ϕ and Φ^{-1} are the standard normal probability distribution and quantile function).

Table 6: UK Banking Sector (Largest Listed Banks): Joint Expected Losses under Adapted SES Methodology, 2005-2017 (In GBP Billions)

Table 6. UK Banking Sector (Largest Listed Banks): Joint Expected Losses under Adapted SES Methodology, 2005-2017 (In GBP Billions)

	Pre-Financial Crisis: June 2005–Aug. 2008	Global Financial Crisis: Aug. 2008–March 2010	Sovereign Debt Crisis: April 2010–Dec. 2011	Post-Crisis: Jan. 2012–Dec. 2017	Entire Time Period
All Banks (Sum)					
Systemic Expected Shortfall (SES) at 95% 1/					
25th percentile	106.1	141.7	31.5	0.0	**0.3**
Median	119.4	148.6	39.4	0.3	**30.4**
75th percentile	254.3	191.0	119.0	24.7	**116.6**
Average	*139.0*	*142.4*	*67.1*	*11.5*	***62.7***
Systemic Value-at-Risk (SVaR) at 95%					
25th percentile	27.0	33.8	32.4	0.0	**0.2**
Median	118.3	163.0	38.9	0.2	**30.9**
75th percentile	253.9	188.7	119.3	25.0	**115.3**
Average	*134.7*	*133.7*	*67.1*	*11.3*	***61.1***
Individual Banks					
Bank 1					
Marginal Expected Shortfall (MES) at 95%					
25th percentile	40.1	36.5	0.0	0.0	**0.0**
Median	46.8	44.4	0.0	0.0	**0.3**
75th percentile	74.1	66.1	27.4	0.3	**41.8**
Average	*49.8*	*43.3*	*10.9*	*7.0*	***20.9***
Marginal Value-at-Risk (MVaR) at 95%					
25th percentile	26.9	0.2	0.0	0.0	**0.0**
Median	46.1	53.5	0.0	0.0	**0.2**
75th percentile	73.7	63.9	27.6	0.3	**41.1**
Average	*48.6*	*39.7*	*10.8*	*7.1*	***20.3***
Bank 2					
Marginal Expected Shortfall (MES) at 95%					
25th percentile	0.0	12.2	0.0	0.0	**0.0**
Median	0.1	18.9	0.0	0.0	**0.0**
75th percentile	70.0	21.7	0.3	0.0	**0.1**
Average	*21.9*	*16.8*	*0.2*	*0.0*	***6.7***
Marginal Value-at-Risk (MVaR) at 95%					
25th percentile	0.0	0.0	0.0	0.0	**0.0**
Median	0.1	19.9	0.0	0.0	**0.0**
75th percentile	69.5	27.2	0.3	0.0	**0.1**
Average	*20.8*	*16.9*	*0.2*	*0.0*	***6.3***
Bank 3					
Marginal Expected Shortfall (MES) at 95%					
25th percentile	0.0	78.5	29.9	0.0	**0.0**
Median	0.0	88.9	33.3	0.0	**0.0**
75th percentile	23.4	91.8	39.2	0.0	**26.5**
Average	*7.5*	*70.5*	*33.0*	*0.0*	***11.6***
Marginal Value-at-Risk (MVaR) at 95%					
25th percentile	0.0	0.3	30.4	0.0	**0.0**
Median	0.0	87.3	33.8	0.0	**0.0**
75th percentile	23.1	91.2	39.8	0.0	**26.6**
Average	*7.1*	*63.3*	*33.9*	*0.0*	***11.9***
Bank 4					
Marginal Expected Shortfall (MES) at 95%					
25th percentile	66.0	2.3	0.0	0.0	**0.0**
Median	72.5	6.6	0.0	0.0	**0.0**
75th percentile	86.1	12.4	61.0	0.6	**61.9**
Average	*59.8*	*11.8*	*23.0*	*4.5*	***23.6***
Marginal Value-at-Risk (MVaR) at 95%					
25th percentile	0.0	5.2	0.0	0.0	**0.0**
Median	71.3	6.6	0.0	0.0	**0.0**
75th percentile	85.4	32.3	61.0	0.4	**60.6**
Average	*58.2*	*13.8*	*22.2*	*4.2*	***22.6***

Source: authors' estimates. Note: 1/ The Systemic Expected Shortfall (SES) aggregates (as weighted sum) the individual losses of all institutions ("marginal expected shortfall" or MES) conditional on a systemic stress event (i.e., the banking sector experiencing losses above a level consistent with its conditional tail expectation (CTE) of 95 percent, estimated over a rolling 120-day window). In applying SES to CCA-generated expected losses, and for a complete comparison with the CoVaR methodology, also a systemic stress level of VaR at 95 percent was chosen as a threshold (which could be termed "Systemic VaR (SVaR)"). Given the concentration of the banking sector, the changes in the total expected losses of all four sample banks indicate the systemic stress level. The CTE at confidence level α=95 percent was approximated via VaR at 98.0 percent (based on the simplifying assumption of normally distributed expected losses, where the z-scores $\varphi(\Phi^{-1}(1-\alpha))/(1-\alpha)$ (for CTE) and $\Phi^{-1}(\alpha)$ (for VaR) are the same, where φ and Φ^{-1} are the standard normal probability distribution and quantile function).

Appendix D - Estimating the Market Value and Volatility of Equity of Unlisted Subsidiaries of Listed Parent Banks

The equity price, E, and equity volatity, σ_E, are essential inputs for the estimation of the market-implied expected losses according to the CCA framework described in equation (26). However, in many countries, equity prices might not be readily observable or usable for significant parts of the financial system, such as unlisted savings and cooperative banks and a signifcant host banking sector, especially through locally incorporated subsidiaries and branches.

To resolve this issue, we specify a two-stage estimation approach based on the fundamental determinants of the price-to-book (PB) ratio of listed banks (i.e., the market value of equity expressed as a multiple of their book value) to derive both the equity price and equity volatility of unlisted banks (and local subsidiaries and branches whose contribution to the parent banks' group-level market valuation is not material).[29]

The PB ratio for $j\epsilon m$ banks at time t is determined by the banks' fundamentals (profitability, asset quality, and credit rating) and general equity market conditions, and is specified by the following panel regression

$$PB_{j,t} = \alpha + \beta_1 RoE_{j,t} + \beta_2 LLR_{j,t} + \beta_3 \triangle EquityMarket_{j,t} + ...$$
$$...\beta_4 Dummy\_GFC_{j,t} + \beta_5 Dummy\_AA_{j,t} + \beta_6 Dummy\_A_{j,t} + \beta_7 Dummy\_BBB_{j,t} + \varepsilon_t \qquad (60)$$

where LLR is the loan-loss reserve, proxied by the ratio of the share of reserves for impaired loans to total loans, $\triangle EquityMarket$ is the percentage change of the relevant (main) stock market index over the selected time horizon consistent with the frequency of available bank data, $Dummy\_GFC$ is the dummy variable defined as one from mid-2008 (and zero otherwise), $Dummy\_AA$, $Dummy\_A$, and $Dummy\_BBB$ take the value of one for banks that are (1) rated "AA-" or higher, (2) rated between "A+" and "A-", and (3) rated between "BBB+" and "BBB-", respectively.

The PB ratio, in turn, is one of several explanatory variables of equity volatility

$$\sigma_{E_{j,t}} = \alpha + \beta_1 PB_{j,t} + \beta_2 \sigma_{EquityMarket_{j,t}} + \beta_3 \ln MarketCap_{j,t} + \beta_4 Leverage_{j,t} + \varepsilon_t, \qquad (61)$$

where $\sigma_{E_{j,t}}$ is the exponentially weighted average annualised volatility, $\sigma_{EquityMarket}$ is the volatility of the main stock market index, $\ln MarketCap$ is the natural logarithm of the bank's market capitalisation, and $Leverage$ is the book leverage ratio (i,e., the ratio between total liabilities and the book value of assets) (Campbell et al. (2001)).[30]

We first obtain the time series of the return on equity, loan-loss reserve, and leverage ratio for all sample banks and relevant banking groups (from their financial disclosure as required by their

[29] This approach was first adopted when the Systemic CCA framework was applied as part of the bank stress test of the IMF's Financial Sector Assessment Programme for Hong Kong SAR (International Monetary Fund (2014)).

[30] If the locally incorporated bank is not separately listed but accounts for a significant portion of total asssets (i.e., 10 percent or above but less than 100 percent as "materiality" threshold), the market-based information of the banking group could reflect the operation of the local bank; however, some adjustments may be needed when using market-based information of the banking group. Thus, the potential equity value could be derived from multiplying the book value of the unlisted local bank by the actual PB ratio of the listed banking group; the equity volatility of the banking group would similarly apply to the unlisted local bank.

prudential regulator) and their credit ratings as well as the PB ratio of all listed banks/banking groups and the book value of equity of all unlisted banks.[31] We supplement the banking information with the required market data, i.e., the annual percentage change of the main stock market index and its volatility. We can then estimate the coefficient values of the explanatory variables in equation (60) for all listed banks/banking groups can obtain $\hat{PB}_{j,t}$ of each unlisted bank by applying the coefficient values to the observed profitability, asset quality, and credit rating. Multiplying the book value of equity of the local bank by $\hat{PB}_{j,t}$ yields the implied equity value, which is then used to derive the quasi-market capitalisation $\ln MarketCap_{j,t}$ as input to equation (61) in order to derive $\hat{\sigma}_{E_{j,t}}$ (with $\hat{PB}_{j,t}$ and the leverage ratio as additional bank-specific inputs).

This approach is applied to two different cases: (i) neither the local bank nor the banking group (if applicable) are listed, or (ii) the local bank is not listed and accounts only for a small portion (i.e., less than 10%) of the total assets of the listed banking group, whose equity prices (and more generally market-based information) has limited relevance for the market-implied default risk of the local bank.

[31] Jordan et al. (2011) examined the PB ratio for 6,604 bank stock observations between December 2006 through June 2009. They relate each bank's PB ratio to several fundamental ratios and find that lower relative costs, higher non-interest income, and lower assets in non-accrual or foreclosed status are associated with higher PB ratios (while controlling for size and other attributes).