BANK OF ENGLAND

Staff Working Paper No. 776

Measuring financial cycle time

Andrew Filardo, Marco Lombardi and Marek Raczko

January 2019

BANK OF ENGLAND

Staff Working Paper No. 776

Measuring financial cycle time

Andrew Filardo,[1] Marco Lombardi[2] and Marek Raczko[3]

Abstract

Motivated by the traditional business cycle approach of Burns and Mitchell (1946), we explore cyclical similarities in financial conditions over time in order to improve our understanding of financial cycles. Looking back at 120 years of data, we find that financial cycles exhibit behaviour characterised by recurrent, endogenous swings in financial conditions, which result in costly booms and busts. Yet the recurrent nature of such swings may not appear so obvious when looking at conventionally plotted time-series data (that is, observed in calendar time). Using the pioneering framework developed by Stock (1987), we offer a new statistical characterisation of the financial cycle using a continuous-time autoregressive model subject to time deformation (ie the difference between the time scale relevant for economic decision-making and conventional calendar time such as months, quarters and years), and test for systematic differences between calendar and a new notion of financial cycle time. We find the time deformation to be statistically significant, and associated with levels of long-term real interest rates, inflation volatility and the perceived riskiness of the macro-financial environment. Implications for statistical modelling, endogenous risk-taking economic behaviour and policy are highlighted.

Key words: Financial cycles, continuous-time autoregressive models, time deformation, behavioural economics, endogenous risk-taking behaviour, central banking.

JEL classification: E32, G01, F32, F34, E58, E71, D80.

---

(1) Bank for International Settlements. Email: andrew.filardo@bis.org
(2) Bank for International Settlements. Email: marco.lombardi@bis.org
(3) Barclays. Email: mraczko@gmail.com

The views expressed in this paper are those of the authors and do not necessarily reflect those of the Bank for International Settlements, the Bank of England or Barclays (Mr Raczko's current employer). Marek Raczko's contributions to this paper were made when he was employed at the Bank of England. We thank Pawel Baranowski, Christian Brownlees, Fabio Canova, Paolo Cavallino, Christian Conrad, Bill English, Mike Joyce, Andrew Meldrum, Benoit Mojon, Jouchi Nakajima, Taejin Park, Pierre Siklos, Simon van Norden and seminar participants at the Bank of Canada, the Bank of England, the Bank for International Settlements, the Joint Research Centre of the European Commission, Cardiff University, the Fifth Asset Pricing Workshop at the University of York, the Annual Conference of the International Association for Applied Econometrics and the 2018 Deutsche Bundesbank-LOEWE Center-SAFE-DIW-IWH Joint Conference.

The Bank's working paper series can be found at www.bankofengland.co.uk/working-paper/staff-working-papers

Bank of England, Threadneedle Street, London, EC2R 8AH
Telephone +44 (0)20 3461 4030 email publications@bankofengland.co.uk

ISSN 1749-9135 (on-line)

1. Introduction

The Great Financial Crisis (GFC) reinvigorated interest in studying financial cycles. One strand of the recent research has focused on understanding the domestic origins of wide swings in financial conditions, namely credit and housing prices. Risk-taking behaviour, in particular, has been emphasised as a key channel that transforms supportive financial conditions into boom-bust cycles (see eg Borio (2013)). Another strand has looked into understanding the global dimension of financial cycles. Waves of global liquidity have been found to interact with domestic financial cycles in ways that generate excessive procyclicality in financial conditions (see eg Claessens et al (2011), Bruno and Shin (2015)).

Despite considerable efforts over the past decade, researchers and policymakers have yet to forge a workhorse model of financial cycles. Modelling efforts have emphasised the behaviour of banks and non-banks as well as the role of financial frictions (see, eg, Lindé (2018)). These frictions are seen as amplifying, amongst other things, exogenous economic and financial shocks. These shocks, under certain circumstances, can result in financial fragility and destabilising financial headwinds which, if large enough, can turn into financial crises. Many proposed models, especially those built on the bedrock of the dynamic stochastic general equilibrium approach, offer interesting insights about particular aspects of boom-bust developments. But no model has broken out of the pack to provide a comprehensive understanding of financial cycles. One reason for this failure is that there remains considerable disagreement about the key mechanisms behind the recent crisis. Some analysts argue, for example, that the GFC was the result of a unique set of developments that exposed weaknesses in the financial regulatory system. Others point to broad similarities with other boom-bust experiences and argue that these similarities suggest boom-bust cycles are an inherent feature of capitalist economies.[1]

Our paper takes a different, but in many ways complementary, approach. We apply the ideas of the classical business cycle literature of Burns and Mitchell (1946) to the financial cycle. Namely, we treat financial booms and busts as endogenous, recurrent phenomena with common features shared across time rather than as unrelated exogenous one-off unique events. Burns and Mitchell ask the fundamental question, "what happens during business cycles?"; in a similar vein, we ask "what happens during financial cycles?". This question pre-supposes that financial cycles share certain commonalities across time that make them, in many respects, all alike.

Having traced the roots of our approach to that of Burns and Mitchell (1946), we find it useful to start by framing the analysis with Burns and Mitchell's famous description of the business cycle:

> *"Business cycles are a type of fluctuation found in the aggregate economic activity of nations that organize their work mainly in business enterprises: a cycle consists*

[1] In terms of policy implications, the former group tends to emphasise strengthening regulatory frameworks to building greater financial resilience; see, eg, the mandates of the Financial Stability Board (http://www.fsb.org/about/mandate/) and the Basel Committee (www.bis.org/bcbs/charter.htm). The latter group tends to focus on improving countercyclical efforts to reduce macro-financial imbalances over time.

of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own."

While the description applies to business cycles, it could be considered apt also for financial cycles. Paraphrasing the quote, financial cycles are a type of fluctuation found in aggregate financial conditions of nations that organise their intermediation in financial markets and financial intermediaries such as banks: a cycle consists of expansions occurring at about the same time in many measures of financial activity, followed by similarly general recessions, contractions (which sometimes become crises) and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration financial cycles vary from more than eight years to 20 or so years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own.

It is important to highlight key differences between financial cycles and business cycles. Graph 1 illustrates the empirical nature of the differences between the two cycles. Statistical features of the financial cycle obviously differ in various respects. First, financial cycles comprise expansions and contractions of financial conditions rather than of the real side of the economy. In this graph, the proxy for the financial cycle is an aggregate of credit and housing prices. Second, the duration of financial cycles varies and is generally longer than the duration of business cycles. Third, the amplitude of the financial cycle varies more than the business cycle. Fourth, the synchronicity of financial and business cycles appears fairly low. The business cycle does not appear to drive the financial cycle. However, there is a relationship between financial bust phases and serious macro-financial disruptions.

These differences should not be interpreted as suggesting that financial cycles have little to do with business cycles. Indeed, cyclical credit conditions played a role in traditional business cycle analysis. Burns and Mitchell (1946), Moore (1961), Eckstein and Sinai (1986) and Zarnowitz (1992) emphasised the dynamics of credit conditions and asset prices when describing the evolution of the stages of the business cycle.[2] At business cycle frequencies, cyclical credit conditions and business cycles were consistently found to be interrelated. The more recent literature on the macro-financial linkages during business cycles has highlighted the role of the so-called financial accelerator (Kyotaki and Moore (1997), Bernanke et al

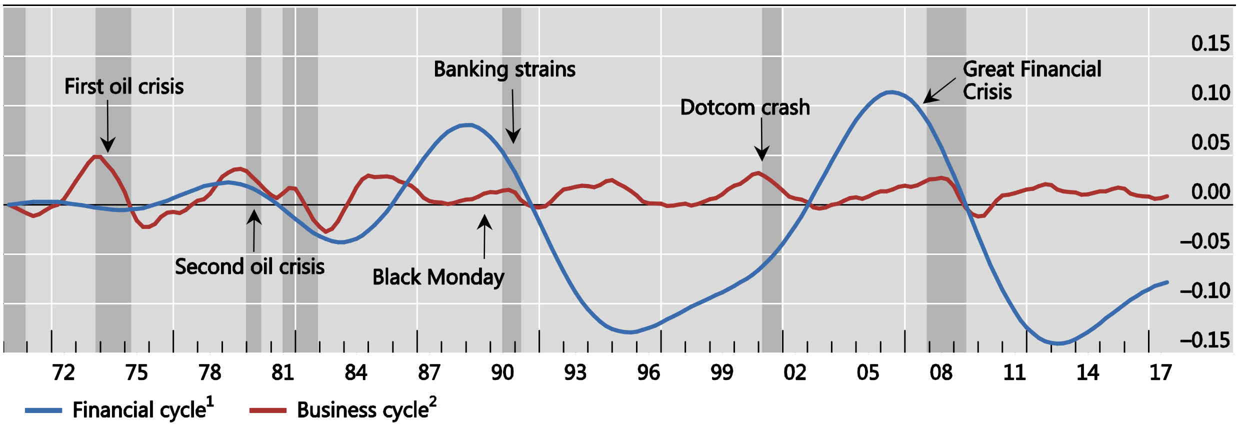
[2] Eckstein and Sinai (1986) find that the financial cycle was an intrinsic part of the business cycle in the post-war period. Financial conditions, especially the health of private sector balance sheets, play dual roles as both a trigger as well as an amplifier of cycles. For households, a key measure of balance sheet strain is the burden of mortgage loan repayments as a share of disposable income and the ratio of financial assets to liabilities; they also emphasised the dominant role of housing in cycles. For firms, the debt service burdens relative to cash flow, the ratio of short-term to total liabilities and the leverage ratio largely characterise the nature of the financial fragilities. And, funding conditions of banks and other financial institutions are also seen as important ingredients of financial cycles. The role of monetary policy was also seen as central, in that it influences the availability of liquidity, debt service burdens and hence financial risk. It should be noted that many of these issues resurfaced after the latest crisis. See Mian and Sufi (2018) for a recent discussion of this perspective. Albuquerque et al (2015) offer an alternative perspective based on long swings in stock prices.

(1999)) and financial frictions (Hall (2013), Duval et al (2017)). However, as pointed out by DeLong and Summers (1986) and Zarnowitz (1992), the historical relationship between financial panics and business cycle contractions is not as close as sometimes thought.

Our study focuses on the longer swings associated with financial cycles, a phenomenon plainly visible in Graph 1. Various features of the financial cycles are notable. The financial cycle does not coincide with the business cycle. So we are not focusing on the amplification mechanism of the conventional financial accelerator but capturing phenomenon of the type described by Borio et al (2012) and Schularick and Taylor (2012). Financial cycles are generally of a longer duration (or, in other words, they are a lower frequency phenomenon) than business cycles, and the duration (and amplitude) changes over time.

In addition, when looking at measures of the financial cycle over an even longer time span, the financial cycles do not appear to be solely regime-dependent, at least in some respects. Since the late 1880s, for example, various exchange rate, monetary, fiscal and regulatory regimes have been in place. This is not to say that these regimes, amongst other things, did not influence the shape of the financial cycle. Certainly, periods of financial repression, for example, tended to influence the shape of the financial cycle (see, eg, Burnside et al (2016)). But through it all, recurrent long swings in financial forces are evident.

The financial and business cycles in the United States — Graph 1



[1] The financial cycle as measured by a frequency-based (bandpass) filters capturing medium-term cycles in real credit, the credit-to-GDP ratio and real house prices. [2] The business cycle as measured by a frequency-based (bandpass) filter capturing fluctuations in real GDP over a period from one to eight years.

Source: M Drehmann, C Borio and K Tsatsaronis, "Characterising the financial cycle: don't lose sight of the medium term!", *BIS Working Papers*, no 380, June 2012.

One challenge in exploring the financial cycle's recurrent nature is the ostensible differences of each cycle over time, as is evident in Graph 1. Conventional time-series plots of financial cycles, on the face of it, suggest that the financial cycles are not at all alike. However, as Burns and Mitchell (1946) emphasised when researching business cycles regularities, conventional time-series plots (in calendar time) may obscure some of important regularities that all cycles share. Put differently, *calendar time* might not be the best way to compare cycles over time.[3]

Indeed, Burns and Mitchell advocated a phase-centric approach to characterising the regularities of business cycles. They did this by defining units of cyclical time as the sequence of phase turning points. So, for example, the expansion phase of a cycle (from trough to peak) would represent one unit of cyclical time, regardless of the elapsed calendar time in that phase. They then summarised changes in economic variables in each of the phase-based time units and assessed the co-movements. Essentially they were stretching and compressing the time series in calendar time so as to transform the data into *business cycle time*. This made it possible to establish clearer stylised facts about the regularity of business cycles. Much of the research that followed from this conjecture emphasised a unique cyclical, rather than calendar, time in understanding business cycles.

Using analogous methods, the aim of our paper is to shed light on the nature of the factors associated with, and the forces behind, the financial cycle. In addition to using the graphical analysis of Burns and Mitchell, we apply the pioneering statistical approach of Stock (1987) in order to assess the extent of the differences between financial cycle time and calendar time, ie technically, we call this *time deformation* in the financial cycle. The statistical approach allows us to characterise the regularities of financial cycles that may arise from differences between calendar time and *financial cycle time*.[4] Technically, we demonstrate how to jointly estimate a continuous-time autoregressive process and the parameters of the non-linear function that describe the relationship between financial cycle time and observable macro-financial variables.[5] The model allows statistical testing for the presence of time deformation and the statistical significance of the related macro-financial variables driving it. We find evidence that low real interest rates, inflation volatility and macro-financial risk environments are associated with financial cycles of longer duration.

Various potential benefits arise from being able to characterise financial conditions in financial cycle time rather than calendar time. From a modelling perspective, if fluctuations in financial conditions exhibit more stable statistical behaviour in financial time than in calendar time, forecasters can take advantage of this timing information and, in principle, produce more accurate predictions of both financial conditions and the conditional probability of a boom ending in a bust. From a more economic point of view, knowing that financial cycles are all alike in financial cycle time, analysts may be able to make more reliable comparisons with past cycles and hence make better inferences about current and future

[3] In another context, Albuquerque et al (2017) find a much stronger correlation between stock market returns and fundamentals than the extant literature by focusing on episodic, not conventional time series, correlations associated with bull and bear markets. They conclude that episode-based time may be a more accurate way to assess relationships than calendar time: "Chronos is the word for calendar time. Kairos refers to a moment of indeterminate time in which something special happens. To account for our findings, we need a model in which the relation between stock returns and consumption growth is different in Chronos and Kairos time."

[4] We assume an endogenous, recurrent but unobserved statistical process reflecting forces associated with the financial cycle, which we model as a time-invariant process in financial cycle time.

[5] See Harvey and Stock (1985), Harvey (1989) and Melino (1994) for technical details.

financial conditions. Policywise, if it is the case that financial cycles are recurrent and driven consistently by endogenous forces, central banks may be able to design policy frameworks which systematically respond to financial developments in a way that moderate financial cycles and thereby improve economic performance.[6]

The rest of the paper is structured as follows. The next section introduces our working definition of the financial cycle. Section 3 outlines the modelling approach of time deformation. Section 4 presents the empirical results. Section 5 offers some interpretations of the type of behaviour leading to time deformation. The final section draws conclusions.

2. Characterising the financial cycle

We define financial cycles as *long* swings found in aggregate measures of financial conditions. These swings are plainly visible in Graph 1, where we plot our working definition of the financial cycle for the United States based on Borio et al (2012). Various features of the financial cycle are notable. First of all, the financial cycle does not coincide with NBER-dated business cycles. Second, swings in the financial cycle are generally longer in duration (or, in other words, of a lower frequency) than those in business cycles; and their amplitude tends to display more variability over time.

Graph 2 points to striking visual evidence of time deformation in the financial cycle going back into the 19th century.[7] One simple way to illustrate this point is to perform a (non-parametric) Burns and Mitchell-type assessment of time deformation. We first identify the peaks and troughs in calendar time with the Bry-Boschan (1971) turning point dating algorithm. Then, we map these turning points of the cycle into the equally-spaced financial cycle time-scale intervals. This mapping is depicted in the lower panel. Note that the differing lengths of the phases in calendar time represent a differing number of observations being mapped into each financial time interval.

Time dilation and compression are evident in the second half of the sample. The post-WWII period witnessed a very long cycle in calendar time, which appears much shorter in financial cycle time. By way of contrast, recent financial cycles appear relatively short in calendar time relative to financial cycle time. Taken together, these findings suggest that

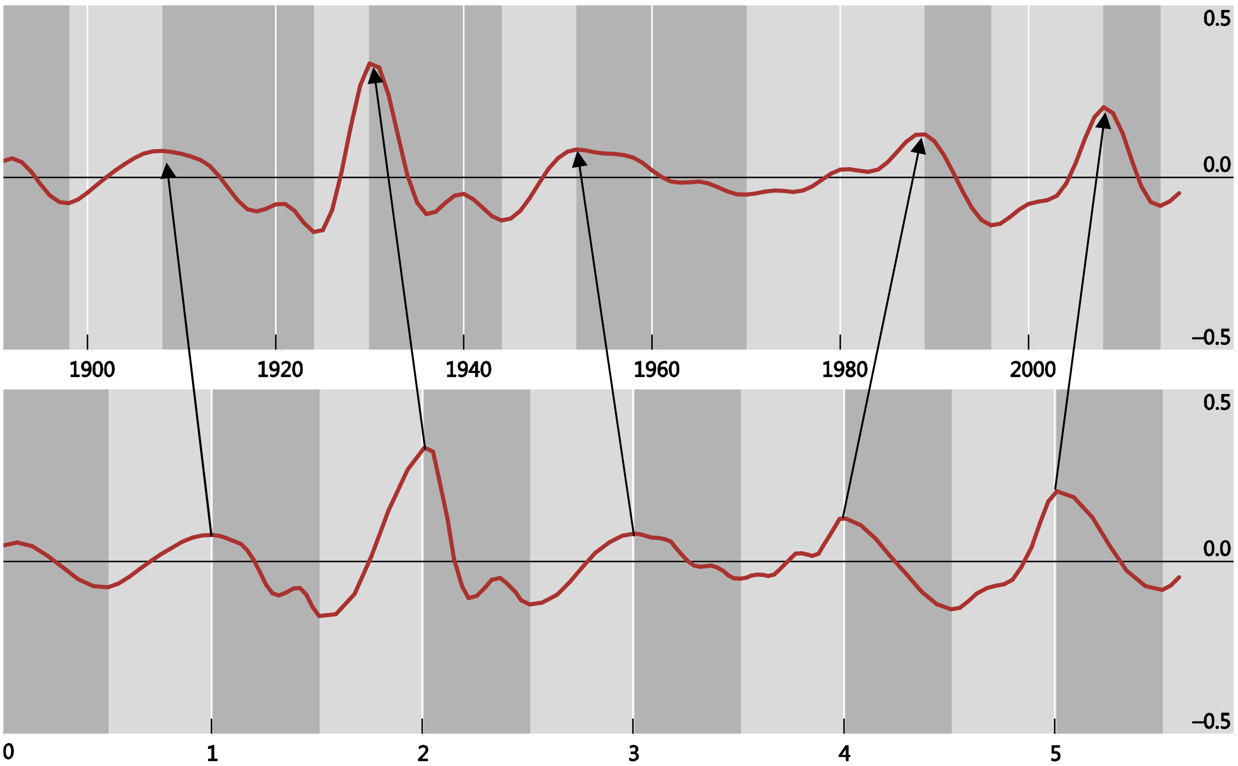
---

[6] See, for example, Filardo and Rungcharoenkitkul (2016), Gourio et al (2017) and Adrian and Duarte (2018) for recent contributions to the monetary policy research debate emphasising the nature of recurrent financial cycles as one of the key justifications for leaning-against-the-wind policies. The largest welfare gains from leaning arise from reshaping the endogenous forces of the financial cycle in a way consistent with the Lucas critique.

[7] US credit to the private non-financial sector (% of potential GDP) is our proxy for the financial cycle, allowing us to extend our annual data series back to 1880. To focus on long financial cycle swings (versus business cycle dynamics), we filter credit using a band-pass filter for frequencies from eight to 32 years. The range is consistent with findings from Claessens et al (2011), Borio et al (2012) and Rünstler and Vlekke (2016). Note that the financial cycle swings reported in the top panel of Graph 2 and in Graph 1 for the period of 1970 to the present are similar; our financial cycle measure is also consistent with alternatives used in the literature (eg Schularick and Taylor (2012)). Further data details are available in the appendix.

swings in financial conditions, at least when crudely dated using the peaks and troughs of the financial cycle, exhibit some form of time deformation.[8]

The financial cycle in calendar and financial cycle time (B-M approach) Graph 2



Credit-to-GDP data. The top panel plots our data in calendar time; the bottom plots the transformed data in financial cycle time, ie based on the stages of the cycle being either expansion or contraction. The time deformation mapping between the two panels is $\Delta g(t) = \sum_{i=1}^{M} \frac{\Delta t}{2T_m} I_{t,m}$; for annual data, Δt =1 and T_m is the number of annual observations in each of the m half cycles. We divide by 2 to normalise each half cycle to equal ½ in financial cycle time and hence 1 for a full cycle.

Source: Authors' calculations.

3. Modelling financial cycle time

The nonparametric Burns and Mitchell approach is subject to the criticism that it is merely highlighting interesting, but possibly spurious, patterns in the data. To put our analysis of time deformation on a more sound statistical footing, we model the financial cycle data using the pioneering approach of Stock (1987, 1988). This allows us to formally estimate and test for time deformation.

[8] Ideally, one could split the cycle into finer stages to enrich the analysis along the lines of Eckstein and Sinai (1986). They identify five distinct stages of a typical financial cycle in the post-war period: boom, credit crunch, recession, reliquefication and recovery. In principle, we could consider finer partitions of calendar-based data. However, these partitions lead to considerable arbitrariness, which raises issues of spurious patterns that could unduly bias inferences. As noted earlier, the Burns and Mitchell-type methodology does not lend itself to formal statistical testing. Our statistical model in many respects addresses these inherent drawbacks of their approach.

We assume financial conditions can be modelled as an *n*-dimensional data vector generated by a continuous-time k^{th}-order process, $\xi(s)$, with time invariant parameters, satisfying the stochastic differential equation:

$$d\left[D^{k-1}\xi(s)\right] = \left[A_1 D^{k-1}\xi(s) + \cdots + A_{k-1}D\xi(s) + A_k\xi(s)\right]ds + d\zeta(s), \tag{1}$$

where A_i are matrices of AR coefficients,[9] D is a differentiation operator, and $\zeta(s)$ is a *n*-dimensional Gaussian white noise process with mean 0 and covariance matrix Ω. Note, the time scale s – the so-called operational time (in our application, financial cycle time) – differs from the calendar time scale (which will be denoted by *t*). As well, the parameters of this process are time invariant with respect to financial cycle time, ie constant with respect to s.

We restrict our attention to a first-order continuous-time auto-regressive (CAR) process that has the following closed-form solution:

$$\xi(s) = e^{A(s-s^-)}\xi(s^-) + \int_{s^-}^{s} e^{A(s-r)}d\zeta(r), \text{ for any } s > s^-. \tag{2}$$

Given that $\xi(s)$ is not directly observable, we must infer it from financial data, Y_t, which is observed in calendar time. In this paper, we assume there is a function *g* which maps calendar time into financial time, ie $s = g(t)$. This yields the equation linking financial cycle time to calendar time observations:

$$Y_t = \xi(g(t)), \qquad t = 1,...,T. \tag{3}$$

A few properties of the g function are worth highlighting. First, a reasonable restriction on the *g* function is the requirement that financial cycle time runs forward and does not run backward. This implies that the g function be monotonically increasing in t. Second, the g function may depend on a set of m observable variables, $\boldsymbol{z}$, which effectively dilate and compress calendar time relative to financial cycle time.

As in Stock (1988), a convenient form for g is an exponential function:

$$\Delta g(t;\boldsymbol{z}_{t-1}) = \exp\left(\boldsymbol{c}'\boldsymbol{z}_{t-1}\right) \Big/ \left[T^{-1}\sum_{t=1}^{T}\exp\left(\boldsymbol{c}'\boldsymbol{z}_{t-1}\right)\right], \tag{4}$$

where $\boldsymbol{c}$ is a m-dimensional vector of parameters. When $\boldsymbol{c}$ = 0, the increments in the g function from $t-1$ to t are unitary irrespective of the values of $\boldsymbol{z}_{t-1}$. In other words, there is no time deformation when the time scales s and t are proportional. The exponential

[9] Note that for a continuous-time AR process to be stationary, the real parts of the roots of *A* are negative.

representation also ensures that calendar time and financial cycle time run forward. As for the z variables, we require them to be pre-determined at time t to simplify the estimation.[10]

Given the *g* function and the normalisation of the A matrix described in Appendix B, the CAR(1) representation in (discrete) calendar time can be written as

$$f_t = e^{\Lambda \Delta g(t;z_{t-1})} f_{t-1} + v_t, \qquad v_t = \int_0^{\Delta g(t;z_{t-1})} e^{\Lambda\left(g(t)-s\right)} d\eta(r). \tag{5}$$

Equation (5) is the state equation governing the evolution of the (unobserved) financial cycle in calendar time. Plugging (5) into (3) and adding a random error term, the measurement equation takes the following form:

$$Y_t = \Gamma f_t + \varepsilon_t, \qquad \varepsilon_t \sim N(0,\Sigma), \tag{6}$$

where Γ is a coefficient matrix capturing the financial cycle's influence on Y_t. The system defined by the state and measurement equations is a standard linear state space, and its parameters–as well as the coefficients c in the *g* function–can be estimated using a Kalman filter.

4. Results

Table 1 reports the baseline results from the time deformation model. We find evidence of time deformation in the financial cycle, with estimates of the model being intuitively plausible and statistically significant at conventional levels of confidence.

The first two lines in the table report the CAR(1) parameters and values of t-student statistics. The continuous-time autoregressive parameter, Λ, is fairly stable across models;[11] the R^2 is roughly 0.85 (not reported in table). The likelihood ratio test indicates that the null hypothesis of no time deformation is rejected at the conventional critical values.[12]

The positive coefficient estimates on the z_{t-1} variables are statistically significant, indicating that a decline in the long-term real interest rate, inflation volatility, the corporate spread and stock-market implied volatility (NVIX; see appendix A for details) dilates the financial cycle in calendar time. In other words, observed financial cycles tend to get longer.

Testing the statistical significance one variable at a time, the corporate spread appears to have the largest influence on time deformation, with the impact of the interest rate and the NVIX being somewhat less significant and the inflation volatility even less so. When we include

[10] We show that this assumption is necessary for estimation via the Kalman filter described in Appendix B.
[11] Note that translating this CAR estimate into an implied discrete time specification yields a large and positive AR(1) parameter.
[12] Our test for the presence of fractional time differencing is rejected (not reported).

all the variables in the g function, the corporate spread still plays a dominant role, with the others smaller but statistically significant.[13]

Estimated CAR(1) model						Table 1
	(1)	(2)	(3)	(4)	(5)	(6)
Λ	-0.83	-0.65	-0.84	-0.83	-0.82	-0.67
	[-3.13]	[-3.07]	[-3.38]	[-3.29]	[-3.33]	[-3.01]
Real long-term rate$_{t-1}$		0.53				0.33
		[6.92]				[3.16]
Inflation volatility$_{t-1}$			0.26			0.30
			[2.50]			[2.17]
Corporate spread$_{t-1}$				0.87		0.56
				[9.36]		[4.76]
NVIX$_{t-1}$					0.54	0.22
					[5.93]	[2.00]
Log likelihood	-722	-747	-730	-761	-741	-771
Test for no time deformation ($\boldsymbol{c}$=0)		50	16	78	38	98
p-value of test		[0.00]	[0.00]	[0.00]	[0.00]	[0.00]

Numbers in brackets report the t-statistics. The test for no time deformation is a likelihood ratio test for the joint significance of the c coefficients in (3). Note that, in the case of continuous-time AR processes, a negative value for Λ corresponds to positive AR coefficient in (discrete) calendar time. The Q defined in Appendix B is estimated to be 0.1.

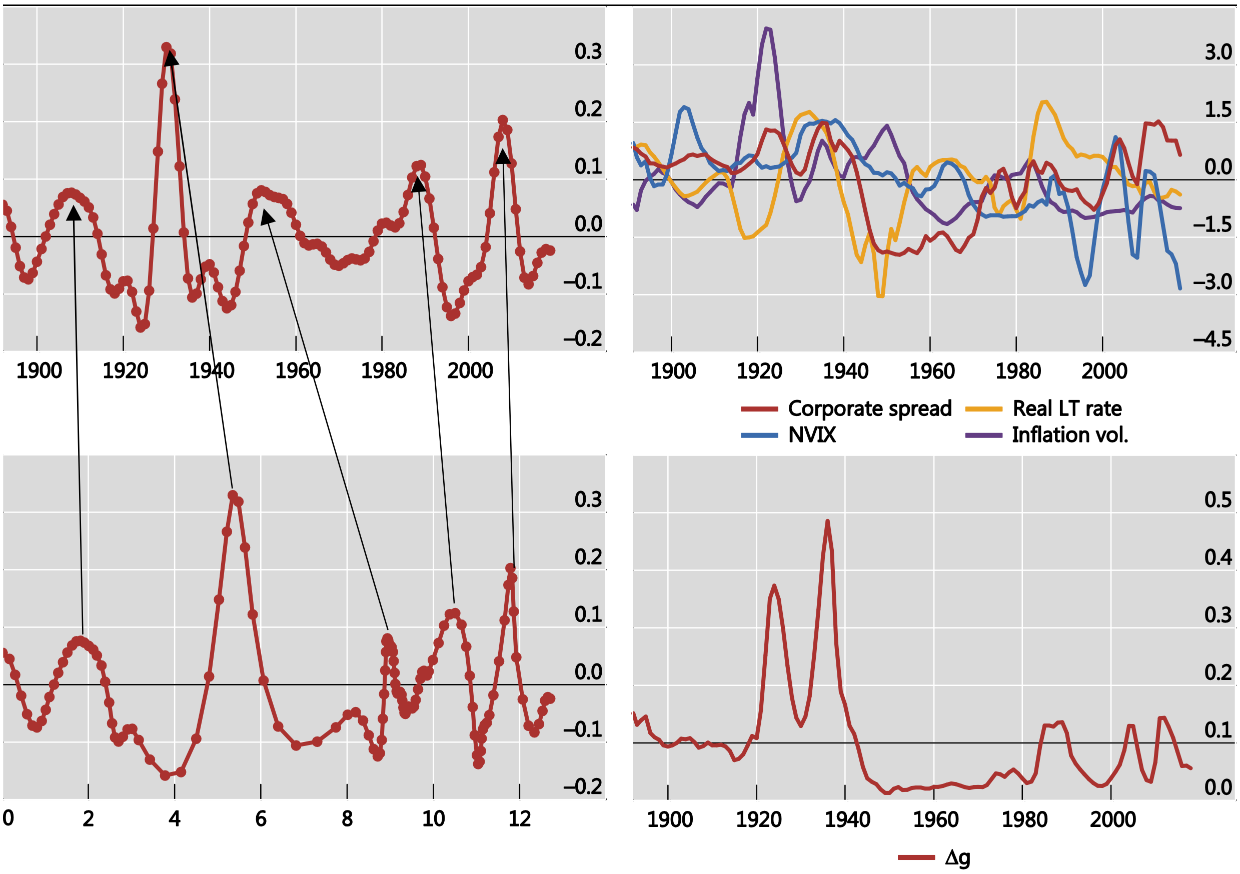
Graph 3 (left-hand panels) shows the observed financial cycle in calendar time (top) and in financial time (bottom) based on our CAR(1) time deformation model in column 6 of Table 1. A few features stand out. First, the early swings in the data in financial cycle time are compressed in calendar time. Second, the first post-World War II swing appears to be considerably dilated; the 50 years of calendar-based data account for only a few periods in financial cycle time. Third, the last two cycles in the panels are roughly equally spaced in calendar and financial time. Finally, comparing the results in this graph to the B-M approach results in Graph 2 displays some interesting differences. The CAR(1) model finds much more evidence of time deformation than the B-M approach; note the arrows matching the peaks in the series differ. This may reflect the limitation of the B-M nonparametric approach which only uses the peaks and the troughs in the observed series to map calendar time into financial cycle time. In Graph 3, all the data are used in the CAR(1) model.

Also note that the estimates of the time deformation function, g, are plotted in the bottom-right panel. We see a clear association between the value of the g function and the pace of calendar time. The lower (higher) is the value of g, the slower (faster) calendar time proceeds relative to financial cycle time. For example, the first post-World War II cycle is

[13] See Gilchrist and Zakrajsek (2012) for evidence on the predictive content of corporate spreads for cyclical activity. Our results suggest corporate spreads also contain information about long swings in financial conditions.

associated with a persistently low value of g. During this period often characterised by financial repression, corporate spreads were below average as were long-term interest rates and inflation volatility (top-right panel).

CAR(1) model results for time deformation — Graph 3



The top-left panel plots the data in calendar time; the bottom-left plots the data in financial cycle time. The top-right panel plots the four series in Δg and the bottom-right plots Δg in calendar time.

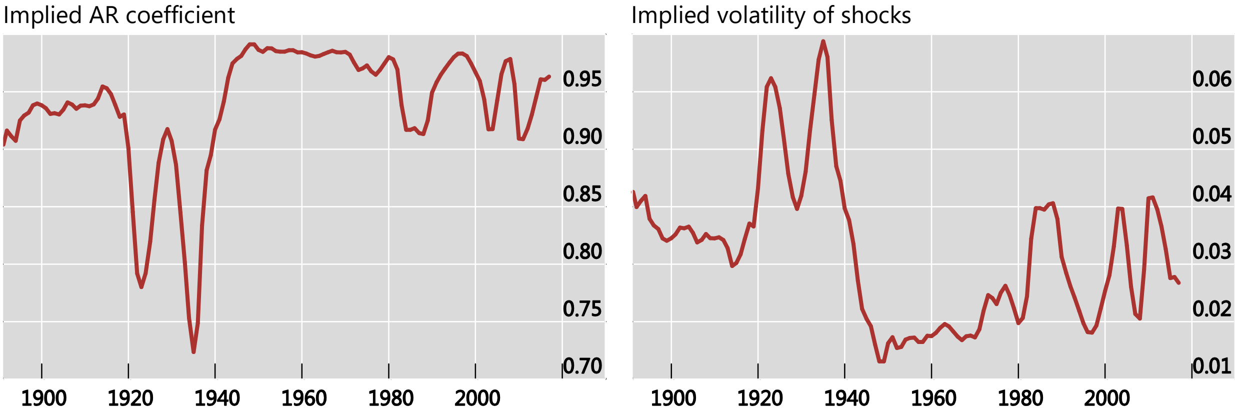
Source: Authors' calculations.

Recent developments in g may shed light on real-time financial market conditions when trying to distinguish developments associated with boom-bust behaviour and with cyclical fluctuations. For example, the estimates of g at the end of the sample period show an upward trend. One narrow interpretation is that the financial cycle accelerating in calendar time and this information could be useful in forecasting. A somewhat broader, more speculative, interpretation would emphasise that very easy financing conditions may be much less related to broad swings in the financial cycle that lead to costly boom-busts and might be more related to phenomena associated with the financial accelerator (ie the mechanism resulting in more procyclical business cycles), which would generate relatively short cycles in financial conditions that do not end in costly crises.[14]

[14] For the United States, this financial accelerator interpretation is consistent with the adoption of financial regulatory reforms aimed at the financial resilience. This would include efforts such as the Dodd-Frank Act and the Basel III Accord. This does not necessarily mean that regulatory actions were the sole factor. Post-GFC, more

Finally, it is important to note the implications for linear model estimation of the financial cycle. Statistically significant time deformation implies a non-stationary time series representation of the financial cycle when observed in calendar time. Technically, a CAR(1) has an approximate *time-varying* parameter AR(1) representation. If the financial cycle is estimated with a linear model, however, the AR coefficient estimators could be seriously biased. To get a sense of the size of the bias, Graph 4 displays the AR(1) coefficient implied by the CAR(1) baseline estimates. Taken at face value, the variation indicates a risk that linear time series models of the financial cycle are subject to considerable misspecification biases.

Implied financial cycle AR(1) and volatility estimates in calendar time — Graph 4



Source: Authors' calculations.

Robustness checks

In this subsection, we look at alternative measures of the financial cycle and consider additional variables that might improve the fit of the g function.

Alternative measures of the financial cycle Consistent with the approach of Borio et al (2012), we look at a multivariate definition of the financial cycle based on the co-movements in different types of financial cycle indicators, namely credit, house prices and equity prices. To this end, we extract the common component from these indicators via the method of principal components, and treat the first principal component as a measure of the financial cycle. Table 2 reports estimates using this principal component approach.

The estimates in Table 2 are similar to those found in Table 1, with some differences in statistical significance. All variables related to time deformation have a positive sign when evaluated one variable at a time (columns 2-5); this implies that increases in these variables (which might be interpreted as evidence of a more risky macro-financial environment) correspond to a faster pace of calendar time relative to financial cycle time. The joint estimation of all the variables in the g function (column 6) underscores the statistical significance of the corporate spread and the real interest rate at conventional test levels.

conservative mindsets of both regulators and those being regulated have arguably made the financial environment less prone to persistent swings that would result in disruptive boom-bust cycles.

Financial cycle defined using the principal component method						Table 2
	(1)	(2)	(3)	(4)	(5)	(6)
Λ	-0.90	-0.87	-0.92	-0.88	-0.99	-0.84
	[-3.24]	[-3.43]	[-3.49]	[-3.40]	[-3.59]	[-3.25]
Real long-term rate $_{t-1}$		0.46				0.17
		[5.52]				[1.59]
Inflation volatility$_{t-1}$			0.16			0.10
			[1.49]			[0.68]
Corporate spread$_{t-1}$				0.91		0.76
				[9.89]		[6.63]
NVIX$_{t-1}$					0.40	0.15
					[4.30]	[1.28]
Log Likelihood	-20.58	-33.31	-21.02	-57.09	-28.40	-60.26
Test for no time deformation (**c**=0)		25.46	0.88	73.02	15.64	79.36
p-value of test		0.00	0.27	0.00	0.00	0.00

The financial cycle measure is the first principal component of credit and asset prices. Numbers in brackets report the t-statistics. The Q defined in Appendix B is estimated to be roughly 3.5 across specifications.

Additional candidate z variables in the g function In Table 3, we extend our baseline model to consider additional variables that may help to capture the nature of time deformation. The first two – slope of the yield curve (ie the long-term nominal yield less the short-term interest rate) and GDP volatility – are alternative risk measures of the macro-financial environment. The coefficient on the slope of the yield curve is negative and statistically significant at the 10% confidence level. The negative sign suggests that as the slope of the yield curve flattens or inverts, calendar time tends to dilate. This evidence is consistent with a reduced risk premia associated with the risk-taking channel during financial booms. Note that inclusion of the yield curve slope reduces the statistical significance of the real interest rate, inflation volatility and the NVIX. These findings support the view that the yield curve is correlated with various developments in the macro-financial environment.

The coefficient on GDP volatility is positive and statistically significant at conventional levels of confidence. The positive sign indicates that great macroeconomic volatility is associated with the compression of calendar time. This is generally consistent with the baseline results that macro-financial risks drive the time deformation.[15]

We also considered business cycle phase interactions with the financial cycle. We find a strong correlation between business cycle recessions and calendar-time deformation. Recessions tend to speed up financial cycles in calendar time. However, some caution is needed in interpreting this result. The ex-post dating of business cycles implies the use of future information, which could influence inferences. Nonetheless, these correlations do

[15] We examined the inflation rate as a driver of time deformation, in part motivated by the experience in the 1970s and early 1980s. The inclusion of inflation, whether included in a specification with the nominal interest rate or not, yielded counterintuitive results. We also constructed an inflation variable uncorrelated with inflation volatility; this did not resolve the apparent puzzle. We leave this conundrum for further research.

point to possible interactions between business cycles and financial cycles that deserve further exploration. [16]

Robustness to alternative drivers of time deformation — Table 3

	(1)	(2)	(3)	(4)	(5)
Λ	-0.67	-0.59	-0.70	-0.75	-0.60
	[-3.01]	[-2.84]	[-3.01]	[-3.04]	[-2.91]
Real long-term rate$_{t-1}$	0.33	0.22	0.38	0.21	0.37
	[3.16]	[1.84]	[3.63]	[1.88]	[3.52]
Inflation volatility$_{t-1}$	0.30	0.17	-0.07	0.45	0.22
	[2.17]	[1.06]	[-0.45]	[3.51]	[1.62]
Corporate spread$_{t-1}$	0.56	0.61	0.81	0.54	0.58
	[4.76]	[5.07]	[6.71]	[4.62]	[5.07]
NVIX$_{t-1}$	0.22	0.14	-0.12	0.22	0.24
	[2.00]	[1.22]	[-0.85]	[2.10]	[2.15]
Slope of yield curve$_{t-1}$		-0.54			
		[-1.78]			
GDP volatility$_{t-1}$			0.60		
			[3.93]		
NBER recession dummy$_t$				1.43	
				[5.18]	
Financial cycle downturn dummy$_t$					0.50
					[2.47]
Log likelihood	-771	-773	-779	-789	-774
Test for no time deformation ($\boldsymbol{c}$=0)		3.17	15.52	34.56	6.14
p-value of test		0.05	0.00	0.00	0.01

Numbers in brackets report the t-statistics. The test for no time deformation is a likelihood ratio test for the joint significance of the c coefficients in (3). The Q defined in Appendix B is estimated to be between 0.01 and 0.02 across specifications.

The coefficient on the financial cycle downturn dummy is positive and statistically significant. Taken at face value, this suggests that time deformation may depend systematically on the phase of the financial cycle. One might conjecture that financial busts naturally lead to an acceleration in calendar time due to the rapid changes in the macro-financial environment. An alternative implication is purely statistical in the sense that financial downturns may entail nonlinear dynamics associated with doom loops and other

[16] Another important feature of empirical financial cycles is the time-varying amplitude. To address potential bias in our estimates from this, we re-ran our baseline model with transformed financial cycle data in which each cycle is re-scaled to have a constant variance (in calendar time). We find a drop in statistical significance of the estimates. However, they generally remain significant (except inflation volatility) and support our conclusion about the association between the risk environment and time deformation.

amplification mechanisms that are not captured in our g function. Such possibilities may also call for a more elaborate statistical models, such as a CAR(2).

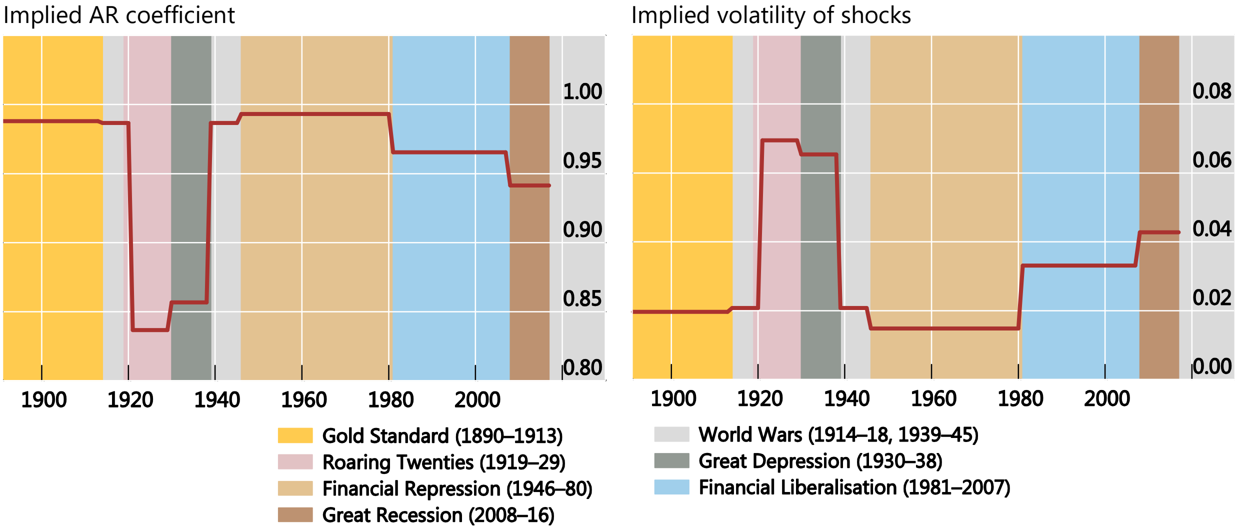
As a final robustness check, we restrict the z variables to be dummy variables which correspond to historical regimes, starting with the Gold Standard period of 1890-1912. The regimes correspond to conventional designations over the past 120 years: Gold Standard, world wars, Roaring Twenties, Great Depression, Financial Repression, Financial Liberalisation and the Great Recession. The coefficient on each dummy variable reflects the average level of time deformation in each era. This test has the benefit of avoiding spurious correlations that may arise when using financial market covariates. Table 4 shows that the average time deformation systematically differs across economic eras. The estimates of Δg during each economic era indicate that calendar time ran slower relative to financial cycle time when below 1 and faster when above 1. Graph 5 displays the implied AR coefficients and variances in calendar time. For example, the Gold Standard and Financial Repression eras exhibit high AR coefficients in periods of low error variances. But the Roaring Twenties, Great Depression and Great Recession eras by contrast were characterised by lower ARs and higher variances.

Average time deformation, by economic era — Table 4

Λ	Gold Standard$_t$	World Wars$_t$	Roaring Twenties$_t$	Great Depression$_t$	Financial Repression$_t$	Financial Liberalisation$_t$	Great Recession$_t$
-0.41	0.29	0.33	4.30	3.73	0.17	0.85	1.46
[-2.55]	(0.10, 0.46)	(0.11, 0.66)	(1.74, 7.16)	(1.42, 6.51)	(0.06, 0.30)	(0.34, 1.41)	(0.45, 3.10)

Numbers in brackets report the t-statistics; those in parentheses are 90% confidence intervals. The test statistic for the test $\boldsymbol{c}$=0 is 165.12 with a p-value of 0.00. The Q defined in Appendix B is estimated to be 0.01. The estimates are scaled so that the threshold between dilation and compression of calendar time is 1.

Implied calendar-time evolution of the parameters for the model with regimes — Graph 5



Source: Authors' calculations.

5. Possible interpretations

Our findings raise questions about the possible mechanisms that could be responsible for endogenous, recurrent financial cycles subject to time deformation. Various economic mechanisms could be at work. We group these into two broad categories: behavioural and informational.[17]

Behavioural economics has recently made concrete strides in building models of agents' cognitive processing based on experimental and experiential data (see eg Gabaix (2016, 2017) and the references therein). This interest in cognitive processing, however, has a long history in macroeconomics and monetary policy.[18] Allais (1966, 1972) introduced the idea of psychological time into the discussion of money demand. He argued that a psychological time scale, reflecting the need to convert past price experiences into expectations about the future, naturally led one to consider stages of the cycle rather than a year as the appropriate unit of time.[19] This strand of the literature emphasising economic (versus calendar) time scales had an impact on the thinking of Friedman and Schwartz (1982) when trying to reconcile the Gibson paradox (ie the relationship between the nominal interest rate and the price *level* in historical data) with the logic behind the Fisher equation.

Other mechanisms have been emphasised in the behavioural literature. A recent paper by Malmendier and Nagel (2016) argues that agents may overweight personal experiences when forming subjective expectations and making decisions over time.[20] In the case of inflation expectations, this *learning-from-experience mechanism* implies that agents rely more heavily on inflation experienced during their lifetimes than on other historical data when forming expectations. The authors conclude that this perspective helps to explain why persistent disagreements about inflation expectations are associated with age cohorts.[21] Looking at labour market dynamics, Storesletten et al (2004) find it useful to use *cyclical-phase-dependent* cross-sectional moments as far back to 1930 as a means to understand labour income risk perceptions in their panel from 1968–93. Studies of financial markets have also emphasised the role of phase-dependent behaviour. For example, Malmendier and Nagel

[17] A third category would be purely statistical, which addresses general issues in nonlinear processes and mixed frequency biases in aggregated data.

[18] Here we recognise the contributions of various authors such as Aliber and Kindleberger (2015), Minsky (1982), Meltzer (2003) and Shiller (2015). They provide narratives about irrational exuberance and pessimism in financial markets. Their qualitative characterisations generally include behavioural assumptions attributed to basic human nature, such as ego, envy, inattention and framing. These views have been regarded more favourably since the GFC.

[19] For a revival in interest in the theories of Allais, see Barthalon (2014). In a different vein, Hall (2017) focuses on time-varying financial discount rates to explain recent cyclical behavior in unemployment rates. Albuquerque et al (2017) also look to time variation in discount rates but focus on high and low frequency changes in investors' time preference to explain the link between fundamentals and stock returns.

[20] Chevillon and Mavroeidis (2017) note that rational agents updating their beliefs by placing different weights on past observations could lead to long memory–in the statistical form of fractional integration–at the aggregate level. Although this is not the same concept of long memory that we are investigating in this paper, extending our approach to this class of fractionally integrated processes could be an interesting avenue for future research.

[21] As they note, this "builds on the psychology evidence on the role of personal experiences and availability bias (Tversky and Kahneman (1973)) rather than on the stochastic properties of macroeconomic variables to explain why data in the distant past is ignored."

(2016) look at whether age cohorts assess financial risks differently because of the macroeconomic experiences that they lived through. They find systematic age-cohort effects on portfolio allocations based on having lived through long periods of either low or high returns. They note that this type of return behaviour has a strong influence on the behaviour of the young, which may help to explain why financial boom-bust cycles are associated with periods of long financial upturns in calendar time; in other words, having never experienced a financial downturn young investors may be particularly prone to endogenous risk-taking behaviour. Bordalo et al (2017a, b) develop a theory of *diagnostic expectations* in macro-financial data, arguing that there is a 'kernel of truth' property where belief updating results in bigger revisions in expectations than constant-gaining learning would suggest. And, Koudijs and Voth (2014) find corroborating historical evidence looking back over the centuries that personal investment experiences, especially large losses, play a disproportionate role in influencing risk tolerance and hence leverage cycles. Finally, the literature on c*onfirmation bias* would naturally be related to the risk-taking channel and stretching of calendar time.[22]

Informational frictions also provide a mechanism through which financial cycles may be subject to time deformation. Such frictions influence the timing of updates by agents when forming subjective expectations. Stock (1987) mentions a number of earlier papers that justify a distinction between calendar time and business cycle time, such as Barro (1970); Jordá (1999) offers a more recent example of agents' (S, s) decision rules and the nature of the time deformation that this can induce in time series data. In the case of financial booms and busts, Zeira (1999) and Burnside et al (2016) highlight informational setups that lead to non-homogenous expectation formation. These models focus on informational overshooting in the sense that agents may put too much weight on optimistic outcomes, only to be disappointed. At a more fundamental level of information processing, the sticky information model of Mankiw and Reis (2002) assumes that information disseminates slowly across an economy and hence results in delayed responses of agents and therefore induces persistence in macroeconomic outcomes. Sims (2010) models rational inattention using Shannon entropy to reflect the finite information processing capacity of economic agents. In his model, the size of shocks and their persistence influences the pace at which agents update their expectations and make decisions. This type of behaviour provides a foundation for time deformation that may help to explain important features of financial booms, such as underestimating adverse tail risks during long periods of favourable macro-financial performance.

It should be also noted that the approach taken in this paper is very much subject to the criticism that it is measurement without theory, similar to that which was initially aimed at Burns and Mitchell.[23] However, our statistical approach does take us down the road from the ad hoc statistical approach by offering a statistical structure that, in principle, could be embedded in a theory of financial cycles. Recent advances in the field of behavioural economics may also present opportunities to connect our results to underlying endogenous

[22] The reduced-form rational belief models for exchange rates of De Grauwe and Grimaldi (2006) are consistent with this mechanism is a world with heterogeneous agents using simple rules of thumb.

[23] See Koopmans (1947) for details of his measurement-without-theory argument.

risk-taking economic behaviour of consumers, firms, financial institutions and investors.[24] In general, we view the findings in this paper as representing a set of empirical observations–so called, stylised facts about financial cycles–for which a more fulsome model of the financial cycle would need to account.

Finally, our empirical results are consistent with models featuring endogenous risk-taking channels. During risk-on periods when imbalances are growing and the financial cycle is in a boom, real interest rates, corporate spreads and real GDP volatility (along with the VIX) tend to be low, which according to our results correspond to periods when calendar time becomes dilated relative to financial cycle time. Conversely, during bust periods, some or all of these risk measures can sharply rise. Empirically, this type of time deformation would result in long financial booms and shorter financial busts of the type that we have seen in the historical data across many countries.

6. Conclusions and policy implications

Our paper demonstrates that the statistical methods initially designed for measuring business cycle time are useful for measuring financial cycle time. We showed that it is possible to jointly estimate the time series behaviour of financial conditions and the nature of the time deformation. As a by-product of this analysis, we produced a new set of stylised facts about financial cycles, which may help motivate theoretical modelling. However, to understand the wider implications of these findings, we need new theoretical models that can capture the mechanisms that generate this type of behaviour. In particular, a model that purports to capture financial cycle dynamics should be able to match the stylised facts about time deformation. Our identification of the variables associated with the extent of the time deformation provides useful clues about what such a model might look like.

With our approach to measure financial cycle time, we have found statistical evidence of significant time deformation in the US financial cycle. The extent of the time deformation appears to be associated with a set of variables measuring subjective risk perceptions. These variables include the level of the long-term interest rate, which has been identified as an important indicator of the strength of risk-taking channel. The other key variables are the volatility of inflation, corporate spreads and the NVIX. These are more direct measures of the riskiness of the macro-financial environment.

The recurrent nature of the financial cycle in financial cycle time also highlights the strength of the underlying forces influencing swings in financial conditions. Over the past 120 years covered by our dataset, there has been a wide range of economic and policy environments that influenced the financial cycle. The United States went from being an

[24] Recent research into GDP-at-risk models stress the role of endogenous risk-taking behavior and bank intermediation frictions in generating heteroskedastic output volatility. These models provide microfoundations of the type which addresses interactions between financial cycles and business cycles. See, for example, Adrian and Duarte (2018) and Adrian et al (2016, 2018). As well, Brunnermeier and Sannikov (2014) argue that low volatility environments lead to increased leverage in an endogenous fashion. Our results are consistent with this view that a low risk environment (characterised by low real rates, corporate spread compression, low inflation volatility and a low VIX) encourages long financial booms.

emerging market to an advanced economy with a highly open financial market. It has gone through the Gold Standard period when inflation was low as well as the 1970s when inflation was high and volatile. Over this long historical period, the price stability credentials of central banks have waxed and waned. As well, fiscal and regulatory policies have varied considerably. Through all this, the financial cycle dynamics remain a constant feature of the economy.

This leads us to speculate that financial cycles are intrinsic to all financially liberalised, market-oriented economies. In this respect, we find attractive those theories that point to the endogenous risk-taking behaviour of agents in the way they process information when forming expectations about the future and especially about risk assessments. This is not a new observation. For example, Minsky (1982) and Aliber and Kindleberger (2015) put forth similar perspectives using historical narratives. The main difference with our study is that we are led to this conclusion by looking at historical data via a formal statistical model emphasising the lens of financial cycle time.

Our findings also shed light on the type of economic mechanisms that might be relevant. Namely, the estimated time deformation is consistent with the information processing and behavioural mechanisms highlighted in the introduction. One facet of these mechanisms that stands out is that they are associated with basic human behaviour and are not regime dependent. In other words, these mechanisms could be seen as reflecting the fundamentals of human behaviour when agents are free to choose from the opportunities that confront them. As such, it may be reasonable to look over long histories to better understand current-day challenges, even though the 19th and early 20th centuries, for example, would appear to be so fundamentally different, say, economically and culturally than today.

With respect to policy implications, our paper provides a new way to analyse the state of financial conditions. For example, easy financial conditions during a period of strong calendar time dilation (relative to financial cycle time) appear to be consistent with rising concerns about the behaviour associated with costly booms and busts. In contrast, easy financial conditions during a period of calendar time compression is more closely associated with models of the financial accelerator, which have less to do with booms and busts and more to do with the amplification role financial conditions can play in the business cycle. Therefore, the information about the financial cycle viewed from the time deformation perspective may help to clarify when policy should 'lean against the wind' and when it simply needs to be more countercyclical.

While our study focuses on past financial cycles, it would be useful in the future to explore the nature of time deformation spillovers from the financial cycle to the business cycle – especially during balance sheet recessions – and to financial cycles in other countries. From a modelling perspective, more attention deserves to be spent on models capturing endogenous financial cycles that are subject to time deformation and the implications for the design of policy frameworks.

Appendices

A. *Data sources*

Measure of financial conditions: US credit to the private non-financial sector (scaled by potential GDP). Annual nominal credit data for 1952-2017 (year-end values) from the BIS; for 1880-1952, we proxy credit data using growth rates of total bank loans growth from Jorda et al (2017). Nominal GDP data for 1880-2013 from Jorda et al. (2017); for 2014-2017, from the US Bureau of Economic Analysis. Potential GDP estimated as the trend from a Hodrick-Prescott filter (smoothing parameter of 100). Finally, we use a band-pass filter for frequencies 8-32 years for smoothing the credit ratio.

Real house price index data for 1950-2017 is taken from Schiller (2015); for the 1880-1945 period, due to poor quality of house price measures, we proxy the real house price index with real equity index (also based on Schiller (2015)); for 1945-1950 we proxy house price index by a weighted average of real house price index and real equity price index. In order to smooth the credit ratio, we use a band-pass filter for frequencies of 8 to 32 years.

Long-term real interest rate: nominal long-term rate less the 10-year centred average rate of annual inflation. Annual long-term interest rate for 1880-2013, from Jorda et al. (2017). For 2014-2017, annual average yield on US Treasury securities (10-year constant maturity) from the Federal Reserve. Annual inflation data for 1880-1913 calculated as year-on-year changes in the General Price Index from the NBER macro-history data. For 1914-2017, year-on-year change in annual CPI from FRED.

Inflation volatility: Centred 5-year moving standard deviation of the annual inflation data.

Corporate spread: Long-term corporate yield less the long-term US treasury yield. For 1880-1918, annual corporate yield from Gordon (1986); from 1919-2017, Moody's annual (seasoned) Baa corporate bond yield from FRED.

Volatility index: stock market implied volatility. For 1986-2017, annual average of the CBOE VXO from FRED; for 1890-1986, the NVIX volatility index from Manela and Moreira (2017).

These candidate variables for the g function: Data lagged one year and smoothed using a one-sided Hodrick-Prescott filter.

B. *Estimation of time deformation via the Kalman filter*

A continuous-time AR processes satisfying equation (1) can be estimated using the Kalman filter (Harvey and Stock, 1985) with the modification proposed by Stock (1988) to allow for time deformation. This section gives further details on the methodology.

We start by re-writing (1) in stacked form, so that the state equation for a general CAR(*k*) process can be expressed as:

(B.1) $$d\bar{\Xi}(s) = A\bar{\Xi}(s)ds + d\zeta(s),$$

where

$$\bar{\Xi}(s) = \begin{bmatrix} \xi(s) \\ D\xi(s) \\ \vdots \\ D^{k-1}\xi(s) \end{bmatrix}, \qquad A = \begin{bmatrix} 0 & & & \\ \vdots & & I & \\ 0 & & & \\ A_k & A_{k-1} & \cdots & A_1 \end{bmatrix}.$$

By diagonalising[25] $A = \Gamma\Lambda\Gamma^{-1}$, premultiplying equation (B.2) by Γ^{-1}, and setting $\Xi(s) = \Gamma^{-1}\bar{\Xi}(s)$, the resulting state equation is

(B.2) $$d\Xi(s) = \Lambda\Xi(s)ds + \Gamma^{-1}d\zeta(s).$$

To simplify notation, the error term can be redefined as $\eta(s) = \Gamma^{-1}\zeta(s)$, with a variance $Q = \Gamma^{-1}\Omega(\breve{\Gamma}^{-1})'$; the inverted hat indicates a matrix of complex conjugates.

This is a first-order system, so it has a closed-form solution akin to equation (2):

(B.3) $$\Xi(s) = e^{\Lambda(s-s^-)}\,\Xi(s^-) + \int_{s^-}^{s} e^{\Lambda(s-r)}d\eta(r).$$

Similar to the CAR(1) example in equation (5), we can express equation (B.4) in calendar time by setting $f(t) = \Xi(g(t))$:

(B.4) $$f_t = e^{\Lambda\Delta g(t;z_{t-1})}f_{t-1} + v_t, \qquad v_t = \int_0^{\Delta g(t;z_{t-1})} e^{\Lambda(g(t)-r)}d\eta(r).$$

Defining

(B.5) $$H_t = e^{\Lambda\Delta g(t;z_{t-1})}$$

and

(B.6) $$Q_t = \int_0^{\Delta g(t;z_{t-1})} e^{\Lambda(g(t)-r)}Q\,e^{\Lambda(g(t)-r)}dr,$$

equation (B.4) can be written in the more familiar format of a linear state equation:

(B.7) $$f_t = H_t f_{t-1} + v_t, \qquad v_t \sim N(0, Q_t).$$

Note that the integral in equation (B.6) depends on the shape of the g function and the assumption that z is pre-determined at time t.[26] Furthermore, under the simplifying assumption that g can be well approximated by a piecewise linear function, that is

[25] This assumes A has distinct characteristic roots.

[26] See Appendix C for a discussion of how to relax this assumption.

(B.8) $$g(\tau) = g(t-1) + (\tau - (t-1))\Delta g(t; z_{t-1}) \quad \forall \tau \in \left]t-1, t\right],$$

the evaluation of the integral is simplified, with the elements of Q_t having the following form:

(B.9) $$Q_{ij,t} = -q_{ij}\left(1 - H_{ii,t}\breve{H}_{jj,t}\right)/\left(\lambda_{ii} + \breve{\lambda}_{jj}\right),$$

where q_{ij} and λ_{ij} are, respectively, the ij-th elements of Q and Λ.

As for the measurement equation, equation (3) can be rewritten in the following form:

(B.10) $$Y_t = \xi(g(t)) + \varepsilon_t, \qquad t = 1, \ldots, T,$$

where ε_t is a n-dimensional white noise process with zero mean, is independent of v_t and has a covariance matrix Σ.

Plugging (B.7) into (B.10), we then obtain

(B.11) $$Y_t = \Gamma f_t + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \Sigma).$$

The system defined by equations (B.7) and (B.11) is a conventional linear state space model. Being so, f_t can be calculated using the Kalman filter and the likelihood can be maximised with respect to the unknown parameters Θ ≡ $[c,\ \Lambda,\ \Gamma,\ Q,\ \Sigma]$. As a byproduct of the Kalman filter, f_t can be updated via one-step-ahead predictions of the latent variable,

(B.12) $$\hat{f}_{t|t-1} = H_t \hat{f}_{t-1}.$$

The covariance matrix of the prediction error of the latent variable $\hat{f}_{t|t-1} - f_t$ is

(B.13) $$P_{t|t-1} = H_t P_{t-1} \breve{H}_t' + Q_t.$$

The one-step ahead prediction of the observed data is

(B.14) $$\hat{Y}_{t|t-1} = \Gamma \hat{f}_{t|t-1}$$

and related prediction error is $e_t = Y_t - \hat{Y}_{t|t-1}$, which has variance

(B.15) $$F_t = \Gamma P_{t|t-1} \breve{\Gamma}' + \Sigma.$$

The estimate of the latent variable is updated by iterating on the following equation:

(B.16) $$\hat{f}_t = \hat{f}_{t|t-1} + P_{t|t-1} \breve{\Gamma}' F_t^{-1} e_t$$

and the covariance matrix of the prediction error updates according to the following equation:

(B.17) $$P_t = P_{t|t-1} - P_{t|t-1}\breve{\Gamma}' F_t^{-1} \Gamma P_{t|t-1}.$$

The filter is initialised using asymptotic averages of the process, that is

$$\hat{f}_{1|0} = 0, \quad P_{ij,1|0} = -q_{ij} \Big/ \left(\lambda_{ii} + \breve{\lambda}_{jj} \right).$$

The resulting log-likelihood function can be then written as

(B.18) $$\ell(\Theta) = \sum_{t=1}^{T} \log\left|F_t\right| + \sum_{t=1}^{T} e_t' F_t^{-1} e_t ,$$

which can be maximised using standard algorithms to obtain parameter estimates and (filtered and smoothed) estimates of the latent process, f_t.

C. *Admissibility of z_t variables and the nature of exogeneity*

In Appendix B, we justify the use of the Kalman filter methodology assuming that z_t is pre-determined with respect to Y_t. In this appendix, we discuss the exogeneity conditions that expand the set of admissible z_t variables for the *g* function. Consider Y_t and z_t as being represented by a general linear model of the economy, which can be express as

(C.1) $$A_0 Y_t + A_1 Y_{t-1} + \cdots + A_p Y_{t-p} + B_0 z_t + \cdots + B_p z_{t-p} + c = u_t ,$$

where u_t is a white noise error term uncorrelated with the past values of Y_t and has a generic covariance matrix S.

Based on Engle et al (1983), z_t is *pre-determined* in equation (C.1) if

(C.2) $$z_t \perp u_{t+i} \qquad \forall i \geq 0;$$

in this case, equation (C.1) has a *weak structural form*. And, z_t is defined as *strictly exogenous* in (C.1) if

(C.3) $$z_t \perp u_{t+i} \qquad \forall i;$$

in this case, equation (C.1) has a *strong structural form*.

In general, such conditions cannot be guaranteed. However, economic theory may impose a set of cross-equation restrictions with implications for the exogeneity conditions. The constraints on A_0 and B_0 are particularly relevant as they relate to the contemporaneous correlation structure of Y_t and z_t; one such useful constraint can be expressed as:

(C.4) $$R_0 \, \mathrm{vec}[A_0 \; B_0] = r_0.$$

In this case with $\Theta \equiv [c,\ A_0, \ldots,\ A_p,\ B_0,\ \ldots,\ B_p]$, we can define a one-to-one transformation $h : \Theta \rightarrow \Psi$ that allows us to impose restrictions on equation (C.4) and to rewrite equation (C.1) in a block-recursive form:

(C.5) $$\begin{bmatrix} \Phi_Y(L) & \Phi_{Yz}(L) \\ \Phi_{zY}(L) & \Phi_z(L) \end{bmatrix} \begin{bmatrix} Y_t \\ z_t \end{bmatrix} = \begin{bmatrix} c_Y \\ c_z \end{bmatrix} + \begin{bmatrix} w_{Y,t} \\ w_{z,t} \end{bmatrix},$$

where $\Phi(L)$ are polynomials in the lag operator and the covariance matrix of the errors is block-diagonal. Under the contemporaneous correlation structure in equation (C.2), z_t is weakly exogenous if and only if

(C.6) $$A_0 \Phi_{Yz0} + B_0 = 0\,;$$

z_t is strictly exogenous if in addition to (C.3)

(C.7) $$\Phi_{zYi} = 0, \qquad i = 1, \ldots, p\,.$$

Condition (C.6) implies a contemporaneous correlation structure that is consistent with the cross-equation restrictions with shocks $w_{Y,t}$ and $w_{z,t}$ being orthogonal. Condition (C.7) implies an additional requirement that z should not Granger-cause Y.

Under these assumptions, the Kalman filter recursion can be rewritten with contemporaneous values of z_t appearing in the g function. Equation (B.4) can be written as

(C.8) $$\vec{f}_t = e^{\Lambda \Delta g(t; z_{t-1} + w_{z,t})} \vec{f}_{t-1} + \vec{v}_t, \qquad \vec{v}_t = \int_0^{\Delta g(t; z_{t-1} + w_{z,t})} e^{\Lambda(g(t) - r)} d\eta(r)\,.$$

Defining

(C.9) $$\overrightarrow{H}_t = e^{\Lambda \Delta g(t; z_{t-1} + w_{z,t})}$$

and

(C.10) $$\vec{Q}_t = \int_0^{\Delta g(t; z_{t-1} + w_{z,t})} e^{\Lambda(g(t) - r)} Q e^{\Lambda(g(t) - r)} dr,$$

the state equation (B.7) can be rewritten as

(C.11) $$\vec{f}_t = \overrightarrow{H}_t \vec{f}_{t-1} + \vec{v}_t, \qquad \vec{v}_t \sim N(0, \vec{Q}_t)\,.$$

Evaluating the upper limit of the integral in (C.10) is stochastic, so $\vec{Q}_t$ cannot be readily computed. But re-writing this in the following form:

and noting the mean of $w_{z,t}$ is zero, we obtain

This derivation highlights the role of the exogeneity assumption; namely, innovations to the Q_t matrix (and hence to the state vector) are orthogonal to those of the observed equation. Furthermore, under the strict exogeneity assumption, the shocks to z_t would influence Y_t only through the changes in the integration limits of equation (C.10).

According to equation (C.13), the prediction error variance of the latent variable is the same as in the case with pre-determined z_t (B.13). As soon as z_t becomes available, however, one would need to update equation (C.13) with the new estimate $\vec{Q}_t$. This would then be used to update the estimate of the state variable as in (B.16).

This appendix has outlined the conditions under which we can extend the Kalman filter approach in the paper to allow for contemporaneous values of $\boldsymbol{z}_t$ in the g function. Weak exogeneity can be secured by imposing theory-based constraints on the contemporaneous correlation structure. However, the additional assumption ensuring strong exogeneity is that of Granger non-causality from Y_t to $\boldsymbol{z}_t$, which is necessary to guarantee that $\boldsymbol{z}_t$ does not directly influence Y_t, but only does so through the time deformation function, g. In practical terms, this assumption can be quite restrictive as it implies that all the potential drivers of the financial cycle only affect it through time deformation.

References

Adrian, T, and F Duarte (2018): "Financial vulnerability and monetary policy", CEPR Discussion Papers, no DP12680, February.

Adrian, T, N Boyarchenko and D Giannone (2017): "Vulnerable growth", Federal Reserve Bank of New York Staff Reports, no 794.

Adrian, T, F Grinberg, N Liang and S Malik (2018): "The term structure of growth-at-risk", IMF Working Papers, no 18/180, August.

Albuquerque, R, M Eichenbaum, D Papanikolaou and S Rebelo (2015): "Long-run bulls and bears", *Journal of Monetary Economics*, 76(S), pp S21-S36.

Aliber, R, and C Kindleberger (2015): *Manias, Panics, and Crashes: A History of Financial Crises*, seventh edition, Palgrave.

Allais, M (1966): "A restatement of the quantity theory of money", *American Economic Review*, vol 56(5), pp 1123-57.

Allais, M (1972): "Forgetfulness and interest", *Journal of Money, Credit and Banking*, vol 4(1), part 1, pp 40-73.

Bank for International Settlements (BIS) (2016): *86th Annual Report*, chapter IV.

Barro, R (1970): "Inflation, the payments period, and the demand for money", *Journal of Political Economy*, vol 78(6), pp 1228-63.

Barthalon, E (2014): *Uncertainty, Expectations and Financial Instability: Reviving Allais's Lost Theory of Psychological Time*, Columbia University Press.

Bernanke, B, M Gertler and S Gilchrist (1999): "The financial accelerator in a quantitative business cycle framework", in (J Taylor and M Woodford, eds) *Handbook of Macroeconomics*, Elsevier, vol 1, chapter 21, pp 1341-93.

Bordalo, P, N Gennaioli, Y Ma and A Shleifer (2017a): "Overreaction in macroeconomic expectations", unpublished Harvard University manuscript, December.

Bordalo, P, A Gennaioli and A Shliefer (2017b): "Diagnostic expectations and credit cycles", *The Journal of Finance*, 73(1), pp 199-227.

Borio, C (2013): "On time, stocks and flows: understanding the global macroeconomic challenges", *National Institute Economic Review*, vol 225(1), pp R3-R13.

Borio, C, P Disyatat, M Drehmann and M Juselius (2017): "Monetary policy, the financial cycle, and ultra-low interest rates," *International Journal of Central Banking*, vol 13(3), pp 55-89.

Borio, C, M Drehmann and K Tsatsaronis (2012): "Characterising the financial cycle: don't lose sight of the medium term!", BIS Working Papers, no 380, June.

Brunnermeier, M, and Y Sannikov (2014): "A macroeconomic model with a financial sector", *American Economic Review*, 104(2), pp 379-421.

Bruno, V, and H-S Shin (2015): "Cross-border banking and global liquidity", *Review of Economic Studies*, vol 82(2), pp 1-30.

Bry, G, and C Boschan (1971): *Cyclical Analysis of Time Series: Procedures and Computer Programs*, National Bureau of Economic Research.

Burns, A, and W Mitchell (1946): *Measuring Business Cycles*, National Bureau of Economic Research.

Burnside, C, M Eichenbaum and S Rebelo (2016): "Understanding booms and busts in housing markets", *Journal of Political Economy*, vol 124(4), pp 1088-147.

Cagan, P (1956): "The monetary dynamics of hyperinflation", in (M Friedman, ed) *Studies in the Quantity Theory of Money*, University of Chicago Press.

Chevillon, G, and S Mavroeidis (2017): "Learning can generate long memory", *Journal of Econometrics*, vol 198(1), pp 1-9.

Claessens, S, M Ayhan Kose and M Terrones (2011): "Financial cycles: what? how? when?", *NBER International Seminar on Macroeconomics*, vol 7(1), pp 303-34.

De Grauwe, P, and M Grimaldi (2006): *The Exchange Rate in a Behavioral Finance Framework*, Princeton University Press.

DeLong, B, and L Summers (1986): "Cyclical variability of economic activity in the United States", in (R Gordon, ed) *The American Business Cycle: Continuity and Change*, University of Chicago Press.

Duval, R, GH Hong and Y Timmer (2017): "Financial frictions and the great productivity slowdown", IMF Working Papers, no 17/129, May.

Eckstein, O, and A Sinai (1986): “The mechanisms of the business cycle in the postwar era”, in (R Gordon, ed) *The American Business Cycle: Continuity and Change*, University of Chicago Press.

Engle, R, D Hendry, and J Richard (1983): “Exogeneity”, *Econometrica*, vol 51(2), pp 277-304.

Filardo, A, and D Guinigundo (2008): “Transparency and communication in monetary policy: a survey of Asian central banks”, paper presented at the BIS-BSP Research Conference, April.

Filardo, A, and P Rungcharoenkitkul (2016): “A quantitative case for leaning against the wind”, BIS Working Papers, no 594, December.

Friedman, M, and A Schwartz (1982): *Monetary Trends in the United States and United Kingdom: Their Relation to Income, Prices, and Interest Rates, 1867–1975*, National Bureau of Economic Research.

Gabaix, X (2016): “A behavioral New Keynesian model”, NBER Working Papers, no 22954, December.

Gabaix, X (2017): “Behavioral inattention”, NBER Working Paper, no 24096, December.

Gilchrist, S, and E Zakrajsek (2012): “Credit spreads and business cycle fluctuations”, *American Economic Review*, vol 102(4), pp 1692-720.

Gordon, R (1986): *The American Business Cycle: Continuity and Change*, University of Chicago Press.

Gourio, F, A Kashyap and J Sim (2017): “The tradeoffs in leaning against the wind”, NBER Working Papers, no 23658, August.

Hall, R (2013): “Financial frictions”, *International Journal of Central Banking*, vol 9(2), pp 155-63.

Hall, R (2017): “High discounts and high unemployment”, *American Economic Review*, vol 107(2), pp 305-30.

Harvey, A (1989): *Forecasting, Structural Time Series Models and the Kalman Filter*, Cambridge University Press.

Harvey, A, and J Stock (1985): “The estimation of higher order continuous time autoregressive models”, *Econometric Theory*, vol 1, pp 97-112.

Jordá, Ò (1999): “Random-time aggregation in partial adjustment models”, *Journal of Business and Economic Statistics*, vol 17(3), pp 382-95.

Jordá, Ò, M Schularick and A Taylor (2017): “Macrofinancial history and the new business cycle facts”, in (M Eichenbaum and J Parker, eds) *NBER Macroeconomics Annual 2016*, vol 31, University of Chicago Press.

Koopmans, T (1947): “Measurement without theory”, *The Review of Economics and Statistics*, vol 29(3), pp 161-72.

Koudijs, P, and H-J Voth (2014): “Leverage and beliefs: personal experience and risk taking in margin lending”, NBER Working Papers, no 19957, March.

Lindé, J (2018): “DSGE models: still useful in policy analysis?”, *Oxford Review of Economic Policy*, vol 34(1–2), pp 269-86.

Lucas, R (1981): *Studies in Business-Cycle Theory*, MIT Press.

Malmendier, U, and S Nagel (2011): "Depression babies: do macroeconomic experiences affect risk taking?", *The Quarterly Journal of Economics*, vol 126(1), pp 373-416.

Malmendier, U, and S Nagel (2016): "Learning from inflation experiences", *The Quarterly Journal of Economics*, vol 131(1), pp 53-87.

Manela, A, and A Moreira (2017): "News implied volatility and disaster concerns", *Journal of Financial Economics*, vol 123(1), pp 137-62.

Mankiw, G, and R Reis (2002): "Sticky information versus sticky prices: a proposal to replace the New Keynesian Phillips curve", *The Quarterly Journal of Economics*, vol 117(4), pp 1295-1328.

Melino, A (1994): "Estimation of continuous-time models in finance", in (C Sims, ed) *Advances in Econometrics: Sixth World Congress*, vol II, Cambridge University Press.

Meltzer, A (2003): "Rational and nonrational bubbles", in (W Hunter, G Kaufman and M Pomerleano, eds) *Asset Price Bubbles: The Implications for Monetary, Regulatory, and International Policies,* MIT Press.

Mian, A, and A Sufi (2018): "Finance and business cycles: the credit-driven household demand channel", NBER Working Papers, no 24322, February.

Minsky, H (1982): *Can "It" Happen Again?*, ME Sharpe.

Moore, G (1961): *Business Cycle Indicators*, Princeton University Press.

Reinhart, C, and K Rogoff (2011): *This Time Is Different: Eight Centuries of Financial Folly*, Princeton University Press.

Rünstler, G, and M Vlekke (2016): "Business, housing and credit cycles", ECB Working Papers, no 1915, June.

Schularick, M, and A Taylor (2012): "Credit booms gone bust: monetary policy, leverage cycles, and financial crises, 1870-2008", *American Economic Review*, vol 102(2), pp 1029-61.

Shiller, R (2015): *Irrational Exuberance*, third edition, Princeton University Press.

Sims, C (2010): "Rational inattention and monetary economics", in (B Friedman and M Woodford, eds) *Handbook of Monetary Economics*, vol 3, Elsevier, pp 155-81.

Stock, J (1987): "Measuring business cycle time", *Journal of Political Economy*, vol 95(6), pp 1240-61.

Stock, J (1988): "Estimating continuous-time processes subject to time deformation: an application to postwar U.S. GNP", *Journal of the American Statistical Association*, vol 83(401), pp 77-85.

Stock, J, and M Watson (1998): "Median unbiased estimation of coefficient variance in a time-varying parameter model", *Journal of the American Statistical Association*, vol 93(441), pp 349-58.

Storesletten, K, C Telmer and A Yaron (2004): "Cyclical dynamics in idiosyncratic labor market risk", *Journal of Political Economy*, vol 112(3), pp 695-717.

Tversky, A, and D Kahneman (1973): "Availability: a heuristic for judging frequency and probability", *Cognitive Psychology*, vol 5(2), pp 207-32.

Zarnowitz, V (1992): *Business Cycles: Theory, History, Indicators, And Forecasting*, University of Chicago Press.

Zeira, J (1999): "Informational overshooting, booms, and crashes", *Journal of Monetary Economics*, vol 43(1), pp 237-57.