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Official demand for US debt: implications for US real rates

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Abstract

We estimate a structural term-structure model of US real rates, where arbitrageurs accommodate demand pressures exerted by domestic and foreign official investors. Official demand affects rates by altering the aggregate price of duration risk, and thereby bond risk premiums. While foreign central banks' demand contributed to reduce long-term real rates mainly in the years prior to the global-financial crisis, the Federal Reserve's demand lowered rates during the QE period. Overall, the two-factor model, augmented to account for changing liquidity conditions, offers a good representation of real rates during the 2001–2016 period; however, we flag some caveats and possible extensions.

Key words: Term structure of real rates, quantitative easing, global imbalances, Bayesian econometrics.

JEL classification: F31, G10.

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1. INTRODUCTION

Since the early 2000s purchases of U.S. Treasury bonds by official investors have reached unprecedented levels becoming an increasingly important tool in central banks' policies. Foreign officials, *i.e.*, foreign central banks, were the first to significantly increase their holdings of U.S. Treasury bonds, as part of their reserve accumulation policies (ECB, 2006). Then, during the recent financial crisis, as policy interest rates approached the zero lower bound (ZLB), the Federal Reserve (Fed) launched the policy of quantitative easing (QE), and as a result the Fed also became an important active investor in the Treasury market. The objective of this unconventional monetary policy (UMP) was to stimulate the economy by reducing long-term interest rates through a series of asset purchase programs. Specifically, the Fed's Large Scale Asset Purchases (LSAPs) mainly focused on longer-term securities, including government bonds and mortgage-backed securities (MBSs).

Two features in particular distinguish foreign and domestic policy makers from a typical U.S. Treasury bond investor. First, official investors stand out for their massive holdings of Treasury securities. Second, their demand displays relatively low price elasticity, in that, it is only slightly sensitive to risk-return considerations (see, *e.g.*, Krishnamurthy and Vissing-Jorgensen, 2012). Of further interest is the rather high duration of the Treasury securities held by official investors. These facts, taken together, lie behind the emerging consensus among market participants, academics and policy makers that such inelastic, large-scale official purchases of U.S. Treasury bonds have qualitatively contributed to lowering U.S. long-term interest rates.

A natural question though is how big is the impact of official demand on yields? We answer this question by estimating a structural arbitrage-free model on U.S. real rates. Specifically, we organize the analysis around the 'preferred-habitat' model proposed by Vayanos and Vila (2009), VV hereafter, in which equilibrium interest rates are determined by the interaction of two different types of investors: those who trade bonds at different maturities for return considerations (arbitrageurs), and those who buy longer-term bonds mainly for reasons other than returns (like official investors).

We estimate this structural model on the term structure of monthly real rates, derived from U.S. Treasury Inflation-Protected Securities (TIPS), over the 2001-2016 period. The term structure is determined by two factors: the short-term real interest rate and excess supply. We specify the excess-supply factor as a function of a number of observable variables, that is, the Fed and foreign official holdings of Treasuries (demand) and the amount of Treasury securities outstanding (supply), each of which is expressed relative to the amount of Treasury securities held by investors with a price-elastic demand for bonds (arbitrageurs). We also account for the changing liquidity conditions in the TIPS market; we do this by allowing for an additional factor, which, although it does not enter directly the bond pricing in the VV model, can still affect the dynamics of real rates. In this way, model tractability is preserved.

We find that the model fits the term structure of U.S. real rates well, across maturities and over time. Demand pressures have a dominant impact on longer-term rates. In contrast, changes in the short-rate factor mainly affect shorter-term real rates, producing almost no impact on very long-term rates. We also find that modeling TIPS liquidity is particularly important to achieve a good model performance during the crisis period. This is the time when TIPS liquidity deteriorated as a result of the short-lived but intense tensions materializing in the repo market (Campbell, Shiller and Viceira, 2009). It is also apparent that the inclusion of the liquidity factor helps estimate the short-rate factor more precisely. Indeed, similar to the short-rate factor, the liquidity component displays a downward-sloping term structure, and hence is important for obtaining an accurate decomposition of the real rates into the expected and term-premium components.

Turning to the analysis of the excess-supply factor extracted from the VV model, we find that our main hypothesis is confirmed, that is, the excess-supply factor moves coherently with measures of demand and supply. Specifically, it increases with supply, whereas it decreases with official demand. In absolute terms, the excess-supply factor loads more on the Fed's holdings than on foreign official holdings and supply, respectively. According to the VV model, this evidence would be coherent with a higher duration of the Fed's holdings, and/or with a lower price sensitivity of Fed's demand. However, to examine the economic relevance of these results, we turn to the price impact analysis.

Although the price impact analysis rests on some model assumptions, it helps shed light on whether the estimated parameter magnitudes are reasonable in an economic sense. We find that the recent purchases of U.S. government debt securities by the Fed and foreign officials have indeed affected the level and dynamics of U.S. real rates. By 2016 foreign purchases of U.S. Treasuries exerted a cumulative negative impact of around 90 basis points on long-term U.S. real rates, which drops to less than 50 basis points at shorter maturities. Furthermore, most of their price impact materializes in the period prior to the crisis, when supply is increasing, but at a much lower pace than foreign official demand. Thus, the net price impact of foreign official demand is also sizable.

As to the Fed price impact during the QE programs, our analysis shows that Fed purchases also contributed to significantly lower real rates. With a focus on LSAP programs, we find that the price impact of the second Large Scale Asset Purchase Program (LSAP2) is larger than that of the other two LSAP programs; this finding also holds when supply is accounted for, *i.e.*, when considering the Fed price impact net of supply. Moreover, the analysis suggests that the Fed proved effective in lowering real rates also during the Maturity Extension Program (MEP), though by a lower extent relative to LSAP2. However, at the time of the MEP, supply pressures also temporarily reverted, contributing to the reduction in real rates. Overall, the net price impact of official purchases varies with the supply of bonds, but also with the risk-bearing capacity of the arbitrageurs, consistently with the VV model.

Our study relates to a number of earlier studies on reserve accumulation by foreign central banks and its impact on U.S. interest rates (*e.g.*, Warnock and Warnock, 2009; Krishnamurthy and Vissing-Jorgensen, 2012; Sierra, 2014), and to those studies trying to quantify the impact of Fed asset purchases on U.S. interest rates (*e.g.*, Gagnon et al., 2011; Krishnamurthy and Vissing-Jorgensen, 2011; Swanson, 2011; D'Amico et al., 2012; Hamilton and Wu, 2012; Meaning and Zhu, 2012; D'Amico and King, 2013). Although these studies agree on the qualitative (*i.e.*, downward) effect of official intervention on U.S.

Treasury yields, the evidence is not unanimous in quantitative terms, as the estimates can vary significantly. In this paper, we bring together these otherwise separate strands of the literature, by jointly analyzing the impact of foreign and domestic official demand pressures on the U.S. Treasury bond markets.

Although a large number of the empirical studies on QE price impacts are motivated by the VV model, to the best of our knowledge, this is the first study to structurally estimate the VV model including observable measures of demand and supply pressures for the U.S., also accounting for the changing liquidity conditions in the TIPS market. Many of the earlier empirical studies investigating the impact of official purchases on the level of yields lack the structural integrity provided by the preferred-habitat model of VV, and/or tend to look at the effects of the foreign and domestic official demands in isolation. Notably, within the framework proposed here, we can estimate the impact of official demand on real rates consistently across maturities, as no-arbitrage is enforced in the VV model.

The VV model is a first important attempt to rationalize the dynamics underlying the impact of quantities on rates within a no-arbitrage model; however, it rests on a number of assumptions which should probably be relaxed to further improve our understanding of such complex dynamics. For example, the sub-sample analysis of the price impact should be taken with caution, as the model parameters are constant and the factor dynamics homoskedastic. However, we also test the robustness of the results along a number of critical dimensions, and we try to flag a number of promising extensions which are not directly addressed in our study. For example, we extend the two-factor VV model and derive its three-factor counterpart, such that the short-rate factor reverts to a stochastic long-run mean using the central tendency process of Balduzzi, Das and Foresi (1998). We show that the main results are robust to the inclusion of this additional factor; it is therefore reasonable to argue that the two-factor version of the VV, augmented to capture the changing liquidity conditions in the TIPS market, provides an empirically good representation of the evolution of U.S. real rates over the 2001-2016 period. We also show that the main results are also largely robust to alternative measures of liquidity conditions and arbitrageurs.

Two further observations are in order. First, the many constraints imposed in the VV model, which largely pertain to its structural form, should help attenuate the weak identification and the downward bias that is pervasive in the estimation of the factor dynamics in small samples (Bauer, Rudebusch and Wu, 2012); indeed, in VV model, the pricing and objective dynamics are strongly tied together. Second, a caveat of this study (but not of the VV model) is that the analysis abstracts from *local* demand/supply effects on yields which might also be at play; this is mainly due to the lack of maturity-specific data on official holdings. The analysis of local supply effects on U.S. interest rates can be found, for example, in D'Amico and King (2013). While their analysis is also inspired by the VV model, their estimates are reduced form. Finally, additional research is needed to shed further light on the price impact exerted by QE forward guidance, which seems to be also relevant in driving real rates.

The remainder of the paper is organized as follows. Section 2 reviews the recent literature on the impact of large Treasury bond investors' demand on interest rates. The model is presented in Section 3, while the data and the econometric methodology are described in Section 4. We subsequently present the main findings in Section 5. The robustness analysis, together with a discussion of the caveats and the remaining open issues, are presented in Section 6. Finally, Section 7 concludes our analysis. The Bayesian algorithm, the two- and three-factor model derivations, and a number of robustness exercises are presented in a separate Online Appendix.

2. LITERATURE REVIEW

An increasingly large body of literature is dedicated to studying the effects of official purchases, foreign and domestic, on U.S. interest rates. Next, we briefly review these largely separated strands of the literature.

The foreign official sector has been playing an important role in the Treasury market since the early 2000s.[1] There are several drivers behind such an unprecedented accumulation of U.S. assets by foreign officials (ECB, 2006). However, regardless of the actual motives driving their purchases, a key feature is that foreign-official institutions pursue objectives that are only slightly sensitive to risk-return considerations. Krishnamurthy and Vissing-Jorgensen (2012), for example, argue that foreign officials' demand for U.S. Treasuries is inelastic; precisely, foreign central banks use the proceeds from their dollar capital inflows, almost 'mechanically', to accumulate more dollar reserves; thus, they buy Treasuries regardless of their prices relative to other assets.

The impact of foreign demand on U.S. interest rates has been mainly analyzed by the macroeconomic literature on 'global imbalances' (see, for example, Caballero, 2006; Caballero, Farhi and Gourinchas, 2008; Mendoza, Quadrini and Rios-Rull, 2009; Caballero and Krishnamurthy, 2009). Available empirical studies are mostly reduced-form (*e.g.*, Warnock and Warnock, 2009), or somewhat inspired by structural models (Krishnamurthy and Vissing-Jorgensen, 2012; Sierra, 2014), and do not provide a unanimous estimate of the effect of foreign intervention on U.S. Treasury yields.[2] This extensive literature, however, lacks the structural integrity of no-arbitrage term-structure models.

A first attempt to estimate foreign demand effects on U.S. bond prices, while imposing the discipline of no-arbitrage, is Kaminska, Vayanos and Zinna (2011). Specifically, to explain how demand for U.S. Treasuries can affect the term-structure of interest rates, their paper builds on the 'limited arbitrage' model of VV, in which, similar to Modigliani and Sutch (1966), investor clienteles with preferences for specific maturities, so-called 'preferred-habitat' investors, play an important role for bond pricing. Specifically, VV set up a formal model of two types of agents: investors with a preferred habitat for specific maturities, and risk-averse arbitragers. The presence of the arbitrageurs guarantees that the term structure of yields is arbitrage-free. While Kaminska, Vayanos and Zinna (2011) take the preferred-habitat model to the data, aggregate demand is modeled as an unobserved factor, and, hence, it is difficult to disentangle the specific impact of foreign official demand; this is also because QE policies and the changing liquidity conditions in the TIPS market are not controlled for.

Rigorous no-arbitrage term-structure models have been more popular for analyzing Fed purchases. For example, to evaluate the effects of the LSAPs, Li and Wei (2013) use a no-arbitrage term-structure model with (excess) supply factors, based on observable supply factors derived from data on private holdings of Treasury debt and agency MBSs. To facilitate the estimation, they assume that supply factors influence Treasury yields predominantly through changes in the term premium, thus focusing implicitly on the scarcity and duration channels. The vast majority of empirical analyses of the LSAP-style operations, however, are based on either event studies (*e.g.*, Gagnon et al., 2011; Krishnamurthy and Vissing-Jorgensen, 2011; and Swanson, 2011; D'Amico and King, 2013), or time series regressions of Treasury yields (and of their components), on demand related variables (*e.g.*, Krishnamurthy and Vissing-Jorgensen, 2011; D'Amico et al., 2012; Meaning and Zhu, 2012).

Many of these papers highlight the importance of the preferred-habitat, or inelastic, nature of Fed demand, but none of the papers estimate a term-structure model with preferred-habitat demand explicitly. Of particular note though is the study of Hamilton and Wu (2012), as the authors derive and estimate a simplified version of the preferred-habitat model of VV on the nominal U.S. yields.[3] Importantly, they show that the maturity structure of the Fed's holdings helps predict bond excess returns, and this predictability adds to that of the level, slope and curvature factors. Furthermore, they argue that, when the economy is at the ZLB, altering the maturity structure of publicly held Treasury debt would be equally effective at lowering long-term yields as buying longer-term assets outright with newly created reserves.

In a related study, Greenwood and Vayanos (2014) examine empirically how the supply and maturity structure of government debt affect bond yields and excess returns. Fundamentally, although their analysis is organized around a simplified version of the VV model – supply shocks are absorbed only by arbitrageurs, so that there is no role for preferred-habitat investors – the results support the main implications of the VV model. However, instead of taking the model directly to the data, they rather test its implications through a series of simple regressions. In a recent paper, Greenwood, Hanson and Vayanos (2015) extend the VV model to account for QE forward guidance.

Although these studies tend to agree on the efficacy of the Fed's asset purchase programs, the range of the estimated impacts on U.S. Treasury yields is quite large: for example, the drop of the nominal ten-year Treasury yield is estimated to be in the 35-160 basis points range. There is also a widespread agreement that the Fed's impact on nominal longer-term yields is mostly felt through the term-premium component (see, *e.g.*, D'Amico et al., 2012; Li and Wei, 2013), suggesting that Fed's purchases affected long-term yields mainly through the scarcity and duration channels. More specifically, the fall of the nominal term premium seems to be largely driven by its real component, *i.e.*, the *real* term premium (Hanson and Stein, 2015; Abrahams et al., 2016).

3. THE MODEL

We build our term-structure model on the limited arbitrage (preferred-habitat) framework of VV. In their model, two types of investor integrate maturity markets: so-called preferred-habitat investors, who have strong preferences for bonds with specific maturities, and arbitrageurs, who do not have maturity preferences but trade bonds of any maturity for return considerations, making the term structure arbitrage free. Arbitrageurs not only deal with the disconnect between the short rate and bond yields, but also bring yields in line with each other, smoothing local demand and supply pressures. However, exactly because arbitrageurs are risk-averse, preferred-habitat demand matters and contributes to determine equilibrium interest rates. The rest of the section introduces the main elements of the model, while the details of the model are provided in the Online Appendix (Section II.1).

The model is set in continuous time, so that the term structure is represented by a continuum of zero-coupon bonds, with bond maturities, τ, in the interval (0;T]. The instantaneous *risk-free rate*, r_t, which is the limit of the spot rate of maturity τ, $\mathrm{R}_{t,\tau}$, when τ goes to zero, follows the Ornstein-Uhlenbeck process

$$dr_t = \kappa_r \left(\overline{r} - r_t\right) dt + \sigma_r dB_{r,t}, \tag{1}$$

where $(\overline{r};\ \kappa_r;\ \sigma_r)$ are positive constants, and $B_{r,t}$ is a Brownian motion. Preferred-habitat investors form maturity clienteles, with the clientele for maturity τ only buying the bond with the same maturity. The *excess demand* for the bond with maturity τ is assumed to be a linear function of the bond's yield $R_{t,\tau}$

$$y_{t,\tau} = \alpha(\tau)\,\tau\,(R_{t,\tau} - \beta_t), \tag{2}$$

where $\alpha(\tau)$ is a function of maturity τ with the only requirement of taking positive values. Similar to VV, we assume the following functional form $\alpha(\tau) = \alpha e^{-\delta\tau}$.[4] The β_t factor captures the part of investors net demand that is price inelastic.[5] Also note that increases in β_t are associated with a decreasing excess demand $y_{t,\tau}$; thus, β_t should increase with supply, and decrease with demand. Similar to the original version of the VV model, we assume that the intercept β_t takes the form

$$\beta_t = \sum_{k=1}^{K} \theta_k \beta_{t,k}, \tag{3}$$

where $\beta_{t,k}$s are multiple demand/supply factors. In particular, VV suggest that these factors could capture changes in the needs of preferred-habitat investors (arising because of changes in policies, central banks' foreign reserve management, pension funds' liabilities or regulation, etc.), or changes in the size or composition of the preferred-habitat investor pool, or changes in the available supply of bonds issued by the government (VV, 2009).[6]

Our hypothesis is that these $\beta_{t,k}$ factors are influenced by the strong accumulation of reserves by foreign officials and large scale bond purchases by the Fed, controlling for the supply of Treasuries. Indeed, as Section 2 documents, official demand for long-term U.S. Treasury securities is high and not fully elastic. In light of these considerations, we assume that β_t comprises a foreign-official demand factor, β_t^{FO}, a separate Fed demand factor, β_t^{FED}, and a supply factor, β_t^{SUP}. It follows that the excess-supply factor β_t is given by

$$\beta_t = \theta_1 \beta_t^{FO} + \theta_2 \beta_t^{FED} + \theta_3 \beta_t^{SUP}. \tag{4}$$

The aggregate excess-supply factor β_t follows the Ornstein-Uhlenbeck process

$$d\beta_t = \kappa_\beta \left(\overline{\beta} - \beta_t\right) dt + \sigma_\beta dB_{\beta,t}, \tag{5}$$

where $\overline{\beta}$ is the unconditional mean, κ_β and σ_β are positive constants, and $B_{\beta,t}$ is a Brownian motion. Therefore, we specify the law of motion of the aggregate excess-supply factor, and not of its components. By doing this, we limit the number of parameters entering the pricing of equilibrium bond yields, mainly to preserve model tractability. As a result, bond pricing depends on the parameters describing the dynamics of two-factors; the short rate of equation (1) and the aggregate excess-supply factor of equation (5). Specifically, while the θ parameters can affect the estimate of β_t, they do not enter directly into the bond pricing recursions.

The presence of the *arbitrageurs* guarantees that bonds with maturities in close proximity trade at similar and coherent prices ruling out arbitrage opportunities; that is, no-arbitrage pricing is preserved in VV. Bond risk premiums compensate the arbitrageurs for the risk inherent in their activity of buying or selling bonds of different maturities. We assume that arbitrageurs' investment strategy follows a mean-variance portfolio optimization, such that the arbitrageurs' optimization problem is given by

$$\max_{\{x_{t,\tau}\}_{\tau \in (0,T]}} \left[E_t(dW_t) - \frac{a}{2} Var_t(dW_t)\right], \tag{6}$$

with a denoting arbitrageurs' risk-aversion coefficient, $x_{t,\tau}$ their dollar investment in the bond with maturity τ, and W_t arbitrageurs' time-t wealth. The arbitrageurs' budget constraint is specified as

$$dW_t = \left(W_t - \int_0^T x_{t,\tau}\right) r_t dt + \int_0^T x_{t,\tau} \frac{dP_{t,\tau}}{P_{t,\tau}}, \tag{7}$$

where $P_{t,\tau}$ is the time-t price of the bond with maturity τ that pays \$1 at time $t+\tau$.

Equilibrium spot rates are affine in the risk factors r_t and β_t,

$$\tau R_{t,\tau} = A_r(\tau) r_t + A_\beta(\tau)\beta_t + C(\tau), \tag{8}$$

and imposing equilibrium $x_{t,\tau} = -y_{t,\tau}$, we can solve for $A_r(\tau)$, $A_\beta(\tau)$ and $C(\tau)$ through a system of linear ODEs.

Finally, the expected *excess return* over the risk free rate at any maturity τ is given by

$$\mu_{t,\tau} - r_t = A_r(\tau)\lambda_{r,t} + A_\beta(\tau)\lambda_{\beta,t}, \tag{9}$$

where $\lambda_{r,t}$ and $\lambda_{\beta,t}$ are the factor prices of risk

$$\begin{aligned} \lambda_{r,t} &= a\sigma_r \int_0^T x_{t,\tau}\left[\sigma_r A_r(\tau) + \rho\sigma_\beta A_\beta(\tau)\right] d\tau, & (10)\\ \lambda_{\beta,t} &= a\sigma_\beta \int_0^T x_{t,\tau}\left[\sigma_\beta A_\beta(\tau) + \rho\sigma_r A_r(\tau)\right] d\tau, & (11) \end{aligned}$$

and ρ is the factor correlation between $B_{r,t}$ and $B_{\beta,t}$. Thus, returns in excess of the risk-free rate are linear functions of the bond specific quantities of risk (the sensitivities to the risk factors), whereas the prices of risk (the expected excess returns that arbitrageurs require as compensation for taking a marginal unit of short-rate and demand/supply risk) are common to all bonds, which is a general consequence of the no-arbitrage assumption.

The economic content of the VV model is in the specifications of the prices of risk that vary with the riskiness of the arbitrageurs' portfolio. Specifically, the arbitrageurs take on/off risk by clearing the price pressures generated by the preferred-habitat investors. In essence, preferred-habitat demand exerts global effects due to arbitrageurs' trading activity, and, for this reason, features directly in the prices of risk equations. Also in the presence of local demand/supply effects, what ultimately matters is how such local shocks are made global by the activity of arbitrageurs. This is why, in the VV model, to provide a good characterization of the effects of demand pressures on rates, it is of primary importance to capture the joint evolution of the prices of risk (Greenwood and Vayanos, 2014).

In sum, demand pressures exerted by official investors affect the prices of risk by altering the riskiness of the arbitrageurs' portfolio. In this way, the model is able to capture both the scarcity and duration channels, as they both work through changes in the prices of risk.[7] This mechanism also justifies the use of a global, *i.e.*, single-maturity, factor to capture the bulk of official demand pressures.

4. DATA AND ECONOMETRIC METHODOLOGY

In this section, we first present the real rate data, and then the observable measures of demand pressures. We proceed with the description of the discretized state-space framework, and the Bayesian methodology.

4.1 Real Interest Rates

Previous research shows that demand pressures on interest rates work mostly through the real term-premium component (*e.g.*, D'Amico et al., 2012). Thus, if we want to estimate the direct impact of official demand, we need to bring the model to real interest rate data.[8] The key source of the data on market real rates is inflation-indexed bonds. In the U.S., inflation-indexed bonds are issued by the U.S. Treasury and their principals are adjusted in line with the consumer prices index (CPI). Since its launch in 1997, the market for Treasury Inflation-Protected Securities (TIPS) has grown considerably, and it now represents the largest and the most liquid market for inflation-indexed bonds. The zero-coupon equivalent U.S. real rates we use in this paper are obtained from the Fed's TIPS-yields estimates, which come from a yield curve based on both on-the-run (newly issued) and off-the-run (previously issued) bonds. Although data on U.S. real yields are available since 1999, we exclude the initial years, when TIPS yields were systematically affected by a lack of liquidity; thus, the analysis spans the period from January 2001 to December 2016.[9]

Turning to the cross-sectional dimension, the inclusion of a sufficient number of maturities is important for achieving precise estimates of the parameters entering the bond pricing; thus, we use real yields of 2-, 5-, 10-, 15- and 20-year maturities. These maturities, which are roughly equally spaced (real rates with maturities shorter than 2 years are not available), well reflect movements at the short, medium and long ends of the term structure. The lack of short-maturity TIPS prior to 2004 implies that real market yields on two-year TIPS are available only from January 2004. The real yield curve is, on average, upward sloping (see Table A1, in the Online Appendix, for yields' summary statistics). However, from 2001 to 2005, long-term real interest rates fell substantially, flattening and eventually even inverting the curve. After a slight recovery in 2006-07, real rates experienced dramatic swings with the onset of the financial crisis. Their odd behavior at the end of 2008, when real rates spiked dramatically, and at some point exceeded nominal rates, coincided with the deflation episode. However, according to Campbell, Shiller and Viceira (2009), this spike could be largely explained by the worsening of the liquidity conditions in the TIPS market; we return to this point later in the section. After 2011, medium- to long-term real interest rates turned negative and stayed deep in negative territory until the spring of 2013, when yields rebounded on indications that the Fed might reduce its bond purchases ('Taper Tantrum'). As to the second moments, short-term yields are more volatile than long-term bond yields, with volatility decreasing gradually with maturity.

According to principal component (PC) analysis, the first two PCs explain more than 99.5 percent of the real yield data (Table A1 in the Online Appendix). The first PC has large explanatory power for short maturities, while the loadings of the second PC increase from large negative values at short maturities to large positive values at long maturities, which is typical of the slope factor. These findings are largely consistent with the two-factor model described in Section 3; there, the first factor describes movements of the unobserved real short interest rate, while the second factor captures official demand pressures. However, the first PC might reflect not only a short-rate factor, but also a liquidity premium; indeed, the term structure of TIPS liquidity premiums seems to be downward sloping (Christensen and Gillan, 2011). We turn to this issue next.

TIPS Liquidity. Although TIPS are widely considered to be less liquid than nominal U.S. Treasury bonds, a consensus on the precise level of the TIPS liquidity premium has yet to develop. Clearly, liquidity was particularly low soon after the U.S. Treasury first issued TIPS. However, the liquidity premium seems to be relatively small and does not display significant variation during normal times (Christensen and Gillan, 2011). During the global financial crisis instead, the spike in real yields was only in part due to the emerging deflationary pressures, with the main explanation being the short-lived but abrupt drop in liquidity that resulted from tensions in the repo market generated by Lehman's default (Campbell, Shiller and Viceira, 2009).

The lack of consensus on the size of the TIPS liquidity premium partly stems from the well-known difficulty in defining and measuring the multifaceted aspects of liquidity (Kyle, 1985). However, it is mainly due to the limited number of precise real-time measures of liquidity conditions in the TIPS market (D'Amico, Kim and Wei, 2018). One measure of (il)liquidity is, however, gaining momentum in the literature: the average absolute fitting errors from the Svensson TIPS yield curve.[10] The fitting errors are large, and liquidity low, when funding constraints are particularly severe, preventing investors from arbitraging away differences from fundamental values, *i.e.*, the fitted yield curve. This measure largely corroborates the considerations above; liquidity conditions in the TIPS market are benign during normal times, but abruptly deteriorate during turbulent times (see Figure IA.5, in the Online Appendix).

In the subsequent analysis, we will use the average fitted errors to account for the changing liquidity conditions in the TIPS market. A third, (il)liquidity factor, which is orthogonal to the r_t and β_t factors, and does not enter the VV bond pricing, will contribute to determine the real rates over time and across maturities. In this way, we can preserve model tractability and yet allow for a liquidity factor.

4.2 Measures of Official Demand Pressures

We now turn to presenting our empirical counterpart to equation (4), linking the β_t factor to observable measures of demand and supply. These measures, standardized by arbitrageurs' holdings of Treasuries, are shown in Figure 1. Next, we discuss each measure in turn, as well as their scaling. First, we proxy foreign official demand, β_t^{FO}, with their holdings of Treasury securities scaled by the total amount of Treasuries held by the arbitrageurs (FO over Arbs Hlds. (b_t^{FO})). Foreign official investors mainly hold medium- to long-term Treasuries. The Treasury International Capital (TIC) System provides data on foreign official holdings of Treasuries at a monthly frequency. However, the raw data suffer from a number of drawbacks. For this reason, we instead use the adjusted foreign holdings provided by Bertaut and Tryon (2007).

Second, we measure the Federal Reserve's demand, β_t^{FED}, with their holdings of Treasury securities scaled by those of arbitrageurs (FED over Arbs Hlds. (b_t^{FED})). While foreign officials mainly hold long-term Treasuries securities, the Fed's portfolio is more diverse, with the Fed also being an active investor in the short-term segment, as part of its money-market operations. In contrast, during the LSAPs, the Fed mostly invested in medium- and long-term maturity bonds. Moreover, excluding short-term bonds from the proxy for the Fed's demand is also important in being able to accurately capture the effects of the MEP. It is therefore important to only consider the Fed's holdings of Treasuries with maturities equal or greater than five years.

Third, although the focus of this study is not on supply as such, to complete the specification of the excess-supply factor, and of the "scarcity channel" in particular, we need to account for the role of supply, β_t^{SUP}. Indeed, the amount of Treasuries outstanding ultimately determines the price impact exerted by the official holdings of Treasuries. Put simply, one would expect demand pressures to be more severe, at times when supply is low. We use the scaled amount of Treasury securities outstanding as proxy for supply (AO over Arbs Hlds. (b_t^{SUP})).

It is necessary to scale the demand and supply variables expressed in dollar values, for a number of reasons. By doing that, for instance, one partly addresses the issue of having particularly persistent regressors, which would otherwise make inference of equation (4) problematic. Moreover, if the variables were in dollar values, as the bond market grows in size, term premia could increase without bound. In order to deflate the demand/supply measures, several existing studies choose to standardize demand/supply variables by the domestic (U.S.) GDP (e.g., Krishnamurthy and Vissing-Jorgensen, 2012;

Greenwood and Vayanos, 2014; and Carvalho and Fidora, 2015). Such scaling is, however, rather arbitrary not being coherent with the VV model, as GDP does not feature in their model. Furthermore, a drawback of such scaling would also be that the standardized observable measures of excess demand/supply could vary due to the changes in GDP, even absent any change in official purchases or supply. Similarly, valuation changes would not be adequately controlled for.

Therefore, we divide all demand/supply variables by the amount of Treasuries held by the arbitrageurs, as we believe that this scaling method is more coherent with the VV model. This is because, in the VV model, the arbitrageurs' risk aversion parameter reflects their risk-bearing capacity, and hence it should be related to the size of their balance sheet, *i.e.*, their Treasury portfolio in this setting. That is, one would expect demand pressures to be more severe (*i.e.*, the official holdings to exert a stronger impact on rates) at times when arbitrageurs have, all else equal, a lower risk-bearing capacity. For this reason, we scale demand variables, and supply to be consistent, using a proxy of the size of the arbitrageur sector.[11]

Clearly, this scaling raises the following question: which types of investors should be regarded as arbitrageurs? We know from Section 3 that arbitrageurs should have price-elastic demand, and thus this characteristic should guide our choice. In the baseline specification, we opt to represent the arbitrageurs' sector by all participants in the Treasury bond market apart from the foreign officials and the Fed, *i.e.*, we include all but the price inelastic official investors. Later on, however, we will also test the robustness of the results using alternative definitions of arbitrageurs (See Section 6.2); in essence, we will try to further narrow down the selection of arbitrageurs, up to the point of regarding as arbitrageurs only shadow banks.

Finally, we note that our empirical measures of demand and supply include both nominal and real bonds. This is mainly due to data availability; for example, TIC data specific to the foreign ownership of TIPS are not available.[12] One could also argue that the two markets have similar safety characteristics, being close substitutes, and the segmentation between the Treasury and TIPS markets during the QE period seems to be contained.[13] As a result, demand pressures could spill over across nominal and real bonds.

4.3 Bayesian Inference

State-space Representation. The VV model can be naturally cast into a state-space framework. The state (or, 'transition') equations describe the evolution of the r_t and β_t factors under the objective probability measure, while the measurement (or, 'observation') equations map these two factors into the observed real rates of selected maturities, $\mathrm{Y}_t = \left[R_t^{2yr}, R_t^{5yr}, R_t^{10yr}, R_t^{15yr}, R_t^{20yr}\right]'$. However, we extend this baseline representation in a number of ways. First, we account for the possibility that liquidity conditions affect the level and dynamics of real rates. As a result, we include an (il)liquidity factor, so that real rates are driven by three factors: the $\mathrm{X}_t = [r_t, \beta_t]'$ factors, and the illiquidity factor l_t. Second, we link the β_t factor to the observable measures of demand and supply described in Section 4.2 (b_t^{FO}, b_t^{FED}, b_t^{SUP}). Third, we include a proxy for the one-month real rate in the estimation to try to better pin down the dynamics of the r_t factor, following Joyce, Kaminska and Lildholdt (2012), among others. The inclusion of a proxy for the one-month real rate is potentially useful, as there is otherwise a large gap in the maturity spectrum of the included real yields, especially in the pre-2004 period, when the 5-year rate is the shortest maturity available in our dataset. Taken together, these model extensions result in the following discretized state-space representation.

`State Equations`:

$$X_{t+\Delta} = G(P_1) + F(P_1)X_t + u_{t+\Delta} \qquad u_t \sim N(0, Q) \tag{12}$$

`Measurement Equations`:

$$Y_{t+\Delta} = f(P)X_{t+\Delta} + \nu l_{t+\Delta} + \varepsilon_{t+\Delta} \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2 I) \tag{13}$$

$$r^o_{t+\Delta} = r_{t+\Delta} + \epsilon_{1,t+\Delta} \qquad \epsilon_{1,t} \sim N(0, \sigma_{1,\epsilon}^2) \tag{14}$$

$$\beta^o_{t+\Delta} = \beta_{t+\Delta} + \epsilon_{2,t+\Delta} \qquad \epsilon_{2,t} \sim N(0, \sigma_{2,\epsilon}^2) \tag{15}$$

where, to simplify the notation, we group the parameters of the VV model as $P_1 = (\rho, \sigma_r, \sigma_\beta, \kappa_r, \kappa_\beta, \overline{r}, \overline{\beta})$ and $P_2 = (a, \alpha, \delta)$, so that $P = (P_1, P_2)$.[14] The system matrices take the form of

$$G(P_1) = \begin{bmatrix} \kappa_r \overline{r} \Delta \\ \kappa_\beta \overline{\beta} \Delta \end{bmatrix} \text{ and } F(P_1) = \begin{bmatrix} 1 - \kappa_r \Delta & 0 \\ 0 & 1 - \kappa_\beta \Delta \end{bmatrix};$$

the factors' variance-covariance matrix is given by

$$Q = \Delta \begin{bmatrix} \sigma_r^2 & \rho \sigma_r \sigma_\beta \\ \rho \sigma_r \sigma_\beta & \sigma_\beta^2 \end{bmatrix},$$

and σ_ε^2 is the common variance of the independent and normally distributed measurement errors, $\varepsilon_{t+\Delta}$. Note that the transition equation is a function only of the parameters P_1, while $f(P)$ is the bond pricing function in the VV model, presented in Section II.1 of the Online Appendix, which maps the β_t and r_t states into the vector of observed yields. Specifically, the model-implied real rates are an affine function of the observed factors with loadings that are complex and highly nonlinear functions of the market price of risk parameters P_2, dynamics parameters P_1, and maturity.

Of particular interest is equation (13), as it implies that the real rate R_t^τ comprises a model-implied rate, $f(P)X_{t+\Delta}$, a liquidity component, $\nu(\tau)l_t$, and a residual pricing error, ε_t.[15] Thus, the liquidity factor l_t is allowed to have maturity specific loadings ν_τ; this feature accords well with the ample evidence suggesting that liquidity varies not only over time, but also across maturities. Equation (13) postulates that, as liquidity declines, an increasing part of real rates is explained by the illiquidity component, and less by the VV model. In this way, the estimated r_t and β_t factors are not forced to capture changes in rates which are not directly related to the VV model, and thus truly reflect the short rate and demand pressures, respectively.[16]

We now turn to describing the additional observation equations (14)-(15). Equation (14) links the unobserved $r_{t+\Delta}$ factor to the proxy for the one-month real rate $r_{t+\Delta}^o$, allowing for measurement error $\epsilon_{1,t+\Delta}$.[17] Equation (15) is the additional measurement equation which connects the unobserved β_t factor to the observed excess-supply factor

$\beta^o_{t+\Delta} = \theta b'_{t+\Delta}$, where $\theta = [\theta_0, \theta_1, \theta_2, \theta_3]$ and $b_{t+\Delta} = [1, b^{FO}_{t+\Delta}, b^{FED}_{t+\Delta}, b^{SUP}_{t+\Delta}]$, which therefore depends on the vector of observed demand/supply variables ($b_{t+\Delta}$) plus a constant term. The error term $\epsilon_{2,t+\Delta}$ may reflect other potential determinants of $\beta_{t+\Delta}$ that are not captured by our foreign and Fed observable factors, or errors in the measurement of such variables.

MCMC Algorithm. While the model could be estimated by maximum likelihood, we resort to Bayesian techniques – specifically to Markov Chain Monte Carlo (MCMC) methods – for a number reasons. First, in our case the likelihood is highly non-linear because bond prices are complex highly nonlinear functions of the parameters, which significantly complicate the numerical optimization. By contrast, Bayesian methods rely on simple block simulations. Second, the draws resulting from the Bayesian algorithm allow us to quantify the uncertainty around post-estimation calculations (such as, *e.g.*, the loadings, term premiums, price impact, etc.), which would be difficult to do by classical methods (Kim and Nelson, 1999). Third, in a Bayesian framework, we can easily specify priors and impose constraints on the parameters (Johannes and Polson, 2004). Instead, parameter constraints may reduce further the performance of optimization algorithms needed in maximum likelihood, compromising the convergence of the optimization.

Given that in our model the observation equation is highly nonlinear in the parameters, the functional form of the density is non-analytic. Therefore, we develop a MCMC algorithm to update the parameters and states which combines a series of slice-sampling steps to draw the P parameters which enter into the bond pricing $f(P)$, and standard Gibbs steps to draw the remaining parameters.[18] Given that real rates are affine functions of normally distributed factors, we use the forward filtering backward sampling of Carter and Kohn (1994) to draw the factors. By combining the prior distribution with the likelihood function, we get the posterior distribution. In particular, we sample model parameters and latent factors from simple blocks of the joint posterior distribution.

Specifically, to implement the MCMC algorithm, we iteratively sample the parameters from these conditional densities:

- $\pi(\gamma|P_{-\gamma}, \sigma^2_\varepsilon, X^T, l^T, Y^T)$, $\forall \gamma \in P$: structural parameters driving bond pricing in

VV model, slice-sampling step;

- $\pi(\nu|P, \sigma_\varepsilon, l^T, X^T, Y^T)$: liquidity loadings in measurement equation, Gibbs step;
- $\pi(X^T|P, \sigma_\varepsilon^2, \nu, \sigma_\epsilon^2, Y^T)$: r_t and β_t states, Carter and Kohn (1994) Gibbs step;
- $\pi(\sigma_\varepsilon^2|P, X^T, \nu, l^T, Y^T)$: real rate pricing error variance, Gibbs step;
- $\pi(\theta|X^T, b^T)$: demand and supply loadings, (HAC) Gibbs step;
- $\pi(\sigma_{1,\epsilon}^2|r^{oT}, r^T)$: variance measurement error in r_t measurement equation, Gibbs step;
- $\pi(\sigma_{2,\epsilon}^2|\theta, b^T, \beta^T)$: variance measurement error in β_t measurement equation, Gibbs step;

The MCMC algorithm, and its implementation, is described in detail in the Online Appendix (Section II.2). Here, we also note that, because arbitrageurs' risk aversion parameter a and the demand elasticity α are not separately identified, we estimate the product of the two ($a\alpha$). Finally, although the priors are uninformative, several parameters are subject to constraints. For example, the factors need to be stationary, and arbitrageurs' risk aversion cannot be negative.

5. ESTIMATION RESULTS

In this section, we first present the estimation output, including the estimated factors, the factor loadings and the decomposition of real rates into the expected-rate and term-premium components. Then, we examine the price impact of official demand pressures.

5.1 Model Estimates and Bond Risk Premiums

Structural-Parameter Estimates. Table 1 presents parameter estimates. Before turning to each parameter, we start by noting that all the state parameters but $\overline{r}$ are statistically significant, and the numerical standard errors and CD diagnostics suggest that the chain has converged. The parameters, κ_r and κ_β, determining the factors' speed of mean reversion are significantly larger than zero, meaning that the state variables are stationary but persistent time series, which is common in term structure models. Moreover, we estimate $\overline{r}$ to be 0.44 percent. This parameter, also called the natural rate, tells us where the short rate should converge to in the very long run, rather than where it should be today. Also for this reason, it is rather difficult to estimate $\overline{r}$ precisely over short samples, as shown by the large credible intervals.[19] Although our estimate is fairly low, it is not statistically different from the evidence found in recent studies (see, *e.g.*, Holston, Laubach and Williams, 2017). The correlation coefficient, ρ, is estimated to be positive, 0.5, which implies that the short-rate factor, which could be interpreted as a real policy rate, and demand pressures are interdependent. In particular, stronger demand pressures (or, equivalently, weaker excess supply) are associated with lower policy rates.[20]

In the VV model, arbitrageurs' risk aversion a determines bond risk premiums. However, the parameter a is not separately identified from α. Nevertheless, the identification of $a\alpha$ should partly come from the sensitivity of long rates to shocks to the short rate. Thus, the product of the two contributes to determine the time variation in term premiums and expected excess returns. We estimate $a\alpha$ to be significantly different from zero, 46. This estimate is therefore consistent with the earlier findings that real bond term premiums are time varying.

The elasticity δ is also a particularly relevant parameter as it determines the shape of the $\alpha(\tau)$ function, which, in turn, maps shocks to the aggregate β_t factor onto net demand at different maturities, $\mathrm{y}_{t,\tau}$. However, differently from β_t, the maturity-specific net demand, $\mathrm{y}_{t,\tau}$, also depends on the elastic (price-sensitive) component of preferred-habitat demand, $\mathrm{R}_{t,\tau}$. We estimate δ to be roughly 0.01, which results in an impact curve $\alpha(\tau)$ that increases almost linearly with maturity, and peaks at (very) long maturities (Figure IA.6, in the Online Appendix). Although a particular form is employed, the figure also shows that this form is sufficiently flexible to account for multiple shapes, as the δ parameter changes. But, to determine the (average) shape of $\mathrm{y}_{t,\tau}$, for a given (average) $\mathrm{R}_{t,\tau}$, we need the estimate of $(\overline{\beta})$ β_t as well as the estimate of δ. As a result, the estimates of $\overline{\beta}$ and δ need to be looked in conjunction and related to the rate of that specific maturity to infer the net excess demand at that maturity τ.[21] Overall, this type of information should be particularly valuable for the U.S. Department of the Treasury, helping inform its debt issuing strategy, but also for issuers of close substitutes to Treasuries, such as (investment-grade) corporate bonds and MBS.[22]

Liquidity Spreads and Model Fit. Next, we turn to analyzing the liquidity spreads, $\nu_\tau l_t$. We find that the ν loadings are positive, strongly statistically significant, and decrease with maturity. Thus, our findings point to the existence of a downward sloping liquidity premium curve for TIPS, which is in line with a number of earlier studies such as, *e.g.*, Christensen and Gillan (2011). Furthermore, we find strong evidence that a drop in liquidity indeed largely explains the short-lived but abrupt spike in real rates observed during the peak of the global financial crisis. During this episode, the liquidity spread is just below 4% at the 2-year maturity, whereas it is less than 1% at the 20-year maturity (Figure 2). D'Amico, Kim and Wei (2018) estimate a liquidity premium of similar magnitude, being roughly 3.5%, after Lehman's bankruptcy.

The inclusion of the liquidity component is clearly important not only to estimate r_t more precisely, and consequently to get an accurate decomposition of the real rates into expected yields and term premiums, but also to achieve a good model fit. In fact, we find that the standard deviation of the yield fitting errors, σ_ε, is small (9.5 basis points); pricing errors do not exceed 25 basis points in absolute value, at any point in time and for any fitted maturity. Thus, our empirical counterpart to the VV model, which also accounts for a liquidity component, delivers a fairly accurate empirical pricing performance.

Smoothed Factors. Model estimation enables us to generate the time series of the two factors, which are presented in Figure 3. Above all, we document that the β factor falls over the sample until the 'Taper Tantrum' episode in 2013, suggesting strong rising demand pressures until then. Furthermore, we find that the factors can behave quite differently during the sample. Until 2004, both factors were decreasing. Subsequently, the short-rate factor picked up, while the excess-supply factor was still decreasing. In the following years, the short-rate grew steadily until the financial crisis started in 2007, whereas that of the β factor was much less pronounced. With the start of the crisis, the short-rate factor experienced a dramatic drop, which was only temporarily interrupted, possibly, by the rising deflationary pressures; it then stabilized somewhat in negative territory, around -3%, before rising again in 2014. In contrast, the β factor resumed its decrease around the end of the crisis, though at a much higher pace than over the earlier part of the sample, falling sharply until 2013. We also find that the estimated short real rate tracks well the observable short-rate proxy during the pre-crisis period. Thus, the crisis and post-crisis periods largely drive the measurement error volatility of 131 basis points (Table 1).[23]

Factor Loadings. The analysis of the factor loadings is useful to shed more light on the response of rates at different maturities to changes in the estimated factors. In the VV model, as shown in equation (8), yields are linear functions of the two unobserved factors with maturity-specific time-invariant factor loadings. Figure 4 shows the factor loadings for different maturities; those on the short-rate factor decrease quickly as time to maturity increases, being negligible for maturities longer than 10 years. In contrast, the coefficients associated with a drop in demand, *i.e.*, with an increase in β_t, are basically zero at very short maturities, reach their maximum around 10 years, and are only slightly lower for the subsequent maturities. Taken together, these findings suggest that the short-rate factor exerts a strong influence on short-term rates and a diminishing effect on long-term rates, while longer rates are mostly driven by the excess-supply factor. This result, coupled with the evidence on the liquidity loadings (showing that liquidity spreads decrease with maturity), explains why the absence of the liquidity component would largely contaminate the estimate of the r_t factor and only slightly that of the β_t factor.

Bond Risk Premiums. The VV model postulates that demand pressures may affect interest rates mainly through the 'scarcity" and 'duration' channels, because excess demand explicitly enters the risk premium specification (see equations (9)-(11)), through the prices of risk. However, to the extent that the r_t and β_t factors are correlated, shocks to demand and supply could also affect the expected short rate. As a result, the signaling channel could also play a role – albeit indirect and of second order – in the VV model. The channels, through which the excess-supply factor affects long-term rates, become more obvious when we look closer at the decomposition of interest rates into the expectation and risk premium components.

Figure 5 shows the decomposition for the 10- and 20-year real rates. The estimated term premiums averaged above 200 basis points in the period up to 2004, while they fell to around 100 basis points in 2005. The reductions in term premiums coincide with two important episodes in the recent history of U.S. long-term rates: (i) the so-called conundrum episode (2004-2005), and (ii) the QE programs, when Fed's purchases aimed at lowering longer-term interest rates.[24] The episode in the post-crisis sample when the term premiums spiked up is associated with Taper Tantrum episode (May-June 2013).[25]

During the conundrum episode (which denotes the period from June 2004 to February 2005), medium-to-long term real rates either did not increase or fell in response to rising policy rates. However, as shown in Figure 5, the behavior of long-term expected rates is not puzzling, as they indeed increased. Long-term real rates did not increase, because at that time demand pressures were also rising and as a consequence the real term premium was decreasing. Thus, on the basis of the VV model, it looks as if the 'conundrum' was never there.

Moreover, it is apparent from Figure 5 that the post-2009 decrease in long-term rates is mainly due to lower expectations of policy rates. In contrast, the consequent decrease in long-term rates, especially from 2011 to 2013, is mainly explained by a decreasing term premium, while medium- to longer-term expected rates stabilized. Thus, the model shows that the term premium on real bonds is extremely important in explaining movements in long real rates during the Fed's policy interventions. As it is evident from the comparison of Figures 3 and 5, term premiums on long real rates strongly co-move with our estimated β_t factor, *i.e.*, especially when demand pressures intensify. These findings, taken together, show that the evolution of the term premium in VV model is largely driven by demand pressures. In what follows, we provide a detailed analysis of the estimated β_t factor.

5.2 β_t-Factor Analysis

We now turn to analyzing the link between the smoothed excess-supply factor, β_t, presented in the previous section, and our observable measures of demand and supply pressures, b_t. We expect these measures will help explain the smoothed β_t factor, so that the θ coefficients should be statistically significant and correctly signed. Specifically, our main *hypothesis* is that the smoothed excess-supply factor, β_t, should decrease with demand and increase with supply. Note that the θ coefficients are estimated jointly with all the other parameters and states and are not subject to any constraint. Therefore, the model is free to determine whether, and to what extent, the smoothed excess-supply factor co-moves with the observable measures of demand and supply.

A further important observation about the θ estimates is in order. Conditional on a draw of the smoothed factor β_t, one can draw the θs from the linear regression model, $\beta_t = \theta b_t' + \epsilon_{2,t+\Delta}$, as the θs do not enter the bond pricing. However, given that the b_t regressors are rather persistent variables, the residuals of the equation are serially correlated. This implies that the parametrization adopted is misspecified. Specifically, by drawing the θs using the unadjusted posterior of a linear regression model, one would underestimate the variance of the distribution of the θ parameters, although its mean would still be correct. There are multiple ways to address this issue; here, we opt for the artificial 'sandwich' posterior method for Bayesian inference proposed by Müller (2013). The original posterior is replaced by an artificial normal posterior centered at the maximum likelihood estimator (MLE) with 'sandwich' covariance matrix, *i.e.*, the HAC-corrected covariance, which is routinely applied in frequentist settings. Müller (2013) shows that i) this method delivers lower asymptotic risk than alternative methods based on a misspecified likelihood; and, ii) it allows us to remain agnostic about the source of misspecification. To make this adjustment operational, we follow Miranda-Agrippino and Ricco (2017). Specifically, this is done by basing the inference about the θ loadings on the artificial Gaussian posterior centered at the MLE but with HAC-corrected covariance matrix.

Table 2 presents the θ estimates, which lend support to our main hypothesis, that is, the size of official holdings, both domestic and foreign relative to that of the arbitrageurs matter for the β_t factor, and hence for long-term Treasury bond yields. The proxy for supply is positively signed, as expected. Indeed, all the estimated θ coefficients are correctly signed and significantly different from zero. It is important to note that one should expect the loadings on the scaled holdings of foreign officials and the Fed (θ_1 and θ_2) to be equal in signs, but not necessarily in magnitudes. This is because the maturity profile of official investors' holdings of bonds might differ, as well as the price elasticity of their demand. Similarly, supply (θ_3) should be positively signed, but again we do not have a prior about its (absolute) magnitude. Table 2 also shows that, in absolute terms, θ_2 is larger than θ_1 and θ_3. Thus, according to the VV model, this would suggest that the Fed holdings of U.S. Treasuries have higher duration than those of foreign officials and of the amount outstanding, and/or that the Fed's demand is less elastic.[26]

The subsequent price-impact analysis, albeit simple, has the benefit of providing a means to evaluate the economic relevance of official demand and supply. However, before turning to the price impact analysis, we note that the observed excess-supply factor, β_t^o, tracks the evolution of β_t well, excluding the first and last years of the sample. Moreover, it seems to capture the slow-moving variation in the β_t factor. This may suggest that our measures of demand are unable to account for the transitory, and/or abrupt, changes in the β factor; we will return to this point later on in Section 6.

5.2.1 Price-Impact Analysis

Thus far, we showed that our measures of demand pressure are qualitatively consistent with the smoothed β_t factor extracted from the VV model. However, the two-factor VV model provides a somewhat stylized representation of the complex dynamics which characterize demand pressures in practice. Thus, a natural question is whether the VV model – despite the limitations discussed in detail in Section 6 – is able to deliver sensible estimates in terms of price impact, and thus can be useful for policy analysis, for example. A nice feature of (our empirical counterpart of) the VV model is that the price impact of each source of demand pressure on rates of any maturity can be easily quantified. This simply consists of mapping, first, the observable demand/supply variables (b_t) onto the smoothed β_t factor through the θ loadings; and, second, the resulting quantity onto the term structure of real rates through $A_\beta(\tau)$. More formally, the overall price impact (PI) of variable $\mathrm{b}^i_{t:T}$ on the rate of maturity τ during the period from t to T is given by

$$\mathrm{PI}(\mathrm{b}^i; \mathrm{R}^\tau; t, T) = \frac{A_\beta(\tau)}{\tau} \times \theta_i \times \left(\mathrm{b}^i_T - \mathrm{b}^i_t\right), \tag{16}$$

where the θ estimates are those reported in Table 2, and $A_\beta(\tau)$ loadings are shown in Figure 4.

Foreign Officials. The price impact estimates reported in Table 3, Panel I, suggest that the holdings of Treasuries by foreign officials indeed exerted a downward pressure on U.S. real rates over the 2001-2016 period; indeed, the cumulative price impact amounts to 92 basis points at the 10-year maturity, and reaches its minimum at the 2-year maturity, 40 basis points, being a direct consequence of the shape of the $A_\beta(\tau)$ curve.

The sub-sample analysis turns out to be particularly interesting. We find that foreign official demand mainly exerted a downward pressure on U.S. real rate before the start of QE, when supply was increasing at a slow pace and arbitrageurs' holdings were stable. Although we find that foreign official demand contributed to push U.S. rates down during the 2004-05 period, consistently with Warnock and Warnock (2009), we document a much more contained drop (of the order of roughly 25 basis points at the 10-year maturity, against their estimate of 80 basis points), during the exact period they focus on. Our estimates are more in line with Greenspan's opinion, who back in 2005 suggested that the foreign buying of U.S. bonds could have depressed U.S. long rates by 'less than 50 basis points'. Also during the QE period, foreign official demand contributed to lower US real rates, but their impact was by no means comparable to that of the Fed, and, at that time, supply was exerting a strong opposite positive effect; in fact, taken together, foreign official demand and supply had a positive price impact, of roughly 65 basis points at the 10-year maturity. Since the end of QE, the overall effect of foreign official demand was of opposite sign, as their holdings stabilized and eventually dropped, against the backdrop of rising holdings by arbitrageurs.

Federal Reserve. Although the Fed's price impact is comparable to that of foreign officials over the full sample, it materializes during the QE period (Panel II, Table 3), when its demand for longer-term bonds surges, and becomes highly inelastic. In the aftermath of QE, the price impact is partly reversed, turning positive; this is because Fed's holdings are constant, due to the reinvestment policy in place, at a time when those of the arbitrageurs are instead increasing.

Table 4 presents the Fed's price impact during the LSAPs and the MEP. The effect of Fed purchases during *LSAP1* is estimated to be about -46 basis points on the 10-year rates and around -20 basis points on the 2-year rate. These estimates are close to previous estimates available in the literature on the impact of LSAP1 on 10-year nominal rates; for example, D'Amico and King (2013) estimate the impact to be roughly equal to -45 basis points, Bomfim and Meyer (2010) to be -60 basis points, and Gagnon et al. (2011) to be between -58 and -91 basis points. We find that the impact of the second round of asset purchases by the Fed on real rates ranges from -45 basis points at the 2-year maturity to -105 basis points at the 10-year rate. The estimated impact of *LSAP2* is therefore somewhat larger, in absolute terms, than those from earlier studies; for example, the estimates by Krishnamurthy and Vissing-Jorgensen (2011), and D'Amico et al. (2012), are of the order of -33 and -55 basis points, respectively. A possible explanation is that our study examines the impact of LSAPs on the real rates, whereas other empirical studies focus on nominal rates. Other studies therefore capture the impact of Fed policies not only on the real component of the nominal yields, but also on inflation expectations and inflation risk premiums.[27] However, the different methodologies employed, as well as the measurement of the Fed demand pressures (*e.g*, the scaling of Fed demand), might also drive the difference in the price impact estimates. Finally, we quantify the *LSAP3* impact on 10-year yields to be around -35 basis points, which is not far from the estimate of -60 basis points of Engen, Laubach and Reifschneider (2015).[28]

To conclude, two further observations are in order. First, we also find that the Fed's exerted a downward pressure on real rates, equal to 42 basis points on 10-year real rates, during the *MEP*. This estimate is larger, in absolute terms, than the -15 basis points suggested earlier by Swanson (2011), but smaller than the -80 basis points estimated by Meaning and Zhu (2012). Second, abstracting from the role of foreign officials, the price impact net of supply of the Fed demand pressures drops (*i.e.*, during LSAP2) and eventually turns positive (*i.e.*, during LSAP1 and LSAP3), whereas it increases during MEP, in absolute terms. This also shows that supply plays a non-negligible role. Somewhat relatedly, when the long-term holdings of the Fed are replaced with the total holdings of Treasuries in the b_t^{FED} vector, we no longer find that the Fed exerted a downward pressure on rates during MEP, with supply accounting for the fall in rates. This lends additional support to the choice of excluding the holdings of short-term Treasuries from our measure of Fed's demand, to being able to adequately capture the 'duration channel'.

Counterfactual Analysis. We complement the analysis above with a simple counterfactual exercise, that is, the interest rates that would have prevailed in the absence of changes in our observed measures of demand and supply pressures. Specifically, we do this by fixing the selected demand/supply measure at its initial 2001 level; then, the wedge between the model yield and the counterfactual yield informs us about the cumulative price impact of the selected measure from 2001 to the date of interest. We do this separately for foreign official demand, Fed demand and supply.

Figure 6 shows the real rate implied by the VV model (solid green line), and the counterfactual rate if the selected demand variable were absent (dotted black line). The top charts show that foreign official demand pressures begin to materialize from late 2003, and revert substantially only towards the end of the sample. The middle charts show instead that the great majority of the Fed's price impact takes place in conjunction with the implementation of the UMPs.

All in all, it is evident that both official investors exerted a downward impact on yields, but also that foreign official pressure started to manifest during the 'conundrum' period, whereas the impact of the Fed is apparent during the QE period. By contrast, the bottom charts show that absent any increase in the amount of Treasury outstanding, *i.e.* supply, real rates would have been lower. That said, to determine the exact timing and magnitude of the price impact of official investors, as well as that of supply, the risk-bearing capacity of the arbitrageurs, and therefore its measurement, also plays an important role, as it is evident from visual inspection of scaled demand and supply variables (Figure 1) vs. their unscaled counterparts (Figure IA.4); we turn to this and other issues in the following section.

6. ROBUSTNESS AND DISCUSSION

In this study, the empirical examination of the effects of QE policies on the term structure of real rates is centered around a structural but parsimonious two-factor model that builds on VV. In what follows, we test the robustness of the analysis along three important dimensions; i) the adjustment for liquidity; ii) the measurement of demand and supply pressures; and, iii) the inclusion of an additional factor, capturing a stochastic central tendency.[29] All in all, we find that the main findings are largely robust. However, it should be also acknowledged that the analysis is subject to a few caveats, and some open issues still remain; thus, to conclude, we discuss in detail some of these issues.

6.1 (Il)liquidity Measures

Measuring liquidity conditions is generally not straightforward, and even more challenging for TIPS. This is because there is a lack of good measures of liquidity specific to the TIPS's market; the few existing measures either are not available for the entire sample, or tend to capture the relative liquidity of the TIPS market instead of its absolute liquidity (D'Amico, Kim and Wei, 2018). In our baseline specification, we use average TIPS fitting errors to control for liquidity, as this measure is free from these shortcomings and is gaining momentum in the literature. We find that this measure works fairly well. Next, we test the robustness of the results by relying on an alternative measure of liquidity: the off/on-the-run spread for Treasuries (Figure IA.10).[30] A long-established empirical regularity is that, on-the-run (the most recently issued) Treasuries trade at a premium relative to off-the-run (the seasoned issued) Treasuries with otherwise comparable characteristics (see, *e.g.*, Fontaine and Garcia, 2012). The reason is that on-the-run Treasuries are more actively traded. The off/on-the-run spread signals poor liquidity conditions at the start of the sample, somewhat worse conditions during the 2008-09 crisis, and better conditions at the end. Overall, it conveys a considerably different picture from that of the average TIPS fitted pricing errors. As a result, regardless of its informativeness, it represents a substantial robustness test for our estimates.

We therefore re-estimate the model using this measure (see Figures IA.11-IA.13). We find that the term structure of the liquidity loadings is concave, with liquidity conditions being particularly poor at the two-year maturity. However, the maximum liquidity spread is 1.5% vs. the 3.7% uncovered earlier on using the average fitted pricing errors. Although this alternative specification yields slightly lower pricing errors, the liquidity spread is by no means sufficient to account for the spike in real rates observed at the peak of the global financial crisis. As a result, during the crisis, the estimate of the short rate is contaminated by liquidity effects, and thus delivers estimates of expected rates and term premiums that are hard to rationalize. In 2009, the expected real rate at the 10-year horizon is projected to spike to 3% and, and, as a consequence, the term premium to drop below zero; soon after, both the expected rate and the term premium revert back, abruptly, to their 2008 values. However, we find that the results in terms of the price impact of the official pressures are largely robust (Table A5), with the estimate of the smoothed excess-supply factor essentially unchanged. This finding actually holds regardless of which liquidity measure is used, as well as absent any control for liquidity (Figure IA.14).[31]

6.2 Linking the β_t Factor to Observable Covariates

In the analysis, we specify the smoothed 'excess-supply' factor, β_t, as a linear function of contemporaneous observable measures of officials' demand and supply. While this simple specification performs rather well, it is subject to a number of potential caveats, which we discuss next, together with additional robustness exercises.

Serially Correlated Residuals. It is apparent that the (b_t) regressors are rather persistent, which results in serially correlated residuals. While the estimates are still consistent, the posterior parameter distribution for the θs might be too narrow, *i.e.*, the credible intervals too tight. To address this concern, we implement the plug-in method based on the HAC variance-covariance matrix of Müller (2013). By doing this, we preserve the relationship between the variables in levels, but allow for a wider posterior distribution. Moreover, the HAC correction can be easily implemented, consisting of a simple plug-in procedure (Müller, 2013). In Table 2, we show that the HAC correction indeed yields higher uncertainty around the estimates, *i.e.* larger standard errors of the draws, relative to the unadjusted posterior (see $\widehat{std}$ vs. std).

QE Forward Guidance. As illustrated in Greenwood, Hanson and Vayanos (2015) forward guidance about QE also can exert a price impact on rates. To examine this additional mechanism through the lens of the VV model, the β_t should be modeled as a two-factor process, where the β_t reverts to a time-varying stochastic mean; and, shocks to the stochastic mean should be interpreted as QE forward guidance. While a formal examination of QE forward guidance is beyond the scope of this study, it is natural to ask if, and to what extent, the smoothed β_t factor reacts to Fed's communication about QE. This might also help explain why at times there are sudden and abrupt movements in β_t on the backdrop of unchanged demand and duration pressures by officials (b_t).

We start from the premise that β_t, which measures excess supply, should increase (decrease) in response to contractionary (expansionary) announcements on QE. In order to investigate this issue, we construct two dummy variables, I_c and I_e, which take the value of 1 in the months of contractionary and expansionary announcements, respectively; we use the same 16 events, of which 4 are contractionary and 12 expansionary, of Greenwood, Hanson and Vayanos (2015). We run off-model regressions which include the announcement dummies (not reported) in addition to the b_t variables, and find that the estimates largely support this hypothesis; I_c and I_e have, respectively, positive and negative signs (I_e is more precisely estimated), while the θ coefficients are basically unchanged. Moreover, the inclusion of such dummies helps account for some of the abrupt changes in β_t, such as during the 'taper-tantrum' episode.

Scaling Demand and Supply. As we briefly discussed in Section 4.2, the scaling of demand and supply variables requires us to take a stand on which types of investors can be regarded as arbitrageurs. We start from the premise that, according to the VV model, arbitrageurs should have price-elastic demands for bonds. Therefore, a natural way to proceed is to select as arbitrageurs all investors except for the price-inelastic official investors (arbs1: AO-FO-FED). We centered the analysis conducted so far around this baseline specification. Here, we consider the following three alternative definitions: i) all investors but official investors and households (arbs2: AO-FO-FED-HH); ii) all investors but official investors, households and foreign private investors (arbs3: AO-FO-FED-HH-FP); and, iii) shadow banks (arbs4: Shadow Banks), as defined in Andolfatto and Spewak (2018); these measures are presented in Figure IA.4.[32]

We re-estimate the whole term structure model changing the demand and supply measures in b_t, that is, by scaling FO, FED and AO by arbs2, arbs3, or arbs4, respectively. We find that the θs are correctly signed. To examine the robustness of the results also in quantitative terms, we focus on the price impact of QE policies (Table A2). We find that the Fed price impact estimates do not differ by more than 20 basis points across specifications. More specifically, the price impacts of Fed policies during LSAP2 fall as we move from arbs1 to arbs4; as a result, the baseline estimate of the price impact of LSAP2 can be regarded as an upper bound. The shadow-bank specification (Panel IV) delivers a lower gross price impact exerted by the Fed during LSAP2, but a higher impact net of supply, and finds no downward pressure exerted by Fed policies during the MEP, which is instead due to supply. Thus, except for these differences, the QE price impact estimates are largely robust, also in quantitative terms.

In sum, despite the specification employed rests on a specific choice of the arbitrageurs, we favored its economic coherence with the VV model. We therefore centered the empirical analysis around this specification, which also turned out to deliver largely robust price-impact estimates, regardless of the specific measure employed for arbitrageurs; however, it is also fair to argue that most of the (qualitative) findings documented in this study do not depend on the specific choice made to scale the observable measures of demand.

6.3 Three-Factor Model: Central-Tendency Process

Thus far, we centered the analysis around a two-factor model; namely, the first factor captured the short-real rate (r_t) and the second factor excess supply (β_t). This choice was dictated by the empirical evidence resulting from the PC analysis, as well as by the desire to preserve model tractability, in light of the non-standard and highly computationally demanding bond pricing, inherent in the VV model. However, a number of studies have recently documented that the natural rate has dropped substantially over time, and, as a consequence, long-run interest-rate expectations have also declined. Therefore, one might argue that the two-factor model described in Section 3 is too restrictive. Indeed, a standard argument is that a constant unconditional mean of r_t implicitly assumes that short-term rate movements are sufficient to predict future short rates; and, as a result, the information contained in long rates is neglected in forming long-term expectations about short rates. In our study, this would translate into the concern that the empirical finding that longer-term rates load exclusively on the β_t factor might be an artifact of the two-factor model set up, rather than a true and novel finding.

We take seriously this potential shortcoming and develop a three-factor version of the VV model, and let the data speak. To do this, we incorporate the two-factor central-tendency process proposed by Balduzzi, Das and Foresi (1998) into the VV model. Specifically, the short rate follows the two-factor central-tendency process

$$dr_t = \kappa_r \left(\overline{r} + \eta_t - r_t\right) dt + \sigma_r dB_{r,t}, \tag{17}$$

$$d\eta_t = -\kappa_\eta \eta_t dt + \sigma_\eta dB_{\eta,t}, \tag{18}$$

where the short-rate, r_t, reverts to the stochastic mean $\overline{r} + \eta_t$, and η_t is modeled as a zero mean Ornstein-Uhlenbeck process. The instantaneous correlation between $B_{r,t}$ and $B_{\eta,t}$ is $\rho_{r,\eta}$. The aggregate excess-supply factor β_t follows the same Ornstein-Uhlenbeck process as in the two factor case. The instantaneous correlations between $B_{\beta,t}$ and $B_{r,t}$ and between $B_{\beta,t}$ and $B_{\eta,t}$ are denoted by $\rho_{r,\beta}$ and $\rho_{\eta,\beta}$, respectively. As in the two-factor model, excess demand for the bond with maturity τ is a linear function of the bond's yield ($R_{t,\tau}$) and aggregate demand (β_t). We then follow the same steps as before to solve the model and derive the bond pricing; the fix-point iteration now consists of solving a system of nine non-linear equations (from the four of the two-factor model), so that the increase in the computational burden is substantial; see Section III, in the Online Appendix.

We take the three-factor model to the real-rate data adapting the previously described MCMC algorithm to account for the additional parameters and states. We find that, despite η_t displaying a downward trend soon after the crisis (as one would expect), the excess-supply factor, β_t, and its factor loadings for long-term maturities, $A_\beta(\tau)$, are essentially unchanged and precisely estimated (Figures IA.16 and IA.17, respectively).[33] This is because both the $A_r(\tau)$ and $A_\eta(\tau)$ loadings take negligible values at longer-term maturities. While $A_r(\tau)$ loads on very short-term rates, $A_\eta(\tau)$ affects somewhat longer maturities, but it still peaks at medium-term maturities and dies out rather quickly thereafter, so that its effect on long-term maturities is small. Put simply, the combined effect of $A_r(\tau)$ and $A_\eta(\tau)$ resemble that of $A_r(\tau)$ uncovered previously for the two-factor model. Table A6 present the parameter estimates. Of particular interest is the structural parameter δ, 0.02, which despite being somewhat higher is not statistically different from the estimate of the two-factor model. These multiple pieces of evidence, coupled with the estimates of the θ loadings, explain why the price impact of official demand pressures, especially on longer-term rates, is robust to including a stochastic central-tendency process (Table A7).[34]

The goal of estimating the three-factor VV model is interesting but challenging due not only to the complexity of solving for the bond prices in VV, but also to the lack of information at the short-end of the real-rate curve. It is well established that to accurately pin down the unconditional short rate a sufficiently long time series of data is needed, and this is even more so for properly identifying the central-tendency process. As a result, it is not surprisingly that we find it to be particularly hard to identify the two factors, r_t and η_t, driving the short real rate dynamic using U.S. real rates. This also casts some doubts over the decompositions of the real rates based on the three-factor model estimates. Moreover, although adding the central-tendency process delivers a more accurate pricing of the real bonds, the key economic insights are essentially unchanged.[35] Rather, the pricing complicates substantially, and the convergence of the MCMC is often not achieved for some parameters.

Taken together, this additional analysis suggests that, for real rates, the two-factor model offers a satisfactory fit and that the main findings documented are not driven by assuming a constant unconditional mean of r_t. Put differently, for real rates, it is more important to add an additional factor that controls for (il)liquidity, rather than adding a central-tendency process to the VV model. That said, the three-factor model, adequately modified to account for the ZLB, can be useful to quantify price pressures on nominal rates.

6.4 Open Issues and Possible Extensions

The *observed* factor, $\beta_t^o = \hat{\theta}\mathrm{b}_t$, seems to capture rather well the low frequency behavior of the *smoothed* β_t factor. However, we observe significant temporary differences between the two. On the one hand, this is partly due to the fact that our measures of demand pressures are rather persistent, whereas longer-term rates (although less than short-term rates) display short-run variability, which is reflected in the smoothed β_t factor; indeed, we find that controlling for QE forward guidance can partly mitigate such discrepancies. On the other hand, under certain circumstances, other investors might display partly inelastic demand for longer-term Treasuries and, therefore, qualify as preferred-habitat. For example, pension fund and life insurer investments are largely driven by considerations other than return, such as liability matching, often resulting from regulatory changes (*e.g.*, Greenwood and Vayanos, 2010). As a result, these institutional investors have largely price inelastic preferences for government securities, and are usually regarded as preferred-habitat investors.

The holdings of Treasuries by commercial banks might also help explain the β factor, and therefore the wedge between β_t^o and β_t. However, how to model commercial banks' demand, relative to other long-term investors, is less straightforward. On the one hand, the modeling could be similar as commercial banks' demand can, at times, be inelastic; this can happen, for example, when their purchases of Treasuries are driven by liquidity and safety needs, induced by (changing) regulatory requirements; U.S. banks have been hoarding Treasuries securities during the 2008-2010 period. On the other hand, commercial banks can also act as the yield-oriented investors of Hanson and Stein (2015). These investors care about their portfolio income and consequently shift their allocations from short- to long-term Treasury based on the current level of the short rate. Thus, their investment in Treasuries is driven by short-termist considerations which make their demand highly elastic; as a result, the yield-oriented investors clearly differ from the long-term investors with inelastic demand of VV. However, the distinction might turn out to be, empirically, less clear cut than one might think: this is because of the functional form of the maturity-specific excess demand $y_{\tau,t}$ which is partly elastic. Thus, we cannot rule out the possibility, based on the current specification, that the estimate αa can partly reflect the activity of commercial banks.[36] That said, it is the inelastic component of commercial banks' demand which can help reduce the wedge between β_t^o and β_t.

Variables capturing market portfolio rebalancing effects could also be important. In a recent paper, Malkhozov et al. (2016) show, both theoretically and empirically, that there are supply effects resulting from the hedging activity of financial intermediaries. At times, when the duration of outstanding MBS drops, financial intermediaries rebalance their hedging portfolios by buying longer-term Treasuries. In this way, they can put further pressure on the interest rates. However, this also brings us to flag another potential shortcoming of the VV model; the model currently contains only one type of asset, *i.e.*, Treasury securities, while, in practice, other assets might also contribute to determine the duration of the arbitrageurs portfolios. Temporary variations in the β_t factor might also be due to changes in market sentiment. During flight-to-safety episodes investors' demand for Treasuries becomes increasingly inelastic. In these circumstances, not only Treasuries are perceived as safe-haven assets, but also specialized investors have fewer possibilities to trade away arbitrage opportunities.

There are other reasons why the VV model could provide a partial representation of the mechanism through which demand pressures can affect rates, which can contaminate the estimate of the unobserved β_t factor; three are worth a mention. First, the VV model assumes that arbitrageurs have mean-variance preferences over wealth. As a result, negative term premiums can be generated solely by the arbitrageurs shorting duration risk. This contrasts with standard no-arbitrage models, where negative term premiums result from bonds being a hedge asset against consumption risk. For example, this consideration suggests that, as deflationary pressures increased in the U.S. as a result of the global financial crisis, TIPS might have turned into riskier assets. This, in turn, might have pushed up the term premium, at times when the UMPs were instead pushing down the term premium. Second, another potential limitation of the model relates to its inability to capture the stochastic volatility of interest rates. While a model can capture fairly well the observed unconditional volatility, it may still mis-represent the unconditional volatility (see, *e.g.*, Collin-Dufresne, Goldstein and Jones, 2009). Third, the VV model assumes that the factors have constant dynamics. Although this is a rather standard assumption in no-arbitrage term-structure models (Christensen and Rudebusch, 2012; Hamilton and Wu, 2012; Christensen, Lopez and Rudebusch, 2015), it might be too restrictive in practice, especially for modeling QE policies.

7. CONCLUDING REMARKS

This paper provides a structural estimation of the impact of official demand pressures on the term structure of U.S. real interest rates. A distinguishing feature of our analysis is that we explicitly estimate the no-arbitrage model of Vayanos and Vila (2009; VV). We organize our analysis around a somewhat simplified, but empirically tractable, version of the VV model, whereby a 'global', single-maturity excess-supply factor, and a short-rate factor determine bond prices. However, to try to better characterize the demand pressures, we focus on the Fed and foreign officials' Treasury portfolios: these investors stand out for their largely inelastic demands for longer-term Treasuries, coupled with large holdings. Although the focus is not on supply as such, it is controlled for, coherently with the VV model. But, because all these variables are expressed in dollar values, they need to be scaled. We use as scaling factor a measure of arbitrageurs risk-bearing capacity, proxied by their holdings of Treasuries, which is also broadly in line with the VV model. Moreover, to estimate the model on real rates, we account for the changing liquidity conditions in the TIPS market. We do this by including a liquidity factor. While this factor does not enter the bond pricing of the VV model, it can still determine the level of real rates, producing maturity-specific effects.

Our empirical counterpart of the VV model performs rather well. The findings lend support to our main hypothesis: the extracted inelastic excess-supply factor co-moves with our observable measures, that is, positively with supply and negatively with official demand. We find that the Fed asset purchase programs, as well as its MEP, have been effective in lowering real rates. Indeed, in the absence of Fed purchases, longer-term yields would have been significantly higher. However, the Fed price pressures were in part attenuated by the rising supply during that period, and were somewhat intensified by the foreign official demand. That said, the foreign official price impact mainly materializes in the period prior to the crisis, and such pressures were only in part attenuated by supply. Overall, official purchases affected real bonds through significant reductions in bond risk premiums.

To conclude, despite subjecting the analysis to a number of robustness tests, some of the findings should be read with caution. The model estimated here clearly offers a simplified representation of complex behavior and interdependence between various players acting in the Treasury market. To better represent the Treasury market, one could enrich the model by introducing other large players, such as pension funds and regulated commercial banks, for example. Alternatively, rather than focusing on aggregate excess-supply within a two-factor model, a more complex dynamic of individual excess-supply factors, capturing also local demand/supply effects, could be considered. On the other hand, the three-factor model derived in this study can prove useful in the examination of the price pressures exerted by UMPs directly on the nominal rates over much longer periods. All these extensions constitute promising avenues for original research on the nexus of central bank policies and financial markets.

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NOTES

[1]In Section I, in the Online Appendix, we provide a description of recent developments in the U.S. Treasury market, with a focus on official demand.

[2]For example, Warnock and Warnock (2009) find that the fall in the 10-year rate associated with foreign official purchases of U.S. Treasuries to be roughly 80 basis points. Krishnamurthy and Vissing-Jorgensen (2012) argue that foreign official purchases, by reducing the supply of safe assets available to the rest of investors, drive up the convenience yield; in fact, they find that, if foreign officials were to sell their holdings, long-term Treasury yields would be raised by 59 basis points relative to the Baa corporate bond yields. Sierra (2014), through a series of forecasting regressions of realized excess returns on measures of net purchases of Treasuries, documents that official flows behave similarly to relative supply shocks. On balance, nonetheless, he finds little role for foreign investors in reducing 10-year yields on U.S. Treasury bonds. See also Bernanke, Reinhart and Sack (2004), Bernanke et al. (2011) and Beltran et al. (2013) on the link between foreign capital flows and Treasury yields.

[3]Hamilton and Wu (2012) first estimate a standard term structure model on Treasury yields, whereby the three standard factors (*i.e., level, slope and curvature*) drive the bond pricing. Notably, they also extend this model to account for the ZLB. Then, they try to bring in some of the preferred-habitat structure of the VV model, in particular, by examining how changes in the maturity structure of outstanding Treasury debt would influence the price of risk. They complete the analysis through a series of auxiliary predictive regressions.

[4]This function maps demand/supply shocks to the global inelastic β factor, relative to the yield of maturity τ, to the partly elastic excess demand for the bond with τ maturity, $y_{t,\tau}$. The proposed functional form takes a hump-shaped pattern, which equals zero for $\tau = 0$ and $\tau = \infty$; this is because the asymptote rate ($\tau = \infty$) should be unaffected by demand shocks in order to maintain the tractability of no-arbitrage assumption (VV, 2009). The location of the peak and the shape of the function are determined by the δ parameter. We would expect the effect of demand/supply shocks to be larger for longer-term maturities. In this way, as also argued by VV, demand/supply shocks would exert a significant impact on the risk premiums, which are more relevant for longer maturities. We will let the data pin down the exact shape of this impact curve.

[5]Greenwood and Vayanos (2014) use an alternative specification of equation (2) where $R_{t,\tau}$ is excluded; in this way, they simplify the model by assuming that supply for each maturity is price inelastic in the absence of arbitrageurs. Such restriction is also driven by the fact that they do not model the demand of preferred-habitat investors, focusing solely on supply effects. In contrast, this restriction would not match the behavior of official demand – the focus of our study – which, in practice, is partly elastic. We therefore opt for the specification of equation (2).

[6]In the original version of the VV model, the use of multiple demand factors is also aimed at capturing local demand effects. However, we abstract from introducing local demand effects due to the lack of maturity-specific data on official holdings, foreign in particular. Furthermore, there is no evidence that official investors have strong preferences among Treasuries of similar maturities. They rather have preferences for maturity segments, *i.e.*, longer- over shorter-term bonds, as reflected in the somewhat long duration of their holdings. These observations lend support to our attempt to center the analysis of official pressures around a *global*, or single-maturity, factor, similar to Greenwood and Vayanos (2014). Analysis of the *local* supply effects of QE on bond-by-bond data is provided, for example, by D'Amico and King (2013).

[7]A distinction between the two channels is beyond the scope of this study, and we therefore refer the reader to other studies for further details (e.g., D'Amico et al., 2012, Meaning and Zhu, 2012). Another prominent channel through which central banks tried to exert downward pressure on rates is the signaling channel (e.g., see, Krishnamurthy and Vissing-Jorgensen, 2011). While the scarcity and duration channels work via changes in bond risk premia, the signaling channel works mainly via changes in expected yields. Central banks' forward guidance can interact with any of these channels (see, e.g., Greenwood, Hanson and Vayanos, 2015).

[8]Modeling real rates rather than nominal rates allows us to organize the analysis around a more parsimonious specification of the VV model, as the inflation dynamics and the ZLB do not need to be modeled explicitly. Clearly, a caveat is that our analysis is silent about the impact of official demand on inflation expectations and risk premiums.

[9]There are two caveats in using TIPS implied real yields. First, TIPS are affected by a three-month indexation lag. However, this effect is estimated to range from basically zero basis points for the one-year TIPS yield to about 4.2 for the 10-year (Grishchenko and Huang, 2012). We therefore do not account for this effect. Second, TIPS yields are also affected by the so-called deflation floor. Indeed, there is an embedded deflation option in the U.S. TIPS, as the TIPS owner receives the greater between the original principal and the inflation adjusted principal at maturity. However, this effect is largely attenuated by the fact that we focus on longer maturity yields (which are affected less) and use a mix of on-the-run and off-the-run TIPS. Moreover, although this effect is partly related to deflationary expectations, the value of the option is usually small (Grishchenko, Vanden and Zhang, 2011).

[10]Hu, Pan and Wang (2013) use a similar measure for nominal bonds, and Grishchenko and Huang (2012) and D'Amico, Kim and Wei (2018) for TIPS. This measure may capture a market-wide liquidity disruption caused by a lack of arbitrage during episodes of severe market stress, such as the peak of the recent 2008-09 financial crisis. As anticipated before, this measure fails to capture other aspects of (il)liquidity; for example, liquidity might be impaired also in normal times when arbitrage is operating well. We refer to D'Amico, Kim and Wei (2018) for a review of the most used measures. Some of these

measures, however, might interact with some of the economic effects present in VV. In this study, we also focus on an alternative measure that, albeit is not specific to the TIPS market, is widely used in the literature and is little contaminated by preferred-habitat effects: the off/on-the-run spread (see Section 6).

[11]As a caveat, the VV model is solved under the assumption of constant risk aversion. However, one could think that investors price bonds as if they believe risk aversion were constant. Alternatively, as stated in Greenwood and Vayanos (2014), "An additional empirical hypothesis follows Proposition 4, which shows that the effect of supply on instantaneous expected returns is increasing in the arbitrageurs' risk-aversion coefficient a. In our model a is constant over time, and Proposition 4 is a comparative statics result. Stepping outside of the model, however, we can interpret Proposition 4 as concerning the effects of time variation in a." Note that they focus only on supply, but this argument naturally extends to demand pressures.

[12]Anecdotal evidence, however, seems to suggest that foreign demand at TIPS auctions has been remarkably strong, averaging around 39 percent (Gongloff, 2010); that said, it might have reduced somewhat in recent years.

[13]Nominal and real rates displayed similar responses to FOMC communications about the Fed's asset purchases (Yellen, 2011), and to the surprise component of LSAP announcements (Rosa, 2012). Also note that the payoffs of Treasury bonds can be replicated, for example, via a combination of TIPS, STRIPS and inflation swaps (*e.g.* Fleckenstein, Longstaff and Lustig, 2014). The segmentation across these markets can, however, intensify when there is less slow moving capital in the economy, such as during the global financial crisis; and, the lack of capital can be more severe for TIPS, which as a consequence can experience a substantial drop in prices (Fleckenstein, Longstaff and Lustig, 2014). This effect, however, is in part captured by our liquidity component. Indeed, the average fitting errors proxy for the slowly moving capital, which is intimately linked with liquidity conditions in the bond market (Hu, Pan and Wang, 2013).

[14]To discretize the continuous-time specification of the factors of equations (1) and (5) we use the Euler scheme. The time step, Δ, is 1/12 for the monthly frequency.

[15]Although specifying the liquidity spread as a linear function of liquidity conditions might appear overly restrictive, especially during the crisis, this concern should be mitigated by the use of price-based liquidity measures, such as the average fitting errors, relative to that of quantity-based measures.

[16]This way of modeling liquidity effects resembles the one adopted by Pan and Singleton (2008) for the pricing of CDS spreads. In their model, the CDS premium composes of the model-implied premium (based on a one-factor reduced-form credit risk model) and the pricing error. Fundamentally, the pricing error volatilities depend on the maturity specific time-t bid-ask spread, thus (il)liquidity varies across maturities and over time. By doing this, they allow for the possibility that the pricing of the credit-

risk model deteriorates as market liquidity conditions worsen. However, we favor the specification of equation (13), relative to that of Pan and Singleton (2008), as it allows us to identify the maturity-specific liquidity component. In contrast, in their specification, the liquidity component is left unexplained, and is captured by the error term. A caveat of both methods is that the market price of liquidity risk is not modeled explicitly, and, as a consequence, one cannot decompose the liquidity spread into its risk and risk premium components, which is, however, beyond the scope of this study.

[17]The proxy of the one-month real rate is calculated as the annualized end-month policy rate, proxied by the 3-month TBill rate, less expected inflation, proxied by the short-term Consensus surveys, *i.e.*, the current year CPI growth rate. This is obviously a proxy because the tenor of the policy rate is not monthly and we are making a reasonable but rather arbitrary assumption about inflation expectations. However, we also experimented with measuring inflation expectations with the (annualized) growth rate of CPI in the previous (or next) month, and with replacing the TBill rate with the Libor and OIS rates.

[18]The slice-sampling method was developed by Neal (2003), and since then has been applied by a number of asset-pricing studies (e.g., Li and Zinna, 2014, 2018, and Zinna, 2016).

[19]As recently highlighted by the Chair of the Fed Board of Governors at that time, Ms. Yellen, "[The natural rate's] value at any point in time cannot be estimated or projected with much precision" (Yellen, 2017).

[20]This finding accounts for a plausible reaction function of the Fed, especially during the UMP regime, when longer lasting Fed bond purchases are coupled with the prolonged expectations of depressed policy rates (the so-called *signaling* channel). However, to fully examine this possibility, one should, for example, allow for the correlation to change over time.

[21]The δ parameter captures the total amount of duration risk borne by arbitrageurs, and, as a result, should contribute to determine the bond risk premiums. As the (average) slope of the yield curve is usually informative about the (average) bond risk premiums, this is the kind of information which should help pin down the δ parameter. On the other hand, this also suggests that the estimate of δ might change somewhat with the sample period considered. However, we find that it is not statistically different from that obtained over the 2001-2012 period.

[22]While the model of VV is directly comparable to ours, VV do not provide an estimate of δ. By contrast, Greenwood and Vayanos (2014), and Greenwood, Hanson and Vayanos (2015) rest on different specifications, so that our estimate of δ is not directly comparable to any of their parameters. However, we note that they calibrate the sensitivity of the dollar supply of bond with maturity τ to the supply factor β_t; and, to do that, they rest on the assumption that changes in β_t can only alter the duration of arbitrageurs' bond portfolio, but not the size of their portfolio, *i.e.* the dollar value of the bonds held. We instead let the data pin down the effect of aggregate demand on the arbitrageurs' portfolio.

[23]The wedge between r_t and r_t^o peaks during the crisis, when uncertainty about expected inflation also

rises, and the differences among the short-real rates proxies widen. For the model to fit both the short real rate proxy and shorter-term real rates, a more complex modelling of the interest-rate factor (e.g. by adding a central tendency process, as done in Section 6.3) is needed. However, when an additional interest-rate factor is included, the evolution of the excess-supply factor, as well as the price impact estimates, remain largely unchanged.

[24]Although the term premium displays a significant drop already around the second half of 2009, it then rises again. A more abrupt and persistent fall in the term premium is evident in the 2011-2013 period.

[25]This spike in the term premium is consistent with the empirical evidence documented in other studies, such as, *e.g.*, Adrian, Crump and Moench (2013). Greenwood, Hanson and Vayanos (2015) show, theoretically, that QE forward guidance can indeed work by altering bond risk premiums, and therefore be responsible for the observed spike in rates associated with the 'Taper Tantrum' episode. We return to this issue in Section 6.

[26]If the holdings were maturity weighted, such as in the construction of 10-year equivalents (e.g., Li and Wei, 2013), then one might argue that differences in θs were driven by the different price sensitivities. However, the lack of data on holdings at the security level does not allow us to convert the total holdings (for foreign officials in particular) to 10-year equivalents. That said, distinguishing between price elasticity vs. maturity profile as the main factor behind the different θs is beyond the scope of this study, but a promising avenue for further research.

[27]According to the various sources, both these components slightly increased during the LSAPs programs, and, as a result, the impact on nominal yields is smaller than the impact on real rates. For example, according to Krishnamurthy and Vissing-Jorgensen (2011), inflation expectations increased as a result of both QE1 and QE2. More specifically, according to the Cleveland Fed estimates, expected inflation alone at 10-year horizon has increased by more than 20 bps in the period from November 2010 to July 2011.

[28]We should acknowledge that our price-impact estimates include the effects exerted by the Fed's reinvestment policy.

[29]These additional results are presented in the separate Online Appendix.

[30]The off/on-the-run spread for TIPS is not available until very recently and is model dependent (Christensen, Lopez and Shultz, 2017). Therefore, we need to recur to its counterpart for nominal bonds, which clearly has the cons of not being specific to the TIPS market. Also for this reason, we center the baseline analysis around the average TIPS curve fitting errors.

[31]The main results for the model with no liquidity controls are presented in Table A4 and Figures IA.7-IA.9. We also experimented with the (inverse of the) turnover in the TIPS market, following, for example, Pflueger and Viceira (2016). However, when we re-estimate the model using this measure, we

find that it fails to capture the spike in real rates. This is possibly because this measure is somewhat more related to aspects of the preferred-habitat demand and supply of the VV model; in addition, it performs less well, as reflected in the larger pricing errors. In this regard, the off/on-the-run spread yields a better performance, as it delivers smaller pricing errors.

[32]While official investors' holdings of Treasuries are available at monthly frequency, those of households, foreign private investors and shadow banks are available, from the Financial Accounts of the U.S., at quarterly frequency. We therefore adopt a linear interpolation to transform such holdings to monthly frequency.

[33]The only material difference being that the peak of the $A_\beta(\tau)$ curve is somewhat shifted to the left relative to the two-factor model counterpart, and as a result β_t becomes even more relevant for medium-term maturities.

[34]In the Online Appendix, to further show the robustness of the results, we report the output obtained from estimating i) the three-factor model with no controls for liquidity (Table A8 and Figures IA.19-IA.21), and ii) the three-factor model where liquidity is controlled for using the on/off the run spread (Table A9 and Figures IA.23-IA.25).

[35]When we add a cap to the short-rate proxy measurement error, thus trying to force the model to more closely fit the proxy, the bond pricing deteriorates somewhat. However, the evidence on the excess-supply factor (and, as a result, also that on the θ loadings and price impact) remains largely unchanged.

[36]The framework used by Greenwood and Vayanos (2014) would help rule out this possibility. But, as stressed earlier on, it is not suitable for the examination of official demand pressures which can also be partly elastic. While it is ex-ante unclear whether the resulting model would still be tractable, a more direct way to shed light on this issue would be to change the specification of the maturity specific excess demand, as follows: $y_{\tau,t} = \alpha_1(\tau)\tau R_{\tau,t} - \alpha_2(\tau)\tau\beta_t$.

Table 1: *Parameter Estimates*

	Panel A: Model Parameters						Panel B: Liquidity Parameters				
	mean	lb	ub	nse	CD		mean	lb	ub	nse	CD
κ_r	0.12	0.11	0.14	0.000	0.59	ν_{2yr}	9.34	8.48	10.21	0.010	-0.11
$\overline{r}$	0.44	-0.27	1.16	0.006	0.76	ν_{5yr}	4.80	4.07	5.53	0.010	0.15
						ν_{10yr}	2.92	2.30	3.54	0.010	0.43
κ_β	0.33	0.28	0.38	0.000	0.78	ν_{15yr}	1.97	1.42	2.54	0.000	0.58
$\overline{\beta}$	1.87	1.64	2.11	0.002	0.69	ν_{20yr}	1.37	0.85	1.88	0.000	0.67
σ_r	1.62	1.53	1.70	0.001	0.59		Panel C: Pricing Errors				
σ_β	0.77	0.73	0.81	0.000	0.88		mean	lb	ub	nse	CD
ρ	51.95	46.27	57.50	0.044	1.40	σ_ε	9.51	9.23	9.80	0.002	0.89
						$\sigma_{1,\epsilon}$	131.2	124.3	138.1	0.057	1.12
$a\alpha$	45.68	42.05	49.05	0.028	0.93						
δ	0.012	0.01	0.02	0.000	0.37						

Note: The table presents posterior means, 68% credible intervals, numerical standard errors (nse), and the absolute value of the convergence diagnostic (CD), as in Geweke (1992), for the estimated structural parameters of the VV model, the liquidity parameters (ν), and the measurement error volatilities of the yields (σ_ε) and short-rate proxy ($\sigma_{1,\epsilon}$). The parameters $\overline{r}$ and $\overline{\beta}$, σ_r, σ_β, and ρ are reported in percent, and σ_ε and $\sigma_{1,\epsilon}$ in basis points. The model is estimated over the period ranging from January 2001 to December 2016 using real rates for the 2-, 5-, 10-, 15- and 20-year maturities at monthly frequency, the (il)liquidity variable and the observable measures of demand and supply presented in Section 4.

Table 2: θ-*Factor Loadings*

	mean	$\widehat{lb}$	$\widehat{ub}$	$\widehat{std}$	std	nse (%)	CD
θ_0	0.008	-0.002	0.017	0.010	0.006	0.010	0.459
θ_1	-0.053	-0.065	-0.042	0.012	0.006	0.010	0.609
θ_2	-0.127	-0.141	-0.113	0.014	0.007	0.010	1.450
θ_3	0.030	0.022	0.037	0.008	0.005	0.000	0.609
	mean	lb	ub	$\widehat{std}$	std	nse	CD
$\sigma_{2,\varepsilon}$	55.10	51.76	58.41	-	3.44	0.028	0.809

Note: The table presents the estimated θ loadings that map the observable measures of demand and supply pressures (b_t) onto the smoothed excess-supply factor (β_t), $\beta_t = \theta b_t' + \epsilon_{2,t+\Delta}$. The loading θ_0 is that associated with the constant term; θ_1 is that associated with foreign officials' holdings of Treasury securities, with maturities greater than or equal to five years, over arbitrageurs' Treasury holdings (b_t^{FO}); θ_2 is that associated with the Fed's holdings of Treasury securities over arbitrageurs' Treasury holdings (b_t^{FED}); and, θ_3 is that associated with amount of Treasury securities outstanding over arbitrageurs' Treasury holdings (b_t^{SUP}). The loadings are estimated within the MCMC algorithm, in the specification where liquidity is controlled for and is proxied by the Average TIPS Curve Fitting Errors. The volatility $\sigma_{2,\epsilon}$ is reported in basis points. We report posterior means, 68% credible intervals and standard deviation of the draws based on HAC correction as $\widehat{lb}$, $\widehat{ub}$ and $\widehat{std}$, the unadjusted as lb, ub and std. We also show the numerical standard errors (nse), and the absolute value of the convergence diagnostic (CD), as in Geweke (1992).

Table 3: *Price Impact*

	Panel I: FO				Panel II: Fed			
	2001-16	pre-QE	QE	post-QE	2001-16	pre-QE	QE	post-QE
2-yr	-39.7	-57.4	-20.0	37.7	-35.7	6.7	-90.9	48.4
5-yr	-72.9	-105.4	-36.6	69.2	-65.6	12.3	-166.9	88.9
10-yr	-92.1	-133.2	-46.3	87.4	-82.9	15.6	-210.8	112.3
15-yr	-92.5	-133.8	-46.5	87.8	-83.3	15.7	-211.7	112.8
20-yr	-86.6	-125.3	-43.5	82.2	-78.0	14.7	-198.4	105.7
	Panel III: AO				Panel IV: Total			
	2001-16	pre-QE	QE	post-QE	2001-16	pre-QE	QE	post-QE
2-yr	36.0	13.4	47.8	-25.1	-39.4	-37.4	-63.1	61.0
5-yr	66.2	24.5	87.7	-46.0	-72.4	-68.6	-115.9	112.1
10-yr	83.6	31.0	110.7	-58.2	-91.4	-86.6	-146.3	141.6
15-yr	83.9	31.1	111.2	-58.4	-91.8	-87.0	-147.0	142.2
20-yr	78.6	29.1	104.2	-54.7	-86.0	-81.5	-137.7	133.2

Note: The table reports the price impact of Foreign Officials (Panel I: FO), of the Fed (Panel II: FED), of supply (Panel III: AO) and the combined impact of FO, FED and AO (Panel IV: Total) on the term structure of U.S. real rates for the selected maturities. As for the sample periods, *2001-16* is the entire sample, from January 2001 to December 2016; *pre-QE* is the period prior to the start of QE, from January 2001 to March 2009; *QE* is the period of QE, from March 2009 to October 2014; *post-QE* is the period after the end of QE, from October 2014 to December 2016. The price impacts are quantified using equation (16) and are reported in basis points.

Table 4: *Fed's QE Price Impact*

	Panel I: FED				
	std	LSAP1	LSAP2	MEP	LSAP3
2yr	-33.0	-19.6	-45.1	-18.0	-14.9
5yr	-60.6	-36.0	-82.7	-33.0	-27.3
10yr	-76.6	-45.5	-104.5	-41.7	-34.5
15yr	-76.9	-45.7	-104.9	-41.9	-34.6
20yr	-72.1	-42.8	-98.3	-39.2	-32.4
	Panel II: AO				
	std	LSAP1	LSAP2	MEP	LSAP3
2yr	19.5	23.7	38.4	-9.2	23.9
5yr	35.8	43.5	70.5	-16.8	44.0
10yr	45.2	54.9	89.0	-21.2	55.5
15yr	45.4	55.2	89.4	-21.3	55.8
20yr	42.5	51.7	83.8	-20.0	52.3

Note: The table reports the price impact of the Fed (Panel I: FED) and of supply (Panel II: AO) on the term structure of U.S. real rates for the selected maturities. The column *std* shows the price impact of one-standard deviation change of the variable at hand. As for the sample periods, *LSAP1* is the first stage of the Fed asset purchase program, from March 2009 to November 2009; *LSAP2* is the second stage of the program, from November 2010 to June 2011; *LSAP3* is the third stage of program, from October 2012 to October 2014; and, *MEP* is the maturity extension program, from September 2011 to June 2012. The price impacts are quantified using equation (16) and are reported in basis points.

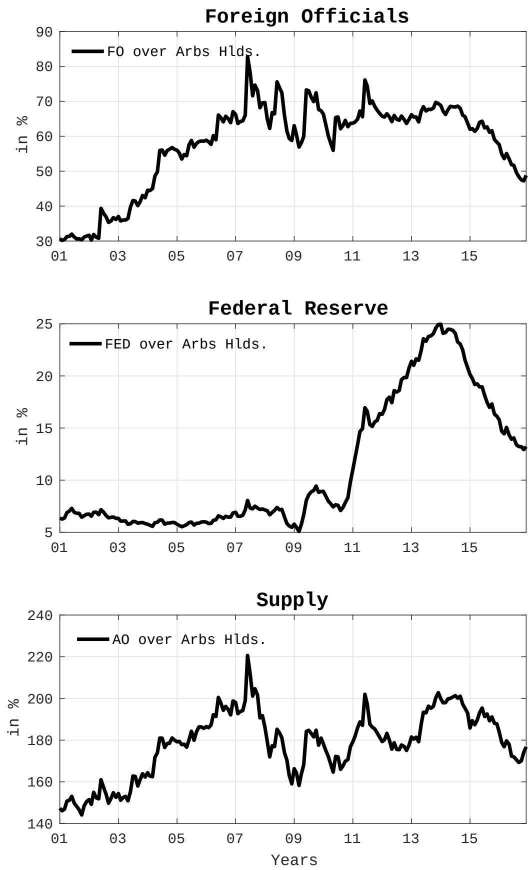

Figure 1: Observable Measures of Demand and Supply

Note: This figure presents the observable measures of demand and supply used in the estimation; foreign officials' (FO) holdings of Treasury securities over arbitrageurs' holdings (*Foreign Officials*; b_t^{FO}); Federal Reserve (FED) holdings of Treasury securities, with maturities greater than or equal to five years, over arbitrageurs' holdings (*Federal Reserve*; b_t^{FED}); and, amount of Treasury securities outstanding (AO) over arbitrageurs' holdings (*Supply*; b_t^{SUP}). The sample period is from January 2001 to December 2016; data are at monthly frequency.

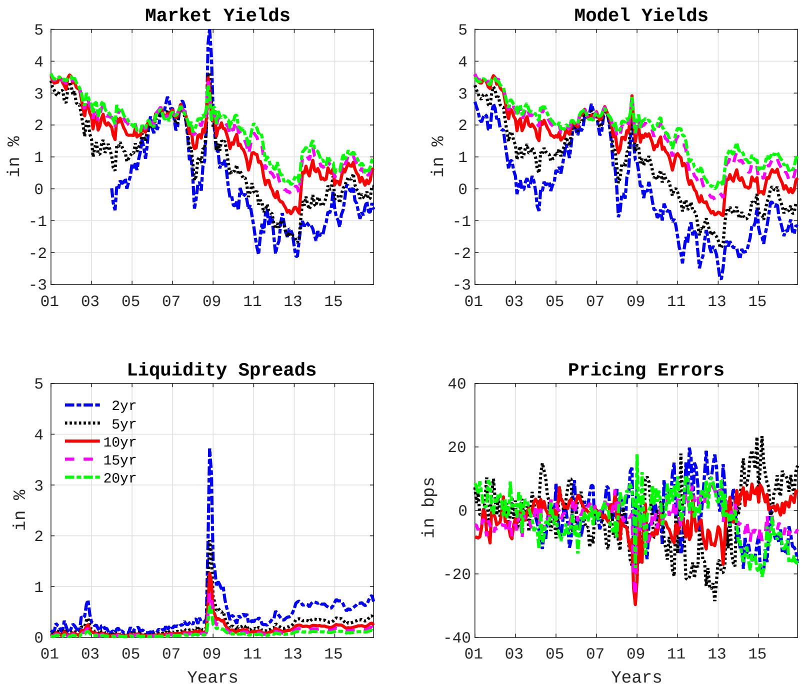

Figure 2: Decomposing Real Rates

Note: The figure shows the observed term structure of real rates for the 2-, 5-, 10-, 15- and 20-year maturities (*Market Yields*); two-factor VV model-implied real rates (*Model Yields*); the liquidity spreads (*Liquidity Spreads*); and, the pricing errors obtained as market yields minus model yields and liquidity spreads (*Pricing Errors*). The (il)liquidity measure used is the Average Fitting Errors. Model-implied rates and liquidity spreads result from the Bayesian estimation of the model presented in Section 4.3. The sample period ranges from January 2001 to December 2016, but the 2-year real rate is available only from January 2004. Data are at monthly frequency.

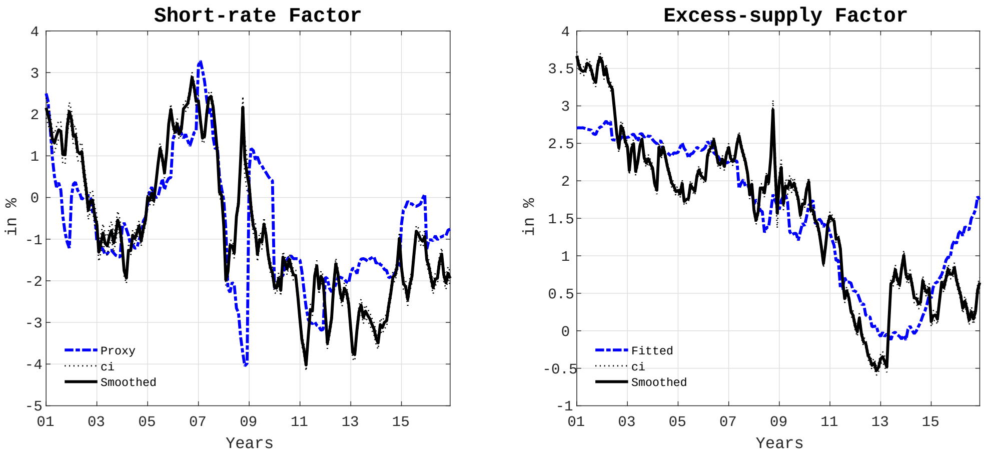

Figure 3: Estimated r_t and β_t Factors

Note: The figure shows the smoothed factors with 68% credible intervals. The left panel plots the short-term real rate factor in black, r_t, and the model-free proxy, which is computed as the 3-month Treasury Bill rate minus the Consensus survey inflation rate for the current year, in blue (*Short-rate Factor*). The right panel plots in black the excess-supply factor, β_t, and the fitted factor based on the observable measures of demand pressures, $\beta_t^o = \hat{\theta}\tilde{B}_t$, in blue (*Excess-supply Factor*). The sample period ranges from January 2001 to December 2016.

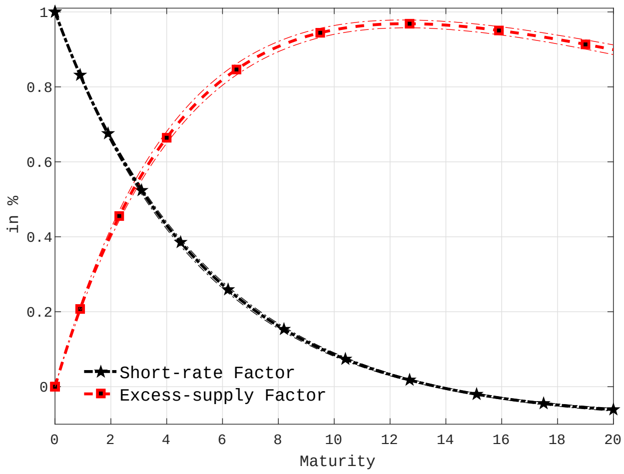

Figure 4: Factor Loadings

Note: This figure shows the effect of a 1% rise in the r_t and β_t factors on the term structure of real rates for maturities ranging from 0 to 20 years. Dotted lines denote the 68% credible intervals.

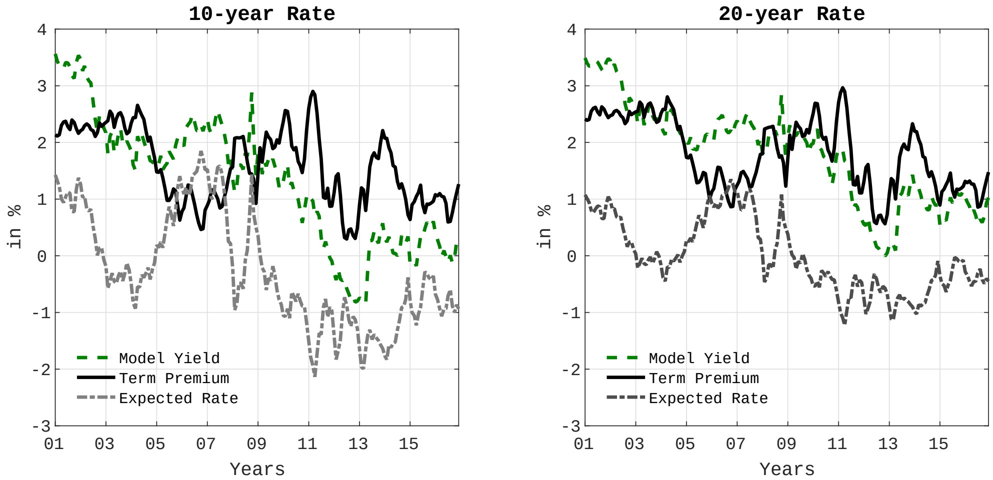

Figure 5: Model-Implied Real Rates, Expected Rates, and Term Premiums

Note: The figure shows the decomposition of the 10- and 20-year model implied real rates into the term-premium and the expected-rate components for the 10- and 20-year maturities. The model rate is the real-rate component obtained from the 2-factor VV model, which therefore does not include the liquidity spread component. The Bayesian algorithm is presented in Section 4.3. The sample period ranges from January 2001 to December 2016.

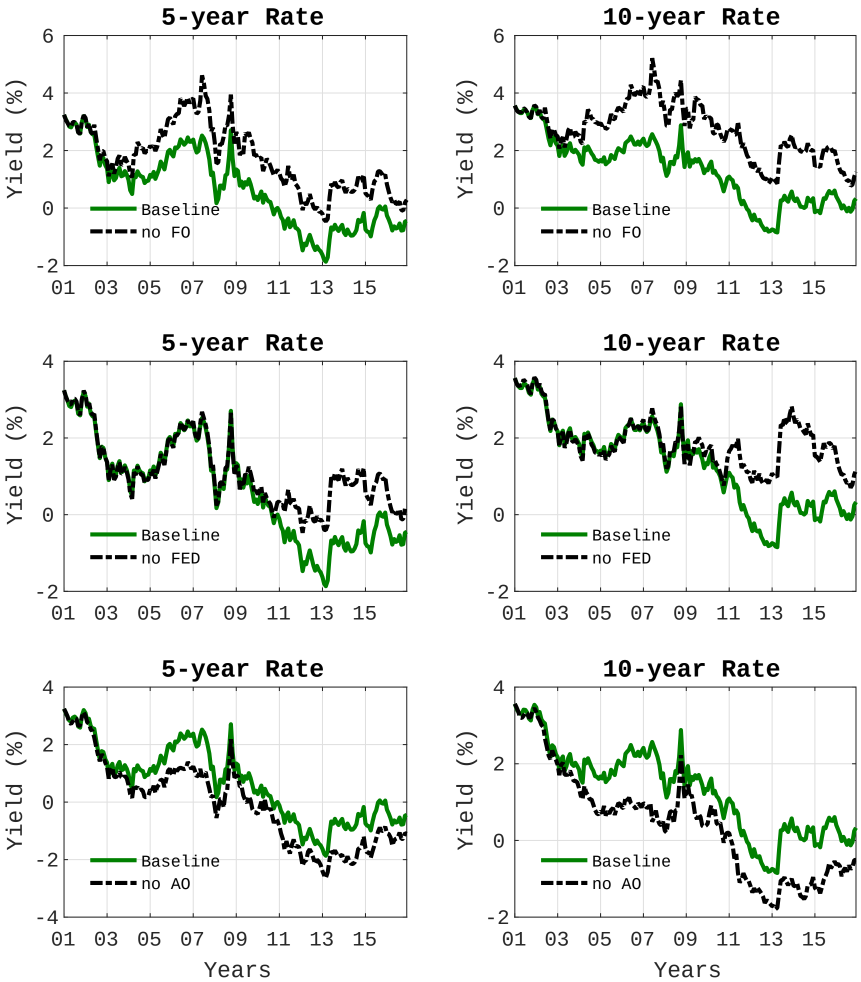

Figure 6: Counterfactual Analysis

Note: The figure presents the counterfactual analysis, showing the cumulative price impacts of foreign officials, Fed purchases and supply on real rates. Model-implied rates, thus abstracting from the liquidity component, are displayed in solid green, *Baseline*, whereas the dotted black lines denote the counterfactual rates, in the absence of official purchases. Specifically, the black line refers to the rate that would have prevailed if official demands, both foreign (no FO) and domestic (no FED), or supply (no AO) were fixed at their January 2001 starting values. The wedge between the Baseline and no FO (no QE or no AO) lines quantifies the cumulative price impact of foreign official (Fed or Supply) price pressures on the 5-year, left panels, and the 10-year, right panels, real rates.