



BANK OF ENGLAND

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## Shareholder risk-taking incentives in the presence of contingent capital

Mahmoud Fatouh<sup>(1)</sup> and Ayowande McCunn<sup>(2)</sup>

### Abstract

This paper presents a model of shareholders' willingness to exert effort to reduce the likelihood of bank distress, and the implications of the presence of contingent convertible (CoCo) bonds in the liabilities structure of a bank. Consistent with the existing literature, we show that the direction of the wealth transfer at the conversion of CoCo bonds determines their impact on shareholder risk-taking incentives. We also find that 'anytime' CoCos (CoCo bonds trigger-able anytime at the discretion of managers) have a minor advantage over regulator CoCo bonds, and that quality of capital requirements can reduce the risk-taking incentives of shareholders. We argue that shareholders can also use manager-specific CoCo bonds to reduce the riskiness of the bank activities. The issuance of such bonds can increase the resilience of individual banks and the whole banking system. Regulators can use restrictions on conversion rates and/or requirements on the quality of capital to address the impact of CoCo bonds issuance on risk-taking incentives.

**Key words:** Risk-taking, CoCo bonds, anytime CoCos, quality of capital requirements, additional Tier 1 capital (AT1), bank manager compensation packages, compensation policy.

**JEL classification:** D81, G21, G28, G30.

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## 1. Introduction

A feature of the great financial crisis was that the levels of capital of many major banks were too low to withstand losses, making public bail-out of failing banks inescapable. Aiming at decreasing the need for such bail-outs without disturbing the financial system, regulators increased capital requirements and sought alternative mechanisms to resolve insolvent financial institutions ([Flannery \(2014\)](#)). In particular, regulators were interested in instruments that could both increase the loss absorbing capacity in times of financial stress, and provide recapitalisation for solvent financial institutions with depleted capital levels (i.e. absorb losses on going concern and gone concern bases). Subordinated (bail-in-able) debt instruments achieve the first but fail the second. They only provide capital when the institution has already become nonviable or a “gone-concern” ([Calello and Ervin \(2010\)](#)). The failure of other debt-like capital in stabilizing banks during the crisis increased the interest in contingent capital instruments, also called contingent convertibles or contingent convertible bonds (CoCo bonds), first suggested by Flannery ([2005](#)). CoCo bonds can convert to equity capital (or have their principal written-down), providing recapitalisation on a going concern basis, unlike bail-in bonds. Banks in some countries have been allowed to use CoCo bonds (*categorized* as Additional Tier 1 (AT1) capital) to partially meet the higher level of capital requirements of Basel III. For instance, in the UK, banks can use such instruments to cover a part (up to 1.5%) of their minimum risk-weighted capital requirements.

Generally, CoCo bonds can be defined through three main contractual features: (1) the type of the CoCo bond, (2) the triggering event and (3) the payoffs of CoCo holders ([Pennacchi et al. \(2014\)](#) and [Hilscher and Raviv \(2014\)](#)). Depending on the loss absorption mechanism (LAM) used, CoCo bonds are either written down (at a predetermined percentage) or converted to equity (at a predetermined conversion ratio) when a predetermined triggering event occurs, and hence can be viewed as a form of crisis insurance, ([Avdjiev et al \(2013\)](#)). Most academics suggest the use of a market-based conversion triggers (such as those based on share price or market capitalisation). However, Basel III standards specify that CoCo bonds can count as Additional Tier 1 (AT1) capital only if they would convert into common equity Tier 1 (CET1) or be written down when the CET1 to risk-weighted assets (RWAs) ratio drops to a triggering level which is not lower than 5.125%.

Several papers shed light on the impact CoCo bonds have on the ex-ante incentives of current shareholders and the managers of the issuing bank. Most authors argue that CoCo bonds that add to the wealth of the initial shareholders increase the risk-taking (or risk-shifting) incentives. Conversely, CoCo bonds whose conversion implies a potential dilution of the current shareholders' value would decrease these incentives (Flannery (2014)). Referring to the optimality argument of debt financing<sup>3</sup>, Koziol and Lawrenz (2011) show that the structure of CoCo bonds delays the transfer of control. This might distort risk-taking incentives, increasing the possibility of financial distress of the issuer. They conclude that CoCo bonds should be used in conjunction with mechanisms that control the risk shifting incentives. Berg and Kaserer (2015) show that CoCo bonds with non-contractible level of risk not only distort risk-taking incentives, but also dis-incentivise the issuance of new equity in a crisis. They also indicate that most CoCo bonds issued have conversion prices that are associated with a wealth transfer from CoCo holders to the current shareholders. Some authors (such as Martynova and Perotti (2012) and Hilscher and Raviv (2014)) claim that appropriately designed CoCo bonds contracts (especially in terms of the conversion ratio) can eradicate the risk-shifting incentives even during times of financial distress. Song and Yang (2016) confirm this by arguing that exogenously-determined (rather than endogenously or issuer-determined) CoCo conversion triggers significantly lower the risk-taking incentives and reduce the overinvestment problem and the debt agency cost. To model the risk-taking incentives, authors generally modify the asset processes<sup>4</sup> to introduce components that reflect asymmetric information between CoCo holders and shareholders and/or managers.

This paper follows a simpler method similar to that of Holmstrom and Tirole (1998). We present a basic model about the moral hazard surrounding shareholders willingness to exert effort that

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<sup>3</sup> Debt financing offers fixed payments in good times but stipulates transfer of ownership in bad times. The threat of losing ownership decreases the risk-taking incentives of the issuer.

<sup>4</sup> The value of assets in most of the CoCo bonds papers follows a stochastic process, mostly in the form of a simple Geometric Brownian Motion (for example, Albul, Jaffee, and Tchisty (2010), Chen, Glasserman and Nouri (2013), Hilscher and Raviv (2014), and Sundaresan and Wang (2015)), or a jump-diffusion process (for example, Pelger (2012), Teneberg (2012), Pennacchi et al. (2014), and Yang and Zhao (2015)).

increases the likelihood of a bank's success. We present a one-shot game and so do not capture the effects of repeated interactions. Such interactions may create reputational constraints, specifically, in the context of anytime CoCos, which can be converted at the discretion of the issuer. Nevertheless, the model is able to identify the key implications of the issuance of CoCo bonds. Our model examines the impact of CoCo bonds issuance on the risk-taking incentives of the issuer. It also investigates the main factors affecting this impact, some of which represent the tools available to the regulator to address risk-taking implications of CoCo bonds issuance. Additionally, we explore the implications of the issuer's discretion over conversion ("anytime" CoCos), capital quality requirements, and misalignment of shareholders' and managers' incentives.

Our analysis draws a number of policy-relevant conclusions. First, consistent with the past literature, the issuance of CoCo bonds can increase the risk-taking incentives if their conversion stipulates a wealth transfer from CoCo holders to shareholders. Conversely, CoCo bonds with wealth transfer from shareholders to CoCo holders reduce these incentives. Second, anytime CoCos are likely to have a minor advantage over regular CoCo bonds as they may convert slightly earlier. However, this would require a properly designed conversion rate that makes it costly for shareholders to wait until the predetermined trigger is hit. Third, by increasing the stake of shareholders in the value of the bank, stricter quality of capital requirements can reduce shareholders' appetite to take excessive risks, when CoCo bonds are issued. Finally, shareholders can also use manager-specific CoCo bonds to reduce the riskiness of the bank activities. The issuance of such bonds can increase the resilience of individual banks and the whole banking system. Regulators can also impose restrictions on conversion rates to address the impact of CoCo bonds issuance on risk-taking incentives.

The remainder of the paper is structured as follows. Section 2 discusses the main features of CoCo bonds. The baseline model is outlined in Section 3. Section 4 presents the solution of the model and the potential implications for the CoCo bonds contracts. Section 5 provides insights for some possible extensions to the baseline model. Section 6 includes concluding remarks.

## 2. CoCo Bonds Features

As mentioned earlier, the type of the CoCo instrument, the triggering event, and the payoff of the holders are the main three features that characterise CoCo bonds contracts. This section discusses these three features and includes a review of the different flavours suggested in the past literature.

### 2.1 Type of the CoCo Bond

The type of the CoCo instrument is determined by its main specifications (such as the face value, interest rate, and maturity date, if not perpetual), and the rights of the issuer and the holders of the bonds before and at maturity. Like with regular bonds, CoCo holders are entitled to a set of interest payments and the face value of the bond on the maturity date (if not perpetual<sup>5</sup>), unless coupons are cancelled<sup>6</sup>, or LAM is triggered (the bonds are converted into common shares or written down). In principle, CoCo bonds conversion/write-down is automatic once the trigger is reached. As a result, the holders of CoCo bonds with market-based triggers might have incentives to induce conversion (through short-selling) if it includes transfer of wealth from shareholders. This is especially true when the market value is close to the triggering level (the death spiral effect). To deal with this problem, some authors suggested modified versions of CoCo bonds. For instance, Pennacchi et al. (2014) propose call option enhanced reverse convertibles (COERCs) that give shareholders of a bank with outstanding CoCo bonds the option to buy back the shares issued at conversion at the face value of the CoCo bonds. Alternatively, De Spiegeleer and Schoutens (2013) prescribe the use of CoCo bonds with multiple triggers each of which initiates the conversion of a part of the bonds outstanding to smooth conversion and reduce the risk of death spirals. Finally, Bolton and Samama (2012) suggest the use of capital access bonds (CoCo bonds with put options) which gives the issuing bank an unconditional right to issue and sell common shares to the bondholders at any time before maturity, at a predetermined exercise price.

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<sup>5</sup> According to CRR Article 52 (1) (g), CoCo bonds have to be perpetual to be considered as additional tier 1 (AT1) capital. As a result, several CoCos issued recently are perpetual with specified (or implied) call schedules.

<sup>6</sup> CRR requires coupon payments on AT1 instruments to be cancellable at the full discretion of the issuing institution, where cancellation doesn't impose any restriction on the institution or impose default upon it (CRR Article 52 (1) (l)). The coupons can also be cancelled at the discretion of the regulator.

## 2.2 The Triggering Event

While the designated Basel III triggering event is a fall in the ratio of CET1 to risk-weighted assets (RWAs) to a pre-specified level not lower than 5.125%, academics have mostly advocated the use of some market-based triggers (Pennacchi et al. (2014)). Generally, CoCo bonds' triggers can be categorized into three main groups. The first group includes accounting-based triggers, in which LAM is activated when some accounting variable (such as equity to RWAs ratio) hits a triggering level. Such triggers, adopted by regulators and some authors<sup>7</sup>, have been criticized due to the failure of accounting variables to reflect economic changes and capture risks in a timely manner, and the CoCo issuer's ability to manipulate accounting ratios to avoid conversion. The second group contains market-based triggers (such as the share price or the market capitalization). Several researchers<sup>8</sup> support the use of such triggers to avoid the problems typical to the use of accounting values, such as the slowness in reflecting economic developments, and issuers' incentives to window-dress the book values to avoid conversion (Flannery (2014)). However, market-based triggers can be criticized on the grounds of potential manipulation by speculators or death spirals (Pennacchi et al. (2014)), and the possible multiplicity or absence of equilibrium price (Sundaresan and Wang (2015)) which make the share price an unreliable conversion trigger. The last group includes discretionary triggers. The LAM of CoCo bonds with this kind of triggers is initiated by the regulator (for example, Credit Suisse has been issuing CoCo bonds to staff<sup>9</sup>), or the bank management<sup>10</sup>. CoCo bonds whose conversion can be triggered by the bank management are similar to the capital access bonds of Bolton and Samama (2012), and are referred to as "anytime" CoCo bonds. Finally, De Spiegeleer and Schoutens (2012) point out that the trigger of any CoCo should be clear, transparent, fixed over time and public.

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<sup>7</sup> See for example Kashyap, Rajan, and Stein (2008), Squam Lake Working Group (2009), and Glasserman and Nouri (2012a).

<sup>8</sup> See for example Flannery (2005), Albul, Jaffee, and Tchisty (2010), Bolton and Samama (2012), Barucci and Del Viva (2013), Bulow and Klemperer (2013), Calomiris and Herring (2013), McDonald (2013), Hilscher and Raviv (2014), and Berg and Kaserer (2015).

<sup>9</sup> Some authors (such as De Spiegeleer and Schoutens (2012)) include a fourth group of triggers which comprises multi-variable triggers. Yet, the inclusion of more than one trigger doesn't change the nature of the triggering event, making that fourth group redundant.

<sup>10</sup> See, Credit Suisse, Capital Instruments: Investor Information (accessed on 1 August 2018): <https://www.credit-suisse.com/corporate/en/investor-relations/financial-and-regulatory-disclosures/regulatory-disclosures/capital-instruments.html>

### 2.3 Payoff of CoCo Holders

The payoff the CoCo holders receive if LAM is triggered relies on the loss absorption approach described in the bond contract. The bonds will be either written down or converted to a number of shares. Under the first approach, the CoCo holders receive no shares and have the principal written down by a pre-specified percentage<sup>11</sup>, or by an amount that is enough to reinstate CET1 capital above the triggering level<sup>12</sup>, or completely wiped out<sup>13</sup>, similar to catastrophe bonds issued by insurance companies (Flannery (2014))<sup>14</sup>. Under the second approach, CoCo holders receive a number of shares in exchange for their bonds. The number of shares per CoCo bond is either pre-specified in the bond contract or determined by dividing the face value of the bond by the conversion price. The use of variable number of shares is associated with two problems. First, a lower share price increases the number of shares CoCo holders receive at conversion. Hence, closer to the trigger, short sellers might have a strong incentive to push the share price of a bank down to force conversion (or to create a death spiral) heavily diluting the wealth of the current shareholders (Hillion and Vermaelen (2004)). This would be the case if the CoCo bonds had share price-based trigger. Moreover, if the issuer of CoCo bonds is hit by a strong negative shock such that the value of its equity falls below the face value of the CoCo bonds, conversion becomes infeasible (Flannery (2009) and Pennacchi (2011)).

### 3. The Baseline model

Our baseline model consists of a basic game in which the shareholders of a bank exert effort to increase the likelihood of the bank's success by controlling the riskiness of activities (the volatility of assets value). The simplified game is designed to pick out key ideas about shareholders' incentives to exert effort and the implications of CoCo bonds issuance for those incentives. It is important to note that the game is used to analyse policy questions and does not intend to capture all practical considerations.

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<sup>11</sup> For instance, Cocos issued by Rabobank (in 2010) have a write down ratio of 75%.

<sup>12</sup> Another batch of Rabobank's Cocos (issued in 2011) specifies a variable write-down that is sufficient to raise the bank's equity capital above the regulatory required level.

<sup>13</sup> Examples include: Cocos issued by Barclays (2012), Credit Suisse (2013), and KBC Bank (2013).

<sup>14</sup> Cocos with write-down LAMs can have a write up/back features which allow the issuer to write up the principal of the bonds in the future when its capital position stabilises.



### 3.1 Players

The model includes four risk-neutral agents, depositors, shareholders, CoCo holders and the government or regulator. Depositors receive the face value of their claim ( $\delta$ ), regardless of the state of the world, as the bank will be bailed out if its value is not sufficient to pay off the face value of deposits. Shareholders manage the bank and receive the residual of the bank's value after paying off the claims of the depositors and the CoCo holders and the tax ( $\tau$ ). The shareholders' payoff relies on the state of the world, but they can reduce the riskiness of the bank business, and increase the probability of success, by exerting a costly effort. The payoff of CoCo holders also depends on the state of world and the covenants of the CoCo bond contract. For simplicity, we assume that they receive  $c_s$  in success state and  $c_f$  in failure state. We also assume that CoCo conversion/write down always happens in the failure state. The regulator imposes capital requirements and provides the possibility of bailing-out.

### 3.2 States of world

There are two states of the world: success (s) and failure (f). Failure is defined as the inability to meet the regulatory capital requirements. Shareholders can affect the probability of the success state by exerting a costly effort. More specifically, shareholders can increase the probability of success from  $p_L$  to  $p_H$  by exerting effort.

### 3.3 Payoffs

The value of the bank relies on the realised state of the world:  $v_s$  in the success state and  $v_f$  in the failure state. Shareholders receive the residual of the realised value after paying off all the claims and the tax. To understand the implications of CoCo bonds issuance for shareholders' incentives to exert effort, we compare between two possible structures of the bank liabilities:

- deposits + equity
- deposits + CoCo bonds + equity

#### 3.3.1 Debt + Equity

In this case, the shares of shareholders and depositors in the value of the bank under the two states of the world would be as follows:

State	Bank value	Shareholders	Depositors
Success	$v_s$	$(1 - \tau).(v_s - \delta)$	$\delta$
Failure	$v_f$	$(1 - \tau).(v_f - \delta)$	$\delta$

While the payoff of depositors is always the same ( $\delta$ ), shareholders can affect their expected payoff by choosing whether to exert a costly effort. By exerting effort, shareholders reduce the riskiness of the bank's assets, and increase the probability of success from  $p_L$  to  $p_H$ . The expected payoff of shareholders if they exerted effort is (where  $b_E$  is the cost of effort):

$$(1 - \tau)[p_H.(v_s - \delta) + (1 - p_H).(v_f - \delta)] - b_E$$

Similarly, the expected payoff of shareholders if they chose not to exert effort is:

$$(1 - \tau)[p_L.(v_s - \delta) + (1 - p_L).(v_f - \delta)]$$

Hence, shareholders would choose to exert effort if the following condition is satisfied:

$$\begin{aligned} (1 - \tau)[p_H.(v_s - \delta) + (1 - p_H).(v_f - \delta)] - b_E &\geq (1 - \tau)[p_L.(v_s - \delta) + (1 - p_L).(v_f - \delta)] \\ \Rightarrow (1 - \tau)(v_s - v_f) &\geq \frac{b_E}{p_H - p_L} \quad (i) \end{aligned}$$

### 3.3.2 Debt + Equity + CoCo Bonds

In this case, shareholders issue CoCo bonds to partially meet the capital requirements imposed by the regulator. We assume that this change in the financing mix does not change the probabilities of success and failure states. The shares of shareholders, CoCo holders and depositors in the realised value of the bank are as follows:

State	Bank value	Shareholders	CoCo holders	Depositors
Success	$v_s$	$(1 - \tau).(v_s - c_s - \delta)$	$c_s$	$\delta$
Failure	$v_f$	$(1 - \tau).(v_f - c_f - \delta)$	$c_f$	$\delta$

The expected payoff of shareholders if they elected to exert effort is:

$$(1 - \tau)[p_H.(v_s - c_s - \delta) + (1 - p_H).(v_f - c_f - \delta)] - b_E$$

Meanwhile, their expected payoff had they chosen not to exerted effort is:

$$(1 - \tau)[p_L.(v_s - c_s - \delta) + (1 - p_L).(v_f - c_f - \delta)]$$

Therefore, shareholders would choose to exert effort if the following condition is satisfied:

$$\Rightarrow (1 - \tau)(v_s - v_f) \geq \frac{b_E}{p_H - p_L} + (1 - \tau)(c_s - c_f) \quad (ii)$$

The term  $(1 - \tau)(c_s - c_f)$  represents the after-tax wealth transfer from CoCo holders to shareholders when CoCo bonds are converted or written down. Equation (ii) shows that shareholders become less likely to exert effort if  $c_s - c_f > 0$ . This is because the payoff of shirking relative to exerting effort is higher in the presence of CoCo bonds. In other words, shareholders would be willing to take higher risks if the CoCo bonds' contract stipulates a wealth transfer from the CoCo holders (i.e. the conversion rate is relatively low). Conversely, CoCo bonds with relatively high conversion rate induce shareholders to take lower risks. This is consistent with Martynova and Perotti (2012) and Hilscher and Raviv (2014) in that CoCo bonds' contracts with appropriately designed conversion ratios can address the risk-taking incentives of shareholders<sup>15</sup>.

As Equation (ii) shows, the likelihood of shareholders shirking (not exerting effort) increases in the difference between  $c_s$  and  $c_f$ , since their loss from the failure of the bank becomes smaller. Therefore, the likelihood of shirking is higher when the CoCo bonds have a write-down LAM.

### 3.4 The conversion rate

In our setup, the CoCo conversion rate affects the payoff of the CoCo holders in the failure state ( $c_f$ ), and hence the wealth transfer from CoCo holders to shareholder at conversion ( $c_s - c_f$ ). Intuitively, if CoCo investors receive less in the failure state, this increases the residual value that accrues to shareholders. More specifically, a lower conversion rate increases this wealth transfer in the failure state, making shirking more attractive to shareholders. This means that shareholders are less likely to exert effort. Meanwhile, a higher conversion rate decreases the wealth transfer ( $c_s - c_f$ ). This results in a lower residual claim of shareholders in the failure state, making them more willing to exert effort. The European regulations (CRR Art 54) set out that the conversion rate needs to be specified in the CoCo bonds contract. It also requires the rate of write-down to be stated for CoCo bonds with write-down LAMs. This means that managers can determine the conversion rate, including deciding whether there should be a write-down. However, regulators, as we will see later, could increase the incentives of shareholders to exert effort by imposing restrictions on the conversion (write-down) rates and/or requirements on quality of capital.

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<sup>15</sup> This also allows us to explore risk-shifting incentives as in Flannery (2014).

### 3.5 The conversion trigger

Since conversion in our model happens only in the failure state, the conversion trigger can be set to the value of the firm in this state ( $v_f$ ). As Equation (ii) shows, what matters from the shareholders' perspective is the value loss if the bank fails (i.e.,  $v_s - v_f$ ). By holding  $v_s$  constant and allowing  $v_f$  to change, we can assess how the conversion trigger changes the incentive of shareholders to exert effort. Specifically, a lower trigger coincides with a lower  $v_f$ . This increases  $v_s - v_f$ , making shareholders more likely to exert effort. In other words, as the difference in the firm value between the success and failure states increases, the relative loss in the failure state increases, inducing shareholders to exert effort to increase the probability of the firm's success. On the other hand, a higher trigger (higher  $v_f$ ), decreases the difference between  $v_s$  and  $v_f$ . This reduces the relative loss in the failure state and makes shareholders less willing to exert effort.

It is important here to note here that while lower conversion triggers increase the shareholders' incentives to exert effort, they may represent a challenge in practice. That is, lower triggers can delay conversion of CoCo bonds until it is too late, affecting the loss absorbing capacity of these bonds.

### 3.6 The tax rate

In Equation (i), an increase in the tax rate reduces the after-tax residual value of the bank, making shareholders less likely to exert effort. In Equation (ii), however, an increase in tax rate would also decrease the after-tax wealth transfer from CoCo holders to shareholders at conversion, partially compensating for the fall in the after-tax value. Thus, to attain the same impact on shareholders' willingness to exert effort, we would need higher changes in the tax rate in the presence of CoCo bonds. As a result, changes in tax rate would have relatively weaker impact on shareholders' preferences. This can be explained by the reduced value of the tax shield of the bank debt due to the convertibility of CoCo bonds.

### 3.7 The cost of effort

Following Holmstrom and Tirole (1998), the cost of effort ( $b_E$ ) acts as a reservation price shareholders have to forgo in order to increase the probability of success from  $p_L$  to  $p_H$ . Any increase in  $b_E$  makes it

less likely for the conditions in (i) and (ii) to be satisfied. In other words, the higher the cost of effort, the lower the shareholders' willingness to exert effort will be.

### 3.8 The riskiness of the bank's business

The difference between  $v_s$  and  $v_f$  in Equations (i) and (ii) can be viewed as a proxy of the riskiness of the bank's assets. That is, larger differences between  $v_s$  and  $v_f$  imply higher volatility of the value of the bank's assets. When  $v_s - v_f$  is greater, the expected gain shareholders forfeit had they not exerted effort rises, making them more likely to exert effort. Hence, other things equal, shareholders of banks with riskier business models are more likely to exert effort to control the bank risk.

### 3.9 The probability of success

As mentioned earlier, the shareholder effort increases the probability of success from  $p_L$  to  $p_H$ . This increase in the probability of success ( $p_H - p_L$ ) can be viewed as the marginal return on shareholders' effort. Accordingly, larger increases in the probability of success, for a given level of effort, make shareholder more likely to exert effort, other things equal.

## 4. Solution

In this section, we assume that there are capital quality requirements, under which shareholders can substitute part of their equity ( $\Delta$ ) by issuing CoCo bonds. We also assume that if CoCo bonds were issued, either shareholders or CoCo holders will exert effort (i.e. the probability of success is always  $p_H$ ). Therefore, these bonds are issued at their fair value, that is:

$$\Delta = p_H \cdot c_s + (1 - p_H) \cdot c_f \quad (\text{iii})$$

### 4.1 What CoCo bond contracts will shareholders write?

By choosing the covenants of the CoCo bond contract, shareholders can determine who will exert effort. To analyse this, we first define shareholders' payoff from issuing CoCo bonds as follows:

$$V(i) = p_H \cdot (1 - \tau) \cdot (v_s + \Delta - \delta - c_s) + (1 - p_H) \cdot (1 - \tau) \cdot (v_f + \Delta - \delta - c_f) - (1 - i) \cdot b_E \quad (\text{iv})$$

Where,  $i \in (0,1)$  is an indicator function, which is equal to 0 if shareholders exert effort and 1 if CoCo holders exert effort. Shareholders select  $i$ ,  $c_s$ , and  $c_f$  to maximise  $V(i)$ , while ensuring that the incentive

compatibility constraint (ICC) for the parity exerting effort is satisfied. Equation (ii) represents the ICC of shareholders. Similarly, the ICC for CoCo holders is ( $b_c$  is the cost of effort for CoCo holders):

$$c_s - c_f \geq \frac{b_c}{p_H - p_L} \quad (v)$$

#### 4.2 CoCo Holders Exert Effort

If CoCo holders exert effort ( $i = 1$ ), the conditions in Equations (iii) and (v) must be satisfied. As a result, the values of  $c_s$  and  $c_f$  will be as follows (the proof is in appendix A1):

$$\begin{cases} c_f \leq \Delta - \frac{p_H}{p_H - p_L} \cdot b_c \\ c_s \geq \Delta + \frac{(1 - p_H)}{p_H - p_L} \cdot b_c \end{cases}$$

Hence, the maximum value for shareholders when  $i = 1$  is:

$$V(1) = p_H \cdot (1 - \tau) \cdot (v_s - \delta) + (1 - p_H) \cdot (1 - \tau) \cdot (v_f - \delta)$$

#### 4.3 Shareholders Exert Effort

Shareholders will be willing to exert effort ( $i = 0$ ) only if they expect to get a share of the increase in the expected value of the bank due to the effort. In other words, they wouldn't be willing to allow the increase in payoff of the CoCo holders ( $c_s - c_f$ ) to exceed the increase in the expected value of the bank. That is:

$$c_s - c_f \leq (p_H - p_L) \cdot (v_s - v_f) \quad (vi)$$

For shareholders to exert effort, the conditions in Equations (iii) and (vi) must be satisfied. As a result, the values of  $c_s$  and  $c_f$  will be as follows (the proof is in appendix A2):

$$\begin{cases} c_f \geq \Delta - p_H \cdot (p_H - p_L) \cdot (v_s - v_f) \\ c_s \leq \Delta + (1 - p_H) \cdot (p_H - p_L) \cdot (v_s - v_f) \end{cases}$$

Hence, the maximum value for shareholders when  $i = 0$  is:

$$V(0) = p_H \cdot (1 - \tau) \cdot (v_s - \delta) + (1 - p_H) \cdot (1 - \tau) \cdot (v_f - \delta) + (p_H - p_L) \cdot (v_s - v_f) - b_E$$

#### 4.4 Who Exerts Effort?

Shareholders will prefer that CoCo holders exert effort when the following condition is met:

$$V(0) \leq V(1) \Leftrightarrow (p_H - p_L) \cdot (v_s - v_f) \leq b_E$$

In other words, shareholders tend to prefer that CoCo holders exert effort if the benefit they get from exerting effort is lower than the cost of this effort. This applies unless effort is costless to shareholders (i.e.  $b_E = 0$ ). As can be inferred from Equation (ii), in this case, shareholders will always exert effort, unless the wealth transfer from CoCo holders to shareholders  $c_s - c_f$  is at least equal to the change in the bank value  $v_s - v_f$  (for instance, CoCo bonds with write-down LAMs).

## 5. Extensions

### 5.1 Anytime CoCos

Some practitioners in the banking industry have proposed the use of anytime CoCos to address the problem of the regular CoCo bonds not triggering in time. These CoCo bonds can be converted at any time at the discretion of the issuer. It has been argued that these CoCo bonds can help in addressing the problem of late conversion of CoCo. This is because the bank management is normally in a better position to assess the future prospects of the bank and, hence, is able to convert CoCo bonds well before it is too late. This form of CoCo bonds can have a benefit to CoCo holders in that they might have a better payoff at conversion if conversion is triggered before the bank accumulates further losses and avoid scenarios similar to that of Banco Popular CoCo holders, whose bonds were entirely written off. Bolton and Samama (2012) argue that allowing issuers to decide when to draw on CoCos is analogous to having a pre-funded capital line in situations where adverse selection problems are large. If the game were repeated, issuer opportunism may be constrained by the threat of punishment by investors in future periods. However, ours is a one-shot game that suggests there is no additional benefit to issuers deciding when to convert.

In our setting, we can introduce the concept of anytime CoCos by considering three scenarios for the payoff of the CoCo holders in the success and failure states, as follows:

(1) $c_s > c_f$	(2) $c_s = c_f$
(3) $c_f > c_s$	

Under the first scenario, shareholders will convert CoCo bonds only in the failure state. As a result, if the predetermined conversion rate results in wealth transfer from CoCo holders to shareholders only at or below the predetermined conversion trigger  $v_f$ , anytime CoCos converge to regular (fixed trigger)

CoCo bonds, as their conversion will not be triggered until the fixed trigger is reached. The payoff of the CoCo holders, under the second scenario, is always the same. Therefore, shareholders will be indifferent between triggering and not triggering CoCo conversion. In other words, CoCo bonds, in this case, will be like conventional bonds. Hence, the presence of CoCo bonds in the financing mix would have no impact on the shareholders' willingness to exert effort.

Under the third scenario, the payoff of the CoCo holders is higher in the failure state compared to the success state, implying a conversion rate that results in a wealth transfer from shareholders to CoCo holders. This can be useful in the case of regular CoCo bonds as it increases the likelihood that shareholders will exert effort. However, anytime CoCos may not be considerably more effective under this scenario. This is because shareholders will have strong incentives to convert the CoCo only very close to the predetermined conversion trigger  $v_f$ . Thus, anytime CoCos are likely to have a minor advantage over the regular CoCo bonds as they can be expected to convert slightly earlier. However, this would require a properly designed conversion rate that makes it costly for shareholders to wait until the predetermined trigger is hit.

## **5.2 Quality of capital requirements**

We introduced the quality of capital requirements in Section 4. These requirements specify the extent to which CoCo bonds can be used to meet the capital requirements of the bank. The current capital requirements allow banks to cover only a part of their Tier 1 capital requirements by issuing CoCo bonds (as AT1 instruments). More precisely, banks can cover up to 25% of their minimum risk-weighted capital requirements using AT1 instruments<sup>16</sup>, but can't use these instruments to cover their capital buffer requirements which have to be completely covered by CET1 (equity) capital<sup>17</sup>.

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<sup>16</sup> Under Basel Pillar 1 requirements, a bank's capital must satisfy a CET1 ratio of 4.5%, and a T1 (CET1 and AT1) ratio of 6%. The UK Pillar 1 requirements are like those of Basel. Additionally, although we ignore them here, Pillar 2 requirements (in the UK) maintain the quality of capital restrictions implied by the Pillar 1 requirements.

<sup>17</sup> These restrictions apply only to the ratio of risk-weighted assets. For the leverage ratio, the finalised Basel III states that the leverage ratio capital requirements (including the minimum and the buffer requirements) can be fully met using T1 capital. However, the current UK leverage ratio framework includes limits on the quality of capital. UK banks subject to the leverage ratio can only cover up to 25% of their minimum leverage ratio



As they increase the stake of shareholders in the value of the bank, more restricting quality of capital requirements are expected to increase their incentives to exert effort. We can conceptualise the quality of capital requirements in our setup as a change in the difference between  $v_s$  and  $v_f$ . That is, more stringent quality of capital requirements widen the difference between  $v_s$  and  $v_f$  and make shareholders more likely to exert effort. This is because the potential loss shareholders could face had they not exerted effort becomes higher. Conversely, less strict quality of capital requirements can decrease shareholders' willingness to exert effort.

### 5.3 The Incentives of the Managers

So far, the analysis has assumed that managers are rewarded in the same way as shareholders. Therefore, the conclusions have been drawn under the assumption that the incentives of shareholders and managers are fully aligned. However, our framework can be used to explore how the incentives of the managers to exert effort as well as the incentives of shareholders to monitor would look like, had managers been compensated in either the same way as depositors or CoCo holders.

#### 5.3.1 Managers' Compensation is State-Independent

Assuming they have a positive cost of effort  $b_M$ , managers would have no incentive to exert effort if they were rewarded with a state-independent payoff (like depositors). There are two possible outcomes here:

- If their ICC in Equation (i) (or (ii)) is satisfied, shareholders monitor managers. Hence, the probability of success will be  $p_H$ , resulting in the following payoffs of shareholders and managers ( $\gamma$  is the fixed compensation of managers):

$$Payoff_{SH} = (1 - \tau)[p_H \cdot (v_s - \gamma - \delta) + (1 - p_H) \cdot (v_f - \gamma - \delta)] - b_E$$

$$Payoff_M = \gamma - b_M$$

- Shareholders don't monitor if their ICC isn't satisfied, the probability of success will be  $p_L$ , and the resulting payoffs are:

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capital requirements using AT1 instruments. All leverage ratio buffer requirements have to be fully met in CET1.

$$Payoff_{SH} = (1 - \tau)[p_L \cdot (v_s - \delta) + (1 - p_L) \cdot (v_f - \delta)]$$

$$Payoff_M = \gamma$$

### 5.3.2 Managers' Are CoCo Holders

On the other hand, if managers were compensated as if they were CoCo holders, their incentive to exert effort will depend on the conversion rate and the corresponding wealth transfer. As discussed earlier, there are three scenarios for the payoffs of the CoCo holders in the success and failure states:

(1) $c_s > c_f$	(2) $c_s = c_f$
(3) $c_f > c_s$	

Scenario 2 implies a state independent payoff analogous to the payoff of depositors. Thus, managers would have no incentive to exert effort under this Scenario. Likewise, managers would be inclined not to exert effort under Scenario 3, since their payoff would be higher in the failure state. Like the first case in 5.3.1, shareholders will monitor the managers if their ICC is satisfied. Otherwise, the probability of success will be always equal to  $p_L$ . Conversely, Scenario 1 entails a wealth transfer from CoCo holders to shareholders. Hence, managers will be willing to exert effort if their cost of effort does not exceed the return from that effort (i.e. the increase in their expected payoff). That is:

$$\begin{aligned} p_H \cdot c_{s,M} + (1 - p_H) \cdot c_{f,M} - b_M &\geq p_L \cdot c_{s,M} + (1 - p_L) \cdot c_{f,M} \\ \Rightarrow c_{s,M} - c_{f,M} &\geq \frac{b_M}{p_H - p_L} \quad (\text{vii}) \end{aligned}$$

This Scenario can hence yield one of three outcomes (the probability of success will be  $p_H$  in the two cases but  $p_L$  in the third):

- Managers exert effort without monitoring (ICC in Equation (vii) is satisfied). The payoffs will be:

$$Payoff_{SH} = (1 - \tau)[p_H \cdot (v_s - c_{s,M} - \delta) + (1 - p_H) \cdot (v_f - c_{f,M} - \delta)]$$

$$Payoff_M = p_H \cdot c_{s,M} + (1 - p_H) \cdot c_{f,M} - b_M$$

- Shareholders monitor managers (ICC in Equation (vii) is not satisfied, but that in Equation (ii) is satisfied). The payoffs will be:

$$Payoff_{SH} = (1 - \tau)[p_H \cdot (v_s - c_{s,M} - \delta) + (1 - p_H) \cdot (v_f - c_{f,M} - \delta)] - b_E$$

$$Payoff_M = p_H \cdot c_{s,M} + (1 - p_H) \cdot c_{f,M} - b_M$$

- Managers do not exert effort and shareholders do not monitor (both ICCs are not satisfied).

The payoffs of will be:

$$Payoff_{SH} = (1 - \tau)[p_L \cdot (v_s - c_{s,M} - \delta) + (1 - p_L) \cdot (v_f - c_{f,M} - \delta)]$$

$$Payoff_M = p_L \cdot c_{s,M} + (1 - p_L) \cdot c_{f,M}$$

Our analysis in this section could be very useful for regulators and investors. Shareholders could design management incentives that make very costly for managers not to exert effort. This could be done by adopting manager-specific CoCo bonds in management remuneration schemes. We think that the issuance of such bonds can enhance the resilience of individual banks and the banking system as a whole. Similarly, regulators can impose restrictions on the conversion rates to address the risk-taking incentives when CoCo bonds are issued.

## 6. Conclusion

Many authors have investigated how the issuance of CoCo bonds affects the *ex-ante* incentives of the current shareholders and the managers of the issuing bank. Unlike previous papers, which have modelled the risk-taking incentives through modified stochastic asset processes, we utilised a simple game similar to that used by Holmstrom and Tirole (1998). The paper presented a model of moral hazard surrounding shareholders' decision to exert costly effort to increase the likelihood of a bank being successful. Specifically, the model is a one-shot game and does not capture the effect of repeated interactions. Repeated interactions may create reputational constraints, especially in the context of anytime CoCos. Despite being simplified, the model was able identify the key implications of the issuance of CoCo bonds. The model inspected how the issuance of CoCo bonds influences the risk-taking incentives of the issuing bank. It also examined the role played by some factors such as the riskiness of the bank business, the conversion rate, and the tax rate in determining the impact on these incentives. The model was extended to consider the implications of some policy factors. This included investigating the effects of capital quality requirements and the misalignment of

shareholders and managers incentives. It also included an assessment of impact of the discretion assumed by the issuer over conversion when anytime CoCos are issued.

Our analysis yielded useful policy-relevant results. First, consistent with the literature, the direction of the wealth transfer at the conversion of CoCo bonds determines their impact on the risk-taking incentives. Specifically, CoCo bonds would increase risk-taking incentives if their conversion transfers wealth from CoCo holders to shareholders and decrease these incentives if the wealth transfer at conversion was from shareholders to CoCo holders. Second, the benefits of anytime CoCos over regular CoCo bonds are likely to be small. They may convert slightly earlier, but this would require a properly designed conversion rate that makes it costly for shareholders to wait until the predetermined trigger is hit. Third, stricter quality of capital requirements can reduce the risk-taking incentives of shareholders. This is because they increase their stake in the value of the bank. Finally, shareholders can also use manager-specific CoCo bonds to reduce the riskiness of the bank activities. The issuance of such bonds might increase the resilience of individual banks and the whole banking system. Regulators can also impose restrictions on conversion rates to address the impact of CoCo bonds issuance on risk-taking incentives.

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## Appendix

### A1. Values of $c_s$ and $c_f$ when CoCo holders exert effort (Section 4.2)

$$\Delta = p_H \cdot c_s + (1 - p_H) \cdot c_f \quad (1)$$

$$c_s - c_f \geq \frac{b_C}{p_H - p_L} \quad (2)$$

- From Equation 1:

$$c_f = \frac{\Delta - p_H \cdot c_s}{(1 - p_H)} \quad (3)$$

- Substituting Equation 3 into Equation 2:

$$\begin{aligned} c_s - \frac{\Delta - p_H \cdot c_s}{(1 - p_H)} &\geq \frac{b_C}{p_H - p_L} \Rightarrow c_s - \left( \frac{p_H \cdot c_s}{(1 - p_H)} \right) \geq \frac{b_C}{p_H - p_L} + \frac{\Delta}{(1 - p_H)} \Rightarrow c_s \cdot \left( 1 + \frac{p_H}{1 - p_H} \right) \geq \frac{b_C}{p_H - p_L} + \frac{\Delta}{(1 - p_H)} \\ &\Rightarrow c_s \cdot \left( \frac{1}{1 - p_H} \right) \geq \frac{b_C}{p_H - p_L} + \frac{\Delta}{(1 - p_H)} \Rightarrow \end{aligned}$$

$$\boxed{c_s \geq \Delta + \frac{(1 - p_H)}{p_H - p_L} \cdot b_C} \quad (4)$$

- On both sides of Equation 4, multiply by  $-p_H$ , add  $\Delta$ , and then divide by  $(1 - p_H)$ :

$$\begin{aligned} c_s \geq \Delta + \frac{(1 - p_H)}{p_H - p_L} \cdot b_C &\Rightarrow -p_H \cdot c_s \leq -p_H \cdot \Delta + \frac{-p_H \cdot (1 - p_H)}{p_H - p_L} \cdot b_C \\ \Rightarrow \Delta - p_H \cdot c_s \leq \Delta - p_H \cdot \Delta + \frac{-p_H \cdot (1 - p_H)}{p_H - p_L} \cdot b_C &\Rightarrow \Delta - p_H \cdot c_s \leq \Delta \cdot (1 - p_H) + \frac{-p_H \cdot (1 - p_H)}{p_H - p_L} \cdot b_C \\ &\Rightarrow \frac{\Delta - p_H \cdot c_s}{(1 - p_H)} \leq \Delta - \frac{p_H \cdot (1 - p_H)}{(p_H - p_L) \cdot (1 - p_H)} \cdot b_C \\ &\Rightarrow \boxed{c_f \leq \Delta - \frac{p_H}{p_H - p_L} \cdot b_C} \end{aligned}$$

### A2. Values of $c_s$ and $c_f$ when shareholders exert effort (Section 4.3)

$$c_s - c_f \leq (p_H - p_L) \cdot (v_s - v_f) \quad (5)$$

- Substituting Equation 3 into Equation 5:

$$\begin{aligned} c_s - \frac{\Delta - p_H \cdot c_s}{(1 - p_H)} &\leq (p_H - p_L) \cdot (v_s - v_f) \Rightarrow c_s \cdot \left( 1 + \frac{p_H}{1 - p_H} \right) \leq (p_H - p_L) \cdot (v_s - v_f) + \frac{\Delta}{(1 - p_H)} \\ &\Rightarrow c_s \cdot \left( \frac{1}{1 - p_H} \right) \leq (p_H - p_L) \cdot (v_s - v_f) + \frac{\Delta}{(1 - p_H)} \Rightarrow \end{aligned}$$

$$\boxed{c_s \leq \Delta + (1 - p_H) \cdot (p_H - p_L) \cdot (v_s - v_f)} \quad (6)$$

- On both sides of Equation 6, multiply by  $-p_H$ , add  $\Delta$ , and then divide by  $(1 - p_H)$ :

$$\begin{aligned} c_s \leq \Delta + (1 - p_H) \cdot (p_H - p_L) \cdot (v_s - v_f) &\Rightarrow -p_H \cdot c_s \geq -p_H \cdot \Delta - p_H \cdot (1 - p_H) \cdot (p_H - p_L) \cdot (v_s - v_f) \\ &\Rightarrow \Delta - p_H \cdot c_s \geq \Delta \cdot (1 - p_H) - p_H \cdot (1 - p_H) \cdot (p_H - p_L) \cdot (v_s - v_f) \\ &\Rightarrow \frac{\Delta - p_H \cdot c_s}{(1 - p_H)} \geq \Delta - \frac{p_H \cdot (1 - p_H) \cdot (p_H - p_L) \cdot (v_s - v_f)}{(1 - p_H)} \cdot b_C \\ &\Rightarrow \boxed{c_f \geq \Delta - p_H \cdot (p_H - p_L) \cdot (v_s - v_f)} \end{aligned}$$