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## The long-run effects of uncertainty shocks

Dario Bonciani<sup>(1)</sup> and Joonseok Jason Oh<sup>(2)</sup>

### Abstract

This paper argues that shocks increasing macroeconomic uncertainty negatively affect economic activity not only in the short but also in the long run. In a sticky-price DSGE model with endogenous growth through investment in R&D, uncertainty shocks lead to a short-term fall in demand because of precautionary savings and rising markups. The decline in the utilised aggregate stock of R&D determines a fall in productivity, which causes a long-term decline in the main macroeconomic aggregates. When households feature Epstein-Zin preferences, they become averse to these long-term risks affecting their consumption process (long-run risk channel), which strongly exacerbates the precautionary savings motive and the overall negative effects of uncertainty shocks.

**Key words:** Uncertainty shocks, R&D, endogenous growth.

**JEL classification:** E32, O40.

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# 1 Introduction

Heightened uncertainty is considered by policymakers and economists as one of the main factors behind the depth of the Great Recession and the subdued recovery (e.g. see [Stock and Watson, 2012](#)). Understanding the channels through which uncertainty propagates to the real economy is therefore relevant both from a research and a policy perspective. In this paper, we study how shocks to uncertainty can have a negative impact on economic activity in the short as well as in the long term.

To motivate that uncertainty may negatively affect economic activity in the long run, in [Figure 1](#), we show how macroeconomic uncertainty is a strong predictor of future low-frequency movements in Total Factor Productivity (TFP). In particular, we compare the backward-looking moving average of macroeconomic uncertainty over the previous 20 quarters and the forward-looking moving average of the TFP growth rate over the next 20 quarters. The uncertainty measure considered is the one proposed by [Jurado et al. \(2015\)](#) and updated by [Ludvigson et al. \(2019\)](#).<sup>1</sup> The measure of TFP growth is taken from [Fernald \(2014\)](#), which is adjusted for capacity utilisation.<sup>2</sup> The left-hand-side and right-hand-side axes relate respectively to uncertainty and TFP growth. Evidently, there is a strong negative correlation between the two series ( $-53.91\%$ ).<sup>3</sup>

This result is consistent with the analysis conducted in the seminal study by [Ramey and Ramey \(1995\)](#), who find that countries with higher volatility have lower mean growth. The evidence provided in [Figure 1](#), while suggestive, does not imply any causality in one direction or the other, nor it excludes the possibility that a third factor is driving both measures. To provide empirical evidence that uncertainty shocks cause a long-run downturn in economic activity, in [section 2](#), we conduct an SVAR analysis for the US. We find that shocks increasing macroeconomic uncertainty induce significant reductions in the main macroeconomic aggregates and in TFP that persist over 40 quarters.

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<sup>1</sup>In [Jurado et al. \(2015\)](#), uncertainty is defined as the common time-varying volatility in the unforecastable component of a large set of macroeconomic time series.

<sup>2</sup>In particular, [Fernald \(2014\)](#) proposes a measure of TFP constructed as a Solow residual, cleansing for variations in factor utilisation, which is an important source of non-technological cyclicalities.

<sup>3</sup>[Section A.1](#) in the appendix shows how the correlation between *uncertainty* and *tfp* varies as we change the window over which we average the two measures.

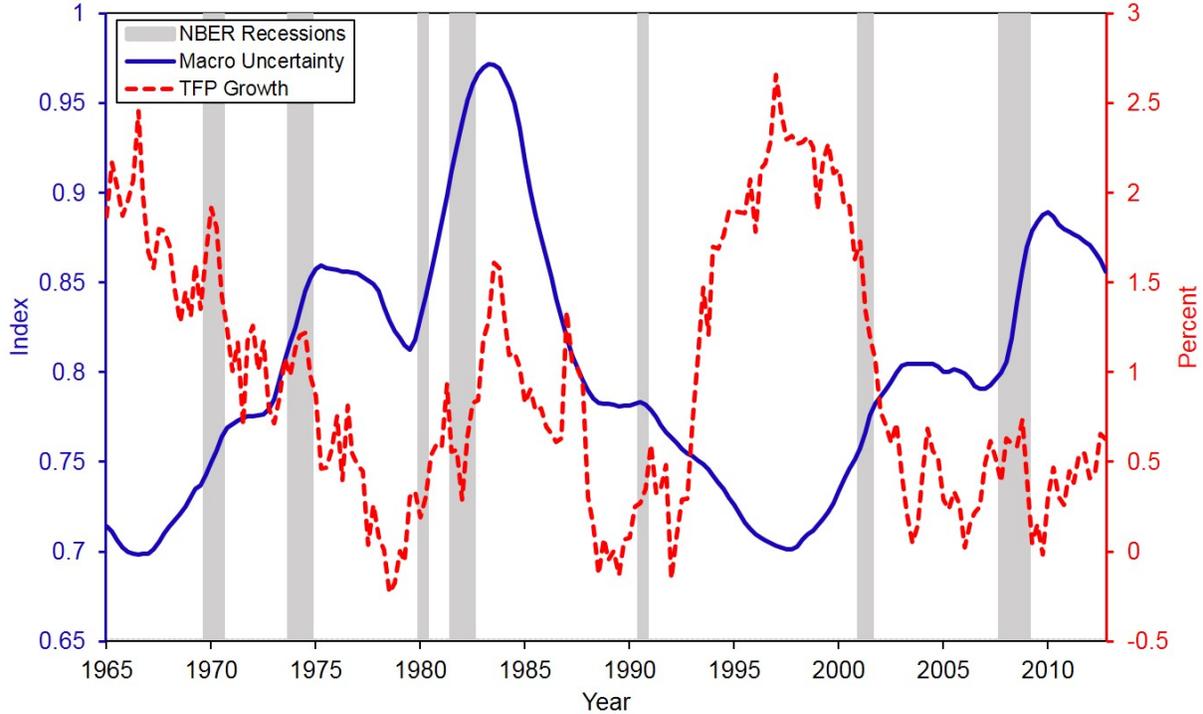


Figure 1: Macro Uncertainty and TFP Growth, U.S.

Note: The solid blue line represents the 5-year backward-looking moving average of the macro uncertainty measure from [Jurado et al. \(2015\)](#) updated by [Ludvigson et al. \(2019\)](#). We use the annual average of their monthly series with  $h = 3$  (i.e., 3-month-ahead uncertainty). The dashed red line represents the 5-year forward-looking moving average of the annualised TFP growth rate from [Fernald \(2014\)](#).

We rationalise these results by estimating a dynamic stochastic general equilibrium (DSGE) model augmented with an endogenous growth mechanism of vertical innovation in the spirit of [Grossman and Helpman \(1991\)](#) and [Aghion and Howitt \(1992\)](#). Productivity has an endogenous component that depends on the aggregate level of R&D services. Spillovers stemming from the accumulation of R&D allow business cycle shocks to affect long-run growth. In this framework, rises in TFP uncertainty cause a fall in output, consumption, and investment in physical capital and R&D. The decrease in the aggregate level of R&D leads to a fall in productivity, and the decline in economic activity becomes therefore permanent. Moreover, we show that when households have recursive preferences and take risks about future long-term growth into account, the precautionary savings motive of households is strongly amplified and the overall effects of uncertainty shocks become quantitatively significant. To highlight the relevance of this “long-run risk” channel, we compare our baseline DSGE model featuring endogenous growth and EZ preferences with alternative model

specifications that do not feature endogenous growth or EZ preferences (or both) and show how the combination of the two elements is necessary to obtain sizable effects of uncertainty shocks.

## 1.1 Related Literature

This work is related to the growing literature on uncertainty shocks, which started with the seminal contribution by [Bloom \(2009\)](#). Numerous papers (e.g. [Bachmann et al., 2013](#); [Born and Pfeifer, 2014](#); [Backus et al., 2015](#); [Fernández-Villaverde et al., 2015](#); [Leduc and Liu, 2016](#); [Basu and Bundick, 2017](#); [Katayama and Kim, 2018](#); [Oh, 2019](#)) have investigated how uncertainty shocks could generate business cycle fluctuations both with empirical and theoretical frameworks. From an empirical perspective, the literature has found that rises in uncertainty can cause a significant fall in economic activity. This result has been found using various measures of uncertainty such as financial volatility indexes ([Bloom, 2009](#)), macroeconomic uncertainty measures ([Jurado et al., 2015](#); [Rossi and Sekhposyan, 2015](#); [Kozeniauskas et al., 2018](#)) or political uncertainty news-based indexes ([Baker et al., 2016](#); [Caldara and Iacoviello, 2018](#)).

The theoretical literature has concentrated on disentangling the potential transmission channels through which uncertainty can affect macroeconomic variables and on quantifying the effects within DSGE models. The main transmission channels that have been discussed in the literature are: (i) the precautionary savings channel, that leads risk-averse agents to reduce consumption and increase labour supply ([Leland, 1968](#) and [Kimball, 1990](#)); (ii) the real options channel, which causes firms to postpone irreversible investments ([Bernanke, 1983](#); [Pindyck, 1991](#); [Bertola and Caballero, 1994](#)); (iii) the precautionary investment channel, for which a higher uncertainty in productivity raises investment, hours, and output if the optimal choices of capital and labour are convex in productivity ([Oi, 1961](#); [Hartman, 1976](#); [Abel, 1983](#)); (iv) the cost of financing channel, for which rises in uncertainty lead to increases in risk premia that in turn make borrowing more costly and therefore reduce investment ([Christiano et al., 2014](#); [Gilchrist et al., 2014](#); [Arellano et al., 2016](#)). While in partial equilibrium these transmission channels have clear-cut effects, they may offset each other in a general equilibrium framework. [Basu and Bundick \(2017\)](#) show that in a model with sticky prices and time-varying markups uncertainty shocks can generate business cycle fluctuations,

i.e. co-movement between output, consumption, and investment.

The literature has provided mixed evidence on the quantitative relevance of uncertainty shocks. With standard business cycle models, the effects of uncertainty shocks tend to be economically insignificant (e.g. [Bachmann and Bayer, 2013](#); [Born and Pfeifer, 2014](#)). The reason for the small effects found in the literature is that the shocks are small and the standard business cycle models are too linear to obtain a significant amplification. Accounting for nonlinearities such as the zero lower bound has been found to be an important source of amplification ([Fernández-Villaverde et al., 2015](#); [Basu and Bundick, 2017](#)). A recent paper by [Bianchi et al. \(2018b\)](#) finds significant effects of uncertainty on both the business cycle and term premia dynamics in an estimated medium-scale Markov-Switching DSGE model. Another strand of the literature has also shown that uncertainty could be amplified in the presence of frictions in the financial sector ([Christiano et al., 2014](#); [Bonciani and van Roye, 2016](#)) or in the labour market ([Leduc and Liu, 2016](#)). In this paper, we consider an additional source of nonlinearity deriving from the aversion of households to long-term risks to their consumption process, in the spirit of the finance literature on long-run risk ([Bansal and Yaron, 2004](#); [Kung and Schmid, 2015](#); [Kung, 2015](#)). This literature has shown how the equity premium puzzle could be solved in models featuring Epstein-Zin preferences and shocks to long-run future consumption growth. Some papers in the literature on uncertainty shocks such as [de Groot et al. \(2018\)](#) also considered New Keynesian models with EZ preferences, but failed to find significant effects of uncertainty shocks, as they abstracted from the long-run risk channel.

By analysing how uncertainty affects economic activity in the long-run, we depart from the previous literature which only focused on the business cycle effects of uncertainty. Hence this work bridges the literature on uncertainty shocks with another relatively recent strand of the literature that analyses the long-run growth impact of business cycle shocks (e.g. [Anzoategui et al., 2019](#); [Bianchi et al., 2018a](#)).

The rest of this paper is organised as follows. Section 2 presents the empirical evidence. In Section 3, we lay out the DSGE model, while in Section 4, we describe the model estimation. In Section 5, we present our results. Last, in Section 6, we provide some concluding remarks.

## 2 SVAR Analysis

In this section, we estimate a Vector Autoregressive (VAR) model for the US economy and analyse the impulse responses (IRFs). In our IRFs analysis, we look at a longer horizon than usually considered in the standard business cycle literature (40 quarters, i.e. 120 months). We identify the shocks with a recursive scheme (i.e. Cholesky identification). The baseline VAR contains 9 variables, entering in the following order: (i) the Standard and Poors 500 index, which is commonly included in the literature to control for movements in the stock market (*S&P500*); (ii) the measure of macroeconomic uncertainty estimated by [Jurado et al. \(2015\)](#) and updated by [Ludvigson et al. \(2019\)](#); (iii) GDP as a measure of aggregate macroeconomic activity (*Output*); (iv) personal consumption in nondurables and services (*Consumption*); (v) durable consumption and private fixed investment excluding R&D investment (*Capital Investment*); (vi) private fixed investment in R&D (*R&D Investment*); (vii) the GDP deflator, as a measure of the price level (*Price*); (viii) the shadow interest rate by [Wu and Xia \(2016\)](#), as a measure of the US monetary policy stance (Interest Rate); (ix) utilisation-adjusted TFP as measured by [Fernald \(2014\)](#) (*TFP*). We take logs of the S&P 500 index and the uncertainty measure, to interpret the IRFs in percentage terms. Output, consumption, capital investment, and R&D investment are expressed in logs, real per capita terms. The ordering described above implies that uncertainty is contemporaneously affected by shocks to the S&P500 index, but not by the other macroeconomic variables. In subsequent periods, however, uncertainty responds to all shocks through its relation with the lags of the variables included in the VAR model. This identification strategy is in line with that in [Bloom \(2009\)](#), [Leduc and Liu \(2016\)](#), and [Basu and Bundick \(2017\)](#). The focus on macroeconomic uncertainty is supported by two recent empirical papers by [Carriero et al. \(2018\)](#) and [Angelini et al. \(2019\)](#) that show that macroeconomic uncertainty can be considered an exogenous source of business cycle fluctuations.

In the baseline framework, data are at a quarterly frequency, spanning the period 1960Q3-2018Q2, and all variables that are available at a higher frequency are averaged over the quarter. We estimate the reduced-form VAR by ordinary least squares:

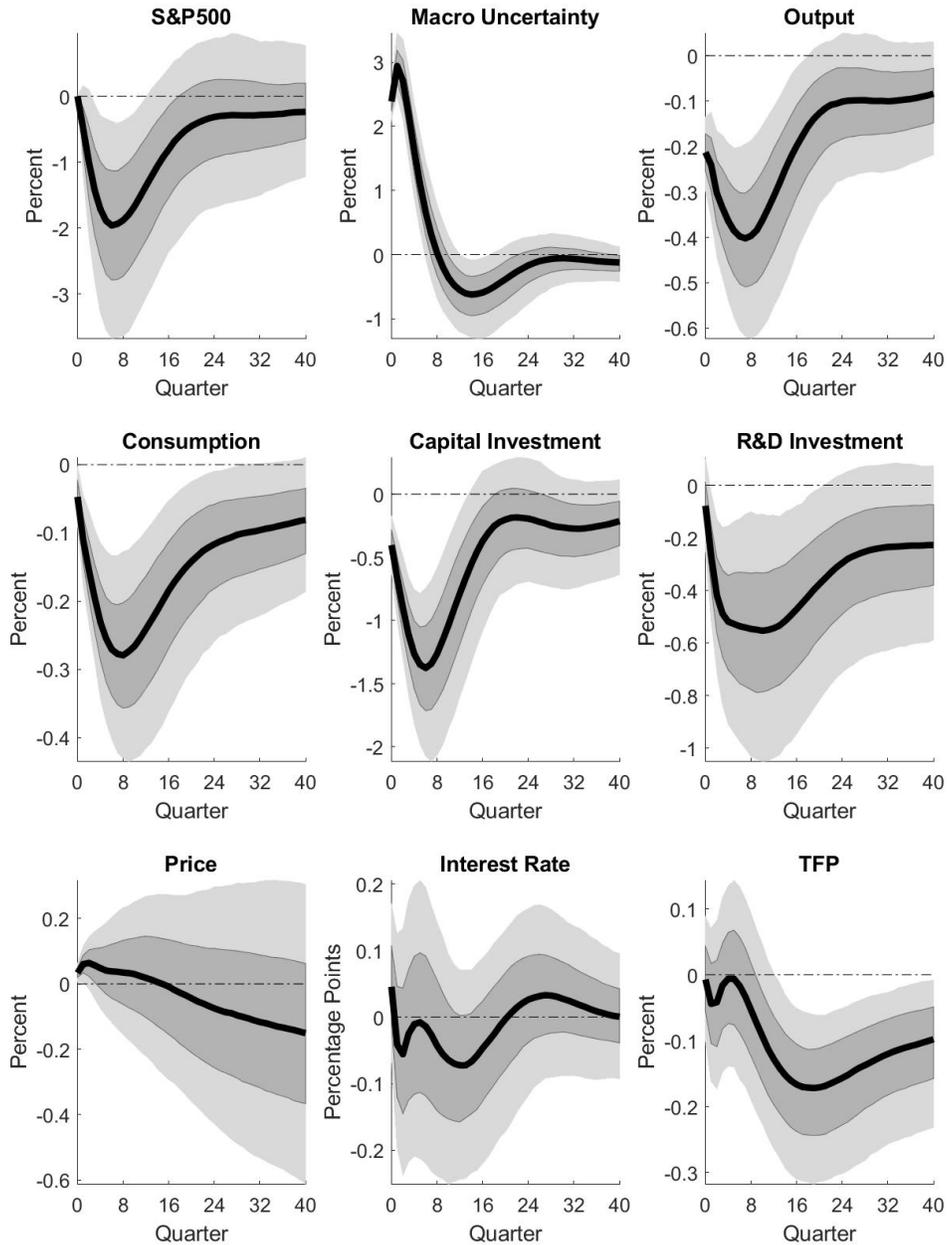


Figure 2: Impulse Responses to a Macro Uncertainty Shock (Baseline VAR)

Note: Variables are in per cent change except for the interest rate, which is in annualised percentage points. Light grey and dark grey shaded areas represent 95 and 68 per cent confidence bands.

$$X_t = c + \sum_{k=1}^L A_k X_{t-k} + e_t \quad (1)$$

where  $X_t$  is the vector of endogenous variables,  $A_k$  is the coefficient matrix for the  $k$ -th lag of  $X_t$  and  $e_t$  is the vector of reduced form innovations, which have zero mean and variance  $\Sigma$ . We include two lags in our VAR, as suggested by the Akaike Information Criterion. All variables in the VAR enter in levels, since differencing or filtering the data discards information about the long-run properties of the data (Canova, 2007; Lütkepohl, 2013).

Figure 2 displays the impulse responses obtained from the VAR. The solid lines are the median responses of the endogenous variables to a one standard deviation uncertainty shock, while the shaded areas represent 68 (dark grey) and 95 (light grey) per cent confidence intervals. Output (real GDP) declines by about 0.4 per cent, while consumption and capital investment fall by 0.3 and 1.5 per cent after 10 quarters. Rises in uncertainty also lead to an initial increase in prices and in the interest rate. Furthermore, the impulse responses show that uncertainty shocks significantly dampen R&D investment and TFP, which fall by approximately 0.6 and 0.2 per cent. Last but not least, all real variables fall in a very persistent manner and do not revert to their trend within 40 quarters.

**Robustness** In the appendix, Section A.2.2, we test the robustness of our baseline results to a variety of changes: (i) we change the ordering of the variables in our model and place uncertainty last in our VAR (Figure A.1); (ii) we include the inverse of the labour share as a measure of markups, in line with Fernández-Villaverde et al. (2015) (Figure A.2); (iii) we consider alternative measures of macroeconomic uncertainty from Rossi and Sekhposyan (2015) (Figures A.3 and A.4); (iv) we increase the number of lags included in the VAR (Figure A.5); (v) we estimate an informationally rich monthly Factor-Augmented VAR (Figures A.6 and A.7); (vi) we estimate both the quarterly and monthly models with data from January 1985 until June 2018 (Figures A.8 and A.9), in order to take into account the regime shift in monetary policy induced by the *Volcker disinflation* (see e.g. Bianchi and Ilut, 2017). In all the robustness exercises, we find the baseline results to be confirmed. Macroeconomic uncertainty shocks lead to very persistent declines in the main macroeconomic aggregates and in total factor productivity. The responses of consumption and TFP tend to be the most persistent and the decline is in most instances significant (68% confidence) for over 40 quarters. For the sake of conciseness, we leave the details of the robustness checks to the

appendix.

### 3 The Model

This section studies the transmission channels of uncertainty shocks in a New-Keynesian DSGE model with endogenous growth through R&D investment. Households have recursive preferences à la [Epstein and Zin \(1989\)](#) (EZ) to separately calibrate the parameters governing relative risk aversion and the elasticity of intertemporal substitution. Moreover, these preferences make households averse to long-term risk about their consumption process ([Bansal and Yaron, 2004](#)). The model features an endogenous growth mechanism of vertical innovation in the spirit of [Grossman and Helpman \(1991\)](#) and [Aghion and Howitt \(1992\)](#), which is introduced as in [Kung \(2015\)](#) and [Bianchi et al. \(2018a\)](#). Uncertainty shocks are modelled assuming that the exogenous component of TFP follows an AR(1) process with stochastic volatility as in [Fernández-Villaverde et al. \(2011\)](#).

#### 3.1 Households

The representative household maximises its lifetime utility choosing consumption  $C_t$ , hours worked  $L_t$ , investment in physical capital  $I_t$  and in R&D  $S_t$ , the rates of utilisation of physical capital  $x_{K,t}$  and R&D  $x_{N,t}$  and next period bond holdings  $b_{t+1}$ . The aggregate stocks of physical capital and R&D are predetermined and denoted by  $K_t$  and  $N_t$ . The parameters  $\psi$  and  $\gamma$  govern the household's elasticity of intertemporal substitution and relative risk aversion. If  $\psi = \frac{1}{\gamma}$  the utility function reduces to the standard power utility. In our case instead, under the assumption  $\gamma \geq \frac{1}{\psi}$ , this type of utility function implies a preference for the early resolution of uncertainty, i.e. households dislike uncertainty over future utility. The problem of the household is formalised as follows:

$$V_t = \max \left[ (1 - \beta) u_t^{1 - \frac{1}{\psi}} + \beta \left( E_t V_{t+1}^{1 - \gamma} \right)^{\frac{1 - 1/\psi}{1 - \gamma}} \right]^{\frac{1}{1 - 1/\psi}}, \quad (2)$$

where  $E_t$  is the conditional expectation operator and  $\beta$  is the subjective discount factor of the households. The term  $u_t$  aggregates consumption and leisure,  $\bar{L} - L_t$  (where  $\bar{L}$  represents the

household's total time endowment), in a Cobb-Douglas fashion:

$$u_t = C_t (\bar{L} - L_t)^\chi. \quad (3)$$

The maximisation problem is subject to the following budget constraint:

$$C_t + I_t + S_t + \frac{\pi_{t+1}}{R_t} b_{t+1} = w_t L_t + r_{K,t} x_{K,t} K_t + r_{N,t} x_{N,t} N_t + b_t + \Pi_t, \quad (4)$$

where  $R_t$  is the nominal return on the risk-free bonds, and  $\pi_t$  is today inflation. Variables  $r_{l,t}$  ( $l = \{K, N\}$ ) are the return on capital (either physical capital or R&D). The aggregate stocks of physical capital and R&D evolve according to the following laws of motion:

$$K_{t+1} = \left(1 - \delta_K (x_{K,t})^{\xi_K}\right) K_t + \Lambda_K \left(\frac{I_t}{K_t}\right) K_t, \quad (5)$$

$$N_{t+1} = \left(1 - \delta_N (x_{N,t})^{\xi_N}\right) N_t + \Lambda_N \left(\frac{S_t}{N_t}\right) N_t, \quad (6)$$

where  $\delta_l$  ( $l = \{K, N\}$ ) is the depreciation rate. Utilisation  $x_{l,t}$  is introduced similarly as in [Neiss and Pappa \(2005\)](#) and enters the laws of motion (5) and (6) nonlinearly with parameter  $\xi_l$ . The function  $\Lambda_l(\cdot)$  represents positive, concave adjustment cost functions, defined as in [Jermann \(1998\)](#):

$$\Lambda_K \left(\frac{I_t}{K_t}\right) = a_{K,1} + \frac{a_{K,2}}{1 - \frac{1}{\tau_K}} \left(\frac{I_t}{K_t}\right)^{1 - \frac{1}{\tau_K}}, \quad (7)$$

$$\Lambda_N \left(\frac{S_t}{N_t}\right) = a_{N,1} + \frac{a_{N,2}}{1 - \frac{1}{\tau_N}} \left(\frac{S_t}{N_t}\right)^{1 - \frac{1}{\tau_N}}. \quad (8)$$

These adjustment costs capture the idea that changing the stocks of capital and R&D rapidly is more costly than changing them slowly. The presence of adjustment costs also implies that the shadow prices of  $K_t$  and  $N_t$  will not be constant. The household's stochastic discount factor derived under the EZ preferences is given by the following condition:

$$M_{t,t+1} = \beta \left(\frac{u_{t+1}}{u_t}\right)^{1 - \frac{1}{\psi}} \left(\frac{C_t}{C_{t+1}}\right) \left(\frac{V_{t+1}}{(E_t V_{t+1}^{1-\gamma})^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi} - \gamma}. \quad (9)$$

### 3.2 Final Goods Firms

The final good  $Y_t$  is produced by aggregating intermediate inputs  $Y_t(i)$  by a constant elasticity of substitution technology:

$$Y_t = \left( \int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad (10)$$

where  $\varepsilon$  is the elasticity of substitution of intermediate goods. The cost-minimisation problem for the final good firm implies that the demand for the intermediate good  $i$  is given by:

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t, \quad (11)$$

where  $P_t(i)$  is the price of the intermediate input. Finally, the zero-profit condition implies that the price index is expressed as:

$$P_t = \left( \int_0^1 Y_t(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}. \quad (12)$$

### 3.3 Intermediate Goods Firms

There exists a continuum of intermediate-goods producing firms indexed by  $i \in (0, 1)$  that rent labour  $L_t(i)$  and services of physical capital  $x_{K,t}(i)K_t(i)$  and R&D  $x_{N,t}(i)N_t(i)$  from the households at the respective prices  $w_t$  (real wage),  $r_t^K$  (rental rate of physical capital), and  $r_t^N$  (rental rate of R&D). These firms act in a monopolistically competitive environment and set their price  $P_t(i)$  facing quadratic adjustment costs à la [Rotemberg \(1982\)](#). Since firms are owned by the households, they discount future profits  $\Pi_{t+j}(i)$  by the stochastic discount factor  $M_{t,t+j}$  defined in Equation (9) and solve the following optimisation problem:

$$\max E_t \sum_{j=0}^{\infty} M_{t,t+j} \Pi_{t+j}(i), \quad (13)$$

$$\Pi_t(i) = \frac{P_t(i)}{P_t} Y_t(i) - w_t L_t(i) - r_{K,t} x_{K,t}(i) K_t(i) - r_{N,t} x_{N,t}(i) N_t(i) - \frac{\phi_P}{2} \left( \frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right)^2 Y_t, \quad (14)$$

$$Y_t(i) = (x_{K,t}(i) K_t(i))^\alpha (Z_t(i) L_t(i))^{1-\alpha}, \quad (15)$$

$$Z_t(i) = A_t (x_{N,t}(i)N_t(i))^\eta (x_{N,t}N_t)^{1-\eta}, \quad (16)$$

$$Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t. \quad (17)$$

Equation (13) and (14) represent the stream of lifetime profits and  $\pi$  is the (non-stochastic) steady state level of inflation. Intermediate-good firm  $i$  produces product  $Y_t(i)$  using a Cobb-Douglas technology as defined in Equation (15). Firm  $i$ 's productivity  $Z_t(i)$  is given by the product of an exogenous component  $A_t$  and an endogenous part that depends both on the amount of R&D services rented by the individual firm  $x_{N,t}(i)N_t(i)$  and on the aggregate level of R&D services  $x_{N,t}N_t$ . The fact that productivity depends on the utilised stock of R&D represents the presence of technological spillovers and captures the idea that accumulated knowledge facilitates the creation of new knowledge. Finally, the parameter  $1 - \eta \in (0, 1)$  governs the degree of technological spillovers over the utilised stock of R&D.

### 3.4 Monetary Authority

The monetary authority sets the nominal rate  $R_t$  following a policy rule à la Taylor (1993). More specifically, we assume that the nominal policy rate depends on deviation of inflation from its non-stochastic steady state and on output growth. The monetary policy rule is formalised as follows:

$$\frac{R_t}{R} = \left( \frac{\pi_t}{\pi} \right)^{\rho_\pi} \left( \frac{\Delta Y_t}{\Delta Y} \right)^{\rho_Y}, \quad (18)$$

where  $R$  and  $\pi$  are the steady-state nominal interest rate and the steady-state inflation respectively and  $\rho_\pi$  and  $\rho_Y$  are the reaction coefficients to inflation and output growth.

### 3.5 Closing the Model

The Rotemberg pricing assumption, as described by Equation (14), implies a symmetric equilibrium, such that all variables  $X_t(i) = X_t$ . Finally, the model closed by the usual resource constraint and

assuming the risk-free bonds are in zero net supply ( $b_t = 0$ ):

$$Y_t = C_t + I_t + S_t + \frac{\phi_P}{2} \left( \frac{\pi_t}{\pi} - 1 \right)^2 Y_t, \quad (19)$$

which states that aggregate output  $Y_t$  is used for expenditure in consumption  $C_t$ , investment in physical capital  $I_t$ , investment in R&D  $S_t$ , and price adjustment costs.

### 3.6 Exogenous Processes

The exogenous component of TFP follows a stationary AR(1) with stochastic volatility (see for example [Fernández-Villaverde et al., 2011](#)):

$$\log A_t = (1 - \rho_A) \log A + \rho_A \log A_{t-1} + \sigma_t^A \varepsilon_t^A, \quad (20)$$

where  $\rho_A$  is the parameter governing the persistence of the TFP shock  $\varepsilon_t^A$ , which is assumed to follow an iid standard normal stochastic process. Similarly, the time-varying standard deviation of the first moment shock,  $\sigma_t^A$ , follows itself a stationary AR(1) process:

$$\log \sigma_t^A = (1 - \rho_{\sigma^A}) \log \sigma^A + \rho_{\sigma^A} \log \sigma_{t-1}^A + \sigma^{\sigma^A} \varepsilon_t^{\sigma^A}. \quad (21)$$

The parameter  $\rho_{\sigma^A}$  measures the persistence of the uncertainty shock. The term  $\varepsilon_t^{\sigma^A}$  is the uncertainty shock, which follows an iid standard normal process.

## 4 Solution, Calibration, and Estimation

### 4.1 Solution Method

In order to induce stationarity, we divide all the trending variables ( $V_t$ ,  $u_t$ ,  $C_t$ ,  $I_t$ ,  $K_t$ ,  $S_t$ ,  $N_t$ ,  $Y_t$ ,  $w_t$ , and  $Z_t$ ) by the aggregate stock of R&D,  $N_t$ .<sup>4</sup> We then solve the model with perturbation methods,

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<sup>4</sup>In appendix [B.1](#), we report the detrended equilibrium conditions.

approximating the policy function to a third-order around its non-stochastic steady state (Adjemian et al., 2011).<sup>5</sup> As emphasized in Fernández-Villaverde et al. (2011), the third-order approximation of the policy function is necessary to analyse the effects of uncertainty shocks independently of the first moment shocks. With lower orders of approximation, in fact, uncertainty shocks either do not matter at all (first-order approximation) or they enter as cross-products with the other state variables (second-order approximation). Furthermore, as discussed in Caldara et al. (2012), perturbation methods for DSGE models with stochastic volatility and recursive preferences are comparable, in terms of accuracy, to global solution methods such as Chebyshev polynomials and value function iteration, while being computationally more efficient.

## 4.2 Calibrated Parameters

Table 1 reports the values of the parameters used for the simulations of the model. Some parameters are calibrated following the literature. In particular, the parameters relating to the household's preferences are specified in line with the long-run risk literature. The discount factor  $\beta$  is set equal to 0.997, while the coefficients of relative risk aversion  $\gamma$  and elasticity of intertemporal substitution  $\psi$  are set to 66 and 1.73, in line with the estimates by van Binsbergen et al. (2012). The risk-aversion parameter is lower than assumed in other works in the literature such as Rudebusch and Swanson (2012), Mumtaz and Theodoridis (2017), and Basu and Bundick (2017, 2018), who used values between 75 and 100. An intertemporal elasticity larger than 1 is also in line with Bansal and Yaron (2004). Similarly as in Neiss and Pappa (2005), the capital and R&D utilisation parameters  $\xi_K$  and  $\xi_N$  are endogenously set to ensure steady-state values of utilisation  $x_K$  and  $x_N$  of 1. The depreciation rate of physical capital is standard in the business cycle literature (0.02), used to match the average capital-investment ratio. The depreciation rate of R&D is set in line with Kung (2015) to 0.0375, which corresponds to an annualised depreciation rate of 15%, a standard value assumed by the Bureau of Labour Statistics in the R&D stock calculations. The share of capital in the production function  $\alpha$  is equal to 0.33 and the demand elasticity  $\varepsilon$  is equal to 6, implying a steady-state markup of 20%. The Rotemberg price adjustment parameter  $\phi_P$  is set to 59.46, which

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<sup>5</sup>The model is solved using Dynare 4.4.3 (MATLAB R2018a). In order to obtain a non-explosive behaviour of the simulations, Dynare relies on the pruning algorithm described in Andreasen et al. (2018).

Table 1: Baseline Quarterly Parameters

Parameter	Description	Value	Source/Target
Households			
$\beta$	Discount factor	0.997	van Binsbergen et al. (2012)
$\psi$	Elasticity of intertemporal substitution	1.73	van Binsbergen et al. (2012)
$\gamma$	Risk aversion	66	van Binsbergen et al. (2012)
$\delta_K$	Capital depreciation rate	0.02	Standard
$\delta_N$	R&D depreciation rate	0.0375	Kung (2015)
$\tau_K$	Capital adjustment cost parameter	<b>7.5036</b>	<b>Estimation</b>
$\tau_N$	R&D adjustment cost parameter	<b>6.2454</b>	<b>Estimation</b>
Firms			
$\alpha$	Power on capital in production	0.33	Standard
$\varepsilon$	Elasticity of substitution between goods	6	20% markup
$\phi_P$	Price adjustment cost parameter	59.46	4Q stickiness
$\eta$	Technological spillovers	0.1	Kung (2015)
Monetary Authority			
$\pi$	Steady-state inflation	1.005	2% annualised inflation rate
$\rho_\pi$	Weight on inflation in policy rule	1.5	Standard
$\rho_Y$	Weight on output in policy rule	0.35	Standard
Exogenous Processes			
$A$	Steady-state productivity	0.2375	1.64% annualised output growth
$\rho_A$	Persistence of productivity Shock	<b>0.6586</b>	<b>Estimation</b>
$\sigma^A$	Volatility of productivity shock	<b>0.0115</b>	<b>Estimation</b>
$\rho_{\sigma^A}$	Persistence of uncertainty Shock	<b>0.8415</b>	<b>Estimation</b>
$\sigma^{\sigma^A}$	Volatility of uncertainty Shock	<b>0.3357</b>	<b>Estimation</b>

to a first order approximation implies a Calvo parameter of 0.75 (i.e. firms, on average, update their price every 4 quarters). The parameter of technological spillovers  $\eta$  is set to 0.1, in order to match the R&D investment rate in the steady state (Kung, 2015; Kung and Schmid, 2015). The Taylor rule coefficients of inflation  $\rho_\pi$  and output growth  $\rho_Y$  are set respectively to 1.5 and 0.35, which are standard values in the New Keynesian literature. The steady state value of productivity  $A$  is calibrated to 0.2375 to match the mean growth rate of output (1.64% annualised).

### 4.3 Estimated Parameters

The parameters that appear in bold in Table 1 are estimated via indirect inference. The basic idea behind the estimation methodology is to find a vector of parameter estimates  $\hat{\lambda}$  that minimises both the distance between the impulse responses of our VAR ( $\hat{r}$ ) and those implied by the DSGE model ( $r$ ), as well as the difference between some key empirical moments ( $\hat{m}$ ) from their counterparts obtained with simulations of our DSGE model ( $m$ ). More formally, the estimation procedure involves solving the following minimisation problem:

$$D = \min_{\lambda} [\hat{r} - r(\lambda)]' W_r^{-1} [\hat{r} - r(\lambda)] + \Omega [\hat{m} - m(\lambda)]' W_m^{-1} [\hat{m} - m(\lambda)], \quad (22)$$

where  $W_j^{-1}$  ( $j \in \{r, m\}$ ) is the inverse of the variance matrix of the moments. In line with Basu and Bundick (2017), the scalar  $\Omega$  is set to roughly equalise the weight on matching impulse responses and moments. The impulse responses we target to match are those of output ( $Y_t$ ), consumption ( $C_t$ ), capital investment ( $I_t$ ), and R&D investment ( $S_t$ ). Moreover, we target the unconditional standard deviations of the growth rates of the variables mentioned above.

Table 2 displays the results of our estimation procedure. As we will further discuss in Section 5.2, aside from our baseline framework, we also consider and estimate three alternative versions of our model. Model B is a version of the model with EZ preferences and no endogenous growth mechanism. In this case, we assume households can invest in physical capital and not in R&D. Model C features the endogenous growth mechanism but no EZ preferences. To this end, we set the RRA parameter  $\gamma$  equal to 2, as common in the business cycle literature, and the EIS equal to  $\frac{1}{\gamma}$ . Model D features neither the endogenous growth mechanism nor EZ preferences. In models B and D, productivity is purely exogenous and the steady-state level of TFP ( $A$ ) is set equal to 1.

As for the baseline case, we estimate the parameters relating to the physical capital and R&D adjustment costs  $\tau_K$  and  $\tau_N$  to be equal to 7.5 and 6.2, in line with the calibrated values used in Kung (2015). We also estimate the parameters of the exogenous processes. For the persistence of the TFP level shock ( $\rho_A$ ), we find a value of 0.66, while for the steady-state level of TFP uncertainty ( $\sigma^A$ ) we obtain a value of 0.012. The autocorrelation of TFP volatility  $\rho_{\sigma^A}$  is estimated to be

Table 2: Empirical and Model-Implied Moments in Macroeconomic Aggregates

	Data	Baseline	Model B	Model C	Model D
Calibrated Parameter					
$\gamma$	-	66	66	2	2
$\psi$	-	1.73	1.73	0.5	0.5
$A$	-	0.2375	1	0.2990	1
Estimated Parameter					
$\tau_K$	-	7.5036	15.3196	1.1159	0.9756
$\tau_N$	-	6.2454	-	0.8076	-
$\rho_A$	-	0.6586	0.9016	0.4586	0.5187
$\sigma^A$	-	0.0115	0.0100	0.1030	0.0538
$\rho_{\sigma^A}$	-	0.8415	0.8781	0.9663	0.9714
$\sigma^{\sigma^A}$	-	0.3357	0.3029	0.1909	0.2367
Unconditional Volatility					
$\Delta Y$	0.82	0.82	0.79	0.89	0.97
$\Delta C$	0.46	0.68	0.49	0.52	0.49
$\Delta I$	2.35	1.49	2.34	2.52	2.55
$\Delta S$	1.31	1.26	-	1.55	-

Note: The lower part of the table compares the empirical standard deviation of the growth rates (log first differences) with those from the models' simulations. Standard deviations are scaled by 100. The empirical sample period is 1960Q3-2018Q2. The baseline model features both EZ preferences and the endogenous growth mechanism. Model B features EZ preferences but no endogenous growth mechanism. Model C features non-recursive CRRA preferences and the endogenous growth mechanism. Last, model D features standard (non-EZ) preferences and no endogenous growth mechanism.

equal to 0.84 and the standard deviation of the volatility shock  $\sigma^{\sigma^A}$  is 0.34. The relatively low persistence of the exogenous component of TFP can be explained by the presence of the endogenous growth mechanism that naturally introduces persistence in the aggregate TFP process. The other parameter estimates for the exogenous processes are broadly consistent with other papers in the literature (e.g. [Born and Pfeifer, 2014](#); [Leduc and Liu, 2016](#)). For the alternative models, we find the parameter estimates to differ substantially from the baseline case. The capital adjustment costs parameter  $\tau_K$  is higher in model B (15.32), while much lower in models C and D (1.12 and 0.98). In order to match the empirical targets, we find that models C and D require a much larger steady-state volatility ( $\sigma^A$ ) with values of 0.1 and 0.05. Compared to the baseline model, in model B we find a much larger persistence of the TFP level shock (0.9) and lower steady-state standard deviation (0.0097), while the parameters of the uncertainty process are broadly similar.

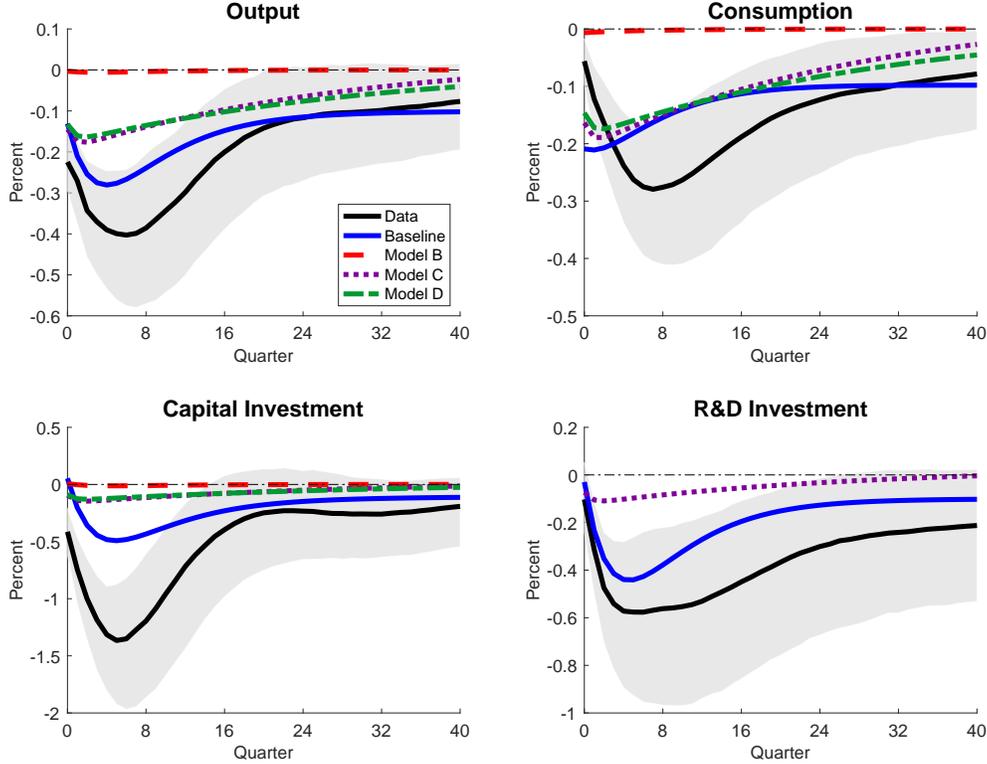


Figure 3: Impulse Responses to Uncertainty Shocks (Estimation)

Note: Variables are in per cent change. Grey shaded area represents 90 per cent confidence bands. Variables are in percentage changes. The baseline model features both EZ preferences and the endogenous growth mechanism. Model B does not feature the endogenous growth mechanism. Model C does not feature EZ preferences, but standard CRRA utility. Model D does not feature either endogenous growth mechanism or EZ preferences.

In terms of fitting the data, the baseline model does a good job both at matching the empirical volatilities as well as the VAR-based IRFs. The model perfectly matches the volatility of output growth (0.82) and it implies volatilities of consumption (0.68) and R&D investment (1.26) that are close to their empirical counterparts (0.46 and 1.31). The standard deviation of investment in physical capital (1.49) is slightly lower than its empirical counterparts (2.35). In Figure 3, we can see how the baseline model is able to replicate the VAR-based IRFs both qualitatively and quantitatively.

In model B, the model-implied standard deviations of output growth (0.79), consumption (0.49), and investment (2.34) are close to those found empirically, yet at the cost of falling short with respect to the impulse responses, which are far smaller than in the VAR. In Models C and D, it is possible to obtain moments and IRFs that are close to their empirical counterparts, yet only with

an unreasonably high steady-state TFP uncertainty. The model-implied volatility of output (0.89 in model C and 0.97 in model D), consumption (0.52 in model C and 0.49 in model D), investment in physical capital (2.52 in model C and 2.55 in model D), and investment in R&D (1.55 in model C) closely match the data. In both models C and D, the IRFs to an uncertainty shock are weaker than the median responses in the VAR, with the exception of consumption, which falls within the 90% confidence bands for most of the time horizon. The IRFs of all the different model specifications are discussed in detail in Section 5.

## 5 Impulse Response Analysis

We now analyse the effects of TFP uncertainty shocks on economic activity using our estimated model. First, we discuss the baseline results. Then, we describe the main transmission channels at play in our model and explain the importance of the long-run risk channel in amplifying the effects of uncertainty shocks.

As mentioned above, because of the endogenous growth mechanism, all real variables have to be detrended before solving the model. The impulse responses of output, consumption, investment in physical capital, and investment in R&D are obtained by adding back the trend. In particular, let  $\hat{x}_t$  be the detrended variable, i.e.  $\hat{x}_t \equiv \log(X_t) - \log(N_t)$ , and let  $\gamma_{N,t} \equiv \frac{N_t}{N_{t-1}}$  be the growth rate of the aggregate stock in R&D. Then the IRF of our variable of interest  $x_t = \log(X_t)$  is calculated as the sum of the IRF of  $\hat{x}_t$  and the cumulative sum of the IRF of  $\gamma_{N,t}$ .

### 5.1 The Effects of TFP Uncertainty Shocks

Figure 4 displays the IRFs to a TFP uncertainty shock, i.e. an exogenous increase in the probability of large (either positive or negative) TFP shocks. As in the empirical section, an uncertainty shock causes a long-run decline in economic activity. In the short term, consumption falls by approximately 0.2, investment in physical capital by 0.5, and R&D Investment,  $S_t$ , by 0.45 per cent. The fall in R&D investment leads to a decline in TFP of about 0.2 per cent, which is

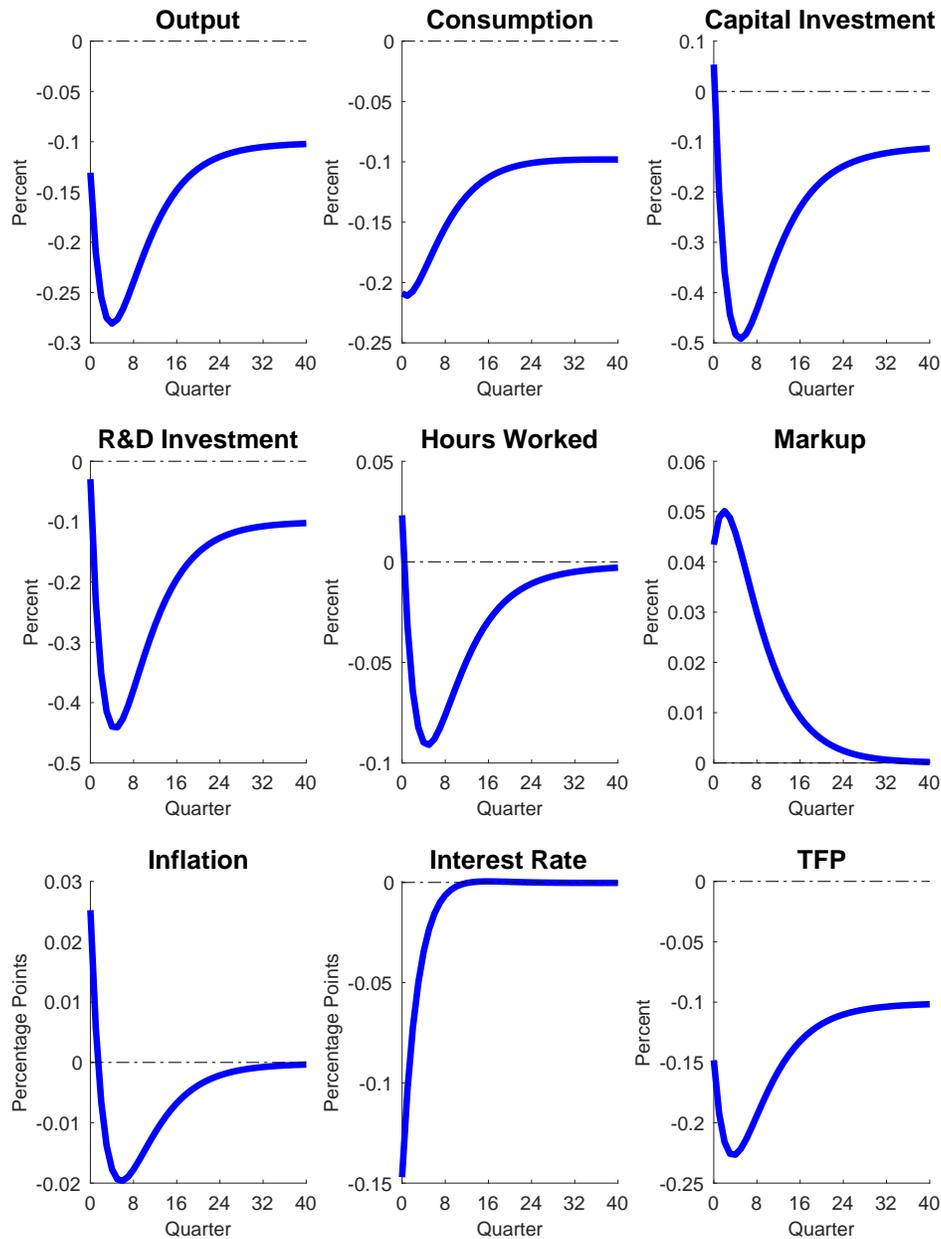


Figure 4: Impulse Responses to Uncertainty Shocks in Baseline Model (Estimation)

Note: Inflation and Interest Rate are expressed in annualised percentage points. All other variables are in percent change.

quantitatively in line with the TFP response in the VAR. Output decreases by approximately 0.3 per cent within the first 8 quarters. The fall in productivity causes an initial rise in inflation, analogously as in the empirical section. The negative effects of the uncertainty shock are partly offset by the reaction of the monetary authority that cuts the interest rate to counteract the strong

fall in output growth. In the long term, TFP, output, consumption, capital investment, and R&D investment remain approximately 0.1 per cent below trend, while the stationary variables (Hours, Inflation, Markup, and Interest Rate) revert back to their steady state within 40 quarters.

## 5.2 Understanding the Transmission Channels

The responses of the endogenous variables described above are due to the interplay of precautionary savings, rising markups, endogenous growth, and long-run risk.

**Precautionary Savings** First, an uncertainty shock leads to a fall in consumption because risk-averse households desire to increase savings for precautionary reasons in order to be able to self-insure against possible negative events occurring in the future. The importance of this channel crucially depends on the degree of relative risk aversion of households. In Figure 5, we show the effect of varying the RRA parameter ( $\gamma$ ) on the transmission of uncertainty shocks. For conciseness, we focus on the effect on output and consumption. We display the effect on the other variables in the appendix (see Figure B.1). When we reduce the parameter from our baseline value (66) to 20, the agents' precautionary motive becomes more subdued and consumption falls less. Conversely, when we increase the parameter from 66 to 100, consumption drops by 0.1 percentage points more than in the baseline scenario.

**Time-Varying Markups** The precautionary motive of households leads to a fall in consumption as well as an increase in labour supply, which reduces nominal marginal costs and wages. When prices are fully flexible, real marginal costs are unaffected by the increase in labour supply and firms' markups remain constant. Since physical capital and R&D are predetermined, the increase in labour supply raises output and we cannot obtain the co-movement between consumption and output, which we find empirically. Under sticky prices instead, markups are time-varying and output is demand-driven in the short term. The fall in consumption for precautionary reasons leads firms to demand less labour, capital services and R&D services. Given that the aggregate stocks of physical capital and R&D are predetermined, we first have a drop in the rates of capital and R&D

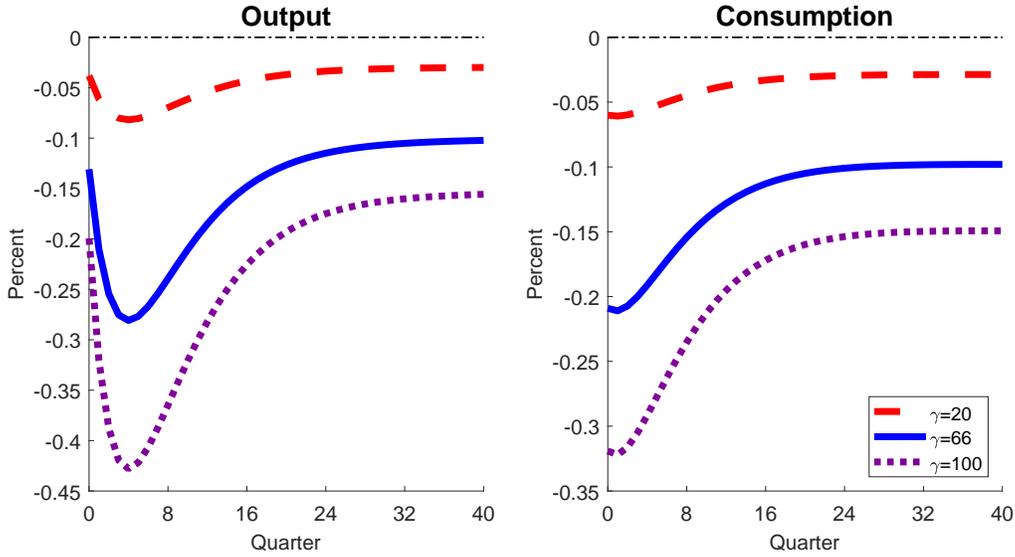


Figure 5: Precautionary Savings

Note: Variables are in percent change.

utilisation and in capital and R&D investment. Hence, when prices are sticky, uncertainty shocks can be a source of business cycle fluctuations, as they cause a drop in all the main macroeconomic aggregates. Figure 6 displays the IRFs to an uncertainty shock when prices are flexible ( $\phi_p$  is set to 0) and in our baseline model ( $\phi_p$  equal to 66). Consistently with Basu and Bundick (2018), in a flexible-price model (red line), uncertainty shocks have expansionary effects on output, while in a sticky price model, we see an increase in markups, which causes a reduction in output. The effect of price stickiness on the other variables is left to the appendix (see Figure B.2).

**Endogenous Growth via R&D** The permanent effects of uncertainty shocks in this theoretical model are due to the endogenous growth mechanism. More specifically, the fall in R&D investment implies a decline in the aggregate stock of R&D, which reduces the accumulation of new ideas and has a negative impact on TFP and long-run growth. To highlight the role of technology spillovers, the top row of Figure 7 compares the transmission of an uncertainty shock under alternative calibration of the spillover parameter  $\eta$ . We find that the larger  $\eta$ , the larger are the effects of an uncertainty shock on R&D investment and hence on TFP. Intuitively, if we consider the

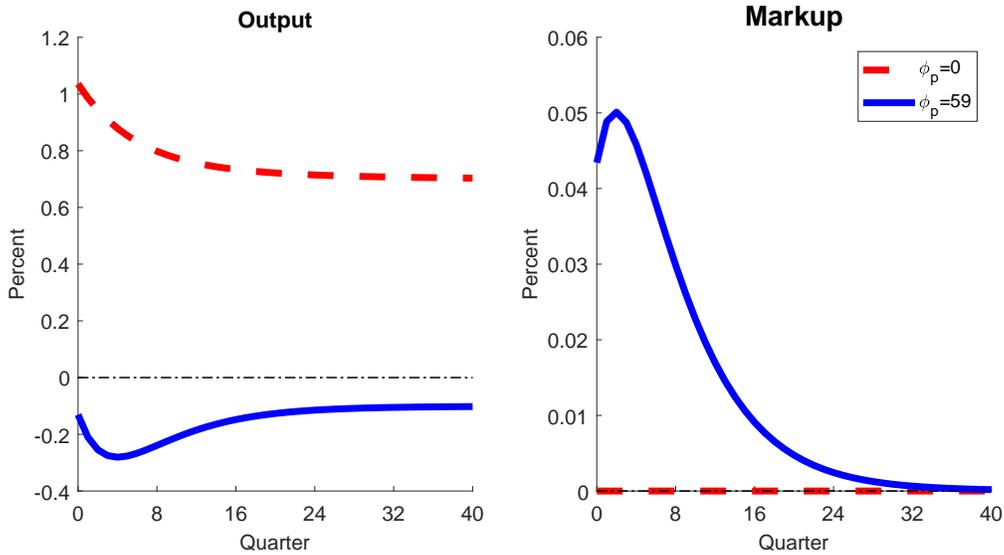


Figure 6: Time-Varying Markups

Note: Variables are in percent change.

extreme case of  $\eta = 0$ , then the endogenous component of TFP would be a pure externality. In other words, the larger  $\eta$ , the more the R&D choice is internalised by the firm. Hence, after an increase in uncertainty firm  $i$ 's demand for R&D will be more affected the larger  $\eta$ . In equilibrium, this leads to a stronger drop in aggregate R&D and therefore a more pronounced decline in TFP.

The degrees of capital and R&D adjustment costs can also affect the demand for R&D and hence influence the transmission of uncertainty shocks in the short and in the long run. The bottom two rows of figure 7 display the effect of an uncertainty shock for different values of the adjustment cost parameters  $\tau_K$  and  $\tau_N$ . For larger values of the adjustment cost parameter (and hence smaller adjustment costs) the model becomes more volatile as the drop in investment becomes more substantial. When we increase  $\tau_K$ , capital investment falls in a more pronounced way and, given input complementarity, this induces a stronger fall in the demand for R&D. Similarly, when we increase  $\tau_N$ , we see a sharper drop in R&D. As R&D falls more substantially, this translates into a larger decline in TFP and more severe effects in the long run on the overall economy. The effect of varying parameters  $\eta$ ,  $\tau_K$ , and  $\tau_N$  on the other variables is shown in the appendix in Figures B.3, B.4, and B.5.

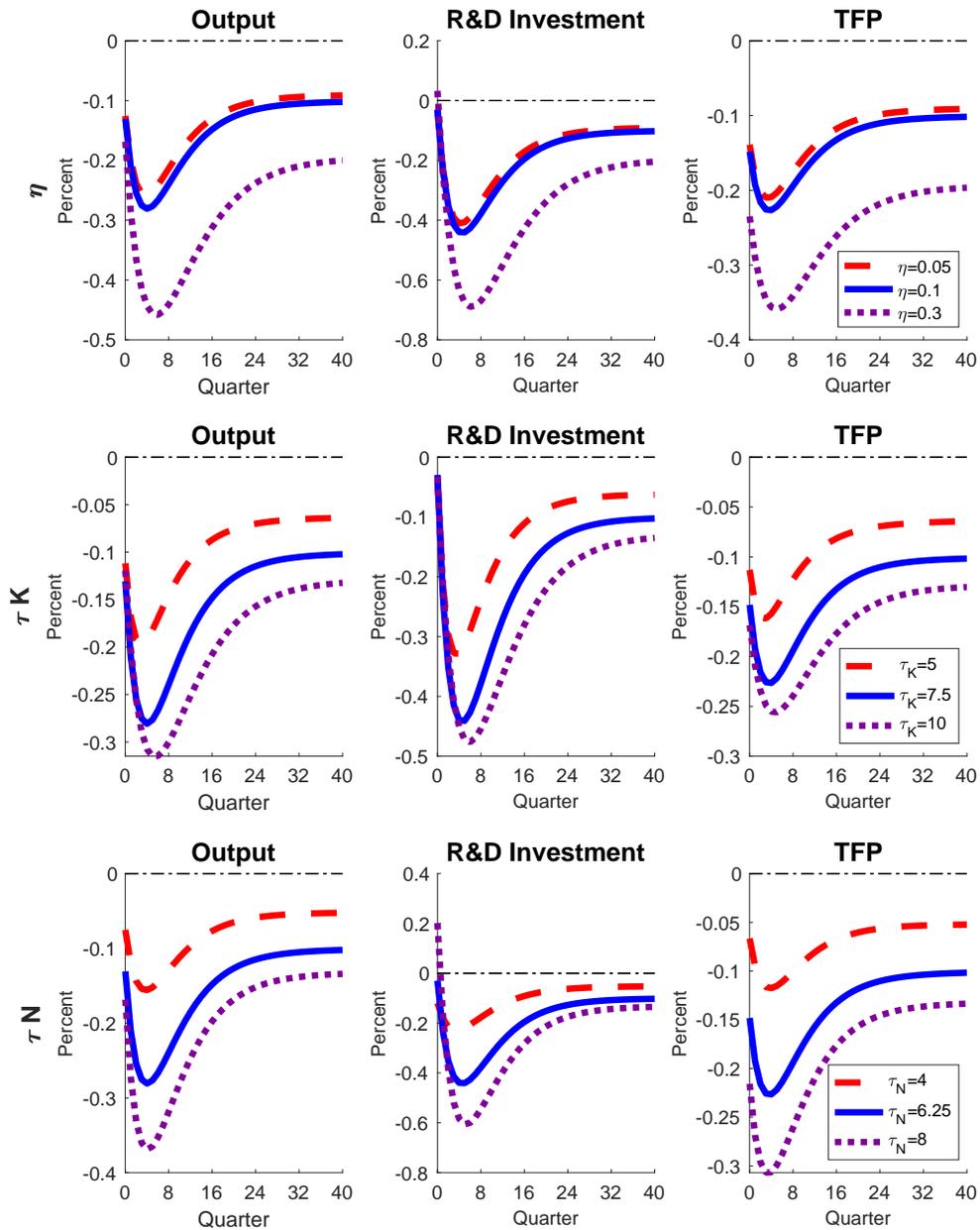


Figure 7: Endogenous Growth via R&D

Note: Variables are in percent change.

**Long-Run Risk** The combination of Epstein-Zin preferences and the endogenous growth mechanism is the main source of amplification of uncertainty shocks in our model. Because of the EZ preferences, households are averse to risks to future long-term growth<sup>6</sup> (Bansal and Yaron, 2004)

<sup>6</sup>This is because, with EZ preferences, the continuation value does not enter linearly in the Bellman equation.

and take therefore into account that shocks in this economy have permanent effects due to the endogenous growth mechanism described above. In other words, when shocks have effects in the long term, households become extremely risk-averse, which exacerbates their precautionary savings motive.

In order to highlight the amplification provided by the long-run risk channel, we analyse the IRFs from the three alternative models previously described: model B that features EZ preferences but no endogenous growth mechanism; model C that features the endogenous growth mechanism but no EZ preferences; finally model D that does not feature either EZ preferences or endogenous growth. Figure 8 displays the IRFs of models B, C, and D. In order to make the responses comparable, we set the parameters of the uncertainty process in these alternative models equal to those in the baseline model.

First, we compare the results from our baseline model and those from the same model without R&D (model B). Given that model B does not feature the endogenous growth mechanism, shocks in this model specification will only be transitory. Comparing the IRFs from Figure 4 to those from model B highlights the importance of long-run risk. The long-run risk channel in the baseline model exacerbates the precautionary savings channel, causing a 200 times larger fall in consumption compared to that in model B. Markups rise approximately 200 times more in our baseline model, which leads to larger drops in investment (100 times more than in model B) and output (150 times more than in model B).

Second, we compare two alternative models with and without R&D in absence of EZ preferences (models C and D). In particular, we consider the standard case in which the EIS parameter  $\psi = \frac{1}{\gamma}$ , where gamma is the RRA parameter. The stochastic discount factor, in this case, writes as:

$$M_{t,t+1} = \beta \left( \frac{u_{t+1}}{u_t} \right)^{1-\gamma} \frac{C_t}{C_{t+1}}. \quad (23)$$

There are two key differences between the stochastic discount factor (SDF) in the baseline model with EZ preferences (Equation, 9) and the one with standard preferences (Equation, 23). First of all, in the standard SDF, one parameter governs both the degree of relative risk aversion and

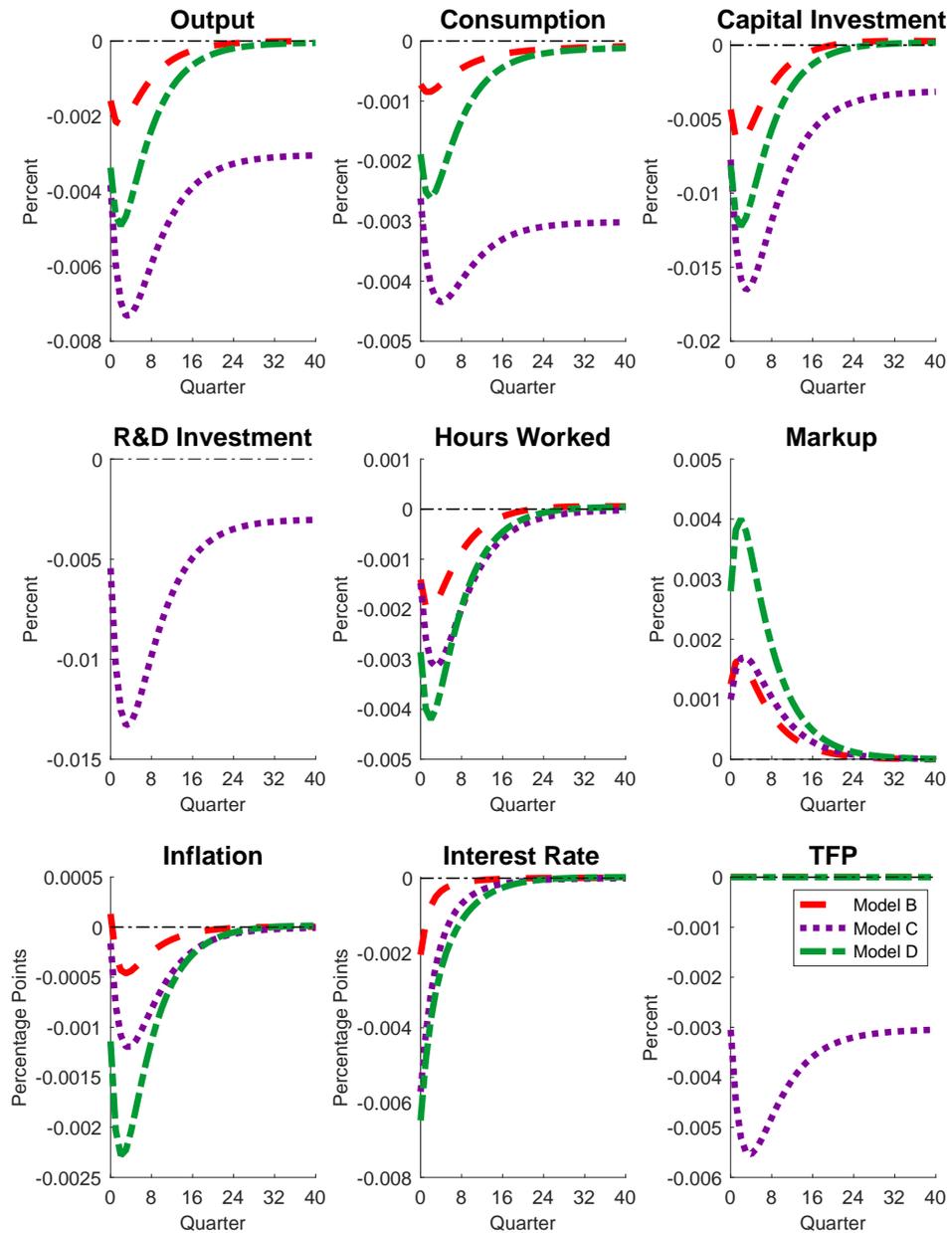


Figure 8: Uncertainty Shock in Model B, C, and D (Baseline Calibration)

Note: Inflation and Interest Rate are expressed in annualised percentage points. All other variables are in per cent change. Model B does not feature R&D and the endogenous growth mechanism. Model C does not feature EZ preferences, but standard CRRA utility. Model D does not feature either endogenous growth mechanism or EZ preferences.

the elasticity of intertemporal substitution. Under EZ preferences instead, we can increase RRA without affecting the EIS.<sup>7</sup> Second, and most importantly, the SDF for non-recursive preferences

<sup>7</sup>The EZ preferences boil down to the standard case when we set  $RRA = 1/EIS$

does not depend on the continuation value  $V_{t+1}$ . With EZ preferences this is not the case, as  $V_{t+1}$  is not additive separable from the instantaneous utility. The fact that  $V_{t+1}$  enters the Bellman equation in a non-linear way captures the idea that agents are averse to fluctuations in  $V_{t+1}$ , i.e. they fear long-run risk. With standard preferences instead, this fear is not accounted for.

As previously mentioned, in models C and D, we fixed the RRA parameter  $\gamma$  to 2, a standard value in the business cycle literature (hence we are implicitly assuming an EIS of 0.5). From Figure 8 we first observe that in models C and D, uncertainty shocks have much smaller effects than in the baseline model. In model C (D), consumption falls approximately 50 (80) times less than in the baseline model, investment drops 30 (40) times less and output 35 (55). Second, unlike for models A and B, the effects of uncertainty shocks in the short term are not significantly different between models C and D. In the first 8 quarters, output falls by 0.007 per cent in model C and 0.005 per cent in model D, consumption by 0.0045 per cent (model C) and 0.0025 (model D), and investment in physical capital drops by 0.016 (model C) and 0.012 (model D) per cent.

As a bottom line, the comparison of the baseline model with model B shows how the presence of long-run risks in our model is crucial to amplify the precautionary savings and the overall effects of uncertainty shocks. Comparing model C and D with the baseline model highlights the importance of assuming that agents take long-run risks into account via EZ preferences. Finally, comparing model C with model D underscores that when households do not feature EZ preferences and do not take long-run risk into account, the presence of an endogenous growth mechanism does not significantly amplify the effects of uncertainty shocks. These three observations are evidence of the importance of the long-run risk channel. In all models in which long-run risk is not accounted for, the effects of an uncertainty shock become negligible.

## 6 Conclusions

In this paper, we argue that shocks to macroeconomic uncertainty have negative long-run effects on economic activity that persist well beyond the business cycle frequency. First, we conduct an SVAR

analysis for the US and find that macroeconomic uncertainty shocks cause a significant decline in consumption, output, investment in physical capital and investment in R&D for over 40 quarters. Moreover, we find that these shocks lead to a persistent decline in total factor productivity. Second, we rationalise the empirical results through the lenses of a sticky-price DSGE model augmented with an endogenous growth mechanism of vertical innovations and recursive preferences à la Epstein-Zin. We find that this framework is able to provide a good fit to the data, both with respect to simple unconditional moments as well as with replicating the IRFs of the VAR. In this model, uncertainty shocks reduce consumption for precautionary reasons and increase markups, which in turn leads to a fall in output and investment in both physical capital and in R&D. The decline in the aggregate stock of R&D induces a fall in productivity that makes the effects of uncertainty shocks permanent. The inclusion of EZ preferences allows us to capture households' aversion to both current and future uncertainty. When faced with permanent risks affecting their future consumption, agents become extremely risk-averse, which significantly exacerbates their precautionary savings motive and the overall negative effects of uncertainty shocks both in the short and in the long run. In particular, we show that this "long-run risk" channel amplifies the effects of uncertainty shocks on the main macroeconomic variables up to 2 orders of magnitude compared to models without either endogenous growth or EZ preferences. In light of our results, we believe future research should focus on further exploring alternative sources of nonlinearities within DSGE models that may be important to quantitatively account for the real effects of uncertainty.

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# Appendices

## A Empirics

### A.1 Correlation between Uncertainty and TFP

In this subsection of the appendix we display the long-run correlations between  $p$  quarters backward-looking moving average of uncertainty and the  $q$  quarters forward-looking moving average of TFP growth. The correlations are calculated controlling for past GDP growth. In practice, we run the following regression:

$$tfp_{t,t+q} = \beta_1 uncertainty_{t-p,t} + \beta_2 gdp_{t-p,t} + \varepsilon_t, \quad (\text{A.1})$$

where  $tfp$ ,  $uncertainty$ , and  $gdp$  are standardised moving averages, so that  $\beta_1$  can be interpreted as a correlation.

Table A.1: Correlation

$q/p$	1	10	20	30	40
<b>1</b>	0.09 (0.27)	-0.20 (0.09)	-0.30 (0.01)	-0.40 (0.00)	-0.49 (0.00)
<b>10</b>	-0.01 (0.83)	-0.16 (0.29)	-0.27 (0.15)	-0.49 (0.02)	-0.60 (0.00)
<b>20</b>	-0.03 (0.54)	-0.19 (0.31)	-0.38 (0.11)	-0.51 (0.03)	-0.56 (0.00)
<b>30</b>	-0.07 (0.25)	-0.33 (0.07)	-0.46 (0.04)	-0.52 (0.01)	-0.48 (0.00)
<b>40</b>	-0.11 (0.05)	-0.35 (0.05)	-0.45 (0.04)	-0.44 (0.02)	-0.39 (0.00)

*Notes:* For each correlation  $(p, q)$  we show the estimate of  $\beta_1$  (upper value) and P-Values based on Newey-West standard errors (lower value).

## A.2 VAR

In this subsection we describe the data sources and present the details and results of our robustness tests for the VAR analysis.

### A.2.1 Data Sources

Table A.2: Data Used in the VAR Analysis

Name	Source	Ticker
Baseline VAR		
S&P 500 Index	Yahoo Finance	GSPC
Macroeconomic Uncertainty	Sydney Ludvigson	
Gross Domestic Product	FRED (BEA)	GDP
Services Consumption	FRED (BEA)	PCES
Nondurables Consumption	FRED (BEA)	PCEND
Services Consumption	FRED (BEA)	PCEDG
Private Residential Fixed Investment	FRED (BEA)	PRFI
Private Nonresidential Fixed Investment	FRED (BEA)	PNFI
Private Fixed Investment R&D	FRED (BEA)	Y006RC1Q027SBEA
GDP Implicit Price Deflator	FRED (BEA)	GDPDEF
Labour Share	FRED	PRS85006173
Shadow Interest Rate	FRBA	
Utilization-Adjusted TFP	FRBSF	
Robustness Exercises		
Alternative Macro Uncertainty	<a href="#">Rossi and Sekhposyan (2015)</a>	
Downside Macro Uncertainty	<a href="#">Rossi and Sekhposyan (2015)</a>	
Macroeconomic Dataset	FRED	FRED-MD
Industrial Production	FRED	INDPRO
Consumer Confidence	FRED (OECD)	CSCICP03USM665S
Consumer Price Index	FRED	CPIAUCSL
Spread Yields BAA - 10yr Treasury	FRED	BAA10Y

### A.2.2 Robustness Exercises

**Uncertainty Ordered Last** First, we change the Cholesky ordering assumed in the baseline setup and allow uncertainty to respond on impact to all the other variables in our model. The other variables instead, will respond only with a quarter lag to an uncertainty shock. The results reported in Figure A.1 confirm those in the baseline VAR. We find a strong persistent decline in all the real macroeconomic variables. The response of prices and interest rate is insignificant throughout the 40 quarters.

**Including a measure of markup** To test the validity of the proposed short-run mechanism, i.e. uncertainty affecting the economy by raising price markups, we include the inverse of the labour share in our VAR. The markup proxy is placed below the macro uncertainty measure, implying that markup shocks do not affect uncertainty on impact. Similarly as in Fernández-Villaverde et al. (2015), in Figure A.2, we find the markup to initially fall, while it immediately rebounds and significantly rises by 0.1 per cent. The other responses are in line with the baseline results, although the response of capital investment and R&D investment become insignificant after approximately 20 quarters.

**Alternative Measure of Uncertainty** We also estimate the VAR above using the measure of macroeconomic uncertainty and downside macroeconomic uncertainty from Rossi and Sekhposyan (2015). They define uncertainty based on the percentile in the historical distribution of forecast errors associated with the realized error. Let  $e_{t+h}$  be the  $h$ -step ahead forecast error of  $y_{t+h}$  defined as  $y_{t+h} - E_t[y_{t+h}]$  and let  $f(e)$  be its forecast error distribution. Uncertainty is then defined as the cumulative distribution  $U_{t+h} = \int_{-\infty}^{e_{t+h}} f(e)de$ . Downside uncertainty is defined as  $U_{t+h}^- = \frac{1}{2} + \max\{\frac{1}{2} - U_{t+h}, 0\}$ . As can be seen in figures A.3 and A.4, the median responses of output, consumption, R&D and TFP are extremely persistent and last well beyond the business cycle frequency, qualitatively and quantitative in line with our baseline results. However, for both alternative measures of macroeconomic uncertainty, the responses in the long-run are less significant than in the baseline case.

**Increase the Number of Lags** We increase the maximum number of lags included in our VAR to 2 to 5 to show that our baseline results are not due to the number of lags included in our VAR, as in Figure [A.5](#).

**FAVARs** There are two potential issues with our baseline specification. The first one relates to the quarterly frequency of the data and the second to the potential insufficient information contained in the model, which would not allow us to uncover the true effects of uncertainty shocks. On the one hand, the exact identification of uncertainty shocks could be undermined by the quarterly-data specification. Furthermore, by using quarterly data, the time-series dimension may not be sufficiently long considering the size of the VAR. In order to overcome these issues, we estimate a monthly-frequency Factor-Augmented VAR (FAVAR) model in the spirit of [Bernanke et al. \(2004\)](#). The factors are extracted as principal components from a large monthly dataset for the US economy, FRED-MD ([McCracken and Ng, 2015](#)), which includes 128 macroeconomic series. We include the first three factors in the VAR, which account for about 55% of the total variance of the data. The FAVAR contains the following variables  $X_t = [f^{(1)}; f^{(2)}; f^{(3)}; S\&P500; Confidence; Uncertainty; IP; C; CPI; FFR; Spread]$ , where  $f^{(1)}$ ,  $f^{(2)}$ ,  $f^{(3)}$ , IP are respectively the three factors and industrial production. We include a measure of consumer confidence from [OECD \(2015\)](#), to avoid that the effects of uncertainty are confounded with the agents' perception of bad economic times. We also include the spread between the yield on BAA corporate bonds and the 10-year constant-maturity treasury bond. *S&P500*, *Confidence*, *Uncertainty*, *IP*, *Consumption*, *CPI* are in logs to interpret the IRFs in percentage changes terms. Figures [A.6](#) and [A.7](#) display the results of the FAVAR, assuming the ordering described above or placing uncertainty last. The responses confirm those found in the smaller quarterly VAR used in the baseline exercise. In particular, the responses in output and consumption fall significantly both in the short and in the long-run. The response of the nominal variables is less clear-cut, with both price and interest rate falling significantly on impact, but quickly becoming insignificant within the first year.

**Post-Volker Sample** Finally, we estimate the baseline quarterly VAR and the monthly FAVAR described above using the sample Jan-1985/Jun-2018 to account for the structural break in mon-

etary policy induced by the *Volker disinflation*. Also in this case, as displayed in figures [A.8](#) and [A.9](#), the responses of output and consumption are extremely persistent and last well beyond the business cycle frequency. Prices significantly decline throughout the 40 quarters (120 months).

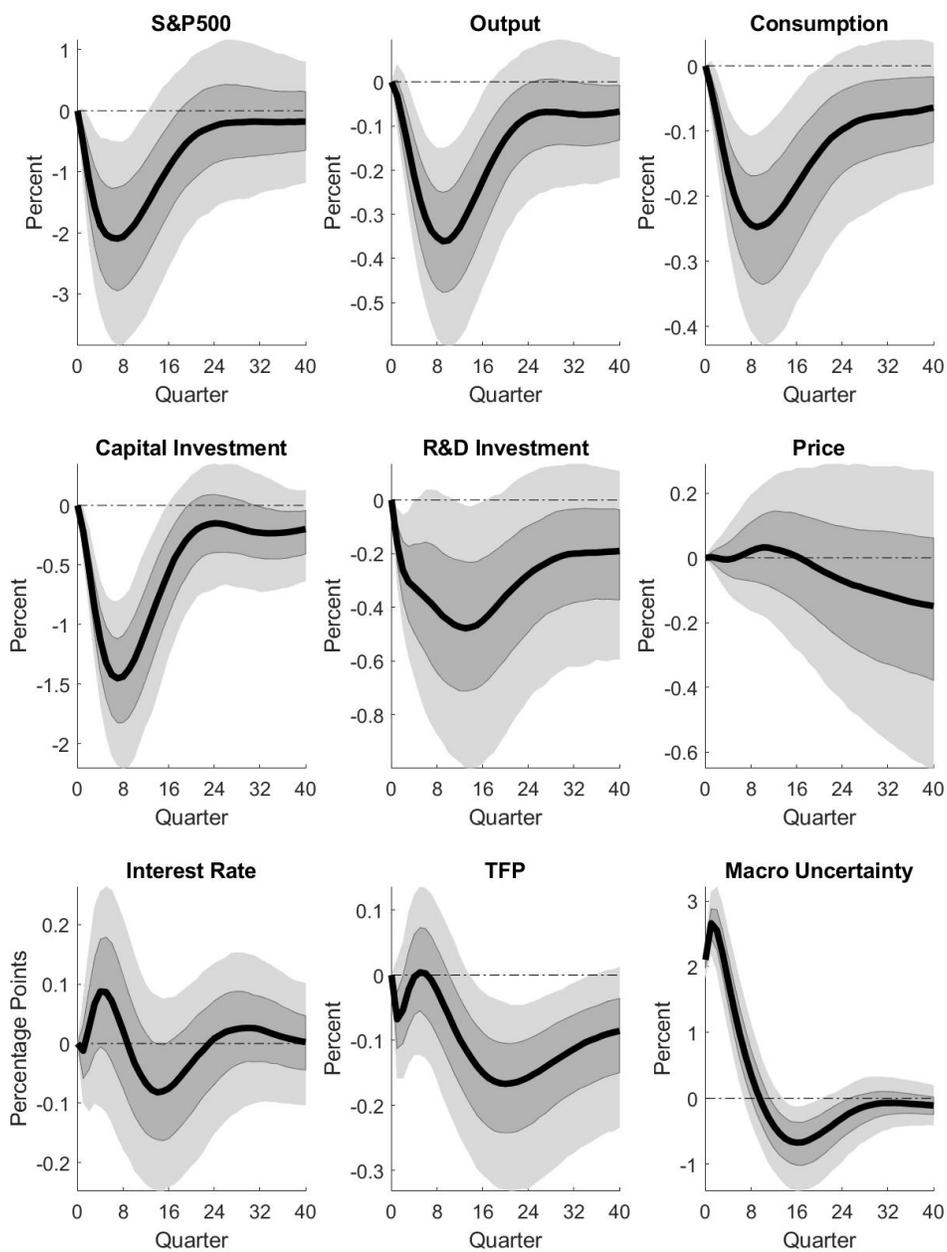


Figure A.1: Uncertainty Ordered Last

Note: Variables are in percentage changes except for the interest rate, which is in annualised percentage points. Light grey and dark grey shaded areas represent 95 and 68 percent confidence bands.

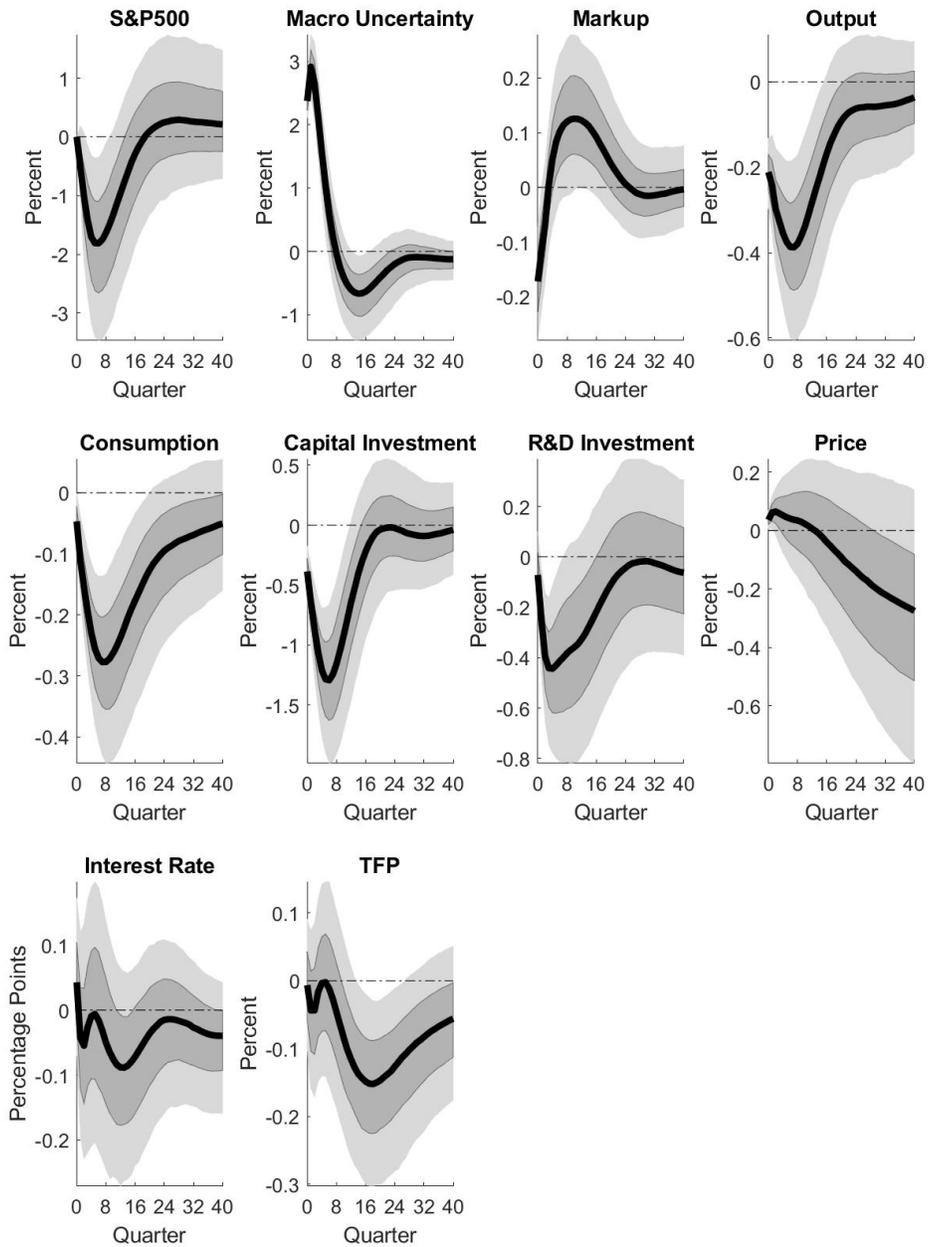


Figure A.2: VAR Including Markups

Note: Variables are in percentage changes except for the interest rate, which is in annualised percentage points. Light grey and dark grey shaded areas represent 95 and 68 percent confidence bands.

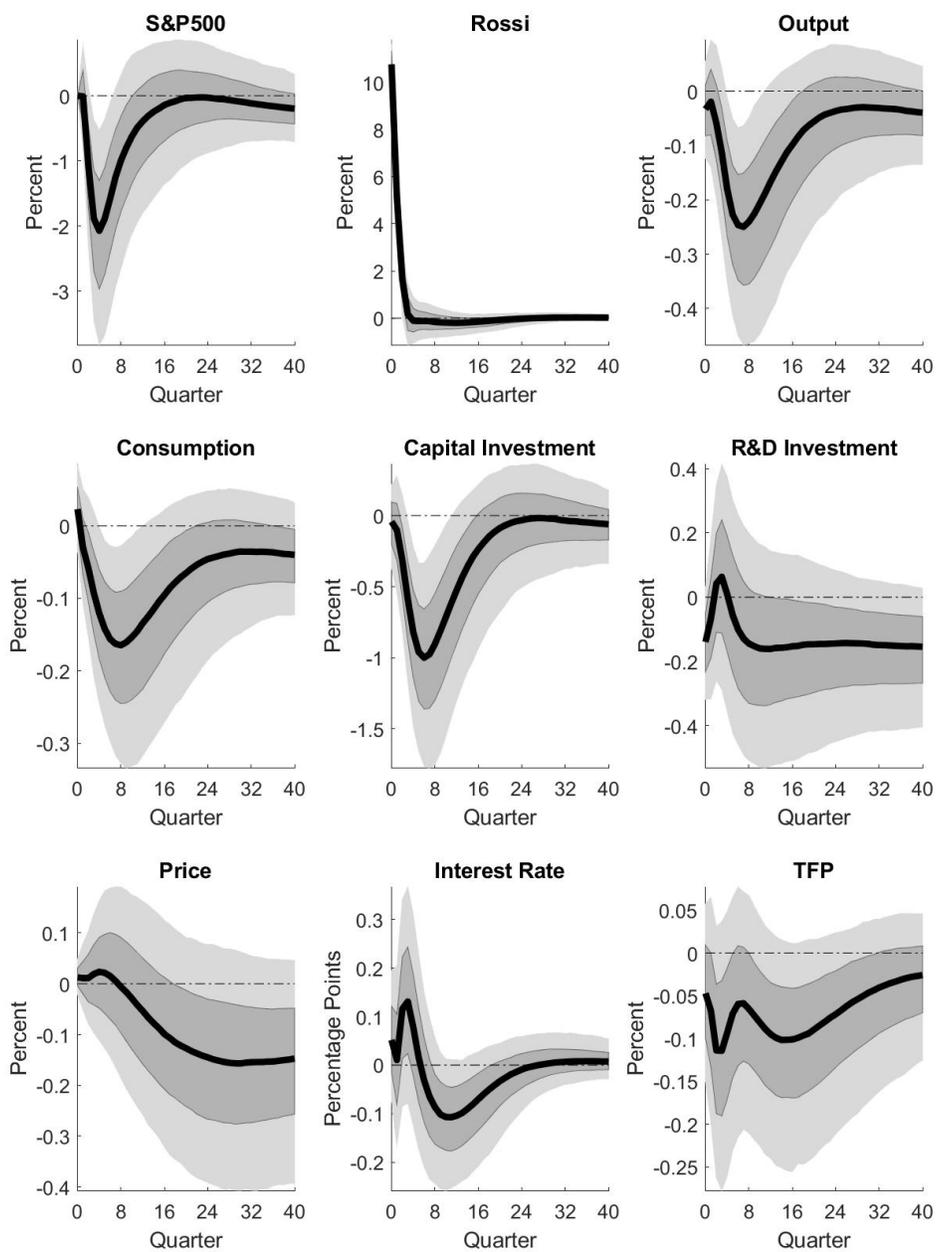


Figure A.3: VAR with Alternative Macro Uncertainty

Note: Variables are in percentage changes except for the interest rate, which is in annualised percentage points. Light grey and dark grey shaded areas represent 95 and 68 percent confidence bands.

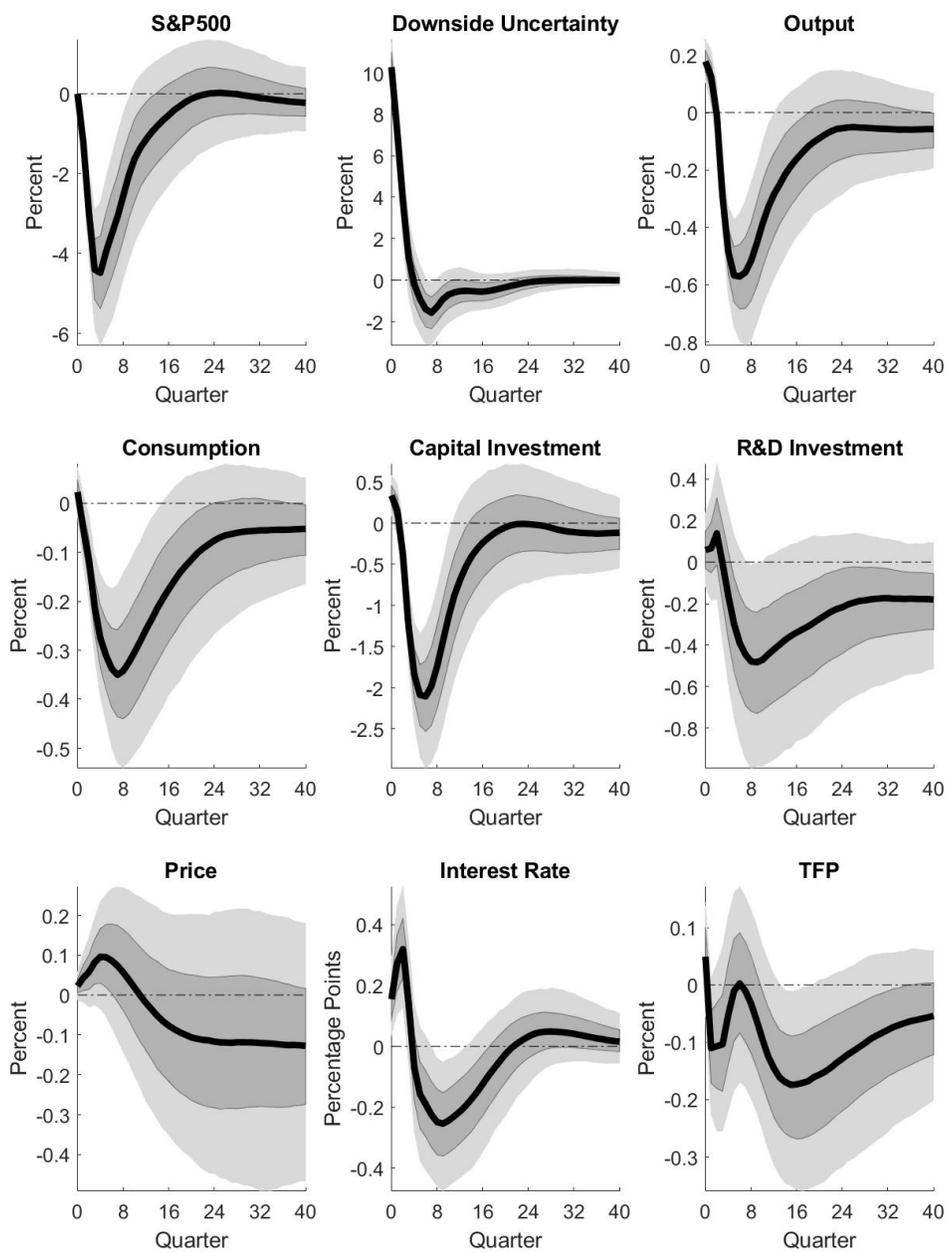


Figure A.4: VAR with Macro Downside Uncertainty

Note: Variables are in percentage changes except for the interest rate, which is in annualised percentage points. Light grey and dark grey shaded areas represent 95 and 68 percent confidence bands.

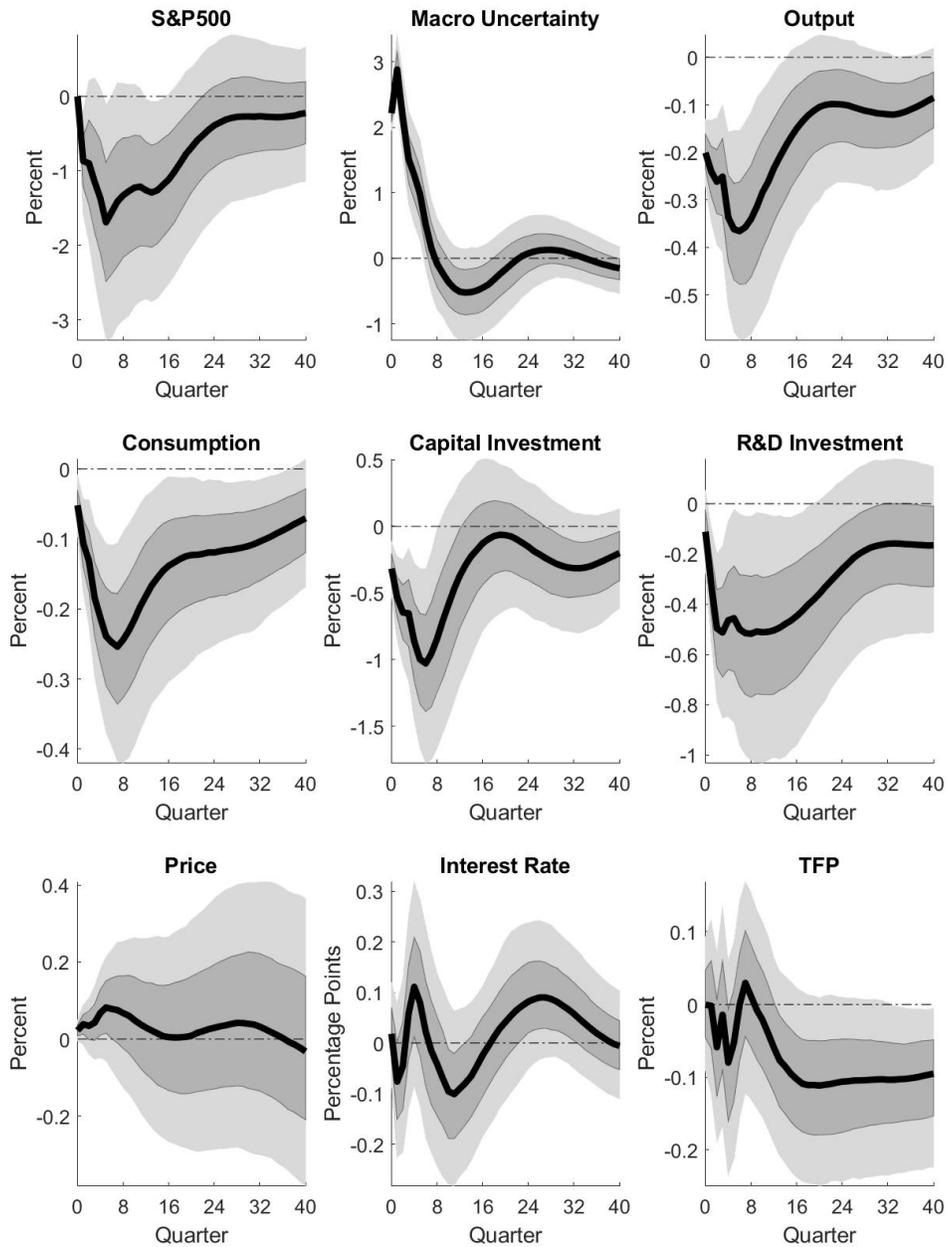


Figure A.5: VAR with 5 lags

Note: Variables are in percentage changes except for the interest rate, which is in annualised percentage points. Light grey and dark grey shaded areas represent 95 and 68 percent confidence bands.

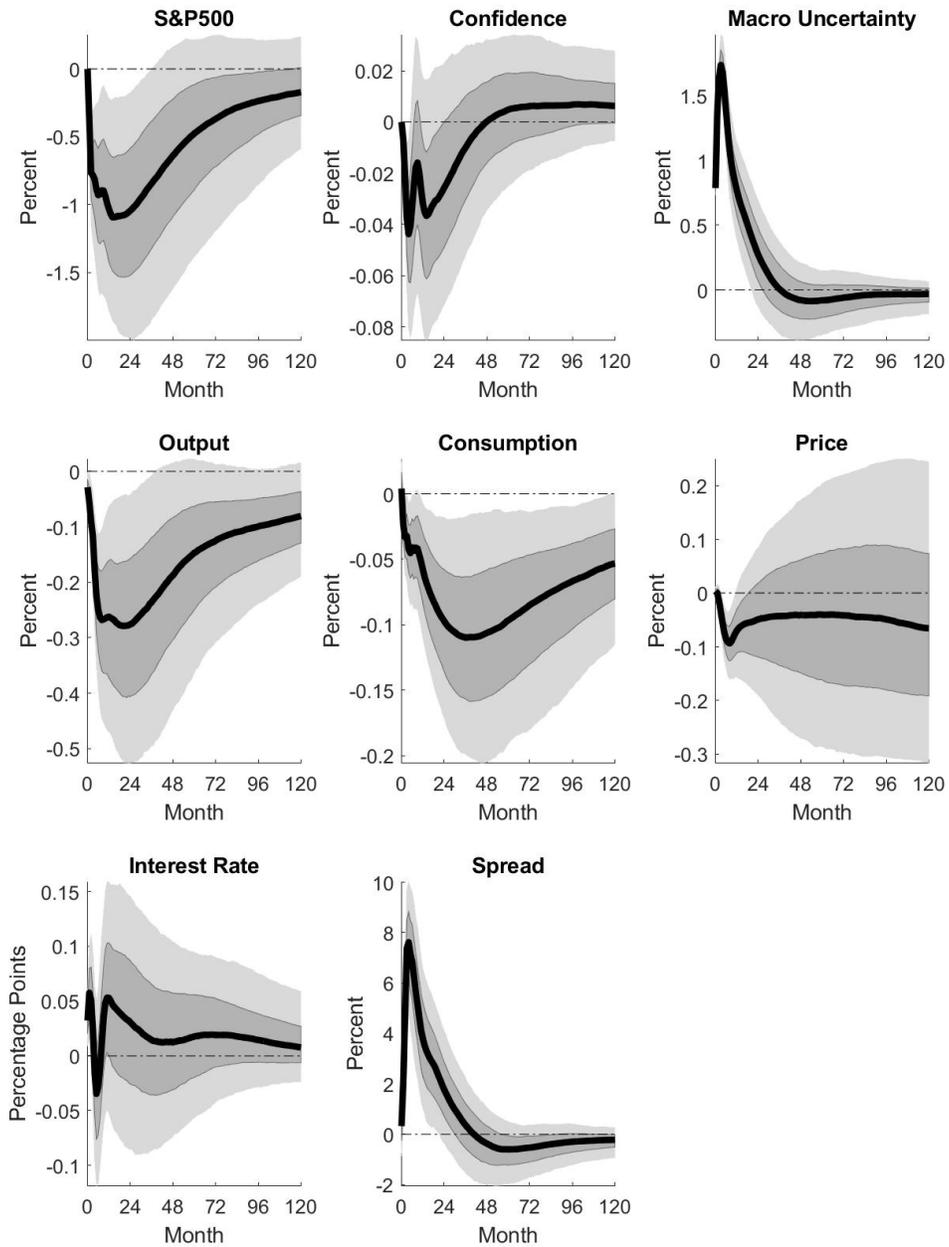


Figure A.6: Monthly FAVAR and Macro Uncertainty

Note: Variables are in percentage changes except for the interest rate, which is in annualised percentage points. Light grey and dark grey shaded areas represent 95 and 68 per cent confidence bands.

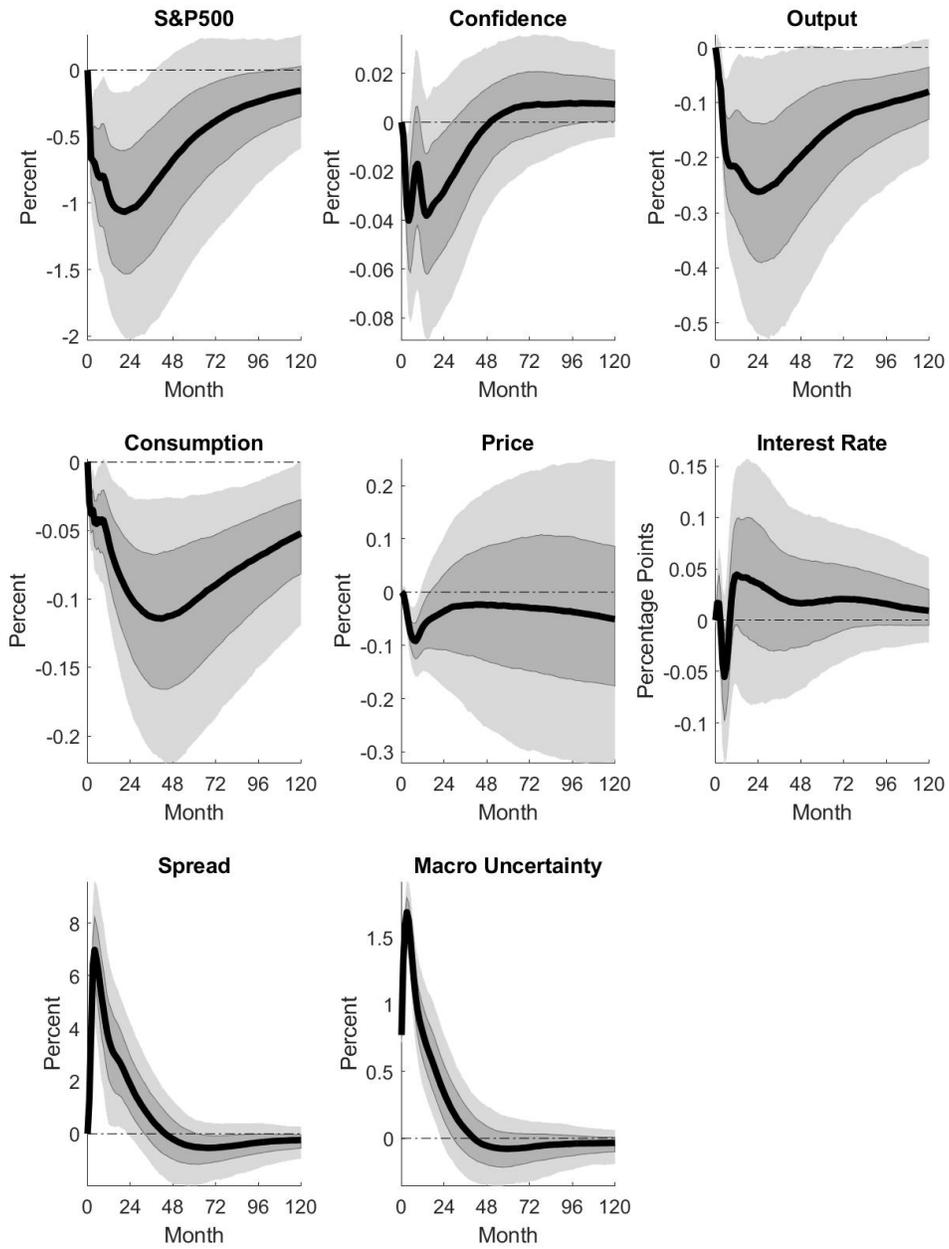


Figure A.7: Monthly FAVAR and Macro Uncertainty Ordered Last

Note: Variables are in percentage changes except for the interest rate, which is in annualised percentage points. Light grey and dark grey shaded areas represent 95 and 68 percent confidence bands.

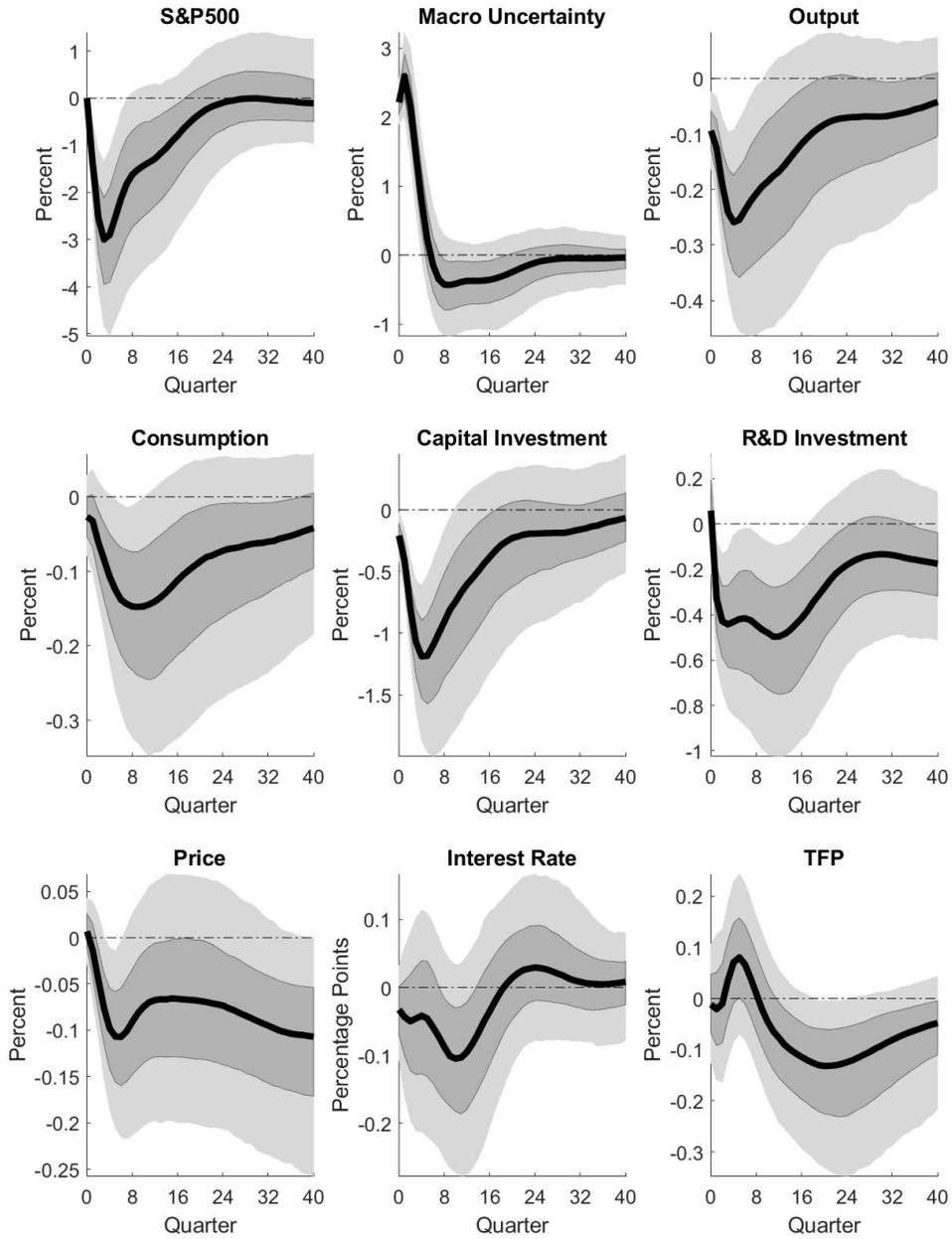


Figure A.8: Quarterly VAR: Time Span 1985Q1 - 2018Q2

Note: Variables are in percentage changes except for the interest rate, which is in annualised percentage points. Light grey and dark grey shaded areas represent 95 and 68 per cent confidence bands.

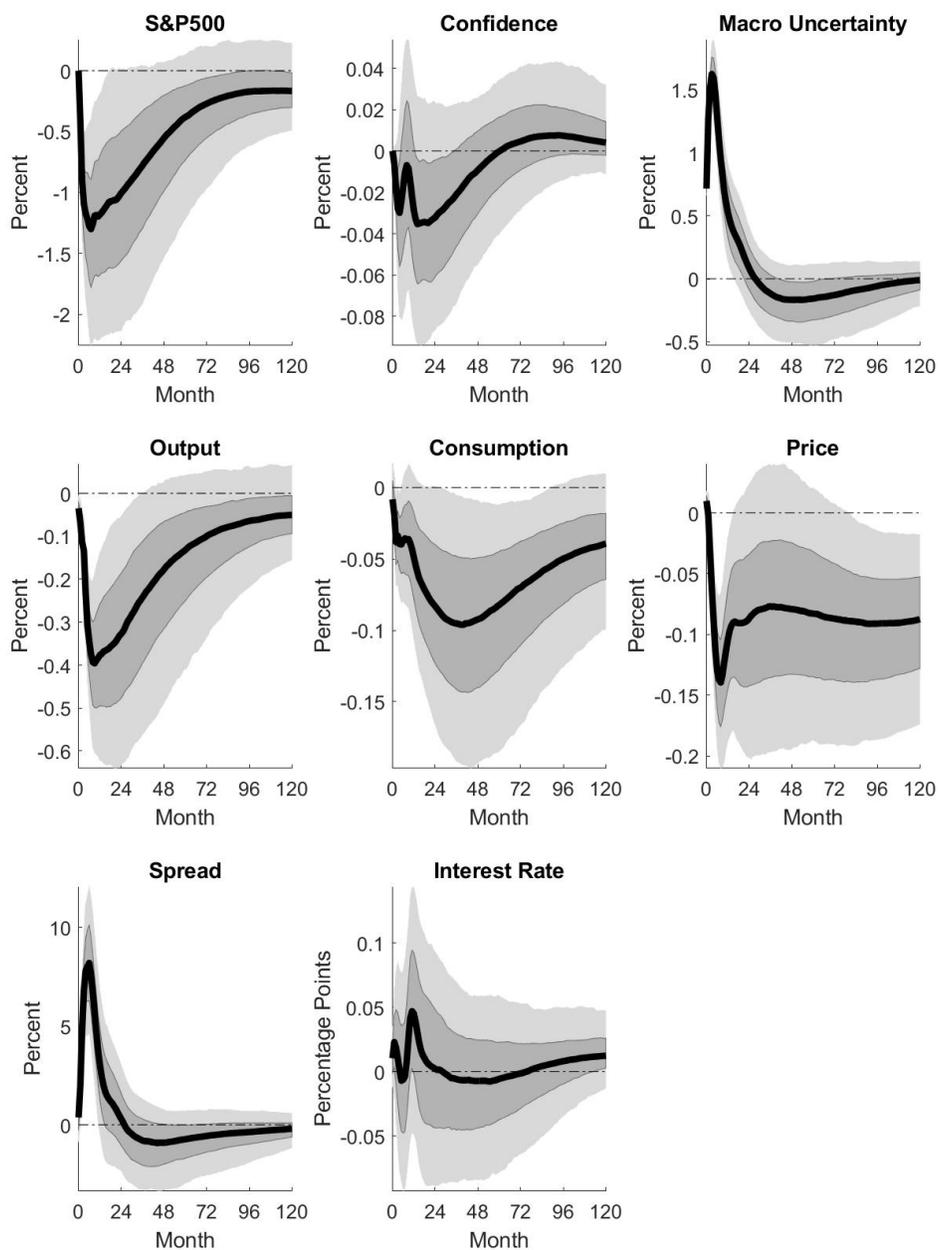


Figure A.9: Monthly FAVAR: Time Span 1985Q1 - 2018Q2

Note: Variables are in percentage changes except for the interest rate, which is in annualised percentage points. Light grey and dark grey shaded areas represent 95 and 68 per cent confidence bands.

## B Model

### B.1 Detrended Model

In order to solve the model in Dynare, we detrend the endogenous variables  $V_t$ ,  $u_t$ ,  $C_t$ ,  $I_t$ ,  $K_t$ ,  $S_t$ ,  $N_t$ ,  $Y_t$ ,  $w_t$ , and  $Z_t$  by  $N_t$ . We define the detrended variables and the growth rate of R&D as  $\hat{X}_t \equiv \frac{X_t}{N_t}$  and  $\gamma_{N,t} \equiv \frac{N_t}{N_{t-1}}$ . The detrended equilibrium conditions are provided below:

$$\hat{V}_t = \left[ (1 - \beta) \hat{u}_t^{1 - \frac{1}{\psi}} + \beta \left( E_t \left( \hat{V}_{t+1} \gamma_{N,t+1} \right)^{1 - \gamma} \right)^{\frac{1 - 1/\psi}{1 - \gamma}} \right]^{\frac{1}{1 - 1/\psi}} \quad (\text{B.1})$$

$$\hat{u}_t = \hat{C}_t (\bar{L} - L_t)^\chi \quad (\text{B.2})$$

$$\gamma_{N,t+1} \hat{K}_{t+1} = \left( 1 - \delta_K (x_{K,t})^{\xi_K} \right) \hat{K}_t + \Lambda_K \left( \frac{\hat{I}_t}{\hat{K}_t} \right) \hat{K}_t \quad (\text{B.3})$$

$$\Lambda_K \left( \frac{\hat{I}_t}{\hat{K}_t} \right) = a_{K,1} + \frac{a_{K,2}}{1 - \frac{1}{\tau_K}} \left( \frac{\hat{I}_t}{\hat{K}_t} \right)^{1 - \frac{1}{\tau_K}} \quad (\text{B.4})$$

$$\Lambda'_{K,t} = a_{K,2} \left( \frac{\hat{I}_t}{\hat{K}_t} \right)^{-\frac{1}{\tau_K}} \quad (\text{B.5})$$

$$\gamma_{N,t+1} = \left( 1 - \delta_N (x_{N,t})^{\xi_N} \right) + \Lambda_N \left( \hat{S}_t \right) \quad (\text{B.6})$$

$$\Lambda_N \left( \hat{S}_t \right) = a_{N,1} + \frac{a_{N,2}}{1 - \frac{1}{\tau_N}} \hat{S}_t^{1 - \frac{1}{\tau_N}} \quad (\text{B.7})$$

$$\Lambda'_{N,t} = a_{N,2} \hat{S}_t^{-\frac{1}{\tau_N}} \quad (\text{B.8})$$

$$M_{t,t+1} = \beta \gamma_{N,t+1}^{-\frac{1}{\psi}} \left( \frac{\hat{u}_{t+1}}{\hat{u}_t} \right)^{1 - \frac{1}{\psi}} \frac{\hat{C}_t}{\hat{C}_{t+1}} \left( \frac{\hat{V}_{t+1}}{\left( E_t \hat{V}_{t+1}^{1 - \gamma} \right)^{\frac{1}{1 - \gamma}}} \right)^{\frac{1}{\psi} - \gamma} \quad (\text{B.9})$$

$$\chi \frac{\hat{C}_t}{\bar{L} - L_t} = \hat{w}_t \quad (\text{B.10})$$

$$1 = q_{K,t} \Lambda'_{K,t} \quad (\text{B.11})$$

$$q_{K,t} = E_t M_{t,t+1} \left( r_{K,t+1} x_{K,t+1} + q_{K,t+1} \left( 1 - \delta_K (x_{K,t+1})^{\xi_K} - \Lambda'_{K,t+1} \frac{\hat{I}_{t+1}}{\hat{K}_{t+1}} + \Lambda_{K,t+1} \right) \right) \quad (\text{B.12})$$

$$r_{K,t} = q_{K,t} \delta_K \xi_K (x_{K,t})^{\xi_K - 1} \quad (\text{B.13})$$

$$1 = q_{N,t} \Lambda'_{N,t} \quad (\text{B.14})$$

$$q_{N,t} = E_t M_{t,t+1} \left( r_{N,t+1} x_{N,t+1} + q_{N,t+1} \left( 1 - \delta_N (x_{N,t+1})^{\xi_N} - \Lambda'_{N,t+1} \hat{S}_{t+1} + \Lambda_{N,t+1} \right) \right) \quad (\text{B.15})$$

$$r_{N,t} = q_{N,t} \delta_N \xi_N (x_{N,t})^{\xi_N - 1} \quad (\text{B.16})$$

$$1 = E_t M_{t,t+1} \frac{R_t}{\pi_{t+1}} \quad (\text{B.17})$$

$$\hat{w}_t = mc_t (1 - \alpha) \frac{\hat{Y}_t}{L_t} \quad (\text{B.18})$$

$$r_{K,t} = mc_t \alpha \frac{\hat{Y}_t}{x_{K,t} \hat{K}_t} \quad (\text{B.19})$$

$$r_{N,t} = mc_t (1 - \alpha) \eta \frac{\hat{Y}_t}{x_{N,t}} \quad (\text{B.20})$$

$$\phi_P \left( \frac{\pi_t}{\pi} - 1 \right) \frac{\pi_t}{\pi} = \phi_P E_t M_{t,t+1} \left( \frac{\pi_{t+1}}{\pi} - 1 \right) \frac{\pi_{t+1}}{\pi} \frac{\hat{Y}_{t+1}}{\hat{Y}_t} \gamma_{N,t+1} + 1 - \varepsilon + \varepsilon mc_t \quad (\text{B.21})$$

$$\frac{R_t}{R} = \left( \frac{\pi_t}{\pi} \right)^{\rho_\pi} \left( \frac{\hat{Y}_t}{\hat{Y}_{t-1}} \frac{\gamma_{N,t}}{\gamma_N} \right)^{\rho_Y} \quad (\text{B.22})$$

$$\hat{Y}_t = \left( x_{K,t} \hat{K}_t \right)^\alpha \left( \hat{Z}_t L_t \right)^{1-\alpha} \quad (\text{B.23})$$

$$\hat{Z}_t = A_t x_{N,t} \quad (\text{B.24})$$

$$\hat{Y}_t = \hat{C}_t + \hat{I}_t + \hat{S}_t + \frac{\phi_P}{2} \left( \frac{\pi_t}{\pi} - 1 \right)^2 \hat{Y}_t \quad (\text{B.25})$$

$$\log A_t = (1 - \rho_A) \log A + \rho_A \log A_{t-1} + \sigma_t^A \varepsilon_t^A \quad (\text{B.26})$$

$$\log \sigma_t^A = (1 - \rho_{\sigma^A}) \log \sigma^A + \rho_{\sigma^A} \log \sigma_{t-1}^A + \sigma^{\sigma^A} \varepsilon_t^{\sigma^A} \quad (\text{B.27})$$

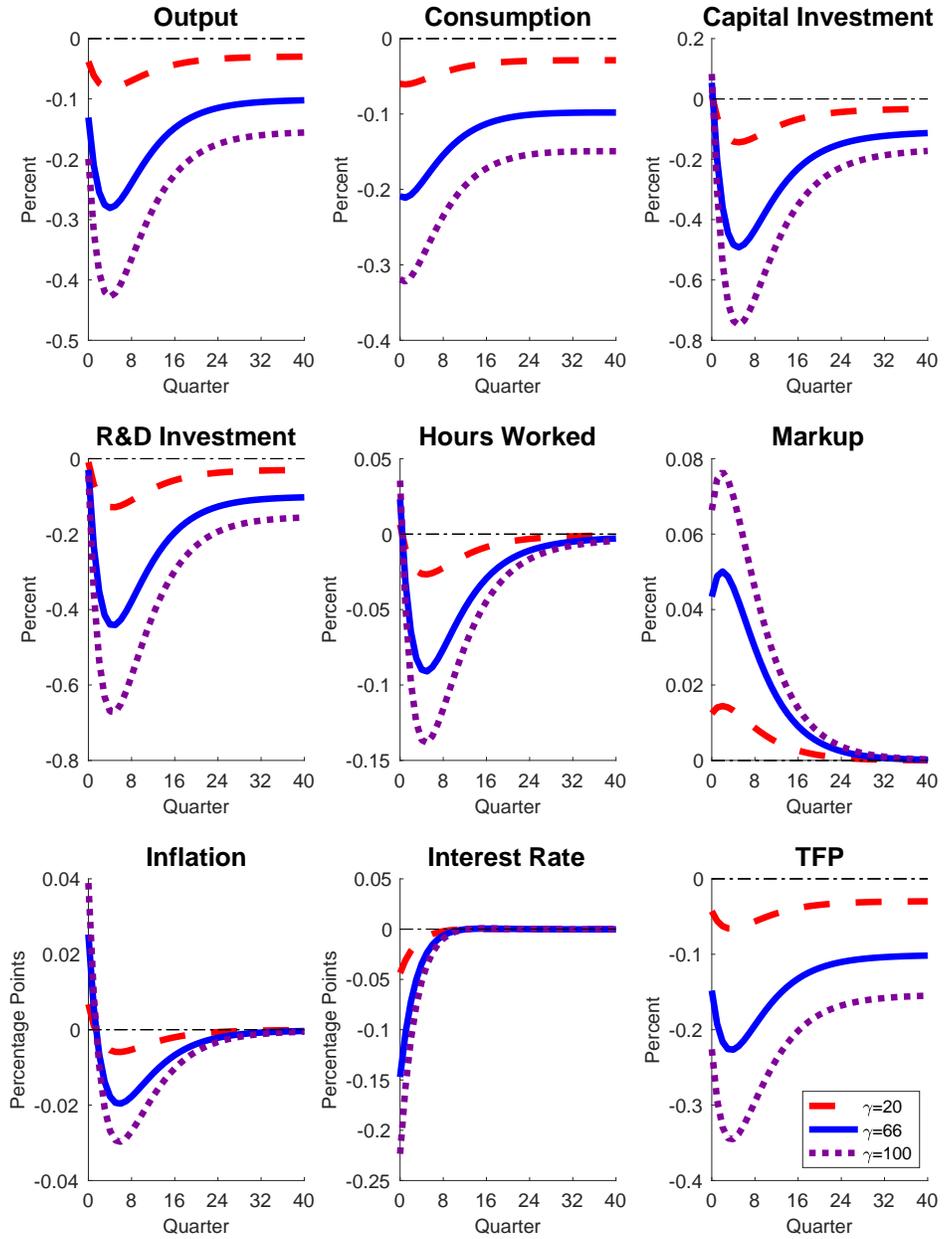


Figure B.1: Uncertainty Shock with Different Levels of Risk Aversion

Note: Inflation and Interest Rate are expressed in annualised percentage points. All other variables are in percent change.

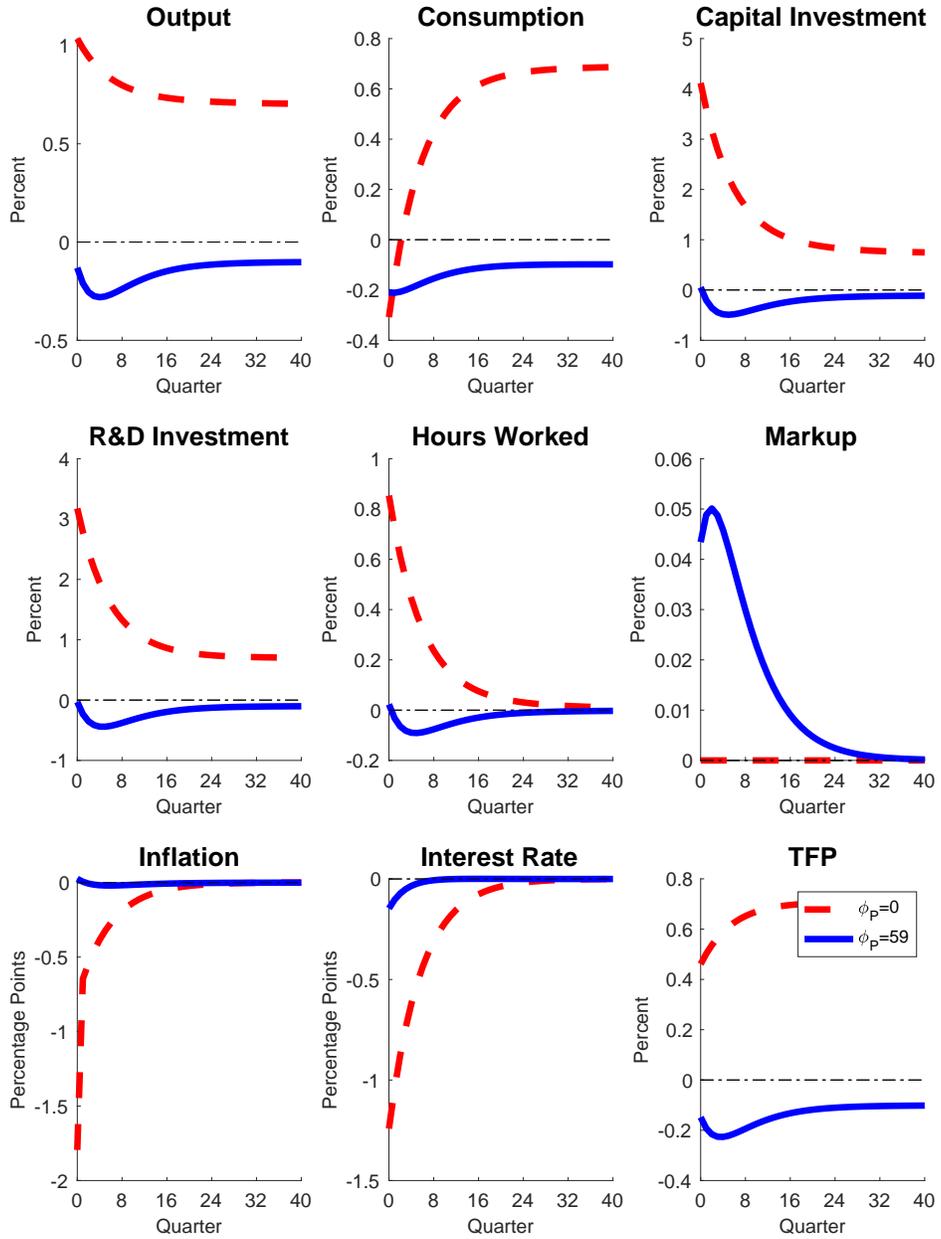


Figure B.2: Uncertainty Shock with Different Levels of Price Stickiness

Note: Inflation and Interest Rate are expressed in annualised percentage points. All other variables are in percent change.

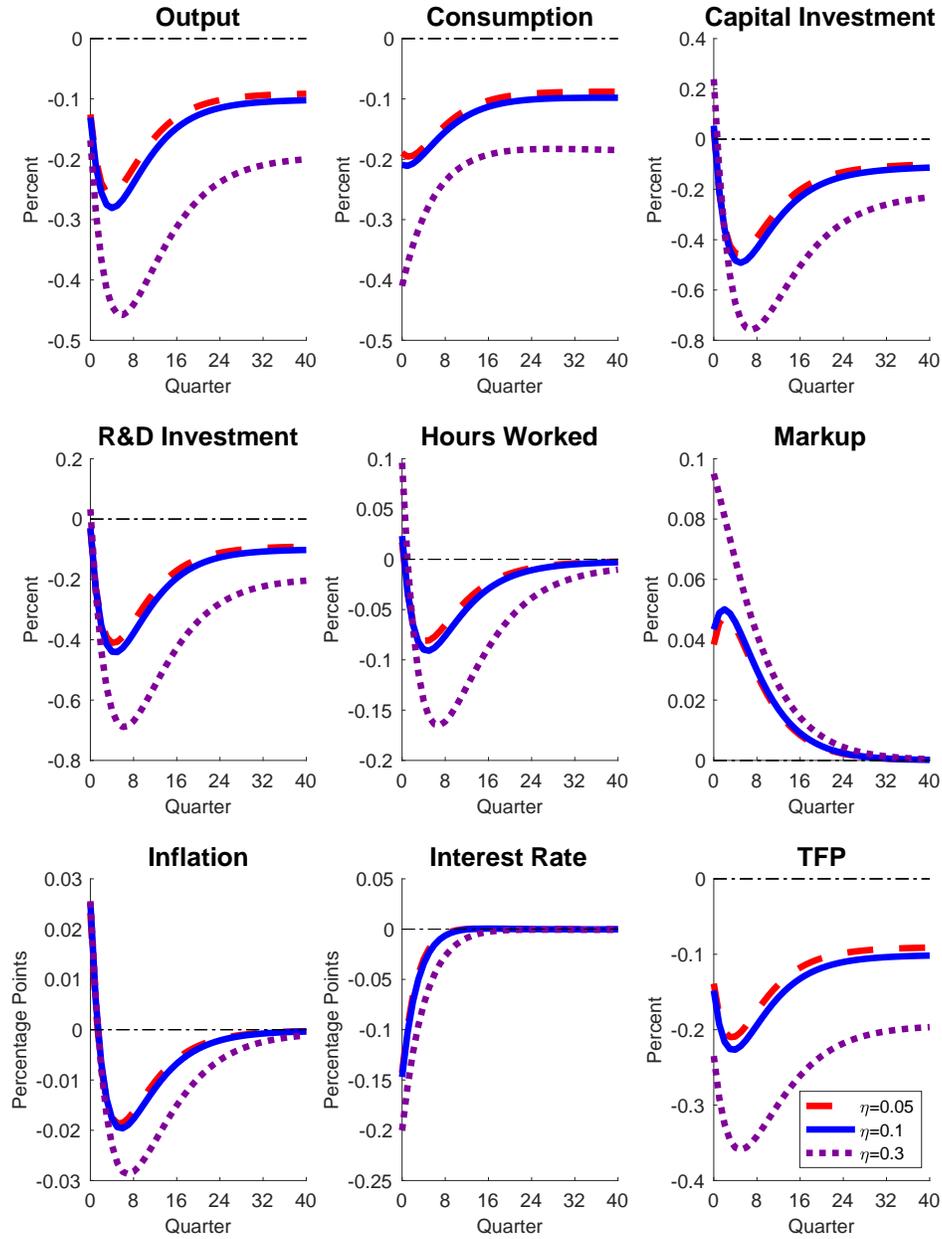


Figure B.3: Uncertainty Shock with Different Levels of Technological Spillovers

Note: Inflation and Interest Rate are expressed in annualised percentage points. All other variables are in percent change.

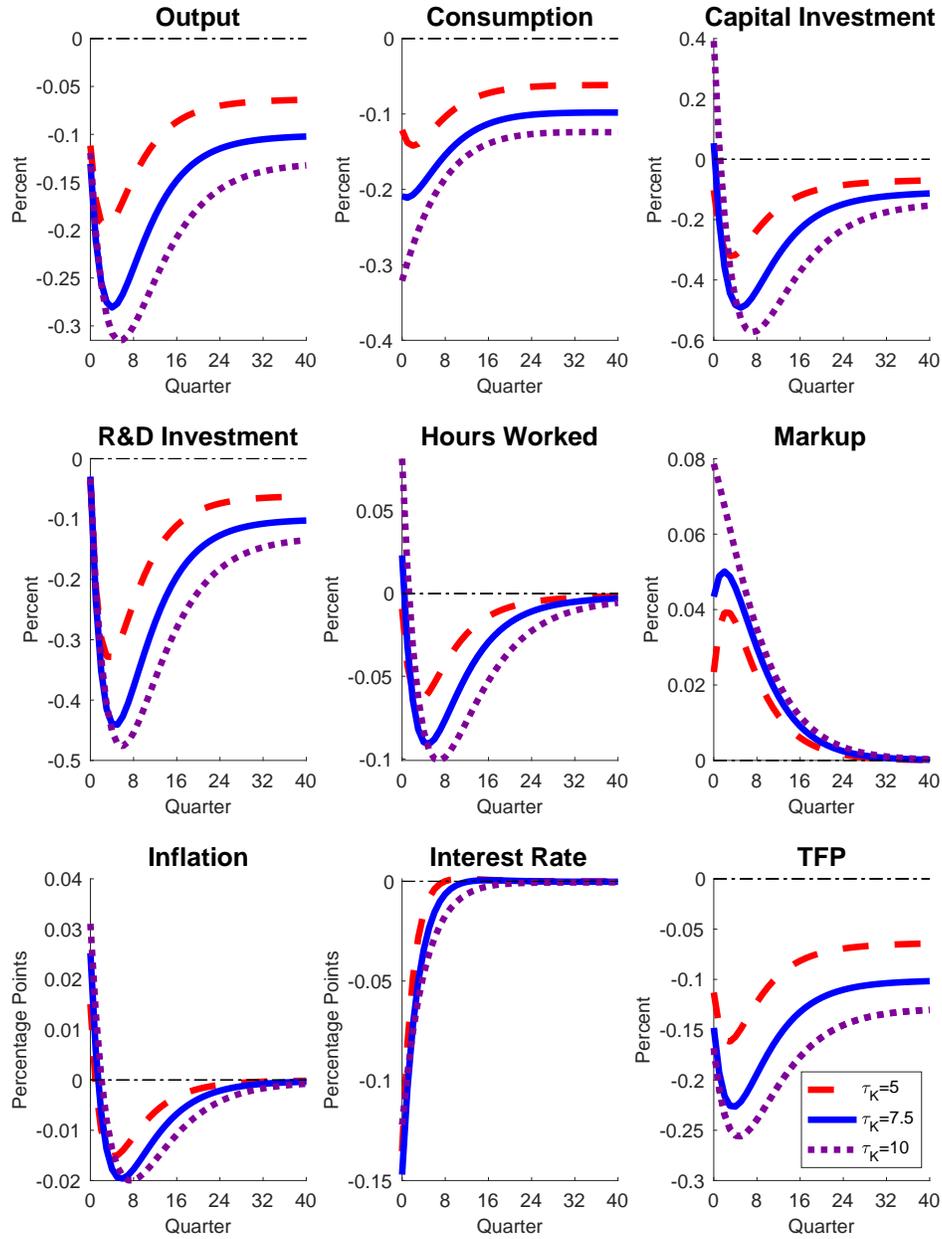


Figure B.4: Uncertainty Shock with Different Levels of Capital Adjustment Costs

Note: Inflation and Interest Rate are expressed in annualised percentage points. All other variables are in percent change.

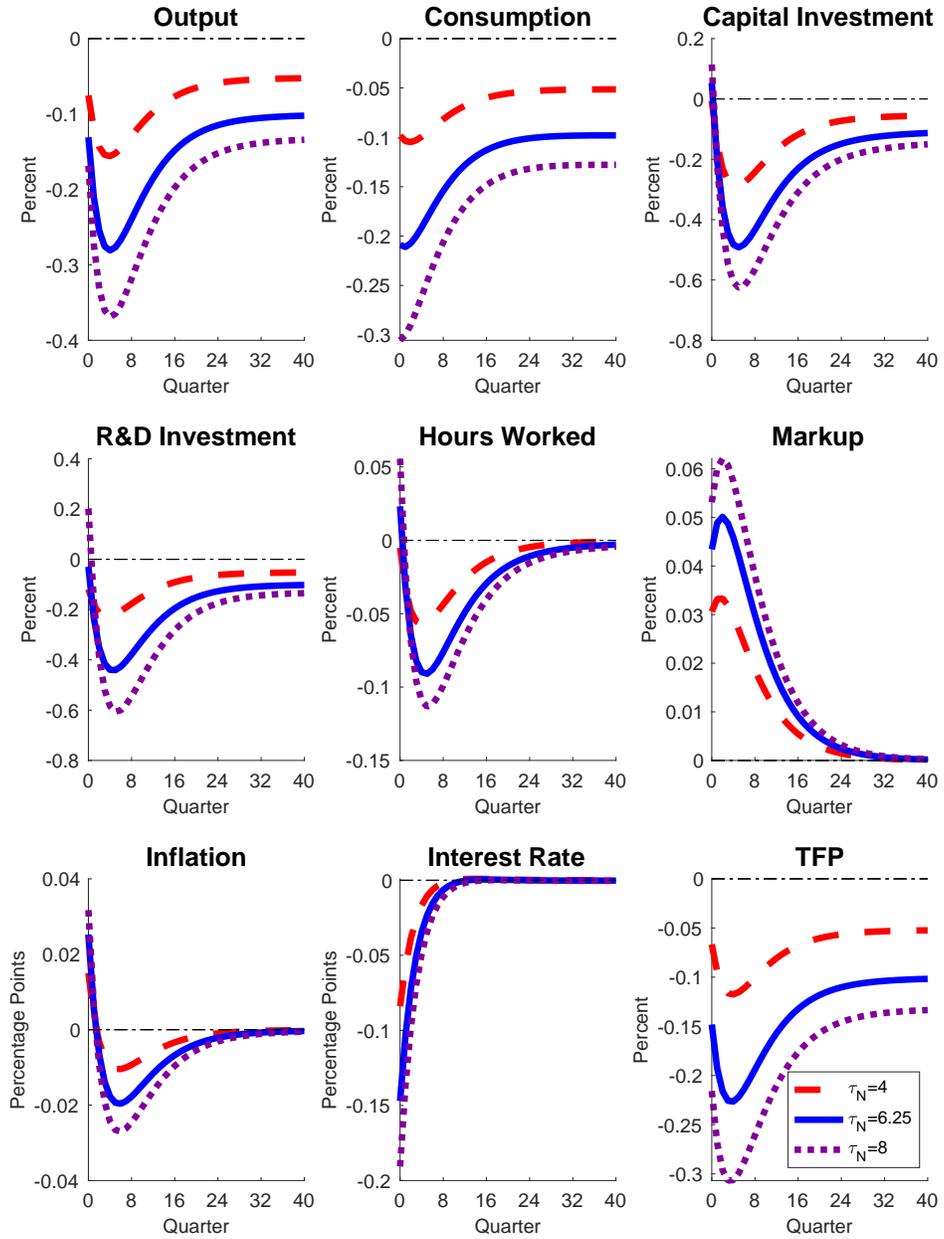


Figure B.5: Uncertainty Shock with Different Levels of R&D Adjustment Costs

Note: Inflation and Interest Rate are expressed in annualised percentage points. All other variables are in percent change.