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Abstract

'Zombie lending' occurs when a lender supports an otherwise insolvent borrower. Recent studies document that zombie lending to European firms has been widespread following the onset of the European sovereign debt crisis. This paper develops a quantitative model to study the impact of these lending practices on the dynamics and financial decisions of firms. In the model, firm liquidations and zombie lending arise endogenously. The model provides a good match to key euro-area firm statistics over the period 2011 to 2014. A key finding is that zombie lending has a substantial impact on borrowing costs, helping more low-productivity firms to survive. This, in turn, causes a drag on aggregate output, investment and productivity. These results suggest that zombie lending practices contributed to the lower output experienced by the euro area following the onset of the sovereign debt crisis.

Key words: Zombie lending, forbearance lending, zombie firms, firm defaults, firm dynamics.

JEL classification: G32, G33, L25.

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1 Introduction

In the late 1990s, around 30 per cent of Japanese firms were in receipt of forbearance lending - or so-called "zombie lending" (Caballero, Hoshi and Kashyap, 2008). These lending practices came to the fore following the 1997 financial crisis in Japan, and were accompanied by a weakened banking sector and a slow recovery. Some argue that forbearance lending was not only a by-product of economic conditions, but in fact contributed to the low output growth experienced by Japan during their lost decade (see, e.g., Peek and Rosengren, 2005, Caballero, Hoshi and Kashyap, 2008).

Fast forward almost 20 years and there are some notable similarities between the Japanese experience and the euro area following the onset of the European sovereign debt crisis and COVID-19 pandemic. Recent studies have uncovered widespread forbearance lending to European firms (see, e.g., Acharya et al., 2019, Banerjee and Hofmann, 2020, Blattner, Farinha and Rebelo, 2018, Adalet McGowan, Andrews and Millot, 2018, Schivardi, Sette and Tabellini, 2017). Weak banks have extended credit to weak firms in order to avoid the declaration of nonperforming loans on their own balance sheets (see, e.g., Acharya et al., 2019, Acharya et al., 2020*b*, Andrews and Petroulakis, 2019, Blattner, Farinha and Rebelo, 2018, Schivardi, Sette and Tabellini, 2017, Storz et al., 2017). In this paper, I study the impact of these lending practices on the financial decisions of firms, firm dynamics and economic performance. To do so, my main contribution is the development of a quantitative model that incorporates a role for forbearance lending.

The vehicle of analysis is a dynamic model of heterogeneous firms in which: (i) firms can obtain a loan from a lender to be used for production and they also have the option to default in each period; (ii) if a firm defaults, it has the option to either liquidate or to obtain loan forbearance; (iii) loan forbearance takes the form of renegotiation over an *ex post* reduction to the outstanding loan repayment, where the extent of the reduction is determined endogenously in a Nash bargaining mechanism; and (iv) lenders face information asymmetry because they observe the overall firm productivity draw but do not know precisely whether the draw belongs to a "low-quality" or "high-quality" firm.

I focus on the euro area after the onset of the European sovereign debt crisis that commenced in 2009, a period marked by high levels of forbearance lending levels. As many as 10 per cent of European firms were in receipt of subsidized bank loans in 2014 (Acharya et al., 2019). As such, the model is calibrated using euro area firm-level statistics over the period 2011 to 2014, primarily obtained from the Amadeus

Database. The calibrated model provides a good match to key firm-level statistics, including the prevalence of forbearance lending and firm defaults.

The quantitative results from the calibrated model provide insights about the characteristics and dynamics of firms in receipt of forbearance lending. Compared to their non-zombie counterparts, these firms have lower sales growth, lower productivity, higher leverage and are smaller in scale. Firms are susceptible to becoming zombies when faced with a sequence of bad productivity shocks; these firms reduce their scale in order to avoid costly equity issuance, and increase their leverage, thereby pushing up their risk of loan forbearance. These results are consistent with the empirical findings of Banerjee and Hofmann (2020).

To evaluate the quantitative impact of forbearance lending, I conduct a counterfactual exercise in which firms have the option to default but no longer have access to any form of loan forbearance. The results of my analysis can be summarized as follows. First, I show that the rate of firm liquidation is only a little higher in the counterfactual scenario with no forbearance lending as compared to the benchmark scenario with forbearance lending. This is because firms behave differently; in the absence of forbearance lending, the risk of liquidation increases because liquidation is now the only default option. In turn, borrowing costs increase, as lenders take account of this increased risk of liquidation in their loan pricing. But firms respond by taking on significantly less leverage, which results in a counterfactual liquidation rate that is little changed from the benchmark scenario.

Second, I find that the averages of firms' growth, investment rates and total factor productivity are higher in the counterfactual scenario with no forbearance lending. A key driver of this result is that there is a larger proportion of high-quality firms in the counterfactual scenario; the forbearance lending that is present in the benchmark scenario helps to prevent the creative destruction of low-quality firms. The combination of these effects results in an increase in aggregate output, investment and total factor productivity in the counterfactual scenario with no forbearance lending. These model results complement several empirical studies that find zombie firm survival can impair aggregate productivity growth and the pace of recovery following recessions (see, e.g., Andrews and Petroulakis, 2019, Blattner, Farinha and Rebelo, 2018, Caballero, Hoshi and Kashyap, 2008, Jorda et al. (2020), Adalet McGowan, Andrews and Millot 2017, Adalet McGowan, Andrews and Millot, 2018). Taken together, my results suggest that forbearance lending practices contributed to the lower output experienced by the euro area following the onset of the European sovereign debt crisis.

I also find that the vast majority of firms in receipt of forbearance lending in the benchmark scenario are low-quality. Several studies show that lower quality firms are more likely to receive some kind of loan forbearance because lenders face zombie lending incentives in order to avoid taking a balance sheet hit due to the write-off of bad loans (see, e.g., Acharya et al., 2019, Andrews and Petroulakis, 2019, Blattner, Farinha and Rebelo, 2018, Giannetti and Simonov, 2013, Schivardi, Sette and Tabellini, 2017, Storz et al., 2017). My model highlights that information asymmetry faced by lenders is another key driver. In the model, lenders do take account of firm-type risk. But because they cannot perfectly predict firm quality, borrowing costs are cheaper than otherwise for a low-quality firm, and vice versa for a high-quality firm. The end result is that credit is (mis-)allocated to low-quality firms that have higher forbearance risks.

It is important to note that the model does not incorporate a role for "zombie lending incentives" by weak banks. Rather, I outline a framework that allows for an evaluation of how zombie lending affects the dynamics of firms. In this respect, I build on existing empirical research that has focused on zombie lending incentives and the consequences of forbearance lending (see, e.g., Acharya et al., 2019, Acharya et al., 2020*a*, Acharya et al., 2020*b*, Andrews and Petroulakis, 2019, Blattner, Farinha and Rebelo, 2018, Bonfim et al., 2020, Caballero, Hoshi and Kashyap, 2008, Schivardi, Sette and Tabellini, 2017, Storz et al., 2017). Compared to those studies, which are largely based on reduced form analyses, this paper provides new insights about the impact of forbearance lending on firm behavior and their dynamics. Moreover, the structural approach facilitates counterfactual analyses to determine the impact of these lending practices.

Additionally, this paper contributes to the literature on factor misallocation, financial frictions and firm dynamics (see, e.g. Bassetto, Cagetti and De Nardi, 2015, Gopinath et al., 2017, Hsieh and Klenow, 2009, Khan and Thomas, 2013, Midrigan and Xu, 2014, Moll, 2014, Restuccia and Rogerson, 2008), which examines how differences in the efficiency of factor allocation can explain observed differences in total factor productivity and output. In these studies, financial frictions typically take the form of incentive compatibility or collateral constraints. I document new evidence about misallocation that arises due to financial frictions as well as endogenous borrowing costs generated by equilibrium default.

The paper also relates to several studies that attempt to model the influence of forbearance lending in Japan. To be more precise, Caballero, Hoshi and Kashyap (2008) outline a stylized model of entry and exit to consider the impact of zombies on firm creation and productivity. Kwon, Narita and Narita (2015), in turn, estimate the impact of zombie lending on aggregate output via a counterfactual analysis that applies the distribution of factor-input wedges estimated for non-zombies to all firms. The present study extends this research by incorporating an explicit role for firm dynamics, loan financing and firm liquidation; these are key factors that are likely to influence any analysis of forbearance lending.

Finally, the paper adds to a growing body of literature that incorporates a role for multiple firm default choices within a dynamic model of heterogeneous firms. In this context, the firm default options in my model are similar to those of Arellano, Bai and Zhang (2012), Corbae and D'Erasmo (2021), Senkal (2014) and Tamayo (2017). The firm equilibrium model in this study, as well as in Corbae and D'Erasmo (2021) and Senkal (2014), builds on other corporate finance models (see, e.g., Cooley and Quadrini, 2001, Hennessy and Whited, 2007), by incorporating endogenous entry similar to that of Hopenhayn and Rogerson (1993) as well as endogenous renegotiation similar to that of Yue (2010). Aside from the fact that I employ the model to study forbearance lending rather than financial development (see, e.g., Arellano, Bai and Zhang, 2012) or U.S. bankruptcy laws (see e.g., Corbae and D'Erasmo (2021), Senkal, 2014 and Tamayo, 2017), another key point of departure from these previous studies is that the model includes a role for information asymmetry about firm-type.[1]

The remainder of this paper is structured as follows. Section 2 outlines the model environment. Section 3 describes the recursive equilibrium of the model. Section 4 discusses the model calibration and analyzes the model equilibrium. Section 5 quantitatively evaluates the impact of forbearance lending on firm dynamics and performance. Section 6 concludes.

2 The Model Economy

2.1 Firms and technology

This section sets out a discrete-time model to study the impact of forbearance lending, which builds on several other firm equilibrium models.[2] In the model, entrepreneurs are infinitely lived. They have access to a pool of risky projects of mass

[1] The model of Tamayo (2017) features a different type of information asymmetry, with the lender being unable to observe any form of firm productivity but having the option to engage in various types of monitoring.

[2] Namely, the model and default options are comparable to those of Arellano, Bai and Zhang (2012), Corbae and D'Erasmo (2021), Senkal (2014) and Tamayo (2017).

one, which are referred to as firms. Each entrepreneur owns at most one firm. An entrepreneur that owns firm j, chooses physical capital k_{jt+1} and a new one period loan contract $(l_{jt+1}, l_{R,jt+1})$ to maximize the expected present value of all current and future dividends:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[d_{jt} \left(1 + \lambda I_{\{d_{jt}<0\}} \right) \right], \tag{1}$$

where $\beta \in (0,1)$ is the discount factor of all firms, λ is a proportional cost of equity issuance, and d_{jt} is the dividend function given by:

$$d_{jt} = y_{jt} - k_{jt+1} + (1-\delta) k_{jt} - l_{R,jt} + l_{jt+1} - \frac{\phi i_{jt}^2}{2k_{jt}} - \chi_c \tag{2}$$

In Equation (2), y_{jt} is output produced by firm j in period t, $\delta \in (0,1)$ is the depreciation rate for physical capital, $k_{jt+1} - (1-\delta) k_{jt}$ is investment i_{jt}, $l_{R,jt}$ is the repayment on the previous period's loan l_{jt}, $\phi i_{jt}^2 / (2k_{jt})$ are capital adjustment costs, and χ_c is a fixed cost of operation that firms must pay to produce. The price of output is normalized to one. Firms produce output according to a decreasing returns to scale production technology $y_{jt} = z_{jt} k_{jt}^{\alpha}$, where z_{jt} is an idiosyncratic productivity process. The probability distribution of firm j's productivity z_{jt} is conditional on the previous realization z_{jt-1} and follows a Markov process given by $f(z_{jt}, z_{jt-1})$.

I assume that firm productivity is a log-normal AR(1) process: $\ln(z_{jt}) = \mu_j (1-\varphi) + \varphi \ln(z_{jt-1}) + \epsilon$, with $|\varphi| < 1$ and $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$. There are two firm types, which differ according to the permanent productivity parameter μ_j. "Low-quality" firms have a permanent productivity μ_b and "high-quality" firms have a permanent productivity μ_g, such that $\mu_b < \mu_g$.

An idle entrepreneur will start a new firm if he receives a project opportunity. Entrants start with zero capital and debt, and decide on the future optimal capital and a future loan before they know their future productivity. Their future productivity is drawn from the stationary probability distribution $g_j(z_{jt+1})$ derived from $f_j(z_{jt+1}, z_{jt})$, for which the permanent productivity μ_j is drawn from a Bernoulli distribution with $\Pr(\mu_j = \mu_b) = 0.5$. The mass of all risky projects available to both entrants and operating firms is always one. As such, a new risky project becomes available for a new entrepreneur to start a firm when an operating entrepreneur decides to liquidate its firm.

2.2 Lenders and model timing

Competitive lenders face information asymmetry when they initially make a loan because they observe z_{jt} but not μ_j. This implies that lenders do not know with certainty whether a firm is low-quality or high-quality, and this is referred to as "firm-type risk". Lenders measure firm-type risk based on their assumed knowledge of the proportion of firms that are low-quality versus high-quality combined with information about the conditional distributions of z_{jt} given μ_j. A full description of this information asymmetry is provided in Section 3.5.

Lenders are risk-neutral and can borrow or lend as much as is needed from international capital markets at a constant interest rate $r > 0$. I denote $\Omega(k_{jt+1}, z_{jt})$ as the set of firm-specific loan schedules available to a firm with next period capital k_{jt+1} and productivity z_{jt}; each contract $(l_{jt+1}, l_{R,jt+1}) \in \Omega(k_{jt+1}, z_{jt})$ maps a current one-period loan l_{jt+1} to a next period repayment amount $l_{R,jt+1}$.

Entrepreneurs have the option to default on the loan amount owing in each period. At the beginning of each period t, the model timing and entrepreneur options are:

1. Productivity z_{jt} is realized, and the state space for incumbent firm j is $\{k_{jt}, l_{R,jt}, z_{jt}, \mu_j\}$.

2. Entrepreneurs choose from the following options for their incumbent firm j, which includes two default options:

   (a) **Continuation**. Firm j continues operating and repays the full loan amount $l_{R,jt}$. The entrepreneur chooses physical capital k_{jt+1} and a new loan contract $(l_{jt+1}, l_{R,jt+1})$.

   (b) **Liquidation**. Firm j defaults on the full loan amount owing, $l_{R,jt}$, and liquidates its assets at a firesale discount $\chi_d < 1$ where $\chi_d k_{jt}$ is the firesale firm recovery value. The entrepreneur receives the liquidation value $\max\{\chi_d k_{jt} - l_{R,jt}, 0\}$; the lender receives $\min\{\chi_d k_{jt}, l_{R,jt}\}$.

   (c) **Loan forbearance**. Firm j obtains loan forbearance, which takes the form of post-default renegotiation over the outstanding loan amount, $l_{R,jt}$. The entrepreneur bargains with the lender over the loan repayment fraction $\psi(k_{jt}, l_{R,jt}, z_{jt}, \mu_j)$ in a Nash bargaining mechanism. The repayment $\psi(k_{jt}, l_{R,jt}, z_{jt}, \mu_j)$ is restricted to the interval $[0, 1]$. When the entrepreneur and the lender agree on the repayment fraction ψ, the entrepreneur repays the reduced loan amount $\psi l_{R,jt}$ in addition to a renegotiating cost proportional to its capital stock $\chi_f k_{jt}$ in the default period.

The entrepreneur continues to operate, chooses physical capital k_{jt+1}, and has access to new loan finance in the default period.

3. **Entrance**. Idle entrepreneurs make an entry decision about whether to start a firm or not. Their initial productivity is drawn from the stationary distribution $g(z_{jt+1})$ derived from $f(z_{jt+1}, z_{jt})$, for which the permanent productivity μ_j is drawn from a Bernoulli distribution with $\Pr(\mu_j = \mu_b) = 0.5$.

Lenders offer a firm-specific loan schedule $\Omega(k_{jt+1}, z_{jt})$ such that they break even in expected value on each loan. The loan schedule incorporates firm-specific liquidation risk, forbearance risk and firm-type risk. The lender's problem is outlined formally in Section 3.5.

3 Recursive Equilibrium

In this section, I define a stationary recursive equilibrium for the model. For all variables, I drop the time and firm subscripts for ease of exposition and employ the notation: $x_{jt-1} = x^-$, $x_{jt} = x$ and $x_{jt+1} = x'$. Given the states $s = (k, l_R, z, \mu)$, the equilibrium is determined by the entrepreneur's policy rules for capital $k'(s)$, for loans $l'_R(s)$, the loan schedule $\Omega(k', z)$, and the default policy. And given any loan schedule, $\Omega(k', z)$, and loan recovery schedule, $\psi(s)$, the entrepreneur solves its firm optimization problem with perfect information about all states s. The lender, however, does not observe μ when initially issuing a loan to an entrepreneur.

3.1 The Firm's Problem

The entrepreneur chooses whether to liquidate the firm, receive loan forbearance or to repay the loan in order to maximize the present value of current and future profits, given the productivity shock z, the permanent productivity μ, the initial capital k and the loan repayment amount l_R. The corresponding value function of the firm is:

$$V(k, l_R, z, \mu) = \max_{\{c,d,f\}} \{V_c(k, l_R, z, \mu), V_d(k, l_R), V_f(k, l_R, z, \mu)\}, \tag{3}$$

where $V_c(k, l_R, z, \mu)$ is the value function if the entrepreneur does not default and continues to operate; $V_d(k, l_R)$ is the value function if the entrepreneur defaults and liquidates the firm; and $V_f(k, l_R, z, \mu)$ is the value function for forbearance lending when the entrepreneur defaults and renegotiates with the lender about the repayment fraction of the original loan.

The decision to default or repay the loan is a period-by-period decision. The value function conditional on the firm not defaulting and continuing to operate is:

$$V_c(k, l_R, z, \mu) = \max_{k', (l', l'_R) \in \Omega} d_c \left(1 + \lambda I_{\{d_c < 0\}}\right) + \beta \int_{z'} V(k', l'_R, z', \mu) f(z', z) \, dz', \tag{4}$$

where $d_c = zk^\alpha - k' + (1-\delta) k - l_R + l' - \frac{\phi i^2}{2k} - \chi_c$ are the dividends for a continuing firm. Here the entrepreneur chooses the new level of optimal k' and a new loan contract (l', l'_R) to maximize the present value of current and future dividends for the firm. The loan schedule offered for new borrowing $\Omega(k', z)$ depends on the firm's choice of k' and on the state z but not on permanent productivity μ.

When the entrepreneur defaults on the loan and liquidates the firm, the value function is:

$$V_d(k, l_R) = \max\{\chi_d k - l_R, 0\}. \tag{5}$$

If the entrepreneur decides to default and obtain loan forbearance, the firm must repay a reduced fraction $\psi(k, l_R, z, \mu)$ of the unpaid loan repayment amount l_R. The recovery rate $\psi(k, l_R, z, \mu)$ is determined endogenously in a Nash bargaining mechanism explained next in Section 3.4. In the period of default, the reduced loan is repaid and the firm continues to have access to new loan finance. This continued access to credit is akin to "evergreening", whereby further credit is extended to a troubled firm. The value function associated with a firm that defaults but repays the agreed reduced fraction of the outstanding loan is:

$$V_f(k, l_R, z, \mu) = \max_{k', (l', l'_R) \in \Omega} d_f \left(1 + \lambda I_{\{d_f < 0\}}\right) + \beta \int_{z'} V(k', l'_R, z', \mu) f(z', z) \, dz', \tag{6}$$

where $d_f = zk^\alpha - k' + (1-\delta) k - \psi(k, l_R, z, \mu) l_R + l' - \frac{\phi i^2}{2k} - \chi_c - \chi_f k$ are the dividends for a firm with forbearance lending, and ψ is the fraction of the unpaid loan repayment amount l_R.

3.2 Default Policies

An entrepreneur's default policy can be characterized by a continuing set $C(k, l_R, \mu)$, a liquidation set $D(k, l_R, \mu)$ and a forbearance lending set $F(k, l_R, \mu)$. These three sets of z's are mutually exclusive and specify when a particular default option is optimal for a given level of capital k and loan contract (l, l_R). They are respectively defined as:

$$C(k, l_R, \mu) = \{z \in Z \;:\; V_c(k, l_R, z, \mu) \geq \max\{V_d(k, l_R), V_f(k, l_R, z, \mu)\}\}, \tag{7}$$

$$D(k, l_R, \mu) = \{z \in Z \;:\; V_d(k, l_R) > \max\{V_c(k, l_R, z, \mu), V_f(k, l_R, z, \mu)\}\}, \tag{8}$$

$$F(k, l_R, \mu) = \{z \in Z \;:\; V_f(k, l_R, z, \mu) > \max\{V_c(k, l_R, z, \mu), V_d(k, l_R)\}\}. \tag{9}$$

3.3 Entrants

When an idle entrepreneur receives a project opportunity, he will attempt to start a new firm. Entrants choose their optimal capital and loan before they know their future productivity or type. Entrants' future productivity is drawn from the stationary distribution $g(z')$ derived from $f(z', z)$, for which permanent productivity μ is drawn from a Bernoulli distribution with $\Pr(\mu = \mu_b) = 0.5$. The value function associated with a potential entrant is given by:

$$V_e = \max_{k', (l', l'_R) \in \Omega} d_e \left(1 + \lambda I_{\{d_e < 0\}}\right) + \int_{z'} V(k', l'_R, z', \mu)\, g(z')\, dz', \tag{10}$$

where $d_e = -k' + l'$, and I assume that the equity issuance costs are the same as for continuing firms.

3.4 The Forbearance Problem

When a firm chooses the option to obtain loan forbearance, this default option takes the form of a bargaining game regarding the fraction $\psi(k, l_R, z, \mu)$ of the outstanding loan repayment l_R. Here the value of the defaulted loan repayment amount l_R is reduced to $\psi(k, l_R, z, \mu)\, l_R$. The value of such an agreement to the firm is equivalent to the present value of all future expected profits of forbearance when the loan recovery rate is $\psi(k, l_R, z, \mu)$, as described by Equation (6). The lender gets the value of the reduced loan repayment $\psi(k, l_R, z, \mu)\, l_R$.

The threat point of the Nash bargaining mechanism is firm liquidation, whereby the entrepreneur receives the liquidation value $V_d(k, l_R, z, \mu)$ and the lender receives the firesale recovery value $\min\{\chi_d k, l_{R,}\}$.

I let $\triangle^f\;(p; k, l_R, z, \mu)$ denote the firm's surplus in the Nash bargaining agreement, which is the difference between the value of accepting the loan recovery rate

$p = \psi(k, l_R, z, \mu)$ and the value of rejecting it, given the firm's capital k, the loan repayment amount l_R, the firm's productivity z, and the firm permanent productivity μ. The firm's surplus is:

$$\triangle^f(p; k, l_R, z, \mu) = V_f(p; k, l_R, z, \mu) - V_d(k, l_R). \tag{11}$$

The firm surplus will differ for a high-quality firm ($\mu = \mu_g$) and a low-quality firm ($\mu = \mu_b$). While the lender faces information asymmetry about firm-type when initially issuing a loan, firm-type is revealed during renegotiation from the firm's surplus. I am therefore using a Nash bargaining mechanism with perfect information to obtain the solution for the repayment fraction $\psi(k, l_R, z, \mu)$.[3]

I let $\triangle^b(p; k, l_R)$ denote the risk-neutral lender's surplus in the bargaining agreement, which is the present vale of the recovered loan repayment. The lender's surplus is:

$$\triangle^b(p; k, l_R) = pl_R - \min\{\chi_d k, l_R\} \tag{12}$$

The lender can extract loan repayments up to the full amount of a firm's cost of default when it has all of the bargaining power. Alternatively, the firm can obtain the maximum reduction of the loan, which is the difference between l_R and the recovery value for the lender, when it has all of the bargaining power. I consider the general case and assume that the borrower has bargaining power θ and the lender has bargaining power $(1-\theta)$. For any capital stock k, loan repayment amount l_R, and productivity shock z, I define the bargaining power set as $\Theta \subset [0, 1]$ for $\theta \in \Theta$ in order to ensure that the renegotiation surplus has a unique optimum.

Given the capital stock k, loan repayment l_R, and productivity shock z: the loan recovery rate $\psi(k, l_R, z, \mu)$ solves the following bargaining problem:

$$\begin{aligned} \psi(k, l_R, z, \mu) = \quad & \underset{p \in [0,1]}{\arg\max} \left[\triangle^f(p; k, l_R, z, \mu)\right]^{\theta} \left[\triangle^b(p; k, l_R)\right]^{1-\theta} \\ \text{s.t.} \quad & \triangle^f(p; k, l_R, z, \mu) \geq 0, \\ & \triangle^b(p; k, l_R) \geq 0. \end{aligned} \tag{13}$$

[3] That said, the Nash bargaining solution when there is perfect information should coincide with a Nash bargaining solution with incomplete information that maximizes the product of the expected surplus of the firm and the lender, given the lender's incomplete information, according to D'Erasmo (2011).

3.5 The Lender's Problem

When initially pricing and offering a loan schedule to a firm, I assume that the lender knows the liquidation risk and forbearance risk for all states but that it does not know μ. Thus, while the lender knows what the liquidation risk is for a low-quality firm, they do not know with certainty whether a given firm is low-quality or high-quality, and I refer to this as "firm-type risk". Lenders assess firm-type risk based on their assumed knowledge of the proportion of firms that are low-quality versus high-quality combined with information about the conditional distributions of z given μ.

More formally, a lender offers a loan schedule $\Omega(k', z)$ to maximize expected profits π_b, which are given by:

$$\begin{aligned} \pi_b(k', l'_R, z) = -l' + & \sum_{j=b,g} \rho(\mu = \mu_j | z) \\ & \times \left\{ \tfrac{1}{1+r} \left[1 - \rho_f(k', l'_R, z, \mu_j) - \rho_d(k', l'_R, z, \mu_j)\right] l'_R \right. \\ & + \tfrac{1}{1+r} \left(\rho_f(k', l'_R, z, \mu_j) \cdot \gamma(k', l'_R, z, \mu_j)\right) l'_R \\ & \left. + \tfrac{1}{1+r} \rho_d(k', l'_R, z, \mu_j) \min\{\chi_d k', l'_R\} \right\} - \xi \end{aligned} \tag{14}$$

where $\rho(\mu = \mu_j | z)$ is the probability that a firm has permanent productivity μ_j given it has observed productivity z (a measure of "firm-type risk"), $\rho_d(k', l'_R, z, \mu)$ is the expected probability of liquidation for a firm, $\rho_f(k', l'_R, z, \mu)$ is the expected probability of forbearance, and $\gamma(k', l'_R, z, \mu)$ is the expected recovery rate, given by the expected proportion of the loans that the creditors can recover, conditional on forbearance.

The lender must pay a fixed cost for each loan, ξ. The first term on the right-hand side of Equation (14) represents the resources that the lender spends today. The remaining three terms on the right-hand side comprise the expected loan repayment amount l'_R, discounted by the risk-free rate and accounting for liquidation risk, forbearance risk, and firm-type risk.

The equilibrium loan schedule $\Omega(k', z)$ comprises all loan contracts (l', l'_R) that allow the lender to break even in expected values, accounting for the liquidation risk, the forbearance risk and the firm-type risk that the lender faces, and is described by:

$$\begin{aligned} l' = \sum_{j=b,g} \rho(\mu = \mu_j | z) & \left\{ \tfrac{1}{1+r} \left[1 - \rho_f(k', l'_R, z, \mu_j) - \rho_d(k', l'_R, z, \mu_j)\right] l'_R \right. \\ & + \tfrac{1}{1+r} \left(\rho_f(k', l'_R, z, \mu_j) \cdot \gamma(k', l'_R, z, \mu_j)\right) l'_R \\ & \left. + \tfrac{1}{1+r} \rho_d(k', l'_R, z, \mu_j) \min\{\chi_d k', l'_R\} \right\} - \xi \end{aligned} \tag{15}$$

where the probability of liquidation, ρ_d, the probability of forbearance, ρ_f, the expected recovery rate, γ, and the probability that a firm has permanent productivity μ_j given z, $\rho(\mu = \mu_j|z)$, are endogenous to the model. The effective interest rate for a loan contract (l', l'_R) is $r_L = l'_R/l' - 1$. The zero profit assumption and the fact that $0 \leq \rho_d \leq 1$, $0 \leq \rho_f \leq 1$ and $0 \leq \gamma \leq 1$ imply that the gross firm effective interest rate lies in the interval $[(1+r), \infty)$.

In equilibrium, the loan schedule $\Omega(k', z)$ must be consistent with the firm's optimization and with expected zero profits of the lender, such that the loan schedule correctly assesses the probability of liquidation, the probability of forbearance of firms, and the probability of firm-type.

The first term in Equation (15) shows that both liquidation risk ρ_d and forbearance risk ρ_f have a first-order effect on the loan schedule because they enter Equation (15) linearly. And the second term shows that the loan recovery rate γ affects the debt schedule through its combined effect with forbearance risk. Additionally, the loan recovery rate has an indirect impact on the *ex ante* liquidation risk, because it affects the second moment of the loan schedule.

The firm-type risk also effects the loan schedule. In comparison to a scenario of perfect information, high-quality firms will face a higher cost of borrowing and vice versa for lower quality firms. These differences in the borrowing costs, driven by the lenders inability to perfectly a firm's permanent productivity, will in turn influence the decision rules of firms.

3.5.1 Liquidation Risk

The liquidation probabilities $\rho_d(k', l'_R, z, \mu)$ are related to the liquidation sets $D(k', l'_R, \mu)$ as follows:

$$\rho_d(k', l'_R, z, \mu) = \int_{D(k', l'_R, \mu)} f(z', z)\, dz'. \tag{16}$$

This implies that when the liquidation sets are empty, $D(k', l'_R, \mu) = \emptyset$, the equilibrium liquidation probabilities $\rho_d(k', l'_R, z, \mu)$ equal zero. The liquidation probabilities $\rho_d(k', l'_R, z, \mu)$ equal one when $D(k', l'_R, \mu) = Z$.

3.5.2 Forbearance Risk

Similarly, the forbearance lending probabilities $\rho_f(k', l'_R, z, \mu)$ are related to the forbearance lending sets $F(k', l'_R, \mu)$ as follows:

$$\rho_f(k', l'_R, z, \mu) = \int_{F(k', l'_R, \mu)} f(z', z)\, dz'. \tag{17}$$

This similarly implies that when forbearance sets are empty, $F(k', l'_R, \mu) = \emptyset$, the equilibrium forbearance probabilities $\rho_f(k', l'_R, z, \mu)$ equal zero. The forbearance probabilities $\rho_f(k', l'_R, z, \mu)$ equal one when $F(k', l'_R, \mu) = Z$.

The expected recovery rate $\gamma(k', l'_R, z, \mu)$ is determined by:

$$\gamma(k', l'_R, z, \mu) = \frac{\int_{F(k', l'_R, \mu)} \psi(s') f(z', z)\, dz'}{\rho_f(k', l'_R, z, \mu)} \tag{18}$$

The numerator of Equation (18) is the proportion of the loan that the lender can expect to recover. The denominator of Equation (18) is the forbearance probability.

3.5.3 Firm-type Risk

I now consider the firm-type risk, which quantifies the information asymmetry faced by the lender. While lenders cannot observe μ, I assume they know: (a) the proportion of firms are low-quality versus high-quality, i.e. $\rho(\mu = \mu_b)$ and $\rho(\mu = \mu_g)$; and (b) they know the conditional distribution of z for a given μ.[4] As such, the lender can determine the probability that a firm has permanent productivity μ_b given z, $\rho(\mu = \mu_b | z)$, by using Bayes' Rule:

$$\rho(\mu = \mu_b | z) = \frac{\rho(z|\mu=\mu_b)\rho(\mu=\mu_b)}{\rho(z|\mu=\mu_b)\rho(\mu=\mu_b) + \rho(z|\mu=\mu_g)\rho(\mu=\mu_g)} \tag{19}$$

where $\rho(\mu = \mu_b)$ in Equation (19) is the proportion of low-quality firms, given by the cross-sectional stationary distributions of firms $\Gamma(s)$ such that:

$$\rho(\mu = \mu_b) = \int \Gamma(k, l_R, z; \mu = \mu_b)\, d(k \times l_R \times z). \tag{20}$$

[4] I denote to the proportion of firms that are low quality, as determined by the cross-sectional distribution of firms, $\rho(\mu = \mu_b)$. This differs from the aforementioned initial probability that a firm is low-quality, $\Pr(\mu = \mu_b) = 0.5$, which is applies to new entrants.

In Equation (19), $\rho(z|\mu = \mu_b)$ is the probability that the observed z is from a log normal distribution with mean μ_b. The initial firm productivity process is distributed such that $\ln(z) \sim \mathcal{N}(\mu, \sigma_\varphi^2)$ for $\sigma_\varphi^2 = \frac{\sigma_\epsilon^2}{1-\varphi^2}$. This initial process can be used to determine the initial conditional probabilities. But the conditional distribution for z is subsequently given by information from the cross-sectional stationary distributions of firms $\Gamma(s)$ and the assumption that the firm productivity process is log-normally distributed. The conditional distribution for z is approximated as follows. First, the mean of the firm productivity process for $\ln(z)$ given $\mu = \mu_b$ as:

$$E(\ln(z)|\mu = \mu_b) = \sum_i \ln(z_i) p_i \tag{21}$$

where the probability of a given z_i is $p_i = \frac{\int \Gamma(k, l_R, z_i, \mu=\mu_b) d(k \times l_R)}{\rho(\mu=\mu_b)}$.[5] The variance of the process is:

$$Var(\ln(z)|\mu = \mu_b) = \sum_i (\ln(z_i) - E(\ln(z)|\mu = \mu_b))^2 p_i. \tag{22}$$

Combined with the initial assumptions about the firm productivity process being log-normal, these results imply that the *conditional* distribution of the firm productivity process is $\ln(z)|\mu = \mu_b \sim \mathcal{N}(E(\ln(z)|\mu = \mu_b), Var(\ln(z)|\mu = \mu_b))$. And so I compute $\rho(\ln(z)|\mu = \mu_b)$ as:

$$\rho(z|\mu = \mu_b) = \rho(\ln(z)|\mu = \mu_b) = 2 \times \Phi\left(-\left|\frac{\ln(z) - E(\ln(z)|\mu = \mu_b)}{\sqrt{Var(\ln(z)|\mu = \mu_b)}}\right|\right) \tag{23}$$

where Φ is the standard normal cumulative distribution function. Expressions for $\rho(\mu = \mu_g)$, $\rho(z|\mu = \mu_g)$ and $\rho(\mu = \mu_g|z)$ are obtained similarly. In equilibrium, the probabilities $\rho(\mu = \mu_b|z)$ are consistent with the firm's decision rules, because they are based on the cross-sectional stationary distribution of firms $\Gamma(s)$, which is determined by the firm's decision rules. This means that the lender can price the firm-type risk such that the loan schedules are consistent with expected zero profits.

[5] In theory, $\ln(z)$ is a continuous random variable. Here I compute the empirical conditional mean and variance based on the discretized shock process, and so are using standard methods normally applied to compute the mean and variance of a discrete random variable.

3.6 The Cross-Sectional Distribution of Firms

The law of motion for the cross-sectional distribution of firms $\Gamma(s)$ is:

$$\begin{aligned}\Gamma(s') = & \int \rho_d(s) Q_e(s') g(z') \Gamma(s) d(k \times l_R \times z \times \mu) \\ & + \int_{C(k',l'_R,\mu)} (1-\rho_d(s)) Q_c(s,s') f(z',z) \Gamma(s) d(k \times l_R \times z \times \mu) \\ & + \int_{F(k',l'_R,\mu)} (1-\rho_d(s)) Q_f(s,s') f(z',z) \Gamma(s) d(k \times l_R \times z \times \mu)\end{aligned} \tag{24}$$

where $C(k', l'_R, \mu)$ is the continuing set, $F(k', l'_R, \mu)$ is the forbearance lending set, $\rho_d(k', l'_R, z, \mu)$ is the liquidation probability, and $Q_c(.)$ denotes a transition function for continuing firms, mapping the current states into future states, given by:

$$Q_c(s,s') = \begin{cases} 1 & \text{if } l'_R(s) = l'_R, k'(s) = k' \\ 0 & \text{otherwise} \end{cases} \tag{25}$$

and $l'_R(s)$ and $k'(s)$ are the optimal policy rules for firms. Similarly, $Q_f(s')$ is 1 if the optimal choice for forbearance lending firms is (k', l'_R) and 0 otherwise. And $Q_e(s')$ is 1 if entrants optimal choice is (k', l'_R) and 0 otherwise.

If an existing firm liquidates, a new project z drawn from the stationary distribution $g(z)$ becomes available to an entrant entrepreneur such that the mass of risky projects is equal to one.

3.7 Stationary Recursive Equilibrium

Definition 1. The stationary recursive equilibrium for this economy, where $s = \{k, l_R, z, \mu\}$ denotes the aggregate states and $V^*(s)$ is the firm's value function, is a set of policy rules for (i) capital holdings $k'^*(s)$, loan contracts $(l'^*(s), l'^*_R(s))$, and the default policy, where the default policy is comprised by the operating sets $C^*(k, l_R, \mu)$, liquidation sets $D^*(k, l_R, \mu)$, and forbearance lending sets $F^*(k, l_R, \mu)$; (ii) the recovery rate $\psi^*(s)$; (iii) the loan schedules $\Omega^*(k', z)$ for the loan contracts; and (iv) the cross-sectional distribution of firms $\Gamma^*(s)$ such that:

1. Given the loan schedule $\Omega^*(k', z)$ and the loan recovery rate $\psi^*(s)$: the value function $V^*(s)$, the policy rules $k'^*(s)$ and $(l'^*(s), l'^*_R(s))$, operating sets $C^*(k, l_R, \mu)$, liquidation sets $D^*(k, l_R, \mu)$, and forbearance lending sets $F^*(k, l_R, \mu)$ are consistent with the firm's optimization problem in Equation(3).

2. Given the loan schedule $\Omega^*(k', z)$ and value function $V^*(s)$: the loan recovery rate $\psi^*(s)$ solves the forbearance problem in Equation (13).

3. Given the loan recovery rate $\psi^*(s)$: the equilibrium loan schedules $\Omega^*(k', z)$ satisfy the lender's expected zero profit condition, and also the liquidation probability $\rho_d^*(k', l_R', z, \mu)$, the forbearance probability $\rho_F^*(k', l_R', z, \mu)$, the expected recovery rate $\gamma^*(k', l_R', z, \mu)$, and the probability that a firm has permanent productivity μ_j given it has observed productivity z $\rho^*(\mu = \mu_j | z)$ are consistent with the firm's default policy and renegotiation agreement.

4. The cross-sectional distribution of firms $\Gamma^*(s)$ is a stationary measure of firms consistent with the firm decision rules and the law of motion for the stochastic variables.

4 Quantitative Analysis

4.1 Calibration

To consider the quantitative implications of the model, I set a model period to one year and calibrate the model to match euro-area data over the period 2011 to 2014. During this period, there was a high prevalence of forbearance lending (Acharya et al., 2019) and low output growth. Table 1 provides a summary of definitions used to compute key variables from the model.

I assume that firm productivity is a log-normal AR(1) process: $\ln(z_{jt}) = \mu_j(1-\varphi) + \varphi \ln(z_{t-1}) + \epsilon$, with $|\varphi| < 1$ and $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$. And that there are two firm types, low-quality firms and high-quality firms, that differ by the permanent productivity parameter μ_j. I use the procedure of Tauchen (1986) to discretize the stochastic shock into a 10-state Markov chain.

The model comprises 15 parameters to be calibrated, which are summarized in

Table 1: Model Definitions for Key Variables

Variable	*Model Definition*
Assets	k
Loan repayment	l_R
Leverage	l_R/k
Loan interest rate	$l_R'/l' - 1$
Investment rate	$(k' - (1-\delta)k)/k^-$
Sales	zk^α
Sales growth	$\left(zk^\alpha - z^-(k^-)^\alpha\right)/z^-(k^-)^\alpha$
Dividend ratio	$I_{\{d>0\}}d/k^-$
Working capital	$(k - l_R)/k^-$

Table 2: Benchmark Parameters

Calibrated Parameters		*Value*
Firm discount rate	β	0.96
Risk-free interest rate	r	0.01
Capital depreciation rate	δ	0.16
Returns to scale	α	0.65
Equity issuance cost	λ	0.30
Capital loss in liquidation	χ_d	0.40
Firm bargaining power	θ	0.40
Autocorrelation of stochastic shock	φ	0.9
Standard deviation of stochastic shock	σ_ϵ	0.118
Low-quality firm permanent productivity	μ_b	0
High-quality firm permanent productivity	μ_g	0.448
Parameters Estimated with SMM		*Value*
Capital adjustment cost	ϕ	0.15
Lender credit cost	ξ	7.00
Fixed operating cost	χ_c	10.75
Renegotiation cost	χ_f	0.20

Table 2. To start with, I select the 11 "Calibrated Parameters" in Table 2 independently of the model equilibrium. The firm discount rate β is set to 0.96, which is standard for an annual RBC model (Arellano, Bai and Zhang, 2012). The risk-free interest rate r is set to 0.01, which is equal to the average interest rate of one-year euro-area government bonds for the period 2011 to 2014. The returns to scale parameter α is set to 0.65, which implies decreasing returns to scale in production, similar as in the Arellano, Bai and Zhang (2012) study based on European firms. The equity issuance cost λ is set to 0.30, following Cooley and Quadrini (2001). The capital loss in liquidation χ_d and the firm bargaining power are both set to 0.40, which coincides with the estimates of Ramey and Shapiro (2001) for the capital loss in liquidation. The autocorrelation φ and standard deviation σ of the stochastic shock are set to 0.90 and 0.118, respectively, which follow **?**.

Next, I use a sample of euro-area firms over the period 2011 to 2014 to estimate several other parameters. A full description of these data is contained in Appendix B. The capital depreciation rate δ is set to 16 per cent per year, which equals the average depreciation rate for the sample. To set the permanent productivity shocks, μ_b and μ_g, I divide the sample into low-quality firms and high-quality firms based on the definition of Acharya et al. (2019). This involves computing for each firm the average interest coverage ratio over the period 2009 to 2011, defined as the ratio of interest expenses to operating income (EBITDA). A firm is categorized as low-quality if its three-year average interest coverage ratio is below the country median and otherwise is considered high-quality. Using these categories, I find the average yearly return on assets for high-quality firms to be around 57 per cent higher than for low-quality firms. Thus, I set μ_b to zero and μ_g to 0.448, which ensures that the average productivity for high-quality firms is around 57 per cent higher than for low-quality firms for the specific productivity process.

Subsequently, I estimate the remaining four parameters in Table 2 by using the simulated method of moments (SMM) to target a set of five moment conditions from the data. More precisely:

$$\hat{\Theta} = \arg\min_{\Theta} \left[m^d - m^s\left(\Theta\right)\right]' W \left[m^d - m^s\left(\Theta\right)\right] \tag{26}$$

where W is a weighting matrix[6], $\Theta = (\phi, \xi, \chi_c, \chi_r)$ are the four parameters to be estimated, m^d are the five targeted moments from the data summarized in the "Data" column of Table 3 and discussed below, and $m^s\left(\Theta\right)$ are the five simulated moments from the model at parameters Θ.

The first target moment is the firm liquidation rate. Standard & Poor's Rating Services (2014, 2016) provide an estimate for the yearly average of this moment for European corporates, which is 3.44 per cent.

The second target moment is the firm forbearance rate. The yearly average value of this moment over the period 2011 to 2014 is approximately 7.5 per cent, according to the estimates of Acharya et al. (2019). These estimates are based on the asset-weighted proportion of European firms that are classified as "zombie firms". Their sample of firms comprises all European privately and publicly traded firms.

The remaining three target moments are: the average and standard deviation of sales growth, and the average of leverage. Appendix B describes how I compute these moments for the sample of euro-area firms.

All of the estimated parameters affect all of the target moments in the model, although some parameters affect some moments more directly. In particular, χ_c is useful to match the firm liquidation rate; χ_f is useful to match the firm forbearance lending rate; ϕ and χ_c is useful to match the average and standard deviation of sales growth; and ξ is useful to match average leverage.

I assess the quantitative performance of the model in Table 3 by comparing the moments from the euro-area data in the "Data" column with those produced by the model in the "Benchmark" column. Overall, the simulated method of moments

[6] I employ the identity matrix as the weighting matrix.

Table 3: Statistical Moments for Euro-Area Data and the Benchmark Model

Targeted Moments	*Data*	*Benchmark*
Firm liquidation rate	3.44%	4.45%
Firm forbearance rate	7.5%	6.79%
Average of sales growth	5.86%	2.00%
Standard deviation of sales growth	16.62%	19.54%
Average of leverage	66.46%	68.69%
Non-targeted Moments	*Data*	*Benchmark*
Average of investment rate	30.47%	18.98%
Average of dividend ratio	3.48%	5.89%
Average of working capital	23.73%	29.99%

Note: The column under Data reports the statistical moments for the euro area, from 2011 to 2014. The column under Benchmark reports the model statistics under the benchmark calibration. All statistical moments are calculated over a one-year period.

estimates a set of four parameter values that produce model moments that are close to the five targeted moments, although it under-predicts the average of sales growth. I also consider how the simulated model can match several non-targeted moments in Table 3. The model underestimates the investment rate and slightly overestimates working capital.

4.2 Loan Schedules and Firm Decisions

Prior to a discussion of the quantitative results, I first provide an overview of how liquidation risk and forbearance lending risk affect firms' loan schedules, and how these subsequently influence firms' choices about future loans and capital. I first consider how firm liquidation risk varies with firms' capital choice k' in Figure 1. Liquidation risk is higher for low-quality firms than for high-quality firms (Panel A). It is also higher for firms with less capital k, and for firms facing lower stochastic shocks (Panel B). In all three instances, the liquidation risk is higher because liquidation is more valuable.

I next consider how the firm forbearance lending risk varies as a function of the loan repayment choice l'_R in Figure 2. The forbearance lending risk is higher for firms with lower levels of physical capital, as well as for firms with larger loan repayment choices (Panel A). It is also higher for low-quality firms than for higher quality firms (Panel B) because high-quality firms are less likely to choose forbearance lending, even when the loan renegotiation generates some loan reduction. And although not depicted here, the forbearance lending risk is higher for firms facing lower stochastic shocks. As with liquidation, the forbearance lending option is more valuable for low-quality firms, and for firms subject to lower stochastic shocks, as well as for firms with larger repayment choices.

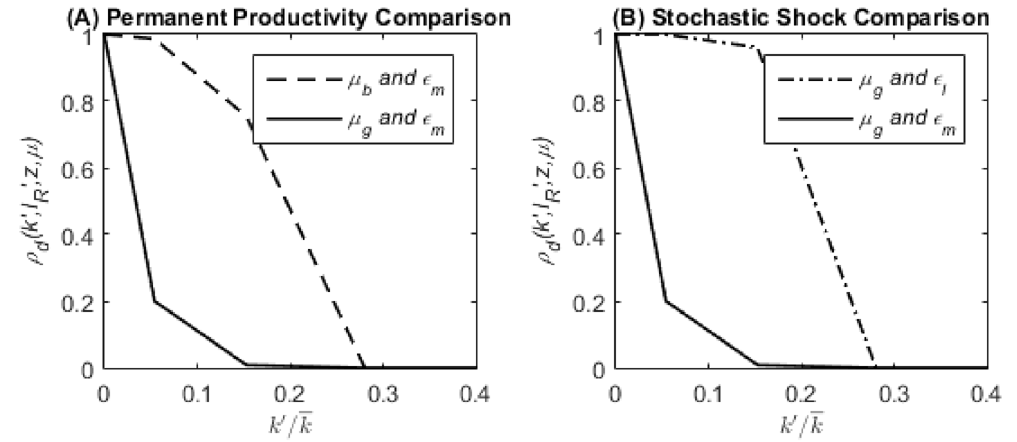

Figure 1: Liquidation Probabilities

Note: Both plots illustrate the liquidation probabilities as a function of the capital choice k' scaled by the average equilibrium capital $\bar{k}$. Panel A depicts two forms , one "high-quality" firm ($\mu = \mu_g$) and one "low-quality" firm ($\mu = \mu_b$), when there is a normal stochastic shock ε_m and the loan repayment choice l_R' equals the average equilibrium choice for all firms. Panel B depicts two firms, one with a low stochastic shock ε_l and one with a normal stochastic shock ε_m, when both firms are high-quality and the loan repayment choice l_R' equals the average equilibrium choice for all firms.

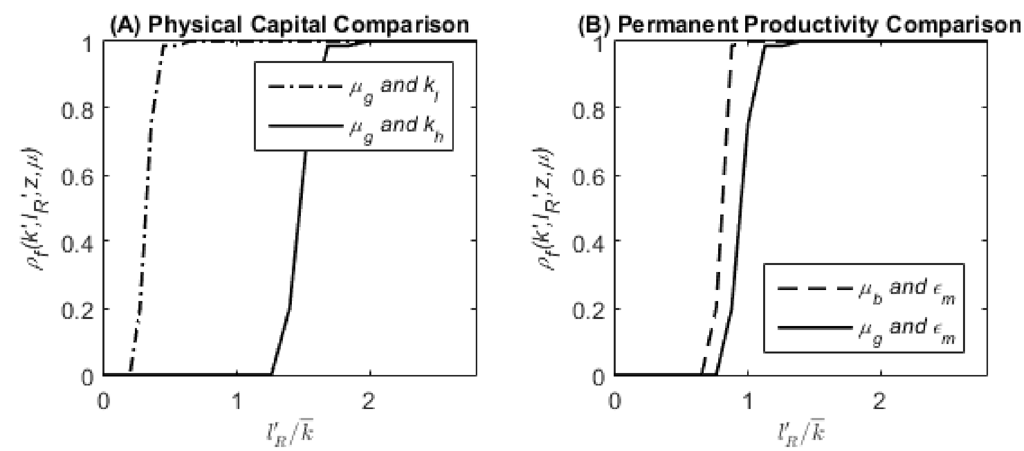

Figure 2: Forbearance Lending Probabilities

Note: Both plots illustrate the forbearance lending probabilities as a function of the loan repayment choice l_R' scaled by the average equilibrium capital $\bar{k}$. Panel A depicts two firms, one with low capital k_l and high capital k_h, where both firms are high-quality ($\mu = \mu_g$), face a normal stochastic shock ε_m and the capital choice k' is equal to the average equilibrium capital, $\bar{k}$. Panel B depicts two firms, one high-quality firm ($\mu = \mu_g$) and one low-quality firm ($\mu = \mu_b$), when there is a normal stochastic shock ε_m and the capital choice k' equals the median value.

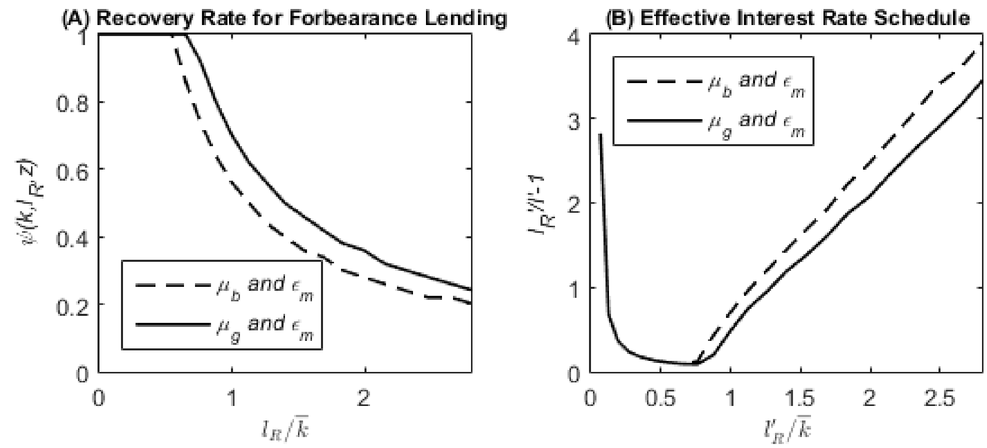

Figure 3: Recovery Rate and Effective Interest Rate Schedule

Note: Panel A plots the equilibrium recovery rates as a function of the loan repayment choice l'_R scaled by the average equilibrium capital $\bar{k}$. Panel B plots the effective interest rate $(l'_R/l' - 1)$ for every contract (l', l'_R) as a function of the loan repayment choice l'_R scaled by the average equilibrium capital $\bar{k}$. Both plots depict two firms, one high-quality firm $(\mu = \mu_g)$ and one low-quality firm $(\mu = \mu_b)$, when there is a normal stochastic shock ε_m and the capital choice k' is equal to the average equilibrium capital $\bar{k}$.

Prior to a consideration of the loan schedules, I consider the loan recovery rates for forbearance lending. Panel A of Figure 3 demonstrates that a firm with a small loan repayment will receive no reduction to its loan. But the loan recovery rate decreases with the loan repayment size. The loan recovery rate is also higher for high-quality firms than for low-quality firms, all else equal. Similarly, and although not depicted here, the loan recovery rate is higher for firms with a more favorable stochastic shock. More generally, a larger loan reduction can increase a firm's *ex ante* forbearance incentives. But the lender anticipates this when pricing the loan schedule, which in turn offsets forbearance incentives.

I now consider the equilibrium loan schedules that arise due to liquidation risk, forbearance risk and firm-type risk. Panel B of Figure 3 illustrates that low-quality firms attract higher interest rates than high-quality firms. This is because they face both higher liquidation risk and forbearance lending risk than high-quality firms. Additionally, the effective interest rate increases with loan repayment size. But small loans also face higher effective interest rates due to the fixed cost of lending.

To consider the firm dynamics of the model, I turn to the policy rules. Panel A of Figure 4 shows that a firm with higher initial capital will choose a larger future capital stock and a smaller future loan. The smaller loan choice is due to a precautionary motive; the firm does not find it optimal to fully utilize its borrowing opportunities because large loans increase liquidation risk and forbearance risk, as well as the risk of

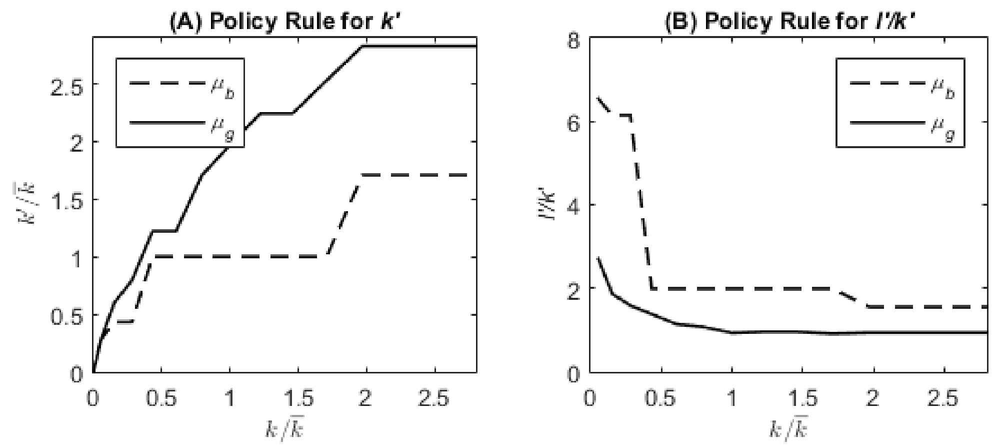

Figure 4: Policy Rules

Note: Panel A plots the optimal capital choice k' as a function of initial capital k. Panel B plots the optimal loan choice relative to the capital choice l'/k' as a function of initial capital k. Both plots depict two firms, one high-quality firm ($\mu = \mu_g$) and one low-quality firm ($\mu = \mu_b$), when there is a high stochastic shock ε_h and the loan repayment l_R equals the average equilibrium choice for all firms. The horizontal axes and vertical axis of Panel A are scaled by the average equilibrium capital $\bar{k}$.

costly equity issuance (Arellano, Bai and Zhang, 2012). With lower levels of capital, the firm decreases its capital and increases its leverage (Panel B).

Loan contracts also influence the way in which firms respond to stochastic shocks. Arellano, Bai and Zhang (2012) note that when firms experience a sequence of low shocks, they will reduce their size in order to avoid equity issuance costs, and increase their loan financing, thus pushing up their leverage and their effective interest rate schedules. And visa versa for when firms experience a sequence of good shocks. Low-quality firms also tend to be smaller and more levered than high-quality firms because their liquidation risk is higher, and so they have more incentives to reduce scale in order to avoid equity issuance costs.

Panel B of Figure 4 shows that high-quality firms tend to be less levered than low-quality firms. In turn, lower leverage also reduces the liquidation risk and forbearance risk of high-quality firms.

4.3 Benchmark Model Results

I now examine the quantitative results for the benchmark model, organized by firm operating status, as reported in Table 4. All model moments are computed from a simulation of the model economy with 1,000 firms over 500 periods, having disregarded an initial 250 periods as a model burn-in.

I first consider the continuing firms in Column (2) of Table 4, which includes

Table 4: Quantitative Model Results by Operating Status

	(1)	(2)	(3)	(4)	(5)
	All Firms	*Continuing*	*Forbearance*	*Liquidating*	*Entrants*
Average of:					
Sales growth	2.00 %	1.07 %	-0.74 %	-10.75 %	15.18 %
Leverage	68.69 %	12.37 %	770.34%	250.86 %	149.26 %
Investment rate	18.98 %	17.65 %	14.17%	-38.82 %	38.40 %
Capital	126.52	139.32	64.42	43.26	27.62
Loan repayment	54.06	40.29	268.11	59.28	38.78
Firm age	46.72	52.89	12.76	22.47	2.37
TFP	1.74	1.79	1.41	0.98	1.63
Proportion of firms:					
In given category	100.00 %	88.77 %	6.79 %	4.45 %	4.45 %
High-quality μ_g	90.28 %	99.65 %	0.13 %	50.02 %	54.69 %

Note: Column (1) reports model statistics for all firms operating with age greater than one. Column (2) reports model statistics for continuing firms, defined as any firm that has existed for more than three periods and does not currently receive loan forbearance. Column (3) reports model statistics for forbearance firms, defined as any firm that has existed for more than three periods and currently receives loan forbearance. Column (4) reports model statistics for liquidating firms, based on the firm statistics in the period preceding firm liquidation. Column (5) reports model statistics for entrant firms, defined as any firm that has existed two periods or less. All model statistics are for the benchmark calibration, in which firms have access to forbearance lending.

all firms that have existed for more than three periods and currently do not receive loan forbearance. Almost all continuing firms are high-quality firms. One reason for this is because high-quality firms are more likely to have higher overall productivity, which makes these firms better able to make a successful entry. Continuing firms also tend to be larger with lower leverage due to precautionary motives, whereby these firms do not find it optimal to exhaust all borrowing opportunities due to the associated increase in liquidation risk and forbearance risk.

I next consider the firms in receipt of forbearance lending in Column (3) of Table 4, which includes all firms that have existed for more than three periods and currently receive loan forbearance. Only 0.13 per cent of firms that receive forbearance lending are high-quality firms. The reason that the majority of firms in receipt of forbearance lending are low-quality relates to the information asymmetry. Specifically, low-quality firms face a higher effective interest rate schedule than high-quality firms due to their higher liquidation and forbearance risk, and lower recovery rates (Figure 3). But the difference between the interest rate schedules of the high-quality firms and low-quality firms is not very large because the lender cannot perfectly predict whether a given firm is high-quality or low-quality, and must instead assign a firm-type probability. The end result is that the interest rate schedule is more favorable for low-quality firms than it would be under perfect information, which makes forbearance lending *ex ante* a more valuable choice for low-quality firms.

The fact low-quality firms have lower overall productivity, on average, also increases their likelihood of receiving loan forbearance. This is because firms facing lower overall productivity, due to a bad stochastic shock or being low-quality or both, are more likely to opt for forbearance lending arrangements as the "continuing to operate" option is less valuable to them. The low overall productivity is consistent with the high level of leverage observed for the forbearance firms; when faced with a sequence of bad productivity shocks, firms will reduce their scale in order to avoid costly equity issuance, and increase their loan financing, thereby pushing up their leverage and effective interest rates. These model results are consistent with Banerjee and Hofmann (2020) who empirically show that in the years before a firm becomes a zombie, they increase leverage and equity issuance to stay afloat. Additionally, the model results are consistent with Goto and Wilbur (2019) and Bargagli-Dtoffi, Riccaboni and Rungi (2020) who empirically find that zombies are more prevalent among smaller firms.

Turning now to the result that firms in receipt of forbearance lending have lower sales growth, there are two factors that explain this. First, these firms are often experiencing a sequence of bad shocks, and this itself will lower sales growth. Second, these firms reduce their scale in response to the sequence of bad shocks, which further contributes to lower sales growth. Once more, these model results are consistent with the empirical findings in Banerjee and Hofmann (2020).

I now consider the liquidating firms in Column (4) of Table 4, which is based on statistics for the period preceding firm liquidation. These firms are equally likely to be high-quality or low-quality firms, although the path to liquidation differs by firm-type. High-quality firms tend to liquidate after being in the market for some time; their liquidation occurs if they experience a sequence of bad stochastic shocks. Low-quality firms, on the other hand, often liquidate after only a few periods only. When low-quality firms do survive for a longer period, this is due to their access to forbearance lending. For both firm-types, firms have high leverage and negative sales growth in the period before they liquidate, which is typical for any firm experiencing a sequence of bad stochastic shocks.

Finally, I consider the firm entrants in Column (5) of Table 4, which includes all firms that have existed for three periods or less. These firms are smaller and have higher sales growth rates, which is consistent with the findings of Arellano, Bai and

Zhang (2012) for firm entrants.

In summary, the evidence in Table 4 suggests that nearly all firms in receipt of forbearance lending are low-quality, which is consistent with the empirical evidence of Acharya et al. (2019).

5 The Impact of Forbearance Lending

5.1 No Forbearance Lending Scenario

I use the calibrated model to examine the quantitative affects of forbearance lending via two counterfactual experiments. The first experiment aims to show how forbearance lending practices influence the financial decisions of firms, as well as firm dynamics and aggregate outcomes. To do this, I consider a counterfactual scenario in which the possibility of forbearance lending is shut down and so firms can only choose between continuing or liquidation. Here the value function of a firm that has the option to default and that starts the current period with capital k, a loan repayment l_R and productivity z is:

$$V\left(k, l_R, z, \mu\right)=\max_{\{c,d\}}\left\{V_c\left(k, l_R, z, \mu\right), V_d\left(k, l_R\right)\right\}, \tag{27}$$

where $V_c\left(k, l_R, z, \mu\right)$ is the value function if the entrepreneur does not default and continues to operate, as defined by Equation (4); and $V_d\left(k, l_R\right)$ is the value function if the entrepreneur defaults and liquidates the firm, as defined by Equation (5). The key departure from the benchmark model is that the value associated with forbearance, $V_f\left(k, l_R, z, \mu\right)$, is no longer an option in the value function.

In this counterfactual scenario, the equilibrium loan schedule $\Omega\left(k', z\right)$ that comprises all loan contracts $\left(l', l'_R\right)$ allowing the lender to break even in expected values, and accounting for the liquidation risk that the lender faces, is described by:

$$\begin{aligned} l'=\textstyle\sum_{j=b,g}\rho\left(\mu=\mu_j|z\right) \quad & \Big\{\tfrac{1}{1+r}\left[1-\rho_d\left(k', l'_R, z, \mu_j\right)\right] l'_R \\ & -\tfrac{1}{1+r}\rho_d\left(k', l'_R, z, \mu_j\right)\min\left\{\chi_d k', l'_R\right\}\Big\}-\xi \end{aligned} \tag{28}$$

To generate the statistical moments for the counterfactual experiments, I employ the definitions outlined in Table 1. For the aggregate statistics, I compute aggregate investment, I, aggregate output, Y, aggregate capital, K, and aggregate total factor

Table 5: Statistical Moments for the Counterfactual Experiments

	(1)	(2)	(3)
Targeted Moments	*Benchmark*	*No Forbearance*	*Perfect Information*
Firm liquidation rate	4.45%	4.76%	5.14%
Firm forbearance rate	6.79%	0.00%	0.03%
Average of sales growth	2.00%	2.27%	3.92%
Standard deviation of sales growth	19.54%	19.40%	24.03%
Average of leverage	68.69%	18.55%	13.10%
Non-targeted Moments	*Benchmark*	*No Forbearance*	*Perfect Information*
Average of investment rate	18.98 %	19.69 %	25.41 %
Average of dividend ratio	5.89 %	5.57 %	6.44 %
Average of working capital	29.99%	78.21%	91.14%
Average of TFP	1.74	1.76	1.79
Aggregate investment	1.000	1.084	1.091
Aggregate TFP	1.000	1.008	1.016
Aggregate output	1.000	1.047	1.050

Note: The column under Data reports the statistical moments for the euro area, from 2011 to 2014. Column (1) reports model statistics under the benchmark calibration, in which firms have access to forbearance lending. Column (2) reports model statistics for the counterfactual experiment in which firms no longer have access to forbearance lending. Column (3) reports model statistics for the counterfactual experiment in which there is no information asymmetry and so lenders can observe firm-type. Aggregate statistics for output, investment and total factor productivity are scaled by the Benchmark Model statistics, and so equal to one for the Benchmark Model. All statistical moments and aggregates are calculated over a one-year period.

productivity, TFP, as follows:

$$\begin{aligned} I &= \int i(s)\Gamma(s)d\left(k \times l_R \times z \times \mu\right) \\ Y &= \int y(s)\Gamma(s)d\left(k \times l_R \times z \times \mu\right) \\ K &= \int k(s)\Gamma(s)d\left(k \times l_R \times z \times \mu\right) \\ TFP &= \frac{Y}{K^{\alpha}} \end{aligned} \tag{29}$$

where $s = (k, l_R, z, \mu)$ are the states, $i = k' - (1-\delta)\,k$ is firm investment, $y = zk^{\alpha}$ is firm output as measured by sales, and k is firm capital.

Table 5 presents statistical moments for the counterfactual experiments, in which Column (2) summarizes the results for the specific counterfactual experiment of no forbearance lending. Table 6 provides a more granular breakdown of the results by firm-type. On the basis of these results, I can conclude that with no forbearance lending: firm liquidation, average sales growth, aggregate total factor productivity (TFP), aggregate investment, and aggregate output are all higher, while average leverage is lower. I discuss these results in more detail below.

First, I focus on the liquidation risk and rate. Although not depicted here, I

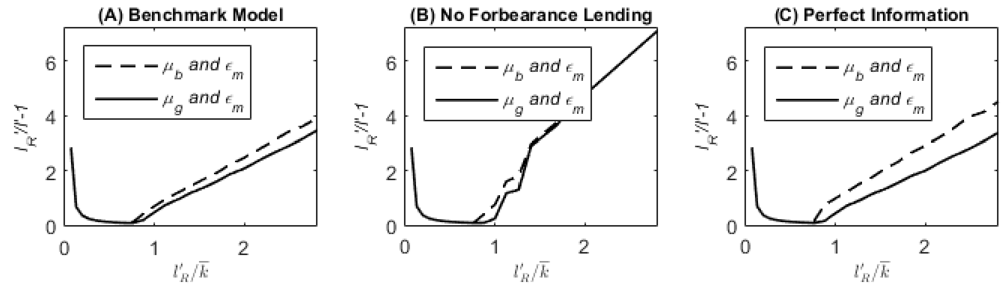

Figure 5: Effective Interest Rate Schedules

Note: Panel A is for the benchmark model with forbearance lending. Panel B is for the counterfactual experiment with no forbearance lending. Panel C is for the counterfactual scenario with perfect information. All figures plot the effective interest rate ($l'_R/l' - 1$) as a function of the loan repayment choice l'_R (scaled by the average equilibrium capital $\bar{k}$). There are for two firms considered, one high-quality firm ($\mu = \mu_g$) and one low-quality firm ($\mu = \mu_b$), when there is a normal stochastic shock ε_m and the capital choice k' is equal to the average equilibrium capital $\bar{k}$.

find that the liquidation *risk* is significantly higher in the counterfactual experiment with no forbearance for otherwise identical firms. But the difference in the actual liquidation *rate* is not very significant (see Table 5). For example, the liquidation rate does not increase to the same extent that forbearance lending occurs - the other available default option - in the benchmark scenario. This is because in the absence of forbearance lending, firms make different equilibrium choices for their capital and loans, which offsets the observed increase in liquidation risk.

Next I consider the equilibrium loan schedules that arise in the counterfactual scenario of no forbearance lending. The equilibrium loan schedule is higher, on average, for firms in the counterfactual experiment than for the benchmark model (see Panel B in Figure 5). This is partly due to the higher liquidation risk of firms in the counterfactual scenario. But the more significant driver of the higher equilibrium loan schedule is the absence of forbearance lending. Forbearance lending risk is priced into the effective loan schedule. But unlike liquidation risk, forbearance lending risk can contribute to more favorable borrowing rates when the associated recovery rate is high because the lender is able to recuperate a much larger proportion of the loan repayment.

The loan schedule results also relate to the observations for overall average leverage. Table 5 shows that overall average leverage is around 73 per cent lower in the counterfactual experiment without forbearance lending. This is again due to the removal of the option for forbearance lending. The high leverage of low-quality firms in receipt of forbearance lending drives up the average leverage in the benchmark model. Additionally, the higher loan schedule in the counterfactual scenario of Panel

B in Figure 5 disincentivizes firms to take on high levels of leverage.

Table 5 also shows that overall average sales growth is around 14 per cent higher in the counterfactual experiment without forbearance lending. The removal of the option of forbearance lending is a key driver of this result. Without access to forbearance lending, low-quality firms default soon after they enter. As such, there is a larger proportion of high-quality firms in the counterfactual scenario, which leads to higher average sales growth. Here low-quality firms in receipt of forbearance lending are no longer lowering the overall average sales growth, as in the benchmark model.

Similarly, Table 6 shows that aggregate investment is around 8 per cent higher in the counterfactual exercise as compared with the benchmark model, while aggregate output is around 5 per cent higher. Aggregate total factor productivity (TFP) is also a little higher. These differences in the aggregate statistics are, once more, due to the lower proportion of low-quality firms brought about by the removal of the option for forbearance lending.

These model results complement the empirical findings of Jorda et al. (2020) who show that for inefficient bankruptcy regimes, the pace of recovery following a corporate credit boom can be significantly dragged down.

These model results complement several empirical studies who show that zombie firm survival - sometimes connected to an inefficient bankruptcy regime - can impair aggregate productivity growth and the pace of recovery following recessions (see, e.g., Andrews and Petroulakis, 2019, Blattner, Farinha and Rebelo, 2018, Caballero, Hoshi and Kashyap, 2008, Jorda et al. (2020), Adalet McGowan, Andrews and Millot 2017, Adalet McGowan, Andrews and Millot, 2018).

It is worth noting that I find evidence of congestion effects in the model, consistent with the findings of Acharya et al. (2019) for European firms and Caballero, Hoshi and Kashyap (2008) for Japanese firms. This can be seen in Column (5) of Table 4 and Table 6, where the TFP for entrants is higher in the benchmark model with forbearance lending (Table 4) than in the counterfactual experiment with no forbearance lending (Panel A of Table 6). And in Column (4) of Table 4 and Table 6, where the TFP for liquidating firms is lower in the benchmark model with forbearance lending than in the counterfactual experiment with no forbearance lending. Together these results suggest that forbearance lending allows firms with lower productivity to continue operating and in so doing, restricts the prospects for new firms to enter. I do not, however, find evidence that the presence of "zombie firms" (firms in receipt of forbearance lending) harmed other "non-zombie firms", as documented by Acharya et al. (2019) and Caballero, Hoshi and Kashyap (2008). Instead, I find

Table 6: Quantitative Model Results for the Counterfactual Experiments

	(1) *All Firms*	(2) *Continuing*	(3) *Forbearance*	(4) *Liquidating*	(5) *Entrants*
Panel A: Counterfactual experiment with no forbearance					
Average of:					
Sales growth	2.27 %	1.03 %	n.a.	-3.34 %	18.36 %
Leverage	18.55 %	18.60 %	n.a.	17.59 %	17.95 %
Investment rate	19.69 %	17.59 %	n.a.	-17.77 %	46.84 %
Capital	134.54	142.68	n.a.	44.00	28.77
Loan repayment	54.28	57.76	n.a.	9.90	9.01
Firm age	50.00	53.68	n.a.	21.12	2.30
TFP	1.76	1.79	n.a.	1.05	1.47
Proportion of firms:					
In given category	100.00 %	95.24 %	n.a.	4.76 %	4.76 %
High-quality μ_g	96.94 %	99.79 %	n.a.	49.69 %	59.89 %
Panel B: Counterfactual experiment with no information asymmetry					
Average of:					
Sales growth	3.92 %	1.87 %	-28.67 %	-5.56 %	109.44 %
Leverage	13.10 %	13.47 %	93.53 %	00.01 %	6.78 %
Investment rate	25.41 %	19.30 %	-38.65%	-32.74 %	352.33 %
Capital	133.96	138.74	262.93	27.05	24.34
Loan repayment	42.68	44.30	243.75	0.01	2.92
Firm age	51.98	54.00	41.45	19.47	2.50
TFP	1.79	1.79	1.46	0.99	1.82
Proportion of firms:					
In given category	100.00 %	94.83 %	0.03 %	5.14 %	5.14 %
High-quality μ_g	99.98 %	100.00 %	100.00 %	49.99 %	99.59 %

Note: Panel A reports model statistics for the counterfactual experiment in which firms no longer have access to forbearance lending. Panel B reports model statistics for the counterfactual experiment in which there is no information asymmetry and so lenders can observe firm-type. Column (1) reports model statistics for all firms operating with age greater than one. Column (2) reports model statistics for continuing firms, defined as any firm that has existed for more than three periods and does not currently receive loan forbearance. Column (3) reports model statistics for forbearance firms, defined as any firm that has existed for more than three periods and currently receives loan forbearance. Column (4) reports model statistics for liquidating firms, based on the firm statistics in the period preceding firm liquidation. Column (5) reports model statistics for entrant firms, defined as any firm that has existed two periods or less. All model statistics are for the benchmark calibration, in which firms have access to forbearance lending.

that the average sales growth, investment and productivity are not significantly different between the benchmark model and counterfactual scenario for continuing firms (Column (2) of Table 4 and Table 6), which is consistent with some of the findings of Schivardi, Sette and Tabellini (2017) and Schivardi, Sette and Tabellini (2020).

5.2 Perfect Information Scenario

I now examine the impact of information asymmetry on forbearance lending by conducting a counterfactual experiment with perfect information. With perfect information, the firm problem remains identical to the baseline scenario but the equilibrium loan schedule changes because the lender can perfectly identify firm type $j \in (b, g)$. The equilibrium loan schedule $\Omega(k', z)$ that comprises all loan contracts (l', l'_R) allowing the lender to break even in expected value is:

$$\begin{aligned} l' = \; & \tfrac{1}{1+r}\left[1 - \rho_f(k', l'_R, z, \mu) - \rho_d(k', l'_R, z, \mu)\right] l'_R \\ & -\tfrac{1}{1+r}\left(\rho_f(k', l'_R, z, \mu) \cdot \gamma(k', l'_R, z, \mu)\right) l'_R \\ & -\tfrac{1}{1+r}\rho_d(k', l'_R, z, \mu) \min\{\chi_d k', l'_R\} - \xi \end{aligned} \tag{30}$$

Overall, Table 5 and Table 6 demonstrate that firm liquidation is higher, forbearance lending rates are lower, sales growth is higher, and leverage is lower under perfect information. I discuss these results in more detail below.

I first consider the impact of perfect information on the liquidation rate. Table 5 shows that the liquidation rate is about 16 per cent higher in the counterfactual experiment with perfect information than in the benchmark scenario with information asymmetry. The higher liquidation rate is mainly driven by higher defaults of entrants, as can be seen by the lower average age of liquidating firms in Table 4 and Table 6 (i.e. 22.47 versus 19.47). Although not depicted here, low-quality firms tend to have a higher liquidation risk in the counterfactual scenario with perfect information and so often liquidate after just one period. Under perfect information, lenders can properly discriminate between low-quality and high-quality firms when pricing loan schedules. The relatively higher loan schedules for low-quality firms reduces the relative value of continuing or forbearance lending, making liquidation typically the most valuable option for these firms.

Next I consider the equilibrium loan schedules that arise in the counterfactual scenario of perfect information. Compared with the benchmark scenario with information asymmetry, the loan schedule for high-quality firms is more favorable under perfect information, and vice versa for low-quality firms (see Panel C of Figure 5). This is because the lender can now perfectly identify firm-type and so more accurately price the loan schedule, and so the gap widens between the loan schedules of the high-quality and low-quality firms.

The loan schedules under perfect information lead to very different outcomes for the type of firms that obtain loan forbearance. With information asymmetry, almost exclusively low-quality firms obtain forbearance lending (see Table 4). With perfect information, only high-quality firms obtain forbearance lending because loan schedules under perfect information are priced such that low-quality firms do not find it optimal to choose forbearance lending (see Table 6). Yet, Table 6 also shows that the overall forbearance rate is much lower in the counterfactual experiment with perfect information. For the given model calibration, firms rarely find it optimal to choose forbearance lending under perfect information.

Average sales growth across all firms is around 96 per cent higher in the counterfactual experiment with perfect information than in the benchmark scenario, and also higher than in the counterfactual experiment with no forbearance lending (see Table 5 and Table 6). With perfect information, low-quality firms face high loan schedules because they have higher liquidation risk and forbearance risk. This contributes to these firms liquidating shortly after they enter. As such, the overall proportion of high-quality firms is 99.98 per cent under perfect information, which is higher than in both the benchmark and the counterfactual scenario of no forbearance lending. Once more, the consequence of a higher proportion of high-quality firms is that overall average sales growth is higher. For similar reasons, aggregate investment, aggregate output and aggregate TFP are respectively 9 per cent, 5 per cent and 2 per cent higher in the counterfactual scenario with perfect information as compared with the benchmark model.

6 Conclusions

In this study, I examine the relationship between forbearance lending, firms' financial decisions and firm dynamics. To do so, I develop a firm equilibrium model that features endogenous liquidations and endogenous forbearance lending. Lenders face information asymmetry because they do not know with certainty whether they are lending to a "low-quality" or "high-quality" firm. The model enables us to consider the net impact of forbearance lending on firm performance and aggregate outcomes by examining whether the costs of forbearance lending, due to credit misallocations from lenders incorrectly assessing a lender to be high-quality, outweigh its benefits, due to refinancing high-quality firms experiencing a negative sequence of shocks. I fit the model to the euro-area economy over the period 2011 to 2014, which represents a period of low output growth and higher levels of forbearance lending.

To examine the impact of forbearance lending, I conduct a counterfactual exercise in which firms still have the option to liquidate but no longer have access to any form of loan forbearance. In the absence of forbearance lending, I find that the average of firms' growth, investment rates and total factor productivity are higher in the counterfactual scenario with no forbearance lending. A key driver of this result is that there is a larger proportion of high-quality firms in the counterfactual scenario; low-quality firms are most likely to receive forbearance lending in the benchmark model, which weakens selection because it prevents these firms from liquidating more quickly. These low-quality firms, kept alive via forbearance lending, lower the aggregate output, investment and total factor productivity and they prevent new, more productive firms from entry.

The fact that the overwhelming majority of firms in receipt of forbearance lending in the model are of low-quality is consistent with the empirical evidence (see, e.g., Acharya et al., 2019). But while previous authors have emphasized the role of zombie lending incentives as a key driver of loan forbearance to low quality firms, my model highlights that information asymmetry faced by lenders is another factor. The results in this study also extend previous empirical findings by using a structural approach to quantifying the impact of forbearance lending on firm dynamics as well as various measures of firm performance and aggregate outcomes.

While the model provides new insights about the impact of forbearance lending on firms' financial decisions and their dynamics, some related questions remain unanswered. Specifically, the model does not include any role for the potential increased unemployment associated with higher firm liquidation rates that may occur in the absence of forbearance lending. A full examination of this potential benefit of forbearance lending presents an exciting avenue for future research. I also do not examine a role for forbearance lending incentives by weak banks, which the empirical literature finds to be important (see, e.g., Acharya et al., 2019, Andrews and Petroulakis, 2019, Blattner, Farinha and Rebelo, 2018, Giannetti and Simonov, 2013, Peek and Rosengren, 2005, Schivardi, Sette and Tabellini, 2017). Instead, I focus on the impact of these lending practices on the dynamics of firms, leaving related theoretical work on this topic open for future investigations.

In reality, I expect lenders to face information asymmetry. Therefore, my results suggest that the output-related costs of forbearance lending associated with misallocating credit to low-quality firms outweigh any output-related benefits associated with refinancing high-quality firms experiencing a sequence of negative shocks. From this perspective, the forbearance lending that has arisen since the onset of the sovereign debt crisis may have contributed the low output observed for the euro area over the same period.

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A Computational Algorithm

I first set grids on the state space of the aggregate states of the economy, $s = \{k, l_R, z, \mu\}$. The productivity shock z is discretized into 10-state Markov chain using the method of Tauchen (1986). The state space of the loan repayment amount l_R and capital k are each discretized into a grid of 15 and 25 points, respectively, between 0 and 400. The loan repayment amount grid and the capital grid are unevenly spaced, using a similar method to Corbae and Quintin (2015). That is, every point of an equally spaced grid between 0 and $400^{2/3}$ is raised to the 3/2 power. As in Corbae and Quintin (2015), this grid contains more points closer to zero and so provides improved numerical performance for our problem.

I solve the model to find the optimal loan schedules, policy functions and default policies using the following algorithm:

1. For all aggregate states of the economy, $s = \{k, l_R, z, \mu\}$, start with an initial guess for the bank loan recovery schedule ψ^0. For all k' and z, start with an initial guess for the loan schedule Ω^{00}, in which all loan contracts (l', l'_R) have the risk-free interest rate. For the probability of firm-type given observed productivity $q^0 = \Pr(\mu = \mu_j | z)$, start with an initial guess by using $\Pr(\mu = \mu_b) = \Pr(\mu = \mu_g) = 0.5$ and $\Pr(z | \mu = \mu_b) = 2 \times \Phi\left(-\left|\frac{\ln(z) - \mu_b}{\sigma_\varphi}\right|\right)$.

2. Given the bank loan recovery schedule ψ^0 and the equilibrium loan schedule Ω^0, use value function iterations to solve for the optimal policy functions for future capital stock $k'(s)$, the new bank loan contract $(l'(s), l'_R(s))$, operating sets $C(k, l_R, \mu)$, liquidation sets $D(k, l_R, \mu)$, and forbearance sets $F(k, l_R, \mu)$. I iterate on the value function until I reach convergence for a given ψ^0 and Ω^0.

3. Given the operating sets, liquidation sets, forbearance sets and q^0: compute the new debt schedule Ω^1 such that lenders break even in expectation, and compare it to loan debt schedule of the previous iteration: Ω^0. If a convergence criterion is met, $\max\{\Omega^0 - \Omega^1\} < \varepsilon$, then assign Ω^1 to Ω^0 and move on to the step 4. Otherwise, update using a Gauss-Seidel algorithm and go back to step 2.

4. Solve the bargaining problem given the converged loan schedule Ω^0 and compute the new bank loan recovery schedule ψ^1 for all aggregate states of the economy, $s = \{k, l_R, z, \mu\}$. If the new bank loan recovery schedule ψ^1 is sufficiently close to ψ^0, stop iterating on ψ. Otherwise, go back to step 2.

5. To compute the stationary cross-sectional distribution, start with a uniform distribution as an initial guess. Then simulate the stationary cross-sectional distribution associated with the set of policy functions obtained above, and such that the mass of all projects always equals one. This implies that the mass of entrants equals the the mass of liquidations. Update q^1 based on $\Pr(\mu = \mu_b)$ and $\Pr(z|\mu = \mu_b)$ from the stationary cross-sectional distribution. If the firm-type probability given productivity q^1 is sufficiently close to q^0, stop iterating on q. Otherwise, go back to step 2.

6. Simulate the model to compute statistics from a model economy of 1,000 firms over 750 periods, using the first 250 periods as a burn-in.

B Data and Parameter Estimation

I use firm-level data to estimate for three parameters in Table 2 independently of the model equilibrium, as well as to estimate most of the model statistics listed in Table 3. I obtain our firm-level data from the Amadeus database, which comprises financial data for both public and private companies in Europe. Our sample comprises all public and private euro-area firms, excluding the financial and government sectors. I collect annual financial data for the period 2009 to 2014.

I clean the dataset in several ways. Following Arellano, Bai and Zhang (2012), I exclude firms in the financial and government sectors. Financial sector firms correspond to NACE Divisions 64 to 67 and government sector firms correspond to NACE Division 84. I furthermore collect data for euro-area firms only. The sample is also restricted to those firms with a reported value for EBIT (that is, earnings before interest and taxes), sales, and fixed assets in at least one of the years from 2012 to 2014. These criteria leave us with around 150,000 firms in 17 countries: all euro-area countries except for Ireland. Table 7 describes how I compute the parameters and model statistics.

Table 7: Definitions of Parameters and Model Statistics from Firm-level Data

Calibrated Parameters	*Definition*
Depreciation	$\frac{\text{Depreciation}_t}{\text{Fixed Assets}_{t-1}}$
Interest Coverage Ratio	$\frac{\text{EBIT}_t}{\text{EBIT}_t - \text{EBT}_t}$

Model Statistics	*Definition*
Sales Growth	$\frac{\text{Sales}_t - \text{Sales}_{t-1}}{\text{Sales}_{t-1}}$
Leverage	$\frac{\text{Total Assets}_t - \text{Total Equity}_t}{\text{Total Assets}_t}$
Working Capital	$\frac{\text{Working Capital}_t}{\text{Total Assets}_{t-1}}$
Investment	$\frac{\text{Fixed Assets}_t - \text{Fixed Assets}_{t-1} + \text{Depreciation}_{t-1}}{\text{Fixed Assets}_{t-1}}$

Note: Each variable is windsorized at the 5 per cent level. The model statistics are computed for each firm in each year from 2011 to 2014. I first take the mean and standard deviation of each statistic in each year. The model statistics are then calculated as the mean of the yearly mean (for the mean of the model statistic) or the mean of the yearly standard deviation (for the standard deviation of the model statistic) over the period 2011 to 2014.