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The real effects of zombie lending in Europe

Belinda Tracey

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Abstract

Around 10% of European firms were in receipt of subsidized bank loans following the peak of the European sovereign debt crisis in 2011. To what extent did such forbearance lending contribute to the subsequent low output growth experienced by the euro area? In this paper, we address this question by developing a quantitative model of firm dynamics in which forbearance lending and firm defaults arise endogenously. The model provides a close approximation to key euro-area firm statistics over the period 2011 to 2014. We evaluate the impact of forbearance lending by considering a counterfactual scenario in which firms no longer have access to loan forbearance. Our key finding is that aggregate output, investment and total factor productivity are higher in the absence of forbearance lending than in the benchmark scenario that includes forbearance lending. This suggests that forbearance lending practices contributed to the low output growth across the euro area following the onset of the sovereign debt crisis.

Key words: Forbearance lending, zombie firms, firm defaults, firm dynamics.

JEL classification: G21, G32, L25.

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(1) Bank of England. Email: belinda.tracey@bankofengland.co.uk

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Bank of England, Threadneedle Street, London, EC2R 8AH
Telephone +44 (0)20 3461 4030 email publications@bankofengland.co.uk

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1 Introduction

Forbearance lending occurs when a lender supports an otherwise insolvent borrower. This type of lending – sometimes referred to as "zombie lending" – can take on many forms, including repayment holidays and evergreening, whereby further credit is extended to a troubled borrower. Japanese banks engaged in extensive forbearance lending to firms during the late 1990s, and this phenomenon is well documented by the literature (see, e.g., Sekine et al., 2003, Peek and Rosengren, 2005, Caballero et al., 2008). More recently, evidence has emerged of widespread forbearance lending to European firms following the sovereign debt crisis that commenced end 2009 (see, e.g., Acharya et al., forthcoming, Adalet McGowan et al., 2017, Schivardi et al., 2017, Blattner et al., 2018). Acharya et al. (forthcoming) estimate that as many as 10 per cent of European firms were in receipt of subsidized bank loans in 2014.[1]

To what extent have these forbearance lending practices contributed to the low output growth experienced by the euro area following the onset of the sovereign debt crisis? In this paper, we address this question by developing a model of firm dynamics that incorporates a role for endogenous forbearance lending. In particular, we investigate the influence of these lending practices on aggregate outcomes and firm dynamics by conducting a counterfactual experiment in which no forbearance lending is permitted.

The existing literature on forbearance lending has emphasized its negative economic costs, and there are good reasons to expect these to be important. To start with, forbearance lending may suppress creative destruction, as well as constrain lending to more productive firms, because bank credit remains tied up in support of otherwise nonviable "zombie firms" (Caballero et al., 2008). Additionally, these lending practices may lower overall productivity because the distribution of firms includes a subset of zombie firms with poor prospects. Following this reasoning, it is entirely possible that forbearance lending to inefficient firms has hampered the economic recovery in the euro area in recent years, as suggested by recent findings (see, e.g., Caballero et al., 2008, Acharya et al., forthcoming, Adalet McGowan et al., 2017, Blattner et al., 2018).

However, there are also potential benefits associated with forbearance lending that make the overall impact of these lending practices ambiguous. For example, forbearance lending could help firms with good long-term prospects survive a period

[1]The study of Acharya et al. (forthcoming) considers all firms with bank-firm relationships in Europe. Our study focuses on firms in the euro area.

of weak demand. While it is too early to ascertain whether such benefits apply to forbearance lending activities in the euro area, Fukuda and Nakamura (2011) provide suggestive evidence that they may have been relevant for the Japanese case. They show that the eventual bankruptcy of Japanese firms who benefited from forbearance lending was rare and that most of these firms recovered in the 2000s. They argue that if these supposed-zombie firms were truly inefficient, the forbearance lending they received would not have prevented their ultimate insolvency. Similarly, Schivardi et al. (2017) conclude that forbearance lending extended to Italian firms did not have wider negative implications for the economy, such as damaging the growth rate of healthier firms. One potential explanation for their findings is that forbearance lending prevents an increase in unemployment, thereby mitigating aggregate demand externalities (Haldane, 2017).

The main contribution of this paper is the development of a structural firm equilibrium model to study whether the costs of forbearance lending, due to lenders misallocating credit, outweigh its benefits, due to lenders refinancing good firms experiencing a sequence of negative shocks. Our vehicle of analysis is a dynamic model of heterogeneous firms in which: (i) firms can obtain a loan from a lender to be used for production and they subsequently have the option to default in each period; (ii) if a firm defaults, it has the option to either liquidate or to obtain loan forbearance; (iii) loan forbearance takes the form of renegotiation over an *ex post* reduction to the outstanding loan repayment, where the extent of the reduction is determined endogenously in a Nash bargaining mechanism; and (iv) lenders face information asymmetry because they observe the overall firm productivity draw but do not know precisely whether the draw belongs to a "low-quality" or "high-quality" firm. Lenders therefore offer firm-specific loan schedules, based on a prediction of liquidation risk, forbearance risk, as well as the "firm-type risk" brought about by the information asymmetry. We use this framework to consider the net impact of forbearance lending on aggregate outcomes and firm dynamics.

We focus our attention on the euro area after the onset of the sovereign debt crisis in 2010, a period marked by higher levels of forbearance levels (Acharya et al., forthcoming). As such, we calibrate our model using euro area firm-level statistics over the period 2011 to 2014, primarily obtained from the Amadeus Database. The calibrated model provides a good match to the key firm-level statistics, including the prevalence of forbearance lending practices and firm defaults. To evaluate the quantitative impact of forbearance lending, we subsequently conduct a counterfactual exercise in which firms have the option to default but no longer have access to any form of loan forbearance. We find that aggregate output is around 4.65 per cent higher in our counterfactual scenario as compared with the benchmark scenario with forbearance lending. We also find that aggregate investment and total factor productivity are respectively 8.44 per cent and 0.76 per cent higher in our counterfactual scenario. A key driver of these results is that there is a larger proportion of high-quality firms in our counterfactual scenario; the forbearance lending that is present in the benchmark scenario prevents low-quality firms from defaulting, which is a drag on aggregate output, investment and total factor productivity.

The reason why primarily low-quality firms access forbearance lending in our benchmark model relates to the information asymmetry between the lenders and firms. The fact that lenders cannot perfectly predict firm quality means that the interest rate schedule is more favorable than otherwise for a low-quality firm, and vice versa for a high-quality firm. The end result is that the vast majority of firms in receipt of forbearance lending are low-quality. Therefore, the costs of forbearance lending, due to allocating credit to low-quality firms, dominate any potential benefits in our model. These findings suggest that forbearance lending practices may have contributed to the recent low growth across the euro area.

Our study contributes to the literature in several ways. First, we go beyond previous work in the zombie lending literature by developing a structural model of forbearance lending. In this respect, we build on previous empirical research that has focused on the consequences of forbearance lending (see, e.g., Caballero et al., 2008, Acharya et al., forthcoming, Adalet McGowan et al., 2017, Blattner et al., 2018). Compared to those studies based largely on reduced form analyses, our paper employs a structural approach that provides new insights about some of the trade-offs between the adverse and beneficial implications of forbearance lending.

Second, our paper relates to several studies that have attempted to model the influence of forbearance lending in Japan. To be more precise, Caballero et al. (2008) outline a stylized model of entry and exit to consider the impact of zombies on firm creation and productivity. Kwon et al. (2015), in turn, estimate the impact of zombie lending on aggregate output for Japan, based on a counterfactual scenario that applies the distribution of factor-input wedges estimated for non-zombies to all firms. We extend this research by incorporating an explicit role for firm dynamics, loan financing and firm liquidation; these are key factors that are likely to influence any analysis of forbearance lending.

Finally, we add to the literature that incorporates a role for multiple firm default choices within a dynamic model of heterogeneous firms. In this context, our model and firm default options are similar to those of Arellano et al. (2012), Senkal (2014), Corbae and D'Erasmo (2017) and Tamayo (2017). While Arellano et al. (2012) focus on the role of financial development on firm dynamics, Senkal (2014), Corbae and D'Erasmo (2017) and Tamayo (2017) consider the impact of U.S. bankruptcy laws. The firm equilibrium model in our study, as well as in Senkal (2014) and Corbae and D'Erasmo (2017), builds on other corporate finance models (see, e.g., Cooley and Quadrini, 2001, Hennessy and Whited, 2007), by incorporating endogenous entry similar to that of Hopenhayn and Rogerson (1993) as well as endogenous renegotiation similar to that of Yue (2010). Aside from the fact that we employ our model to study forbearance lending rather financial development or U.S. bankruptcy laws, another key point of departure from these previous studies is that we incorporate a role for information asymmetry about firm-type in our model.[2]

The remainder of this paper is structured as follows. Section 2 outlines the model environment. Section 3 describes the recursive equilibrium of the model. Section 4 discusses the model calibration and analyzes the model equilibrium. Section 5 quantitatively evaluates the impact of forbearance lending on firm dynamics and performance. Section 6 concludes.

2 The Model Economy

2.1 Firms and technology

This section sets out a discrete-time, quantitative model for our study of forbearance lending, which builds on several other firm equilibrium models.[3] In the model, entrepreneurs are infinitely lived. They have access to a pool of risky projects of mass one, which are referred to as firms. Each entrepreneur owns at most one firm. An entrepreneur that owns firm j chooses physical capital k_{jt+1} and a new one period loan contract $(l_{jt+1}, l_{R,jt+1})$ to maximize the expected present value of all current and future dividends:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[d_{jt} \left(1 + \gamma I_{\{d_{jt}<0\}} \right) \right], \tag{1}$$

[2]The model of Tamayo (2017) features a different type of information asymmetry, with the lender being unable to observe any form of firm productivity but having the option to engage in various types of monitoring.

[3]Namely, the model and default options are comparable to those of Arellano et al. (2012), Senkal (2014), Corbae and D'Erasmo (2017) and Tamayo (2017).

where $\beta \in (0,1)$ is the discount factor of all firms, γ is a proportional cost of equity issuance, and d_{jt} is the dividend function given by:

$$d_{jt} = y_{jt} - k_{jt+1} + (1-\delta)\,k_{jt} - l_{R,jt} + l_{jt+1} - \frac{\phi i_{jt}^2}{2k_{jt}} - \chi_c \tag{2}$$

where y_{jt} is output produced by the firm, $\delta \in (0,1)$ is the depreciation rate for physical capital, $k_{jt+1} - (1-\delta)\,k_{jt}$ is investment i_{jt}, $l_{R,jt}$ is the loan repayment on the previous period's loan l_{jt}, $\phi i_{jt}^2/\left(2k_{jt}\right)$ are capital adjustment costs, and χ_c is a fixed cost of operation that firms must pay to produce. The price of output is normalized to one. Firms produce output according to a decreasing returns to scale production technology $y_{jt} = z_{jt}k_{jt}^{\alpha}$, where z_{jt} is an idiosyncratic productivity process. The probability distribution of firm j's productivity z_{jt} is conditional on the previous realization z_{jt-1} and follows a Markov process given by $f\left(z_{jt}, z_{jt-1}\right)$.

We assume that firm productivity is a log-normal AR(1) process: $\ln\left(z_{jt}\right) = \mu_j\left(1-\varphi\right) + \varphi \ln\left(z_{jt-1}\right) + \epsilon$, with $|\varphi| < 1$ and $\epsilon \sim \mathcal{N}\left(0, \sigma_\epsilon^2\right)$. There are two firm types j, which differ according to the permanent productivity parameter μ_j. "Low-quality" firms have a permanent productivity μ_b and "high-quality" firms have a permanent productivity μ_g, such that $\mu_b < \mu_g$. This firm type distinction is important for our consideration of forbearance lending. We distinguish between forbearance lending to a low-quality firm and forbearance lending a high-quality firm that is experiencing a negatives series of shocks.

One way to describe the dividend function in Equation (2) is that firm j uses both internal and external funds in order to finance investment and capital adjustment costs, as well as to generate dividends. The internal funds consist of the firm's output net of the debt repayment, $y_{jt} - l_{R,jt}$. The firm obtains external funds by obtaining a one-period loan from a lender each period, l_{jt+1}. When there are negative dividends, $d_{jt} < 0$, the entrepreneur can obtain resources from equity holders by paying a proportional cost γ.

An idle entrepreneur will start a new firm if he receives a project opportunity. Entrants start with zero capital and debt, and decide on the future optimal capital and a future loan before they know their future productivity. Their future productivity is drawn from the stationary probability distribution $g\left(z_{jt+1}\right)$ derived from $f\left(z_{jt+1}, z_{jt}\right)$, for which the permanent productivity μ_j is drawn from a Bernoulli distribution with $\Pr\left(\mu_j = \mu_b\right) = \Pr\left(\mu_j = \mu_g\right) = 0.5$. The mass of all risky projects available to both entrants and operating firms is always equal to one. Thus, if an operating entrepreneur decides to liquidate its firm, a new risky project becomes available for a new entrepreneur to start a firm.

2.2 Lenders

Competitive lenders face information asymmetry when they initially make the loan because they observe z_{jt} but not μ_j. This implies that lenders do not know with certainty whether a firm is low-quality or high-quality, and we refer to this as "firm-type risk". Lenders measure firm-type risk based on their assumed knowledge of the proportion of firms that are low-quality versus high-quality combined with information about the conditional distributions of z_{jt} given μ_j. We provide a full description of this information asymmetry in Section 3.5.

More formally, lenders are risk-neutral and can borrow or lend as much as is needed from international capital markets at a constant interest rate $r > 0$. We denote $\Omega\left(k_{jt+1}, z_{jt}\right)$ as the set of firm-specific loan schedules available to a firm with next period capital k_{jt+1} and productivity z_{jt}; each contract $\left(l_{jt+1}, l_{R,jt+1}\right) \in \Omega\left(k_{jt+1}, z_{jt}\right)$ maps a current one-period loan l_{jt+1} to a next period repayment amount $l_{R,jt+1}$.

An entrepreneur has the option to default each period. There are two default options:

1. **Liquidation**. Firm j liquidates its assets at a firesale discount $\chi_d < 1$ and so $\chi_d k_{jt}$ is the firesale firm recovery value. The entrepreneur receives $\max\left\{\chi_d k_{jt} - l_{R,jt}, 0\right\}$; the lender receives $\min\left\{\chi_d k_{jt}, l_{R,jt}\right\}$.

2. **Loan forbearance**. Loan forbearance takes the form of post-default renegotiation over the outstanding loan amount. The lender and the entrepreneur bargain over the loan repayment fraction $\psi\left(k_{jt}, l_{R,jt}, z_{jt}, \mu_j\right)$ in a Nash bargaining mechanism. The repayment $\psi\left(k_{jt}, l_{R,jt}, z_{jt}, \mu_j\right)$ is restricted to the interval $[0, 1]$. The entrepreneur repays the reduced loan amount $\psi l_{R,jt}$ in the default period, as well as a renegotiating cost proportional to its capital stock $\chi_f k_{jt}$. The entrepreneur continues to operate and to have access to new loan finance in the default period.

Lenders offer a firm-specific loan schedule $\Omega\left(k_{jt+1}, z_{jt}\right)$ such that they break even in expected value on each loan. The loan schedule incorporates firm-specific liquidation risk and forbearance risk. The loan schedule also incorporates the firm-type risk, which accounts for the information asymmetry but still ensures that the zero expected profit condition will hold. The lender's problem is outlined formally in Section 3.5.

In theory, the loan schedule could give rise to both separating contracts and pooling contracts in equilibrium. For any pooling contract in which the lender makes zero profits, high-quality firms must subsidize low-quality firms because both firm-types pay the same price but low-quality firms have a higher probability of default (D'Erasmo, 2011). Thus, where possible, the lender has an incentive to create separating contracts that attract only attract high-quality firms. One way to create such a separating contract is to offer high-quality firms slightly less than "full insurance" at a price that is more favorable than the pooling contract (D'Erasmo, 2011). In general, the lender will offer a pooling contract *unless* a separating contract exists that a high-quality firm will strictly prefer (over the pooling contract) but that a low-quality firm will not strictly prefer (D'Erasmo, 2011).[4]

2.3 Timing

At the beginning of time t, the model timing is as follows:

1. Productivity z_{jt} is realized, and the state space for incumbent firm j is $\{k_{jt}, l_{R,jt}, z_{jt}, \mu_j\}$.

2. Entrepreneurs choose from the following options for their incumbent firm j:

   (a) to continue operating and repay the full loan amount $l_{R,jt}$. The entrepreneur chooses physical capital k_{jt+1} and a new loan contract $(l_{jt+1}, l_{R,jt+1})$.

   (b) to default on the full loan amount owing, $l_{R,jt}$, and liquidate the firm. Here the entrepreneur receives the liquidation value $\max\{\chi_d k_{jt} - l_{R,jt}, 0\}$.

   (c) to obtain loan forbearance by renegotiating over the outstanding loan amount, $l_{R,jt}$. The entrepreneur bargains with the lender over the loan repayment fraction $\psi(k_{jt}, l_{R,jt}, z_{jt}, \mu_j)$. When the entrepreneur and the lender agree on the repayment fraction, the firm repays $\psi(k_{jt}, l_{R,jt}, z_{jt}, \mu_j)\, l_{R,jt}$. It chooses physical capital k_{jt+1}, and continues to have access to new loan finance in the default period.

3. Idle entrepreneurs make an entry decision about whether to start a firm or not. Their initial productivity is drawn from the stationary distribution $g(z_{jt+1})$ derived from $f(z_{jt+1}, z_{jt})$, for which the permanent productivity μ_j is drawn from a Bernoulli distribution with $\Pr(\mu_j = \mu_b) = \Pr(\mu_j = \mu_g) = 0.5$.

[4] See D'Erasmo (2011) for a more detailed discussion of pooling contracts and separating contracts in an incomplete information setting.

3 Recursive Equilibrium

In this section, we define a stationary recursive equilibrium for the model. For all variables, we drop the time and firm subscripts for ease of exposition and employ the notation: $x_{jt-1} = x^{-}$, $x_{jt} = x$ and $x_{jt+1} = x'$. We denote the optimal policy rules for the entrepreneur for capital and loans as $k'(s)$ and $l'_R(s)$, respectively, for the states $s = (k, l_R, z, \mu)$. Given the states $s = (k, l_R, z, \mu)$, the equilibrium is determined by the policy rules for the entrepreneur, $k'(s)$ and $l'_R(s)$, and the loan schedule for loan contracts, $\Omega(k', z)$. And given any loan schedule for loan contracts, $\Omega(k', z)$, and debt recovery schedule, $\psi(s)$, the entrepreneur solves its firm optimization problem. Note that the entrepreneur solves its firm optimization problem with perfect information about all states $s = (k, l_R, z, \mu)$. But the lender does not observe μ when initially issuing a loan to an entrepreneur.

3.1 The Firm's Problem

The firm chooses whether to default or repay its loan in order to maximize the present value of current and future profits, given the productivity shock z, the permanent productivity μ, the initial capital k and the loan repayment amount l_R. Recall there are two firm types, low-quality and high-quality, which differ by their permanent productivity μ. Their permanent productivity μ also represents the mean of their log-normal shock process z.

If the entrepreneur chooses to default, it can either liquidate the firm or receive loan forbearance by renegotiating the repayment fraction of the original loan. Alternatively, the entrepreneur can choose to repay the loan and continue operating. If the entrepreneur chooses to continue operating, the firm must also choose its new level of capital k' and a new loan contract (l', l'_R). The loan schedule offered for new borrowing $\Omega(k', z)$ depends on the firm's choice of k' and on the state z but not on permanent productivity μ, which is understood by the firm. The value function for a firm that has the option to default and that starts the current period with capital k, a loan repayment l_R and productivity z is:

$$V(k, l_R, z, \mu) = \max_{\{c,d,f\}} \{V_c(k, l_R, z, \mu), V_d(k, l_R), V_f(k, l_R, z, \mu)\}, \tag{3}$$

where $V_c(k, l_R, z, \mu)$ is the value function if the entrepreneur does not default and continues to operate; $V_d(k, l_R)$ is the value function if the entrepreneur defaults and liquidates the firm; and $V_f(k, l_R, z, \mu)$ is the value function for forbearance lending when the entrepreneur defaults and renegotiates with the lender about the repayment fraction of the original loan.

The decision to default or repay the loan is a period-by-period decision. The value function conditional on the firm not defaulting and continuing to operate is:

$$\begin{aligned} V_c(k, l_R, z, \mu) = \quad & \max_{k', (l', l'_R) \in \Omega} d_c \left(1 + \gamma I_{\{d_c < 0\}}\right) \\ & + \beta \int_{z'} V(k', l'_R, z', \mu) f(z', z) \, dz', \end{aligned} \tag{4}$$

where $d_c = zk^\alpha - k' + (1-\delta)k - l_R + l' - \frac{\phi i^2}{2k} - \chi_c$ are the dividends for a continuing firm, and γ is the proportional equity issuance cost. Here the entrepreneur chooses the optimal k' and (l', l'_R) to maximize the present value of current and future dividends for the firm.

When the entrepreneur defaults on the loan and liquidates the firm, the value function is:

$$V_d(k, l_R) = \max\{\chi_d k - l_R, 0\}, \tag{5}$$

If the entrepreneur decides to default and obtain loan forbearance, the firm must repay a reduced fraction $\psi(k, l_R, z, \mu)$ of the unpaid loan repayment amount l_R. The recovery rate $\psi(k, l_R, z, \mu)$ is determined endogenously in a Nash bargaining mechanism explained in the Section 3.4. In the period of default, the reduced loan is repaid and the firm continues to have access to new loan finance. This continued access to credit is akin to "evergreening". The value function associated with a firm that defaults but repays the agreed reduced fraction of the outstanding loan is:

$$\begin{aligned} V_f(k, l_R, z, \mu) = \quad & \max_{k', (l', l'_R) \in \Omega} d_f \left(1 + \gamma I_{\{d_f < 0\}}\right) \\ & + \beta \int_{z'} V(k', l'_R, z', \mu) f(z', z) \, dz', \end{aligned} \tag{6}$$

where $d_f = zk^\alpha - k' + (1-\delta)k - \psi(k, l_R, z, \mu) l_R + l' - \frac{\phi I^2}{2K} - \chi_c - \chi_f k$ are the dividends for a firm with forbearance lending, γ is the proportional equity issuance cost, and ψ is the fraction of the unpaid loan repayment amount l_R.

3.2 Default Policies

An entrepreneur's default policy can be characterized by a continuing set, a liquidation set and a forbearance lending set. The continuing set, $C(k, l_R, \mu)$, is the set of z's for which continued operation is optimal when capital is k and the loan contract is (l, l_R), such that:

$$C(k, l_R, \mu) = \left\{ z \in Z \ : \ V_c(k, l_R, z, \mu) \geq \max_{\{d,f\}} \{V_d(k, l_R), V_f(k, l_R, z, \mu)\} \right\}. \quad (7)$$

The liquidation set $D(k, l_R, \mu)$, is the set of z's for which loan default and liquidation is optimal when capital is k and the loan contract is (l, l_R), such that:

$$D(k, l_R, \mu) = \left\{ z \in Z \ : \ V_d(k, l_R) > \max_{\{c,f\}} \{V_c(k, l_R, z, \mu), V_f(k, l_R, z, \mu)\} \right\}. \quad (8)$$

And finally, the forbearance lending set, $F(k, l_R, \mu)$, is the set of productivity shocks z's for which loan default and renegotiation is optimal when capital is k and the loan contract is (l, l_R), such that:

$$F(k, l_R, \mu) = \left\{ z \in Z \ : \ V_f(k, l_R, z, \mu) > \max_{\{c,d\}} \{V_c(k, l_R, z, \mu), V_d(k, l_R)\} \right\}. \quad (9)$$

These three sets that comprise an entrepreneur's default policy are mutually exclusive. That is, for example, $C(k, l_R, \mu) = \tilde{D}(k, l_R, \mu) \bigcup \tilde{F}(k, l_R, \mu)$.

3.3 Entrants

When an idle entrepreneur receives a project opportunity, he will attempt to start a new firm. Entrants choose their optimal capital and loan before they know their future productivity. Entrants future productivity is drawn from the stationary distribution $g(z')$ derived from $f(z', z)$, for which permanent productivity μ is drawn from a Bernoulli distribution with $\Pr(\mu = \mu_b) = \Pr(\mu = \mu_g) = 0.5$. The value function associated with a potential entrant is given by:

$$V_e = \max_{k', (l', l'_R) \in \Omega} d_e \left(1 + \gamma I_{\{d_e < 0\}}\right) + \int_{z'} V(k', l'_R, z', \mu)\, g(z')\, dz', \quad (10)$$

where $d_e = -k' + l'$, and we assume that the equity issuance costs are the same for the initial period as for continuing firms.

3.4 The Forbearance Problem

If a firm chooses the option to obtain loan forbearance following a default, forbearance lending takes the form of a bargaining game regarding the fraction $\psi(k, l_R, z, \mu)$ of the outstanding loan repayment l_R. Here the value of the defaulted loan repayment amount l_R is reduced to $\psi(k, l_R, z, \mu)\, l_R$. The value of such an agreement to the firm is described by Equation (6), which is the present value of all future expected profits of forbearance when the loan recovery rate is $\psi(k, l_R, z, \mu)$. The lender gets the value of the reduced loan repayment $\psi(k, l_R, z, \mu)\, l_R$.

The threat point of the Nash bargaining mechanism is firm liquidation, whereby the entrepreneur receives the liquidation value $V_d(k, l_R, z, \mu)$ and the lender receives the firesale recovery value $\min\{\chi_d k, l_{R,}\}$.

We let $\triangle^f\ (p; k, l_R, z, \mu)$ denote the firm's surplus in the Nash bargaining agreement, which is the difference between the value of accepting the loan recovery rate p and the value of rejecting it, given the firm's capital k, the loan repayment amount l_R, the firm's productivity z, and the firm permanent productivity μ. The firm's surplus is:

$$\triangle^f\ (p; k, l_R, z, \mu) = \quad V_f\,(p; k, l_R, z, \mu) - V_d\,(k, l_R) \tag{11}$$

The firm surplus will differ for a high-quality firm (with permanent productivity μ_g) and a low-quality firm (with permanent productivity μ_b). While the lender faces information asymmetry about firm-type when initially issuing a loan, firm-type is revealed during renegotiation from the firm's surplus. As such, we are using a Nash bargaining mechanism with perfect information to obtain the solution for the repayment fraction $\psi(k, l_R, z, \mu)$.[5]

We let $\triangle^b\ (p; k, l_R)$ denote the risk-neutral lender's surplus in the bargaining agreement, which is the present vale of the recovered loan repayment. The lender's surplus is:

$$\triangle^b\ (p; k, l_R) = p l_R - \min\{\chi_d k, l_R\} \tag{12}$$

The lender can extract loan repayments up to the full amount of a firm's cost of default when it has all of the bargaining power. Alternatively, the firm can obtain a

[5] That said, D'Erasmo (2011) claims that the Nash bargaining solution when there is perfect information should coincide with a Nash bargaining solution with incomplete information that maximizes the product of the expected surplus of the firm and the lender, given the lender's incomplete information.

complete reduction of its loan when it has all of the bargaining power. We consider the general case and assume that the borrower has bargaining power θ and the lender has bargaining power $(1-\theta)$. For any capital stock k, loan repayment amount l_R, and productivity shock z, we define the bargaining power set as $\Theta \subset [0,1]$ for $\theta \in \Theta$ in order to ensure that the renegotiation surplus has a unique optimum.

Given the capital stock k, loan repayment amount l_R, and productivity shock z, the loan recovery rate $\psi(k, l_R, z, \mu)$ solves the following bargaining problem:

$$\begin{aligned} \psi(k, l_R, z, \mu) = \quad & \underset{p \in [0,1]}{\arg\max} \left[\triangle^f(p; k, l_R, z, \mu)\right]^\theta \left[\triangle^b(p; k, l_R)\right]^{1-\theta} \\ \text{s.t.} \quad & \triangle^f(p; k, l_R, z, \mu) \geq 0, \\ & \triangle^b(p; k, l_R) \geq 0. \end{aligned} \tag{13}$$

3.5 The Lender's Problem

When initially pricing and offering a loan schedule to a firm, we assume that the lender knows the liquidation risk and forbearance risk for all states. That is, the lender knows the difference in liquidation risk between a low-quality firm (with permanent productivity μ_b) and a high-quality firm (with permanent productivity μ_g). But the lender does not observe μ when initially issuing the loan. Thus, while they know what the liquidation risk is for a low-quality firm, they do not know with certainty that it is a low-risk firm or high-risk firm, and we refer to this as "firm-type risk". Lenders can measure firm-type risk based on their assumed knowledge of the proportion of firms that are low-quality versus high-quality combined with information about the conditional distributions of z_{jt} given μ_j.

More formally, the representative lender offers a loan schedule $\Omega(k', z)$ to maximize expected profits π_b, which are given by:

$$\begin{aligned} \pi_b(k', l'_R, z) = -l' + \quad & \textstyle\sum_{j=b,g} \rho(\mu = \mu_j | z) \\ & \times \left\{ \tfrac{1}{1+r} \left[1 - \rho_f(k', l'_R, z, \mu_j) - \rho_d(k', l'_R, z, \mu_j)\right] l'_R \right. \\ & + \tfrac{1}{1+r} \left(\rho_f(k', l'_R, z, \mu_j) \cdot \gamma(k', l'_R, z, \mu_j)\right) l'_R \\ & \left. + \tfrac{1}{1+r} \rho_d(k', l'_R, z, \mu_j) \min\{\chi_d k', l'_R\} \right\} - \xi \end{aligned} \tag{14}$$

where $\rho(\mu = \mu_j | z)$ is the probability that a firm has permanent productivity μ_j given it has observed productivity z (our measure of "firm-type risk"), $\rho_d(k', l'_R, z, \mu)$ is the expected probability of liquidation for a firm with capital stock k', a loan repayment amount l'_R, in state z, and with permanent productivity μ, $\rho_f(k', l'_R, z, \mu)$ is the expected probability of forbearance, and $\gamma(k', l'_R, z, \mu)$ is the expected recovery rate, given by the expected proportion of the loans that the creditors can recover, conditional on forbearance.

The lender must pay a fixed cost for each loan, ξ. The first term on the right-hand side of Equation (14) represents the resources that the lender spends today. The remaining three terms on the right-hand side comprise the expected loan repayment amount l'_R, discounted by the risk-free rate and accounting for liquidation risk, forbearance risk, and firm-type risk.

The equilibrium loan schedule $\Omega(k', z)$ comprises all loan contracts (l', l'_R) that allow the lender to break even in expected values, accounting for both the liquidation risk, the forbearance risk and the firm-type risk that the lender faces, and is described by:

$$\begin{aligned} l' = \textstyle\sum_{j=b,g} \rho(\mu = \mu_j | z) \quad & \Big\{ \tfrac{1}{1+r} \left[1 - \rho_f(k', l'_R, z, \mu_j) - \rho_d(k', l'_R, z, \mu_j)\right] l'_R \\ & + \tfrac{1}{1+r} \left(\rho_f(k', l'_R, z, \mu_j) \cdot \gamma(k', l'_R, z, \mu_j)\right) l'_R \\ & + \tfrac{1}{1+r} \rho_d(k', l'_R, z, \mu_j) \min\{\chi_d k', l'_R\} \Big\} - \xi \end{aligned} \tag{15}$$

where the probability of liquidation, $\rho_d(k', l'_R, z, \mu)$, the probability of forbearance, $\rho_f(k', l'_R, z, \mu)$, the expected recovery rate, $\gamma(k', l'_R, z, \mu)$, and the probability that a firm has permanent productivity μ_j given z, $\rho(\mu = \mu_j|z)$, are endogenous to the model. The effective interest rate for a loan contract (l', l'_R) is $r_L = l'_R/l' - 1$. The zero profit assumption and the fact that $0 \leq \rho_d \leq 1$, $0 \leq \rho_f \leq 1$ and $0 \leq \gamma \leq 1$ imply that the gross firm effective interest rate lies in the interval $[(1+r), \infty)$.

In equilibrium, the loan schedule $\Omega(k', z)$ must be consistent with the firm's optimization and with expected zero profits of the lender, such that the loan schedule correctly assesses the probability of liquidation, the probability of forbearance of firms, and the probability of firm-type (i.e. the probability of being a low- or a high-quality firm).

The first term in Equation (15) shows that both liquidation risk ρ_d and forbearance risk ρ_f have a first-order effect on the loan schedule because they enter Equation (15) linearly. And the second term shows that the loan recovery rate γ affects the debt schedule through its combined effect with forbearance risk. Additionally, the loan recovery rate has an indirect impact on the *ex ante* liquidation risk, because it affects the second moment of the loan schedule.

The firm-type risk also has an effect on the loan schedule. Because lenders cannot perfectly identify a firm's permanent productivity, high-quality firms will face a higher cost of borrowing but lower quality firms will face a lower cost of borrowing, both in comparison to a situation of perfect information. And these differences in the cost of borrowing will in turn influence the decision rules of firms.

3.5.1 Liquidation Risk

The liquidation probabilities $\rho_d(k', l'_R, z, \mu)$ are related to the liquidation sets $D(k', l'_R, \mu)$ as follows:

$$\rho_d(k', l'_R, z, \mu) = \int_{D\left(k', l'_R, \mu\right)} f(z', z)\, dz'. \tag{16}$$

This implies that when the liquidation sets are empty, $D(k', l'_R, \mu) = \emptyset$, the equilibrium liquidation probabilities $\rho_d(k', l'_R, z, \mu)$ equal zero. This is because with capital k' and loan repayment amount l'_R, a firm will not liquidate for any realization of the productivity shocks. The liquidation probabilities $\rho_d(k', l'_R, z, \mu)$ equal one when $D(k', l'_R, \mu) = Z$.

3.5.2 Forbearance Risk

Similarly, the forbearance lending probabilities $\rho_f(k', l'_R, z, \mu)$ are related to the forbearance lending sets $F(k', l'_R, \mu)$ as follows:

$$\rho_f(k', l'_R, z, \mu) = \int_{F\left(k', l'_R, \mu\right)} f(z', z)\, dz'. \tag{17}$$

This similarly implies that when forbearance sets are empty, $F(k', l'_R, \mu) = \emptyset$, the equilibrium forbearance probabilities $\rho_f(k', l'_R, z, \mu)$ equal zero. This is because with capital k' and loan repayment amount l'_R, a firm not choose to renegotiate their loan for any realization of the productivity shocks. The forbearance probabilities $\rho_f(k', l'_R, z, \mu)$ equal one when $F(k', l'_R, \mu) = Z$.

The expected recovery rate $\gamma(k', l'_R, z, \mu)$ is determined by:

$$\gamma(k', l'_R, z, \mu) = \frac{\int_{F\left(k', l'_R, \mu\right)} \psi(s') f(z', z)\, dz'}{\rho_f(k', l'_R, z, \mu)} \tag{18}$$

The numerator of Equation (18) is the proportion of the loan that the lender can expect to recover. The denominator of Equation (18) is the forbearance probability.

3.5.3 Firm-type Risk

We now consider the firm-type risk, which is a quantification of the information asymmetry faced by the lender. This is necessary for the zero profits condition. While lenders cannot observe μ, we assume they know: (a) the proportion of firms are low-quality versus high-quality, i.e. $\rho(\mu = \mu_b)$ and $\rho(\mu = \mu_g)$; and (b) they know the conditional distribution of z for a given μ.[6] As such, the lender can determine the probability that a firm has permanent productivity μ_b given z, $\rho(\mu = \mu_b|z)$, by using Bayes' Rule:

$$\rho(\mu = \mu_b|z) = \frac{\rho(z|\mu=\mu_b)\rho(\mu=\mu_b)}{\rho(z|\mu=\mu_b)\rho(\mu=\mu_b)+\rho(z|\mu=\mu_g)\rho(\mu=\mu_g)} \tag{19}$$

where $\rho(\mu = \mu_b)$ in Equation (19) is the proportion of low-quality firms, given by the cross-sectional stationary distributions of firms $\Gamma(s)$ such that:

$$\rho(\mu = \mu_b) = \int \Gamma(k, l_R, z; \mu = \mu_b)\, d(k \times l_R \times z). \tag{20}$$

$\rho(z|\mu = \mu_b)$ in Equation (19) the probability that the observed z is from a log normal distribution with mean μ_b. The initial firm productivity process is distributed such that $\ln(z) \sim \mathcal{N}(\mu, \sigma_\varphi^2)$ for $\sigma_\varphi^2 = \dfrac{\sigma_\epsilon^2}{1-\varphi^2}$. This initial process can be used to determine the initial conditional probabilities. But the conditional distribution for z is subsequently given by information from the cross-sectional stationary distributions of firms $\Gamma(s)$ and our assumption that the firm productivity process is log-normally distributed. The conditional distribution for z is approximated as follows. First, the mean of the firm productivity process for $\ln(z)$ given $\mu = \mu_b$ as:

$$E(\ln(z)|\mu = \mu_b) = \sum_i \ln(z_i)\, p_i \tag{21}$$

where the probability of a given z_i is $p_i = \frac{\int \Gamma(k,l_R,z_i,\mu=\mu_b)d(k \times l_R)}{\rho(\mu=\mu_b)}$.[7] The variance of the

[6] We denote to the proportion of firms that are low quality, as determined by the cross-sectional distribution of firms, $\rho(\mu = \mu_b)$. This differs from the aforementioned initial probability that a firm is low-quality, $\Pr(\mu = \mu_b) = 0.5$, which is applies to new entrants.

[7] In theory, $\ln(z)$ is a continuous random variable. Here we compute the empirical conditional mean and variance based on the discretized shock process, and so are using standard methods normally applied to compute the mean and variance of a discrete random variable.

process is:

$$Var\left(\ln\left(z\right)|\mu=\mu_b\right)=\sum_i\left(\ln\left(z_i\right)-E\left(\ln\left(z\right)|\mu=\mu_b\right)\right)^2 p_i. \tag{22}$$

Combined with our initial assumptions about the firm productivity process being log-normal, these results imply that the *conditional* distribution of the firm productivity process is $\ln\left(z\right)|\mu=\mu_b\sim\mathcal{N}\left(E\left(\ln\left(z\right)|\mu=\mu_b\right),Var\left(\ln\left(z\right)|\mu=\mu_b\right)\right)$. And so we can compute $\rho\left(\ln\left(z\right)|\mu=\mu_b\right)$ as:

$$\rho\left(z|\mu=\mu_b\right)=\rho\left(\ln\left(z\right)|\mu=\mu_b\right)=2\times\Phi\left(-\left|\frac{\ln\left(z\right)-E\left(\ln\left(z\right)|\mu=\mu_b\right)}{\sqrt{Var\left(\ln\left(z\right)|\mu=\mu_b\right)}}\right|\right) \tag{23}$$

where Φ is the standard normal cumulative distribution function. Expressions for $\rho\left(\mu=\mu_g\right)$, $\rho\left(z|\mu=\mu_g\right)$ and $\rho\left(\mu=\mu_g|z\right)$ are obtained similarly. In equilibrium, the probabilities $\rho\left(\mu=\mu_b|z\right)$ are consistent with the firm's decision rules, because they are based on the cross-sectional stationary distributions of firms $\Gamma\left(s\right)$, which is determined by the firm's decision rules. This means that the lender can price the firm-type risk such that the loan schedules are consistent with expected zero profits.

3.6 The Cross-Sectional Distribution of Firms

The law of motion for the cross-sectional distribution of firms $\Gamma\left(s\right)$ is:

$$\begin{aligned}\Gamma\left(s'\right)=\;&\int\rho_d\left(s\right)Q_e\left(s'\right)g\left(z'\right)\Gamma\left(s\right)d\left(k\times l_R\times z\times\mu\right)\\&+\int_{C\left(k',l'_R,\mu\right)}\left(1-\rho_d\left(s\right)\right)Q_c\left(s,s'\right)f\left(z',z\right)\Gamma\left(s\right)d\left(k\times l_R\times z\times\mu\right)\\&+\int_{F\left(k',l'_R,\mu\right)}\left(1-\rho_d\left(s\right)\right)Q_f\left(s,s'\right)f\left(z',z\right)\Gamma\left(s\right)d\left(k\times l_R\times z\times\mu\right)\end{aligned} \tag{24}$$

where $C\left(k',l'_R,\mu\right)$ is the continuing set, $F\left(k',l'_R,\mu\right)$ is the forbearance lending set, $\rho_d\left(k',l'_R,z,\mu\right)$ is the liquidation probability, and $Q_c\left(.\right)$ denotes a transition function for continuing firms, mapping the current states into future states, given by:

$$Q_c\left(s,s'\right)=\begin{cases}1 & \text{if } l'_R\left(s\right)=l'_R,k'\left(s\right)=k'\\0 & \text{otherwise}\end{cases} \tag{25}$$

and $l'_R\left(s\right)$ and $k'\left(s\right)$ are the optimal policy rules for firms. Similarly, $Q_f\left(s'\right)$ is 1 if the optimal choice for forbearance lending firms is $\left(k',l'_R\right)$ and 0 otherwise. And $Q_e\left(s'\right)$ is 1 if entrants optimal choice is $\left(k',l'_R\right)$ and 0 otherwise.

If an existing firm liquidates, a new project z drawn from the stationary distribution $g(z)$ becomes available to an entrant entrepreneur such that the mass of risky projects is equal to one.

3.7 Stationary Recursive Equilibrium

Here we define a stationary recursive equilibrium for the economy. Let $s = \{k, l_R, z, \mu\}$ be the aggregate states for the economy.

Definition 1. The stationary recursive equilibrium for this economy is a set of policy rules for (i) the firm's value function $V^*(s)$, capital holdings $k'^*(s)$, loan contracts $(l'^*(s), l_R'^*(s))$, and the default policy, where the default policy is comprised by the operating sets $C^*(k, l_R, \mu)$, liquidation sets $D^*(k, l_R, \mu)$, and forbearance lending sets $F^*(k, l_R, \mu)$; (ii) the recovery rate $\psi^*(s)$; (iii) the loan schedules $\Omega^*(k', z)$ for the loan contracts; and (iv) the cross-sectional distribution of firms $\Gamma^*(s)$ such that:

1. Given the loan schedule $\Omega^*(k', z)$ and the loan recovery rate $\psi^*(s)$: the value function $V^*(s)$, the policy rules $k'^*(s)$ and $(l'^*(s), l_R'^*(s))$, operating sets $C^*(k, l_R, \mu)$, liquidation sets $D^*(k, l_R, \mu)$, and forbearance lending sets $F^*(k, l_R, \mu)$ are consistent with the firm's optimization problem in Equation(3).

2. Given the loan schedule $\Omega^*(k', z)$ and value function $V^*(s)$: the loan recovery rate $\psi^*(s)$ solves the forbearance problem in Equation (13).

3. Given the loan recovery rate $\psi^*(s)$: the equilibrium loan schedules $\Omega^*(k', z)$ satisfy the lender's expected zero profit condition, and also the liquidation probability $\rho_d^*(k', l_R', z, \mu)$, the forbearance probability $\rho_F^*(k', l_R', z, \mu)$, the expected recovery rate $\gamma^*(k', l_R', z, \mu)$, and the probability that a firm has permanent productivity μ_j given it has observed productivity z $\rho^*(\mu = \mu_j | z)$ are consistent with the firm's default policy and renegotiation agreement.

4. The cross-sectional distribution of firms $\Gamma^*(s)$ is a stationary measure of firms consistent with the firm decision rules and the law of motion for the stochastic variables.

4 Quantitative Analysis

4.1 Calibration

To consider the quantitative implications of the model, we set a model period to be one year and calibrate the model to match euro area data over the period 2011 to 2014. During this period, there was a high prevalence of forbearance lending (Acharya et al., forthcoming) and low output growth . Table 1 provides a summary of definitions that we use to compute key variables from our model.

As previously noted, we assume that firm productivity is a log-normal AR(1) process: $\ln(z_{jt}) = \mu_j(1-\varphi) + \varphi \ln(z_{t-1}) + \epsilon$, with $|\varphi| < 1$ and $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$. There are two firm types i, which differ according to the permanent productivity parameter μ_i. Low-quality firms have a permanent productivity μ_b and high-quality firms have a permanent productivity μ_g, such that $\mu_b < \mu_g$. Entrant firms draw their initial permanent productivity μ_j from a Bernoulli distribution with $\Pr(\mu_j = \mu_b) = \Pr(\mu_j = \mu_g) = 0.5$. We use the procedure of Tauchen (1986) to discretize the stochastic shock into a 10-state Markov chain.

The model comprises 15 parameters to be calibrated, which are summarized in Table 2. We first select the 11 "Calibrated Parameters" in Table 2 independently of the model equilibrium. To start with, the firm discount rate β is set to 0.96, which is standard for an annual RBC model according to Arellano et al. (2012). The risk-free interest rate r is set to 0.01, which is equal to the average interest rate of one-year euro area government bonds for the period 2011 to 2014.

We next use a sample of euro area firms over the period 2011 to 2014 to estimate several other parameters. A full description of these data is contained in Appendix B. The capital depreciation rate δ is set to 16 per cent per year, which equals the average depreciation rate for our sample. The returns to scale parameter α is set to 0.65, which implies decreasing returns to scale in production, similar as in the Arellano et al. (2012) study based on European firms. The equity issuance cost γ is set to 0.30, following Cooley and Quadrini (2001). The capital loss in liquidation χ_d and the firm bargaining power are both set to 0.40, which coincides with the estimates of Ramey and Shapiro (2001) for the capital loss in liquidation. The autocorrelation φ and

Table 1: Model Definitions for Key Variables

Variable	*Model Definition*
Assets	k
Loan repayment	l_R
Leverage	l_R/k
Loan interest rate	$l'_R/l' - 1$
Investment rate	$(k' - (1-\delta)k)/k^-$
Sales	zk^α
Sales growth	$\left(zk^\alpha - z^-(k^-)^\alpha\right)/z^-(k^-)^\alpha$
Working capital	$(k - l_R)/k^-$

Table 2: Benchmark Parameters

Calibrated Parameters		*Value*
Firm discount rate	β	0.96
Risk-free interest rate	r	0.01
Capital depreciation rate	δ	0.16
Returns to scale	α	0.65
Equity issuance cost	γ	0.30
Capital loss in liquidation	χ_d	0.40
Firm bargaining power	θ	0.40
Autocorrelation of stochastic shock	φ	0.9
Standard deviation of stochastic shock	σ_ϵ	0.118
Low-quality firm permanent productivity	μ_b	0
High-quality firm permanent productivity	μ_g	0.448
Parameters Estimated with SMM		*Value*
Capital adjustment cost	ϕ	0.15
Lender credit cost	ξ	7.00
Fixed operating cost	χ_c	10.75
Renegotiation cost	χ_f	0.20

standard deviation σ of the stochastic shock are set to 0.90 and 0.118, respectively, which follow Corbae and D'Erasmo (2017).

Finally, we use our sample of euro area firms to set the value of the low and high permanent productivity shocks, μ_b and μ_g. To do this, we first divide our sample into low-quality firms and high-quality firms based on the definition of Acharya et al. (forthcoming). This involves computing for each firm the average interest coverage ratio over the period 2009 to 2011, defined as the ratio of interest expenses to EBITDA (that is, earnings before interest, taxes and amortization). A firm is then categorized as low-quality if its three-year average interest coverage ratio is below the country median and otherwise the firm is considered high-quality. Using these categories, we find the average yearly return on assets for high-quality firms over the period 2011 to 2014 to be around 57 per cent higher than for low-quality firms. Thus, we set μ_b to zero and μ_g to 0.448, which ensures that the average productivity for high-quality firms is around 57 per cent higher than for low-quality firms.

Subsequently, we estimate the remaining four parameters in Table 2 by using the simulated method of moments (SMM) to target a set of five moment conditions from the data. More precisely,

$$\hat{\Theta} = \arg\min_{\Theta} \left[m^d - m^s(\Theta)\right]' W \left[m^d - m^s(\Theta)\right] \tag{26}$$

Table 3: Statistical Moments in the Euro Area Data and in the Benchmark Model

Targeted Moments	*Data*	*Benchmark*
Firm liquidation rate	3.44%	4.45%
Firm forbearance rate	7.5%	6.79%
Average of sales growth	5.86%	2.00%
Standard deviation of sales growth	16.62%	19.54%
Average of leverage	66.46%	68.69%
Non-targeted Moments	*Data*	*Benchmark*
Standard deviation of leverage	21.70%	268.59%
Average of working capital	23.73%	29.99%
Standard deviation of working capital	21.68%	257.72%

Note: The column under Data reports the statistical moments for the euro area, from 2011 to 2014. The column under Benchmark reports the model statistics under the benchmark calibration, in which firms have access to forbearance lending. All statistical moments are calculated over a one-year period.

where W is a weighting matrix[8], $\Theta = (\phi, \xi, \chi_c, \chi_r)$ are the four parameters to be estimated, m^d are the five targeted moments from the data summarized in the "Data" column of Table 3 and discussed below, and $m^s(\Theta)$ are the five simulated moments from the model at parameters Θ.

Our first target moment is the firm liquidation rate. An estimate for the yearly average of this moment over the period 2011 to 2014 is 3.44 per cent, according to the default rate by value of European corporates (including non-rated corporates) of Standard & Poor's Rating Services (2014, 2016).

Our second target moment is the firm forbearance rate. The yearly average value of this moment over the period 2011 to 2014 is approximately 7.5 per cent, according to estimates of Acharya et al. (forthcoming). These estimates are based on the asset-weighted proportion of European firms that are classified as "zombie firms".[9] Their sample of firms comprises all privately and publicly traded firms from all EU counties.

Our remaining three target moments are similar to several moments of Arellano et al. (2012) for their firm equilibrium model: the average and standard deviation of sales growth, and the average of leverage. We obtain these moments from our sample of euro area firms over the period 2011 to 2014. A description of how we compute each data moment is also contained in Appendix B.

All of the estimated parameters affect all of the target moments in the model, although some parameters affect some moments more directly. In particular, χ_c is

[8]We employ the identity matrix as our weighting matrix.

[9]There are three criteria to classify a firm as a "zombie firm", one being that they estimate the firm is in receipt of subsidized bank credit.

useful to match the firm liquidation rate; χ_f is useful to match the firm forbearance lending rate; ϕ and χ_c is useful to match the average and standard deviation of sales growth; and ξ is useful to match the average of leverage.

We assess the quantitative performance of the model in Table 3 by comparing the moments from the euro area data in the "Data" column with those produced by the model in the "Benchmark" column. We describe how we compute the statistical moments for the data in Appendix B. We compute the statistical moments for the model based on the definitions outlined in Table 1.

Overall, the simulated method of moments estimates a set of four parameter values that produce model moments that are close to the five targeted moments, although it under-predicts the average of sales growth. We also consider how the simulated model can match several non-targeted moments in Table 3. The model provides a good match to the working capital as observed in the data. Conversely, the model greatly overestimates the standard deviation of leverage and working capital as observed in the data. This is a general feature of models that include loan renegotiation, because there is a large difference between the leverage of firms that engage in loan renegotiation versus those that do not. By contrast, models with liquidation only are able to match the standard deviation of leverage rather well (see, e.g., Arellano et al., 2012).

4.2 Equilibrium Loan Schedules and Firm Decisions

Here we consider the equilibrium default policies, loan schedules and policy functions for the fitted model. We first consider firm liquidation risk. Figure 1 plots the liquidation probabilities as a function of the capital choice k'. Panel (A) of Figure 1 plots the liquidation probabilities for a high-quality firm (with permanent productivity μ_g) and a low-quality firm (with permanent productivity μ_b) when there is a normal stochastic shock ε_m and the loan repayment choice l'_R equals the average equilibrium choice for all firms (i.e. across both the low-quality and high-quality firms). The horizontal axis is scaled by the average equilibrium capital $\bar{k}$ for all firms.

For both types of firms, the liquidation probability is lower for a firm with more capital k, because more capital makes liquidation less valuable. Similarly, the liquidation probability is lower for the high-quality firm than for the low-quality firm because liquidation is less valuable for the former. For high capital and all but the worst stochastic shocks, the liquidation probability is close to zero for high-quality firms.

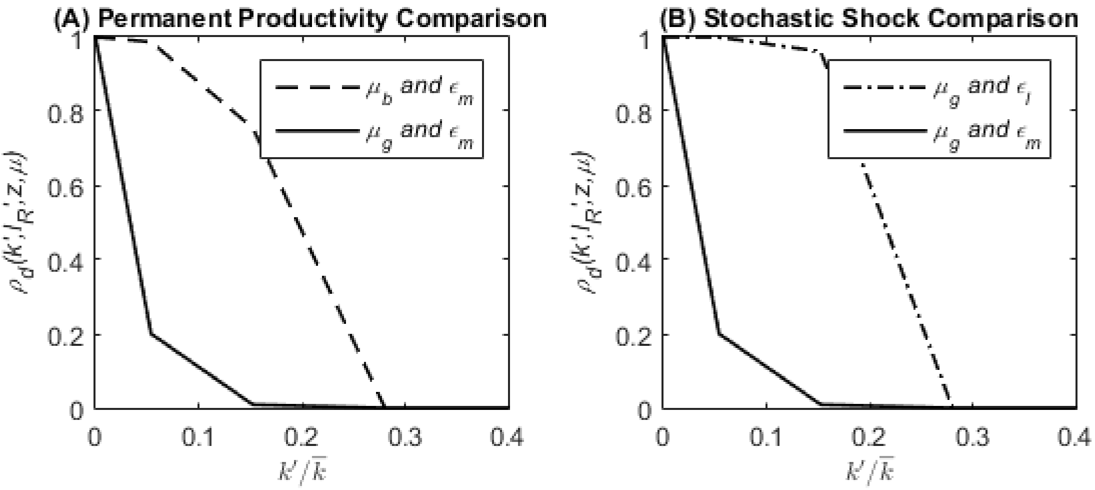

Figure 1: Liquidation Probabilities

Note: Panel (A) plots the liquidation probabilities as a function of the capital choice k' for two firms, one "high-quality" firm (with permanent productivity μ_g) and one "low-quality" firm (with permanent productivity μ_b), when there is a normal stochastic shock ε_m and the loan repayment choice l'_R equals the average equilibrium choice for all firms. Panel (B) plots the liquidation probabilities as a function of the capital choice k' for two firms with a low stochastic shock ε_l and a normal stochastic shock ε_m when both firms are high-quality (with permanent productivityμ_g) and the loan repayment choice l'_R equals the average equilibrium choice for all firms. The horizontal axes for capital choice are scaled by the average equilibrium capital $\bar{k}$.

Panel (B) of Figure 1 plots the liquidation probabilities for two firms with a low stochastic shock ε_l and a normal stochastic shock ε_m when both firms are high-quality and the loan repayment choice l'_R equals the average equilibrium choice for all firms. The liquidation probability is lower for the normal (or around the medium) stochastic shock ε_m than the low stochastic shock ε_l because liquidation is less valuable for that firm. In general, liquidation is more valuable, and so liquidation risk is higher, for a firm facing a lower stochastic shock.

We next consider the firm forbearance lending risk. Figure 2 plots the forbearance lending probabilities as a function of the loan repayment choice l'_R scaled by the average equilibrium capital $\bar{k}$. Panel (A) of Figure 2 plots the forbearance lending probabilities for two firms: one with low capital k_l and the other with high capital k_h. Both firms are high-quality, they face a normal stochastic shock ε_m and their capital choice k' is equal to the average equilibrium capital $\bar{k}$. The forbearance lending probability is higher for firms with lower levels of physical capital. The forbearance lending probability also higher for firms with larger loan repayment choices.

Panel (B) of Figure 2 plots the forbearance lending probabilities for two firms, one high-quality firm and one low-quality firm, when there is a normal stochastic shock ε_m and the capital choice k' equals the average equilibrium capital $\bar{k}$. The

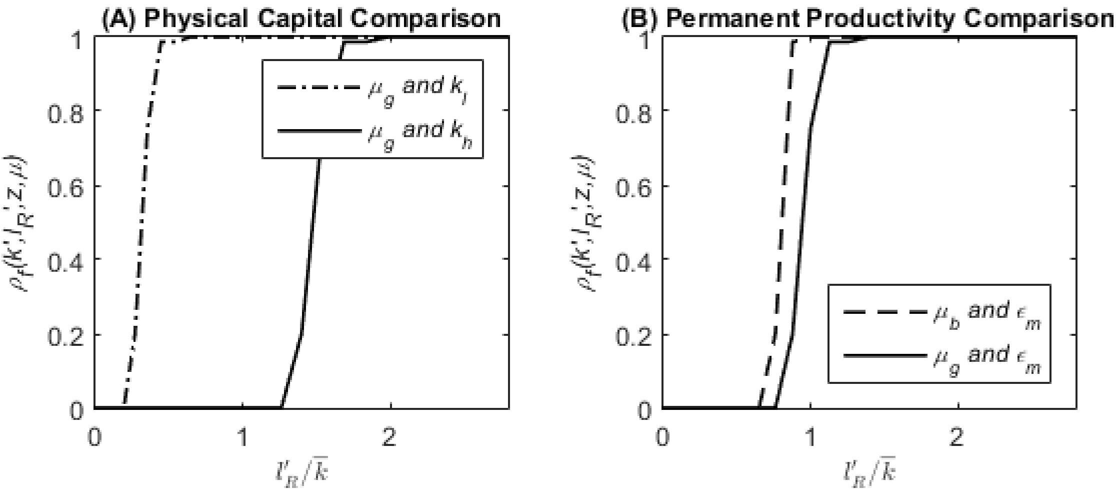

Figure 2: Forbearance Lending Probabilities

Note: Panel (A) plots the forbearance lending probabilities as a function of the loan repayment choice l_R' scaled by the average equilibrium capital $\bar{k}$ for two firms with low capital k_l and high capital k_h, when both firms are high-quality (with permanent productivityμ_g), face a normal stochastic shock ε_m and the capital choice k' is equal to the average equilibrium capital, $\bar{k}$. Panel (B) plots the forbearance lending probabilities as a function of the loan repayment choice l_R' scaled by the average equilibrium capital $\bar{k}$ for two firms, one high-quality firm (with permanent productivity μ_g) and one low-quality firm (with permanent productivity μ_b), when there is a normal stochastic shock ε_m and the capital choice k' equals the median value. The horizontal axes for the loan repayment choice l_R' are scaled by the average equilibrium capital $\bar{k}$.

forbearance lending probability is greater for low-quality firms than for high-quality firms. The high-quality firms are less likely to choose forbearance lending, even if the loan renegotiation may generate some loan reduction. The same trend holds when both types of firms face either a low stochastic shock or a normal stochastic shock, i.e. the forbearance lending probability is higher for low-quality firms than for high-quality firms. Although not depicted here, the forbearance lending probability is higher for lower stochastic shocks, holding all else equal. As with liquidation, forbearance lending is more valuable for low-quality firms, and for firms subject to lower stochastic shocks, as well as for firms with larger repayment choices.

Prior to a consideration of the equilibrium loan schedule, we consider the equilibrium loan recovery rate for forbearance lending. Panel (A) of Figure 3 plots the equilibrium recovery rates as a function of the outstanding loan repayment l_R scaled by the average equilibrium capital $\bar{k}$ for two firms, one high-quality firm and one low-quality firm, when there is a normal stochastic shock ε_m and with capital k equal to the average equilibrium capital, $\bar{k}$.

A firm with a small loan repayment will receive no reduction to its loan. Subsequently, the loan recovery rate decreases as the loan repayment size increases. The

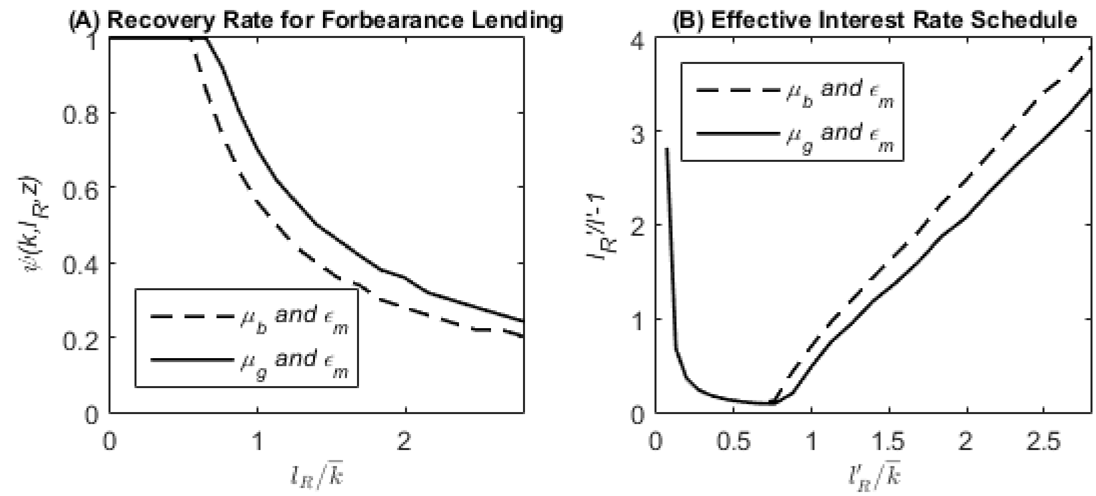

Figure 3: Recovery Rate and Effective Interest Rate Schedule

Note: Panel (A) plots the equilibrium recovery rates. Panel (B) plots the effective interest rate ($l_R'/l' - 1$) for every contract (l', l_R'). Both plots are as a function of the loan repayment choice l_R' scaled by the average equilibrium capital $\bar{k}$ and for two firms, one high-quality firm (i.e. with permanent productivity μ_g) and one low-quality firm (with permanent productivity μ_b), when there is a normal stochastic shock ε_m and the capital choice k' is equal to the average equilibrium capital $\bar{k}$. The horizontal axes are scaled by the average equilibrium capital $\bar{k}$.

loan recovery rate is also higher for high-quality firms than for low-quality firms, all else equal. Similarly, and although not depicted here, the loan recovery rate is higher for firms with a more favorable stochastic shock. For the same loan size, a firm that obtains loan forbearance with a more favorable stochastic shock needs to pay back a higher proportion of the loan.

We now consider the equilibrium loan schedules that arise due to liquidation risk, forbearance risk and firm-type risk. Panel (B) of Figure 3 plots the effective interest rate ($l_R'/l' - 1$) for every contract (l', l_R') as a function of the loan repayment choice l_R' scaled by the average equilibrium capital $\bar{k}$ for the same two firms as in Panel (A). The plot shows that low-quality firms attract higher interest rates. This is due to both the higher liquidation risk and the lower recovery rate or a low-quality firm. In general, a higher loan reduction, as is the case for a low-quality firm, as well as a firm experiencing a bad stochastic shock, can increase a firm's *ex ante* forbearance incentive. But the lender anticipates this when pricing the loan schedule, which in turn offsets forbearance incentives. We also note that the effective interest rate increases with loan repayment size. Yet, small loans also face a high effective interest rate due to the fixed cost of lending.

To consider the firm dynamics of the model, we turn to the policy rules. Figure 4 plots the decision rules as a function of initial capital k for two firms, one high-

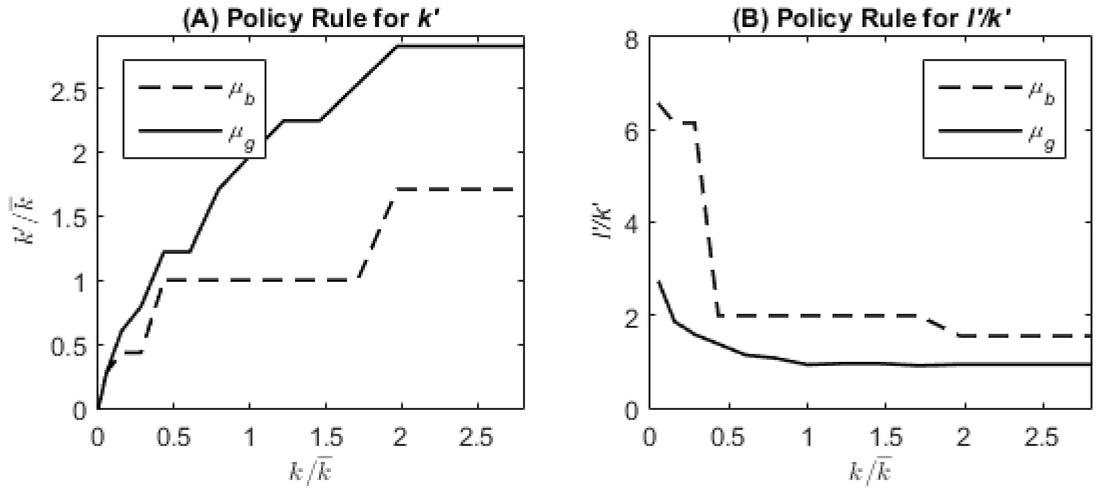

Figure 4: Policy Rules

Note: Panel (A) plots the optimal capital choice k'. Panel (B) plots the optimal loan choice relative to the capital choice l'/k'. All plots as a function of initial capital k for two firms, one high-quality firm (with permanent productivity μ_g) and one low-quality firm (with permanent productivity μ_b), when there is a high stochastic shock ε_h and the loan repayment l_R equals the average equilibrium choice for all firms. The horizontal axes and vertical axis of Panel (A) are scaled by the average equilibrium capital $\bar{k}$.

quality firm and one low-quality firm, when there is a high stochastic shock ε_h and the loan repayment l_R equals the average equilibrium choice for all firms. Panel (A) plots the optimal capital choice k'. Panel (B) plots the optimal loan choice relative to the capital choice l'/k'.

Figure 4 shows that with higher initial capital, the firm chooses a larger future capital stock and a smaller future loan. The smaller loan choice is due to a "precautionary motive"; the firm does not find it optimal to fully utilize its borrowing opportunities, because large loans increase liquidation risk and forbearance risk, as well as the risk of costly equity issuance (Arellano et al., 2012). With lower levels of capital, the firm decreases its capital and increases its leverage (Panel (B)).

Loan contracts also influence the way in which firms respond to stochastic shocks. Related to this, Arellano et al. (2012) note that when firms experience a sequence of low shocks, they will reduce their size in order to avoid equity issuance costs, and increase their loan financing, thus pushing up their leverage and their effective interest rate schedules. Conversely, when they experience a sequence of good shocks, they will enlarge their size and reduce their loan financing, which lowers their effective interest rate schedules. Additionally, low-quality firms tend to be smaller and more levered than high-quality firms because their liquidation risk is higher, so they have more incentives to reduce scale their in order to avoid equity issuance costs.

Panel (B) of Figure 4 shows that high-quality firms tend to be less levered than low-quality firms. In general, this lowers the liquidation risk and forbearance risk of high-quality firms, as shown in Figure 1 and Figure 2, respectively. And more generally, the loan contracts appear to separate types in equilibrium, as can be seen by the different effective interest rates offered to high-quality firms in Panel (B) of Figure 3. Although, where the liquidation and liquidation risk is low, there appears to be some pooling contracts in equilibrium. This can be seen in Panel (B) of Figure 3, where for smaller future loan choices, the effective interest rate schedule is identical for high-quality and low-quality firms.

4.3 Benchmark Model Results

We now consider the statistical moments for the benchmark model, organized by firm operating status, as reported in Table 4. All model moments are computed from a simulation of the model economy with 1,000 firms over 500 periods.[10] Statistics for all firms in Column (1) of Table 4 are based on any firm operating in a period. The continuing firm statistics in Column (2) are based on firms that have existed for more than three periods and currently do not receive to loan forbearance. The forbearance firm statistics in Column (3) are based on firms that have existed for more than three periods and currently receive loan forbearance. The liquidating firm statistics in Column (4) are based on firm statistics in the period preceding firm liquidation. The entrants firm statistics in Column (5) are based on firms that have existed for three periods or less.

We first consider the continuing firms in Column (2) of Table 4. Almost all continuing firms are high-quality firms. One reason for this observation is that high-quality firms are more likely to make a successful entry. This is because high-quality firms are more likely to have higher overall productivity, which makes these firms better able to enter and not immediately liquidate. And this differential in successful entry by firm-type contributes to continuing firms being predominately high-quality. Continuing firms also tend to be larger with lower leverage due to the aforementioned precautionary motives, in which the continuing firms do not find it optimal to exhaust all borrowing opportunities due to the associated increase in liquidation risk and forbearance risk.

We next consider the firms in receipt of forbearance lending in Column (3) of Table 4. Only 0.13 per cent of firms that receive forbearance lending are high-

[10] In fact, we simulate the model over 750 periods, and disregard the first 250 periods as the model burn-in.

Table 4: Quantitative Model Results by Operating Status

	(1) *All Firms*	(2) *Continuing*	(3) *Forbearance*	(4) *Liquidating*	(5) *Entrants*
Average of:					
Sales growth	2.00 %	1.07 %	-0.74 %	-10.75 %	15.18 %
Leverage	68.69 %	12.37 %	770.34%	250.86 %	149.26 %
Investment rate	18.98 %	17.65 %	14.17%	-38.82 %	38.40 %
Capital	126.52	139.32	64.42	43.26	27.62
Loan repayment	54.06	40.29	268.11	59.28	38.78
Firm age	46.72	52.89	12.76	22.47	2.37
Total factor productivity	1.74	1.79	1.41	0.98	1.63
Proportion of firms:					
In given category	100.00 %	88.77 %	6.79 %	4.45 %	4.45 %
High-quality, i.e. $\mu = \mu_g$	90.28 %	99.65 %	0.13 %	50.02 %	54.69 %

Note: Column (1) reports model statistics for all firms operating with age greater than one. Column (2) reports model statistics for continuing firms, defined as any firm that has existed for more than three periods and does not currently receive loan forbearance. Column (3) reports model statistics for forbearance firms, defined as any firm that has existed for more than three periods and currently receives loan forbearance. Column (4) reports model statistics for liquidating firms, based on the firm statistics in the period preceding firm liquidation. Column (5) reports model statistics for entrant firms, defined as any firm that has existed two periods or less. All model statistics are for the benchmark calibration, in which firms have access to forbearance lending.

quality firms. The reason that the majority of firms in receipt of forbearance lending are low-quality relates to the information asymmetry. Specifically, *ceteris paribus* low-quality firms face a higher effective interest rate schedule than high-quality firms due to their higher liquidation and forbearance risk, and lower recovery rates (Figure 3). But the difference between the interest rate schedules of the high-quality firms and low-quality firms is not very large. This is because the lender cannot perfectly predict whether a given firm is high-quality or low-quality, but assigns a probability to a firm being low or high-quality. The end result is that the lender offers interest rate schedules that separates firms by type. But the interest rate schedule is more favorable for low-quality firms than it would have been under perfect information.[11] The relatively lower effective interest rate schedule makes forbearance lending *ex ante*

[11] To expand on this, when a high-quality firm facing a lower stochastic shock has the same overall productivity as a low-quality firm experiencing a higher stochastic shock, the lender will offer both firms the same interest rate schedule. But if a high-quality firm and a low-quality firm both experience the *same* stochastic shock, the overall productivity of the high productivity firm will be higher, and so the interest rate schedule will be more favorable for this firm. But it will still be less favorable than in the perfect information world, because the lender will still assign some probability to the high-quality firm being low-quality. The size of this probability will depend on the overall productivity, where lower values are assigned a higher probability of being low-quality firm.

a more valuable choice for low-quality firms. And vice versa for high-quality firms. As such, the majority of firms in receipt of forbearance lending are low-quality.

The fact low-quality firms have lower overall productivity, on average, also increases their likelihood of receiving loan forbearance. This is because firms facing lower overall productivity, due to a bad stochastic shock or being low-quality or both, are more likely to opt for forbearance lending arrangements as continuing to operate is less valuable to them. The low overall productivity is consistent with the high level of leverage observed for the forbearance firms: as mentioned above, when faced with a sequence of bad shocks, firms will reduce their scale in order to avoid costly equity issuance, and increase their loan financing, thereby pushing up their leverage and effective interest rates.

There are two factors that explain why firms in receipt of forbearance lending have lower sales growth. First, these firms are often experiencing a sequence of bad shocks, and this itself will lower sales growth. Second, these firms reduce their scale in response to the sequence of bad shocks, which further contributes to lower sales growth.

We now consider the liquidating firms in Column (4) of Table 4. These firms are equally likely to be high-quality or low-quality firms, although the path to liquidation differs by firm-type. High-quality firms tend to liquidate after being in the market for some time; their liquidation occurs after they experience a sequence of bad stochastic shocks at some point. Low-quality firms, on the other hand, often liquidate quite quickly, after a few periods only. In other instances, low-quality firms survive for around 12 years due to their access to forbearance lending, but at some point they also experience a sequence of shocks that makes liquidation a more valuable option than forbearance lending. For both firm-types, firms have high leverage and negative sales growth in the period before they liquidate, which is as we would expect for any firm experiencing a sequence of bad stochastic shocks.

Finally, we consider the entrant firms in Column (5) of Table 4. These firms are smaller and have higher sales growth rates, which is consistent with the findings of Arellano et al. (2012) for firm entrants.

In summary, the evidence in Table 4 suggests that the nearly all firms in receipt of forbearance lending are low-quality. This is consistent with the empirical evidence of Acharya et al. (forthcoming), who find that low-quality firms are more likely to be in receipt of subsidized loans. In the next section we quantify the impact of this forbearance lending on firm dynamics and aggregate outcomes.

5 The Quantitative Impact of Forbearance Lending

We consider the quantitative affects of forbearance lending by conducting two counterfactual experiments. We first consider a counterfactual experiment in which the possibility of forbearance lending is shut down and so firms can only choose between continuing or liquidation.[12] This experiment aims to show how forbearance lending practices affect firm dynamics and aggregate outcomes. We next consider the impact of information asymmetry by conducting a counterfactual experiment in which the lender can identify firm-type. Below we outline each counterfactual scenario more formally and consider the results of each experiment in turn.

5.1 No Forbearance Lending Scenario

For the first experiment of no forbearance lending, the value function of a firm that has the option to default and that starts the current period with capital k, a loan repayment l_R and productivity z is:

$$V\left(k, l_R, z, \mu\right) = \max_{\{c,d\}} \left\{V_c\left(k, l_R, z, \mu\right), V_d\left(k, l_R\right)\right\}, \tag{27}$$

where $V_c\left(k, l_R, z, \mu\right)$ is the value function if the entrepreneur does not default and continues to operate, as defined by Equation (4); and $V_d\left(k, l_R\right)$ is the value function if the entrepreneur defaults and liquidates the firm, as defined by Equation (5). Thus, the key departure from the benchmark model is that the value associated with forbearance, $V_f\left(k, l_R, z, \mu\right)$, no longer enters the value function as an option.

Similar to the problem above, an idle entrepreneur will attempt to start a new firm when she receives a project opportunity, which occurs when another entrepreneur liquidates their firm. The entrant problem is defined by Equation (10).

In this scenario, the equilibrium loan schedule $\Omega\left(k', z\right)$ that comprises all loan contracts $\left(l', l'_R\right)$ allowing the lender to break even in expected values, and accounting for the liquidation risk that the lender faces, is described by:

$$\begin{aligned} l' = \textstyle\sum_{j=b,g} \rho\left(\mu = \mu_j | z\right) \quad & \left\{\tfrac{1}{1+r+\xi}\left[1 - \rho_f\left(k', l'_R, z, \mu_j\right) - \rho_d\left(k', l'_R, z, \mu_j\right)\right] l'_R \right. \\ & \left. -\tfrac{1}{1+r+\xi}\rho_d\left(k', l'_R, z, \mu_j\right) \min\left\{\chi_d k', l'_R\right\}\right\} \end{aligned} \tag{28}$$

To generate the statistical moments for the benchmark model and for the counterfactual experiments, we employ the definitions outlined in Table 1. For the corresponding aggregate statistics, we compute aggregate investment, I, aggregate output,

[12] Tamayo (2015) conducts a similar counterfactual exercise to examine the impact of different US bankruptcy options.

Y, aggregate capital, K, and aggregate total factor productivity, TFP, as follows:

$$\begin{aligned} I &= \int i(s)\Gamma(s)d\left(k \times l_R \times z \times \mu\right) \\ Y &= \int y(s)\Gamma(s)d\left(k \times l_R \times z \times \mu\right) \\ K &= \int k(s)\Gamma(s)d\left(k \times l_R \times z \times \mu\right) \\ TFP &= \frac{Y}{K^{\alpha}} \end{aligned} \tag{29}$$

where $s = (k, l_R, z, \mu)$ are the states, $i = k' - (1 - \delta)\, k$ is firm investment, $y = zk^{\alpha}$ is firm output as measured by sales, and k is firm capital.

Table 5 presents statistical moments and aggregates for the benchmark model and for the counterfactual experiments, in which Column (2) summarizes the results for the specific counterfactual experiment of no forbearance lending. Table 6 provides a more granular breakdown of the results by firm-type. On the basis of these results, we can conclude that with no forbearance lending: firm liquidation, average sales growth, aggregate total factor productivity (TFP), aggregate investment, and aggregate output are all higher, while average leverage leverage is lower. We discuss these results in more detail below.

First, we focus on the liquidation rate. Figure 5 plots the liquidation probabilities as a function of the capital choice k' for two firms, one high-quality firm (with high permanent productivity μ_g) and one low-quality firm (with permanent productivity μ_b), when there is a normal stochastic shock ε_m and the loan repayment choice l'_R equals the average equilibrium choice for all firms in the benchmark model. Panel (A) is for the benchmark model and Panel (B) is for the counterfactual experiment without forbearance lending. Panel (C) will be discussed below in Section 5.2.

Table 5 shows that the liquidation rate is around 6.97 per cent higher in the counterfactual experiment with no forbearance lending. Similarly, Figure 5 shows that the liquidation risk is higher for otherwise identical firms. But the liquidation rate in this counterfactual experiment is not significantly higher than for the benchmark model; that is, it does not increase to the same extent that forbearance lending occurs in the benchmark scenario. This is because in the absence of forbearance lending, firms make different equilibrium choices for their capital and loans, which offsets any increase in liquidation risk.

Next we consider the equilibrium loan schedules that arise in the counterfactual scenario of no forbearance lending. Figure 6 plots the effective interest rate $(l'_R/l' - 1)$ for every contract (l', l'_R) as a function of the loan repayment choice l'_R (scaled by the average equilibrium capital $\bar{k}$ for the benchmark model) for two firms, one high-quality firm and one low-quality firm, when both firms face a normal stochastic

Table 5: Statistical Moments and Aggregates in the Benchmark Model and for the Counterfactual Experiments

	(1)	(2)	(3)
Targeted Moments	*Benchmark*	*No Forbearance*	*Perfect Information*
Firm liquidation rate	4.45%	4.76%	5.14%
Firm forbearance rate	6.79%	0.00%	0.03%
Average of sales growth	2.00%	2.27%	3.92%
Standard deviation of sales growth	19.54%	19.40%	24.03%
Average of leverage	68.69%	18.55%	13.10%
Non-targeted Moments and Aggregates	*Benchmark*	*No Forbearance*	*Perfect Information*
Standard deviation of leverage	268.59%	41.48%	31.65%
Average of working capital	29.99%	78.21%	91.14%
Average of investment rate	18.98 %	19.69 %	25.41 %
Average of total factor productivity	1.74	1.76	1.79
Aggregate investment	1.000	1.084	1.091
Aggregate total factor productivity	1.000	1.008	1.016
Aggregate output	1.000	1.047	1.050

Note: The column under Data reports the statistical moments for the euro area, from 2011 to 2014. Column (1) reports model statistics under the benchmark calibration, in which firms have access to forbearance lending. Column (2) reports model statistics for the counterfactual experiment in which firms no longer have access to forbearance lending. Column (3) reports model statistics for the counterfactual experiment in which there is no information asymmetry and so lenders can observe firm-type. Aggregate statistics for output, investment and total factor productivity are scaled by the Benchmark Model statistics, and so equal to one for the Benchmark Model. All statistical moments and aggregates are calculated over a one-year period.

shock ε_m and the capital choice k' is equal to the average equilibrium capital $\bar{k}$ for the benchmark model. Panel (A) is for the benchmark model and Panel (B) is for the counterfactual experiment without forbearance lending. We will return to Panel (C) below in Section 5.2.

The equilibrium loan schedule is higher, on average, for firms in the counterfactual experiment than for the benchmark model. This is partly due to the higher liquidation risk of firms in the counterfactual scenario (see Figure 5). But the more significant driver of the higher equilibrium loan schedule is the absence of forbearance lending. Forbearance lending risk is priced into the effective loan schedule. But relative to liquidation risk, forbearance lending risk can in fact contribute to more favorable borrowing rates when the associated recovery rate is high because the lender is able to recuperate a much larger proportion of the loan repayment.

The loan schedule results also relate to our observations for overall average leverage. Table 5 shows that overall average leverage is around 72.99 per cent lower in the counterfactual experiment without forbearance lending. Once more, this is due to

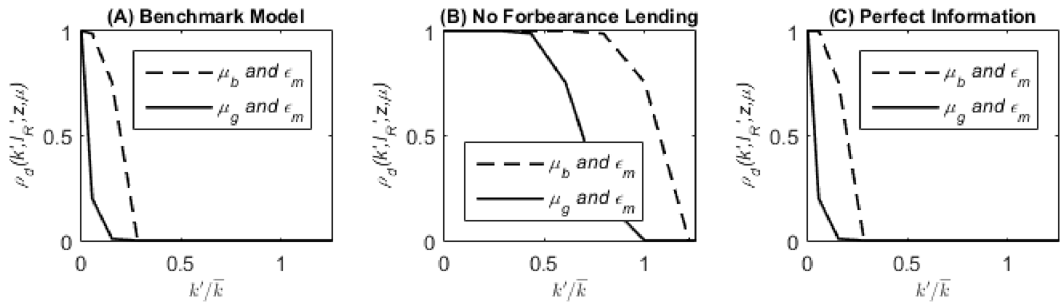

Figure 5: Liquidation Probabilities

Note: Panel (A) is for the benchmark model with forbearance lending. Panel (B) is for the counterfactual experiment with no forbearance lending. Panel (C) is for the counterfactual scenario with perfect information. All figures plot the liquidation probabilities as a function of the capital choice k' for two firms, one high-quality firm (i.e. with high permanent productivity μ_g) and one low-quality firm (i.e. with permanent productivity μ_b), when there is a normal stochastic shock ε_m and the loan repayment choice l_R' equals the average equilibrium choice for all firms in the benchmark model. The horizontal axes for capital choice are scaled by the average equilibrium capital $\bar{k}$.

the removal of the option for forbearance lending. The high leverage of low-quality firms in receipt of forbearance lending drives up the average leverage in the benchmark model. Additionally, the higher loan schedule in the counterfactual scenario of Panel (B) in Figure 6 also disincentivizes firms to take on high levels of leverage.

Table 5 also shows that overall average sales growth is around 13.50 per cent higher in the counterfactual experiment without forbearance lending. The removal of the option of forbearance lending is a key driver of this result. Without access to forbearance lending, the low-quality firms default soon after they enter. As such, the overall proportion of high-quality firms is higher in the counterfactual scenario, i.e. 96.94 per cent of all firms are high-quality versus 90.28 per cent in the benchmark model. A consequence of a higher proportion of high-quality firms is that average sales growth is higher. Low-quality firms in receipt of forbearance lending are no longer lowering the overall average sales growth, as in the benchmark model.

Similarly, aggregate investment is around 8.44 per cent higher in the counterfactual exercise as compared with the benchmark model, as can be seen in Table 6, and aggregate output is around 4.65 per cent higher. Aggregate total factor productivity (TFP) is also a little higher. These differences in the aggregate statistics are, once more, due to the removal of the option for forbearance lending and the resulting reduction in the proportion of low-quality firms.

It is worth noting that we find evidence of congestion effects in our model, consistent with the findings of Acharya et al. (forthcoming) for European firms and Caballero et al. (2008) for Japanese firms. This can be seen in Column (5) of Table

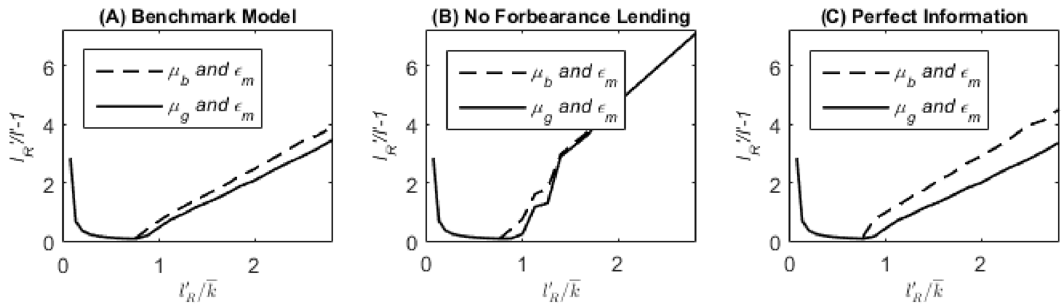

Figure 6: Effective Interest Rate Schedules

Note: Panel (A) is for the benchmark model with forbearance lending. Panel (B) is for the counterfactual experiment with no forbearance lending. Panel (C) is for the counterfactual scenario with perfect information. All figures plot the effective interest rate ($l'_R/l' - 1$) for every contract (l', l'_R). Both plots are as a function of the loan repayment choice l'_R (scaled by the average equilibrium capital $\bar{k}$) and for two firms, one high-quality firm (i.e. with high permanent productivity μ_g) and one low-quality firm (i.e. with permanent productivity μ_b), when there is a normal stochastic shock ε_m and the capital choice k' is equal to the average equilibrium capital $\bar{k}$. The horizontal axes are scaled by the average equilibrium capital $\bar{k}$.

6, where the TFP for entrants is higher in the benchmark model with forbearance lending (Panel (A)) than in the counterfactual experiment with no forbearance lending (Panel (B)). And in Column (4) of Table 6, where the TFP for liquidating firms is lower in the benchmark model with forbearance lending than in the counterfactual experiment with no forbearance lending. Together these results suggest that forbearance lending allowed firms with lower productivity to continue operating and in so doing, restricted the prospects for new firms to enter. We do not, however, find evidence that the presence of "zombie firms" (firms in receipt of forbearance lending) harmed other "non-zombie firms", as documented by Acharya et al. (forthcoming) and Caballero et al. (2008). Instead, we find that the average sales growth, investment and productivity are not significantly different between the benchmark model and counterfactual scenario for continuing firms (Column (2) of Table 6), which is consistent with some of the findings of Schivardi et al. (2017).

5.2 Perfect Information Scenario

We now consider the impact of information asymmetry by conducting a counterfactual experiment with perfect information. With perfect information, the firm problem remains identical to the baseline scenario but the equilibrium loan schedule changes because the lender can perfectly identify firm type $i \in (b, g)$. The equilibrium loan schedule $\Omega(k', z)$ that comprises all loan contracts (l', l'_R) allowing the lender to break even in expected value is:

$$
\begin{aligned}
l' = \quad & \tfrac{1}{1+r}\left[1-\rho_f\left(k', l'_R, z, \mu\right)-\rho_d\left(k', l'_R, z, \mu\right)\right] l'_R \\
& -\tfrac{1}{1+r}\left(\rho_f\left(k', l'_R, z, \mu\right) \cdot \gamma\left(k', l'_R, z, \mu\right)\right) l'_R \\
& -\tfrac{1}{1+r}\rho_d\left(k', l'_R, z, \mu\right) \min\left\{\chi_d k', l'_R\right\}-\xi
\end{aligned} \tag{30}
$$

Overall, Table 5 and Table 6 demonstrate that firm liquidation is higher, forbearance lending rates are lower, sales growth is higher, and leverage is lower under perfect information. We discuss these results in more detail below.

We first consider the liquidation rate under perfect information. Table 5 shows that the liquidation rate is about 15.51 per cent higher in the counterfactual experiment with perfect information than in the benchmark scenario. The higher liquidation rate is mainly driven by higher defaults of entrant firms, as can be seen by the lower average age of liquidating firms in Table 6 (i.e. 19.47 versus 22.47). In particular, low-quality firms now often liquidate after just one period. Under perfect information, lenders can properly discriminate between low-quality and high-quality firms when pricing loan schedules. The higher loan schedules for low-quality firms reduce their relative value of continuing or forbearance lending. Liquidating is typically the most valuable option for these firms.

Next we consider the equilibrium loan schedules that arise in the counterfactual scenario of perfect information. Panel (C) of Figure 6 is the equilibrium loan schedule for the counterfactual experiment with perfect information. Compared with Panel (A), the loan schedule for the high-quality firms is more favorable under perfect information, and vice versa for low-quality firms. This is because the lender can now perfectly identify firm-type and so more accurately price liquidation risk and forbearance risk. The information asymmetry had the effect of narrowing the gap between the loan schedules of the high-quality and low-quality firms.

The loan schedules under perfect information lead to very different outcomes for the type of firms that obtain loan forbearance, as shown in Table 6. With information asymmetry, almost exclusively low-quality firms obtain forbearance lending. As previously mentioned, this is because the loan schedule under information asymmetry is more favorable for low-quality firms, which therefore incentivized these firms to choose this option. With perfect information, only high-quality firms obtain forbearance lending. Loan schedules under perfect information are priced such that low-quality firms do not find it optimal to choose forbearance lending. Yet, Table 5 also shows that the overall forbearance rate is much lower in the counterfactual experiment with perfect information. For the given model parameters, firms rarely find it optimal to choose forbearance lending under perfect information.

Table 6: Quantitative Model Results by Operating Status for the Benchmark Model and the Counterfactual Experiments

	(1) *All Firms*	(2) *Continuing*	(3) *Forbearance*	(4) *Liquidating*	(5) *Entrants*
Panel A: Benchmark Model					
Average of:					
Sales growth	2.00 %	1.07 %	-0.74 %	-10.75 %	15.18 %
Leverage	68.69 %	12.37 %	770.34%	250.86 %	149.26 %
Investment rate	18.98 %	17.65 %	14.17%	-38.82 %	38.40 %
Capital	126.52	139.32	64.42	43.26	27.62
Loan repayment	54.06	40.29	268.11	59.28	38.78
Firm age	46.72	52.89	12.76	22.47	2.37
Total factor productivity	1.74	1.79	1.41	0.98	1.63
Proportion of firms:					
In given category	100.00 %	88.77 %	6.79 %	4.45 %	4.45 %
High-quality, i.e. $\mu = \mu_g$	90.28 %	99.65 %	0.13 %	50.02 %	54.69 %
Panel B: Counterfactual experiment with no forbearance					
Average of:					
Sales growth	2.27 %	1.03 %	n.a.	-3.34 %	18.36 %
Leverage	18.55 %	18.60 %	n.a.	17.59 %	17.95 %
Investment rate	19.69 %	17.59 %	n.a.	-17.77 %	46.84 %
Capital	134.54	142.68	n.a.	44.00	28.77
Loan repayment	54.28	57.76	n.a.	9.90	9.01
Firm age	50.00	53.68	n.a.	21.12	2.30
Total factor productivity	1.76	1.79	n.a.	1.05	1.47
Proportion of firms:					
In given category	100.00 %	95.24 %	n.a.	4.76 %	4.76 %
High-quality, i.e. $\mu = \mu_g$	96.94 %	99.79 %	n.a.	49.69 %	59.89 %
Panel C: Counterfactual experiment with no information asymmetry					
Average of:					
Sales growth	3.92 %	1.87 %	-28.67 %	-5.56 %	109.44 %
Leverage	13.10 %	13.47 %	93.53 %	00.01 %	6.78 %
Investment rate	25.41 %	19.30 %	-38.65%	-32.74 %	352.33 %
Capital	133.96	138.74	262.93	27.05	24.34
Loan repayment	42.68	44.30	243.75	0.01	2.92
Firm age	51.98	54.00	41.45	19.47	2.50
Total factor productivity	1.79	1.79	1.46	0.99	1.82
Proportion of firms:					
In given category	100.00 %	94.83 %	0.03 %	5.14 %	5.14 %
High-quality, i.e. $\mu = \mu_g$	99.98 %	100.00 %	100.00 %	49.99 %	99.59 %

Note: Panel A reports model statistics under the benchmark calibration. Panel B reports model statistics for the counterfactual experiment in which firms no longer have access to forbearance lending. Panel C reports model statistics for the counterfactual experiment in which there is no information asymmetry and so lenders can observe firm-type. Within each panel, Column (1) reports model statistics for all firms operating with age greater than one. Column (2) reports model statistics for continuing firms, defined as any firm that has existed for more than three periods and does not currently receive loan forbearance. Column (3) reports model statistics for forbearance firms, defined as any firm that has existed for more than three periods and currently receives loan forbearance. Column (4) reports model statistics for liquidating firms, based on the firm statistics in the period preceding firm liquidation. Column (5) reports model statistics for entrant firms, defined as any firm that has existed two periods or less.

Table 6 shows that average sales growth across all firms is around 96.00 per cent higher in the counterfactual experiment with perfect information than in the benchmark scenario, and also higher than in the counterfactual experiment with no forbearance lending. With perfect information, low-quality firms face high loan schedules because they have higher liquidation risk and forbearance risk. This contributes to these firms liquidating shortly after they enter. This results in the overall proportion of high-quality firms being 99.98 per cent under perfect information, which is higher than in both the benchmark and the counterfactual scenario of no forbearance lending. Once more, the consequence of a higher proportion of high-quality firms is that overall average sales growth is higher. For similar reasons, aggregate investment, aggregate output and aggregate TFP are respectively 9.13 per cent, 5.02 per cent and 1.66 per cent higher in the counterfactual scenario with perfect information as compared with the benchmark model.

6 Conclusions

This study examines the relationship between forbearance lending, firm dynamics and aggregate outcomes. To do so, we develop a firm equilibrium model that features endogenous default and endogenous forbearance lending. Lenders face information asymmetry because they do not know with certainty whether they are lending to a "low-quality" or "high-quality" firm. Our model enables us to consider the net impact of forbearance lending on firm performance and aggregate outcomes by examining whether the costs of forbearance lending, due to credit misallocations from lenders incorrectly assessing a lender to be high-quality, outweigh its benefits, due to refinancing high-quality firms experiencing a negative sequence of shocks. We fit our model to the euro area economy over the period 2011 to 2014, which represents a period of low output growth and higher levels of forbearance lending.

To examine the impact of forbearance lending, we conduct a counterfactual exercise in which firms still have the option to liquidate but no longer have access to any form of loan forbearance. In the absence of forbearance lending, we find that aggregate output is around 4.65 per cent higher in our counterfactual scenario than in the benchmark scenario fitted to the euro area. We also find that aggregate investment and total factor productivity are 8.44 per cent and 0.76 per cent higher in our counterfactual scenario, respectively. A key driver of this result is that there is a larger proportion of high-quality firms in our counterfactual scenario; the forbearance lending present in the benchmark model prevents low-quality firms from defaulting more quickly. We find that low-quality firms are most likely to receive forbearance lending in our benchmark model. These low-quality firms, kept alive via forbearance lending, lower the aggregate output, investment and total factor productivity.

The fact that the overwhelming majority of firms in receipt of forbearance lending in our model are of low-quality is consistent with the empirical evidence of Acharya et al. (forthcoming), who find that low-quality firms are more likely to be in receipt of subsidized loans. Our results extend their empirical findings by quantifying the impact of forbearance lending on various measures of firm performance and aggregate outcomes. That said, we do not find evidence that "zombie firms" harm "non-zombie firms" in our model, as documented by Acharya et al. (forthcoming) for European firms and Caballero et al. (2008) for Japanese firms. In an additional counterfactual exercise with perfect information, we also show that only high-quality firms access forbearance lending. In this scenario, lenders know firm-type and this leads to an even higher proportion of high-quality firms among all firms. As such, average sales growth, aggregate investment, output and total factor productivity are higher than in the benchmark model with information asymmetry.

While our model provides new insights about some of the trade-offs of forbearance lending, it does not capture all of these. Specifically, our model does not include any role for the potential increased unemployment associated with higher default rates that may occur in the absence of forbearance lending. However, we do not find that default rates increase significantly in the absence of forbearance lending, which suggests a smaller role for this potential channel. Nonetheless, a full examination of whether the costs of forbearance lending, due to lenders misallocating credit, outweigh the benefits, due to preventing an increase in unemployment, presents an exciting avenue for future research. We also do not examine a role for forbearance lending incentives by weak banks, which the empirical literature finds to be important (see, e.g., Peek and Rosengren (2005), Schivardi et al. (2017), Blattner et al. (2018)). Instead, we focus on the impact of these lending practices on economic performance, leaving any related theoretical work on this topic open for future investigations.

We stress that in reality, we do expect lenders to face information asymmetry. Therefore, our results suggest that the output-related costs of forbearance lending associated with misallocating credit to low-quality firms most likely outweigh any output-related benefits associated with refinancing high-quality firms experiencing a sequence of negative shocks. From this perspective, forbearance lending may have contributed to the low output growth observed for the euro area since the onset of the sovereign debt crisis.

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A Computational Algorithm

We first set grids on the state space of the aggregate states of the economy, $s = \{k, l_R, z, \mu\}$. The productivity shock z is discretized into 10-state Markov chain using the method of Tauchen (1986). The state space of the loan repayment amount l_R and capital k are each discretized into a grid of 15 and 25 points, respectively, between 0 and 400. The loan repayment amount grid and the capital grid are unevenly spaced, using a similar method to Corbae and Quintin (2015). That is, every point of an equally spaced grid between 0 and $400^{2/3}$ is raised to the 3/2 power. As in Corbae and Quintin (2015), this grid contains more points closer to zero and so provides improved numerical performance for our problem.

We solve the model to find the optimal loan schedules, policy functions and default policies using the following algorithm:

1. For all aggregate states of the economy, $s = \{k, l_R, z, \mu\}$, start with an initial guess for the bank loan recovery schedule ψ^0. For all k' and z, start with an initial guess for the loan schedule Ω^{00}, in which all loan contracts (l', l'_R) have the risk-free interest rate. For the probability of firm-type given observed productivity $q^0 = \Pr(\mu = \mu_j | z)$, start with an initial guess by using $\Pr(\mu = \mu_b) = \Pr(\mu = \mu_g) = 0.5$ and $\Pr(z | \mu = \mu_b) = 2 \times \Phi\left(-\left|\frac{\ln(z) - \mu_b}{\sigma_\varphi}\right|\right)$.

2. Given the bank loan recovery schedule ψ^0 and the equilibrium loan schedule Ω^0, use value function iterations to solve for the optimal policy functions for future capital stock $k'(s)$, the new bank loan contract $(l'(s), l'_R(s))$, operating sets $C(k, l_R, \mu)$, liquidation sets $D(k, l_R, \mu)$, and forbearance sets $F(k, l_R, \mu)$. We iterate on the value function until we reach convergence for a given ψ^0 and Ω^0.

3. Given the operating sets, liquidation sets, forbearance sets and q^0: compute the new debt schedule Ω^1 such that lenders break even in expectation, and compare it to loan debt schedule of the previous iteration: Ω^0. If a convergence criterion is met, $\max\{\Omega^0 - \Omega^1\} < \varepsilon$, then assign Ω^1 to Ω^0 and move on to the step 4. Otherwise, update using a Gauss-Seidel algorithm and go back to step 2.

4. Solve the bargaining problem given the converged loan schedule Ω^0 and compute the new bank loan recovery schedule ψ^1 for all aggregate states of the economy, $s = \{k, l_R, z, \mu\}$. If the new bank loan recovery schedule ψ^1 is sufficiently close to ψ^0, stop iterating on ψ. Otherwise, go back to step 2.

5. To compute the stationary cross-sectional distribution, start with a uniform distribution as an initial guess. Then simulate the stationary cross-sectional distribution associated with the set of policy functions obtained above, and such that the mass of all projects always equals one. This implies that the mass of entrants equals the the mass of liquidations. Update q^1 based on $\Pr(\mu = \mu_b)$ and $\Pr(z|\mu = \mu_b)$ from the stationary cross-sectional distribution. If the firm-type probability given productivity q^1 is sufficiently close to q^0, stop iterating on q. Otherwise, go back to step 2.

6. Simulate the model to compute statistics from a model economy of 1,000 firms over 750 periods, using the first 250 periods as a burn-in.

B Data and Parameter Estimation

We use firm-level data to estimate for three parameters in Table 2 independently of the model equilibrium, as well as to estimate most of the model statistics listed in Table 3. We obtain our firm-level data from the Amadeus database, which comprises financial data for both public and private companies in Europe. Our sample comprises all public and private euro area firms, excluding the financial and government sectors. We collect annual financial data for the period 2009 to 2014.

We clean the dataset in several ways. Following Arellano et al. (2012), we exclude firms in the financial and government sectors. Financial sector firms correspond to NACE Divisions 64 to 67 and government sector firms correspond to NACE Division 84. We furthermore collect data for euro area firms only. The sample is also restricted to those firms with a reported value for EBIT (that is, earnings before interest and taxes), sales, and fixed assets in at least one of the years from 2012 to 2014. These criteria leave us with around 150,000 firms in 17 countries: all euro area countries except for Ireland. Table 7 describes how we compute the parameters and model statistics.

Table 7: Definitions of Parameters and Model Statistics from Firm-level Data

Calibrated Parameters	*Definition*
Depreciation	$\frac{\text{Depreciation}_t}{\text{Fixed Assets}_{t-1}}$
Interest Coverage Ratio	$\frac{\text{EBIT}_t}{\text{EBIT}_t - \text{EBT}_t}$

Model Statistics	*Definition*
Sales Growth	$\frac{\text{Sales}_t - \text{Sales}_{t-1}}{\text{Sales}_{t-1}}$
Leverage	$\frac{\text{Total Assets}_t - \text{Total Equity}_t}{\text{Total Assets}_t}$
Working Capital	$\frac{\text{Working Capital}_t}{\text{Total Assets}_{t-1}}$
Investment	$\frac{\text{Fixed Assets}_t - \text{Fixed Assets}_{t-1} + \text{Depreciation}_{t-1}}{\text{Fixed Assets}_{t-1}}$

Note: Each variable is windsorized at the 5 per cent level. The model statistics are computed for each firm in each year from 2011 to 2014. We first take the mean and standard deviation of each statistic in each year. The model statistics are then calculated as the mean of the yearly mean (for the mean of the model statistic) or the mean of the yearly standard deviation (for the standard deviation of the model statistic) over the period 2011 to 2014.