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Compositional nature of firm growth and aggregate fluctuations
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Abstract

This paper studies firm dynamics over the business cycle. I present evidence from the United Kingdom that more rapidly growing firms are born in expansions than in recessions. Using administrative records from Census data, I find that this observation also holds for the last four recessions in the United States. I also present suggestive evidence that financial frictions play an important role in determining the types of firms that are born at different stages of the business cycle. I then develop a general equilibrium model in which firms choose their managers' span of control at birth. Firms that choose larger spans of control grow faster and eventually get to be larger, and in this sense have a larger target size. Financial frictions in the form of collateral constraints slow the rate at which firms reach their target size. It takes firms longer to get up to scale when collateral constraints tighten; therefore, businesses with the largest target size are affected disproportionately more. Thus, fewer entrepreneurs find it profitable to choose larger projects when financial conditions deteriorate. Using Bayesian methods, I estimate the model using micro and aggregate data from the United Kingdom. I find that financial shocks account for over 80% of fluctuations in the formation of businesses with a large target size, and TFP and labour wedge shocks account for the remaining 20%. An independently estimated version of the model with no choice over the span of control needs larger aggregate shocks in order to account for the same data series, suggesting that the intensive margin of business formation is important at business cycle frequencies. The model with the choice over the span of control generates an empirically relevant and non-targeted collapse in the right tail of the cumulative growth distribution among firms started in recessions, while the model without such a choice does not. The paper also discusses implications for micro-targeted government stimulus policies.

Key words: Business cycles, firm dynamics.

JEL classification: E23, E32, H25.

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1 Introduction

In this paper, I study the business formation and growth of young enterprises over the business cycle. It is well known that the entry rate falls in recessions. This paper puts forward an idea that recessions particularly discourage the formation of rapidly growing enterprises, which results in relatively few rapidly growing and relatively more slowly growing new firms (*compositional effect*). I provide evidence for the compositional effect in the data by showing that fewer rapidly growing firms are started during economic downturns. I also provide suggestive evidence that rapidly growing firms are more financially constrained than slower growing enterprises. Therefore, access to financing can play an important role in determining which firm types are started at different stages of the business cycle. I then develop a general equilibrium model with financial frictions in which entrepreneurs choose the optimal size of their projects upon entry. I use the model to quantitatively assess the compositional effect and study the implications for the government stimulus policy.

Studying firm dynamics and business formation is important, since the behavior of young firms can have sizable aggregate implications. New enterprises constitute a small fraction of employment and fixed assets, but they contribute disproportionately to aggregate job creation and investment (Decker, Haltiwanger, Jarmin and Miranda, 2014; Gonzalez-Uribe and Paravisini, 2019).[1] In fact, young enterprises in the United States accounted for one-half of the overall decline in employment during the Great Recession (Sedláček and Sterk, 2017).[2] Therefore, a reduction in the number of rapidly growing firms during recessions can have a large impact on aggregate investment and employment growth. Moreover, provided that the compositional shift from rapidly growing to more slowly growing enterprises in a recession determines the subsequent growth of firms, the *intensive* margin of business formation can have a long-lasting impact on the economy and delay the recovery in the aftermath of economic crises.

I start by documenting a substantial heterogeneity in the subsequent growth rates of firms after they are born. I use firm-level data from the United Kingdom to show that the mean cumulative growth rate of firms that are born in expansions (*expansionary firms*) is larger than that of firms started in recessions (*recessionary firms*). I further show that the observed difference in mean cumulative growth rates is primarily driven by the collapse in

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[1]As for the U.S. manufacturing sector, Eslava, Haltiwanger and Pinzon (2019) find that in the absence of entry, incumbents' share of employment would have shrunk by 40% over the course of a decade.

[2]In the data, the growth rate of young firms fell stronger than that of mature enterprises during the last 3 aggregate contractions in the U.S. (Figure B6). Thus, it is likely that the importance of young businesses for the aggregate job creation is not limited to the Great Recession.

the right tail of the cumulative growth rate distribution, whereas I find the left tail to be stable.

I also apply a clustering algorithm proposed by Bonhomme and Manresa (2015), which optimally assigns firms to groups so that businesses within each group share similar life cycle profiles. The advantage of this method is that it implements the optimal grouping, taking the effect of observables into account. Since the life cycles of firms can be affected by aggregate conditions, I implement the clustering algorithm controlling for sequences of aggregate shocks each firm went through over its tenure. I measure aggregate shocks as a cyclical component of GDP series from HP-filtering. I find that the mass of recessionary firms assigned to the group with the most rapidly growing businesses is significantly smaller compared with that of expansionary firms. I repeat this exercise controlling for both aggregate shocks and a measure of balance sheet strength—financial leverage—and find similar results.

My results for the United Kingdom are based on a firm-level accounting dataset, which covers only one recessionary episode (the 2008-2009 crisis). I additionally use administrative records from the U.S. Census Bureau's Longitudinal Business Database (LBD) to show that each of the last four recessions in the United States saw a collapse in the right tail of firms' growth distribution.[3] The LBD is the most comprehensive source of information on U.S. businesses and spans the time period 1976-2016.[4] The extensive coverage of the LBD allows me to document this finding across several recessions.

I then provide suggestive evidence that rapidly growing firms are more financially constrained relative to more slowly growing enterprises. I exploit a unique feature of the U.K. data—one can observe a measure of the residential property of firm owners[5]—to study the response of firm-level investment to idiosyncratic fluctuations in the residential real estate of their owners. I find that firms with high subsequent growth rates are more responsive to idiosyncratic fluctuations in housing wealth than more slowly growing enterprises. Following a large empirical literature on the collateral effects of firm financing (*e.g.*, Chaney, Sraer and Thesmar, 2012; Bahaj, Foulis and Pinter, 2018), I interpret this finding as strong, suggestive evidence that fast growing firms are more financially constrained (as identified through the collateral channel) than slow growing businesses.[6] This finding also implies that financial

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[3]The list of recessions includes 1980-1982, 1990-1991, 2001-2002 and 2008-2009.

[4]The LBD covers all sectors of the U.S. economy and encompasses all businesses with at least one paid employee (Jarmin and Miranda, 2002).

[5]The data provide information on the residential property of firm *directors*—a group of people (or a single person) responsible for running and promoting the firm. However, over 70% of directors are shareholders in corresponding firms (Bahaj, Foulis and Pinter, 2018).

[6]Davis and Haltiwanger (2019) draw on the LBD and document that movements in local housing prices

frictions may affect the type of firms that are born at different stages of the business cycle.

I then build a general equilibrium model of firm dynamics over the business cycle, in which the behavior of incumbents is formalized similarly to that in Khan and Thomas (2013): heterogeneous firms need physical capital to produce, and they accumulate it subject to a collateralized borrowing constraint. I extend their model along several dimensions. First, I assume that firms differ in their managers' spans of control. Firms that have larger spans of control grow faster and eventually get to be larger, and in this sense have a larger target size.[7] The model's assumption that the life cycle of projects is to a certain extent determined by an ex ante fixed component captures the idea that adjusting organizational capabilities is costly (see, for example, Hannan and Freeman, 1984 and Henderson and Clark, 1990).[8]

Second, potential entrants can *target* their start-up attempts toward projects of different optimal size. This assumption allows my model to capture both margins of business formation. On the *extensive* margin, economic crises can reduce the number of entering firms (Clementi, Khan, Palazzo and Thomas, 2015; Clementi and Palazzo, 2016). On the *intensive* margin, relatively few firms with a large span of control can get started during recessions. The focus of this paper is on the latter channel (the compositional shift), but the model is designed to capture both channels. Thus, my model differs from classic industry dynamics frameworks (Jovanovic, 1982; Hopenhayn, 1992) in that entrepreneurs can *a priori* decide how large their businesses can be.[9] The ability of entrepreneurs to control the target size of their businesses also finds support in survey data: a significant fraction of business owners have no intention to grow their business to a large size, and some of them start firms for non-pecuniary reasons (Hurst and Pugsley, 2011).

Motivated by the empirical evidence, I assume that firms are subject to financial frictions. In particular, firms can finance their investment expenditures using three sources: retained

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affect firm creation. This paper makes a related point using another measure of financial shocks: the value of the residential real estate of the firm's directors.

[7]The idea that firms differ in their ex ante type is particularly attractive in light of a recent study by Sedláček, Pugsley and Sterk (2018) in which they decompose firms' growth profiles into a fixed "business quality" component and a sequence of ex post shocks. This approach reveals that ex ante heterogeneity is an important determinant of firms' life cycles. Firms grow large not only because of pure luck (a sequence of high demand, low cost shocks, and so on), but also, and more importantly, because the very idea behind the business plan was good.

[8]In the model, firms are subject to idiosyncratic productivity shocks. Therefore, firms' life cycles are allowed to be different *within* types.

[9]Such a theoretical construct also allows me to break the tight link between size and age in classic models, where young firms are less productive and small at the beginning, but over time their productivity mean-reverts and makes them grow (Gavazza, Mongey and Violante, 2018). In the data, roughly half of 10-year-old firms hire less than 10 workers—a size close to 6 employees hired by a typical entrant (Karahan, Pugsley and Şahin, 2019).

earnings, debt financing and equity issuance.[10] Debt financing is subject to a collateral constraint, and issuing equity is costly. As I discuss in Section 3, both frictions are necessary to make firms financially constrained. Finally, my model has three aggregate stochastic processes: TFP, financial shocks and labor wedge shocks, which I estimate using Bayesian methods on 40 years of quarterly data from the United Kingdom.

I make several findings in this paper. First, I decompose fluctuations in the entry intensity of firm types over time into contributions of different aggregate shocks and find that the financial shock accounts for 82% of fluctuations in the entry intensity of fast growing businesses. Aggregate TFP and labor wedge shocks account for the remaining 20%. As for more slowly growing businesses, financial shock accounts for only 5% of fluctuations in the entry intensity of such firms. This finding suggests that rapidly growing businesses are particularly sensitive to aggregate financial conditions.

Second, my model can account for two-thirds of the collapse in the right tail of the cumulative growth distribution during the 2008-2009 recession, even though it was not directly targeted in the estimation process. To demonstrate that this feature of the data arises because of the intensive margin of firm entry, I develop and independently estimate a version of the model in which the composition of entrants is business cycle invariant. I show that such a model cannot account for the collapse in the right tail of the cumulative growth distribution.

Third, I find that the compositional effect is quantitatively large at business cycle frequencies. To this end, I confront the predictions of my baseline model and the model with no compositional effect during the Great Recession episode. I find that the model with no intensive margin of business formation explains 25% less of the drop in investment and hours observed during the crisis, suggesting that the model needs larger shocks to account for the same dynamics of aggregate economic series. The key reason is that adverse aggregate shocks affect rapidly growing firms disproportionately more than more slowly growing firms, precisely because it takes the former more time and resources to get up to scale than the latter. In the model, bad aggregate conditions discourage potential entrants from pursuing financially demanding projects, and they switch to less ambitious ideas. The relative entry of projects with a large target size declines and leads to a compositional shift, for which I find support in the data. Given that businesses with a high span of managerial control

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[10]The typical assumption in firm dynamics models with financial frictions is that firms have to deliver a non-negative dividend stream (see, *e.g.,* Khan and Thomas, 2013; Ottonello and Winberry, 2018). In this paper, however, I relax this assumption because equity financing is non-negligible among young firms (Robb and Robinson, 2014). Moreover, small and old firms behave differently with respect to their capital structure over the business cycle (Begenau and Salomao, 2018): large and old firms substitute between debt and equity, whereas small firms (and young ones) have procyclical debt and equity financing policies.

account for a significant share of aggregate investment and employment, a smaller fraction of such firms makes the recession deeper.[11] The model without an endogenous choice of project types does not capture this amplification effect, which forces the model to have larger shocks in order to account for the same aggregate dynamics. This finding, coupled with the result that formation of rapidly growing businesses is particularly sensitive to financial conditions, suggests a potentially important role for government stimulus policies.

Fourth, I argue that the amplification mechanism is relevant. To do so, I compare the predictions of the two models with respect to a key non-targeted series: the extensive margin of business formation (the entry rate). I show that the model with an intensive margin that is business cycle invariant predicts a counterfactually larger decline in the entry rate during the financial crisis, whereas the prediction of the baseline model is much closer to the data. Intuitively, this occurs because the benchmark model can deliver an empirically realistic fall in macroeconomic aggregates during downturns by reducing the entry intensity of rapidly growing firms, which is associated with a small overall decline in the entry rate (there are relatively few fast growing projects as compared with slow growing ones). The model without compositional effects induces an equal decline in entry of all project types, which translates into a larger fall in the entry rate overall.

Finally, I study the welfare consequences of government policies. The first case I consider is based on the assumption of full information on behalf of the government; it knows the assignment of firms across the types and targets the policy toward firms with the highest growth potential. The second case assumes that the government is agnostic about the firm-type assignment; therefore, it applies the policy to all firm types. The particular policy I consider takes the form of a reduction in the cost of entry at the expense of lower tax rebate to the household. I find that welfare benefits *(i)* are large in the first case (about 0.3% of lifetime consumption) and *(ii)* are negligible in the second case. This highlights the importance of the micro-targeted government policies as they help achieve welfare gains due to cost-efficiency.

Related literature This paper contributes to understanding the effect of aggregate economic conditions upon firms' inception on their subsequent growth. On a conceptual level, this paper is most closely related to the studies by Moreira (2016) and Sedláček and Sterk (2017), although they abstract from financial constraints. Ouyang (2009) studies the scar-

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[11]On a related matter, Eslava, Haltiwanger and Pinzon (2019) compare firm dynamism in the U.S. and Colombia, and find that in Colombia employment growth of an average firm is slower due to a less enthusiastic contribution of fast-growing firms.

ring and cleansing effects of recessions. In her model, entering businesses are uncertain about their productivity, and learn it over time. Economic downturns reduce the amount of time firms can stay in the market learning their productivity, which forces some high-quality businesses to exit as they cannot bear to learn as long as during good times. My paper extends the aforementioned studies by introducing financial frictions and studying how aggregate economic conditions affect the subsequent growth of firms. Albert and Caggese (2019) study a multi-country entrepreneurial survey and find that adverse financial shocks discourage the entry of fast growing businesses. My paper complements that study by quantifying the importance of the intensive margin of firm entry for the business cycle in a model with heterogeneous firms.

My paper contributes to the literature that attempts to measure the impact of access to financing on entrepreneurship. I use a structural general equilibrium model to interpret the empirical evidence from the firm-level data, whereas the existing literature typically draws on the household-side data. Such literature indirectly infers how much potential and existing entrepreneurs would like to borrow and at which price. While some studies find evidence of borrowing constraints (see, *e.g.*, Gentry and Hubbard, 2004 for an empirical investigation, and Evans and Jovanovic, 1989; Buera, 2006, and Cagetti and De Nardi, 2006 for estimated structural models), others argue that liquidity constraints might not be such a strong force in discouraging business formation, at least in the United States (see Hurst and Lusardi, 2004 for a notable example).

Finally, this paper contributes to the literature on financial shocks at business cycle frequencies. In particular, I develop a model in which firms are financially constrained and have *permanent* heterogeneity in the optimal size. To the best of my knowledge, my paper is the first to study the transmission of financial shocks in such a framework. In an influential study, Jermann and Quadrini (2012) find that standard real business cycle models cannot account for aggregate fluctuations in financial flows; thus, they highlight the importance of financial shocks. Chari (2014) argues that financial frictions in representative firm models—when brought to the data to match aggregate series of financial flows—will have no effect. The reason is that, in the aggregate, available funds for firms exceed aggregate investment expenditures. He suggests that any quantitatively successful model with financial frictions has to feature firm heterogeneity. This idea was further developed in a quantitative study by Zetlin-Jones and Shourideh (2017), which shows that financial shocks transmit through their impact on private—rather than public—firms. My paper, in turn, argues that firms with larger spans of control are more sensitive to aggregate financial conditions, and, therefore,

financial shocks transmit through the response of the rapidly growing enterprises.[12]

The rest of the paper is organized as follows. Section 2 presents the empirical results. In Section 3, I lay out a heterogeneous agent model of firm dynamics with endogenous entry and financial frictions, which I bring to the data in Section 4. Section 5 provides the main results. Section 6 explores policy implications, and Section 7 concludes.

2 Empirical Results

This section describes my empirical results. I first show that recessionary cohorts of firms grow slower than expansionary ones because of fewer fast growing enterprises. Second, I use fluctuations in firm directors' residential wealth as an identified financial shock to argue that rapidly growing businesses are more financially constrained than slow growing ones.

2.1 Data

Here I provide a brief description of the data. Subsection 2.1.1 introduces the U.K. data, and subsection 2.1.2 - the U.S. data. Please refer to Appendix A.1 for a more detailed discussion of the data sources.

2.1.1 U.K. data

Firm-Level Data   The key empirical results presented in this paper are based on a large panel dataset of firms' financial accounts called Financial Analysis Made Easy (FAME), provided by the Bureau van Dijk (BvD).[13] The data covers the corporate universe of U.K. firms for the period 1995-2017, and encompasses approximately 1.5mln private and public firms per year. The data includes both the firms' balance sheet (assets and liabilities, debt structure, issued capital) and income statements (operating profit, turnover, cost of sales, etc.). Information on firms' directors—a group of people (or a single person) responsible for running and promoting a firm—is also reported.

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[12]I also find that a TFP shock affects all firm types proportionately, and therefore permanent heterogeneity in growth profiles does not play an important role for the propagation of aggregate productivity shocks. This is related to findings of Clementi, Khan, Palazzo and Thomas (2015) and Smirnyagin (2018) in that the effects of TFP shocks in rich heterogeneous firms and simplified models are similar.

[13]This dataset is different from commonly used Orbis and Amadeus in that it covers only U.K. registered firms. FAME is a live panel meaning that its information is accurate only at the moment of filing and not for historical reference. In order to improve the coverage, identify entry and exit of firms, multiple vintages of FAME have been combined - see Bahaj, Foulis and Pinter (2018) for a detailed explanation of this process.

Real Estate Price Data   Residential housing data comes from the Land Registry's Price Paid dataset (covering England and Wales) and the Registers of Scotland. These datasets cover the universe of residential property transactions since 1995. BvD provides residential addresses of firms' directors[14], enabling one to measure the residential wealth of directors whose property was bought or sold at some point after 1995.[15]

Sample Selection   Standard cleaning procedures were applied to the raw data. Financial and real estate sectors (FIRE), as well as firms which do not comply with Companies' Act, were excluded. Outliers and observations for which the balance sheet identity did not hold were dropped. See Appendix A.2 for details as well as for summary statistics (Table A1).

2.1.2   U.S. Data

My data source for the U.S. is the LBD, which is an administrative panel dataset that covers the universe of non-farm establishments in the U.S. private sector with at least one paid employee (Jarmin and Miranda, 2002). The unit of observation in the LBD is an establishment, which is defined as a single physical location where business is conducted.[16] The LBD is a set of annual snapshots of the U.S. private sector (with establishment-level longitudinal identifiers); currently, the LBD includes annual observations from 1976 to 2016. I perform my analysis at the establishment-level in order to avoid difficulties associated with the firm-level data (*e.g.* firms operating several establishments in different locations and industries). However, given that my focus is on young businesses, few entering firms operate more than 1 establishment. Besides, establishment managers have a substantial independence in making hiring and investment decisions in the U.S. (Bloom, Sadun and Van Reenen, 2012). I nevertheless check that my results still hold at the firm level.

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[14]By UK law, directors must report several pieces of information (such as their name, date of birth and residential address among other things) when they register a firm. BvD contains this information.

[15]Strictly speaking, not all directors are complete owners of their houses. Bahaj, Foulis and Pinter (2018) made use of the Product Sale Database (PSD)—an administrative data on all residential mortgages since 2005 at origination and on the stock of outstanding mortgages in 2015. The PSD is provided by the UK Financial Conduct Authority. The FCA Product Sales Data include regulated mortgage contracts only, and therefore exclude other regulated home finance products such as home purchase plans and home reversions, and unregulated products such as second charge lending and buy-to-let mortgages. PSD contains information on the borrower's date of birth and the mortgaged property's full postcode. Therefore, by linking directors' home values with mortgage data, it is possible to construct a measure of *home equity*. They find that results remain largely unaffected when residential *equity* is used instead of residential real estate. This is not surprising given that approximately 90% of directors are homeowners.

[16] It is possible to aggregate the establishment-level information to the firm-level, by using appropriate firm identifiers. The aggregation process is associated with several well-known issues (a new firm identifier emerges in the LBD if, for example, a firm merges with another firm). See Appendix A.1 for a discussion on how to aggregate data to firm-level in a way robust to ownership and control changes.

2.2 Compositional Effect of Recessions

In this subsection, I document that recessionary cohorts are growing slower than expansionary ones, and this is driven by the feature that there are fewer rapidly growing firms started in recessions.

2.2.1 Evidence from the U.K.

The U.K. economy went through the Great Recession in 2008-2009, and was expanding in 2006-2007. Figure 1 plots the mean, the median and the top decile of the cumulative growth distribution for expansionary (born in 2006-2007) and recessionary (born in 2008-2009) cohorts. According to Panel A, expansionary firms grew on average by 35% between ages 1 and 5, and recessionary—by only 8%.

The observed difference in the average growth between cohorts is mainly driven by the right tail of the cumulative growth distribution. In particular, Panel B shows that the median firm in recessionary and expansionary cohorts grew at a similar rate of 4%. However, the 90th percentile appears to be strikingly different (Panel C): the top decile of firms born in the expansion grew 3 times faster over the course of 5 years, as compared to the top 10% of businesses entered in the recession.

FIGURE 1: MEAN, MEDIAN AND TOP DECILE OF CUMULATIVE GROWTH OVER THE FIRST 5 YEARS OF TENURE BY YEAR OF FIRM BIRTH

(A) Mean   (B) Median   (C) P90
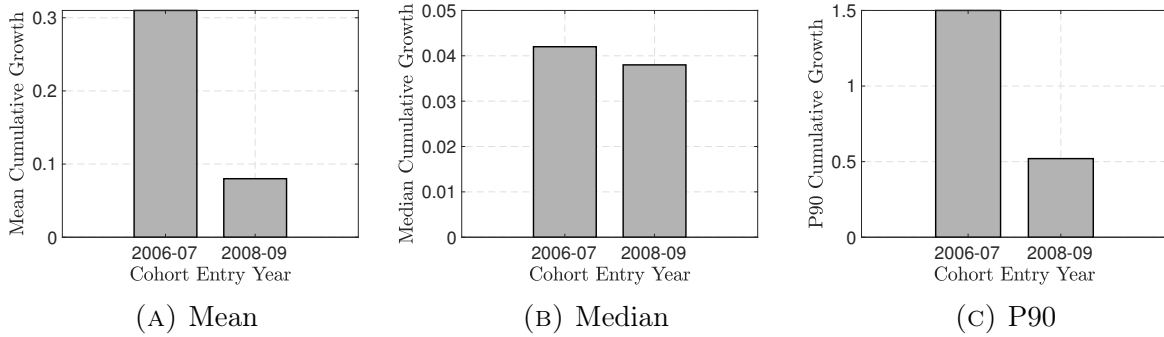
Notes: Figure 1 contains 3 panels. Panel A plots the average cumulative growth, Panel B—the median cumulative growth, and Panel C—the top decile of cumulative growth. Cumulative growth is in terms of employment, and is defined as $\log\left(\frac{y_{i5}}{y_{i1}}\right)$, where y_{i5} and y_{i1} are numbers of employees at firm i at ages 1 and 5, respectively. Source: BvD.

Another way to illustrate the collapse of the right tail of the firms' growth distribution for recessionary firms is to look at the distribution of growth rates by firm age (see Figure B4 in Appendix B). Several observations are in order. First, expansionary cohorts grow faster,

since the distribution of growth rates is positively skewed relative to recessionary cohorts. At the same time, the median and the bottom decile of the growth rate distribution look remarkably similar across cohorts, confirming Figure 1.[17]

Disentangling Aggregate Effects and Firms' Growth Profiles Overall, the data shows that recessionary cohorts grow slower than expansionary ones because of fewer rapidly growing firms. One potential concern with Figures 1 and B4 is that they ignore aggregate effects: the fast growth of expansionary firms can be fueled by favorable aggregate conditions.

FIGURE 2: BONHOMME AND MANRESA (2015) GROUPED FIXED EFFECTS ESTIMATION

(A) Controlling for business cycle

(B) Controlling for business cycle and leverage
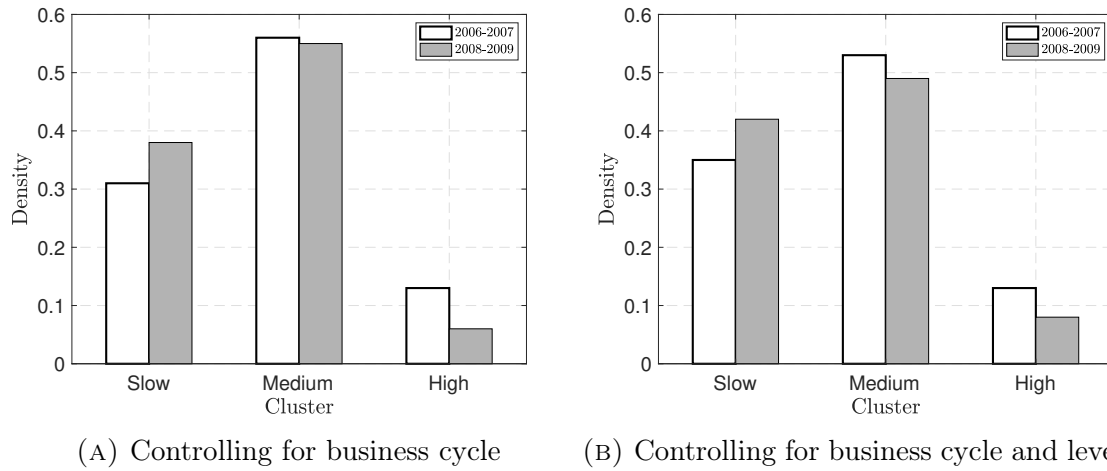
Notes: Figure 2 plots the distribution of firms across clusters based on the grouped fixed effects estimator by Bonhomme and Manresa (2015) . White bars correspond to the firms started in 2006-2007, and gray ones—to the firms which entered in 2008-2009. The following linear model is considered:

$$y_{ia} = \alpha_{g_i a} + x'_{ia}\theta + \varepsilon_{ia}, \quad i = 1, \ldots, N, \quad a = \overline{1, 8},$$

where y_{ia} is the real book value of assets of firm i of age a, and x_{ia} is a vector of control variables. Panel A plots the resulting distribution of firms across clusters when x_{ia} contains the cyclical component of GDP from the HP filter with smoothing parameter 100 (annual frequency). The vector of controls in Panel B also includes the liability-based leverage of firms (ratio of total liabilities to total assets). Clustering was performed on a 20% random subsample of manufacturing firms, which I tracked for up to 8 years. The choice of age 8 as an upper bound was dictated by the panel's duration: this is the maximal observable age of firms born in 2009. The exercise was repeated numerous times in order to ensure that the results are robust to different draws. See Appendix A.4 for further details. Source: BvD.

In order to assuage this concern, I apply a *grouped fixed effects* clustering algorithm developed by Bonhomme and Manresa (2015) (see Appendix A.4 for details). The benefit of this approach is that it optimally assigns firms into the pre-specified number of groups

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[17]This pattern cannot be explained by productivity differences: Table C1 in Appendix C shows that, if anything, recessionary entrants were on average more productive than expansionary ones (relative to incumbents).

based on how similar firms' time profiles of observables look like after the effect of controls has been taken out. By applying this estimator to the firm-level data, I can assign firms into growth types (fast, medium and slow) *controlling* for the sequence of aggregate shocks each firm went through.

I experiment with 2 different sets of controls. The first one includes only the cyclical (from HP filtering) component of GDP in order to control for the business cycle (Panel A in Figure 2). The second one adds a leverage ratio[18] as a proxy for firms' financial conditions (Panel B). In both cases, I obtain broadly similar results: the mass of rapidly growing firms is roughly 40% smaller in recessionary cohorts, which is reflected in a relatively large mass of slow types.[19]

2.3   Evidence from the U.S.

The results sourced from the U.K. data are based on one recessionary episode—the financial crisis of 2008-2009, which was particularly deep and prolonged. In this subsection, I show that this feature of the data—the right tail of the cumulative growth distribution is lower among firms entered during economic downturns—has occurred during the last 4 recessions in the U.S.

I rely on the administrative data from the LBD—a comprehensive dataset on the U.S. private sector businesses housed by the U.S. Census Bureau. According to Figure 3, the left tail of the cumulative growth distribution in the U.S.—defined as the difference between the median and the bottom 10th percentile—is very stable across the entire sample. At the same time, the right tail (difference between the 90th and 50th percentiles) fluctuates over time, and falls during the NBER recessions. Consistent with the U.K. evidence, the Great Recession caused a large collapse in the right tail, potentially reflecting a financial nature of that economic crisis.[20]

In order to study the impact of initial aggregate conditions on the subsequent growth of firms, I fit several linear probability models in Appendix C. In particular, I estimate the effect of being born in a recession on the probability for the establishment to exhibit a 5-year cumulative growth rate above the 10th (50th and 90th) percentile of the corresponding

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[18]I use a liability-based leverage: a ratio of total liabilities to total assets.

[19]In Appendix A.4, I report the average growth within each firm type. For example, "high" type firms grew 5 times faster between ages 1 and 5 than "slow" type businesses.

[20]These patterns are unlikely to be driven by an more intensive exit of firms started in recessions. I calculate the survival rates at 3-, 5- and 7-year horizons and find them to fluctuate very little over time (Figure B5).

distribution. I find that the fact that an establishment was started during the NBER recession has no effect on the odds of having a cumulative growth rate above the 10th percentile (Table C2). At the same time, the adverse aggregate conditions upon inception reduce the probability of the cumulative growth rate to exceed the 50- and 90th percentiles by 2pp and 5pp, respectively (Tables C3 and C4). The results are robust to the inclusion of a rich set of controls, such as industry, region and type of operation fixed effects.

FIGURE 3: TOP AND BOTTOM TAILS OF CUMULATIVE GROWTH DISTRIBUTION IN THE U.S.
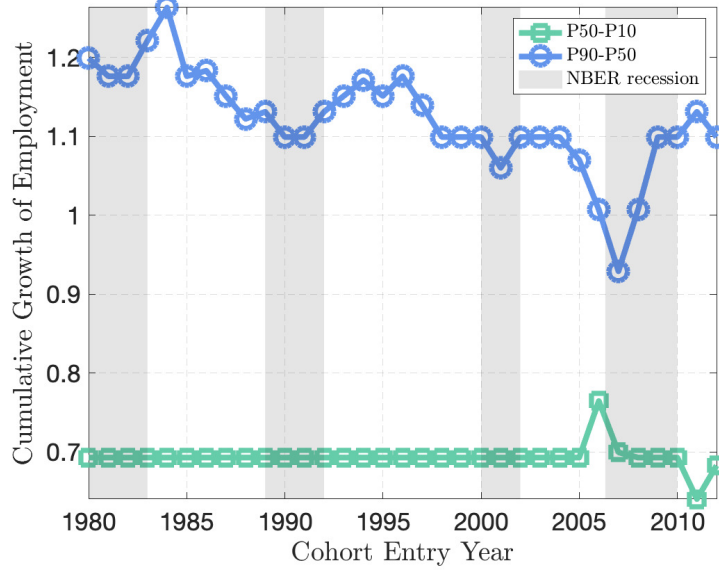
Notes: Figure 3 plots 2 lines. The blue line with circle markers is the right tail of the cumulative growth distribution. The green line with square markers is the left tail of the cumulative growth distribution. The right tail is the difference between the 90th and 50th percentiles of the cumulative growth distribution among establishments born in a specific year. Analogously, the left tail is the difference between the 50th and 10th percentiles. Cumulative growth of each establishment is measured as a log change in employment between ages 1 and 5. The vertical gray bars represent NBER recession dates. Data source: LBD.

2.4 Heterogeneous Response to Collateral Shocks

This section studies the investment response of fast and slow growing firms to fluctuations in the residential real estate of their directors. Following a large empirical literature (Chaney, Sraer and Thesmar, 2012; Bahaj, Foulis and Pinter, 2018 among others), I interpret my findings as strong, suggestive evidence of an extent to which different firm types are financially constrained. I first describe the classification of firms into growth types, and subsequently lay out my empirical strategy.

Assignment of Firms to Growth Groups I follow a parsimonious approach and group firms based on how their growth rate of total assets—a uniformly reported measure of size— relates to the growth rates of their peers (same industry firms born in the same year). I assign a firm to the "fast" type if it grew faster than its median peer in at least half of the years of its tenure.[21] I assign a firm to the "slow" type if it grew slower than its median peer in at least half of the years of its tenure. In Appendix A, I run several robustness checks with respect to this classification: in particular, I redefine the thresholds (40- and 60th percentiles instead of the median), increase the number of years a firm has to be above (below) its median peer in order to be classified as a "fast" ("slow") type, and drop firms which did not survive through age 5.

Empirical Strategy Once I obtain the assignment of firms into groups, I study the response of investment of different firm types to residential collateral shocks. Following Bahaj, Foulis and Pinter (2018), instead of measuring residential property of firm directors in every year independently, the real estate of each firm's director is fixed at its 2002 level, and subsequently rolled forward based on the local housing price index. This approach is advantageous as it isolates fluctuations in residential wealth from potentially endogenous decisions of directors to move into bigger/smaller houses depending on the performance of their firms. Therefore, throughout this section, directors' residential real estate for firm i at time t is measured as:

$$Residential\ RE_{it} = |D_i| \frac{\sum_{d \in \tilde{D}_i} L^d_{i,2002} L^P_{h_d,t}}{|\widetilde{D}_i|}, \tag{1}$$

where $L^d_{i,2002}$ is the estimated value of a house where the director d working at firm i lived in 2002, and $L^P_{h_d,t}$ is the house price index of the region h_d where that director lived in 2002 (with a normalization $L^P_{h_d,2002} = 1$). According to Equation (1), the residential real estate for firm i is the average value of property across matched directors $\tilde{D}_i$, multiplied by the total number of directors D_i.[22] The benchmark specification takes the following form:

$$Investment_{it} = \alpha_i + \delta_{kt} + \mu_{lt} + \sum_{j \in \boldsymbol{J}} \eta_j \times \mathbf{1}_{\{i \in j\}} \times Residential\ RE_{it} + \gamma \times controls_{it} + \varepsilon_{it}, \tag{2}$$

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[21]Technically, I compute median growth rates of total assets in 3 dimensional cells over age, 2-digit SIC code and year. Subsequently, for each firm, I calculate the number of years it grew faster than its peers, and divide it by the total number of years this firm was observed in the panel. I classify a firm into "fast" type if the resulting fraction exceeds 0.5. Similarly, I assign a firm to the "slow" type if it grew slower than its median peer in at least 50% of the periods in which it was observed in the sample.

[22]As described in Bahaj, Foulis and Pinter (2018), the residential property of roughly 60% of directors was successfully valued.

where *Investment* is the change in fixed assets plus depreciation, and $\boldsymbol{J} = \{\text{slow, fast}\}$. α_i, δ_{kt} and μ_{lt} are firm, region-time and industry-time fixed effects. Indicator function $\mathbf{1}_{\{i \in j\}}$ takes a value of 1 if a firm i was assigned to group j, and 0 otherwise. Standard errors are clustered at the level of the firm's region. All monetary variables are scaled by lagged fixed assets[23], which provides a pound-to-pound interpretation of the coefficients. Thus, coefficients of interest $\{\eta_j\}_{j \in \boldsymbol{J}}$ show by how many pounds investment of type j firms will change if their directors' residential wealth appreciates by £1.

Identification   Before getting to the results, it is worth discussing some potential endo-geneity concerns. The firm fixed effect α_i absorbs any time-invariant omitted factors which affect firm's behavior. The list of such factors includes the initial values of directors' homes $L_{i,2002}^d$, as well as the number and composition of directors in 2002. It is also the case that $L_{h_d,t}^P$—the house price index for each director's region—is typically correlated with the firm's real estate price index $L_{j,t}^P$. In turn, $L_{j,t}^P$ could affect the firm's investment opportunities; for example, by way of fueling local consumption (Mian and Sufi, 2014). I include region-time fixed effects δ_{kt} in order to address this. Following Chaney, Sraer and Thesmar (2012), the vector of controls includes firm-level characteristics: a measure of the balance sheet strength (financial leverage), and a measure of cash flow (operating profits). As Bahaj, Foulis and Pinter (2018) point out, residential real estate does not naturally scale up with the firm's size as, for instance, corporate real estate would; therefore, the vector of controls further includes the inverse of lagged fixed assets in order to eliminate any spurious size effects.

Results   Table C5 reports my baseline results. It shows that the investment of rapidly growing firms is more responsive to idiosyncratic fluctuations in directors' real estate than that of more slowly growing businesses. In particular, according to the tightest specification considered in Column (8), a £1 appreciation of directors' residential housing is associated with a 1.3p (pence) increase in a fast type firms' investment, and with only 0.6p for the slow type enterprises. This suggests that fast growing firms are more financially constrained than slow growing firms since the collateral shock leads to a twice bigger investment response among the former group.

Table C6 splits the data into young (under the age of 5) and old subsamples (Columns 2 and 3), and shows that young businesses respond stronger to collateral shocks as compared to more mature enterprises (point estimates are now 1.4p and 1p for fast and slow types,

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[23]Chaney, Sraer and Thesmar (2012) also use lagged fixed assets as a scaling variable.

respectively). This is consistent with an idea that young businesses are usually below their target size, grow fast, and exhibit high investment demand. Furthermore, Columns (4) and (5) in Table C6 show that larger firms (≥ 50 employees) respond stronger to fluctuations in residential wealth, potentially reflecting the higher investment expenditures such businesses need to undertake.[24]

Arguably, one needs to observe a firm long enough in order to properly classify it. For that reason, Table C7 reports the estimates of the baseline specification for firms which survive through age 5. I find that results are barely affected. Finally, in Table C8, I check how robust estimates are to alternative groupings of firms. I find that the estimates are largely unaffected when the requirement to spend half of tenure above (below) the median peer is increased to 75% (Column 2), or when the threshold is shifted from the median to 40th percentile for the slow type and the the 60th percentile for the fast type (Column 3). Finally, I re-estimate the model on the largest possible sample with no controls and total asset growth as a dependent variable (Column 4), and qualitatively confirm my baseline results.

2.5   Taking Stock

The first part of the critical empirical evidence presented in this section established that recessionary cohorts of firms grow slower than expansionary ones. In particular, I documented a collapse of the right tail of cumulative growth distribution among firms born during recessions, which implies that there are fewer rapidly growing firms started during economic contractions.

The second part of this section studied the investment response of fast and slow growing businesses to idiosyncratic fluctuations in the residential wealth of their owners. I found that rapidly growing enterprises respond stronger to collateral shocks than slower growing businesses, which suggests that the former are more financially constrained than the latter.

Next, I build a structural model of firm dynamics which is useful to interpret these empirical findings.

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[24]The subsamples with respect to size are constructed based on the time-average of employment within each firm. Therefore, a firm might be in the "large" subsample but be small (< 50 employees) at some point.

3 Model

In this section, I develop a heterogeneous firms real business cycle model with financial frictions. The model builds on Khan and Thomas (2013) and extends it along several meaningful dimensions. First, since the focus of this paper is on business formation, it is important to have a realistic description of young firms' financing decisions. In particular, in the model I allow firms to issue equity—an important source of funding for young firms (Robb and Robinson, 2014). Second, I introduce heterogeneity in firms' growth profiles. This feature of the model will allow me to study how and why the composition of startups changes over the cycle, and to quantify the aggregate implications of such compositional shifts. Finally, potential entrants will get to *choose* the project type upon entry, and, therefore, determine the target long-run size of their businesses. This modeling choice differentiates this paper from classic papers on industry dynamics (Jovanovic, 1982; Hopenhayn, 1992). The entry mechanism in this paper adapts tools from the search literature, and is reminiscent of that in Sedláček and Sterk (2017).

3.1 Environment

Time in the model is discrete and runs forever: $t = 0, 1, ...$. The economy is populated by 4 types of agents: potential entrepreneurs, incumbent firms, a government, and a representative household. Next, I describe the physical environment.

There is a finite number of projects types $J \in \mathbb{N} \setminus \{0, 1\}$, each of them indexed by a parameter μ_j.[25] This parameter characterizes the optimal size of the firm at the steady-state, which is formally incorporated into the firms' production function as a Lucas (1978) span of control:

$$y_{ijt} = Z_t e^{z_{it}} [k_{ijt}^\alpha n_{ijt}^\nu]^{\mu_j}, \tag{3}$$

where k_{ijt} and n_{ijt} denote capital and labor inputs, and y_{ijt} stays for the output of type-j firm i at time t. The permanent firm-level heterogeneity captured by μ_j will allow me to study compositional shifts in business formation over the cycle. The assumption that the life cycle of projects is determined by an ex ante fixed component captures the idea that organizational capabilities are costly to adjust (Hannan and Freeman, 1984; Henderson and Clark, 1990). Parameters $\{\mu_j\}$ will be estimated in Section 4 to generate a realistic firm-size distribution.

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[25] Hereafter, $\boldsymbol{J}$ will denote a set $\{1, \ldots, J\}$.

Parameters α and ν are assumed to be strictly positive, with their sum being less than 1, $\alpha + \nu < 1$. The decreasing returns technology implies that every firm has a finite target size. The production function (3) is scaled by 2 factors: a time-varying idiosyncratic productivity z_{it} and an aggregate TFP Z_t. I postpone the description of aggregate shocks until Subsection 4.4, and at this point only specify the process for idiosyncratic productivity, which is assumed to be an AR(1):

$$z_{it} = \rho_z z_{it-1} + \varepsilon_{it}, \quad \varepsilon_{it} \sim \mathcal{N}(0, \sigma_z^2), \tag{4}$$

where ε_{it} is i.i.d. across time and space.

FIGURE 4: STRUCTURE OF THE MODEL
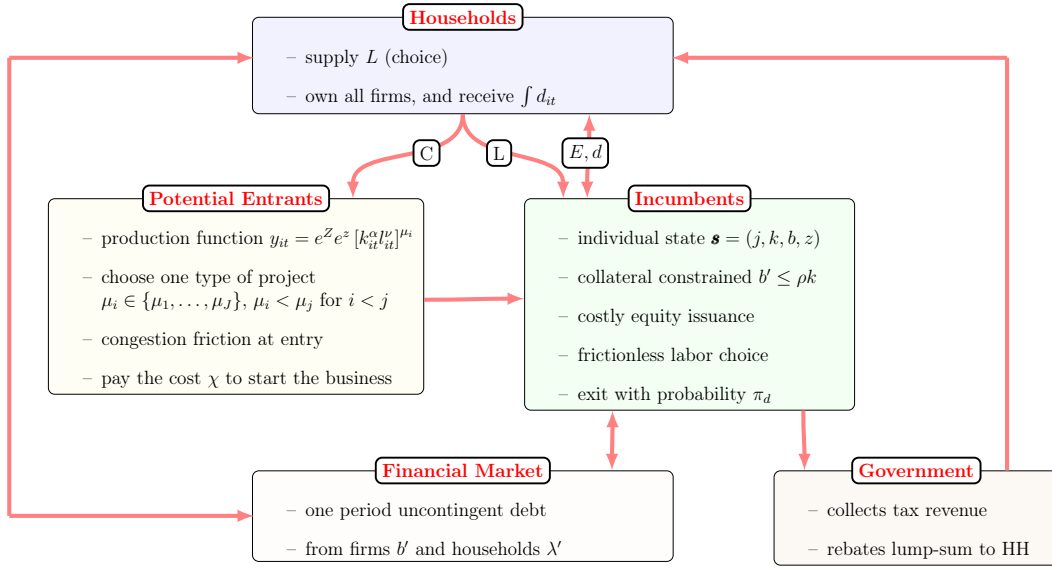
All firms are owned by a representative household, and their objective is to maximize the discounted stream of dividends. In order to finance investment expenses, firms can use internal and external funds, each of them subject to a friction. On one hand, firms can finance investment by way of reducing current payments to shareholders, or may even raise equity if necessary (in this case, firms bear additional costs). On the other hand, firms can also borrow external funds from a competitive financial intermediary; this channel is also subject to a friction. Following a wide body of literature (Khan and Thomas, 2013; Zetlin-Jones and Shourideh, 2017 among others), I assume that the amount of external borrowing is limited by a fraction of the firms' installed capital—their collateral. I provide a formal description of firms' problem in Subsection 3.2. Both frictions are necessary in equilibrium: without the collateral constraint, firms can obtain any level of capital by issuing debt. With zero equity issuance cost, firms can instantaneously get to the optimal size by way of financing their

investment through equity issuance—even if the collateral constraint binds.

FIGURE 5: TIMELINE OF THE MODEL
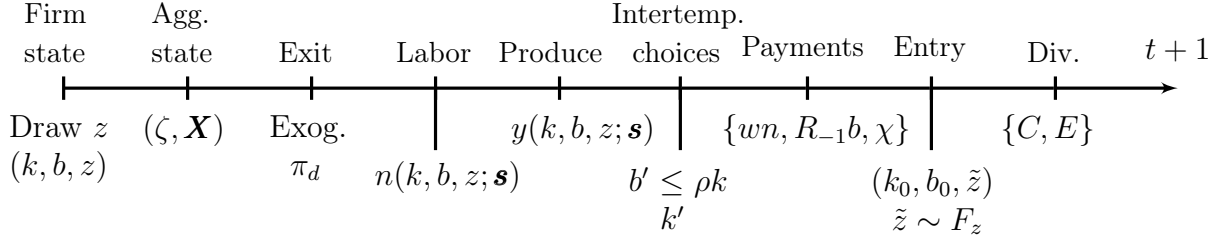
Every period the economy is confronted with a large mass of ex-ante identical potential entrants. In order to make an entry attempt, they have to pay a cost χ denominated in units of the final good. They subsequently endogenously choose a project type μ_j. The entry process is subject to a coordination friction, meaning that some entry attempts will be unsuccessful. Successful entrants are endowed with an exogenous amount of capital k_0 and debt b_0. They draw their initial idiosyncratic productivity z from a distribution $F(z)$. Subsection 3.3 describes the entry process in full detail.

The tax authority (government) levies a tax on firms' operating profits and rebates the proceedings back to households in a lump-sum manner. Figure 4 graphically shows the environment of the model.

The timing of the events within a period is as follows:

1. the aggregate state is realized;

2. each incumbent firm observes the realization of an exit shock, which is a Bernoulli random variable with parameter π_d. Firms that received the shock have to exit the economy at the end of the period, after the production stage takes place. Other firms may continue into the next period;

3. the production stage takes place: firms choose the optimal labor input and produce. At this point, firms which received an exit shock leave the economy. The rest of firms make intertemporal decisions k' and b';

4. the influx of new firms (successful potential entrants) enters the economy;

5. representative household consumes.

The verbal description of events is summarized in Figure 5. Next, I provide the recursive formulations of the individual optimization problems.

3.2 Incumbent Firms

The state vector of the incumbent firm contains four elements: physical capital k, financial position b, idiosyncratic productivity z, and its type j. Since the model features aggregate uncertainty, the state vector $\boldsymbol{s}$ includes both the aggregate shocks $\boldsymbol{X}$ and the distribution of firms over the state space ζ. Therefore, the aggregate state vector is $\boldsymbol{s} = (\zeta, \boldsymbol{X})$. In order to streamline the exposition, I postpone the discussion of aggregate shocks until Subsection 4.4.

At the start of the period, each firm first learns the realization of an exit shock, which is i.i.d. across time and space. The value of the firm at the beginning of the period and prior to the realization of an exit shock is:

$$v_j^0(k, b, z; \boldsymbol{s}) = \pi_d v_j^1(k, b, z; \boldsymbol{s}) + (1 - \pi_d) v_j^2(k, b, z; \boldsymbol{s}), \tag{5}$$

where $v_j^1(\cdot)$ and $v_j^2(\cdot)$ are the values the firm attains, conditional on the realization of an exit shock.

Firms can use several sources to finance their investment expenditures. First, businesses can use retained earnings Π—funds left after they sell output, pay the wagebill and the interest on their outstanding debt. Second, they can use debt financing and borrow b' (throughout the paper it is understood that $b' > 0$ means borrowing, and $b' < 0$ corresponds to savings). Finally, firms can raise funds by issuing equity E. Negative values of E correspond to the case when the firm pays out dividends, while positive values are interpreted as equity issuance. The firm's budget constraint takes the following form:

$$i_j(k, b, z; \boldsymbol{s}) = \Pi_j(k, b, z; \boldsymbol{s}) + E_j(k, b, z; \boldsymbol{s}) + b_j'(k, b, z; \boldsymbol{s}) - b, \tag{6}$$

where subscript j denotes the firm's "type", and the left-hand side variable $i_j(\cdot)$ is the firm's investment choice. Operating profit $\Pi_j(\cdot)$ is formally defined as:

$$\Pi_j(k, b, z; \boldsymbol{s}) = \max_{n \in \mathbb{R}_+} (1 - \tau) \left[y(k, n, z; \boldsymbol{s}) - wn - (R(\boldsymbol{s}_{-1}) - 1) b \right] + \tau \delta k, \tag{7}$$

where $R(\boldsymbol{s}_{-1})$ is a gross interest rate from the preceding period and τ is the tax rate. Physical capital accumulation process takes the form:

$$k_j'(k, b, z; \boldsymbol{s}) = (1 - \delta)k + i_j(k, b, z; \boldsymbol{s}) - \Phi(k_j', k, b, z; \boldsymbol{s}). \tag{8}$$

Function $\Phi(\cdot)$ in Equation (8) denotes the capital adjustment costs. I introduce these costs in order to match important moments of the investment rate distribution. I assume that function $\Phi(\cdot)$ takes a standard quadratic form:

$$\Phi(k'_j, k, b, z; \boldsymbol{s}) = \frac{\varphi^K}{2} \left(\frac{k'_j(k, b, z; \boldsymbol{s})}{k} - 1 \right)^2 k, \tag{9}$$

where parameter φ^K governs the extent to which adjustment costs prevail in the model.

It is instructive to continue the exposition of the model with the problem of a firm which did not receive an exit shock. Such firm is allowed to continue into the next period after choosing a new level of capital k' and issuing a new level of debt b'. The recursive formulation of a continuing firm's problem is:

$$v_j^2(k, b, z; \boldsymbol{s}) = \max_{k', b'} -E_j(k, b, z; \boldsymbol{s}) - C(E_j(k, b, z; \boldsymbol{s})) + \mathbb{E}\left[d(\boldsymbol{s}'|\boldsymbol{s}) v_j^0(k', b', z'|\boldsymbol{s}') \right] \tag{10}$$

$$i_j(k, b, z; \boldsymbol{s}) = \Pi_j(k, b, z; \boldsymbol{s}) + E_j(k, b, z; \boldsymbol{s}) + b'_j(k, b, z; \boldsymbol{s}) - b,$$

$$b'_j(k, b, z; \boldsymbol{s}) \leq \rho k, \tag{11}$$

$$\boldsymbol{s}' \sim \Gamma(\boldsymbol{s}),$$

where the first constraint is the budget constraint, the second is the collateral constraint, and the third is the law of motion for the aggregate state.

Function $C(\cdot)$ in Equation (10) captures the equity issuance cost. Parameter ρ in the collateral constraint governs the tightness of financial frictions in the economy: high values of ρ allow firms to pledge a larger fraction of currently installed physical capital, and, therefore, borrow more. Conversely, with smaller values of ρ, firms need larger amounts of capital in order to be able to borrow the same.

Since all firms in this economy belong to the household, the future stream of dividends in Equation (10) is priced according to the stochastic discount factor $d(\boldsymbol{s}'|\boldsymbol{s})$, which converts the value of future resources in terms of current ones.

Now I turn to the problem of a firm which is forced to exit at the end of the period. Such a firm produces in the current period, sells its undepreciated capital, pays the outstanding debt, and leaves the economy. Exiting firms do not make any intertemporal choices, their value is then simply given by:

$$v_j^1(k, b, z; \boldsymbol{s}) = \Pi_j(k, b, z; \boldsymbol{s}) + (1 - \delta)k - b, \tag{12}$$

where $\Pi_j(\cdot)$ is defined as in Equation (7).

3.3 Entry

This paper departs from a wide body of literature on firm dynamics in that potential entrants can *choose* a type of business opportunity upon entry. To operationalize this, I introduce a coordination friction which is reminiscent of the literature on directed search, and is formulated similarly to Sedláček and Sterk (2017).

Let $\{\psi^j\}_{j\in \boldsymbol{J}}$ be a distribution of available to potential entrants business opportunities of types $\boldsymbol{J}$ in period t. In order to enter, an aspiring potential entrant has to pay a cost χ denominated in units of the final good. This entrance cost is designed to capture expenditures associated with market research, developing a business plan and alike. Upon paying the cost, a potential entrant gets to choose one of projects $j \in \boldsymbol{J}$, and, subsequently, has a chance to seize one of the available ideas ψ^j. Therefore, parameters $\{\psi^j\}$ can be thought of governing the relative probability of success of starting different project types.

Given a coordination friction, not all business opportunities are seized, while others are seized by several aspiring entrepreneurs. This friction is modeled by a matching function which returns the mass of successful entrants of type j, m_t^j:

$$m_t^j = \left(e_t^j\right)^{\phi} \left(\psi^j\right)^{1-\phi}, \tag{13}$$

where e_t^j is the mass of potential entrants who decided to pursue a project of type j, and $\phi \in (0,1)$ is an elasticity of successful entrants with respect to startup attempts.

While the equilibrium of the model will be laid out later, it is convenient to formulate one equilibrium condition here. Potential entrants are indifferent with respect to which project to start in equilibrium; this consideration gives rise to a set of free-entry conditions (one for each type). The free-entry condition states that the cost of starting a business has to match the associated expected benefit:

$$\underbrace{\chi}_{\text{cost of entry}} = \underbrace{\frac{m_t^j}{e_t^j}}_{\text{success probability}} \underbrace{\int_z v_j^0(k_0, b_0, z; \boldsymbol{s}) \mathrm{d}F(z)}_{\text{exp. value of type-}j\text{ project}} \qquad \forall j \in \boldsymbol{J} \quad z \sim F(z), \tag{14}$$

where the ratio on the right-hand side is a success probability of starting a project of type j.

Business cycle fluctuations (shifts in elements of $\boldsymbol{s}$) trigger a change in the expected benefit of starting and running projects; as discussed above, different project types can be

affected differentially. But the free-entry condition (14) ensures that the success probability of projects with a relatively high value adjusts correspondingly (downwards), so that the right-hand side of (14) remains unchanged.

By way of combining the matching friction (13) with free-entry conditions (14), and defining $\tilde{v}_j^0(k_0, b_0; \boldsymbol{s}) := \int_z v_j^0(k_0, b_0, z; \boldsymbol{s}) \mathrm{d}F(z)$, one can derive the following equation for the mass of successful entrants of each type:

$$m_t^j = \chi^{\frac{\phi}{\phi-1}} \left[\tilde{v}_j^0(k_0, b_0; \boldsymbol{s}) \right]^{\frac{\phi}{1-\phi}} \psi^j \quad \forall j \in \boldsymbol{J}. \tag{15}$$

The right-hand side of Equation (15) contains a product of 3 terms. Given that $\phi < 1$, larger entry costs χ reduce the mass of entrants m_t^j. Furthermore, an increase in the value $\tilde{v}_j^0(\cdot)$ and/or project availability ψ^j stimulates the entry of type-j projects. Critically, Equation (15) shows how time-varying aggregate conditions affect entry decisions of potential entrepreneurs; it is also clear that if the value $\tilde{v}_j^0$ responds differently across the firm types to the same aggregate shock, one should expect a differential change in the intensity of entry of those firm types.[26]

3.4   Households

The economy is populated by a unit mass of identical households. Each household consumes, supplies labor, and saves into a risk-free bond λ' and firms' shares $\omega(k, b, z, j)$. The price of current shares is $\rho_0(\cdot)$, and the purchase price of new shares is $\rho_1(\cdot)$. The net risk-free interest rate is $q_0(\boldsymbol{s})^{-1} - 1$. The household's dynamic programming problem is:

$$W(\omega, \lambda; \boldsymbol{s}) = \max_{c, n, \lambda', \omega'} \left[U(c, n) + \beta \mathbb{E} W(\omega', \lambda'; \boldsymbol{s}') \right] \tag{16}$$

subject to the budget constraint and the law of motion for the aggregate state:

$$c + q_0(\boldsymbol{s})\lambda' + \int_{K \times B \times Z \times \boldsymbol{J}} \rho_1(k', b', z', j; \boldsymbol{s}) \mathrm{d}\omega' \leq w(\boldsymbol{s})n + \lambda + \int_{K \times B \times Z \times \boldsymbol{J}} \rho_0(k, b, z, j; \boldsymbol{s}) \mathrm{d}\omega + T,$$

$$\boldsymbol{s}' \sim \Gamma(\boldsymbol{s}).$$

---

[26]My model does not capture "waiting" decisions of potential entrepreneurs; in other words, they do not postpone entry decisions for later when aggregate conditions deteriorate. In order to introduce such a mechanism, one needs to study business formation through the lens of occupation choice models, and this is beyond the scope of this paper. However, based on the evidence from the Business Formation Statistics (BFS)—a new dataset developed by the U.S. Census—the waiting time to form businesses declined during the Great Recession in the U.S. (see Figure B3 in Appendix B). This implies that, if anything, the financial crisis did not generate a pronounced delayed entry effect.

The right-hand side of the budget constraint represents resources available to the household: it consists of firms' shares coming from the previous period, tax rebate from the government T, as well as labor income. The left-hand side shows that a part of these resources is consumed, and the rest is reinvested into new firm shares as well as into a risk-free bond.

Let $C(\omega, \lambda; \boldsymbol{s})$ be the household's consumption policy function, and $N(\omega, \lambda; \boldsymbol{s})$ be a labor supply policy function. Also, let $\Xi(k', b', z', \omega, \lambda, j; \boldsymbol{s})$ be a number of shares purchased in firms of type j which start tomorrow with capital k', debt b', idiosyncratic productivity z'. Finally, $\Lambda(\omega, \lambda)$ is the policy function with respect to a risk-free bond.

3.5   Equilibrium

Let Σ_K, Σ_B and Σ_Z be Borel sigma algebras over K, B and Z. The state space is $S = K \times B \times Z$ with (k, b, z) being an element of that space. Let Σ_S be the sigma algebra on the state space with typical set $\mathcal{S} = \mathcal{K} \times \mathcal{B} \times \mathcal{Z}$, and (S, Σ_S) be the corresponding measurable space. Let $\zeta_j : \Sigma_S \to [0, 1]$ be a distribution of type-j firms across the state space at the beginning of the period after idiosyncratic uncertainty has been revealed.

A *recursive competitive equilibrium* for this economy is a collection of functions:

$$\left\{ v_j^0, v_j^1, v_j^2, k_j', b_j', n_j, w, \rho_0, \rho_1, W, C, N, \Lambda, \Xi, d, m^j, e^j, \Gamma \right\}_{j \in \{1, \dots, J\}},$$

such that:

1. W solves the household's problem (16), and (C, N, Ξ, Λ) are the associated policy functions,

2. $\{v_j^0, v_j^1, v_j^2\}$ solve the firms' problem (5)-(12), and $\{k_j', b_j', n_j\}$ are the corresponding policy functions,

3. $\{m^j\}$ satisfy the free-entry conditions (14),

4. consistency condition satisfies $\forall (k, b, z, j) \in K \times B \times Z \times \boldsymbol{J}$

$$\Xi(k', b', z', \{\zeta\}, \lambda, j; \boldsymbol{s}) = \zeta_j'(k', b', z'),$$

5. labor market clears

$$N(\{\zeta\}, \lambda; \boldsymbol{s}) = \sum_{j=1}^{J} \int_S n_j(k, b, z; \boldsymbol{s}) \mathrm{d}\zeta_j,$$

6. stochastic discount factor satisfies

$$d(\boldsymbol{s}'|\boldsymbol{s}) = \beta \frac{U_C'(C(\{\zeta'\}, \lambda', \boldsymbol{s}'), N(\{\zeta'\}, \lambda', \boldsymbol{s}'))}{U_C'(C(\{\zeta\}, \lambda, \boldsymbol{s}), N(\{\zeta\}, \lambda, \boldsymbol{s}))},$$

7. goods market clears

$$C(\{\zeta\}, \lambda; \boldsymbol{s}) = \sum_{j=1}^{J} \int_S \left[e^z [k_j^\alpha n_j^\nu]^{\mu_j} - (1 - \pi_d)(k_j' - (1 - \delta)k_j) + \right.$$
$$\left. + \pi_d((1 - \delta)k_j) - m^j k_0 \right] \mathrm{d}\zeta_j - \sum_{j=1}^{J} \chi e^j,$$

8. government runs a balanced budget (transfers = tax revenue)

$$T = \tau \sum_{j=1}^{J} \int_S [y_j - wn_j - (R(\boldsymbol{s}_{-1}) - 1)b_j) - \delta k_j] \mathrm{d}\zeta_j$$

9. bonds market clears (by Walras law),

10. the law of motion for the aggregate state vector is consistent with firms' policy functions.

Following Khan and Thomas (2008), in Appendix D.1 I discuss how to combine household's and firms' programming problems for the efficient computation of the equilibrium.

3.6   Solution Method

In general, it is difficult to solve for the recursive competitive equilibrium in models of the type laid out in Section 3, since the firms' policy functions depend on the aggregate state vector $\boldsymbol{s}$, which includes an infinitely dimensional object—distribution of firms across the state space. The standard method in the literature to solve such models is Krusell and Smith (1998). In particular, for the model laid out in Section 3, the method of Krusell and Smith (1998) requires stipulating the law of motion for the marginal utility and future mean capital stock as functions of a finite number of moments of the cross-sectional distribution. The resulting forecasting rule would include many level and cross-sectional terms, rendering the approach both impractical and slow. Since the model has to be solved many times in order to estimate aggregate processes, the solution procedure must be fast.

This reasoning motivates me to follow an alternative approach - perturbation method of Reiter (2009): it involves solving the firms' decisions rules globally at the deterministic steady-state, and then perturbing the solution with respect to aggregate shocks. This approach, therefore, preserves the full non-linearity of the firms' policy rules with respect to idiosyncratic states, and perturbs these policies linearly with respect to aggregate shocks (Mongey and Williams, 2016). Further details on the solution method are relegated to Appendix D.

FIGURE 6: DECISION RULES AT STEADY-STATE

(A) Decisions at binding coll. constraint, μ_L

(B) Decisions at binding coll. constraint, μ_H

(C) Decisions at $\bar{k} = \mathbb{E}[k]$, μ_L

(D) Decisions at $\bar{k} = \mathbb{E}[k]$, μ_H
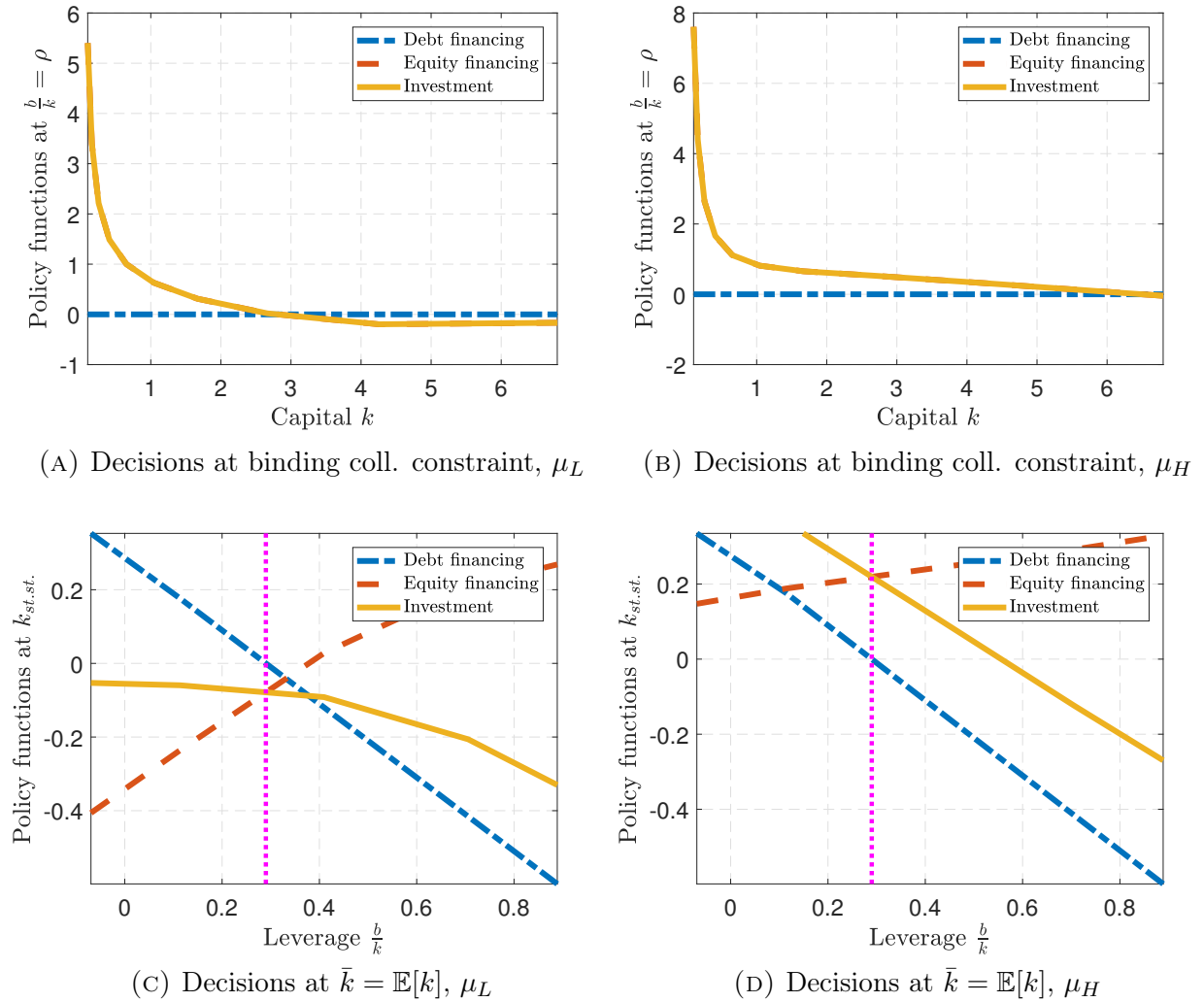
Notes: Figure 6 consists of 4 panels. Panels (A) and (B) plot decision rules when collateral constraint (11) binds, and Panels (C) and (D) plot decision rules when firms' capital is fixed at the cross-sectional mean. Panels (A) and (C) correspond to $\mu = \mu_L$, while Panels (B) and (D)—to $\mu = \mu_H$. Vertical pink dotted lines in Panels (C) and (D) correspond to the calibrated tightness of the collateral constraint (parameter ρ). All decision rules are computed before profits ($\Pi = 0$), and are normalized by the idiosyncratic level of capital.

3.7 Characterization of Steady-State

In order to demonstrate the workings of the model, in this section I plot the firms' decision rules at the calibrated steady-state (aggregate shocks are turned off). Given the dimensionality of the problem, I follow Guo (2019) and first show how different firms finance their investment expenditures when they can no longer issue debt (collateral constraint (11) binds), and subsequently plot firms' decisions by tracing out leverage ratios at the cross-sectional mean level of capital $\bar{k} = \mathbb{E}[k]$.

To facilitate the visual inspection, here I focus on the lowest (μ_L) and highest (μ_H) types only. Idiosyncratic productivity is fixed at the unconditional mean ($z = 0$). The policy functions for investment demand, debt financing and equity financing are depicted in Figure 6. The top row plots the decisions for when the collateral constraint binds, and the bottom row—at the cross-sectional mean level of capital $\bar{k}$.

Along the size dimension (top row), there is a significant variation in investment and equity financing policies, but little variation in debt financing policies. Given the decreasing returns to scale technology (3) and mean-reverting productivity process (4), firms have a finite optimal target size. Therefore, small firms, which are further away from the optimal size, exhibit higher demand for investment. Conversely, large firms are at or around the optimal size, and their investment demand is small or even slightly negative. This is the main reason why the investment policy is decreasing with size. Given that the collateral constraint binds, firms cannot issue debt to finance their investment; therefore, businesses finance their investment expenditures primarily through equity issuance (corresponding lines lie on top of each other). It is also straightforward to see that firms with a larger span of control have uniformly higher investment rates, and their target size (determined by the point where investment demand crosses 0) is larger.

Along the leverage ratio dimension (bottom row in Figure 6), there is a significant variation in financing policies. The cross-sectional mean level of capital is above the target size of low-type firms, but below the target size of high-type businesses. This observation is reflected in the way decision rules look like for these firm types: investment expenditures are negative for μ_L and positive for μ_H firms. With an increase in the leverage ratio, the residual capacity of debt financing decreases, so the debt financing declines and the equity financing increases correspondingly.[27] Due to the equity issuance cost, firms do not raise

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[27]In the data, young firms issue significant amounts of debt (see Figure B1 in Appendix B). In line with the model's predictions, equity issuance is most pronounced in the first several years of firm tenure (Figure B2).

equity one-to-one with investment demand, and investment decreases with the leverage ratio. Businesses with a larger span of control reduce their investment demand more than low type firms as their leverage rises. Besides, while μ_H firms issue equity (red dashed line is in positive region across all leverage ratios), μ_L firms pay out dividends when leverage ratio is relatively low, and start issuing equity only when they become highly leveraged.

In the next section, I bring the model to the data.

4 Parametrization and Estimation

In my quantitative exercise, I set number of types equal $J = 3$, which is both computationally feasible and aligns with empirical investigation from Section 2.

Strategy I split all parameters into 3 groups. Parameters in the first group are standard in macro literature, and I assign values to them without solving the model (Subsection 4.1). The second group contains parameters which govern the behavior of the model economy at the steady-state; I assign values to these parameters so that the model-generated moments match a set of empirical targets (Subsection 4.2). In Subsection 4.3, I show that the model performs well along the dimensions that were not directly targeted at the estimation step.

The last group consists of persistence and variance parameters for aggregate stochastic processes, which I jointly estimate using Bayesian methods in Subsection 4.4. Finally, before presenting the main results of the paper in Section 5, Subsection 4.5 validates the model by way of comparing the model-implied business cycle statistics against the U.K. data.

4.1 Fixed Parameters

The period in the model is one quarter, which is a suitable frequency to study business cycles. Therefore, I set the discount factor $\beta = 0.98$. I set labor share $\nu = 0.67$ and capital share $\alpha = 0.33$, and then estimate $\{\mu_j\}_{j=1}^J$ in Subsection 4.2. I will restrict the values of $\{\mu_j\}$ to be less than 1 in order to guarantee decreasing returns to scale (and a finite target size).

I set depreciation rate $\delta = 0.025$ so that the aggregate investment is 10% per annum. Tax rate is set equal to 0.24, which is the average corporate tax rate in the U.K. over the last decade. Entering firms start with zero initial debt $b_0 = 0$. Quarterly exit probability is set to 0.03 in order to get an annual 12% exit rate.

TABLE 1: FIXED PARAMETERS

Parameter	Description	Value
β	Discount factor	0.98
α	Capital share	0.33
ν	Labor share	0.67
δ	Depreciation rate	0.025
b_0	Initial debt position	0.00
τ	Tax rate	0.24
π_d	Exit probability	0.03

I let the instantaneous utility function of the household be separable between consumption and labor:

$$U(C(\boldsymbol{s}), N(\boldsymbol{s})) = \frac{C(\boldsymbol{s})^{1-\eta}}{1-\eta} - A\frac{N(\boldsymbol{s})^{\zeta}}{\zeta}, \tag{17}$$

where Frisch elasticity of substitution is assumed to be infinite ($\zeta = 1$). Parameter A is the disutility from labor, and is estimated in Section 4 to make the household devote a third of its time endowment to market work. Following Bloom, Floetotto, Jaimovich, Saporta-Eksten and Terry (2018), I also assume a log consumption utility function ($\eta = 1$). With this, the utility function (17) implies that the following equilibrium condition must hold:

$$w(\boldsymbol{s}) = AN(\boldsymbol{s})^{\zeta-1}C(\boldsymbol{s})^{\eta} = AC(\boldsymbol{s}). \tag{18}$$

Equation (18) represents a link between equilibrium aggregate consumption and the wage rate, and allows for the efficient computation of equilibrium. Table 1 summarizes the pre-set parameters.

4.2   Internally Estimated Parameters

In this section, I estimate parameters which govern the behavior of the model at the steady-state. In what follows, I provide a heuristic identification argument that justifies the choice of the target moments. Even though every targeted moment is simultaneously affected by all the parameters, in this section I discuss each of them in relation to the parameter for which, intuitively, that moment yields the most identification power. For the most part I follow literature in picking the moments to target.

Since the firm-level data comes at annual frequency, and the model period is set to a quarter, I have to make the model- and data-based moments comparable. To do so, at each step of the estimation procedure, I draw a panel from the model and subsequently aggregate

the data to annual frequency in order to compute the corresponding moments.

The elasticity of the matching function ϕ is informative about how volatile firm entry is over the business cycle (see Equation (14)); I, therefore, target the relative volatility of the entry rate with respect to GDP.[28] Parameter χ is picked so that the total amount of resources devoted to entry equals 7.3% of GDP at the steady-state (Sedláček and Sterk, 2017). Collateral constraint parameter ρ directly affects the degree to which firms are financially constrained in the model; I, therefore, require the model to deliver the mean leverage ratio of 0.27 as in the data.

<div align="center">

TABLE 2: ESTIMATED PARAMETERS

</div>

Parameter	Description	Value	Target	Data	Model
A	Preference for leisure	2.04	N	—	0.33
ϕ	Elast. of match. fun.	0.72	$\frac{\sigma(\text{entry})}{\sigma(Y)}$	2.78	2.29
χ	Entry cost,%	0.32	$\frac{\sum_{j=1}^{J} \chi e_j}{Y}$	0.07	0.12
ρ	Tightness of coll. const.	0.31	$\mathbb{E}\left[\frac{b}{k}\right]$	0.27	0.27
σ_z	Std of idios. inn.	0.03	$\mathbb{E}\left[\frac{i}{k}\right]$	0.29	0.30
ρ_z	Pers. of idios. inn.	0.98	$\sigma\left[\frac{i}{k}\right]$	0.68	0.71
φ^E	Equity iss. cost	0.06	$\mathbb{E}\left[\frac{E}{k}\middle|\text{age} < 5\right]$	0.06	0.05
φ^K	Adjustment cost	0.13	$\mathbb{E}\left[\left|\frac{i}{k}\right| > 0.2\right]$	0.36	0.42
k_0	Capital endowment	0.20	$\frac{\mathbb{E}[n|\text{age}=0]}{\mathbb{E}[n]}$	0.32	0.32
μ_2	Medium DRS	0.80	Emp. share $n < 50$	0.37	0.37
$\Delta\mu$	DRS spread	0.05	Emp. share $n \geq 250$	0.48	0.25
ψ_1	Availability of low type	0.40	Firm share $n < 50$	0.95	0.95
ψ_2	Availability of med. type	0.33	Firm share $n \geq 250$	0.005	0.005

Idiosyncratic productivity process parameters ρ_z and σ_z along with a capital adjustment cost parameter φ_K affect investment decisions of firms the most. I, therefore, include the first two moments of the investment rate distribution as well as the frequency of investment spikes[29] in the set of empirical targets.

I assume that the cost of issuing equity is cubic in Equation (10):

$$C(E_j) = \varphi^E [\max\{0, E_j\}]^3.$$

Such formulation implies that firms find it increasingly hard to raise equity. From the technical perspective, I found that a cubic function smooths out the kink, which results in

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[28]I obtain quarterly data from OECD: it reports the index of firm entry between 2006Q1 and 2018Q2, with normalization of 1 for year 2007.

[29]Investment spike is a situation when the firm's investment rate exceeds 20% in absolute value.

a more stable solution. Equity issuance cost φ^E is chosen so that the model generates an average (among firms up to age 5) issued equity-to-capital ratio of 0.06.

Capital endowment of successful entrants k_0 is set to make entrants' size 32% of the average incumbent's size (in terms of employment). New firms are assumed to draw initial idiosyncratic productivity from a time-invariant normal distribution $F(z)$ with a mean -0.1 and variance σ_z.[30]

Span of control parameters $\{\mu_j\}_{j=1}^J$ are chosen to generate a skewed firm-size distribution (see Figure A1 in Appendix A.3). In particular, I target the share of firms with less than 50 and more than 250 employees. Finally, availability parameters $\{\psi_j\}_{j=1}^J$ are picked to match the share of aggregate employment in firms with less than 50 employees, and share of employment in firms with at least 250 workers. Table 2 reports the estimates for the structural parameters.

4.3   Model Validation

In this section, I first explore the firms' average life cycle profiles. Subsequently, I check how well the model performs with respect to the moments not directly targeted in estimation: specifically, I will be looking at the investment and financial heterogeneity, as well as at the distribution of employment growth rates. Finally, I show that, in the model, rapidly growing firms are more responsive to collateral shocks than slowly growing businesses, mirroring findings from Section 2.

Average life cycle   Figure 7 plots the central to this paper dimensions of firms' life cycle profiles. Panel A shows how different target sizes are across firm types; while the average low type firm ends up being around 3 times bigger than a newborn business (in terms of labor), the average high type firm takes off pretty fast and exceed the size of entrants by a factor of 13 about 5 years into their tenure.[31] It is also visible that it takes around 5 quarters for a low-type firm to reach its target size, while the high type business is still growing even 5 years out.

The patterns observed in Panel A translate into growth rate terms in Panel B, where the mean growth rate of labor is depicted. It illustrates how the high type businesses exhibit high (relative to low and medium type firms) growth rates throughout their life cycles.

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[30]Entering businesses are estimated to be 10% less productive than incumbents (see Table C1). Lee and Mukoyama (2015) obtain similar findings using Census data.

[31]Note that the relative size of firms is calculated with respect to employment in the first *quarter* of their tenure.

Growth in terms of labor is accompanied by capital accumulation through investment (Panel C). Entering businesses are endowed with low physical capital holdings (around 10% of the level of an average incumbent), and, therefore, exhibit high investment rates early in their tenure. High investment expenditures are financed through 3 sources (retained earnings, debt and equity issuance). Panel D shows that young firms rely a lot of equity financing, and start paying out dividends only several years after entry (it takes longer for high type firms as their target size is larger).

FIGURE 7: AVERAGE LIFE CYCLE PROFILES

(A) Size relative to age 0

(B) Mean labor growth rate

(C) Mean investment rate

(D) Mean equity financing
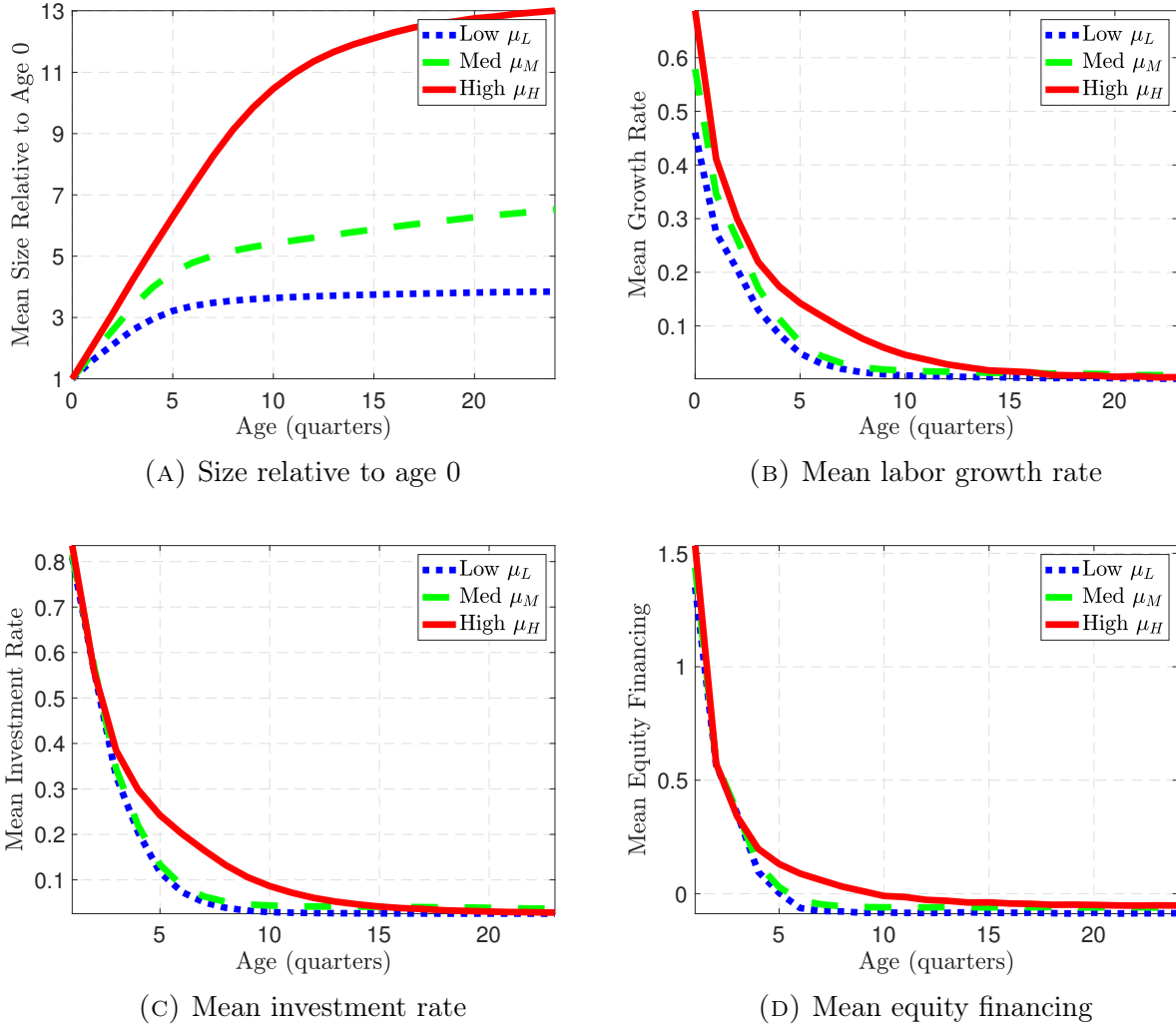
Notes: Figure 7 plots the average (across idiosyncratic productivity) life cycle profiles of firms in the first 25 quarters after their birth. Panels A, B, C and D plot the mean size (in terms of labor) relative to entrants, the mean growth rate of labor, the mean investment rate, and the mean equity financing, respectively. The growth rate of labor is calculated as $\Delta_t = (n_t - n_{t-1})/\frac{1}{2}(n_t + n_{t-1})$. The equity financing is normalized by the firm's current size (capital).

Figure 7 is key for understanding the compositional effects I study in this paper. Provided that it takes high type firms substantially more time (and resources—see Panel D) to get up to scale as compared to low type businesses, tightening of the collateral constraint affects the former stronger than the latter. At the extreme, consider a firm for which its target size coincides with its initial size—such firm will not be affected by collateral shocks at all. Therefore, financial shock is expected to affect μ_H firms stronger than other firm types.

TFP shocks change target sizes of all firm types *proportionately*, therefore, equally affecting the values of low, medium and high target size businesses. Thus, the impact of the aggregate productivity shock is expected to affect similarly the entry intensity of different firm types.

Investment and leverage heterogeneity In order to characterize the investment and leverage heterogeneity in the model—moments not directly targeted in the calibration—I follow Ottonello and Winberry (2018) and compare (auto)correlations of investment rates and leverage in the model-simulated panel against the data.

I compute statistics in a balanced panel conditional on observing firms for 10 years.[32] Overall, Table 3 shows that the model is broadly consistent with the investment and financial heterogeneity observed in the data. Even though I targeted only the first moment of the leverage distribution and the first two moments of the investment distribution, the model does a reasonable job in picking up the joint heterogeneity.

TABLE 3: INVESTMENT AND LEVERAGE HETEROGENEITY

Moment	Description	Data	Model
Investment heterogeneity			
$\mathbb{E}\left[\frac{i}{k}\right]$	Mean investment rate (targeted)	0.29	0.30
$\sigma\left[\frac{i}{k}\right]$	Std investment rate (targeted)	0.68	0.71
$\rho\left(\frac{i}{k}, \frac{i}{k}_{-1}\right)$	Autocorrelation investment rate	0.12	0.58
Joint heterogeneity			
$\rho\left(\frac{b}{k}, \frac{b}{k}_{-1}\right)$	Autocorrelation leverage	0.76	0.84
$\rho\left(\frac{i}{k}, \frac{b}{k}\right)$	Correlation of investment and leverage	-0.15	-0.51

Notes: Table 3 reports the statistics regarding the cross-sectional distribution of investment rates and leverage ratios in the data (BvD) and in the model (steady-state). Model-generated data has been aggregated to annual frequency. A balanced 10-year long panel in considered. The first two moments reported (mean and std. of investment rates) have been targeted during the estimation step in Section 4.

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[32]Empirical moments are computed on a balanced panel for years 2001-2010.

Growth Rates Distribution   Figure 8 displays a distribution of employment growth rates in the model and in the data. In order to comply with the empirical counterpart, the model-generated data was aggregated to annual frequency.

Figure 8 shows that the model-implied distribution of employment growth rates looks reasonably similar to its empirical analog, even though it was not directly targeted at the parametrization step. The model generates a spike at the bin corresponding to growth rates between -5% and 5%, as in the data. The size of the spike is very close to the one observed in the data.

FIGURE 8: DISTRIBUTION OF EMPLOYMENT GROWTH RATES
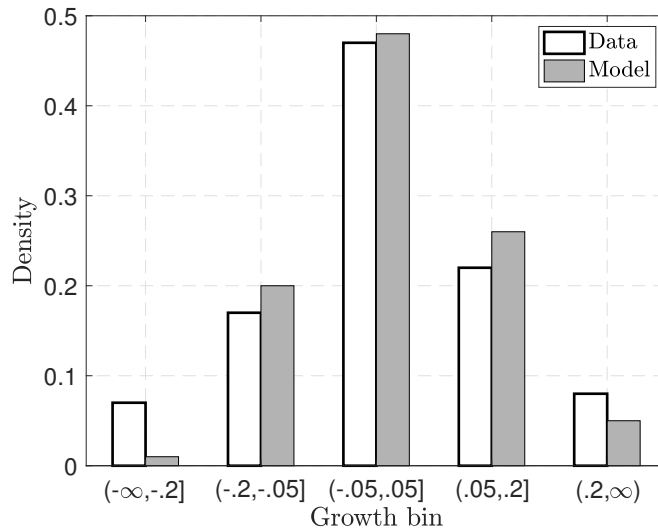
Notes: Figure 8 plots the empirical and the model-generated employment growth distribution. Simulated data was aggregated to annual frequency. Underlying data are 10-year long balanced panels. Employment growth of firm i at time t was calculated as $2(n_{it} - n_{it-1})/(n_{it} + n_{it-1})$. Data source: BvD.

Heterogeneity in Responsiveness to Collateral Shocks   In Section 2, I argued that fast growing businesses are more financially constrained than slow growing firms by projecting firms' investment on the orthogonal idiosyncratic fluctuations in the residential wealth of firms' directors. Clearly, it is not feasible to run the exactly same regression on the model-simulated data; however, it is possible to study the *relative* responsiveness of firms' investment to fluctuations in their collateral.

Given the formulation of the financial constraint in Equation (11), I mimic fluctuations in the collateral by generating independent (across time and space) shocks to firms' capital. In particular, I simulate a panel of firms without collateral shocks, but at the same time I also record in each period what the investment of each firm would have been had they

experienced a shock to capital. Let k_{it} be the capital stock of firm i at time t. I assume that collateral shocks are drawn from the uniform distribution with the support $[-k_{it}, k_{it}]$, so that the after-shock capital holding is non-negative. I classify firms into fast and slow growing types exactly as it was described in Subsection 2.4. I then regress investment after a collateral shock on an interaction of the shock's size and firm's type (including the same controls as in the benchmark empirical specification (2)).

TABLE 4: HETEROGENEOUS RESPONSE TO COLLATERAL SHOCKS

	Data	Model	
		With controls	Without controls
Slow	0.006	0.55	0.56
Fast	0.013	1.00	1.01
Ratio	**2.2**	**1.8**	**1.8**
Firm FE	Yes	Yes	Yes
Time FE	Yes	Yes	Yes

Notes: Table 4 reports OLS estimates from projecting firms' investment on the interaction of the collateral shock and firm growth group. Classification of firms into slow and fast types was performed as in Subsection 2.4. The vector of controls includes firms' profits, the inverse scale and the leverage. Variables were lagged and normalized by the idiosyncratic level of capital. Firm and time fixed effects were included.

Table 4 shows that, in the model, fast growing firms are also more responsive to collateral shocks, providing validation for the theoretical framework developed in Section 3. In particular, fast growing firms are 1.8 times more responsive to collateral shocks than slow growing businesses; this is close to the estimate of 2.2 observed in the data. Next, I proceed with the estimation of aggregate shocks.

4.4 Estimation of Aggregate Shocks

I introduce 3 exogenous aggregate stochastic processes: the first governs the tightness of the collateral constraint ρ, the second affects the disutility of labor A, and the third is an aggregate productivity shock Z. I assume that shocks follow AR(1) processes in logs:

$$\log \widetilde{X}_{t+1} = \rho_X \log \widetilde{X}_t + \varepsilon_{X,t+1}, \tag{19}$$
$$\varepsilon_{X,t+1} \sim \mathcal{N}(0, \sigma_X^2)$$
$$X \in \{\rho, A, Z\},$$

where $X_t = X_{ss} \times \widetilde{X}_t$. The collateral shock directly affects the ease with which firms in the model can issue debt and finance their investment. The model developed in Section

3 features frictionless labor markets; therefore, in order to account for the distressed labor markets during recessions, I introduce shocks to the disutility of labor A—it is a parsimonious way to model disruptions originating in the labor market. Finally, aggregate TFP shock is a reduced-form way to account for alternative, deeper mechanisms, not captured by the benchmark model (Kehoe, Midrigan and Pastorino, 2018).[33]

TABLE 5: ESTIMATION OF AGGREGATE SHOCKS

Param.	Type	Prior			Posterior (**flexible**)			Posterior (**fixed**)		
		Mean	Std		Mode	5%	95%	Mode	5%	95%
Persistence										
ρ_z	Beta	0.5	0.05		0.964	0.964	0.995	0.996	0.981	0.996
ρ_r	Beta	0.5	0.05		0.858	0.851	0.913	0.889	0.885	0.968
ρ_n	Beta	0.5	0.05		0.989	0.954	0.995	0.983	0.925	0.997
Standard deviation										
σ_z	Inv. Gamma	0.25	0.06		0.009	0.008	0.011	0.008	0.008	0.009
σ_r	Inv. Gamma	0.25	0.06		0.101	0.089	0.113	0.096	0.088	0.117
σ_n	Inv. Gamma	0.25	0.06		0.007	0.007	0.008	0.007	0.007	0.008

Notes: Table 5 reports the results of the Bayesian estimation of aggregate exogenous stochastic processes. Each process is characterized by its persistence and the standard deviation of innovations—see Equation (19). Table reports the priors for each parameter as well as the mode and 95% confidence bounds based on 10000 draws from the posterior distribution. See text and Appendix D.4 for more details on the estimation procedure. "Flexible" refers to a benchmark model, and "fixed" - to a model with a fixed composition of firm types.

I use employment, GDP and investment expenditures aggregate series for the U.K. economy in order to estimate the persistence and volatility of 3 stochastic processes (see Figure D1 in the Appendix). I make a standard in the literature choice[34] and use the Beta and Inverse gamma distributions as priors for persistence and volatility parameters, respectively. Table 5 reports the estimation results. Anticipating a quantitative assessment of the intensive margin of business formation in Section 5, I independently estimate 2 models: a benchmark model along with a fixed composition version of the model (to be described in Subsection 5.1).

Identification How does the aggregate data identify the parameters of the exogenous stochastic processes? Figure 9 plots the impulse-response functions for 3 aggregate stochas-

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[33]Chari, Kehoe and McGrattan (2007) develop a business cycle accounting procedure and argue that an empirically successful business cycle model should feature mechanisms which would manifest themselves as labor and efficiency wedges in a *prototype* model. I introduce these wedges directly into my model by including labor disutility and TFP shocks.

[34]See, for example, An and Schorfheide (2007).

FIGURE 9: IMPULSE-RESPONSE FUNCTIONS TO AGGREGATE SHOCKS

(A) TFP shock, aggregates

(B) Financial shock, aggregates

(C) Disutility shock, aggregates

(D) TFP shock, entry

(E) Financial shock, entry

(F) Disutility shock, entry
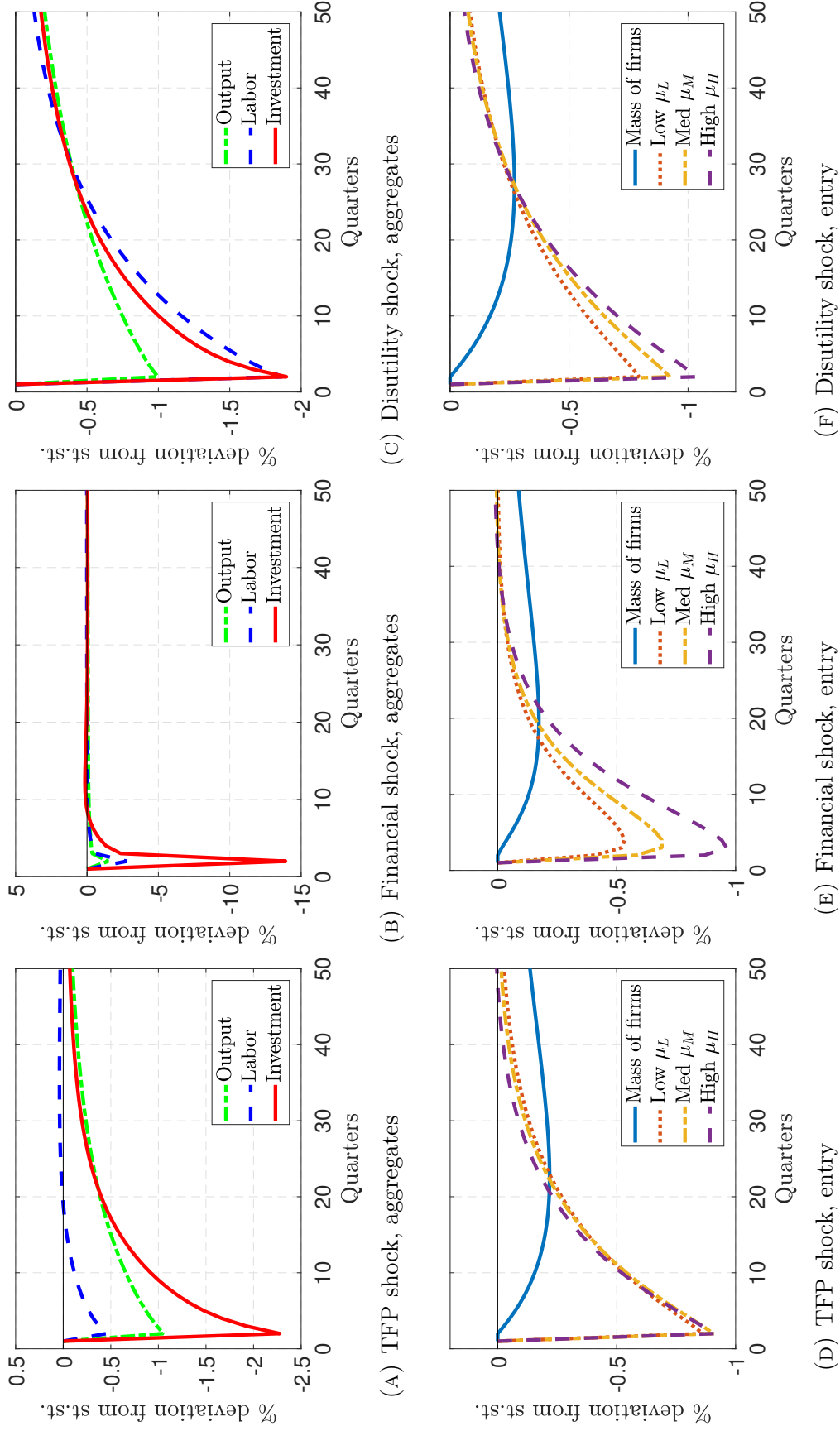
Notes: Figure 9 plots impulse-response functions to innovations in TFP (panels A and B), financial (panels C and D), and labor disutility (panels E and F) stochastic processes. Size of shocks was calibrated to generate a 1% decline in output upon impact. Persistence and volatility of exogenous stochastic processes are as in Table 5.

tic processes considered.[35] For each shock, it separately plots the response of key macro aggregates - output, hours and investment (top row), and the intensity of entry for each type (bottom row). For comparability reasons, the size of each shock was set to generate a 1% decline in output upon impact.

Each shock has different quantitative implications for the behavior of macroeconomic aggregates. It is clear that the financial shock affects investment stronger than output and hours. At the same time, the labor disutility shock leads to an equally strong decline in hours and investment. The aggregate productivity shock affects output stronger than hours, which is the opposite to the effect of the labor shock. Therefore, the identification of shocks comes from the way aggregate series move relative to each other over the cycle.

TABLE 6: UNCONDITIONAL BUSINESS CYCLE STATISTICS

	Volatility			Cyclicality	
Statistic	**Data**	**Model**	**Statistic**	**Data**	**Model**
$\sigma(Y_t)$	1.38%	1.67%			
$\sigma(C_t)/\sigma(Y_t)$	0.91	0.64	$\rho(C_t, Y_t)$	0.81	0.86
$\sigma(I_t)/\sigma(Y_t)$	3.20	2.72	$\rho(I_t, Y_t)$	0.79	0.81
$\sigma(N_t)/\sigma(Y_t)$	1.20	0.98	$\rho(N_t, Y_t)$	0.74	0.75

Notes: Table 6 reports the empirical and model-generated unconditional business cycle statistics. The aggregate data has quarterly frequency and spans the period 1975Q1-2016Q4. Consumption is "Real Consumption Expenditures in the U.K.", Investment is "Real Investment Expenditures in the U.K.", Output corresponds to "Real GDP for U.K.", and Hours worked is "Total actual hours worked". The data comes from the ONS and FRED databases. "Model" refers to an estimated model from Section 3. Prior to computing the statistics, the data was logged and HP-filtered with the smoothing parameter of 1600.

Aggregate Shocks and Business Formation   Aggregate shocks have important implications for the formation of different firm types. Panel D in Figure 9 shows that a negative TFP shock leads to a nearly identical drop in the entry of low, medium and high type firms. This occurs precisely because the TFP shock affects the expected benefit *proportionately* across the types according to Equation (14), resulting in a similar response in the mass of successful entrants. Financial and labor disutility shocks affect high-type firms *disproportionately* more—given that such businesses require more resources to get up to scale—and depress their entry intensity more as compared to other firms with a lower span of control.

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[35]Aggregate processes have the estimated persistence and volatility parameters from Table 5.

4.5 Business Cycle Statistics

As an additional validation check, I compute standard unconditional business cycle statistics in the model and data (volatility and cyclicality), and report them in Table 6. I find that my model does a good job in picking up the key business cycle moments of the U.K. economy. Investment is almost 3 times more volatile than GDP, while consumption is less volatile. The model-implied volatility of hours is lower than in the data, which is a common issue that many business cycle models share.

On the cyclicality side, consumption, investment and hours are found to be strongly positively correlated with output, in line with the data. I also find that the model-based cyclicality moments are quantitatively very close to empirical ones.

5 Compositional Effects and Business Cycle

In this section, I study the quantitative implications of compositional shifts for the U.K. business cycle. I start by introducing in Subsection 5.1 a version of the baseline model which features no intensive margin of firm entry over the business cycle. I then show in Subsection 5.2 that the benchmark model with the intensive margin of business entry can generate a collapse of the right tail of the cumulative growth distribution in recession, while the model without this mechanism cannot. Subsequently, I assess the quantitative relevance of the intensive margin mechanism by way of comparing the performance of the two estimated versions of the model during the Great Recession episode in Subsection 5.3. I then argue that the mechanism is empirically relevant in Subsection 5.4. Finally, in Subsection 5.5, I quantify the contribution of each of exogenous aggregate forces in driving the intensive margin of business formation.

5.1 Fixed Composition Model

A natural version of the model laid out in Section 3 but without compositional shifts can be obtained by modifying the timing of the entry problem described in Subsection 3.3. In particular, I now assume that potential entrants still know the distribution of *type availability* (parameters $\{\psi^j\}_{j \in \boldsymbol{J}}$), but can no longer target their entry efforts to any particular type. Instead, each potential entrant is assigned the type randomly—according to the *induced*

availability probability mass function[36]—upon successful entry. Under this formulation, the distribution of entrants across types will mechanically be business cycle invariant.

The new free-entry condition takes the following form:

$$\underbrace{\chi}_{\text{cost of entry}} = \underbrace{\frac{m_t}{e_t}}_{\text{success probability}} \underbrace{\int_z \sum_{j \in \boldsymbol{J}} \psi^j v_j^0(k_0, b_0, z; \boldsymbol{s}) \mathrm{d}F(z).}_{\text{exp. value of entry}} \tag{20}$$

The difference between (14) and (20) is in the integrand on the right-hand side: now potential entrants cannot know which project they will end up operating, and so in equilibrium they have to balance the entry cost χ with an expected (over types) value of running a firm. Therefore, it is the extensive (overall mass of entrants)—not the intensive (distribution of firms over types)—margin of firm entry which is allowed to fluctuate over time. Therefore, the difference in predictions between the two models is fully accounted by the compositional effect.

5.2 Collapse of the Right Tail of Cumulative Growth Distribution

I now illustrate that the benchmark model is able to generate the collapse in the right tail of the cumulative growth distribution, while the model with no compositional effect cannot. To do so, I simulate both models during the Great Recession episode, and calculate the moments of the cumulative growth distribution based on the model-generated panels.

TABLE 7: MOMENTS OF THE CUMULATIVE GROWTH: MODEL VERSUS DATA

	Data	Full model	Fixed comp. model
ΔP90	-66%	-45%	-8%
ΔP50	-13%	-5%	-7%

Notes: Table 7 reports the by how many percent the median and the top 10th percentile of the cumulative growth distribution change in recession (2008-2009) relative to the expansion (2006-2007). "Data" corresponds to the BvD evidence from Section 2. See Subsection 5.1 for the description of the fixed composition model.

Table 7 shows that the full model generates a collapse in the right tail of the cumulative growth distribution, which is about two thirds of the size observed in the data. Note that this change was not targeted at the estimation step.

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[36]In particular, this means that in a fixed composition model, the share of new firms of type $j \in \boldsymbol{J}$ among entrants is $\frac{\psi_j}{\sum_{j=1}^J \psi_j}$. These shares are business-cycle invariant, since the parameters $\{\psi_j\}$ are scalars.

The fixed composition model does not generate any sizable change in the right tail because the median and the top 10th percentile declined similarly in a recession. Intuitively, this occurs because in that version of the model, there is no protraction mechanism which can make the effect of a recession persistent. Even though adverse aggregate shocks also affect high type firms stronger in the fixed composition model, these firms get up to full potential when aggregate conditions improve. As a result, I do not detect any sizable change in the right tail of the 5 year-long growth in this version of the model.

5.3 Great Recession and the Intensive Margin of Firm Entry

In order to assess the quantitative importance of compositional changes, I now compare the predictions of two versions of the model during the Great Recession episode. Using the Kalman smoother, I obtain sequences of exogenous aggregate processes from the benchmark model. Subsequently, using the finite representation of the model (see Appendix D.4 for further details), I simulate two sets of endogenous states: one for the benchmark and one for the fixed composition model. Effectively, I will be comparing the predictions of the two models by feeding in the identical series of aggregate shocks.

FIGURE 10: GREAT RECESSION: DATA VERSUS MODEL (2007Q4 = 0)

(A) Investment   (B) Hours
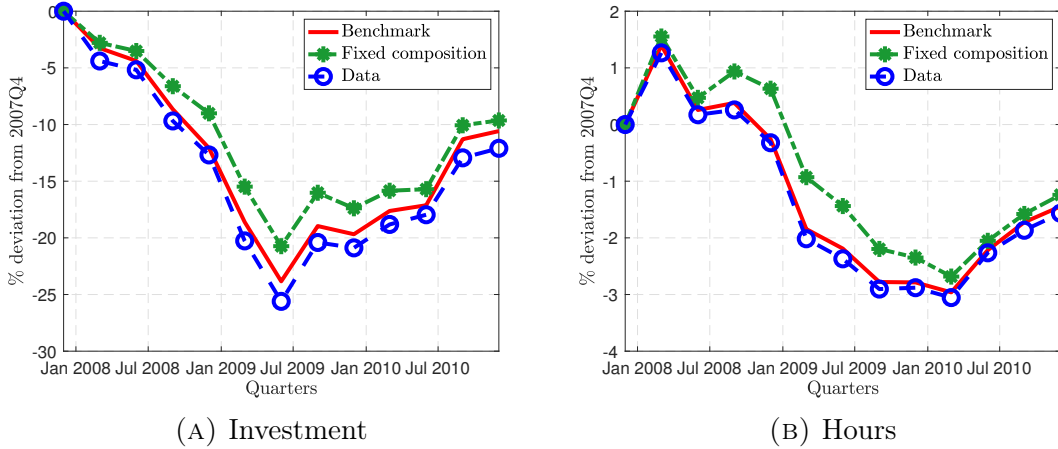
Notes: Figure 10 contains 2 panels. Panel A corresponds to aggregate investment, and panel B - to aggregate hours. Each panel has 3 lines. The blue dashed line is data, the red solid line corresponds to the benchmark model with flexible composition, and the green dash-dotted line—to a version of the model with fixed composition. Each series represents the percentage deviation from the corresponding level in 2007Q4.

Figure 10 plots the recovered series of aggregate investment and hours for the two versions of the model along with the data during the Great Recession episode. By construction, the benchmark model almost fully accounts for the data, since I fed in the shocks filtered through

that model. In order to facilitate the visual inspection, I plot the deviation of aggregates relative to the last quarter of 2007. I find that the fixed composition model requires larger shocks in order to explain the same dynamics of macro aggregates. Around mid 2009—when the economy reached the trough of the contraction—benchmark model falls roughly 25% deeper than the model with no compositional shifts. After the trough of the recession, the series corresponding to the two models get closer to each other, potentially reflecting the restored intensity of the high type business formation. I investigate this mechanism in full detail in the next subsection.

5.4   Discussion of the Amplification Mechanism

An important question is whether we actually need an amplification mechanism; in other words, maybe shocks were indeed bigger than what the benchmark model implies?

To this end, I look at the key non-targeted dimension of model performance—the extensive margin of business entry—and confront it with the data. Panel A in Figure 11 plots 3 lines over the 12 quarters period starting from 2008Q1. The solid red line is the benchmark model, the green line with star markers is the fixed composition model, and the dashed blue line with circle markers is data.

FIGURE 11: GREAT RECESSION: EXTENSIVE AND INTENSIVE MARGINS OF BUSINESS FORMATION

(A) Extensive Margin of Entry   (B) Intensive Margin of Entry
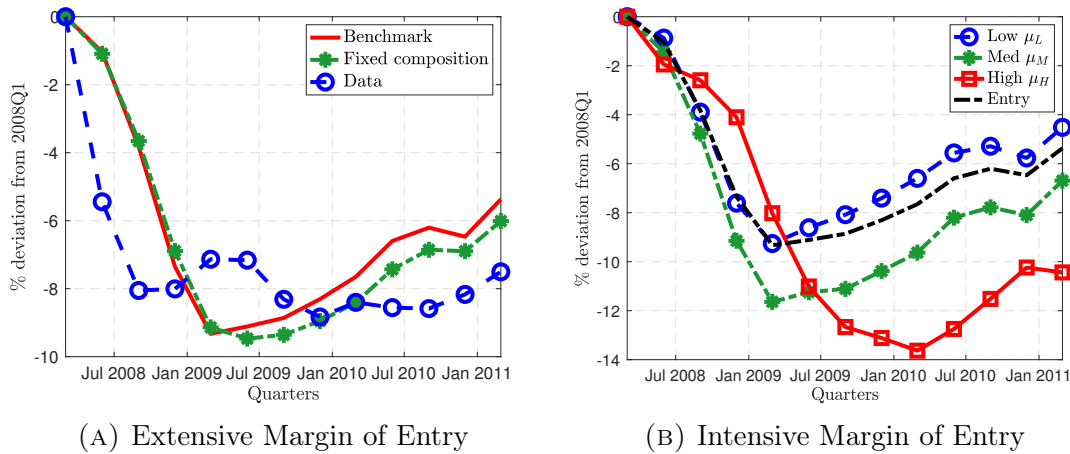
Notes: Figure 11 contains 2 panels. Panel A plots the extensive margin of business formation for the benchmark model (red solid), the fixed composition model (green dot dashed) and in the data (blue dashed). Panel B unpacks the extensive margin (black dot-dashed) into the entry intensity of the low type (blue with circle markers), middle type (green with star markers) and high type (red with square markers). Data source: OECD.

According to Panel A of Figure 11, both models exhibit a delayed—relative to the data—

drop in the entry rate. However, it is also clear that the benchmark model delivers a more realistic dynamics of business formation rate during the Great Recession as compared to the fixed composition model. This implies that the fixed composition model needs to compress the extensive margin of firm entry counterfactually more in order to explain the same series of investment, output and hours, suggesting that the amplification effect of the intensive margin of business formation is empirically relevant.

The key reason behind the observed discrepancy between the models is that, intuitively, the full model has two degrees of freedom (intensive and extensive margins of firm entry) to account for the dynamics of aggregate series, while the fixed composition model has only one. Panel B in Figure 11 shows that the entry intensity of high type firms declines stronger during the Great Recessions as compared to low and medium type businesses. Therefore, the full model can account for the fall in macroeconomic aggregates during the recession by reducing the entry intensity of rapidly growing firms, which is associated with a small overall decline in the entry rate (there are relatively few fast growing projects as compared to slow growing ones). The model without compositional effects induces an equal decline in entry of all project types, which translates into an overall larger fall in the entry rate.

FIGURE 12: VARIANCE DECOMPOSITION
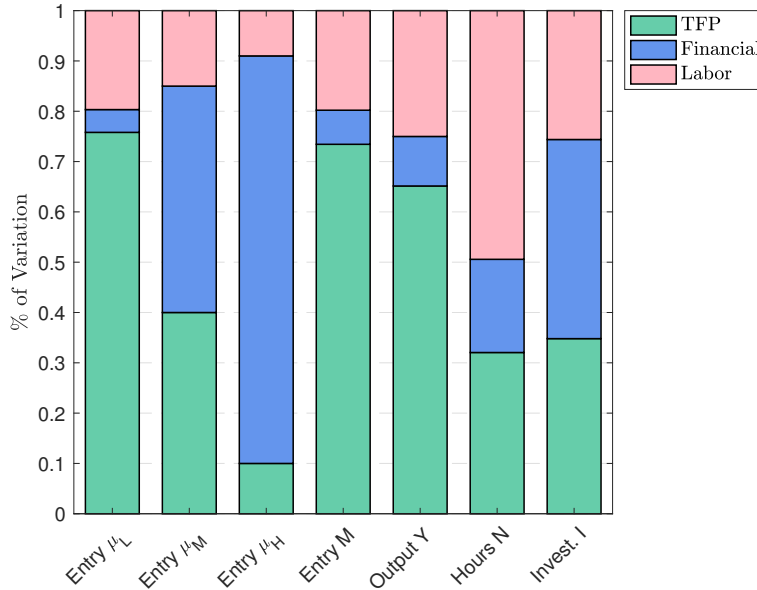
Notes: Figure 12 reports the decomposition of aggregate fluctuations of several variables across the 3 aggregate shocks considered. Each bar is split into green (TFP), blue (financial) and pink (labor wedge) parts; the height of each part equals the share of the overall variance accounted by each shock. The following variables are considered: entry of low types ("entry μ_L"), entry of medium types ("entry μ_M"), entry of high types ("entry μ_H"), overall entry ("entry M"), output ("Y"), hours ("N") and investment ("I").

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5.5 Variance Decomposition of Aggregate Shocks

Which aggregate shocks are driving the intensive margin of business formation over the cycle? This question is important for two reasons. First, as was shown in Subsection 5.3, aggregate implications of compositional effects are quantitatively sound. Second, one needs to evaluate the role financial shocks play relative to other aggregate forces in order to assess the room for government intervention.

Figure 12 reports the results of the variance decomposition of business formation of low, medium and high types along with several key macro aggregates with respect to aggregate forces. The financial shock accounts for about 82% of variation in the formation of the high types. TFP and labor wedge shocks account for the remaining 18%. As for the low type firms, TFP shock explains almost 80% in the entry intensity of these firms. Remarkably, the relative contribution of the financial shock declines substantially from 82% down to 5%.

This suggests that the formation of rapidly growing firms is very sensitive to conditions on financial markets. This finding coupled with the quantitative soundness of compositional shifts documented in Subsection 5.3 hints at potentially large benefits from government stimulus policies. Next, I turn to the quantitative assessment of such policies.

6   Policy Implications

In this section, I study the policy implications of my model. First, I illustrate that the policy which stimulates the formation of all firm types by reducing the cost of entry during aggregate contractions does not yield any welfare benefits; however, the government can generate a sizable welfare improvement if it targets the policy toward businesses of the highest growth potential. I argue that it occurs due to cost-inefficiency of the non-targeted policy.

In reality, however, the firm type is not observable. Thus, I explore the welfare benefits associated with the policy targeted at firms with a high leverage ratio—the object which is observable by the policymaker. The goal of this exercise is to illustrate the magnitude of welfare benefits associated with this feasible micro-targeted government policy, rather than to strongly advocate for a particular implementation.

Subsection 6.1 explains the welfare criterion I use to evaluate different policies. I highlight the importance of the micro-targeted policies in Subsection 6.2. Finally, Subsection 6.3 describes the countercyclical policy targeted at high leveraged firms.

6.1 Welfare Calculation

I use Lucas (1987) measure to evaluate welfare changes associated with policy interventions. This criterion calculates the percent of additional lifetime consumption that must be endowed at all future dates and states to a representative household under no policy so that the expected welfare is the same as in the economy where policy is implemented. Technically, the welfare criterion takes the following form:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[U(\widehat{C}(1 + x), \widehat{N}) \right] = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[U(\widetilde{C}, \widetilde{N}) \right], \tag{21}$$

where hats denote a status-quo allocation (no policy), and wiggles—an outcome under the policy considered. Parameter x governs the percent of lifetime consumption which makes the household in the status-quo economy indifferent between the two economies. In other words, I measure welfare benefits in consumption equivalent units.

FIGURE 13: WELFARE ASSESSMENT OF ENTRY COST REDUCTION POLICY
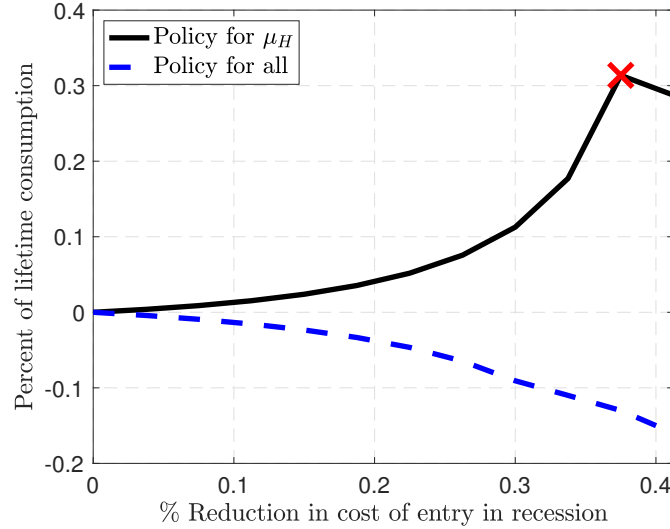
Notes: Figure 13 summarizes the welfare benefits associated with the entry cost reduction government policy. The solid black line corresponds to the policy targeted toward μ_H firms. The blue dashed line corresponds to the policy applied to all firm types. Refer to Subsection 6.2 for details.

6.2 Subsidizing the Entry Cost

The policy I consider takes the form of a reduction in the cost of entry in recessions. In particular, the cost of starting a business becomes time-varying, and takes the following

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form:

$$\widetilde{\chi}_t = \chi \times \left(1 - d \times \mathbf{1}_{\{Y_t < Y_{ss}\}}\right), \quad d \in [0, 1], \tag{22}$$

where d is the share of the cost subsidized by the government. Note that this policy will reduce the tax rebate to the household; therefore, such a stimulus policy will yield welfare gains as long as the benefits associated with a more enthusiastic business formation exceed the lower tax rebate from the government.

The quantitative results of this policy are summarized in Figure 13. I find that if the government is able to target this policy toward μ_H firms only, the welfare gains can reach 0.3% of lifetime consumption. Intuitively, this occurs because there are very few high type firms, but which collectively account for a significant share of investment and employment growth. Therefore, the government can generate a welfare improvement by devoting a small fraction of resources to subsidizing the formation of such businesses.

In stark contrast with the targeted policy case, my simulations show no welfare benefits associated with the policy which subsidizes the formation of all firm types. This occurs because such a policy is not cost-efficient: the welfare benefits associated with stimulating the formation of low and medium type businesses do not outweigh the loss in the tax revenue.

TABLE 8: WELFARE ASSESSMENT OF GOVERNMENT COUNTERCYCLICAL POLICY, % OF LIFETIME CONSUMPTION

	$\iota = 0.2$	$\iota = 0.25$
$\xi = \mathbb{E}[n] \times w_{ss}$	0.12	0.17
$\xi = 1.5 \times \mathbb{E}[n] \times w_{ss}$	0.13	0.21

Notes: Table 8 reports the results of welfare assessment of the policy described in Subsection 6.3. The reported numbers are in percent of lifetime consumption (see Subsection 6.1 for details). w_{ss} is the equilibrium wage rate in the steady state.

6.3   Subsidizing Leveraged Firms

Next, I consider the policy which takes the form of a subsidy to highly leveraged firms in recessions. Formally, the new budget constraint of a firm becomes:

$$i_j(k, b, z; \boldsymbol{s}) = \Pi_j(k, b, z; \boldsymbol{s}) + E_j(k, b, z; \boldsymbol{s}) + \Delta b_j(k, b, z; \boldsymbol{s}) + \xi \mathbf{1}_{\{Y_t < Y_{ss}\}} \mathbf{1}_{\{\frac{b}{k} \geq \iota\}}, \tag{23}$$

According to Equation (23), in a recession—whenever output falls below the steady-state level $Y_t < Y_{ss}$—the highly levered firms (those with leverage ratio exceeding ι) get additional resources of ξ. By varying parameters ι and ξ, I explore the generosity of the proposed

policy.

Table 8 reports the results. I find the largest welfare gains when the government targets the policy towards the most constrained firms ($\iota = 0.25$). I also find that welfare gains fall more when the size of the policy is large ($\xi = 1.5\mathbb{E}[n]w_{ss}$), and the government applies it to less constrained firms. This mirrors the cost-efficiency logic from the previous subsection.

7 Conclusion

Motivated by the empirical evidence that there are less fast growing firms started in recessions than in expansions, and that rapidly growing firms are more financially constrained, I build and estimate a general equilibrium model of firm dynamics where these patterns arise naturally. The two key model ingredients are the ability of potential entrepreneurs to target their startup efforts towards project of different target size, and that firms' growth is hampered by financial frictions. The estimated model shows that the intensive margin of firm entry is quantitatively pronounced at business cycle frequencies, and that financial shocks account for a large share of fluctuations in the formation of rapidly growing firms. I also discuss the implications for the government stimulus policy.

I see several fruitful avenues for future research. While this paper studied the intensive margin from the perspective of the industry dynamics literature, new insights can be obtained by looking at this phenomenon through the lens of models with occupational choice. For instance, such frameworks can speak to the labor market implications of the compositional effect, and, thus, lead to new policy prescriptions.

Second, more empirical work is needed to better understand the decisions of entrepreneurs to start firms with different growth potential. Individual labor market experiences as well as educational, family and demographic factors can all play an important role in this process; new empirical evidence can help inform structural models.

Finally, my model can have important financial market implications. For example, by way of incorporating firms' default decisions into my model, one can study firm-level spreads and, therefore, the risk dimension of business formation. Such a model can shed light on, for example, the behavior of stock prices for innovative (high growth) firms (Pástor and Veronesi, 2009). I leave these issues to future research.

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Table of Contents for Appendix

A Empirical Appendix

This appendix provides further details for the empirical part of the paper, including data background, sample selection and additional empirical results, referenced in the main text.

A.1 Bureau van Dijk Data (BvD) - Background

The objective of this section is to provide a brief institutional background for the firm-level accounting data in the U.K. Please see Bahaj, Foulis and Pinter (2018) for full details on data construction.

Companies House is the Registrar of firms in the U.K. At the end of a fiscal year, every firm must prepare a set of statutory annual accounts which they file with Companies House. This information includes the balance sheet, as well as profit and loss accounts. All limited firms are required to file, but reporting standards differ across firms of different sizes (see Companies House guide for details). Firms must file accounts every 18 months. Firms which do not report every 12 months are excluded from the analysis.

Companies House is the original source of the data, but the direct source is BvD, which provides a workable interface to access it. For the U.K., BvD provides microdata through the product called FAME (Financial Analysis Made Easy). This is a separate product from commonly used Amadeus and Orbis - also provided by BvD - which cover firms at the European and Global levels, respectively. The data comes on a monthly basis in DVD (and, subsequently, Blu-Ray) discs. A disc contains a snapshot of the FAME database for the U.K. firms in a particular month. The database is updated on the monthly basis, and incorporate new information as firms file new accounts, or when firms conduct report-driven filings (for example, if director's information changes).

The discs were sampled at 6 month frequency for two reasons. First, as has been mentioned above, firms might have irregular filing periods or may conduct event-driven irregular filings, therefore, biannual sampling mitigates this issue. Second, this frequency has been chosen to balance the capacity required to store the information and the amount of new information which is gained by adding additional discs. Increasing the frequency beyond biannual was found to bring very little extra information.

Each firm in the UK is assigned a unique Companies House Registration Number (CRN) upon formation which stays with the firm throughout its lifetime. The CRN may change if Companies House chooses to adopt a new numbering format, but this did not happen over the sample period. This firm-level identifier was used to construct a panel from multiple

vintages of the FAME dataset.

A.2   Selection Criteria and Summary Statistics

The sample was constructed to satisfy the following selection criteria:

1. only observations with "Live" status were retained;

2. observations with extreme growth rates of real total assets were dropped (i.e., those in 1st or 99th percentile of the corresponding distribution of growth rates);

3. similarly, observations with extreme investment rates were excluded. Investment rate for firm i in year t is calculated as follows:

$$irate_{it} = \frac{\Delta \texttt{fixed assets}_{it} + \texttt{depreciation}_{it}}{\texttt{fixed assets}_{it-1}},$$

   where $\texttt{fixed assets}_{it}$ is the firm's i amount of fixed assets at the end of period t;

4. drop firms which do not comply with Companies' Act: "Economic European Interest Grouping", "Guarantees", "Industrial Provident", "Limited Liability Partnership", "Not Companies Act", "Royal Charter", "Unlimited", "Public Investment Trust", "Limited Partnership" and "Charity";

5. exclude firms in services, financial and real estate sectors ($\texttt{sic} > 5$).

6. drop observations with negative leverage, or with leverage exceeding 1. I consider 3 different financial leverages:

   (a) short-term leverage, which is a ratio of short-term debt and overdrafts to total assets;

   (b) long-term leverage, defined as a ratio of long-term debt to total assets;

   (c) full liability-based leverage:

$$leverage_{it} = \frac{(\texttt{total assets}_{it} - \texttt{shareholders' funds}_{it})}{\texttt{total assets}_{it}}.$$

7. observations with negative $\texttt{turnover}$ and/or $\texttt{total assets}$ were also dropped.

	Empl.	Collat.	ST lev.	LT lev.	Total lev.	Age
	Private firms					
Mean	103	0.30	0.19	0.17	0.29	12
Bottom 25%	10	0.07	0.04	0.04	0.13	3
Median	44	0.21	0.12	0.11	0.26	7
Top 25%	97	0.47	0.27	0.24	0.42	16
	Public firms					
Mean	737	0.39	0.12	0.13	0.24	26
Bottom 25%	32	0.17	0.02	0.03	0.11	9
Median	113	0.36	0.07	0.09	0.21	19
Top 25%	628	0.58	0.16	0.19	0.34	37

Notes: Table A1 reports the descriptive statistics. All nominal variables have been deflated with the CPI (2014 being base year). Table reports summary statistics for private and public firms separately. Private firms include "Private Limited" category. Public firms include the following categories: "Public, Quoted", "Public, Not Quoted", "Public, Quoted OFEX", "Public AIM". `Collateral` is the ratio of fixed assets to total assets. `ST leverage` is short-term debt and overdrafts divided by total assets. `LT leverage` is long-term debt to total assets. `Total leverage` is a sum of `short-term debt`, `overdrafts` and `long-term debt` divided by total assets. All ratios are winsorized at 1/99 percentiles.

In the end, I am left with $\approx$ 7mln (firm, year) observations, which corresponds to approximately 1.5mln distinct firms. Table A1 reports summary statistics for the final dataset.

A.3 Firm-Size Distribution

The dataset covers the corporate universe in the U.K., but as it was mentioned earlier in Appendix A.1, reporting requirements vary dramatically by the size of the firm. In general, larger firms have to report more information. The `total assets` variable — the book value of firm's assets — is reported uniformly in the data, but `employment` is available for only about 10% of observations.

In order to circumvent reporting issues and gauge how the sample firm-size distribution looks like, I follow Haltiwanger, Jarmin, Kulick and Miranda (2017) and Dinlersoz, Kalemli-Ozcan, Hyatt and Penciakova (2018) and construct weights by way of fitting probit regressions. In particular, I split all (firm, year) couples into one of 3 groups: entering, continuing and exiting businesses. Subsequently, within each group I fit a probit model with left-hand side variable R_{it} being an indicator which takes a value of 1 if firm i reports employment in year t, and 0 otherwise:

– continuing firms

$$R_{it} = \alpha + \gamma_1 log(TotalAssets_{it}) + \gamma_2 Age_{it} + \gamma_3 D10_{it} + \gamma_4 AG_{it} + ind + \varepsilon_{it}$$

– births

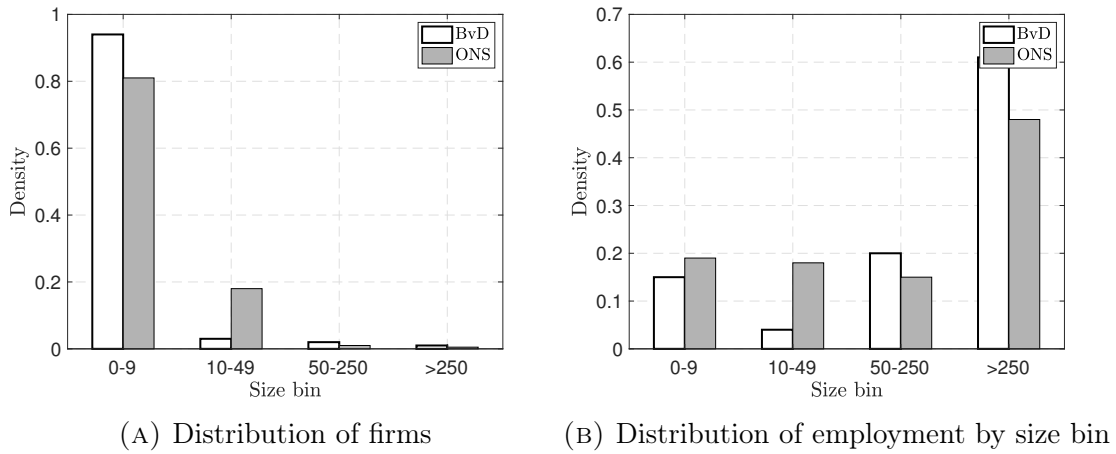$$R_{it} = \alpha + \gamma_1 log(TotalAssets_{it}) + ind + \varepsilon_{it}$$

– deaths

$$R_{it} = \alpha + \gamma_1 log(TotalAssets_{it}) + \gamma_2 Age_{it} + \gamma_3 D10_{it} + ind + \varepsilon_{it},$$

where $D10$ is an indicator whether the firm is older that 10 years, AG is asset growth rate (7 groups), and ind is 2-digit SIC industry dummy.

FIGURE A1: FIRM-SIZE DISTRIBUTION IN 2014: BvD AND ONS

(A) Distribution of firms   (B) Distribution of employment by size bin

Notes: Figure A1 contains 2 panels. Panel A plots the distribution of firms across size bins: ONS data is gray, BvD is white. Size bins are in terms of employment. Panel B reports distribution of employment across size groups. See Appendix A.3 for details on how firm-size distribution was constructed in BvD data.

Figure A1 is constructed using weights which are the inverse of predicted probabilities from probit regressions. The data gets reasonably close to the official firm-size distribution: it captures correctly the share of small firms in population, as well as their share of the overall employment.

A.4   Bonhomme and Manresa (2015) Grouped Fixed Effects Estimator

This appendix describes the application of Bonhomme and Manresa (2015) grouped fixed effect estimator (GFE) on BvD data. This clustering algorithm optimally assigns firms into the pre-specified number of homogeneous groups after controlling for observables. Specifically, let y_{it} be a some observable of firm i at time t. Let g_i denote the group firm i belongs

to (the total number of groups is pre-set by researcher). Bonhomme and Manresa (2015) consider the following linear model:

$$y_{it} = \alpha_{g_i t} + x'_{it}\theta + \varepsilon_{it}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T. \tag{24}$$

In Equation (24), x_{it} is a vector of covariates whose effect one wishes to partial out. As a result, units whose time profiles of observable - net of impact of covariates - are most similar, are grouped together in estimation.

There were several features of the data and estimator itself that determined the nature of this exercise for the purposes of this project. First of all, GFE is a computationally intensive procedure which is suitable for relatively small panels. This feature of the algorithm rendered its application on the entire BvD data impossible. To circumvent this issue, I followed "trial and error" approach and found the largest sample of firms I could work with. In particular, GFE was applied on a 20% random subsample of manufacturing firms born between years 2006 and 2009. GFE was applied numerous times in order to ensure that results are consistently similar across different draws.

Since small firms face different from big firms reporting requirements in the U.K., few entrants report employment consistently throughout their life cycle. Therefore, uniformly reported total assets were used as a measure of size. I also restricted analysis to firms reaching the age of 8.[37] This was done for two reasons. First, the model developed in Section 3 views long-term differences in firm growth as a consequence of permanent heterogeneity. In order to pick up this heterogeneity in the data, one needs to observe firms for sufficiently long period of time. Second, longer time dimension improves the performance of GFE.

Finally, clustering was performed based on the life cycle of firms' sizes. In practice, I found that clustering based on growth rates is very unstable. Moreover, Table A2 shows that even though firms were clustered based on their size, the groups preserve ranking with respect to cumulative growth. This is not surprising given that for young firms growth and size are highly correlated.

Figure 2 in main text reports the results. In panel A, vector of controls includes a cyclical component of GDP. Panel B adds liability-based leverage into vector of controls. I consistently find a shift toward less fast-growing firms started during recessionary years 2008-2009.

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[37]This is the longest life cycle I can observe for a firm born in 2009, given that BvD spans years through 2017. No noticeable differences were found when a different sample was used (firms surviving through age of 5).

TABLE A2: AVERAGE 5-YEAR CUMULATIVE GROWTH BY CLUSTER

	$\mathbb{E}\left[\log(y_{age5}) - \log(y_{age1})\right]$
Slow	0.12
Medium	0.45
High	0.58

Notes: Table A2 reports the average growth of total assets between ages 1 and 5 for firms clustered into "slow", "medium" and "high" groups using Bonhomme and Manresa (2015) grouped fixed effects estimator. See Appendix A.4 for further details on application of that algorithm.

A.5 Longitudinal Business Database (LBD)

LBD is the most comprehensive panel dataset covering the universe of U.S. businesses and spanning the years 1976-2016.[38] The unit of observation in LBD is an establishment, which is defined as a single physical location where business operations take place. A firm is then defined as a set of establishments that are under common ownership or control.

LBD is based on several sources, such as the Business Register (also known as the Standard Statistical Establishment Listing - SSEL), Economic Censuses, and surveys. The LBD offers the most reliable and complete data on births, deaths, and age of establishments operating in the US. There are several data issues potentially leading to measurement errors in identification of business formation (for example, non-administratively registered establishments may not be correctly identified, or gaps in the records of establishments). See Jarmin and Miranda, 2002) for more details on which efforts have been undertaken to mitigate this issues in the process of construction of the LBD.

A.5.1 Identification of Birth and Death

The unit of observation in LBD is an establishment, and variable `lbdnum` - which is robust to mergers and acquisitions - is used to track establishments over time.

Establishments The age of an establishment is measured as the number of years elapsed since the first year that establishment appeared in the data.[39] Since LBD contains establishments with at least one paid employee, it might be the case that some businesses are older than what can be measured in the data (if that establishment had no paid employees in the

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[38]Years 2017 and 2018 will soon become available.

[39]Age cannot be measured for establishments born prior to 1976 - the first year covered by the LBD. For that reason, in my empirical exercise I exclude the cohort of 1976. Besides, (Moscarini and Postel-Vinay, 2012) argue that there are concerns with the first couple of LBD cohorts; I, therefore, use cohorts starting from 1978.

first several years of operation and then evolved to an employer business).

Firms Identification of firm birth and death is associated with the construction of firm linkages over time. I follow a standard approach in the literature which is robust to ownership changes and acquisitions (Haltiwanger, Jarmin and Miranda, 2013). A new firm identifier emerges in the LBD either because a new firm is born or because an existing businesses undergoes a change of ownership and control (e.g. merger and acquisition, divestitures). A new firm is registered when all of its establishments are of age 0. Accordingly, when a new firm identifier arises through a merger of two preexisting firms, it is not treated as a firm birth and is assigned the age of the oldest continuing establishment of the newly combined business. The firms are then allowed to age naturally regardless of mergers and acquisitions as long as the ownership and control does not change. A firm death is determined when a firm identifier disappears and all associated establishments cease operations and exit.

A.5.2 Employment

Establishments Employment (LBD variable `emp`) is defined as the number of full- and part-time employees[40] as of March 12th of each calendar year.

Firms Firms can own a single establishment or many establishments, which may span multiple geographic areas and industries. I compute firm-level characteristics based on the characteristics of the set of establishments of the firm. Naturally, firm-level employment is calculated as the sum of employment across the establishments belonging to that firm.

A.5.3 Industry

The LBD also includes detailed information on industry classification. The period of analysis coincides with the transition from SIC to NAICS industry classifications, which leads to classification issues. I, therefore, use a consistent NAICS 2002 industry classification variable constructed by Fort and Klimek (2016) (variable `fk_naics12`).

Firms frequently encompass establishments from several industries. For that reason, I assign to the firm an industry of its largest (in terms of employment) establishment.

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[40]Including employees who are on paid sick leave, holidays, and vacations. The reported number also includes salaried officers and executives of corporations, but it excludes sole proprietors and partners of unincorporated businesses.

B   Figures

This section contains additional figures referenced in the main text.

FIGURE B1: SHORT- AND LONG-TERM LEVERAGE BY AGE

(A) Short-term leverage
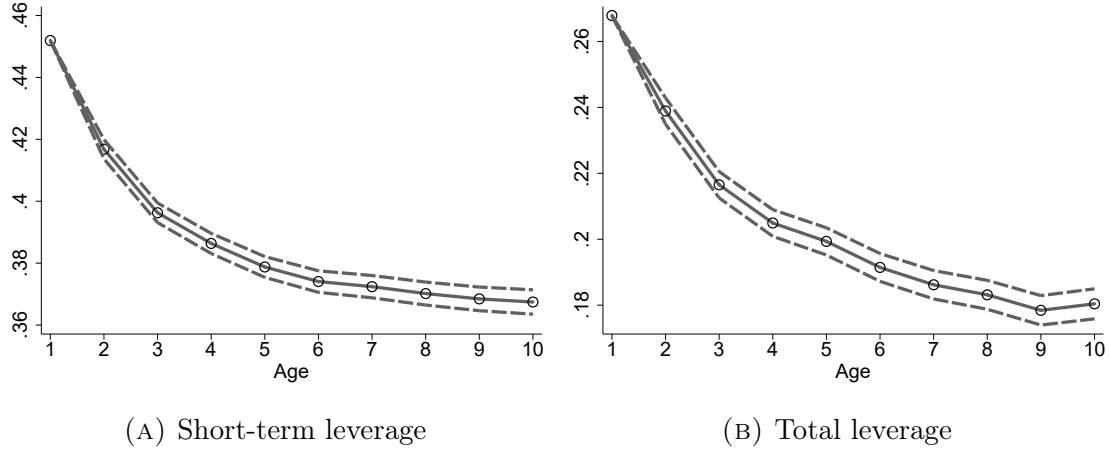
(B) Total leverage

Notes: Figure B1 plots the estimated age fixed effects (plus constant) from projecting short-term leverage (defined as a ratio of short-term debt and overdrafts to total assets) and long-term leverage (long-term debt to total assets) on firms' log assets, collateral, turnover, age dummies and a full set of industry-year fixed effects. All ratios were winsorized at 2/98 percentiles. Source: BvD.

FIGURE B2: DEBT AND EQUITY FINANCING BY AGE

(A) Debt financing
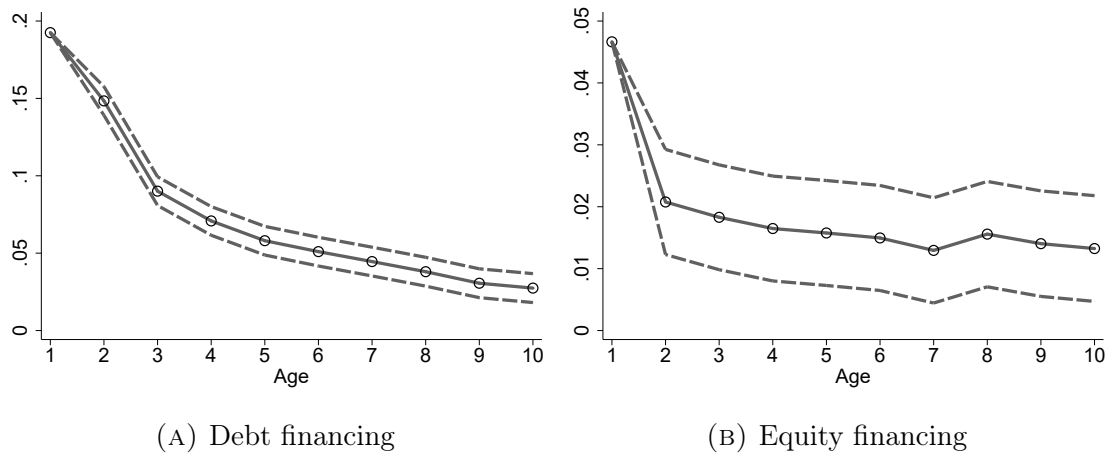
(B) Equity financing

Notes: Figure B2 plots the estimated age fixed effects (plus constant) from projecting a change in total liabilities (Panel A) and equity financing (Panel B) on age dummies and a full set of industry-year fixed effects. Equity financing is defined as a change in issued capital and share premium account. All ratios were winsorized at 2/98 percentiles. Source: BvD.

FIGURE B3: BUSINESS APPLICATIONS AND TIME TO BUSINESS FORMATION IN THE U.S.

(A) Business Applications

(B) Time to Business Formation
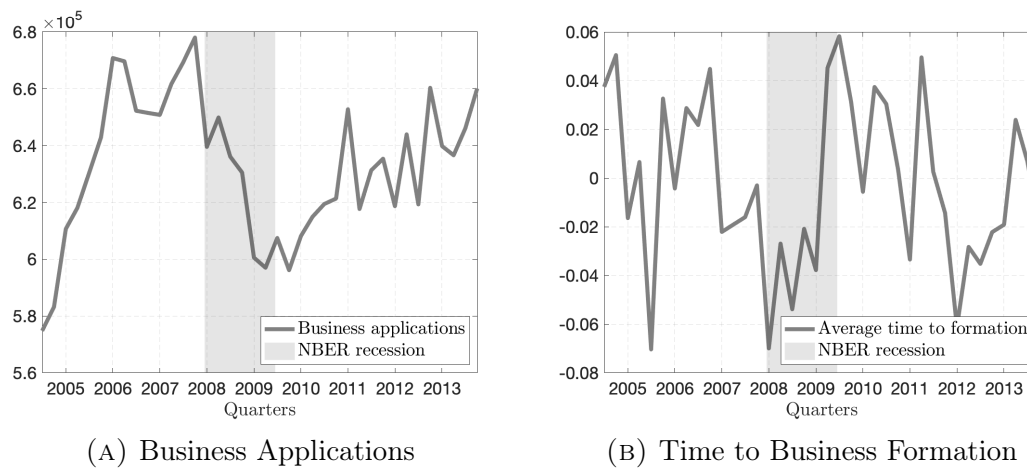
Notes: Figure B3 consists of 2 panels. Panel A plots the number of business application in the U.S., while Panel B plots the average time between application and business formation in the U.S. The data is from the Business Formation Statistics and spans the period 2004Q3 to 2013Q4 at quarterly frequency. Business application is identified as a filing of the IRS Form SS-4. Time to business formation is an average (in quarters) time between the filing of the SS-4 form and the first quarter when positive payroll is recorded. Panel B plots the detrended series (linear time trend is subtracted).
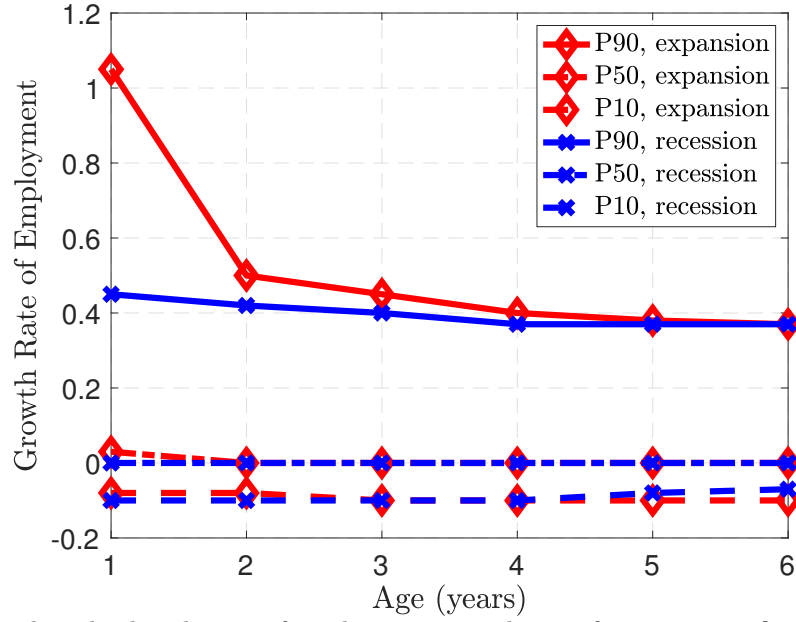
Notes: Figure B4 plots the distribution of employment growth rates for continuing firms born in the recession (2008-2009) and in the expansion (2006-2007). Dashed lines correspond to the 10th percentile of the employment growth distribution (traced by age), the dot-dashed lines mark the median growth, and solid lines - the 90th percentile. Blue lines with circle markers denote recessionary cohorts, and red lines with triangle markers - expansionary ones. Growth rate of firm i at time t is computed as $2(E_{it} - E_{it-1})/(E_{it} + E_{it-1})$. Source: BvD.
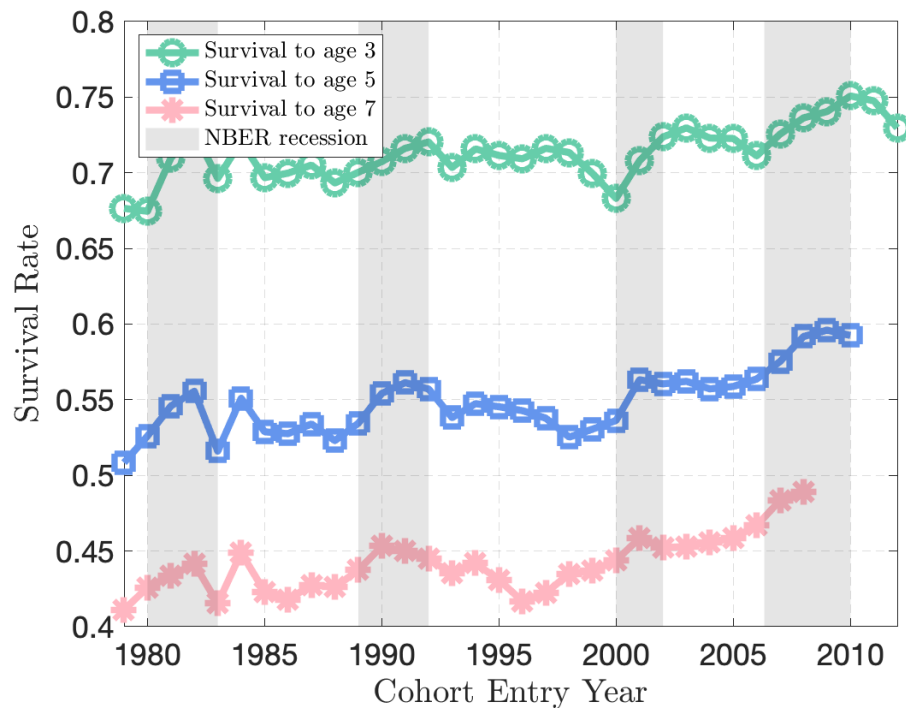
Notes: Figure B5 plots the survivla rates in the U.S. private sector. The line with circle markers corresponds to survival rates to the age of 3, the line with square markers—to the age of 5, and the line with star markers—to the age of 7. The survival rate is calculated as a number of establishments which make it to a specific age divided by the total number of establishments started in a specific year. The gray vertical bars represent NBER recessions. Data source: LBD.
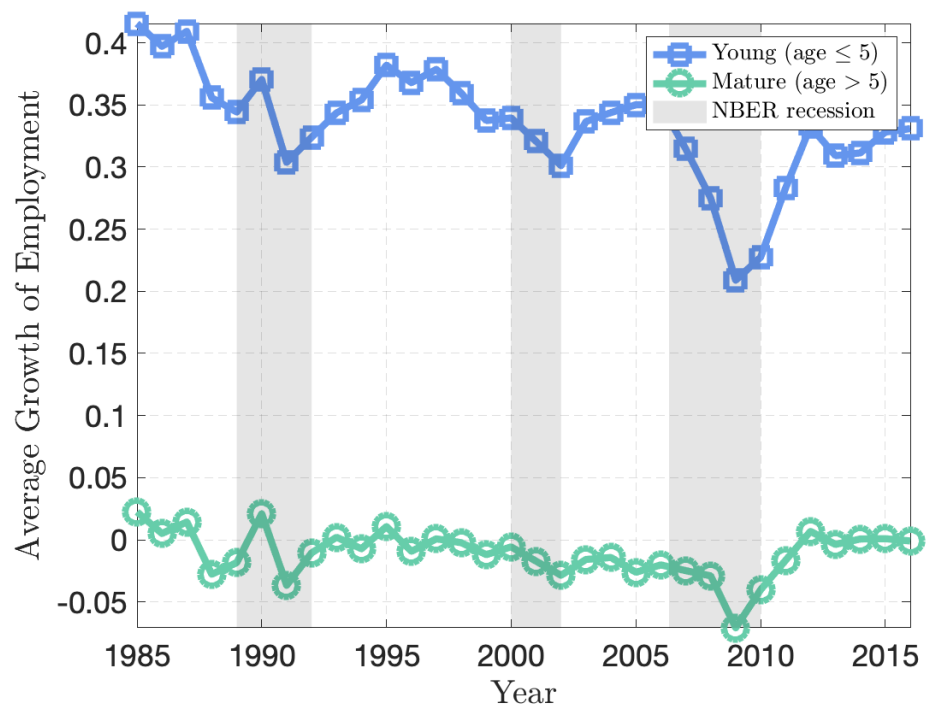
Notes: Figure B6 plots the average growth of employment for young (age under 5 years) and old (age above 5 years) establishments. The average growth rate is employment-weighted. The growth rate is defined as a one-year log change in employment. The vertical gray bars represent NBER recession dates. Data source: LBD.

C Tables

<div align="center">

TABLE C1: RELATIVE PRODUCTIVITY OF ENTRANTS

</div>

	Labor productivity		TFP	
	Manufacturing	All	Manufacturing	All
2005-2006	0.92	0.88	0.91	0.90
2007-2009	0.93	0.90	0.94	0.92
2010-2012	0.95	0.90	0.96	0.92

Notes: Table C1 reports the relative productivity of entrants with respect to incumbents for all industries considered (see sample selection criteria in Appendix A.2) and for manufacturing sector separately. Two productivity measures are considered: labor productivity and TFP (defined as a residual from production function). Entrants are firms of ages 1 and 2, incumbents are firms older than 2. Labor productivity is calculated as log turnover minus log of number of employees. Production function was estimated for each 1-digit industry using Olley and Pakes (1996) methodology. Fixed assets were used as a measure of capital, and investment expenditures were used as a proxy for unobserved productivity.

TABLE C2: INITIAL AGGREGATE CONDITIONS AND PROBABILITY OF CUMULATIVE GROWTH TO BE ABOVE THE 10TH PERCENTILE: U.S. DATA

	(1)	(2)	(3)	(4)	(5)
$\mathbf{1}_{\{Recession\}}$	0.0008***	0.0009***	0.0023***	0.0026***	-0.0012***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Const.	0.9180***	0.9181***	0.9180***	0.9180***	0.9208***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
N (in 1000's)	3752	3744	3576	3569	2358
Adj. R^2	0.01	0.01	0.01	0.01	0.01
Industry FE	Yes	Yes	Yes	Yes	Yes
Region FE	No	Yes	Yes	Yes	Yes
Legal Form of Organization FE	No	No	Yes	Yes	Yes
Multi-Unit Status FE	No	No	No	Yes	Yes
Type of Operation FE	No	No	No	No	Yes

Notes: Table C2 reports the OLS estimates of the following linear probability model:

$$\mathbf{1}_{\{c.growth_{it} \geq P10\}} = \beta \mathbf{1}_{\{entered\ in\ recession\}} + controls + \varepsilon_{it},$$

where the dependent variable equals 1 if the 5-year cumulative growth of establishment i is above the 10th percentile, and 0 otherwise. The key independent variable is a binary variable, which takes a value of 1 if establishment i entered in a recession, and 0 otherwise. Recessionary years are defined according to the NBER classification. The vector of controls includes the following categorical variables: 4-digit NAICS (imputed from a 6-digit NAICS variable `fk_naics12` constructed by Fort and Klimek, 2016), county FIPS, legal form of organization (LBD variable `lfo`), multi-unit status (LBD variable `mu`), and type of operation (LBD variable `toc`). The cumulative growth for each establishment i is calculated as a log change in employment between ages 1 and 5. Subsequently, the 10th percentile of the cumulative growth distribution is constructed based on the pooled sample of cumulative growth rates across all establishment for which the cumulative growth is defined. Data source: LBD.

Table C3: Initial Aggregate Conditions and Probability of Cumulative Growth to Be above the 50th Percentile: U.S. Data

	(1)	(2)	(3)	(4)	(5)
$\mathbf{1}_{\{\text{Recession}\}}$	-0.0041***	-0.0040***	-0.0043***	-0.0038***	-0.0087***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Const.	0.4794***	0.4794***	0.4794***	0.4792***	0.4719***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
N (in 1000's)	3752	3744	3576	3569	2358
Adj. R^2	0.01	0.01	0.01	0.01	0.02
Industry FE	Yes	Yes	Yes	Yes	Yes
Region FE	No	Yes	Yes	Yes	Yes
Legal Form of Organization FE	No	No	Yes	Yes	Yes
Multi-Unit Status FE	No	No	No	Yes	Yes
Type of Operation FE	No	No	No	No	Yes

Notes: Table C3 reports the OLS estimates of the following linear probability model:

$$\mathbf{1}_{\{\text{c.growth}_{it} \geq P50\}} = \beta \mathbf{1}_{\{\text{entered in recession}\}} + controls + \varepsilon_{it},$$

where the dependent variable equals 1 if the 5-year cumulative growth of establishment i is above the 50th percentile, and 0 otherwise. The key independent variable is a binary variable, which takes a value of 1 if establishment i entered in a recession, and 0 otherwise. Recessionary years are defined according to the NBER classification. The vector of controls includes the following categorical variables: 4-digit NAICS (imputed from a 6-digit NAICS variable `fk_naics12` constructed by Fort and Klimek, 2016), county FIPS, legal form of organization (LBD variable `lfo`), multi-unit status (LBD variable `mu`), and type of operation (LBD variable `toc`). The cumulative growth for each establishment i is calculated as a log change in employment between ages 1 and 5. Subsequently, the 50th percentile of the cumulative growth distribution is constructed based on the pooled sample of cumulative growth rates across all establishment for which the cumulative growth is defined. Data source: LBD.

TABLE C4: INITIAL AGGREGATE CONDITIONS AND PROBABILITY OF CUMULATIVE GROWTH TO BE ABOVE THE 90TH PERCENTILE: U.S. DATA

	(1)	(2)	(3)	(4)	(5)
$\mathbf{1}_{\{\text{Recession}\}}$	-0.0039***	-0.0039***	-0.0046***	-0.0043***	-0.0053***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)
Const.	0.0947***	0.0947***	0.0948***	0.0946***	0.0957***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
N (in 1000's)	3752	3744	3576	3569	2358
Adj. R^2	0.01	0.01	0.01	0.02	0.02
Industry FE	Yes	Yes	Yes	Yes	Yes
Region FE	No	Yes	Yes	Yes	Yes
Legal Form of Organization FE	No	No	Yes	Yes	Yes
Multi-Unit Status FE	No	No	No	Yes	Yes
Type of Operation FE	No	No	No	No	Yes

Notes: Table C4 reports the OLS estimates of the following linear probability model:

$$\mathbf{1}_{\{\text{c.growth}_{it} \geq P90\}} = \beta \mathbf{1}_{\{\text{entered in recession}\}} + controls + \varepsilon_{it},$$

where the dependent variable equals 1 if the 5-year cumulative growth of establishment i is above the 90th percentile, and 0 otherwise. The key independent variable is a binary variable, which takes a value of 1 if establishment i entered in a recession, and 0 otherwise. Recessionary years are defined according to the NBER classification. The vector of controls includes the following categorical variables: 4-digit NAICS (imputed from a 6-digit NAICS variable `fk_naics12` constructed by Fort and Klimek, 2016), county FIPS, legal form of organization (LBD variable `lfo`), multi-unit status (LBD variable `mu`), and type of operation (LBD variable `toc`). The cumulative growth for each establishment i is calculated as a log change in employment between ages 1 and 5. Subsequently, the 90th percentile of the cumulative growth distribution is constructed based on the pooled sample of cumulative growth rates across all establishment for which the cumulative growth is defined. Data source: LBD.

Table C5: Investment and Residential Wealth of Directors

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Slow	0.0004	0.0160***	0.0189***	0.0191***	0.0189***	0.0059***	0.0062***	0.0062***
	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Fast	0.0082***	0.0261***	0.0269***	0.0271***	0.0270***	0.0148***	0.0126***	0.0130***
	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Leverage					-0.1847***	-0.1790***	-0.1492***	-0.1500***
					(0.016)	(0.016)	(0.015)	(0.015)
Inv. scale						20.2029***	18.4466***	18.3335***
						(1.091)	(1.031)	(1.029)
Profit							0.0766***	0.0774***
							(0.003)	(0.003)
HPI								-0.0001
								(0.000)
Const.	0.1371***							
	(0.002)							
N	108013	105109	105072	105004	104998	104998	104998	104072
Adj. R^2	0.03	0.18	0.20	0.20	0.20	0.21	0.23	0.23
Firm FE	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry-time FE	No	No	Yes	Yes	Yes	Yes	Yes	Yes
Region-time FE	No	No	No	Yes	Yes	Yes	Yes	Yes

Notes: Table C5 reports OLS estimates of Equation (2). The sample covers reporting UK firms over the period 2002-2014. The dependent variable is investment (change in fixed assets plus depreciation). Residential RE is the total value of residential property held by directors of the firm, holding the composition of directors and their properties fixed in 2002, updating the value through time with changes in their respective regional house price indices, as defined in Equation (1). Leverage is a liability-based leverage, which is a ratio of total liabilities to total assets. All of these variables (except leverage) are scaled by the lag of firm "Fixed Assets". All ratios are winsorized at the median ± 5 times interquantile range. Standard errors, clustered by firm region, are in parentheses. Column (1) reports the effect of residential real estate on firms' investment without other controls. Column(2) adds firm fixed effects. Column (3) further adds industry-time (2-digit SIC classification) fixed effects. Column (4) adds region-year fixed effects. Column (5) adds firms' leverage. Column (6) additionally controls for potentially spurious size effects and includes the inverse of (lagged) fixed assets. Columns (7) and (8) further include firms' profits and a firm-region house price index. *, **, *** denote statistical significance at 10, 5, and 1 percent levels, respectively.

TABLE C6: COLLATERAL CHANNEL OF RESIDENTIAL PROPERTY: SPLIT BY AGE AND SIZE

	Baseline	Age group		Size group	
		≤ 5 years	> 5 years	≤ 50 emp.	> 50 emp.
	(1)	(2)	(3)	(4)	(5)
Slow	0.0062***	0.0108*	0.0065***	0.0067***	0.0173***
	(0.001)	(0.006)	(0.001)	(0.002)	(0.003)
Fast	0.0130***	0.0140**	0.0135***	0.0105***	0.0209***
	(0.001)	(0.006)	(0.001)	(0.001)	(0.003)
Leverage	-0.1500***	-0.2224***	-0.1588***	-0.1463***	-0.1767***
	(0.015)	(0.085)	(0.017)	(0.032)	(0.021)
Inv. scale	18.3335***	32.8427***	18.4368***	21.3020***	14.3574***
	(1.029)	(4.274)	(1.109)	(1.555)	(5.373)
Profit	0.0774***	0.0873***	0.0762***	0.0545***	0.1065***
	(0.003)	(0.018)	(0.003)	(0.005)	(0.005)
HPI	-0.0001	-0.0004	-0.0001	0.0002	-0.0002
	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)
N	104072	5019	99844	30382	52329
Adj. R^2	0.23	0.38	0.22	0.23	0.20
Firm FE	Yes	Yes	Yes	Yes	Yes
Industry-time FE	Yes	Yes	Yes	Yes	Yes
Region-time FE	Yes	Yes	Yes	Yes	Yes

Notes: Table C6 reports OLS estimates of Equation (2) for different sample splits. The full sample covers reporting UK firms over the period 2002-2014. The dependent variable is investment (change in fixed assets plus depreciation). Residential RE is the total value of residential property held by directors of the firm, holding the composition of directors and their properties fixed in 2002, updating the value through time with changes in their respective regional house price indices, as defined in Equation (1). Leverage is a liability-based leverage, which is a ratio of total liabilities to total assets. All of these variables are scaled by the lag of firm "Fixed Assets". All ratios are winsorized at the median ± 5 times interquantile range. Column (1) provides baseline estimates. Columns (2) and (3) report estimates of Equation (2) for young (ages 1-5) and old (ages 5+) subsamples. Columns (4) and (5) report estimates for small (less than 50 employees) and large (more than 50 employees) subsamples. Size is determined as a time-series average of firm-level employment. Standard errors, clustered by firm region, are in parentheses. *, **, *** denote statistical significance at 10, 5, and 1 percent levels, respectively.

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Table C7: Investment and Residential Wealth of Directors, Conditional on Reaching Age 5

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Slow	0.0005*	0.0162***	0.0191***	0.0193***	0.0191***	0.0060***	0.0063***	0.0064***
	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Fast	0.0082***	0.0260***	0.0269***	0.0271***	0.0270***	0.0148***	0.0126***	0.0130***
	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Leverage					-0.1852***	-0.1794***	-0.1496***	-0.1503***
					(0.016)	(0.016)	(0.015)	(0.015)
Inv. scale						20.2223***	18.4575***	18.3352***
						(1.123)	(1.064)	(1.061)
Profit							0.0769***	0.0776***
							(0.003)	(0.003)
HPI								-0.0001
								(0.000)
Const.	0.1370***							
	(0.002)							
N	107767	104949	104912	104844	104837	104837	104837	103916
Adj. R^2	0.03	0.18	0.20	0.20	0.20	0.21	0.23	0.23
Firm FE	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Industry-time FE	No	No	Yes	Yes	Yes	Yes	Yes	Yes
Region-time FE	No	No	No	Yes	Yes	Yes	Yes	Yes

Notes: Table C7 reports OLS estimates of Equation (2). The sample covers reporting UK firms over the period 2002-2014. The dependent variable is investment (change in fixed assets plus depreciation). Residential RE is the total value of residential property held by directors of the firm, holding the composition of directors and their properties fixed in 2002, updating the value through time with changes in their respective regional house price indices, as defined in Equation (1). Leverage is a liability-based leverage, which is a ratio of total liabilities to total assets. All of these variables (except leverage) are scaled by the lag of firm "Fixed Assets". All ratios are winsorized at the median ± 5 times interquantile range. Standard errors, clustered by firm region, are in parentheses. Column (1) reports the effect of residential real estate on firms' investment excluding other controls. Column(2) adds firm fixed effects. Column (3) further adds industry-time (2-digit SIC classification) fixed effects. Column (4) adds region-year fixed effects. Column (5) adds firms' leverage. Column (6) additionally controls for potentially spurious size effects and includes the inverse of (lagged) fixed assets. Columns (7) and (8) further include firms' profits and firm-region house price index. *, **, *** denote statistical significance at 10, 5, and 1 percent levels, respectively.

Table C8: Investment and Residential Wealth of Directors: Robustness of Firms' Assignemnt

	(1)	(2)	(3)	(4)
Slow	0.0064***	0.0052	0.0043**	0.0045***
	(0.001)	(0.004)	(0.002)	(0.001)
Fast	0.0130***	0.0184***	0.0139***	0.0370***
	(0.001)	(0.003)	(0.001)	(0.001)
Leverage	-0.1503***	-0.2308***	-0.1716***	
	(0.015)	(0.035)	(0.021)	
Inv. scale	18.3352***	14.9576***	17.4575***	
	(1.061)	(3.906)	(1.667)	
Profit	0.0776***	0.0984***	0.0874***	
	(0.003)	(0.009)	(0.006)	
HPI	-0.0001	-0.0004	-0.0003	
	(0.000)	(0.001)	(0.000)	
N	103916	16353	46983	439113
Adj. R^2	0.23	0.29	0.24	0.12
Firm FE	Yes	Yes	Yes	Yes
Industry-time FE	Yes	Yes	Yes	Yes
Region-time FE	Yes	Yes	Yes	Yes

Notes: Table C8 reports OLS estimates of Equation (2). The sample covers reporting UK firms over the period 2002-2014. The dependent variable in Columns (1) - (3) is investment (change in fixed assets plus depreciation) and change in Total Assets in Column (4). Residential RE is the total value of residential property held by directors of the firm, holding the composition of directors and their properties fixed in 2002, updating the value through time with changes in their respective regional house price indices, as defined in Equation (1). Leverage is a liability-based leverage, which is a ratio of total liabilities to total assets. All of these variables (except leverage) are scaled by the lag of firm "Fixed Assets". All ratios are winsorized at the median ± 5 times interquantile range. Standard errors, clustered by firm region, are in parentheses. Column (1) is the baseline. Column(2) reports estimates for when the firm has to spend more than 75% of its tenure above the median growth rate in order to be classified as "high" type. Column (3) reports estimates when the threshold is moved from 50th percentile to 30th and 60th for "slow" and "fast" type, respectively. Column (4) corresponds to the case with no controls and growth rate of total assets as a dependent variable. *, **, *** denote statistical significance at 10, 5, and 1 percent levels, respectively.

D Model Appendix

The computation of the model can be broadly divided into 3 parts: (1) simplification of programming problems by way of combining household's and firms' optimization problems, (2) computation of the model at the steady-state, and (3) solving for the model with aggregate fluctuations using perturbation techniques. In what follows, I lay out the key details of the numerical algorithm. To facilitate the exposition, I assume there is only one type of firms $J = 1$.

D.1 Analysis of the Model

The model outlined in Section 3 incorporates optimization problems for three distinct types of agents: representative household, incumbent firms and potential entrants. This implies that, first, I need to solve 3 programming problems, and then make sure that the agents' decisions are consistent with each other, and markets clear. Fortunately, it is possible to combine the optimality conditions for the households and the firms' Bellman equations, and thus reduce the computational complexity of the problem at hand. Using $C(\boldsymbol{s})$ and $N(\boldsymbol{s})$ to denote the market clearing values of household consumption and hours worked it is straightforward to show that market-clearing requires:

1. the real wage w be equal to the household marginal rate of substitution between leisure and consumption:
$$w(\boldsymbol{s}) = \frac{U_2'(C(\boldsymbol{s}), 1 - N(\boldsymbol{s}))}{U_1'(C(\boldsymbol{s}), 1 - N(\boldsymbol{s}))};$$

2. the risk-free bond price q_0^{-1} be equal to the expected gross real interest rate:
$$q_0(\boldsymbol{s}) = \beta \mathbb{E}\left[\frac{U_1'(C(\boldsymbol{s}'), 1 - N(\boldsymbol{s}'))}{U_1'(C(\boldsymbol{s}), 1 - N(\boldsymbol{s}))}\right];$$

3. firms' state-contingent discount factors be consistent with the household marginal rate of substitution between consumption across states:
$$d(\boldsymbol{s}'|\boldsymbol{s}) = \beta \frac{U_1'(C(\boldsymbol{s}'), 1 - N(\boldsymbol{s}'))}{U_1'(C(\boldsymbol{s}), 1 - N(\boldsymbol{s}))}.$$

Following Khan, Senga and Thomas (2014), I compute for the recursive competitive equilibrium effectively substituting the equilibrium implications of household optimization into the recursive problems faced by the firms. Let $p(\boldsymbol{s})$ be the marginal utility of the

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household with respect to equilibrium consumption $C(\boldsymbol{s})$. Then equations (5) - (12) can be rewritten by way of expressing each firm's value in terms of the marginal utility of the household.

$$V_j^0(k, b, z; \boldsymbol{s}) = \pi_d V_j^1(k, b, z; \boldsymbol{s}) + (1 - \pi_d) V_j^2(k, b, z; \boldsymbol{s}). \tag{25}$$

$$V_j^2(k, b, z; \boldsymbol{s}) = \max_{k' \geq 0, b' \in \mathcal{B}} p(\boldsymbol{s})[-E_j(k, b, z; \boldsymbol{s}) - \varphi^E \max\{E_j, 0\}^3] + \beta \mathbb{E}\left[V_j^0(k', b', z' | \boldsymbol{s}')\right]. \tag{26}$$

$$V_j^1(k, b, z; \boldsymbol{s}) = p(\boldsymbol{s})[\Pi_j(k, b, z; \boldsymbol{s}) + (1 - \delta)k - b]. \tag{27}$$

Next, I lay out the algorithm which I used to solve for the equilibrium.

D.2 Steady-State

I use collocation methods to solve the firm's functional equations (25)-(27). In practice, I use Chebyshev polynomials to approximate the value functions.

I set up a grid of collocation nodes $\mathcal{K} \times \mathcal{B} \times \mathcal{Z}$, with N_i nodes in each dimension, $i \in \{\mathcal{K}, \mathcal{B}, \mathcal{Z}\}$. Throughout the algorithm, I compute expectations with respect to idiosyncratic productivity shocks using Gauss-Hermite quadrature. The computation of the stationary state of the model proceeds in the following 4 steps:

1. guess the equilibrium wage rate, w;

2. solve for individual decision rules (k', b');

3. given the decision rules, compute stationary histogram (distribution of firms over the state space);

4. compute the excess demand on the labor market. If it exceeds some prespecified tolerance, adjust the wage guess correspondingly and go back to Step 2. Otherwise, terminate.

D.2.1 Approximation of Value Functions

I approximate 2 (normalized by the household's marginal utility) value functions: $V^0(\cdot)$ and $V^2(\cdot)$, which are defined in (25) and (26), respectively. In particular, I represent these value

functions as weighted sums of orthogonal polynomials:

$$
\begin{cases}
V^0(k,b,z) & = \sum_{i,j,k=1,1,1}^{N_{\mathcal{K}},N_{\mathcal{B}},N_{\mathcal{Z}}} \theta_0^{ijk} T^i(k) T^j(b) T^k(z) \\
V^1(k,b,z) & = \sum_{i,j,k=1,1,1}^{N_{\mathcal{K}},N_{\mathcal{B}},N_{\mathcal{Z}}} \theta_1^{ijk} T^i(k) T^j(b) T^k(z),
\end{cases}
$$

where $\{\theta_0^{i,j,k}, \theta_1^{i,j,k}\}$ are approximation coefficients, and $T^i(\cdot)$ is the Chebyshev polynomial of order i.

I use collocation method to simultaneously solve for $\{\theta_0^{i,j,k}, \theta_1^{i,j,k}\}$. Collocation method requires setting the residual equation to hold exactly at $N = N_{\mathcal{K}} \times N_{\mathcal{B}} \times N_{\mathcal{Z}}$ points ; therefore, I essentially solve for $2 \times N$ unknown coefficients. I compute the basis matrices for Chebyshev polynomials using Miranda and Fackler (2002) Compecon toolbox. Subsequently, I solve for a vector of unknown coefficients using Newton's method. A much slower alternative is to iterate on the value function. Given the current guess of coefficients, I solve for the optimal policies $k'(k,b,z)$ and $b'(k,b,z)$ using nested vectorized golden search. After I solve for the policy function, I recompute decision rules on a finer grid, and, subsequently, compute the stationary distribution.

D.2.2 Stationary Distribution

When I solve for a stationary distribution, I iterate on a mapping using firms' decisions rules:

$$
L' = \boldsymbol{Q}'L + L^e,
$$

where L is a current distribution of incumbents across the state space, and L^e is a distribution of successful entrants. Matrix $\boldsymbol{Q}$ is a transition matrix, which determines how mass of firms shifts in the (k,b,z)-space. It is a direct product of three transition matrices $\boldsymbol{Q}_k, \boldsymbol{Q}_b$ and $\boldsymbol{Q}_z$:

$$
\boldsymbol{Q} = \boldsymbol{Q}_k \odot \boldsymbol{Q}_b \odot \boldsymbol{Q}_z,
$$

which govern the shift of mass along k- , b- and z-dimensions, respectively. While $\boldsymbol{Q}_z$ is completely determined by the exogenous stochastic process (4), matrices $\boldsymbol{Q}_k$ and $\boldsymbol{Q}_b$ are constructed so that the model generates an unbiased distribution in term of aggregates.[41] More precisely, element (i,j) of the transition matrix $\boldsymbol{Q}_x$ with $x \in \{k,b\}$ informs which fraction of firms with the current idiosyncratic state (k,b,z) will end up having x_j tomorrow.

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[41]See Young (2010) for more details.

Therefore, this entry of the matrix is computed as:

$$\boldsymbol{Q}_x(i,j) = \left[\mathbf{1}_{x' \in [x_{j-1}, x_j]} \frac{x' - x_j}{x_j - x_{j-1}} + \mathbf{1}_{x' \in [x_j, x_{j+1}]} \frac{x_{j+1} - x'}{x_{j+1} - x_j}\right].$$

Tensor product of matrices $\boldsymbol{Q}_k$, $\boldsymbol{Q}_b$ and $\boldsymbol{Q}_z$ is computed using the `dprod` function from Miranda and Fackler (2002) toolkit.

D.3 Model with Aggregate Shocks

Once a finite representation of the system at the steady-state is obtained, I can write down equilibrium conditions as a system of difference equations, where some equations are backward-looking (e.g., the evolution of the distribution), and forward-looking (Bellman equations). Following a standard approach of using the expectation errors in forward-looking equations (denoted η_{t+1}), the equilibrium with aggregate uncertainty can be written as the finite non-linear system

$$\Gamma(\Theta_t, \Theta_{t+1}, \eta_{t+1}, \varepsilon_{t+1}) = \mathbf{0}, \tag{28}$$

where the vector Θ_t contains state and jump variables (such as histogram and collocation parameters for value functions approximation), and ε_{t+1} is a vector of Gaussian disturbances to exogenous aggregate stochastic processes. The vector Θ_t also contains an $M \times 1$ vector $\boldsymbol{g}_t$, which collects observables used in the estimation step. This vector is general can be a non-linear function of other elements of Θ_t.

With this representation at hand, the solution of the steady-state boils down to solving for the value of $\bar{\Theta}$ when aggregate shocks are turned-off; that is, it has to satisfy

$$\Gamma(\bar{\Theta}, \bar{\Theta}, \mathbf{0}, \mathbf{0}) = \mathbf{0}.$$

Subsequently, one can express (28) in terms of log deviations from the steady state, $\hat{\Theta}_t = \log(\Theta_t) - \log(\bar{\Theta})$, and take a first-order Taylor expansion. This delivers a linear system of equations, which provides a SVAR representation of the model:

$$\Gamma_0 \hat{\Theta}_{t+1} = \Gamma_1 \hat{\Theta}_t + \Psi \varepsilon_{t+1}. \tag{29}$$

The matrices Γ_0 and Γ_1 contain first-order partial derivatives of equilibrium conditions with respect to elements of Θ_t, which are computed numerically using automatic differentiation.[42]

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[42]I use `myAD` toolkit written by SeHyoun Ahn, which is available at https://github.com/sehyoun/

I solve the resulting system of equations (29) using the rational expectations solver `Gensys` provided by Sims (2002).

FIGURE D1: U.K. QUARTERLY AGGREGATE DATA, 1975Q1-2016Q4
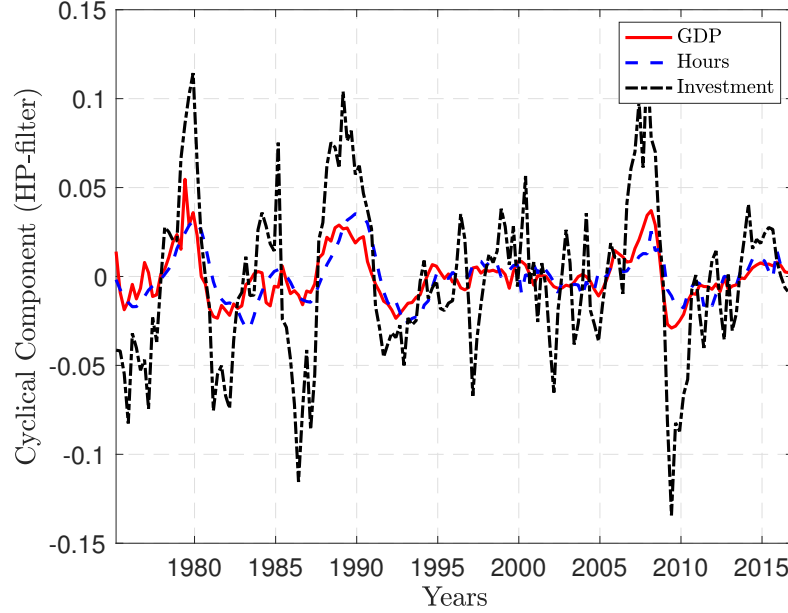
Notes: Figure D1 plots 3 aggregate series used for Bayesian estimation: real GDP, total annual hours worked, and investment expenditures. Series have been logged and HP-filtered with smoothing parameter 1600. Nominal values have been converted to real using CPI index (2015 being base year). Source: Office for National Statistics.

D.4 Estimation

In this section, I describe the estimation procedure of $\Omega = \{\rho_x, \sigma_x\}_{x \in \{z,r,n\}}$ - parameters of aggregate stochastic processes introduced in Subsection 4.4.

The solution to Equation (29) along with measurement equation form the following system of equations:

$$\hat{\Theta}_{t+1} = A(\Omega)\hat{\Theta}_t + B(\Omega)\varepsilon_{t+1}$$
$$Y_t = C\hat{\Theta}_t + D\zeta_t,$$

where $A(\Omega)$ describes the evolution of the model's state and $B(\Omega)$ is an impact matrix. The second equation is a measurement equation: it relates the observable series $\{Y_t\}_{t=1}^{T}$ to a latent state $\{\hat{\Theta}_t\}_{t=1}^{T}$. With the representation above, one can compute the likelihood of any

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`MATLABAutoDiff`

sequence of $\{Y_t\}_{t=1}^T$ using Kalman filter (see An and Schorfheide (2007) and Mongey and Williams (2016) for the description of that procedure).

Given a current draw of parameters Ω, let $P\left(\{Y_t\}_{t=1}^T|\Omega\right)$ denote the likelihood of the observed data. The posterior can be computed by combining the likelihood with the prior:

$$P\left(\Omega|\{Y_t\}_{t=1}^T\right) \propto P\left(\{Y_t\}_{t=1}^T|\Omega\right)P(\Omega).$$

In order to quantify the uncertainty about parameter estimates, I characterize the posterior by drawing from it using Markov Chain Monte Carlo; I use Metropolis-Hastings algorithm to accomplish this step.

I estimate parameters of 3 exogenous stochastic processes (6 parameters in total) using aggregate series for GDP, hours worked and investment expenditures. The data is quarterly and spans the time period 1975Q1:2016Q4 (168 data points in total) – see Figure D1. Table 5 in main text reports the prior distributions used in the estimation step, and characterizes the posteriors: their modes and 5%, 95% bounds.