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Staff Working Paper No. 875 Financial stress and the debt structure David Gauthier

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Abstract

This paper identifies shocks to credit conditions based on aggregate firms' debt composition. I develop a model where firms fund production with bonds and loans. Only financial shocks imply opposite movements in the two types of debt as firms adjust their debt composition to new credit conditions. I use this result to inform a sign-restriction VAR and identify the sources of US business cycles. Financial shocks account for a third of output fluctuations. I construct an index of financial stress to test the identification strategy.

Key words: Business cycles, financial shocks, firm funding, sign restrictions.

JEL classification: C11, E32, E44, G21.

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1. Introduction

What is the footprint of a shock to firms' credit conditions? Addressing this question is key to extricate the sources of the business cycle. It is also arduous since virtually all shocks propagate through credit conditions. This paper identifies financial shocks based on firms' funding choices. I construct a model where firms finance production with bank loans and bonds. In this economy, shocks to credit conditions modify firms' funding decisions and trigger opposite movements in the two forms of debt. I use this result to inform a sign-restriction VAR and isolate economic disturbances that imply opposite responses in bonds and loans. I dub these shocks the financial shocks. They explain a large share of the US business cycle.

Various papers have studied the role of financial shocks in economic fluctuations.¹ This paper offers a strategy designed to avoid the pitfalls characteristic of financial shock identification. Several reasons explain the difficulty to establish causal links between the financial sector and the rest of the economy. First, financial variables are procyclical and forward-looking, making it arduous to separate financial shocks from economic cycles with standard recursive identification schemes.² Second, because financial stress can result in credit rationing rather than in price changes, using statistical indices of financial stress and spreads to account for firms' credit conditions can be misleading.³ Third, structural models such as DSGE models used to identify the sources of economic fluctuations do not always qualitatively distinguish shocks to credit conditions from other macroeconomic shocks, rendering the identification very sensitive to the model structure.

I use firms' funding decisions as a proxy for credit conditions in place of the more usual spreads

¹Examples based on DSGE models include Gilchrist, Otiz, and Zakrajsek (2009), Nolan and Thoenissen (2009), Jermann and Quadrini (2012), Christiano, Motto, and Rostagno (2014) and Ajello (2016). Studies using long-run time series include Reinhart and Rogoff (2009) and Schularick and Taylor (2012). For vector autoregression evidence, see Gilchrist and Zakrajšek (2012), Meeks (2012), Fornari and Stracca (2012), Bassett et al. (2014), Prieto, Eickmeier, and Marcellino (2016), Caldara, Fuentes-Albero, Gilchrist, and Zakrajsek (2016) and Furlanetto, Ravazzolo, and Sarferaz (2017). For univariate forecasting specifications, see López-Salido, Stein, and Zakrajšek (2017).

²See Mumtaz, Pinter, and Theodoridis (2018) for a critical review of financial shock identification with structural VAR models.

³As pointed out in Kashyap, Stein, and Wilcox (1993), Stock and Watson (2012), Caldara, Fuentes-Albero, Gilchrist, and Zakrajsek (2016) and Romer and Romer (2017). See also Chari, Christiano, and Kehoe (2008) and Cohen-Cole, Duygan-Bump, Fillat, and Montoriol-Garriga (2008) on the use of prices versus quantities to identify the financial nature of a crisis.

and asset prices.⁴ To study how bonds and loans respond to macroeconomic shocks, I augment the workhorse New Keynesian (NK) model with the mechanism of debt choice from De Fiore and Uhlig (2011, 2015). The model assumes the existence of banks more efficient than markets to reduce asymmetric information problems but also more costly.⁵ Based on a prior predictive analysis, I find that only financial shocks generate opposite movements in loans and bonds on impact. On the other hand, supply, monetary, and other demand shocks generate comovements in the *two* types of debt. Why is that so? The reason is that in response to a financial shock, firms adjust their funding to the new credit conditions and substitute the most efficient type of debt for the other. In contrast, adverse non-financial shocks imply that both types of debt become less desirable for firms. This triggers a simultaneous fall in bonds and loans.

In a second step, I use the qualitative predictions of the modified NK framework to inform a sign-restriction VAR model estimated with aggregate US corporate firm balance-sheet data. Three key results emerge from this identification strategy. First, financial shocks account for a large share of the US business cycle. Second, these shocks are identified around precise events such as the Japanese crisis, the LTCM crisis, and the Great Recession. Third, the financial shocks I obtain resemble financial shocks estimated based on more constrained identification techniques or richer data set.

In the final part of the paper, I estimate the modified NK model so as to minimize the distance between its impulse responses and those from the VAR model. The method allows to estimate the model shock by shock, depending on the restrictions imposed for the VAR estimation.⁶ I find that the modified NK model can reproduce both qualitative and quantitative features implied by the subset of identified VARs. This is true for all types of shock. The estimated model is used to

⁴This method is reminiscent of Kashyap, Stein, and Wilcox (1993) who use debt composition to identify the credit channel of monetary policy. Example of papers that use firms' debt composition to proxy credit conditions include Adrian, Colla, and Song Shin (2013), Becker and Ivashina (2014) and Altavilla, Darracq Pariès, and Nicoletti (2015),.

⁵Closely related, Repullo and Suarez (2000) develop a partial equilibrium model where banks with high monitoring intensity are the only possible source of funds for firms with low net worth. Crouzet (2018) constructs a model where banks provide flexible debt contracts to producing firms. He finds the latter substitute bonds for loans in response to financial shocks.

⁶This stands in contrast to a full information approach that would discard some of the estimated shocks based on purely statistical ground.

construct a measure of financial stress for the past 30 years. The index highly correlates with other measures of financial stress and is predictive of the bond spread.⁷

The rest of the paper is organized as follows. Section 2 introduces the modified NK model, section 3 presents the calibration of the model and discusses its properties. Section 4 lays out the sign-restriction VAR model. Section 5 estimates the modified NK model and provides out-of-sample exercises. Section 6 concludes.

2. Debt Arbitrage in a New Keynesian Model

The model is populated by three types of agents. Households consume, work and save, firms use capital and labor to produce final goods, financial intermediaries channel funds from households to the productive sector.⁸

2.1. Households

The model assumes a large number of identical and competitive households. A representative household maximizes its utility function defined as:

$$E_0 \sum_{t=0}^{\infty} \beta^t \zeta_t^C \left\{ \log(C_t) - \psi_H \frac{H_t^{1+\sigma_H}}{1+\sigma_H} \right\},\tag{1}$$

where C_t is consumption, $\zeta_t^C > 0$ is a preference shock, $\sigma_H > 1$ is the inverse Frisch elasticity of labor supply and ψ_H is a weighting parameter for labor desutility. Each household is subject to the budget constraint:

$$p_t C_t + p_t D_t + q_t^K K_t \le w_t H_t + R_t p_{t-1} D_{t-1} + \left(q_t^K (1 - \delta) + p_t r_t^K \right) K_{t-1} + O_t.$$
(2)

Households spend on consumption of the final goods priced at p_t and capital K_t purchased from capital installers at price q_t^K . Revenues come from selling labor H_t at a nominal wage w_t . Real deposits D_{t-1} are remunerated at a gross nominal rate R_t . Each period, households supply capital

⁷Importantly, no data on the cost of credit is used.

⁸Section 1 of the appendix provides a detailed derivation of the model and lists the full set of equations.

 K_t to entrepreneurs at a competitive rental rate r_t^K . Depreciated past period capital is sold back to capital installers. Variable O_t corresponds to transfers from entrepreneurs.

Capital Installers.—Capital installers buy investment goods I_t from the final good producer and turn it into installed capital sold to households in a competitive market at price q_t^K . They maximize the sum of their profits discounted with household stochastic discount factor $\beta^t \zeta_t^C \tilde{\Lambda}_t$,

$$E_0 \sum_{t=0}^{\infty} \beta^t \zeta_t^C \tilde{\Lambda}_t \left\{ q_t^K K_t - p_t I_t \right\},\tag{3}$$

using the following capital accumulation technology:

$$K_{t} = (1 - \delta)K_{t-1} + \left[1 - S\left(\zeta_{t}^{I}\frac{I_{t}}{I_{t-1}}\right)\right]I_{t}.$$
(4)

Here, $0 < \delta < 1$ is the depreciation rate of capital, *S*(.) is an increasing adjustment cost function and ζ_t^I is a shock to the marginal efficiency of investment in producing capital.

2.2. Firms

Firms produce final goods using capital and labor inputs. I follow Gali (2010) in assuming a three-sector structure for firms. Entrepreneurs produce homogeneous goods transformed by monopolistically competitive retailers into intermediate goods. The final good producers combine intermediate goods to produce homogeneous final goods sold to households in competitive markets.

2.2.1 Entrepreneurs

Entrepreneurs are heterogeneous agents modeled as in De Fiore and Uhlig (2011). Each period entrepreneurs have the option to contract with a financial intermediary to fund working capital and produce homogeneous goods sold to intermediate producers. Because there exist different types of financial intermediaries, entrepreneurs can select the form of debt they prefer depending on their own characteristics.

Production.—A continuum of risk-neutral entrepreneurs $e \in [0, 1]$ operate in competitive markets. An entrepreneur *e* produces goods Y_{et}^E with capital and labor inputs using the following Cobb-Douglas technology:

$$Y_{et}^E = \varepsilon_{et}^E A_t K_{et-1}^{\alpha} H_{et}^{1-\alpha}, \tag{5}$$

where K_{et} and H_{et} denote respectively capital and labor inputs used for production. Variable A_t corresponds to a technology shock and ε_{et}^E is a sequence of independent idiosyncratic shock realizations.

Entrepreneurs are subject to a debt constraint. An entrepreneur starts the period *t* with net worth N_{et} that corresponds to the sum of past period profits minus dividends transferred to households. Each period entrepreneur *e* rents capital inputs and purchases labor paid at a real wage $\tilde{w}_t = w_t/p_t$ using funds X_{et} :

$$X_{et} \ge r_t^K K_{et} + \tilde{w}_t H_{et},\tag{6}$$

where X_{et} is the sum of the entrepreneur's net worth and external debt \bar{D}_{et} :

$$X_{et} = N_{et} + \bar{D}_{et}.$$
(7)

To obtain external funds \bar{D}_{et} from a financial intermediary, an entrepreneur must pledges her net worth according to the leverage constraint:

$$X_{et} = \xi N_{et},\tag{8}$$

where ξ is a parameter that pins down entrepreneur leverage.⁹ Production Y_{et}^E is sold to retailers at a competitive price p_t^E . The problem of an entrepreneur given available funds X_{et} is to choose the

⁹Similar to De Fiore and Uhlig (2011) and in contrast to the standard debt contracts from the canonical model of Bernanke, Gertler, and Gilchrist (1999), one need to assume a fixed leverage for entrepreneurs to obtain an interior solution to the borrowing decision problem. The reason is that entrepreneurs have different credit worthinesses. In the practical case where the distribution of ε_{et}^{E} is bounded, optimal leverage would imply a corner solution with all available funds going to the best entrepreneur.

combination of capital and labor inputs maximizing her real profits,

$$\left(p_t^E/p_t\right)Y_{et}^E - r_t^K K_{et} - \tilde{w}_t H_{et},\tag{9}$$

subject to the debt constraint defined in equation (6). The solution to the problem of the entrepreneur implies the first order conditions:

$$X_{et} = r_t^K K_{et} + \tilde{w}_t H_{et}, \tag{10}$$

$$\frac{r_t^k}{\tilde{w}_t} = \frac{\alpha Y_{et}/K_{et}}{(1-\alpha)Y_{et}/H_{et}},\tag{11}$$

where equation (10) implies that entrepreneurs use all available debt to fund production and equation (11) that they equalize the relative cost of the production factors to the ratio of their marginal productivities. Using these optimality conditions together with equation (5), it is possible to express individual production for each entrepreneur as:

$$Y_{et}^E = \frac{\varepsilon_{et}^E X_{et}}{s_t},\tag{12}$$

where s_t is the aggregate component of the entrepreneur's marginal cost of production expressed in terms of the final good and defined as,

$$s_t = \frac{1}{A_t} \left(\frac{p_t}{p_t^E} \right) \left(\frac{r_t^K}{\alpha} \right)^{\alpha} \left(\frac{\tilde{w}_t}{1 - \alpha} \right)^{1 - \alpha}.$$
(13)

For later use, it is also convenient to define $q_t = \frac{1}{s_t}$, where q_t is a measure of the aggregate entrepreneurial markup over input costs.¹⁰

Idiosyncrasy.—Before production takes place, each entrepreneur gets hit by a series of successive idiosyncratic productivity shocks that determine whether she produces or not and her preferred type of financial intermediary.

¹⁰Here s_t must not be confused with the marginal cost of the intermediate good producer, $\tilde{p}_t^E = \frac{p_t^E}{p_t}$, which is taken as given by entrepreneurs.

Three successive idiosyncratic shocks are considered. First, a shock $\varepsilon_{1,et}$ is publicly observed and creates heterogeneity in the productivity of entrepreneurs. This shock realizes together with aggregate shocks and before entrepreneurs contract with financial intermediaries. Second, a shock $\varepsilon_{2,et}$ occurs after financial contracts are set and is observed only by bank-funded entrepreneurs and their banks. This shock creates a rationale for choosing intermediated finance over direct finance.¹¹ A third shock $\varepsilon_{3,et}$ is privately observed by entrepreneurs and realizes just before production takes place. This final shock justifies the existence of risky debt contracts between entrepreneurs and financial intermediaries. Both privately observed shocks $\varepsilon_{2,et}$ and $\varepsilon_{3,et}$ can be monitored at a cost by financial intermediaries.

After the first idiosyncratic shock $\varepsilon_{1,et}$ is realized, each entrepreneur decides whether she wants to produce and if so selects her optimal source of funds. Entrepreneurs have the option to contract with banks to decrease their production risk. To do so they must pay a share τ_b of their net worth used to resolve part of their productivity uncertainty. A bank-funded entrepreneur e pays $\tau_b N_{et}$ to observe the realization of $\varepsilon_{2,et}$ and to share it with her bank. Before production takes place and based on the realization of $\varepsilon_{2,et}$, bank-funded entrepreneurs have the possibility to renegotiate their debt contract. In this case they recover their pledged net worth and abstain from production. An entrepreneur can also choose to fund from markets in which case she produces regardless of her productivity. The net worth of an entrepreneur after having contracted with a financial intermediary of type $f \in \{b, c\}$, b for bank and c for market is:

$$N_{et}^{f} = \begin{cases} N_{et} & , if bond financing\\ (1 - \tau_b)N_{et} & , if loan financing. \end{cases}$$
(14)

At the end of period t and after shock $\varepsilon_{3,et}$ is privately observed, producing entrepreneurs rent capital K_{et} and hire labor H_{et} from households. They produce, sell output to retailers and use their net worth and funds obtained from financial intermediaries to repay production factors. In a final stage entrepreneurs reimburse their financial intermediary. This is done conditional on the

¹¹In the rest of the paper, I use interchangeably intermediated debt or bank loan and direct debt or bond.

realization of their residual uncertain productivity ω_{et}^{f} defined as:

$$\omega_{et}^{f} = \begin{cases} \varepsilon_{2,et} \varepsilon_{3,et} & \text{, if bond financing} \\ \varepsilon_{3,et} & \text{, if loan financing.} \end{cases}$$
(15)

The realization of ω_{et}^{f} is kept private unless the financial intermediary decides to monitor the defaulting entrepreneur. In this case a fraction μ_{f} of seized assets is lost in the monitoring process. Entrepreneurs abstaining from production simply rent out their initial net worth as capital until the end of the period.

In application, I assume that idiosyncratic shocks $\varepsilon_{1,et}$, $\varepsilon_{2,et}$ and $\varepsilon_{3,et}$ follow independent lognormal distributions with unit means and respective variances σ_1^2 , $\sigma_2^2 + v_t$ and $\sigma_3^2 - v_t$. Here v_t is a zero-mean shock shifting the relative share of idiosyncratic productivity that bank-funded entrepreneurs can observe and transmit to their bank. Denoting σ_t^f the standard deviation of the residual uncertainty productivity factor $\omega_{e,t}^f$ conditional on entrepreneur's funding decision yields:

$$\sigma_t^f = \begin{cases} \sqrt{\sigma_2^2 + \sigma_3^2} & \text{, if bond financing} \\ \sqrt{\sigma_3^2 - v_t} & \text{, if loan financing.} \end{cases}$$
(16)

Notice that this specification implies that the standard deviation of entrepreneurs' productivity prior to their funding decision – which also corresponds to the standard deviation of productivity conditional on funding with bonds – is left unchanged after a shock v_t .

Financial Contracts.—The model assumes a continuum of risk-neutral financial intermediaries of each type, bank *b* or market *c*, able to fully diversify risk among entrepreneurs. Both fund using deposits from households remunerated at the nominal rate R_t . After the realization of the first two idiosyncratic shocks, an entrepreneur *e* and a financial intermediary of type *f* agree on a standard

debt contract conditional on ε_{et}^{f} , the expected productivity of the contracting entrepreneur, where:

$$\varepsilon_{et}^{f} = \begin{cases} \varepsilon_{1,et} &, \text{ if bond financing} \\ \varepsilon_{1,et}\varepsilon_{2,et} &, \text{ if loan financing.} \end{cases}$$
(17)

Given an optimal threshold $\bar{\omega}_{et}^{f}$ for ω_{et}^{f} under which monitoring occurs, the expected share of final output accruing to a contracting entrepreneur is:

$$v(\bar{\omega}_{et}^f, \sigma_t^f) = \int_{\bar{\omega}_{et}^f}^{\infty} (\omega - \bar{\omega}_{et}^f) \varphi(\omega, \sigma_t^f) d\omega, \qquad (18)$$

and the expected share of final output accruing to a lender of type f is:

$$g(\bar{\omega}_{et}^f, \sigma_t^f) = \int_0^{\bar{\omega}_{et}^f} (1 - \mu_f) \omega \varphi(\omega, \sigma) d\omega + \bar{\omega}_{et}^f [1 - \Phi(\bar{\omega}_{et}^f, \sigma_t^f)], \tag{19}$$

where $\varphi(\omega_{et}^f, \sigma_t^f)$ and $\Phi(\omega_{et}^f, \sigma_t^f)$ correspond respectively to the distribution and cumulative density functions of ω_{et}^f implied by the distributional assumptions on idiosyncratic shock distributions. Here the first and second terms on the right hand side correspond respectively to revenues seized from monitored entrepreneurs and payments from non-defaulting entrepreneurs.

The optimal debt contract chosen by entrepreneur *e* sets a threshold $\bar{\omega}_{et}^{f}$ under which monitoring occurs and maximizing the expected fixed repayment $\varepsilon_{et}^{f} \bar{\omega}_{et}^{f} X_{et} q_{t}$ paid to the financial intermediary. The problem of the entrepreneur is subject to the debt constraint from equation (6) and,

$$\varepsilon_{et}^{f} q_{t} g^{f}(\bar{\omega}_{et}^{f}, \sigma_{t}^{f}) X_{et} \ge (X_{et} - N_{et}^{f}) R_{t},$$

$$(20)$$

$$v(\bar{\omega}_{et}^f, \sigma_t^f) + g^f(\bar{\omega}_{et}^f, \sigma_t^f) \le 1 - G_{\omega}^f(\bar{\omega}_{et}^f, \sigma_t^f), \tag{21}$$

$$\varepsilon_{et}^f q_t v(\bar{\omega}_{et}^f, \sigma_t^f) X_{et} \ge N_{et}^f, \tag{22}$$

where $G_{\omega}^{f}(\bar{\omega}_{et}^{f}, \sigma_{t}^{f}) = \mu_{f} \int_{0}^{\bar{\omega}_{et}^{f}} \omega \varphi(\omega, \sigma_{t}^{f}) d\omega$ denotes the share of output lost to monitoring. Equa-

tion (20) implies that the financial intermediaries' expected returns must exceed repayment to households, equation (21) ensures the feasibility of the debt contract, and equation (22) guarantees entrepreneur's willingness to borrow from a financial intermediary. Notice that because the problem of the entrepreneur is linear in net worth, the optimal solution implies that each entrepreneur invests all or none of her net worth.

Under optimal contracts and assuming free entry for financial intermediaries such that equation (20) is always binding, optimal thresholds $\bar{\omega}_{et}^{f}$ are given as the minimal solution to:

$$g^{f}(\bar{\omega}_{et}^{f}, \sigma_{t}^{f}) = \left(\frac{\xi - 1}{\xi}\right) \frac{R_{t}}{\varepsilon_{et}^{f} q_{t}} \quad for f \in \{b, c\}.$$

$$(23)$$

These equations implicitly define thresholds $\bar{\omega}_{et}^{f}$ as functions of aggregate variables q_t , R_t , v_t and idiosyncratic expected idiosyncratic productivity ε_{et}^{f} such that:

1

$$\bar{\omega}_{et}^{f} = \begin{cases} \bar{\omega}^{c}(\varepsilon_{1,et}, q_{t}, R_{t}) &, \text{ if bond financing} \\ \bar{\omega}^{b}(\varepsilon_{1,et}\varepsilon_{2,et}, q_{t}, R_{t}, v_{t}) &, \text{ if loan financing,} \end{cases}$$
(24)

where it can be seen from equation (23) that both thresholds $\bar{\omega}_{et}^{f}$ for $f \in \{b, c\}$ are increasing in R_t and decreasing in q_t , v_t and ε_{et}^{f} .

Funding Choices.—Following De Fiore and Uhlig (2011) it is possible to show the existence and uniqueness of thresholds in the realizations of idiosyncratic productivity shocks to characterize entrepreneurs' funding decisions.

First, consider an entrepreneur *e* having contracted with a bank in period *t*. After the second idiosyncratic shock $\varepsilon_{2,et}$ is observed this entrepreneur decides to proceed with a loan only if her expected profit from producing is higher than the opportunity cost of producing, what corresponds to her net worth. The total expected return for a bank-funded entrepreneur is given by

 $V^{d}(\varepsilon_{1,et}, \varepsilon_{2,et}, q_t, R_t, v_t) N_{et}^{b}$ where:

$$V^{d}(\varepsilon_{1},\varepsilon_{2},q,R,\nu) = \varepsilon_{1}\varepsilon_{2}q\nu(\bar{\omega}^{b}(\varepsilon_{1}\varepsilon_{2},q,R,\nu))\xi.$$
⁽²⁵⁾

Conditional on the realizations of $\varepsilon_{1,et}$ and aggregate variables q_t , R_t and v_t , entrepreneur e proceeds with bank finance only if the realization of $\varepsilon_{2,et}$ is higher than a threshold $\bar{\varepsilon}_d(\varepsilon_{1,et}, q_t, R_t, v_t)$ implicitly defined by:

$$1 = V^d(\varepsilon_{1,et}, \bar{\varepsilon}_{d,et}, q_t, R_t, \nu_t).$$
(26)

This equation implies that the threshold $\bar{\varepsilon}_d$ is increasing in $\varepsilon_{1,et}$, q_t and v_t and decreasing in R_t . The funding decision of an entrepreneur having observed $\varepsilon_{1,et}$ is deduced similarly by comparing her expected payoffs conditional on her funding choice. The expected payoff for an entrepreneur proceeding with bank finance conditional on the realization of $\varepsilon_{1,et}$ is $V^b(\varepsilon_{1,et}, q_t, R_t, v_t)N^b_{et}$, where:

$$V^{b}(\varepsilon_{1}, q, R, \nu) = \int_{\bar{\varepsilon}_{d}} V^{d}(\varepsilon_{1}, \varepsilon_{2}, q, R, \nu) \Phi(d\varepsilon_{2}) + \Phi(\bar{\varepsilon}_{d}(\varepsilon_{1}, q, R, \nu)).$$
(27)

Here the two terms on the right hand side correspond respectively to the expected returns for producing and abstaining bank-financed entrepreneurs. Similarly, the expected payoff for an entrepreneur proceeding with bond finance after having observed $\varepsilon_{1,et}$ is $V^c(\varepsilon_{1,et}, q_t, R_t)N_{et}^c$, where:

$$V^{c}(\varepsilon_{1},q,r) = \varepsilon_{1}qv(\bar{\omega}^{c}(\varepsilon_{1},q,E))\xi.$$
⁽²⁸⁾

Finally, the expected total payoff for an entrepreneur abstaining from production is N_{et} . Based on the realization of $\varepsilon_{1,et}$ each entrepreneur selects the funding option delivering the maximum expected payoff $V(\varepsilon_{1,et}, q_t, R_t)N_{et}$ such that:

$$V(\varepsilon_1, q, R, \nu) = max\{1, (1 - \tau_b)V^b(\varepsilon_1, q, R, \nu), V^c(\varepsilon_1, q, R)\}.$$
(29)

Under the conditions that $\frac{\partial V^b(.)}{\partial \varepsilon_1} \ge 0$ and $\frac{\partial V^c(.)}{\partial \varepsilon_1} > \frac{\partial V^b(.)}{\partial \varepsilon_1}$, it can be shown that there exists

a unique threshold $\bar{\varepsilon}^b$ for the first idiosyncratic shock ε_1 implicitly defined by the condition $V^b(\bar{\varepsilon}_{b,t}, q_t, R_t, v_t) = 1$ and under which entrepreneurs do not raise external finance. Because this cutoff point depends only on aggregate variables such that $\bar{\varepsilon}_{b,t} = \bar{\varepsilon}_b(q_t, R_t, v_t)$, it is identical across all entrepreneurs. Similarly, there exists a unique threshold $\bar{\varepsilon}_c$ for ε_1 , implicitly defined by the condition $V^b(\bar{\varepsilon}_{c,t}, q_t, R_t, v_t) = V^c(\bar{\varepsilon}_{c,t}, q_t, R_t)$ such that $\bar{\varepsilon}_{c,t} = \bar{\varepsilon}_c(q_t, R_t, v_t)$ and above which entrepreneurs prefer to fund from markets. Conditional on q_t , R_t , and v_t entrepreneurs split into four distinct sets mapping the realization of their first idiosyncratic productivity shock $\varepsilon_{1,et}$ to their optimal funding decision:

$$s_t^a = \Phi\left(\bar{\varepsilon}^b(q_t, R_t, \nu_t)\right),\tag{30}$$

$$s_t^b = \Phi\left(\bar{\varepsilon}^c(q_t, R_t, \nu_t)\right) - \Phi\left(\bar{\varepsilon}^b(q_t, R_t, \nu_t)\right),\tag{31}$$

$$s_t^c = 1 - \Phi\left(\bar{\varepsilon}^c(q_t, R_t, \nu_t)\right),\tag{32}$$

$$s_t^{bp} = \int_{\varepsilon^b(q_t, R_t, \nu_t)}^{\varepsilon^c(q_t, R_t, \nu_t)} \int_{\varepsilon^d(\varepsilon_1, q_t, R_t, \nu_t)} \Phi(d\varepsilon_2) \Phi(d\varepsilon_1),$$
(33)

where s_t^a , s_t^b , s_t^c and s_t^{bp} denote respectively the shares of entrepreneurs abstaining from production, contracting with banks, proceeding with bonds and proceeding with bank loans.¹²

Aggregation.—Rearranging the first order conditions (10) and (11) and integrating across entrepreneurs yields aggregate capital and labor demands.¹³

$$K_t = \alpha \frac{X_t}{r_t^K},\tag{34}$$

$$H_t = (1 - \alpha) \frac{X_t}{\tilde{w}_t}.$$
(35)

¹²The presentation of the spreads and default rates for the different types of entrepreneur is relegated to section 1 of the appendix.

¹³In what follows, I write aggregate counterparts of individual variables without subscript *e*. For a generic variable Z_{et} , its aggregate counterpart Z_t is defined as $Z_t = \int_0^1 Z_{et} de$.

Next, I integrate over the entrepreneur production functions defined in equations (5) to obtain entrepreneur aggregate production:

$$Y_t^E = \int_0^1 Y_{et}^E de,$$

$$= \frac{\psi_t^Y \xi N_t}{s_t}.$$
(36)

Here, N_t corresponds to aggregate entrepreneur net worth and variable ψ_t^Y aggregates the productivity of the different entrepreneurs into a single productivity factor.

Aggregate funds available to entrepreneurs X_t are obtained using the shares of bank-funded and market-funded entrepreneurs,

$$X_t = \left[(1 - \tau_b) s_t^{bp} + s_t^c \right] \xi N_t.$$
(37)

The level of aggregate external debt \overline{D}_t corresponds to the volumes of bond B_t and loan L_t raised by entrepreneurs:

$$\bar{D}_t = B_t + L_t, \tag{38}$$

with,

$$B_t = (\xi - 1) \, s_t^c N_t, \tag{39}$$

$$L_t = (\xi - 1) s_t^{bp} (1 - \tau_b) N_t.$$
(40)

and where equilibrium on the debt market implies that $\bar{D}_t = D_t$. Each period, a share $1 - \gamma$ of entrepreneurs' past period profits is transferred to households as dividends O_t . The rest of the profits is accumulated as net worth such that:

$$N_t = \gamma \psi_{t-1}^V N_{t-1},\tag{41}$$

where the variable ψ_t^V aggregates profits over the different types of entrepreneurs. Accordingly, dividends redistributed to households evolve as:

$$O_t = (1 - \gamma)\psi_{t-1}^V N_{t-1}.$$
(42)

Denoting y_t^M the resources consumed in bank-specific information acquisition costs and in monitoring costs,

$$y_t^M = \left[\tau_b s_t^b + \psi_t^M \xi q_t\right] N_t,\tag{43}$$

where ψ_t^M aggregates monitoring costs over all defaulting entrepreneurs.¹⁴

2.2.2 Retailers

Retailers are monopolistically competitive firms indexed by $j \in [0, 1]$. They produce differentiated goods Y_{jt} using a linear homogeneous technology,

$$Y_{jt} = Y_{jt}^E, (44)$$

where Y_{jt}^E is the quantity of goods used by retailers *j* as an input and purchased to entrepreneurs in competitive markets at price p_t^E . Assuming Calvo staggered price contracts, $1 - \xi_p$ denotes the probability for a retailer to be able to readjust her price each period. Retailers unable to reoptimize their prices follow an indexation rule defined as: $p_{jt} = (\pi)^{\iota_p} (\pi_{t-1})^{1-\iota_p} p_{jt-1}$, where ι_p is a parameter and π corresponds to steady-state inflation.

2.2.3 Final Good Producers

A representative final good producer combines intermediate goods Y_{jt} into homogeneous final goods Y_t using the following technology:

$$Y_t = \int_0^1 \left[Y_{jt}^{\frac{1}{\lambda_p}} \right]^{\lambda_p}, \quad \lambda_p > 1,$$
(45)

¹⁴Definitions for the aggregators ψ_t^Y, ψ_t^V and ψ_t^M are given in section 1 of the appendix.

where λ_p is a markup over the intermediate good price p_t^E . The first order conditions for profit maximization by final good producers imply the following demand schedule:

$$p_{jt} = p_t \left(\frac{Y_{jt}}{Y_t}\right)^{\frac{\lambda_p}{\lambda_p - 1}}, \quad j \in [0, 1],$$

$$(46)$$

where p_{jt} is the price of good Y_{jt} and p_t is the price of the final good which satisfies the following relation:

$$p_{t} = \left[\int_{0}^{1} p_{jt}^{\frac{1}{1-\lambda_{p}}} dj \right]^{1-\lambda_{p}}.$$
(47)

2.3. Aggregates and Monetary Authority

The aggregate resource constraint of the economy writes:

$$Y_t = C_t + I_t + y_t^M. (48)$$

A monetary authority sets the nominal interest rate according to a Taylor rule expressed in linearized form as:

$$R_t - R = \rho_p \left(R_{t-1} - R \right) + (1 - \rho_p) \left[\alpha_\pi \left(E \pi_{t+1} - \pi \right) + \frac{\alpha_{\Delta Y}}{4} g_{Y,t} \right] + \frac{1}{400} \epsilon_t^p, \tag{49}$$

where ϵ_t^p is a monetary policy shock expressed in annual percentage points, and ρ_p is a smoothing parameter in the policy rule. Here, $R_t - R$ is the deviation of the nominal interest rate, R_t , from its steady-state value R. Parameters α_{π} and $\alpha_{\Delta Y}$ are coefficients on the quarterly rate of expected inflation $E\pi_{t+1} - \pi$ and on output quarterly growth rate $g_{Y,t}$.

2.4. Shock Processes

The model includes four different shock processes, A_t , ζ_t^C , ζ_t^I , and v_t . The first three shocks correspond respectively to technology, preference and investment shocks. The shock v_t is a bank-efficiency shock that shifts the ability of banks to reduce firm asymmetric information problem, its properties are discussed later. All shocks follow standard autoregressive processes of degree one.

A generic exogenous variable x_t writes as:

$$log\left(\frac{x_t}{x}\right) = \rho_x log\left(\frac{x_{t-1}}{x}\right) + \epsilon_t^x \text{ and } \epsilon_t^x \sim N\left(0, \sigma_x\right).$$

In addition, exogenous shifts in monetary policy are captured by innovations ϵ_t^p which are assumed i.i.d and normally distributed. The model is linearized and simulated locally around its steady state. The next section discusses the calibration of the model.

3. Calibration and Model Properties

3.1. Model Calibration

I use a calibrated version of the model to investigate the evolution of firms' debt structure in response to the different types of aggregate shocks. There are 21 parameters in total.¹⁵ Most of the parameters are standard in the DSGE literature and calibrated with conservative values.

Param.	Description	Value
α	Capital share	0.37
β	Discount factor	0.995
δ	Depreciation rate	0.025
λ_p	Price markup	1.2
$\dot{\psi_H}$	Labor disutility	0.55
σ_{H}	Frisch elasticity	1
$ au_y$	Retailers subsidy	0.167
$\alpha_{\Delta y}$	Taylor rule output coefficient	0.5
α_{π}	Taylor rule inflation coefficient	2
$ ho_p$	Taylor rule smoothing	0.7
ξ_p	Calvo price stickiness	0.5
ι_p	Price indexation on inflation target	0.5
<i>Ś</i> "	Invest. adjustment cost curvature	3

Table 1: Calibrated Parameters

Parameter α is set at 0.37 to target a labor share of 63 percent as observed for US non-financial corporate firms in Karabarbounis and Neiman (2014). The depreciation rate δ is 0.025 to obtain an

¹⁵Not including parameters characterizing the different shocks processes. For exposition purposes all autocorrelation coefficients are set to 0.9 and shock variances are set to imply output responses of similar magnitudes for the different shocks. The shocks defined in 2.4 are centered around one.

Variable	Description	Model	Data
L/B	Loan-to-bond ratio	0.44	0.44
\bar{D}/N	Debt-to-equity ratio	0.47	0.47
Δ^c	Risk premium for bonds	2.96	2.97
Δ^b	Risk premium for loans	2.01	2.01
F^{c}	Delinquency rate for bonds	3.43	3.43
F^b	Delinquency rate for loans	2.86	2.86

Table 2: Financial Facts - Model vs Data

Note: Default rates and risk premia are expressed in annualized percentage points.

annual rate of capital depreciation of 10 percent. The household discount factor β at 0.995 implies a policy rate of 4 percent, equal to the average annualized federal funds rate observed between 1985Q1 and 2018Q1. The price markup λ_p is 1.2 to match the average markup observed in the US between 1980 and 2013 by De Loecker, Eeckhout, and Unger (2020). The subsidy rate on intermediate goods τ_Y is set at 0.17 to equate the price of the intermediate goods with the price of the final goods.¹⁶ I set the inverse Frisch elasticity σ_H to 1 and the labor disutility parameter ψ_H to 0.68 in order to normalize steady-state hours to unity. Parameters for the Taylor rule, price stickiness and investment cost curvatures are calibrated so as to lie within the posterior densities obtained from medium-scale New-Keynesian models estimated for the US on samples covering the past thirty years.¹⁷ Calibration for these parameters is summarized in table 1.

Parameters for the financial sector and the idiosyncratic productivity distributions are less usual and are calibrated to jointly match the characteristics of intermediated and direct debt for US non-financial corporate firms over the period 1987Q1 to 2016Q3. Table 2 displays the targeted financial variables and their model counterparts. Calibration for the financial parameters is summarized in table 3. The loan-to-bond and debt-to-equity ratios are computed using data from the Flow of Funds Accounts for non-financial US corporate firms. Their average values amount respectively to 0.44 and 0.47. The risk premium for loans corresponds to the spread between the interest rate for

¹⁶Because profit maximization for the final good producer under flexible prices yields $p_t = \lambda_p (1 - \tau_Y) p_t^E$, in steady state this implies $\tau_Y = 1 - \frac{1}{\lambda_p}$.

¹⁷See for instance Smets and Wouters (2007), Justiniano, Primiceri, and Tambalotti (2011), Christiano, Motto, and Rostagno (2014) and Bécard and Gauthier (2020).

commercial and industrial loans and the federal funds rate.¹⁸ I obtain a mean annualized spread of 2 percent. For the bond risk premium, I use Moody's Aaa corporate bond yield minus the federal funds rate which is equal to 2.97 percent. The corporate rate of default for loans corresponds to the delinquency rate on commercial and industrial loans at 2.86 percent. Finally, the default rate for corporate bonds is inferred from Emery and Cantor (2005) who show that the default rate for bonds is 20 percent higher on average than the default rate for loans.¹⁹ The model is able to accurately replicate the above financial facts.

Param.	Description	Value
$ au_b$	Bank intermediation costs	0.0347
ξ	Pledgeable fraction of networth	2.19
$1 - \gamma$	Dividend rate	0.262
μ_b	Bank monitoring cost	0.83
μ_c	Market monitoring cost	0.249
σ_1	Idiosyncratic shock dispersion	0.385
σ_2	Idiosyncratic shock dispersion	0.197
σ_3	Idiosyncratic shock dispersion	0.316

Table 3: Calibrated Parameters - Financial

3.2. Firm Funding Decisions

Before presenting the dynamic implications of the model, I describe the link between entrepreneurs' expected productivity and their funding decisions in the static model. The upper panel in figure 1 displays entrepreneurs' expected profits for the different funding options, conditional on the realizations of the first idiosyncratic shock ε_1 . The lower panel displays the density of this shock. The grey, orange, and blue areas correspond respectively to the shares of entrepreneurs abstaining from production, contracting with banks and funding from markets.

Entrepreneurs with intermediate expected productivity contract with banks while those with high expected productivity prefer to fund from markets. The reason is that entrepreneurs with low expected productivity have a higher probability of default and prefer to hedge their net worth against

¹⁸The series are taken from the Survey on Term Business Lending.

¹⁹This study covers the period 1995 to 2003. Their results are confirmed by more recent evidence presented in Lonski (2018).

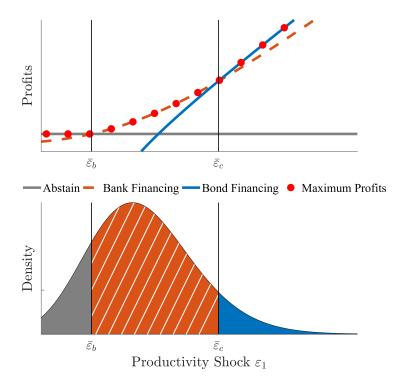


Figure 1: Funding Decisions

risk by *not* producing or by entering into renegotiable contracts with banks. On the other hand, entrepreneurs with high productivity and low risk of default are better off funding from markets and avoiding intermediation costs.²⁰ Notice also that because entrepreneurs' profits are a monotonic function of their net worth, the model rules out simultaneous funding from markets and banks.²¹

Finally, while the model assumes a constant leverage for entrepreneurs, it is important to notice that equity for non-financial corporate US firms has been stable compared to their debt composition. Figure A1 in the appendix plots the ratios of assets-to-debt, assets-to-loans and assets-to-bonds between 1985 and 2018. While the ratio of assets-to-debt stays around its mean, both assets-to-

Note: The first panel corresponds to the expected profits of entrepreneurs depending on their funding choice and conditional on the realization of the first idiosyncratic shock ε_1 . The second panel displays the density of this shock.

²⁰This mapping between entrepreneurs' expected productivity and their funding decision is coherent with evidence presented in Denis and Mihov (2003). Using firm-level data for US corporations, they show that the credit quality of the issuer is the primary determinant of firm debt structure with most productive firms funding from markets and firms with lower credit quality funding from banks. Adrian, Colla, and Song Shin (2013) also stress the importance of credit quality as a determinant of firms' debt structure.

²¹This implicit assumption of debt specialization is backed by the evidence presented in Colla, Ippolito, and Li (2013) who show that 85 percent of US-listed firms have recourse only to one type of debt.

loans and assets-to-bonds appear very volatile, wavering from simple to double over the period considered.

3.3. Model Dynamics and the Debt Structure

This section presents the dynamic implications of the different aggregate shocks. The key result is that only the responses in loans and bonds allow to qualitatively distinguish financial shocks from other macroeconomic shocks.

3.3.1 The Bank-Efficiency Shock

I start with the presentation of a positive bank-efficiency shock v_t . Figure 2 displays impulse responses for the main variables. This shock increases the share of idiosyncratic productivity banks can observe among their borrowers. Because financial contracts imply that banks only take on downside risk, the lower dispersion in the productivity of bank-funded entrepreneurs implies a higher share of output accruing to banks. Due to competition among financial intermediaries, bankfunded entrepreneurs can pledge a lower fraction of their profits and increase their expected payoff. In contrast, the expected payoff for abstaining and market-funded entrepreneurs is unchanged. Entrepreneurs that were indifferent between not producing and contracting with a bank or between contracting with a bank and borrowing from markets now favor bank finance.

With the share of market-funded entrepreneurs decreasing and the share of bank-funded entrepreneurs rising - the extreme case being if none of the entrepreneurs switching to bank finance decide to proceed with their loan – the bank-efficiency shock implies opposite movements in the shares of bank and bond-funded entrepreneurs. Overall, the aggregate level of debt increases as the proportion of abstaining entrepreneurs switching to bank finance and proceeding with their loan outweighs the share of entrepreneurs switching from market to bank finance and *not* proceeding with their loan. As funds available to entrepreneurs move up, demand for labor and capital increases together with the wage and the capital rental rate. Entrepreneurs' marginal cost of production goes up. Output, investment, consumption and hours increase, along with capital price, goods price, and

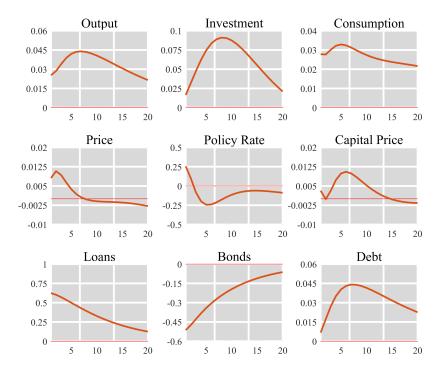


Figure 2: Responses to a Bank-Efficiency Shock v_t

Note: All series are expressed in deviation from steady state in percentage points. Policy rate response is expressed in basis points.

the policy rate. The increase in the policy rate and in the marginal cost of production pushes up funding and production costs and dampens the rise in aggregate debt. On the other hand, because entrepreneurs' aggregate profits react positively to the fall in aggregate uncertainty triggered by the shock, aggregate net worth increases and feeds up next period borrowing through the leverage constraint. Because only the least productive of market-funded entrepreneurs switch to bank funding, the risk borne by bond holders also declines. This leads to a fall in the risk premia for the two types of debt. Overall the bank-efficiency shock pushes firms to substitute loans for bonds and triggers positive responses in output, investment and consumption.²²

²²Here I focus on a bank-efficiency shock v_t but other financial shocks embedded in the model have similar qualitative implications. This is the case for instance for an exogenous shock to the financial intermediation costs τ_b . As for the bank-efficiency shock, this shock implies a simultaneous increase in output and loans and a fall in bonds.

3.3.2 Macroeconomic Shocks

Without detailing impulse responses for other shocks, it is important to notice that non-financial shocks transmit differently to entrepreneurs' funding decisions relative to financial shocks. Figure

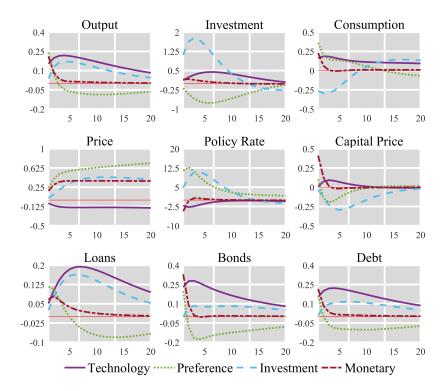


Figure 3: Responses to Non-Financial Shocks

Note: All series are expressed in deviation from steady state in percentage points. Policy rate response is expressed in basis points.

3 presents impulse responses following technology, preference, investment, and monetary shocks. First, notice that the introduction of debt arbitrage in the NK framework does not modify the qualitative implications of the model. The signs of the impulse responses for non-financial shocks correspond to those described in Straub and Peersman (2006). Importantly, all these shocks generate comovements in output, loans and bonds. Two effects are at play here. Regardless of the type of shock hitting the economy, entrepreneurs must produce more for output to increase. Non-financial shocks do not impact directly credit conditions, instead they modify aggregate entrepreneurial markup either by decreasing input costs or by increasing firms' productivity. Following a nonfinancial shock entrepreneurs' profitability increases. This pushes up net worth and increases demand for the two types of debt. Loans and bonds increase altogether. On the other hand, the increase in the profitability of entrepreneurs reduces their production risk and modifies their funding decisions. Some entrepreneurs abstaining from production are better off producing after the shock is realized. Accordingly, the shares of entrepreneurs abstaining from production or not proceeding with their bank loan decrease. On the other hand, some entrepreneurs that were contracting with a bank prior to the shock now prefer to avoid intermediation costs and switch to market finance. Overall the share of entrepreneurs abstaining from production decreases and both the shares of market-funded entrepreneurs and entrepreneurs proceeding with their bank loans increase. Following non-financial shocks, both bond and loan volumes comove with output.

Section 2 of the appendix presents sensitivity tests for the different impulse responses shown here. The signs of the responses for output, loans and bonds to financial and other aggregate shocks are robust to various parameter specifications. Comparing impulse responses for the different types of shock, there exists no robust qualitative differences between demand and financial shocks other than the response of bonds. In the next section, I use the qualitative features of the NK model to inform a sign-restriction VAR and identify financial shocks based on loan and bond dynamics.

4. Empirical Analysis

This section presents the results from a sign-restriction VAR model used to identify financial shocks and evaluate their business cycle implications.

4.1. The Sign-Restriction VAR

I use the qualitative predictions of the modified NK model to inform a sign-restriction Bayesian VAR. The model is estimated with US quarterly data for the period 1985Q1 to 2018Q1. The data set includes the gross domestic product, the GDP implicit price deflator, the ratio of investment-over-GDP and the annualized effective federal funds rate. I take outstanding loan and bond volumes for corporate non-financial firms to track the evolution of aggregate debt composition. Loan series

includes loans from depository institutions and mortgage loans. Bond series includes both bonds and commercial papers. All series are seasonally adjusted and expressed in log-levels except the federal funds rate which is in level.²³

The model is estimated using the Jeffreys' prior with a lag order of two what minimizes the Bayesian information criterion and the Hannan-Quinn information criterion.²⁴ The estimation of the model involves two separate steps. First, I estimate a reduced form Bayesian VAR model. Second, I use the algorithm presented in Arias, Rubio-Ramirez, and Waggoner (2018) to generate candidate impulse responses and retain models satisfying the imposed sign-restrictions until a sufficient number of draws are obtained.²⁵ I consider five types of structural shocks identified

	Supply	Demand	Investment	Monetary	Financial
Output	+	+	+	+	+
Price	-	+	+	+	?
Policy Rate	?	+	+	-	?
Invest. / Output	?	-	+	?	?
Loans	+	+	+	+	+
Bonds	+	+	+	+	-

 Table 4: Sign Restrictions

Note: Sign restrictions imposed. The restrictions are imposed on impact only. The presence of a question mark indicates the absence of restriction.

based on the signs of the impulse responses on impact for the different variables. A sixth shock is left unrestricted to add a degree of freedom to the estimation. The restrictions imposed and the series used are chosen so as to classify shocks into five broad categories - supply, demand, investment, monetary and financial. These capture most of the shocks found in the business cycle literature as well as the shocks present in the modified NK model.²⁶ The sign-restrictions imposed

²³I also estimate the model using the shadow rate from Wu and Xia (2016). The results are presented in section 4 of the appendix. They are robust to this alternative specification.

²⁴The model is also estimated with a lag order of four. Impulse responses for the different shocks are robust to this modification. The share of output and price variance explained by demand shocks slightly increases relative to supply shocks.

²⁵The following results are based on a subsample of 2000 draws. Section 3 of the appendix contains a more detailed presentation of the data set and the econometric methods used to estimate the model.

²⁶The sign-restrictions imposed for non-financial variables also lies in the intervals of robust impulse responses derived by Canova and Paustian (2011) based on a variety of DSGE models. This is true except for the response of the

are summarized in table 4. Supply shocks are identified as implying opposite movements in output and prices. Demand and investment shocks generate comovements in output and prices and have respectively negative and positive impacts on the investment-to-output ratio. Monetary shocks generate opposite responses in the policy rate and in output and prices. Finally, all these shocks generate comovements in output, loans, and bonds.

Financial shocks are identified as the only type of shock that can simultaneously generate comovements in output and loans and opposite movements in output and bonds. Importantly, financial shocks need *not* to be identified as demand shocks. This restriction is commonly imposed to identify financial shocks in sign-restriction VAR but at odds with recent evidence.²⁷ As I do not impose restrictions on the responses of prices, interest rate and the investment-to-output ratio conditional to a financial shock, these can be used as a simple test for the overidentifying predictions of the VAR model.

4.2. Empirical Results

This section presents the results from the structural VAR model, I focus on the characteristics of financial shocks and how they relate to financial shocks identified with different econometric methods.

4.2.1 What Financial Shocks Do

Figure 4 displays the median impulse responses following a one standard deviation financial shock. The response of output is short-lived with a duration close to 10 quarters before returning to zero. While left unrestricted, the impact on the investment-to-output ratio is positive and twice as strong as for output with a similarly short duration. In comparison, the impact on loans takes more than 15 quarters to fade out and is nearly 5 times stronger than for output. Its maximum impact is

policy rate to a supply shock I leave unrestricted. This is to take into account the fact that the response of the policy rate to a supply shock hinges on the degree of price rigidity, as shown by Peersman and Straub (2009).

²⁷See for instance Gilchrist, Schoenle, Sim, and Zakrajšek (2017) who show that financial disturbances can induce constrained firms to raise prices following adverse financial shocks and Angeletos, Collard, and Dellas (2019) who find that shocks driving output fluctuations are orthogonal to the ones responsible for price dynamics.

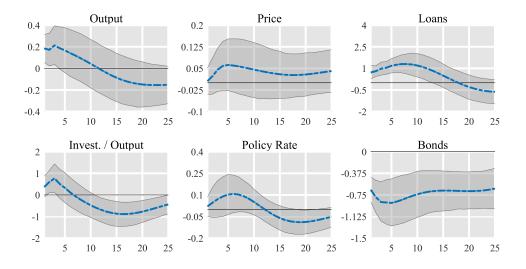


Figure 4: Responses to a Financial Shock

Note: Median impulse responses to a one standard deviation financial shock. The grey lines correspond to the 16th and 84th quantiles. All series are expressed in percentage points. The policy rate is annualized.

reached after 10 quarters with a value close to 2 percent. The fall in bonds is twice weaker than the increase in loans and peaks more rapidly after only 5 quarters. The federal funds rate exhibits a large positive hump-shaped response dying out after 10 quarters. I find the response of prices to be weak and positive. The responses of the policy rate and prices are consistent with a large body of empirical and theoretical evidence.²⁸

While financial shocks are identified restricting only the responses of output, loans and bonds, the responses obtained for the investment-to-output ratio, the policy rate and the price level all match dynamics implied by financial shocks from various DSGE model.²⁹ Impulse responses for the other shocks are displayed in section 6 of the appendix.

²⁸Schularick and Taylor (2012) present international evidence of aggressive monetary policy in response to financial shocks during the postwar era. Using a set of estimated DSGE models, Cesa-Bianchi and Sokol (2017) find that the policy rate systematically decreases in response to adverse financial shocks. Gertler and Karadi (2011) also show that expansionary financial shocks can relax firms' borrowing constraints, pushing up demand and leading to inflationary pressures.

²⁹See for instance Gertler and Kiyotaki (2010), Christiano, Motto, and Rostagno (2014) and Boissay, Collard, and Smets (2016).

4.2.2 Financial Shocks and the Business Cycle

Figure 5 displays the median historical shock decomposition for the output growth rate. Even though financial shocks play an important role over the estimation period, all three recessions contained in the sample are associated with different types of perturbations.

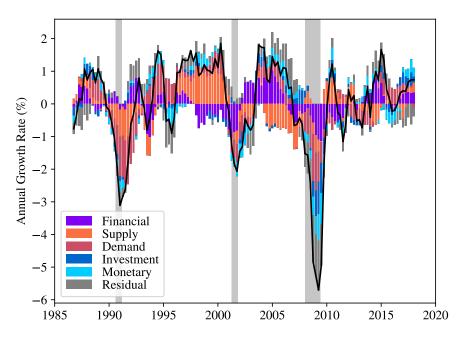


Figure 5: Historical Shock Decomposition for Output

Note: Contribution of the different structural shocks to output fluctuations. Grey areas correspond to NBER recession dates.

According to the model estimates, the outset of the 90's recession is dominated by a combination of demand and supply shocks increasing from 1990 onward.³⁰ In contrast, financial shocks weight down on output growth in 1993 and 1998. The two periods coincide with the Japanese bank crisis and the Russian crisis, described respectively by Peek and Rosengren (2000) and Chava and Purnanandam (2011) as examples of credit supply shocks affecting non-financial firms via their negative impact on US bank equity. The recession of the early 2000s is also associated with financial

³⁰Walsh (1993) and Blanchard (1993) stress the strong role of adverse demand shocks in the early 90's recession. The role of oil shocks and of the Iraq war in the 90's recession is more controverted. Kilian and Vigfusson (2017) find a significant impact of oil shocks on US activity when using net oil price - the difference of oil price with its peak value over the 12 previous months, instead of a standard linear model. Hamilton (2009) studies the impact of oil shocks on the auto industry between 1990Q1 and 2007Q4. He finds a significant impact of oil shocks during the 90's recession.

as well as monetary and demand factors.³¹ Perhaps more surprising, the model attributes only a limited role to financial shocks during the Great Recession. The initial fall in output is explained mainly by supply-side disturbances with a strong role for demand and monetary factors.³² The moderate role attributed to financial shocks during the Great Recession also echoes Mian and Sufi (2015) who cast doubt on the importance of financial stress experienced by non-financial firms during the Great Recession.³³ Here, financial shocks start weighing down on activity by the end of 2008. This is consistent with Ivashina and Scharfstein (2010) who show that the beginning of the Great Recession was accompanied by an increase in commercial and industrial loans as corporate borrowers drew on their existing credit lines in reaction to expected financial stress.

	Financial	Supply	Demand	Investment	Monetary
Output	39.91	21.29	19.21	4.8	12.13
Price	5.35	52.0	31.4	5.54	5.44
Policy Rate	18.11	0.8	49.02	8.16	23.4
Invest. / Output	33.44	9.44	12.19	9.96	32.61
Loans	42.92	2.54	26.88	7.03	17.43
Bonds	62.17	19.4	8.75	5.31	4.27

Table 5: Variance Contributions

Note: Contributions of the structural shocks to the business-cycle volatility of the model observables. The table does not display the residual shock to save space. Business cycle frequency includes cycles between 6 and 32 quarters obtained using the model spectrum.

Table 5 shows the contributions of the different shocks to the business-cycle volatility of the model observables. Financial shocks are the main driver of the business cycle. They account for nearly 40 percent of fluctuations in output, and respectively 43 and 62 percent of loan and bond fluctuations but bear little implication for prices. Both supply and demand shocks have a sizable role for fluctuations in output and in the price level.

³¹Caldara, Fuentes-Albero, Gilchrist, and Zakrajsek (2016) find that the fall of industrial production of the early 2000s is entirely attributed to financial exogenous perturbations.

³²This view of the crisis is consistent with the results from Stock and Watson (2012). They estimate a dynamic factor model and find that the Great Recession is best explained by heterogeneous sources where oil shocks account for the initial slowdown, financial and demand shocks explain the bulk of the recession and a subsequent drag is added by an effectively tight conventional monetary policy arising from the zero lower bound.

³³Using survey data from the National Federation of Independent Businesses, they show that the fraction of small businesses citing financing or interest rates as a main concern never rose above 5 percent between 2007 and 2009. Examples of possible concerns include "poor sales," "regulation and taxes," or "financing and interest rates".

To verify that the characteristics of the estimated financial shocks do not hinge on the sign-restrictions imposed for the responses of price, interest rate and investment, I re-estimate the VAR model but restricting only the responses of output, loans and bonds. I also add a measure of credit spread to alleviate risks of non-invertibility and verify the implications of financial shocks for credit costs.³⁴ Section 5 of the appendix presents the results for this alternative specification. The characteristics of the financial shocks are identical to those obtained in the fully specified model.

5. Putting the Model to the Test

In this final section, I use an estimated version of the modified NK model to investigate how financial shocks identified using aggregate debt composition relate to measures of financial stress such as the corporate bond spread.

5.1. Impulse Response Matching

The estimation procedure consists in minimizing the distance between the median impulse responses implied by the structural VAR and by the modified NK model. I estimate a total of 22 parameters which are listed in table A2 of the appendix. Writing θ the vector that contains the estimated parameters, its estimator θ^* is obtained as the solution of:

$$\theta^* = \underset{\theta}{\operatorname{argmin}} \left[\hat{\Psi} - \bar{\Psi}(\theta) \right]' V^{-1} \left[\hat{\Psi} - \bar{\Psi}(\theta) \right].$$
(50)

Here, $\hat{\Psi}$ is a vector that contains the median impulse responses obtained from the VAR model, $\bar{\Psi}(\theta)$ contains the impulse responses from the NK model and V is a diagonal matrix with the variances of the empirical impulse responses stacked along its main diagonal. I consider a horizon of 25 periods for the five different structural shocks and the six different variables. This implies that $\bar{\Psi}(\theta)$ is a 750 column vector. Figure 6 displays impulse responses to a financial shock for the estimated NK model and the VAR model. The modified NK model is able to reproduce both qualitative and

³⁴I consider two types of shocks, non-financial shocks that imply comovements in output, loans and bonds and financial shocks specified as above.

quantitative features of the VAR model for all types of shock with parameter values in line with those obtained from medium-scale DSGE models estimated with US data.³⁵ Impulse responses for

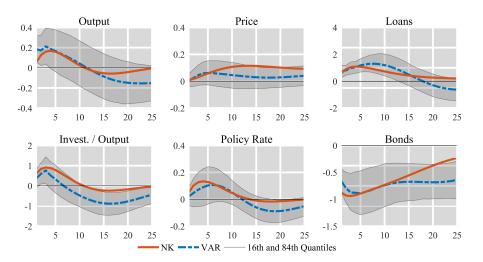


Figure 6: Impacts of a Financial Shock in the VAR and NK Models

Note: Median impulse responses to a one standard deviation financial shock. The grey lines correspond to the 16th and 84th quantiles for the VAR model. All series are expressed in percentage points. The policy rate is annualized.

the other shocks are provided in section 6 of the appendix.

5.2. Financial Shocks and the Bond Spread

Going back to the question of whether corporate debt choices can help to identify financial shocks, I investigate the relevance of the identification strategy based on two criteria. First, does the identification method yield financial shocks that resemble measures of financial stress as experienced by non-financial firms? Second, do firm funding decisions help to predict disruptions in the financial system? To address these questions, I proceed as follows. I assume that the estimated NK model is the true data generating process and use it to recover the structural shocks implied by the data set.³⁶

Figure 7 plots the financial shock process v_t obtained from the modified NK model and Moody's

³⁵See for instance Gilchrist, Otiz, and Zakrajsek (2009), Christiano, Trabandt, and Walentin (2010), Del Negro, Giannoni, and Schorfheide (2015) and Bécard and Gauthier (2020).

³⁶I use the same data as for the estimation of the VAR model. Series for output, loans, bonds, and price level are stationarized using a first-difference filter. Because there are only five types of shocks in the NK model, I assume distinct measurement errors for each of the different series as in Bianchi, Kung, and Morales (2019). Importantly, the properties of the financial shocks presented hereafter are robust to the exclusion of series other than loans and bonds.

seasoned Aaa corporate bond yield minus the federal funds rate.³⁷ The financial shock process resembles the bond spread. The two series are correlated at 0.67 over the whole sample.The proximity between the two series indicates that the modified NK model inherits the quantitative properties of the sign-restriction VAR and also that the identification method can capture financial stress based on aggregate firms' funding choices.³⁸

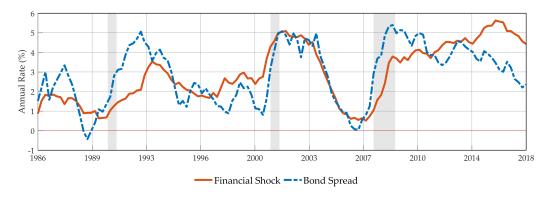


Figure 7: Financial Stress and the Bond Spread

Note: The orange line corresponds to minus the estimate of the updated v_t process. The blue line corresponds to the Moody's seasoned Aaa corporate bond minus federal funds rate. Grey areas correspond to NBER recession dates.

Finally, I investigate whether financial shocks can help to predict changes in the bond spread. Table 6 displays the result from Granger-causality tests at different lag orders. The hypothesis that financial shocks do not Granger cause the bond spread is rejected for all specifications. This exercise highlights the importance of firms' funding decisions for the evolution of borrowing costs. This also reflects the finding of Adrian, Colla, and Song Shin (2013) that the rise observed in the bond spread during the Great Recession was mostly the results of firms substituting bonds for loans.

³⁷All computations presented here are done using Dynare.

³⁸Figure A10 in the appendix plots financial shocks from the VAR model together with the updated bank efficiency shocks from the NK model. The two series are correlated at 0.66.

H_0 : Financial Shocks Do Not Cause Bond Spreads				
Lags	1	2	3	4
P-values	0.001900	0.013107	0.000691	0.002322

Table 6: Granger Causality Test

Note: Granger causality is inferred based on likelihood ratio test. The financial shocks correspond to bank efficiency shocks ϵ_t^{γ} obtained using a Kalman filter.

6. Conclusion

I include a mechanism of debt arbitrage into a New Keynesian model to investigate the evolution of firms' debt structure in response to various macroeconomic shocks. The model implies that only financial shocks produce opposite movements in bonds and loans. In contrast, other macroeconomic shocks generate comovements in the two types of debt. I use these results to inform a sign-restrictions VAR estimated with US data. Financial shocks account for a large share of the business cycle. I estimate the modified NK model using impulse response matching methods. The NK model can replicate the quantitative implications of the structural VAR for all types of shock. Finally, I use the estimated model to construct a measure of financial stress for the US and test the identification strategy.

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Appendix to "Financial Stress and the Debt Structure"

David Gauthier

This appendix is divided into five sections. Section 1 derives the full model and lists all equilibrium conditions. Section 2 provides sensitivity tests for the predictions of the modified NK model. Section 3 presents the VAR methodology and the data used in the VAR estimation. Section 5 presents the results from a VAR model where only financial and non-financial shocks are identified. Section 6 gives additional results from the IRF matching estimation.

1. Model Derivation

This section provides the derivation of the model and lists all the equations.

1.1. Households

A representative household decides its optimal level of consumption C_t , capital K_t and deposit D_t in order to maximize utility defined as:

$$E_0 \sum_{t=0}^{\infty} \beta^t \zeta_t^C \left\{ \log(C_t) - \psi_H \frac{H_t^{1+\sigma_H}}{1+\sigma_H} \right\}.$$

The budget constraint writes as:

$$p_t C_t + p_t D_t + q_t^K K_t \le w_t H_t + p_{t-1} R_t D_{t-1} + \left[q_t^K (1-\delta) + p_t r_t^K \right] K_{t-1} + O_t.$$
(1)

The Lagrangian associated to the households' problem can be written as:

$$\begin{aligned} \mathcal{L} &= E_0 \sum_{t=0}^{\infty} (\beta)^t \zeta_t^C \left\{ \log(C_t) - \psi_H \frac{H_t^{1+\sigma_H}}{1+\sigma_H} \right. \\ &+ \tilde{\Lambda}_t \left(w_t H_t + p_{t-1} R_t D_{t-1} + \left[q_t^K (1-\delta) + p_t r_t^K \right] K_{t-1} + O_t - p_t C_t - p_t D_t - q_t^K K_t \right) \right\}. \end{aligned}$$

The first-order condition with respect to consumption C_t is:

$$\zeta_t^C \tilde{\Lambda}_t p_t = \frac{\zeta_t^C}{C_t}.$$
(2)

The first-order condition with respect to labor H_t is:

$$\psi_H H_t^{\sigma_H} = w_t \tilde{\Lambda}_t. \tag{3}$$

The first-order condition with respect to risk-free deposits D_t is:

$$\zeta_t^C \tilde{\Lambda}_t p_t = \beta E_t \zeta_{t+1}^C \tilde{\Lambda}_{t+1} p_{t+1} \frac{R_{t+1}}{\pi_{t+1}}.$$
(4)

Households supply capital K_t to entrepreneurs. The first-order condition with respect to capital K_t is:

$$\zeta_t^C \tilde{\Lambda}_t = \beta E_t \zeta_{t+1}^C \tilde{\Lambda}_{t+1} R_{t+1}^K, \tag{5}$$

with,

$$R_{t+1}^{K} = \frac{q_{t+1}^{K}(1-\delta) + r_{t+1}^{K}p_{t+1}}{q_{t}^{K}}.$$
(6)

1.2. Capital Installer

The capital installer selects its optimal level of investment I_t to maximize the sum of its profits discounted with households' stochastic discount factor:

$$E_0 \sum_{t=0}^{\infty} \beta^t \zeta_t^C \tilde{\Lambda}_t \left\{ q_t^K K_t - p_t I_t \right\},\,$$

and using the following technology:

$$K_t = (1 - \delta)K_{t-1} + \left[1 - S\left(\zeta_t^I \frac{I_t}{I_{t-1}}\right)\right]I_t.$$

The first order condition for profit maximization with respect to I_t writes:

$$\begin{aligned} \zeta_t^C \tilde{\Lambda}_t q_t^K \left[1 - S \left(\zeta_t^I \frac{I_t}{I_{t-1}} \right) - \zeta_t^I \frac{I_t}{I_{t-1}} S' \left(\zeta_t^I \frac{I_t}{I_{t-1}} \right) \right] - \zeta_t^C \tilde{\Lambda}_t p_t \\ + \beta \zeta_{t+1}^C \tilde{\Lambda}_{t+1} q_{t+1}^K \zeta_{t+1}^I \left(\frac{I_{t+1}}{I_t} \right)^2 S' \left(\zeta_{t+1}^I \frac{I_{t+1}}{I_t} \right) = 0. \end{aligned}$$

$$(7)$$

1.3. Firms

I follow Gali (2010) in assuming a three-sector structure for good producers. Firms in the final goods sector produce differentiated goods using entrepreneurs production bought in competitive markets. The former are subject to nominal rigidity introduced via staggered-price contracts à la Calvo.

1.3.1 Entrepreneurs

Entrepreneurs produce intermediate goods using capital and labor obtained from the households. There exists a continuum $e \in [0, 1]$ of entrepreneurs operating in competitive markets. An entrepreneur e enters the period with net worth N_{et} pledged to obtain debt X_{et} . Debt is used to fund working capital and is a fixed proportion of the net worth:

$$X_{et} = \xi N_{et}.\tag{8}$$

Here ξ is a parameter that corresponds to entrepreneurs leverage. Entrepreneur *e* sells production Y_{et}^E at a competitive price p_t^E to retailers, where Y_{et}^E is produced using the following Cobb-Douglas technology:

$$Y_{et}^E = \varepsilon_{et} A_t K_{et}^{\alpha} H_{et}^{1-\alpha}, \tag{9}$$

where K_{et} and H_{et} are capital and labor input used to produce. Variable A_t is a technology shock and ε_{et} is a sequence of idiosyncratic shock realizations. An entrepreneur is constrained on her capital inputs K_{et} and labor inputs H_{et} relative to her debt capacity X_{et} according to the following debt constraint:

$$X_{et} \ge r_t^K K_{et} + \tilde{w}_t H_{et}.$$
(10)

An entrepreneur *e* maximizes her real profits defined as,

$$\frac{p_t^E Y_{et}^E}{p_t} - r_t^K K_{et} - \tilde{w}_t H_{et},\tag{11}$$

by choosing optimal inputs K_{et} and H_{et} for a given level of debt X_{et} and subject to the debt constraint defined in equation (10). The first order conditions for the optimization problem of the entrepreneur can be written as:

$$\alpha X_{et} = r_t^K K_{et},\tag{12}$$

$$(1-\alpha)X_{et} = \tilde{w}_t H_{et}.$$
(13)

Defining s_t the aggregate component of the marginal cost of production expressed in terms of the final goods implies:

$$s_t = \frac{1}{A_t} \left(\frac{p_t}{p_t^E} \right) \left(\frac{\tilde{w}_t}{1 - \alpha} \right)^{1 - \alpha} \left(\frac{r_t^K}{\alpha} \right)^{\alpha}.$$
 (14)

Idiosyncratic Shocks.—Each period, an entrepreneur e is hit by a sequence of three idiosyncratic shocks. I summarize here the characteristics of the successive shocks:

- Shock $\varepsilon_{1,et}$: Publicly-observed, realizes along aggregate shocks for all entrepreneurs. This shock creates heterogeneity in entrepreneurs' productivity.
- Shock $\varepsilon_{2,et}$: Publicly-observed, only observed by bank-financed entrepreneurs. This shock is the rationale for choosing bank finance over the less expensive bond finance.
- Shock $\varepsilon_{3,et}$: Privately-observed, can be monitored at a cost by financial intermediaries. This shock creates a rationale for the existence of risky debt contract.

Financial Variables.—Using the productivity thresholds $\bar{\varepsilon}_t^b$ and $\bar{\varepsilon}_t^c$, it is possible to express entrepreneur average risk premia and default rates conditional on their funding decisions. Denoting respectively ψ_t^{Mb} and ψ_t^{Mc} the default rates for bank-funded and market-funded entrepreneurs gives:

$$\psi_t^{Mb} = \int_{\bar{\varepsilon}_b(q_t, R_t, v_t)}^{\bar{\varepsilon}_c(q_t, R_t, v_t)} \int_{\bar{\varepsilon}_d(\varepsilon_1, q_t, R_t, v_t)} \Phi(\bar{\omega}^b(\varepsilon_1 \varepsilon_2, q_t, R_t, v_t)) \Phi(d\varepsilon_2) \Phi(d\varepsilon_1),$$
(15)

$$\psi_t^{Mc} = \int_{\bar{\varepsilon}_c(q_t, R_t, v_t)} \Phi(\bar{\omega}^c(\varepsilon_1, q_t, R_t, v_t)) \Phi(d\varepsilon_1).$$
(16)

With the expected fixed repayment for the financial intermediary being $\varepsilon_{et}^f \bar{\omega}_{et}^f q_t$ per unit of fund X_{et} , the credit spread for entrepreneur *e* writes:

$$\Lambda_{e,t}^{f} = \frac{\xi}{\xi - 1} \frac{q_t \varepsilon_{e,t}^{f} \bar{\omega}_{e,t}^{f}}{R_t} - 1.$$
(17)

Denoting ψ_t^{rb} and ψ_t^{rc} the aggregate credit spreads paid respectively by bank-funded and marketfunded entrepreneurs yields:

$$\psi_t^{rb} = \int_{\bar{\varepsilon}_b(q_t, R_t, v_t)}^{\bar{\varepsilon}_c(q_t, R_t, v_t)} \int_{\bar{\varepsilon}_d(\varepsilon_1, q_t, R_t, v_t)} \left\{ \frac{\xi}{\xi - 1} \frac{\varepsilon_1 \varepsilon_2 \bar{\omega}_{e,t}^b q_t}{R_t} - 1 \right\} \Phi(d\varepsilon_2) \Phi(d\varepsilon_1), \tag{18}$$

$$\psi_t^{rc} = \int_{\bar{\varepsilon}_c(q_t, R_t, v_t)} \left\{ \frac{\xi}{\xi - 1} \frac{\varepsilon_1 \bar{\omega}_{e,t}^c q_t}{R_t} - 1 \right\} \Phi(d\varepsilon_1).$$
(19)

Finally, it is possible to express Λ_t^b and Λ_t^c the average spreads for bank-funded and bond-funded entrepreneurs as:

$$\Lambda_t^b = \frac{\psi_t^{rb}(q_t, R_t, \nu_t)}{s_t^{bp}},\tag{20}$$

$$\Lambda_t^c = \frac{\psi_t^{rc}(q_t, R_t, v_t)}{s_t^c}.$$
(21)

Aggregate Production.—The expected output for entrepreneur *e* at the time of contracting with a financial intermediary writes as:

$$Y_{et}^E = \varepsilon_{et} K_{et}^{\alpha} H_{et}^{1-\alpha}.$$
 (22)

Using the first-order conditions from equations (12) and (13), entrepreneur individual production writes as: (E)

$$\begin{split} Y_{et}^{E} &= \varepsilon_{et} \left(\frac{p_{t}^{E}}{p_{t}} \right) K_{et}^{\alpha} H_{et}^{1-\alpha}, \\ &= \varepsilon_{et} \left(\frac{p_{t}^{E}}{p_{t}} \right) A_{t} \left(\alpha \frac{X_{et}}{r_{t}^{K}} \right)^{\alpha} \left((1-\alpha) \frac{X_{et}}{\tilde{w}_{t}} \right)^{1-\alpha}, \\ &= \varepsilon_{et} \left(\frac{p_{t}^{E}}{p_{t}} \right) A_{t} X_{et} \left(\frac{\alpha}{r_{t}^{K}} \right)^{\alpha} \left(\frac{1-\alpha}{\tilde{w}_{t}} \right)^{1-\alpha}, \\ &= \frac{\varepsilon_{et}^{E} X_{et}}{s_{t}}. \end{split}$$

Defining $\psi_t^Y = \int_0^1 \varepsilon_{et}^E de$, the aggregate production is obtained as:

$$Y_t^E = \int_0^1 Y_{et}^E,$$
$$= \frac{\psi_t^Y \xi N_t}{s_t}.$$

Where N_t is the aggregate net worth and ψ_t^Y aggregates the realizations of the different idiosyncratic productivity shocks.

1.4. Retailers

Retailers are monopolistically competitive firms indexed by $j \in [0, 1]$. They produce differentiated final good Y_{jt} with the following technology:

$$Y_{jt} = Y_{jt}^E,$$

where Y_{jt}^E is the quantity of intermediate good used by retailers *j* as an input and purchased to entrepreneurs *j* in a competitive market at price p_t^E . Assuming price-staggered contracts as in Calvo (1983), $1 - \xi_p$ is defined as the probability for a retailer to be able to reset its price each period. Defining p_{jt} the price of a firm j in period t:

$$p_{jt} = \begin{cases} p_t^* & \text{if adjusts, with probability } 1 - \xi_p, \\ p_{jt-1}\bar{\pi}_t & \text{if does not adjust, with probability } \xi_p. \end{cases}$$

Here $\bar{\pi}_t$ is the inflation rate for retailers not adjusting their prices. The model assumes some degree of price indexation expressed as a combination of steady-state inflation π and past period inflation π_{t_1} , hence $\bar{\pi}_t$ can be written as:

$$\bar{\pi}_t = \pi_{t-1}^{\iota_p} \pi^{1-\iota_p}.$$
(23)

The nominal flow of profits for a retailer j in period t + s is:

$$p_{jt+s}Y_{jt+s} - (1 - \tau_y)p_{t+s}^E Y_{jt+s}^E,$$
(24)

with τ_y a subsidy rate. Accordingly the net present value of its profits is:

$$E_t \sum_{s=0}^{\infty} (\beta \xi_p)^s \zeta_t^C \tilde{\Lambda}_{t+s} p_{t+s} \left[\frac{p_{jt+s}}{p_{t+s}} Y_{jt+s} - (1-\tau_y) \frac{p_{t+s}^E}{p_{t+s}} Y_{jt+s}^E \right],$$

where $\zeta_t^C \tilde{\Lambda}_t$ is the multiplier used in the household's budget constraint. Taking into account the demand curve of final goods producer from equation (30), retailer profits rewrite as:

$$E_t \sum_{s=0}^{\infty} (\beta \xi_p)^s \zeta_t^C \tilde{\Lambda}_{t+s} p_{t+s} \left[\left(\frac{p_{jt+s}}{p_{t+s}} \right)^{\frac{1}{1-\lambda_p}} Y_{t+s} - (1-\tau_y) \frac{p_{t+s}^E}{p_{t+s}} \left(\frac{p_{jt+s}}{p_{t+s}} \right)^{\frac{\lambda_p}{1-\lambda_p}} Y_{t+s} \right].$$

Here p_{jt+s} denotes the price of a firm in period t + s that sets $p_{jt} = p_{jt}^*$ in t and does not reoptimize between t + 1, ..., t + s. Using the indexing rule of non-adjusters,

$$p_{jt+s} = p_{jt+s-1}\bar{\pi}_{t+s} \\ = p_{jt}\bar{\pi}_{t+1}\bar{\pi}_{t+2}...\bar{\pi}_{t+s},$$

similarly,

$$p_{t+s} = p_{t+s-1}\pi_{t+s} = p_t\pi_{t+1}\pi_{t+2}...\pi_{t+s}.$$

Accordingly it is possible to write,

$$\frac{p_{jt+s}}{p_{t+s}} = \frac{p_{jt}}{p_t} M_t^s,$$

where,

$$M_t^s = \begin{cases} \frac{\bar{\pi}_{t+s} \dots \bar{\pi}_{t+1}}{\pi_{t+s} \dots \pi_{t+1}}, & \text{if } s > 0 \\ 1 & \text{if } s = 0. \end{cases}$$

Finally, the net present value of retailer real profits can be expressed as:

$$E_t \sum_{s=0}^{\infty} (\beta \xi_p)^s \zeta_t^C \tilde{\Lambda}_{t+s} p_{t+s} Y_{t+s} \left[\left(M_t^s \frac{p_{jt}}{p_t} \right)^{\frac{1}{1-\lambda_p}} - (1-\tau_y) \frac{p_{t+s}^E}{p_{t+s}} \left(M_t^s \frac{p_{jt}}{p_t} \right)^{\frac{\lambda_p}{1-\lambda_p}} \right].$$

Because firms able to set their price in period t all face the same problem, they have the same solution and set the same price written p_t^* . Accordingly, the first-order condition for maximizing the net discounted sum of profits is:

$$E_t \sum_{s=0}^{\infty} (\beta \xi_p)^s \Psi_{t+s} p_t^* \frac{\lambda_p}{1-\lambda_p} \left[M_t^s \frac{p_t^*}{p_t} - \lambda_p (1-\tau_y) \frac{p_{t+s}^E}{p_{t+s}} \right] = 0,$$

where Ψ_{t+s} is exogenous from the point of view of the firm:

$$\Psi_{t+s} = \zeta_t^C \tilde{\Lambda}_{t+s} p_{t+s} Y_{t+s} \left(M_t^s \right)^{\frac{\lambda_p}{1-\lambda_p}}.$$

Rearranging the previous condition yields the optimal price for a reoptimizing firm:

$$p_t^* = \lambda_p \frac{E_t \sum_{s=0}^{\infty} (\beta \xi_p)^s \Psi_{t+s} (1 - \tau_y) \frac{p_{t+s}^p}{p_{t+s}}}{E_t \sum_{s=0}^{\infty} (\beta \xi_p)^s \Psi_{t+s} M_t^s} = \frac{K_{p,t}}{F_{p,t}}$$

Where auxiliary variables $K_{p,t}$ and $F_{p,t}$ are defined as:

$$K_{p,t} = (1 - \tau_y)\lambda_p E_t \sum_{s=0}^{\infty} (\beta \xi_p)^s \Psi_{t+s} \frac{p_{t+s}^E}{p_{t+s}},$$

$$F_{p,t} = E_t \sum_{s=0}^{\infty} (\beta \xi_p)^s \Psi_{t+s} M_t^s.$$

Rewriting the previous definitions:

$$E_t \left[\zeta_t^C \tilde{\Lambda}_t p_t Y_t + \beta \xi_p \left(\frac{\bar{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_p}} F_{p,t+1} - F_{p,t} \right] = 0,$$
(25)

$$E_t \left[\lambda_p (1 - \tau_y) \frac{p_t^E}{p_t} \zeta_t^C \tilde{\Lambda}_t p_t Y_t + \beta \xi_p \left(\frac{\bar{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{\lambda_p}{1 - \lambda_p}} K_{p,t+1} - K_{p,t} \right] = 0.$$
(26)

The aggregate price index writes:

$$p_{t} = \left[\int_{0}^{1} p_{jt}^{\frac{1}{1-\lambda_{p}}} dj \right]^{1-\lambda_{p}},$$

$$= \left[\int_{j \text{ adj}} p_{jt}^{\frac{1}{1-\lambda_{p}}} dj + \int_{j \text{ dont adj}} p_{jt}^{\frac{1}{1-\lambda_{p}}} dj \right]^{1-\lambda_{p}},$$

$$= \left[\int_{j \text{ adj}} p^{*\frac{1}{1-\lambda_{p}}} dj + \bar{\pi}_{t}^{\frac{1}{1-\lambda_{p}}} \int_{j \text{ dont adj}} p_{jt-1}^{\frac{1}{1-\lambda_{p}}} dj \right]^{1-\lambda_{p}},$$

$$= \left[(1-\xi_{p}) p_{t}^{*\frac{1}{1-\lambda_{p}}} + \pi_{t}^{*\frac{1}{1-\lambda_{p}}} \xi_{p} \int_{j} p_{jt-1}^{\frac{1}{1-\lambda_{p}}} dj \right]^{1-\lambda_{p}}.$$
(27)

Accordingly inflation can be written as:

$$\pi_{t} = \left[(1 - \xi_{p}) p_{t}^{* \frac{1}{1 - \lambda_{p}}} \pi_{t}^{\frac{1}{1 - \lambda_{p}}} + \xi_{p} \bar{\pi}_{t}^{\frac{1}{1 - \lambda_{p}}} \right]^{1 - \lambda_{p}},$$

$$= \left[\frac{\xi_{p}}{1 - (1 - \xi_{p}) p_{t}^{* \frac{1}{1 - \lambda_{p}}}} \right]^{1 - \lambda_{p}} \bar{\pi}_{t},$$
(28)

and the aggregate price index is:

$$p_{t}^{*} = \left[\frac{1 - \xi_{p} \left(\frac{\bar{\pi}_{t}}{\pi_{t}}\right)^{\frac{1}{1-\lambda_{p}}}}{1 - \xi_{p}}\right]^{1-\lambda_{p}}.$$
(29)

1.5. Final Goods Producers

A representative final good producer manufactures homogeneous final goods using technology:

$$Y_t = \int_0^1 \left[Y_{jt}^{\frac{1}{\lambda_p}} \right]^{\lambda_p} dj, \lambda_p > 1.$$

The first order conditions for profit maximization by final good producers are:

$$p_{jt} = p_t \left(\frac{Y_{jt}}{Y_t}\right)^{\frac{\lambda_p}{\lambda_p - 1}}, \quad \text{for} j \in [0, 1].$$
(30)

Finally the price of final goods satisfies the following relation:

$$p_t = \left[\int_0^1 p_{jt}^{\frac{1}{1-\lambda_p}} dj\right]^{1-\lambda_p}.$$
(31)

1.6. Adjustment Cost Functions

The investment adjustment cost function is taken from Christiano, Motto, and Rostagno (2014) and writes:

$$S(\eta_t) = \frac{1}{2} \left[\exp\left(\sqrt{S''/2}(\eta_t - \eta)\right) + \exp\left(-\sqrt{S''/2}(\eta_t - \eta)\right) - 2 \right],$$
(32)

where $\eta_t = \zeta_t^I I_t / I_{t-1}$. This implies $S(\eta) = S'(\eta) = 0$ and $S''(\eta) = S''$ which is a parameter.

Summary of Equilibrium Conditions

For convenience let us define $\tilde{q}_t^K = \frac{q_t^K}{p_t}$, $\tilde{p}_t^E = \frac{p_t^E}{p_t}$, $\tilde{w}_t = \frac{w_t}{p_t}$ and, $\Lambda_t = \tilde{\Lambda}_t p_t$.

Prices

First-order condition 1 price:

$$E_t \left[\zeta_t^C \Lambda_t Y_t + \beta \xi_p \left(\frac{\bar{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_p}} F_{p,t+1} - F_{p,t} \right] = 0$$
(1)

First-order condition 2 price:

$$E_t \left[(1 - \tau_Y) \lambda_p \tilde{p}_t^E \zeta_t^C \Lambda_t Y_t + \beta \xi_p \left(\frac{\bar{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{\lambda_p}{1 - \lambda_p}} K_{p,t+1} - K_{p,t} \right] = 0$$
(2)

Aggregate price index:

$$p_t^* = \left[\frac{1 - \xi_p \left(\frac{\bar{\pi}_t}{\pi_t}\right)^{\frac{1}{1-\lambda_p}}}{1 - \xi_p}\right]^{1-\lambda_p} \tag{3}$$

Households

Households' resource constraint:

$$C_{t} + D_{t} + \tilde{q}_{t}^{K} K_{t} = \tilde{w}_{t} H_{t} + \frac{R_{t}}{\pi_{t}} D_{t-1} + \left[\tilde{q}_{t}^{K} (1-\delta) + r_{t}^{K} \right] K_{t-1} + O_{t}$$
(4)

First-order condition consumption:

$$\zeta_t^C \Lambda_t = \frac{\zeta_t^C}{C_t} \tag{5}$$

First-order condition labor:

$$\psi_H H_t^{\sigma_H} = \tilde{w}_t \Lambda_t. \tag{6}$$

First-order condition deposit:

$$\zeta_t^C \Lambda_t = \beta E_t \zeta_{t+1}^C \Lambda_{t+1} \frac{R_{t+1}}{\pi_{t+1}}$$
(7)

Capital returns:

$$R_{t+1}^{K} = \pi_{t+1} \frac{\tilde{q}_{t+1}^{K}(1-\delta) + r_{t+1}^{k}}{\tilde{q}_{t}^{K}}$$
(8)

First-order condition capital:

$$\zeta_t^C \Lambda_t = \beta E_t \zeta_{t+1}^C \Lambda_{t+1} R_{t+1}^K \tag{9}$$

Capital accumulation:

$$K_{t} = (1 - \delta)K_{t-1} + \left[1 - S\left(\zeta_{t}^{I}\frac{I_{t}}{I_{t-1}}\right)\right]I_{t}$$
(10)

First-order condition investment:

$$\zeta_t^C \Lambda_t \tilde{q}_t^K \left[1 - S\left(\zeta_t^I \frac{I_t}{I_{t-1}}\right) - \zeta_t^I \frac{I_t}{I_{t-1}} S'\left(\zeta_t^I \frac{I_t}{I_{t-1}}\right) \right] - \zeta_t^C \Lambda_t + \beta \zeta_{t+1}^C \Lambda_{t+1} \tilde{q}_{t+1}^K \zeta_{t+1}^I \left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\zeta_{t+1}^I \frac{I_{t+1}}{I_t}\right) = 0$$

$$\tag{11}$$

Entrepreneurs

Aggregate production:

$$Y_t = \frac{\psi_t^Y \xi N_t}{s_t} \tag{12}$$

First-order condition capital:

$$\alpha X_t = r_t^K K_t \tag{13}$$

First-order condition labor

$$(1 - \alpha)X_t = \tilde{w}_t H_t \tag{14}$$

Marginal cost:

$$s_t = \frac{1}{A_t \tilde{p}_t^E} \left(\frac{r_t^K}{\alpha}\right)^{\alpha} \left(\frac{\tilde{w}_t}{1-\alpha}\right)^{1-\alpha}$$
(15)

Entrepreneur dividends:

$$O_t = (1 - \gamma)\psi_{t-1}^V n_{t-1}$$
(16)

Entrepreneur net worth:

$$N_t = \gamma \psi_{t-1}^V N_{t-1} \tag{17}$$

Aggregates

Aggregate resource constraint:

$$Y_t = C_t + I_t + y_t^M \tag{18}$$

Aggregate profits:

$$\psi_t^V = \int V(\varepsilon_1, q_t, R_t, v_t) \Phi(d\varepsilon_1)$$
(19)

$$\psi_t^V = s^a + \int_{\bar{\varepsilon}_b(q_t, R_t, v_t)}^{\bar{\varepsilon}_c(q_t, R_t, v_t)} V^b(\varepsilon, q_t, R_t, v_t) \Phi(d\varepsilon_1) + \int_{\bar{\varepsilon}_c(q_t, R_t, v_t)} V^c(\varepsilon_1, q, R) \Phi(d\varepsilon_1)$$
(20)

Aggregate productivity:

$$\psi_t^Y = (1 - \tau_b) \int_{\bar{\varepsilon}_b(q_l, R_l, \nu_l)}^{\bar{\varepsilon}_c(q_l, R_l, \nu_l)} \varepsilon_1 \int_{\bar{\varepsilon}_d(\varepsilon_1, q_l, R_l, \nu_l)} \varepsilon_2 \Phi(d\varepsilon_2) \Phi(d\varepsilon_1) + \int_{\bar{\varepsilon}_c(q_l, R_l, \nu_l)} \varepsilon_1 \Phi(d\varepsilon_1)$$
(21)

Aggregate default:

$$\psi_t^M = (1 - \tau_b)\mu_b \psi_t^{Mb} + \mu_c \psi_t^{Mc}$$
(22)

$$\psi_t^{Mb} = \int_{\bar{\varepsilon}_b(q_t, R_t, \nu_t)}^{\bar{\varepsilon}_c(q_t, R_t, \nu_t)} \int_{\bar{\varepsilon}_d(\varepsilon_1, q_t, R_t, \nu_t)} \Phi(\bar{\omega}^b(\varepsilon_1 \varepsilon_2, q_t, R_t, \nu_t)) \Phi(d\varepsilon_2) \Phi(d\varepsilon_1)$$
(23)

$$\psi_t^{M_c} = \int_{\bar{\varepsilon}_c(q_t, R_t, v_t)} \Phi(\bar{\omega}^c(\varepsilon_1, q_t, R_t, v_t)) \Phi(d\varepsilon_1)$$
(24)

Monetary Policy

$$R_t - R = \rho_p (R_{t-1} - R) + (1 - \rho_p) \left(\alpha_\pi \left(E \pi_{t+1} - \pi \right) + \frac{\alpha_{\Delta Y}}{4} g_{Y,t} \right) + \frac{1}{400} \varepsilon_t^p$$
(25)

Miscellaneous

$$S(\eta_t) = \frac{1}{2} \left\{ \exp\left[\sqrt{S''/2}(\eta_t - \eta)\right] + \exp\left[-\sqrt{S''/2}(\eta_t - \eta)\right] - 2 \right\}$$
(26)

Log-Linearised Equations

Prices

First-order condition 1 price:

$$E_t \left[\zeta_t^C \Lambda_t Y_t + \beta \xi_p \left(\frac{\bar{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_p}} F_{p,t+1} - F_{p,t} \right] = 0$$
(1)

$$(1 - \beta \xi_p)(\hat{\Lambda}_t + \hat{\zeta}_t^C + \hat{Y}_t) + \left[\left(\frac{1}{1 - \lambda_p} \right) (\hat{\pi}_{t+1} - \hat{\pi}_{t+1}) + \hat{F}_{p,t+1} \right] = \hat{F}_{p,t}$$
(2)

First-order condition 2 price:

$$E_t \left[(1 - \tau_Y) \lambda_p \tilde{p}_t^E \zeta_t^C \Lambda_t Y_t + \beta \xi_p \left(\frac{\bar{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{\lambda_p}{1 - \lambda_p}} K_{p,t+1} - K_{p,t} \right] = 0$$
(3)

$$(1 - \beta \xi_p) \left[\hat{p}_t^E + \hat{\Lambda}_t + \hat{\zeta}_t^C + \hat{Y}_t \right] + \beta \xi_p \left[\frac{\lambda_p}{1 - \lambda_p} (\hat{\pi}_{t+1} - \hat{\pi}_{t+1}) + \hat{K}_{p,t+1} \right] = \hat{K}_{p,t}$$
(4)

Aggregate price index:

$$\frac{\hat{K}_{p,t}}{\hat{F}_{p,t}} = \left[\frac{1 - \xi_p \left(\frac{\bar{\pi}_t}{\pi_t}\right)^{\frac{1}{1-\lambda_p}}}{1 - \xi_p}\right]^{1-\lambda_p} \tag{5}$$

$$K_{p,t} - F_{p,t} = \frac{\xi_p}{1 - \xi_p} \left[\hat{\pi}_t - \hat{\pi}_t \right]$$
(6)

Households

Households' resource constraint (not required):

$$C_t + D_t + \tilde{q}_t^K K_t = \tilde{w}_t H_t + \frac{R_t}{\pi_t} D_{t-1} + \tilde{q}_t^K \frac{(1 + r_t^K - \delta)}{\pi_t} K_{t-1} + O_t$$
(7)

First-order condition consumption:

$$\zeta_t^C \Lambda_t = \frac{\zeta_t^C}{C_t} \tag{8}$$

$$\hat{\Lambda}_t = -\hat{C}_t \tag{9}$$

First-order condition labor:

$$\psi_H H_t^{\sigma_H} = \tilde{w}_t \Lambda_t. \tag{10}$$

$$\sigma_H \hat{H}_t = \hat{\tilde{w}}_t + \hat{\Lambda}_t. \tag{11}$$

First-order condition deposit:

$$\zeta_t^C \Lambda_t = \beta E_t \zeta_{t+1}^C \Lambda_{t+1} \frac{R_{t+1}}{\pi_{t+1}}$$
(12)

$$\hat{\zeta}_t + \hat{\Lambda}_t = \hat{\zeta}_{t+1}^C + \hat{\Lambda}_{t+1} + \hat{R}_{t+1} - \hat{\pi}_{t+1}$$
(13)

Capital returns:

$$\frac{R_{t+1}^K}{\pi_{t+1}} = \frac{\tilde{q}_{t+1}^K (1-\delta)}{\tilde{q}_t^K}$$
(14)

$$\hat{R}_{t+1}^{K} - \hat{\pi}_{t+1} = \frac{\hat{\hat{q}}_{t+1}^{K}(1-\delta) + r^{K}\hat{R}_{t+1}^{K}}{R^{K}} - \hat{\hat{q}}_{t}^{K}$$
(15)

First-order condition capital:

$$\zeta_t^C \Lambda_t = \beta E_t \zeta_{t+1}^C \Lambda_{t+1} R_{t+1}^K$$
(16)

$$\hat{\zeta}_{t}^{C} + \hat{\Lambda}_{t} = \hat{\zeta}_{t+1}^{C} + \hat{\Lambda}_{t+1} + \hat{R}_{t+1}^{K} - \hat{\pi}_{t+1}$$
(17)

Capital accumulation:

$$K_{t} = (1 - \delta)K_{t-1} + \left[1 - S\left(\zeta_{t}^{I}\frac{I_{t}}{I_{t-1}}\right)\right]I_{t}$$
(18)

$$\hat{K}_t = (1-\delta)\hat{K}_{t-1} + \delta\hat{I}_t \tag{19}$$

First-order condition investment:

$$\zeta_t^C \Lambda_t \tilde{q}_t^K \left[1 - S\left(\zeta_t^I \frac{I_t}{I_{t-1}}\right) - \zeta_t^I \frac{I_t}{I_{t-1}} S'\left(\zeta_t^I \frac{I_t}{I_{t-1}}\right) \right] - \zeta_t^C \Lambda_t + \beta \zeta_{t+1}^C \Lambda_{t+1} \tilde{q}_{t+1}^K \zeta_{t+1}^I \left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\zeta_{t+1}^I \frac{I_{t+1}}{I_t}\right) = 0$$

$$\tag{20}$$

$$\hat{q}_{t}^{K} = S'' \left[-\hat{I}_{t-1} + (1+\beta)\hat{I}_{t} + \hat{\zeta}_{t}^{I} - \beta\hat{I}_{t+1} - \beta\hat{\zeta}_{t+1}^{I} \right]$$
(21)

Entrepreneurs

Aggregate production:

$$Y_t = \frac{\psi_t^Y \xi N_t}{s_t} \tag{22}$$

$$\hat{Y}_t = \hat{\psi}_t^y + \hat{N}_t - \hat{s}_t \tag{23}$$

First-order condition capital:

$$\alpha X_t = r_t^K K_t \tag{24}$$

$$\hat{X}_t = \hat{r}_t^K + \hat{K}_t \tag{25}$$

First-order condition labor:

$$(1 - \alpha)X_t = \tilde{w}_t H_t \tag{26}$$

$$\hat{X}_t = \hat{w}_t + \hat{H}_t \tag{27}$$

Marginal cost:

$$s_t = \frac{1}{A_t \tilde{p}_t^E} \left(\frac{r_t^K}{\alpha}\right)^{\alpha} \left(\frac{\tilde{w}_t}{1-\alpha}\right)^{1-\alpha}$$
(28)

$$\hat{s}_t = (1 - \alpha)\hat{\hat{w}}_t + \alpha \hat{r}_t^K - \hat{A}_t - \hat{\tilde{p}}_t^E$$
(29)

Entrepreneur dividends:

$$O_t = (1 - \gamma)\psi_{t-1}^V N_{t-1}$$
(30)

$$\hat{O}_t = \hat{\psi}_{t-1}^V + \hat{N}_{t-1} \tag{31}$$

Entrepreneur networth:

$$N_t = \gamma \psi_{t-1}^V N_{t-1} \tag{32}$$

$$\hat{N}_t = \hat{\psi}_{t-1}^V + \hat{N}_{t-1} \tag{33}$$

Aggregates

Resource constraint:

$$Y_t = C_t + I_t + y_t^M \tag{34}$$

$$\hat{Y}_t = \frac{C}{Y}\hat{C}_t + \frac{I}{Y}\hat{I}_t + \frac{y^M}{Y}\hat{y}_t^M$$
(35)

Debt equilibrium:

$$D_{t} = \left[(1 - \tau_{b}) s_{t}^{bp} + s_{t}^{c} \right] (\xi - 1) N_{t}$$
(36)

Entrepreneur's funding:

$$X_t = \left[(1 - \tau_b) s_t^{bp} + s_t^c \right] \xi N_t \tag{37}$$

Profits:

$$\psi_t^V = \int V(\varepsilon_1, q_t, R_t, \nu_t) \Phi(d\varepsilon_1)$$
(38)

$$\psi_t^V = s^a + \int_{\bar{\varepsilon}_b(q_t, R_t, v_t)}^{\bar{\varepsilon}_c(q_t, R_t, v_t)} V^b(\varepsilon, q_t, R_t, v_t) \Phi(d\varepsilon_1) + \int_{\bar{\varepsilon}_c(q_t, R_t, v_t)} V^c(\varepsilon_1(q_t, R_t, v_t)) \Phi(d\varepsilon_1)$$
(39)

Productivity:

$$\psi_t^Y = (1 - \tau^b) \int_{\bar{\varepsilon}_b(q_t, R_t, \nu_t)}^{\bar{\varepsilon}_c(q_t, R_t, \nu_t)} \varepsilon_1 \int_{\bar{\varepsilon}_d(\varepsilon_1, q_t, R_t, \nu_t)} \varepsilon_2 \Phi(d\varepsilon_2) \Phi(d\varepsilon_1) + \int_{\bar{\varepsilon}_c(q_t, R_t, \nu_t)} \varepsilon_1 \Phi(d\varepsilon_1)$$
(40)

Monitoring costs:

$$\psi_t^M = (1 - \tau^b) \mu^b \psi_t^{Mb} + \mu^c \psi_t^{Mc}$$
(41)

$$\psi_t^{Mb} = \int_{\bar{\varepsilon}_b(q_t, R_t, \nu_t)}^{\bar{\varepsilon}_c(q_t, R_t, \nu_t)} \int_{\bar{\varepsilon}_d(\varepsilon_1, q_t, R_t, \nu_t)} \Phi(\bar{\omega}^b(\varepsilon_1 \varepsilon_2, q_t, R_t, \nu_t)) \Phi(d\varepsilon_2) \Phi(d\varepsilon_1)$$
(42)

$$\psi_t^{Mc} = \int_{\bar{\varepsilon}_c(q_t, R_t, v_t)} \Phi(\bar{\omega}^c(\varepsilon_1, q_t, R_t, v_t)) \Phi(d\varepsilon_1)$$
(43)

Monetary Policy

$$R_t - R = \rho_p (R_{t-1} - R) + (1 - \rho_p) \left(\alpha_\pi \left(E \pi_{t+1} - \pi \right) + \frac{\alpha_{\Delta Y}}{4} g_{Y,t} \right) + \frac{1}{400} \varepsilon_t^p$$
(44)

Miscellaneous

$$S(\eta_t) = \frac{1}{2} \left\{ \exp\left[\sqrt{S''/2}(\eta_t - \eta)\right] + \exp\left[-\sqrt{S''/2}(\eta_t - \eta)\right] - 2 \right\}$$
(45)

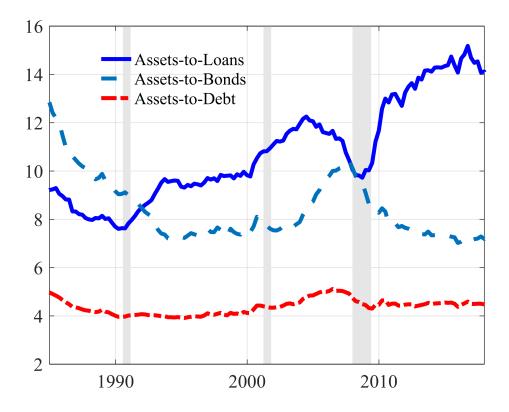


Figure A1: Debt Composition and Firm Leverage

Note: This figure plots the ratios of assets-over-bonds, assets-over-loans and assets-over-total debt for US non-financial corporate firms. The bond series corresponds to the sum of commercial papers and bonds. All series are obtained from FRED.

2. Sensitivity Analysis

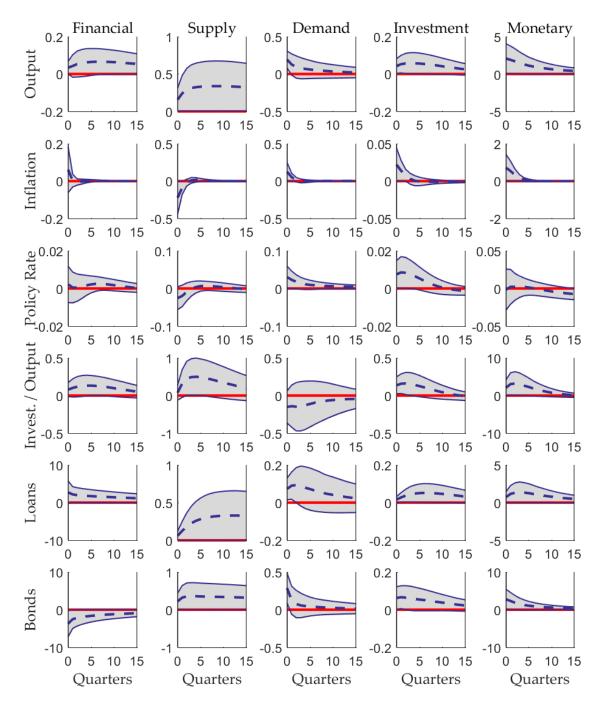


Figure A2: Robustness Test - Impulse Responses

Note: Impulse response functions for the NK model. The grey area corresponds to the IRFs for different calibrations. The highest and lowest two percentiles are trimmed out to remove responses when the model approaches instability. The dashed lines correspond to the means of the total set of IRFs. A total of 100000 sets of parameter are drawn from the uniform distributions displayed in figure A3. Inflation is shown here instead of prices to ease readability.

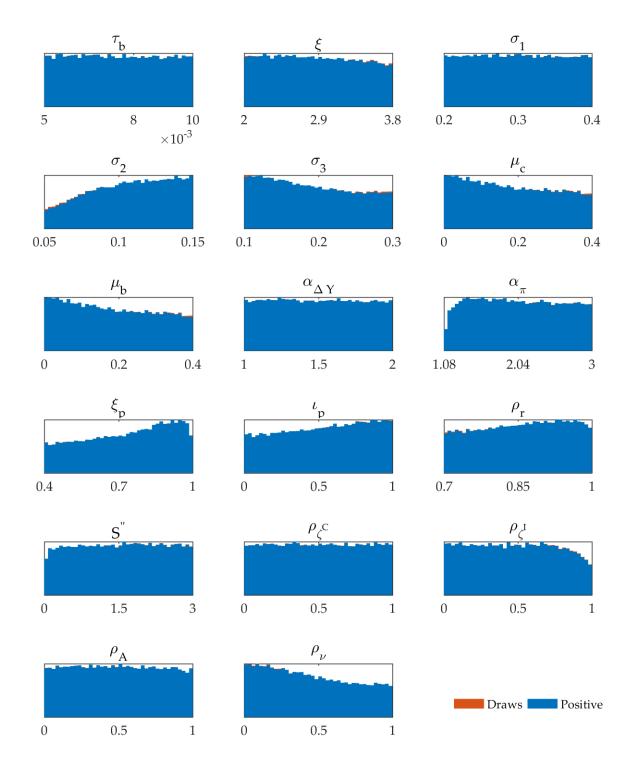


Figure A3: Parameter Acceptance.

Note: This graph plots parameters drawn from uniform distributions and implying model determinacy and a positive response of output. A total of 100000 draws are realized. The supports of the distributions are given by the x-axis. Parameter draws implying opposite movements in bonds and loans for financial shocks and comovements in bonds and loans for all other shocks are marked as 'Positive'.

3. Time-Series Analysis

3.1. Bayesian VAR

This subsection gives an overview of the methods used to compute the reduced form VAR model, a complete description of the Bayesian VAR methodology can be found in Kilian and Lütkepohl (2017).

Consider the following reduced form VAR of order *p*:

$$y_t = c + \sum_{i=1}^p B_i y_{t-i} + u_t,$$
 (1)

where y_t is a $N \times 1$ vector containing the N endogenous variables, c a $N \times 1$ vector of constant, B_i for i = 1, ..., p are $N \times N$ parameter matrices. The vector u_t is a $N \times 1$ vector of prediction errors with $u_t \sim N(0, \Sigma)$ and Σ a variance-covariance matrix. Defining matrices Y, B, U and X such that $Y = [y_1 \ ... \ y_T]'$, $B = [c \ B1 \ ... \ Bp]'$, $U = [u_1 \ ... \ u_T]'$ and,

	1	y'_0	y'_1		<i>y</i> ′_ <i>p</i>	
<i>X</i> =	:	÷	÷	÷	:	,
	1	y'_{T-1}	y'_1	•••	y'_{T-p}	

the VAR model defined in (1) rewrites as Y = XB + U. Vectorising this equation yields:

$$y = (I_N \otimes X)\beta + u, \tag{2}$$

where y = vec(Y), $\beta = vec(B)$ and u = vec(U). Here vec() denotes column wise vectorisation operator. The error term u is assumed to follow a normal distribution with a zero mean and a variance-covariance matrix $\Sigma \otimes I_T$. Accordingly, the likelihood function in B and Σ can be expressed as:

$$L(B,\Sigma) \propto |\Sigma|^{-\frac{T}{2}} exp\left[-\frac{1}{2}\left(\beta - \hat{\beta}\right)'\left(\Sigma^{-1} \otimes X'X\right)\left(\beta - \hat{\beta}\right)\right] exp\left[-\frac{1}{2}tr\left(\Sigma^{-1}S\right)\right],\tag{3}$$

where $S = \left[\left(Y - X\hat{B} \right)' \left(Y - X\hat{B} \right) \right]$ and $\hat{\beta} = vec(\hat{B})$ and $\hat{B} = (X'X)^{-1}X'Y$. I use the Jeffreys' prior distribution for *B* and Σ which is proportional to $|\Sigma|^{\frac{-(n+1)}{2}}$. Following Kadiyala and Karlsson (1997) the joint posterior density for *B* and Σ can be written as:

$$p(B,\Sigma|Y,X) \propto |\Sigma|^{-\frac{T+n+1}{2}} exp\left[-\frac{1}{2}\left(\beta - \hat{\beta}\right)'\left(\Sigma^{-1} \otimes X'X\right)\left(\beta - \hat{\beta}\right)\right] exp\left[-\frac{1}{2}tr\left(\Sigma^{-1}S\right)\right].$$
(4)

Where it is possible to draw β conditional on Σ from:

$$\beta|\Sigma, Y, X \sim N(\hat{\beta}, \Sigma \otimes (X'X)^{-1}), \tag{5}$$

and to draw Σ from:

$$\Sigma|Y, X \sim IW(S, z),\tag{6}$$

where $z = (T - N) \times (p - 1)$.

3.2. Sign-Restriction Algorithm

This subsection sketches the method used to characterize the subset of structural VAR models satisfying the imposed sign restrictions and drawn from the previous distribution of models. While various identification schemes are available, the identification of a VAR model with sign restrictions allows to identify structural shocks with a minimal and qualitative set of hypotheses.¹

The algorithm used in this paper is developed in Arias, Rubio-Ramirez, and Waggoner (2018), the method is as follows. It is possible to express the vector of prediction error u_t as a combination of structural innovations ε_t where $u_t = D\varepsilon_t$ and $\varepsilon_t \sim N(0, I_N)$ with I_N an identity matrix and Da non-singular parameter matrix such that $DD' = \Sigma$. To construct the matrix D, one first draw candidates β and Σ using the posterior distributions given by expressions (5) and (6). The next step involves computing a random orthogonal matrix Q drawn from $N(0, I_N)$. This is achieved by drawing a matrix W from $N(0, I_N)$ further transformed into an orthogonal Q matrix using the QR

¹Advantages of sign-restriction methods are detailed in Uhlig (2005), see Fry and Pagan (2011) for a more critical treatment.

factorization. The matrix *D* is computed as the product matrix of *P* and *Q*, where *P* corresponds to the lower-triangular Cholesky decomposition of Σ . The following step is to compute the impulse responses implied by the coefficient matrices β and *D* for the different structural shocks ε_t . The draws for β , Σ and *W* that imply impulse responses satisfying the sign restrictions are kept. The same process is repeated until a sufficient number of draws are obtained. The set of structural models gathered allows to characterize the distributions of models derived from the reduced form VAR satisfying the sign restrictions imposed.

3.3. Data

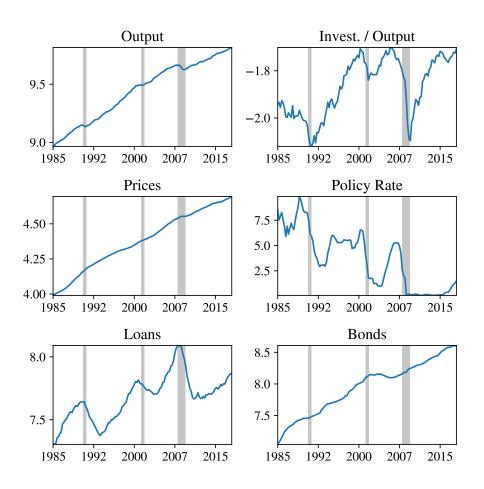


Figure A4: Data used for the SR-VAR estimation

Note: All series are expressed in log-level except the policy rate which is expressed in annual percentage points. GDP, investment, as well loan and bond volumes are expressed in real terms. Prices correspond to the GDP deflator.

Mnemonic	Description	Unit	Source			
A. Macroeconomic Series						
GDP	Gross domestic product	\$bn	BEA			
GDPDEF	Gross domestic product: implicit price deflator	idx	BEA			
GPDI	Gross private domestic investment	\$bn	BEA			
FEDFUNDS	Effective federal funds rate	%	BOG			
B. Financial Series						
CBLBSNNCB	Nonfinancial corporate business: corporate bonds	\$bn	BOG			
CPLBSNNCB	Nonfinancial corporate business: commercial paper	\$bn	BOG			
NCBLL	Nonfinancial corporate business: loans	\$bn	BOG			
AAAFFM	Moody's Aaa corporate bond yield Minus Federal Funds Rate	%	Moody			

Notes: BEA: Bureau of Economic Analysis; BOG: Board of Governors.

Table A1: Data Sources

4. Alternative Dataset

This section presents results for the model estimated using the shadow rate from Wu and Xia (2016) instead of the fed funds rate. Figure A5 presents the impulse response following a financial shock. Figure A6 presents the historical shock decomposition for the different observables. The results obtained are nearly identical to the ones obtain in the baseline specification.



Figure A5: Responses to a Financial Shock

Note: Median impulse responses to a one standard deviation financial shock. The grey lines correspond to the 16th and 84th quantiles. All series are expressed in percentage points. The policy rate is annualized.

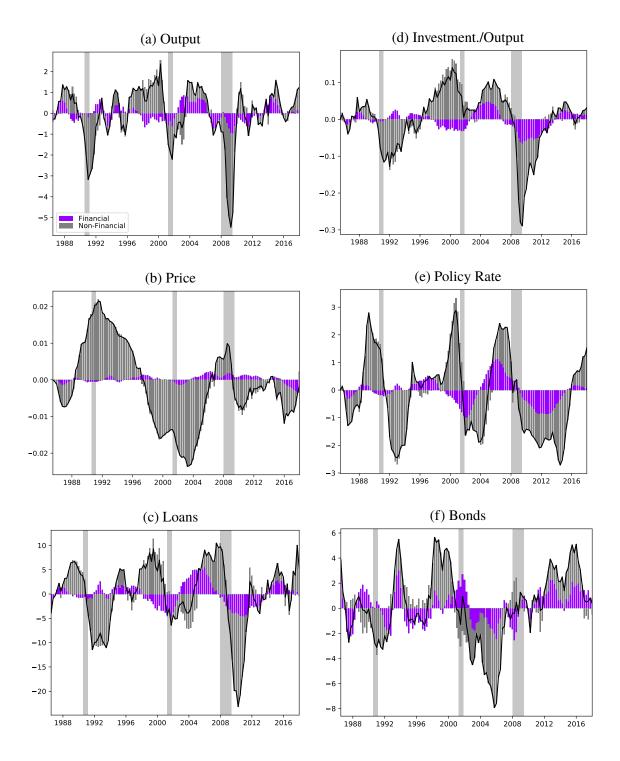


Figure A6: Historical Shock Decomposition

Note: Impact of financial and non-financial shocks. Output, loans and bonds are expressed in annualized growth, all variables are expressed without the constant term. The policy rate is annualized.

5. Alternative Identification

This section presents results from a VAR model identified restricting only the responses of output, loans and bonds. Only two types of structural shocks are considered here, financial shocks that imply comovements in output and loans and opposite movements in bonds, and non-financial shocks that imply comovements in output, loans *and* bonds. I also include Moody's Aaa corporate bond yield minus federal funds rate in the dataset.

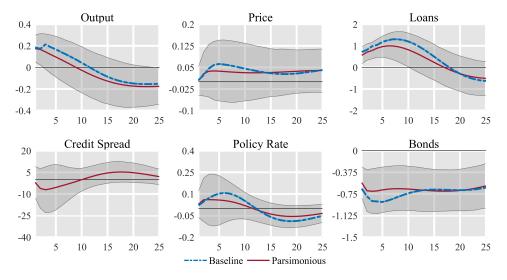


Figure A7: Responses to a Financial Shock

Note: Median impulse responses to a one standard deviation financial shock. The grey lines correspond to the 16th and 84th quantiles. All series are expressed in percentage points. Credit spread and the policy rate are annualized.

Figure A7 shows the impulse responses following a one standard deviation financial shocks. The blue line corresponds to the model when only financial and non-financial shocks are identified, the purple line corresponds to the full specification. The characteristics of the financial shocks implied by the two different sets of restrictions are very close. Figure A8 displays the historical shock decomposition for the different variables. The historical shock decomposition for output is robust to this alternative identification. The fluctuations implied by financial shocks are close to what is obtained in the more constrained model. This is true also for the ratio of investment-to-output which is not displayed here to save space.

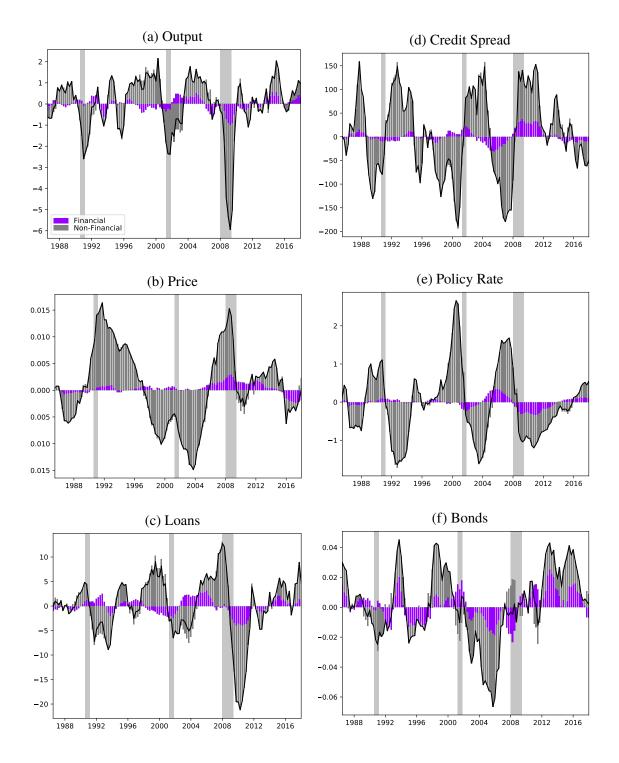


Figure A8: Historical Shock Decomposition

Note: Impact of financial and non-financial shocks. Output, loans and bonds are expressed in annualized growth, all variables are expressed without the constant term. Credit spread and the policy rate are annualized. Credit spread is expressed in basis points.

6. Impulse Response Matching

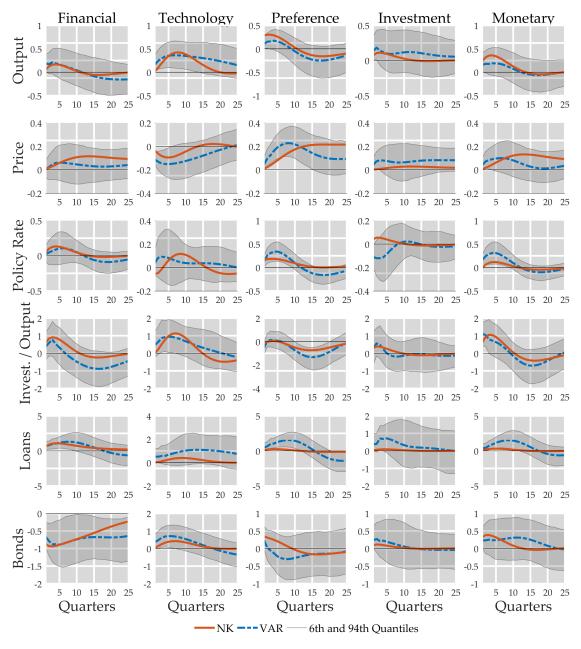


Figure A9: Robust Responses

Note: Median impulse responses to a one standard deviation shock, the grey lines correspond to the 6th and 94th quantiles. All series are expressed in percentage points. The policy rate is annualized. The dash blue lines correspond to the median responses from the VAR model, the orange lines correspond to responses from the NK model.

Param.	Description	Mode
τ_b	Bank intermediation costs	0.005
ξ	Pledgeable fraction of networth	3.7
μ_b	Monitoring cost for loans	0.22
μ_c	Monitoring cost for bonds	0.33
σ_1	Idiosyncratic shock dispersion	0.34
σ_2	Idiosyncratic shock dispersion	0.1
σ_3	Idiosyncratic shock dispersion	0.2
$\alpha_{\Delta Y}$	Taylor rule output coefficient	2.3
α_{π}	Taylor rule inflation coefficient	2.5
ρ_p	Taylor rule smoothing	0.79
ξ_p	Calvo price stickiness	0.94
ι_p	Price indexation on inflation target	0.035
<i>Ś</i> "	Invest. adjustment cost curvature	0.2
ρ_{ζ^C}	Autocorr. preference	0.94
ρζι	Autocorr. MEI	0.57
ρ_A	Autocorr. stationary technology	0.14
$ ho_{v}$	Autocorr. financial	0.93
$\sigma_{\zeta}c$	SD preference	0.0079
σ_{ζ^I}	SD MEI	0.0086
σ_A	SD stationary technology	0.083
σ_{ν}	SD financial	0.07
$\sigma_{arepsilon_p}$	SD monetary policy	0.15

Note: This table displays the parameters minimizing the distance between the impulse responses from the NK model and from the median impulse responses implied by the BVAR.

Table A2: Estimated Parameters

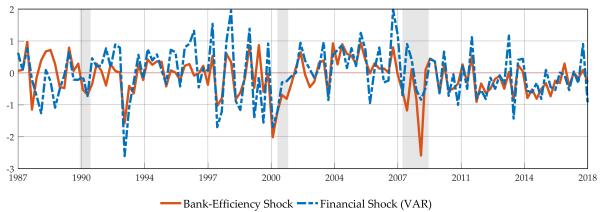


Figure A10: Financial Shocks - NK vs VAR

Note: The orange line corresponds the estimate of the updated bank efficiency shocks. The blue line corresponds to the mean of the financial shocks estimated in the VAR model. Grey areas correspond to NBER recession dates. Correlation between the two series is 0.66.

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