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Monetary policy inertia and the paradox of flexibility

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Abstract

This paper revisits the paradox of flexibility, ie, the result that, in a liquidity trap, greater price flexibility amplifies output volatility in response to negative demand shocks. We argue this paradox is the consequence of a failure of standard models to correctly characterise monetary policy and that allowing for a smooth adjustment of the shadow policy rate eliminates the paradox and produces output responses to a negative demand shock that are in line with those under optimal monetary policy. The reason is that, under an inertial policy, a decline in the shadow rate implies that the future actual policy rate will remain relatively low, which increases expectations about the economic outlook and inflation. The rise in inflation expectations reduces the real rate, thereby sustaining real activity. As we raise the degree of price flexibility, a negative demand shock causes a sharper fall in the shadow rate and incr ease in inflation expectations, which leads to a more significant drop in the real rate and, hence, a milder decline in the output gap.

Key words: Interest rate smoothing, liquidity trap, zero lower bound, paradox of flexibility.

JEL classification: E32, E52, E61.

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1 Introduction

Since the Global Financial Crisis in 2008, nominal interest rates in the U.S. and the Euro Area have approached the zero lower bound (ZLB) and warranted for unconventional monetary policies. Eggertsson and Krugman (2012) point out that at the ZLB, in a standard New Keynesian (NK) model, higher price flexibility is destabilising and raises the volatility of output in response to adverse demand shocks (*paradox of flexibility*). The reason is that in a liquidity trap, negative demand shocks can lead to a significant fall in inflation expectations and a deflationary spiral. As the nominal policy rate is stuck at zero, the decline in inflation expectations increases the real rate that causes a sharp drop in real activity. Under flexible prices, the drop in prices and expectations becomes substantially larger than under sticky prices, which amplifies these adverse effects.

One key determinant of this result is the monetary policy rule. The previous literature has studied the paradox of flexibility using models in which the policy rate is given by a truncated Taylor rule without an interest rate smoothing term. In this paper, we show, using a simple three-equation NK model, that smoothing the shadow rate, i.e., the hypothetical policy rate that would prevail in the absence of a ZLB constraint, can substantially mitigate the effects of adverse demand shocks at the ZLB.[1] First, when we consider a Taylor-type rule where the notional interest rate is adjusted smoothly, demand shocks lead to milder output losses. Second, we do not find any paradox of flexibility. In other words, we find that when prices are more flexible, adverse demand shocks have a smaller impact on the output gap. To explain these results, we conduct both an analytical exercise, based on a two-period version of the model, as well as numerical one using the infinite-period model.

The reason why an inertial monetary policy produces the results above is that smoothing the shadow rate introduces history dependence in the policy reaction function. As a result, at the ZLB, a fall in the shadow rate due to a negative demand shock implies that the future actual policy rate will be relatively low compared to that in the absence of shadow rate inertia. In other words, during a liquidity trap, by reducing the shadow rate, the central bank is de facto committing to keeping the actual policy rate lower for longer than if it was following a rule with no inertia. Introducing the lagged shadow rate into the policy rule is one way to characterise such a "lower-for-longer" policy, which Billi and Galí (2020) define as a form of *forward guidance*.[2] When prices are flexible, the shadow rate falls significantly more, given the initial slump in inflation,

[1]It bears noting that some papers in the literature, such as Wu and Xia (2016), label as shadow rate an estimated policy rate that summarises the actual monetary policy stance taking into account all the various unconventional monetary policies.

[2]Other approaches to modelling forward guidance policies are described, for example, in Del Negro et al. (2015), McKay et al. (2016), McKay et al. (2017), and Sims and Wu (2020). de Groot and Haas (2019), for example, highlight a similar forward

which reinforces the forward guidance channel, thereby avoiding deflationary spirals and sharply increasing inflation expectations. As the actual policy rate is at its lower bound, the rise in inflation expectations causes a significant fall in the real rate, which mitigates the contraction in the output gap. When this channel is absent, i.e., in the absence of shadow rate inertia, the central bank in the simple NK model is not able to credibly counteract the effects of an adverse demand shock while in a liquidity trap. Therefore greater price flexibility leads to sharper declines in inflation, inflation expectations, and the output gap compared to a model with more rigid prices. Moreover, we find that the inertial policy can correct for the paradox of flexibility even if we mitigate the Forward Guidance Puzzle, following McKay et al. (2017) and Gabaix (2020). Finally, we show that when the central bank smooths the actual rate rather than the shadow rate, its ability to sustain demand and manage expectations is significantly impaired during a liquidity trap. This type of inertial policy, in fact, misses the forward guidance channel mentioned above and, for this reason, cannot correct the paradox of flexibility.

The motivation for focusing on an inertial Taylor rule is threefold. First, the previous literature (e.g., Smets and Wouters, 2003, 2007; Christiano et al., 2014) has provided substantial evidence in support of the presence of the lagged policy rate in the monetary policy rule. Typical estimates for the smoothing coefficient are positive and large, usually above 0.8. Second, the presence of policy inertia is consistent with statements from central banks around the world, indicating their intention to keep interest rates close to zero for a considerable period after the economic recovery strengthens (Hills and Nakata, 2018). If the central bank adjusts its policy rate based only on current inflation and economic conditions, it will raise the policy rate as soon as economic activity starts to recover. If instead, the policy rule features a large weight on the lagged shadow policy rate, then the central bank will adjust the actual policy rate sluggishly. Third, the literature has highlighted how history dependence of monetary policy through policy rate inertia could resolve other policy paradoxes that arise in the New Keynesian model when the nominal rate is at the ZLB. For example, Hills and Nakata (2018) shows that government spending multipliers become significantly smaller under an inertial monetary policy rule. Similarly, Roulleau-Pasdeloup (2018) shows how a commitment policy, implying a similar history dependence, substantially mitigates the positive impact of government spending on inflation.

Related Literature This paper is strictly related to the literature addressing the paradox of flexibility. Eggertsson and Krugman (2012) highlight the potential risks of policies aimed at increasing wage and price flexibility when the policy rate is at the ZLB. Kiley (2016) re-assesses the paradox of flexibility, as well as

guidance (signalling) channel implied in negative reserve rates.

other standard puzzles that arise in the simple NK model in a ZLB environment. He shows that these puzzles are not present in a sticky-information model. Bhattarai et al. (2018) find that flexible prices amplify output volatility for demand shocks if monetary policy does not respond sufficiently to inflation, and they often reduce welfare even under optimal monetary policy if full efficiency cannot be achieved. Eggertsson and Garga (2019) show that sticky information makes the paradox of toil and the paradox of flexibility more severe. Finally, Billi and Galí (2020) show that the ZLB amplifies the adverse welfare effects of wage flexibility.

This paper is also related to a few papers addressing the benefits of interest rate smoothing for monetary policy. Woodford (2003b) shows in the context of a simple model that monetary policy inertia can be optimal, in the sense of minimising a loss function that penalises inflation variations, the output gap, and changes in the interest rate. This paper suggests that the sluggish interest rate adjustment, by steering private-sector expectations of future policy, can be desirable even if the reduction of the magnitude of interest-rate changes is not a social objective. Hills and Nakata (2018) show that smoothing of the shadow policy rate in the interest rate feedback rule significantly reduces the government spending multiplier when the policy rate is at the ZLB. They explain that in an economy with policy inertia, increased inflation and output led by higher government spending during a recession speeds up the return of the policy rate to the steady state after the recession ends. This, in turn, has an impact on expectations, which mitigate the expansionary effects of government spending during the recession. Nakata and Schmidt (2019) find that including interest rate smoothing in the objective function of a discretionary central bank improves the welfare of an economy with an occasionally binding ZLB. Through expectations, the temporary overheating of the economy associated with a low-for-long interest rate policy mitigates the declines in inflation and output when the lower bound constraint is binding. de Groot and Haas (2019) highlight a novel signalling channel of negative interest rates. They show that in a framework where deposit rates are constrained by a ZLB, and the central bank adjusts the nominal policy rate smoothly, negative policy rates signal lower future deposit rates boosting aggregate demand and net worth.[3]

The remainder of the paper is structured as follows. Section 2 provides a brief description of the model. In Section 3, we provide an analytical explanation of why shadow rate smoothing can correct the paradox based on a two-period version of the model. In Section 4, we conduct a numerical exercise and comment on the results. Finally, in Section 5, we provide some concluding remarks.

[3] Other papers analysing the role of monetary policy at the ZLB include, for example, Nakov (2008), Cochrane (2017), Nakata (2017), and Masolo and Winant (2019).

2 The Model

For our exercise, we consider the basic "three-equation" NK model as in Woodford (2003a) and Galí (2015). The output gap, x_t, is defined as the difference between output in the sticky-price model and its flexible-price counterpart, whereas the other variables are expressed in absolute deviations from their steady-state values. In particular, π_t represents the inflation rate, i_t is the nominal interest rate set by the monetary authority and r_t^n is the natural rate of interest, which is assumed to be exogenous. Finally, we label the shadow or notional policy rate as $i_t^\star$. The three main equations of the model are given by the dynamic IS equation, Equation (1) and the NK Phillips curve, Equation (2):

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - r_t^n\right), \tag{1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t, \tag{2}$$

where $\kappa \equiv \frac{(1-\theta)(1-\beta\theta)(\sigma+\varphi)}{\theta(1+\varphi\varepsilon)}$. θ is the Calvo price rigidity parameter, β is the household's discount factor, ε is the intermediate good demand elasticity, σ is the coefficient of relative risk aversion, and φ is the inverse labor supply elasticity. The model is closed by a monetary policy rule, which is defined by Equation (3) and Equation (4):

$$i_t = \max\left\{i_t^\star, 0\right\}, \tag{3}$$

$$i_t^\star = \rho_i i_{t-1}^\star + (1-\rho_i)\left(r_t^n + \phi_\pi \pi_t\right). \tag{4}$$

While the nominal rate, i_t, is bounded from below, the shadow (or notional) rate is not. The central bank sets its shadow rate $i_t^\star$ in response to deviations of the natural rate and the inflation rate from their steady-state values. Moreover, we assume that the monetary authority has a preference for smoothing the shadow rate, which is given by the autoregressive component in Equation (4). In Section 4, we explain in great detail the importance of including this smoothing component and compare this to the optimal monetary policy under commitment:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[-\frac{1}{2}\left(\pi_t^2 + \frac{\kappa}{\varepsilon} x_t^2\right)\right], \tag{5}$$

subject to Equation (1) and Equation (2).

3 Analytical Results

3.1 Shadow Rate Smoothing

In this section, we examine analytically the implications of inertial policy to an exogenous reduction in the natural rate, which drives the actual policy rate to the ZLB. We do so through the lenses of a two-period version of the model, which allows us to provide intuition about the role of inertial policy in mitigating the effects of negative demand shocks during a liquidity trap and correcting the paradox of flexibility. The two-period version of the model assumes that $r_1^n < r_2^n = r$ such that $i_1 = 0$ and $i_2 > 0$. In other words, we assume that in time $t = 1$ the natural rate is below its steady-state level $r = \frac{1}{\beta} - 1$. The model equations for periods 1 and 2 write as follows:

$$x_1 = x_2 - \frac{1}{\sigma}\left(i_1 - \pi_2 - r_1^n\right), \tag{6}$$

$$x_2 = -\frac{1}{\sigma}\left(i_2 - r_2^n\right), \tag{7}$$

$$\pi_1 = \beta\pi_2 + \kappa x_1, \tag{8}$$

$$\pi_2 = \kappa x_2, \tag{9}$$

$$i_1 = 0, \tag{10}$$

$$i_1^\star = \rho_i i_0^\star + (1-\rho_i)\left(r_1^n + \phi_\pi \pi_1\right) < 0, \tag{11}$$

$$i_2 = \rho_i i_1^\star + (1-\rho_i)\left(r_2^n + \phi_\pi \pi_2\right) > 0. \tag{12}$$

In the model without policy inertia ($\rho_i = 0$), a decline in the natural rate causes a contemporaneous fall in the output gap (x_1) through the dynamic IS equation, Equation (6). The fall in the output gap leads to a decline in the inflation rate (π_1) via the NK Phillips curve, Equation (8). In this case, raising price flexibility (i.e., increasing κ), would increase the responsiveness of inflation to the contemporaneous change in the output gap, but would not affect the response of the output gap. Obviously, in the absence of policy inertia, the decline in the shadow rate in period 1 does not affect the interest rate in period 2 and hence the other variables.

When the monetary policy authority follows an inertial policy rule ($\rho_i > 0$), instead, a fall in the shadow rate in period 1, due to the fall in the natural rate, reduces the policy rate in period 2 (i_2). The decline in i_2 directly increases the future output gap x_2 via Equation (7) and indirectly future inflation π_2, via Equation

(9). The rise in π_2 has a positive effect on π_1, via Equation (8), and on x_1 via Equation (6). The positive contemporaneous effect of a reduction in the period-1 shadow rate on the output gap puts additional upward pressure on inflation via Equation (8). Under the inertial policy, raising price flexibility (i.e. increasing κ), amplifies the response of π_2 to the increase in x_2. The stronger rise in π_2, in turn, has a positive effect on π_1 and x_1. In addition, it also amplifies the response of π_1 to x_1, which puts further upward pressure on period-1 inflation. From the discussion above, we can conclude that under monetary policy inertia: i) a reduction in the shadow rate mitigates the adverse effects of the period-1 fall in the natural rate by increasing period-2 inflation rate, and hence reducing the period-1 real rate; ii) increasing price flexibility mitigates the fall in x_1 caused by the drop in the natural rate. This is because π_2 reacts more strongly to the increase in x_2, driven by the reduction in period-2 policy rate i_2; iii) increasing price flexibility has two positive effects on π_1. First, it amplifies the response of π_2 to an increase in x_2, thereby putting upward pressure on π_1. Second, greater price flexibility amplifies the response of π_1 to an increase in x_1, which is in turn positively affected by the fall in the real rate and, therefore, the rise in π_2.

From the equations above, we derive the aggregate demand (AD) and aggregate supply (AS) curves for period 1, given by Equation (13) and Equation (14):

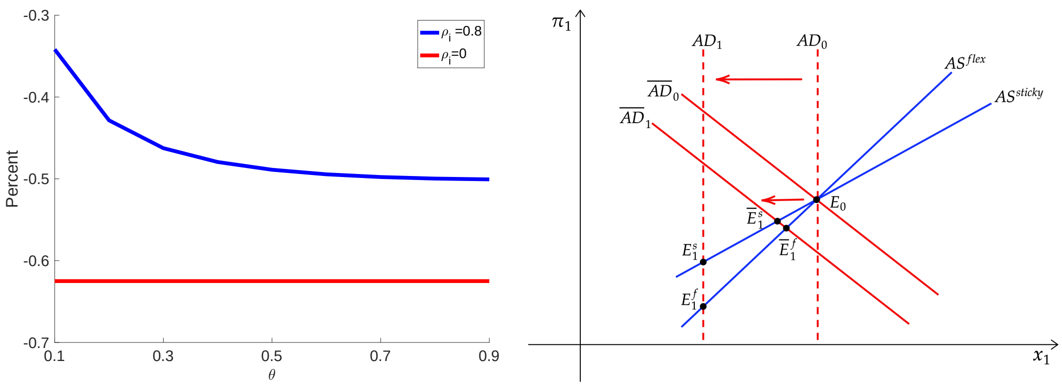
$$x_1 = -\zeta_\pi \pi_1 + \zeta_{rn} r_1^n + \zeta_r r, \tag{13}$$

$$\pi_1 = \xi_x x_1 - \xi_r r_1^n + \xi_r r, \tag{14}$$

where $\zeta_\pi \equiv \left(1 + \frac{\kappa}{\sigma}\right) \frac{\rho_i(1-\rho_i)\phi_\pi}{\sigma+\kappa(1-\rho_i)\phi_\pi} \geq 0$, $\zeta_{rn} \equiv \frac{1}{\sigma} - \frac{\zeta_\pi}{\phi_\pi} > 0$, $\zeta_r \equiv \frac{\zeta_\pi}{\phi_\pi} \geq 0$, $\xi_x \equiv \frac{\kappa(\sigma+\kappa(1-\rho_i)\phi_\pi)}{\sigma+\kappa(1-\rho_i)\phi_\pi(1+\beta\rho_i)} > 0$, and $\xi_r \equiv \frac{\beta\kappa\rho_i(1-\rho_i)}{\sigma+\kappa(1-\rho_i)\phi_\pi(1+\beta\rho_i)} \geq 0$.[4] In the absence of policy inertia (i.e., $\rho_i = 0$), x_1 does not depend on π_1. In other words, the AD curve is vertical. If $\rho_i > 0$ instead, x_1 depends negatively on current inflation π_1, and the AD becomes downward sloping. Moreover, an inertial policy also implies a smaller shift in the AD due to a change in r_1^n and a flattening of the AS curve, since ζ_{rn} and ξ_x are smaller for $\rho_i > 0$ than for $\rho_i = 0$. By equalising demand and supply, it is easy to show analytically that the equilibrium output gap x_1^* under an inertial policy is smaller, in absolute value, than the one in the absence of inertia. In other words, smoothing mitigates the negative effects of the lower natural rate:

$$(x_1^*)_{\rho_i>0} \; = \; \left(\frac{\zeta_\pi\xi_r + \zeta_{rn}}{1+\zeta_\pi\xi_x}\right) r_1^n + \left(\frac{\zeta_r - \zeta_\pi\xi_r}{1+\zeta_\pi\xi_x}\right) r \; < \; (x_1^*)_{\rho_i=0} \; = \; \frac{1}{\sigma} r_1^n \; . \tag{15}$$

[4] We report the full derivations in Appendix A and B.



(a) Output Gap (x_1^*)

(b) AS-AD

Figure 1: Paradox of Flexibility in a Two-Period Model

Note: The left panel shows the equilibrium output gap x_1^* for different degrees of price stickiness, under monetary policy inertia (blue line) and the absence of inertia (red line). The right panel displays a simplified illustration of the paradox flexibility in the NK model with and without policy inertia. We abstract from: i) changes in the slope of the AS curve because of policy inertia; ii) the shift of the AS curve due to the decline in the natural rate; iii) changes in the slope of the AD curve due to changes in price rigidity. Aggregate demand curves feature kinks because of the ZLB constraint. $\overline{AD}$ is the aggregate demand when the central bank uses an inertial policy. AD is aggregate demand in a liquidity trap without policy inertia. AS is aggregate supply. The y-axis is inflation (π), while the x-axis is the output gap (x).

Figure 1(a) shows how the period-1 equilibrium output gap x_1 varies for different degrees of price stickiness with the two alternative policies. Under an inertial policy (blue line), the equilibrium output gap becomes more negative as price stickiness increases. In other words, as explained above, with such a policy, the paradox of flexibility does not occur. When the monetary authority follows a policy rule without inertia (red line), instead, reducing price rigidity does not affect the equilibrium output. It should be noted that in the two-period version of the model, the paradox of flexibility takes a weaker form than in the infinite-horizon model. As discussed in Eggertsson and Krugman (2012), in an infinite-horizon model during a liquidity trap, demand tends to be upward-sloping rather than vertical. Therefore, in such a context, higher price flexibility will be detrimental for economic activity. For the exercise displayed in Figure 1(a), we consider the same parameter values as in section 4, which are reported in Table 1. In this section, however, as discussed above, we assume that the fall in the natural rate lasts only for one period. We assume that $r_1^n = -0.0125$, i.e., -5% in annualised terms, which leads $i_1 = 0$ and $i_2 > 0$.

In Figure 1(b), we display the results in a diagram. To simplify the graphical analysis, the figure abstracts from: i) the change in the slope of the AS curve because of policy inertia; ii) the shift of the AS curve due to the decline in the natural rate[5]; iii) the change in the slope of the AD curve due to changes in price rigidity. The aggregate demand curves, both with and without inertial policy, feature a kink due to the presence

[5]The size of the shift is determined by the parameter ξ_r, which is a small number.

of the ZLB constraint. In the absence of policy inertia, a section of the aggregate demand (green line) is vertical due to the binding ZLB constraint. The negative rate shock shifts the aggregate demand curve from AD_0 to AD_1. Higher price flexibility causes a steepening of the AS curve. Given the two alternative AS curves for sticky and less sticky (flexible) prices, the equilibrium moves from the initial point E_0 to either E_1^s (sticky prices) or E_1^f (flexible prices). Since both equilibria lie on the vertical section of AD_1, an increase in price flexibility leads to a significant drop in inflation, as measured by the distance between E_1^s and E_1^f, whereas the output gap remains unaffected. Under policy inertia, the aggregate demand curves (red line) is downward-sloping even when the actual nominal rate is at the ZLB.[6] The fall in the natural rate causes a relatively mild shift (determined by the parameter ζ_{rn}) in the downward-sloping demand curves, from $\overline{AD}_0$ to $\overline{AD}_1$. In this case, the new equilibria are $\overline{E}_1^s$ and $\overline{E}_1^f$. In this case, inflation and output fall less than in the case without policy inertia. Moreover, the flexible-price equilibrium is characterised by lower inflation but a larger output gap. Hence, under policy inertia, the paradox of flexibility disappears because the demand curve becomes downward sloping.

3.2 Actual Rate Smoothing

If we remove Equation (11) and replace Equation (12) with:

$$i_2 = \rho_i i_1 + (1-\rho_i)(r_2^n + \phi_\pi \pi_2) = (1-\rho_i)(r_2^n + \phi_\pi \pi_2), \tag{16}$$

the interest rate in period 2 (i_2) does not depend on period-1 inflation (π_1) and natural rate (r_1^n). For this reason, aggregate demand will be vertical in this case, as shown below in equation. The rate i_2 will however be lower than the case without smoothing. The analytical expressions for the aggregate demand, supply, and equilibrium output gap are given by:

$$x_1 = \frac{1}{\sigma} r_1^n + \left(1 + \frac{\kappa}{\sigma}\right)\left(\frac{\rho_i}{\sigma + (1-\rho_i)\,\phi_\pi \kappa}\right) r, \tag{17}$$

$$\pi_1 = \kappa x_1 + \frac{\beta \kappa \rho_i}{\sigma + (1-\rho_i)\,\phi_\pi \kappa} r, \tag{18}$$

$$x_1^* = \frac{1}{\sigma} r_1^n + \left(1 + \frac{\kappa}{\sigma}\right)\left(\frac{\rho_i}{\sigma + (1-\rho_i)\,\phi_\pi \kappa}\right) r. \tag{19}$$

From Equation (19), we see that, with actual rate inertia, a decrease in θ (implying an increase in κ and increase in price flexibility), still has a positive effect on the equilibrium output gap, by increasing the weight

[6]Because of policy inertia the AD curve features a milder change in the slope when the actual policy rate is at the ZLB compared to the case without policy inertia.

on the constant term (the steady-state value of the natural rate). However, comparing the AD (Equation, 17) and AS (Equation, 18) curves with Equation (13) and Equation (14), it is clear how, under actual rate smoothing, the positive effect of increasing price flexibility is weaker than under shadow rate smoothing. This is because, with shadow rate smoothing, price flexibility implies a downward-sloping demand and a more muted impact of r_1^n. With actual rate smoothing, demand is vertical, and the shift in demand due to the decline in the natural rate is the same as in the absence of policy inertia. Nevertheless, as mentioned above, in the two-period model, even with actual rate inertia, price flexibility can be beneficial for economic activity via the impact on the constant (intercept) term.

By contrast, in Section 4.5, we show that the beneficial effects of price flexibility are not present in the infinite-horizon model, where the liquidity-trap aggregate demand is upward sloping rather than vertical. In such a context, the numerical exercise shows indeed that the actual rate inertial policy cannot correct the paradox of flexibility, i.e., price flexibility does not mitigate the response of x_1 to a decline in the natural rate.

4 Numerical Results

In this section, we consider a numerical exercise based on the infinite-period model. We parameterise the model using values that are standard in the literature, as listed in Table 1. Regarding the interest rate smoothing parameter, we choose a baseline value of 0.8, which is in line with previous estimates. For example, typical values for the US (Smets and Wouters, 2007; Coibion and Gorodnichenko, 2012; Christiano et al., 2014) lie between 0.81 and 0.85. Evidence for other countries like the Euro Area and Sweden (e.g., Smets and Wouters, 2003; Adolfson et al., 2008; Christiano et al., 2011; Coenen et al., 2018) find values ranging from 0.82 to 0.96.

In line with the literature (e.g., Galí, 2015), we solve the model with the ZLB constraint, using a perfect foresight solution (Adjemian et al., 2011).[7] The experiment we conduct consists in simulating a substantial drop in the natural rate, from 1.2 to -5 per cent, in annualised terms, for 8 quarters, which pushes the policy rate to the ZLB. We then analyse the response of our model variables under different monetary policy rules, with and without policy inertia, and under optimal monetary policy, for different levels of price rigidity.

[7] Solving the model with a piecewise linear approximation as suggested by Guerrieri and Iacoviello (2015) delivers results that are in line with the perfect foresight solution.

Table 1: Quarterly Calibration

Parameter	Description	Value
β	Discount factor	0.997
σ	Risk aversion	2
φ	Inverse labor supply elasticity	1
ε	Elasticity of substitution between goods	10
θ	Probability of keeping price unchanged	0.75
ρ_i	Interest rate smoothing	$\{0, 0.8\}$
ϕ_π	Coefficient on inflation	1.5
r^n	Long-run natural real rate $(1/\beta - 1)$	0.003
r^n_L	Natural real rate in period 1 to 8	-0.0125

4.1 The Paradox of Flexibility

In the left column of Figure 2, we show how the impact responses of the output gap and inflation to the negative natural rate shock described above vary for different values of price rigidity when the smoothing parameter in Equation (4) is equal to zero. In line with Eggertsson and Krugman (2012), we find that the output gap and inflation are dramatically more responsive for low levels of price rigidity.[8] The reason is that the fall in inflation cannot be effectively counteracted by the central bank when the policy rate is at the ZLB, which causes inflation expectations to fall and a deflationary spiral. When prices are stickier, inflation reacts less to the shock, and the overall effect is, therefore, more muted than under flexible prices. As the price rigidity parameter approaches zero (flexible prices), the drops in inflation and the output gap become increasingly more severe, highlighting the significant consequences a liquidity trap can have.

4.2 The Role of Monetary Policy

The right columns of Figure 2 shows the same exercise for the case of optimal policy under commitment, as defined by Equation (5). In this case, we see that higher price rigidity increases the effect on the output gap and inflation. This result highlights that the source of the paradox of flexibility is a failure of standard monetary policy at the ZLB. In line with the analytical results described in Section 3, the central column of Figure 2 shows that by simply including smoothing in the shadow policy, the paradox disappears. Similarly, as for the optimal policy case, the impact response of the output gap is amplified for higher levels of price rigidity. To understand the mechanism at play, in Figure 3, we compare the full paths of the model variables to a negative natural rate shock for two different values of price rigidity. In the central column, we display the response under shadow rate inertia, while in the left column, we show the path without inertia. The

[8]It bears noting that in the two-period version of the model, increasing flexibility does not affect the response of the output gap at all.

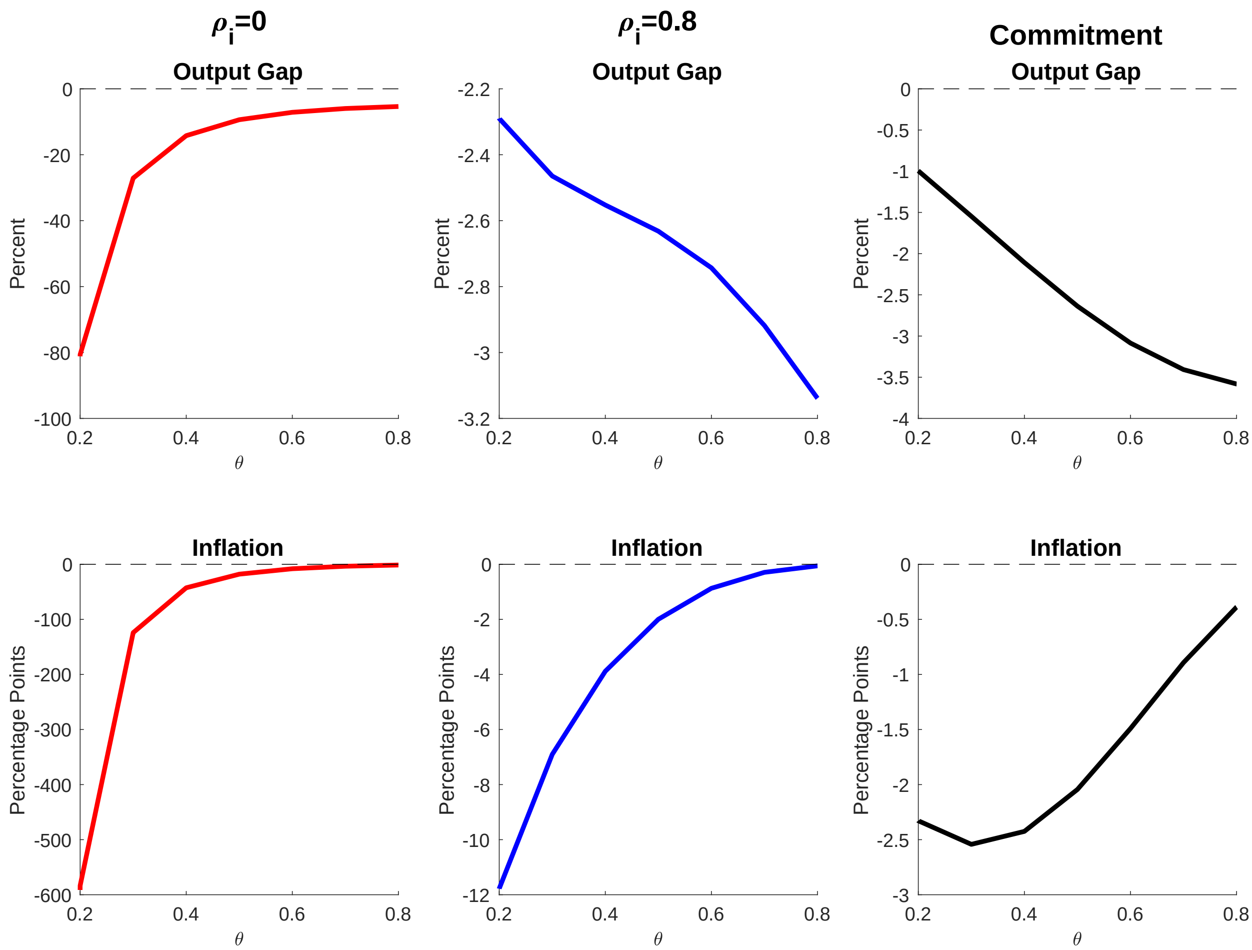


Figure 2: Impact Responses for Different Degrees of Price Stickiness

Note: The figure shows the impact response to a negative shock to the natural interest rate for different values of the price stickiness parameter. The shock reduces the natural interest rate from 1.2 to -5 per cent for eight quarters and leads the policy rate to hit the ZLB. Inflation is expressed in annualised terms.

right column shows the responses under commitment.

The key takeaways from the figure are: (i) when the central bank smooths the shadow rate, the fall in the output gap is more acute and persistent for higher values of price rigidity; (ii) the magnitude of the declines in inflation and the output gap is far more muted in the model with policy inertia, than without; (iii) the adjustment in the actual policy rate as the shock unwinds is slower in the model with shadow rate inertia than in the model without. Similarly, under optimal monetary policy, the policy rate is kept at zero for longer. (iv) In the model with shadow rate inertia and high price rigidity, inflation expectations increase, as reflected in the decline of the real rate. Despite the initial increase, the drop in the real rate is even larger when prices are relatively more flexible. In the model without inertia, instead, inflation expectations drop

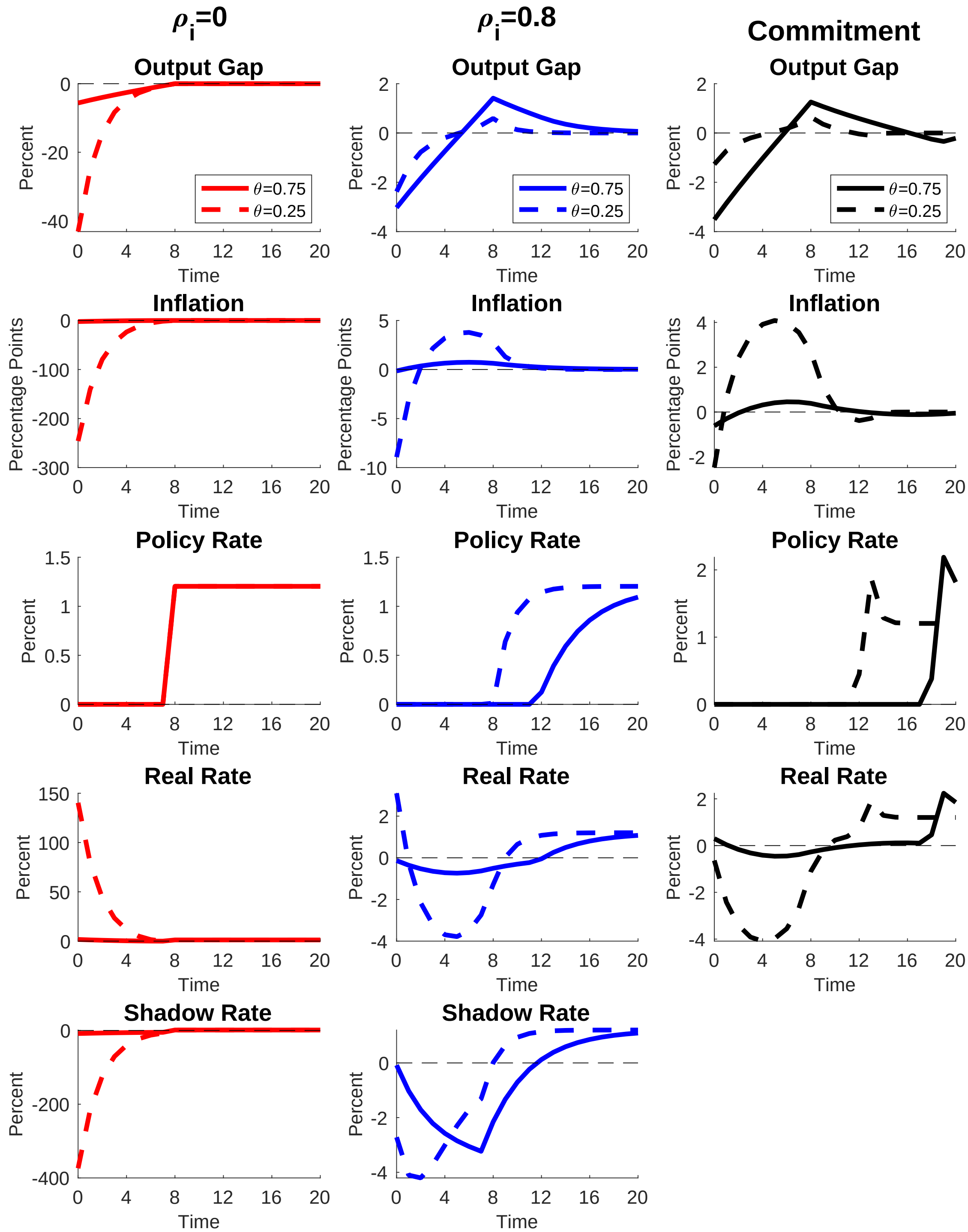


Figure 3: Responses to a Natural Rate Shock: Stickiness and Inertial Policy

Note: The figure shows the impact responses to a negative shock to the natural interest rate for different values of the interest rate smoothing parameter. The shock reduces the natural interest rate from 1.2 to −5 per cent for eight quarters and leads the policy rate to hit the ZLB. Inflation, policy rate, shadow rate, and real rate are expressed in annualised terms. Inflation is expressed in absolute deviation from steady state.

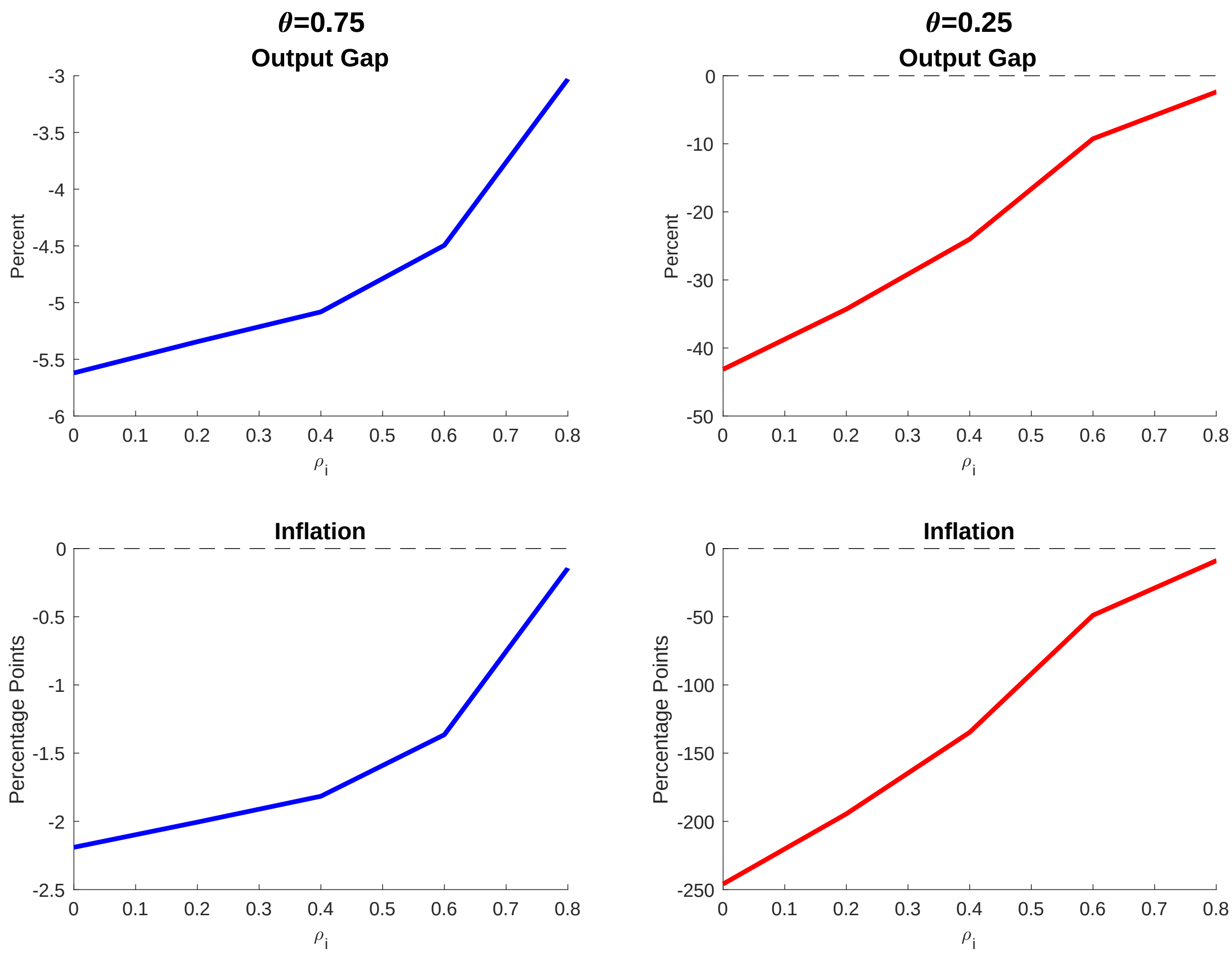


Figure 4: Impact Responses for Different Degrees of Interest Rate Smoothing

Note: The figure shows the impact responses to a negative shock to the natural interest rate for different values of the interest rate smoothing parameter. The shock reduces the natural interest rate from 1.2 to -5 per cent for eight quarters and leads the policy rate to hit the ZLB. Inflation is expressed in annualised terms.

significantly, leading to a substantial increase in the real rate. (v) The responses of the output gap and the real rate are similar in the model with policy inertia and under optimal monetary policy. Moreover, in both models, we see an overshoot in inflation.

4.3 The Forward Guidance Channel and Commitment

In the model with policy inertia, a low shadow rate today signals that the shadow rate and the policy rate will stay lower for longer, which significantly counteracts the sharp fall in inflation expectations and hinders the severe deflation featured in the model without policy inertia. This channel becomes more effective for higher degrees of monetary policy inertia, as displayed in Figure 4. The figure shows how the impact response of the output gap and inflation becomes more muted for larger values of the smoothing parameter. Finally,

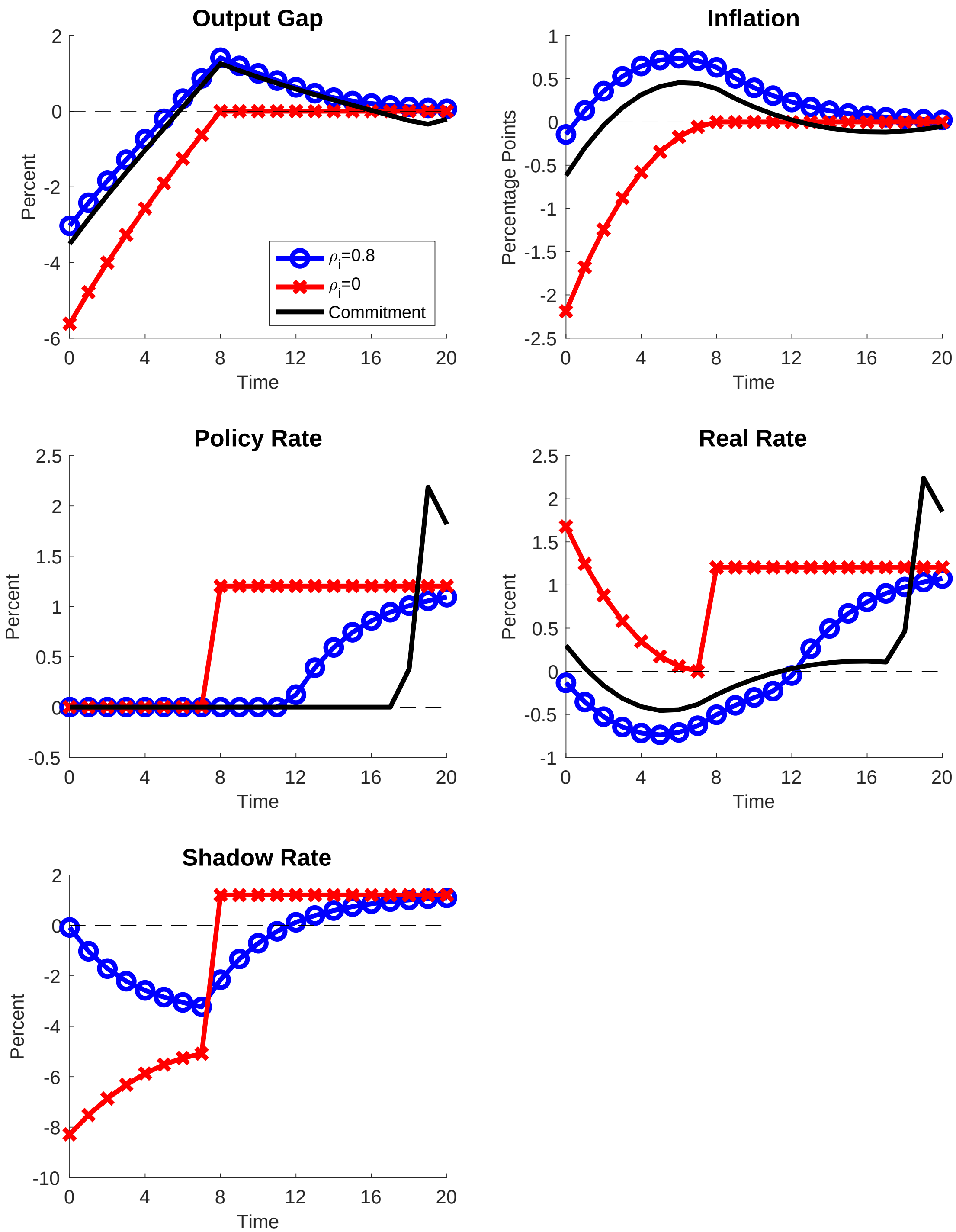


Figure 5: Importance of the Forward Guidance Channel

Note: The figure shows the responses to a negative shock to the natural interest rate for different monetary policies. The shock reduces the natural interest rate from 1.2 to -5 per cent for eight quarters and leads the policy rate to hit the ZLB. Inflation, policy rate, shadow rate, and real rate are expressed in annualised terms. Inflation is expressed in absolute deviation from steady state.

Figure 5 compares the policy rules with and without inertia (blue and red lines) in the shadow rate to the optimal policy under commitment (black line) for the case of sticky prices (i.e., $\theta = 0.75$). Again, the simple Taylor-rule case without shadow rate smoothing implies a stronger decline in both inflation and inflation expectations, as reflected in the sharp rise in the real rate. The increase in the real rate leads to a more severe contraction in the output gap, which is about twice more pronounced than under shadow rate smoothing or the optimal policy. The difference implied by the two policies is even starker when looking at inflation. We see how under policy inertia, the policy rate remains low for long, slowly reverting to its steady-state. This result is similar to the optimal policy case, where the central bank keeps the policy rate at zero for 12 quarters, instead of eight, before abruptly adjusting it. Therefore under an inertial policy rule, the central bank, by reducing the shadow rate, is de facto committing to keeping the policy rate low in the future, which can be interpreted as a form of forward guidance (e.g., Billi and Galí, 2020).

4.4 Mitigating the Forward Guidance Puzzle

As highlighted in Del Negro et al. (2015), standard monetary DSGE models are subject to the "forward guidance puzzle", i.e., the effects of forward guidance become increasingly larger with the horizon of the intended change in the policy rate. To address this concern, we consider two robustness checks. First, we follow the solution suggested by McKay et al. (2017), which consists in introducing discounting terms in the dynamic IS equation. In their paper, this is the result of income risk and borrowing constraints. In our model, this implies changing Equation (1) as follow

$$x_t = \zeta_1 E_t x_{t+1} - \frac{\zeta_2}{\sigma} \left(i_t - E_t \pi_{t+1} - r_t^n \right), \tag{20}$$

where $\zeta_1 = 0.97$, $\zeta_2 = 0.75$ as in McKay et al. (2017).[9] Second, Gabaix (2020) considers a discounted version of the NK Phillips curve, derived from the bounded rationality of firms. We therefore consider the following modified NK Phillips curve:

$$\pi_t = \zeta_3 \beta E_t \pi_{t+1} + \kappa x_t, \tag{21}$$

where $\zeta_3 = 0.8$ in line with Gabaix (2020).

In Figure 6, we display the results from Figure 2 under three alternative model specifications: i) with the discounted dynamic IS equation from Equation (20); ii) with the discounted NK Phillips curve from Equation

[9] Gabaix (2020) considers an alternative specification of the discounted dynamic IS equation from the bounded rationality of households, that the discounting parameter is associated only with expected output gap. The key results of our analysis are robust to this alternative specification of the discounted dynamic IS equation.

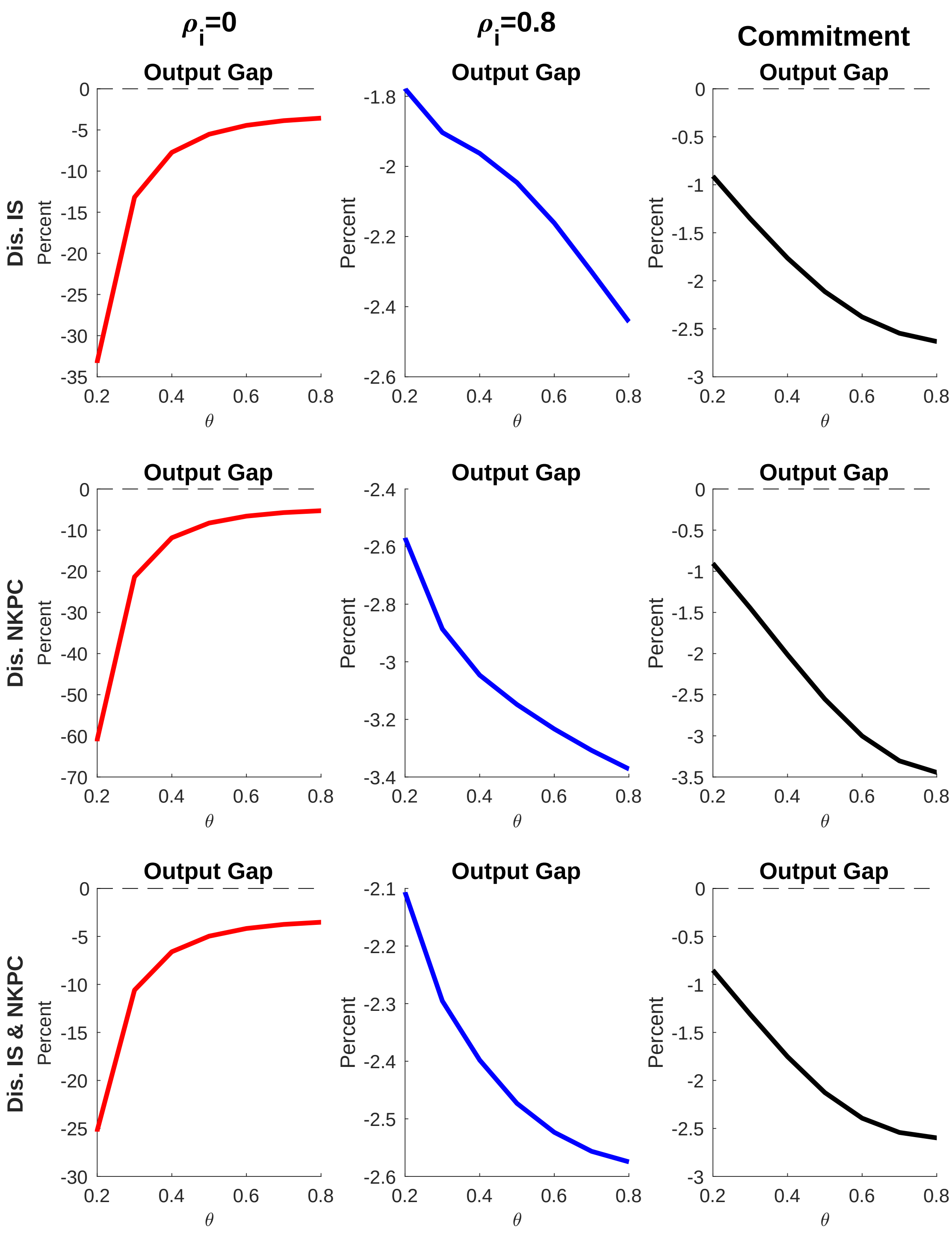


Figure 6: Mitigating the Forward Guidance Puzzle

Note: The figure shows the impact response to a negative shock to the natural interest rate for different values of the price stickiness parameter. The shock reduces the natural interest rate from 1.2 to -5 per cent for eight quarters and leads the policy rate to hit the ZLB. Each row represents an alternative model specification: i) with a discounted dynamic IS equation (Dis. IS); ii) with a discounted NK Phillips curve (Dis. NKPC); iii) with both discounted equations.

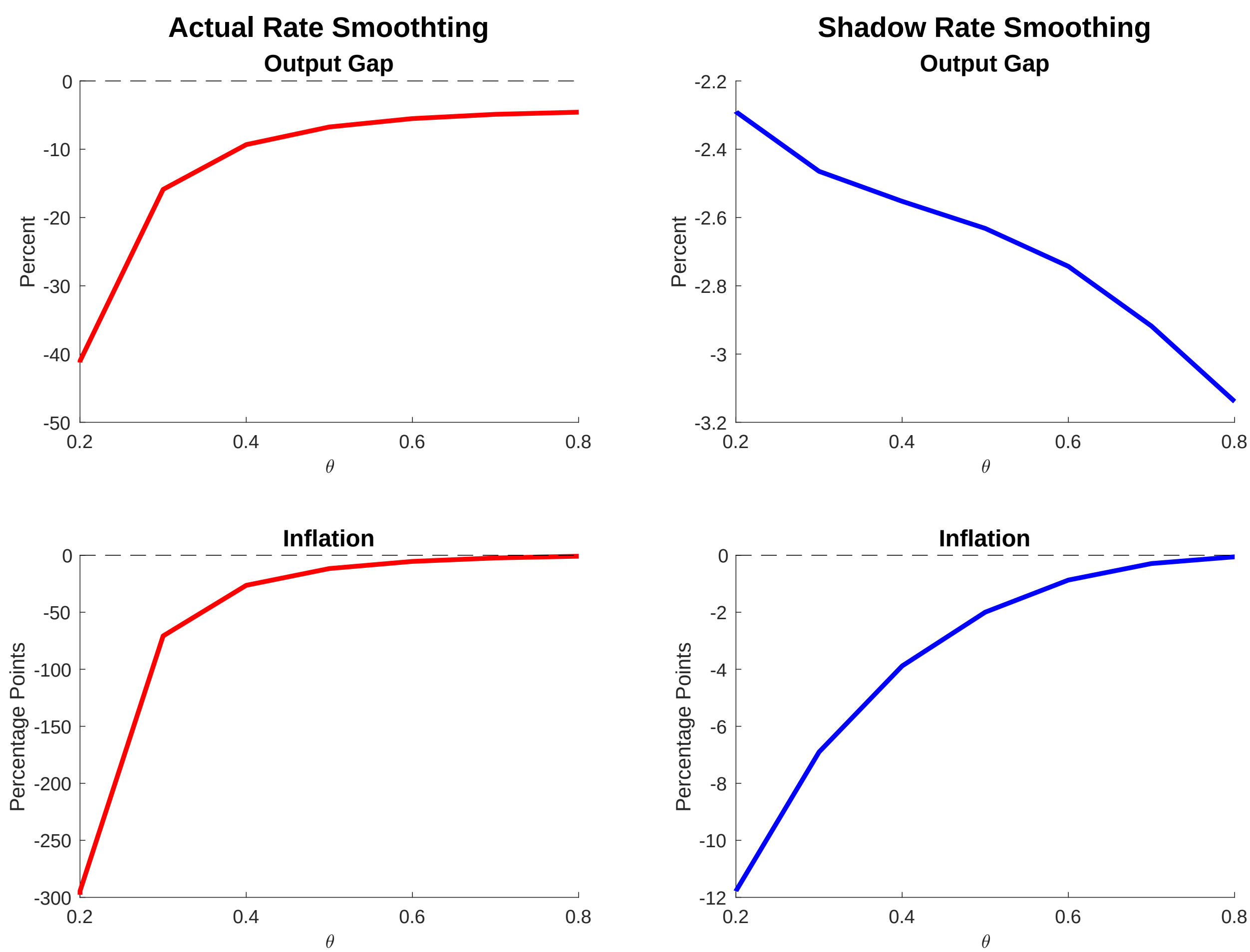


Figure 7: Impact Responses for Different Degrees of Price Stickiness: Actual Rate Smoothing

Note: The figure shows the impact responses to a negative shock to the natural interest rate for different values of the price stickiness parameter. The shock reduces the natural interest rate from 1.2 to -5 per cent for eight quarters and leads the policy rate to hit the ZLB. Inflation is expressed in annualised terms.

(21); iii) with both discounted equations. Even by mitigating the power of forward guidance, the inertial policy, as well as optimal policy under commitment, can correct the paradox of flexibility.

4.5 Actual Rate Smoothing

Similarly as in the analytical section, we compare the model with shadow rate smoothing to the model with actual rate smoothing. In particular, we consider the following policy rule:

$$i_t = \max\left\{i_t^\star, 0\right\}, \tag{22}$$

$$i_t^\star = \rho_i i_{t-1} + (1-\rho_i)\left(r_t^n + \phi_\pi \pi_t\right). \tag{23}$$

In this case, when the economy is in a liquidity trap, and $i_t = i_t^\star = 0$, the stance of monetary policy does not respond to current economic conditions as in the shadow-rate-smoothing case. After a negative demand shock, under actual rate smoothing, agents are aware that the future interest rate increase will be sluggish but not dependant on today's economic conditions. Therefore, the forward guidance channel is absent in this kind of policy reaction function, and the ability of monetary policy to sustain demand is significantly weaker. Figure 7 compares the impact response of the output gap and inflation to a negative natural rate shock for different degrees of price rigidity, under different types of policy inertia. For this exercise, we assume the same inertia parameter, $\rho_i = 0.8$. The panels on the left-hand side show how the actual rate smoothing policy is not able to correct the paradox of flexibility. In fact, as we decrease the price rigidity parameter θ, the decline in the output gap becomes more substantial.

5 Conclusion

In this paper, we show that the paradox of flexibility, which occurs when deflationary shocks hit the economy during a liquidity trap, is the result of a failure of standard models to correctly characterise monetary policy at the ZLB. We find that this paradox can be easily circumvented by allowing the monetary authority to smoothly adjust the shadow rate, i.e., by including an autoregressive term in the shadow rate's Taylor rule. In this case, a fall in the shadow rate implies a lower-for-longer actual policy rate (forward guidance channel), which stimulates economic activity and counteracts the deflationary spirals, which would occur in the absence of inertia. When this policy is in place, the responses to adverse demand shocks are larger under sticky prices than under flexible prices, and the overall effects are significantly more muted compared to an economy without an inertial monetary policy. Furthermore, we find the outcome of such inertial monetary policy to be in line to that under an optimal policy with commitment. In both cases, the policy rate does not immediately revert to its initial steady state when the negative shock unwinds. Finally, we argue that standard inertial policy, smoothing the actual policy rate rather than the shadow rate, cannot correct the paradox because it misses the forward guidance channel. The results in this paper call for caution when drawing policy recommendations based on a simple model with a ZLB constraint. In such a context, the credibility of the central bank and the ability to manage expectations are crucial to avoid seemingly paradoxical results.

Appendices

A Derivation of AD Curve

To derive the AD curve, defined in Equation (13), we need to combine equations (6), (7), (9), (10), (11), and (12). First, combining (11) and (12) with the assumption $i_0^\star = r_2^n = r$, we get:

$$i_2 = \rho_i \left(\rho_i r + (1-\rho_i)\left(r_1^n + \phi_\pi \pi_1\right)\right) + (1-\rho_i)\left(r + \phi_\pi \pi_2\right). \tag{A.1}$$

We then can combine it with (7) and (9):

$$x_2 = -\frac{1}{\sigma}\left(\rho_i \left(\rho_i r + (1-\rho_i)\left(r_1^n + \phi_\pi \pi_1\right)\right) + (1-\rho_i)\left(r + \phi_\pi \kappa x_2\right) - r\right). \tag{A.2}$$

Rearranging (A.2):

$$x_2 = -\frac{\rho_i (1-\rho_i)\left(r_1^n + \phi_\pi \pi_1 - r\right)}{\sigma + (1-\rho_i)\phi_\pi \kappa}. \tag{A.3}$$

Moreover, from (6), (9), and (10) we have:

$$x_1 = x_2 - \frac{1}{\sigma}\left(0 - \kappa x_2 - r_1^n\right). \tag{A.4}$$

The expression above can be rewritten as:

$$x_1 = \left(1 + \frac{\kappa}{\sigma}\right) x_2 + \frac{1}{\sigma} r_1^n. \tag{A.5}$$

We then combine (A.3) and (A.5):

$$x_1 = -\left(1 + \frac{\kappa}{\sigma}\right)\frac{\rho_i (1-\rho_i)\left(r_1^n + \phi_\pi \pi_1 - r\right)}{\sigma + (1-\rho_i)\phi_\pi \kappa} + \frac{1}{\sigma} r_1^n. \tag{A.6}$$

Rearranging gives us the AD curve:

$$x_1 = -\left(1 + \frac{\kappa}{\sigma}\right)\frac{\rho_i (1-\rho_i)\phi_\pi}{\sigma + (1-\rho_i)\phi_\pi \kappa}\pi_1 + \left(\frac{1}{\sigma} - \left(1 + \frac{\kappa}{\sigma}\right)\frac{\rho_i (1-\rho_i)}{\sigma + (1-\rho_i)\phi_\pi \kappa}\right) r_1^n + \left(1 + \frac{\kappa}{\sigma}\right)\frac{\rho_i (1-\rho_i)}{\sigma + (1-\rho_i)\phi_\pi \kappa} r. \tag{A.7}$$

B Derivation of AS Curve

To derive AS curve, defined in Equation (14), we need to combine equations (8), (9), and (A.3). We start by using (8) and (9) to obtain:

$$\pi_1 = \beta\kappa x_2 + \kappa x_1. \tag{B.1}$$

We then combine the equation above with (A.3):

$$\pi_1 = -\beta\kappa \frac{\rho_i\left(1-\rho_i\right)\left(r_1^n + \phi_\pi \pi_1 - r\right)}{\sigma + \left(1-\rho_i\right)\phi_\pi \kappa} + \kappa x_1. \tag{B.2}$$

Finally, rearranging the expression above provides the AS curve:

$$\pi_1 = \frac{\kappa\left(\sigma + \kappa\left(1-\rho_i\right)\phi_\pi\right)}{\sigma + \kappa\left(1-\rho_i\right)\phi_\pi\left(1+\beta\rho_i\right)} x_1 - \frac{\beta\kappa\rho_i\left(1-\rho_i\right)}{\sigma + \kappa\left(1-\rho_i\right)\phi_\pi\left(1+\beta\rho_i\right)} r_1^n + \frac{\beta\kappa\rho_i\left(1-\rho_i\right)}{\sigma + \kappa\left(1-\rho_i\right)\phi_\pi\left(1+\beta\rho_i\right)} r. \tag{B.3}$$

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