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Abstract

This paper evaluates whether recent advances in modelling the extensive margin of exports contribute to our understanding of export fluctuations over the business cycle. Using US and euro-area data, we estimate a general equilibrium model in which the extensive margin of exports varies over the business cycle. A comparison of its performance to two similar models that differ in their modelling of the extensive margin of exports shows that, while recent advances in modelling the extensive margin of trade help replicate exports dynamics, this is not the result of a good fit to the observed extensive margin of exports: the model-implied extensive margin of exports varies considerably more than the data suggests.

Key words: Export dynamics, heterogeneous firms, extensive margin of trade, international business cycles.

JEL classification: F41, F44, E32.

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1 Introduction

A vaste literature on export dynamics - both empirical and theoretical - testifies of the gaps in our understanding of these dynamics. In particular, as pointed out by Engel and Wang (2011), standard open economy models cannot replicate the observed behaviour of exports. In this paper, we investigate whether recent advances in modelling the extensive margin of exports contribute to our understanding of business cycle and aggregate export dynamics in a general equilibrium framework where business cycles arise as a consequence of different types of shocks.

Our aim is to evaluate the performance of a model in which the extensive margin of exports varies both as a result of changes in the number of varieties produced and as the result of changes in the share of those varieties which is exported.[1] The model is motivated by empirical evidence pointing out that the number of varieties available within an economy as well as the share of those varieties which is exported change over the business cycle. For instance, Broda and Weinstein (2010) find that the number of new varieties introduced by US firms is positively correlated to their sales and consumption. And Ghironi and Melitz (2007) show that, in the US, the number of exported varieties is positively correlated to exports, while Devereux and Hnatkovska (2012) show that changes in the share of traded relative to non-traded output is positively correlated to GDP growth in a range of OECD countries including the US. The model we evaluate therefore endogenises, first, the firms' choice whether to produce and, second, whether to export or only supply the varieties they produce to the domestic market. The extensive margin of exports can thus potentially vary through two channels: as a result of a change in the share of varieties that are exported, and/or as a result of a change in the overall number of varieties produced in the economy.

We build on an important strand of literature which seeks to introduce these two channels into standard open economy general equilibrium models. The seminal paper within that literature is Ghironi and Melitz (2005) who develop an open economy general equilibrium model in which firms are heterogeneous with respect to their individual levels of productivity as in Melitz (2003). Firms have to pay a sunk entry cost to start producing their product variety, hence, the varieties available within an economy fluctuates over the business cycle. Moreover, because firms face fixed and variable export costs, only the most productive firms will find it profitable to export their product variety. As a result, not only the set of varieties available within an economy changes with the economic conditions, but also the share of those varieties that are exported. Building on that model, we introduce nominal rigidities in the form of staggered wage setting and monetary policy.

We evaluate the model's ability - and in particular the implications of allowing the extensive margin of exports to vary as a result of variations in the number of varieties produced and/or in the share of varieties exported - to

[1] The model we evaluate thus differs from Alessandria and Choi (2007) who focus on variations of the extensive margin of exports happening only as a consequence of variations in the share of varieties which are exported.

improve our understanding of US business cycles and of export dynamics. Different from previous evaluations of this framework, such as Ghironi and Melitz (2007), we take into account that business cycle fluctuations arise not only from productivity shocks but also from monetary policy shocks, preference shocks and shocks to the uncovered interest rate parity (UIP) condition. We investigate the implications of our modelling of the extensive margin of exports by comparing three models that differ only in features crucial to the extensive margin of exports. In the first model, the extensive margin of exports varies as a result of endogenous variations in both the number of produced varieties and the share of exported varieties; in the second model, the extensive margin of exports varies only as a result of endogenous variation in the number of produced varieties; and in the third model, the extensive margin of exports is constant. We calibrate the structural parameters of the models using empirical evidence provided in the literature. We then estimate the parameters related to the stochastic shock processes using data for the US and the euro area with Bayesian estimation methods. We compare the three models' overall performance as well as their ability to help us understand US export dynamics.

The comparison of the three models reveals that the models which allow the extensive margin of exports to vary in most cases fit the data best and always get closer to replicating observed export dynamics. But we also find that these models do not replicate data on the extensive margin of exports well: they considerably over-predict fluctuations in the extensive margin of exports. It thus appears that adding more channels through which export dynamics may vary improves the model's export performance, though this is not the result of an improvement in our understanding of the dynamics of the extensive margin of exports over the business cycle.

There is a vast international macroeconomics literature that builds on the framework developed in Ghironi and Melitz (2005), as do we. These authors analyse the implications of their model for international prices, focusing in particular on the Harrod-Balassa-Samuelson effect. The framework has been used to study a number of different topics, ranging from the elasticity of exports and imports to relative prices (Ruhl (2008)) to the consequences of trade integration (Cacciatore and Ghironi (2013)) and optimal monetary policy (Cooke (2016)). Our paper is more closely related to the strand of literature that studies the framework's ability to improve our understanding of business cycles and of export dynamics. Ghironi and Melitz (2007) find that, in the face of productivity shocks, their model can replicate the link between the number of varieties traded across countries and aggregate trade levels, as well as some cross-correlations of trade with GDP. Devereux and Hnatkovska (2012) find that including an extensive margin for trade allows to better replicate observed changes in the share of traded output over the business cycle and helps explain the consumption risk sharing puzzle.[2] However, they also find that their model with an extensive margin of

[2] Note that in the model developed by Devereux and Hnatkovska (2012), the extensive margin of trade only changes as a result of movements in the share of traded versus non-traded goods, not through variations in the overall number of produced goods as this number is constant.

trade does not out-perform a similar model without an extensive margin when it comes to matching moments and does mixed at explaining trade dynamics and their relation to GDP. Both those papers rely on productivity shocks being the source of business cycle fluctuations. To the best of our knowledge, the framework has not been evaluated with respect to its performance in matching business cycle properties and more specifically trade fluctuations in an environment where shocks other than to productivity can also cause business cycle fluctuations.

Our findings are important as we show that allowing for more channels through which business cycle fluctuations affect exports helps the model fit export dynamics. However, we also find that modelling the extensive margin of exports as in Ghironi and Melitz (2005) does not seem to improve our understanding of the dynamics of the extensive margin of exports over the business cycle, in response to an array of estimated shocks. In other words, the mechanisms leading to changes in the extensive margin of trade emphasized in Ghironi and Melitz (2005), while important in the face of productivity shocks as argued in Ghironi and Melitz (2007), seem less relevant when other shocks also affect export dynamics. More research on the dynamics of the extensive margin of exports in response to different types of shocks is needed to make progress in our understanding of the extensive margin of exports and aggregate export dynamics over the business cycle.

In the next section, we present the three versions of our framework that we use to understand whether modelling the extensive margin of exports as in Ghironi and Melitz (2005) improves our understanding of business cycles and export dynamics. In Section 3, we present our estimation strategy and the data used for the estimation exercises. We then discuss how well the models fit the data and in particular exports in Section 4. We also briefly discuss the robustness of our results before concluding.

2 Models

In the following, we first present our benchmark model, Model-1, which allows for variations in the extensive margin of trade as a result of endogenous variations in the number of varieties produced and in the share of exported varieties, as is the case in Ghironi and Melitz (2005). We then present a variant, Model-2, which allows for some variation in the extensive margin of exports, but only as a result of endogenous variation in the number of varieties produced (not in the share exported). Finally, we present Model-3 which does not allow for any variation in the extensive margin of exports. A comparison of these models will help us understand the implications of modelling the extensive margin of trade as in Ghironi and Melitz (2005). Different from previous literature, our focus is on understanding the implications for the dynamics of exports in an estimated model with a range of business cycle shocks.

2.1 Benchmark Model: Model-1

The benchmark model we present here closely follows Ghironi and Melitz (2005), but expands that model by introducing nominal rigidities and monetary policy. It is a two country general equilibrium model in which firms are heterogeneous with respect to their relative productivity levels. Firms face fixed and variable costs of exporting so only more productive firms are able to export. This structure allows variations both at the extensive and intensive margins of trade. Financial markets are incomplete at the international level. Different from Ghironi and Melitz (2005), we introduce nominal rigidities in labour markets such that there is a role for monetary policy in our framework and such that nominal shocks may drive the business cycle alongside real shocks. In particular, wages are staggered a la Calvo (1983) in both countries. And monetary policy is conducted according to a Taylor type rule. We will denote the Foreign country variables with an asterisk (*). In our presentation of the model, we focus on the economic agents in country H, but analogous relations hold for the Foreign agents.

2.1.1 Households

The representative home household's lifetime utility function can be expressed as a function of consumption (C_t) and labour (L_t):

$$U_t = \mathrm{E}_t \sum_{t=0}^{\infty} \beta_t \, \chi_t \left[\frac{C_t^{1-\gamma} - 1}{1-\gamma} - \kappa \, \frac{L_t^{1+\eta}}{1+\eta} \right] \tag{1}$$

where E_t denotes the expectations at time t, β is the discount factor, $1/\gamma$ is the intertemporal elasticity of substitution and η is the inverse of the Frisch elasticity of labour supply. χ is a preference shock which affects the discount factor. It follows an AR(1) process:

$$ln(\chi_t) \; = \; \rho_\chi \, ln(\chi_{t-1}) \; + \; \varepsilon_{\chi,t} \tag{2}$$

where $0 \leq \rho_\chi < 1$ and $\varepsilon_{\chi,t} \sim N(0, \sigma_\chi^2)$.

Consumers can consume differentiated varieties (ω) defined over a continuum set of varieties, Ω:

$$C_t = \left[\int_{\omega \epsilon \Omega} c_t(\omega)^{\frac{\theta-1}{\theta}} \, d\omega \right]^{\frac{\theta}{\theta-1}}$$

such that θ denotes the elasticity of substitution between the differentiated varieties. Each period only a subset of varieties are actually available for consumption and that subset is allowed to be different across countries and vary over time.

The corresponding consumer price index for the Home economy is:

$$P_t = \left[\int_{\omega \in \Omega} p_t(\omega)^{1-\theta} d\omega \right]^{\frac{1}{1-\theta}}$$

The optimal allocation of nominal expenditure of the representative household in the Home country for each differentiated variety ω yields the following demand function:

$$c_t(\omega) = \left(\frac{p_t(\omega)}{P_t} \right)^{-\theta} C_t$$

2.1.2 Firms

A continuum of monopolistically competitive firms in each country produce differentiated varieties $\omega \in \Omega$. To start producing i.e. to enter the market, firms need to pay a sunk cost (f_E) in the form of a labour requirement which is equal to $w_t f_E / Z_t$ in real terms. w_t is the real wage and Z_t is the aggregate productivity in the Home country. Once a firm has entered the market, it draws its productivity from a common distribution $G(z)$ where $z \in [z_{min}, \infty)$. This productivity does not vary over the life of the firm. A firm produces in every period until it is hit by an exogenous "death" shock. In every period, firms face this death shock with probability $\delta \in (0, 1)$, which is independent from their relative productivity. The production function of firms has constant returns to scale with labour being the only production input:

$$Y_t(\omega) = Z_t z(\omega) l_t(\omega)$$

where $Y_t(\omega)$ denote the production of variety ω and l_t is the amount of labour required to produce that. Z_t is the country-specific productivity level, and $z(\omega)$ denotes the productivity level specific to variety ω. The aggregate technology shock, Z_t, has the following stochastic form:

$$ln(Z_t) = \rho_z \, ln(Z_{t-1}) + \varepsilon_{z,t}$$

where $0 \leq \rho_z < 1$ and $\varepsilon_{z,t} \sim N(0, \sigma_z^2)$.

Once firms decide to enter, they produce and sell in the domestic market. Firms can also export, and every period firms decide whether they will do so. To export in a given period, firms need to pay a fixed export cost (f_X) in effective labour units as well as an iceberg cost, τ.[3] Firms decide to export if they extract positive profits from exporting, and this depends on their relative productivity and demand conditions. Only the most productive firms – who can afford this fixed cost as well as the variable iceberg cost – will export.

Each firm produces one variety ω with associated productivity level z, and we index variables by z in the rest of the paper rather than by ω. Each firm maximises profits subject to a downward sloping demand curve. Prices are flexible, and therefore set as a mark-up over marginal costs. The profit maximisation problem of the firms and the pricing decisions are similar to those derived in Ghironi and Melitz (2005) and we refer the reader to the discussion therein for more details.

All firms have the choice to export, but only those with a relative productivity z above a cutoff level ensuring non-negative profits from exporting, $z_{X,t} = inf\{z : \Pi_t^X(z) > 0\}$, will do so. The exporting firms sell their goods both in local and foreign markets. So, while all firms can export, a firm with productivity between z_{min} and the export cutoff level, $z_{X,t}$, will decide to serve only the local market.[4] The export productivity cutoff level $z_{X,t}$ varies with economic conditions, and therefore so does the share of exporting firms or, equivalently, the share of exported varieties. The size of the non-traded sector relative to the traded sector is thus determined endogenously. In particular, given a mass $N_{D,t}$ of firms in the Home country, $N_{X,t} = [1 - G(z_{X,t})]N_{D,t}$ of them also export. This structure is symmetric in the Foreign country; $z^*_{X,t}$ fluctuates endogenously in an isomorphic way.

Following Ghironi and Melitz (2005), we assume that relative productivity is drawn from a Pareto distribution with lower bound z_{min} and shape parameter k which is higher than $\theta - 1$[5]: $G(z) = 1 - (z_{min}/z)^k$. It follows that average productivity of all Home firms and of exporting firms are $\tilde{z}_D = \phi z_{min}$ and $\tilde{z}_{X,t} = \phi z_{X,t}$ respectively, where $\phi \equiv \{k/(k - (\theta - 1))\}^{1/(\theta-1)}$. So the number of Home exporting firms, the extensive margin of exports, can be written as:

$$N_{X,t} = [1 - G(z_{X,t})]N_{D,t} = \left(\frac{z_{min}}{z_{X,t}}\right)^k N_{D,t}$$

[3] The fixed per period costs differs from the sunk costs analysed e.g. in Alessandria and Choi (2007) to the extent these sunk costs cannot be spread out across the periods during which the firm exports, e.g. through borrowing.

[4] The lower bound for idiosyncratic productivity, z_{min}, is below $z_{X,t}$.

[5] Ghironi and Melitz (2005) assume Pareto distribution for firm productivity as this distribution fits firm level data quite well. A Pareto distribution is a skewed and heavy-tailed distribution. As it is heavy-tailed for a finite mean and variance the shape parameter needs to be sufficiently high: $k > 1$ ensures a finite mean and $k > 2$ ensures a finite variance. In our case the mean firm size/sale will be finite when $k/\theta - 1 > 1$.

Using productivity averages we can rewrite the share of exporters as:

$$\frac{N_{X,t}}{N_{D,t}} = \left(\frac{z_{min}\phi}{\tilde{z}_{X,t}}\right)^k \tag{3}$$

Firms enter the market at time t and start their production at $t+1$, so some of these new entrants will die (with probability δ) before starting the production at the end of period t. The total number of firms at period t in the Home country will be equal to the previous period's new entrants, $N_{E,t-1}$, and established firms, $N_{D,t-1}$, who survived from the previous period:

$$N_{D,t} = (1-\delta)(N_{D,t-1} + N_{E,t-1}) \tag{4}$$

The free entry condition implies that firms will enter until the average firm value, i.e. the expected present discounted value of future profits denoted $\tilde{v}_t$, is equal to the entry cost:

$$\tilde{v}_t = w_t f_E / Z_t \tag{5}$$

We refer the reader to Ghironi and Melitz (2005) for the derivation of other useful firm averages and to Appendix B for the full list of equilibrium equations relating to firms.

2.1.3 Budget Constraint

Households finance their expenditure through labour income and holdings of home and foreign denominated bonds. In addition households buy the shares from a mutual fund and receive dividends in return, x_t . Households can trade these shares domestically. Each period the mutual fund pays the entry costs, collects the profits and distributes them to the owners of the shares. We assume that labour income is subsidised at a constant rate, σ. International asset markets are incomplete in the sense that households are able to trade only nominal bonds. We follow Benigno (2001) in modelling the incomplete asset market structure. Households in the Home country can hold two kinds of nominal bonds; one is denominated in units of the home currency and the other is denominated in the foreign currency. However, the bonds issued by the Home country are not traded internationally for simplicity. Households in the Home country face an additional cost when they take a position in the foreign asset market. As discussed in Schmitt-Grohe and Uribe (2003), we thus avoid non-stationarity in the model, as the cost function $\Theta(.)$ ensures a stationary distribution of wealth across countries.[6] The budget constraint in real terms is:

$$\begin{aligned} C_t + B_{H,t+1} + \tilde{v}_t(N_{D,t} + N_{E,t})x_{t+1} + \frac{Q_t B_{F,t+1}}{\Theta(Q_t B_{F,t+1})} + TX_t = (1 + r_t)B_{H,t} + (1 + \sigma)w_t L_t \\ + Q_t(1 + r_t^*)B_{F,t} + (\widetilde{\Pi}_t + \tilde{v}_t)N_{D,t}x_t \end{aligned} \tag{6}$$

where $B_{H,t}$ and $B_{F,t}$ are home and foreign currency nominated bonds, $(1+r_t)$ and $(1+r_t^*)$ are the gross real interest rates paid on these respective bonds. TX_t denotes the lump sum taxes paid by the household, Q_t is the real exchange rate and $\widetilde{\Pi}_t$ denotes the average profit of Home firms.

Households make the intertemporal decision by maximising (1) subject to (6). This yields the following Euler equations for bonds and share holdings respectively and, combined with the analogous foreign conditions, the UIP condition:

$$1 = \mathrm{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{\chi_{t+1}}{\chi_t} \right) (1 + r_{t+1}) \right] \tag{7}$$

$$\tilde{v}_t = \mathrm{E}_t \left[\beta(1-\delta) \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{\chi_{t+1}}{\chi_t} \right) (\tilde{v}_{t+1} + \widetilde{\Pi}_{t+1}) \right] \tag{8}$$

$$1 = \beta(1 + r_{t+1}^*)\Theta(Q_t B_{F,t+1})\mathrm{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{\chi_{t+1}}{\chi_t} \right) \frac{Q_{t+1}}{Q_t} \right] \tag{9}$$

Given the rejection of the UIP condition at the empirical level[7], we introduce an exogenous shock denoted ψ_t to Equation 9:

$$1 = \beta\,\psi_t(1 + r_{t+1}^*)\Theta(Q_t B_{F,t+1})\mathrm{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{\chi_{t+1}}{\chi_t} \right) \frac{Q_{t+1}}{Q_t} \right] \tag{10}$$

where ψ_t has the following autoregressive process:

$$ln(\psi_t) \;=\; \rho_\psi\, ln(\psi_{t-1}) \;+\; \varepsilon_{\psi,t}$$

with $0 \leq \rho_\psi < 1$ and $\varepsilon_{\psi,t} \sim N(0, \sigma_\psi^2)$.

[6] In order to have a well-behaved steady state in the model, we impose the following restrictions on the cost function: $\Theta(.)$ is a differentiable decreasing function in the neighbourhood of steady state level of net foreign assets and when the net foreign assets are in the steady state level ($B_{F,t} = 0$), the cost function is equal to 1 ($\Theta(0) = 1$). See, Benigno (2001) for a more detailed explanation.

[7] See, for instance, Lewis (1995).

The situation of foreign households is analogous.

2.1.4 Labour Supply and Wage Setting

Expanding on Ghironi and Melitz (2005), we introduce nominal rigidities through labour market frictions. We assume that there is monopolistic competition among households in the labour market, in the sense that households offer differentiated labour services and we assume that wages are set in a staggered fashion. These assumptions imply that monetary policy shocks may contribute to the business cycle. The modelling choice is motivated by Christiano et al. (2005) who find that Calvo-style nominal wage rigidities are crucial in order to account for the estimated response to a monetary policy shock. It is also in line with research such as Olivei and Tenreyro (2007) and Olivei and Tenreyro (2010) showing that wage rigidities and in particular staggered wage setting plays an important role for the transmission of monetary policy.

Following Erceg et al. (2000), an "employment agency" combines individual households' supply using a Dixit-Stiglitz function:

$$L_t = \left[\int_0^1 L_t(i)^{\frac{\theta_w - 1}{\theta_w}} di \right]^{\frac{\theta_w}{\theta_w - 1}}$$

where $\theta_w > 1$ is the elasticity of substitution between labour inputs.

The aggregate nominal wage index in the Home country can be defined as:

$$W_t = \left[\int_0^1 W_t(i)^{1-\theta_w} di \right]^{\frac{1}{1-\theta_w}}$$

The cost minimisation problem of producers gives a downward sloping labour demand curve. The total demand for household i's labour services by all firms is:

$$L_t(i) = \left[\frac{W_t(i)}{W_t} \right]^{-\theta_w} L_t \tag{11}$$

Wages are staggered á la Calvo (1983); in a given period $(1-\xi)$ of households are able to adjust their wages. To choose the optimum wage $\widetilde{W}_t(i)$, households maximise the expected lifetime utility (1) subject to the budget constraint and the labour demand curve (11). The first order condition for this nominal wage setting problem is:

$$\sum_{k=0}^{\infty} (\beta\xi)^k E_t \left[L_{t+k}(i) U_C(C_{t,t+k}) \left((1+\sigma) \frac{\widetilde{W}_t(i)}{P_{t+k}} - \frac{\theta_w}{\theta_w - 1} MRS_{t,t+k} \right) \right] = 0 \tag{12}$$

where $MRS_{t,t+k}$ is the marginal rate of substitution between consumption and labour in period $t+k$ for the household resetting the wage in period t, i.e. $MRS_{t,t+k} \equiv -\dfrac{U_L(L_{t,t+k})}{U_C(C_{t,t+k})}$.

When wages are flexible ($\xi \to 0$), the real wage multiplied by the subsidy will be equal to the mark-up over the marginal rate of substitution:

$$(1+\sigma)\frac{W_t}{P_t} = \frac{\theta_w}{\theta_w - 1} MRS_{t,t+k} \tag{13}$$

In order to ensure a perfectly competitive labour market in the steady state, we assume that the subsidy cancels out the monopolistic distortion in steady state, implying that $(1+\sigma) = \dfrac{\theta_w}{\theta_w - 1}$. The structure is symmetric across countries.

2.1.5 Monetary Policy

The monetary policy instrument is the nominal interest rate paid on bonds. We assume that monetary policy is conducted using a Taylor type rule which targets the domestic inflation rate, denoted π (π^* in country F) and the growth rate of output, denoted Y (Y^* in country F), and that monetary policy is subject to shocks denoted ε_m and ε^*_m.

$$\frac{i_t}{i} = \left(\frac{i_{t-1}}{i}\right)^{\Gamma_{i-1}} \left(\pi_t^{\Gamma_\pi}\right)^{1-\Gamma_{i-1}} \left[\left(\frac{Y_t}{Y_{t-1}}\right)^{\Gamma_y}\right]^{1-\Gamma_{i-1}} exp(\varepsilon_{m,t}) \tag{14}$$

$$\frac{i^*_t}{i^*} = \left(\frac{i^*_{t-1}}{i^*}\right)^{\Gamma^*_{i-1}} \left(\pi_t^{*\Gamma^*_\pi}\right)^{1-\Gamma^*_{i-1}} \left[\left(\frac{Y^*_t}{Y^*_{t-1}}\right)^{\Gamma^*_y}\right]^{1-\Gamma^*_{i-1}} exp(\varepsilon^*_{m,t}) \tag{15}$$

where $\varepsilon_{m,t} \sim N(0, \sigma_m^2)$ and $\varepsilon^*_{m,t} \sim N(0, \sigma_m^{*^2})$. The persistence of the nominal interest rate is determined by Γ_{i-1} (Γ^*_{i-1} in F) and its response to the domestic inflation rate is determined by Γ_π (Γ^*_π in F) while that to output growth by Γ^*_y (Γ^*_y in F).

2.1.6 Market Clearing and the Definition of Some International Variables

Aggregating across households' budget constraints shows that revenue from production (labour income and profits) and bond holdings is invested in new firms as well as used for consumption of domestic and imported goods and bond purchases:

$$w_t L_t + N_{D,t}\widetilde{\Pi}_t = C_t + N_{E,t}\tilde{v}_t + \frac{Q_t B_{F,t+1}}{\Theta(Q_t B_{F,t+1})} - Q_t(1+r^*_t)B_{F,t}$$

The labour market clearing condition can be computed by adding labour demand by the average firm supplying only the domestic market and labour demand by the average exporting firm and by adding the entry cost of f_E/Z_t of labour:

$$L_t = \frac{\theta-1}{w_t} N_{D,t}\, \widetilde{\Pi}_t^D + \frac{\theta-1}{w_t} N_{X,t}\, \widetilde{\Pi}_t^X + \frac{\theta}{Z_t} N_{X,t} f_X + \frac{1}{Z_t} N_{E,t} f_E \tag{16}$$

We define the current account as the change in claims on foreign agents:

$$CA_t = \frac{Q_t B_{F,t}}{\Theta(Q_t B_{F,t})} - (1 + r_t^*) Q_t B_{F,t-1} \tag{17}$$

The total output produced in the Home country is equal to the sum of home consumption, investment plus net exports:

$$Y_t = C_t + C_{X,t} - C_{X,t}^* + N_{E,t} v_t \tag{18}$$

Given our interest in trade dynamics, we also define total exports and imports of the Home country. We measure them as a ratio of output:

$$\frac{X_t}{Y_t} = \frac{Q_t\, N_{X,t} (\tilde{\rho}_{X,t})^{1-\theta} C_t^*}{Y_t} \tag{19}$$

$$\frac{M_t}{Y_t} = \frac{N_{X,t}^* (\tilde{\rho}_{X,t}^*)^{1-\theta} C_t}{Y_t} \tag{20}$$

where X_t and M_t are the exports and imports of the Home country. Both of them are expressed in the home currency.

The real exchange rate we have been using so far is a welfare-based real exchange rate defined as $Q_t \equiv \frac{S_t\, P_t^*}{P_t}$, which includes a variety effect. To see this, note that we can rewrite the price indices as $P_t = N_t^{1/1-\theta} \tilde{P}_t$ and $P_t^* = (N_t^*)^{1/1-\theta} \tilde{P}_t^*$ where the average prices $\tilde{P}_t$ and $\tilde{P}_t^*$ correspond to the data reported in the official statistics. To be consistent with official statistics, we construct a CPI-based real exchange rate which discounts the impact arising from changes in the varieties consumed by using the average prices: $\tilde{Q}_t = \frac{S_t\, \tilde{P}_t^*}{\tilde{P}_t}$. Details on the derivation of this CPI-based real exchange rate feature in Ghironi and Melitz (2005). Note that an increase in the real exchange rate, both for $\tilde{Q}$ and Q, means a depreciation.

2.2 Model With Fixed Share of Exporters: Model-2

One of the key features of our benchmark model, Model-1, is that exports vary both at the intensive and extensive margin. In particular, the extensive margin of trade fluctuates through two key channels: First, through the endogenous entry of firms producing new varieties of which some are exported, and second, through the endogenous determination of the share of exported varieties. Instead, in our second model, which we present here, we do not allow for the share of exported varieties to vary. Therefore, while the extensive margin may still vary as a result of variations in the overall number of varieties produced, it does not vary as a result of changes in the share of varieties exported. Comparing the performance of this model to our benchmark model will allow us to evaluate the importance of fluctuations in the share of exported varieties for our understanding of export dynamics over the business cycle. We here discuss the features of Model-2 which differ from those of Model-1.

In Model-2, the share of exported varieties, $\frac{N_{X,t}}{N_{D,t}}$, does not vary. As is evident from Equation (3), this implies that the relative export productivity threshold is constant and equal to its steady state level: $\tilde{z}_{X,t} = \bar{\tilde{z}}_X \forall t$. The share of exported varieties is constant and equal to:

$$\frac{N_{X,t}}{N_{D,t}} = \left(\frac{z_{min}\phi}{\bar{\tilde{z}}_X}\right)^k \tag{21}$$

The number of exporters is thus not determined by the zero profit export cut-off condition but by $N_{X,t} = \left(\frac{z_{min}\phi}{\bar{\tilde{z}}_X}\right)^k N_{D,t}$. This equation shows that, in this second model, exports vary at the extensive margin only as a consequence of variations in the number of new varieties being introduced into the economy (affecting $N_{D,t}$), not through changes in the share of varieties exported.

For the remaining equilibrium conditions see Model-1.

2.3 Model Without Extensive Export Margin: Model-3

In our benchmark model, the number of exporting firms varies not only because the share of varieties exported varies but also because the number of domestic firms varies over the business cycle. In order to understand the impact of the second channel on the extensive margin of exports and on overall export dynamics, we compare the performance of our benchmark model (Model-1) and of the same model without an endogenous export share (Model-2) to that of a similar model without any endogenous firm entry and thus no extensive margin of exports. This model, which we label Model-3, is similar to Model-2, but ensures that the number of firms in the domestic economy is fixed as well as is the share of those firms which export. This means that the number of domestic firms is constant as well as is the number of varieties exported. Only the intensive margin of exports may vary.

In this model, the free entry condition, Equation (5), no longer holds. Instead, that equation is replaced by a condition ensuring that the number of firm entrants is equal to the number of firm deaths:

$$N_{E,t} = \left(\frac{\delta}{1-\delta}\right) N_{D,t} \tag{22}$$

The remaining equilibrium equations are similar to those for Model-2.

3 Estimation

We investigate how well our models can explain business cycle features and in particular the dynamics of US exports to the euro area. We solve the models by log-linearising the system of equilibrium equations around a symmetric steady state. We calibrate the structural parameters of the model and estimate with Bayesian methods the variance and persistence of shocks using data for the US and the euro area. Contrary to previous studies, we consider the models' performance in the face of a set of shocks, including but not limited to productivity shocks. Allowing for different types of shocks to drive the models' dynamics enables the models to fit the data of importance to export dynamics. In particular, allowing for monetary policy shocks and shocks to the uncovered interest parity condition allows us to match real exchange rate dynamics, which together with (foreign) demand shocks driving foreign consumption are important determinants of exports in our models.[8]

3.1 Calibration

In order to allow for comparison with previous literature, we calibrate the structural parameters of our three models to match the calibration chosen by Ghironi and Melitz (2005). For the parameters present in our model but not in Ghironi and Melitz (2005), we follow other papers with models of the US and euro area.

We present the calibrated parameters in Table 1. In our benchmark calibration, we assume that the deep parameters are symmetric across regions. We set the discount factor to 0.99 so that the steady state interest rate is 4% per year. The value of the coefficient of risk aversion is set to 1.5 and the inverse of the Frisch elasticity is set to 2. The choice of the value of these parameters are standard in the DSGE literature. The value of the elasticity of substitution between differentiated labour inputs is in line with Erceg et al. (2000) and imply a 33% mark-up. We assume that the average

[8] To illustrate the relevance of the different shocks, Figures 8-10 in Appendix E depicts the estimated shock decomposition of exports in each of the three models.

duration of wage contracts is four quarters implying $\xi = 0.75$. The cost of intermediation in the international bond markets is calibrated following Ghironi and Melitz (2005).

We follow Rabanal and Tuesta (2010) in our calibration of the Taylor rule parameters. We rely on their estimation as their model is estimated on US-euro area data and their policy rule has the same functional form. The interest rate smoothing parameter is 0.79, the coefficient related to inflation is 1.3 and that related to output growth is 0.75.

Table 1: Parameter Values

β	0.99	Discount factor
γ	1.5	Degree of risk aversion
η	2	Inverse Frisch elasticity of labour supply
θ	3.8	Elasticity of substitution between goods
θ_w	4	Elasticity of substitution between labour inputs
ξ	0.75	Duration of wages
ω	0.0025	Cost of international financial intermediation
Γ_i	0.79	Interest rate smoothing parameter
Γ_π	1.3	Interest rate sensitivity to inflation
Γ_y	0.75	Interest rate sensitivity to output growth
δ	0.025	Probability of firm death
k	3.4	Dispersion of the productivity distribution
z_{min}	1	Lower productivity bound
fe	1	Entry cost
fx	0.0085	Fixed export cost
τ	1.3	Per unit export cost

Regarding the calibration of parameters related to firm dynamics, we follow Ghironi and Melitz (2005). In particular, we follow their calibration of the probability of firm death and of the productivity distribution across firms. The value chosen for the elasticity of substitution parameter ($\theta = 3.8$) implies a dispersion parameter (k) of 3.4 given a standard deviation of sales equal to 1.67 as reported in Bernard et al. (2003).[9] We normalise z_{min} and f_E to 1. We compute the fixed exporting cost to be consistent with a steady state share of exporting firms equal to 21%, implying that f_X is equal to 0.0085 as in the steady state calibration of Ghironi and Melitz (2005). We fix the value of the iceberg cost, τ, to 1.3. In Model-2 and Model-3, the productivity threshold is fixed. We set the value of z_x to its steady state value in Model-1. Hence the share of exporters in Model-2 and Model-3 matches the steady state share of exporters in Model-1.

After our main analysis using the benchmark calibration presented above, we conduct sensitivity analysis around several parameters. In particular, we focus on the degree of wage flexibility, the per unit export cost and the degree of substitutability between differentiated goods, all of which could be important for the models' overall performance and

[9] This then implies: $1.67 = 1/(k + 1 - \theta)$.

in particular their export performance.

3.2 Estimation Method and Data

We estimate the persistence and variance of shocks by using Bayesian estimation techniques. To obtain the posterior distributions, we run two parallel chains of 500000 replications of the Metropolis Hastings algorithm with an acceptance rate around 25-30% in all three estimations. We discard the first half of the draws. We observe the convergence through Brooks and Gelman (1998) statistics.

The estimation uses quarterly data for the US and euro area over the period 1984Q1- 2014Q4. We use data on GDP, consumption and CPI inflation for each region as well as data on the bilateral real exchange rate.[10] To be consistent with the model specification, we take the log of the data variables, and we demean interest rates and exchange rates while we de-trend output and consumption using the HP-filter.[11] The details on data sources and plots of the observables used in the estimation feature in Appendix A.

3.3 Priors

Our estimation allows for the shock processes to be different across the two regions. Table 2 summarises the prior distributions of the estimated parameters. As the persistence of the shock processes are bounded between 0 and 1, our prior assumes a beta distribution. We set the prior mean of the distribution to 0.85 with a standard deviation of 0.1. The standard deviation of the shocks are assumed to follow an inverse gamma distribution with a mean of 0.1 and a standard deviation of 2.[12]

3.4 Posteriors

The estimation results are reported in Table 2. We present the posterior mean and standard deviation as well as the mode and the 5th and 95th percentiles of the posterior distribution. In Appendix B, we report the posterior distribution of the estimated parameters for all three models (see Figures 5, 6 and 7).

In line with previous literature, our estimation shows a high degree of persistence of productivity shocks. The preference shocks are also relatively persistent though the estimated degree of persistence depends on the model features: Model-3 identifies less persistence in the preference shocks than Model-1 and Model-2. The shocks to the

[10] Since we measure everything in terms of domestic prices in the model, we compute the real GDP data by dividing the nominal GDP by the CPI of the corresponding country. Note that the model counterpart of the real exchange rate data is the log-linearised CPI-based real exchange rate ($\tilde{q}_t$) that we derived in Section 2.1.6.

[11] As we are working with quarterly data, we use 1600 for the smoothing parameter of the HP-filter.

[12] Our prior choice is relatively standard. See, for instance, Smets and Wouters (2004) for the persistence parameters and see, Jacob and Peersman (2013) for the shock variances.

UIP condition are consistently less persistent than the other shocks, as should be expected given frequently observed deviations from UIP.

Turning to the estimations of the shocks' standard deviations, we note that the posterior estimates of the standard deviations of the shocks are relatively high for all models, but especially so for Model-3. This implies that Model-3 does not fit the data very well, and so to get close to fitting the data, large (sometimes off-setting) shocks are required. This likely reflects the fact that in Model-3 some variables are fixed and the inherent volatility of the model is therefore limited. To compensate for limited inherent volatility, the shocks, especially the preference and the UIP shocks, are estimated to be very volatile. We interpret this as a first indication that Model-3 faces difficulties in accounting for observed dynamics in real variables and exchange rates simultaneously, and that Model-1 and Model-2 which allow for more flexibility seem to fare better at doing so.

Finally, it is worth noting that, in line with Smets and Wouters (2004) and Lubik and Schorfheide (2006), the estimated parameter values are fairly similar across the two regions in all three models.

Table 2: Prior and Posterior Distributions

AR(1) Coefficients:			ρ_z	ρ_z^*	ρ_χ	ρ_χ^*	ρ_ψ		
	Prior	Density[1]	B	B	B	B	B		
		Mean	0.85	0.85	0.85	0.85	0.85		
		Std.dev.	0.10	0.10	0.10	0.10	0.10		
Posteriors	Model-1	Mode	0.99	0.99	0.98	0.98	0.63		
		Std.dev.	0.002	0.002	0.006	0.006	0.026		
		Mean	0.99	0.99	0.98	0.98	0.63		
		[5,95]	[0.99, 0.99]	[0.99, 0.99]	[0.96, 0.99]	[0.97, 0.99]	[0.58, 0.67]		
	Model-2	Mode	0.99	0.99	0.97	0.97	0.49		
		Std.dev.	0.002	0.001	0.009	0.009	0.039		
		Mean	0.99	0.99	0.97	0.97	0.47		
		[5,95]	[0.99, 0.99]	[0.96, 0.99]	[0.96, 0.99]	[0.96, 0.99]	[0.41, 0.54]		
	Model-3	Mode	0.98	0.99	0.89	0.89	0.59		
		Std.dev.	0.013	0.009	0.017	0.016	0.031		
		Mean	0.97	0.98	0.89	0.89	0.59		
		[5,95]	[0.95, 0.99]	[0.97, 0.99]	[0.87, 0.92]	[0.86, 0.92]	[0.55, 0.64]		
Standard Deviations:[2]			ε_z	ε_z^*	ε_χ	ε_χ^*	ε_ψ	ε_m	ε_m^*
	Prior	Density	IG	IG	IG	IG	IG	IG	IG
		Mean	0.10	0.10	0.10	0.10	0.10	0.10	0.10
		Std.dev.	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Posteriors	Model-1	Mode	2.14	2.18	4.18	3.64	4.85	0.32	0.34
		Std.dev.	0.138	0.154	1.208	1.144	0.569	0.023	0.023
		Mean	2.16	2.21	4.51	4.20	5.05	0.33	0.34
		[5,95]	[1.93, 2.38]	[1.98, 2.44]	[2.06, 7.14]	[1.70, 7.02]	[4.05, 5.99]	[0.29, 0.37]	[0.30, 0.38]
	Model-2	Mode	2.73	2.77	3.20	2.62	10.9	0.42	0.44
		Std.dev.	0.172	0.190	1.544	0.740	1.490	0.034	0.032
		Mean	2.76	2.80	4.21	3.11	11.53	0.42	0.44
		[5,95]	[2.47, 3.04]	[2.51, 3.09]	[1.74, 7.21]	[1.42, 5.02]	[9.02, 13.95]	[0.37, 0.47]	[0.38, 0.49]
	Model-3	Mode	2.48	2.56	51.27	50.76	51.46	11.19	11.20
		Std.dev.	0.164	0.173	3.318	3.315	6.078	0.732	0.735
		Mean	2.55	2.61	52.10	51.53	51.79	11.28	11.30
		[5,95]	[2.26, 2.82]	[2.32, 2.89]	[46.4, 57.5]	[45.9, 56.8]	[42.6, 60.8]	[10.0, 12.4]	[10.0, 12.4]

[1] B stands for beta and IG stands for inverse gamma distributions.
[2] Standard deviation of shocks are in percentages.

4 Model performance and export dynamics

4.1 Business cycle statistics

In this section, we analyse how well Model-1 – a model that incorporates an extensive margin of trade through both creation of new varieties and firms' endogenous decision to trade those new as well as existing varieties – performs over the business cycle. In particular, we are interested in understanding whether endogenising the introduction of new varieties in the economy and the change in the share of exported varieties allow to better replicate observed export dynamics and therefore business cycle statistics. We start by comparing the performance of Model-1 which has these features with Model-2, where the share is fixed and where the extensive margin only varies through the introduction of new varieties into the domestic market. We then compare these two models in which the extensive margin of trade varies over the business cycle with Model-3, where both the share of exported varieties and the overall number of varieties are fixed so that the extensive margin does not vary.

We first asses the overall fit of each model by comparing their marginal likelihood (see Table 3). Model-1 does best in terms of overall fit to the data since it has the highest likelihood statistics. The ranking of the likelihood functions shows that allowing for variations in the share of exported varieties matters for the model's overall performance as does variations in the number of firms entering the domestic market to produce new varieties. Model-3 has the lowest likelihood, suggesting the importance of incorporating a more complex trade structure for the model dynamics to match the data.

Table 3: Model comparison of marginal likelihoods

Model-1	**Model-2**	**Model-3**
-1978.6	-2110.7	-3354.7

To elaborate on our comparison, we present selected second moments obtained from the HP-filtered data and compare them with the corresponding theoretical moments obtained from each of the models in Table 4. We first note that all three models have difficulties replicating the observed volatility of the real exchange rate, as do many other international RBC models.[13] Moreover, Model-3 implies a higher volatility of consumption than of the real exchange rate. Our data shows that the exports to GDP ratio is 4 times as volatile as GDP and it's correlation with GDP is 0.35. This is consistent with Engel and Wang (2011) who document that exports are highly volatile and pro-cyclical. All

[13] See Heathcote and Perri (2002).

three models do relatively well in replicating the volatility of exports relative to GDP, with Model-2 outperforming the other models, but only Model-3 can account for the pro-cyclicality of exports relative to GDP though the predicted correlation is double the observed one. As a result of the exchange rate being a main determinant of export dynamics in all three models, exports have a very high correlation with the real exchange rate in the models, much higher than in the data. Allowing for variation in the extensive margin of exports does not seem to significantly affect that relationship.

The three models have a similar predictive performance when it comes to the auto-correlations of variables. In particular, Table 4 shows that they under-predict the persistence of exports as well as other variables.

We conclude that while the models with a more complex trade structure (Model-1 and Model-2) overall do better at matching the data, this does not necessarily translate into a better performance of the model in terms of the business cycle statistics related to exports: while the more complex models match the volatility of exports to GDP better, their exports-GDP correlation is counter-factual and the exports auto-correlation is too low.

Table 4: Selected Second Moments

	Data	**Model-1**	**Model-2**	**Model-3**
Std.dev.s (σ)				
RER	5.09	0.83	0.67	0.59
US Consumption	0.76	0.43	0.44	0.70
US Exports/GDP	4.44	3.88	4.54	3.17
Cross-Correlations				
US Exports-GDP	0.35	-0.56	-0.58	0.70
US Exports-RER	0.20	0.95	0.95	0.96
Autocorrelations				
US Exports	0.78	0.47	0.33	0.50
US GDP	0.81	0.66	0.63	0.60
US Consumption	0.89	0.68	0.62	0.68
RER	0.80	0.47	0.35	0.53

Note: The moments are calculated for the US and euro area data for the period 1999:Q1-2014:Q4. All series are HP-filtered and in logarithms. The model counterpart of the statics are obtained by using the posterior mean. We also HP-filter the model variables. The standard deviation of variables are reported relative to the standard deviation of the US GDP.

4.2 Model Fit to Export Dynamics

4.2.1 Export Determinants

A natural next step in our model comparison is to analyse more in detail the models' performance in replicating observed export dynamics. In our models, the determinants of export fluctuations are summarised by Equation (19) which in log-linear terms can be written as:

$$\hat{X}_t = \hat{Q}_t + (1-\theta)\tilde{\rho}_{X,t} + \hat{N}_{X,t} + \hat{C}_t^* \tag{23}$$

As in standard international RBC models, exports change when the demand for existing exported goods, $\hat{C}_t^*$, and prices - export prices and exchange rate movements, $\tilde{\rho}_{X,t}$ and $\hat{Q}_t$ - change, but also as a result of changes in the number of exported varieties, $\hat{N}_{X,t}$.

More specifically, a depreciation of the exchange rate, i.e. an increase in Q_t, increases the demand for exports. The effect of changes in export prices depend on the value of the elasticity of substitution, θ. As long as θ is greater than 1, exports will be negatively related to its price.

On the demand side, Equation (23) shows that in Model-1 and Model-2, foreign demand varies as a result of the extensive margin of exports and not only as a consequence of changes in foreign consumption demand for previously exported varieties. Exports rise when there is an increase in the foreign consumption demand but also when there are more varieties exported. Thus these models incorporate a richer theoretical structure of trade. In the next section, we will investigate whether this structure contributes to our understanding of aggregate export fluctuations.

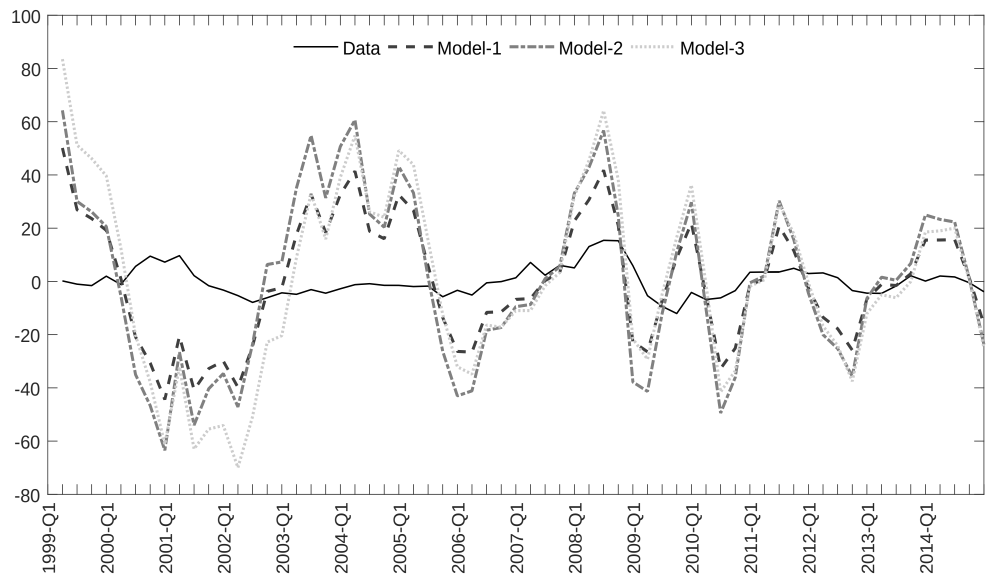
4.2.2 Export Dynamics (Benchmark Calibration)

We now analyse how well Model-1 predicts export fluctuations over the business cycle relative to Model-2 and Model-3. It is important to note that the models are estimated using information on the real exchange rate and on selected real variables but not exports or any other variables related to trade. Given that, we evaluate the models' ability to replicate exports data. We compare our three models' export performance so as to understand whether endogenising the share of exported varieties and the number of produced varieties allows the model to better replicate observed export dynamics. That is, in comparing our three models, we evaluate the role of these two channels in our understanding of US export dynamics.

To analyse the models' export prediction performance, we present the smoothed estimates (i.e. the two-sided predicted values) of the exports to GDP ratio that we obtain through the Kalman filter, along with the US exports to

GDP ratio from the data in Figure 1.[14] The figure shows that all three models have difficulties predicting the exports data. In particular, all three models over-predict the fluctuations in exports and the predictive performance of the three models only differs slightly. This results from the real exchange rate being significantly volatile in the data and all three models predicting a very high correlation between the exports to GDP ratio and the real exchange rate (see Table 4).

Figure 1: Two-sided Predicted Export Values and Actual exports



Note: "Data" is the percent deviation from the HP-filtered trend ratio of nominal US exports to the euro area to nominal US GDP. We also HP-filter the two-sided predicted export values that we obtain from our estimations. The bilateral exports data is only available since the launch of the euro. To match the periods between the data and Kalman filtered estimates, we drop the data points prior to 1999.

In order the quantify the deviations between the predicted and observed values for exports we calculate the root mean square error (RMSE) statistics, see Table 5. The RMSE statistics show that Model-1 predicts exports the best as it does the overall data (see Table 3). The ranking of the RMSE statistics follow our marginal likelihood comparison; the third model has the highest RMSE. However, the difference between the second and the third model is only 2%, hence Model-2 predicts the exports only marginally better compared with Model-3. This seems to indicate that while endogenous variety creation improves the export fit marginally, what may matter more for the model's ability to

[14] The bilateral exports data are only available in nominal terms. What we observe in the data is then:

$$\frac{X_t}{Y_t} = \frac{S_t\, N_{X,t}(\tilde{\rho}_{X,t})^{1-\theta} C_t^*}{P_t Y_t}$$

This could cause measurement problems since in the model we express all variables relative to the CPI. But we evade this issue as a result of the observational equivalence between the above equation and Equation 19.

replicate overall export dynamics over the business cycle is that the share of exported varieties can vary.

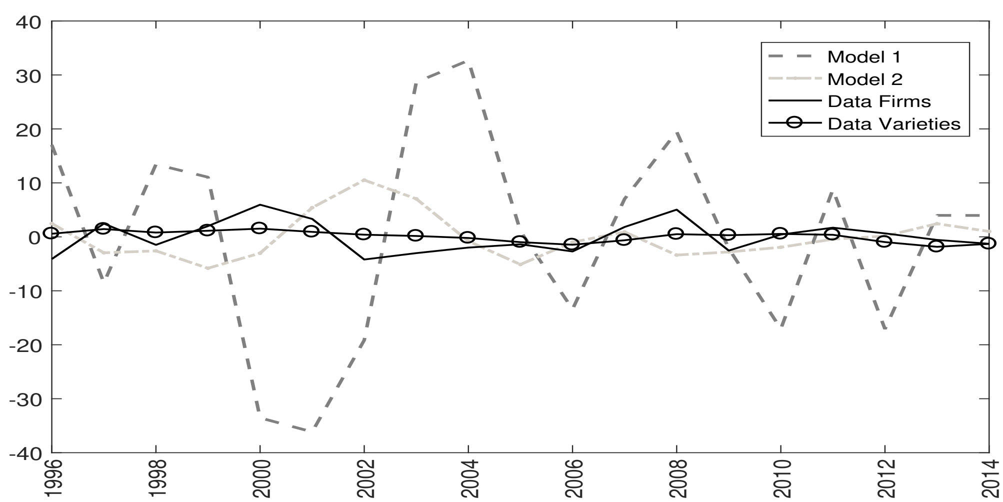
Table 5: Model Comparison of export dynamics

Statistics	**Model-1**	**Model-2**	**Model-3**
RMSE-Exports	22.35	31.44	33.16

Our analysis shows that including variations in the share of exported varieties as well as the total number of varieties within an economy helps the model fit the data and get a little closer to replicating the observed dynamics of exports. It thus appears that incorporating an extensive margin of trade through the channels we consider here improves the model's performance. However, our modelling of the extensive margin of exports does not make substantial progress in our understanding of US export dynamics over the period considered.

To investigate whether Model-1 performs best as a result of actually accounting for the dynamics of the extensive margin, i.e. of N_X, through the two distinct channels previously discussed, we now check whether Model-1 does best at replicating the dynamics of the extensive margin of exports. This allows us to understand whether the performance of Model-1 is related to a better modelling of the extensive margin of trade.

Figure 2: Two-sided Predicted Number of Exporters and Actual Number of Exporters and Exported Varieties



Note: "Data Firms" is the deviations from the HP-filtered trend number of US exporters. As the data is at annual frequency we use 6.25 for the smoothing parameter of the HP-filter. "Data Varieties" is the proportion of 6-digit varieties exported within a 4-digit industry. We take logs and demean this data as it does not carry a trend. We converted the two-sided predicted export values that we obtain from our estimations from quarterly to annual frequency by taking averages and then HP-filtered them. The data on number of exporters is available only after 1996. To match the periods between the data and Kalman filtered estimates, we drop the data points prior to 1996.

To check this we use data for the number of exporters in the US as well as the number of exported varieties and compare these to the two-sided predicted values of $N_{X,t}$ obtained from the estimations of Model-1 and Model-2. That is, we use two sources of data to approximate N_X in our model: The first is the number of exporting firms in the US from the County Business Patterns Census 1996-2014 and Profile of US exporting and importing companies 1996-2014, US Census Bureau;[15] the second is the number of 6-digit varieties exported from the US to the euro area according to UN Comtrade.[16] We consider both those data sources to approximate N_X since, in our models, every firm exports a variety so firm and variety are used interchangeably.

Figure 2 shows that neither Model-1 nor Model-2 comes close to replicating the dynamics of the extensive margin reported in the data, whether that related to the number of exporting firms ('Data Firms') or that related to the numer of exported varieties ('Data Varieties'), and that Model-1 implies an extensive margin of exports that is significantly more volatile than that observed in the data.

We're thus forced to conclude, that under the benchmark calibration which is in line with previous literature and in particular with Ghironi and Melitz (2005), the introduction and our modelling of the extensive margin improves the model fit and the prediction of export dynamics, but this is not the result of an improvement in the performance of the model in relation to the extensive margin of exports. In other words, though more complex trade structures help the model fit the data and to some extent observed export dynamics, this does not result from a good modelling of the extensive margin of exports.

4.3 The Role of the Elasticity of Substitution Between Goods

The elasticity of substitution between traded goods is very important in shaping the domestic and the international transmission of shocks and the dynamics of exports. This has been emphasized in e.g. Corsetti et al. (2008), Heathcote and Perri (2002) or Hjortsoe (2016). Our benchmark calibration, which follows Ghironi and Melitz (2005) and uses a value for the intratemporal elasticity of substitution between goods (θ) of 3.8, implies a relatively high mark-up over marginal costs (around 35%).[17] This elasticity of substitution between traded goods also applies to domestic goods in our model, and while a high elasticity seems reasonable when considering the substitutability among domestically produced goods, it might be too high when applied to the substitutability between domestic and foreign goods. In particular, it could be considered too high in relation to fluctuations over the business cycle, as e.g. argued in Ruhl

[15] We used the total number of exporters, but the data for only the exporters in the manufacturing sector looks very similar.

[16] Because the total number of varieties potentially exported may vary over time (as classification systems used have varied over time), we de-base the number of 6-digit varieties exported by the number of 6-digit varieties which exist in each 4-digit industry at each point in time. This ensures that the variation in our number of varieties does not simply result from a change in the classification system.

[17] It is worth noting that even though the implied mark-up over marginal costs is quite high, the mark-up over average costs is lower due to the presence of fixed export costs. See, Ghironi and Melitz (2005) for further details.

(2008).

We therefore test the sensitivity of our results to the benchmark calibration of the elasticity of substitution.[18] We do so by re-estimating each of the models varying the value of the elasticity of substitution from 2 to 7 with a 0.1 interval.[19]

In line with e.g. Corsetti et al. (2008), we find that a model with a relatively simple trade structure such as Model-3 fits the data better the lower the trade elasticity within the range considered: the likelihood of the model increases as θ decreases and is relatively sensitive to that parameter value. This is shown in the first subplot in Figure 3. However, this is not the case for the models with a more complex trade structure which allow for the extensive margin of trade to vary: Model-1 and Model-2 both see their likelihood increasing with the trade elasticity.

While a low trade elasticity increases the likelihood of Model-3 and reduces the likelihood of Model-1 and Model-2, it consistently increases the export performance of all three models as shown in the second subplot: the RMSE of exports increase with the trade elasticity. Similarly, the lower the trade elasticity the better the fit of the extensive margin of exports as measured by the RMSE, see the third subplot.

Importantly, our main results that Model-1's export performance is better than Model-2's export performance which is better than Model-3's performance is independent of the trade elasticity. Similarly, the fact that this does not result from a better fit to the extensive margin holds for all trade elasticities considered.

4.4 Further Robustness Checks

To ensure that our results are not dependent on the chosen combination of parameter values, and in particular do not depend on parameters which could potentially be important for export dynamics, we here report the robustness of our results to two other key parameters. In particular, we repeat our estimations and statistical analysis but vary respectively the parameter values determining the duration of wage contracts and the variable trade costs (i.e. iceberg costs).[20] We use the same priors and data for estimation as those used in our benchmark analysis.[21]

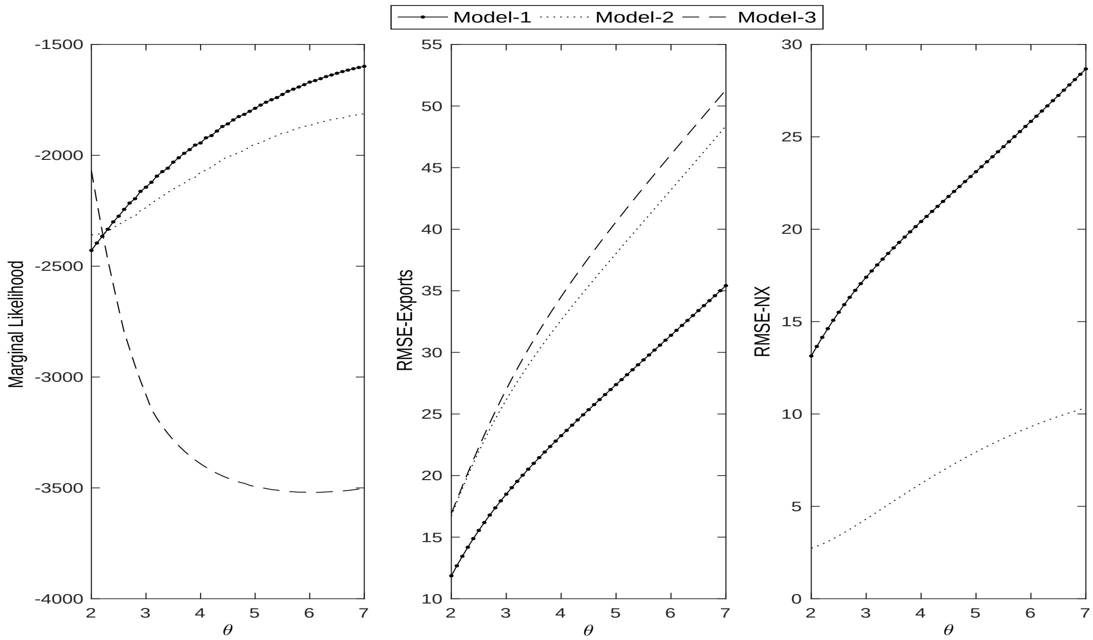
[18] We obtain the posterior distributions by running two parallel chains of 100000 replications of the Metropolis Hastings algorithm. We reduce the number of draws to limit the computation time.

[19] In our benchmark calibration, we ensure that the standard deviation of the log plant sales is consistent with the findings of Bernard et al. (2003). Consistently, we adjust the value of the dispersion parameter (k) so that the standard deviation remains at its initial value. We also make sure that the steady state share of exporters match the data -21%- by adjusting f_X and the steady state values of z_X accordingly.

[20] We also performed robustness checks on the remaining calibrated parameters. Specifically, we have checked the sensitivity of our results to the calibration of the degree of risk aversion, the Frisch elasticity of labour supply, the degree of Calvo wage stickiness, the elasticity of substitution between differentiated labour inputs, the probability of firm death and the monetary policy parameters by carrying out sensitivity analyses for each of these parameters similarly to that for the trade elasticity in Section 4.3. For each parameter, we consider a wide range of plausible values, consistently with Hjortsoe et al. (2018). Our results are robust to the findings presented in the benchmark analysis. In all parameterisations considered, Model-1 has the best overall fit to the data and to exports, but this is not due to a better model fit to the extensive margin of exports. The results of this exercise are available upon request from the authors. Note that we do not perform sensitivity analysis regarding the fixed export costs, f_x, as we adjust it's value such that we match the steady state share of UK exporting firms.

[21] We do not report the posterior statistics and the one-sided prediction results, but they are available on request from the authors.

Figure 3: Statistics for different elasticities of substitution (θ)



Note: The first panel shows the likelihood of each model for different values of θ, the second panel shows that RMSE of exports and the third panel depicts the RMSE of the extensive margin of exports.

Our first robustness exercise consists in varying the average duration of wage contracts, a proxy for labour market flexibility in the model. Given that it can be argued that the labour market in the euro area is less flexible than in the US,[22] we increase the probability of not being able to renegotiate wages (ξ^*) to 0.9 in the Foreign country. This implies that the average duration of wage contracts in the euro area is 10 quarters while it remains 4 quarters in the US. Results from this exercise show that our findings are robust to the degree of rigidities in the labour market (see the first rows of Table 6). As under the benchmark calibration, Model-1 fits the overall features of the data best and also fits the exports dynamics best. However, as previously, that improvement does not arise from a good modelling of the extensive margin of trade.

Our second robustness check consists in investigating the role of variable trade costs, i.e. iceberg costs. Both the steady state level and variations in exports are influenced by this cost and the introduction of such a cost is an important contribution of the framework developed by Melitz (2003). It is also one of the key features that differentiate this model from a standard open economy DSGE model. To see the implications of the value of iceberg costs (τ, τ^*) on our results, we decrease the value from 1.3 to 1.1 meaning that variable export costs amount to 10% of overall costs.

[22] Even though, generally US labour markets are considered to be more flexible than they are in the euro area, the estimates of the Calvo parameter for wages in some cases found to be higher in the US than in the euro area. See, for instance, Smets and Wouters (2004).

The likelihood of the three models with lower iceberg costs as well as the RMSE of exports and the extensive margin of exports are reported in the second block of Table 6. It shows that the conclusions of our analysis is not dependent on the parameterisation of iceberg costs.

Table 6: Model Comparison: Robustness check

Statistics	**Model-1**	**Model-2**	**Model-3**
Models with asymmetric wage stickiness			
Marginal Likelihood	-1890.35	-2028.74	-3371.16
RMSE-Exports	22.36	31.44	33.16
RMSE-NX	19.86	5.85	-
RMSE-NX/ND	19.94	-	-
Models with low variable export costs			
Marginal Likelihood	-2331.86	-2491.86	-4015.01
RMSE-Exports	26.4	39.8	47.4
RMSE-NX	24.7	8.14	-
RMSE-NX/ND	25.0	-	-

5 Conclusion

This paper has aimed to shed some light on the extent to which recent advances in modelling the extensive margin of trade in general equilibrium models is contributing to our understanding of export dynamics over the business cycle. We find that models which incorporate an extensive margin of trade in the spirit of Melitz (2003) and Ghironi and Melitz (2005) perform better in replicating business cycle dynamics as well as export dynamics. However, the improvement is not substantial and importantly does not result from an improvement in our understanding of the underlying dynamics of the extensive margin of exports in response to an array of different types of shocks. We find that these models, when estimated, over-predict the volatility of the extensive margin of exports.

We're therefore forced to conclude that substantial progress in our understanding of the extensive margin of trade, and in particular how the determinants may differ in response to different types of shocks, is still very much needed. We hope that this paper will spark further research in this area and help improve our understanding of not only the extensive margin of trade but ultimately overall export dynamics over the business cycle.

Appendices

Appendix A Data

A.1 Data used in estimation

For the US, we take the quarterly real consumption data from the Bureau of Economic Analysis (BEA). We obtain the US CPI series from the OECD Main Economic Indicators (MEI) database. Nominal US GDP data comes from the OECD, Quarterly National accounts (QNA). It is at current prices in quarterly levels and seasonally adjusted. We take the GDP at current prices and convert it to real GDP by using the US CPI to be consistent with the model deflating method. All data for the euro area as well as the bilateral nominal exchange rate come from the Area Wide model (AWM) (see, Fagan et al. (2001)). The nominal exchange rate is defined as euro per US dollar in the AWM database. Since in the model nominal exchange rate is the Home currency price of the Foreign currency and the US is the Home country, we take the inverse of the series. The population data (for people aged 15 - 64) for the US are taken from the OECD, while historical population data and projections for the euro area are from EUROSTAT. Both population series are available at annual frequency; we transform them into quarterly data by using linear interpolation. To express output and consumption in per capita terms, we divide them by population. The series transformed as explained figure below.

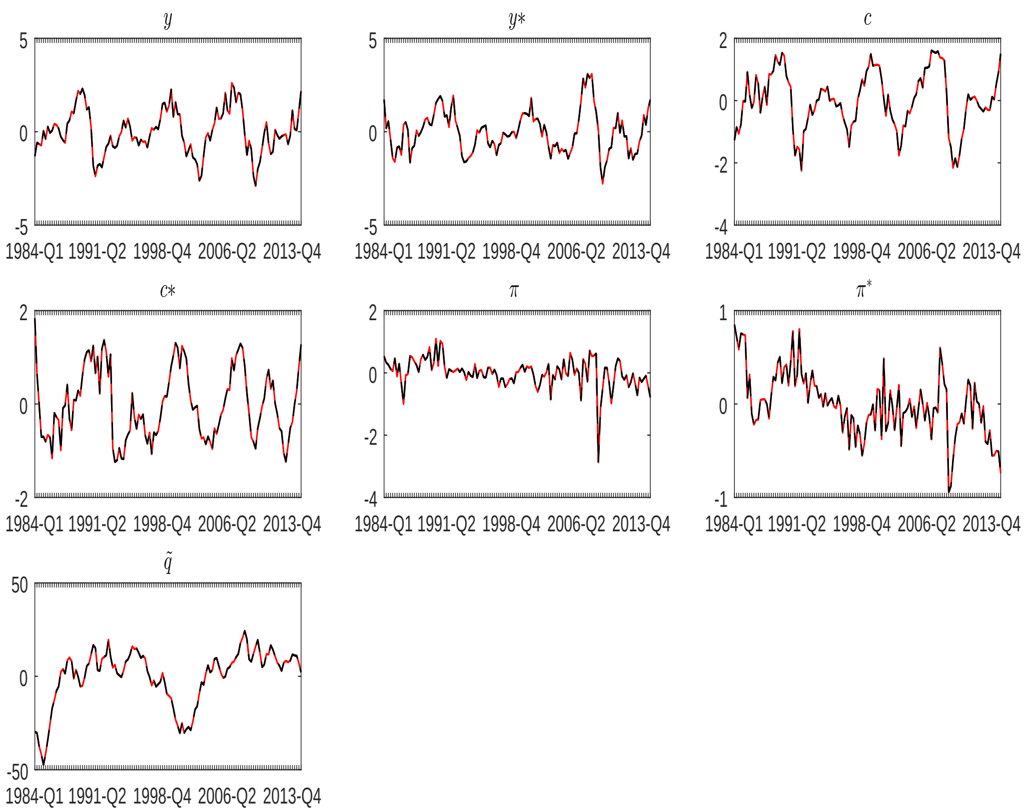
A.2 Additional data

Quarterly bilateral export data is taken from the US Census Bureau. The series are reported in US dollars. We seasonally adjust them by using the X-13 ARIMA method.

The data on the number of US exporting firms comes from the US Census Bureau (see, 'Country Business Patterns Census 1996-2014' and 'Profile of US exporting and importing companies 1996-2014').

To calculate the variation in exported varieties over time, we use the UN-Comtrade data on exports from US to the euro area at 6-digit level.

Figure 4: Data Series



Note: All series are in natural logarithms. c is the consumption of the US, $c*$ is the consumption of the euro area, y is the output of the US, $y*$ is the output of the euro area, π and π^* are the inflation of the US and euro area respectively, finally, $\tilde{q}$ is the bilateral real exchange rate. Consumption and output for both countries are HP-filtered. Inflation and real exchange rate series are demeaned. All series are expressed in percentages.

Appendix B Additional Tables and Figures

Figure 5: Model-1: Prior and Posterior Distributions

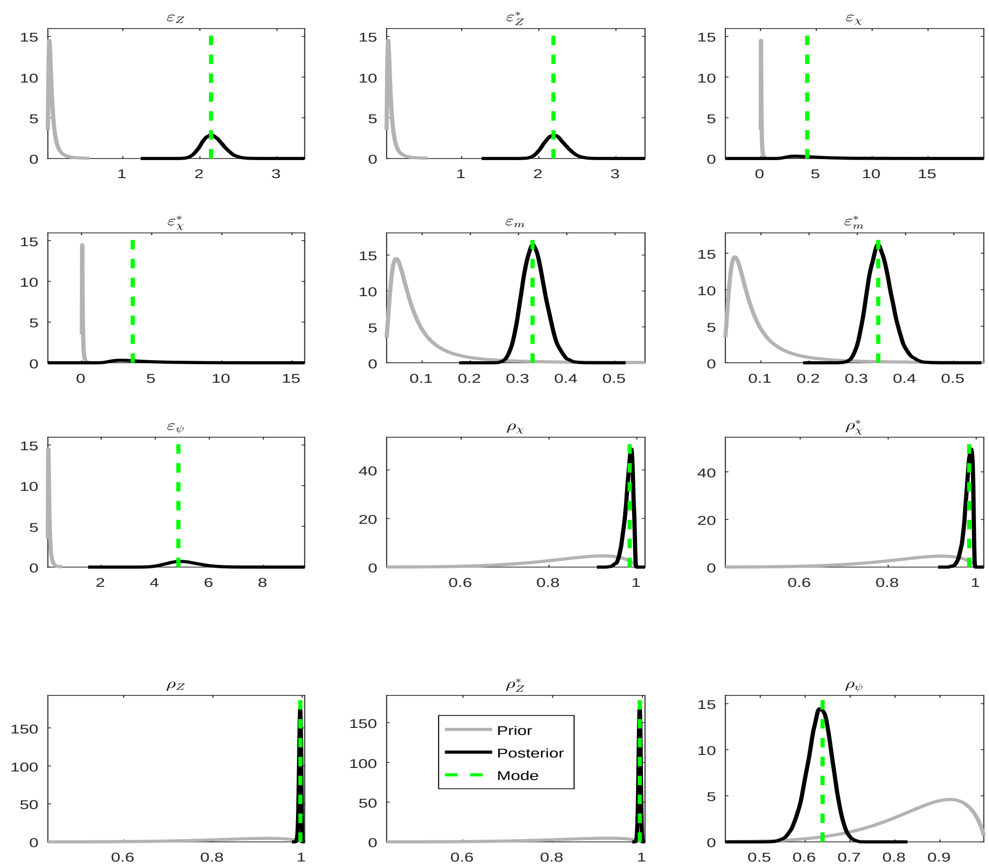


Figure 6: Model-2: Prior and Posterior Distributions

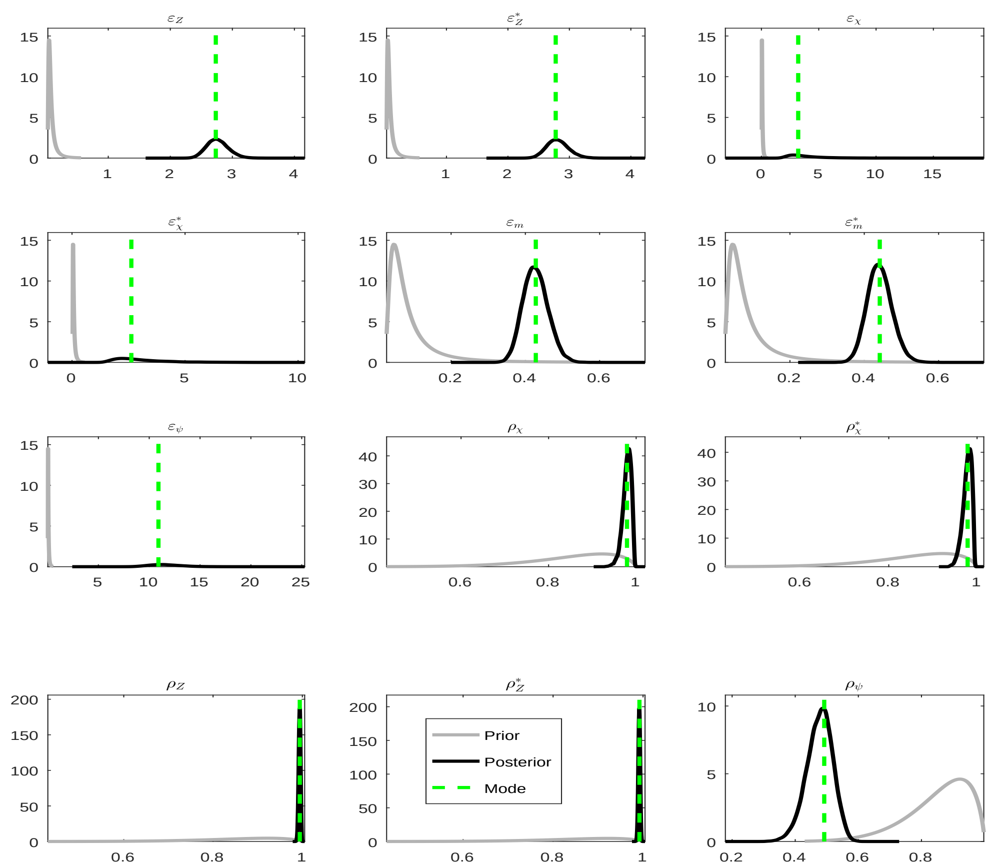


Figure 7: Model-3: Prior and Posterior Distributions

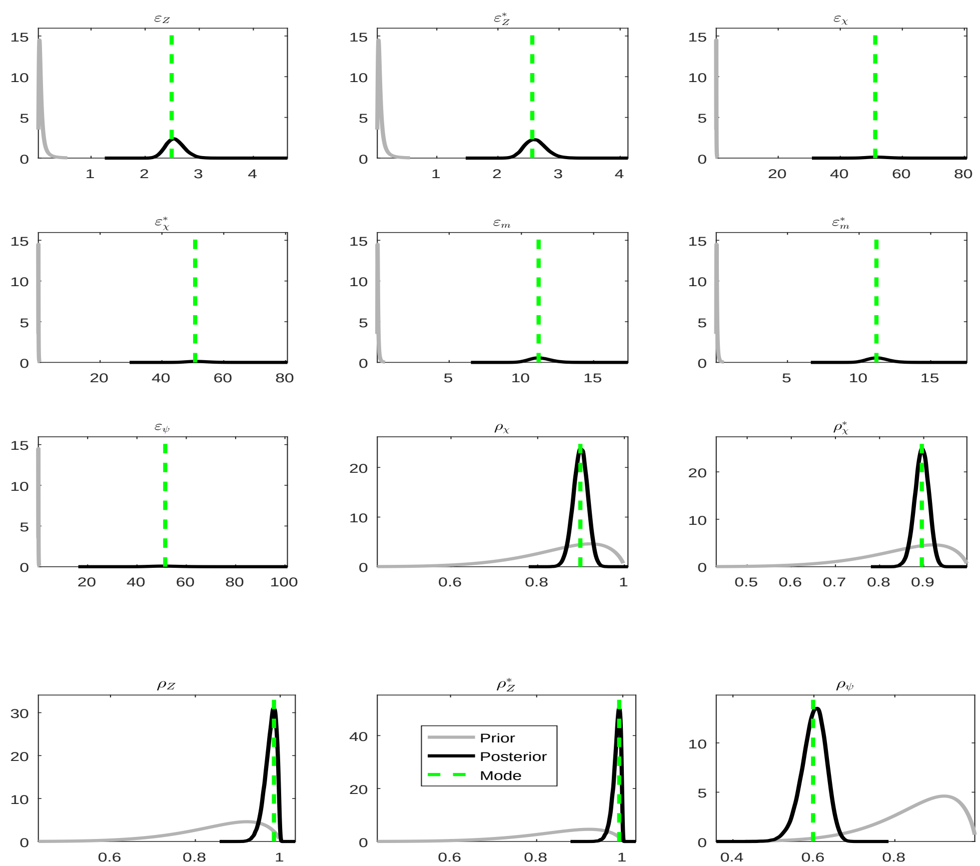


Figure 8: Model-1: Shock Decomposition of Exports

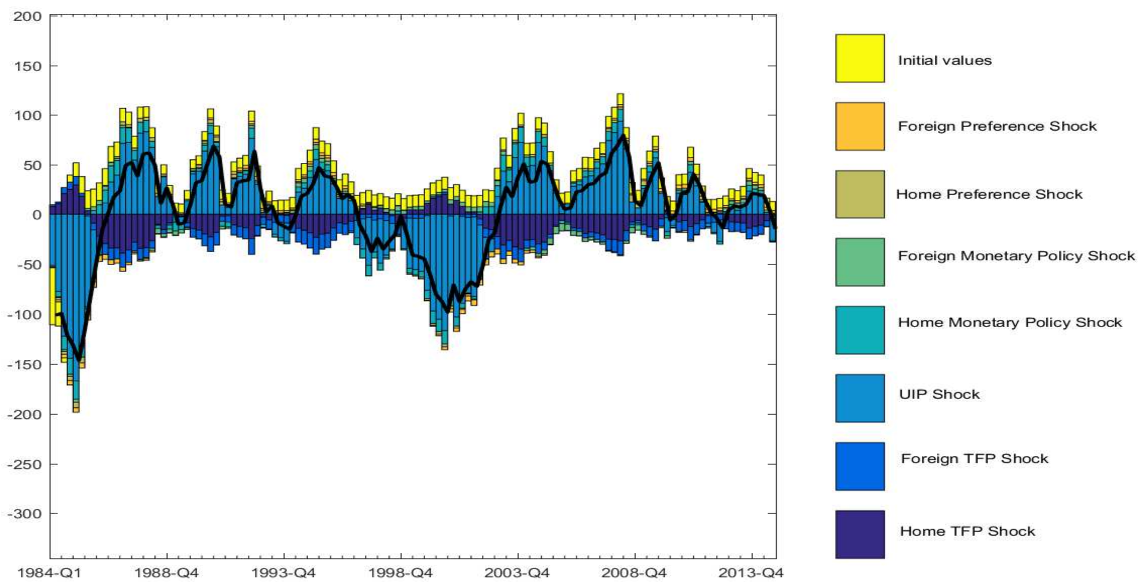


Figure 9: Model-2: Shock Decomposition of Exports

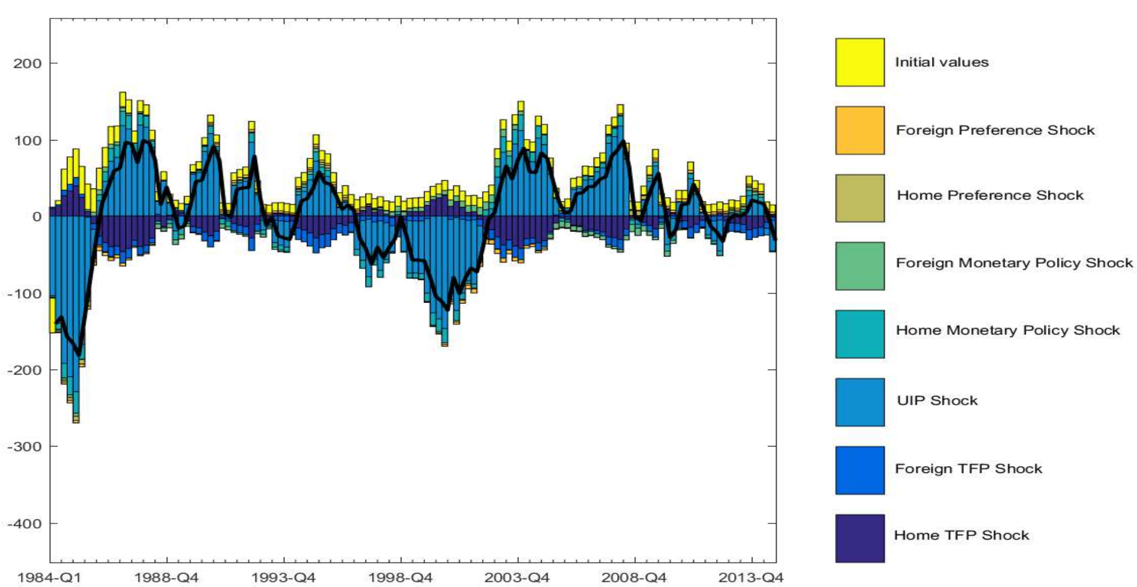
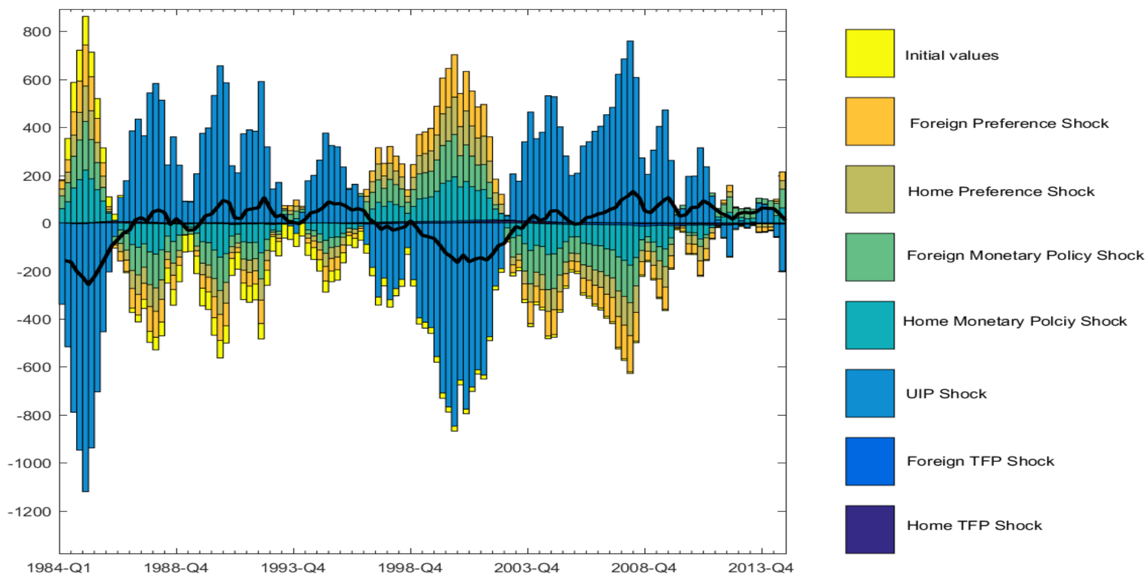


Figure 10: Model-3: Shock Decomposition of Exports



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