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Abstract

We build a semi-structural New Keynesian model with financial frictions to study the drivers of macroeconomic tail risk ('GDP-at-Risk'). We analyse the empirically observed fat left tail of the GDP distribution by modelling three key non-linearities emphasised in the literature: 1) an effective lower bound on nominal interest rates, 2) a credit crunch in bank credit supply when bank capital depletes, and 3) deleveraging by borrowers when debt service burdens become excessive. We obtain three key results. First, our model generates a significantly fat-tailed distribution of GDP – a finding that is absent in most linear New Keynesian and RBC models. Second, we show how these constraints interact with each other. We find that an economy prone to debt deleveraging will experience significantly more credit crunch and effective lower bound episodes than otherwise. Moreover, as the effective lower bound becomes more proximate, the frequency of credit crunch episodes increases significantly. As a rule of thumb, we find that each 50 basis point decline in monetary policy headroom requires additional capital buffers of 1% of assets or 2%–2.5% points lower debt service burdens to hold the risk level constant. Third, we use the model to generate a historical decomposition of GDP-at-Risk for the United Kingdom. The implied risk outlook deteriorates significantly in the run-up to the Global Financial Crisis, driven by depleted capital buffers and increasing debt burdens. Since then, GDP-at-Risk has remained elevated, with greater bank resilience and lower debt offset by the limited capacity of monetary policy to cushion adverse shocks.

Key words: Financial crises, bank capital, debt deleveraging, macroprudential policy, effective lower bound, GDP-at-Risk.

JEL classification: G01, G28.

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1. Introduction

Monitoring risks to financial stability is now a core part of many central banks' mandates. A typical risk monitoring framework is focused on identifying build-ups of vulnerabilities in the financial system – examples of which include accumulations of debt in different sectors, deteriorating lending standards, asset valuations that appear elevated by normal metrics, increasing leverage, and funding structures that rely on asset markets remaining liquid.[1] Despite the progress that has been made in this area, there remains no acceptable method for aggregating these developments into an overall judgement of the outlook for financial stability – an analogue of the role played by inflation forecasts in monetary policy analysis. Without this, financial stability analysis is blunted and lacks objective criteria to evaluate the effectiveness of macroprudential policies.

For this reason, one of the most exciting developments in financial stability research in recent years is the notion that we might quantify the significance of developments in the financial system in terms of what they imply for tail risks to the macroeconomy. This so-called 'GDP-at-Risk' approach opens up the possibility of adding discipline to central banks' financial stability monitoring efforts.[2] If developments in the financial system can be quantified in the common currency of what they imply for tail risks to the economy, risks can be compared directly and mitigating efforts can target areas of greatest threat. There is also the prospect that such an approach might aid central banks' efforts to communicate risks to the public, and provide a quantitative anchor for calibrating macroprudential tools such as the countercyclical capital buffer (CCyB).

In this paper, we present a novel semi-structural model of GDP-at-Risk. 'Semi-structural' because our model does not provide a micro-founded description of the behaviour of optimising agents; instead we aim to provide a plausible macro-level description of how certain nonlinear constraints interact with macro-financial dynamics to generate tail risk. The constraints we focus on – the protagonists in our model – are: first, an effective lower bound (ELB) on policy rates which reduces the capacity of the central bank to cushion shocks; second, a credit crunch in bank credit supply that takes effect when bank equity capital is sufficiently depleted; and third, deleveraging by borrowers when debt service ratio (DSR) burdens become excessive. The mechanisms modelled by these constraints have taken centre stage in developments in the global economy over the past decade and have featured prominently in macroeconomic research since the crisis.[3] A key contribution of this paper is that we are able to capture simultaneously the joint impact of these constraints, in a framework that is both simple and transparent.

A structural model is an important complement to the hitherto purely empirical work on GDP-at-Risk. It allows researchers to conduct 'what if' scenarios such as: how much larger would

[1]Financial Stability Reports such as those published by the Bank of England, the Federal Reserve and the European Central Bank provide good examples of the approach.

[2]See Cecchetti and Schoenholtz (2018) for discussion of the concept and Adrian et al (2018), Adrian et al (2019), and Aikman et al. (2019) for recent empirical analysis. See Cecchetti (2006) and De Nicolo and Lucchetta (2012) for early expositions of this approach.

[3]For example, on the effective lower bound, see Kiley and Roberts (2017) and Bernanke (2017); on credit crunch dynamics, see Gertler and Kiyotaki (2015), Zhu (2008), and Karmakar (2016); and on household leverage, see Mian and Sufi (2010).

GDP-at-Risk be in a low-for-long environment where the effective lower bound is a more proximate concern? In addition, it permits an assessment of the quantitative significance of the constraints as drivers of GDP-at-Risk, including via indirect effects in a general equilibrium setting. And while not the focus of this paper, a structural approach offers the prospect of allowing researchers to assess alternative policy strategies in mitigating risk, such as the benefits of high-static vs dynamically adjusting capital buffers.

We present three key results. First, we show that the model generates a distribution of GDP that is significantly fat-tailed – a feature that is absent from standard New Keynesian models. The 5th percentile outturn is a recession twice as severe as that occurring in a linear unconstrained version of the model, which provides a natural benchmark for thinking about the *normal* level of tail risk we might expect if the economy were sufficiently far from its constraints. GDP-at-Risk in our model is also increasing (in absolute terms) over a policy-relevant time horizon, consistent with empirical evidence.

Second, we explore in detail the role of the three constraints in amplifying tail risk. Employing a 'Shapley value' approach, we find that the bank capital and DSR constraints make only a modest contribution to macroeconomic risk conditional on current capital buffers and debt levels. The effective lower bound makes a material contribution, however. We find an interesting interplay between these constraints: an economy prone to debt deleveraging will experience substantially more frequent credit crunch and effective lower bound episodes; moreover, low capital buffers make ELB episodes more frequent and vice versa. As a rule of thumb, we find each 50 basis point decline in monetary policy headroom requires additional capital buffers of 1% of assets or 2-2.5% points lower debt service ratios in order to hold risk constant.

Third, we use the model to generate a historical decomposition of GDP-at-Risk for the United Kingdom over the past two decades. We show that the depletion in bank capital buffers and the build-up in private sector nonfinancial debt contributed to the worsening of the growth outlook in the run up to the Global Financial Crisis. In the post-crisis environment, the limited space for monetary policy to cushion adverse shocks exerts a material impact, leading GDP-at-Risk to remain elevated for several years after the crisis. Our assessment contrasts with those based purely on macro-financial indicators alone (such as the Basel credit-to-GDP gap), which typically point to risks being subdued in the aftermath of the crisis. Despite the elevated risk level, by 2014 the marginal contribution of the bank capital constraint to risk is approximately zero. This suggests the capital buffers by this date – 2.3% of assets or approximately 6.5% of risk-weighted assets (with an average risk weight of 35%) – were of an adequate magnitude to absorb the 1-in-20 year credit losses the system might face.

The policy implications of these findings are three-fold. First, our results suggest that the macro-financial scenarios used in bank stress tests should be made more severe than 'normal' because of the proximity of the effective lower bound. Second, we find significant spillovers from the DSR constraint to the frequency of ELB and credit crunch episodes, suggesting there are wider macroeconomic stability benefits to reducing debt deleveraging risks. Third, our results have implications for the level of capital buffers required to maintain stability. We find the marginal benefits of increasing usable capital buffers beyond around 2.5% of assets is negligible in terms of reducing the 5% level of GDP-at-Risk over the next 3-to-5 years. There is an important caveat to this statement, however. If there are frictions to maintaining capital buffers at their current level – associated with chronic low profitability in the banking system, for example – then a longer-term perspective is required. In longer-run simulations, our model suggests that in a time of ultra-low interest rates, the marginal benefits of increasing capital buffers remain positive for buffers up to around 4% of assets or 11.5% of risk-weighted assets (with an average risk weight of 35%).

Related literature. A large literature has documented the decline of GDP volatility before the financial crisis of 2008 (see, e.g., Blanchard and Simon (2001); Giannone et al. (2008)). In contrast, the current emerging literature on GDP-at-Risk goes beyond the second moment of GDP growth and analyses the whole conditional distribution of GDP growth.

Existing research has, empirically, quantified how easy current financial conditions can lead to heightened tail risks in the medium term. Adrian et al. (2018) was one of the first papers to demonstrate the term structure of growth-at-risk. Using panel quantile regressions for 11 advanced and 10 emerging market economies, they show that the conditional distribution of GDP growth depends on financial conditions, with growth-at-risk (GaR) – defined as growth at the lower 5th percentile. Aikman et al. (2019) also use a quantile regression approach to examine how the downside risk to growth over the medium term is affected by a set of macroprudential indicators. The authors find that credit booms, property price booms and wide current account deficits each pose material downside risks to growth at horizons of 3 to 5 years and that such risks can be mitigated by increasing the capitalisation of the banking system. Other papers also study the policy implications of borrower-based macroprudential policy measures for GDP-at-Risk (Duprey and Ueberfeldt (2018), Galan (2020), Franta and Gambarcorta (2020)). Adrian et al. (2019) study the conditional distribution of GDP growth as a function of economic and financial conditions. They show that the lower quantiles of GDP growth vary significantly with financial conditions while the upper quantiles remain stable over time. They also argue that amplification mechanisms in the financial sector generate the observed growth vulnerability dynamics. Loria et al. (2019) study the distribution of GDP growth-shifts in response to shocks and find that contractionary shocks disproportionately increase downside risk. The empirical literature is quite novel and enlightening. However, as mentioned above, it has one limitation in that it does not allow us to conduct what if exercises (e.g. how do macro-critical non-linearities affect the tail of the GDP distribution?)

Our paper takes a first step to address this gap in the literature by constructing a semi-structural model featuring crucial non-linearities observed in the data. Adrian and Duarte (2017) present a parsimonious macroeconomic framework for incorporating financial vulnerabilities in monetary policy, starting from the standard New Keynesian models of Woodford (2003) and Gali (2008). Vulnerabilities in the model give rise to a sharply negatively skewed unconditional output distribution, and therefore, captures movements in the conditional GDP distribution that correspond to the downside risk of growth. Duprey nad Ueberfeldt (2020) also propose a small scale DSGE model of growth at risk that relies on risk-shifting behaviour. Our model adds to this emerging theoretical strand of research by explicitly modelling three key non-linearities that are analysed in DSGE models i.e., the debt deleveraging channel, the credit crunch channel and the zero lower bound.

The paper is structured as follows. Section 2 presents the model alongside the three occasionally-binding constraints. Section 3 presents the dataset employed and explains our calibration strategy. In Section 4, we present the main results of the paper. Section 5 analyses spillovers across the constraints. Section 6 uses the model to analyse a historical decomposition of GDP-at-Risk for the United Kingdom. Section 7 concludes with directions for future research.

2. A simple model of GDP-at-Risk

We augment the standard New Keynesian model, similar to that described in Woodford (2003) and Galí (2008), to include nonlinearities associated with three occasionally-binding constraints: (a) an effective lower bound on interest rates, (b) the potential that banks may restrict lending sharply when their capital position becomes impaired, and (c) the potential that the household and nonfinancial corporate sectors delever sharply when their debt service burdens become too large. We first outline the basic structure of the model, before explaining how these constraints are modelled.

The model consists of six equations, which govern the evolution of six endogenous variables: the output gap, y_t, the inflation rate, π_t, the nominal interest rate, r_t, the loan spread, s_t, the level of private nonfinancial sector credit, b_t, and the banking system's aggregate capital ratio, k_t. All variables are defined in terms of deviations from their respective steady states.

Aggregate demand, inflation and interest rates in period t are determined in conventional fashion by 'IS' and 'Phillips' curves and a Taylor rule respectively:

$$y_t = \theta^y y_{t-1} - \theta^r (r_t - \pi_{t-1} + s_t) + \epsilon_t^y + \epsilon_t^d \tag{1}$$

$$\pi_t = \beta^\pi \pi_{t-1} + \beta^y y_{t-1} + \beta^s s_{t-1} + \epsilon_t^\pi \tag{2}$$

$$r_t = \max\left[\bar{r}, (1-\phi^r)(\phi^\pi \pi_t + \phi^y y_t) + \phi^r r_{t-1} + \epsilon_t^r\right] \tag{3}$$

where ϵ_t^y, ϵ_t^π and ϵ_t^r are shocks to output, inflation and the interest rate respectively; ϵ^d is a deleveraging shock, discussed further in the next section. We assume these shocks are orthogonal and follow first-order autoregressive processes $\epsilon_t^i = \rho^i \epsilon_{t-1}^i + u_t^i$, where $u_t^i \sim iid(0, \sigma_i^2)$ for $i = y, d, \pi, r$. $\bar{r}$ is the feasible lower bound on the central bank's monetary policy rate.

Following Curdia and Woodford (2010, 2016) among others, we introduce a role for fluctuations in credit spreads in driving macroeconomic variables. Higher credit spreads push down on aggregate demand via the IS equation by increasing the interest rates facing households and firms wishing to borrow for consumption and investment. Higher spreads also enter the Phillips curve and act as an endogenous 'cost-push' shock, reducing the economy's productive capacity. This channel could be given a number of structural interpretations. In models where financial frictions exist between households and lenders, such as Curdia and Woodford (2010), a cost-push effect such as this derives from the impact of higher loan spreads in distorting households' labour supply decisions. In models where firms face binding credit constraints, such as Gertler and Karadi (2011), higher loan spreads lead to capital shallowing, reducing labour productivity. They also increase the cost of financing firms' working capital needs, as in Carlstrom, Fuerst and Paustian (2010).

The financial side of our model consists of equations for credit demand, borrowing spreads and the evolution of banks' capital ratios:

$$b_t = \gamma^y y_t + \gamma^b b_{t-1} - \gamma^r (r_t - \pi_{t-1} + s_t) + \epsilon_t^b + \epsilon_t^d \tag{4}$$

$$s_t = f^s s_{t-1} + f^b b_t - \tilde{f}^k k_{t-1} + \epsilon_t^s \tag{5}$$

$$k_t = \delta^k k_{t-1} - \delta^r \Delta r_t + \delta^s (r_{t-1} + s_{t-1}) + \epsilon_t^k \tag{6}$$

where ϵ_t^b and ϵ_t^s are exogenous shocks to credit demand and the loan spread respectively and follow first-order autoregressive processes $\epsilon_t^j = \rho^j \epsilon_{t-1}^j + u_t^j$, where $u_t^j \sim iid(0, \sigma_j^2)$ for $j = b, s$.

Real credit demand, which we interpret as reflecting both credit provided by banks and non-banks, including via financial markets, is increasing in contemporaneous output (with elasticity γ^y) and decreasing in the real interest rate inclusive of the credit spread, with persistence γ^b. The supply of finance is assumed to be influenced by the level of bank capital: if capital ratios are eroded, the supply curve shifts leftwards, as governed by the regime-dependent parameter $\tilde{f}^k$, increasing the borrowing spread, s_t. Borrowing spreads are also assumed to be subject to exogenous shocks, ϵ^s. We interpret these shocks are reflecting shifts in investor risk appetite and in the risk bearing capacity of key nonbank intermediaries in financial markets.

Finally, the banking system's capital-to-asset ratio is a key state variable of the model. We assume banks' capital ratios are increasing with a lag in the *level* of nominal interest rates plus spread, but decreasing in the *change* in nominal interest rates. This captures the notion that the banking system benefits in the short-run from a reduction in interest rates, but a period of sustained low rates impairs net interest income.[4] Banks also suffer unexpected write-offs ϵ_t^k on their asset portfolios, the intensity of which varies procyclically according to $\epsilon_t^k = \nu^y y_{t-1} + \rho^k \epsilon_{t-1}^k + u_t^k$.

2.1. Thresholds governing nonlinear dynamics

The model features three distinct thresholds associated with the effective lower bound on nominal interest rates, $\bar{r}$, the level of the banking system's capital ratio, $\bar{k}$, and the debt-service ratio (DSR) of the private nonfinancial sector, $\bar{dsr}$, giving a total of seven potential regimes governing the economy's dynamics.

[4] A richer version of this mechanism would capture a 'reversal rate' effect, by which, beyond a certain point, reductions in interest rates would contract the supply of bank finance (see Brunnermeier and Koby (2019))

Effective lower bound (ELB) regime. We assume the central bank's instrument for monetary policy, the short-term nominal interest rate, is bounded below by $\bar{r}$, limiting the extent to which it can be reduced to stimulate economic activity. We do not consider the potential for quantitative easing or other extraordinary monetary policies to be implemented when short-term nominal interest rates hit this ELB given the ongoing debate about the degree to which these policies are a substitute for traditional interest rate policy (see e.g., Karadi and Nakov (2021)).

Credit crunch regime. We assume credit supply becomes highly sensitive to banks' capital positions when their effective capital buffer is exhausted and k falls below the threshold $\bar{k}$. That is:

$$\tilde{f}^k = \begin{cases} f^{\tilde{k}H} & \text{if } k_{t-1} \leq \bar{k} \\ f^{\tilde{k}L} & \text{otherwise} \end{cases} \tag{7}$$

where $f^{\tilde{k}H} \gg f^{\tilde{k}L}$. We interpret $\bar{k}$ as the effective deleveraging point for the bank, which in general will lie above its minimum regulatory capital ratio, reflecting both precautionary behaviour on the part of a bank and constraints placed on it by market participants.

This assumption is consistent with models that view banks as profit-maximising subject to an occasionally-binding constraint on their capital ratio. For instance, Van den Heuvel (2002) studies a bank's choice of new lending and dividends to maximise its profits subject a regulatory constraint on its minimum capital. A bank in this model will choose to maintain a buffer of excess capital over and above its regulatory minimum to lower the expected cost of future capital inadequacy. The model predicts that the lending decisions of an abundantly-capitalised bank will be close to those of an unconstrained bank, with small changes in capital having little impact on new lending. By contrast, when excess capital is low, the bank will choose to cut new lending aggressively in response to shocks to write offs, even when the constraint itself remains formally slack. Other models that feature a nonlinear relationship between credit supply and bank equity capital include He and Krishnamurthy (2019), Holden, Levine and Swarbrick (2019), von Peter (2009) and Valencia (2014).

Deleveraging regime. We assume the private sector reduces spending sharply to pay down debt when its debt service burden – interest payments on debt outstanding relative to income – exceeds the threshold $\bar{dsr}$.[5] That is, deleveraging occurs when:

$$DSR_t \equiv r_t + s_t + b_t - y_t \geq \bar{dsr} \tag{8}$$

If borrowers attempt to delever by reducing spending en masse, and assuming, as is plausible, that borrowers have a higher marginal propensity to consume than savers, there will be an aggregate demand externality (Eggertsson and Krugman (2012), Korinek and Simsek (2016), Farhi and Werning (2016)). We capture this by introducing a shock, ϵ_t^d, which reduces both credit demand and overall

[5] One feature of expressing the debt limit in terms of the debt service burden is that debt capacity is endogenous to the level of interest rates. A fall in interest rates allows borrowers to systain a high debt to income ratio and vice versa.

aggregate demand in period t. The decline in income initially frustrates the attempt to reduce the debt burden. But over time, the stock of debt falls by more than income in response to this shock because the long-run income elasticity of credit demand exceeds one.

We assume the size of the deleveraging shock in period t is given by the gap betwen DSR_{t-1} and $\bar{dsr}$. That is, the private sector will aim to reduce its debt level by the amount required to return the debt service ratio to the threshold in period t, holding interest rates and overall output fixed:

$$u_t^d = \begin{cases} \bar{dsr} - (r_{t-1} + s_{t-1} + b_{t-1} - y_{t-1}) & \text{if } DSR_{t-1} \geq \bar{dsr} \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

We treat deleveraging as a persistent process that continues to depress the economy for mutliple periods after it occurs: $\epsilon_t^d = \rho^d \epsilon_{t-1}^d + u_t^d$. This ad hoc feature allows us to capture the well-documented persistence of deleveraging episodes (see e.g., Buttiglione et al. (2014)).[6]

3. Calibration

3.1. Data set

For most of the analysis in the paper, we use six quarterly data series from 1992-2014: real GDP, inflation, Bank Rate, total credit (capturing borrowing by the non-financial private sector, from banks and non-banks), lending spreads, and the capital ratio of the UK banking system. Figure A.1 in the Appendix plots the raw time series used in the analysis.[7]

Our measure of bank capital is the tangible common equity ratio (TCE), which is tangible common equity normalised by total tangible assets. Tangible common equity is defined as common equity minus preference shares and intangible assets.[8]

Our series on lending spreads is a weighted average of spreads on three different funding categories: household mortgage credit, consumer credit and corporate credit. The weights, which are consistent with those reported in the February 2014 Inflation Report of the Bank of England, are as follows: household credit is attributed a weight of 0.7 while corporate credit is assigned a weight of 0.3; within household credit, mortgage spreads have a weight of 0.75 (comprising deposit spreads 0.5, low-LTV spreads 0.21, and high LTV spreads 0.04) while consumer credit spreads have a weight of 0.25. Within corporate credit, spreads on small and medium enterprise loans have a weight of 0.37 while those on bond issues by large businesses are attributed a weight of 0.63.

For the purpose of calibrating model parameters, we demean all variables and, with the exception of the loan spread series, pass them through a one-sided Hodrick-Prescott filter with a smoothing parameter of 1,600.

[6] The effects of this shock are qualitatively similar if we treat it as iid by setting $\rho^d = 0$.

[7] We choose to end this exercise in 2014 because the Bank of England introduced limits on permissible loan to income ratios from July of that year; assessing the impact of these constraints on the risk level is beyond the scope of our analysis in this paper.

[8] The TCE measure is strongly correlated with other measures of banking system leverage. It has a correlation of 0.75 with the Bank of England's leverage indicator for the United Kingdom.

3.2. Calibration approach

In high-level terms, our calibration approach is to (a) fix some parameters based on commonly accepted values in the literature, and (b) choose others to match a set of relevant target moments, including standard deviations and cross-correlations of endogenous variables, but also impulse responses and empirical correlations drawn from other studies. We do not attempt to match quantiles of the historical distribution of detrended output in the UK as part of the calibration; tails of this distribution are rare events, and historical outturns are influenced by a range of policy responses that are outside our model (e.g., fiscal relief packages).

Table 1 records calibrated values of the model's parameters. We fix parameters of the Taylor rule and the slope of the Phillips curve to match posterior mean estimates used in the the Bank of England's main macro-forecasting model, COMPASS (as reported in Burgess et al. (2013)). To ensure the baseline linear model without constraints is dynamically stable, we set the endogenous persistence parameters to 0.5, with the exception of inflation persistence, which is set at 0.35. We set γ^y, the short-run income elasticity of credit demand, to 1; together with our assumption for γ^b, this implies a long-run income elasticity of 2. This is consistent with the approximate doubling in the UK's credit-to-GDP ratio over the time period considered (the ratio increased from 68% in 1992 to 141% in 2014).

To simulate the model, we first write the system in matrix form:

$$A_t Y_t = B_t Y_{t-1} + C_t e_t \tag{10}$$

where Y_t is the vector of n endogenous variables in period t, e_t is the vector of exogenous disturbances, A_t is the nxn (invertible) matrix of contemporaneous coefficients, B_t is the nxn matrix of lagged coefficients, and C_t is the matrix of loadings on the shock processes. The matrices A, B and C are state-dependent and determined by which of the seven regimes the economy was operating in last period. To solve the system, we write the model in companion form $Y_t = F_t Y_{t-1} + G_t z_t$ where $F_t \equiv A_t^{-1} B_t$ and $G_t \equiv A_t^{-1} C_t$. For dynamic stability, we require that the moduli of the eigenvalues of F_t all lie within the unit circle.

3.2.1 Target moments

We choose $f^{\tilde{k}H}$, the parameter governing the strength of the bank credit crunch, by assuming that when the capital constraint binds the banking system attempts to keep its leverage ratio constant. This implies that each £1 loan loss is met with a $\frac{1}{\bar{k}}$ contraction in bank credit supply. That is, if $\bar{k} = 5\%$, the banking system will reduce its lending by 20% ($= 1/0.05$) for each % point reduction in its leverage ratio. In mapping this to $f^{\tilde{k}H}$ we further scale this parameter by 50% to reflect the fact that banks account for approximately half of total UK financial system assets (Bank of England,

Table 1: *Calibration of the model's parameters*

Parameter	Description	Value
Macro block:		
θ^y	Persistence of aggregate demand in the IS curve	0.5
θ^r	Interest elasticity of aggregate demand	-0.45
β^π	Persistence of inflation in Phillips curve	0.35
β^y	Slope of the (structural) Phillips curve	0.3
β^s	Impact of spreads on inflation	0.1
ϕ^π	Taylor rule coefficient on inflation	1.497
ϕ^y	Taylor rule coefficient on output gap	0.1512
ϕ^r	Taylor rule coefficient on inflation	0.6
Financial block:		
γ^y	Short-run income elasticity of credit demand	1
γ^b	Persistence of credit demand	0.5
γ^r	Short-run interest elasticity of credit demand	-0.1
f^s	Persistence in loan spread	0.5
f^b	(Inverse) elast of credit supply wrt loan spread	0.05
$\tilde{f}^k$	Elasticity of loan spread wrt bank capital ratio	0.07 (baseline, f^{kL})
		$0.5 f_b/k_{t-1}$ (credit crunch, f^{kH})[a]
δ^k	Persistence of bank capital ratio	0.5
δ^r	(Semi) elast of capital wrt change in nominal interest rate	0.05
δ^s	(Semi) elast of capital wrt level of nominal interest rate	0.01
ν^y	Elasticity of banks' write-offs wrt to output	0.033

Notes: This table presents calibrated values of the model's parameters. (a) We bound $f_{kH} \leq 0.5 f_b/0.02$ to avoid bank deleveraging becoming unboundedly large as k approaches 0.

2016).[9]

We calibrate other key parameters of the model to match certain moments observed in the data and generated by other models in the literature. In particular, we choose the following specific calibration targets:

– First, the responses of output and inflation to an exogenous monetary policy shock in COM-

[9]The macroeconomic implications of this calibration can be compared with Guerrieri et al. (2015), who compare the impact of bank capital shocks across five dynamic general equilibrium models. The transfer shock they consider is worth the equivalent of 7.5% of GDP. Calibrating this for the UK (GDP c. £2tr; Core Tier 1 capital c. £350bn) means a shock of around £150bn, which would wipe out around 40% of banks' capital base. With an average leverage ratio of 3.8% in our dataset, this means a shock to ϵ^k of 1.5% points. Output in our model falls by 2.6% in response to this shock, with credit spreads increasing by 480 bps. The average peak responses reported in Guerrieri et al. (2015) are -2.4% for output and 130 basis points for credit spreads, with ranges in each case of (-0.5%,-5%) and (0,350).

PASS (Burgess et al. (2013));

– Second, the response of banks' net interest margin following an exogenous monetary policy shock in Alessandri and Nelson (2015);

– Third, the relationship between GDP, impairments and bank capital in the Bank of England's annual stress tests of the UK banking system (Bank of England (2018));

– Fourth, the relationship between ex ante household debt growth and ex post GDP declines around financial crisis episodes reported in Aikman et al. (2019).

Table 2 compares the performance of the model to these target moments. In each case, we report peak responses over the relevant horizon. Overall, the fit of the model is reasonable, although there are trade-offs in which moments to hit.

Responses to a 25bps monetary policy shock are close to those in reported in Burgess et al. (2013), with an almost identical output response, but a somewhat smaller impact on inflation (-0.05 % points vs -1.1 % points). To calibrate the relationship between interest rates and banks' net interest margins (as governed by δ_r and δ_s), we match the results reported in Alessandri and Nelson (2015), whereby positive shocks to interest rates initially depress banks' NIMs (as funding reprices more quickly than loans) before eventually exerting a modest positive impact.

To examine the relationship between macro variables and banking system losses and capital, we simulate a macroeconomic stress chosen to match the scenario used by the Bank of England in its 2018 stress test (Bank of England (2018)). In this test, UK output contracts by 4.7%, Bank Rate is assumed to rise to 4%, and investment grade bond spreads increase by 380bps. In our scenario, we apply a combination of IS, Phillips curve and loan spread shocks calibrated to deliver similar responses: our scenario has output contracting by 4.8%, Bank Rate rising to 4.5% and lending spreads rising by almost 340bps. In response to these shocks, the banking system's leverage ratio declines by 1.15% points, closely matching the -1.1% impact on bank capital in the 2018 stress test. However, we find a markedly smaller impact on impairments, which increase by only 0.6% points in our model compared with 1.4% points in the stress test.

3.2.2 Shock processes

We end this section by discussing our approach to calibrating the shock processes – parameters of these processes are reported in Table A.1 in the Appendix. We first generate 5000 simulations from the model of 140 quarters, discarding the first 40 quarters from each. This gives us 5000 samples of simulated data of equivalent length to our dataset.

Each simulation is initialised at the model's steady state. We then compute a variety of statistics from each simulated path, and take the average across all 5000 samples. We calibrate the thresholds relating to the effective lower bound, the credit crunch the debt-deleveraging regimes as follows. We set $\bar{r} = -3.5$, consistent with an effective lower bound of 0% and an average value of Bank Rate over the sample of 3.5%. We set $\bar{k} = -1.5$, reflecting the idea that the UK economy experienced a

Table 2: *Comparison of the model with target moments*

Description	Target	Model	Source
(a) Monetary transmission mechanism:			
Response of inflation to +25 bps Bank Rate	-0.11pp	-0.05pp	Burgess et al. (2013)
Response of output to +25 bps Bank Rate	-0.14%	-0.13%	Burgess et al. (2013)
(b) Sensitivity of net interest margin (NIM) to interest rates:			
SR response of NIM to +100 bps Bank Rate	-0.04%	-0.05%	Alessandri and Nelson (2015)
LR response of NIM to +100 bps Bank Rate	0.004%	0.008%	Alessandri and Nelson (2015)
(c) Relationship between output, impairments and bank capital:			
Response of impairments in the BoE stress test	1.40pp	0.61pp	Bank of England (2018)
Response of bank capital in the BoE stress test	-1.10pp	-1.15pp	Bank of England (2018)
(d) Response of output to household deleveraging:			
Debt and recession severity	-0.37	-0.29	Aikman et al. (2019)
Debt service and recession severity	-0.29	-0.50	Drehmann and Juselius (2012)

Notes: This table compares the performance of the model with target moments as described in the text.

Table 3: *Comparison of the model with UK dataset*

Description	Data	Model
Standard deviations of model variables:		
Std. dev. of output (%)	1.30	1.18
Std. dev. of inflation (% pts)	0.82	0.57
Std. dev. of Bank Rate (% pts)	1.05	1.69
Std. dev. of loan spread (% pts)	0.81	0.77
Std. dev. of credit (%)	3.18	3.53
Std. dev. of bank capital (% pts)	0.28	0.63
Correlations with output:		
Inflation	-0.04	0.37
Bank Rate	0.66	0.42
Loan spread	-0.12	-0.20
Credit	0.19	0.35
Bank capital	-0.35	0.36

Notes: This table compares the standard deviations and output correlations in the data versus those in our model simulations.

credit crunch between 2007-2009, a period when our measure of the banking system's leverage ratio was 1.5% points below its mean. Following similar logic, we set $\bar{dsr} = 10$ as our measure of the debt-service ratio was 10% above its sample mean in these years.

We set the persistence parameter of each shock such that the model's autocorrelation function matches that of the data – see Figure A.3 in the Appendix presents the results of this exercise. We set the variance of the shock processes by attempting to match the standard deviations of each variable with those in the data; in doing so we also put weight on correlations of endogenous variables with output in the model and data. Table 3 reports these statistics. Overall, the standard deviations of GDP, inflation, the loan spread and credit are all close to those in the data. However, we overestimate those of Bank Rate (1.69 vs 1.05) and bank capital (0.63 vs 0.28).

The table also shows correlations of these variables with output. The model matches the cyclicality of Bank Rate, loan spreads and credit well. However, the correlations it generates of inflation and bank capital with output are not close. Inflation is uncorrelated with our measure of detrended output empirically, whereas our model sees these variables as intrinsically linked via the Phillips curve. Consistent with empirical evidence reported in Adrian and Shin (2010), the banking system leverage ratio has a negative correlation with output in the data reflecting the fact that banks operated with ever decreasing capital cushions in the boom prior to the crisis and were forced by regulators to raise capital significantly during the slump. In the model, bank capital is instead procylical – credit losses in the model are declining in output, while net interest income is increasing in output reflecting the beahviour of interest rates.

4. Baseline results

In this section, we report key results from the baseline model. We first plot the density of output in long-run simulations; we then show how tail risk in the model varies over a policy-relevant time horizon.[10]

4.1. Long-run stochastic simulations to explore determinants of GDP-at-Risk

We begin the description of the model's properties by analysing long-run stochastic simulations. This allows us to explore the dynamics of the model in a general setting where the impact of particular assumptions about initial conditions is diminished.

To characterise tail risk, we focus on the q-th percentile of GDP, which we refer to interchangeably as $GaR^q(y)$ or the q-% level of GDP-at-Risk. To measure this, we simulate the model a large number of times and report the average value across simulations. That is, if N is the number of simulations, the 5% level of GDP-at-Risk would be given by:

$$GaR^5(y) \equiv \sum_{n=1}^{N} \frac{1}{N} q^5(y_n) \tag{11}$$

where $q^5(y_n)$ is the 5th percentile of output in simulation $n \in N$. We have also explored an alternative measure of tail risk – the 'expected loss' of output, defined as the expected value of output conditional on output being below its q-th percentile. The results we report are qualitatively unaffected by which metric we focus on.

In general, the model can become unstable in these longer simulations. When monetary policy is constrained at the ELB, banks are restricting credit and debt deleveraging is underway, there is no stabilising force and the economy can be trapped in a situation where unboundedly bad outcomes are possible. To avert this, we assume a bank recapitalisation plan is implemented when the banking system's leverage ratio approaches 0% (falls 5% pts below its steady state level in our baseline calibration). This could be interpreted as a bail-in of the banking system's private creditors or as a taxpayer-funded equity injection. We calibrate this package by assuming it wipes out the current level of bad debt and pushes the system's leverage ratio above the credit crunch threshold. That is, if rc_t is the recapitalisation package and $\bar{\bar{\bar{k}}}$ is the leverage ratio at which the plan is implemented, we have $rc_t = \bar{k} - \bar{\bar{\bar{k}}} - \nu^y y_{t-1}$.[11] Figure A.6 in the appendix illustrates this mechanism.

We centre the results around a baseline simulation in which the economy's steady state values of nominal interest rates, bank capital buffers and private nonfinancial sector debt imply the following distances to respective constraints:

[10] Figures A.4 and A.5 in the appendix present impulse responses from the baseline version of the model alongside the linear no-constraints benchmark, which will give readers a further information about how the model behaves.

[11] We also experimented with a fiscal stimulus package implemented when output falls 10% below trend ($\bar{y} = -10$). If the size of this fiscal package is $f_t = x - (y_t - \bar{y})$, such a scheme would prevent extreme drops in output and establish a floor for the drop in output relative to trend. Our analysis below is unaffected by the inclusion of such a package, although more extreme tails of the output distribution would be affected.

– *Effective lower bound:* The economy has 3% points of headroom above the effective lower bound, consistent with an estimate of R^* of around 1% (Bank of England (2018)), a 2% inflation target and a 0% floor;

– *Credit crunch:* There is 2% points of leverage ratio headroom before the banking system delevers sharply. Given that the UK banking system had a leverage ratio of 5.4% in 2017Q4 in our dataset, this implies banks will rapidly delever at a leverage ratio of 3.4%. With an average risk weight of 35%, our calibration implies the banking system can absorb losses of almost 6% of risk-weighted assets (i.e. $\frac{2}{0.35}$) before sharp deleveraging occurs;

– *Debt deleveraging:* There is capacity for a 10% increase in debt service burden before private sector debt-deleveraging occurs.

Figure 1a plots the density of output in these simulations against a version of the model with all constraints switched off ('linear model'). The distribution of output in the model is unimodal with a heavy left-hand tail: the probability of large declines in output is significantly greater than that in the symmetric Gaussian distribution generated by the linear model: the 5% level of GDP-at-Risk is -2.8% in the baseline model vs -1.7% in the linear model. By comparison, the 5th percentile of the output gap in the historical data used to calibrate the model is -2.9%. In the simulation, the ELB binds 11.1% of the time, the bank capital constraint binds 1.8% of the time and the DSR constraint binds 1.9% of the time.

4.2. Downside risks to GDP over a typical stress test horizon

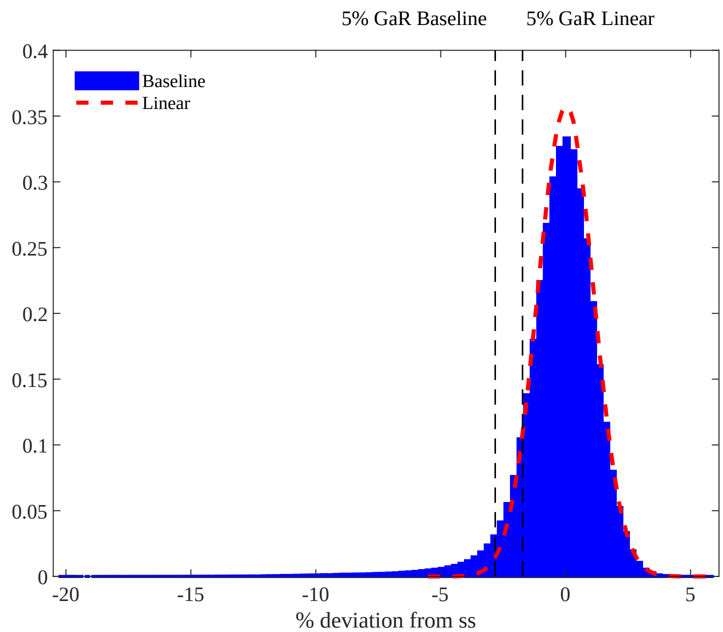
We can also analyse the model-generated distribution of output at particular horizons. We focus here on horizons macroprudential policymakers typically use to inform their decisions, which we take to be out to 20 quarters ahead.[12]

In Figure 1b, we report the locus of 5th percentiles of these distributions from quarter 1 to quarter 20 – the 'term structure' of tail risk to use the terminology of Adrian et al. (2019). GDP-at-Risk is increasing (in absolute terms) in the horizon considered. This reflects two factors. First, via the basic properties of stationary autoregressive processes, the dispersion in outcomes for GDP will increase for an initial transition period in the simulation before reaching its long-run variance. Reflecting this, the 5th percentile of GDP deteriorates sharply for the first few quarters even in the case of the underlying linear unconstrained model. Second, the probability of hitting constraints is increasing in the simulation horizon. Given the level of interest rates, capital buffers and debt service costs assumed in our calibration, these constraints only influence macroeconomic risk at this confidence level at horizons beyond 8 quarters. Evidently it takes time for endogenous tail risks to emerge in the model if the economy is initially in a situation where vulnerabilities are low.

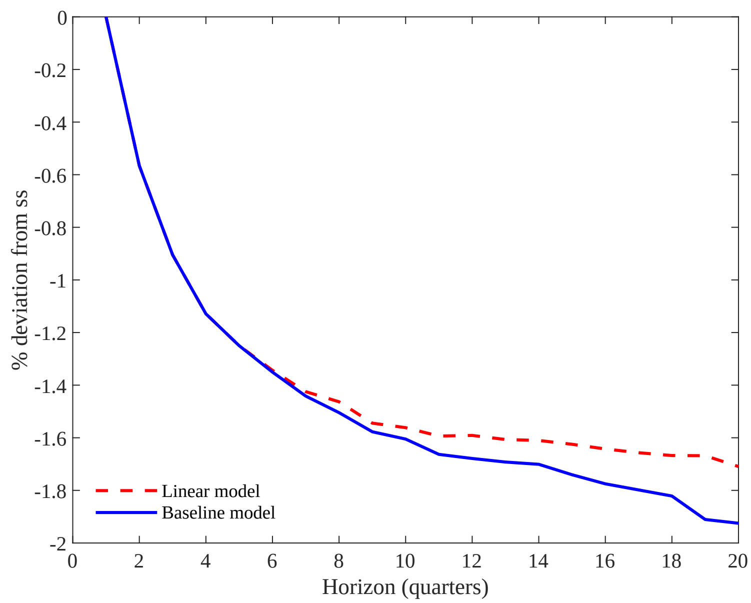
[12]For example, the Bank of England's Annual Cyclical Scenario stress test provides scenario variables out to 20 quarters ahead; the Fed's Comprehensive Capital Analysis and Review stress tests provide scenario variables out 13 quarters ahead. This is also consistent with empirical literature on early warning indicators. For instance, Schularick and Taylor (2012) find that a credit boom is indicative of a heightened risk of a banking crisis up to 5 years ahead.

Figure 1: *GDP at risk in the baseline model*

(a) *The density of GDP*



(b) *The term structure of GDP-at-Risk*



Notes: Figure (a) presents the density of output in the baseline model versus that in a linear benchmark economy where constraints have been switched off. It shows the kernel density estimate of the distribution of output across these 5000 one-hundred year simulations of the model. Figure (b) presents the 5-year term structure of GDP-at-Risk in the model. It shows the locus of 5th percentiles of output at horizons 1 to 20 quarters (shown on the x-axis), obtained from 5000 5-year model simulations.

5. Impact of constraints

In this section, we delve further into the drivers of GDP-at-Risk by analysing the role of the three constraints. In particular, we ask three questions: First, what is the contribution of each to tail risk in our baseline calibration? Second, how do the constraints interact with one another? If one becomes more proximate, what impact does that have on the others? Third, how substitutable are the constraints from a risk perspective? We take each question in turn.

5.1. Contributions to overall tail risk

Given the nonlinear structure of our model, in general the impact of any constraint will depend on the state of the economy, including the proximity of the other constraints considered. A simple approach of adding up the standalone impact of each constraint will therefore not adequately reflect their contributions to the overall risk level. Similarly, an approach of turning on constraints sequentially and recording their marginal impact will yield contributions that are sensitive to the order in which this is done.

Recognising this, we proceed by applying a methodology developed in game theory under which the contribution of a player to an overall outcome is given by her average marginal contribution across all possible subgroups in which she participates. In our context, implementing this 'Shapley value' approach (Shapley (1953)) amounts to simulating all combinations in which constraints are applied and assigning each constraint its average contribution across these combinations – see Appendix B for details.

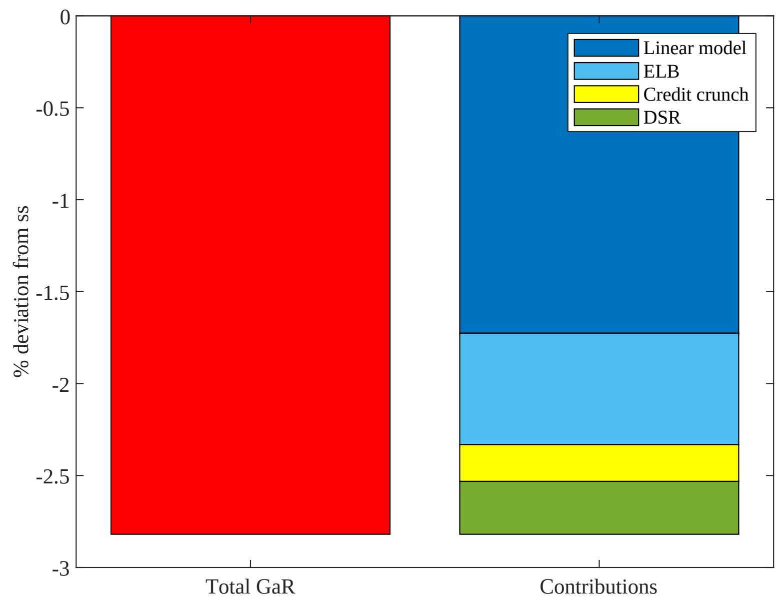
Figure 2 presents the results of this exercise. The left-hand column (red) is the 5% GDP-at-Risk level generated by our model over a large number of one-hundred year simulations – a risk level of -2.8%. The right-hand column shows the Shapley-value contributions of the three nonlinear constraints, plus the baseline linear model. The effective lower bound makes the largest contribution to tail risk, accounting for a full 0.6% points of the fall in output at the fifth percentile (shown in light blue). The contributions of the debt service constraint and bank capital constraint are 0.3% points (green) and 0.2% points (yellow) respectively. These contributions are all incremental to tail risk in the linear model of -1.7% (dark blue).

These contributions are sensitive to the assumed proximity of each constraint. In particular, the relatively smaller contribution of the bank capital constraint reflects the fact that bank capital buffers are currently large by historical standards. In section 6, we use our model to explore how these contributions have fluctuated in importance over recent UK macro-financial history.

5.2. Interactions between constraints

Given the structure of our model, when we change the proximity of one constraint, its impact on the overall risk level will in part reflect how it interacts with other constraints. For instance, a reduction in monetary policy headroom vis-a-vis the ELB will amplify the severity of credit crunch episodes and so part of the impact of the ELB constraint on the overall risk level will in part reflect this spillover.

Figure 2: *Contributions to GDP-at-Risk in the baseline model*



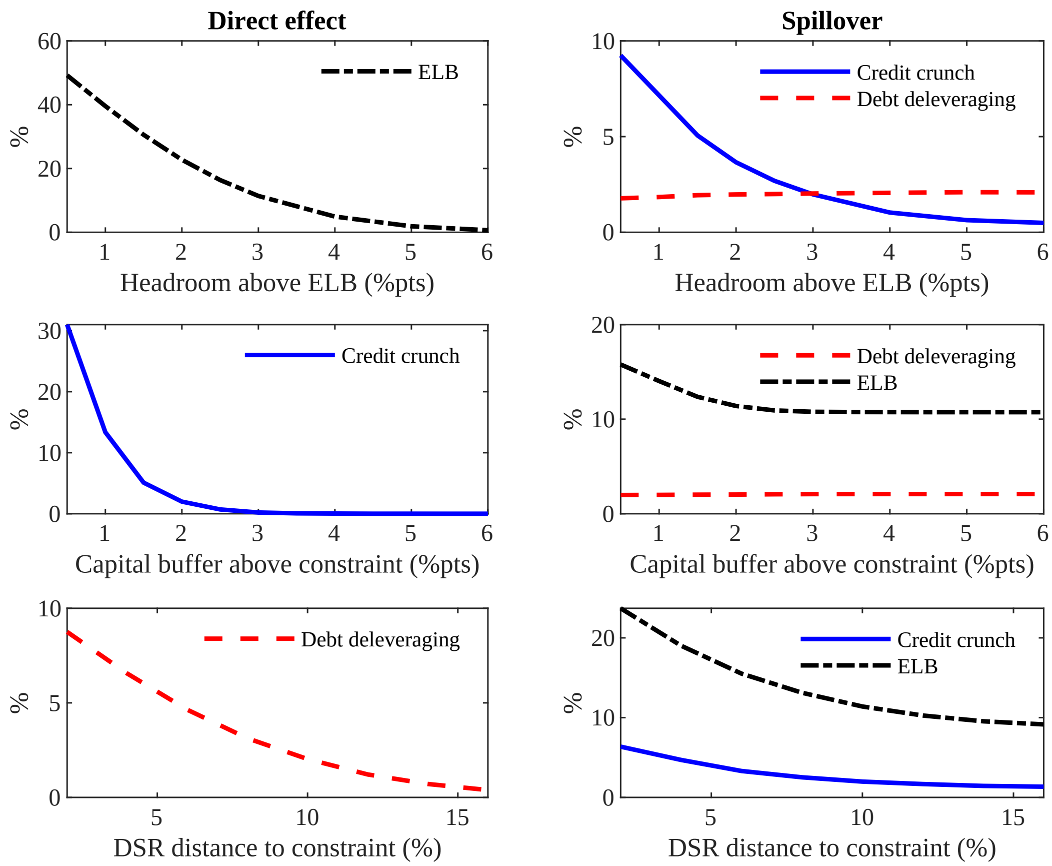
Notes: This figure presents Shapley value contributions of the three nonlinear constraints to the model's 5% level of GDP-at-Risk generated across 5000 one-hundred year simulations. See Appendix B for details of the method.

Figure 3 provides a method for gauging the magnitude of these *cross-constraint spillovers*. The vertical axis on each plot shows the proportion of quarters (in %) with which each constraint binds on average across simulations. The horizontal axis shows the proximity of the bank capital, DSR and ELB constraints.

The top row shows the impact of changing headroom over the ELB constraint. As monetary policy headroom shrinks, the frequency of ELB episodes unsurprisingly increases. There is a material spillover to credit crunch episodes, as shown in the right-hand panel – intuitively when the capacity of the central bank to cushion falls in output diminishes, the result is deeper recessions and hence larger credit losses for banks. There is no impact on deleveraging risk, however. The reason for this is that in a credit crunch, the model predicts a rapid fall in the stock of outstanding credit, alleviating debt service pressure.

The second row performs the same exercise vis-a-vis the bank capital constraint. As buffers shrink, we observe an increase in the frequency of credit crunches. The relationship is highly nonlinear: the marginal impact of an erosion in capital buffers is greater when buffers are already low, with an inflexion point when usable capital falls to around 1.5% of assets (c. 4.5% of risk-weighted assets). The spillovers created are modest, however. The frequency of ELB episodes begins to increase once buffers hit 2% of assets. But there is no impact on the frequency of debt deleveraging episodes. The reason for this is that while loan spreads jump and income falls in credit crunch episodes, the impact on debt service is dominated by falls in Bank Rate and in the stock of credit.[13]

Figure 3: *Frequency of each constraint binding*



Notes: The figure plots the frequencies with which constraints bind as a function of the economy's headroom over its constraints. We simulate the model for 440 quarters, discarding the first 40 and repeating the process 1000 times. We report in the y-axis the average % of simulated quarters for which each constraint binds. The x-axis is the degree of headroom vis-a-vis the stated constraint assumed in the steady state. The first column ('Direct effect') shows the impact of making each constraint more proximate on the frequency with which it binds. The second column ('Spillover') shows the impact on the frequency of other constraints binding.

The bottom row examines spillovers created by the debt-deleveraging threshold. As the DSR constraint becomes more proximate, debt delevereraging frequency increases. Cross-constraint spillovers are substantial in this case – credit crunch and ELB episodes both increase in frequency significantly. The reason is straightforward: recessions become more frequent and more severe when the economy operates closer to the DSR constraint. So a given lower bound on interest rates will be hit more frequently, and banks will suffer larger credit losses, depleting their capital to the credit

[13]A richer model would capture slow adjustment of the stock of credit in such circumstances, creating the potential for a larger spillover from capital buffers to debt deleveraging risk.

crunch threshold more often.

Discussion. To summarise the conclusions from this spillover analysis: Low capital buffers make ELB episodes somewhat more frequent and low equilibrium real interest rates make credit crunch episodes somewhat more likely, but an economy more prone to debt deleveraging will experience substantially more frequent credit crunch and ELB episodes.

An immediate policy implication is that the returns to addressing vulnerabilities that lead to debt delevering (e.g. via borrower-based macroprudential measures) can be material – doing so reduces both debt deleveraging risk itself and the risk of pathologies associated with bank credit crunches and the ELB. Another implication relates to the design of bank stress tests. All else equal, these tests should be made more severe the greater is the proximity of the ELB and the larger the risk of debt-deleveraging.

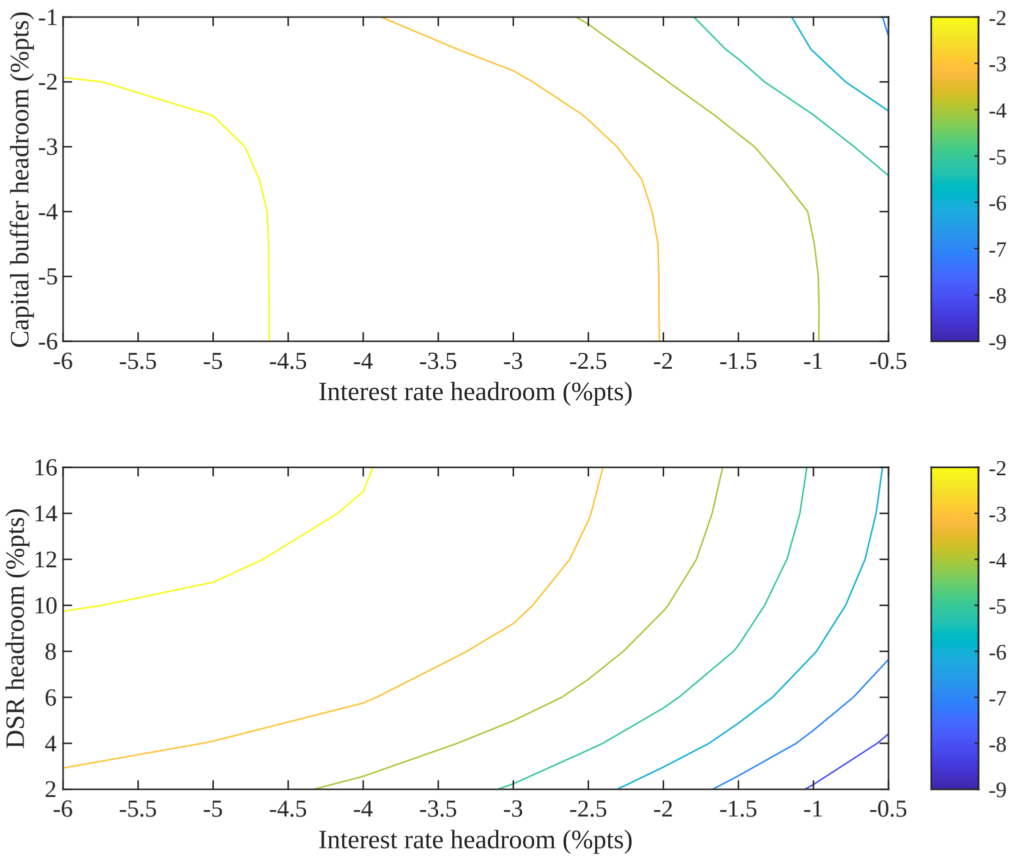
5.3. The 'marginal rate of risk substitution'

We conclude this section by assessing the extent to which the impact of a change in the proximity of one constraint can be offset by making another more distant – a concept we refer to as the *marginal rate of risk substitution*. Figure 4 illustrates this concept via 'iso-risk' curves – that is, combinations of constraints that generate the same level of GDP-at-Risk holding everything else equal. We focus on the marginal rate of risk transformation between interest rate headroom and the other two constraints, holding one of which constant at its benchmark level in each plot.

The iso-risk curves in the upper panel slope downwards, reflecting the fact that diminished interest rate headroom must be accompanied by larger capital buffers if the overall risk level is to remain constant. Within the vicinity of our baseline calibration (interest rate headroom of 3% pts, capital buffers of 2% of assets), the slope of the curve is approximately -2, implying as a rule of thumb that a 1% pt increase in capital buffers is required to 'offset' the impact of interest rate headroom declining by 50bps. The contours become near-vertical once buffers exceed 4% of assets: capital buffers at this level are large enough to absorb losses so increases beyond this have no attenuating impact on risk. It is notable that the contours become more tightly spaced in the north-east region of the plot, indicating that the risk level is increasing more steeply in this region.

The lower panel illustrates the marginal rate of risk substitution with respect to the debt service constraint. Here, the lines slope upwards as, for a given risk level, the economy requires larger headroom vis-a-vis the DSR constraint when interest rate headroom falls. Their convexity indicates that DSR headroom becomes less effective as a risk-mitigant as that headroom increases. In the vicinity of the baseline calibration (DSR headroom of 10%, interest rate headroom of 3% points), the slope of the line is around 4-5, indicating that a 2-2.5% increase in DSR headroom is required to 'offset' the risk impact of interest rate headroom declining by 50bps.

Figure 4: *Iso-risk curves*



Notes: The figure plots combinations of constraint settings that deliver the same level of GDP-at-Risk. The upper panel sets out the marginal rate of risk transformation between interest rate headroom and the level of capital buffers for a given level of DSR headroom (assumed to be 10%, as per the benchmark calibration). The lower panel does likewise for interest rate headroom and headroom vis-a-vis the debt-service constraint for a given level of capital buffer (assumed to be 2% of assets, as per the benchmark calibration). The colourbar shows the corresponding level of GDP-at-Risk at which each contour is drawn, ranging from -2% to -9%.

6. Analysing the drivers of GDP-at-Risk in recent decades

In this final section, we use the model to produce an estimate of fluctuations in GDP-at-Risk in the pre-and post-crisis period, alongside the role played by the constraints we consider. The exercise serves two purposes: first it allows us to gauge the relative importance of different factors in the lead-up to the global financial crisis; second, it provides a useful plausibility check on the behaviour of the model.

To proceed, we first express each threshold in terms of the *level* of the relevant variable, allowing us to capture changes in the proximity of each constraint over the period. For the effective lower bound, we assume a threshold of 0% for the nominal interest rate. To calibrate the bank capital threshold, we appeal to the time series for banks' leverage ratios and choose a threshold consistent with the UK economy as having experienced a credit crunch during the global financial crisis. This implies setting the leverage ratio threshold at 2.3%. We take a similar approach for calibrating the debt-deleveraging threshold. We choose a DSR threshold value of 20%, consistent with debt deleveraging having begun in fourth quarter of 2007.

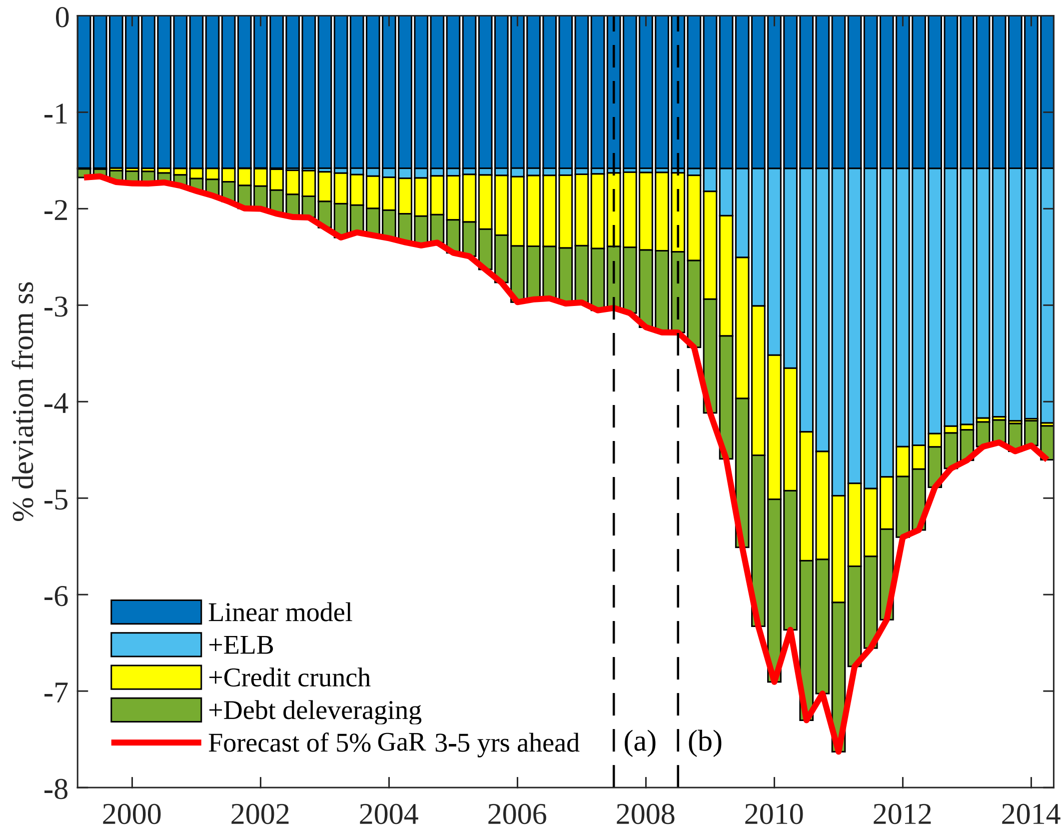
To map from model projections – which describe cyclical fluctuations – into level projections, we add in the trend component for each series. For example, to assess whether the bank capital constraint binds n periods ahead, we check whether $k^c_{t+n} + k^T_{t+n} \leq \bar{k}$, where the k^c_{t+n} is the output of the model. The trend component, which is estimated via a HP filter as described in section 3, is assumed to remain unchanged in the projection, $k^T_{t+n} = k^T_t$. For the debt service ratio, we introduce a measurement error term to explain the deviation between the model-implied DSR and the cyclical component of the DSR time series. We use an AR(1) model to project this measurement error forwards in our simulations, where the persistence parameter is estimated to be 0.92.

We initialise the model using realised data outturns in 1999Q4 and simulate 20 quarters forward, repeating this process 5000 times.[14] Our estimate of projected GDP-at-Risk for 1999Q4 is then given by the average 5th percentile of output across these simulations between quarters 12 and 20. We then iterate forward one quarter to 2000Q1 and repeat the process. We hold the distribution of shocks fixed at each iteration – this reflects the notion that shocks are unpredictable events, whereas changes in the state of the economy are observable and can be conditioned on.

Figure 5 presents the time series of UK GDP-at-Risk obtained by applying this method. The red line presents the full model-generated GDP-at-Risk projection at each date, alongside contributions of the effective lower bound, credit crunch and debt deleveraging constraints. This is the worst recession that might reasonably be expected over the period (up to a given confidence level); it is not a forecast of the most likely path of the economy. The linear model (dark blue bars) provides a useful benchmark for this exercise – it gives the *normal* level of tail risk we might expect if the economy were sufficiently far from the constraints that they can effectively be ignored. This is constant at -1.8% because projections from linear model at this horizon are unresponsive to initial conditions, the effects of which die away.

[14] The DSR series we use is from the Bank of International Settlements, and it covers debt service payments of the UK's private non-financial sector. Figure A.2 in the Appendix plots the raw time series. The series begins in 1999Q4, constraining us to begin the exercise on this date.

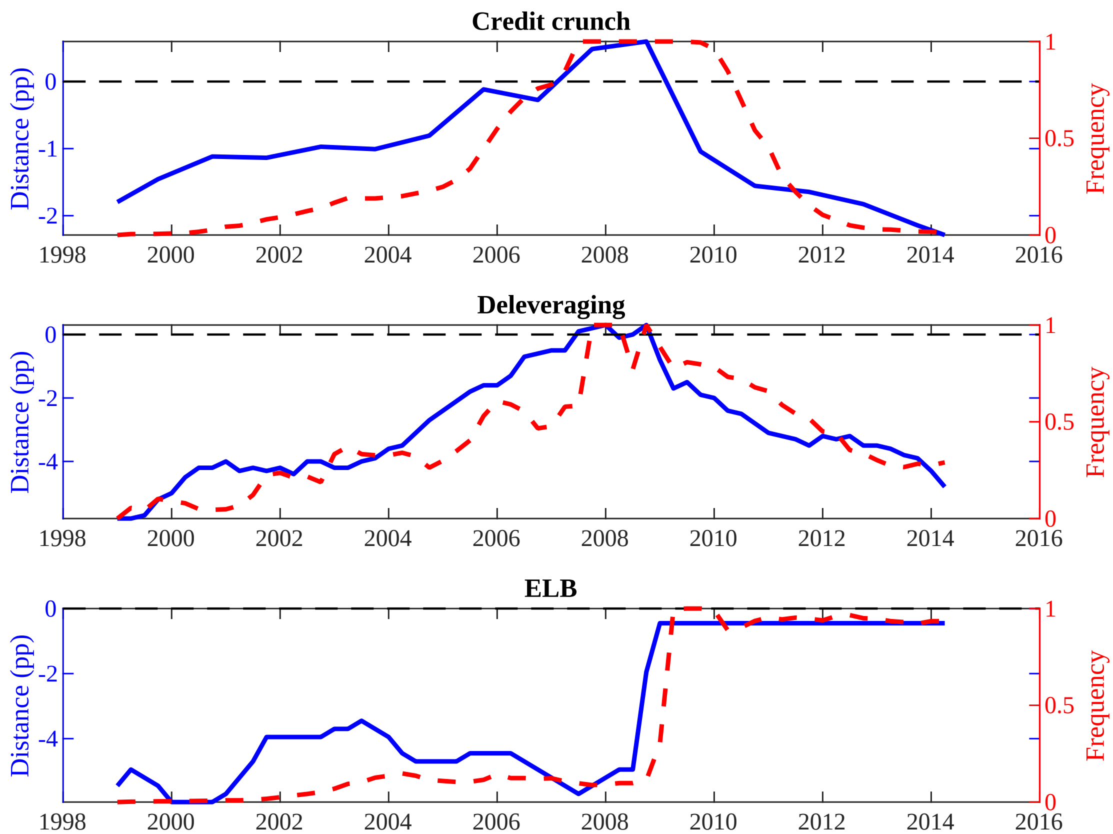
Figure 5: *Historical decomposition of GDP-at-Risk for the United Kingdom, 1999-2014*



Notes: This figure presents the model's projected 5% level of GDP-at-Risk 3-5 years ahead in the United Kingdom between 1999 and 2014. For each date, we condition the model on data outturns and simulate 20 quarters forward, repeating the process 5000 times. GDP-at-Risk for each date is given by the average 5th percentile of output across quarters 12 to 20, averaged across simulations. The bars show the Shapley value contributions of the three nonlinear constraints alongside the baseline linear model. The dashed vertical lines (a) and (b) denote two key dates during the crisis: Northern Rock's liquidity support from the Bank of England (14 September 2007) and Lehman Brothers' filing for Chapter 11 bankruptcy protection (15 September 2008) respectively.

Viewed through the lens of the model, the risk outlook began to deteriorate in the early 2000s, driven by an erosion in banks' capital buffers and an increasing debt burden in the private nonfinancial sector. The likelihood of hitting the bank capital threshold and of breaching the debt service burden threshold therefore grew, as per the rising contributions of the credit crunch and debt deleveraging bars until the global financial crisis. By the first quarter of 2007, a time when the UK economy was booming, the model projects a 5% GDP-at-Risk level over the subsequent 3-5 years of around -3% – almost double the normal risk level. This is below the realised peak decline in GDP of -5.3% in 2009 Q1, however.

Figure 6: *Proximity of each constraint and frequency with which each binds*



Notes: The blue (solid) lines present the evolution of bank capital, the debt service ratio and nominal interest rates in the data relative to their thresholds – the distance to each constraint is shown on the left-hand axis. We inverted distances vis-a-vis the bank capital and ELB constraints so that an inccrease in each plot indicates a move closer to the constraint. The red (dashed) lines show the frequencies with which each constraint binds across simulations. We record a constraint as binding in simulation s if it binds in at least one quarter of the 20 quarter projection. The chart plots average frequencies across 5000 simulations at each date.

After the collapse of Lehman Brothers, Bank Rate was reduced sharply. While this supported the economy in the short-run, it also reduced the scope for the central bank to cushion future shocks. GDP-at-Risk in the medium term therefore deteriorated. Consistent with this, the contribution of the effective lower bound can be seen to grow significantly from late 2008 onwards, leading to a peak risk level of around -7.5% in 2011. Thereafter, the risk outlook began to improve, driven by the long and gradual rebuild of bank capital buffers via the Basel III process, alongside the post-crisis decline in debt service costs. The risk level settled at around -4.5% by mid-2014, with the limited room for further interest rate stimulus being the main cause of higher-than-normal GDP-at-Risk.

This risk profile contrasts significantly with those derived from macro-financial indicators such as the Basel credit-to-GDP gap (see also Aikman *et al.* (2018) which analyses a large number of risk indicators for the United Kingdom). Those measures typically peak during the crisis and then recede sharply. Because our model takes explicit account of the effective lower bound and its interaction with financial constraints, the risk level in our model grows significantly in the years following Lehman Brothers' failure as the ELB becomes an important constraint on monetary policy. The implication is that the UK economy was at risk of a further crisis-level contraction over this period.

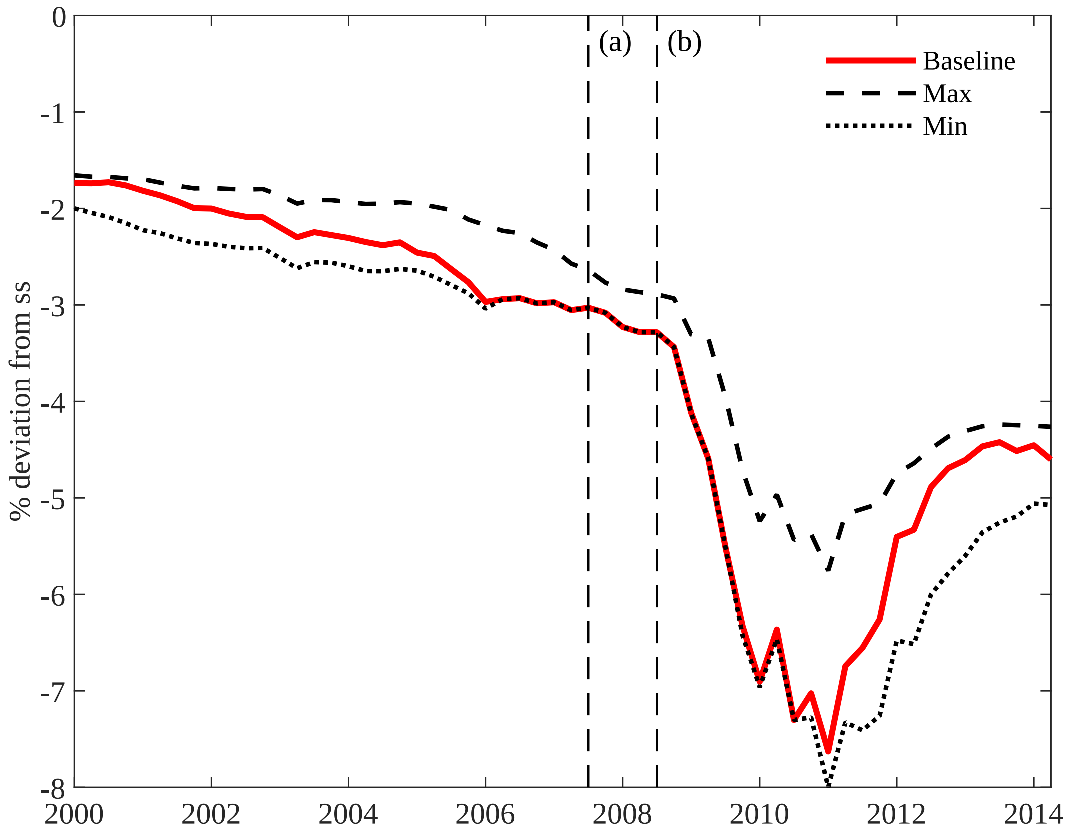
Figure 6 provides a complementary way to visualise these developments; it shows the frequency with which each constraint binds in the simulations, plotted against fluctuations in the proximity of each constraint over our sample. We invert the proximities of the bank capital and ELB constraints so that an increase shows a closer constraint in each case. As is intuitive, the frequency of each constraint binding follows a similar path to its proximity in the data. We can also see the impact of spillovers from the ELB to financial constraints: between 2009 and around 2011, the frequencies of credit crunch and deleveraging episodes remains elevated despite substantial improvements in bank capital buffers and in the debt service ratio.

These precise risk estimates are unsurprisingly sensitive to the thresholds assumed for the credit crunch and debt deleveraging regimes – thresholds that are unobservable and over which there is significant uncertainty. To explore the impact of alternative plausible thresholds, we repeat the simulations above but vary the credit crunch threshold $\bar{k}$ by $\pm 1\%$ points and the DSR threshold by $\pm 2\%$. Figure 7 plots the range of resulting GDP-at-Risk profiles, alongside the baseline estimate repeated from Figure 5. The broad profile of growing risks in the mid-2000s, which then increase materially after Lehman's bankruptcy before receding is robust to these alternative threshold values. The scale of risk estimates is sensitive, however – particularly in the 2009-2012 period. Reassuringly, the variance of the risk estimates diminishes greatly by the end of the period considered.

Policy implications. Our results suggest that the macro-financial scenarios used in bank stress tests should be more severe than 'normal' because of the proximity of the effective lower bound. Despite this, it is noteworthy that by the end of the period higher capital buffers have little attenuating impact – the yellow bars in Figure 5 fall to zero by the end of the period. The policy implications of this are subtle and worth drawing out. It implies that capital buffers of around 2.5% of assets (the 2014 level) are sufficient to absorb the losses that are likely to materialise at this confidence level over a 5 year horizon. Even though the loss distribution will be more adverse than normal given the proximity of the ELB, buffers of this size are adequate to allow the banking system to remain above the credit crunch threshold over this horizon. This contrasts with the finding reported in section 5.3, where higher capital buffers remained effective in reducing risk up to buffer levels of around 4% of assets.

The thought experiments underpinning these exercises differ. The historical decomposition exercise is similar to central banks' macro stress testing programmes. It asks: given the current state of the economy, how is the banking system likely to fare over the next few years if there were a severe adverse shock? For the iso-risk curves, we instead ask: what risk level is it reasonable to expect in the long-run if the steady state capital buffer were a little higher/lower?

Figure 7: *Sensitivity of GDP-at-Risk to alternative constraint thresholds*



Notes: The red (solid) line is the 5% GDP-at-Risk projection 3-to-5 years ahead conditional on realised data outturns and a given distribution of shocks. See the note to Figure 5. The black dashed and dotted lines are the maximum and minimum GDP-at-Risk values each quarter across 4 alternative simulations with $\bar{k} = 2.3\% \pm 1$ and $\bar{dsr} = 20\% \pm 2$. The dashed vertical lines (a) and (b) denote the dates when Northern Rock sought liquidity support from the Bank of England (14 September 2007) and when Lehman Brothers filed for Chapter 11 bankruptcy protection (15 September 2008) respectively.

Our results point to a possible tension between short-term and longer-term resilience assessments. While the stress testing perspective tells us buffers are adequate today, it misses the potential for risks to accumulate, such that today's adequate buffer might be insufficient 2-3 years from now. This would not be a concern if policymakers were freely able to adjust buffers in future as warranted by the risk level. But if there were constraints on this process – either economic constraints caused by chronic low profitability in the banking system such that organic capital generation is not possible, or political constraints associated with the power of the banking lobby – then stress test results need to be interpreted with caution.

7. Conclusion

This paper presents a semi-structural New Keynesian model augmented with three key non-linearities observed in the data: the effective lower bound, household deleveraging, and credit crunch.

The model generates a fat left tail of the GDP distribution and a term-structure of GDP-at-Risk showing that tail risks worsen over longer time horizons. We also show how these constraints interact. Our results show that an economy prone to debt deleveraging will experience significantly more credit crunch and ELB episodes than otherwise. And as the effective lower bound becomes more proximate, the frequency of credit crunch episodes increases significantly. We also use the model to generate a historical decomposition of GDP-at-Risk for the United Kingdom. The implied risk outlook deteriorates significantly in the run-up to the financial crisis, driven by depleted capital buffers and increasing debt burdens. Since then, GDP-at-Risk has remained elevated, with greater bank resilience and lower debt offset by the limited capacity of monetary policy to cushion adverse shocks.

Our results have a number of policy implications. First, our results suggest that the macro-financial scenarios used in bank stress tests should be made more severe than 'normal' because of the proximity of the effective lower bound. Second, there are wider macroeconomic stability benefits to reducing debt deleveraging risks. Third, our results imply that there are marginal benefits of increasing usable capital buffers beyond a certain point in terms of reducing the tail risk over the next 3-to-5 years. The last statement includes a caveat in that over a sufficiently long period of time, in a time of ultra-low interest rates, the marginal benefits of increasing capital buffers remains positive.

In the ongoing work, we are exploring extending the model along a number of interesting dimensions. One natural area of interest is including a role for countercyclical macroprudential tools such as the CCyB, and whether a strategy of adjusting such tools based on forecasts of the risk level is desirable or not.

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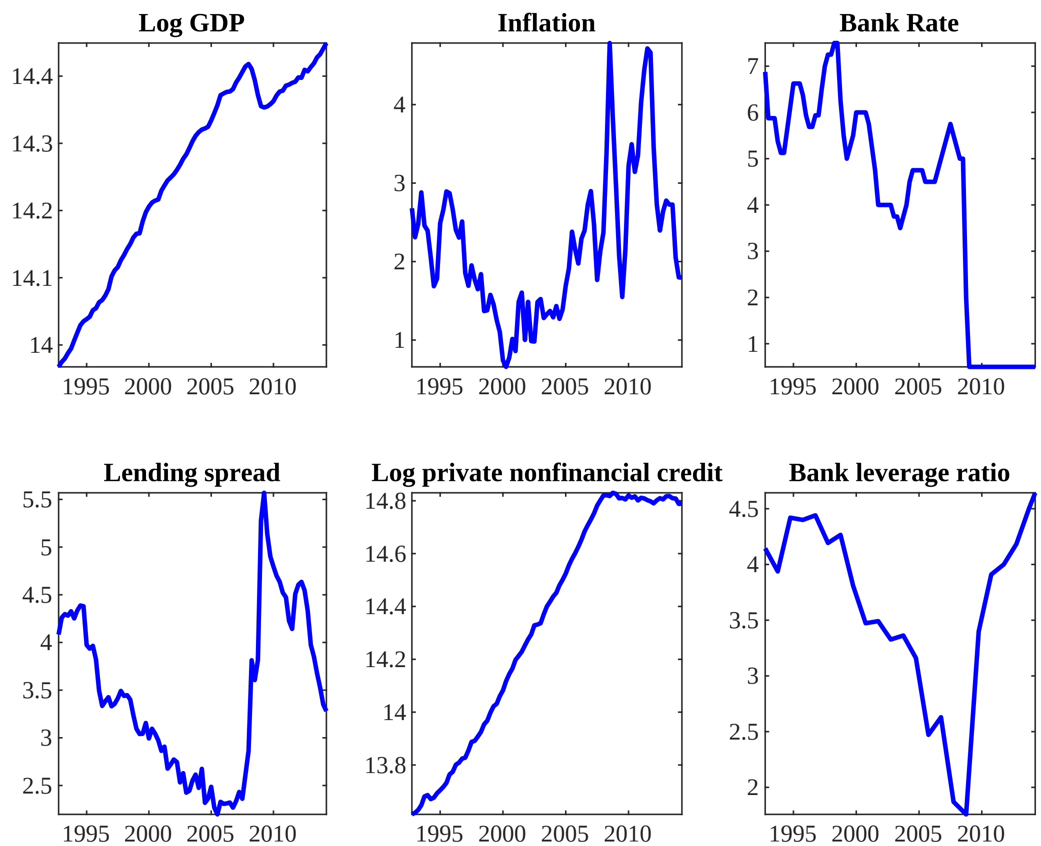
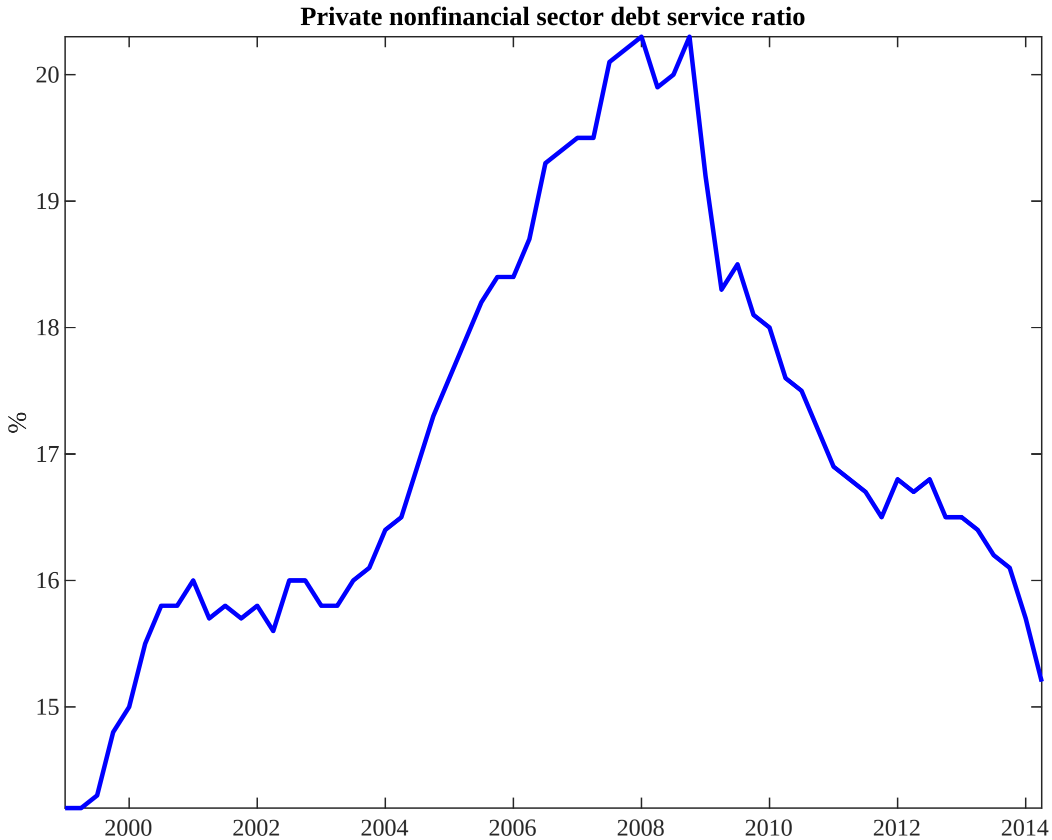
Appendix A: Supplementary charts and tables

Figure A.1: *UK time series*



Notes: This figure reports time series of the data series used in the analysis .

Figure A.2: *UK debt service ratio*



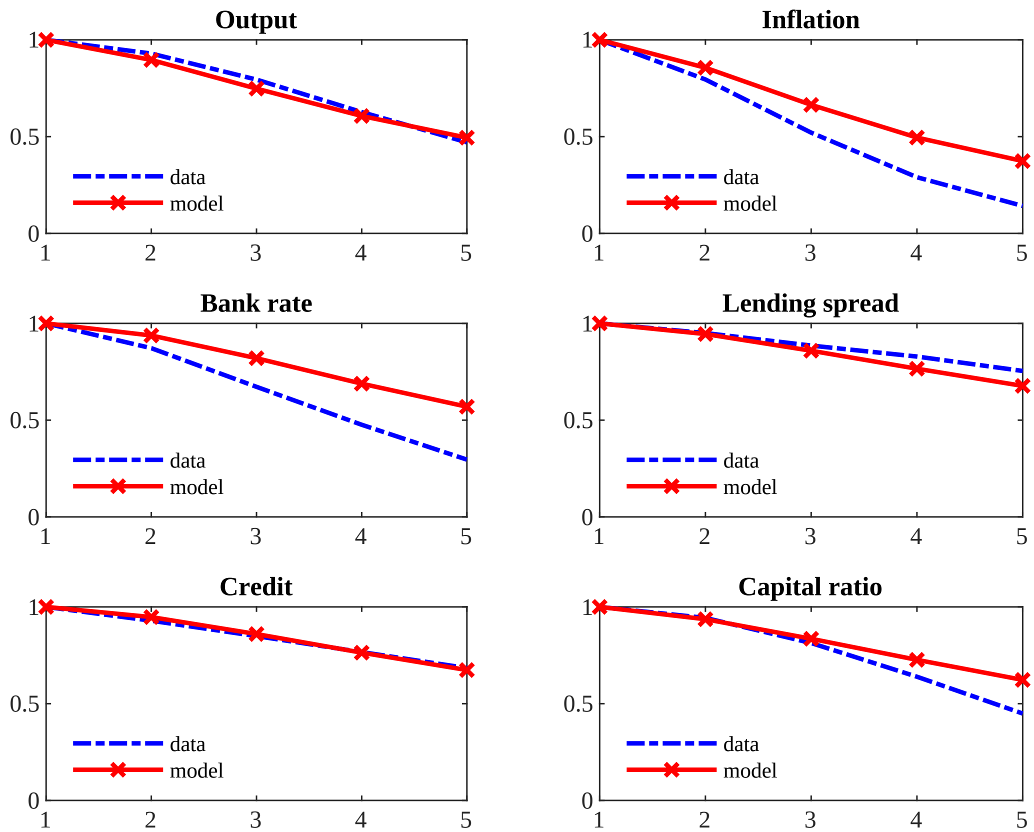
Notes: This figure shows debt service payments relative to income in the UK's private nonfinancial sector. Source: Bank of International Settlements.

Table A.1: *Calibration of the model's shock processes*

Parameter	Description	Value
ρ^y	Persistence of aggregate demand shock	0.95
ρ^π	Persistence of cost-push shock	0.8
ρ^r	Persistence of monetary policy shock	0
ρ^s	Persistence of loan spread shock	0.95
ρ^b	Persistence of credit demand shock	0.99
ρ^u	Persistence of write-off shock	0.85
ρ^d	Persistence of debt-deleveraging shock	0.9
σ_y^2	Variance of aggregate demand shock	0.25^2
σ_π^2	Variance of cost-push shock	0.2^2
σ_r^2	Variance of monetary policy shock	0.1^2
σ_s^2	Variance of loan spread shock	0.1^2
σ_b^2	Variance of credit demand shock	0.75
σ_u^2	Variance of write-off shock	0.15^2

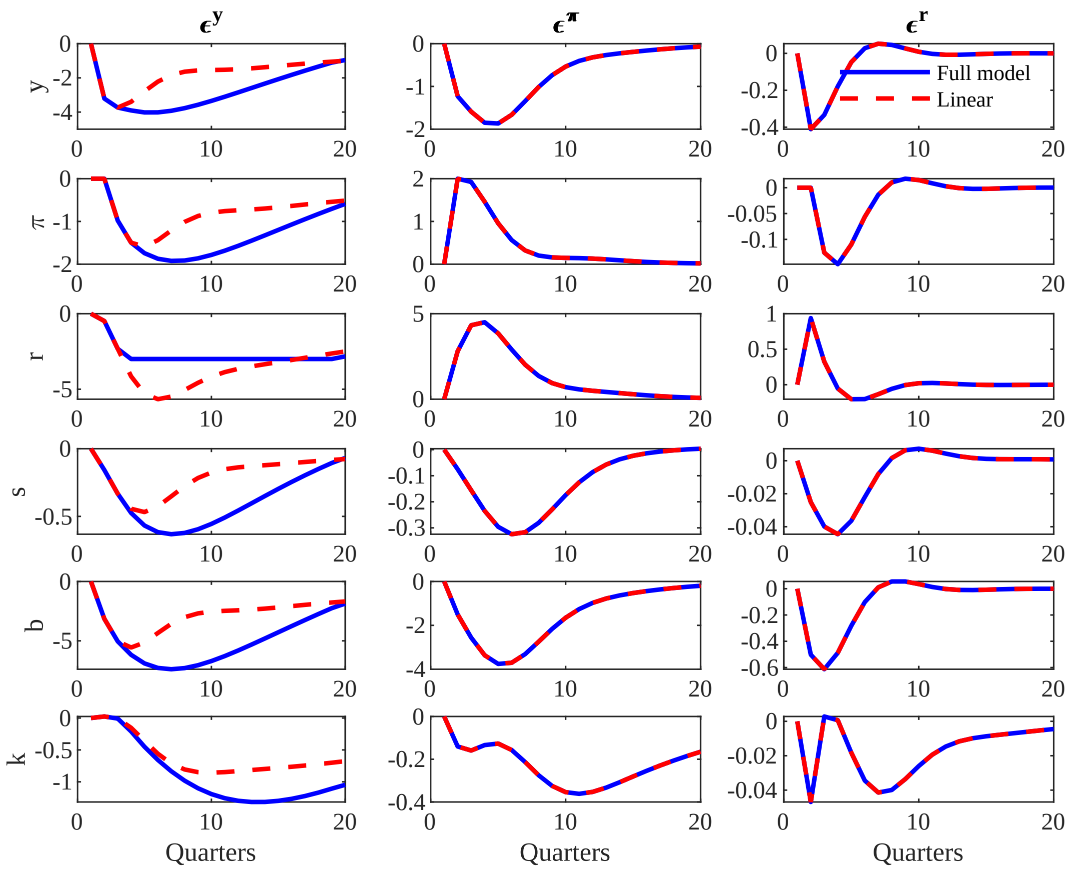
Notes: This table presents calibrated values of the model's shock processes.

Figure A.3: *Autocorrelations of endogenous variables, model vs data*



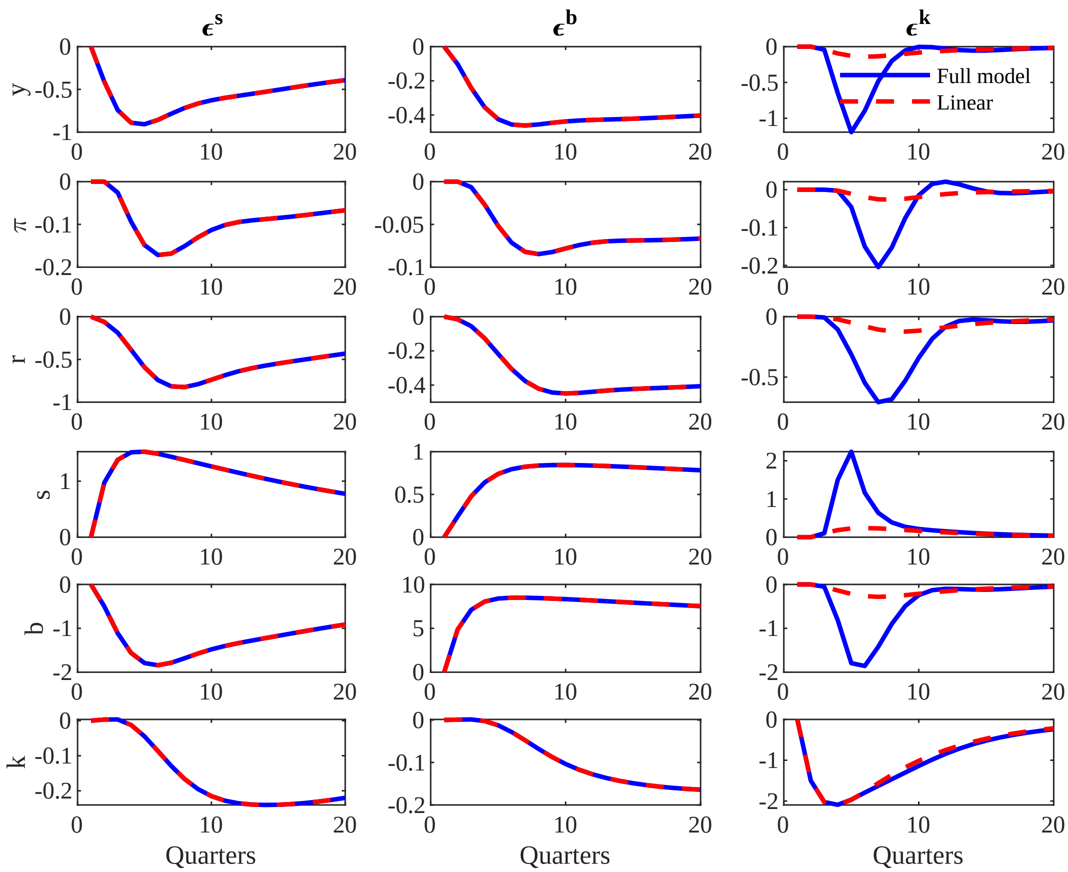
Notes: This figure presents autocorrelation functions in the model versus data.

Figure A.4: *Impulse responses to shocks*



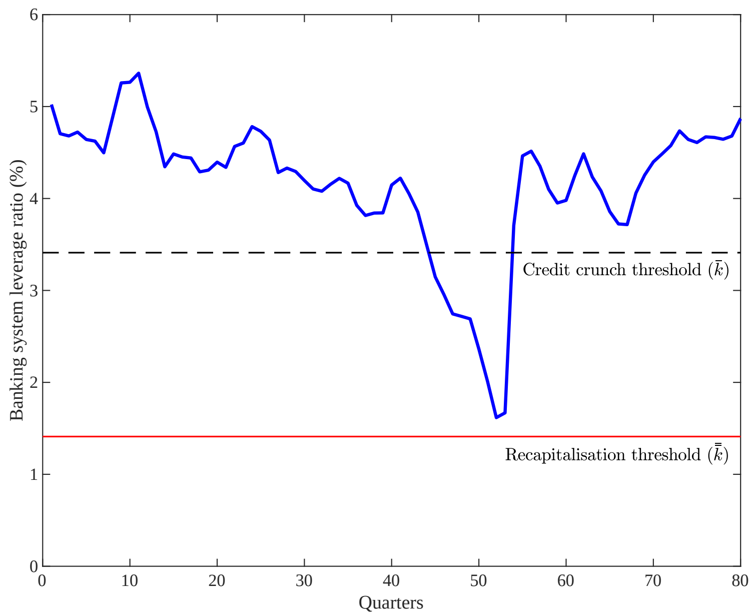
Notes: This figure presents impulse responses of the model's endogenous variables (rows) following shocks to output (ϵ^y), inflation (ϵ^π), and interest rates (ϵ^r) (columns). The blue solid lines show full model responses; red dashed lines show responses in the linear no-constraints model. The y-axis reports % responses for output and credit, and %pt responses for the other variables. The calibration of the steady state is that of the baseline economy, with $\bar{r} = -3$, $\bar{k} = -2$, $\bar{dsr} = 10$.

Figure A.5: *Impulse responses to shocks*



Notes: This figure presents impulse responses of the model's endogenous variables (rows) following shocks to loan spreads (ϵ^s), credit demand (ϵ^b), and bank capital (ϵ^k) (columns). The blue solid lines show full model responses; red dashed lines show responses in the linear no-constraints model. The y-axis reports % responses for output and credit, and %pt responses for the other variables. The calibration of the steady state is that of the baseline economy, with $\bar{r} = -3$, $\bar{k} = -2$, $\bar{dsr} = 10$.

Figure A.6: *Example of dynamics of bank leverage with recapitalisation*



Notes: This figure illustrates the impact of a recapitalisation on the dynamics on bank leverage in one simulation run. We transform the model's raw output, which is expressed as a %pt deviation from steady state, to levels of the leverage ratio by assuming the UK banking system was at steady state in 2017Q4 with a leverage ratio of 5.4%. In the simulation, the banking system suffers mounting credit losses eroding its equity cushion such that its credit crunch threshold is breached. Thereafter, the banking system reduces credit availability sharply, weakening economic activity and increasing credit losses further. This adverse feedback loop is finally halted in period 52 when a recapitalisation occurs, preventing what would otherwise be a breach of the point of non-viability. Once recapitalised, bank credit conditions normalise and the economy begins to recovers.

Appendix B: Shapley value methodology

In this appendix, we set out details of the Shapley value contributions analysis used in sections 5.1 and 6. Implementing this requires that we simulate all combinations in which constraints are applied and assigning each constraint its average contribution across these combinations.

In particular, if we start with the linear model as the baseline, there are $3! = 6$ possible orders in which the nonlinear constraints can be applied. These are: (ELB, CC, DSR), (ELB, DSR, CC), (CC, ELB, DSR), (CC, DSR, ELB), (DSR, ELB, CC), and (DSR, CC, ELB).

The next step is to calculate the marginal impact on the overall risk level of applying each constraint in each of the above orderings. These are obtained by comparing nested versions of the model, which isolate the impact of the constraint in question. For instance, when the constraints are ordered (ELB, CC, DSR), we must first compare the ELB only model with the linear model to obtain the ELB's impact; then the ELB plus credit crunch model to the ELB only model (model 1) to obtain the bank capital constraint's impact; and third, the ELB plus credit crunch plus DSR constraint model to the ELB plus credit crunch model for the DSR's impact.

The table below lists the full set of cases. Shapley value contributions are given by the mean contributions of each constraint, averaging across the six rows. :

Table B.1: *Calculation of Shapley value contributions*

		Marginal contribution of:		
Constraint order	Linear model	ELB	Bank capital constraint	DSR constraint
(ELB, CC, DSR)	Linear	Model (1)-Linear	Model (2)-Model (1)	Model (3)-Model (2)
(ELB, DSR, CC)	Linear	Model (1)-Linear	Model (3)-Model (4)	Model (4)-Model (1)
(CC, ELB, DSR)	Linear	Model (2)-Model (5)	Model (5)-Linear	Model (3)-Model (2)
(CC, DSR, ELB)	Linear	Model (3)-Model (6)	Model (5)-Linear	Model (6)-Model (5)
(DSR, ELB, CC)	Linear	Model (4)-Model (7)	Model (3)-Model (4)	Model (7)-Linear
(DSR, CC, ELB)	Linear	Model (3)-Model (6)	Model (6)-Model (7)	Model (7)-Linear
Shapley value:	Mean	Mean	Mean	Mean

Notes: The models are as follows: Model 1 = ELB only; Model 2 = ELB + Credit crunch; Model 3 = ELB + Credit crunch + DSR; Model 4 = ELB + DSR; Model 5 = Credit crunch only; Model 6 = Credit crunch + DSR; Model 7 = DSR only. The terminology "ELB only" should be taken to mean that all constraints other than the effective lower bound are sufficiently distant that they can be ignored, and so on.