BANK OF ENGLAND

Staff Working Paper No. 956

Banks' internal capital allocation, the leverage ratio requirement and risk-taking

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May 2022

This is an updated version of the Staff Working Paper originally published on 17 December 2021

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Banks' internal capital allocation, the leverage ratio requirement and risk-taking

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Abstract

This paper examines how banks' asset risk is affected by the level (ie group or business unit) at which regulatory requirements are applied. We develop a theoretical model and calibrate it to UK banks. Our main finding is that the impact differs depending on which regulatory constraint is binding at the group level. If that is the leverage ratio requirement, then the allocation of regulatory constraints to business units either maintains or decreases the riskiness of banks' investment portfolios. However, if the risk-weighted requirement is the binding constraint at the group level, applying regulatory requirements at the business unit level can lead to banks selecting riskier asset portfolios as optimal. We also find that the impact on banks' asset risk differs across bank business models.

Key words: Leverage ratio requirement, risk-weighted capital requirements, banks' internal capital allocation.

JEL classification: G21, G28.

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We are grateful for helpful comments from Staff Working Paper referees Tirupam Goel and Bill Francis; from discussants Lenka Fiala, George Pennacchi, Adrian Pop and from Arnoud Boot, Tom Crossland, Sebastian Gryglewicz, Aakriti Mathur, Ieva Sakalauskaite, Arzu Uluc, Razvan Vlahu, Matthew Willison as well as participants at 2021 FDIC/JFSR Bank Research Conference, WinE EEA 2021, EFMA 2021, GdRE Symposium on Money, Banking and Finance 2021, and Bank of England seminars. All remaining errors are of our own. The views expressed in this paper are those of the authors, and not necessarily those of the Bank of England or its Committees.

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ISSN 1749-9135 (on-line)

1 Introduction

The Great Financial Crisis 2007-2009 triggered a substantial overhaul of the prudential framework that took more than a decade in the making with last reforms completed in December 2019. The focus of regulatory authorities around the world has been shifted, since then, into evaluating the effectiveness of the Basel III reforms and identifying any unintended consequences. One prominent reform, which attracts great interest in policy evaluation exercises as well as in the academic literature, is the introduction of the leverage ratio (LR) requirement to complement the risk-weighted (RW) requirements and make the whole regulatory capital framework more robust.

The literature focuses so far on the impact of the LR requirement on banks' asset compositions. To the best of our knowledge, no formal analysis has been done to understand how that effect depends on the business levels (e.g. group level vs. business-unit level) at which the LR requirement is applied. This question is however very relevant since, as shown in recent supervisory reviews (see Bajaj et al. (2018)), some banks take account of the LR requirement within their internal capital allocation process to determine the amount of equity capital needed to support the activities of each business unit. Such practice implies that, although the LR requirement is legally imposed at the group level, some banks actually choose to apply it at the business-unit level by requiring each individual business unit to be allocated enough equity capital to comply with this requirement. We hereafter refer to the latter as the allocation of regulatory constraints to business units or interchangeably, the application of regulatory constraints at the business-unit level. This behaviour raises concern about its implications for the effectiveness of the prudential framework.

This paper aims to fill that gap by formally examining how the allocation of regulatory constraints to business units by banks impacts their investment decisions. To do so, we construct a model of banks' optimal investments in the presence of multiple capital constraints and multiple business units. We then investigate, both analytically and via calibrated simulations, how banks' investments across business units differ depending on the level at which they apply regulatory constraints.

Among banks' businesses considered as more likely to be affected by the LR requirement, repurchase agreements (repos) are drawing significant attention given the key role of repo markets in facilitating the flow of cash and securities around the financial system. There exists early evidence (e.g. BIS CGFS (2017); Duffie (2018); Allahrakha et al. (2018)) that the LR requirement had some negative impact on banks' incentives to undertake repo intermediation in both Europe and the US. But the underlying mechanism behind that impact is not clear, especially when this requirement does not appear to be the binding constraint at the group consolidated level for most of banks. Our model will offer an explanation of how this impact can arise from the banks' practice of applying regulatory requirements at a lower business level through their internal capital allocation process.

Our model features a risk-neutral banking group that runs two business units. One unit has higher non-risk adjusted returns but is also riskier, while the other has lower returns and is less risky. Motivated by the evidence on the impact of the LR requirement on banks' repo activities, we model the low-risk business as repo while the high-risk business in our set-up resembles a lending business. The bank finances its activities with debt and a fixed amount of equity capital that will be allocated across its two business units. It is subject to two requirements, namely the RW capital requirement and the LR requirement, which are legally applied at the group level.

To capture banks' internal capital allocation practices, we consider the case where the banking group in our model chooses to allocate equity capital such that each business unit must be assigned enough equity capital to make them comply with both regulatory constraints. We compare the resulting bank's optimal investments in this case to those in the case where the bank just applies two regulatory requirements at the group level. Our focus is on how the riskiness of the bank's asset portfolio - the bank's asset risk - is different between the two cases.

We start with characterising analytically the impact of applying regulatory constraints at the business unit level on the bank's asset risk. Then we calibrate the model using data on a sample of UK banks. The simulation of the calibrated model verifies the empirical relevance of our analytical findings and complements them with the analysis on how the impact of allocating down requirements varies across business models.

The main analytical insight from our model is that the effect on the bank's asset risk differs depending on which of the two regulatory constraints binds at the group level. If the banking group is bound by the LR requirement, allocating regulatory requirements to business units will either leave the riskiness of its investments unchanged or reduce it. However, if the RW requirement is the binding constraint at the group level, then under certain conditions, allocating requirements down can result in an increase of the bank's asset risk by inducing the bank to invest relatively less in repo and more in lending.

To understand the intuition, note first that the bank's asset risk can change with the application level of regulatory constraints if the latter affects the bank's *relative* incentives to invest across two businesses. Second, the incentives to invest in each business depend on the cost in terms of required equity capital for such an investment, which in turn is determined by the regulatory constraint that binds. Therefore, the impact of allocating constraints down differs on whether the banking group is LR-bound or RW-bound, as the application level affects the binding constraint differently in these two situations.

More specifically, consider first the situation where the banking group is bound by the LR requirement. When the two requirements are applied at the group level, the binding constraint is the LR and thus, the bank's incentives to invest in two business units are determined by their *marginal leverage cost* - the additional equity capital required by the LR requirement for each additional unit of investment. When those requirements are applied at the business-unit level, two interesting cases arise depending on the average risk weight (ARW) of each unit on a stand-alone basis: either both units are constrained by the LR or they are both bound by the RW requirement. The asset risk of the bank does not change in the former case, compared to the group-level application, since the bank's investments are also determined by the marginal leverage cost of the two businesses. The asset risk decreases in the latter case since investing in repos now becomes relatively more attractive for the bank than investing in lending. This is in turn due to two reasons. First, the cost of investments is now determined by the *marginal RW capital cost* - the additional equity capital required by the RW requirement for each additional unit of investment. Second, repo activities generally incur higher leverage cost but lower RW capital cost than lending activities.

When the banking group is bound by the RW requirement, the bank's asset risk can increase following the allocation of requirements to business units under conditions that make investing in the repo business relatively less attractive than in the case of group-level application. One of the conditions is that, when constraints are applied at the business-unit level, the RW capital requirement binds for lending business but the LR requirement binds for the repo business. Another condition is that the marginal leverage cost of the repo business is higher that its marginal RW capital cost.

To complement our theoretical analysis, we take the model to the data using several data sources that contain information on lending and repo activities of a sample of UK banks. The numerical simulation confirms the empirical relevance of our analytical insights. We find that the allocation of constraints leads to an increase in asset risk of the average bank in our sample when only the RW requirement binds at the group level.

To understand the role of the business model, we examine whether the impact of the allocation of the constraints differs across banks with different business models. We therefore classify the UK banks in our sample into two types of banks, namely retail and wholesale banks. We recalibrate the model to each type of banks. From our calibration, we observe that the lending business of retail banks is riskier than that of wholesale banks. When rerunning the simulations for each business model, we find that there is a stark difference in the impact of the allocation of constraints on the asset risk between the two types, especially for the case where only the RW constraint binds at the group level. Precisely, we find in that case that allocating constraints down results in an increase in the asset risk of retail banks but a decrease in the asset risk of wholesale banks.

Related literature To the best of our knowledge, this paper is the first to address formally how the application level of regulatory constraints affects banks' asset compositions. Its insights are related to three main strands of literature.

The first strand of literature assesses the impact of the LR requirement on banks' risk-taking.[1] Contributions in the literature include, among others, Kiema and Jokivuolle (2014) and Acosta-Smith et al. (2020). The focus of Kiema and Jokivuolle (2014) is on the model risk argument. They find that the shift in risk-taking does not affect the aggregate risk profile and stability of the whole banking system, as banks re-shuffle the loans: banks focusing on low-risk lending will shift towards more high-risk lending, while the high-risk lending banks will reallocate part of their portfolio to low-risk investments. Acosta-Smith et al. (2020) focus on the complementary role of the LR requirement as compared to risk-based capital requirements. They find that the introduction of the LR leads to an increase in banks' risk-taking if equity is sufficiently costly, or if banks are bound by the LR. They confirm these results empirically for a large panel of European banks. Choi et al. (2018) find similar empirical evidence for the US, where banks shift towards riskier investments but the shift is counterbalanced by increased capital, leading to no change in overall risk. Our paper adds to this literature by investigating how the impact of the LR on banks' investments depends on the business level at which it is applied by banks.

The second strand of literature examines the optimal capital allocation approach within complex financial institutions. One focus of this literature is on the choice between the Risk Adjusted Returns on Capital (RAROC) and Economic Value Added (EVA) as measures of relative profitability across different business units (Ita, 2017; Khaykin et al., 2017). Some others analyse how the choice of internal hurdle rate affects banks' investments. For example, Krüger et al. (2015) show empirically that the allocation based on firm-wide cost of capital leads to under-investment in safer businesses and over-investment in relatively riskier ones. Papers such as Perold (2005) instead discuss how accounting for diversification benefits between different businesses can reduce banks' economic capital needs. The most related to our paper is Goel et al. (2020), which studies how banks' internal capital allocation makes shocks to one banking activity spill over to another activity. They assume that regulatory

[1] Other strands of the literature analyse how the LR affects banks' incentives to truthfully report their riskiness (see Blum (2008)) and how it interacts with the RW requirement to affect the business cycle (see Gambacorta and Karmakarb (2018)).

constraints are applied at the business-unit level. Our paper however examines how the banks' practice of applying regulatory constraints at the lower business level through their internal capital allocation process affect their investment decisions.

Finally, there are studies looking at the impact of the LR requirement on banks' incentives to undertake market making businesses, especially repo activities. In a recent study on the market liquidity in the UK gilt market, Bicu-Lieb et al. (2020) find that a decrease in the repo liquidity coincides with the introduction of the leverage ratio requirement. Kotidis and Van Horen (2019) also analyse the UK gilt repo market. They find that repo dealers constrained by the LR decrease initially the transacted volumes with smaller clients, but this effect was temporary. The repo dealers who were not affected by the LR requirement took over the smaller clients, and later on the affected dealers increased haircuts on reverse repo transactions, in order to pass through increased costs of regulation. Our paper offers an explanation on the mechanism underlying the impact of the LR on repo business found in these empirical analyses.

The organisation of the paper is as follows. In Section 2 we set out our theoretical model. Section 3 presents our main analytical insights. Then in Section 4 we calibrate our model to the UK banks and explain our numerical simulations in Section 5. In Section 6 we break down our UK bank sample into different business models and discuss our new simulation results in this context. Finally Section 7 concludes.

2 The model

We consider a banking group that is funded by a fixed amount of equity capital K and by debts of gross interest rate R. It runs two business units, one yielding higher non-risk adjusted returns but is riskier than the other. Although our results will hold for any combination of two businesses that have those characteristics, in this paper, we model the riskier business as a lending business and the safer business as a repo business.

Two business units The lending unit grants loans to customers. We denote the bank's ex-ante gross interest income from loans by $G(L)$ where L is the total value of granted loans. We capture the fact that granting loans is a risky business by assuming that ex-post some borrowers default and cannot fully repay their loans. Let $\tilde{Z}$ be the random variable that represents the losses per unit of loans. Therefore, the bank's ex-post lending revenue is equal to $G(L) - ZL$ where Z is the realised value of $\tilde{Z}$. We assume that $\tilde{Z}$ is distributed according to the distribution H_Z, h_Z with expected value equal to μ_Z.

The repo unit owns a stock of government bonds of value X with coupon c. It uses this inventory to raise collateralised funding to finance bond trading activities or to act as an intermediary entering into repo transactions with some counterparties and offsetting reverse repos with others. We denote the ex-ante income from those activities by $F(X)$. We capture the risk of the repo business by assuming that ex-post the bank could suffer losses equal to $\tilde{\varepsilon}X$ due to, for example, unpaid repayments by reverse repo counterparties or losses from trading activities. The distribution of $\tilde{\varepsilon}$ is characterised by $H_\varepsilon, h_\varepsilon$ with expected value μ_ε.

We make the following assumptions on the profitability and riskiness of the two business units.

Assumption 1. *Functions $G(.)$ and $F(.)$ satisfy the following conditions:*

$$G(0) = 0; \quad G'(.) > 0 \quad and \quad G''(.) < 0$$

$$F(0) = 0; \quad F'(.) > 0 \quad and \quad F''(.) < 0$$

Assumption 1 implies that both lending and repo businesses have diminishing marginal returns. For the lending business, this property can be explained by the fact that the loan interest rate is a decreasing function of loan size. For the repo business, this can be due to the fact that the interest rate of reverse repos is less sensitive to the transactional amount than the repo rate, which in turn can come from the market power of banks in both activities.

Assumption 2. *The rank of profitability between two business units is as follows:*

$$G'(y) - \mu_Z > F'(y) + c - \mu_\varepsilon \quad for\ all \quad y \leq max(X^*, L^*)$$

where

$$X^* = \underset{y}{\operatorname{argmax}}\,[F(y) + cy - \mu_\varepsilon y - Ry] \quad \textit{and} \quad L^* = \underset{y}{\operatorname{argmax}}\,[G(y) - \mu_Z y - Ry]$$

Assumption 2 indicates that lending business is more profitable than repo business on a non risk-adjusted basis. Note that X^* and L^* defined in this assumption represent the size of, respectively, the repo and lending businesses so that the expected profits from those activities are maximised when their cost of funding is R. The bank will never grant more loans than L^* and never hold an inventory of government bonds of value higher than X^*.

Assumption 3. *Two random variables $\tilde{Z}$ and $\tilde{\varepsilon}$ are independently distributed and ranked as follows:*

$$VaR_{1-q}(\tilde{Z}) \geq VaR_{1-q}(\tilde{\varepsilon})$$

where $VaR_{1-q}(\tilde{Y})$ denotes the Value at Risk (VaR) of a random variable $\tilde{Y}$ at confidence level $1-q$, which is defined as:

$$VaR_{1-q}(\tilde{Y}) \equiv inf\left\{y : \mathbb{P}(\tilde{Y} \geq y) \leq q\right\} \tag{1}$$

Assumption 3 states a ranking between two random variables $\tilde{Z}$ and $\tilde{\varepsilon}$ based on the VaR measure. It implies that lending business is riskier on a stand-alone basis than repo business.

Regulatory constraints and internal capital allocation The bank is subject to two regulatory constraints, namely the LR requirement and the RW capital requirement. In line with the principle underlying the Basel requirements, we formulate the RW capital requirement using the VaR constraint. Among the total equity capital K that the bank has, K_L will be allocated to support the lending business while K_X is allocated to the repo business.

Before explaining how regulatory constraints look like depending on the level at which the bank applies them, it is useful to introduce some notations. We denote by $\tilde{\Pi}_L$ and $\tilde{\Pi}_X$ the profit of, respectively, the lending and repo units. $\tilde{\Pi}_L$ and $\tilde{\Pi}_X$ can therefore be written as follows:

$$\tilde{\Pi}_L = G(L) - \tilde{Z}L - R(L - K_L) \quad \text{and} \quad \tilde{\Pi}_X = F(X) + cX - \tilde{\varepsilon}X - R(X - K_X)$$

The overall profit of the whole banking group is thus equal to $\tilde{\Pi}_L + \tilde{\Pi}_X$.

When regulatory constraints are applied at the group level, the RW requirement can be written as follows:

$$\mathbb{P}\left(\tilde{\Pi}_L + \tilde{\Pi}_X \leq 0\right) \leq a \tag{2}$$

In words, Constraint (2) states that the probability for the total losses of the bank's asset portfolio being higher than its equity capital is lower than a. After some algebra, it can be rewritten as[2]:

$$K \geq \frac{VaR_{1-a}(\tilde{Z}L + \tilde{\varepsilon}X) - \Pi(L, X)}{R} \tag{3}$$

where

$$\Pi(L, X) = \underbrace{G(L) - RL}_{\equiv \Pi_L(L)} + \underbrace{F(X) + cX - RX}_{\equiv \Pi_X(X)}$$

In the Basel III framework, the right hand side (RHS) of Constraint (3) is equivalent to the product of the RW capital requirements and the risk-weighted assets (RWAs) of the bank at the group level. We denote the former by γ and the latter by RWA^G. RWA^G can thus be proxied in our model by:

$$RWA^G(L, X) = \frac{VaR_{1-a}(\tilde{Z}L + \tilde{\varepsilon}X) - \Pi(L, X)}{\gamma R}$$

The LR requirement is expressed in terms of the ratio of equity capital over leverage exposure. The leverage exposure of the lending unit is equal to its size L. For the repo unit,

[2]See Appendix A.1 for the detailed derivation.

because of different possible regulatory treatments of repo activities, its leverage exposure can be a multiple of its size X. For example, when the bank runs a matched repo book, if all reverse repo transactions are not eligible to netting, due to the requirement that securities sold as collateral cannot be removed from the bank's balance sheet, the leverage exposure of the repo business will be equal to $2X$. To capture this characteristic of repo activities, we assume that the leverage exposure of the repo unit equals αX where $\alpha \in [1, 2]$.[3] The LR requirement at the group level is therefore as follows:

$$K \geq \chi(L + \alpha X) \tag{4}$$

where χ is the required LR.

When the bank chooses to allocate regulatory constraints down to its business units, the allocated capital K_L and K_X are determined so that both business units have enough equity capital to comply with both regulatory constraints individually. Therefore, K_L is such that the lending business has to satisfy:

$$\mathbb{P}\left(\tilde{\Pi}_L \leq 0\right) \leq a \quad \text{and} \quad K_L \geq \chi L$$

while K_X is determined so that the repo business satisfies:

$$\mathbb{P}\left(\tilde{\Pi}_X \leq 0\right) \leq a \quad \text{and} \quad K_X \geq \chi \alpha X$$

The two individual VaR constraints can similarly be expressed in terms of RWA^L and RWA^X - the RWAs of, respectively, the lending and repo businesses on a stand-alone basis - as follows:

$$K_L \geq \gamma RWA^L(L) \quad \text{where} \quad RWA^L(L) = \frac{VaR_{1-a}(\tilde{Z}L) - \Pi_L(L)}{\gamma R} \tag{5}$$

$$K_X \geq \gamma RWA^X(X) \quad \text{where} \quad RWA^X(X) = \frac{VaR_{1-a}(\tilde{\varepsilon}X) - \Pi_X(X)}{\gamma R} \tag{6}$$

[3]Note that results stated in the following analysis will apply to other types of low risk and low return businesses when α is set to 1.

Measures of bank asset risk There exist several possible measures for the riskiness of the bank's assets.[4]

In the regulatory world, banks' asset risk is usually measured by the so-called average risk weight (ARW), which is defined as the ratio of RWAs over leverage exposure. In our model, the ARW of the banking group can be computed as

$$ARW^G(L, X) = \frac{RWA^G(L, X)}{L + \alpha X}$$

while the ARW of each business unit calculated as

$$ARW^L(L) = \frac{RWA^L(L)}{L} \quad \text{and} \quad ARW^X(X) = \frac{RWA^X(X)}{\alpha X}$$

In the portfolio theory, a simple measure of the riskiness of an asset portfolio is the variance of its returns. In our set up, since we assume that $\tilde{Z}$ and $\tilde{\varepsilon}$ are independently distributed, the variance of the returns of the bank's asset portfolio, denoted by σ_p^2, is computed as follows

$$\sigma_p^2 = L^2\sigma_Z^2 + X^2\sigma_\varepsilon^2 \tag{7}$$

where σ_Z^2 and σ_ε^2 denote the variance of, respectively, $\tilde{Z}$ and $\tilde{\varepsilon}$.

Given that in our model, the bank runs two businesses with one riskier than the other, an intuitive measure of the riskiness of the bank's assets is the fraction of the bank's total balance sheet devoted to lending business - the riskier one. We denote by w this fraction, i.e.

$$w = \frac{L}{L + X} \tag{8}$$

In the following, we will use w as our main measure of bank asset risk since it also intuitively captures the bank's rebalancing portfolio actions and facilitates the analytical

[4]Note the difference between banks' asset risk and banks' total risk or banks' overall resilience. The overall resilience will decrease when the asset risk increases if the increase in asset risk is not dominated by a decrease in bank funding risk due to, for example, lower leverage.

derivations. As will become clear later, in the equilibrium, when w increases, the ARW of the banking group also increases, which implies that our insights are robust across these two measures of bank asset risk. In Appendix A.2, we discuss why it is without loss of generality to focus on the cases where the variance of the returns of the bank's asset portfolio σ_p^2 is increasing with w. Hence, our insights also hold if bank asset risk is measured by the variance of asset portfolio's returns.

3 Analysis

We analyse in this section the bank's optimal investments. Our main objective is to investigate how the bank's asset risk, measured by w is affected by the level at which the two regulatory constraints are applied. To do so, we will compare the bank's optimal investments in each business unit between the case where the two constraints are applied at the group level and the case in which both business units have to individually comply with both regulatory constraints. To get started, we first formulate the bank's profit-maximisation problem for each of these two scenarios. Then, we will examine how the bank's investment decisions differ between them.

3.1 Bank's optimisation problems

Optimisation problem with constraints applied at the group level When all constraints are applied at the group level, the bank's optimisation problem, denoted as $\wp^G$, can be written as follows:

$$Problem \quad \wp^G: \qquad \operatorname*{Max}_{L,X} \quad \mathbb{E}\left[\tilde{\Pi}_L + \tilde{\Pi}_X\right]$$

subject to Constraints (3) and (4).

To facilitate the examination of how the bank's asset risk w would change depending on the application level of the two regulatory constraints, we reformulate Problem $\wp^G$ by changing the bank's decision variables from (L, X) to w and the bank's total balance sheet size $S = L + X$. After expressing L and X in terms of S and w, Problem $\wp^G$ could be rewritten as:

$$\underset{S,w}{\text{Max}} \quad \{\Pi(w,S) - \mu_Z wS - \mu_\varepsilon(1-w)S + RK\}$$

subject to

$$K \geq \gamma RWA^G(w,S) = \frac{VaR_{1-a}(\tilde{Z}w + (1-w)\tilde{\varepsilon})S - \Pi(w,S)}{R} \tag{9}$$

$$K \geq \chi\,(wS + \alpha(1-w)S) \tag{10}$$

Optimisation problem with constraints allocated down to business units When the bank allocates two constraints to its business units, the bank's optimisation problem, denoted as $\wp^B$, is as follows:

$$Problem \quad \wp^B: \qquad \underset{L,X}{\text{Max}} \quad \mathbb{E}\left[\tilde{\Pi}_L + \tilde{\Pi}_X\right]$$

subject to Constraints (5), (6) as well as the following two LR requirements:

$$K_L \geq \chi L \quad \text{and} \quad K_X \geq \chi\alpha X$$

and the internal capital allocation constraint:

$$K \geq K_L + K_X$$

After reformulating Problem $\wp^B$ in terms of w and S, we get:

$$\underset{S,w}{\text{Max}} \quad \{\Pi(w,S) - \mu_Z wS - \mu_\varepsilon(1-w)S + RK\}$$

subject to

$$K_L \geq \gamma RWA^L(w,S) = \frac{VaR_{1-a}(\tilde{Z}w)S - \Pi_L(w,S)}{R} \tag{11}$$

$$K_X \geq \gamma RWA^X(w,S) = \frac{VaR_{1-a}((1-w)\tilde{\varepsilon})S - \Pi_X(w,S)}{R} \tag{12}$$

$$K_L \geq \chi w S \tag{13}$$

$$K_X \geq \chi \alpha (1-w) S \tag{14}$$

$$K \geq K_L + K_X \tag{15}$$

3.2 Bank's optimal investments

We are now equipped to compare the bank's investments, especially the bank's asset risk, between the two above scenarios. Denote by (w^G, S^G) and (w^B, S^B) the solutions to, respectively, Problem $\wp^G$ and $\wp^B$.

To get a first intuition on how investment decisions of the bank differ between the two cases, let us compare the constraints of Problem $\wp^G$ to those of Problem $\wp^B$. Clearly, we see that the group-level LR constraint is weakly looser than business unit-level LR constraints. The group-level RW constraint is also looser than the business unit-level one and the gap can be expressed in terms of Div defined as follows:

$$Div = VaR_{1-a}(\tilde{Z}w) + VaR_{1-a}((1-w)\tilde{\varepsilon}) - VaR_{1-a}(\tilde{Z}w + (1-w)\tilde{\varepsilon}) \tag{16}$$

Div represents the diversification benefit per unit of size to the bank if it applies the RW constraint at the group level.

These first observations imply that applying regulatory constraints at the business unit level will weakly reduce the set of investment opportunities available to the bank. The following proposition highlights the efficiency losses resulting from allocating both constraints down to business units.

Proposition 1. *Efficiency losses:*

$$S^B \leq S^G$$

Proof. It is the direct consequence of the fact that constraints of Problem $\wp^B$ are weakly tighter than those of Problem $\wp^G$. □

We now turn to the impact on the bank's asset risk. Before characterising it, we state in the lemma below the relationship between w and ARW^G - two possible measures of the bank's asset risk.

Lemma 1. *In the equilibrium, ARW^G increases with w*

Proof. See Appendix A.4. □

Lemma 1 implies that the following insights, which focus on how w changes depending on the application level of the two regulatory constraints, will also apply to ARW^G as a measure of the bank's asset risk. We will consider two cases where the bank is bound at the group level either by the LR requirement or by the RW requirement. It can happen that both constraints bind at the group level at the same time. But given that that case is a knife-edge case, we do not analyse it in this section.[5]

LR-constrained bank The bank is bound by the LR requirement at the group level when Constraint (10) is tighter than Constraint (9). We state in the following proposition our first results related to the bank's asset risk.

Proposition 2. *When the bank is bound by the LR requirement at the group level (i.e. $ARW^G(w^G, S^G) < \frac{\chi}{\gamma}$), we have:*

1. *$w_B = w_G$ if the following two conditions hold globally*

$$ARW^L(w,S) < \frac{\chi}{\gamma} \quad \text{and} \quad ARW^X(w,S) < \frac{\chi}{\gamma}$$

2. *$w_B < w_G$ if the following two conditions hold globally*

$$ARW^L(w,S) > \frac{\chi}{\gamma} \quad \text{and} \quad ARW^X(w,S) > \frac{\chi}{\gamma}$$

Proof. See Appendix A.5 □

[5]The case where no constraints bind is also not interesting. That is because in that case, the bank's optimal investments are the unconstrained optimum which won't be affected by the application level of constraints.

Proposition 2 states conditions under which the allocation of constraints down to business units either leaves unchanged the asset risk of the LR-constrained bank or reduces it. Those conditions relate the ARW of each business unit to the so-called critical average risk weight (CRW) that equals $\frac{\chi}{\gamma}$ - the ratio of required LR over required RW capital ratio.

Specifically, the first part of the proposition corresponds to the case where both units will be bound by the LR requirement when regulatory constraints are allocated down since their ARW is lower than the CRW. The second part specifies the impact on the bank's asset risk when both units will be bound by the RW constraint. Note that the situation where the group is bound by the LR but business units bound by the RW constraint can happen if all three inequalities below can be satisfied simultaneously:

$$\frac{VaR_{1-a}(\tilde{Z}w)S - \Pi_L(w,S)}{R} > \chi w S \tag{17}$$

$$\frac{VaR_{1-a}((1-w)\tilde{\varepsilon})S - \Pi_X(w,S)}{R} > \chi\alpha(1-w)S \tag{18}$$

and

$$\chi w S + \chi\alpha(1-w)S > \frac{VaR_{1-a}(\tilde{Z}w + (1-w)\tilde{\varepsilon})S - \Pi(w,S)}{R} \tag{19}$$

The necessary condition for Inequalities (17), (18) and (19) being compatible with each other is as follows:

$$VaR_{1-a}(\tilde{Z}w)S + VaR_{1-a}((1-w)\tilde{\varepsilon})S > VaR_{1-a}(\tilde{Z}w + (1-w)\tilde{\varepsilon})S \tag{20}$$

or, in words, the diversification benefit Div is high enough.

To get the intuition underlying the two results stated in Proposition 2, it is useful to compare the first order conditions (FOC) that characterise w^G and w^B. w^G is indeed determined by:

$$\big[G'(w^G S^G) - \mu_Z - R\big] - \big[F'((1-w^G)S^G) + c - \mu_\varepsilon - R\big] = \lambda_{LR}\left(\overbrace{\underbrace{\chi}_{\substack{\text{marginal leverage}\\ \text{cost of lending}}} - \underbrace{\alpha\chi}_{\substack{\text{marginal leverage}\\ \text{cost of repo}}}}^{\leq 0}\right) \tag{21}$$

w^B is in turn determined by:

$$\big[G'(w^B S^B) - \mu_Z - R\big] - \big[F'((1-w^B)S^B) + c - \mu_\varepsilon - R\big] = \lambda^L_{LR}\left(\overbrace{\underbrace{\chi}_{\substack{\text{marginal leverage}\\ \text{cost of lending}}} - \underbrace{\alpha\chi}_{\substack{\text{marginal leverage}\\ \text{cost of repo}}}}^{\leq 0}\right) \tag{22}$$

when both ARW^L and ARW^X are below CRW and by:

$$\big[G'(w^B S^B) - \mu_Z - R\big] - \big[F'((1-w^B)S^B) + c - \mu_\varepsilon - R\big] = \lambda^L_{VaR}\left[\overbrace{\underbrace{\frac{\partial(\gamma RWA^L)}{\partial L}}_{\substack{\text{marginal RW}\\ \text{capital cost of lending}}} - \underbrace{\frac{\partial(\gamma RWA^X)}{\partial X}}_{\substack{\text{marginal RW}\\ \text{capital cost of repo}}}}^{\geq 0}\right] \tag{23}$$

when both ARW^L and ARW^X are above CRW.[6]

All three Equations (21), (22) and (23) equate, on the left hand side (LHS), the marginal benefit of reallocating investment from repo business to lending business with its marginal cost on the right hand side (RHS). The former is the increase in the bank's expected marginal profit due to higher profitability of lending business while the latter is measured in terms of marginal changes in required equity capital. Comparing those equations, we see that what

[6] λ_{LR}, λ^L_{LR} and λ^L_{VaR} are the shadow price of, respectively, the group LR, the business-unit LR and the business-unit RW requirements.

drives the difference between w^G and w^B is the marginal cost.

When both units have ARW lower than the CRW, applying regulatory constraints at the business unit level does not affect the bank's asset risk compared to the case of group-level application. That is because, as shown in Equations (21) and (22), the bank's investments are determined by the difference in the marginal leverage cost between two businesses in both cases of application level.

When both units have ARW greater than the CRW, in comparison to the group-level application (Equation (21)), the business unit-level application (Equation (23)) leads to higher marginal costs of rebalancing investment portfolio from repo toward lending. In other words, with business-level application, investments in repo is relatively more attractive than lending since repo business incurs lower marginal RW capital cost but higher marginal leverage cost than lending business. Therefore, when constraints are applied down, the banking group reduces its risk-taking.

RW-constrained bank We now turn to the case where the RW constraint binds at the group level. This is equivalent to Constraint (10) being looser than Constraint (9) or $ARW^G(w^G, S^G) > \frac{\chi}{\gamma}$. The following proposition formally states three conditions that will make an increase in the asset risk of the RW-constrained bank more likely to occur when regulatory constraints are allocated down to business units.

Proposition 3. *When the bank is bound by the RW constraint at the group level (i.e. $ARW^G(w^G, S^G) > \frac{\chi}{\gamma}$), it can happen that $w^B > w^G$ if the following conditions are satisfied globally:*

1. $ARW^L \geq \frac{\chi}{\gamma}$ *and* $ARW^X \leq \frac{\chi}{\gamma}$
2. $\chi\alpha \geq \frac{\partial(\gamma RWA^X)}{\partial X}$
3. $\frac{\partial Div}{\partial w} = VaR_{1-a}(\tilde{Z}) - VaR_{1-a}(\tilde{\varepsilon}) - \frac{\partial VaR_{1-a}(\tilde{Z}w+\tilde{\varepsilon}(1-w))}{\partial w} < 0$

Proof. See Appendix A.6 □

Similarly to the case of the LR-constrained bank, the impact of allocating down regulatory constraints to business units on the asset risk of the RW-constrained bank depends on the ARW of each business unit. Proposition 3 considers the case where, as implied by the first condition, the lending unit is bound by the RW requirement while the repo unit is bound by the LR requirement. In this situation, Proposition 3 indicates that if, following the second condition, the marginal leverage cost of repo business is higher than its marginal RW capital cost and if, as stated by the third condition, the diversification benefit Div is decreasing with the share of the lending business in the bank's total balance sheet, then changing the application level from group to business unit can lead to an increase of the asset risk of the RW-constrained bank.

To understand why the three conditions specified in Proposition 3 make an increase in the asset risk of the RW-constrained bank more likely to occur when regulatory constraints are allocated down to business units, it is again useful to compare the two FOCs that determine w^G and w^B. For a RW-constrained bank, if $ARW^L \geq \frac{\chi}{\gamma}$ and $ARW^X \leq \frac{\chi}{\gamma}$, then w^G and w^B are characterised respectively by:

$$\left[G'(w^G S^G) - \mu_Z - R\right] - \left[F'((1-w^G)S^G) + c - \mu_\varepsilon - R\right] = \lambda_{VaR}\left[\underbrace{\frac{\partial(\gamma RWA^L)}{\partial L}}_{\substack{\text{marginal RW}\\ \text{capital cost of lending}}} - \underbrace{\frac{\partial(\gamma RWA^X)}{\partial X}}_{\substack{\text{marginal RW}\\ \text{capital cost of repo}}} - \frac{\partial Div}{\partial w}\right] \tag{24}$$

and by

$$\left[G'(w^B S^B) - \mu_Z - R\right] - \left[F'((1-w^B)S^B) + c - \mu_\varepsilon - R\right] = \lambda^L_{VaR}\left[\underbrace{\frac{\partial(\gamma RWA^L)}{\partial L}}_{\substack{\text{marginal RW}\\ \text{capital cost of lending}}} - \underbrace{\chi\alpha}_{\substack{\text{marginal leverage}\\ \text{cost of repo}}}\right] \tag{25}$$

Equation (24) and (25) also equate the marginal benefit of reallocating one unit of investment from repo business to lending business with its marginal cost. We can see that if the last two conditions of Proposition 3 are satisfied, then the term in the square bracket on the RHS of Equation (24) is higher than the corresponding term on the RHS of Equation (25). This in turn means that the marginal cost of that rebalancing action can be higher in the case of group-level application than in the case of business unit-level application. If so, the bank will prefer to invest relatively more in the lending business in the latter case than in the former one. As a consequence, the bank's asset risk is higher when regulatory constraints are allocated down to business units. Note that the three conditions stated in Proposition 3 are not sufficient condition for the increase in the bank's asset risk since the RHS of two Equations (24) and (25) also depend on the shadow price of the regulatory constraints.

4 Model calibration

In the previous analysis, we intentionally kept our theoretical setup very general to emphasise the generality of our insights. That generality however implies that we cannot characterise analytically all the possible changes in the bank's investments following the allocation of regulatory constraints to its business units. In this section we calibrate our model to data for banks in the UK, to complement those analytical insights with numerical simulations. The calibration exercise is also helpful to verify the empirical relevance of our previous analytical findings.

We first set out additional parametric assumptions that we make to take the model to the data. We then describes the data used for the calibration and explain our calibration procedure.

Parametric assumptions The bank's ex-ante gross interest income from loans $G(L)$ is naturally the product of the loan volume L and the gross interest rate charged on loans. We assume that the interest rate is a decreasing function of the loan volume: $g_1 + g_2L$ where

$g_1 > 0$ and $g_2 < 0$. Therefore, we have:

$$G(L) = (g_1 + g_2 L)L$$

In line with the literature, we also assume that the losses per unit of loans $\tilde{Z}$ are log-normally distributed with parameter μ_Z^{log} and σ_Z^{log}.

For the repo income, as explained below, since we just have data on short-term repo and reverse repo transactions secured against UK government bonds, we will focus here on this relatively safer type of repo activities and on the role of the repo unit as a market maker. This in turn has two implications. First, $F(X)$ will be the revenue from reverse repo activities net of repo funding cost. We assume that the interest rates charged on both repos and reverse repos depend on the transactional amount, which implies that $F(X)$ can be written as:

$$F(X) = \underbrace{(d_1 + \varepsilon_1 X)X}_{\text{reverse repo revenue}} - \underbrace{(d_2 + \varepsilon_2 X)X}_{\text{repo cost}}.$$

We further denote $\beta_1 = d_1 - d_2$ and $\beta_2 = \varepsilon_1 - \varepsilon_2$ where $\beta_1 > 0$ and $\beta_2 < 0$. Second, we assume that repo business is riskless, i.e. repo losses $\tilde{\varepsilon}$ are equal to zero with probability 1. Table 1 summarises the set of parameters that need to be calibrated.

Table 1: Parameters to be calibrated

Parameters	Description
a	VaR confidence level
χ	Leverage ratio requirement
c	Coupon on government bond
R	Bank's borrowing cost
g_1	Marginal return on loan
g_2	Curvature of loan return
μ_Z^{log}	Lognormal parameter of loan losses
σ_Z^{log}	Lognormal parameter of loan losses
β_1	Marginal return on repo
β_2	Curvature of return on repo

Data To calibrate the model, we use three main data sources. The first dataset is daily yield rates for the 15-year UK government bond retrieved from Factset. Second, we collect information on performance analysis, asset quality and balance sheet of 15 UK banks for which semi-annual data is available in S&P Market Intelligence (S&P MI) from 2015 to 2018. Our last source of data is the confidential Sterling Money Market Data (SMMD) of the Bank of England. It contains daily, transactional level data on repo and reverse repo transactions with maturity of up to one year that are denominated in GBP and secured against UK government-issues securities. The repo and reverse repo transactions reported in this dataset cover 95% of total turnover of the market. They are executed by institutions with a significant proportion of total activity in the market among which there are 5 UK banks. Table 2 reports the variables that we use in these datasets for our calibration.

Table 2: Data sources

Variable description	**Timespan**	**Frequency**	**Data source**
Gross loans to customers	2015-2018	Semi annual	S&P MI
Impaired loans	2015-2018	Semi annual	S&P MI
Net interest margin	2015-2018	Semi annual	S&P MI
Cost of funds	2015-2018	Semi annual	S&P MI
Yield 15Y UK gilt	2015-2018	Daily	Factset
Repo Transaction Nominal Amount	2017-2019	Daily	SMMD
Repo interest rate	2017-2019	Daily	SMMD
Reverse repo Transaction Nominal Amount	2017-2019	Daily	SMMD
Reverse repo interest rate	2017-2019	Daily	SMMD

Calibration methods We set a series of parameters individually. In line with the Basel III risk-weighted capital requirements and the leverage ratio requirement, we set the VaR confidence level a to be equal to 0.001 and the minimum leverage ratio χ equal 3%. We proxy the coupon on government bonds with the 15Y UK gilt yield, as the average of daily yields over the entire period. We set the bank's borrowing cost R to be the average cost of funds of all banks in our sample.

To estimate the distribution parameters μ_Z^{log} and σ_Z^{log} of the random variable $\tilde{Z}$, we proxy its realised value by the amount of impaired loans per unit of total loans. Then we use the maximum likelihood estimation to fit the lognormal distribution of Z with the distribution of impaired loans.

We employ the least square fitting method to derive parameters g_1 and g_2 that underlies the function of gross lending income from net interest margin reported in our datasets. To do so, we first express the net interest margin of bank i at time t - denoted by $IM_{i,t}$ - via g_1 and g_2 as follows:

$$IM_{i,t} = g_1 + g_2 L_{i,t} - Z_{i,t} - R_{i,t}$$

where $L_{i,t}$ is gross loans to customers; $Z_{i,t}$ is the realised value of impaired loans to total loans and $R_{i,t}$ is the cost of funds - all variables are observed in the data. g_1 and g_2 then can be obtained by estimating the following regression equation:

$$y_{i,t} = g_1 + g_2 L_{i,t} + \eta_{i,t}$$

where $y_{i,t} = IM_{i,t} + Z_{i,t} + R_{i,t}$ and $\eta_{i,t}$ is error term. Both coefficients g_1, g_2 derived from the regression are statistically significant at, respectively, 1% and 5% level.

Similarly, to estimate the repo income, we regress the repo and reverse repo interest rate - denoted by $f_{i,t}^{repo}$ and $f_{i,t}^{reverse}$ respectively - reported for each transaction on the borrowing amount of that transaction using the equations:

$$f_{i,t}^{reverse} = d_1 + \varepsilon_1 X_{i,t}^{reverse} + \nu_{i,t} \quad \text{and} \quad f_{i,t}^{repo} = d_2 + \varepsilon_2 X_{i,t}^{repo} + \upsilon_{i,t}$$

Both regressions give statistically significant coefficients at 1% level. Afterward, we calculate the marginal return on repo β_1 as equal to $d_1 - d_2$ and the curvature of repo return β_2 as $\varepsilon_1 - \varepsilon_2$. Table 3 reports the calibrated value for all parameters.[7]

In Figure 1 we display the characteristics of the two business units of our calibrated bank. As seen in the left panel, consistent with Assumption 2, the marginal returns on lending are higher compared to the repo returns. They also decrease at a much slower rate compared

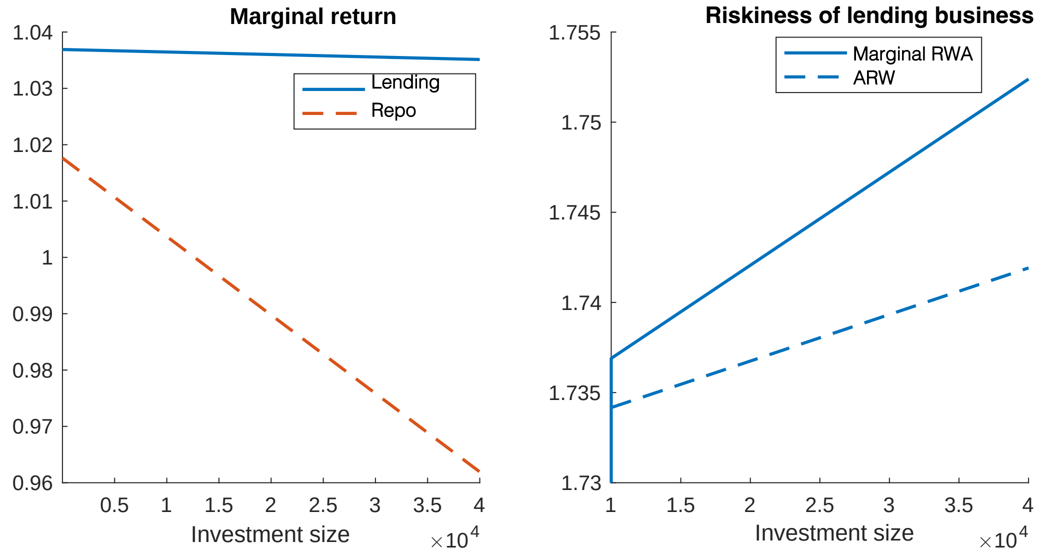
[7] Values of parameters are reported, when appropriate, in terms of billion GBP.

Table 3: Calibration to UK banks

Description	Parameters	Calibrated Value
VaR confidence level	a	0.001
Leverage requirement	χ	0.03
Coupon on government bond	c	1.0172
Bank's borrowing cost	R	1.0114
Lending unit		
Marginal return on loan	g_1	1.0356
Curvature of loan return	g_2	$-2.22 \cdot 10^{-5}$
Log-normal parameter of Z	μ_Z^{log}	-4.568
(Mean Z)		(0.015)
Log-normal parameter of Z	σ_Z^{log}	0.913
(Standard deviation Z)		(0.018)
Repo unit		
Marginal return on repo business	β_1	0.000427
Diminishing return parameter	β_2	$-6.943 \cdot 10^{-4}$

to the repo ones, as the investment size increases. In terms of riskiness, we observe from the right panel that the ARW of our calibrated lending business are globally higher than $\frac{\chi}{\gamma}$ which is equal to 0.35.

Figure 1: Characteristics of two business units



Note: This figure displays main risk and return characteristics of the two business units of our calibrated bank. The left panel shows the marginal returns of repo and lending. The right panel shows two riskiness measures of the lending business, namely the marginal RWA and ARW.

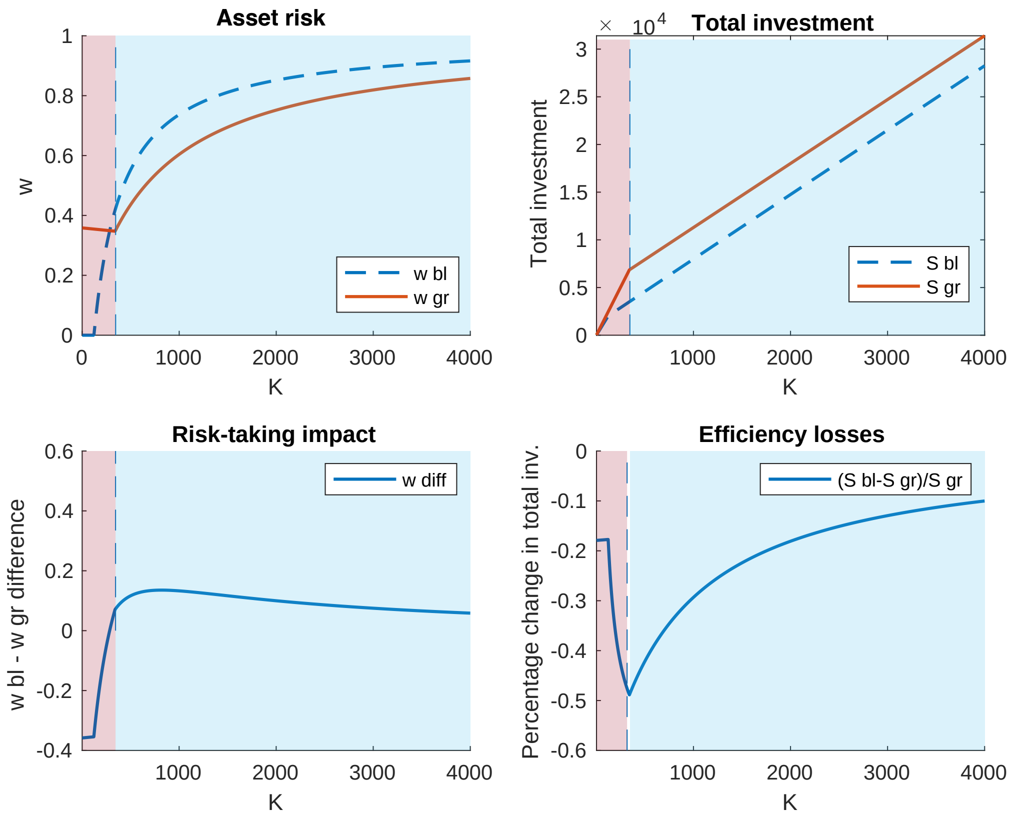
5 Numerical simulations

Using the calibrated parameter values, we solve numerically, for different values of the bank's initial equity K, the two optimisation problems $\wp^G$ and $\wp^B$ as defined in Section 3.1. Note that depending on the value of K, the bank can be bound at the group level either by both LR and RW constraints or by only the RW constraint. Figure 2 compares the bank's optimal investments between the case where all constraints are applied at the group level and the case in which both business units have to comply with both constraints individually.

We can see that the allocation of constraints leads to efficiency losses since the total investments are reduced (see bottom right panel). As explained in the Section 3, these losses are due to the fact that, when allocating regulatory constraints to its business units, the bank cannot exploit the diversification of its investment portfolio to increase the total size of the portfolio for each level of capital resource.

In term of asset risk impact, we can observe from the top left panel that this impact

Figure 2: Bank's optimal investments



Note: This figure compares the bank's optimal investments in two cases:(i) when both regulatory constraints are applied at the group level and (ii) when the bank allocates both constraints down to its business units. In the two top panels, the red solid lines represent bank's choices for (i), while the blue dashed lines stand for bank's choices under (ii). The two bottom panels show the difference, between the two cases, in the bank's total investments (bottom right panel) and in the bank's asset risk (bottom left panel). For all panels, the dark pink area corresponds to the situation where both LR and RW constrains bind at the group level while in the light blue area, only RW constrain binds at the group level.

depends on whether the bank is constrained only by the RW constrains at the group level (light blue area) or by both RW and LR constraints (dark pink area). When only the RW constraint binds at the group level, requiring all business units to comply with both regulatory constraints will lead to a investment distortion in the sense that the bank will invest relatively more into riskier business - lending. This in turn will increase the overall asset risk of the bank. When both constraints bind at the group level, the impact on the bank's asset risk is somehow ambiguous. When K is very small, the bank's asset risk decreases but when K is above a certain threshold, it increases following the allocation of constraints.

6 Role of business model

As highlighted in the analytical part, the impact of the allocation of constraints on banks' investment decisions depends on the specific characteristics of their businesses such as the riskiness. We therefore expect that this impact can vary with banks' business model. In this section, we examine this potential effect of business model. We first classify the 15 UK banks in our S&P MI dataset into different business models. Then we recalibrate the lending business for each type and run numerical simulations. Note that since the limited number of UK banks in the SMMD database that we use to calibrate the repo unit does not allow us to have a meaningful business model classification, we focus here on the consequences of the difference in lending business characteristics and funding costs between business models.

6.1 Business model classification and calibration

Our business model classification relies on the methodology proposed in Roengpitya et al. (2014). They use a statistical clustering method based on various ratios of banks' balance sheet which are informative on the bank business model. They find that retail-funded banks have a high share of gross loans and rely more on stable sources of funding, such as deposits. The wholesale-funded banks have a lower percentage of funding coming from deposits, but a higher share of inter-bank liabilities compared to retail banks. Lastly, the capital markets-orientated banks have a much higher percentage in trading assets and liabilities compared to the previous two types. The last type of banks has the highest ratio of inter-bank borrowing as percentage of total assets and also display a lower reliance on stable funding. The paper reports average values of these ratios to total assets, and we use them as a benchmark to construct the selection criteria for our sample.

Due to limited data availability, compared to the Roengpitya et al. (2014), we use a restricted version of their selected ratios, and we adjust downwards the threshold criteria to match our sample. Our criteria include: the ratio of customer deposits to total liabilities for the stable source of funding ratio, the ratio of assets held for trading to total assets as a measure of tradable assets, loans to banks as fraction of total assets for our inter-bank lending measure, and bank deposits to total liabilities as the bank deposit ratio. Classifying these ratios based on observed bank characteristics from the Roengpitya et al. (2014), we find 9 retail-funded, 5 wholesale-funded and one capital markets-oriented bank. Having one bank only in one group would not permit for a meaningful comparison, so we aggregate the wholesale with the capital markets oriented bank, giving us a sample split into 9 retail-funded, and 6 wholesale-funded and capital markets-orientated banks, to which we refer from now on as wholesale banks. Table 4 reports some characteristics of each business model.

Table 4: Business model descriptives

Description	Values	
	Retail	**Wholesale**
Interest rate on unsecured debt	0.0129 (47)	0.009 (31)
Leverage ratio	0.0549 (55)	0.0529 (43)
Fully loaded risk-weighted capital ratio	0.253 (43)	0.185 (32)
Loans to total assets	0.7649 (63)	0.619 (43)
Percentage of impaired loans to total loan size	1.11% (53)	2.04% (28)

The number of observations is in brackets, unless otherwise stated.

We recalibrate the model to the two different categories of banks. We report the calibrated values of different parameters for each type of banks in Table 5. We observe that wholesale banks have lower costs of funding than retail banks.

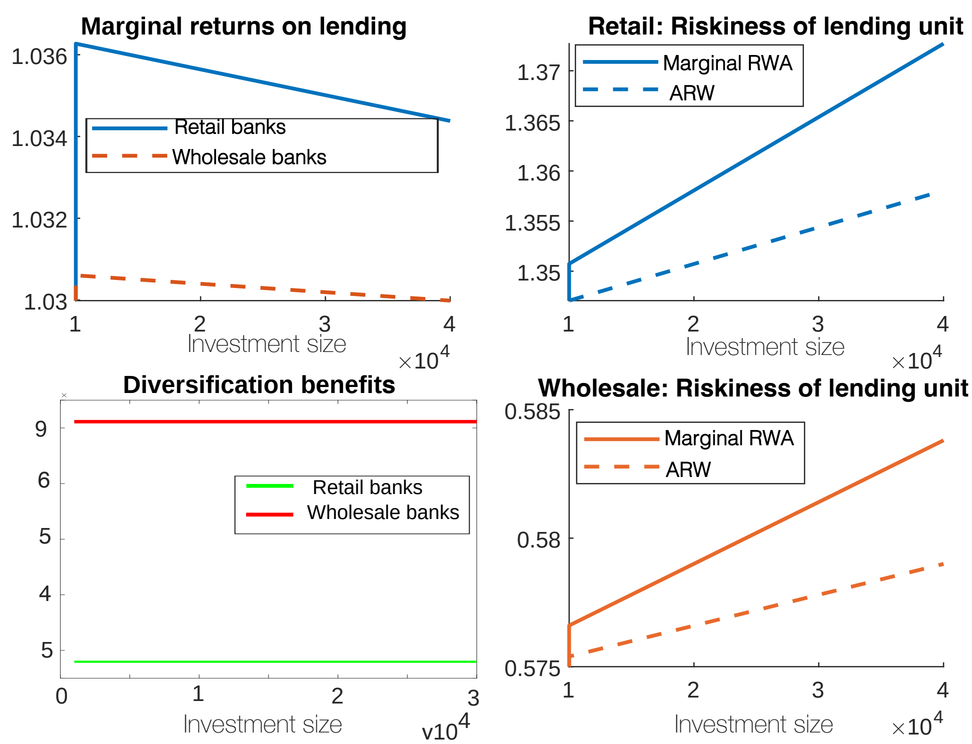
Further, in Figure 3, we compare the two business models in terms of returns and riskiness of their lending business. As seen in the top left panel, the marginal returns of lending are higher for retail banks and decrease at a higher speed compared to wholesale banks. The two panels on the right show that the lending business of retail banks is riskier than that of

Table 5: Calibration across business models

Description	Parameters	**Retail**	**Wholesale**
Bank's borrowing cost	R	0.0129	0.009
	Lending		
Marginal return on loan	g_1	1.0369	1.03081
Curvature of loan return	g_2	$-3.15 \cdot 10^{-5}$	$-1.03 \cdot 10^{-5}$
Log-normal parameter of Z	μ_Z^{log}	-4.885	-3.97
(Mean Z)		(0.0118)	(0.0207)
Log-normal parameter of Z	σ_Z^{log}	0.945	0.429
(Standard deviation Z)		(0.0142)	(0.0093)

wholesale banks.

Figure 3: Characteristics of lending business across business models



Note: This figure compares some main characteristics of lending business across business models. The top left panel shows the marginal returns on lending. The bottom left panel shows, as a function of lending size, the diversification benefits defined as the difference between RWA^L and RWA^G. In the two right panels, we represent the ARW and the marginal RWA as a function of investment in lending.

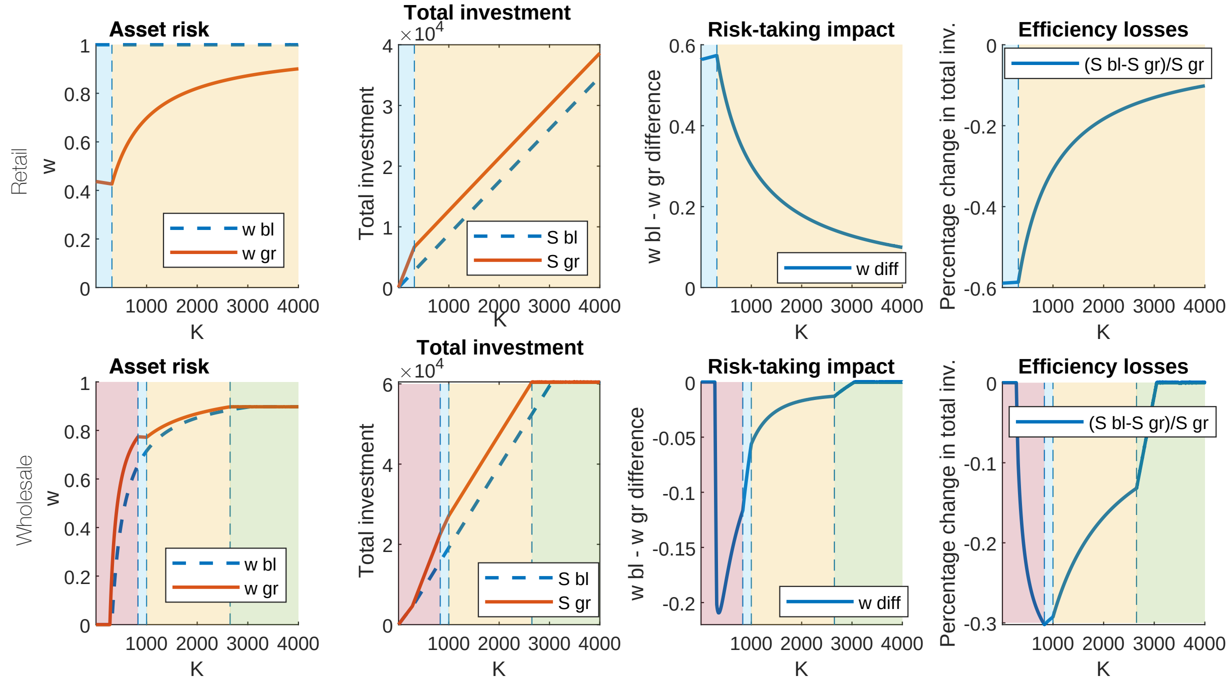
6.2 Numerical simulations for different business models

We now run simulations for each business model. Figure 4 compares the optimal investments of retail and wholesale banks between the case where all constraints are applied at the group level and the case in which both constraints are allocated down to business units.

Two main observations are in order here. First the situation in which the leverage constraint binds at the group level happens only with wholesale banks, but not for retail banks. Note also that since in these simulations, we assume that the repo business is riskless, the ARW of this business for both types of banks is lower than the CRW which is equal to 0.35. This difference can therefore be explained by the fact that the ARW of the lending business for both types of banks is higher than the CRW but the diversification benefits of wholesale banks are higher than those of retail banks as shown in the bottom left panel of Figure 3.

Second, there is a stark difference in the impact of the allocation of constraints on the banks' asset risk between retail and wholesale banks when only the RW constraint binds at the group level (beige area in Figure 4). Precisely, in this case, while the allocation of constraints results in an increase in the asset risk for retail banks, it brings about a decrease in the asset risk for wholesale banks.

Figure 4: Optimal investments: comparison across business models



Note: This figure compares the optimal investments of retail banks (first row) and wholesale banks (second row) in two cases: (i) both regulatory constraints are applied at the group level and (ii) the bank allocates both constraints to its business units. In the first two columns, the red solid lines represent bank's choices in the first case while the blue dashed lines stand for bank's choices in the second case. The panels in the third and fourth columns represent the difference in, respectively, the bank asset risk and banks' total investments between the two cases. For all panels, the dark pink area corresponds to the situation where only the LR constraint binds at the group level; the light blue area to the case in which both LR and RW constrains bind; the beige area to the case where only RW constraint binds and finally the green area is when no constraints bind.

7 Conclusion

In this paper we assess how banks' asset risk depends on the level at which they apply regulatory requirements. We develop a model where a banking group has two business units: a riskier one which yields higher returns and a less risky one which has lower returns. We refer to these units as lending and repo business units, respectively. We also calibrate the model to UK banks and evaluate the implications of bank business models for the impact of application level on asset risk.

We find that the effect on the bank's asset risk differs depending on which of the two regulatory constraints binds at the group level. If the banking group is bound by the LR requirement, allocating regulatory requirements to business units will either leave the riskiness of its investments unchanged or reduce it. However, if the RW requirement is the binding constraint at the group level, then under certain conditions, applying requirements at the business-unit level can result in an increase of the bank's asset risk by inducing the bank to invest relatively less in repo and more in lending.

When calibrating the model to UK banks, we get some additional insights on how the way banks treat regulatory requirements within their internal regulatory capital allocation process impacts asset risk. If only the RW requirement binds at the group level, consistent with the analytical finding, we find that the bank's investments are distorted with a relatively higher investments in the riskier unit. Both the efficiency losses and risk-taking effects diminish as the bank's equity capital increases. If the banking group is bound by the LR and the RW requirements simultaneously, the results are ambiguous in terms of asset risk. Finally, when we calibrate the model based on different business models, we find that applying requirements at the business-unit level leads to an increase in risk-taking for retail banks but not for wholesale banks.

In terms of policy implications, our paper offers useful insights on the impact of regulatory measures can depend on the level at which banks apply those regulatory constraints. First, we highlight two potential costs when banks apply them at low business levels: it can make banks increase their asset risk and decrease their overall investments. This has immediate implications for low-risk, low-margin markets such as repo market, as banks will decrease their activities in those markets. Second, we find that one size does not fit all, and the impact on asset risk differs across bank business models.

References

Acosta-Smith, J., Grill, M., and Lang, J. H. (2020). The leverage ratio, risk-taking and bank stability. *Journal of Financial Stability*, Article nr. 100833.

Allahrakha, M., Cetina, J., and Munyan, B. (2018). Do higher capital standards always reduce bank risk? the impact of the Basel leverage ratio on the US triparty repo market. *Journal of Financial Intermediation*, 34:3–16.

Bajaj, R., Binmore, A., Dasgupta, R., and Vo, Q.-A. (2018). Banks internal capital markets: how do banks allocate capital internally? *Bank of England Quarterly Bulletin Q2*.

Bicu-Lieb, A., Chen, L., and Elliott, D. (2020). The leverage ratio and liquidity in the gilt and gilt repo markets. *Journal of Financial Markets*, 48:Article nr. 100510.

BIS CGFS (2017). Repo market functioning report. *Committee on the Global Financial System Papers*, no 59.

Blum, J. M. (2008). Why Basel II may need a leverage ratio restriction. *Journal of Banking & Finance*, 32(8):1699–1707.

Choi, D. B., Holcomb, M. R., and Morgan, D. P. (2018). Leverage limits and bank risk: New evidence on an old question. Technical Report No. 856, Staff Report Federal Reserve Bank of New York, New York NY.

Duffie, D. (2018). Financial regulatory reform after the crisis: An assessment. *Management Science*, 64(10):4835–4857.

Gambacorta, L. and Karmakarb, S. (2018). Leverage and risk-weighted capital requirements. *International Journal of Central Banking*, 14(5):153–191.

Goel, T., Lewrick, U., and Tarashev, N. (2020). Bank capital allocation under multiple constraints. *Journal of Financial Intermediation*, 44:Article nr. 100844.

Ita, A. (2017). Capital allocation in large banks-a renewed look at practice. *Available at DOI http://dx.doi.org/10.2139/ssrn.2726165.*

Khaykin, I., Koyluoglu, U., Elliott, D., and Spicer, C. (2017). Balancing complex and competing constraints on capital, liquidity and funding. *Oliver Wyman. Financial Resource Management. Technical report.*

Kiema, I. and Jokivuolle, E. (2014). Does a leverage ratio requirement increase bank stability? *Journal of Banking & Finance*, 39:240–254.

Kotidis, A. and Van Horen, N. (2019). Repo market functioning: The role of capital regulation. *CEPR Discussion Paper No. DP13090.*

Krüger, P., Landier, A., and Thesmar, D. (2015). The WACC fallacy: The real effects of using a unique discount rate. *The Journal of Finance*, 70(3):1253–1285.

Perold, A. F. (2005). Capital allocation in financial firms. *Journal of Applied Corporate Finance*, 17(3):110–118.

Roengpitya, R., Tarashev, N. A., and Tsatsaronis, K. (2014). Bank business models. *BIS Quarterly Review December.*

A Appendix

A.1 Derivation of Constraint (3) - the RW constraint at the group level

Given that

$$\tilde{\Pi}_L = G(L) - \tilde{Z}L - R(L - K_L)$$

and

$$\tilde{\Pi}_X = F(X) + cX - \tilde{\varepsilon}X - R(X - K_X)$$

we can write $\mathbb{P}\left(\tilde{\Pi}_L + \tilde{\Pi}_X \leq 0\right) \leq a$ as:

$$\mathbb{P}\Big(G(L) + F(X) + cX - R(X + L - K) \leq \tilde{Z}L + \tilde{\varepsilon}X\Big) \leq a \tag{A.1}$$

Using Definition (1), Inequality (A.1) is equivalent to

$$VaR_{1-a}(\tilde{Z}L + \tilde{\varepsilon}X) \leq G(L) + F(X) + cX - R(X + L - K)$$

or

$$K \geq \frac{VaR_{1-a}(\tilde{Z}L + \tilde{\varepsilon}X) - [G(L) - RL + F(X) + cX - RX]}{R} \tag{A.2}$$

A.2 Variance of asset returns as a measure of bank asset risk

Using Expression (7), we could rewrite the variance of the returns of the bank's asset portfolio σ_p^2 as a function of w as follows:

$$\sigma_p^2 = S^2 w^2 \sigma_Z^2 + S^2 (1-w)^2 \sigma_\varepsilon^2$$

We thus obtain:

$$\frac{\partial \sigma_p^2}{\partial w} = 2S^2\left(w\sigma_Z^2 - (1-w)\sigma_\varepsilon^2\right) \quad \text{and} \quad \frac{\partial^2 \sigma_p^2}{\partial w^2} \geq 0$$

Denote by $\underline{w}$ the value of w when $\frac{\partial \sigma_p^2}{\partial w} = 0$, i.e.

$$\underline{w} = \frac{\sigma_\epsilon^2}{\sigma_Z^2 + \sigma_\varepsilon^2}$$

Since $\sigma_\varepsilon^2 < \sigma_Z^2$, we have $\underline{w} < 0.5$.

We observe that σ_p^2 is decreasing with w when $w \in (0, \underline{w})$ and increasing with w when $w \in [\underline{w}, 1]$. Therefore, the results for w as a measure of the bank asset risk also hold for σ_p^2 when $w \in [\underline{w}, 1]$. We believe that focusing on the interval $[\underline{w}, 1]$ is without loss of generality due to two reasons. First, if σ_ε^2 is low enough, $\underline{w}$ is closing to 0. Second, $[\underline{w}, 1]$ would be the more empirically relevant interval for the share of risky businesses in the banks' balance sheet. Indeed, based on our data about balance sheet information of a sample of UK banks, we find that the share of lending business for those banks is generally above 0.5.

A.3 Derivation of the first order conditions (FOCs) for Problems $\wp^G$ and $\wp^B$

The Lagrangian for Problem $\wp^G$ reads:

$$\begin{aligned}\Lambda_g = \quad & \Pi(w,S) - \mu_Z wS - \mu_\varepsilon(1-w)S + RK + \lambda_{VaR}\Big(K - \frac{VaR_{1-a}(\tilde{Z}w + (1-w)\tilde{\varepsilon})S - \Pi(w,S)}{R}\Big) \\ & + \lambda_{LR}(K - \chi\big(w + \alpha(1-w)\big)S)\end{aligned}$$

where λ_{VaR} and λ_{LR} are the Lagrange multiplier for, respectively, the group-level RW constraint and the group-level LR constraint. The FOC that determines w^G is as follows:

$$\frac{\partial \Pi(S,w)}{\partial w} - \mu_Z S + \mu_\varepsilon S - \lambda_{VaR}\frac{\partial(\gamma RWA^G(w,S))}{\partial w} - \lambda_{LR}\chi(1-\alpha)S = 0 \qquad \text{(A.3)}$$

Similarly, the Lagrangian for Problem $\wp^B$ reads:

$$\begin{aligned}\Lambda_b = \quad & \Pi(S,w) - \mu_Z wS - \mu_\varepsilon(1-w)S + RK + \lambda^L_{VaR}\Big(K_L - \frac{VaR_{1-a}(\tilde{Z}w)S - G(wS) + RwS}{R}\Big) \\ & + \lambda^X_{VaR}\Big(K_X - \frac{VaR_{1-a}(\tilde{\varepsilon}(1-w))S - F\big((1-w)S\big) - c(1-w)S + R(1-w)S}{R}\Big) \\ & + \lambda^L_{LR}(K_L - \chi wS) + \lambda^X_{LR}(K_X - \chi\alpha(1-w)S) + \lambda_K(K - K_L - K_X)\end{aligned}$$

where $\lambda^L_{VaR}, \lambda^X_{VaR}, \lambda^L_{LR}, \lambda^X_{LR}$ and λ_K are the Lagrange multipliers of corresponding constraints. The FOC for w^B is written as follows:

$$\begin{aligned}\frac{\partial\Pi(S,w)}{\partial w} - \mu_Z S + \mu_\varepsilon S - \lambda^L_{VaR}\frac{\partial(\gamma RWA^L(w,S))}{\partial w} & \\ - \lambda^X_{VaR}\frac{\partial(\gamma RWA^X(w,S))}{\partial w} - \lambda^L_{LR}\chi S + \lambda^X_{LR}\chi\alpha S = 0 & \quad \text{(A.4)}\end{aligned}$$

A.4 Proof of Lemma 1

Note that the ARW of the banking group - ARW^G - is computed as:

$$ARW^G(w,S) = \frac{RWA^G(w,S)}{wS + \alpha(1-w)S} \quad \text{(A.5)}$$

We see that since α is greater than or equal to 1, the denominator of the RHS of Expression (A.5) is weakly decreasing with w. We will now examine how $RWA^G(w,S)$ changes with w.

From Equation (A.3), we see that the FOC that determines w at the group level can be written as follows:

$$\frac{\partial\Pi(w,S)}{\partial w} - (\mu_Z - \mu_\epsilon)S = \gamma\lambda_{VaR}\frac{\partial RWA^G}{\partial w} - \chi\lambda_{LR}(\alpha-1)S \quad \text{(A.6)}$$

Since the bank will always choose w such that the LHS of Equation (A.6) is non negative, in the equilibrium we have:

$$\gamma\lambda_{VaR}\frac{\partial RWA^G}{\partial w} - \chi\lambda_{LR}(\alpha-1)S \geq 0$$

which implies that in the equilibrium $\frac{\partial RWA^G}{\partial w} \geq 0$ since $\alpha \geq 1$.

Hence, in the equilibrium, the numerator of the RHS of Expression (A.5) is increasing with w, which in turn implies that in the equilibrium, ARW^G increases with w.

A.5 Proof of Proposition 2

- *First, we prove that if both business units have ARW below $\frac{\chi}{\gamma}$, then the asset risk of the LR-constrained bank does not change with the application level of requirements.*

To prove the above, we will establish that the solution (w^G, S^G) to Problem $\wp^G$ will also be the solution to Problem $\wp^B$ if the three following conditions are satisfied:

$$\chi(wS + \alpha(1-w)S) > \gamma RWA^G \tag{A.7}$$

as well as

$$\frac{RWA^L}{wS} \leq \frac{\chi}{\gamma} \quad \text{and} \quad \frac{RWA^X}{\alpha(1-w)S} \leq \frac{\chi}{\gamma} \tag{A.8}$$

Indeed, since (w^G, S^G) is the solution to Problem $\wp^G$ when Condition (A.7) is satisfied, we have:

$$K = \chi(w^G S^G + \alpha(1-w^G)S^G) \tag{A.9}$$

When two conditions in (A.8) hold, the relevant constraints for Problem $\wp^B$ will be Constraints (13), (14) and (15). Clearly, (w^G, S^G) that satisfies Equality (A.9) will also satisfy all Constraints (13), (14) and (15) where we simply choose $K_L = \chi w^G S^G$ and $K_X = \chi\alpha(1-w^G)S^G$. This in turn implies that (w^G, S^G) belong to the feasible set of Problem $\wp^B$. Since the feasible set of Problem $\wp^B$ is smaller than that of Problem $\wp^G$, (w^G, S^G) are also the solution to Problem $\wp^B$.

- *We now prove that if both business units have ARW greater than $\frac{\chi}{\gamma}$, then the asset risk of the LR-constrained bank will decrease when regulatory constraints are allocated down to business units.*

To prove this, we will compare the FOCs that determine w^G and w^B. When the LR requirement is the binding constraint at the group level, we have $\lambda_{VaR} = 0$ and $\lambda_{LR} \geq 0$. Therefore, based on Equation (A.3), after some algebra, we see that in this case, w^G is determined by the following equation:

$$\left[G'(w^G S^G) - \mu_Z - R\right] - \left[F'((1 - w^G)S^G) + c - \mu_\varepsilon - R\right] = \lambda_{LR} \left(\chi - \alpha\chi\right) \tag{A.10}$$

In relation to w^B, when $ARW^L \geq \frac{\chi}{\gamma}$ and $ARW^X \geq \frac{\chi}{\gamma}$, Constraint (11) is tighter than Constraint (13) and Constraint (12) is tighter than Constraint (14). The binding constraints in Problem $\wp^B$ will thus be Constraints (11) and (12), which implies $\lambda^L_{LR} = \lambda^X_{LR} = 0$.

Based on the Lagrangian for Problem $\wp^B$ explained in Appendix A.3, the two FOCs for K_L and K_X are as follows:

$$\lambda^L_{VaR} + \lambda^L_{LR} - \lambda_K = 0 \quad \text{and} \quad \lambda^X_{VaR} + \lambda^X_{LR} - \lambda_K = 0$$

which means:

$$\lambda^L_{VaR} + \lambda^L_{LR} = \lambda^X_{VaR} + \lambda^X_{LR} = \lambda_K \tag{A.11}$$

Therefore, we obtain

$$\begin{cases} \lambda^L_{LR} = \lambda^X_{LR} = 0 \\ \lambda^L_{VaR} = \lambda^X_{VaR} \geq 0 \end{cases} \tag{A.12}$$

Plugging Result (A.12) into the FOC (A.4), after some algebra, we obtain the equation that characterises w^B as follows:

$$\left[G'(w^B S^B) - \mu_Z - R\right] - \left[F'((1 - w^B)S^B) + c - \mu_\varepsilon - R\right] = \lambda^L_{VaR} \left[\frac{\partial(\gamma RWA^L)}{\partial L} - \frac{\partial(\gamma RWA^X)}{\partial X}\right] \tag{A.13}$$

Note that the LHS of Equations (A.10) and (A.13) is a decreasing function of w. Moreover the RHS of Equation (A.10) is non positive while the RHS of Equation (A.13) is non negative. These all together imply that $w^B < w^G$

A.6 Proof of proposition 3

Since the bank is bound by the RW constraint at the group level, we have $\lambda_{LR} = 0$ and $\lambda_{VaR} \geq 0$. From Equation (A.3), we see that w^G is characterised by the following equation:

$$\big[G'(w^G S^G) - \mu_Z - R\big] - \big[F'((1-w^G)S^G) + c - \mu_\varepsilon - R\big] = \lambda_{VaR}\left[\frac{\partial(\gamma RWA^L)}{\partial L} - \frac{\partial(\gamma RWA^X)}{\partial X} - \frac{\partial Div}{\partial w}\right] \quad \text{(A.14)}$$

Regarding w^B, when $ARW^L > \frac{\chi}{\gamma}$ and $ARW^X < \frac{\chi}{\gamma}$, Constraint (11) is tighter than Constraint (13) and Constraint (12) is looser than Constraint (14). The binding constraints in Problem $\wp^B$ will thus be Constraints (11) and (14), which implies $\lambda^L_{LR} = 0$ and $\lambda^X_{VaR} = 0$. Using Result (A.11), we thus have $\lambda^L_{VaR} = \lambda^X_{LR} \geq 0$. This in turn implies that w^B is determined as follows:

$$\big[G'(w^B S^B) - \mu_Z - R\big] - \big[F'((1-w^B)S^B) + c - \mu_\varepsilon - R\big] = \lambda^L_{VaR}\left[\frac{\partial(\gamma RWA^L)}{\partial L} - \chi\alpha\right] \quad \text{(A.15)}$$

Therefore, if the three conditions stated in Proposition 3 are satisfied, it can happen that the RHS of Equation (A.14) is greater than that of Equation (A.15). In that case $w^G < w^B$ since the LHS of the two equations is decreasing with w.