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Income inequality, mortgage debt and house prices

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Abstract

This paper studies housing and credit market implications of increasing income inequality and discusses how a low interest rate environment can alter its consequences. I develop an analytical general equilibrium model with a novel borrower risk composition channel of income inequality. Following a rise in income inequality house prices and mortgage debt decline, and aggregate default risk increases. I then show that low real rates mitigate the depressing effect of inequality on house prices at the cost of amplifying the aggregate default risk. Using a panel of US states and instrumental variables approach, I verify the model's predictions.

Key words: Income inequality, mortgage lending, mortgage default, house prices, real interest rates, risk taking, shift-share instruments.

JEL classification: D31, E44, E58, G21, R21.

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In recent decades the United States has experienced a steady increase in income inequality. This rise in inequality has been accompanied with upward trends in real house prices and real mortgage debt - albeit a boom-bust episode around the Great Recession (Figure 1, left panel). The popular interpretation of these aggregate trends was that rising income inequality might have contributed to the accumulation of debt by the easening of lending conditions, particularly among low income households, and to the risk build-up in the credit markets (Rajan (2010)).[1] Mortgage delinquencies indeed started rising prior to the Great Recession peaking at levels well above that of early 1990s recession and precipitated a crash in the financial markets (Figure 1, right panel).[2]

While income inequality increased, real rates declined in the U.S. since 1980s - a development through declining mortgage rates is expected to contribute to upwards trends in mortgage debt and house prices. In this paper, I study the dynamics of mortgage debt, house prices and mortgage delinquencies in the presence of rising income inequality and declining real rates. I use both empirical and theoretical strategies to isolate the effect of rising inequality from that of declining rates. I find that rising inequality and declining rates operate in opposite directions: while declining rates, unsurprisingly, increases house prices and mortgage debt through lower mortgage rates; rising income inequality decreases house prices and mortgage debt. On the empirical side, I exploit variation in inequality from US states and counties to control for the effect of declining real rates, among other common trends, and use instrumental variables approach to estimate causal effects. On the theoretical side, I develop an analytical model with a rich form of borrower heterogeneity which is an important characteristic of the data (Bartscher, Kuhn, Schularick and Steins (2020), Albanesi, De Giorgi and Nosal (2017) Adelino, Schoar and Severino (2016), Foote, Loewenstein and Willen (2016)) and is overlooked in borrower - saver frameworks relating household debt to inequality (Kumhof, Rancière and Winant (2015), Mian, Straub and Sufi (2020a)). I show that rising income inequality in isolation depresses house prices and mortgage debt through a novel borrower composition channel of income inequality.

The first contribution of this paper is to document new cross-sectional facts regarding growth in income inequality, house prices, and mortgage credit. Figure 2 plots the

[1] Kumhof, Rancière and Winant (2015) show that the period preceding the Great Depression was also characterised by concurrent upward trends in household debt and income inequality.

[2] Delinquent loans are those past due thirty days or more and still accruing interest as well as those in nonaccrual status. They are measured as a percentage of end-of-period loans.

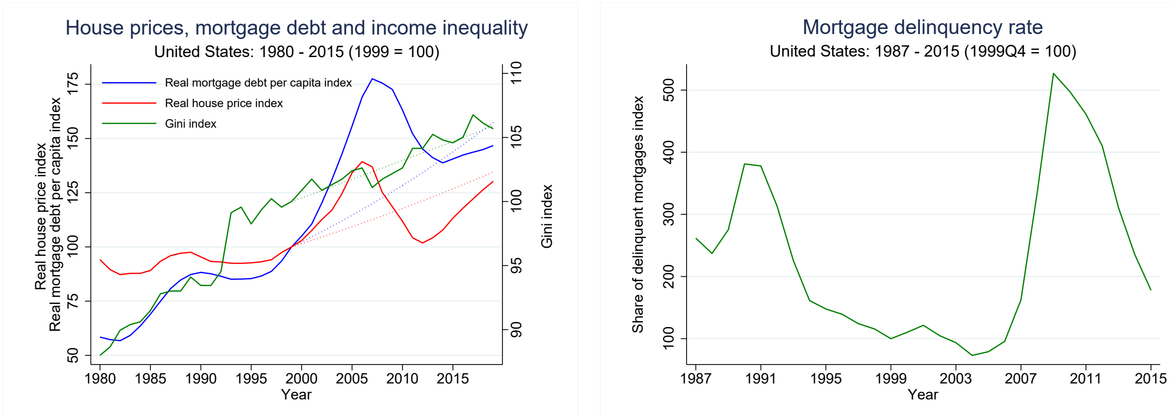


Figure 1: Income inequality, real house prices, mortgage debt and mortgage delinquencies in the US

Source: US Census Bureau, US Flow of Funds, Federal Housing and Finance Agency, Federal Financial Institutions Examination Council

partial correlation of the Gini coefficient with house prices, mortgage debt and mortgage delinquencies between 1999 and 2011 in US counties, controlling for a variety of county characteristics.[3] The figure shows that counties which experienced a greater increase in income inequality had lower house price growth, lower mortgage debt growth and a greater increase in the delinquency rate over the same period.[4] For both house prices and mortgage debt, the cross-sectional relationships are at odds with the aggregate trends in Figure 1. The positive association between income inequality and delinquency suggests a channel through which higher inequality may have increased risk in credit markets via its effect on mortgage delinquencies, however, this is not through higher debt and house prices.

While cross-sectional analysis has the advantage of controlling for common trends and provides more robust relationships than the aggregate trends, it is silent about the channel through which income inequality might affect housing and credit markets. The second contribution of this paper is to construct a structural model with closed form solutions which can be used to study the effects of income inequality and real rates independent of each other. The model has the minimal ingredients to derive tractable general equilibrium results. Heterogeneous income households borrow to

[3]The sample is guided by mortgage data availability at the county level.

[4]In the online appendix I show that the negative relationship between house prices and income inequality has been prevalent since 1989. I also show that the negative association of income inequality with house price and mortgage debt is robust within different terciles of initial subprime borrower share and of Saiz (2010) housing supply elasticity, which controls for county characteristics that might correlate with lending practices or house price growth expectations (Mian and Sufi (2009)).

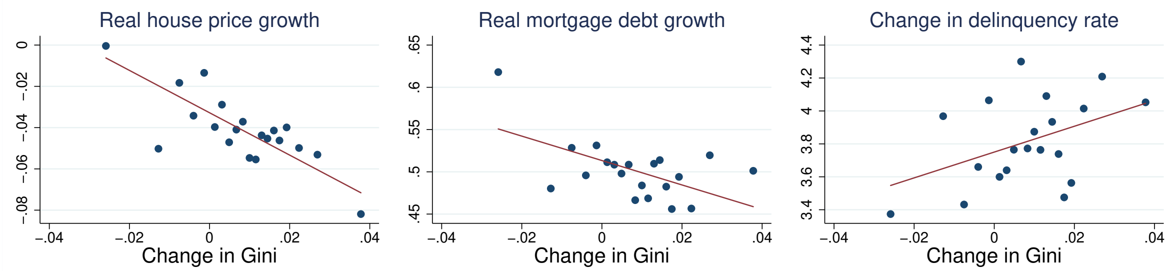


Figure 2: Changes in income inequality, real house price growth, mortgage debt growth and change in mortgage delinquency rate over US counties between the years 1999 and 2011

Source: US Census Bureau, New York Fed Consumer Credit Panel, Federal Housing and Finance Agency, Bureau of Labor Statistics.

Note: Binned scatter plots. All growth rates and changes are calculated between the years 1999 and 2011. Control variables in the plots are state fixed effects, mean income growth, population growth, the share of subprime borrowers in 2000, median income in 1999, and the number of households in 1999.

finance their housing purchases and they may later default on their mortgage payments if doing so implies higher utility than repayment. The key ingredient of the model is that households select from a menu of mortgage contracts with different default risk. The lenders price mortgages and set borrowing limits consistent with the borrowers' expected default probability.

Cost of borrowing increases with debt and decreases with the size of housing as it serves as collateral in the case of default. Households at different points in the income distribution make different contract choices. The model features an income cut-off that defines the marginal risk taking borrower. That is, borrowers with incomes below the marginal borrower select into having mortgage default risk, which translates into high cost of borrowing, low down-payments, and small housing consumption. A rise in the Gini coefficient increases the share of households that opt for mortgages with default risk and this depresses housing demand. That is, in income inequality operates through a borrower composition channel in which a rise is associated with increased risk taking. I also find that a rise in top incomes shares leads to a decline in mortgage debt as households with low incomes select into high loan-to-income ratio mortgages compared to high income borrowers.

While the model predictions are consistent with the cross-sectional evidence presented in Figure 2, the question of why house prices and mortgage debt have been increasing with income inequality in the aggregate data remains. Another secular trend for the U.S. over the same time period is declining real interest rates. In the model, a decline in the real interest rate leads the mortgage interest rate and down-payment to decline for all mortgage contracts. Borrowers then demand larger houses, which increases house prices. Declining real interest rates can thus overturn the negative effect of increasing inequality on house prices and mortgage debt, and allow the model to match the aggregate trends. However, this further increases the aggregate default risk in the economy. A fall in the real rates translates into lower down-payments and higher housing consumption for all contracts, however, due to the collateral value of houses in mortgage pricing the pass through is stronger in the presence of default risk. This raises the income cut-off that defines risk taking, or equivalently, the marginal risky borrower comes from upper quantiles of the income distribution. Aggregate default risk increases as rates decline, amplifying the effect of rising income inequality.[5] This paper therefore also contributes to the literature on the risk-taking channel of low interest rates by providing a mechanism which operates through the housing market.[6]

To verify the model's predictions, I turn to a panel of US states with longest data availability at annual frequency. Using this data allows me to provide further evidence on the cross-sectional facts that motivate the theoretical model and test also the interaction predictions of the model akin second derivatives.[7] In order to study the causal effect of rising income inequality, I derive a shift-share instrument for the Gini coefficient that measures the exposure of local income distribution to differential wage growth across industries according to initial patterns of industry specialization. The shift component of my instrument is to predict the local industry wages by iterating forward 1990 levels with the national growth in industry wages. While differential wage growth in industries can arise from developments like import penetration and skill-biased technical change, my approach abstracts from the initial source and considers the national wage developments as the shift. The exposure component of my instrument is 2-year lagged local industry employment shares. Therefore, different than the

[5] Adelino, Schoar and Severino (2016) show that middle-income, high-income, and prime borrowers all sharply increased their share of delinquencies in the Great Recession. The risk composition channel of my paper then suggests that declines in the real rate might have caused selection into mortgage default risk in the upper quantiles of the income distribution.

[6] DellAriccia, Laeven and Marquez (2014) present a theoretical model of bank-risk taking. They show that, when bank capital cannot adjust, a decrease in the real interest rate can increase risk-taking. However, this results depends on the shape of an exogenous loan demand. Sheedy (2018) studies the financial stability implications of expansionary monetary policy through housing and mortgage markets.

[7] I use annual data from 1992 to 2015 for house prices, and from 2003 to 2015 for mortgage credit and delinquency at state level. Inequality data is based on IRS tax returns. County level inequality data is from American Community Surveys and reliable measures are available every three years after 2005. This makes the time dimension of the county panel data too short to study the interaction between income inequality and real rates.

canonical shift-share instruments à la Bartik (1991), I derive a higher moment instrument not a weighted average exposure instrument. A state that has a predetermined specialization in industries that are on the decline might then experience a decline in income inequality when measured with my instrument. In order to further ensure that the instrument is not subject to reverse causality, I exclude the industries that are related to housing and credit markets in calculating local wage Gini coefficient.[8] I estimate specifications which include state fixed effects to control for any time invariant state characteristics and time fixed effects to control for macroeconomic developments. This identification strategy thus controls for time varying demand and supply related developments in credit and housing markets that are common to all US states. These include secular decline in real rates, financial liberalization and eased credit access among others bringing me closer to a set-up that can test the risk composition channel that lies in the heart of the model. Finally, state level data allows me to study the effect of real rates that is exogenous to local inequality developments as depicted in my model. This is because national real rates are less likely to fall because of by local inequality increases.[9]

I find that one percentage point increase in the Gini coefficient leads to 2.2% decline in real house prices, 0.34 percentage point increase in the share of delinquent mortgages, and 1.4 % decline in real mortgage debt per capita.[10] I then examine how changes in the long-term real interest rate alter the responses of these variables to changes in income inequality. Results confirm the theoretical predictions. I show that one percentage point decline in real rates mitigates the inequality elasticity by half for the house prices and amplifies it 1.3 folds for the mortgage delinquency rate.

In this paper, rising inequality and declining real rates are taken as exogenous developments.[11] Therefore, my theoretical findings can be interpreted as: to the extent

[8]I exclude NAICS sectors construction, finance and insurance and real estate and rental and leasing, and keep only wages from privately owned enterprises.

[9]I exclude the states with the highest GDP levels from the sample to further ensure that national real rates are exogenous to local developments and find the same qualitative and similar quantitative results.These states are California, Texas, New York, Florida and Illinois, and comprise about 40% of national GDP as of 2020.

[10]A back of the envelope calculation then suggests that absent the rise in income inequality from 2003 to 2015, real house prices would be about 7.7% higher, mortgage delinquency rate would be 1.2 percentage points lower and real mortgage debt would be 5% higher in the mean US state.

[11]The determinants of declining real rates are beyond the scope of current paper. Population aging (Eggertsson, Mehrotra and Robbins (2019)), income inequality (Auclert and Rognlie (2018)), the global saving glut (Bernanke (2005), the decline in the price of capital goods (Sajedi and Thwaites (2016)) are among proposed explanations.

that rising inequality contributes to the decline in real rates, this would contribute to the positive effect of declining rates on house prices and mortgage debt, and amplify the aggregate default risk effect further.[12]

While my paper presents a borrower risk composition channel, similar to Rajan (2010) several studies suggested a credit supply channel of income inequality where higher inequality gives rise both to a rise in debt and a decline in borrowing rates. Closely related to my paper is the seminal work of Kumhof, Rancière and Winant (2015) that employs a two-agent borrower-saver DSGE model with endogenous probability of a financial crisis. The authors show that a rise in top income shares increases savings of top income households, which are channeled to bottom income households as more debt and raise the risk of a financial crisis. Concurrently with higher inequality, return to savings, or equivalently, the cost of borrowing declines suggesting a credit supply channel of income inequality. In a recent contribution using a borrower-saver DSGE model Mian, Straub and Sufi (2020a) study rising income inequality and financial deregulation as two secular forces that can give rise to a high debt and low rates environment.[13] My paper differs from these studies by deriving analytical results and by modeling lenders as foreigners, rather than the top income households. Mian, Straub and Sufi (2020b) show that for the U.S. the magnitude of the household liabilities held by foreigners is similar to that of the savings of top 1% of the income distribution. Therefore, a decline in real rates in my model can be interpreted as a result of global savings glut rather than the savings glut of the rich, as the case of Mian, Straub and Sufi (2020a). My paper complements these theoretical analyses by introducing endogenous house prices, and by having a particular focus on mortgage debt and mortgage defaults as the measures of risk in credit markets.[14]

[12]Rachel and Summers (2019) compares alternative explanations and finds that rising inequality accounts for about 0.6 percent of the 3 percent decline in real rates since 1970 - a value similar to the findings of Auclert and Rognlie (2018). The highest estimate for the contribution of rising inequality to declining in real rates is suggested as 1 percent by Straub (2018).

[13]Both Kumhof, Rancière and Winant (2015) and Mian, Straub and Sufi (2020a) assume preference structures that give rise to high marginal propensity to save for top income earners and focus on the equilibrium in which bottom earners borrow and top earners save. To derive analytical solutions, I assume quasilinear preferences similar to Justiniano, Primiceri and Tambalotti (2015), this gives rise to loan-to-income ratio that decreases with income level. That is, in my model relative to their incomes mortgage borrowing of top income households approach to zero.

[14]Schularick and Taylor (2012) and Mian, Sufi and Verner (2017) document the role of credit growth in the occurrence of financial crises. Jorda, Schularick and Taylor (2016) find that the growth of mortgage credit in particular has been an increasingly important determinant of financial stability. Corbae and Quintin (2015), Chatterjee and Eyigungor (2015), Hedlund (2016) and Campbell and Cocco (2015) among others study the relationship between house price changes and foreclosures

Several empirical studies have examined the relationship between income inequality and household debt.[15] Most studies in this literature use country level data and reach different conclusions. For instance, Bordo and Meissner (2012) find no evidence of a rise in the top income share leading to credit booms, whereas Perugini, Hölscher and Collie (2016) find a positive relationship between income concentration and private sector debt. Stefani (2020) instead uses multiple micro data sources and shows that rising income inequality is associated with higher mortgage debt accumulation among homeowners, a similar result to that of Bartscher, Kuhn, Schularick and Steins (2020) that home equity extraction in response to rising house prices among middle income households drive the aggregate debt dynamics. Coibion, Gorodnichenko, Kudlyak and Mondragon (2014) finds that low income households in high income inequality areas borrow less compared to similar households in low income inequality areas, negating the demand side explanations relating income inequality to household debt.[16]

Another contribution of my paper relative to empirical studies relating inequality developments to housing and credit markets is to study causal effects using cross-sectional identification.[17] In constructing the instruments I follow an approach similar to Bartik (1991). Main line of departure in this paper is that I construct an instrument for the Gini coefficient, a higher moment related variable, rather than a mean variable of interest, like local labor demand.

Layout. The rest of this paper is organized as follows. Section 1 presents an equilibrium model of housing and mortgage markets. Section 2 verifies the model's predictions through panel data analysis. Section 3 concludes.

employing a quantitative model. Mian, Sufi and Trebbi (2015) document that mortgage foreclosures had significant effects on house prices and employment.

[15]Note that, while Mian, Straub and Sufi (2020b) provides empirical evidence on a relationship between income inequality and savings of the top income households, the authors do not test the relationship between household debt and inequality as in my paper. They find that a higher top income share is positively correlated with higher savings at the top of the income distribution by exploiting state level variation in income inequality as in my paper.

[16]Credit demand explanations of rising income inequality include, among others, keeping up with the rich (Veblen (1899)), expenditure cascades (Frank, Levine and Dijk (2014)) and precautionary savings against income risk (Iacoviello (2008), Nakajima (2005)).

[17]Several papers employ identification strategies based on geographical variation. This line of research for mortgage debt and house prices was initiated by Mian and Sufi (2009), and many papers have used similar techniques. See Nakamura and Steinsson (2017) for a discussion of the use of regional variation for identification in macroeconomics, and its applications in areas other than household credit and house prices.

1 An analytical model of housing and mortgage markets

To the best of my knowledge, this paper is the first to use a structural model to study the responses of housing and credit markets to changes in income inequality in general equilibrium. The model implies a borrower risk composition channel, which is a novel channel compared to credit demand and credit supply channels of income inequality. First, I present the partial equilibrium in the mortgage market that takes house prices and real rates as given. The lenders offer a menu of mortgage contracts to heterogeneous income borrowers. This mortgage menu is a price schedule for different levels of debt and housing pledged as collateral that is consistent with the expected default risk. Borrowers across the income distribution choose different mortgage contracts and demand different size houses internalizing the effect of their choices on the cost of borrowing. Therefore changes in the income distribution affects the borrower pool through the endogenous selection of borrowers into different mortgage types. Changes in house prices also affect the mortgage selection and thus affect the risk profile of the borrower pool.

Second, I present the partial equilibrium in the housing market that takes borrowers' mortgage choices as given. Aggregate demand for housing will again vary with the changes in the income distribution through the housing demand associated to different mortgage types.

Market clearing in housing and mortgage markets ensure market clearing in the non-durable consumption good market, where total consumption of lenders and borrowers equal total endowment of the consumption good net of costs associated with borrower default. The general equilibrium of the model is borrowers' optimal mortgage choice in period-1, their housing choices in both period-1 and period-2, lenders' consumption choices in both period-1 and period-2, house price levels that clear the housing market, and loan prices defined in period-1.

Environment. The model has two periods $t = 1, 2$. There is a continuum of borrowers who differ in their period-1 endowment income. A measure $\psi(y_{1i})$ of borrowers receive endowment income y_{1i}, and the income distribution is denoted by Ψ. Endowment income in period-2 is

$$y_{2i} = \omega y_{1i}. \tag{1}$$

where ω is an aggregate income growth shock which renders this income uncertain. The distribution of income growth shocks is denoted by Ω. The distribution of initial endowment incomes is similar to a skill distribution in a production economy, and ω as a stochastic aggregate labor productivity growth. For simplicity, this set-up here abstracts from idiosyncratic risk and income mobility.[18]

In addition to their endowment income, each household receives a symmetric housing endowment of h. Therefore, the only exogenous type of inequality is that of income. Households borrow in period-1. In period-2, they observe their income and decide whether to repay their loan. Borrowers derive utility from non-durable consumption and housing consumption in both periods. The consumption good is the numeraire and p_t is the house price in period $t = 1, 2$. House price is determined by market clearing conditions in both periods.

Borrowers. Borrowers maximize their lifetime utility, which is derived from non-durable good and housing consumption in both periods.

$$\max_{h_{1i},d_{1i},c_{1i},c_{2i},h_{2i}} U_1(c_{1i}, h_{1i}) + \beta \mathbb{E}_1 U(c_{2i}, h_{2i}), \tag{2}$$

subject to the following budget constraints

$$c_{1i} + p_1 h_{1i} = y_{1i} + q(y_{1i}, d_{1i}, h_{1i}) d_{1i} + p_1 h, \tag{3}$$

$$c_{2i} + p_2(\omega) h_{2i} = \begin{cases} \omega y_{1i} - d_{1i} + p_2(\omega) h_{1i} & \text{if repay } d_{1i}, \\ \xi \omega y_{1i} & \text{if default on } d_{1i}. \end{cases} \tag{4}$$

where h_{ti} is the housing consumption and c_{ti} is the non-durable consumption in period $t = 1, 2$, and d_{1i} is the mortgage repayment that is borrowed in period 1 to be repaid in period 2. Period-1 budget constraint (3) states that period-1 consumption, c_{1i}, and housing expenditure, $p_1 h_{1i}$ are financed by endowment income, the value of the initial housing endowment, $p_1 h$, and a mortgage loan. For each unit of debt d_{1i} to be repaid in period-2, the lender gives the borrower $q(y_{1i}, d_{1i}, h_{1i}) d_{1i}$ units of consumption good in period-1 and how loan price $q(y_{1i}, d_{1i}, h_{1i})$ is set by the lenders is described in the next section. Borrowers internalize the effect of their choices of housing consumption and debt on the loan price.

[18] Thus in my model the rank of a household in the income distribution remains the same over her life-time. This is consistent with the finding of Guvenen, Kaplan, Song and Weidner (2017) that income inequality is persistent over the life cycle in the US.

In period-2, total resources available for consumption and housing depend on the optimal default decision of the borrower as shown in period-2 budget constraint (4). If the borrower repays, she uses both her endowment income and financial income generated from selling her house. On the other hand, if she defaults she faces a fractional dead-weight income loss and recovers only ξ fraction of her period-2 endowment income.[19] Mortgage debt d_{1i} is non-recourse and thus the lender cannot confiscate borrower income. Similar to Kumhof, Rancière and Winant (2015) I also assume that independent of their income levels, borrowers incur a utility cost κ if they default.

Let U^d_{2i} and U^r_{2i} denote maximum utility under default and repayment, respectively, in period-2 achieved subject to relevant budget constraints defined in (4). The rational default rule can then be defined as:

$$\mathbb{1}_{2i}(\omega, y_{1i}, d_{1i}, h_{1i}) = \begin{cases} 1 & \text{if } U^d_{2i} - \kappa > U^r_{2i}, \\ 0 & \text{otherwise.} \end{cases}$$

The default rule takes a value of one for income y_{1i}, mortgage debt d_{1i} and housing h_{1i} if the borrower chooses to default at this point in the state space. There is no information asymmetry, so lenders use the same default rule when they price loans. To simplify notation, I henceforth use $\mathbb{1}_{2i}$ in place of $\mathbb{1}_{2i}(\omega, y_{1i}, d_{1i}, h_{1i})$ and q_i in place of $q_i(y_{1i}, d_{1i}, h_{1i})$.

Lenders. Lenders are perfectly competitive, risk neutral and are deep pocketed foreigners endowed with the same non-durable consumption good as borrowers. Let y^*_t denote the non-durable consumption good endowment of the lenders in periods $t = \{1, 2\}$. Lenders consume the non-durable consumption good only and do not derive utility from housing that borrowers consume. They discount future consumption at the risk-free rate R^f. As is standard in the literature, lenders are more patient than households, $\frac{1}{R^f} > \beta$. Their preferences can thus be represented as:

$$\max_{c^*_1, c^*_2} c^*_1 + \frac{1}{R^f} \mathbb{E}_1 c^*_2, \tag{5}$$

The lenders use their total endowment to issue mortgage debt to borrowers and to consume the non-durable consumption good in period-1. Therefore, savings of the lenders is the sum of mortgage debt of each borrower i in period-1.

[19] Eaton and Gersovitz (1981), Arellano (2008) and Kumhof, Rancière and Winant (2015) also assume income losses in the case of default. This captures the effect of default penalties outside of asset forfeiture, such as a negative effect on the borrowers credit history.

$$c_1^* + \int q_i d_{1i} d\Psi = y_1^*, \tag{6}$$

Houses serves as collateral. If a borrower defaults, the lender seizes their house, sells it back to the households and receives $\theta p_2(\omega)$ per unit of housing, where θ is the recovery rate and $p_2(\omega)$ is the relative house price when the income growth realization is ω.

$$c_2^* = y_2^* + \int (1 - \mathbb{1}_{2i}) d_{1i} + \mathbb{1}_{2i} \theta p_2(\omega) h_{1i} d\Psi. \tag{7}$$

In period-2 lenders consume their endowment income and the return to their savings in the form of mortgage debt, which is the sum of debt repayment, and return to foreclosed houses in case of default.

Note that in the absence of mortgage lending, expected life-time utility of lenders is $y_1^* + \frac{1}{R^f}\mathbb{E}y_2^*$. Lenders set the price of the mortgage loan d_{1i} in period-1 at $q_i(y_{1i}, d_{1i}, h_{1i})$ such that lending to each borrower i increases their life-time utility:

$$q_i d_{1i} \leq \frac{1}{R^f}\mathbb{E}\left((1 - \mathbb{1}_{2i}) d_{1i} + \mathbb{1}_{2i} \theta p_2(\omega) h_{1i}\right) \tag{8}$$

As the lenders face competition at the borrower level, they can make zero-expected life-time utility gain from lending d_{1i} at price q_i and thus the inequality above binds. Rearranging the condition for any positive amount of d_{1i} gives the loan pricing schedule as:

$$q(y_{1i}, d_{1i}, h_{1i}) = \frac{1}{R^f} E\left(1 - \mathbb{1}_{2i} + \mathbb{1}_{2i} \frac{\theta p_2(\omega) h_{1i}}{d_{1i}}\right) \tag{9}$$

If the borrower repays the loan irrespective of the realization of the income growth shock, that is $\mathbb{E}_1 \mathbb{1}_{2i} = 0$, then the loan price is equal to the lenders' discount rate. I refer to any contract with a combination of debt and housing collateral such that the borrower will always repay the mortgage as risk-free.

When the borrower strategically defaults under certain income growth realizations, $\mathbb{E}_1 \mathbb{1}_{2i} > 0$, the lenders price this risk. If there was no collateral, as is the case with models of sovereign default a la Eaton and Gersovitz (1981), the price would set as the lenders' discount factor adjusted by the default probability at $\frac{\mathbb{E}_1 \mathbb{1}_{2i}}{R^f}$. The presence of collateral gives rise to a loan spread that is lower that the default risk, and is endogenous to the amount of collateral and debt. Unsurprisingly, a high loan-to-value ratio, $\frac{d_{1i}}{p_1 h_{1i}}$, leads to a low loan price and thus a high spread over the risk-free rate.

Characterization of the general equilibrium. The general equilibrium of the model is defined as market clearing in non-durable consumption, housing and mortgage markets. In the mortgage market, borrowers and lenders take house prices as given, and across the income distribution borrowers make different mortgage debt choices. The mortgage market clears loan-by-loan in a manner that is consistent with the loan pricing schedule.

In the housing market, contract choices are taken as given and the aggregate demand for housing varies with mortgage market conditions. Housing demand is the aggregate of housing consumption choices across the income distribution and total demand for housing among borrowers equal total supply at price p_t in each period $t \in \{1, 2\}$. In period-2, if a lender possesses a foreclosed property, she sells it back to he borrowers of period-1. Therefore, total housing supply remains the same across periods.

Non-durable consumption good market also clears. Total endowment of non-durable consumption good is absorbed by borrowers and lenders in period-1 and non-default states of period-2. In default states of period-2, part of non-durable consumption good endowment is lost in the form of borrower income, a fraction of $1-\xi$. Lenders also lose income due to borrower default. For per unit of housing unit, from selling a foreclosed house lenders recover $1-\theta$ units of the numeraire consumption good. I first describe the mortgage market equilibrium and then the housing market equilibrium. These allows me to define for the general equilibrium of the model as consumption good market clears by Walras law. Having characterized the general equilibrium, I study the effects of income inequality and its interaction with interest rate environment.

I solve the model by backward induction. I first solve for the optimal housing and consumption choices in period-2 and find market clearing house price. Then, define the period-1 choices of the borrowers, where they choose their housing and debt levels, internalizing the loan price schedule (9) and budget constraints (3) and (4). Both lenders and borrowers take into account market clearing house price in period-2, while making their period-1 decisions.

Functional forms. In order to derive closed-form solutions, I make three assumptions regarding functional forms. First, similar to Justiniano, Primiceri and Tambalotti (2015) preferences are quasi-linear in non-durable consumption. This implies housing expenditure independent of income level in both periods and simplifies aggregation of housing demand across the income distribution.

$$U(c_{it}, h_{it}) = c_{it} + \phi_t \ln(h_{it}). \tag{10}$$

Second, preference for housing in period-2, ϕ_2, is exogenous and vary positively with the income growth realization, i.e. $\phi^i \geq \phi^j$ for all $\omega^i \geq \omega^j$. This assumption allows housing expenditure and its share in total household expenditure to change with income growth.[20]

Finally, I assume that income growth risk can take two values

$$\omega = \begin{cases} \omega^H & \text{, with probability } \nu, \\ \omega^L & \text{, with probability } 1 - \nu. \end{cases}$$

ν is the probability of high income growth. This assumption simplifies the loan price schedule $q_i(y_{1i}, d_{1i}, h_{1i})$ and the default rule $\mathbb{1}_{2i}$, which will be described in detail in the next section. Moreover, it simplifies the problem into choosing between two mortgage risk types and makes analytical solutions possible.

As far as the income distribution is concerned, I consider two commonly used distributions: Pareto and log-normal, both of which give rise to the same qualitative results.

1.1 Partial equilibrium in the mortgage market in period-1

I solve for mortgage market equilibrium in period-1 through backward induction. I begin with the rational default decision of borrowers in period-2. Optimal default rule in period-2 depends on the optimal housing and non-durable consumption choice of borrowers. Having defined the housing consumption choices, I derive market clearing house price in period-2 under each income growth realization. I then move to the period-1 decisions of both lenders and borrowers.

1.1.1 Default/Repayment decision and housing market clearing in period-2

In period-2 the borrower makes a rational default decision based on utility from default and repayment under optimal non-durable and housing consumption. Before

[20] Absent housing preference shock quasi-linear preferences imply that housing demand and thus house prices in period-2 takes the same value across all states of income growth, a result that is inconsistent with business cycle dynamics.

making the optimal default decision, she observes income growth realization ω and housing preference ϕ, where $\omega \in \{\omega^H, \omega^L\}$ and $\phi_2 \in \{\phi_2^L, \phi_2^H\}$. Period-2 optimization problem for borrower i that has period-1 income, debt and housing choices $\{y_{1i}, h_{1i}, d_{1i}\}$ is defined as:

$$\max_{c_{2i}, h_{2i}} c_{2i} + \phi_2 \ln(h_{2i})$$

subject to relevant budget constraint (4) for x $\in$ {repay, default}. Under each income growth realization $j \in \{L, H\}$ and repayment/default option, interior solutions for non-durable consumption and housing choices in period-2 is then given as:

$$c_{2i} = \begin{cases} \omega^j y_{1i} + p_2 h_{1i} - d_{1i} - \phi_2 & \text{if repay} \\ \xi \omega^j y_{1i} - \phi_2 & \text{if default} \end{cases} \tag{11}$$

$$h_{2i} = \frac{\phi_2^j}{p_2} \qquad \text{both repay \& default} \tag{12}$$

Remember that lenders do not derive utility from housing consumption, therefore total demand for houses in period-2 is the aggregate across the borrower income distribution. Total supply of houses remains at h as lenders sell the foreclosed houses back to households. Therefore in period-2 each household consumes h unit of housing under each income growth realization but total housing expenditure ϕ_2 varies with it.

$$\int h_{2i} d\Psi = \frac{\phi^j}{p_2} \int d\Psi = h, \tag{13}$$

Therefore, market clearing house price in period-2 is:

$$p_2^j = \frac{\phi_2^j}{h}. \tag{14}$$

with $p_2(\omega^H) \geq p_2(\omega^L)$ and $h_{2i} = h$ for each borrower i.

Using housing market clearing and optimal consumption choices under default and repayment, now I turn to optimal default/ repayment choice by comparing implied maximum utility levels in period-2 for an income growth realization ω.

$$U_2^r = \omega y_{1i} + \frac{\phi_2}{h} h_{1i} - d_{1i} - \phi_2 + \phi_2 \ln(h),$$

$$U_2^d = \xi \omega y_{1i} - \phi_2 + \phi_2 \ln(h) - \kappa.$$

Optimal default rule for an income growth realization ω can be written as:

$$\mathbb{1}_{2i}(\omega, y_{1i}, d_{1i}, h_{1i}) = \begin{cases} 1 & \text{if } d_{1i} \geq (1-\xi)y_{1i}\omega + \frac{\phi_2}{h}h_{1i} - \kappa, \\ 0 & \text{otherwise.} \end{cases} \tag{15}$$

A borrower defaults on her loan if (i) value of the house in period-2 is too low, (ii) her income is too low or (iii) both, in which negative home equity is a necessary but not sufficient condition for default. Borrowers may find it optimal to repay even if they are underwater due to the costs associated with default. This is consistent with the findings of Gerardi, Herkenhoff, Ohanian and Willen (2017) that borrowers remain current on their mortgage debt even when they are underwater.[21]

1.1.2 Loan pricing by the lenders in period-1

Borrowers' default rule (15) gives rise to a debt threshold such that any amount of debt below this threshold will be repaid under each income growth realization. Debt thresholds associated with low and high income growth $\bar{d}_i^L$ and $\bar{d}_i^H$, respectively, defined as:

$$\begin{aligned} \bar{d}_i^L &= (1-\xi)y_{1i}\omega^L + \frac{\phi_2^L}{h}h_{1i} - \kappa, \\ \bar{d}_i^H &= (1-\xi)y_{1i}\omega^H + \frac{\phi_2^H}{h}h_{1i} - \kappa. \end{aligned} \tag{16}$$

Note that $\bar{d}_i^L \leq \bar{d}_i^H$. The loan pricing schedule for borrower i with income y_{1i} who wants to purchase house of size h_{1i} is given by:

$$q(y_{1i}, d_{1i}, h_{1i}) = \begin{cases} \frac{1}{R^f} & \text{if } d_{1i} \leq \bar{d}_i^L, \\ \frac{1}{R^f}\left(\nu + (1-\nu)\theta\frac{\phi_2^L}{h}\frac{h_{1i}}{d_{1i}}\right) & \text{if } \bar{d}_i^L < d_{1i}, \leq \bar{d}_i^H, \\ 0 & \text{otherwise.} \end{cases} \tag{17}$$

If $d_{1i} \leq \bar{d}_i^L$, then the borrower repays under both realizations of income growth and the loan is thus priced at the lender's discount rate. I label any mortgage loan with these properties as *risk-free loans*. If $d_{1i} > \bar{d}_i^H$, then the borrower will always default on the loan. I assume that lenders will not issue loans with these prospects, so the

[21] See Gete and Reher (2016) and Jeske, Krueger and Mitman (2013) for models with one period mortgage loans with rational default decision. Both papers assume that borrowers default when they are underwater and there is no utility or economic cost of default. Among others, Foote, Gerardi and Willen (2008) provide empirical evidence of double-trigger defaults. See Foote and Willen (2017) for a review of mortgage default research.

price is set to zero. Therefore, for each borrower, debt limit is $\bar{d}_{1i}^{H}$.

For debt levels between these debt thresholds, the middle case, the borrower repays only when aggregate income growth is high and thus the probability of repayment is ν. I label such loans as *risky loans*. If income growth is low, the borrower defaults and the lender seizes the house that serves as the collateral. Loan price offer for a given contract choice of (h_{1i}, d_{1i}) then decreases with the risk-free rate, and increases with the probability of repayment ν, house price recovery rate θ and period-2 house price level $\frac{\phi_2^L}{h}$.

Note that the debt threshold of a risky loan $\bar{d}_i^H$ increases at a higher rate with both income and housing size compared to a risk-free loan. Moreover, under a risky loan not only the mortgage debt limit but also the mortgage price increases with housing size. As I show later in the paper, the heterogeneous effect of housing choice for loan price across loan types leads lower real rates to affect risky loans to a higher extent than the risk-free loans.

Relation to the exogenous lending constraint models. In my paper rewriting the maximum debt limit $\bar{d}_{1i}^{H}$ as a function of loan-to-value (LTV) and loan-to-income (LTI) ratios reads[22]

$$LTV_i \leq L\bar{T}V_i = \frac{\phi_2^H}{hp_1}\left(1 - \frac{(1-\xi)\omega^H}{LTI_i}\right)^{-1}. \tag{18}$$

Therefore, in my model LTV limit $L\bar{T}V_i$ decreases with the LTI ratio. A high LTV loan is issued as long as it has a low LTI ratio.[23] It also implies that in a regulatory environment with exogenous LTI^* and LTV^* limits, for different households a different constraint might bind. That is, $L\bar{T}V_i$ can be lower than the regulatory LTV^* limit if LTI_i is constrained by the regulatory limit LTI^*.[24]

[22]Here

$$LTI_i = \frac{d_{1i}+\kappa}{y_{1i}}, LTV_i = \frac{d_{1i}+\kappa}{h_{1i}p_1}$$

[23]Similarly, a high LTI loan limit is offered if the borrowing is with a low LTV. Rewriting (18) gives the LTI limit

$$LTI_i \leq L\bar{T}I_i = (1-\xi)\omega^H\left(1 - \frac{\phi_2^H}{hp_1}\frac{1}{LTV_i}\right)^{-1}. \tag{19}$$

[24]See Greenwald (2018) for a model with exogenously determined loan-to-value and payment-to-income constraints. The paper abstracts from endogenous default and shows that limiting payment-to-income ratios rather than the loan-to-income ratios is more effective prudential tool in limiting debt cycles.

A pure LTV constraint that is extensively studied in the literature arises as a special case of my model with no income cost of default, $\xi = 1$. While it's not the focus of this paper, the framework here provides a micro foundation for the relaxation of LTV constraints: an increase in lenders' house price expectations. Kaplan, Mitman and Violante (2017) show that an increase in the exogenous LTV limit has limited effect on house prices unless it is accompanied by an increase in house price expectations, which in my model arises from higher preference for housing consumption in period-2. Within the framework of my model LTV limits themselves are endogenous to house price expectation and this may amplify the effect of lenders' beliefs on house prices and leverage.

Next I characterize optimal debt and housing choices in period-1, and discuss the implications of the optimal portfolio choice for endogenous aggregate default risk in the economy.

1.1.3 Mortgage debt and housing consumption choice across the income distribution in period-1

In period-1 borrowers choose between taking a risky and a risk-free loan based on utility implied by each type of loan taking period-1 house price as given. I show below that if real rates is high enough households with low income choose to take a risky loan and those with high income take a risk-free loan. The utility benefit of a risky loans is that the borrowers make low down-payment. However, with a risky loan borrowers buy small houses as it is costly to borrow and also since they repay less frequently they make low financial income in period-2 from selling it. These translates into low utility from period-1 housing consumption and period-2 non-durable consumption. Remember that in period-2, housing consumption is symmetric across income growth realizations and across households at h.[25]

When making optimal housing and mortgage debt choice in period-1, borrowers internalize the effect of their housing choice on (i) the loan price and debt limits (17), (ii) period-1 non-durable consumption from the budget constraint (3) and finally (iv) optimal period-2 non-durable consumption defined in (11). Appendix A presents borrower optimization conditional on mortgage risk choice. I discuss only the results within the main text.

[25] Therefore, housing consumption inequality is only temporary in the model. Housing endowment in the beginning of period-1 and at the end of period-2 is h. Housing consumption might deviate from h in the presence of mortgage lending only in period-1.

As preferences are linear in consumption and borrowers are less patient than lenders, mortgage debt limits (16) bind for both risky and risk-free contracts. That is, borrowers find mortgage debt utility enhancing relative to not borrowing.

Result 1 *For both risk-free and risky mortgage contracts it is optimal to borrow maximum amount of debt that is consistent with the associated default risk:*

$$\begin{aligned} d_{1i}^{R} &= (1-\xi)\omega^{H} y_{1i} + \tfrac{\phi_2^H}{hp_1} h_1^R - \kappa, \\ d_{1i}^{NR} &= (1-\xi)\omega^{L} y_{1i} + \tfrac{\phi_2^L}{hp_1} h_1^{NR} - \kappa. \end{aligned} \tag{20}$$

As is the case with period-2 housing consumption, housing consumption under any loan type does not depend of borrowers' income level. Housing consumption under risk-free and risky loans are denoted as h_1^{NR} and h_1^R, respectively, and are defined as:

Result 2 *Period-1 housing consumption under a risky loan,* h_1^R*, and under a risk-free loan,* h_1^{NR} *are defined as:*

$$\begin{aligned} h_1^{NR} &= \phi\left(p_1 - \tfrac{1}{R^f}\tfrac{\phi_2^L}{h} - \beta\nu\left(\tfrac{\phi_2^H}{h} - \tfrac{\phi_2^L}{h}\right)\right)^{-1}, \\ h_1^{R} &= \phi\left(p_1 - \tfrac{1}{R^f}\left(\nu\tfrac{\phi_2^H}{h} + \theta(1-\nu)\tfrac{\phi_2^L}{h}\right)\right)^{-1}. \end{aligned} \tag{21}$$

Housing choice is a result of the trade-off between period-1 marginal housing utility and the sum of period-1 and period-2 marginal non-durable consumption utility. Through period-1 budget constraint p_1 is the marginal cost of housing in terms of period-1 non-durable consumption. In states of the world that households repay their loans, an additional marginal utility cost of housing arises in the form of period-2 non-durable consumption as debt increases with house size. Expressions following p_1 includes this period-2 marginal consumption cost and also marginal utility benefits from an additional unit of housing via (i) higher mortgage debt through raising the binding debt limit, (ii) higher period-2 consumption due to financial income from selling a house and (iii) for risky loans only, higher mortgage price.[26] Both higher debt

[26] At an interior solution, the first order condition in terms of h_{1i} is:

$$\underbrace{\frac{\phi}{h_1}}_{\text{marginal period-1 housing utility}} = \underbrace{p_1 + B}_{\text{marginal period-1 and period-2 non-durable consumption utility}}.$$

where B is the terms following p_1 in (21). Note that in the absence of mortgages, p_1 governs the marginal rate of substitution between period-1 housing consumption and period-1 non-durable consumption.

limit and higher mortgage price decreases the down-payment, the difference between house value and the mortgage loan, which increases period-1 non-durable consumption. Having described optimal choices for each borrower under each default risk option, I now describe how different income borrowers sort into mortgage contracts taking house price p_1 as given.

Proposition 1 *Let* $\gamma = (1-\xi)\left(\frac{\omega^L - \nu\omega^H}{R^f} + \beta\nu(\omega^H - \omega^L)\right)$. *There exists a unique income cut-off* $\bar{y}$

$$\bar{y} = \frac{1}{\gamma}\left((1-\nu)\left(\frac{1}{R^f} - \beta\right)\kappa - \phi \ln\left(\frac{h_1^{NR}}{h_1^R}\right)\right)$$

such that borrowers with income less than $\bar{y}$ *take risky loans as long as risk-free rate is sufficiently high*

$$\Gamma_{1i}^R = \begin{cases} 1 & \text{if } y_{1i} \leq \bar{y} \\ 0 & \text{if } y_{1i} > \bar{y} \end{cases} \quad \text{as long as} \quad R^f \geq \frac{1}{\beta}\frac{\nu\omega^H - \omega^L}{\omega^H - \omega^L}$$

Across the income distribution, different contract choices arise due to a trade-off faced by borrowers which has three components. Conditional on choosing a risky loan, a borrower makes (i) a lower down-payment in period-1 (Lemma 1), (ii) has lower period-1 housing consumption (Lemma 2) and (iii) has lower mean period-2 consumption (Lemma 3) compared to a risk-free loan.

Figure 3 represents Proposition 1 graphically. It plots the life-time non-durable consumption gain from a risky loan, solid line, against the housing utility cost, dashed line. The slope of the total consumption gain line is $-\gamma$ defined in Proposition 1. Low income borrowers opt for risky loans as long as the gain from a low down-payment exceeds the costs of lower period-1 housing and mean period-2 non-durable consumption. This is true when the utility cost of default κ is sufficiently high, so that the intercept of the consumption gain is above that of the housing utility loss. This is the benchmark specification that I use to study the consequences of rising income inequality and declining real rates.[27]

[27] Three other cases are possible. First, if the real interest rate is low, then the consumption gain schedule is upward sloping and it is optimal to choose a risky loan irrespective of income. Second, if κ is small and the risk-free rate is low, then only the risk-free contract exists in equilibrium. Finally, if κ is small and the risk-free rate is high, then low income borrowers will opt for risk-free loans and high income borrowers risky loans. The last case may arise at business cycle frequency. Adelino, Schoar and Severino (2016) show that middle-income, high-income, and prime borrowers all sharply increased their share of delinquencies in the recent crisis. Since the focus of the current paper is the

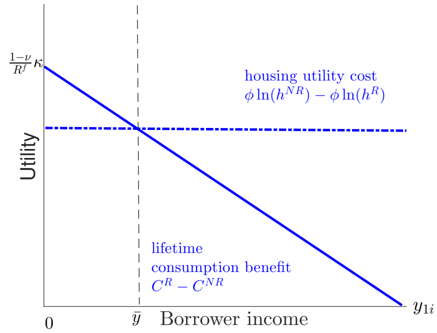


Figure 3: Utility trade-off in period-1: costs and benefits of a risky loan

Note: The diagram plots the utility costs and benefits from switching to a risky loan for different levels of borrower income. For borrowers with income below $\bar{y}$, the utility benefits exceed the utility costs.

Lemma 1 *The down-payment of a risky loan is lower than a risk-free loan at all points in the income distribution. A sufficient condition is* $\frac{\nu\omega^H}{\omega^L} \geq 1$*.*

Down-payments, $h_{1i}p_1 - q_{1i}d_{1i}$, under both types of loans decline with probability of debt repayment and the mortgage debt amount. From the definition of debt thresholds (16) it is straightforward to see that there are two components to debt and thus down-payment: (i) income variant and (ii) income invariant. If the sufficiency condition in Lemma 1 is satisfied, then both these components are lower for a given level of income under a risky-loan compared to a risk-free loan. At the logical extreme of zero income, down-payment is $(1-\nu)\kappa$ under a risky loan and κ under a risk-free loan. Note that for low income borrowers, the income invariant component of down-payment gain is better than losing a fraction of endowment income in period-2 and makes it worth taking a risky loan.[28] As income increases, the down-payment gain under a risky loan becomes less and less important.

Lemma 2 *Period-1 housing consumption is higher under a risk-free contract compared to a risky contract,* $h_1^{NR} \geq h_1^R$*, as long as loan recovery rate is sufficiently low:*

$$\theta \leq \theta^{max} \quad where \quad \theta^{max} = 1 - (1 - \beta R^f)\left(\frac{\phi_2^H}{\phi_2^L} - 1\right)\frac{\nu}{1-\nu}.$$

long-run determinants of house price and credit developments, I leave this interesting case for future research. The constraint on real rate in Proposition 1 ensures that the real rate is sufficiently high, that is $\gamma > 0$.

[28]This is the source of the down-payment gain from a risky loan for a borrower with low income.

If the loan recovery rate is sufficiently low, marginal benefit of pledging one more unit of housing and raising the mortgage price is not high compared to higher period-2 average financial income net of debt repayment. That is, borrowers that repay more frequently have stronger incentives to buy larger houses and tilt consumption from period-1 to period-2 by making larger down-payment in period-1. While borrowers that repay less frequently tilt consumption from period-2 to period-1 by making lower down-payment and lower average financial income per unit of housing purchased in period-1.

The third component of the contract choice is the mean utility derived from period-2 consumption. Under a risky loan non-durable consumption in period-2 is lower for two reasons. First, when aggregate income growth is low, the borrower defaults and loses $1 - \xi$ share of her endowment. Second, her average financial income from selling her house is lower as she repays and sells her house less frequently.

Lemma 3 *Mean period-2 consumption is higher under a risk-free loan than a risky loan across the income distribution.*

1.2 General Equilibrium

The equilibrium of the model is given by quantities $\{c_{1i},\ c_1^*,\ h_1^R,\ h_1^{NR},\ d_{1i}^R,\ d_{1i}^{NR},\ c_{2i}^R\ ,\ c_{2i}^{NR},\ c_2^*,\ h_2^R,\ h_2^{NR}\ \}$, prices $\{q_i, p_1, p_2^H, p_2^L\}$ and contract type choice Γ_{1i}^R for each household i such that

1. Borrowers optimize by solving problem 2 with associated decision rules $\{c_{1i},\ h_1^R,\ h_1^{NR},\ d_{1i}^R,\ d_{1i}^{NR},\ c_{2i}^R\ ,\ c_{2i}^{NR},\ h_2^R,\ h_2^{NR},\ \Gamma_{1i}^R\ \}$.

2. The mortgage market clears loan-by-loan with loan prices defined by equation 17 and decision rules $\{c_{1i},\ c_1^*,\ h_1^R,\ h_1^{NR},\ d_{1i}^R,\ d_{1i}^{NR},\ c_{2i}^R\ ,\ c_{2i}^{NR},\ c_2^*,\ h_2^R,\ h_2^{NR}\ \}$

3. The housing market clears at price p_1 in period-1 and at price p_2^j for income growth realization $j \in \{L, H\}$ in period-2:

$$\int (\Gamma_{1i}^R h_t^R + (1 - \Gamma_{1i}^R) h_t^{NR}) d\Psi = h.$$

$$p_2^j = \frac{\phi^j}{h}.$$

4. The non-durable consumption good market clears:

$$y_1^* + \int y_{1i} d\Psi = \int c_{1i} d\Psi + c_1^*.$$

$$y_2^* + \int y_{2i}(\omega^H) d\Psi = c_2^*(\omega^H) + \int c_{2i} d\Psi.$$

$$y_2^* + \int y_{2i}(\omega^L) d\Psi = c_2^*(\omega^L) + \int c_{2i} d\Psi + \int \Gamma_1^R \left((1-\xi) y_{2i}(\omega^L) + (1-\theta) h_1^R \frac{\phi^L}{h} \right) d\Psi.$$

Note that in period-2 total endowment of the numeraire good is allocated only to borrower and lender consumption in the absence of default, that is, under high income growth realization. Under low income growth realization, some of the total endowment is lost in the form of household income loss and foreclosed housing values. The latter enters the endowment constraint because lenders sell the foreclosed houses and fund their period-2 non-durable consumption. The difference between lenders' income and consumption of the non-durable good is their net exports.

Graphical representation of general equilibrium in period-1.

Remark 1 *The general equilibrium of the model in period-1 can be represented in* (p_1, S) *space as follows:*

The locus of (p_1, S) *consistent with housing market clearing is* HH*:*

$$S h_1^R(p_1) + (1-S) h_1^{NR}(p_1) = h \quad (HH)$$

The locus of (p_1, S) *consistent with mortgage market clearing is* MM*:*

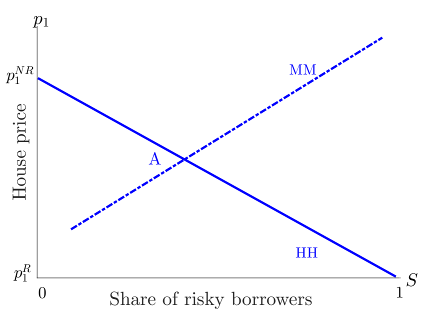
$$S = \Psi(\bar{y}(p_1)) \quad (MM)$$

where $\Psi(\bar{y}(p_1))$ *is the share of borrowers with income less than* $\bar{y}$*, and thus* S *is the share of risky borrowers.*

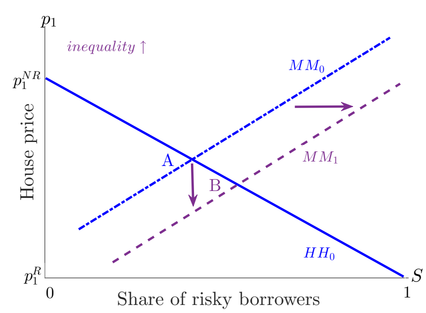
- *The HH curve is downward sloping in S*
- *The MM curve is upward sloping in S*

Figure 4a represents the general equilibrium of the model with house prices p_1 on the y-axis and the share of risky borrowers S on the x-axis. The HH curve has intercept p^{NR}. This corresponds to the case where all borrowers choose risk-free loan, housing demand is high and thus the equilibrium house price is at its highest level. As the share of risky borrowers increases, the total demand for housing declines. Thus, the house price declines along the HH curve.

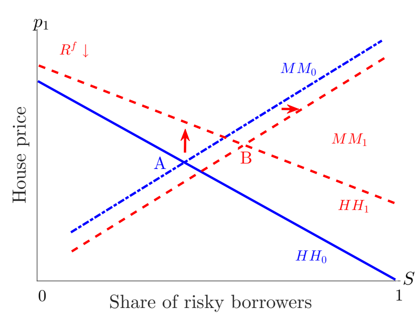
The MM curve depicts how the share of risky borrowers changes with the house price, which is taken as given in the mortgage market. As house price increases, the housing consumption cost of a risky loan decreases. This implies that it is optimal for a higher share of borrowers to choose a risky loan. That is, $\bar{y}$ increases. This is because h_1^{NR} has a higher price elasticity than h_1^R. Thus a risky loan is less costly in terms of housing consumption at high price levels.



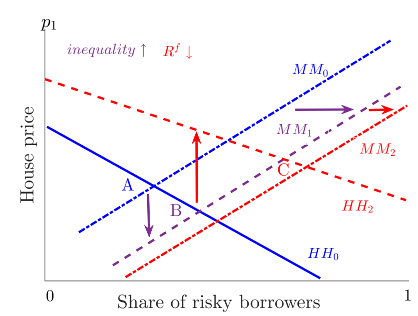
(a) General equilibrium



(b) General equilibrium effect of a rise in income inequality



(c) General equilibrium effect of a decline in real rates



(d) General equilibrium effect of a rise in income inequality and a decline in real rates

Figure 4: Graphical representation of general equilibrium and the effect of high income inequality and low real rates in period-1

Note: The HH curve represents equilibrium in the housing market and the MM curve represents equilibrium in the mortgage market in period-1.

A change in the income distribution from Ψ to $\tilde{\Psi}$ lead only MM to shift, which then implies a movement along HH. As show in Figure 4b a rise in income inequality increases the share of risky borrowers in the economy and depresses house prices. This is consistent with the cross-sectional finding motivating the analysis. However, Figure 4c displays that a change in the risk-free rate shifts both the HH and MM schedules leading to a rise in both house prices and share of risky borrowers. Higher income inequality and lower reals rates together in Figure 4d unambiguously increase the share of risky borrowers in the economy. However, ambiguous effect on house prices suggest that declining real rates in the key to reconcile aggregate trends in house prices in Figure 1 and cross-sectional facts in Figure 2. Subsequent sections present the formal derivation of the results before I present panel analyses to test these predictions.

1.3 The general equilibrium effect of an increase in income inequality: matching the cross-sectional facts

I now study the general equilibrium effect of a mean-preserving increase in income inequality. I hold mean income constant in order to isolate the effect of an increase in inequality.[29] I show that a mean-preserving increase in income inequality leads to a decline in equilibrium house prices and an increase in the share of risky borrowers. This result is depicted in Figure 4b.

The intuition for this result is as follows. A mean-preserving increase in income inequality means that incomes decline for the lower percentiles of the distribution. The share of borrowers with incomes below $\bar{y}$ thus rises. I consider Pareto and log-normal income distributions, two empirically plausible parametric income distributions for which it is possible to derive an analytical result for the change in the share of risky borrowers.

1.3.1 Income Distribution: Pareto

The Pareto distribution is characterized by two parameters: a scale parameter y_{min} and a shape parameter α. The mean, the Gini coefficient and the cumulative density function Ψ of the Pareto distribution are:

$$Mean = \frac{\alpha}{\alpha - 1} y_{min}, \quad Gini = \frac{1}{2\alpha - 1}, \quad \Psi(y) = 1 - \left(\frac{y_{min}}{y}\right)^{\alpha}$$

[29] See, for instance, Blinder (1975) and Auclert and Rognlie (2018) for applications to consumption demand.

The scale parameter affects only the mean of the distribution, whereas the shape parameter affects both the mean and the Gini coefficient. In order to understand the impact of income inequality alone, I consider changes in income inequality with mean income held constant. To achieve this, I vary y_{min} with α. Let mean income be fixed at $\bar{M}$, then the share of borrowers with income below $\bar{y}$ is:

$$\Psi(\bar{y}) = 1 - \left(\frac{\bar{M}}{\bar{y}}\frac{\alpha - 1}{\alpha}\right)^{\alpha}$$

Proposition 2 *A mean-preserving increase in income inequality under a Pareto income distribution increases the share of risky borrowers in the economy*

$$\frac{\partial \Psi(\bar{y})}{\partial Gini} > 0 \qquad \textit{as long as} \qquad \Psi(\bar{y}) \leq 1 - exp\left(-\frac{1}{\alpha - 1}\right)^{\alpha}$$

Note that as α increases, feasible values for $\Psi(\bar{y})$ declines and thus the lowest upper bound is given by:

$$\lim_{\alpha \to \infty} 1 - exp\left(-\frac{1}{\alpha - 1}\right)^{\alpha} = 1 - exp(-1) = 0.63$$

For a Gini coefficient as low as 0.1, the sufficiency condition is $\Psi(\bar{y}) \leq 0.7$. For the Gini coefficient levels of the early 1990s in the US, the sufficient condition is much weaker. A mean preserving change in income inequality leads to an increase in the share of risky borrowers in the economy if the current state of the income inequality is around that in 1990s and the share of risky borrowers in the economy is less than around 95% - a sufficiency condition that is highly likely to be satisfied. Since the Pareto distribution is widely used to study incomes in the upper tail, rather than the whole distribution. For robustness, I also consider the log-normal income distribution.

1.3.2 Income Distribution: Log-normal

The log-normal distribution is characterized by parameters μ and σ^2. The mean, the Gini coefficient and the cumulative distribution function of the log-normal distribution are given by[30]

$$Mean = exp\left(\mu + \frac{\sigma^2}{2}\right), \quad Gini = erf\left(\frac{\sigma}{2}\right), \quad \Psi(y) = \frac{1}{2} + \frac{1}{2}erf\left(\frac{\ln(y) - \mu}{\sqrt{2}\sigma}\right)$$

Similar to the Pareto distribution, one parameter, μ, affects only the mean income, while another, σ^2, affects both mean income and the Gini coefficient. As before, a rise mean preserving increase in the Gini coefficient is achieved by varying μ with σ. Let $\bar{M}$ be the fixed mean income level, then the share of borrowers with income below $\bar{y}$ is defined as:

$$\Psi(\bar{y}) = \frac{1}{2} + \frac{1}{2}erf\left(\frac{\ln(\bar{y}) - \ln(\bar{M})}{\sqrt{2}\sigma} + \frac{\sigma}{2\sqrt{2}}\right)$$

Proposition 3 *A mean-preserving increase in income inequality under a log-normal income distribution increases the share of risky borrowers in the economy*

$$\frac{\partial\Psi(\bar{y})}{\partial Gini} > 0 \qquad \textit{as long as} \qquad \bar{y} \leq e^{\sigma^2} median$$

For the log-normal distribution, a mean preserving increase in income inequality leads income at percentiles that are much above the median to decline. Given the rates of defaults in the data, the sufficient condition is likely to hold.[31]

1.4 The general equilibrium effect of a decline in the risk-free rate

This section analyses the impact of a decrease in the risk-free rate on equilibrium house price and aggregate default risk in period-1. Unlike an increase in income inequality, a change in the risk-free rate affects partial equilibrium in both the housing and mortgage markets. That is, both the HH and MM loci shift.

A decline in the risk-free rate increases loan prices. Therefore, down-payments fall. This enables borrowers to increase housing consumption under any contract type, so the HH curve shifts outwards. For a given share of risky borrowers, equilibrium house

[30] where $erf(x) = \frac{1}{\pi}\int_{-x}^{x} e^{-t^2} dt$.

[31] Under log-normal distribution a change in the Gini coefficient is associated with a rise in the share of risky borrowers as long as the initial share of risky borrowers is less than one half.

price increases. The differential response of h_1^R and h_1^{NR} to a change in R^f determines the slope of the new housing market clearing condition. If the relative increase in housing consumption is higher under a risky loan, then the HH curve flattens. Lemma 5 shows that this is the case as long as loan recovery rate is sufficiently high. The response of the housing market equilibrium is shown by the red dashed HH line in Figure 4c.

A lower interest rate affects the mortgage market as the changes in down-payments and housing consumption affect mortgage choice. Under a risky contract, the down-payment declines more (Lemma 4) and the relative increase in housing consumption is higher (Lemma 5). Changes in the real rate does not affect period-2 mean non-durable consumption. Therefore, a decline in the real rate leads to a rise in the consumption benefit of a risky loan increases, and a fall in the utility cost from lower housing consumption. For a given price of housing, the income cut-off rises. An increase in the share of risky borrowers in the mortgage market leads the MM to shift to the right in Figure 4c.

Lemma 4 *A decline in the risk-free rate decreases the down-payment more for a risky loan than a risk-free loan. A sufficient condition is* $\frac{\nu\omega^H}{\omega^L} \geq 1$

Lemma 5 *There exists a loan recovery rate* $\underline{\theta}$ *such that, for any loan recovery rate above* $\underline{\theta}$

1. *The semi-elasticity of housing demand is higher under a risky loan compared to a risk-free loan:*

$$\left|\frac{\partial \ln(h_1^R)}{\partial R^f}\right| \geq \left|\frac{\partial \ln(h_1^{NR})}{\partial R^f}\right|$$

2. *The HH curve flattens following a decline in the risk-free rate.*

The following proposition describes the impact of a change in the risk-free rate on the mortgage market equilibrium. It combines the findings of Lemma 5 and Lemma 4.

Proposition 4 *Holding the price of housing constant, a decline in the risk-free rate increases the share of borrowers with a risky loan*

$$\frac{\partial \Psi(\bar{y}(p_1))}{\partial R^f} < 0$$

The general equilibrium effect of a decline in the risk-free rate is an increase in aggregate mortgage default risk. The effect on the house price depends on the relative shifts of the MM and the HH curves.

1.5 Reconciling cross-sectional facts with aggregate trends

This section analyses the joint effect of rising income inequality and declining real interest rate. In particular, I show that a decline in the real interest rate is necessary to match the observed aggregate trends in income inequality and house prices. Figure 4d adds the effects of a real interest rate decline to those of an increase in inequality which were depicted in Figure 4a. An increase in income inequality moves the economy from A to B, which is consistent with the cross-sectional stylized facts provided earlier. A decline in the risk-free rate then moves equilibrium from point B to C. The decline in the risk-free rate might overturn the negative effect of income inequality on house prices. This is accompanied with an increase in the share of risky borrowers in the economy. A lower risk-free rate stimulates housing consumption across the income distribution. However, the effect is stronger for the borrowers with risky loans. This mitigates the effect on house prices.

Using the optimal mortgage contract choice across the income distribution and associated debt levels, aggregate mortgage debt in the economy can be written as:

$$
\begin{aligned}
D = & \int^{\bar{y}} \bar{d}^H_{1i}(y_{1i}) d\Psi + \int_{\bar{y}} \bar{d}^L_{1i}(y_{1i}) d\Psi \\
= & \bar{M}(1-\xi)\left(\omega^L + (\omega^H - \omega^L)\underbrace{\int^{\bar{y}} \frac{y_{1i}}{\bar{M}} d\Psi}_{\text{income share}}\right) + \phi^L + (\phi^H - \phi^L)\underbrace{\frac{\Psi(\bar{y}) h^R_1}{h}}_{\text{housing share}} - \kappa. \qquad (22)
\end{aligned}
$$

where $\bar{M}$ is mean income. Therefore, aggregate debt in the economy increases with the mean income, and the income and housing wealth shares of risky borrowers. The intuition is that all else equal marginal propensity to borrow out of income is higher under a risky loan, $\frac{\partial d^R_i}{\partial y_{1i}} > \frac{\partial d^{NR}_i}{\partial y_{1i}}$. When a lower share of aggregate income is allocated to low income borrowers, average propensity to borrow in the economy declines and this depresses aggregate mortgage debt. Therefore, the novel borrower decomposition channel of income inequality in my model is consistent with the motivating facts presented in Figure 2 and is at odds with the aggregate trends, which motivates both credit demand and supply explanations of rising income inequality. In particular, adding borrower heterogeneity and isolating the effect of income inequality from declining real rates gives opposite predictions to the two-agent models of Kumhof, Rancière and Winant (2015) and Mian, Straub and Sufi (2020a). In the empirical analysis, I show that an increase in top income shares is associated with a decline in real mortgage debt.[32]

Result 3 *Aggregate mortgage debt decreases with income inequality.*

Note also that low income borrowers have higher marginal propensity to borrow against housing collateral, $\frac{\partial d_i^R}{\partial h} > \frac{\partial d_i^{NR}}{\partial h}$. When the share of housing wealth owned by the low-income/risky households in equilibrium increases, aggregate debt also increases. In other words, a given housing stock in the economy is owned with higher leverage.

As shown in Proposition 4 a decline in real rates increases the share of borrowers with a risky loan and leads to a higher average propensity to borrow against income in the economy, for a given level of mean income, this increases mortgage debt. To put differently, risk- taking channel of low real rates gives rise to an increase in mortgage debt to income ratio for a given level of house prices. Therefore, declining real rates might overturn the negative effect of income inequality on mortgage debt. However, the extent of it is an empirical question as declining real rates might increase or decrease the housing wealth share of the risky/ low-income households depending on the equilibrium house price effect. In the panel regressions below I use home-ownership rate as a control variable to capture the changes in housing wealth share of low income borrowers.

2 Verifying the model's predictions: a panel instrumental variables analysis of US States

In this paper, I show that the isolated effect of income inequality operates through a borrower decomposition channel and its implications are at odds with aggregate trends that motivate most studies. While model predictions are consistent with novel stylized

[32] I provide support for this result also in the county level data. In the online appendix, I show that a rise in top income shares is negatively associated with real mortgage debt growth. That is, the counties that experienced the highest increase in the income share of top 20 and 5 percent of the income distribution have experienced the smallest mortgage debt growth.

facts presented in Figure 2, these facts as well are not identified relations. In this section, I use a cross-sectional identification approach using a panel of US states and suggest an instrumental variable for income inequality.[33] The instrument measures the state level exposure to aggregate developments in the labor market. Note that the model studies a rise in income inequality that is (i) orthogonal to housing and mortgage market developments and (ii) does not lead to a change in the supply of credit, in particular a decline in real rates as is the case with Kumhof, Rancière and Winant (2015) and Mian, Straub and Sufi (2020a). The instrumental variable approach guarantees that (i) is satisfied. In the panel regressions, I use the national real rate developments and thus the identifying assumption is that local inequality developments do not affect national real rate dynamics. To further ensure that my analysis satisfies (ii), I exclude five states with the highest GDP as a robustness exercise, as these states might be driving the domestic component of total supply of credit in the U.S.. These states are California, Texas, New York, Florida and Illinois and comprise about 40% of national GDP as of 2020. I use the following regression specifications:

$$Y_{st} = \alpha_s + \alpha_t + \beta_1 Gini_{st} + \Gamma X_{st} + \epsilon_{st} \tag{23}$$

where Y_{st} is the outcome variable, α_s and α_t are state and time fixed effects, and X_{st} is a vector of time-varying state level covariates.

In order to test whether interest rate environment alters the relationship between income inequality housing and mortgage variables, I also estimate a specification which includes the interaction of the Gini coefficient with the real interest rate.

$$Y_{st} = \alpha_s + \alpha_t + \beta_2 Gini_{st} + \mu Rate_t \times Gini_{st} + \Gamma X_{st} + \epsilon_{st} \tag{24}$$

State fixed effects control for time invariant developments that might affect both income inequality and local labor, housing and credit market dynamics. Using year fixed effects controls any macroeconomic development that might affect the local equilibrium, including the direct effect of declining real rates, which might arise from a global savings glut (Bernanke (2005)), and savings glut of the rich (Mian, Straub and Sufi (2020b)) among other explanations. Therefore, using year fixed effects is key to test the borrower risk composition channel in isolation from the credit supply channel of

[33] Nakamura and Steinsson (2017) for identification in empirical macroeconomics and other studies that employ cross-sectional identification similar to my paper.

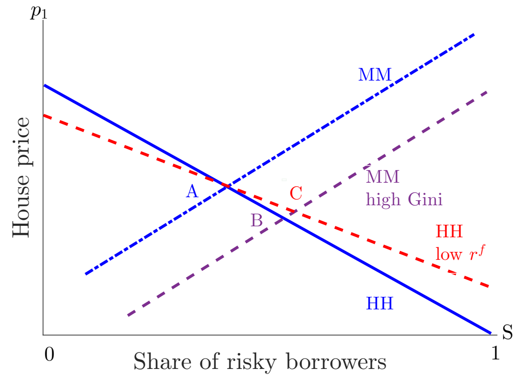


Figure 5: Equilibrium impact of rising inequality in high and low interest rate environments controlling for the direct effect of lower rates on house prices and share of risky borrowers

Note: The HH curve represents equilibrium in the housing market and the MM curve represents equilibrium in the mortgage market in period-1. The slope of the HH curve increases with the real rate and a mean-preserving change in income inequality shifts MM to MMhigh Gini.

income inequality.

Figure 5 provides model based guidance regarding the interaction between inequality and the interest rate environment. The figure is a variant of Figure 4c with the direct effects of the decline in the risk-free rate is eliminated or, within the context of the regression specification, averaged out by year fixed effect α_t. State fixed effect α_s ensures that the comparison of high and low rates is coming from within state variations in income inequality and other variables. Therefore the analysis is akin a difference-in-differences approach with continuous treatment. The model thus predicts that for real house prices $\beta_1 < 0$ and $\mu < 0$, and for aggregate default risk $\beta_1 > 0$ and $\mu < 0$.

Next I describe the data I use and the choice of control variables $X_{s,t}$, then describe the construction the Gini coefficient instrument and finally present the 2SLS estimation results.

Data and summary statistics. In my empirical analysis I use a panel of US States, which includes measures of house prices, mortgage debt, mortgage delinquency, mean income and population and summary statistics are reported in Table 1. The data set covers the years between 1992-2015 for all variables other than the mortgage variables. Mortgage data is available to me only after 2003 at the state level. It covers all US states, except for Alaska, and the District of Columbia.

The Gini coefficient and top income shares are constructed by Mark Frank from individual tax filing data available through the Internal Revenue Service website.[34] Table 1 shows that from 1992 to 2015 mean state income inequality increased by about 4 basis points and 4 percentage points using the Gini coefficient and top 5% income shares, respectively. About two thirds of the rise in the Gini took place between the years 2003 and 2015. Looking at any year cross-section we see that interquartile range is above 3 basis points suggesting that there is comparable variation in the Gini coefficient both across time and across states.

I use the Federal Housing and Financing Agency (FHFA) all transactions index (1980 = 100) to measure house prices. The FHFA constructs this index by reviewing repeat mortgage transactions, both purchase and refinancing, on properties whose mortgages were securitized or bought by Fannie Mae or Freddie Mac. Full sample panel of Table 1 shows that average real house price is around 18% above 1980 levels with a high level heterogeneity across years and states, where the 75^{th} percentile has above 35% higher prices than 1980 level. Year-by-year cross-sections in the lower panels of the table shows that mean state experienced about 25% growth between 1992 and 2015, no growth between 2003 and 2015 consistent with the boom-bust episode displayed in Figure 1.

Household debt data is from Consumer Credit Panel/Equifax (CCP) data available at the state level from the New York Federal Reserve website.[35] I use the per capita balance of mortgage debt excluding home equity lines of credit as my measure of mortgage debt and mortgage delinquencies are the percent of the mortgage debt balance that has been delinquent for more than ninety days. Table 1 shows that on average about 3% of mortgages are delinquent and vary significantly in the full sample. From 2003 to 2015 mean delinquency rate increases 1.6 folds and this is accompanied by a 7% rise in real mortgage debt per capita.

Data on resident populations, the 10-year treasury constant maturity rate, the number of new private housing permits authorized, and mean adjusted gross income are available from the St Louis Federal Reserve Bank.[36] I use the CPI-UR-S series

[34]See Frank (2014) for the construction of Gini and other income inequality measure for US states. Inequality data is available through http://www.shsu.edu/eco_mwf/inequality.html or World Inequality Database. I use the data from Mark Frank's website, which did not have the Wyoming 2012 data error as in WID website data at the time of data download.

[35]https://www.newyorkfed.org/microeconomics/databank.html.For a detailed description of this dataset see Lee and der Klaauw (2010).

[36]10-year treasury constant maturity rate is from Board of Governors of the Federal Reserve System. New private housing permits authorized is from US Bureau of the Census and U.S. Department of Housing and Urban Development. Mean adjusted gross income series are from US Bureau of the

Table 1: Summary statistics

	Gini coefficient	Top 5% income share (pp)	Real house price index (log)	Real mortage debt (log)	Mortgage delinquency rate (bp)	Real mean income (log)	Population (log)	New housing permits (log)	Home-ownership rate (%)	Real 10-year rate (%)
Full sample										
mean	0.589	0.131	0.18	4.74	3.05	5.43	8.17	9.31	68.10	1.90
sd	0.036	0.041	0.29	0.37	2.65	0.17	1.02	1.15	6.43	1.27
p25	0.562	0.103	-0.02	4.49	1.36	5.31	7.41	8.50	65.80	0.82
p75	0.610	0.145	0.34	5.02	3.96	5.53	8.80	10.16	72.10	3.21
min	0.521	0.066	-0.50	3.75	0.30	5.01	6.14	6.36	35.00	-0.63
max	0.711	0.312	1.15	5.68	20.74	5.99	10.57	12.28	81.30	3.66
N	1200	1200	1200	649	650	1200	1200	1200	1200	
1992										
mean	0.567	0.101	-0.01			5.28	8.05	9.41	65.78	3.21
sd	0.024	0.022	0.26			0.13	1.02	1.10	6.72	
p25	0.550	0.090	-0.19			5.20	7.19	8.58	64.30	
p75	0.583	0.109	0.17			5.37	8.70	10.26	70.00	
min	0.528	0.067	-0.50			5.01	6.14	7.05	35.00	
max	0.617	0.184	0.55			5.63	10.34	11.29	73.80	
N	50	50	50			50	50	50	50	
2003										
mean	0.576	0.120	0.24	4.58	1.23	5.44	8.18	9.71	69.90	1.56
sd	0.025	0.030	0.27	0.35	0.53	0.15	1.03	1.07	6.33	
p25	0.556	0.103	0.06	4.37	0.83	5.31	7.46	9.09	68.00	
p75	0.599	0.123	0.40	4.86	1.59	5.56	8.77	10.42	73.70	
min	0.536	0.082	-0.24	3.78	0.41	5.17	6.22	7.60	43.00	
max	0.629	0.208	1.01	5.22	2.53	5.86	10.47	12.00	78.10	
N	50	50	50	49	50	50	50	50	50	
2015										
mean	0.610	0.147	0.24	4.65	2.00	5.51	8.28	9.09	65.53	-0.05
sd	0.036	0.042	0.28	0.33	1.02	0.16	1.02	1.03	6.10	
p25	0.584	0.124	0.02	4.44	1.23	5.40	7.52	8.39	62.90	
p75	0.620	0.157	0.45	4.87	2.36	5.64	8.88	9.55	69.90	
min	0.548	0.097	-0.21	3.97	0.59	5.21	6.37	6.79	40.40	
max	0.707	0.278	0.99	5.37	5.43	5.92	10.57	11.60	74.90	
N	50	50	50	50	50	50	50	50	50	

Note: BLS CPI-UR-S (1980 = 100) series is used when calculating real variables. Real 10-year rate is 10-year constant maturity treasury rate minus 10-year ahead inflation forecasts from Livingston Survey. Mortgage delinquency rate is reported in basis points.

from Bureau of Labor Statistics to deflate the house price index, mortgage debt and mean adjusted gross income.[37] I construct long-term real rates by subtracting 10-year inflation forecasts from Livingston Survey from the 10-year treasury constant maturity rate. I find the annual forecast by taking the average of quarterly forecasts. This

Census.

[37]This series is considered to be the most detailed and systematic consistent CPI series available. This is important as my data starts before 2000 where the series had a methodological change.

measure of real rates declines about 3.25 percentage points between 1992 and 2015.[38]

Instrumental variables are calculated using Quarterly Census of Employment and Wages from BLS. Regional data at different NAICS dissaggreation levels is available from 1990. I use employment shares and wage growth of NAICS 2- to 6-digit industries to calculate shift-share type instruments for the Gini coefficient to study the causal effect of income inequality.

Choice of controls in the regressions. As mentioned above, my regressions include both state and year fixed effects. Year fixed effects capture changes in aggregate variables that might confound the effect of income inequality. For example, declining real interest rates, business cycles, or an increase in the aggregate supply of credit. State fixed effects control for any time-invariant heterogeneity across states. This includes any cultural, social, historical, geographic and other conditions that remained constant within the study period.

I include variables that vary by state over time in order to control for confounding effects in the state-by-time dimension. Changes in inequality might be correlated with changes in other variables that directly affect housing demand. For example, real mean income and population. A rise in income inequality can result from changes in the different quintiles of the income distribution which might give rise to an increase or a decrease in the mean income. Therefore, to analyze whether income inequality is an independent vector explaining the developments in the outcome variable, I control for mean income. Including population controls directly for aggregate housing demand and also indirectly for changes in the demographics of a state, which could affect preferences for home-ownership and thus housing demand. Demographics can also affect the borrower pool. While state fixed effects can control for the time invariant component of borrower quality, including population can be considered as an indirect control for this type of change. Moreover, changes in demographics can affect the income distribution in a state. Depending on the relative incomes of movers and residents, mean income and income inequality might increase or decrease. I include the home-ownership rate in the model to control for cyclical changes in housing demand that can arise from various sources such as an increase in house price expectations or easier access to mortgage lending. If the access to lending is not increasing homogeneously across the income distribution, its effect might confound that of income inequality. Finally, I introduce

[38] My results are also robust to using 10-year real rates of Aruoba (2014) and are available upon request.

a measure of a change in housing supply that cannot be captured by the state fixed effects. Developers may wish to build more houses when incomes are increasing. This might confound the effect of income inequality especially if potential buyers from some points of the income distribution fare better than others.

Constructing shift-share instruments for the Gini coefficient. My instrument is akin to canonical shift-share instrument of Bartik (1991). To derive causal interpretations, similar to the model presented above, one needs a measure of income inequality that is not affected by housing and credit market developments in state s and time t and thus not subject to reverse causality. The use of the instruments also addresses potential measurement error issues that can be associated with calculating the Gini coefficient and omitted variable bias that can derive developments in both income inequality and local housing and mortgage markets.

To this end, I calculate the local industrial wage Gini coefficient by using the predetermined industry shares in state s and the national growth rate in industrial wages to predict local wages in time t. I consider wages and salaries in privately owned establishments in calculating state income inequality instrument. The shift I consider is then heterogeneous developments in wages across sectors, which can arise from any development that can affect labor demand or supply[39] and the share is the employment share of a given sector in state s. In what follows I explain why I consider wages from privately owned establishments as the type of income, describe how I measure national wage growth for a given sector and how I calculate the instrument. Then introduce the regression framework and discuss the control variables.

Using wages from privately owned establishments have several advantages. First, they exclude rental income and thus income inequality does not mechanically rise between homeowners and renters following an increase in rents or house prices. Second, this measure does not include mortgage related tax exemptions that might link developments in mortgage market or mortgage policies to income inequality between mortgage holders and non-holders. Third, I exclude wages of federal or local government owned establishment in which wages might reflect changes in local property tax incomes and thus can be affected by local housing market developments. Last, when calculating wage inequality, I also exclude sectors that are directly related to housing and credit

[39]These include heterogeneous effects of skill-biased technical change or China's import penetration effect.

like construction, finance and insurance and real estate.[40]

The endogenous regressor in specifications (23) and (24) Gini coefficient from the IRS grouped income data is calculated as:

$$Gini_{st} = \frac{1}{2\bar{M}_{s,t}} \sum_{i=1}^{N} \sum_{j=1}^{N} \psi_{ist}\psi_{jst}|y_{ist} - y_{jst}|. \tag{25}$$

where ψ_{ist} is the population share of the income level y_i in state s and year t. Then, let f_{kst} and w_{kst} represent industry share and wages of sector k in state s in period t, and $\bar{M}_{s,t}$ denote mean state income, then the Gini coefficient from grouped industry wages is calculated as:

$$Gini^{wage}_{st} = \frac{1}{2\bar{W}_{s,t}} \sum_{i=1}^{K} \sum_{j=1}^{K} f_{ist}f_{jst}|w_{ist} - w_{jst}|. \tag{26}$$

where $\bar{W}_{s,t}$ is the mean wage in state s in year t. Note that this wage inequality measure cannot be used as an instrument as both local industry wages w_{kst} and employment shares f_{kst} should be affected by local housing and credit market dynamics causing the same endogeneity issues as the income Gini coefficient discussed above.

In constructing the instrument I follow the literature and use predetermined exposure to wage growth as 2 year-lagged industry shares. While higher lags can be used, doing so would make the estimation sample shorter for the house price regressions. In deriving the shift component of the instrument, i.e. national growth in industry wages, I follow a leave-one-out strategy for each state. That is, for each state I calculate the national growth as the mean of industrial wage growth in sector k at time t excluding state s. This approach addresses the finite sample bias that comes from using own-observation information. Let Δw_{kt} denote gross national wage growth, then the industrial wage at industry k state s and year t is predicted as:

$$\tilde{w}_{kst} = w_{ks1990}\Pi^{t}_{\tau=0}\Delta w_{k1990+\tau}. \tag{27}$$

[40] Therefore, I exclude sectors under NAICS supersectors 23, 52 and 53.

Then my instrument for the Gini coefficient is:

$$Gini_{st}^{IV} = \frac{1}{2\bar{\tilde{W}}_{s,t}} \sum_{i=1}^{K} \sum_{j=1}^{K} f_{ist-2} f_{jst-2} |\tilde{w}_{ist} - \tilde{w}_{jst}|. \tag{28}$$

The data on industry wages and employment shares are available at different level of industry aggregations. I calculate the wage inequality Gini coefficient instrument for 2- to 6-digit industries and thus I develop five instruments using the definition (28). Note that, the higher the disaggregation, the more likely an industry is removed from Gini instrument calculations if it is produced only in that state due to leave-one-out strategy in calculation national industry wage growth rates.

For the interest of space, I report first step-regression estimates in the online appendix. Within text I present the second-step estimation results and first-step test statistics. I consider two sets of instruments (i) all five instruments derived from 2- to 6-digit industries and (ii) only the 6-digit instrument. I choose these two groups because in different samples, different group of instruments have the best first-step statistics that reject the presence of weak- and under-identification.[41] In the 1992-2015 sample using all five instruments imply rejection of weak and under-identification of both Kleibergen and Paap (2006) and Sanderson and Windmeijer (2016). Therefore, when discussing the effect of income inequality on house prices, I discuss the regression results with all of the instruments. Remember that, mortgage variables are available after 2003. I find that for this shorter sample, using the highest digit instrument provides better first-step statistics in terms of weak and under-identification tests. Therefore, when studying the effect of income inequality on mortgage delinquency rate and real mortgage debt per capita, I focus on the just-identified 2SLS regressions.

In the specifications with over-identification, i.e. more instruments than endogenous variables, I report the over-identification test results in addition to weak- and under-identification results. Hansen J-test displays the p-value for the test of over-identifying restrictions where the joint null hypothesis that the instruments are valid instruments and that the excluded instruments are correctly excluded from the estimated equation.

[41]In 2SLS regressions of (23) the Gini coefficient is instrumented with the instruments calculated according to definition (26) and in regressions (24) both the Gini coefficient and its real rate interaction are endogenous. The latter specification includes Gini instruments interacted with the real rate as additional instruments. When Gini is the only endogenous regressor using either set of instruments reject under- and weak- identification tests of both Kleibergen and Paap (2006) and Sanderson and Windmeijer (2016). Therefore, reporting findings allow me to show that the 2SLS results are qualitatively robust to using different instruments.

Included instruments are the control variables and year fixed effects and excluded instruments are Gini coefficients calculated using NAICS industries. A rejection casts doubt on the validity of the instruments and all my regressions fail to reject the null hypothesis with p-values above 0.2.

Result 1: House prices decline with income inequality and low rates mitigate this effect

Table 2 shows that consisted with model predictions depicted in Figure 5, (i) income inequality and real house prices are negatively related and (ii) a decline in real rates mitigates this effect. The first two columns present the panel OLS results while the remaining columns show the 2SLS regression output and thus the causal effect of income inequality on real houses prices. Columns (3) - (6) consider all of the states in my sample, while columns (7)-(10) exclude 5 highest GDP states from the sample as a robustness check.

The OLS regression in column (1) suggests that one percentage point rise in income inequality is associated with 1.6% decline in real house prices keeping real mean income, population, new housing supply and home-ownership rate constant. Moving from the 25^{th} percentile to 75^{th} percentile of real rates in the sample, makes the slope a third steeper.[42]

Columns (3) and (4) presents 2SLS regression results and shows that the causal effect of income inequality on house prices and the mitigating effect of real rates are economically stronger than the OLS estimates.[43] One percentage point increase in income inequality leads to 2.25% percent decline in house prices. A back of the envelope calculation suggests that 4.3 basis points rise in the Gini coefficient for the mean US state from 1992 to 2015 depressed the real house prices by 9.7 percent, which is about 40% of the realized growth of 25%. Moreover, one percentage point lower real rates decreases the Gini elasticity of house prices by half, and moving from the 25^{th} percentile to 75^{th} percentile of real rates in the sample makes the slope more than 2 folds steeper. Finally, columns (7) and (8) show that when I exclude the highest GDP states from my sample, the results do not change qualitatively or quantitatively.

[42]The interquartile range of real rates in the sample is 2.39 and multiplying this with the estimated μ gives and additional effect of 5.5%. This is a third of estimated coefficient of Gini in column (1).

[43]As mentioned above, I report results from columns (3) and (4) instead of columns (5) and (6), as they have better first-step statistics of under-identification test of both Kleibergen and Paap (2006) and Sanderson and Windmeijer (2016).

Table 2: Dependent variable: Log Real FHFA House Price Index
Panel: US States from 1992 to 2015

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Gini	-1.61***	-1.34***	-2.25***	-1.93*	-1.23	-1.34	-2.10**	-1.11	-0.89	-0.50
	(0.24)	(0.31)	(0.85)	(1.03)	(1.28)	(1.28)	(0.83)	(1.10)	(1.28)	(1.58)
Gini × Real rate		-0.23*		-1.08**		-0.44		-1.36*		-0.76
		(0.13)		(0.54)		(1.09)		(0.77)		(2.01)
Observations	1200	1200	1200	1200	1200	1200	1080	1080	1080	1080
Number of states	50	50	50	50	50	50	45	45	45	45
Number of included instruments			27	27	27	27	27	27	27	27
Number of excluded instruments			5	10	1	2	5	10	1	2
Degree of overidentification			4	8	0	0	4	8	0	0
Instrument NAICS digits			2-to-6	2-to-6	6	6	2-to-6	2-to-6	6	6
Hansen J-test (p-value)			0.30	0.94			0.35	0.84		
Under-id KP (p-value)			0.00	0.04	0.00	0.06	0.00	0.08	0.01	0.22
Under-id SW (Gini)(p-value)			0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
Under-id SW(Gini × Real rate)(p-value)				0.01		0.14		0.04		0.32
Weak-id KP (Wald rk F stat)			10.43	3.18	14.22	1.78	10.32	2.75	12.60	0.64
Weak-id SW (Gini)(F stat)			10.43	7.82	14.22	13.15	10.32	5.46	12.60	3.15
Weak-id SW (Gini)(p-value)			0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02
Weak-id SW (Gini× Real rate)(F stat)				2.49		2.07		1.82		0.95
Weak-id SW (Gini × Real rate)(p-value)				0.03		0.16		0.09		0.33
R-squared within	0.81	0.82								

*p<0.1, **p<0.05, ***p<0.01.
All regressions include year and state fixed effects, and control variables log real mean state income, log population, log new housing permits and homeownership rate. Errors in parentheses clustered at the state level. BLS CPI-UR-S series is used when calculating real variables. Real 10-year rate is 10-year constant maturity treasury rate minus 10-year ahead inflation forecasts from Livingston Survey. Columns (1) and (2) are panel OLS regression results. Columns (3) to (10) are two-stage least squares (2SLS) results. Columns (1)-(8) use full sample and columns (9) and (10) exclude 5 highest GDP states: California, Texas, New York, Florida and Illinois from the sample. In columns (3), (5), (7) and (9) only the Gini coefficient is an endogenous regressor, while in columns (4),(6),(8) and (10) both the Gini coefficient and its interaction with 10-year real rate are endogenous. Two groups of Gini coefficient and real rate interaction instruments are use in the 2SLS regressions: (i) 6-digit NAICS and (ii) all five of 2- to 6-digit NAICS industries. Columns (5) and (9) use only the highest digit instrument and columns (6) and (10) use its real rate interaction in addition. Columns (3) and (7) use all digits and columns (4) and (8) use their real rate interaction in addition. Hansen J-test displays the p-value for the test of overidentifying restrictions where the joint null hypothesis is that the instruments are valid instruments and that the excluded instruments are correctly excluded from the estimated equation. Included instruments are the control variables and year fixed effects and excluded instruments are Gini coefficients calculated using NAICS industries. A rejection casts doubt on the validity of the instruments. This statistic is not calculated for the regressions with the highest digit instrument as they are exact-identified. For over- and under-identification tests I present statistics from Kleibergen and Paap (2006)(KP) and Sanderson and Windmeijer (2016)(SW). The null hypothesis of the KP LM test is that the structural equation is under-identified and for the weak instruments Wald rk test the null hypothesis is that instruments are (jointly) weak. The SW statistics are tests of underidentification and weak identification, respectively, of individual endogenous regressors. They are constructed by partialling-out linear projections of the remaining endogenous regressors. In the case of one endogenous regressor, SW and KP weak identification statistics are identical. For the SW test the critical values for hypothesis testing are available and p-values are reported in the table.

Result 2: Mortgage delinquency rate increases with income inequality and this effect is stronger in low interest rate environments

The first row of Table 3 displays that a rise in income inequality is associated with a rise in mortgage delinquencies and the second row shows that low real rates amplify the effect of income inequality as predicted by the model in Figure 5. Similar to the case with real house prices, estimates from the 2SLS suggest a stronger causal effect than the OLS estimates. That is, column (5) shows that one percentage point

Table 3: Dependent variable: Percent of Mortgage Debt Balance 90+ Days Delinquent
Panel: US States from 2003 to 2015

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Gini	0.13**	0.20***	0.31**	0.30**	0.34**	0.44	0.41***	0.34**	0.45***	0.72**
	(0.05)	(0.06)	(0.15)	(0.13)	(0.16)	(0.27)	(0.15)	(0.16)	(0.15)	(0.32)
Gini × Real rate		-0.09*		-0.21**		-0.45*		-0.19*		-0.52
		(0.04)		(0.10)		(0.27)		(0.11)		(0.34)
Observations	650	650	650	650	650	650	585	585	585	585
Number of states	50	50	50	50	50	50	45	45	45	45
Number of included instruments			16	16	16	16	16	16	16	16
Number of excluded instruments			5	10	1	2	5	10	1	2
Degree of overidentification			4	8	0	0	4	8	0	0
Instrument NAICS digits			2-to-6	2-to-6	6	6	2-to-6	2-to-6	6	6
Hansen J-test (p-value)			0.41	0.60			0.57	0.44		
Under-id KP (rk LM stat)			11.87	9.91	10.23	2.90	9.66	12.00	8.17	2.47
Under-id KP (p-value)			0.04	0.36	0.00	0.09	0.09	0.21	0.00	0.12
Under-id SW (Gini)(p-value)			0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.02
Under-id SW(Gini × Real rate)(p-value)				0.02		0.03		0.00		0.06
Weak-id KP (Wald rk F stat)			4.73	1.84	22.17	1.60	3.21	2.79	14.88	1.38
Weak-id SW (Gini)(F stat)			4.73	3.56	22.17	8.79	3.21	3.67	14.88	4.89
Weak-id SW (Gini)(p-value)			0.00	0.00	0.00	0.01	0.02	0.00	0.00	0.03
Weak-id SW (Gini× Real rate)(F stat)				2.09		4.61		3.11		3.48
Weak-id SW (Gini × Real rate)(p-value)				0.05		0.04		0.01		0.07
R-squared within	0.68	0.69								

*p<0.1, **p<0.05, ***p<0.01.
All regressions include year and state fixed effects, and control variables log real mean state income, log population, log new housing permits and homeownership rate. Errors in parentheses clustered at the state level. BLS CPI-UR-S series is used when calculating real variables. Real 10-year rate is 10-year constant maturity treasury rate minus 10-year ahead inflation forecasts from Livingston Survey. Columns (1) and (2) are panel OLS regression results. Columns (3) to (10) are two-stage least squares (2SLS) results. Columns (1)-(8) use full sample and columns (9) and (10) exclude 5 highest GDP states: California, Texas, New York, Florida and Illinois from the sample. In columns (3), (5), (7) and (9) only the Gini coefficient is an endogenous regressor, while in columns (4),(6),(8) and (10) both the Gini coefficient and its interaction with 10-year real rate are endogenous. Two groups of Gini coefficient and real rate interaction instruments are use in the 2SLS regressions: (i) 6-digit NAICS and (ii) all five of 2- to 6-digit NAICS industries. Columns (5) and (9) use only the highest digit instrument and columns (6) and (10) use its real rate interaction in addition. Columns (3) and (7) use all digits and columns (4) and (8) use their real rate interaction in addition. Hansen J-test displays the p-value for the test of overidentifying restrictions where the joint null hypothesis is that the instruments are valid instruments and that the excluded instruments are correctly excluded from the estimated equation. Included instruments are the control variables and year fixed effects and excluded instruments are Gini coefficients calculated using NAICS industries. A rejection casts doubt on the validity of the instruments. This statistic is not calculated for the regressions with the highest digit instrument as they are exact-identified. For over- and under-identification tests I present statistics from Kleibergen and Paap (2006)(KP) and Sanderson and Windmeijer (2016)(SW). The null hypothesis of the KP LM test is that the structural equation is under-identified and for the weak instruments Wald rk test the null hypothesis is that instruments are (jointly) weak. The SW statistics are tests of underidentification and weak identification, respectively, of individual endogenous regressors. They are constructed by partialling-out linear projections of the remaining endogenous regressors. In the case of one endogenous regressor, SW and KP weak identification statistics are identical. For the SW test the critical values for hypothesis testing are available and p-values are reported in the table.

increase in the Gini coefficient leads to 0.34 percentage points increase in the mortgage delinquency rate, which is about three times the OLS association in column (1). From 2003 to 2015 mean US state has experienced 3.5 basis points increase in the Gini coefficient, a back of the envelope calculation suggests that this gives rise to 1.2 basis points rise in the share of delinquent mortgages, and explains the 160% of the 0.77 basis points increase. Therefore other developments like rising mean income must have compensated for the effect of rising income inequality in this period. Column (6) shows that in a one percent lower interest rate environment, the effect of income inequality on mortgage delinquencies is twice as high. As real rates declined by 1.6% between 2003 and 2015, this suggests over time income inequality elasticity of mortgage delinquencies has increased significantly, making the financial stability implications of rising income inequality worse of a problem.

Result 3: Mortgage debt declines with income inequality

Finally, I analyse how a rise in income inequality is associated with mortgage debt. In Figure 2 I show that a rise in the Gini coefficient is negatively associated with mortgage debt growth. In this section I provide robustness for this result and estimate the causal effect even though I don't present theoretical predictions with respect to the Gini coefficient. Remember that my model has theoretical predictions that relate top income shares to mortgage debt similar to Rajan (2010), Kumhof, Rancière and Winant (2015) and Mian, Straub and Sufi (2020a). These studies suggest a positive correlation between top income shares and household debt that operates through a credit supply channel of income inequality. In contrast, my model predicts that risky/low-income households have higher propensity to borrow against their income and thus when a lower share of income goes to these borrowers, average propensity to borrow and mortgage debt declines. I also present the results from the regression where I use the top 5% income shares as the explanatory variable, however, these regressions are not causal in the sense that I don't develop an instrument for the top income shares. I present the findings in Tables 4 and 5 using the Gini coefficient and the top 5% income share respectively. The first row of Table 4 and the first column of Table 5 both show that a rise in income inequality is associated with a decline in real mortgage debt independent of how income inequality is measured. The evidence using top income shares still should not be interpreted as a rejection of the credit supply theories of income inequality as the supply side of the mortgage market does not face the same geographic segmentation as the housing market. This is particularly the case for the period I consider as it is after the deregulation in the banking sector in the US that allowed nationwide interstate banking.[44] Year fixed effects in my empirical specification allows me to control for any time varying development that would give rise to an increase in the availability of credit at the national level, including that of rising income inequality at the national level as is the case discussed in Kumhof, Rancière and Winant (2015).

[44]The Riegle-Neal Act allowed the banks hold nationwide branch networks after mid-1997.

Table 4: Dependent variable: Log Real Mortgage Debt per Capita (excluding HELOC)
Panel: US States from 2003 to 2015

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Gini	-1.01***	-1.12***	-1.74**	-0.98	-1.38*	-1.53*	-1.42*	-0.35	-0.96	-1.20
	(0.24)	(0.33)	(0.78)	(0.63)	(0.76)	(0.92)	(0.85)	(0.79)	(0.82)	(1.34)
Gini × Real rate		0.12		-0.17		0.75		-0.58		0.47
		(0.19)		(0.67)		(0.89)		(0.73)		(1.10)
Observations	649	649	649	649	649	649	584	584	584	584
Number of states	50	50	50	50	50	50	45	45	45	45
Number of included instruments			16	16	16	16	16	16	16	16
Number of excluded instruments			5	10	1	2	5	10	1	2
Degree of overidentification			4	8	0	0	4	8	0	0
Instrument NAICS digits			2-to-6	2-to-6	6	6	2-to-6	2-to-6	6	6
Hansen J-test (p-value)			0.59	0.23			0.43	0.49		
Under-id KP (p-value)			0.04	0.36	0.00	0.09	0.09	0.21	0.00	0.12
Under-id SW (Gini)(p-value)			0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.02
Under-id SW(Gini × Real rate)(p-value)				0.02		0.03		0.00		0.06
Weak-id KP (Wald rk F stat)			4.73	1.85	22.14	1.60	3.20	2.82	14.87	1.38
Weak-id SW (Gini)(F stat)			4.73	3.58	22.14	8.80	3.20	3.69	14.87	4.90
Weak-id SW (Gini)(p-value)			0.00	0.00	0.00	0.01	0.02	0.00	0.00	0.03
Weak-id SW (Gini × Real rate)(F stat)				2.10		4.61		3.14		3.48
Weak-id SW (Gini × Real rate)(p-value)				0.05		0.04		0.01		0.07
R-squared within	0.79	0.79								

*p<0.1, **p<0.05, ***p<0.01.
All regressions include year and state fixed effects, and control variables log real mean state income, log population, log new housing permits and homeownership rate. Errors in parentheses clustered at the state level. BLS CPI-UR-S series is used when calculating real variables. Real 10-year rate is 10-year constant maturity treasury rate minus 10-year ahead inflation forecasts from Livingston Survey. Columns (1) and (2) are panel OLS regression results. Columns (3) to (10) are two-stage least squares (2SLS) results. Columns (1)-(8) use full sample and columns (9) and (10) exclude 5 highest GDP states: California, Texas, New York, Florida and Illinois from the sample. In columns (3), (5), (7) and (9) only the Gini coefficient is an endogenous regressor, while in columns (4),(6),(8) and (10) both the Gini coefficient and its interaction with 10-year real rate are endogenous. Two groups of Gini coefficient and real rate interaction instruments are use in the 2SLS regressions: (i) 6-digit NAICS and (ii) all five of 2- to 6-digit NAICS industries. Columns (5) and (9) use only the highest digit instrument and columns (6) and (10) use its real rate interaction in addition. Columns (3) and (7) use all digits and columns (4) and (8) use their real rate interaction in addition. Hansen J-test displays the p-value for the test of overidentifying restrictions where the joint null hypothesis is that the instruments are valid instruments and that the excluded instruments are correctly excluded from the estimated equation. Included instruments are the control variables and year fixed effects and excluded instruments are Gini coefficients calculated using NAICS industries. A rejection casts doubt on the validity of the instruments. This statistic is not calculated for the regressions with the highest digit instrument as they are exact-identified. For over- and under-identification tests I present statistics from Kleibergen and Paap (2006)(KP) and Sanderson and Windmeijer (2016)(SW). The null hypothesis of the KP LM test is that the structural equation is under-identified and for the weak instruments Wald rk test the null hypothesis is that instruments are (jointly) weak. The SW statistics are tests of underidentification and weak identification, respectively, of individual endogenous regressors. They are constructed by partialling-out linear projections of the remaining endogenous regressors. In the case of one endogenous regressor, SW and KP weak identification statistics are identical. For the SW test the critical values for hypothesis testing are available and p-values are reported in the table.

Another common message from the estimation results presented in Tables 4 and 5 is that interest rate environment does not alter the association of income inequality and mortgage debt, that is, the interaction term is statistically insignificant.

As far as the causal effect of income inequality on mortgage debt is concerned, column (5) of Table 4 shows that one basis point rise in the Gini coefficient is associates with about 1.4% decline in real mortgage debt per capita. With 3.5 basis points rise in the Gini coefficient for the mean US state in the sample period, absent this effect

Table 5: Top 5% income share, real house prices, real mortgage debt and mortgage delinquencies

	Log real mortgage debt		Log real house price index		Mortgage delinquencies	
	(1)	(2)	(3)	(4)	(5)	(6)
Top 5% share	-2.74***	-2.78***	-2.01***	-1.87***	-0.11	-0.06
	(0.31)	(0.34)	(0.41)	(0.42)	(0.09)	(0.10)
Top 5% share × Real rate		0.09		-0.11		-0.10**
		(0.12)		(0.11)		(0.03)
Years	2003-2015	2003-2015	1992-2003	1992-2003	2003-2015	2003-2015
Observations	649	649	1200	1200	650	650
R-squared within	0.82	0.82	0.81	0.81	0.67	0.69

*p<0.1, **p<0.05, ***p<0.01.
All regressions include year and state fixed effects, and control variables log real mean state income, log population, log new housing permits and homeownership rate. Errors in parentheses clustered at the state level. BLS CPI-UR-S series is used when calculating real variables. Real 10-year rate is 10-year constant maturity treasury rate minus 10-year ahead inflation forecasts from Livingston Survey.

real mortgage debt per capita growth could have been about 12% instead of 7%.

3 Conclusions

Income inequality, house prices and household debt have increased enormously in the U.S. in the last few decades. During the same period, real interest rates declined to historically low levels. This paper adopts empirical and theoretical strategies that disentangle the effect of income inequality from declining real rates on housing and credit markets. I show that declining real rates and rising income inequality work in opposite directions. I develop an analytical general equilibrium model with heterogeneous income borrowers which presents a novel borrower risk decomposition channel of income inequality. I then test the model's predictions using a cross-sectional identification strategy and the causal interpretations are due to a shift-share instrument á la Bartik (1991) for the Gini coefficient.

I find that, in isolation, rising income inequality is associated with declines in real house prices and mortgage debt, but a rise in mortgage delinquencies. The key theoretical mechanism is that households with heterogeneous incomes are offered a menu of mortgage contracts with different default risk. A rise in income inequality alters the borrower pool for the worse - a higher share of borrowers find it optimal to select into high risk loans, borrow costly and demand for housing is depressed. These borrowers also have high propensity to borrow against their income and when a smaller share of income is allocated to them, average propensity to borrow and thus aggregate mortgage debt declines.

While house price and debt dynamics are positively correlated with income inequality in aggregate data, I show that the model's predictions hold for a panel of US states. That is, the cross sectional and aggregate trends are at odds. I show that declining real rates are central to reconciling the cross-sectional and aggregate correlations as they can overturn the negative effect of income inequality on house prices and mortgage debt. However, this leads to a rise in mortgage delinquencies and amplifies the effect of income inequality on mortgage default risk. While it is not surprising that an expansion in credit supply boosts mortgage debt and thus house prices, heterogeneous transmission of low rates across the income distribution through a risk taking channel is another contribution of this paper. I show that in a low interest rate environment mortgage default risk increases because borrowers higher at the income distribution than a high interest rate environment selects into taking mortgage default risk.

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Mathematical appendix for Income Inequality, Mortgage Debt and House Prices

A Borrower optimization

In this section I define the optimal period-1 decisions consistent with a risk-free contract and with a risky contract, where both borrowers and lenders take into consideration optimal period-2 decisions of the borrowers. Figure A.1 show the relationship between loan prices, mortgage debt d_{1i} and housing collateral h_{1i}. The left panel shows that for a given level of housing collateral when debt is sufficiently low, the lenders do not ask for any risk premium and as the debt increases the borrowers switch from a risk-free loan to a risky loan. The risk premium increases with debt amount. The right panel shows the effect of a rise in housing collateral pledged. Naturally, effect on loan prices is present only for the risky loans while debt limits increase with both types of loans. For a given amount of debt, if the borrower is pledging a larger collateral, the lender asks smaller risk premium. The heterogeneous and stronger pass through of low real rates to risky loans comes from the collateral value of houses on loan prices. Borrowers internalize these effects when making their optimal debt and housing choices.

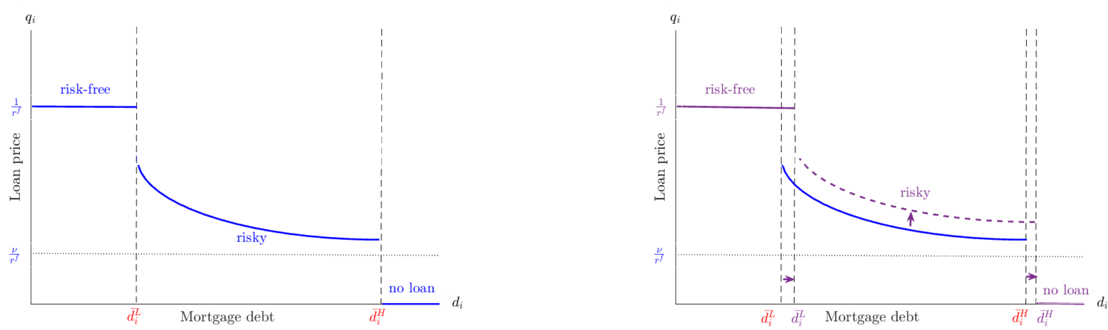

Figure A.1: Graphical representation of loan pricing (17) and the impact of housing collateral on loan pricing

The left panel shows the loan pricing equation for a given level of income y_{1i} and h_{1i} and the right panel show the effect of a rise in h_{1i} on loan prices.

Below c_2^r and c_2^d are consumption in period-2 under repayment and default, respectively.

A.1 Risk-free Loan

Borrower solves the following optimization problem if she were to take on a risk-free loan:

$$\max_{h_{1i},d_{1i}} c_{1i}+\phi\ln(h_{1i})+\beta\nu c_{2i}^r(\omega^H)+\phi^H\nu\ln(h_{2i}^r(\omega^H))+\beta(1-\nu)c_{2i}^r(\omega^L)+\phi^L(1-\nu)\ln(h_{2i}^r(\omega^L))$$

subject to

$$d_{1i} \leq \bar{d}_i^L = h_{1i}p_2^L + (1-\xi)y_{1i}\omega^L - \kappa$$

$$d_{1i} \geq 0$$

$$q(y_{1i}, d_{1i}, h_{1i}) = \frac{1}{R^f}$$

where

$$c_{1i} = y_{1i} + q(y_{1i}, d_{1i}, h_{1i})d_{1i} - h_{1i}p_1 + hp_1$$

and for each $j \in \{L, H\}$

$$c_{2i}^r(\omega^j, y_{1i}, h_{1i}, d_{1i}) = y_{1i}\omega^j - d_{1i} + p_2^j h_{1i} - \phi_2^j$$

$$h_{2i}^r(\omega^j, y_{1i}, h_{1i}, d_{1i}) = h$$

$$p_2^j = \frac{\phi^j}{h}$$

Since the borrower repays the debt under each income growth realization, the price she pays is the lenders' discount rate $q(y_{1i}, d_{1i}, h_{1i}) = \frac{1}{R^f}$. The debt constraint ensures that borrowing is low enough to be paid under low income growth realization and non-negative debt constraint ensures that households borrow. First order conditions are as follows:

$$d_{1i}: \qquad \lambda_1 = -\beta + \frac{1}{R^f} + \lambda_2$$

$$h_{1i}: \quad \frac{\phi}{h_{1i}} = \underbrace{p_1}_{\substack{\text{marginal utility}\\ \text{period-1 non-durable consumption}}} - \lambda_1 \underbrace{p_2^L}_{\substack{\text{utility benefit}\\ \text{higher debt limit}}} - \underbrace{\beta(\nu p_2^H + (1-\nu)p_2^L)}_{\substack{\text{marginal utility}\\ \text{period-2 non-durable consumption}}}$$

Since borrowers are assumed to be impatient, i.e. $\beta \leq \frac{1}{R^f}$, first order condition with respect to mortgage debt implies that maximum debt constraint binds in equilibrium.

Borrowers' optimal choices under the risk-free contract is then

$$d_{1i} = \bar{d}_i^L, \lambda_1^{NR} = \frac{1}{R^f} - \beta, \lambda_2^{NR} = 0$$

Using the equilibrium value of the Lagrange multiplier λ_1 and market clearing house price in period-2 under high and low income growth realizations gives housing demand under a risky free loan as:

$$h_1^{NR} = \frac{\phi}{p_1 - \frac{1}{R^f}\frac{\phi^L}{h} - \beta\nu(\frac{\phi^H}{h} - \frac{\phi^L}{h})}$$

Note that each borrower that takes out a risk-free loan consumes the same amount of housing. This results from log-linear preferences assumed in order to simplify the aggregation in the housing market.

$$c_{1i}^{NR} = p_1 h + y_{1i} - \underbrace{(h_1^{NR} p_1 - \frac{1}{R^f}((1-\xi)\omega^L y_{1i} - \kappa + h_1^{NR}\frac{\phi^L}{h})}_{down-payment}$$

Thus using the definition of h_1^{NR} down-payment under a risk-free loan is:

$$down{-}payment = h_1^{NR} p_1 - q_{1i}^{NR} d_{1i}^{NR} = \phi \underbrace{\frac{p_1 - \frac{1}{R^f}\frac{\phi^L}{h}}{p_1 - \frac{1}{R^f}\frac{\phi^L}{h} - \beta\nu(\frac{\phi^H}{h} - \frac{\phi^L}{h})}}_{\geq 1} - \frac{1}{R^f}((1-\xi)\omega^L y_{1i} - \nu)$$

and expected period-2 consumption is:

$$c_{2i}^{NR} = (\nu\omega^H + (\nu+\xi)\omega^L) y_{1i} + h_1^{NR}\nu(\frac{\phi^H}{h} - \frac{\phi^L}{h}) - (\nu\phi^H + (1-\nu)\phi^L)$$

Discounted lifetime utility derived from non-durable consumption is then:

$$C^{NR} = c_{1i}^{NR} + \beta c_{2i}^{NR} = p_1 h - \frac{1}{R^f}\kappa - \phi + y_{1i}\left(1 + \beta(\nu\omega^H + (\nu+\xi)\omega^L) + \frac{(1-\xi)\omega^L}{R^f}\right) - \beta(\nu\phi^H + (1-\nu)\phi^L)$$

Expected life-time utility from a risk-free loan is then

$$U^{NR} = C^{NR} + \phi\ln(h_1^{NR}) + \beta(\nu\phi^H\ln(h) + (1-\nu)\phi^L\ln(h))$$

A.2 Risky Loan

Borrower solves the following optimization problem if she takes a risky loan:

$$\max_{h_{1i},d_{1i}} c_{1i}+\phi\ln(h_{1i})+\beta\nu c_{2i}^{r}(\omega^{H})+\phi^{H}\nu\ln(h_{2i}^{r}(\omega^{H}))+\beta(1-\nu)c_{2i}^{d}(\omega^{L})+\phi^{L}(1-\nu)\ln(h_{2i}^{d}(\omega^{L}))$$

subject to

$$d_{1i}\leq\bar{d}_{i}^{H}=h_{1i}p_{2}^{H}+(1-\xi)y_{1i}\omega^{H}-\kappa$$

$$d_{1i}\geq\bar{d}_{i}^{L}=h_{1i}p_{2}^{L}+(1-\xi)y_{1i}\omega^{L}-\kappa$$

$$q(y_{1i},d_{1i},h_{1i})=\frac{1}{R^{f}}\left\{\nu+(1-\nu)\frac{\theta p_{2}^{L}h_{1i}}{d_{1i}}\right\}$$

where

$$c_{1i}=y_{1i}+q(y_{1i},d_{1i},h_{1i})d_{1i}-h_{1i}p_{1}+hp_{1}$$

$$c_{2i}^{r}(\omega^{H},y_{1i},h_{1i},d_{1i})=y_{1i}\omega^{H}-d_{1i}+p_{2}^{H}h_{1i}-\phi^{H}$$

$$c_{2i}^{d}(\omega^{L},y_{1i},h_{1i},d_{1i})=\xi y_{1i}\omega-\phi^{L}$$

and for each $x\in\{r,d\}$ and $j\in\{L,H\}$

$$h_{2i}^{x}(\omega^{j},y_{1i},h_{1i},d_{1i})=h$$

$$p_{2}^{j}=\frac{\phi^{j}}{h}$$

The first constraint is to ensure that borrower can repay the loan under high income growth realization. The second constraint is imposed so that loan pricing is consistent with borrower choice. That is, if borrower takes an amount less than the low debt level constraint, she can repay it under low income growth realization as well and the correct loan price is then lenders' discount rate. First order conditions are as follows:

$$d_{1i}: \qquad \lambda_1 = \nu\left(-\beta + \frac{1}{R^f}\right) + \lambda_2$$

$$h_{1i}: \quad \frac{\phi}{h_{1i}} = \underbrace{p_1}_{\substack{\text{marginal utility}\\ \text{period-1 non-durable consumption}}} - \underbrace{\frac{1}{R^f}(1-\nu)\theta p_2^L}_{\substack{\text{period-1 utility benefit}\\ \text{higher loan price}}} - \lambda_1 \underbrace{p_2^H}_{\substack{\text{utility benefit}\\ \text{higher upper debt limit}}}$$
$$+\lambda_2 \underbrace{p_2^L}_{\substack{\text{utility benefit}\\ \text{higher lower debt limit}}} - \beta \underbrace{\nu p_2^H}_{\substack{\text{marginal utility}\\ \text{period-2 non-durable consumption}}}$$

Borrower impatience again implies that it is optimal to take on the largest loan that she can repay, i.e. $\lambda_1 = \nu\left(-\beta + \frac{1}{R^f}\right) > 0$. Therefore under a risky contract it is optimal to have

$$d_{1i} = \bar{d}_i^H, \lambda_1^R = \nu\left(-\beta + \frac{1}{R^f}\right), \lambda_2^R = 0$$

$$h_1^R = \frac{\phi}{p_1 - \frac{1}{R^f}(\nu p_2^H + \theta(1-\nu)p_2^L)}$$

$$c_{1i}^R = p_1 h + y_{1i} - \underbrace{(h_1^R p_1 - \frac{1}{R^f}((1-\xi)\nu\omega^H y_{1i} - \kappa\nu + h_1^R(\theta(1-\nu)p_2^L + \nu p_2^H))}_{down-payment}$$

Using the equilibrium value of h_1^R, down-payment under a risky loan is:

$$down - payment = h_1^R p_1 - q_{1i}^R d_{1i}^R = \phi - \frac{1}{R^f}((1-\xi)\nu\omega^H y_{1i} - \kappa\nu)$$

and period-2 consumption is:

$$c_{2i}^R = \xi((1-\nu)\omega^L + \nu\omega^H)y_{1i} - (\nu\phi^H + (1-\nu)\phi^L)$$

Lifetime utility derived from non-durable consumption is

$$C^R = c_{1i}^R + \beta c_{2i}^R = p_1 h - \frac{\nu}{R^f}\kappa - \phi - \beta(\nu\phi^H + (1-\nu)\phi^L) + y_1\left(1 + \beta\xi(\nu\omega^H + (1-\nu)\omega^L) + \frac{(1-\xi)\nu\omega^H}{R^f}\right)$$

Expected life-time utility from a risky loan is then

$$U^R = C^R + \phi \ln(h_1^R) + \beta(\nu\phi^H \ln(h) + (1-\nu)\phi^L \ln(h)) - \beta(1-\nu)\kappa$$

B Partial equilibrium of the mortgage market

Proposition 1 *Let* $\gamma = (1-\xi)\left(\frac{\omega^L - \nu\omega^H}{R^f} + \beta\nu(\omega^H - \omega^L)\right)$*. There exists a unique income cut-off* $\bar{y}$

$$\bar{y} = \frac{1}{\gamma}\left((1-\nu)\left(\frac{1}{R^f} - \beta\right)\kappa - \phi\ln\left(\frac{h_1^{NR}}{h_1^R}\right)\right)$$

such that borrowers with income less than $\bar{y}$ *take risky loans as long as risk-free rate is sufficiently high*

$$\Gamma_{1i}^R = \begin{cases} 1 & \text{if } y_{1i} \leq \bar{y} \\ 0 & \text{if } y_{1i} > \bar{y} \end{cases} \quad \text{as long as} \quad R^f \geq \frac{1}{\beta}\frac{\nu\omega^H - \omega^L}{\omega^H - \omega^L}$$

Proof. It is optimal to take a risky loan if:

$$U^R - U^{NR} = -\phi\ln\left(\frac{h^{NR}}{h^R}\right) + (1-\nu)\left(\frac{1}{R^f} - \beta\right)\kappa - y_1(1-\xi)\left\{\frac{\omega^L - \nu\omega^H}{R^f} + \beta\nu(\omega^H - \omega^L)\right\} \geq 0$$

$$(1-\nu)\left(\frac{1}{R^f} - \beta\right)\kappa - \phi\ln\left(\frac{h^{NR}}{h^R}\right) - y_1\gamma \geq 0$$

Thus, if $\gamma > 0$ borrowers with income less than $\bar{y}$ choose a risky loan. This is satisfied when

$$R^f \geq \frac{1}{\beta}\frac{\nu\omega^H - \omega^L}{\omega^H - \omega^L}$$

■

Lemma 1 *The down-payment of a risky loan is lower than a risk-free loan at all points in the income distribution. A sufficient condition is* $\frac{\nu\omega^H}{\omega^L} \geq 1$*.*

Proof. Under a risk-free loan

$$down - payment^{NR} = \phi \underbrace{\frac{p_1 - \frac{1}{R^f}\frac{\phi^L}{h}}{p_1 - \frac{1}{R^f}\frac{\phi^L}{h} - \beta\nu(\frac{\phi^H}{h} - \frac{\phi^L}{h})}}_{\geq 1} - \frac{1}{R^f}((1-\xi)\omega^L y_{1i} - \nu)$$

Under a risky loan

$$down - payment^R = \phi - \frac{1}{R^f}((1-\xi)\nu\omega^H y_{1i} - \kappa\nu)$$

Therefore, a sufficient condition for low down-payment across the income distribution is $\frac{\nu\omega^H}{\omega^L} \geq 1$. Note that, for low income borrowers, this condition does not need to hold, i.e. when $y_{1i} = 0$ for instance. ■

Lemma 2 *Period-1 housing consumption is higher under a risk-free contract compared to a risky contract, $h_1^{NR} \geq h_1^R$, as long as loan recovery rate is sufficiently low:*

$$\theta \leq \theta^{max} \quad where \quad \theta^{max} = 1 - (1-\beta R^f)\left(\frac{\phi_2^H}{\phi_2^L} - 1\right)\frac{\nu}{1-\nu}.$$

Proof. $h^{NR} - h^R \geq 0$ if $\frac{\phi}{h^{NR}} - \frac{\phi}{h^R} \leq 0$

$$\frac{\phi}{h^{NR}} - \frac{\phi}{h^R} = \nu\left(\frac{1}{R^f} - \beta\right)\frac{\phi^H}{h} - \left(\frac{1}{R^f} - \beta\right)\frac{\phi^L}{h} - (1-\nu)\left(\beta - \frac{\theta}{R^f}\right)\frac{\phi^L}{h}$$

Thus housing consumption under a risk-free contract is higher than that of a risky contract as long as:

$$\theta \leq 1 - (1-\beta R^f)\left(\frac{\phi^H}{\phi^L} - 1\right)\frac{\nu}{1-\nu}$$

■

Lemma 3 *Mean period-2 consumption is higher under a risk-free loan than a risky loan across the income distribution.*

Proof.

$$c_{2i}^R = \xi((1-\nu)\omega^L + \nu\omega^H)y_{1i}$$

$$c_{2i}^{NR} = (\nu\omega^H + (\nu+\xi)\omega^L)y_{1i} + h^{NR}\nu\left(\frac{\phi^H}{h} - \frac{\phi^L}{h}\right)$$

■

C General equilibrium representation

Remark 1 *The general equilibrium of the model in period-1 can be represented in (p_1, S) space as follows:*

The locus of (p_1, S) consistent with housing market clearing is HH:

$$Sh_1^R(p_1) + (1 - S)h_1^{NR}(p_1) = h \quad (HH)$$

The locus of (p_1, S) consistent with mortgage market clearing is MM:

$$S = \Psi(\bar{y}(p_1)) \quad (MM)$$

where $\Psi(\bar{y}(p_1))$ is the share of borrowers with income less than $\bar{y}$, and thus S is the share of risky borrowers.

- *The HH curve is downward sloping in S*
- *The MM curve is upward sloping in S*

Proof.

- The HH curve: Since $h^{NR} > h^R$, then as S increases, total housing demand declines and thus house prices needs to decline for housing market to clear at quantity h.

$$\frac{\partial p_1}{\partial S} < 0$$

- The MM curve:

$$\frac{\partial S}{\partial p_1} = \frac{\partial S}{\partial \ln(h_1^{NR}/h_1^R)} \frac{\partial \ln(h_1^{NR}/h_1^R)}{\partial p_1}$$

First partial derivative is negative as relative increase in housing consumption under a risk free contract discourages taking a risky contract and thus share of risky borrowers decline. Second partial derivative is also negative as price elasticity of housing consumption is higher under a risk-free contract.

$$\frac{\partial \ln(h_1^{NR})}{\partial p_1} - \frac{\partial \ln(h_1^R)}{\partial p_1} = -\frac{h_1^{NR}}{\phi} + \frac{h_1^R}{\phi} < 0$$

as $h_1^{NR} \geq h_1^R$.

■

D General equilibrium effects of a change in income inequality

D.1 Pareto income distribution

Proposition 2 *A mean-preserving increase in income inequality under a Pareto income distribution increases the share of risky borrowers in the economy*

$$\frac{\partial \Psi(\bar{y})}{\partial Gini} > 0 \qquad \textit{as long as} \qquad \Psi(\bar{y}) \leq 1 - exp\left(-\frac{1}{\alpha - 1}\right)^{\alpha}$$

Proof.

$$\frac{\partial \Psi(\bar{y})}{\partial Gini} = \underbrace{\frac{\partial \Psi(\bar{y})}{\partial \alpha}}_{\leq 0} \underbrace{\frac{\partial \alpha}{\partial Gini}}_{<0} > 0$$

First,

$$\frac{\partial \Psi(\bar{y})}{\partial \alpha} = -\frac{1 - \Psi(\bar{y})}{\alpha - 1} \left(\frac{\alpha - 1}{\alpha} \ln(1 - \Psi(\bar{y})) + 1\right)$$

$$\frac{\partial \Psi(\bar{y})}{\partial \alpha} \leq 0 \quad if \quad \Psi(\bar{y}) \leq 1 - exp\left(-\frac{1}{\alpha - 1}\right)^{\alpha}$$

Second, using the definition of Gini coefficient for Pareto distribution

$$\alpha = \frac{1}{2}\left(\frac{1}{Gini} + 1\right)$$

$$\frac{\partial \alpha}{\partial Gini} < 0$$

■

D.2 Log-normal income distribution

Proposition 3 *A mean-preserving increase in income inequality under a log-normal income distribution increases the share of risky borrowers in the economy*

$$\frac{\partial \Psi(\bar{y})}{\partial Gini} > 0 \qquad \textit{as long as} \qquad \bar{y} \leq e^{\sigma^2} median$$

Proof.

$$\frac{\partial \Psi(\bar{y})}{\partial Gini} = \frac{\partial \Psi(\bar{y})}{\partial \sigma} \frac{\partial \sigma}{\partial Gini}$$

$$\frac{\partial Gini}{\partial \sigma} = \frac{e^{-\frac{\sigma^2}{4}}}{\sqrt{\pi}} > 0$$

Let $x = \frac{\ln(\bar{y}) - \ln(\bar{M})}{\sqrt{2}\sigma} + \frac{\sigma}{2\sqrt{2}}$

$$\frac{\partial \Psi(\bar{y})}{\partial \sigma} = \frac{1}{2} erf'(x) \frac{\partial x}{\partial \sigma}$$

$$\frac{\partial x}{\partial \sigma} = -\left(\frac{\ln(\bar{y}) - \ln(\bar{M})}{\sqrt{2}\sigma^2}\right) + \frac{1}{2\sqrt{2}}$$

Since $erf(x)$ is increasing in x, then $\frac{\partial x}{\partial \sigma}$ positive if $\bar{y} \leq \bar{M} e^{\frac{\sigma^2}{2}} = e^{\mu + \sigma^2}$, since median is e^{μ}, then the sufficiency condition can be written as

$$\frac{\partial \Psi(\bar{y})}{\partial \sigma} \geq 0 \quad \text{if} \quad \bar{y} \leq e^{\sigma^2} median$$

■

E General equilibrium effects of a change in real rates

Lemma 4 *A decline in the risk-free rate decreases the down-payment more for a risky loan than a risk-free loan. A sufficient condition is* $\frac{\nu \omega^H}{\omega^L} \geq 1$

Proof.

$$\frac{\partial down - payment^R}{\partial R^f} = \frac{1}{(R^f)^2}((1-\xi)\nu\omega^H y_{1i} - \kappa\nu) > 0$$

$$\frac{\partial down - payment^{NR}}{\partial R^f} = \frac{1}{(R^f)^2}((1-\xi)\omega^L y_{1i} - \nu) > 0$$

Similar to the case in Lemma 1 $\frac{\pi\omega^H}{\omega^L} \geq 1$ is a sufficient condition. ■

Lemma 5 *There exists a loan recovery rate* $\underline{\theta}$ *such that, for any loan recovery rate above* $\underline{\theta}$

1. *The semi-elasticity of housing demand is higher under a risky loan compared to a risk-free loan:*
$$\left|\frac{\partial \ln(h_1^R)}{\partial R^f}\right| \geq \left|\frac{\partial \ln(h_1^{NR})}{\partial R^f}\right|$$
2. *The HH curve flattens following a decline in the risk-free rate.*

Proof.

1. $$\epsilon^{NR} = \frac{\partial \ln(h_1^{NR})}{\partial R^f} = -\frac{\phi^L}{h}\frac{h^{NR}}{\phi(R^f)^2} < 0$$
$$\epsilon^R = \frac{\partial \ln(h_1^R)}{\partial R^f} = -(\theta(1-\nu)\frac{\phi^L}{h} + \nu\frac{\phi^H}{h})\frac{h^R}{\phi(R^f)^2} < 0$$

   Note that as θ increases ϵ^R increases monotonically. Let $\underline{\theta}$ denote the value at which $\epsilon^R(\underline{\theta}) = \epsilon^{NR}$. That is
$$\left(\underline{\theta}(1-\nu) + \nu\frac{\phi^H}{\phi^L}\right)\frac{h^R}{h^{NR}} = 1$$

   It needs to be proven that the set $[\bar{\theta}, \theta^{max}]$ is nonempty. I prove by contradiction. Suppose $\underline{\theta} = \varepsilon + \theta^{max}$ with $\varepsilon > 0$ and thus from Lemma 2 $h^R > h^{NR}$. Then
$$\underline{\theta}(1-\nu) + \nu\frac{\phi^H}{\phi^L} = \frac{h^{NR}}{h^R} < 1$$
   Since $\underline{\theta} = \theta^{max} + \varepsilon$, left hand-side becomes
$$(1-\nu)\varepsilon + 1 + \beta R^f\left(\frac{\phi^H}{\phi^L} - 1\right) > 1$$
   contradiction. Thus $\underline{\theta} < \theta^{max}$.

2. The HH curve flattens if
$$\frac{\partial p^R}{\partial R^f} / \frac{\partial p^{NR}}{\partial R^f} \geq 1$$
   where p^R is market clearing price when $\Psi(\bar{y}) = 1$ and p^{NR} is market clearing price when $\Psi(\bar{y}) = 0$. Optimality conditions imply the following prices:
$$p^R = \frac{\phi}{h} + \frac{1}{R^f}(\theta(1-\nu)\frac{\phi^L}{h} + \nu\frac{\phi^H}{h})$$

$$p^{NR} = \frac{\phi}{h} + \frac{1}{R^f}\frac{\phi^L}{h}$$

Then

$$\frac{\partial p^R}{\partial R^f} = -\frac{1}{(R^f)^2}(\theta(1-\nu)\frac{\phi^L}{h} + \nu\frac{\phi^H}{h}) = \epsilon^R\frac{\phi}{h^R}$$

$$\frac{\partial p^{NR}}{\partial R^f} = -\frac{1}{(R^f)^2}\frac{\phi^L}{h} = \epsilon^{NR}\frac{\phi}{h^{NR}}$$

Thus

$$\frac{\partial p^R}{\partial R^f} / \frac{\partial p^{NR}}{\partial R^f} = \frac{\epsilon^R}{\epsilon^{NR}}\frac{h^{NR}}{h^R} \geq 1$$

Therefore, following a decline in the risk-free rate the HH curve flattens.

■

Proposition 4 *Holding the price of housing constant, a decline in the risk-free rate increases the share of borrowers with a risky loan*

$$\frac{\partial \Psi(\bar{y}(p_1))}{\partial R^f} < 0$$

Proof. Remember from Proposition 1 that the income cut-off for the risky loan is given by:

$$\bar{y} = \frac{1}{\gamma}\left\{(1-\nu)\left(\frac{1}{R^f} - \beta\right)\kappa - \phi\ln\left(\frac{h^{NR}}{h^R}\right)\right\}$$

where

$$\gamma = (1-\xi)\left\{\frac{\omega^L - \nu\omega^H}{R^f} + \beta\nu(\omega^H - \omega^L)\right\}$$

Thus

$$\frac{\partial \gamma}{\partial R^f} = -\frac{\omega^L - \nu\omega^H}{(R^f)^2} > 0$$

Therefore, as R^f increases using the results from Lemma 5 and Lemma 4, $\bar{y}$ declines. Thus, share of risky borrowers decline with the risk-free rate. ■

Online appendix for Income Inequality, Mortgage Debt and House Prices