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Optimal monetary policy mix at the zero lower bound

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Abstract

Long-term asset purchases carried out by central banks increase the consumption volatility of households holding long-term debt. For this reason, monetary authorities should not just aim at stabilising inflation and the output gap but also mitigate the volatility of their balance sheet. In response to negative demand shocks at the zero lower bound (ZLB), the optimal monetary policy consists of a mix of forward guidance and mild adjustments in the balance sheet. The presence of balance-sheet policies reduces the optimal ZLB duration and significantly improves social welfare. Mitigating the effectiveness of forward guidance calls for a more substantial balance-sheet expansion and a shorter ZLB duration. If a central bank only aims to stabilise inflation and the output gap, welfare losses are significantly larger than under the optimal policy and balance-sheet policies only improve welfare if the weight on output-gap stabilisation is relatively large. Last, simple implementable policy rules can achieve welfare outcomes close to those under the optimal policy.

Key words: Optimal monetary policy, unconventional monetary policy, quantitative easing, forward guidance.

JEL classification: E52, E58, E61.

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1 Introduction

In the wake of the 2008 Global Financial Crisis, nominal interest rates in the US and other advanced economies have approached the zero lower bound (ZLB). As a response, central banks have resorted to various forms of unconventional policies. The most prominent of which have been large-scale asset purchases and communications about the future path of the policy rate, commonly known, respectively, as quantitative easing (QE) and forward guidance (FG).

While the literature has analysed these policies separately (see, e.g., Sims et al., 2021, on QE or Bilbiie, 2019, on FG), there is relatively little work on the optimal mix of these two policies. In this paper, we fill this gap by studying optimal policy at the ZLB in the context of a tractable two-agent New Keynesian (TANK) model along the lines of Sims et al. (2021), in which the central bank can carry out both QE and FG. The model features patient and impatient households, short and long-term bonds, and a frictional financial sector that allows QE to impact the real economy. Differently from the original model, we assume that households feature bounded rationality (Gabaix, 2020), which mitigates the excessive power of FG under rational expectations (Forward Guidance Puzzle, Del Negro et al., 2015). In its log-linearised form, the model boils down to a simple extension of the basic three-equation model (Woodford, 2003), which allows accounting for balance-sheet policies. Unlike conventional monetary policy, which only affects aggregate demand, QE, in this framework, shifts both aggregate demand and supply, and its ability to stabilise inflation is, therefore, relatively muted. However, QE is comparatively effective at stabilising the output gap. In this context, we derive the social welfare loss function. In contrast with the canonical three-equation model, where the welfare loss is a function of inflation and output-gap volatility, in our model, this also includes the volatility of the central bank's real long-term bond holdings. In other words, in this model, social welfare implies a preference for low volatility of the central bank's balance sheet. The intuition behind this result is that changes in the central bank's balance sheet increase the volatility of consumption of impatient households, who hold long-term debt.

Following a negative demand shock that drives the policy rate to the ZLB, the optimal monetary policy under commitment consists of a combination of a low-for-long policy (i.e., FG) and balance-sheet policies. Specifically, FG sustains current demand by boosting expectations about inflation and the output gap. A mild increase in the size of the balance sheet (i.e., QE) further eases the initial drop in demand, and a subsequent contraction (i.e., Quantitative Tightening or QT) mitigates the overshoots in prices and real activity caused by FG. The presence of balance-sheet policies reduces the optimal duration of the ZLB required to stabilise inflation and the output gap. Moreover, compared to the case where the central bank only relies on FG (i.e., assuming the size of the central bank's balance sheet is constant), the optimal policy mix significantly improves welfare by further mitigating the rise in inflation and output-gap volatility. These welfare gains outweigh the costs of higher volatility of the central bank's balance sheet. When the central bank is not able to commit (i.e., under discretion), instead, the central bank cannot carry out forward guidance and, therefore, lifts off the short-term rate as soon as the ZLB constraint is not binding. In the absence of balance-sheet policies, the optimal policy under discretion coincides with a strict inflation-targeting rule for the short-term rate and leads to a severe recession at the ZLB in response to a negative demand shock. Also in this case, balance-sheet policies can help mitigate the decline in inflation and the output gap. However, the welfare losses under discretion are substantially larger than under commitment, both with and without balance-sheet policies.

We show that the power of FG significantly affects the optimal monetary policy under commitment. We do so by controlling the degree of cognitive discounting (see, e.g., Nakata et al., 2019). In the absence of balance-sheet policies, a weaker FG calls for a longer ZLB duration to stabilise inflation and the output gap sufficiently. In other words, if FG is relatively weak due to households' myopia, the central bank should maintain the short-term rate at zero longer than under fully rational expectations. In the presence of balance-sheet policies, instead, a weaker FG calls for a stronger balance sheet expansion and a shorter ZLB duration. To put it in another way, as FG becomes less powerful, the central bank partially substitutes FG with balance sheet policies in order to stimulate demand.

To further highlight the trade-off between mitigating inflation, output-gap, and balance-sheet volatility, we consider the case in which the central bank targets a simplified loss function, only accounting for inflation and output-gap volatility, as in the canonical three-equation model. An important reason to consider this case is that, in practice, central banks' mandates entail a limited set of objectives, such as stabilising inflation and (sometimes) a measure of economic slack (Svensson, 2010 and Debortoli et al., 2019). We find that, in the case of a simple ad-hoc loss function, the benefits of balance-sheet policies depend on the relative weight associated with output-gap stabilisation. When this weight is large (i.e., low weight on inflation), QE is beneficial since, as mentioned above, output-gap stabilisation only requires small adjustments in the central bank's balance sheet. Hence, this policy leads to a relatively mild increase in balance-sheet volatility and minor welfare losses. When the relative weight on the output gap is small (i.e., a large weight on inflation), the required adjustment in the central bank's balance sheet is much larger since QE is not as inflationary as conventional monetary policy or FG. As a result, a small weight on the output gap in the central bank's objective function leads the monetary policy authority to substantially increase the size of its balance sheet, causing large welfare losses due to higher balance-sheet volatility. It is also important to highlight that a policy aimed at minimising a simplified loss function performs significantly worse than the optimal policy under commitment.

Finally, we consider the case where the central bank sets the short-term interest rate and the size of its balance sheet according to simple policy rules. If the central bank sets the short-term rate according to an inflation-targeting rule (with zero weight on the output gap), welfare improves as we increase the responsiveness to inflation. In the case of a strict inflation-targeting rule (i.e., the weight associate with inflation goes to infinity) for the short-term rate, we find that the central bank's balance sheet should not respond to inflation and only to the output gap. Following such a policy mix outperforms the optimal policy under discretion in terms of welfare losses. We also consider the case where the central bank sets the short-term rate according to a price-level-targeting rule. Also in this case, welfare improves as we increase the responsiveness to the price level. Under a price-level-targeting rule for the short-term interest rate, the central bank's balance sheet should only respond to the output gap. We find that a policy mix consisting of a strict price-level-targeting rule for the short-term rate and a flexible output-gap-targeting rule for the balance sheet delivers welfare outcomes close to the optimal monetary policy under commitment.

From a policy perspective, our results underscore that expanding the set of tools at the central bank's disposal can create further trade-offs beyond the canonical one between inflation and the output gap. Our model suggests that a mild adjustment in the central bank's balance sheet may be preferable over large abrupt changes.

Related Literature This paper is strictly related to the strand of the macroeconomic literature studying the optimal conduct of monetary policy when nominal short-term rates are at the ZLB. Eggertsson and Woodford (2003) examines the implications of the ZLB on the ability of a central bank to contrast deflation. They show that a credible commitment to the right sort of history-dependent policy can largely mitigate the distortions created by the ZLB. Jung et al. (2005) shows that at the ZLB, the optimal monetary policy response implies policy inertia, i.e., a zero interest rate policy should be continued for a while even after the natural rate returns to a positive level. Adam and Billi (2006, 2007) study optimal monetary policy under commitment and discretion at the ZLB. Agents anticipate the possibility of reaching the lower bound in the future, which amplifies the effects of adverse shocks before the ZLB is actually reached. This result calls for a more aggressive response by the central bank. Under discretionary monetary policy, output losses and deflation are much more sizable than under commitment. Bilbiie (2019) studies how long a central bank should keep interest rates at a low level after a liquidity trap ends. The paper argues that the optimal duration is approximately half the time the economy spent in a liquidity trap. Nakata et al. (2019) shows that in a framework where the stimulating ability of forward guidance is relatively muted, and the economy is in a liquidity trap, the monetary policy authority should commit to keeping the policy rate at zero for a significantly long time.[1]

Our work also relates to the literature on optimal monetary policy in models with heterogeneous agents. In particular, Bilbiie (2008) studies, among other things, optimal monetary policy in the context of a stylised TANK model, in which a share of the agents is hand-to-mouth, i.e., they have limited participation in asset markets. Cúrdia and Woodford (2016) and Nisticò (2016) study optimal monetary policy in models with infrequent participation and borrowers and savers. Challe (2020) analyses optimal monetary policy in a heterogeneous-agent New Keynesian (HANK) model with labour market frictions and idiosyncratic unemployment risk. In such a context, contractionary cost-push or productivity shocks lead to a rise in precautionary savings and a fall in inflation, calling for an accommodative monetary policy. Acharya et al. (2020) studies optimal monetary policy in a HANK framework, where the central bank's objective function accounts for consumption inequality, besides stabilising output and inflation. When income risk is countercyclical, they find that policy curtails the fall in output in recessions to alleviate the increase in inequality. Bilbiie and Ragot (2021) shows that price stability is not optimal when households self-insure against idiosyncratic risk using scarcely available liquid assets.

Our paper builds on the literature analysing QE within DSGE models. Gertler and Karadi (2011), Gertler and Karadi (2013), and Carlstrom et al. (2017) among others incorporated QE into medium-scale DSGE models. Cui and Sterk (2021) analyse the impact of QE in a New Keynesian model with heterogeneous agents and find that QE is highly stimulative and successfully mitigated the drop in demand during the Great Recession. However, their paper suggests that QE could, as a byproduct, significantly increase inequality and thereby reduce welfare. Lee (2021) develops a HANK model to study how QE affects household welfare across the wealth distribution. Sims and Wu (2021) builds on Gertler and Karadi (2013)'s modelling of the financial sector to analyse the impact and interaction of the main forms of unconventional monetary policy (QE, FG, and negative interest rates). Sims et al. (2021) develops a tractable four-equation New Keynesian model that accounts for balance-sheet policies, whereas Sims and Wu (2020a) deploys this

[1] A non-exhaustive list of papers dealing with monetary policy at the ZLB are Nakov (2008), Christiano et al. (2011), Nakata (2017), Nakata and Schmidt (2019), Masolo and Winant (2019), and Bonciani and Oh (2021).

framework to study the degree of substitutability between conventional monetary policy and QE. Sims and Wu (2020b) shows that in times where the production sector is facing significant cash-flow shortages, such as the COVID-19 crisis, QE should be aimed at lending to non-financial corporations rather than banks.

Finally, a closely related paper to ours is Harrison (2017), which studies optimal QE in a model with portfolio adjustment costs. The paper focuses on optimal policy under discretion and models QE such that it is a perfect substitute for conventional monetary policy. In our paper, instead, we study the optimal monetary policy both under commitment and under discretion. QE affects both the demand and the supply-side of the economy and, as a result, is an imperfect substitute for conventional monetary policy. By studying optimal monetary policy under commitment at the ZLB, we analyse the interactions between FG and QE and show the implications of mitigating the power of FG. Furthermore, we study the cases where the central bank targets simplified loss functions or follows simple policy rules.

The remainder of the paper is structured as follows. In Section 2, we describe the basic model. Section 3 studies the welfare implications of optimal monetary policy under commitment and under discretion. In Section 4, we consider the case where the central bank targets a simpler (ad-hoc) loss function rather than the welfare-based loss function. Section 5 studies the case where the central bank sets the short-term interest rate and the size of its balance sheet according to simple policy rules. Finally, Section 6 presents some concluding remarks.

2 Model

Our model is based on Sims et al. (2021), which extends the basic three-equation framework to allow for QE. In its nonlinear form, the model includes two types of agents, patient and impatient, and financial intermediaries (modelled along the lines of Gertler and Karadi, 2011) subject to a risk-weighted leverage constraint. It features short and long-term bonds, which, combined with the credit frictions, allows QE to affect real activity. Besides setting the short-term interest rate, the central bank can purchase long-term bonds by expanding its balance sheet. The increase in the price of long-term bonds, as a consequence of the unconventional monetary policy, relaxes the financial intermediary's leverage constraint, easing the supply of credit. Therefore, in such a framework, QE is equivalent to an increase in credit supply. By affecting both aggregate demand and supply, QE is less inflationary and only an imperfect substitute of conventional monetary policy (and FG), which, instead, just affects aggregate demand.

For the purpose of our analysis, we deviate from the original model in two ways. First, to match empirically plausible inflation dynamics at the ZLB, we assume industry-specific labour markets as in Woodford (2003), rather than aggregate labour markets. As highlighted in Woodford (2003), using industry-specific labour markets instead of economy-wide factor markets has important implications for real rigidities. Secondly, we assume that households feature boundedly-rational expectations to alleviate the forward guidance puzzle (Del Negro et al., 2015) as in Gabaix (2020).[2] For the sake of conciseness, we leave the derivations and the details of the model to Appendixes A and B.

The first two equations of the linear version of the model are a discounted IS curve and a New Keynesian Phillips curve (NKPC), augmented to account for changes in the central bank's balance sheet:

$$x_t = (1-\alpha)\, E_t x_{t+1} - \frac{1-z}{\sigma}\left(r_t^s - E_t \pi_{t+1} - r_t^n\right) + z\bar{b}_{cb}\left(qe_t - (1-\alpha)\, E_t qe_{t+1}\right), \tag{1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \gamma\left(\chi + \frac{\sigma}{1-z}\right) x_t - \frac{\gamma\sigma z \bar{b}_{cb}}{1-z} qe_t. \tag{2}$$

Lowercase variables denote log deviations around the deterministic steady state. π_t is inflation and $x_t \equiv y_t - y_t^f$ denotes the output gap, where y_t^f is the level of output that arises in the flexible-price version of the model absent balance-sheet policies. r_t^s is the nominal policy rate, whereas qe_t denotes the real value of the central bank's long-term bond holdings. r_t^n captures exogenous fluctuations in the natural real rate of interest, which can also be interpreted as a demand shock. This shock is commonly used in the literature to achieve a binding ZLB constraint.

The parameter $\alpha \in [0,1]$ captures the degree of cognitive discounting under the assumption of bounded rationality of households. In particular, when $\alpha = 0$, households feature rational expectations. The parameter σ represents the inverse elasticity of intertemporal substitution, β is the discount factor of the patient households, and χ is the inverse labour supply elasticity. γ is a convolution of deep parameters defined as $\gamma \equiv \frac{(1-\phi)(1-\phi\beta)}{\phi(1+\chi\varepsilon)}$, where ϕ is the price adjustment probability (Calvo, 1983) and ε is the elasticity of the intermediate good's demand. The parameter z is the share of impatient households, and $\bar{b}_{cb}$ is the steady-state share of long-term bonds that the central bank holds.

[2] Other important contributions on solving the forward guidance puzzle are McKay et al. (2016, 2017), Haberis et al. (2019), Woodford (2019), Cole (2020), and McKay and Wieland (2021).

3 Optimal Monetary Policy

This section presents the welfare loss function implied by the model and studies two types of possible optimal monetary policy conducts. Specifically, we define the optimal monetary policies under discretion and commitment. Then, we discuss welfare implications and dynamic responses to a negative demand shock.

3.1 Welfare-Theoretic Loss Function

To understand the relevant policy trade-offs, we derive a second-order approximation to the aggregate welfare loss function (see Appendix C), similarly as in Woodford (2003). We assume that in steady state a subsidy corrects the distortion stemming from monopolistic competition and that trend inflation is equal to zero. Deriving the aggregate welfare loss function entails calculating the welfare of the two households and weighting them by their shares in the population. The resulting welfare function writes as:

$$\mathbb{W} = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \mathbb{L}_t + t.i.p. + O\left(||\xi||^3\right), \tag{3}$$

where $t.i.p.$ stands for the terms independent of monetary policy, and $O\left(||\xi||^3\right)$ denotes all relevant terms that are of third or higher order. We express welfare losses in terms of the equivalent permanent consumption decline, i.e., as a fraction of steady-state consumption. The period loss function $\mathbb{L}_t$ is equal to:

$$\mathbb{L}_t = \underbrace{\frac{\varepsilon}{\gamma} {\pi_t}^2 + \left(\chi + \frac{\sigma}{1-z}\right) {x_t}^2}_{\text{Standard}} + \underbrace{\frac{\sigma z \bar{b}_{cb}^2}{1-z} q{e_t}^2}_{\text{Balance Sheet}}. \tag{4}$$

The first two terms denote the welfare loss from higher inflation and output-gap volatility. These are standard components in the loss function obtained in the representative agent New Keynesian (RANK) model. The last term is the volatility of the central bank's balance sheet size. In other words, this model's welfare loss function also implies a social preference for small deviations in the central bank's balance sheet from its steady state.

The loss function above can also be written as:[3]

$$\mathbb{L}_t = \underbrace{\frac{\varepsilon}{\gamma} {\pi_t}^2 + \left(\chi + \frac{\sigma}{1-z}\right) {x_t}^2}_{\text{Standard}} + \underbrace{\frac{\sigma z}{1-z} {c_{b,t}}^2}_{\text{Inequality}}. \tag{5}$$

[3] This result stems from the "full bailout" assumption, i.e., each period transfers from the patient to the impatient households payoff their outstanding debt obligations. Because of this assumption, the real value of long-term bonds equals the impatient households' consumption $C_{b,t} = \frac{Q_t}{P_t} B_t = QE_t + \frac{Q_t}{P_t} B_{FI,t} = QE_t + t.i.p.$ The long-term bonds held by the financial intermediaries $B_{FI,t}$ are assumed to be exogenous and constant. They are therefore not affected by monetary policy interventions.

The last term represents the welfare loss caused by higher volatility of consumption for the impatient household. This term stems from the TANK structure of the model and is usually interpreted as the welfare loss due to inequality (see, e.g., Bilbiie and Ragot, 2021).

3.2 Optimal Monetary Policy under Discretion

Under discretion, then central bank seeks to minimise the period-t social welfare loss:

$$\min_{x_t,\pi_t,r_t^s,qe_t} \frac{1}{2}\mathbb{L}_t, \tag{6}$$

subject to Equations (1), (2), and the ZLB constraint on the policy rate:

$$r_t^s \geq -\frac{R^s - 1}{R^s}. \tag{7}$$

The instantaneous welfare loss function $\mathbb{L}_t$ is defined in Equation (4), and is a function of inflation volatility ${\pi_t}^2$, output-gap volatility ${x_t}^2$, and long-term bond holdings volatility ${qe_t}^2$. In other words, in addition to stabilising inflation and the output gap, the central bank also seeks to avoid excessively large changes in the size of its balance sheet. Appendix D provides further details on the optimisation problem and the associated first-order conditions.

3.3 Optimal Monetary Policy under Commitment

Under commitment, the central bank seeks to maximise welfare by minimising the expected discounted lifetime social welfare loss:

$$\min_{\{x_t,\pi_t,r_t^s,qe_t\}_{t=0}^{\infty}} \frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\mathbb{L}_t, \tag{8}$$

subject to Equations (1), (2), and (7). Further details on the optimisation problem and the associated first-order conditions are provided in Appendix E. Unlike the optimal policy under discretion, commitment allows the central bank to carry out forward guidance, in addition to balance-sheet policies.

3.4 Calibration and Solution Method

For the numerical exercises, we parameterise the model using standard values in the literature, as listed in Table 1. The cognitive discounting parameter α is set to 0.2, in line with Gabaix (2020). Some parameters specific to the four-equation model are taken from Sims et al. (2021). In particular, we set the share of impatient households z to 0.33. The steady-state share of central bank's long-term bond holdings, $\bar{b}_{cb}$, is set

Table 1: Baseline Quarterly Calibration

Parameter	Description	Value
α	Cognitive discounting parameter	0.20
β	Discount factor of patient households	0.997
σ	Inverse elasticity of intertemporal substitution	1.00
χ	Inverse labour supply elasticity	1.00
z	Share of impatient households	0.33
$\bar{b}_{cb}$	Steady-state share of central bank's long-term bond holdings	0.3
ε	CES parameter	11.00
ϕ	Probability of keeping price unchanged	0.75
ρ_n	Natural rate shock persistence	0.83
σ_n	Natural rate shock volatility	0.0501

to 0.3. The shock's persistence and size are calibrated to $\rho_n = 0.83$ and $\sigma_n = 0.0501$. Under the optimal discretionary policy in the absence of QE, this calibration implies that the shock induces a 10 per-cent fall in the output gap, a 3 percentage-point decline in inflation, and the ZLB constraint binds for 16 quarters. We solve the model with a perfect foresight algorithm using the Levenberg-Marquardt mixed complementarity problem solver (Adjemian et al., 2011).

3.5 Numerical Results under Optimal Monetary Policies

3.5.1 Welfare

In Table 2, we present the welfare implications of the optimal monetary policies, under discretion (OMP-D) and commitment (OMP-C), following an exogenous decline in the natural rate. Each column represents a different policy. For each type of policy, we consider both the cases with and without balance-sheet policies (denoted as QE for conciseness). Each row of Table 2 presents the change in aggregate welfare and its subcomponents, i.e., changes in welfare due to variations in inflation, the output gap, and the balance sheet.

The OMP-D policy without QE leads to the largest welfare losses (in consumption-equivalent terms). As conventional monetary policy is unavailable due to the binding ZLB, the central bank cannot counteract the adverse effects of the negative demand shock. The welfare losses stem from a large decline in inflation (and increase in its volatility) and the output gap. Allowing for balance-sheet policies significantly improves welfare by attenuating both inflation and output-gap volatility. Under OMP-C, both with or without balance sheet policies, welfare losses are significantly smaller than under OMP-D. The presence of QE substantially mitigates the welfare losses from inflation and the output gap. The improved stabilisation of macroeconomic

Table 2: Evaluation of the Optimal Policy Mix

	OMP-D		OMP-C	
Welfare	w/ QE	w/o QE	w/ QE	w/o QE
Aggregate	−9.89%	−15.02%	−4.90%	−7.80%
Inflation	−7.69%	−11.63%	−2.60%	−5.09%
Output Gap	−0.99%	−3.40%	−0.71%	−2.71%
Balance Sheet	−1.21%	0%	−1.59%	0%

Note: We evaluate the welfare implications of the optimal policy under discretion (OMP-D) and commitment (OMP-C) with (w/) and without (w/o) balance sheet policies.

conditions comes at the expense of higher balance-sheet volatility, which instead negatively affects welfare.

3.5.2 Impulse Response Functions under Discretion

As a benchmark, we first present the dynamic responses to an exogenous decline in the natural rate when the central banks follows the OMP-D, both with and without QE. Figure 1 displays the responses of the main variables in our model. The negative demand shock causes a significant decline in the output gap and inflation. The fall in inflation leads the central bank to cut its policy rate, which hits the ZLB constraint and stays there for 16 quarters. In the absence of balance-sheet policies (dashed red line), the central bank cannot respond to the fall in inflation due to the binding ZLB, which causes inflation expectations to fall and the real rate to increase, worsening the decline in demand. When we allow for balance-sheet policies (solid blue line), the central bank can respond to the declining demand by carrying out QE. Since QE affects both the demand (IS curve) and the supply-side (NKPC) of the economy, it is relatively more powerful at sustaining the output gap than inflation. As a result, the policy substantially mitigates the drop in the output gap and (to a lesser extent) eases the fall in inflation.

Finally, it bears noting that, under discretion, the duration of the ZLB is independent of the presence of balance-sheet policies, as the central bank finds it optimal to raise the policy rate as soon as the ZLB constraint is not binding anymore.

3.5.3 Impulse Response Functions under Commitment

Figure 2 displays the dynamic responses of key model variables to a negative demand shock under the OMP-C. In the absence of balance-sheet policies (dashed red line), the central bank can only carry out FG, i.e., commit to keeping the nominal policy rate lower for longer than implied by current macroeconomic conditions. Compared to the OMP-D, the policy rate is kept at zero for further 13 quarters. FG has the

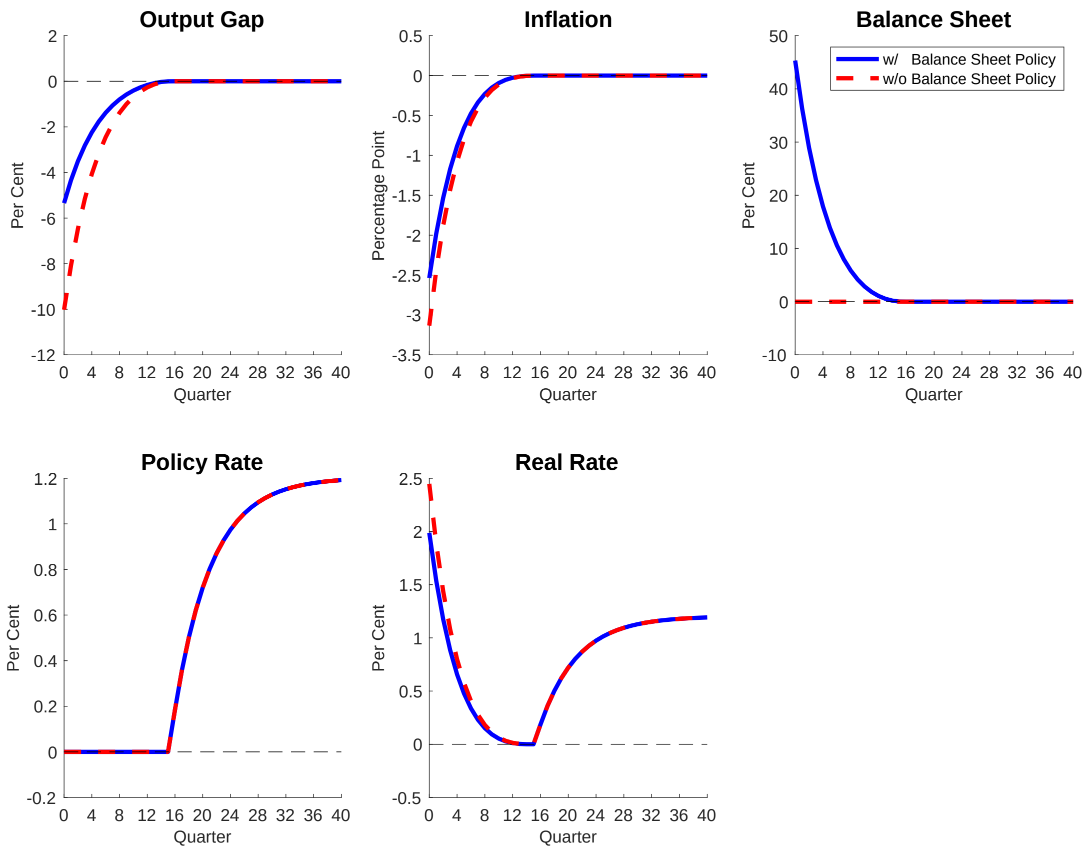

Figure 1: Optimal Monetary Policy under Discretion

Note: The figure displays the model responses to a negative natural rate shock when the central bank conducts the optimal policy under discretion discretion. The output gap and balance sheet are expressed in per-cent deviation from their steady-state values. Inflation is expressed in annualised percentage-points deviation from steady state. The policy rate and the real rate are annualised.

effect of boosting inflation expectations, which lowers the real rate and eases the drop in the output gap. In particular, the fall in the output gap is about one percentage point smaller than under discretion. The initial drop in inflation is also about one percentage point more muted.

When the monetary policy authority can carry out balance-sheet policies (solid blue line), additionally to FG, it can partially substitute FG with a balance-sheet expansion. As a result, it responds to the fall in inflation and output by keeping the nominal rate at zero for 27 quarters, 3 quarters less than the case without QE. The central bank expands its balance sheet by about 40 per cent in response to the shock. As inflation and the output gap overshoot due to FG, the central bank reduces its long-term bond holdings (QT) in subsequent periods. The central bank's balance-sheet policy mitigates the drop in output by about four percentage points compared to the case without QE. Inflation also declines less on impact by about 0.5

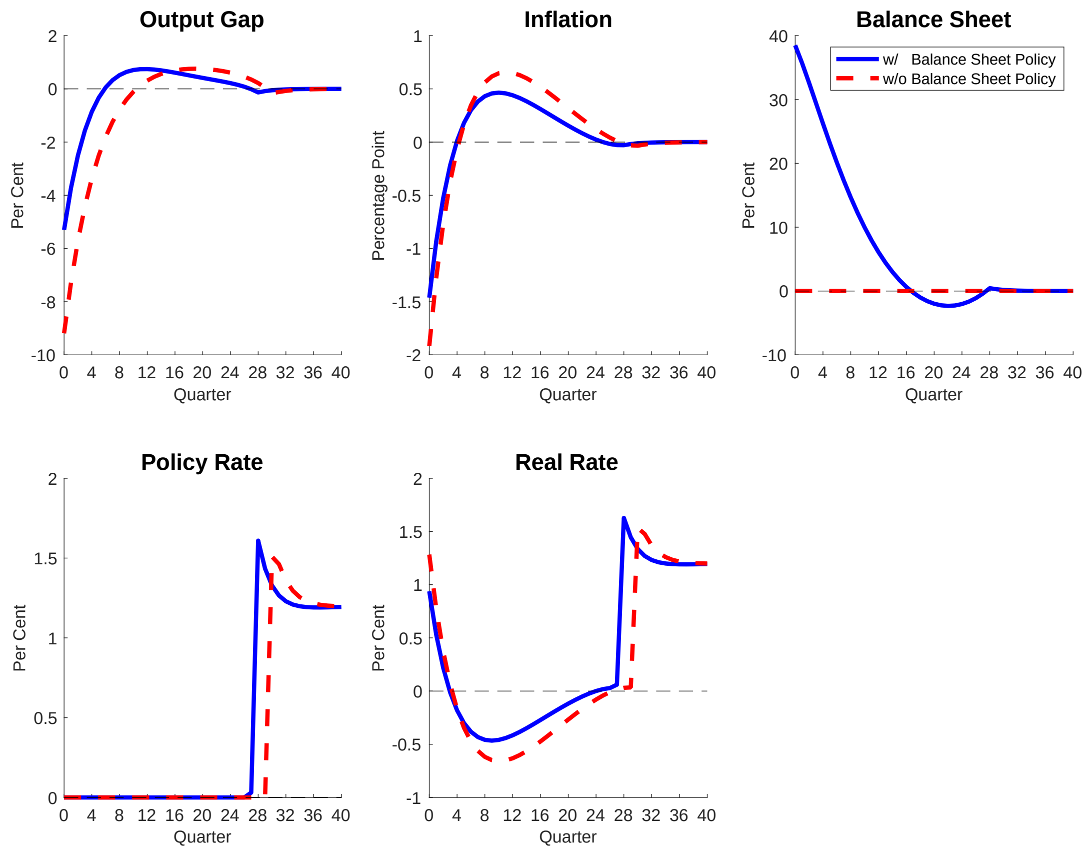

Figure 2: Optimal Monetary Policy under Commitment

Note: The figure displays the model responses to a negative natural rate shock under the optimal monetary policy under commitment. The shock is calibrated to cause a 10 per-cent fall in the output gap and a 3 percentage-point decline in inflation when the central bank conducts the optimal policy under discretion absent QE. The output gap and balance sheet are expressed in per-cent deviation from their steady-state values. Inflation is expressed in annualised percentage-points deviation from steady state. The policy rate and the real rate are annualised.

percentage points and overshoots less.

3.6 The Power of Forward Guidance

In order to assess how the power of forward guidance affects the optimal monetary policy mix under commitment, we analyse how the dynamic responses vary for different values of the cognitive discounting parameter α. Figures 3 and 4 display the results without and with balance-sheet policies respectively. In particular, as α increases, the greater the departure from rational expectations ($\alpha = 0$) and the weaker is the forward guidance puzzle. Conditional on the value of α, we adjust the persistence ρ_n and size σ_n of the natural rate shock such that the output gap and inflation respectively fall by 10 per cent and 3 percentage

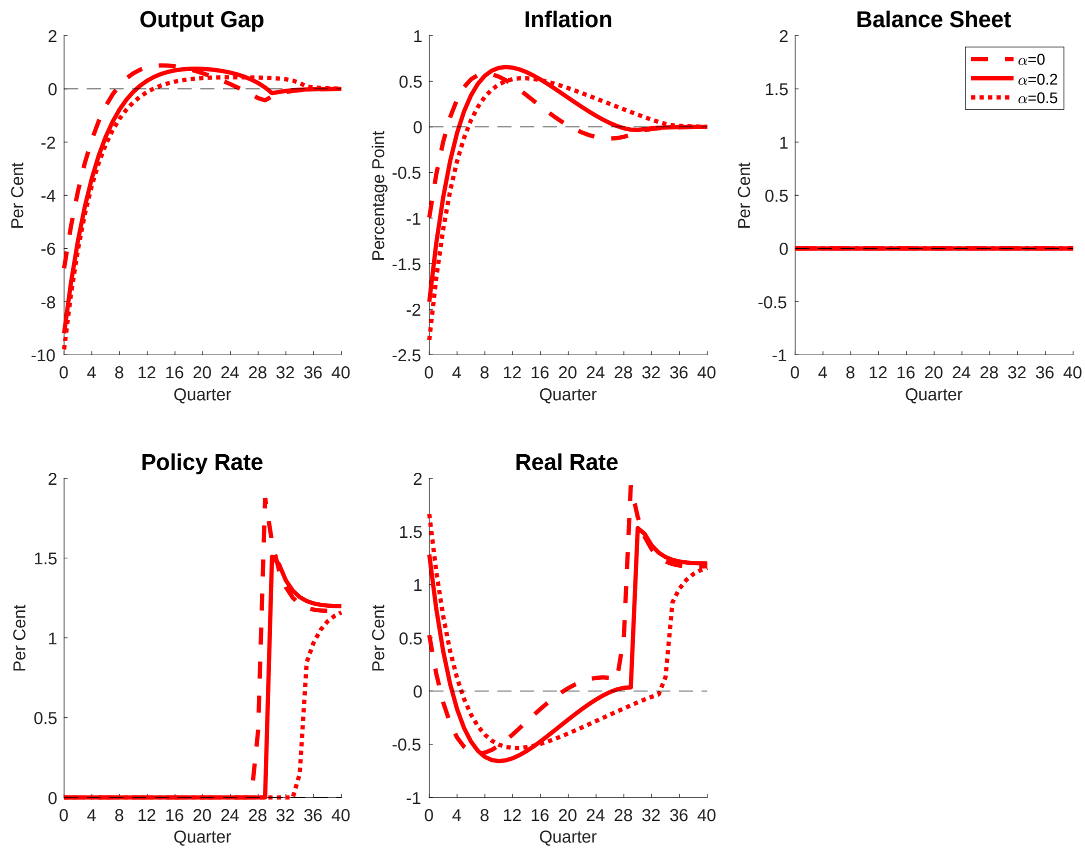

Figure 3: Adjusting the Power of Forward Guidance in the Absence of Balance-Sheet Policies

Note: The figure displays the model responses to a negative natural rate shock under the optimal monetary policy under commitment. The shock is calibrated to cause a 10 per-cent fall in the output gap and a 3 percentage-point decline in inflation when the central bank conducts the optimal policy under discretion absent QE. The output gap and balance sheet are expressed in per-cent deviation from their steady-state values. Inflation is expressed in annualised percentage-points deviation from steady state. The policy rate and the real rate are annualised.

points and the ZLB constraint binds for 16 quarters when the central bank follows the OMP-D without QE.[4]

When the central bank does not deploy balance-sheet policies (Figure 3), the duration of the ZLB is longer, the weaker forward guidance is (i.e., for larger α). Intuitively, this is because more forward guidance is required to ease the fall in the output gap and inflation. This result is in line with Nakata et al. (2019).

By contrast, in the presence of balance-sheet policies (Figure 4), we find that a weaker FG implies a shorter ZLB duration (i.e., less FG) and a stronger expansion in the central bank's balance sheet. In other words, when forward guidance is relatively weak, balance-sheet policies become relatively more effective at stabilising

[4]The approach of keeping the severity of the crisis constant as one varies the model's parameter values is adopted by Boneva et al. (2016), Hills and Nakata (2018), and Nakata et al. (2019).

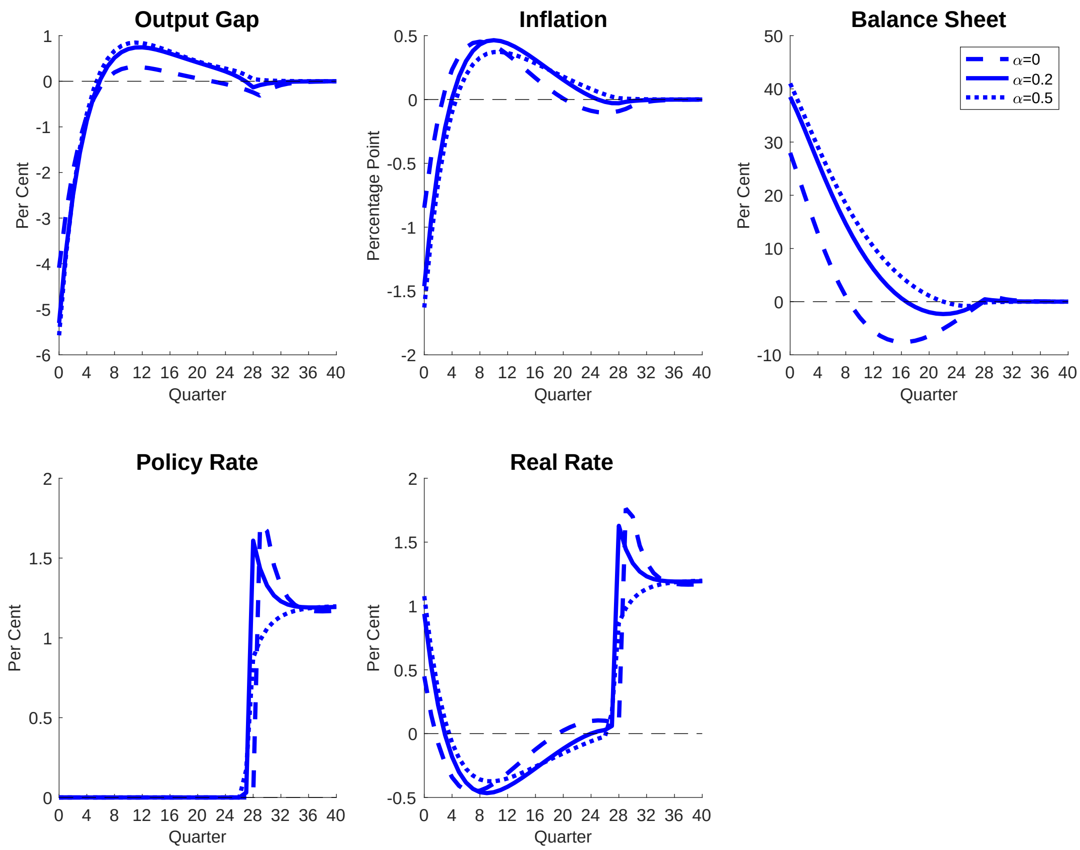

Figure 4: Adjusting the Power of Forward Guidance in the Presence of Balance-Sheet Policies

Note: The figure displays the model responses to a negative natural rate shock under the optimal monetary policy under commitment. The shock is calibrated to cause a 10 per-cent fall in the output gap and a 3 percentage-point decline in inflation when the central bank conducts the optimal policy under discretion absent QE. The output gap and balance sheet are expressed in per-cent deviation from their steady-state values. Inflation is expressed in annualised percentage-points deviation from steady state. The policy rate and the real rate are annualised.

macroeconomic conditions. Consequently, the central bank finds it optimal to carry out more QE and lift off earlier the policy rate.

4 Simple Mandates

4.1 Simplified Objective Function

In this section, we analyse the consequences of the central bank following a simple mandate, aiming to minimise a weighted average of inflation and output-gap volatility. In other words, we study the implications of omitting the balance-sheet volatility from the policy objective function. The reason for this exercise is that, in line with the RANK literature, central banks around the world have mostly focused on inflation and output-gap stabilisation. Under the simple mandate, the central bank has the following objective function:

$$\min_{\{x_t,\pi_t,r_t^s,qe_t\}_{t=0}^{\infty}} \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \mathbb{L}_t^{sm}, \tag{9}$$

$$\mathbb{L}_t^{sm} = {\pi_t}^2 + \vartheta {x_t}^2, \tag{10}$$

subject to Equations (1), (2), and (7). ϑ is the relatively weight the monetary policy authority puts on output-gap stabilisation. Equation (10) is the relevant objective function for a welfare-maximising central bank in the RANK literature. However, as discussed above, the simple-mandate does not not coincide with the optimal policy objective in the four-equation model, defined above in Equation (4). It is important to note that, although the objective of the central bank is Equation (10), Equation (4) is still the relevant welfare measure we use to assess the policy implications. Further details on the optimisation problem and the associated first-order conditions are provided in Appendix F.

4.2 Numerical Results under Simple Mandates

4.2.1 Welfare

We study the welfare implications of changing ϑ in the simple loss function (10) in response to an exogenous decline in the natural rate. Two particular cases of interest are the simple mandate with $\vartheta = \frac{\gamma}{\varepsilon}\left(\chi + \frac{\sigma}{1-z}\right) \approx 0.007$, in line with the OMP, and the simple mandate with $\vartheta = 1/16 = 0.0625$, i.e., with equal weight on annualised inflation and the output gap. We label the two policies SM-O (O for optimal weight) and SM-D (D for dual mandate).

Figure 5 shows how aggregate welfare and its subcomponents varies for different values of ϑ. Dashed red (solid blue) lines represent the case in the absence (presence) of balance-sheet policies.[5] The red (blue) circle represents the policy under SM-O (SM-D). In the absence of balance-sheet policies, a larger ϑ (i.e., smaller weight on inflation) worsens aggregate welfare. For larger values of ϑ, the central bank increasingly focuses on stabilising the output gap and allows more inflation volatility. This results in a stronger decline in the inflation component of welfare and a smaller decline in the output-gap component. However, the fall in the inflation component of welfare significantly outweighs the improvement in the output-gap component.

In the presence of balance-sheet policies, welfare increases with ϑ. As mentioned above, QE is less effective

[5] It is important to note that the central bank's objective function differs from the social welfare function. The latter, in fact, includes QE volatility.

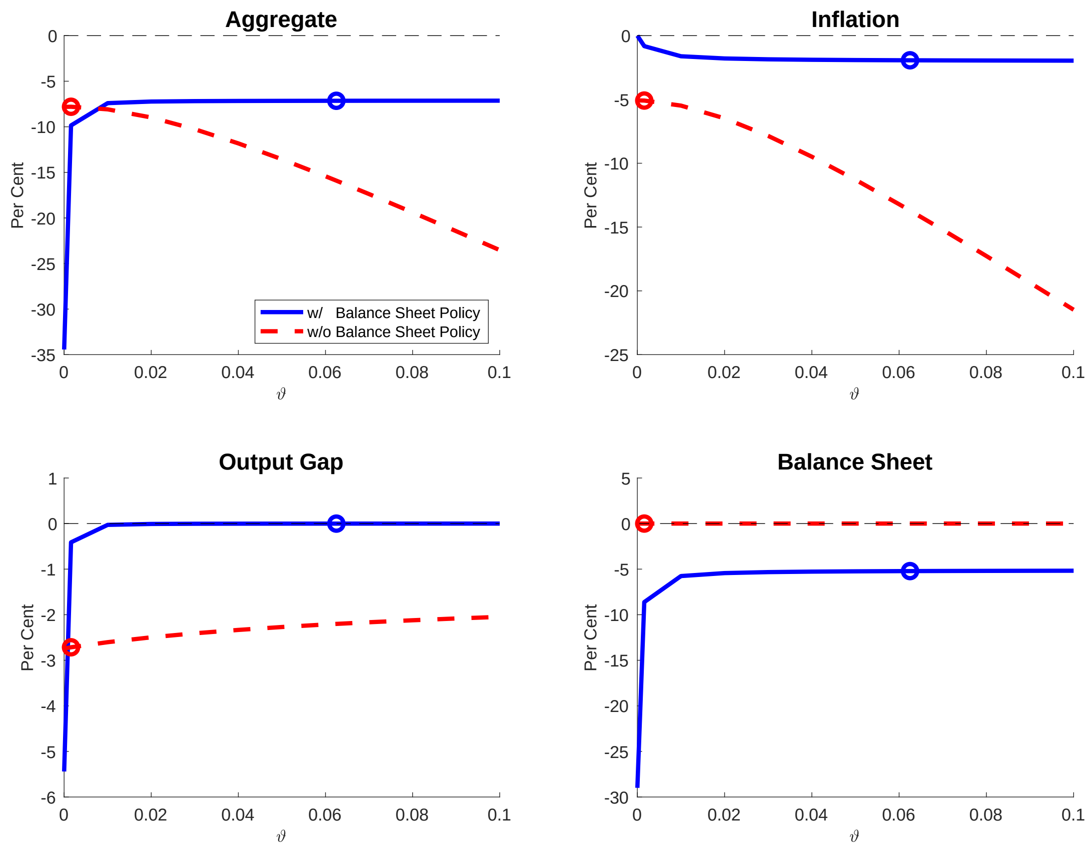

Figure 5: Welfare under a Simple Mandate

Note: The figure displays the aggregate welfare and its subcomponents for different weight on the output gap in the central bank's objective function. We assume that the central bank follows a simple mandate, aiming to stabilise only inflation and the output gap. The red dot represents $\vartheta = \frac{\gamma}{\varepsilon}\left(\chi + \frac{\sigma}{1-z}\right)$, as under the OMP. The blue dot represents $\vartheta = 1/16$, i.e., when the central bank places equal weight on the output gap and annual inflation.

than conventional policy (and FG) at stabilising inflation and more effective at stabilising the output gap. A small ϑ (i.e., a large weight on inflation) requires a significant balance-sheet expansion, which increases QE volatility and, therefore, worsens the QE component of welfare. Moreover, if ϑ is very small, the strong rise in QE boosts the output gap, causing a rise in its volatility (negatively affecting welfare). For larger values of ϑ, the central bank needs to expand its balance sheet significantly less. At the cost of slightly higher inflation volatility, the larger ϑ implies a significantly smaller increase in the volatility of the output gap and QE.

Last, Table 3 reports the welfare implications of the SM-O or SM-D mandates. Under SM-O, ϑ is relatively small, and balance-sheet policies cause larger welfare losses due to higher QE volatility. By contrast, under SM-D, ϑ is larger than under SM-O, and balance-sheet policies improve aggregate welfare, as the gains from

Table 3: Evaluation of the Simple-Mandate Policy Mix

	SM-O		SM-D	
Welfare	w/ QE	w/o QE	w/ QE	w/o QE
Aggregate	−9.85%	−7.80%	−7.15%	−15.91%
Inflation	−0.82%	−5.09%	−1.93%	−13.71%
Output Gap	−0.41%	−2.71%	0%	−2.20%
Balance Sheet	−8.62%	−0%	−5.21%	0%

Note: We evaluate the welfare implications of the simple-mandate policies with optimal (SM-O) and equal weight (SM-D) on annual inflation and the output gap. We consider both the cases with (w/) and without (w/o) balance sheet policies.

stabilising the output gap and inflation outweigh the welfare costs from higher QE volatility. It bears noting that both mandates perform significantly worse than the OMP-C with balance-sheet policies.

4.2.2 Impulse Response Functions under Optimal Weighting

Figure 6 displays the dynamic responses of key model variables to a negative demand shock under SM-O. In the absence of balance-sheet policies (dashed red line), the responses are the same as those in Figure 2. When the monetary policy authority can carry out balance-sheet policies (solid blue line), additionally to FG, it aggressively increases its long-term bond holdings. This is because the central bank puts a large weight on inflation stabilisation. This is reflected in a milder drop in inflation by about one percentage point compared to the scenario without balance-sheet policies. Furthermore, the strong balance-sheet expansion has the effect of fully offsetting the negative impact on the output gap, which increases on impact.

4.2.3 Impulse Response Functions under Equal Weighting

Figure 7 displays the dynamic responses of key model variables to a negative demand shock under SM-D. In this case, the central bank puts a larger weight on output-gap stabilisation than the SM-O. In the absence of balance-sheet policies (dashed red line), the policy rate is kept at zero for 23 quarters, 7 quarters longer than under the OMP-D policy without QE and 7 quarters less than under SM-O without QE. At the end of the ZLB, the central bank adjusts the policy rate gradually. The FG policy and the gradual lift-off of the policy rate boost inflation and inflation expectations. As a consequence, the real rate declines, which mitigates the drop in the output gap. Under this policy, the output gap declines by 8 per cent on impact, one percentage point less than under SM-O without QE. When we allow for QE (solid blue line), the central bank aggressively expands its balance sheet by about 90 per cent (10 percentage points less than under SM-O). This policy slightly increases the output gap, while inflation declines by about 1.2 percentage points. It is important to notice that, under this policy, the central bank is more concerned about stabilis-

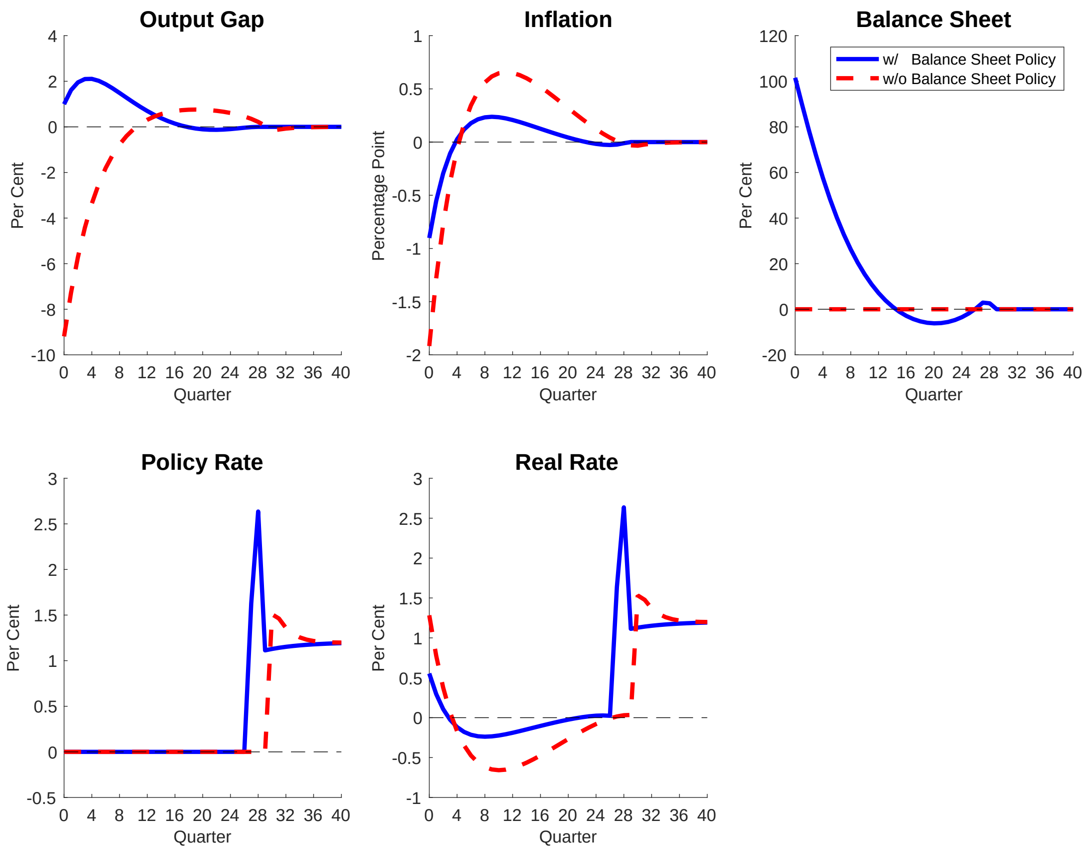

Figure 6: Simple Mandate with Optimal Weight on Annual Inflation and Output Gap

Note: The figure displays the model responses to a negative natural rate shock under the SM-Optimal policy, i.e., $\vartheta = \frac{\gamma}{\varepsilon}\left(\chi + \frac{\sigma}{1-z}\right)$. The shock is calibrated to cause a 10 per-cent fall in the output gap and a 3 percentage-point decline in inflation when the central bank conducts the optimal policy under discretion absent QE. The output gap and balance sheet are expressed in per-cent deviation from their steady-state values. Inflation is expressed in annualised percentage-points deviation from steady state. The policy rate and the real rate are annualised.

ing the output gap than under SM-O. To this end, after the ZLB, the central bank raises the interest rate significantly more (nearly five percentage points) compared to the SM-O case (2.7 percentage-point increase).

Compared to the OMP-C discussed in the previous section, both SM-O and SM-D imply a significantly larger balance-sheet expansion, as the central bank does not internalise the welfare costs associated with higher QE volatility. Unlike the OMP-C case, the output gap increases on impact, and the fall in inflation is more muted.

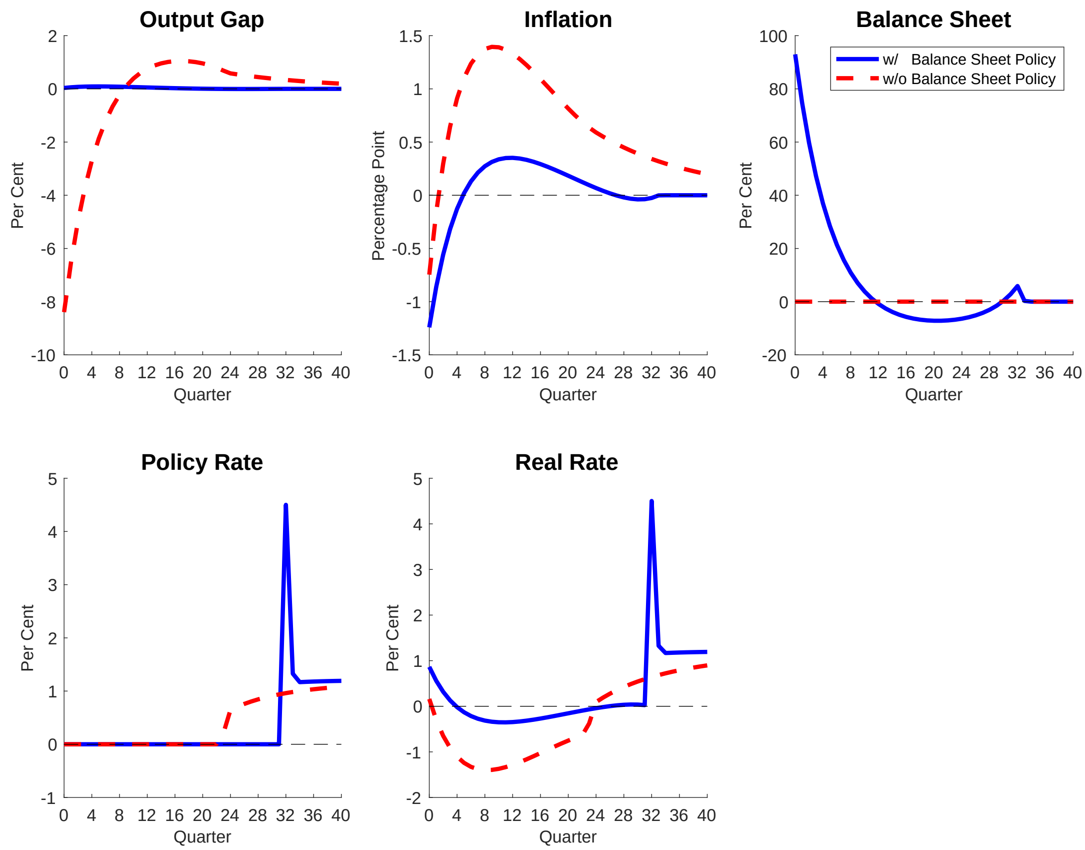

Figure 7: Simple Mandate with Equal Weight on Annual Inflation and Output Gap

Note: The figure displays the model responses to a negative natural rate shock under the Dual Mandate, i.e., $\vartheta = 1/16$. The shock is calibrated to cause a 10 per-cent fall in the output gap and a 3 percentage-point decline in inflation when the central bank conducts the optimal policy under discretion absent QE. The output gap and balance sheet are expressed in per-cent deviation from their steady-state values. Inflation is expressed in annualised percentage-points deviation from steady state. The policy rate and the real rate are annualised.

5 Welfare Analysis under Policy Rules

This section studies the welfare implications of the central bank following simple policy rules for setting the short-term interest rate and the real value of its balance sheet. We then compare the outcomes to those under the optimal monetary policies. First, we consider the possibility that the central bank sets the short-term interest rate following an inflation-targeting rule:

$$r_t^{\star} = \eta_\pi \pi_t. \tag{11}$$

The second rule we consider is a price-level-targeting rule:

$$r_t^{\star} = \eta_p p_t. \tag{12}$$

Table 4: Evaluation of Simple Rules

	QE Rule		Welfare			
	ξ_π	ξ_x	Aggregate	Inflation	Output Gap	Balance Sheet
OMP-D	-	-	-9.89%	-7.69%	-0.99%	-1.21%
OMP-C	-	-	-4.90%	-2.60%	-0.71%	-1.59%
SI-IT Rule						
$\eta_\pi = 1.5$	50	50	-14.93%	-9.47%	-0.05%	-5.40%
$\eta_\pi = 5.0$	50	29	-11.92%	-7.60%	-0.12%	-4.20%
$\eta_\pi = +\infty$	0	28	-8.90%	-5.30%	-0.23%	-3.14%
SI-PLT Rule						
$\eta_p = 1.5$	0	13	-5.66%	-3.47%	-0.55%	-1.65%
$\eta_p = 5.0$	0	13	-5.58%	-3.40%	-0.55%	-1.64%
$\eta_p = +\infty$	0	13	-5.54%	-3.36%	-0.55%	-1.64%

Note: Conditional on the parameters η_π or η_p, we select the values of ξ_π and ξ_x that maximise aggregate welfare. We consider natural values of ξ_π and ξ_x on the interval $[0, 50]$.

where $\pi_t \equiv p_t - p_{t-1}$. Both rules are subject to the ZLB constraint:

$$r_t^s = \max\left\{r_t^\star, -\frac{R^s - 1}{R^s}\right\}. \tag{13}$$

It is important to notice that, unlike Equation (11), Equation (12) implies history dependence in the short-term interest rate. In other words, under price-level-targeting, a fall in inflation today implies that the central bank will keep its rate lower-for-longer and allow for an overshoot in inflation in the future. In addition to the policy rule governing the short-term interest rate, the central bank chooses the amount of long-term bond holdings according to the following rule:

$$qe_t = -\xi_\pi \pi_t - \xi_x x_t. \tag{14}$$

Next, we consider the welfare implications of combining rule (14) with either (11) or (12) for different values of η_π, η_p, ξ_π, and ξ_x.[6]

Table 4 reports how welfare changes in response to a negative demand shock under different policy-rule combinations and for different parameterisations. In particular, for each value of η_π or η_p, we report the QE-rule coefficients that minimise the welfare losses. We label the mix of Equations (11) and (14) as SI-IT (Short-term Interest rate, Inflation Targeting) and the mix of Equations (12) and (14) as SI-PLT (Short-

[6]Conditional on η_π or η_p, we select the welfare-maximising combination of ξ_π and ξ_x. To this end, we perform a grid-search on a discrete interval $[0, 50]$ with unit steps.

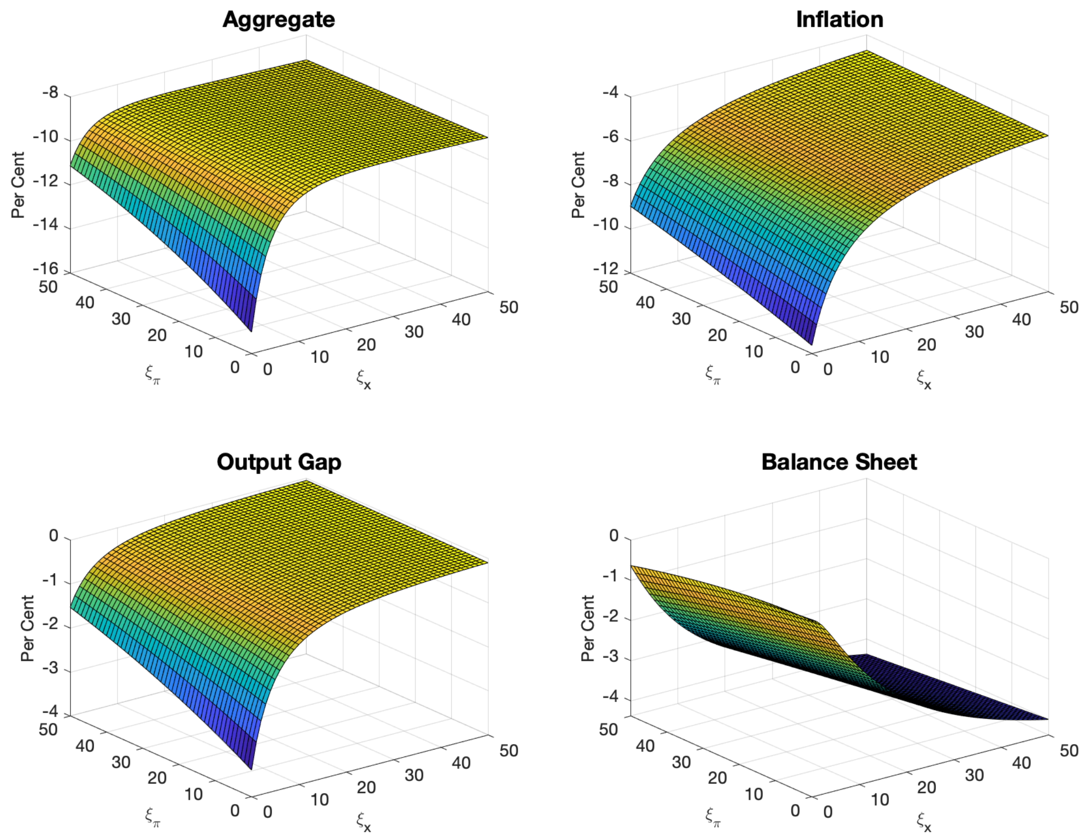

Figure 8: Strict Inflation-Targeting Rule and Welfare

Note: The figure displays the aggregate welfare and its subcomponents when the central bank sets the short-term interest rate following strict inflation-targeting rule and adjusts its balance sheet according to a policy rule with parameters ξ_π and ξ_x.

term Interest rate, Price Level Targeting). Under SI-IT, a higher value of η_π significantly improves welfare. Moreover, as we increase η_π, the coefficients in the QE rule become smaller, indicating that the role of balance-sheet policies becomes relatively less important for welfare. When $\eta_\pi \to \infty$, balance-sheet policies should only be targeting the output gap. In this case, we find that the welfare we can achieve is significantly lower than with the OMP-C (see Table 2). However, this policy mix performs better than the OMP-D. This is because the simple implementable rules represent a form of commitment.

Under SI-PLT, the optimal QE-rule coefficients are $\xi_\pi = 0$ and $\xi_x = 10$. We find that higher values of η_p mitigate the welfare losses by reducing the volatility of inflation and, to a lesser extent, that of the balance sheet. We also note that SI-PLT significantly outperforms SI-IT in terms of minimising welfare costs. This is because of the history-dependence implied by the price-level-targeting rule (see, e.g., Hills and Nakata, 2018). It also bears noting that SI-PLT, combined with the QE rule, can bring the welfare losses significantly

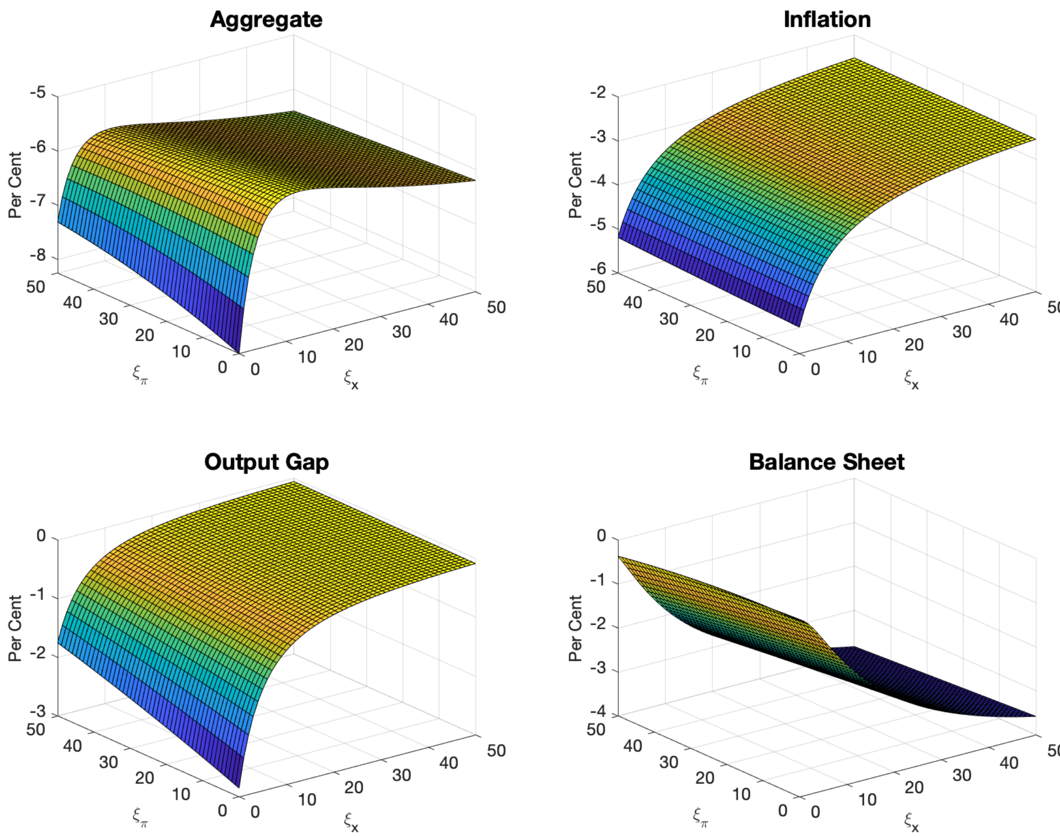

Figure 9: Strict Price-Level-Targeting Rule and Welfare

Note: The figure displays the aggregate welfare and its subcomponents when the central bank sets the short-term interest rate following strict price-level-targeting rule and adjusts its balance sheet according to a policy rule with parameters ξ_π and ξ_x.

closer to those with OMP-C.

Finally, Figures 8 and 9 display how aggregate welfare and its subcomponents change, as we vary the parameters in the QE rule ξ_π and ξ_x. In particular, the first figure considers the case of an SI-IT rule with $\eta_\pi \to \infty$, i.e., the short-term is set following a strict inflation-targeting rule. Instead, the second figure represents the case of an SI-PLT with $\eta_p \to \infty$, i.e., the short-term is set according to a strict price-level-targeting rule. In Figure 8, we see that increasing ξ_π and ξ_x can sharply increase welfare (for small values of ξ_x). However, for larger values of these coefficients, aggregate welfare becomes relatively irresponsive to changes in the coefficients. This result arises because increasing further ξ_x and ξ_π causes higher balance-sheet volatility, which partially offsets the welfare gains from output-gap and inflation stabilisation. Figure 9 shows how aggregate welfare is maximised for $\xi_\pi = 0$ and $\xi_\pi = 13$. Despite attenuating inflation and output-gap volatility, setting $\xi_\pi > 0$ and $\xi_x > 13$ would reduce welfare due to a rise in the volatility of the central bank's balance sheet. Therefore, the exercises discussed in this section highlight once more the trade-off between stabilising inflation, the output gap, and the size of the central bank's balance sheet.

6 Concluding Remarks

In this paper, we study the optimal monetary policy conduct at the ZLB when the central bank can carry out both forward guidance and balance-sheet policies. We do so through the lenses of a stylised model, which allows us to derive the second-order approximation to the social welfare loss function. Unlike the canonical result, where welfare is a negative function of the volatility of inflation and the output gap, we show that our model also implies a social preference for low volatility of the central bank's balance sheet. The reason is that the central bank's long-term assets purchases affect asset prices and increase the consumption volatility of long-term debt holders. In this context, we show that, following a negative demand shock at the ZLB, the optimal monetary policy under commitment implies a mix of FG and mild adjustments in the balance sheet size. Specifically, FG boosts expectations about inflation and the output gap, an initial increase in the size of the balance sheet further eases the initial drop in demand, and a subsequent contraction mitigates the overshoots in prices and real activity. The presence of balance-sheet policies reduces the optimal duration of the ZLB required to stabilise inflation and the output gap. Under discretion, instead, the central bank is unable to carry out FG and can only rely on balance-sheet policies to stabilise inflation and the output gap. Compared to the optimal policy under commitment, the absence of FG leads to lower inflation and output-gap stabilisation and a stronger increase in the central bank's balance sheet. As a result, under discretion, welfare losses are significantly larger.

The optimal quantity of QE and ZLB duration crucially depend on how powerful FG is. In particular, when households are assumed not to react as strongly to FG as under rational expectations, balance-sheet policies become comparatively more effective at stabilising inflation and the output gap. Therefore, the central bank should expand its balance sheet more aggressively and lift off the short-term rate earlier.

When the central bank only aims to stabilise inflation and the output gap, rather than to maximise social welfare, we find that balance-sheet policies are beneficial only if the relative weight on the output gap is large. Since, in our model, balance-sheet policies are relatively less effective at stabilising inflation than the output gap, a smaller weight on the output gap (i.e., large weight on inflation) requires a stronger balance-sheet expansion in response to a negative demand shock. As a result, a smaller weight on the output gap leads to higher balance-sheet volatility, negatively impacting welfare.

Last, we consider the welfare implications of the central bank following simple policy rules. In this case, we show that a central bank setting the short-term rate according to a strict price-level-targeting rule and the balance sheet following a flexible output-gap-targeting rule can achieve welfare outcomes close to the optimal policy under commitment.

The results presented in this paper highlight that the introduction of unconventional monetary policies in the central bank's toolkit can potentially bring further trade-offs beyond the standard inflation/output-gap considerations. Understanding these trade-offs is important from policy and research perspectives, especially considering the secular decline in interest rates and inflation, which have increased the probability of binding ZLB periods.

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Appendices

A Full Nonlinear Model

The model we consider follows Sims et al. (2021). Unlike their paper, and in line with Woodford (2003), we assume industry-specific labour in the utility function, rather than aggregate labour. We assume there is a share z of patient households and $1-z$ of impatient households, whereas in the original model z represents the impatient household's steady-state share of consumption. This assumption allows us to simplify the calculation of the aggregate welfare function, as detailed in Section C. This assumption does not affect the equilibrium conditions.

A.1 Patient Households

A representative patient household maximises its discounted lifetime utility:

$$\max E_0^{BR} \sum_{t=0}^{\infty} \beta^t \left[\frac{{C_{s,t}}^{1-\sigma} - 1}{1-\sigma} - \psi \int_0^1 \frac{L_{s,t}(i)^{1+\chi}}{1+\chi} di \right], \tag{A.1}$$

where E_0^{BR} is the subjective (behavioural) expectation operator, and $C_{s,t}$ is a Dixit-Stiglitz aggregate:

$$C_{s,t} = \left[\int_0^1 C_{s,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{A.2}$$

and $L_{s,t}(i)$ is the quantity of labour supplied to the firm producing good i. The parameter σ is the coefficient of relative risk aversion, ε is the demand elasticity of good i, χ is the inverse of the Frisch elasticity of labour, ψ is a normalising constant, and β is the discount factor.

The household demand for good i is given by:

$$C_{s,t}(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_{s,t}, \tag{A.3}$$

where $P_t(i)$ is the price of good i. The aggregate price level P_t therefore writes as:

$$P_t \equiv \left[\int_0^1 P_t(i)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}, \tag{A.4}$$

so that:

$$P_t C_{s,t} = \int_0^1 P_t(i) C_{s,t}(i) di. \tag{A.5}$$

The patient household maximises its expected discounted lifetime utility (A.1) subject to the following budget constraint:

$$\begin{aligned}\int_0^1 P_t(i)C_{s,t}(i)di + \frac{S_t}{1-z} \leq R^s_{t-1}\frac{S_{t-1}}{1-z} + \int_0^1 W_t(i)L_{s,t}(i)di + \int_0^1 \frac{D_t(i)}{1-z} + P_t\frac{D_{FI,t}}{1-z} + P_t\frac{T_t}{1-z} \\ -P_t\frac{X_{b,t}}{1-z} - P_t\frac{X_{FI,t}}{1-z},\end{aligned} \tag{A.6}$$

where S_t is a one period risk-free bond, paying a gross nominal interest rate R^s_t. $W_t(i)$ is the nominal wage rate in the ith industry in the economy and $D_t(i)$ are the nominal profits from the sale of good i. The household owns the financial intermediaries and receives dividends $D_{FI,t}$. T_t is a lump-sum transfer from the central bank. Finally, $X_{b,t}$ and $X_{FI,t}$ are transfers to the impatient households and the financial intermediaries. The resulting optimality conditions are standard:

$$1 = E^{BR}_t \Lambda^s_{t,t+1} \frac{R^s_t}{\Pi_{t+1}}, \tag{A.7}$$

$$\Lambda^s_{t-1,t} = \beta \left(\frac{C_{s,t}}{C_{s,t-1}} \right)^{-\sigma}, \tag{A.8}$$

where $\Lambda^s_{t-1,t}$ is the patient household's stochastic discount factor. $\Pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate.

A.2 Impatient Households

The impatient household can borrow/save with long term bonds B_t. Similarly as in Woodford (2001), long-term bonds are modelled as perpetuities with geometrically decaying coupon payments. The decaying rate of the coupon payments is denoted by $\kappa \in [0, 1]$. The agent that issues the bond in time t needs to pay 1, κ, κ^2, ... in the following periods. The new bond issuance CB_t equals:

$$CB_t = B_t - \kappa B_{t-1}. \tag{A.9}$$

Given the market price of newly issued bonds Q_t, the total value of the bond portfolio equals $Q_t B_t$. Moreover, define the gross return on the long bond as:

$$R^b_t = \frac{1 + \kappa Q_t}{Q_{t-1}}, \tag{A.10}$$

and the gross yield-to-maturity as:

$$Q_t = \frac{1}{RL_t^b} + \frac{\kappa}{RL_t^{b2}} + \frac{\kappa^2}{RL_t^{b3}} + \cdots, \tag{A.11}$$

Consequently,

$$RL_t^b = \frac{1}{Q_t} + \kappa. \tag{A.12}$$

The impatient household does not work and derives utility only from its consumption C_t^b. It maximises its lifetime utility:

$$\max E_0^{BR} \sum_{t=0}^{\infty} {\beta_b}^t \left[\frac{{C_{b,t}}^{1-\sigma} - 1}{1-\sigma} \right], \tag{A.13}$$

subject to a budget constraint choosing $C_{b,t}$ and B_t:

$$P_t C_{b,t} + \frac{B_{t-1}}{z} \leq Q_t \left(\frac{B_t}{z} - \kappa \frac{B_{t-1}}{z} \right) + P_t \frac{X_{b,t}}{z}. \tag{A.14}$$

The optimality condition for the impatient households is, therefore:

$$1 = E_t^{BR} \Lambda_{t,t+1}^b \frac{R_{t+1}^b}{\Pi_{t+1}}, \tag{A.15}$$

where $\Lambda_{t-1,t}^b$ denotes stochastic discount factor, defined as:

$$\Lambda_{t-1,t}^b = \beta_b \left(\frac{C_{b,t}}{C_{b,t-1}} \right)^{-\sigma}. \tag{A.16}$$

A.3 Financial Intermediaries

A representative financial intermediary is born each period and exits the industry in the subsequent period. It receives an exogenous amount of net worth from the patient household, $P_t X_{FI,t}$, which equals:

$$P_t X_{FI,t} = P_t \bar{X}^{FI} + \kappa Q_t B_{FI,t-1}. \tag{A.17}$$

$\bar{X}^{FI}$ is a fixed amount of new equity, whereas $\kappa Q_t B_{FI,t-1}$ is the value of outstanding long-bonds inherited from past intermediaries. The balance sheet of the financial intermediary is given by:

$$Q_t B_{FI,t} + RE_{FI,t} = S_{FI,t} + P_t X_{FI,t}. \tag{A.18}$$

where the left-hand side are the assets (long-term lending to impatient households $Q_t B_{FI,t}$ and reserves $RE_{FI,t}$), whereas the right-hand side are the liabilities (short-term deposits from the patient household $S_{FI,t}$ and the transfer $P_t X_{FI,t}$). The financial intermediary, pays interest R_t^s on the deposits, earns interest, R_t^{re}, on its reserves, and earns a gross return R_{t+1}^b on long-term bonds.

When the financial intermediary exits the market, gives dividends $P_{t+1} D_{FI,t+1}$ (in nominal terms) to the patient household:

$$P_{t+1} D_{FI,t+1} = \left(R_{t+1}^b - R_t^s\right) Q_t B_{FI,t} + \left(R_t^{re} - R_t^s\right) RE_{FI,t} + R_t^s P_t X_{FI,t}. \tag{A.19}$$

In time t the financial intermediary maximises the expected $t+1$ dividends, discounted by the nominal stochastic discount factor of the patient household $\frac{\Lambda_{t,t+1}}{\Pi_{t+1}}$, subject to a leverage constraint:

$$Q_t B_{FI,t} \leq \Theta P_t \bar{X}^{FI}. \tag{A.20}$$

The condition states that the value of the long-term loans to the impatient households cannot be larger than a multiple Θ of the value of its equity. The first-order conditions with respect to $B_{FI,t}$ and $RE_{FI,t}$ write as:

$$E_t \Lambda_{t,t+1} \frac{R_{t+1}^b - R_t^s}{\Pi_{t+1}} = \Omega_t, \tag{A.21}$$

$$E_t \Lambda_{t,t+1} \frac{R_t^{re} - R_t^s}{\Pi_{t+1}} = 0, \tag{A.22}$$

where Ω_t is the Lagrangian multiplier associated with the leverage constraint.

A.4 Production

A monopolistically competitive firm produces good i using the following production function:

$$Y_t(i) = A_t L_t(i) = A_t \left(1 - z\right) L_{s,t}(i). \tag{A.23}$$

Each firm faces a downward-sloping demand function given by:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t. \tag{A.24}$$

Following Woodford (2003), the labour employed by each monopolistically competitive firm corresponds to a particular type of the labour variety supplied by the households. The firm takes the wage rate as given and its period profits are given by:

$$D_t(i) = P_t(i) \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t - (1-\tau) \frac{W_t^I}{A_t} \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t, \tag{A.25}$$

where W_t^I should be interpreted as an industry specific wage for good variety i. W_t^I can then be related to the price level of good i via the first-order labour condition of the household as:

$$\frac{W_t^I}{P_t} = \psi \left(\frac{L_t(i)}{1-z} \right)^{\chi} C_t{}^{\sigma} = \psi \left(\frac{\frac{Y_t}{A_t} \left(\frac{P_t^I}{P_t} \right)^{-\varepsilon}}{1-z} \right)^{\chi} C_t{}^{\sigma}, \tag{A.26}$$

where P_t^I is the industry-wide common price. We write then the period profit function of a firm producing good i as $D\left(P_t(i), P_t^I, P_t, Y_t\right)$.

As in Calvo (1983), a fraction $1-\phi$ of randomly picked firms can reset their price. Let P_t^* be the optimal reset price in period t. A supplier that changes its price in period t chooses its newly-adjusted price $P_t(i)$ to maximise the its expected discounted lifetime profits, taking as given the industry level wage W^I, expressed in terms of P_t^I:

$$E_t \sum_{j=0}^{\infty} \phi^j \Lambda_{t,t+j} \frac{D\left(P_t(i), P_{t+j}^I, P_{t+j}, Y_{t+j}\right)}{P_{t+j}}, \tag{A.27}$$

The first-order condition for optimal price setting is:

$$E_t \sum_{j=0}^{\infty} \phi^j \Lambda_{t,t+j} \left((1-\varepsilon) \left(\frac{P_t^*}{P_{t+j}} \right)^{1-\varepsilon} Y_{t+j} + \varepsilon (1-\tau) \psi \left(\frac{\frac{Y_{t+j}}{A_{t+j}} \left(\frac{P_{t+j}^I}{P_{t+j}} \right)^{-\varepsilon}}{1-z} \right)^{\chi} C_{s,t+j}{}^{\sigma} \left(\frac{P_t^*}{P_{t+j}} \right)^{-\varepsilon} \frac{Y_{t+j}}{A_{t+j}} \right) = 0. \tag{A.28}$$

Following Woodford (2003), all firms in industry I reset the price in period t:

$$E_t \sum_{j=0}^{\infty} \phi^j \Lambda_{t,t+j} \left((1-\varepsilon) \left(\frac{P_t^*}{P_{t+j}} \right)^{1-\varepsilon} Y_{t+j} + \varepsilon (1-\tau) \psi \left(\frac{\frac{Y_{t+j}}{A_{t+j}} \left(\frac{P_t^*}{P_{t+j}} \right)^{-\varepsilon}}{1-z} \right)^{\chi} C_{s,t+j}{}^{\sigma} \left(\frac{P_t^*}{P_{t+j}} \right)^{-\varepsilon} \frac{Y_{t+j}}{A_{t+j}} \right) = 0. \tag{A.29}$$

This implies that the optimal reset price is:

$$\frac{P_t^*}{P_t} = \left(\frac{\varepsilon\,(1-\tau)}{\varepsilon-1} \frac{E_t \sum_{j=0}^{\infty} \phi^j \Lambda_{t,t+j} \psi (1-z)^{-\chi} {C_{s,t+j}}^{\sigma} \left(\frac{P_t}{P_{t+j}}\right)^{-\varepsilon(1+\chi)} \left(\frac{Y_{t+j}}{A_{t+j}}\right)^{1+\chi}}{E_t \sum_{j=0}^{\infty} \phi^j \Lambda_{t,t+j} \left(\frac{P_t}{P_{t+j}}\right)^{1-\varepsilon} Y_{t+j}} \right)^{\frac{1}{1+\chi\varepsilon}}, \tag{A.30}$$

where P_t indicates the aggregate price level. We can re-write the expression in recursive form:

$$p_t^* = \left(\frac{\varepsilon\,(1-\tau)}{\varepsilon-1} \frac{F_{1,t}}{F_{2,t}} \right)^{\frac{1}{1+\chi\varepsilon}}, \tag{A.31}$$

$$F_{1,t} = \psi\,(1-z)^{-\chi} \left(\frac{Y_t}{A_t}\right)^{1+\chi} + \phi\beta E_t {\Pi_{t+1}}^{\varepsilon(1+\chi)} F_{1,t+1}, \tag{A.32}$$

$$F_{2,t} = {C_{s,t}}^{-\sigma} Y_t + \phi\beta E_t {\Pi_{t+1}}^{\varepsilon-1} F_{2,t+1}, \tag{A.33}$$

where inflation Π_t evolves according to:

$$\phi {\Pi_t}^{\varepsilon-1} = 1 - (1-\phi)\, {p_t^*}^{1-\varepsilon}. \tag{A.34}$$

A.5 Central Bank

The monetary authority creates reserves to finance the purchase of long bonds $B_{cb,t}$. Its balance sheet, therefore, writes as:

$$Q_t B_{cb,t} = RE_t. \tag{A.35}$$

The real value of long-term bonds held by the central bank is denoted as:

$$QE_t = Q_t b_{cb,t}, \tag{A.36}$$

where $b_{cb,t} = \frac{B_{cb,t}}{P_t}$. Potential profits made by the central bank are then transferred lump-sum to the patient households:

$$P_t T_t = R_t^b Q_{t-1} B_{cb,t-1} - R_{t-1}^{re} RE_{t-1}. \tag{A.37}$$

A.6 Aggregation and Equilibrium

Market clearing requires the following conditions:

$$RE_t = RE_{FI,t}, \tag{A.38}$$

$$S_t = S_{FI,t}, \tag{A.39}$$

$$B_t = B_{FI,t} + B_{cb,t}, \tag{A.40}$$

$$Y_t = C_t = (1 - z)\, C_{s,t} + z C_{b,t}, \tag{A.41}$$

$$P_t X_{b,t} = (1 + \kappa Q_t)\, B_{t-1}, \tag{A.42}$$

$$P_t C_{b,t} = Q_t \frac{B_t}{z}. \tag{A.43}$$

The output gap X_t is simply defined as:

$$X_t = \frac{Y_t}{Y_t^f}. \tag{A.44}$$

where Y_t^f is the level of output arising in the flexible-price version of the model absent balance-sheet policies. The logarithm of the productivity shocks is assumed to follow AR(1) process:

$$\log A_t = \rho_a \log A_{t-1} + \sigma_a \epsilon_t^a, \tag{A.45}$$

B Full Linearised Model

B.1 Rational Inattention

In order to mitigate the forward guidance puzzle highlighted in Del Negro et al. (2015), we assume that households are partially myopic as in Gabaix (2020). By reacting myopically to distant events, such as future interest rate changes, forward guidance becomes significantly less powerful than in the canonical rational-expectations NK model. In particular, for any variable $z(X_t)$ with $z(0) = 0$, we have that, for all $k \geq 0$:

$$E_t^{BR} z(X_{t+k}) = (1-\alpha)^k E_t z(X_{t+k}), \tag{B.1}$$

where E_t^{BR} is the subjective (behavioural) expectation operator, and E_t is the rational one. $\alpha \in [0,1]$ captures the degree of attention to the future and, when $\alpha = 0$, agents have rational expectations.

We assume that both patient and impatient households are affected by cognitive discounting and their linearised Euler equations write as:

$$c_{s,t} = E_t^{BR} c_{s,t+1} - \frac{1}{\sigma}\left(r_t^s - E_t \pi_{t+1}\right), \tag{B.2}$$

$$c_{b,t} = E_t^{BR} c_{b,t+1} - \frac{1}{\sigma}\left(E_t^{BR} r_{t+1}^b - E_t^{BR} \pi_{t+1}\right). \tag{B.3}$$

Using Equation (B.1), we can re-write the Euler equations above as:

$$c_{s,t} = (1-\alpha) E_t c_{s,t+1} - \frac{1}{\sigma}\left(r_t^s - E_t \pi_{t+1}\right), \tag{B.4}$$

$$c_{b,t} = (1-\alpha) E_t c_{b,t+1} - \frac{1-\alpha}{\sigma}\left(E_t r_{t+1}^b - E_t \pi_{t+1}\right). \tag{B.5}$$

B.2 System of Equations

The equilibrium conditions in log-linearised form are summarised below. The following variables denote percentage change deviations from their steady-state values, e.g., $c_{s,t} \equiv \frac{C_{s,t} - C_s}{C_s}$. We use the "hat" notation when the variable in the nonlinear model is labelled with a lower-case letter, e.g., $\widehat{b}_{FI,t} \equiv \frac{b_{FI,t} - b_{FI}}{b_{FI}}$.

$$c_{s,t} = (1-\alpha) E_t c_{s,t+1} - \frac{1}{\sigma}\left(r_t^s - E_t \pi_{t+1}\right), \tag{B.6}$$

$$r_t^b = \frac{\kappa}{R^b} q_t - q_{t-1}, \tag{B.7}$$

$$rl_t^b = -\frac{1}{1+\kappa Q} q_t, \tag{B.8}$$

$$c_{b,t} = (1-\alpha) E_t c_{b,t+1} - \frac{1-\alpha}{\sigma} \left(E_t r_{t+1}^b - E_t \pi_{t+1} \right), \tag{B.9}$$

$$q_t + \widehat{b}_{FI,t} = 0, \tag{B.10}$$

$$Q b_{FI} (1-\kappa) q_t + Q b_{FI} \widehat{b}_{FI,t} - \kappa Q b_{FI} \widehat{b}_{FI,t-1} + \kappa Q b_{FI} \pi_t + re\widehat{re}_t = s\widehat{s}_t, \tag{B.11}$$

$$-\sigma (E_t c_{s,t+1} - c_{s,t}) - E_t \pi_{t+1} + \frac{R^b}{sp} E_t r_{t+1}^b - \frac{R^s}{sp} r_t^s = \omega_t, \tag{B.12}$$

$$r_t^{re} = r_t^s, \tag{B.13}$$

$$\widehat{p}_t^* = \frac{1}{1+\chi\varepsilon} (f_{1,t} - f_{2,t}), \tag{B.14}$$

$$f_{1,t} = (1-\phi\beta)(1+\chi)(y_t - a_t) + \phi\beta\varepsilon (1+\chi) E_t \pi_{t+1} + \phi\beta E_t f_{1,t+1}, \tag{B.15}$$

$$f_{2,t} = -(1-\phi\beta)\sigma c_{s,t} + (1-\phi\beta) y_t + \phi\beta(\varepsilon - 1) E_t \pi_{t+1} + \phi\beta E_t f_{2,t+1}, \tag{B.16}$$

$$(1-z) c_{s,t} + z c_{b,t} = y_t, \tag{B.17}$$

$$\pi_t = \frac{1-\phi}{\phi} \widehat{p}_t^*, \tag{B.18}$$

$$q_t + \widehat{b}_{cb,t} = re_t, \tag{B.19}$$

$$\widehat{b}_t = \frac{b_{FI}}{b} \widehat{b}_{FI,t} + \frac{b_{cb}}{b} \widehat{b}_{cb,t}, \tag{B.20}$$

$$c_{b,t} = q_t + \widehat{b}_t, \tag{B.21}$$

$$qe_t = \widehat{re}_t, \tag{B.22}$$

$$x_t = y_t - y_t^f, \tag{B.23}$$

$$y_t^f = \frac{(1+\chi)(1-z)}{\chi(1-z)+\sigma} a_t, \tag{B.24}$$

$$r_t^n = -\frac{\sigma(1-(1-\alpha)\rho_a)(1+\chi)}{\chi(1-z)+\sigma} a_t \tag{B.25}$$

$$a_t = \rho_a a_{t-1} + \sigma_a \epsilon_t^a. \tag{B.26}$$

B.3 Deriving the IS Curve

First, begin by adding the Euler equations of the patient and impatient households, i.e., Equations (B.6) and (B.9):

$$(1-z)\, c_{s,t} + z c_{b,t} = (1-\alpha)\, E_t \left((1-z)\, c_{s,t+1} + z c_{b,t+1}\right) - \frac{1-z}{\sigma}\left(r_t^s - E_t \pi_{t+1}\right) - \frac{z\,(1-\alpha)}{\sigma}\left(E_t r_{t+1}^b - E_t \pi_{t+1}\right). \tag{B.27}$$

Using the resource constraint, Equation (B.17), we get a first version of the IS curve:

$$y_t = (1-\alpha)\, E_t y_{t+1} - \frac{1-z}{\sigma}\left(r_t^s - E_t \pi_{t+1}\right) - \frac{z\,(1-\alpha)}{\sigma}\left(E_t r_{t+1}^b - E_t \pi_{t+1}\right). \tag{B.28}$$

Second, combine equations (B.21) and (B.20):

$$c_{b,t} = q_t + \bar{b}_{FI}\hat{b}_{FI,t} + \bar{b}_{cb}\hat{b}_{cb,t} = \bar{b}_{FI}\left(q_t + \hat{b}_{FI,t}\right) + \bar{b}_{cb}\left(q_t + \hat{b}_{cb,t}\right). \tag{B.29}$$

where $\bar{b}_{FI} \equiv \frac{b_{FI}}{b}$ and $\bar{b}_{cb} \equiv \frac{b_{cb}}{b}$. Using equations (B.10) and (B.19), the last equation rewrites as:

$$c_{b,t} = \bar{b}_{cb} q e_t. \tag{B.30}$$

Using this in the child's Euler equation (Equation (B.9)) we have:

$$\bar{b}_{cb}\left((1-\alpha)\, E_t q e_{t+1} - q e_t\right) = \frac{1-\alpha}{\sigma}\left(E_t r_{t+1}^b - E_t \pi_{t+1}\right). \tag{B.31}$$

This last result can be used to rewrite the IS curve as a function of qe_t:

$$y_t = (1-\alpha)\, E_t y_{t+1} - \frac{1-z}{\sigma}\left(r_t^s - E_t \pi_{t+1}\right) + z\bar{b}_{cb}\left(q e_t - (1-\alpha)\, E_t q e_{t+1}\right). \tag{B.32}$$

The latter can be written in terms of the output gap:

$$x_t = (1-\alpha)\, E_t x_{t+1} - \frac{1-z}{\sigma}\left(r_t^s - E_t \pi_{t+1} - r_t^n\right) + z\bar{b}_{cb}\left(q e_t - (1-\alpha)\, E_t q e_{t+1}\right). \tag{B.33}$$

B.4 Deriving the NKPC

Combine Equations (B.14)-(B.16), and (B.18) to obtain:

$$\pi_t = \frac{(1-\phi)\,(1-\phi\beta)}{(1+\chi\varepsilon)\,\phi}\left(\chi y_t - (1+\chi)\, a_t + \sigma c_{s,t}\right) + \beta E_t \pi_{t+1}. \tag{B.34}$$

Define $\gamma \equiv \frac{(1-\phi)(1-\phi\beta)}{(1+\chi\varepsilon)\phi}$ and use the resource constraint, Equation (B.17), to replace $c_{s,t}$:

$$\pi_t = \gamma\left(\left(\chi + \frac{\sigma}{1-z}\right)y_t - (1+\chi)\,a_t - \frac{\sigma z}{1-z}c_{b,t}\right) + \beta E_t \pi_{t+1}. \tag{B.35}$$

Use Equation (B.30) to write the NKPC as a function of qe_t:

$$\pi_t = \gamma\left(\left(\chi + \frac{\sigma}{1-z}\right)y_t - (1+\chi)\,a_t - \frac{\sigma z \bar{b}_{cb}}{1-z}qe_t\right) + \beta E_t \pi_{t+1}. \tag{B.36}$$

Finally, use Equations (B.23) and (B.24) to replace output and productivity with the output gap:

$$\pi_t = \beta E_t \pi_{t+1} + \gamma\left(\chi + \frac{\sigma}{1-z}\right)x_t - \frac{\gamma\sigma z \bar{b}_{cb}}{1-z}qe_t. \tag{B.37}$$

C Utility-Based Welfare Criterion

We follow Woodford (2003) in deriving the utility-based loss function. We take a Taylor expansion of each term of the following utility function:

$$\overline{U}_t \equiv U\left(C_{s,t}, C_{b,t}\right)+\int_0^1 V\left(L_t(i)\right)di = (1-z)\left(\frac{{C_{s,t}}^{1-\sigma}-1}{1-\sigma}\right)+z\left(\frac{{C_{b,t}}^{1-\sigma}-1}{1-\sigma}\right)-(1-z)\,\psi\int_0^1\frac{\left(\frac{L_t(i)}{1-z}\right)^{1+\chi}}{1+\chi}di. \tag{C.1}$$

Recall that the economy's resource constraint is:

$$Y_t = C_t = (1-z)\,C_{s,t} + zC_{b,t}. \tag{C.2}$$

We can rewrite:

$$U\left(C_{s,t}, C_{b,t}\right) = U\left(Y_t, C_{b,t}\right) = (1-z)\left(\frac{\left(\frac{Y_t - zC_{b,t}}{1-z}\right)^{1-\sigma}-1}{1-\sigma}\right) + z\left(\frac{{C_{b,t}}^{1-\sigma}-1}{1-\sigma}\right). \tag{C.3}$$

Taking a second-order expansion around the steady state, we obtain

$$\begin{aligned} U\left(Y_t, C_{b,t}\right) = U\left(Y, C_b\right) + U_Y\left(Y_t - Y\right) + \frac{1}{2}U_{YY}\left(Y_t-Y\right)^2 + U_{C_b}\left(C_{b,t}-C_b\right) + \frac{1}{2}U_{C_bC_b}\left(C_{b,t}-C_b\right)^2 \\ + t.i.p. + O\left(||\xi||^3\right), \end{aligned} \tag{C.4}$$

where $O\left(||\xi||^3\right)$ represents all relevant terms that are of third or higher order, and $t.i.p.$ denotes all the terms independent of monetary policy. Then, we take a second-order Taylor expansion of Y_t and $C_{b,t}$:

$$Y_t = Y\left(1 + y_t + \frac{1}{2}{y_t}^2\right) + O\left(||\xi||^3\right), \tag{C.5}$$

$$C_{b,t} = C_b\left(1 + c_{b,t} + \frac{1}{2}{c_{b,t}}^2\right) + O\left(||\xi||^3\right), \tag{C.6}$$

where $y_t \equiv \log Y_t - \log Y$ and $c_{b,t} \equiv \log C_{b,t} - \log C_b$. This implies:

$$Y_t - Y = Yy_t + \frac{1}{2}Y{y_t}^2 + O\left(||\xi||^3\right), \tag{C.7}$$

$$C_{b,t} - C_b = C_b c_{b,t} + \frac{1}{2}C_b {c_{b,t}}^2 + O\left(||\xi||^3\right). \tag{C.8}$$

Substituting Equations (C.7) and (C.8) into Equation (C.4) gives:

$$U\left(Y_t, C_{b,t}\right) = U\left(Y, C_b\right) + U_Y Y y_t + \frac{1}{2} U_Y Y {y_t}^2 + \frac{1}{2} U_{YY} Y^2 {y_t}^2 + U_{C_b} C_b c_{b,t} + \frac{1}{2} U_{C_b} C_b {c_{b,t}}^2 + \frac{1}{2} U_{C_b C_b} {C_b}^2 {c_{b,t}}^2 + t.i.p. + O\left(||\xi||^3\right). \tag{C.9}$$

Note that $U(Y, C_b)$ is independent of monetary policy. We rewrite (C.9) as:

$$U\left(Y_t, C_{b,t}\right) = U_Y Y \left(y_t + \frac{1}{2} {y_t}^2 + \frac{1}{2} \frac{U_{YY} Y}{U_Y} {y_t}^2 + \frac{U_{C_b} C_b}{U_Y Y} \left(c_{b,t} + \frac{1}{2} {c_{b,t}}^2 \right) + \frac{1}{2} \frac{U_{C_b C_b} {C_b}^2}{U_Y Y} {c_{b,t}}^2 \right) + t.i.p. + O\left(||\xi||^3\right). \tag{C.10}$$

From the utility function, we have $\frac{U_{YY} Y}{U_Y} = -\frac{\sigma}{1-z}$, $\frac{U_{C_b} C_b}{U_Y Y} = 0$, and $\frac{U_{C_b C_b} {V_b}^2}{U_Y Y} = -\frac{\sigma z}{1-z}$. Thus, we obtain:

$$U\left(Y_t, C_{b,t}\right) = U_Y Y \left(y_t + \frac{1}{2} \left(1 - \frac{\sigma}{1-z} \right) {y_t}^2 - \frac{1}{2} \frac{\sigma z}{1-z} {c_{b,t}}^2 \right) + t.i.p. + O\left(||\xi||^3\right). \tag{C.11}$$

Using the resource constraint, we know $U_C C = U_Y Y$. Finally, we then rewrite:

$$\frac{U\left(C_{s,t}, C_{b,t}\right)}{U_C C} = y_t + \frac{1}{2} \left(1 - \frac{\sigma}{1-z} \right) {y_t}^2 - \frac{1}{2} \frac{\sigma z}{1-z} {c_{b,t}}^2 + t.i.p. + O\left(||\xi||^3\right). \tag{C.12}$$

Now, we also take a second-order Taylor expansion of $V\left(L_t(i)\right)$.

$$V\left(L_t(i)\right) = V(L) + V_L \left(L_t(i) - L\right) + V_{LL} \left(L_t(i) - L\right)^2 + t.i.p. + O\left(||\xi||^3\right). \tag{C.13}$$

The second-order approximation of $L_t(i)$ is:

$$L_t(i) = L \left(1 + l_t(i) + \frac{1}{2} l_t(i)^2 \right) + O\left(||\xi||^3\right), \tag{C.14}$$

where $l_t(i) \equiv \log L_t(i) - \log L$. This implies:

$$L_t(i) - L = L l_t(i) + \frac{1}{2} L l_t(i)^2 + O\left(||\xi||^3\right), \tag{C.15}$$

Substituting Equation (C.15) into Equation (C.13) gives:

$$V\left(L_t(i)\right) = V(L) + V_L L l_t(i) + \frac{1}{2} V_L L l_t(i)^2 + \frac{1}{2} V_{LL} L^2 l_t(i)^2 + t.i.p. + O\left(||\xi||^3\right). \tag{C.16}$$

Note that $V(L)$ is independent of monetary policy. We rewrite (C.16) as:

$$V\left(L_t(i)\right) = V_L L \left(l_t(i) + \frac{1}{2} l_t(i)^2 + \frac{1}{2} \frac{V_{LL} L}{V_L} l_t(i)^2 \right) + t.i.p. + O\left(||\xi||^3\right). \tag{C.17}$$

Since $\frac{V_{LL} L}{V_L} = \chi$, we rewrite Equation (C.17) as:

$$V\left(L_t(i)\right) = V_L L \left(l_t(i) + \frac{1}{2}\left(1+\chi\right) l_t(i)^2 \right) + t.i.p. + O\left(||\xi||^3\right). \tag{C.18}$$

From the production function, we have:

$$l_t(i) = y_t(i) - a_t. \tag{C.19}$$

Substituting Equation (C.19) into Equation (C.18), we obtain:

$$V\left(L_t(i)\right) = V_L L \left(y_t(i) - a_t + \frac{1}{2}\left(1+\chi\right)\left(y_t(i) - a_t\right)^2 \right) + t.i.p. + O\left(||\xi||^3\right). \tag{C.20}$$

$$V\left(L_t(i)\right) = V_L L \left(y_t(i) + \frac{1}{2}\left(1+\chi\right) y_t(i)^2 - \left(1+\chi\right) a_t y_t(i) \right) + t.i.p. + O\left(||\xi||^3\right). \tag{C.21}$$

By integrating Equation (C.21), we obtain:

$$\int_0^1 V\left(L_t(i)\right) di = V_L L \left(E_i y_t(i) + \frac{1}{2}\left(1+\chi\right)\left(\left(E_i y_t(i)\right)^2 + var_i y_t(i)\right) - \left(1+\chi\right) a_t E_i y_t(i) \right) + t.i.p. + O\left(||\xi||^3\right). \tag{C.22}$$

Taking a second-order approximation of the aggregators gives:

$$y_t = E_i y_t(i) + \frac{1}{2}\left(1 - \frac{1}{\varepsilon}\right) var_i y_t(i) + O\left(||\xi||^3\right), \tag{C.23}$$

which implies

$$E_i y_t(i) = y_t - \frac{1}{2}\left(1 - \frac{1}{\varepsilon}\right) var_i y_t(i) + O\left(||\xi||^3\right), \tag{C.24}$$

$$\left(E_i y_t(i)\right)^2 = {y_t}^2 + O\left(||\xi||^3\right). \tag{C.25}$$

We substitute Equations (C.24) and (C.25) into Equation (C.22) obtaining:

$$\int_0^1 V\left(L_t(i)\right) di = V_L L \left(y_t + \frac{1}{2}\left(1+\chi\right) {y_t}^2 - \left(1+\chi\right) a_t y_t + \frac{1}{2}\left(\chi + \frac{1}{\varepsilon}\right) var_i y_t(i) \right) + t.i.p. + O\left(||\xi||^3\right). \tag{C.26}$$

Now, recall that $L = \frac{Y}{A} = Y = C$. From the household's labour supply relation, we have:

$$-\frac{V_L L}{U_C C} = 1, \tag{C.27}$$

Then, we rewrite:

$$\frac{\int_0^1 V\left(L_t(i)\right) di}{U_C C} = -y_t - \frac{1}{2}\left(1+\chi\right) {y_t}^2 + \left(1+\chi\right) a_t y_t - \frac{1}{2}\left(\chi + \frac{1}{\varepsilon}\right) var_i y_t(i) + t.i.p. + O\left(||\xi||^3\right). \tag{C.28}$$

Combining Equations (C.12) and (C.28), we finally obtain:

$$\begin{aligned}\frac{\overline{U}_t}{\overline{U}_C C} \equiv \frac{U\left(C_{s,t}, C_{b,t}\right) + \int_0^1 V\left(L_t(i)\right) di}{U_C C} &= -\frac{1}{2}\left(\chi + \frac{\sigma}{1-z}\right) {y_t}^2 - \frac{1}{2}\frac{\sigma z}{1-z} {c_{b,t}}^2 + \left(1+\chi\right) a_t y_t \\ &\quad -\frac{1}{2}\left(\chi + \frac{1}{\varepsilon}\right) var_i y_t(i) + t.i.p. + O\left(||\xi||^3\right).\end{aligned} \tag{C.29}$$

We have the potential level of output:

$$y_t^f = \frac{\left(1+\chi\right)\left(1-z\right)}{\chi\left(1-z\right) + \sigma} a_t. \tag{C.30}$$

Combining Equations (C.30) and (C.29), we obtain:

$$\begin{aligned}\frac{\overline{U}_t}{\overline{U}_C C} = -\frac{1}{2}\left(\chi + \frac{\sigma}{1-z}\right) {y_t}^2 - \frac{1}{2}\frac{\sigma z}{1-z} {c_{b,t}}^2 + \left(\chi + \frac{\sigma}{1-z}\right) y_t^f y_t - \frac{1}{2}\left(\chi + \frac{1}{\varepsilon}\right) var_i y_t(i) \\ + t.i.p. + O\left(||\xi||^3\right).\end{aligned} \tag{C.31}$$

Then, we can rewrite:

$$\begin{aligned}\frac{\overline{U}_t}{\overline{U}_C C} = -\frac{1}{2}\left(\chi + \frac{\sigma}{1-z}\right)\left({y_t}^2 - 2 y_t^f y_t + {y_t^f}^2 - {y_t^f}^2\right) - \frac{1}{2}\frac{\sigma z}{1-z} {c_{b,t}}^2 - \frac{1}{2}\left(\chi + \frac{1}{\varepsilon}\right) var_i y_t(i) \\ + t.i.p. + O\left(||\xi||^3\right).\end{aligned} \tag{C.32}$$

Since ${y_t^f}^2$ belongs to $t.i.p.$ and $x \equiv y_t - y_t^f$, we obtain:

$$\frac{\overline{U}_t}{\overline{U}_C C} = -\frac{1}{2}\left(\chi + \frac{\sigma}{1-z}\right) {x_t}^2 - \frac{1}{2}\frac{\sigma z}{1-z} {c_{b,t}}^2 - \frac{1}{2}\left(\chi + \frac{1}{\varepsilon}\right) var_i y_t(i) + t.i.p. + O\left(||\xi||^3\right). \tag{C.33}$$

We take the expected discounted sum over time, we obtain:

$$\mathbb{W} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\overline{U}_t}{\overline{U}_C C} = E_0 \sum_{t=0}^{\infty} \beta^t \left[-\frac{1}{2} \left(\chi + \frac{\sigma}{1-z} \right) {x_t}^2 - \frac{1}{2} \frac{\sigma z}{1-z} {c_{b,t}}^2 - \frac{1}{2} \left(\chi + \frac{1}{\varepsilon} \right) var_i y_t(i) \right] + t.i.p. + O\left(||\xi||^3\right). \tag{C.34}$$

The demand for $Y_t(i)$ is given by:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t. \tag{C.35}$$

Then, we get:

$$y_t(i) = -\varepsilon \left(p_t(i) - p_t \right) + y_t. \tag{C.36}$$

This implies that:

$$var_i y_t(i) = \varepsilon^2 var_i p_t(i), \tag{C.37}$$

where $\Delta_t \equiv var_i p_t(i)$ is a measure of price dispersion. When prices are staggered as in the discrete time Calvo (1983) fashion, Woodford (2003) shows that:

$$\Delta_t = \phi \Delta_{t-1} + \frac{\phi}{1-\phi} {\pi_t}^2 + O\left(||\xi||^3\right) = \phi^{t+1} \Delta_{-1} + \sum_{k=0}^{t} \phi^{t-k} \left(\frac{\phi}{1-\phi} \right) {\pi_k}^2 + O\left(||\xi||^3\right). \tag{C.38}$$

If a new policy is conducted from $t \geq 0$, the first term, $\phi^{t+1}\Delta_{-1}$ is independent of policy. If we take the discounted sum over time, we obtain:

$$\sum_{t=0}^{\infty} \beta^t \Delta_t = \frac{\phi}{(1-\phi)(1-\phi\beta)} \sum_{t=0}^{\infty} \beta^t {\pi_t}^2 + t.i.p. + O\left(||\xi||^3\right). \tag{C.39}$$

Now, we consider:

$$\mathbb{W} = E_0 \sum_{t=0}^{\infty} \beta^t \left[-\frac{1}{2} \left(\chi + \frac{\sigma}{1-z} \right) {x_t}^2 - \frac{1}{2} \frac{\sigma z}{1-z} {c_{b,t}}^2 - \frac{1}{2} \left(\chi + \frac{1}{\varepsilon} \right) \varepsilon^2 \Delta_t \right] + t.i.p. + O\left(||\xi||^3\right). \tag{C.40}$$

Therefore, we can rewrite:

$$\mathbb{W} = E_0 \sum_{t=0}^{\infty} \beta^t \left[-\frac{1}{2} \left(\chi + \frac{\sigma}{1-z} \right) {x_t}^2 - \frac{1}{2} \frac{\sigma z}{1-z} {c_{b,t}}^2 - \frac{1}{2} \frac{\phi\varepsilon\left(1+\chi\varepsilon\right)}{(1-\phi)(1-\phi\beta)} {\pi_t}^2 \right] + t.i.p. + O\left(||\xi||^3\right). \tag{C.41}$$

Now, using market clearing conditions:

$$C_{b,t} = Q_t \frac{b_t}{z}, \tag{C.42}$$

$$b_t = b_{FI,t} + b_{cb,t}, \tag{C.43}$$

$$Q_t b_{FI,t} = \Theta \bar{X}^{FI}, \tag{C.44}$$

$$Q_t b_{cb,t} = QE_t, \tag{C.45}$$

we have:

$$C_{b,t} = \frac{\Theta \bar{X}^{FI} + QE_t}{z}. \tag{C.46}$$

Then, we can rewrite:

$$C_{b,t} - V_b = \frac{QE_t - QE}{z}. \tag{C.47}$$

We take a second-order Taylor expansion of QE_t:

$$QE_t = QE\left(1 + qe_t + \frac{1}{2}{qe_t}^2\right) + O\left(||\xi||^3\right), \tag{C.48}$$

where $qe_t \equiv \log QE_t - \log QE$. Substituting Equations (C.8) and (C.48) into Equation (C.47) gives:

$$c_{b,t} + \frac{1}{2}{c_{b,t}}^2 = \bar{b}_{cb}\left(qe_t + \frac{1}{2}{qe_t}^2\right) + O\left(||\xi||^3\right). \tag{C.49}$$

This implies:

$${c_{b,t}}^2 = \left(\bar{b}_{cb} qe_t\right)^2 + O\left(||\xi||^3\right). \tag{C.50}$$

Substituting Equation (C.50) into Equation (C.41) gives:

$$\mathbb{W} = E_0 \sum_{t=0}^{\infty} \beta^t \left[-\frac{1}{2}\left(\chi + \frac{\sigma}{1-z}\right){x_t}^2 - \frac{1}{2}\frac{\sigma z \bar{b}_{cb}^2}{1-z}{qe_t}^2 - \frac{1}{2}\frac{\phi\varepsilon\left(1+\chi\varepsilon\right)}{\left(1-\phi\right)\left(1-\phi\beta\right)}{\pi_t}^2\right] + t.i.p. + O\left(||\xi||^3\right). \tag{C.51}$$

Finally, we then rearrange:

$$\mathbb{W} = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\phi\varepsilon\left(1+\chi\varepsilon\right)}{\left(1-\phi\right)\left(1-\phi\beta\right)}{\pi_t}^2 + \left(\chi + \frac{\sigma}{1-z}\right){x_t}^2 + \frac{\sigma z \bar{b}_{cb}^2}{1-z}{qe_t}^2\right] + t.i.p. + O\left(||\xi||^3\right). \tag{C.52}$$

The average welfare loss per period is thus given as:

$$\mathbb{L}_t = \frac{\phi\varepsilon\left(1+\chi\varepsilon\right)}{\left(1-\phi\right)\left(1-\phi\beta\right)}{\pi_t}^2 + \left(\chi + \frac{\sigma}{1-z}\right){x_t}^2 + \frac{\sigma z \bar{b}_{cb}^2}{1-z}{qe_t}^2. \tag{C.53}$$

D Optimal Monetary Policy under Discretion

The central bank is assumed to choose x_t, π_t, r_t^s, qe_t in order to minimise the period losses:

$$\frac{1}{2}\left(\frac{\varepsilon}{\gamma}{\pi_t}^2 + \left(\chi + \frac{\sigma}{1-z}\right){x_t}^2 + \frac{\sigma z \bar{b}_{cb}^2}{1-z}{qe_t}^2\right), \tag{D.1}$$

subject to the sequences of constraints and $r^s \geq 0$:

$$x_t = (1-\alpha) E_t x_{t+1} - \frac{1-z}{\sigma}\left(r_t^s - E_t\pi_{t+1} - r_t^n\right) + z\bar{b}_{cb}\left(qe_t - (1-\alpha) E_t qe_{t+1}\right), \tag{D.2}$$

$$\pi_t = \beta E_t \pi_{t+1} + \gamma\left(\chi + \frac{\sigma}{1-z}\right) x_t - \frac{\gamma\sigma z\bar{b}_{cb}}{1-z} qe_t, \tag{D.3}$$

where the terms $E_t x_{t+1}$, $E_t\pi_{t+1}$, r_t^n, and $E_t qe_{t+1}$ are taken as given by the central bank. The Lagrangian for the above problem takes the form:

$$\begin{aligned} L = \frac{1}{2}\left(\frac{\varepsilon}{\gamma}{\pi_t}^2 + \left(\chi + \frac{\sigma}{1-z}\right){x_t}^2 + \frac{\sigma z \bar{b}_{cb}^2}{1-z}{qe_t}^2\right) \\ +\xi_{1,t}\left(x_t - (1-\alpha) x_{t+1} + \frac{1-z}{\sigma}\left(r_t^s - \pi_{t+1} - r_t^n\right) - z\bar{b}_{cb}\left(qe_t - (1-\alpha) qe_{t+1}\right)\right) \\ +\xi_{2,t}\left(\pi_t - \beta\pi_{t+1} - \gamma\left(\chi + \frac{\sigma}{1-z}\right) x_t + \frac{\gamma\sigma z\bar{b}_{cb}}{1-z} qe_t\right), \end{aligned} \tag{D.4}$$

where $\xi_{1,t}, \xi_{2,t}$ are Lagrangian multipliers.

Differentiating the Lagrangian with respect to x_t, π_t, r_t^s, and qe_t yields the optimality conditions:

$$\frac{\partial L}{\partial x_t} = \left(\chi + \frac{\sigma}{1-z}\right) x_t + \xi_{1,t} - \gamma\left(\chi + \frac{\sigma}{1-z}\right)\xi_{2,t} = 0, \tag{D.5}$$

$$\frac{\partial L}{\partial \pi_t} = \frac{\varepsilon}{\gamma}\pi_t + \xi_{2,t} = 0, \tag{D.6}$$

$$\frac{\partial L}{\partial r_t^s}\left(r_t^s + \frac{R^s-1}{R^s}\right) = \frac{1-z}{\sigma}\xi_{1,t}\left(r_t^s + \frac{R^s-1}{R^s}\right) = 0, \quad \xi_{1,t} \geq 0 \text{ and } r_t^s \geq -\frac{R^s-1}{R^s}, \tag{D.7}$$

$$\frac{\partial L}{\partial qe_t} = \frac{\sigma z\bar{b}_{cb}^2}{1-z} qe_t - z\bar{b}_{cb}\xi_{1,t} + \frac{\gamma\sigma z\bar{b}_{cb}}{1-z}\xi_{2,t} = 0, \tag{D.8}$$

that must hold for $t = 0, 1, 2, \ldots$ and where $\xi_{1,-1} = \xi_{2,-1} = 0$, because Equations (D.2) and (D.3) corresponding to period -1 is not an effective constraint for the central bank choosing its optimal plan in period 0. In sum, the equilibrium conditions under the optimal discretionary policy are then given by Equations (D.2), (D.3), (D.5), (D.6), (D.7), and (D.8).

E Optimal Monetary Policy under Commitment

The central bank is assumed to choose a state-contingent sequence $\{x_t, \pi_t, r_t^s, qe_t\}_{t=0}^{\infty}$ that minimises:

$$\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left(\frac{\varepsilon}{\gamma}{\pi_t}^2+\left(\chi+\frac{\sigma}{1-z}\right){x_t}^2+\frac{\sigma z\bar{b}_{cb}^2}{1-z}{qe_t}^2\right), \tag{E.1}$$

subject to the sequences of constraints and $r^s \geq 0$:

$$x_t=(1-\alpha)E_t x_{t+1}-\frac{1-z}{\sigma}\left(r_t^s-E_t\pi_{t+1}-r_t^n\right)+z\bar{b}_{cb}\left(qe_t-(1-\alpha)E_t qe_{t+1}\right), \tag{E.2}$$

$$\pi_t=\beta E_t\pi_{t+1}+\gamma\left(\chi+\frac{\sigma}{1-z}\right)x_t-\frac{\gamma\sigma z\bar{b}_{cb}}{1-z}qe_t, \tag{E.3}$$

In order to solve the previous problem it is useful to write down the associated Lagrangian, which is given by:

$$\begin{aligned}L=E_0\sum_{t=0}^{\infty}\beta^t\Bigg[&\frac{1}{2}\left(\frac{\varepsilon}{\gamma}{\pi_t}^2+\left(\chi+\frac{\sigma}{1-z}\right){x_t}^2+\frac{\sigma z\bar{b}_{cb}^2}{1-z}{qe_t}^2\right)\\ &+\xi_{1,t}\left(x_t-(1-\alpha)x_{t+1}+\frac{1-z}{\sigma}\left(r_t^s-\pi_{t+1}-r_t^n\right)-z\bar{b}_{cb}\left(qe_t-(1-\alpha)qe_{t+1}\right)\right)\\ &+\xi_{2,t}\left(\pi_t-\beta\pi_{t+1}-\gamma\left(\chi+\frac{\sigma}{1-z}\right)x_t+\frac{\gamma\sigma z\bar{b}_{cb}}{1-z}qe_t\right)\Bigg],\end{aligned} \tag{E.4}$$

where $\{\xi_{1,t}, \xi_{2,t}\}_{t=0}^{\infty}$ are sequences of Lagrangian multipliers, and where the law of iterated expectations has been used to eliminate the conditional expectation that appeared in each constraint.

Differentiating the Lagrangian with respect to x_t, π_t, r_t^s, and qe_t yields the optimality conditions:

$$\frac{\partial L}{\partial x_t}=\left(\chi+\frac{\sigma}{1-z}\right)x_t+\xi_{1,t}-\gamma\left(\chi+\frac{\sigma}{1-z}\right)\xi_{2,t}-\frac{1-\alpha}{\beta}\xi_{1,t-1}=0, \tag{E.5}$$

$$\frac{\partial L}{\partial \pi_t}=\frac{\varepsilon}{\gamma}\pi_t+\xi_{2,t}-\frac{1-z}{\beta\sigma}\xi_{1,t-1}-\xi_{2,t-1}=0, \tag{E.6}$$

$$\frac{\partial L}{\partial r_t^s}\left(r_t^s+\frac{R^s-1}{R^s}\right)=\frac{1-z}{\sigma}\xi_{1,t}\left(r_t^s+\frac{R^s-1}{R^s}\right)=0,\quad \xi_{1,t}\geq 0 \text{ and } r_t^s\geq-\frac{R^s-1}{R^s}, \tag{E.7}$$

$$\frac{\partial L}{\partial qe_t}=\frac{\sigma z\bar{b}_{cb}^2}{1-z}qe_t-z\bar{b}_{cb}\xi_{1,t}+\frac{\gamma\sigma z\bar{b}_{cb}}{1-z}\xi_{2,t}+\frac{z\bar{b}_{cb}(1-\alpha)}{\beta}\xi_{1,t-1}=0, \tag{E.8}$$

that must hold for $t = 0, 1, 2, \ldots$ and where $\xi_{1,-1} = \xi_{2,-1} = 0$, because Equations (E.2) and (E.3) corresponding to period -1 is not an effective constraint for the central bank choosing its optimal plan in period 0. In sum, the equilibrium conditions under the optimal commitment policy are then given by Equations (E.2), (E.3), (E.5), (E.6), (E.7), and (E.8).

F Simple Mandate

The central bank is assumed to choose a state-contingent sequence $\{x_t, \pi_t, r_t^s, qe_t\}_{t=0}^{\infty}$ that minimises:

$$\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left({\pi_t}^2+\vartheta {x_t}^2\right), \tag{F.1}$$

subject to the sequences of constraints and $r^s \geq 0$:

$$x_t = (1-\alpha)\,E_t x_{t+1} - \frac{1-z}{\sigma}\left(r_t^s - E_t\pi_{t+1} - r_t^n\right) + z\bar{b}_{cb}\left(qe_t - (1-\alpha)\,E_t qe_{t+1}\right), \tag{F.2}$$

$$\pi_t = \beta E_t \pi_{t+1} + \gamma\left(\chi + \frac{\sigma}{1-z}\right)x_t - \frac{\gamma\sigma z\bar{b}_{cb}}{1-z}qe_t, \tag{F.3}$$

In order to solve the previous problem it is useful to write down the associated Lagrangian, which is given by:

$$\begin{aligned} L = E_0\sum_{t=0}^{\infty}\beta^t\Bigg[\frac{1}{2}\left({\pi_t}^2+\vartheta {x_t}^2\right)& \\ +\xi_{1,t}\left(x_t-(1-\alpha)\,x_{t+1}+\frac{1-z}{\sigma}\left(r_t^s-\pi_{t+1}-r_t^n\right)-z\bar{b}_{cb}\left(qe_t-(1-\alpha)\,qe_{t+1}\right)\right)& \\ +\xi_{2,t}\left(\pi_t-\beta\pi_{t+1}-\gamma\left(\chi+\frac{\sigma}{1-z}\right)x_t+\frac{\gamma\sigma z\bar{b}_{cb}}{1-z}qe_t\right)\Bigg]&, \end{aligned} \tag{F.4}$$

where $\{\xi_{1,t}, \xi_{2,t}\}_{t=0}^{\infty}$ are sequences of Lagrangian multipliers, and where the law of iterated expectations has been used to eliminate the conditional expectation that appeared in each constraint.

Differentiating the Lagrangian with respect to x_t, π_t, r_t^s, and qe_t yields the optimality conditions:

$$\frac{\partial L}{\partial x_t} = \vartheta x_t + \xi_{1,t} - \gamma\left(\chi+\frac{\sigma}{1-z}\right)\xi_{2,t} - \frac{1-\alpha}{\beta}\xi_{1,t-1} = 0, \tag{F.5}$$

$$\frac{\partial L}{\partial \pi_t} = \pi_t + \xi_{2,t} - \frac{1-z}{\beta\sigma}\xi_{1,t-1} - \xi_{2,t-1} = 0, \tag{F.6}$$

$$\frac{\partial L}{\partial r_t^s}\left(r_t^s+\frac{R^s-1}{R^s}\right) = \frac{1-z}{\sigma}\xi_{1,t}\left(r_t^s+\frac{R^s-1}{R^s}\right) = 0, \quad \xi_{1,t}\geq 0 \text{ and } r_t^s \geq -\frac{R^s-1}{R^s}, \tag{F.7}$$

$$\frac{\partial L}{\partial qe_t} = -z\bar{b}_{cb}\xi_{1,t} + \frac{\gamma\sigma z\bar{b}_{cb}}{1-z}\xi_{2,t} + \frac{z\bar{b}_{cb}(1-\alpha)}{\beta}\xi_{1,t-1} = 0, \tag{F.8}$$

that must hold for $t = 0, 1, 2, \ldots$ and where $\xi_{1,-1} = \xi_{2,-1} = 0$, because Equations (F.2) and (F.3) corresponding to period -1 is not an effective constraint for the central bank choosing its optimal plan in period 0. In sum, the equilibrium conditions under the optimal policy are then given by Equations (F.2), (F.3), (F.5), (F.6), (F.7), and (F.8).