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Abstract

Standard New Keynesian models deliver puzzling results at the effective lower bound of short-term interest rates: greater price flexibility amplifies the fall in output in response to adverse demand shocks; labour tax cuts are contractionary; the multipliers of wasteful government spending are large. These outcomes stem from a failure to characterise monetary policy correctly. Both analytically and numerically, we show that allowing the central bank to respond to inflation with quantitative easing (QE) can resolve all these paradoxes. In quantitative terms, mild adjustments to the central bank's balance sheet are sufficient to obtain results more in line with conventional wisdom.

Key words: Policy paradoxes, unconventional monetary policy, quantitative easing, liquidity trap, effective lower bound.

JEL classification: E52, E58, E62, E63.

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1 Introduction

Since the Global Financial Crisis in 2008, nominal interest rates in the US and other advanced economies have approached the effective lower bound (ELB). This fact has naturally motivated a vast literature to analyse monetary and fiscal policies through the lenses of the standard New Keynesian (NK) model when the nominal policy rate is at the ELB.[1] Some papers (e.g., Eggertsson, 2010a, Eggertsson and Krugman, 2012, and Bhattarai et al., 2018) have highlighted how NK models imply seemingly paradoxical results when the ELB constraint is binding. More specifically, if the central bank cannot further reduce the policy rate to stabilise inflation, a fall in prices leads to a decline in inflation expectations, a rise in the real rate, and a fall in output. In such a context, greater price flexibility exacerbates the fall in inflation expectations and amplifies the drop in output in response to an adverse demand shock (the paradox of flexibility). Distortionary labour tax cuts, reducing inflation and inflation expectations, cause an increase in the real rate and a fall in output (the paradox of toil). Government spending, by increasing inflation and inflation expectations, reduces the real rate and, therefore, tends to have larger positive effects on output than away from the ELB (Christiano et al., 2011).

In this paper, we argue that all these results are not a fundamental flaw of the New Keynesian framework with sticky prices (as argued by Kiley, 2016) but rather a failure of standard models to characterise monetary policy in a liquidity trap correctly, i.e., monetary policy is completely unable to respond to shocks when the nominal rate is at the ELB. However, in practice, all the leading central banks have been deploying various forms of unconventional policies in response to nominal policy rates reaching their ELB. The most prominent unconventional measure has been large-scale asset purchases, commonly known as quantitative easing (QE). Figure 1 shows how, over the period from November 2008 to October 2014, when the effective federal funds rate (dotted black line) was at its ELB, the FED significantly increased the size of its balance sheet (solid blue line) by purchasing longer-term securities. This policy raised the price of long-term government and corporate bonds and a fall in their yields (dashed lines).[2]

Through the lenses of a tractable NK model along the lines of Sims et al. (2020), we analyse both analytically and numerically if, and under what conditions, QE can resolve the puzzling results that characterise the NK framework at the ELB. We show that, if QE follows an endogenous rule reacting to deviations in

[1] See, e.g., Nakov (2008), Nakata (2017), Masolo and Winant (2019), and Nakata et al. (2019).

[2] The effectiveness of QE and other unconventional monetary policies remains an open empirical question. For example, Debortoli et al. (2019) provide evidence of the efficacy of unconventional monetary policies and the "irrelevance" of the zero lower bound on the nominal policy rate, whereas, other papers (e.g., Fabo et al., 2020 and Giansante et al., 2020) argue that the contribution of QE to boosting lending and economic activity may have been quite limited.

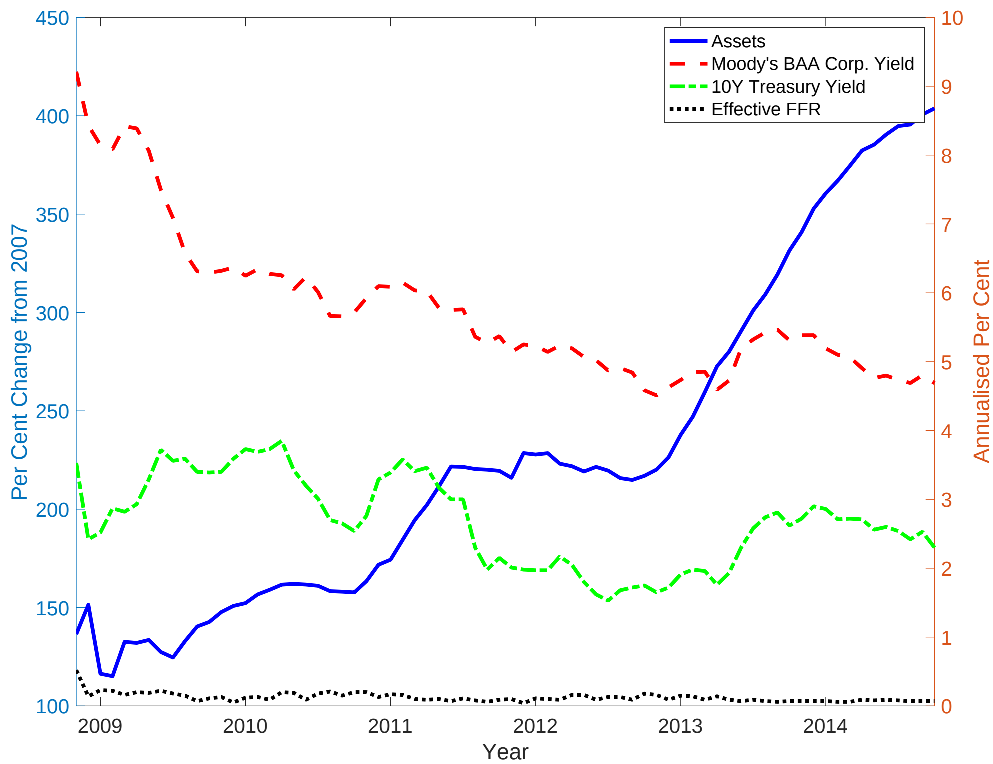


Figure 1: FED's Balance Sheet and Long-Term Bond Yields

Note: The figure displays the assets held by the FED (blue solid line), the ten year Treasury yield (red dashed line), Moody's Seasoned Baa Corporate Bond Yield (green dashed line) and the effective federal funds rate (black dotted line). Assets are expressed in percentage change from December 2007, which is taken as a reference point.

inflation from target, a mild adjustment in QE is sufficient to resolve the paradoxes, so that price flexibility mitigates the fall in output to an adverse demand shock, distortionary labour tax cuts become expansionary, and government spending multipliers are significantly smaller. We highlight that the results above are conditional on the policy rule and particularly rely on the central bank targeting inflation. When the monetary policy authority solely reacts to the output gap, deploying QE cannot resolve the paradoxes of flexibility and toil, whereas this policy can substantially mitigate the size of government spending multipliers.

Our numerical results show that: (i) when the inflation parameter in the QE rule is at least 20, implying an increase in real long-term bond holdings by about 11.6 per cent and a fall in their yield-to-maturity by 0.6 annualised percentage points in response to a 2.2 annualised percentage-point decrease in inflation, the paradox of flexibility disappears. Under this condition, in fact, the increase in QE is strong enough to offset the rise in the real rate due to the negative demand shock, thereby mitigating the fall in the output gap. In such a context, increasing flexibility amplifies the initial drop in inflation and induces a larger increase in QE, which further reduces the drop in the output gap. (ii) When the weight on inflation in the QE rule is at least 46 (i.e., a 28.8 per cent increase in QE and a 1.1 percentage-point decrease in the long-term bond yields in response to a 2.2 annualised percentage-point decrease in inflation), the increase in QE in response to (deflationary) labour tax cuts is sufficient to offset the rise in the real rate and increase output. (iii) Similarly, when the QE parameter is at least 46, the contraction in QE due to (inflationary) government spending shocks, is strong enough to counteract the fall in the real rate, thereby significantly mitigating the increase in output and making the multiplier smaller than one.

The results in this paper underscore the critical role of QE, in particular, and unconventional monetary policies, in general, in relaxing the constraints imposed by the ELB on nominal interest rates. We also show that accounting for the various policy tools adopted by central banks since the Global Financial Crisis is key to assess the effectiveness of policies aimed at reducing nominal rigidities, as well as fiscal policies, such as tax cuts and increases in government spending.

Related Literature Our paper is strictly related to the literature on policy paradoxes in the New Keynesian model in a liquidity trap. In particular, Eggertsson and Krugman (2012) highlight how at the ELB price flexibility increases real activity volatility in response to deflationary shocks, whereas Eggertsson (2010a) points out the existence of the paradox of toil. Christiano et al. (2011), Eggertsson (2010b), and Woodford (2011) report large government spending multipliers when the economy is in a liquidity trap because monetary policy cannot counteract the increase in inflation and the output gap. The analysis in our paper is closely related to Kiley (2016), who revisits the paradoxes mentioned above and argues that by replacing the sticky-price assumption with sticky information à la Mankiw and Reis (2002) these results disappear. Eggertsson and Garga (2019), however, show that Kiley (2016)'s findings hold only when the policy rate is exogenously pegged for a specified period. Furthermore, concerning the paradox of toil and the sizable government spending multipliers, one critical assumption is that the peg duration is longer than the duration of the fiscal shock. When the ELB constraint is hit because of some fundamental shock or the duration of the fiscal shock matches that of the ELB, then the sticky-information model ends up exacerbating these paradoxes. Cochrane (2017) argues that the paradoxes arising in NK models with a binding ELB are strongly affected by the equilibrium selection and that equilibria, which bound initial jumps, such as those implied by fiscal considerations, do not have these puzzling implications. In our paper, we argue that these results rely on the counterfactual assumption that monetary policy is completely unable to react to macroeconomic shocks when the nominal rate is at its ELB and that relaxing this assumption removes the paradoxes mentioned above.

Our paper closely relates to the papers suggesting solutions to the paradoxes by correcting the monetary policy rule. Hills and Nakata (2018) and Bonciani and Oh (2020) show that in an NK model where the central bank smoothly adjusts its shadow policy rate, the government spending multiplier at the ELB becomes significantly smaller, and the paradox of flexibility disappears. The reason is that, under an inertial monetary policy, a fall (rise) in the notional rate signals that the actual policy rate is going to stay lower (higher) for longer. Therefore, this policy rule can be interpreted as a form of forward guidance, which significantly mitigates the constraints imposed by the ELB. Wu and Zhang (2019) show that a model, in which the shadow policy rate summarises various unconventional monetary policies, does not present the paradox of toil and large government spending multipliers. Our work departs from these papers, as we provide the qualitative and quantitative conditions necessary for QE to solve the three paradoxes mentioned above.

Our work builds on the literature analysing QE within DSGE models. Gertler and Karadi (2011), Gertler and Karadi (2013), and Carlstrom et al. (2017) among others incorporated QE into medium-scale DSGE models. Cui and Sterk (2018) analyse the impact of QE in an NK model with heterogeneous agents and find that QE is highly stimulative and successfully mitigated the drop in demand during the Great Recession. However, their paper suggests that QE could, as a byproduct, significantly increase inequality and thereby reduce welfare. Sims and Wu (2020b) build on Gertler and Karadi (2013)'s modelling of the financial sector to analyse the impact and interaction of the main forms of unconventional monetary policy (QE, forward guidance, and negative interest rates). Sims et al. (2020) develops a tractable four-equation NK model that accounts for QE, whereas Sims and Wu (2020a) deploys this framework to study the degree of substitutability between conventional monetary policy and QE. Finally, Sims and Wu (2020c) show that in times where the production sector is facing significant cash-flow shortages, such as the COVID-19 crisis, QE should be aimed at lending to non-financial corporations ("Main Street QE") rather than banks ("Wall Street QE").

The remainder of the paper is structured as follows. In Section 2, we describe the NK model with QE. Section 3 presents analytical results, based on a two-period version of the model, and provides a graphical intuition. In Section 4, we set out our numerical analysis. Finally, in Section 5, we provide some concluding remarks.

2 The Model

We consider a model based on Sims et al. (2020), which extends the basic three-equation framework to allow for QE. In its nonlinear form, the model includes two types of agents, patient and impatient, and financial intermediaries (modelled along the lines of Gertler and Karadi, 2011) subject to a risk-weighted leverage constraint. It features short and long-term bonds, which, combined with the credit frictions, allows QE to affect real activity. Besides setting the short-term interest rate, the central bank can purchase long-term bonds, thereby expanding its balance sheet. The increase in the price of long-term bonds, as a consequence of the unconventional monetary policy, relaxes the financial intermediary's leverage constraint, which eases credit supply. Therefore, in such a framework, QE is equivalent to a positive credit supply. By affecting both aggregate demand and aggregate supply, QE is only an imperfect substitute of conventional monetary policy, which, instead, just affects aggregate demand. For the purpose of our analysis on the NK policy paradoxes, we augment this model to account for distortionary labour income taxes and government spending, and abstract from credit shocks. Furthermore, we make a similar modelling assumption about the slope of the New Keynesian Phillips curve (NKPC) following Woodford (2003).[3] For the sake of conciseness, we leave the details of the nonlinear model to Appendix A and its log-linearised version to Appendix B.

The log-linearised model boils down to four equations (1)-(4). Equations (1) and (2) represent the dynamic IS curve and the NKPC. The other main equations are a strict inflation targeting rule when the policy rate is above its ELB, Equation (3), and a QE rule, Equation (4).

$$x_t = E_t x_{t+1} - \frac{(1-s_g)(1-z)}{\sigma}\left(r_t^s - E_t\pi_{t+1} - r_t^n\right) + g_t - E_t g_{t+1} + (1-s_g)\, z\bar{b}^{cb}\left(qe_t - E_t qe_{t+1}\right), \tag{1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \left(\gamma\chi + \frac{\gamma\sigma}{(1-s_g)(1-z)}\right)x_t - \frac{\gamma\sigma}{(1-s_g)(1-z)}g_t + \frac{\gamma}{1-T}\tau_t - \frac{\gamma\sigma z\bar{b}^{cb}}{1-z}qe_t, \tag{2}$$

$$\pi_t = 0, \quad \text{s.t.} \quad r_t^s > -\frac{R^s-1}{R^s}, \tag{3}$$

$$qe_t = -\vartheta\pi_t. \tag{4}$$

Lowercase variables denote log deviations around the non-stochastic steady state. π_t is inflation and $x_t = y_t - y_t^f$ denotes the output gap, where y_t^f is the potential level of output, i.e., the equilibrium level of output that would arise in a model with flexible prices. r_t^s is the nominal policy rate, whereas qe_t denotes the real value of the central bank's long-term bond holdings. We assume that the natural rate of interest, r_t^n, the distortionary labour tax rate, τ_t, and the government consumption share of output, g_t, follow exogenous processes.[4] Similarly as in Eggertsson (2010b), we are implicitly assuming that Ricardian equivalence holds and lump-sum tax adjustments clear the government budget in response to changes in distortionary labour

[3] While in the version of the three-equation model in Galí (2015), the utility function includes aggregate labour, in Woodford (2003) the utility function has industry-specific labour.

[4] In particular, $g_t \equiv \frac{G_t - G}{Y}$ and $\tau_t \equiv T_t - T$, where G_t and T_t are the levels of government spending and labour tax rate. G, T, and Y are the steady-state values of government spending, labour tax rate, and output level.

taxes or government spending.

The parameter σ represents the inverse elasticity of intertemporal substitution, β is the discount factor of the patient households, and χ is the inverse labour supply elasticity. γ is a convolution of deep parameters defined as $\gamma \equiv \frac{(1-\phi)(1-\phi\beta)}{\phi(1+\chi\epsilon)}$, where ϕ is the price adjustment probability (Calvo, 1983) and ϵ is the elasticity of the intermediate good's demand. The parameter z is the share of total consumption going to the impatient household, $\bar{b}^{cb}$ is the steady-state share of long-term bonds that are held by the central bank, T the steady-state labour income tax rate and s_g represents the steady-state government spending to output ratio. Finally, the coefficient ϑ determines the responsiveness of the central bank's asset purchases to inflation.

It is important to stress again that a critical difference between QE and conventional monetary policy is that qe_t enters both the dynamic IS curve (1) and the NKPC (2). On the one hand, an increase in qe_t affects demand positively, by entering the IS with a positive sign, thereby putting upward pressure on both output and inflation. On the other hand, because of the negative sign in the NKPC, a rise in qe_t also curbs inflation and positively impacts output. Because of these conflicting channels, an increase in QE tends to be less inflationary than a cut in the policy rate, which only enters the IS curve (Sims et al., 2020).

3 Analytical Results

3.1 Assumptions

Our analytical results are based on a two-period version of the model, making the following simplifying assumptions:

1. Shock: $r_1^n < 0$, $r_1^s = -\frac{R^s-1}{R^s}$, $r_2^n = 0$,
2. Fiscal policy: $(\tau_1, g_1) = (\tau_1, g_1)$, $(\tau_2, g_2) = (0, 0)$,
3. Short-term interest rate policy (Strict inflation targeting): $\pi_2 = 0$,
4. QE: $qe_1 = -\vartheta\pi_1$,
5. Perfect foresight: $E_t x_{t+1} = x_{t+1}$, $E_t \pi_{t+1} = \pi_{t+1}$.

Under the assumptions above, the two-period version of the model writes as:

$$x_1 = x_2 - \frac{(1-s_g)(1-z)}{\sigma}\left(r_1^s - \pi_2 - r_1^n\right) + g_1 - g_2 + (1-s_g)\, z\bar{b}^{cb}\left(qe_1 - qe_2\right), \tag{5}$$

$$\pi_1 = \beta\pi_2 + \left(\gamma\chi + \frac{\gamma\sigma}{(1-s_g)(1-z)}\right)x_1 - \frac{\gamma\sigma}{(1-s_g)(1-z)}g_1 + \frac{\gamma}{1-T}\tau_1 - \frac{\gamma\sigma z\bar{b}^{cb}}{1-z}qe_1, \tag{6}$$

$$r_1^s = -\frac{R^s-1}{R^s}, \tag{7}$$

$$qe_1 = -\vartheta\pi_1, \tag{8}$$

$$x_2 = \pi_2 = r_2^s = qe_2 = \tau_2 = g_2 = 0. \tag{9}$$

From Equation (5), (7), (8), and (9) we can obtain the aggregate demand (AD) curves with ($\vartheta > 0$) and without ($\vartheta = 0$) QE, as expressed by Equation (10) and (11):

$$x_1 = \frac{(1-s_g)(1-z)}{\sigma}\left(\frac{R^s-1}{R^s} + r_1^n\right) + g_1 - (1-s_g)\,z\bar{b}^{cb}\vartheta\pi_1, \tag{10}$$

$$x_1 = \frac{(1-s_g)(1-z)}{\sigma}\left(\frac{R^s-1}{R^s} + r_1^n\right) + g_1. \tag{11}$$

We note from Equation (11) that aggregate demand does not depend on inflation (vertical demand) when there is no QE in place. When the central bank can deploy QE to respond to inflation, instead, aggregate demand, defined by Equation (10), is a negative function of inflation and is therefore represented by a downward-sloping curve. Equations (12) and (13), obtained by combining Equations (6), (8), and (9), describe the aggregate supply (AS) curves with and without QE:

$$\pi_1 = \left[\left(\gamma\chi + \frac{\gamma\sigma}{(1-s_g)(1-z)}\right)x_1 - \frac{\gamma\sigma}{(1-s_g)(1-z)}g_1 + \frac{\gamma}{1-T}\tau_1\right] \Big/ \left[1 - \frac{\gamma\sigma z\bar{b}^{cb}\vartheta}{1-z}\right], \tag{12}$$

$$\pi_1 = \left(\gamma\chi + \frac{\gamma\sigma}{(1-s_g)(1-z)}\right)x_1 - \frac{\gamma\sigma}{(1-s_g)(1-z)}g_1 + \frac{\gamma T}{1-T}\tau_1. \tag{13}$$

In both cases, the resulting AS curves are upward-sloping. In Equation (12), the coefficient multiplying x_1 is larger than in Equation (13), meaning that QE implies a steeper AS curve.[5]

Given the AD and AS curves above, we derive the equilibrium output gap $x_1^\star$:

$$\begin{aligned}(x_1^\star)_{\vartheta>0} = \Bigg[&\frac{(1-s_g)(1-z)}{\sigma}\left(\frac{R^s-1}{R^s} + r_1^n\right) + \left(1 + \frac{(1-s_g)\,z\bar{b}^{cb}\vartheta}{1-\frac{\gamma\sigma z\bar{b}^{cb}\vartheta}{1-z}}\frac{\gamma\sigma}{(1-s_g)(1-z)}\right)g_1 \\ &- \frac{(1-s_g)\,z\bar{b}^{cb}\vartheta}{1-\frac{\gamma\sigma z\bar{b}^{cb}\vartheta}{1-z}}\frac{\gamma}{1-T}\tau_1\Bigg] \Big/ \left[1 + \frac{(1-s_g)\,z\bar{b}^{cb}\vartheta}{1-\frac{\gamma\sigma z\bar{b}^{cb}\vartheta}{1-z}}\left(\gamma\chi + \frac{\gamma\sigma}{(1-s_g)(1-z)}\right)\right],\end{aligned} \tag{14}$$

[5] More precisely, QE implies a steeper AS curve if and only if $0 < \frac{\gamma\sigma z\bar{b}^{cb}\vartheta}{1-z} < 1$.

$$(x_1^\star)_{\vartheta=0} = \frac{(1-s_g)(1-z)}{\sigma}\left(\frac{R^s-1}{R^s}+r_1^n\right)+g_1. \tag{15}$$

We use these equations to derive the impact of varying price flexibility and solve for the fiscal multipliers. It bears noting that, in the simple NK model with a one-period deterministic fundamental shock, policy paradoxes take a weaker form (see also, e.g., Eggertsson and Garga, 2019). In particular, raising price flexibility and cutting labour taxes do not affect output, whereas increases in government spending raise output one-for-one.

3.2 The Paradox of Flexibility

From Equations (14) and (15) we can then analytically show that the period-1 equilibrium output gap $x_1^\star$ is smaller, in absolute value, when the central bank deploys QE to stabilise inflation ($\vartheta > 0$) compared to the case where it does not ($\vartheta = 0$). In other words, QE mitigates the decline in the output gap. This result is expressed in Equation (16) with the assumption that $(\tau_1, g_1) = (0,0)$:[6]

$$\begin{aligned}\left|(x_1^\star)_{\vartheta>0}\right| &= \left|\left[\frac{(1-s_g)(1-z)}{\sigma}\left(\frac{R^s-1}{R^s}+r_1^n\right)\right] \Big/ \left[1+\frac{(1-s_g)\,\gamma z\bar{b}^{cb}\vartheta}{1-\frac{\gamma\sigma z\bar{b}^{cb}\vartheta}{1-z}}\left(\chi+\frac{\sigma}{(1-s_g)(1-z)}\right)\right]\right| \\ &< \left|(x_1^\star)_{\vartheta=0}\right| = \left|\frac{(1-s_g)(1-z)}{\sigma}\left(\frac{i}{1+i}+r_1^n\right)\right|. \end{aligned} \tag{16}$$

As derived in Equation (17), when $\vartheta > 0$, the absolute value of x_1 decreases in γ, and, therefore, increases in the Calvo parameter ϕ.[7] This means that, when the central bank engages in QE as a mean to stabilise prices, greater price flexibility reduces the size of the fall in the output gap:

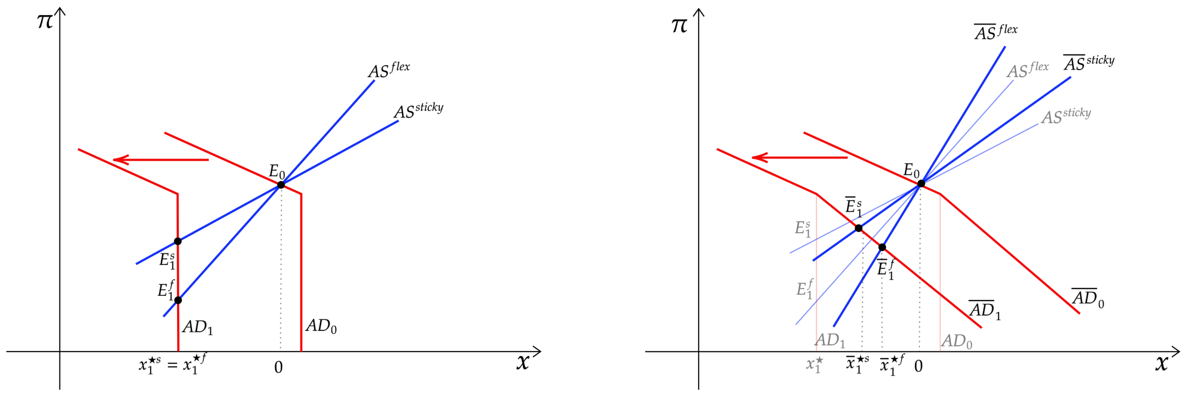
$$\left(\frac{d\left|x_1^\star\right|}{d\gamma}\right)_{\vartheta>0} = -\frac{\frac{(1-s_g)(1-z)}{\sigma}\left|\frac{R^s-1}{R^s}+r_1^n\right|(1-s_g)\,z\bar{b}^{cb}\vartheta\left(\chi+\frac{\sigma}{(1-s_g)(1-z)}\right)}{\left[(1-s_g)\,z\bar{b}^{cb}\vartheta\left(\gamma\chi+\frac{\gamma\sigma}{(1-s_g)(1-z)}\right)+1-\frac{\gamma\sigma z\bar{b}^{cb}\vartheta}{1-z}\right]^2} < 0. \tag{17}$$

Equation (18) shows that, when $\vartheta = 0$, instead, an increase in γ (i.e., a decrease in ϕ) does not affect the equilibrium output gap x_1. In other words, in the two-period model and the absence of QE, greater price flexibility does not affect the size of the fall in the output gap due to the adverse demand shock:

$$\left(\frac{d\left|x_1^\star\right|}{d\gamma}\right)_{\vartheta=0} = 0. \tag{18}$$

To understand the intuition behind the paradox of flexibility and how QE can correct it, consider first the model equations (5)-(9). The fall in the natural rate causes a drop in x_1 via the IS curve, which, in turn,

[6] Fiscal variables are only necessary for the analysis of the paradox of toil and the large government spending multiplier.
[7] For $0 < \phi < 1$, γ is a negative function of ϕ.



(a) Without QE

(b) With QE

Figure 2: The Paradox of Flexibility and QE

Note: The figures display the effect of a negative rate shock that causes the ELB constraint to bind. AD and AS represent aggregate demand and supply in the absence of QE, whereas $\overline{AD}$ and $\overline{AS}$ are aggregate demand and supply when the central bank can use QE. Superscript $flex$ and $sticky$ denote different degrees of price stickiness. The y-axis is inflation (π), while the x-axis is the output gap (x).

reduces π_1. An increase in flexibility (i.e., an increase in γ) amplifies the fall in inflation π_1. The response of the output gap instead, depends on the policy response. In the absence of QE, i.e., $qe_1 = 0$, the monetary policy authority cannot respond to the fall in π_1 by reducing r_1^s and is therefore unable to affect the real rate and hence the output gap x_1. As a result, the response of x_1 is independent of price flexibility. When the central bank can respond to inflation by increasing qe_1, the larger is the fall in inflation (because of the increase in price flexibility), the more substantial the increase in qe_1, which has a positive effect on x_1. Therefore, with QE, price flexibility mitigates the drop in the output gap caused by the fall in demand.

The two panels of Figure 2 show the intuition above in an AD-AS diagram. We display the effect of a negative natural rate shock that leads the nominal policy rate to hit its ELB constraint. Given the ELB constraint, the aggregate demand curves display a kink[8] and slope change for some low value of inflation. The sections of the AD curves when the nominal policy rate is at its ELB are defined by Equations (10) and (11). In the absence of QE (left panel), the AD curve is vertical when the economy is in a liquidity trap (see Equation, 11). An exogenous decline in the natural rate shifts AD_0 (aggregate demand at time 0) to the left (AD_1). An increase in price flexibility makes the aggregate supply curve steeper, i.e., AS^{sticky} becomes AS^{flex}. The two equilibria under stickier and more flexible prices are labelled respectively E_1^s, and E_1^f and both points lie on the vertical section of AD_1. Hence, in the simplified two-period version of the model,

[8]It bears noting that the section of the AD, where the interest rate is larger than zero, should be horizontal. For illustrative purposes, we assume it is downward sloping.

greater price flexibility implies a stronger fall in prices, whereas the fall in the output gap is unaffected, as shown analytically in Equation (18). Under QE instead (right panel), the AD curve is not vertical when the nominal rate is at its ELB (see Equation, 10) and its slope depends on how strongly QE responds to inflation. The initial shock shifts the downward-sloping demand curve $\overline{AD}_0$ to the left ($\overline{AD}_1$). Similarly, as above, greater price flexibility makes the AS curve steeper, i.e., $\overline{AS}^{sticky}$ becomes $\overline{AS}^{flex}$.[9] Hence, the resulting equilibria under stickier and more flexible prices are now given by points $\overline{E}_1^s$ and $\overline{E}_1^f$. Hence, when the central bank can carry out QE, the exogenous fall in demand causes a more substantial decline in the output gap when prices are relatively stickier.

3.3 The Paradox of Toil

From Equation (14) we can obtain an analytical expression of the response of the output gap ($x_1^\star$) to a decrease in distortionary labour taxes (τ_1). When the monetary authority actively responds to inflation with QE, a reduction in taxes increases the output gap, as shown in Equation (19):

$$\left(\frac{dx_1^\star}{d\tau_1}\right)_{\vartheta>0} = -\left[\frac{(1-s_g)\, z\bar{b}^{cb}\vartheta}{1-\frac{\gamma\sigma z\bar{b}^{cb}\vartheta}{1-z}}\frac{\gamma}{1-T}\right] \Big/ \left[1+\frac{(1-s_g)\, z\bar{b}^{cb}\vartheta}{1-\frac{\gamma\sigma z\bar{b}^{cb}\vartheta}{1-z}}\left(\gamma\chi+\frac{\gamma\sigma}{(1-s_g)\,(1-z)}\right)\right] < 0. \tag{19}$$

Moreover, the stronger the reaction of the monetary authority to inflation with QE (the larger ϑ) the more negative the effects of taxes on the output gap:

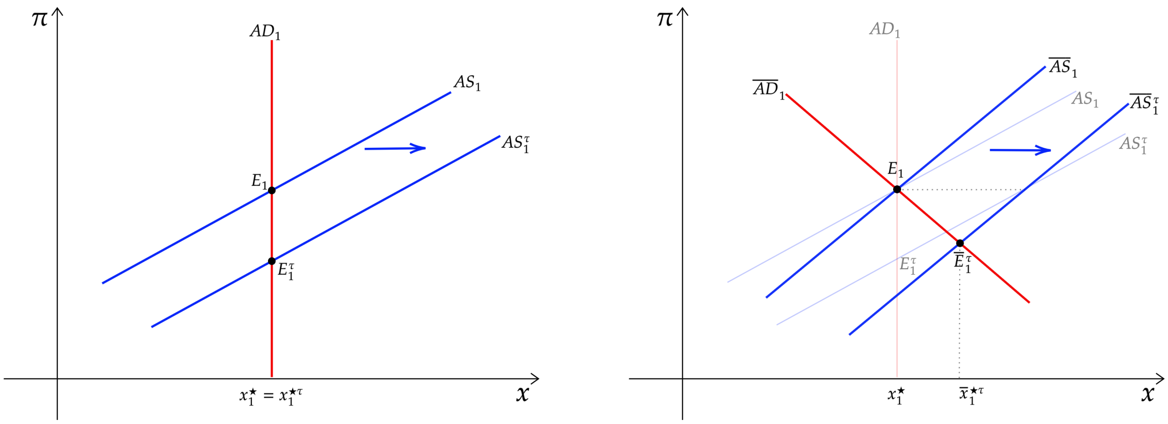
$$\frac{d\left(\frac{dx_1^\star}{d\tau_1}\right)_{\vartheta>0}}{d\vartheta} = -\frac{(1-s_g)\, z\bar{b}^{cb}\frac{\gamma}{1-T}}{\left[(1-s_g)\, z\bar{b}^{cb}\vartheta\left(\gamma\chi+\frac{\gamma\sigma}{(1-s_g)(1-z)}\right)+1-\frac{\gamma\sigma z\bar{b}^{cb}\vartheta}{1-z}\right]^2} < 0. \tag{20}$$

Instead, when the monetary policy authority does not respond to inflation with QE ($\vartheta = 0$), the paradox of toil occurs. In particular, in this two-period setting, a tax cut does not have any impact on the output gap:

$$\left(\frac{dx_1^\star}{d\tau_1}\right)_{\vartheta=0} = 0. \tag{21}$$

To understand the intuition behind these results, consider consider first the model equations (5)-(9). A reduction in τ_1 causes a drop in period-1 inflation, π_1, via the NKPC equation. In the absence of QE (i.e., $qe_1 = 0$), the cut in the labour tax rate has no effect on x_1, as can be seen from the IS equation. The reason is that the central bank cannot further reduce the interest rate r_1^s to respond to the lower inflation rate π_1 and has, therefore, no control over the real rate and cannot boost real activity x_1. When the central bank

[9] It is important to note that in the presence of QE, the AS curves are steeper than in the case without QE, as shown in Equation (12).



(a) Without QE

(b) With QE

Figure 3: The Paradox of Toil and QE

Note: The figures display the effect of a distortionary labour tax cut at the ELB. AD and AS represent aggregate demand and supply in the absence of QE, whereas $\overline{AD}$ and $\overline{AS}$ are aggregate demand and supply when the central bank can use QE. The y-axis is inflation (π), while the x-axis is the output gap (x).

can carry out QE, instead, the fall in π_1 leads to a rise in qe_1, which boosts the output gap x_1. Thus, under QE, labour tax cuts become expansionary.

The two panels of Figure 3 show graphically why the paradox of toil does not occur under QE. In particular, they display the effects of a cut in distortionary labour taxes when the policy rate is at its ELB (period 1). A tax cut is a positive supply shock, as it incentivises people to work more, putting downward pressure on real wages and inflation. In the absence of QE (left panel), the cut in labour taxes shifts the aggregate supply curve, AS_1, to the right (AS_1^τ). Given that the vertical aggregate demand AD_1 is unaffected, the equilibrium E_1^τ features a lower inflation rate, compared to the initial state E_1, but the same level of the output gap. Under QE, the aggregate demand curve is downward-sloping ($\overline{AD}_1$) and the aggregate supply curve, $\overline{AS}_1$, is steeper than without QE. The shift in $\overline{AS}_1$ to the right ($\overline{AS}_1^\tau$) leads the new equilibrium, $\overline{E}_1^\tau$, to be characterised by lower inflation and a larger output gap than E_1.

3.4 Large Government Spending Multipliers

Similarly as above, from Equation (14) we can analytically derive the government spending multiplier as the derivative of the output gap $x_1^\star$ with respect to government spending g_1. When the monetary policy authority uses QE to stabilise inflation ($\vartheta > 0$), the resulting multiplier is smaller than one, as derived in

Equation (22):

$$\left(\frac{dx_1^\star}{dg_1}\right)_{\vartheta>0} = \left[1+\frac{(1-s_g)\,z\bar{b}^{cb}\vartheta}{1-\frac{\gamma\sigma z\bar{b}^{cb}\vartheta}{1-z}}\frac{\gamma\sigma}{(1-s_g)\,(1-z)}\right] \Big/ \left[1+\frac{(1-s_g)\,z\bar{b}^{cb}\vartheta}{1-\frac{\gamma\sigma z\bar{b}^{cb}\vartheta}{1-z}}\left(\gamma\chi+\frac{\gamma\sigma}{(1-s_g)\,(1-z)}\right)\right] < 1. \tag{22}$$

Equation (23) represents the derivative of the government spending multiplier with respect to the weight on inflation in the QE rule (ϑ). This shows how a stronger monetary policy response to inflation through QE reduces the size of the multiplier:

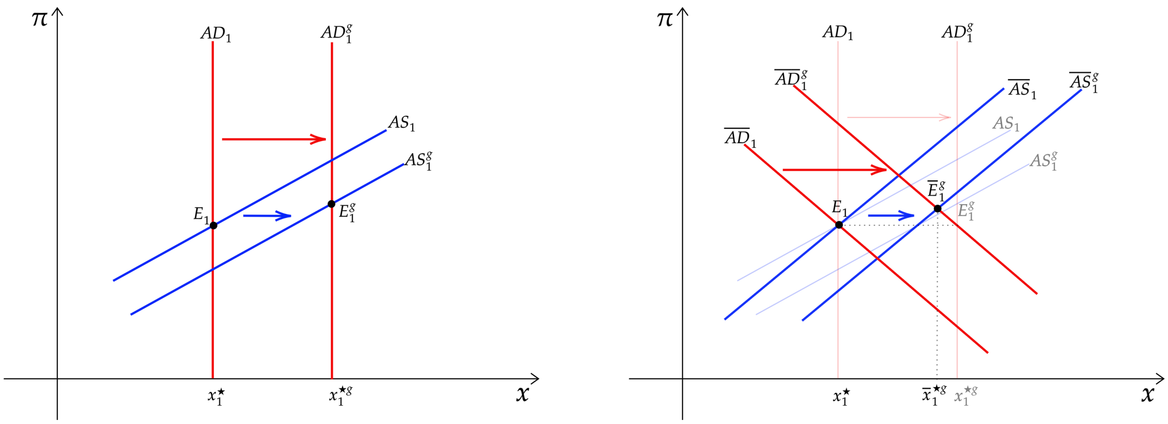
$$\frac{d\left(\frac{dx_1^\star}{dg_1}\right)_{\vartheta>0}}{d\vartheta} = -\frac{(1-s_g)\,z\bar{b}^{cb}\gamma\chi}{\left[(1-s_g)\,z\bar{b}^{cb}\vartheta\left(\gamma\chi+\frac{\gamma\sigma}{(1-s_g)(1-z)}\right)+1-\frac{\gamma\sigma z\bar{b}^{cb}\vartheta}{1-z}\right]^2} < 0. \tag{23}$$

When the monetary policy authority does not stabilise inflation using QE ($\vartheta = 0$), the resulting multiplier is exactly one:

$$\left(\frac{dx_1^\star}{dg_1}\right)_{\vartheta=0} = 1. \tag{24}$$

To gain intuition behind the results above, consider first the model equations (5)-(9). An increase in g_1 has a positive effect on x_1 via the IS equation. As can be seen from the NKPC equation, the expansion in government spending has both a negative direct effect on inflation and a stronger positive indirect effect via x_1. In the absence of QE ($qe_1 = 0$), the monetary policy authority cannot react to the rise in π_1, since the nominal rate is at its ELB. Therefore, the real rate remains unaffected and does not crowd out the impact of the government spending shock. Thus, x_1 increases one-for-one with g_1. When the central bank can respond to the rise in inflation with QE, the decrease in qe_1 puts downward pressure on x_1 and mitigates the effect of the government spending shock. Therefore, under QE, government spending multipliers will be smaller than in a standard liquidity trap, where the central bank is unable to respond to the rise in inflation.

The two panels of Figure 4 explain graphically why government spending multipliers are smaller when the monetary policy authority can deploy QE to respond to changes in inflation. In the absence of QE (left panel), the government spending shock leads the vertical AD curve (AD_1) to shift to the right. The new AD curve is labelled AD_1^g. Furthermore, to a lesser extent, the shock also shifts the AS curve (AS_1) to the right (AS_1^g). The reason is that government spending takes away resources from private consumption, which pushes agents to work more to make up for it partially. This channel shifts out labour supply and reduces real wages. The new equilibrium is given by point E_1^g, the intersection of AD_1^g and AS_1^g. When QE is in place (right panel), the government spending shock shifts the downward-sloping curve $\overline{AD}_1$ to the right ($\overline{AD}_1^g$). The AS curve ($\overline{AS}_1$) is slightly steeper as a result of QE and shifts to the right ($\overline{AS}_1^g$). The new



(a) Without QE

(b) With QE

Figure 4: Large Government Spending Multipliers and QE

Note: The figures display the effect of a positive government spending shock at the ELB. AD and AS represent aggregate demand and supply in the absence of QE, whereas $\overline{AD}$ and $\overline{AS}$ are aggregate demand and supply when the central bank can use QE. The y-axis is inflation (π), while the x-axis is the output gap (x).

resulting equilibrium is now $\overline{E}_1^g$. Comparing equilibria E_1^g and $\overline{E}_1^g$, we can see that the government spending shock increases output by less when the demand curve is downward-sloping (i.e., under QE). Under QE, the effects on inflation will depend on how steep the AS and AD curves are.

3.5 QE Targeting the Output Gap

The results above show that, if QE is used as a tool to stabilise inflation, the paradoxes can be resolved. If the monetary policy authority, instead, uses QE to adjust the output gap, i.e.:

$$qe_1 = -\vartheta^x x_1, \tag{25}$$

then the AD and AS equations for period 1 write as:

$$x_1 = \frac{\frac{(1-s_g)(1-z)}{\sigma}\left(\frac{R^s-1}{R^s} + r_1^n\right) + g_1}{1 + (1-s_g)\, z\bar{b}^{cb}\vartheta^x}, \tag{26}$$

$$\pi_1 = \left(\gamma\chi + \frac{\gamma\sigma}{(1-s_g)\,(1-z)} + \frac{\gamma\sigma z\bar{b}^{cb}\vartheta^x}{1-z}\right)x_1 - \frac{\gamma\sigma}{(1-s_g)\,(1-z)}g_1 + \frac{\gamma}{1-T}\tau_1. \tag{27}$$

It is clear from the equations above, that the presence of QE in this case does not affect the slope of the AD curve, which remains vertical (in the π-x diagram), like in a liquidity trap. The AS curve, instead, becomes steeper. Since the AD curve is vertical, the equilibrium output gap is simply determined by the AD curve:

$$(x_1^\star)_{\vartheta^x>0} = \frac{\frac{(1-s_g)(1-z)}{\sigma}\left(\frac{R^s-1}{R^s} + r_1^n\right) + g_1}{1 + (1-s_g)\, z\bar{b}^{cb}\vartheta^x}. \tag{28}$$

Similarly as in the absence of QE, an increase in γ (equivalent to a reduction in the price rigidity parameter) does not affect the equilibrium output gap $x_1^\star$. Therefore, greater price flexibility does not affect the magnitude of the output-gap response to a fall in the natural rate:

$$\left(\frac{d\,|x_1^\star|}{d\gamma}\right)_{\vartheta^x>0} = \left(\frac{d\,|x_1^\star|}{d\gamma}\right)_{\vartheta^x=0} = 0. \tag{29}$$

Moreover, the equilibrium output is independent of the labour tax shock τ_1:

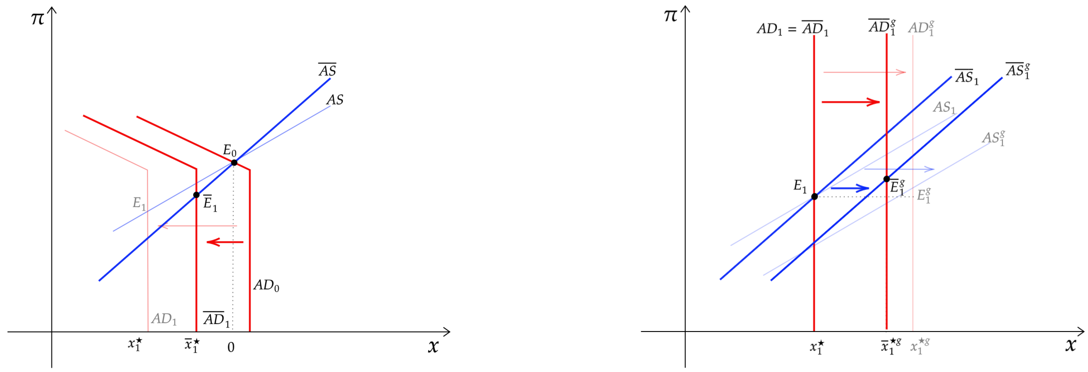
$$\left(\frac{dx_1^\star}{d\tau_1}\right)_{\vartheta^x>0} = \left(\frac{dx_1^\star}{d\tau_1}\right)_{\vartheta^x=0} = 0. \tag{30}$$

In other words, when QE is set according to a policy rule as in (25), the paradox of flexibility and the paradox of toil cannot be resolved, just like in the standard liquidity trap case without QE. After a fall in the natural rate, x_1 drops via the IS curve, which, in turn, reduces π_1. An increase in flexibility (i.e., an increase in γ) amplifies the fall in inflation π_1. Given that monetary policy does not respond to inflation, the decline in x_1 will be independent of the price rigidity. It bears noting that the drop in x_1 calls for a monetary policy expansion via QE, which mitigates the overall drop in the output gap. This effect can be seen graphically in the left panel of Figure 5. Under QE, the fall in demand ($AD_0 \to \overline{AD}_1$) is more muted than in the absence of QE ($AD_0 \to AD_1$). The intuition why the alternative QE rule cannot correct the paradox of toil is analogous to the case without QE discussed previously. A distortionary labour tax reduces inflation π_1. Since the central bank does not adjust QE in response to the fall in inflation, the tax cut cannot boost the output gap.

Unlike for the two paradoxes discussed above, the alternative QE rule can reduce the size of the government spending multiplier. We show this formally below:

$$\left(\frac{dx_1^\star}{dg_1}\right)_{\vartheta^x>0} = \frac{1}{1 + (1-s_g)\, z\bar{b}^{cb}\vartheta^x} < 1, \tag{31}$$

$$\frac{d\left(\frac{dx_1^\star}{dg_1}\right)_{\vartheta^X>0}}{d\vartheta^x} = -\frac{(1-s_g)\, z\bar{b}^{cb}}{\left[1 + (1-s_g)\, z\bar{b}^{cb}\vartheta^x\right]^2} < 0, \tag{32}$$



(a) Demand Shock

(b) Government Spending Shock

Figure 5: Alternative QE Rule

Note: Figure (a) displays the effect of a negative natural real rate that causes the ELB constraint to bind. Figure (b) displays the effect of a positive government spending shock under an output-targeting QE rule. AD and AS represent aggregate demand and supply in the absence of QE, whereas $\overline{AD}$ and $\overline{AS}$ are aggregate demand and supply when the central bank can use QE. The y-axis is inflation (π), while the x-axis is the output gap (x).

$$\left(\frac{dx_1^\star}{dg_1}\right)_{\vartheta^x=0} = 1. \tag{33}$$

Intuitively, the reason is that the government spending shock positively affects the output gap by boosting aggregate demand and aggregate supply. The rise in the output gap calls for a reduction in QE, which mitigates the expansionary effects of the shock. We display this result in the right panel of Figure 5. In the presence of QE, the shifts in aggregate demand ($\overline{AD}_1 \rightarrow \overline{AD}_1^g$) and aggregate supply ($\overline{AS}_1 \rightarrow \overline{AS}_1^g$) are more muted than in the absence of QE ($AD_1 \rightarrow AD_1^g$ and $AS_1 \rightarrow AS_1^g$).

4 Numerical Analysis

4.1 Calibration and Solution

In this section, we consider a numerical exercise based on the infinite-period model. We parameterise the model using standard values in the literature, as listed in Table 1. Some parameters specific to the four-equation model are taken from Sims et al. (2020). In particular, we set the consumption share of the impatient household z to 0.33. The steady-state share of central bank's long-term bond holdings, $\bar{b}^{cb}$, is set to 0.3. The rest of the parameters are in line with Eggertsson and Garga (2019). In particular, the standard deviation of the natural rate shock σ_{rn} is set to -0.0364 so to generate a 10 per cent drop in output and annualised 2.2 percentage-point drop in inflation in the model without QE, whereas the persistence parameter ($\rho_{rn} = 0.85$) is calibrated to keep the ELB constraint binding for 16 quarters.

Table 1: Baseline Quarterly Calibration

Parameter	Description	Value
β	Discount factor of patient households	0.997
z	Consumption share of impatient households	0.33
σ	Inverse elasticity of intertemporal substitution	1.032
χ	Inverse labour supply elasticity	1.7415
$\bar{b}^{cb}$	Steady-state share of central bank's long-term bond holdings	0.3
ϵ	CES parameter	13.6012
ϕ	Probability of keeping price unchanged	0.75
s_g	Steady-state government spending to output ratio	0.2
T	Steady-state labour income tax rate	0.1
ρ^{rn}	Makes ELB bind for around 16 quarters	0.85
σ^{rn}	Generates 10% drop in output, 2.2%p drop in inflation	-0.0364

In line with the literature, we solve the model with the ELB constraint, using a perfect foresight solution (Adjemian et al., 2011). The experiment we conduct consists of simulating a substantial drop in the natural rate that causes the ELB to bind for 16 quarters. We then analyse the responses of our model variables to variations in the price rigidity parameter (the paradox of flexibility), a 16 quarter exogenous decreases in the labour income tax rate (the paradox of toil), and a 16 quarter exogenous increases in government spending under different monetary policy rules, with and without QE. In particular, when we study the effect of varying the price rigidity parameter, we fix the shock size such that it causes a decline in the output gap by 10 per cent and inflation by 2.2 percentage points when the Calvo parameter, ϕ, is equal to 0.75 in the economy without QE. Instead, when we analyse the effects of labour tax or government spending shocks, we adjust the size of the natural rate shock such that the inflation rate drops by 2.2 percentage points, in annualised terms. Keeping the inflation rate constant allows us to conveniently calculate the increase in QE for each level of ϑ (i.e., the weight on inflation in the QE rule).[10] In Section 4.6, we repeat the same numerical exercises described above under the assumption that the central bank sets QE according to an output-gap-targeting policy rule.

4.2 The Paradox of Flexibility

Figure 6 shows the impact response of the output gap to an exogenous reduction in the natural rate of interest for different degrees of price stickiness when the policy rate is at its ELB. Each curve represents a different level of responsiveness to inflation in the QE rule. When the monetary authority does not respond sufficiently to inflation, namely for $\vartheta = 0$ or $\vartheta = 10$, the paradox of flexibility arises, i.e., greater price

[10] It is important to note that, given the first-order approximation of the model, the initial level of the output gap does not affect the size of the fiscal multiplier.

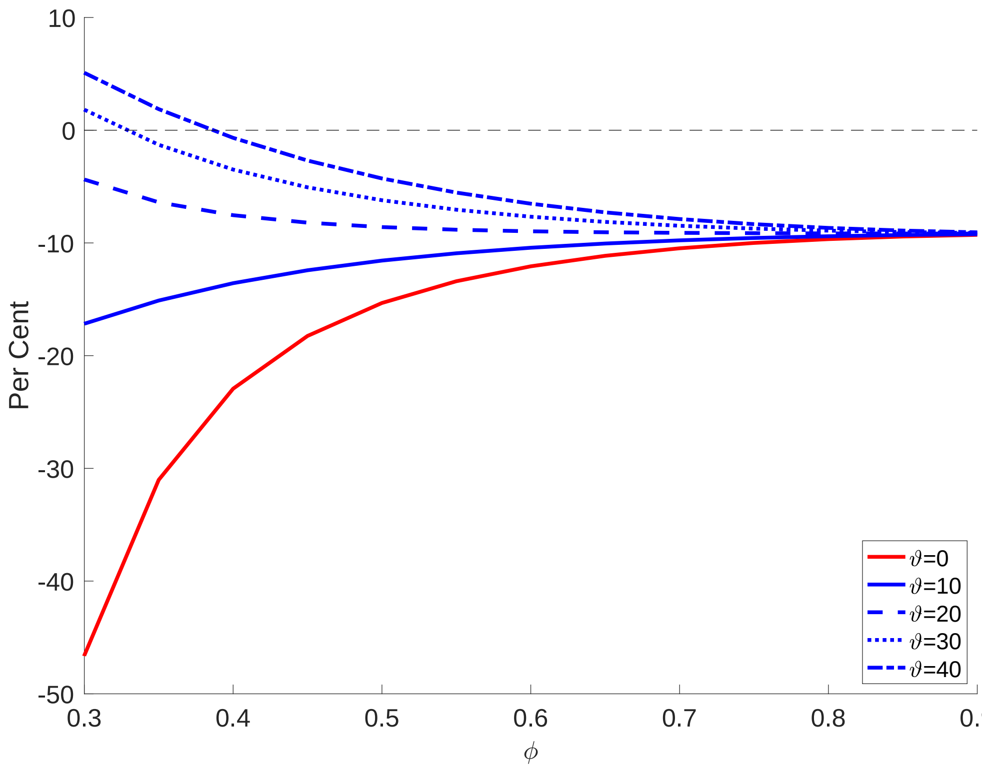


Figure 6: The Effects of Price Flexibility under Different QE Rules

Note: The figure shows the impact response of the output gap to a negative shock to the natural interest rate for different values of the price stickiness parameter. The shock causes the nominal policy rate to bind for 16 quarters. Each curve represents a different value of ϑ, i.e., the responsiveness of QE to inflation.

flexibility amplifies the drop in the output gap. As pointed out by Eggertsson and Krugman (2012) and Eggertsson and Garga (2019), greater price flexibility leads to a sharp drop in inflation expectations after an adverse demand shock, which increases the real rate and causes the output gap to fall more significantly than in the case with stickier prices. When $\vartheta \geq 20$, monetary policy can effectively counteract the fall in inflation and inflation expectations (and hence the rise in the real rate) by increasing QE.[11] With greater price flexibility, inflation initially drops more, causing a larger increase in QE, which mitigates the fall in output. Thus, provided a strong enough reaction of QE to inflation, the paradox of flexibility does not occur any longer, and higher price flexibility mitigates the initial drop in the output gap.

4.3 The Paradox of Toil

Figure 7 displays how the labour income tax multiplier varies depending on the weight on inflation in the QE rule (ϑ) and compares the case with a binding ELB constraint to the unconstrained case. In the scenario

[11] After an annualised 2.2 percentage-point drop in inflation, $\vartheta = 20$ implies an approximate increase in QE by $\exp(20 \times 2.2/400) - 1 = 11.6$ per cent.

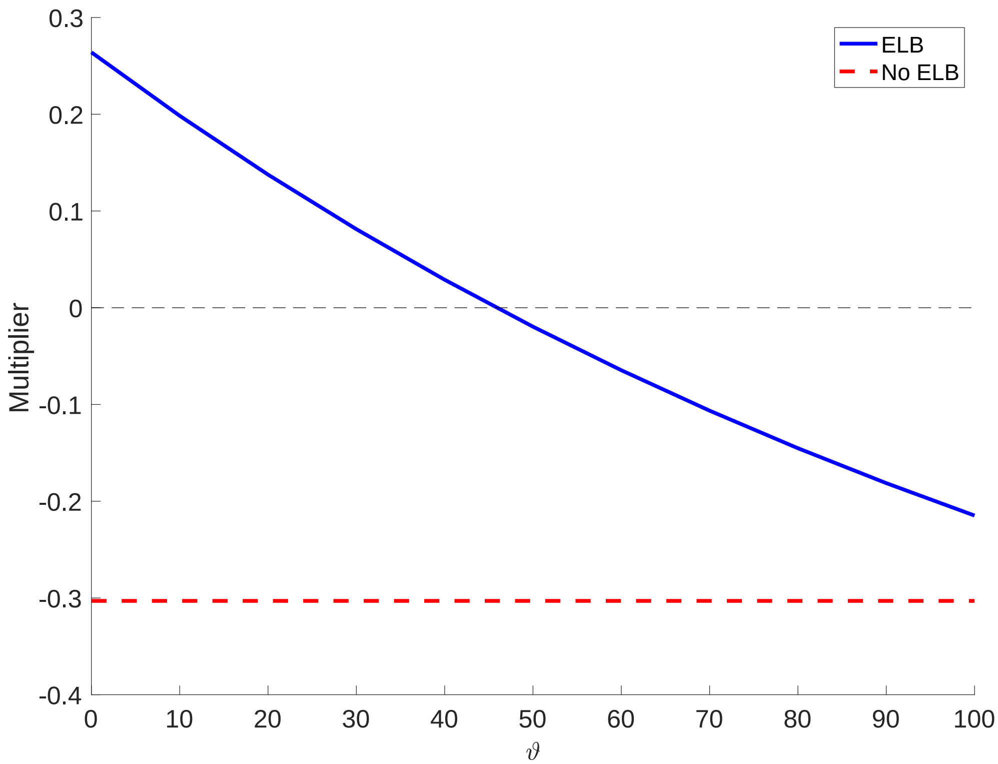


Figure 7: Labour Tax Multipliers under Different QE Rules

Note: The figure shows the impact labour income tax multipliers to a negative shock to the natural interest rate for different levels of ϑ, i.e., the responsiveness of QE to inflation. The blue solid line represents the case, where the nominal interest rate is at the ELB due to a negative natural rate shock. While the red dashed line represents the case where nominal rate is away from the ELB.

without ELB constraint, we assume that the central bank only uses conventional monetary policy. In such a case, a decrease in the labour income tax rate leads to a rise in output. In particular, the labour income tax multiplier is equal to -0.3 and is independent of ϑ. The drop in the nominal rate leads to a fall in the real rate, which causes the output gap to rise. When the policy rate is at its ELB, instead, and conventional monetary policy is unavailable, the paradox of toil arises. In this case, the fall in inflation and inflation expectations causes the real rate to increase and the output gap to fall. In the absence of QE ($\vartheta = 0$), the tax multiplier is positive and equal to 0.26, which means that a reduction in the labour income tax decreases output.[12] A larger weight on inflation in the QE rule mitigates the paradox of toil, and, when $\vartheta \geq 46$ the labour tax cut becomes expansionary. More specifically, given the 2.2 annualised percentage-point decrease in the inflation rate, a value of $\vartheta \geq 46$ implies an increase in QE by greater than 28.8 per cent, which is strong enough to offset the fall in inflation expectations and the rise in the real rate.[13]

[12] The multiplier is the ratio of the impact response of output and the impact response of the tax rate. Hence, if both the tax rate and output fall, the multiplier will be positive.

[13] Given an annualised 2.2 percentage-point drop in inflation, $\vartheta = 46$ means an approximate increase in QE by $\exp(46 \times 2.2/400) - 1 = 28.8$ per cent.

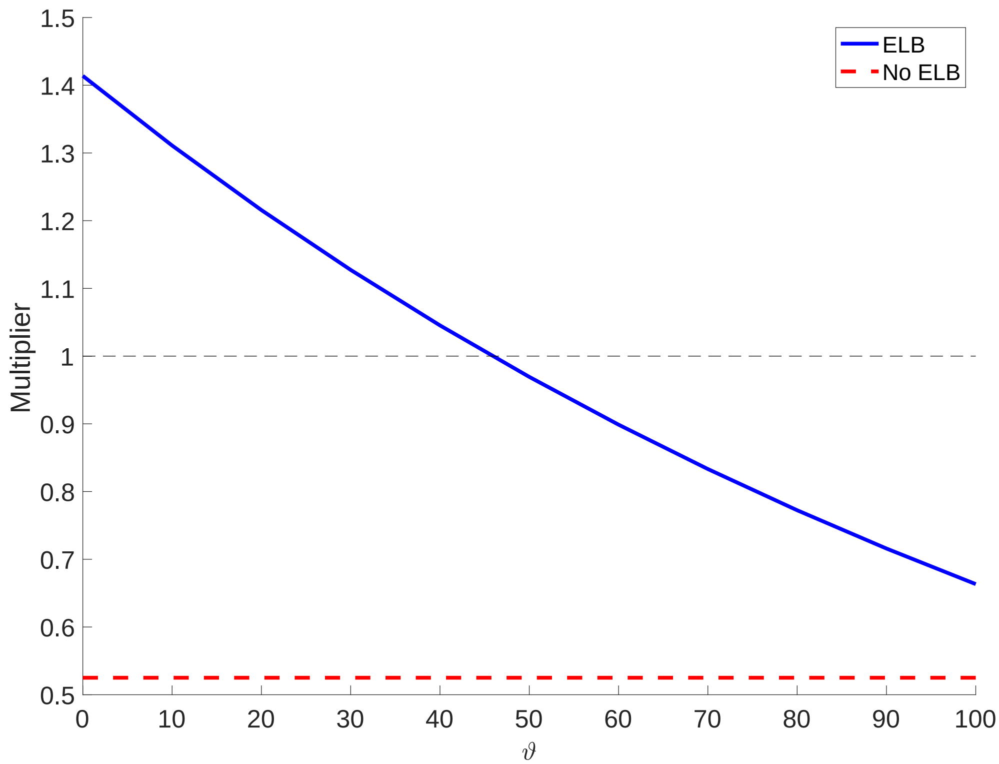


Figure 8: Government Spending Multipliers under Different QE Rules

Note: The figure shows the impact government spending multipliers to a negative shock to the natural interest rate for different levels of ϑ, i.e., the responsiveness of QE to inflation. The blue solid line represents the case, where the nominal interest rate is at the ELB due to a negative natural rate shock. While the red dashed line represents the case where nominal rate is away from the ELB.

4.4 Large Government Spending Multipliers

Similarly, as for the paradox of toil, Figure 8 displays how the multiplier associated with an increase in government spending varies depending on the weight on inflation in the QE rule (ϑ), and compares the case where the ELB constraint on the nominal policy rate is binding to the unconstrained case. When the policy rate is not constrained by the lower bound and we assume that the central bank only deploys conventional monetary policy, the government spending multiplier is significantly smaller than one and equal to 0.53. The rise in the nominal policy rate increases the real rate, which dampens the rise in the output gap. At the ELB, if the central bank does not sufficiently react to inflation via QE, the government spending multiplier can be significantly larger than one because of the rise in inflation expectations and the fall in the real rate. In particular, when $\vartheta = 0$, the government spending multiplier is equal to 1.41. For larger values of ϑ instead, the reduction in QE (quantitative tightening), in response to the rise in inflation, counteracts the fall in the real rate and when $\vartheta \geq 46$ the multiplier becomes smaller than one.

4.5 Relationship between QE and Long-Term Yield

To facilitate the interpretation of the results described above, we now discuss the relationship between the real value of long-term bonds held by the central bank, i.e., qe_t, and the yield-to-maturity of long-term bonds rl_t^b. As shown in Appendix, the yield is a negative function of the bond price (q_t). The monetary authority, by purchasing long-term bonds, increases their price and compresses their yields. Formally, through the lenses of the model, the negative relationship between qe_t and rl_t^b can be expressed as:

$$\left(1 + \frac{\kappa}{R^b - \kappa}\right) rl_t^b - \frac{\kappa}{R^b - \kappa} E_t rl_{t+1}^b - E_t \pi_{t+1} = -\sigma \bar{b}^{cb} \left(qe_t - E_t qe_{t+1}\right). \tag{34}$$

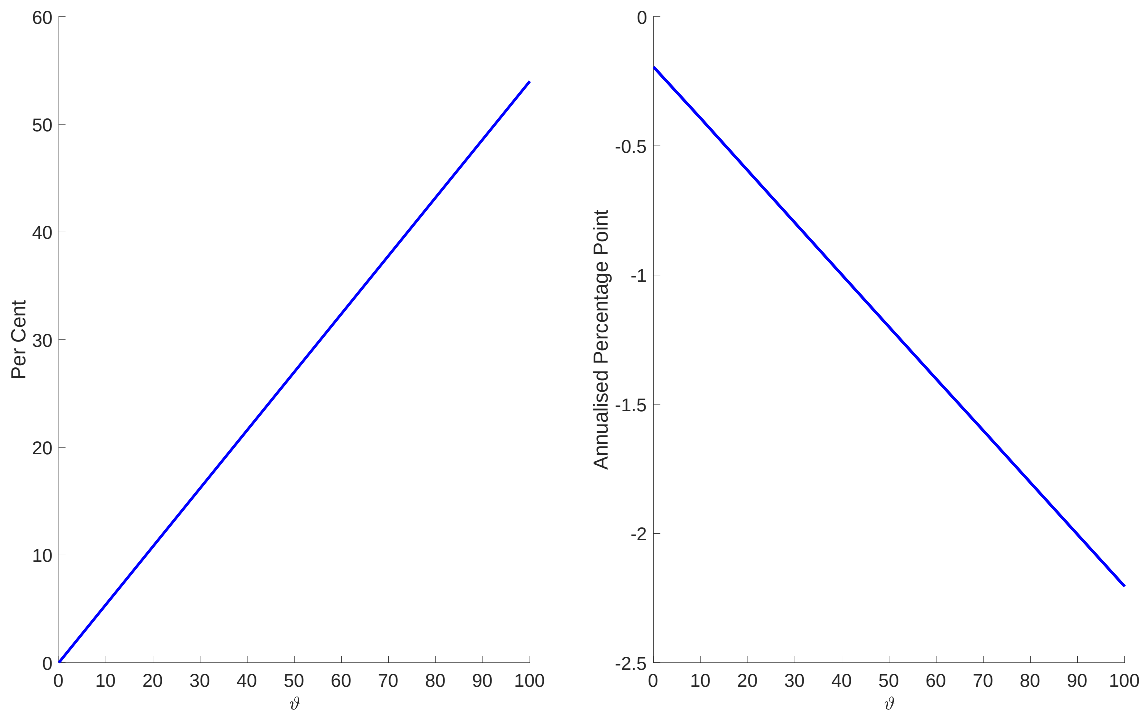
The parameter κ is the decaying rate of the coupon payments on long-term bonds and is set to $1 - \frac{1}{40}$ to imply an average maturity of the long-term bonds of 10 years. R^b is the steady-state value of the gross return on long-term bonds, which is set equal to 1.008, i.e., 200 annualised basis points higher than the short term interest rate ($R^s = \frac{1}{\beta} = 1.003$).

Consider the effect of a negative demand shock that leads to an annualised 2.2 percentage-point drop in inflation and causes the ELB to bind for 16 quarters. As shown in Figure 9, the monetary authority responds to the fall in inflation by increasing QE. The increase in QE is proportional to the parameter ϑ since we keep the fall in inflation fixed. The rise in QE compresses the long-term yield. Therefore for larger ϑ, we have a more sizable fall in the yield. For example, given a value of $\vartheta = 20$, necessary to solve the paradox of flexibility, the fall in demand implies an 11.6 per cent increase in QE and a reduction in the yield by 0.6 annualised percentage points. When $\vartheta = 46$, i.e., the value that solves the fiscal paradoxes, QE increases by 28.8 per cent, and the yield falls by 1.13 annualised percentage points.

Compared to the fall in inflation by 2.2 annualised percentage points, the decline in the yield necessary to remove the fiscal paradoxes is relatively mild. In normal times, for example, under a conventional Taylor rule and a standard weight on inflation of 1.5, a 2.2 percentage-point drop in inflation would imply a 3.3 annualised percentage-point decrease in the policy rate.

Wu and Zhang (2019) suggest that a 1 per cent increase in QE translates into a decrease in the shadow policy rate by 0.018 annualised percentage points.[14] Based on this rule-of-thumb value, the 11.6 per cent increase in QE necessary to solve the paradox of flexibility would imply a reduction of the shadow policy rate

[14] Their value is obtained by regressing the shadow rate by Wu and Xia (2016) on the nominal value of the Federal Reserve's assets. Using the real value of QE, deflated by the GDP deflator, delivers almost identical results.



(a) QE

(b) Long-Term Bond Yield

Figure 9: QE and Yield-to-Maturity after a Negative Demand Shock

Note: The figure shows the impact responses of QE and the nominal yield to maturity to a negative shock to the natural interest rate for different levels of ϑ, i.e., indicates the responsiveness of QE to inflation.

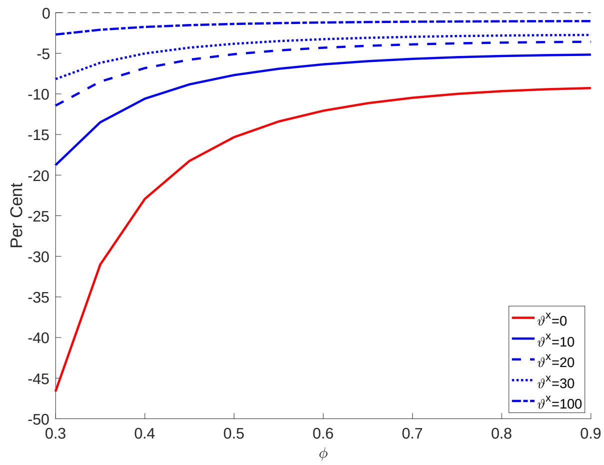
by just 0.21 annualised percentage points for a 2.2 percentage-point decline in inflation. Similarly, the rise in QE necessary to remove the fiscal paradoxes (28.8 per cent), would translate in a 0.52 percentage-point fall in the shadow rate. Such a back-of-the-envelope calculation suggests that the monetary policy response required to resolve the NK paradoxes is small.

4.6 QE Targeting the Output Gap

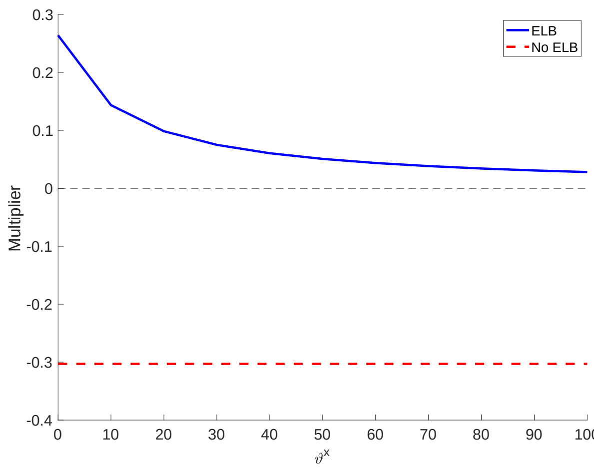
To highlight the importance of inflation-targeting for solving the paradox of flexibility and the paradox of toil, we conduct similar experiments under the alternative QE rule discussed in Section 3.5. In particular, we assume that, at the ELB, the monetary policy authority adjusts QE targeting the output gap rather than inflation:

$$qe_t = -\vartheta^x x_t. \tag{35}$$

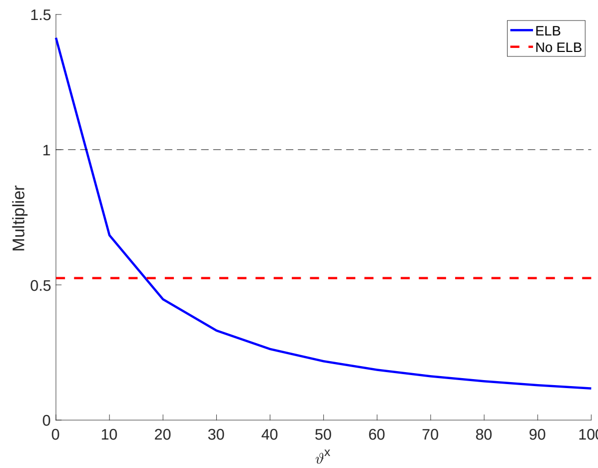
In Figure 10(a), we show the impact response of the output gap to a fall in the natural rate, for varying degrees of price flexibility. In line with the analytical part, regardless of the weight on the output gap, we



(a) The Effects of Price Flexibility



(b) Labour Tax Multipliers



(c) Government Spending Multipliers

Figure 10: NK Paradoxes under an Output-Gap-Targeting QE Rule

Note: The top panel displays the impact response of the output gap to a fall in the natural interest rate for different values of the price stickiness parameter. Each curve represents a different value of ϑ^x. The lower panels show the impact fiscal multipliers for different values of ϑ^x with and without ELB constraint.

find that lower price stickiness amplifies the drop in the output gap. As discussed above, this result stems from the larger drop in inflation and inflation expectations. Since the central bank does not respond to inflation, the fall in inflation expectations is not counteracted by stimulative monetary policy interventions, and output drops more significantly under flexible prices.

In Figure 10(b), we compare the impact multiplier of a labour tax rate hike at the ELB with that in the absence of an ELB constraint. At the ELB, the alternative QE rule cannot correct the paradox of toil, i.e., a tax cut is contractionary. This is because the labour tax rate cut causes a fall in inflation and inflation expectations, which is not counteracted by the central bank. The fall in inflation expectations increases the real rate and depresses real activity.

Unlike the other two paradoxes, the alternative rule is very effective in mitigating the size of the government spending multiplier. Figure 10(c) shows that at the ELB, the government spending multiplier becomes lower than one for a value of $\vartheta^x \geq 8$. The reason is that under the alternative policy rule, the central bank decreases QE in response to the rise in output induced by the fiscal expansion. The reduction in QE mitigates the expansion in aggregate demand and aggregate supply, and therefore the multiplier drops quickly below one.

5 Conclusion

In this paper, we reconsider three common results associated with NK models and the effective lower bound of the nominal policy rate: the paradox of flexibility, the paradox of toil, and the large government spending multipliers. We argue that these results originate from a failure of standard models to characterise monetary policy at the ELB correctly. Through the lenses of a tractable four-equation DSGE model (Sims et al., 2020), we show both analytically and numerically how accounting for QE as a tool to stabilise inflation at the ELB can correct these seemingly paradoxical results. Despite QE being an imperfect substitute of conventional monetary policy, we find that if the central bank responds strongly enough to deviations in inflation from target, greater price flexibility mitigates the fall in the output gap due to a negative demand shock. In particular, after an initial decline by 2.2 annualised percentage points in inflation, QE should increase by about 11 per cent for the paradox of flexibility to disappear. An increase in QE by about 29 per cent, given the same fall inflation, makes a labour income tax cut expansionary and brings the government spending multiplier below one.

We highlight that the adjustments in QE required to resolve these paradoxes are relatively mild. Moreover, it bears noting that the resolution of the paradoxes is conditional on the type of monetary policy rule. When the central bank only responds to the output gap, the paradox of flexibility and the paradox of toil cannot be corrected. In other words, an inflation-targeting rule is necessary to resolve all three paradoxes.

The results presented in our paper underscore the critical role of QE, in particular, and unconventional monetary policies, in general, in relaxing the constraints imposed by the ELB on nominal interest rates. Accounting for the various policy tools, which central banks have adopted since the Global Financial Crisis, is key to assessing the effectiveness of policies aimed at reducing nominal rigidities or fiscal policies, such as labour tax cuts and increases in government spending, in times of low interest rates.

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Appendices

A The Full Nonlinear Model

The model we consider follows Sims et al. (2020). However, there are two main different modelling assumptions. First, our model, in line with Woodford (2003), assumes industry-specific labour in the utility function, rather than aggregate labour. Second, our model also includes a fiscal sector, with labour tax and government spending shocks.

A.1 Patient Households

A representative patient household maximises its discounted lifetime utility:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{{C_t}^{1-\sigma} - 1}{1-\sigma} - \psi \int_0^1 \frac{L_t\,(i)^{1+\chi}}{1+\chi} di \right], \tag{A.1}$$

where C_t is a Dixit-Stiglitz aggregate:

$$C_t = \left[\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{A.2}$$

and $L_t(i)$ is the quantity of labour supplied to the firm producing good i. The parameter σ is the coefficient of relative risk aversion, $\epsilon > 1$ is the demand elasticity of good i, $\chi > 0$ is the inverse of the Frisch elasticity of labour, ψ is a normalising constant, and β is the discount factor.

The household demand for good i is given by:

$$C_t(i) = C_t \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon}, \tag{A.3}$$

where $P_t(i)$ is the price of good i. The aggregate price level P_t therefore writes as:

$$P_t \equiv \left[\int_0^1 P_t(i)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}, \tag{A.4}$$

so that:

$$P_t C_t = \int_0^1 P_t(i) C_t(i) di. \tag{A.5}$$

The patient household maximises its expected discounted lifetime utility (A.1) subject to the following budget constraint:

$$\int_0^1 P_t(i)C_t(i)di + S_t \leq R^s_{t-1}\zeta_{t-1}S_{t-1} + (1-T_t)\int_0^1 W_t(i)L_t(i)di + \int_0^1 D_t(i) + P_tD_t^{FI} \\ + P_tT_t^{FA} + P_tT_t^{cb} - P_tX_t^b - P_tX_t^{FI}, \tag{A.6}$$

where S_t is a one period risk-free bond, paying a gross nominal interest rate R^s_t. $W_t(i)$ is the nominal wage rate in the ith industry in the economy and $D_t(i)$ are the nominal profits from the sale of good i. The household owns the financial intermediaries and receives dividends D_t^{FI}. T_t^{FA} and T_t^{cb} are lump-sum transfers from the fiscal authority and the central bank, whereas T_t is a labour tax. ζ_t is the risk-premium shock. Finally, X_t^b and X_t^{FI} are transfers to the impatient households and the financial intermediaries. The resulting optimality conditions are standard:

$$1 = E_t\Lambda_{t,t+1}\frac{R^s_t\zeta_t}{\Pi_{t+1}}, \tag{A.7}$$

$$\Lambda_{t-1,t} = \beta\left(\frac{C_t}{C_{t-1}}\right)^{-\sigma}, \tag{A.8}$$

where $\Lambda_{t-1,t}$ is the patient household's stochastic discount factor. $\Pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate.

A.2 Impatient Households

The impatient household can borrow/save with long term bonds B_t. Similarly as in Woodford (2001), long-term bonds are modelled as perpetuities with geometrically decaying coupon payments. The decaying rate of the coupon payments is denoted by $\kappa \in [0,1]$. The agent that issues the bond in time t needs to pay 1, κ, κ^2, ... in the following periods. The new bond issuance CB_t equals:

$$CB_t = B_t - \kappa B_{t-1}. \tag{A.9}$$

Given the market price of newly issued bonds Q_t, the total value of the bond portfolio equals Q_tB_t. Moreover, define the gross return on the long bond as:

$$R^b_t = \frac{1+\kappa Q_t}{Q_{t-1}}, \tag{A.10}$$

and the gross yield-to-maturity as:

$$Q_t = \frac{1}{RL^b_t} + \frac{\kappa}{{RL^b_t}^2} + \frac{\kappa^2}{{RL^b_t}^3} + \cdots, \tag{A.11}$$

Consequently,

$$RL_t^b = \frac{1}{Q_t} + \kappa. \tag{A.12}$$

The impatient household does not work and derives utility only from its consumption C_t^b. It maximises its lifetime utility:

$$\max E_0 \sum_{t=0}^{\infty} {\beta^b}^t \left[\frac{{C_t^b}^{1-\sigma} - 1}{1-\sigma} \right], \tag{A.13}$$

subject to a budget constraint choosing C_t^b and B_t:

$$P_t C_t^b + B_{t-1} \leq Q_t \left(B_t - \kappa B_{t-1} \right) + P_t X_t^b. \tag{A.14}$$

The optimality condition for the impatient households is, therefore:

$$1 = E_t \Lambda_{t,t+1}^b \frac{R_{t+1}^b}{\Pi_{t+1}}, \tag{A.15}$$

where $\Lambda_{t-1,t}^b$ denotes stochastic discount factor, defined as:

$$\Lambda_{t-1,t}^b = \beta^b \left(\frac{C_t^b}{C_{t-1}^b} \right)^{-\sigma}. \tag{A.16}$$

A.3 Financial Intermediaries

A representative financial intermediary is born each period and exits the industry in the subsequent period. It receives an exogenous amount of net worth from the patient household, $P_t X_t^{FI}$, which equals:

$$P_t X_t^{FI} = P_t \bar{X}^{FI} + \kappa Q_t B_{t-1}^{FI}. \tag{A.17}$$

$\bar{X}^{FI}$ is a fixed amount of new equity, whereas $\kappa Q_t B_{t-1}^{FI}$ is the value of outstanding long-bonds inherited from past intermediaries. The balance sheet of the financial intermediary is given by:

$$Q_t B_t^{FI} + RE_t^{FI} = S_t^{FI} + P_t X_t^{FI}. \tag{A.18}$$

where the left-hand side are the assets (long-term lending to impatient households $Q_t B_t^{FI}$ and reserves RE_t^{FI}), whereas the right-hand side are the liabilities (short-term deposits from the patient household S_t^{FI} and the transfer $P_t X_t^{FI}$). The financial intermediary, pays interest R_t^s on the deposits, earns interest, R_t^{re}, on its reserves, and earns a gross return R_{t+1}^b on long-term bonds.

When the financial intermediary exits the market, gives dividends $P_{t+1}D_{t+1}^{FI}$ (in nominal terms) to the patient household:

$$P_{t+1}D_{t+1}^{FI} = \left(R_{t+1}^{b} - R_t^s\right) Q_t B_t^{FI} + \left(R_t^{re} - R_t^s\right) RE_t^{FI} + R_t^s P_t X_t^{FI}. \tag{A.19}$$

In time t the financial intermediary maximises the expected $t+1$ dividends, discounted by the stochastic discount factor $\frac{\Lambda_{t,t+1}}{\Pi_{t+1}}$, subject to a leverage constraint:

$$Q_t B_t^{FI} \leq \Theta P_t \bar{X}^{FI}. \tag{A.20}$$

The condition states that the value of the long-term loans to the impatient households cannot be larger than a multiple Θ of the value of its equity. The first order conditions with respect to B_t^{FI} and RE_t^{FI} write as:

$$E_t \Lambda_{t,t+1} \frac{R_{t+1}^{b} - R_t^s}{\Pi_{t+1}} = \Omega_t, \tag{A.21}$$

$$E_t \Lambda_{t,t+1} \frac{R_t^{re} - R_t^s}{\Pi_{t+1}} = 0, \tag{A.22}$$

where Ω_t is the Lagrangian multiplier associated with the leverage constraint.

A.4 Production

A monopolistically competitive firm produces good i using the following production function:

$$Y_t(i) = L_t(i). \tag{A.23}$$

Each firm faces a downward-sloping demand function given by:

$$Y_t(i) = Y_t \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}. \tag{A.24}$$

Following Woodford (2003), the labour employed by each monopolistically competitive firm corresponds to a particular type of the labour variety supplied by the households. The firm takes the wage rate as given and its period profits are given by:

$$D_t(i) = P_t(i) \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t - W_t^I \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t, \tag{A.25}$$

where W_t^I should be interpreted as an industry specific wage for good variety i. W_t^I can then be related to the price level of good i via the first order labour condition of the household as:

$$\frac{W_t^I}{P_t} = \frac{\psi N_t(i)^\chi {C_t}^\sigma}{1-\tau_t} = \frac{\psi \left(Y_t \left(\frac{P_t^I}{P_t} \right)^{-\epsilon} \right)^\chi {C_t}^\sigma}{1-\tau_t}, \tag{A.26}$$

where P_t^I is the industry-wide common price. We write then the period profit function of a firm producing good i as $D\left(P_t(i), P_t^I, P_t, Y_t\right)$.

As in Calvo (1983), a fraction $1-\phi$ of randomly picked firms can reset their price. Let P_t^* be the optimal reset price in period t. A supplier that changes its price in period t chooses its newly-adjusted price $P_t(i)$ to maximise the its expected discounted lifetime profits, taking as given the industry level wage W^I, expressed in terms of P_t^I:

$$E_t \sum_{j=0}^{\infty} \phi^j \Lambda_{t,t+j} \frac{D\left(P_t(i), P_{t+j}^I, P_{t+j}, Y_{t+j}\right)}{P_{t+j}}, \tag{A.27}$$

The first-order condition for optimal price setting is:

$$E_t \sum_{j=0}^{\infty} \phi^j \Lambda_{t,t+j} \left((1-\epsilon) \left(\frac{P_t^*}{P_{t+j}} \right)^{1-\epsilon} Y_{t+j} + \epsilon \frac{\psi \left(Y_{t+j} \left(\frac{P_{t+j}^I}{P_{t+j}} \right)^{-\epsilon} \right)^\chi {C_{t+j}}^\sigma}{1-T_{t+j}} \left(\frac{P_t^*}{P_{t+j}} \right)^{-\epsilon} Y_{t+j} \right) = 0. \tag{A.28}$$

Following Woodford (2003), all firms in industry I reset the price in period t:

$$E_t \sum_{j=0}^{\infty} \phi^j \Lambda_{t,t+j} \left((1-\epsilon) \left(\frac{P_t^*}{P_{t+j}} \right)^{1-\epsilon} Y_{t+j} + \epsilon \frac{\psi \left(Y_{t+j} \left(\frac{P_t^*}{P_{t+j}} \right)^{-\epsilon} \right)^\chi {C_{t+j}}^\sigma}{1-T_{t+j}} \left(\frac{P_t^*}{P_{t+j}} \right)^{-\epsilon} Y_{t+j} \right) = 0. \tag{A.29}$$

This implies that the optimal reset price is:

$$\frac{P_t^*}{P_t} = \left(\frac{\epsilon}{\epsilon-1} \frac{E_t \sum_{j=0}^{\infty} \phi^j \Lambda_{t,t+j} \frac{\psi}{1-T_{t+j}} {C_{t+j}}^\sigma \left(\frac{P_t}{P_{t+j}} \right)^{-\epsilon(1+\chi)} {Y_{t+j}}^{1+\chi}}{E_t \sum_{j=0}^{\infty} \phi^j \Lambda_{t,t+j} \left(\frac{P_t}{P_{t+j}} \right)^{1-\epsilon} Y_{t+j}} \right)^{\frac{1}{1+\chi\epsilon}}, \tag{A.30}$$

where P_t indicates the aggregate price level. We can re-write the expression in recursive form:

$$p_t^* = \left(\frac{\epsilon}{\epsilon-1} \frac{F_{1,t}}{F_{2,t}} \right)^{\frac{1}{1+\chi\epsilon}}, \tag{A.31}$$

$$F_{1,t} = \frac{\psi}{1 - T_t} {Y_t}^{1+\chi} + \phi \beta E_t {\Pi_{t+1}}^{\epsilon(1+\chi)} F_{1,t+1}, \tag{A.32}$$

$$F_{2,t} = {C_t}^{-\sigma} Y_t + \phi \beta E_t {\Pi_{t+1}}^{\epsilon - 1} F_{2,t+1}, \tag{A.33}$$

where inflation Π_t evolves according to:

$$\phi {\Pi_t}^{\epsilon - 1} = 1 - (1 - \phi) \, {p_t^*}^{1-\epsilon}. \tag{A.34}$$

A.5 Central Bank

The monetary authority creates reserves to finance the purchase of long bonds B_t^{cb}. Its balance sheet, therefore, writes as:

$$Q_t B_t^{cb} = RE_t. \tag{A.35}$$

The real value of long-term bonds held by the central bank is denoted as:

$$QE_t = Q_t b_t^{cb}, \tag{A.36}$$

where $b_t^{cb} = \frac{B_t^{cb}}{P_t}$. Potential profits made by the central bank are then transferred lump-sum to the patient households:

$$P_t T_t^{cb} = R_t^b Q_{t-1} B_{t-1}^{cb} - R_{t-1}^{re} RE_{t-1}. \tag{A.37}$$

The monetary authority sets the short-term interest rate R_t^s according to the following rule à la Taylor (1993):

$$\log R_t^s = \max \left\{ 0, \log R^s + \phi_\pi \left(\log \Pi_t - \log \Pi \right) \right\}. \tag{A.38}$$

In particular, away from the ELB, i.e., when $R^s > 1$, the central bank sets the interest rate, such that $\Pi_t = \Pi$. This strict inflation-targeting rule implies $\phi_\pi \to +\infty$. At the ELB ($R^s = 1$), the monetary authority chooses QE_t using a similar policy rule:

$$\log QE_t - \log QE = -\vartheta \left(\log \Pi_t - \log \Pi \right). \tag{A.39}$$

A.6 Fiscal Authority

The government can levy distortionary labour taxes T_t from the patient households and consumes G_t. To keep a balanced budget, the government makes use of lump-sum transfer adjustments T_t^{FA}. The government's budget constraint therefore writes as:

$$T_t \int_0^1 W_t(i) L_t(i) di = P_t G_t + P_t T_t^{FA}. \tag{A.40}$$

A.7 Aggregation and Equilibrium

Market clearing requires the following conditions:

$$RE_t = RE_t^{FI}, \tag{A.41}$$

$$S_t = S_t^{FI}, \tag{A.42}$$

$$B_t = B_t^{FI} + B_t^{cb}, \tag{A.43}$$

$$Y_t = C_t + C_t^b + G_t, \tag{A.44}$$

$$P_t X_t^b = (1 + \kappa Q_t) B_{t-1}, \tag{A.45}$$

$$P_t C_t^b = Q_t B_t. \tag{A.46}$$

Given that we abstract from productivity shocks, the level of output that arises in the flexible-price version of the model is constant and equal to the steady-state value, i.e. $Y_t^f = Y$. It follows that the output gap X_t is simply defined as:

$$X_t = \frac{Y_t}{Y_t^f} = \frac{Y_t}{Y}. \tag{A.47}$$

From the flexible-price version of the mode, we obtain the natural rate of interest:

$$R_t^n = \frac{\Pi}{\beta \zeta_t}, \tag{A.48}$$

and is therefore exogenously determined by the following process:

$$\log R_t^n = (1 - \rho^{rn}) \log R^n + \rho^{rn} \log R_{t-1}^n + \sigma^{rn} \varepsilon_t^{rn}. \tag{A.49}$$

B Equilibrium Conditions of the Linearised Model

The equilibrium conditions in log-linearised form are summarised below. The following variables denote percentage change deviations from their steady-state values, e.g., $x_t \equiv \frac{X_t - X}{X}$. We use the "hat" notation when the variable in the nonlinear model is labelled with a lower-case letter, e.g., $\widehat{x}_t \equiv \frac{x_t - x}{x}$.

$$\lambda_{t-1,t} = -\sigma \left(c_t - c_{t-1} \right), \tag{B.1}$$

$$0 = E_t \lambda_{t,t+1} + r_t^s - E_t \pi_{t+1} + \widehat{\zeta}_t, \tag{B.2}$$

$$\lambda_{t-1,t}^b = -\sigma \left(c_t^b - c_{t-1}^b \right), \tag{B.3}$$

$$r_t^b = \frac{\kappa}{R^b} q_t - q_{t-1}, \tag{B.4}$$

$$rl_t^b = -\frac{1}{1 + \kappa Q} q_t, \tag{B.5}$$

$$0 = E_t \lambda_{t,t+1}^b + E_t r_{t+1}^b - E_t \pi_{t+1}, \tag{B.6}$$

$$q_t + \widehat{b}_t^{FI} = 0, \tag{B.7}$$

$$Qb^{FI} \left(1 - \kappa \right) q_t + Qb^{FI} \widehat{b}_t^{FI} - \kappa Q b^{FI} \widehat{b}_{t-1}^{FI} + \kappa Q b^{FI} \pi_t + re\widehat{re}_t = s\widehat{s}_t, \tag{B.8}$$

$$E_t \lambda_{t,t+1} - E_t \pi_{t+1} + \frac{R^b}{sp} E_t r_{t+1}^b - \frac{R^s}{sp} r_t^s = \omega_t, \tag{B.9}$$

$$r_t^{re} = r_t^s, \tag{B.10}$$

$$\widehat{p}_t^* = \frac{1}{1 + \chi \epsilon} \left(f_{1,t} - f_{2,t} \right), \tag{B.11}$$

$$f_{1,t} = -\frac{(1 - \phi\beta)}{1 - T} \tau_t + (1 - \phi\beta)(1 + \chi) y_t + \phi\beta\epsilon (1 + \chi) E_t \pi_{t+1} + \phi\beta E_t f_{1,t+1}, \tag{B.12}$$

$$f_{2,t} = -(1 - \phi\beta) \sigma c_t + (1 - \phi\beta) y_t + \phi\beta (\epsilon - 1) E_t \pi_{t+1} + \phi\beta E_t f_{2,t+1}, \tag{B.13}$$

$$(1 - s_g)(1 - z) c_t + (1 - s_g) z c_t^b + g_t = y_t, \tag{B.14}$$

$$\pi_t = \frac{1 - \phi}{\phi} \widehat{p}_t^*, \tag{B.15}$$

$$q_t + \widehat{b}_t^{cb} = re_t, \tag{B.16}$$

$$\widehat{b}_t = \frac{b^{FI}}{b} \widehat{b}_t^{FI} + \frac{b^{cb}}{b} \widehat{b}_t^{cb}, \tag{B.17}$$

$$c_t^b = q_t + \widehat{b}_t, \tag{B.18}$$

$$qe_t = -\vartheta \pi_t, \tag{B.19}$$

$$r_t^{re} = \max\left\{ -\frac{R^s - 1}{R^s}, \phi_\pi \pi_t \right\}, \tag{B.20}$$

$$qe_t = \widehat{re}_t, \tag{B.21}$$

$$x_t = y_t - y_t^f. \tag{B.22}$$

$$\widehat{\zeta}_t = -r_t^n, \tag{B.23}$$

$$r_t^n = \rho^{rn} r_{t-1}^n + \sigma^{rn} \varepsilon_t^{rn}. \tag{B.24}$$