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## Abstract

Banking crises have severe short and long-term consequences. We develop a general equilibrium model with financial frictions and endogenous growth in which macroprudential policy supports economic activity and productivity growth by strengthening bank's resilience to adverse financial shocks. The improved intermediation capacity of a safer banking system leads to a higher steady state growth rate. The optimal bank capital ratio of 18% increases welfare by 6.7%, 14 times more than in the case without endogenous growth. When the economy enters a liquidity trap, the effects of financial disruptions and thus the benefits of macroprudential policy are even more significant.


**Key words:** Slow recoveries, endogenous growth, financial stability, macroprudential policy.

**JEL classification:** E32, E44, E52, G01, G18.

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# 1 Introduction

This paper studies the role of macro-prudential policy (MacroPru) in alleviating the short- and long-term consequences of adverse financial shocks. In the aftermath of the 2007/08 Financial Crisis, the recovery of real GDP per capita was slow and a reversion to the pre-crisis trend did not occur in many advanced economies. These permanent losses in output, relative to the pre-crisis trend, have been attributed to an endogenous decline in productivity growth due to a decline in productivity enhancing-investments and a reduction in the adoption speed of new technologies (e.g. Anzoategui et al. (2019)). These findings point to the importance of thinking about innovation and productivity growth as endogenous outcomes. Following the emergence of models that blend short-term business cycle and medium-term growth dynamics, recent studies have analysed the implications for monetary policy in an environment with endogenous growth (e.g. Moran and Queralto (2018) and Ikeda and Kurozumi (2019)). Despite the empirical link between financial disruptions and slow recoveries, the literature has not yet addressed the potential long-term benefits of macro-prudential policy within a DSGE setup. To the best of our knowledge, this paper is the first to study macro-prudential policy in a medium-scale model with financial frictions and endogenous growth. In most of the theoretical literature the welfare gains from MacroPru are found to be relatively small, because most models struggle to generate slow recoveries and permanent effects following financial disruptions. In our work, we show that the welfare gains from MacroPru can be much larger if one considers not only the stabilisation of output fluctuations around a given growth path but also the stabilisation of the growth path itself.

To motivate our theoretical investigation, we first provide empirical evidence on the negative relationship between banking crises, innovation and productivity. To this end, we estimate local projections based on panel data for 24 advanced economies.

In our theoretical model, the financial sector is built following Gertler et al. (2012) (henceforth GKQ). Financial intermediaries (FIs) have access to short-term debt and outside equity, making their risk exposure an endogenous choice. FIs take asset prices as given, and their reliance on outside equity is inefficiently low. In this context, we model macro-prudential policy as a subsidy on outside equity, which increases the resilience of FIs to changes in asset prices and mitigates the financial accelerator mechanism through which shocks propagate. The second key feature of our model is an endogenous growth mechanism of vertical innovation in the spirit of Grossman and Helpman (1991) and Aghion and Howitt (1992). Productivity has an endogenous component that depends on the aggregate level of intangible capital services. Spillovers stemming from the accumulation of intangible capital allow business cycle shocks to affect long-run growth. In this framework, financial shocks cause a substantial fall in output and investment in both physical and intangible capital. The latter leads to a temporary fall in productivity growth and a decline in output without a full recovery. By facilitating the flow of credit towards physical and intangible investment, macro-prudential policy has a positive impact on productivity growth and the long-term level of real activity.

The key results of our work are as follows. First, we show that welfare-maximising macro-prudential policy can prevent more than half of the permanent output losses in response to financial shocks.

Second, we show that the optimised macro-prudential policy is associated with a bank capital ratio of 18%, 4pp higher than in a version of the model with exogenous growth.

Third, our paper is the first to find large welfare gains from macro-prudential policy. We find consumption-equivalent welfare gains of roughly 7%, much larger than what was previously found in the literature (GKQ find 0.28%). The intuition behind this result is that optimised macro-prudential policy permanently increases the economy's growth rate. This has a strong effect on our measure of welfare, the household lifetime-utility.

Finally, when considering the possibility of a liquidity trap, the benefits from macro-prudential policy in an endogenous growth context are even larger. Since optimised macro-prudential policy increases the resilience of the financial system and the productivity growth rate and hence the 'steady state' real interest rate, it reduces the frequency and the severity of ZLB episodes.

Our results highlight that taking medium-term economic prospects into account is crucial to design appropriate macro-prudential policies. These policies can play an important role in promoting both financial stability and productivity growth. It bears noting, however, that an excessively tight regulation of FIs is inefficient and hampers potential productivity growth.

**Related Literature** Our paper is closely related to three strands of literature: (i) the literature on endogenous growth and slow recoveries, (ii) the literature on endogenous bank balance sheet determination with debt and equity as available forms of bank finance, and (iii) the literature on the interaction between macro-prudential policy and monetary policy.

There are two common approaches to modelling endogenous growth in the literature. The first one is the expanding variety approach ('horizontal innovation') pioneered by Romer (1986) and Romer (1990). Comin and Gertler (2006) and incorporate this endogenous growth mechanism into an otherwise standard business cycle macro model. A recent body of work builds on this paper to study the slow recovery we have observed after the Great Financial Crisis. In particular, Anzoategui et al. (2019) find that a significant fraction of the post-Great Recession fall in productivity was an endogenous phenomenon, suggesting that demand factors played an essential role in the post-crisis slowdown of capacity growth. Benigno and Fornaro (2018) analyse how animal spirits can generate a long-lasting liquidity trap in a New Keynesian growth model with multiple equilibria. Queralto (2020) includes endogenous growth in a model with financial frictions and finds that financial frictions significantly amplify the medium-run TFP and output losses following a crisis. Moran and Queralto (2018) use a New Keynesian model with endogenous growth and find significant TFP losses due to the constraints on monetary policy imposed by the zero lower bound (ZLB). Ikeda and Kurozumi (2019) develop a model with endogenous TFP growth in which adverse financial shocks can induce a slow recovery. In their model, a welfare-maximising monetary policy rule features a strong response to output, and the welfare gain from output stabilisation is much larger than when TFP follows an exogenous trend. Ma (2020) studies the impact of macro-prudential policy through the lenses of a small open economy model with endogenous growth and occasionally binding collateral constraints. The paper finds that optimal macro-prudential policy reduces the probability of crises by two thirds at the cost of slightly lowering average growth. Whereas Ma (2020)'s real, partial-equilibrium framework applies to emerging economies and the conduct of macro-prudential policy via capital controls, we develop a New Keynesian DSGE model to describe advanced economies and macro-prudential policy via bank capital regulation.

The second approach to modelling endogenous growth is that of (Schumpeterian) creative destruction ('vertical innovation'), pioneered by Aghion and Howitt (1992). Two recent papers have considered this approach in otherwise standard business cycle models. Kung (2015) includes such endogenous growth mechanism into a basic New Keynesian model with recursive preference to rationalise key stylised facts in bond markets. Bianchi et al. (2019) builds on this paper to analyse the effects of financial shocks and show that they can lead to significant slowdowns in productivity growth. Because of its tractability, our analysis considers an endogenous growth mechanism of vertical innovation similar to these papers, which we build into a medium-scale DSGE with frictions in the banking sector.

A growing literature provides evidence on the negative impact of tighter financial conditions on innovation and productivity. Among others, Aghion et al. (2010) shows that liquidity shocks move firms away from long-term productivity-enhancing investments if credit constraints are tight. de Ridder (2016) uses a linked lender-borrower dataset on 522 U.S. companies responsible for 58% of industrial research and development and finds that tight credit conditions reduced intangible investment between 2010 and 2015. Huber (2018) estimates a significant fall in innovation, firm-size and productivity after a large German bank reduced its lending to firms. The paper also finds that output and employment remained persistently low even after bank lending had returned to normal. Li (2011) shows that financially constrained $R\&D$-intensive firms are more likely to suspend $R\&D$ projects. Schmitz (2020) shows that small and young innovative firms are significantly affected by credit tightness, which amplifies the adverse effects of financial crises on innovation.

Concerning the modelling of the financial sector, our paper is closely related to Gertler et al. (2012) (henceforth GKQ), de Groot (2014) and Liu (2016). We build on GKQ who develop a real DSGE model in which banks can fund themselves with non-state-contingent debt and state-contingent equity. Bank equity increases the bank's resilience against adverse shocks but, if increased too much, it may tighten the bank's intermediation capacity. de Groot (2014) builds on GKQ and develops a monetary extension of their framework to examine how monetary policy affects the riskiness of banks' balance sheets. Liu (2016) studies the welfare gains from various macro-prudential policy rules in the framework developed by GKQ. In our paper, instead, we extend the analysis in GKQ to account for the long-term benefits of macro-prudential policy.

**Roadmap** The remainder of the paper is organised as follows. In Section 2 we provide some empirical evidence that banking crises are followed by a sharp fall in output, $R\&D$ activity and productivity growth. In Section 3 we introduce the model. In Section 4, we discuss how a contraction in financial intermediation leads to a decline productivity growth and how this gives rise to a slow recovery and permanent losses in output. In Section 5, we discuss policy options and in Section 6 we conclude.

## 2 The Effects of Banking Crises on Productivity and Output

To motivate our theoretical investigation, we present some international empirical evidence on the negative relationship between banking crises, innovation and productivity. We show how banking crises are associated with significantly larger downturns in output, innovation, proxied by R&D, and total factor productivity compared to other recessions. To this end, we estimate local projections à la Jorda (2005) based on panel regressions, using annual data from 1970 to 2018 for 24 advanced economies. A detailed description of the data can be found in the Appendix in Section A.

**The Local Projection Method** One key feature of the local projection method is its robustness to model misspecifications, as it does not impose dynamic restrictions on the impulse responses. The model we use to estimate our local projections is the following fixed-effects regression:

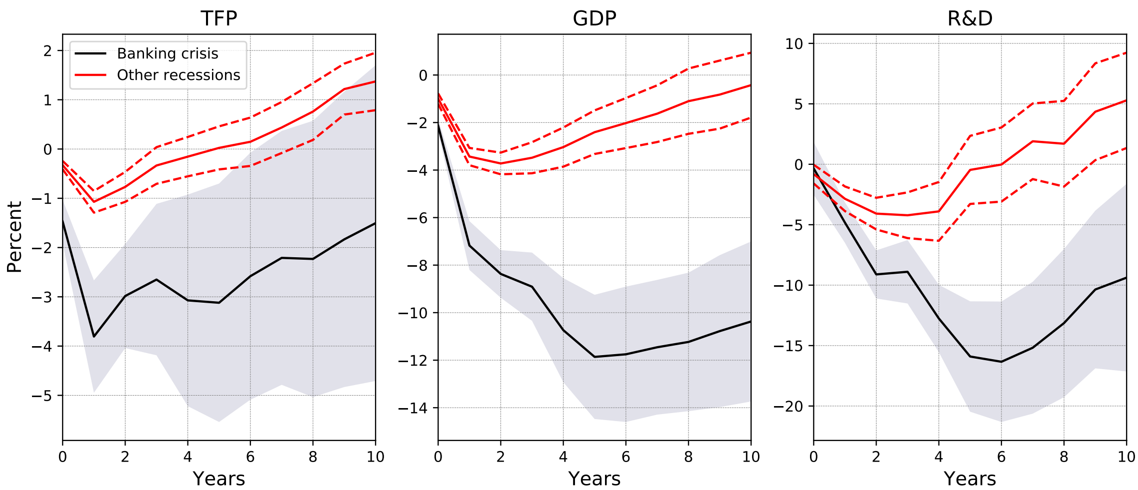
$$Y_{i,t+h} - Y_{i,t-1} = \alpha_{i,h} + \gamma_{i,h}\, \mathrm{D}_{i,t} + \Gamma_{i,h}\,(L)\, \mathrm{D}_{i,t-1} + \varepsilon_{i,t+h} \qquad \text{for } h = 0, 1, 2, ..., H \tag{2.1}$$

where $\alpha_{i,h}$ are country $i$'s fixed effects, $Y$ is the dependent variable in log-level, $\Gamma_{i,h}\,(L)$ is a lag polynomial of order 5 and $\mathrm{D}_i$ is the dummy variable of "Banking crisis" or "Other recessions", as defined in section

A. For example, projecting $Y_{t+2}$ onto the variables on the right-hand side, we obtain the estimate $\hat{\gamma}_2$. This is the effect of a banking crisis on $Y$ two-years ahead, that is orthogonal to the other variables on the right-hand side of the equation. Estimating $H$ regressions for each response variable $Y$ of interest gives us the sequence of local projections $(\hat{\gamma}_h)_{h=0}^{H}$, where an horizon of $H = 10$ years is considered.

**Results** Figure 1 displays the responses of TFP, GDP and R&D to a banking crisis or a recession. The black and red solid lines represent the local projections to a one-unit increase in the "banking crisis" dummy or the "other recessions". The grey shaded areas and red dashed lines are 68% confidence bands. The first panel shows how banking crises are associated with a stronger and more persistent decline in TFP compared to other recessions. TFP declines within the first two years by about 4% and does not revert to its initial trend within the 10-year horizon. Other recessions, instead, induce a 1% decline in TFP, which reverts to its trend within five years and slightly overshoots afterwards. The difference in responses between banking crises and other recessions is more striking for GDP. In this case, we find a very persistent and severe decline in GDP by 12% after five years. In normal recessions, GDP declines three times less (about 4%) and recovers within the 10-year horizon. Finally, we find a significant and persistent fall in business expenditure in R&D by more than 15% following a banking crisis. In other recessions, R&D declines by less than 5%, recovering within five years and rebounding afterwards. The estimated responses are quantitatively in line with Queralto (2020), except for the drop in R&D after a banking crisis, which is only half as strong in our case, possibly due to the different set of countries under consideration.[1]

FIGURE 1: IMPULSE RESPONSES TO BANKING CRISES AND OTHER RECESSIONS




Note: Black and red solid lines are local projections estimated on a dynamic panel regression with country fixed effects. Grey shaded areas and red dashed lines are 68% Driscoll-Kraay HAC-robust confidence bands.

[1]We consider our empirical analysis as complementary to that conducted in Queralto (2020), though we differ with respect to the methodology and our data. In particular, Queralto (2020) follows the methodology by Cerra and Saxena (2008) and considers both emerging market economies and advanced economies.

# 3 The Model

## 3.1 Key Features

We develop a dynamic stochastic general equilibrium model with a financial sector and macro-prudential policy in the spirit of GKQ. We extend their framework by including an endogenous growth mechanism of vertical innovation à la Grossman and Helpman (1991) and Aghion and Howitt (1992), which we introduce along the lines of Kung (2015) and Bianchi et al. (2019). More specifically, in equilibrium, the labour-augmenting productivity of intermediate output firms is dependent on the (utilised) aggregate level of intangible knowledge capital. The additional presence of this new form of capital implies that the production function will feature increasing returns to scale. It follows that the growth rate of the real variables in the model will depend on the rate of accumulation of knowledge capital in the economy.

While it is true that *R&D* projects are also financed internally, the empirical evidence suggests that a tightening of credit conditions leads to a decline in *R&D* finance which clearly indicates that external finance does matter, as described in Section 1.

## 3.2 Households

There is a continuum of identical households defined on the unit interval. Within each household there are $1 - f$ 'workers' and $f$ 'financial intermediaries'. Workers supply labour and return their wages to the household. Financial intermediaries channel funds to non-financial firms and transfer the associated profit back to the household. Within each household there is perfect consumption insurance. Households can only save by supplying funds to financial intermediaries. In addition to non-state-contingent deposits $D_t$ the household can also save by purchasing state-contingent outside equity $E_t$ from financial intermediaries. The household lifetime utility $\mathcal{V}_t$ is given by the expected, discounted sum of period utilities $\mathcal{U}$ following the specification by Greenwood et al. (1988)

$$\mathcal{V}_t = \mathbf{E}_t \sum_{k=0}^{\infty} \beta^{t+k} \mathcal{U}_{t+k}, \quad \mathcal{U}_t = \frac{1}{1-\gamma} \left( C_t - h\Gamma_t C_{t-1} - \vartheta_t \frac{L_t^{1+\varphi}}{1+\varphi} \right)^{1-\gamma}, \quad \vartheta_t = \chi N_t.$$

The source of growth in our model is the accumulation of *intangible knowledge capital* $N_t$ via research and development (R&D). Its gross growth rate is defined as $\Gamma_t \equiv N_t / N_{t-1}$. We assume that the weight associated with the dis-utility of providing labour, $\vartheta_t$, grows at the same rate as the economy itself.

$C_t$ denotes the household's consumption basket, $L_t$ denotes labour supply. $\beta$ denotes the household discount factor, $\gamma > 1$ denotes the degree of risk-aversion, $\varphi$ denotes the inverse Frisch elasticity, $\chi$ is the weight parameter associated with the dis-utility of labour supply and $h$ is the parameter characterising internal habit formation[2]. Households choose their consumption $C_t$, labour supply $L_t$, nominal riskless deposits $D_t$ and outside equity $E_t$ issued by the financial intermediary in order to maximise their life-time utility subject to the sequence of budget constraints

$$P_t C_t + Q_t^E E_t + D_t \quad = \quad W_t L_t + \Xi_t - T_t + Q_{t-1}^E R_t^E E_{t-1} + R_{t-1}^D D_{t-1}.$$

$W_t$ is the nominal wage, $\Xi_t$ is a real transfer of net profits from the financial intermediaries and monopolistically competitive firms to the household[3], and $T_t$ is a real lump-sum tax transfer. $\mathcal{R}_t^E$ denotes the

[2] This specification of habit formation allows us to detrend the period utility function.

[3] These transfers matter since intermediaries enter and exit in this economy. Exiting intermediaries transfer a dividend payment

flow returns at time $t$ from one unit of equity and will be defined when discussing financial intermediaries. $Q_t^E$ is the associated price of outside equity. Each unit of outside equity $E_t$ is a claim to the future return on the portfolio of assets that the financial intermediary holds. $R_t^D$ is the nominal gross deposit rate, while $\Pi_t \equiv P_t/P_{t-1}$ is represents the gross inflation rate. Combining the household's inter-temporal optimality conditions gives rise to a standard no-arbitrage condition between investing in bank deposits $D_t$ and in bank issued outside equity $E_t$

$$0 \quad = \quad \mathbf{E}_t \left[ \Lambda_{t,t+1} \Pi_{t+1}^{-1} \left( R_t^D - R_{t+1}^E \right) \right] \tag{3.1}$$

where we defined the stochastic discount factor as the discount factor $\beta$ multiplied by the ratio of marginal utilities of consumption $\Lambda_{t,t+1} \equiv \beta \mathcal{U}_{C,t+1}/\mathcal{U}_{C,t}$ and the gross return on investing in bank outside equity as $R_t^E$. A detailed derivation and full statement of the household optimality conditions can be found in Appendix B.

## 3.3 Non-financial Firms

The three non-financial firms in this model are: final output producers, intermediate output producers and capital producers.

**Final Output Producers** Perfectly competitive final output producers purchase varieties of intermediate outputs $Y_{m,t}$, $m \in [0,1]$ at price $P_{m,t}$ and aggregate them into a final output $Y_t$. The demand schedule for intermediate output varieties is given by

$$Y_{m,t} = \left( \frac{P_{m,t}}{P_t} \right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} Y_t, \quad \text{where} \quad P_t \equiv \left( \int_0^1 (P_{m,t})^{\frac{1}{1-\mathcal{M}}} dm \right)^{1-\mathcal{M}},$$

where $\mathcal{M} \equiv \epsilon(\epsilon-1)^{-1}$ is the markup that intermediate output producers charge on top of their marginal costs, while $\epsilon$ is the elasticity of substitution between intermediate goods.

**Intermediate Output Producers** Intermediate output producers operate in a monopolitiscally competitive market. Each variety $Y_{m,t}$ is produced according to the production function

$$Y_{m,t} = \left( \varepsilon_t^K U_{m,t}^K K_{m,t} \right)^{\alpha} \left( \mathcal{X}_{m,t}^{LAP} L_{m,t} \right)^{1-\alpha}. \tag{3.2}$$

where $\varepsilon_t^K$ denotes a physical capital quality shock. The production inputs are physical capital $K_{m,t}$, intangible capital $N_{m,t}$ and labour $L_{m,t}$. The parameter $\alpha$ denotes the share of physical capital, $U_{m,t}^i$, $i \in [N,K]$ represents the degree of capital utilisation and $\mathcal{X}_{m,t}^{LAP}$ is labour-augmenting productivity at the firm level

$$\mathcal{X}_{m,t}^{LAP} = \left( U_{m,t}^N N_{m,t} \right)^{\eta} \left( U_t^N N_t \right)^{1-\eta}. \tag{3.3}$$

Firm $m$'s labour-augmenting productivity depends therefore both on its chosen amount of utilised intangible capital $U_{m,t}^N N_{m,t}$, as well as on the aggregate level $U_t^N N_t$. The parameter $\eta \in (0,1)$ drives the degree of knowledge spillovers affecting the individual firm's productivity. If we set $\eta$ to be a small

to the household, while newly entering intermediaries receive a 'start-up' endowment.

number (implying large spillovers), productivity would be almost exogenous from the firm's perspective. In equilibrium, this would imply a smaller endogenous response of productivity to the shocks affecting our economy. It also bears noting that the endogenous growth in this model stems from the production function (3.2) featuring increasing returns to scale.

At the end of period $t-1$, firms order capital $\{K_{m,t}, N_{m,t}\}$ for use in production in the subsequent period $t$. To purchase this capital they need funding from a financial intermediary. There is frictionless intermediation between the intermediate output producer and the financial intermediary since the former is able to issue state-contingent claims on its capital. The price of such a claim is equal to the price of the underlying capital so that $K_{m,t} = B^K_{m,t-1}$ and $N_{m,t} = B^N_{m,t-1}$. These claims on capital, $B^K_{m,t}$ and $B^N_{m,t}$ can be interpreted as corporate bonds or commercial paper. After aggregate shocks have materialized at the beginning of period $t$ and production has taken place, intermediate output firms sell the remaining (non-depreciated) capital on the open market to capital goods producers at price $Q^i_t$, $i \in [K, N]$ who then conduct capital refurbishment and investment. The firm's optimality conditions are standard and give rise to demand schedules for labour and capital

$$W_t = \frac{MC_{m,t}}{\tau^{\mathcal{M}}_t}(1-\alpha)\frac{Y_{m,t}}{L_{m,t}} \tag{3.4}$$

$$R^N_t = \frac{\mathcal{R}^N_{m,t} + (1-\delta^N_t)Q^N_t}{Q^N_{t-1}}, \quad R^K_t = \frac{\mathcal{R}^K_{m,t} + (1-\delta^K_t)Q^K_t \varepsilon^K_t}{Q^K_{t-1}} \tag{3.5}$$

$$\mathcal{R}^N_{m,t} \equiv \frac{MC_{m,t}}{\tau^{\mathcal{M}}_t}(1-\alpha)\eta\frac{Y_{m,t}}{N_{m,t}}, \quad \mathcal{R}^K_{m,t} \equiv \frac{MC_{m,t}}{\tau^{\mathcal{M}}_t}\alpha\frac{Y_{m,t}}{K_{m,t}}. \tag{3.6}$$

$MC_{m,t}$ denotes the nominal marginal cost of producing one more unit of final output and $\tau^{\mathcal{M}}_t = \tau^{\mathcal{M}}\varepsilon^{\mathcal{M}}_t$ is a subsidy to correct for the distortions associated with monopolistic competition. We allow for a shock $\varepsilon^{\mathcal{M}}_t$ to marginal costs that is isomorphic to a price markup shock. The labour demand schedule (3.4) equates the wage paid by the firm to the marginal product of labour multiplied by the tax-adjusted marginal cost factor. We introduce the auxiliary variables $\mathcal{R}^K_{m,t}$ and $\mathcal{R}^N_{m,t}$, which can be interpreted as the net returns on physical and intangible capital and which are given by the respective marginal products of capital. The gross returns on capital, $R^K_t$ and $R^N_t$, are given by the sum of the net return and the re-selling value, relative to the purchasing value of capital in the previous period.

Intermediate output producers maximise profits by choosing their price. Each period, with probability $\phi_P$, a firm may not be allowed to reset its price. In this case, prices are indexed to a combination of previous-period inflation and steady-state inflation where $ind_P \in [0,1]$ is the weight attached to $\Pi_{t-1}$. We state the remaining optimality conditions in Appendix B.

**Capital Producers** There are two types of perfectly competitive capital producers $i \in [K, N]$, refurbishing physical and intangible capital, respectively, subject to convex investment adjustment costs $\psi_{I^i}$

$$\max_{I^i_t} \mathbf{E}_t \left[ \sum_{k=0}^{\infty} \beta^{t+k} \frac{\mathcal{U}_{C,t+k}}{\mathcal{U}_{C,t}} \frac{P_t}{P_{t+k}} \left\{ Q^i_{t+k} I^i_{t+k} - \left[ 1 + \frac{\psi_{I^i}}{2} \left( \frac{I^i_{t+k}}{I^i_{t+k-1}} - \bar{\Gamma} \right)^2 \right] P_{t+k} I^i_{t+k} \right\} \right],$$

where $\bar{\Gamma}$ is the gross growth rate of investment on the balanced growth path (BGP)[4] of the economy and where physical and intangible capital investment, $I_t^K$ and $I_t^N$ respectively, are given by

$$I_t^K = K_{t+1} - \left[1 - \delta_t^K\right] K_t \varepsilon_t^K, \quad I_t^N = N_{t+1} - \left[1 - \delta_t^N\right] N_t. \tag{3.7}$$

For both types $i \in [K, N]$ the optimality condition can be interpreted as a capital supply schedule

$$Q_t^i = 1 + \frac{\psi_{I^i}}{2}\left(\frac{I_t^i}{I_{t-1}^i} - \bar{\Gamma}\right)^2 + \left(\frac{I_t^i}{I_{t-1}^i}\right)\psi_{I^i}\left(\frac{I_t^i}{I_{t-1}^i} - \bar{\Gamma}\right) - \mathbf{E}_t \Lambda_{t,t+1}\left(\frac{I_{t+1}^i}{I_t^i}\right)^2 \psi_{I^i}\left(\frac{I_{t+1}^i}{I_t^i} - \bar{\Gamma}\right). \tag{3.8}$$

### 3.4 Financial Intermediaries

Financial intermediaries collect funds from households and lend to firms to finance their investment in physical and intangible capital. The portfolio of assets of a financial intermediary $j$ consists of (i) reserve holdings $RE_{j,t}$ (ii) claims on physical capital $B_{j,t}^K$ and (iii) claims on intangible capital $B_{j,t}^N$. The portfolio is either funded by net worth $NW_{j,t}$, by outside equity $Q_t^E E_{j,t}$, or by non-state-contingent deposits $D_{j,t}$

$$RE_{j,t} + \left(1 + \tau_t^N\right) Q_t^N B_{j,t}^N + \left(1 + \tau_t^K\right) Q_t^K B_{j,t}^K = NW_{j,t} + \left(1 + \tau_t^E\right) Q_t^E E_{j,t} + D_{j,t}.$$

We will describe the macro-prudential taxation scheme[5] $\{\tau_t^K, \tau_t^N, \tau_t^E\}$ on the intermediary's assets and outside equity issuance in detail in Section 3.6. In contrast to $E_{j,t}$ and $D_{j,t}$, net worth $NW_{j,t}$ is raised internally via the accumulation of retained earnings

$$NW_{j,t} = R_{t-1}^D RE_{t-1} + R_t^N Q_{t-1}^N B_{j,t-1}^N + R_t^K Q_{t-1}^K B_{j,t-1}^K - R_t^E Q_{t-1}^E E_{j,t-1} - R_{t-1}^D D_{j,t-1}.$$

The nominal franchise value, $V_{j,t}$, is the expected payout from the terminal nominal net worth $NW_{j,t}$

$$V_{j,t} \quad = \quad \mathbf{E}_t \left[\sum_{\tau=t+1}^{\infty} (1 - \sigma_t)\sigma_t^{\tau-t-1} \Lambda_{t,\tau} \Pi_{t,\tau}^{-1} NW_{b,\tau}\right],$$

where $\sigma_t = \sigma \varepsilon_t^\sigma$ denotes the survival rate of the intermediary. The shock process that affects the intermediary 'survival rate', $\varepsilon_t^\sigma$, is the key financial disturbance in our model.

Following GKQ we introduce a moral hazard problem in order to limit the ability of intermediaries to expand their balance sheet and maximise their terminal dividend value. It is assumed that intermediaries are able to abscond with a fraction $\Theta_t$ of their assets. Households therefore incentivise intermediaries not to divert assets by limiting the funding of intermediaries such that their franchise value $V_{j,t}$ is at least as large as the asset stock that can be diverted

$$V_{j,t} \quad \geq \quad \Theta_t \left(\Delta^K Q_t^K B_{j,t}^K + Q_t^N B_{j,t}^{R\&D}\right), \quad 0 < \Delta^K < 1.$$

Note that we allow for different liquidation values of physical and intangible capital claims via the risk-weight $\Delta^K < 1$. We assume that physical capital claims are less risky than $R\&D$ capital claims. We furthermore assume that the fraction of assets that the financial intermediary can steal depends on the

[4] We abstract from sector-specific trends and assume that all expenditure components in this economy grow at the same rate.

[5] We assume that the subsidy on outside equity is entirely financed by the revenue raised via the taxation of intermediary assets so that $\tau_t^N Q_t^N B_{j,t}^N + \tau_t^K Q_t^K B_{j,t}^K = \tau_t^E Q_t^E E_{j,t}$. Moreover, we assume that the tax rates on the assets are set according to their relative risk-profiles so that $\tau_t^K = \Delta \tau_t^N$ which implies $\tau_t^N \left(\Delta^K Q_t^K B_{j,t}^K + Q_t^N B_{j,t}^N\right) = \tau_t^E Q_t^E E_{j,t} \Leftrightarrow \tau_t^N = \tau_t^E X_{j,t}$.

liability composition. Following Calomiris and Kahn (1991) and GKQ, we argue that the more (outside) equity an intermediary uses to finance itself, the more difficult it is to monitor its balance sheet. In terms of repayments and returns, debt is assumed to be more transparent and can, therefore, serve as a 'disciplining device'. As in GKQ, the diversion rate $\Theta$ is an increasing function of equity and decreasing in debt

$$\Theta_t = \theta\left(1+\omega_1 X_{j,t}+\frac{\omega_2}{2}X_{j,t}^2\right) \tag{3.9}$$

where the intermediary's risk-weighted equity-to-asset ratio is given by

$$X_{j,t} \equiv \frac{Q_t^E E_{j,t}}{\Delta^K Q_t^K B_{j,t}^K + Q_t^N B_{j,t}^N}, \quad X_{j,t} \in (0,1)$$

and where $\omega_1 < 0$ and $\omega_2 > 0$. The calibration[6] of the parameters $\omega_1, \omega_2$ will be such that the marginal diversion rate is positive $\Theta'(X_{j,t}) = (\omega_1 + \omega_2 X_{j,t}) > 0$ in the risk-adjusted BGP ('steady state' with balanced growth). We define the intermediary's risk-weighted leverage ratio as follows

$$\phi_{j,t} \equiv \frac{\Delta^K Q_t^K B_{j,t}^K + Q_t^N B_{j,t}^N}{NW_{j,t}}. \tag{3.10}$$

The intertemporal optimality conditions of the intermediary and the associated auxiliary definitions are

$$\Delta^K \mu_{j,t}^{B^N} \equiv \mathbf{E}_t\left[\Omega_{j,t+1}\Lambda_{t,t+1}\Pi_{t+1}^{-1}\left(R_{t+1}^K - R_t^D\right)\right] \tag{3.11}$$

$$\mu_{j,t}^{B^N} \equiv \mathbf{E}_t\left[\Omega_{j,t+1}\Lambda_{t,t+1}\Pi_{t+1}^{-1}\left(R_{t+1}^N - R_t^D\right)\right] \tag{3.12}$$

$$\mu_{j,t}^{E} \equiv \mathbf{E}_t\left[\Omega_{j,t+1}\Lambda_{t,t+1}\Pi_{t+1}^{-1}\left(R_t^D - R_{t+1}^E\right)\right] \tag{3.13}$$

$$\nu_{j,t} \equiv \mathbf{E}_t\left[\Omega_{j,t+1}\Lambda_{t,t+1}\Pi_{t+1}^{-1} R_t^D\right], \tag{3.14}$$

where $\mu_{j,t}^{B^N}$ denotes the excess returns from investing in intangible capital claims over the cost of issuing deposits. $\mu_{j,t}^{E}$ denotes the excess funding cost from issuing deposits over issuing equity and $\nu_{j,t}$ denotes the cost of issuing deposits. $\Omega_{j,t}$ can be interpreted as the shadow price of net worth

$$\Omega_{j,t} = (1-\sigma_t) + \sigma_t\left[\nu_{j,t} + \phi_{j,t}\left(\mu_{j,t}^{B^N} + \phi_{j,t} X_{j,t} \mu_{j,t}^{E}\right)\right]. \tag{3.15}$$

Given its net worth, the intermediary has to decide on the quantity of its assets and the share of the portfolio of assets funded by equity. Combining the first-order conditions of the intermediary delivers

$$\frac{\mu_{j,t}^{B^N} + \mu_{j,t}^{E} X_{j,t}}{\Theta(X_{j,t})} = \frac{\mu_{j,t}^{E} + \nu_{j,t}\tau_t^E}{\Theta'(X_{j,t})}. \tag{3.16}$$

Equation (3.16) describes the equality between the benefit-cost ratio of having more assets (LHS) and the benefit-cost ratio of increasing outside equity issuance (RHS) to finance the portfolio increase. Note that the intermediary faces a trade-off when using outside equity. On the one hand, outside equity provides a

[6] The reason for allowing for a negative $\omega_1$ is that GKQ want to calibrate a sufficiently high level of equity financing to match the respective data counterpart. Crucially, at the margin, $\Theta'(X_{j,t}) > 0$.

hedging value for the intermediary since it is state-contingent and therefore tied to the return on assets.[7] On the other hand, issuing outside equity is assumed to increase the fraction $\Theta$ and will therefore tighten the overall borrowing capacity of the financial intermediary.[8] Assuming that the incentive constraint is always binding, we derive the intermediary's optimal leverage ratio

$$\phi_{j,t} = \frac{\nu_{j,t}}{\Theta_t - \left(\mu^{BN}_{j,t} + X_{j,t}\mu^{E}_{j,t}\right)} \tag{3.17}$$

which is increasing in those elements that raise the franchise value of the intermediary. We normalise the intermediary's return on outside equity such that it entitles the household to the return on one unit of the intermediary's portfolio of assets. The gross return on equity is thus given by

$$R^E_t = \frac{\mathcal{R}^E_t + Q^E_t \varepsilon^E_t}{Q^E_{t-1}} \tag{3.18}$$

where the flow return on equity $\mathcal{R}^E_t$ is equal to the flow return on total capital, which, in turn, is a weighted average of the flow return on physical and on intangible capital

$$\mathcal{R}^E_t = R^K_t \frac{K_t}{\mathcal{K}_t} + R^N_t \frac{N_t}{\mathcal{K}_t}, \quad \mathcal{K}_t \equiv \left((K_t)^\alpha (N_t)^{\eta(1-\alpha)}\right)^{1/(\alpha+\eta(1-\alpha))}, \quad \varepsilon^E_t \equiv \left(\varepsilon^K_t\right)^{\alpha/(\alpha+\eta(1-\alpha))} \tag{3.19}$$

Total capital $\mathcal{K}_t$ is defined as a composite between physical and intangible capital and the shock to equity returns $\varepsilon^E_t$ is a scaling of the shock to physical capital quality. A detailed derivation can be found in Appendix B.

## 3.5 Monetary Policy

The central bank sets the short-term nominal gross interest rate on deposits $R^D_t$ according to a simple Taylor-type rule, subject to a ZLB constraint

$$\frac{R^{TR}_t}{\bar{R}^{TR}} = \left(\frac{R^{TR}_{t-1}}{\bar{R}^{TR}}\right)^{\rho_{RTR}} \left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\kappa_\Pi} \left(\frac{MC_t}{\overline{MC}}\right)^{\kappa_Y}\right]^{1-\rho_{RTR}} \varepsilon^{MP}_t \tag{3.20}$$

$$R^D_t = \max\left(1, R^{TR}_t\right). \tag{3.21}$$

When the ZLB constraint is not binding, the central bank responds to deviations of inflation from target and to a proxy of the output gap, assumed to be the marginal cost relative to the flexible price marginal cost. $\kappa_\Pi$ and $\kappa_Y$ are the respective reaction coefficients. The parameter $\rho_{RTR}$ captures the gradual adjustment of the policy instrument. Finally, $\varepsilon^{MP}_t$ represents a monetary policy shock.

## 3.6 Macro-prudential Policy

Since asset prices enter the intermediary's incentive compatibility constraint, a pecuniary externality arises. The use of state-contingent outside equity dampens fluctuations that transmit via shocks on asset returns through the intermediary's net worth (or shocks to net worth), to lending to firms and

[7] Also note that the intermediary uses discount factor $\Lambda_{t,t+1}\Pi^{-1}_{t+1}\Omega_{t+1}$ to discount it's returns on assets and equity. The presence of $\Omega_{t+1}$ makes the intermediary's discount factor more volatile and therefore more risk-averse than the household. Consequently, using state-contingent equity provides hedging value for the intermediary.

[8] Note that if one assumes that $\Theta$ is a constant, unresponsive to equity $E_{j,t}$, then intermediaries would prefer to exclusively fund themselves with state-contingent outside equity and their net worth would not at all respond to asset returns.

investment. Financial intermediaries take asset prices as given. As a result, they ignore the stabilisation benefits that outside equity finance would have on asset prices. The failure to recognise the external benefits of outside equity issuance constitutes an inefficiency which warrants a macro-prudential policy intervention.

Macro-prudential policy is implemented via a subsidy, $\tau_t^E$, on outside equity. This subsidy is financed by taxing the intermediary's asset holdings while taking into account the relative asset risk profiles: $\tau_t^K = \Delta^K \tau_t^N$[9]. Subsidising equity issuance increases its relative attractiveness over deposit finance. We assume that $\tau_t^E$ responds to the inverse of the shadow cost of deposits

$$\tau_t^E \quad = \quad \kappa_\nu \nu_t^{-1}. \tag{3.22}$$

The reaction coefficient $\kappa_\nu$ governs the stance of macro-prudential policy. If the shadow cost of deposits $\nu_t$ is low, the macro-prudential subsidy $\tau_t^E$ will be high, and vice versa. By increasing the macro-prudential subsidy $\tau_t^E$, the regulator provides an incentive for intermediaries to issue more outside equity even though the shadow cost of deposits is low.

## 3.7 Market Clearing, Aggregation and Equilibrium Definition

The model is closed with market clearing conditions for for securities, capital and labour. In equilibrium, all financial intermediaries have the same risk-weighted equity-to-asset and leverage ratios. Aggregate net worth is the sum of the net worth of 'old' intermediaries, $NW_t^o$, and of the 'new' intermediaries, $NW_t^y = \xi$, $NW_t = \sigma_t NW_t^o + \xi$. The aggregate net worth of those who did not exit, $NW_t^o$, is the difference of earnings on assets net of the cost of funding. Combining this with the balance sheet identity one can derive an equation that pins down the law of motion of aggregate net worth as follows

$$\begin{aligned} NW_t \quad = \quad & \sigma_t \Big[ Q_{t-1}^K B_{t-1}^K \left( R_t^K - R_{t-1}^D \right) + Q_{t-1}^N B_{t-1}^N \left( R_t^N - R_{t-1}^D \right) \\ & + X_{t-1} \phi_{t-1} \left( R_{t-1}^D - R_{t-1}^E \right) NW_{t-1} + R_{t-1}^D NW_{t-1} \Big] + \xi. \end{aligned} \tag{3.23}$$

In order to induce stationarity, we divide all real quantities by the aggregate stock of $N_t$ capital, all nominal quantities are detrended by dividing by $P_t N_t$ and nominal prices are expressed in real terms by dividing by $P_t$. For example, detrended output $\hat{Y}_t$, real detrended net worth $\widehat{NW}_t$ and the real price of capital $\hat{Q}_t^K$ are given by $\hat{Y}_t \equiv Y_t / N_t$, $\widehat{NW}_t \equiv NW_t / (P_t N_t)$, and $\hat{Q}_t^K \equiv Q_t^K / P_t$. Consider the detrended version[10] of the intangible capital accumulation equation and note that growth $\Gamma_t$ is endogenous in this model

$$\Gamma_{t+1} \quad = \quad \hat{I}_t^N + \left[ 1 - \delta_t^N \right], \quad \text{where} \quad \Gamma_t \equiv \frac{N_t}{N_{t-1}}.$$

The exogenous processes for physical capital quality ($\log \varepsilon_t^K$), markups ($\log \varepsilon_t^{\mathcal{M}}$), the survival rate

[9] Refer to Appendix Section B for details and implications.

[10] In the Appendix in Section C there is a detailed derivation of how all the above derived equations were detrended.

($\log \varepsilon_t^{\sigma}$) and monetary policy ($\log \varepsilon_t^{MP}$) follow standard AR(1) processes

$$
\begin{aligned}
\log \varepsilon_t^{K} &= \rho_K \log \varepsilon_{t-1}^{K} + \varsigma_K \eta_t^{K} && (3.24)\\
\log \varepsilon_t^{\mathcal{M}} &= \rho_{\mathcal{M}} \log \varepsilon_{t-1}^{\mathcal{M}} + \varsigma_{\mathcal{M}} \eta_t^{\mathcal{M}} && (3.25)\\
\log \varepsilon_t^{\sigma} &= \rho_{\sigma} \log \varepsilon_{t-1}^{\sigma} + \varsigma_{\sigma} \eta_t^{\sigma} && (3.26)\\
\log \varepsilon_t^{MP} &= \rho_{MP} \log \varepsilon_{t-1}^{MP} + \varsigma_{MP} \eta_t^{MP} && (3.27)
\end{aligned}
$$

with persistence $\rho_j$ and standard deviation $\varsigma_j$ for shock $j$, respectively. In the Appendix in Section D we state the complete set of stationary equilibrium conditions.

In the analysis that follows below, we compare our baseline model to an exogenous growth counterpart and to a model with endogenous growth but without financial frictions. In the case of exogenous growth, it holds that $\Gamma_t = \bar{\Gamma}$, which can be thought of as a limiting case in which the investment adjustment cost for intangible capital investment is infinitely high $\psi_{I^N} \to \infty$. All variables associated with intangible capital become constants. In the model without financial frictions, we can disregard the equations associated to the banking block, (3.9)-(3.19) and all spreads become zero.

## 3.8 Model Solution and Calibration

**Model Solution** A key element of our model is the endogenous liability choice of financial intermediaries between non-state contingent deposits $D_t$ and state-contingent outside equity $E_t$. In order to capture these effects, the model is solved by taking a first-order Taylor approximation around the risk-adjusted BGP[11] as in GKQ and de Groot (2014). The risk-adjusted BGP refers to the 'point where agents choose to stay at a given date if they expect future risk and if the realisation of shocks is 0 at this date' (Coeurdacier et al. (2011), refer to Appendix E for details).

The reason for solving the model around the risk-adjusted BGP rather than using the standard deterministic BGP is to account for the stabilizing properties of banks' equity. When solved around the deterministic BGP, the model implies that banks have no advantage to fund with equity over using debt. This is because shocks are expected to be zero in the future. As banks anticipate only one possible future state for the economy, banks prefer to use cheap debt rather than to go for expensive state-contingent equity. In contrast, allowing for future risk in the computation of the BGP enables us to express the level of risk as a function of banks' liabilities. The fact that banks do not fully internalize the benefits of funding with equity, as they take asset prices as given, motivates macro-prudential policy in our model. A better capitalised banking system with a higher intermediation capacity is able to channel more funds from lenders to borrowers and thereby facilitates higher levels of investment in physical and intangible capital. Thus, solving around a risk-adjusted BGP will allow us to capture the benefits of macro-prudential policy by accounting for the reduced volatility.

To further illustrate the implications of the risk-adjusted BGP, recall the household's no-abritrage relationship between debt and equity (3.1)

$$
0 = \mathbf{E}_t \left[ \Lambda_{t,t+1} \Pi_{t+1}^{-1} \left( R_t^{D} - R_{t+1}^{E} \right) \right]
$$

[11] The deterministic BGP refers to an equilibrium in which the economy is not hit by any shocks. In models without growth this equilibrium would be referred to as the deterministic steady state. In the analysis that follows below we use the subscript $_{bgp}$ to refer to the risk-adjusted balanced growth path.

and the financial intermediaries optimality condition for choosing debt versus equity finance (3.13)

$$\mu_t^E = E_t\left[\Lambda_{t,t+1}\Omega_{t+1}\Pi_{t+1}^{-1}\left(R_t^D - R_{t+1}^E\right)\right].$$

In a conventional deterministic BGP equilibrium, these two equations would imply that $\mu^E_{bgp,d} = 0$, so that the excess value of using equity is zero. In the risk-adjusted BGP equilibrium, we would instead have

$$\begin{aligned} 0 &= \left[\Lambda_{bgp,r}\Pi^{-1}_{bgp,r}\left(R^D_{bgp,r} - R^E_{bgp,r}\right)\right] + M_1 \\ \mu^E_{bgp,r} &= \left[\Lambda_{bgp,r}\Omega_{bgp,r}\Pi^{-1}_{bgp,r}\left(R^D_{bgp,r} - R^E_{bgp,r}\right)\right] + M_2. \end{aligned}$$

The intermediary's excess value of using equity is then given by $\mu^E_{bgp,r} = -\Omega_{bgp,r}M_1 + M_2$, which is positive as long as $\Omega_{bgp,r}M_1 < M_2$. The risk-adjustments $M_1$ and $M_2$ are functions of the covariances between the household's and the financial intermediary's stochastic discount factors, and the return on equity. Note that the household discounts the spread between deposits and equity at $\Lambda_{t,t+1}$ while the intermediary discounts at $\Lambda_{t,t+1}\Omega_{t+1}$. Under our calibration, the financial intermediary's augmented SDF is more volatile so that the intermediary is more risk-averse. $M_2 > \Omega_{bgp,r}M_1$ and $\mu^E_{bgp,r} > 0$ implies that the intermediary receives hedging value by substituting debt with equity. The substitution towards equity is constrained by the increased divertibility, as discussed above. Furthermore, the use of outside equity is inefficiently low since the atomistic intermediary takes asset prices as given.

**Calibration** Table 1 presents the calibration of our baseline model. The five parameters related to the households $\{\gamma, \beta, h, \chi, \varphi\}$ take values that are standard in the literature. In line with Kung and Schmid (2015), we calibrate the balanced-growth-path (BGP) value of employment such that the steady-state share of $R\&D$ investment to GDP is roughly 2.3%, which implies a value for the knowledge spillover parameter of $\eta = 0.0491$.

We calibrate the annual net growth rate of the economy to be 1.6%. This implies a quarterly gross growth rate of $\bar{\Gamma} = 1.004$, in line with the average real per-capita GDP growth rate of the US economy since the early 1980s. The investment adjustment cost parameters are calibrated to match the volatilities of investment. The depreciation rate of intangible capital is set to $\delta^N_{bgp} = 0.0375$, consistent with the value used by the US Bureau of Labour Statistics in the $R\&D$ stock calculations. The capital share $\alpha$, the physical capital depreciation rate $\delta^K_{bgp}$, the elasticity of substitution $\epsilon$, the Calvo price adjustment cost parameter $\psi_P$, and the indexation to past inflation $ind_P$ are all calibrated to standard values.

Regarding the parameters related to the financial sector, we target a leverage ratio of roughly 6 and average spreads for physical and intangible capital that correspond to investment-grade and high-yield corporate bond spreads. For physical capital, we target the average spread between 'Moody's Seasoned Aaa' corporate bond yield and the federal funds rate, while for intangible capital we target the average spread between the 'ICE BofA BB US High Yield Index Effective' yield and the federal funds rate. The latter is a 'high yield' corporate bond spread, and it is available since 1997. The ratio between these average spreads is $\Delta^K = 0.63$, and it indicates the relative risk between these two types of bonds.

We calibrate the absconding rate for intangible capital to be higher than in most models[12], $\theta = 0.9127$,

[12]In Queralto (2020) and Ikeda and Kurozumi (2019)'s Q-type model in the Appendix, the collateral coefficient for knowledge capital is calibrated to less than one third, implying a high liquidation value of more than 2/3. In Bianchi et al. (2019) the authors incorporate a financial friction a la Jermann and Quadrini (2012). They assume a liquidation value of 0, which would correspond to $\theta = 1$ in our model. However, it is questionable whether intangible capital has indeed a liquidation value of exactly 0. Moreover,

TABLE 1: PARAMETER VALUES

| Parameter | Definition | Value | Source/Target |
|---|---|---|---|
| *Households* | | | |
| $\gamma$ | Household Risk Aversion | 2.0000 | Literature |
| $\beta$ | Household Discount Factor | 0.9990 | Literature; annual net nominal rate $r_{bgp} \approx 3.7\%$ |
| $h$ | Habit formation parameter | 0.6000 | Literature; Volatility of $C$ |
| $\chi$ | Utility Weight of Labour | 1.6318 | $L_{bgp} \approx 0.6$ so that $I^N_{bgp}/Y_{bgp} \approx 2.3\%$ |
| $\varphi$ | Inverse Frisch Elasticity | 1.0000 | Literature |
| *Endogenous Growth* | | | |
| $\eta$ | Knowledge Spillover parameter | 0.0508 | Match $I^N_{bgp}/Y_{bgp} \approx 2.3\%$ |
| $\bar{\Gamma}$ | BGP Gross Growth Rate | 1.0040 | 1.6 % BGP annual net growth |
| *Non-financial Firms* | | | |
| $\alpha$ | Capital share | 0.3300 | Literature; Capital/Labour Shares |
| $\psi_{IK}$ | $K$ Investment Adjustment Cost | 0.4000 | Volatility of $I^K$ |
| $\psi_{IN}$ | $N$ Investment Adjustment Cost | 2.0000 | Moran and Queralto (2018); Volatility of $I^N$ |
| $\bar{\delta}^K$ | Constant $K$ Depreciation parameter | 0.0154 | Target $\delta^K_{bgp} = 0.0250$ |
| $\bar{\delta}^N$ | Constant $N$ Depreciation parameter | 0.0303 | Target $\delta^N_{bgp} = 0.0375$ |
| $b_{U,K}$ | $K$ Depreciation Sensitivity | 0.0379 | Target $\delta^K_{bgp} = 0.0250$ |
| $b_{U,N}$ | $N$ Depreciation Sensitivity | 0.0593 | Target $\delta^N_{bgp} = 0.0375$ |
| $\zeta_{U,K}$ | $K$ Depreciation Elasticity | 7.2000 | Gertler and Karadi (2011) |
| $\zeta_{U,N}$ | $N$ Depreciation Elasticity | 7.2000 | Gertler and Karadi (2011) |
| $\epsilon$ | Substitution Elasticity | 11.0000 | Markup of 10% |
| $\psi_P$ | Calvo Price Adjustment | 0.7500 | Literature; average lifetime of prices |
| $ind_P$ | Price Indexation to $\Pi_{t-1}$ | 0.2000 | Literature; Volatility of $\Pi$ |
| *Financial Intermediaries* | | | |
| $\sigma$ | Survival Rate | 0.9300 | Gertler et al. (2020) , target leverage $\approx 6$ |
| $\xi$ | Transfer to entering FI | 0.0108 | Target $SpreadK_{bgp} \approx 0.0075$ |
| $\theta$ | Absconding coefficient for $N$ capital | 0.9127 | Target $SpreadRnD_{bgp} \approx 0.0115$ |
| $\Delta^K$ | Risk-weight on $K$ capital claims | 0.6300 | Avg Spread Ratio: (AAA-FFR)/(BofABB-FFR) |
| $\omega_1$ | Asset Diversion Parameter 1 | -0.7500 | de Groot (2014), target $X_{bgp} \approx 6\%$ |
| $\omega_2$ | Asset Diversion Parameter 2 | 12.7334 | de Groot (2014), target $X_{bgp} \approx 6\%$ |
| *Macro-prudential Policy* | | | |
| $\kappa_\nu\,(\kappa^*_\nu)$ | Equity Subsidy Sensitivity to $\nu^{-1}$ | 0.0000 (0.0760) | Find welfare-maximising value |
| *Monetary Policy* | | | |
| $\kappa_\Pi$ | Interest Rate Sensitivity to Inflation | 1.5000 | Literature |
| $\kappa_Y$ | Interest Rate Sensitivity to Output | 0.1250 | Literature |
| $\rho_{RTR}$ | Interest Rate Smoothing | 0.7000 | Literature |
| $\Pi^*$ | Inflation Target | 1.0000 | similar to Sims and Wu (2020), so ZLB can bind |
| *Shock Processes* | | | |
| $\rho_{\mathcal{M}}$ | Persistence of Markup Shock | 0.7000 | Similar to Queralto (2020); ensure $\Pi \uparrow$ on impact |
| $\rho_\sigma$ | Persistence of Survival Rate Shock | 0.0000 | Ensure spreads increase on impact |
| $\rho_{CQ}$ | Persistence of K Capital Quality Shock | 0.0000 | Similar to GKQ |
| $\rho_{MP}$ | Persistence of MP Shock | 0.0000 | Literature, Persistence via $\rho_{RTR}$ |
| $\varsigma_{\mathcal{M}}$ | St Dev of Markup Shock | 0.0100 | Similar to Queralto (2020); match volatility of $Y$ |
| $\varsigma_\sigma$ | St Dev of Survival Rate Shock | 0.0500 | Similar to Coenen et al. (2018), ensure $\mu^E_{bgp} > 0$ |
| $\varsigma_{MP}$ | St Dev of MP Shock | 0.0018 | $\approx$ 25 Basis point increase in $R^D$ on impact |
| $\varsigma_{CQ}$ | St Dev of K Capital Quality Shock | 0.008 | Similar to GKQ |

which implies a liquidation value of intangible capital of less than 9%. The parameters $\omega_1$ and $\omega_2$ are calibrated to deliver an equity-to-asset ratio of $X \approx 0.06$. This corresponds to a 6% 'capital adequacy ratio (CAR)' in the unregulated risk-adjusted BGP equilibrium.[13] We will then vary the macro-prudential sensitivity parameter $\kappa_\nu$ and select the value that maximises the household's lifetime value $\hat{\mathcal{V}}_{BGP}$.

The monetary policy parameters $\kappa_\Pi, \kappa_Y, \rho_{RTR}$ are calibrated to standard values commonly found in the literature. We assume that there is no inflation along the BGP so that $\bar{\Pi} = 1$. This assumption, together with our assumptions on $\beta$ and $\bar{\Gamma}$, implies an annual nominal net interest rate of $r_{BGP} \approx 3.7\%$. This ensures that the nominal interest rate can occasionally hit the ZLB.

We calibrate the markup shock following the specification used in Queralto (2020), Comin and

in contrast to the Jermann and Quadrini (2012)-type model, the GKQ-type banking model cannot be solved for values of $\theta$ too close to unity.

[13] A capital ratio of 6% in an unregulated economy is not implausible when one considers that the first Basel accord of 1988 stipulated that 'Banks with an international presence are required to hold capital equal to 8% of their risk-weighted assets (RWA)'.

Gertler (2006) and Galí et al. (2007). The monetary policy shock will induce a persistent response via the interest rate smoothing parameter $\rho_{RTR}$, whereas the survival rate shock via the inertial evolution of net worth. Moreover, positive persistence values for the survival rate shock, even small values, lead to a fall in the spread on impact, which is at odds with the data. We then set the standard deviation of these shocks to match the volatilities of output, consumption, investment, prices and spreads. The standard deviation of the markup shock (0.01) is broadly in line with the value used in Queralto (2020). The standard deviation of the monetary policy shock is chosen such that the nominal interest rate increases by roughly 25 basis points on impact. The value assigned to standard deviation of the survival rate shock ($\varsigma_\sigma = 0.05$)[14] is in line with the value used in Coenen et al. (2018).

# 4 Slow Recoveries and Permanent Losses in Output

In Figure 2, we display the impulse-response functions (IRFs) of key variables to (i) a markup shock (upper row), a (ii) monetary policy shock (second row), a financial intermediary survival rate shock (third row) and a $K$ capital quality shock. In order to highlight how endogenous growth and the financial frictions affect the transmission of the shocks, we compare the IRFs across different models. In particular, we consider three alternative specifications: the baseline model with endogenous growth and financial frictions (blue-solid line), a version of the model with exogenous growth and financial frictions (green-circled line), and, finally, a model with endogenous growth but without financial frictions (purple-dotted line). It bears noting that the responses of output are expressed in percentage deviations from the initial BGP.

All four shocks lead to a contraction in output, growth rate and net worth, and an increase in the spread. Unlike the monetary policy and the survival rate shock, the markup shock is associated with an initial increase in inflation.

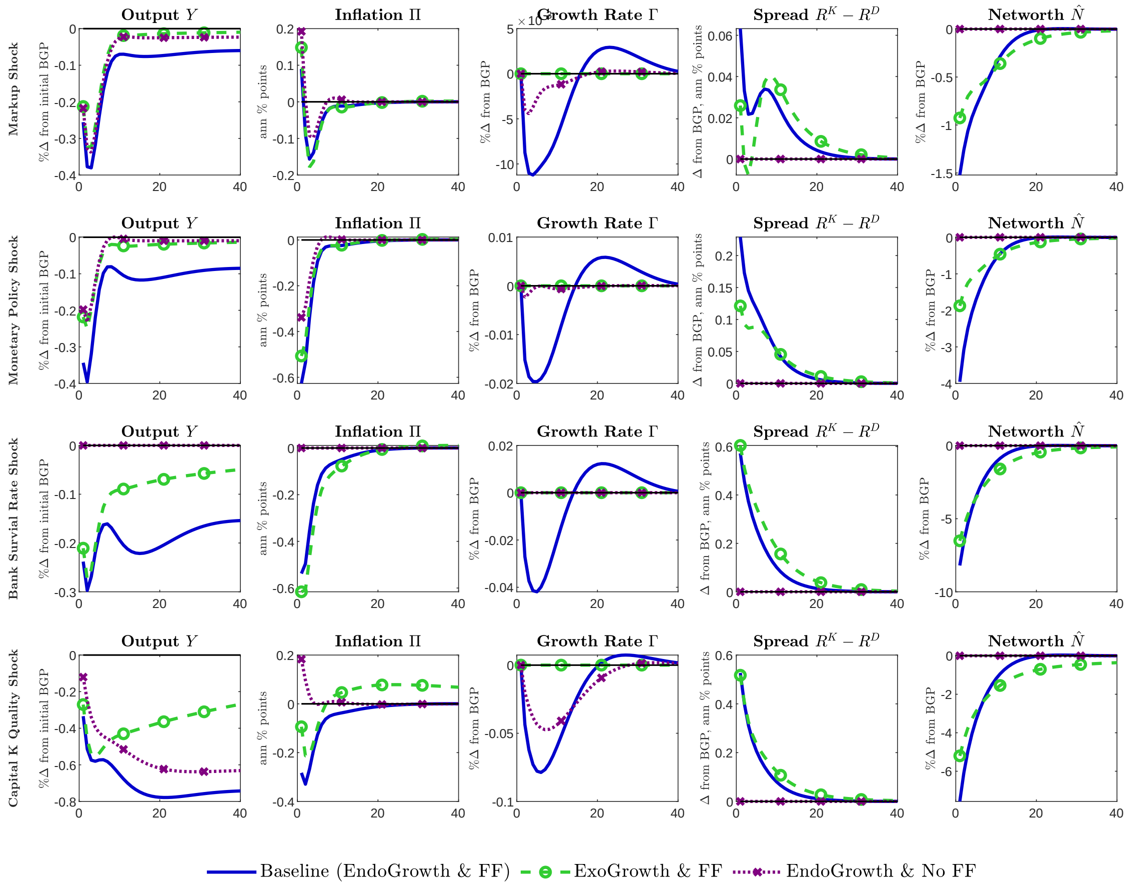
In the baseline model, the economic contraction leads to a decline in investment, in both physical and intangible capital. The fall in intangible capital investment, in turn, triggers a decline in productivity growth $\Gamma$. For this reason, in our baseline model, the level of output does not return to its initial (no-shock) BGP. The permanent losses in the output level are particularly evident in response to a bank survival rate shock, which causes the most significant contraction in intangible capital and the productivity growth rate. In other words, the bank survival rate shock induces a slow recovery and permanent losses in output, qualitatively in line with the empirical evidence. By contrast, despite causing a large initial drop in output, the long-term effects of markup and monetary policy shocks tend to be more muted due to a relatively milder fall in the growth rate.

To highlight how endogenous growth affects the transmission of the shocks, compare our baseline results to those from the exogenous-growth specification. In the model with exogenous growth, the shocks considered above tend to have a similar impact effect on output as in the baseline case. However, in the long-run, the level of output tends to return to its initial BGP, since shock leaves the productivity growth rate unaffected.

Comparing our baseline model to the specification without financial frictions allows us to underscore the amplification provided by the financial sector. In particular, we note how financial frictions strongly amplify the response of the growth rate. The reason is that, in our baseline model, a contractionary shock leads to a rise in the spread and a credit tightening. This has particularly severe consequences

[14] In the model with exogenous growth we decrease the standard deviation to $\varsigma_\sigma = 0.03$ in order to generate similar IRFs of the $K$ Spread for both, the endogenous and the exogenous growth model.

Figure 2: IRFs for Markup, Monetary Policy, Bank Survival Rate and K Capital Quality Shock




Note: The upper row depicts the IRFs for a (+1stdev) markup shock, the middle row for a (+1stdev) monetary policy shock and the lower row for a (-1stdev) survival rate shock. The blue-straight line depicts the responses of the baseline model, with endogenous growth and financial frictions, the green-circled line depicts the responses for an exogenous growth model with financial frictions and the purple-crossed line depicts the responses for an endogenous growth model without financial frictions.

for both forms of investment. Since the endogenous growth engine is linked to the dynamics of intangible investment, financial frictions cause adverse shocks to have relatively severe consequences on the productivity growth rate and hence larger permanent output losses.

# 5 Macro-prudential Policy Analysis

In this section, we study how macro-prudential policy can mitigate the slow recovery and permanent output losses in response to negative shocks. This type of policy is particularly effective in counteracting the adverse effects of financial shocks, which have the largest long-term consequences. We first provide some intuition on how our assumption regarding endogenous growth affects the welfare-based policy analysis. We then discuss the analysis of welfare-optimising macro-prudential policy and its static and dynamic implications. Finally, we discuss how macro-prudential policy can be a useful tool to avoid the effective lower bound on the nominal interest rate.

The key result that emerges from our analysis is that the optimised macro-prudential policy in an environment with endogenous growth is associated with a higher bank capital ratio and larger welfare gains than under exogenous growth.

## 5.1 Welfare under Endogenous Growth

Throughout this paper, we use the household's lifetime value $\mathcal{V}$ as the relevant welfare metric. Under the assumption of GHH-type preferences we can derive[15] an expression of the stationary lifetime value $\hat{\mathcal{V}}_t$ as a function of the stationary period utility $\hat{\mathcal{U}}_t$ and the gross growth rate of the economy $\Gamma_t \equiv N_t/N_{t-1}$

$$\mathcal{V}_t \quad = \quad \mathcal{U}_t + \beta \mathbf{E}_t \mathcal{V}_{t+1} \quad \Leftrightarrow \quad \hat{\mathcal{V}}_t \equiv \frac{\mathcal{V}_t}{N_t^{1-\gamma}} = \left\{ \hat{\mathcal{U}}_t + \beta \mathbf{E}_t \Gamma_{t+1}^{1-\gamma} \hat{\mathcal{V}}_{t+1} \right\}.$$

It bears noting that under our assumption of GHH-type preferences, the period utility and the lifetime utility will actually take negative values. The policy maker's objective would then be to minimise the value of lifetime "disutility".

The BGP value of the household's lifetime utility $\hat{\mathcal{V}}_{bgp}$ is then given by

$$\hat{\mathcal{V}}_{bgp} \quad = \quad \hat{\mathcal{U}}_{bgp} \left(1 - \tilde{\beta}_{bgp}\right)^{-1}, \; \tilde{\beta}_{bgp} \equiv \beta \Gamma_{bgp}^{1-\gamma}, \; \gamma > 1.$$

Since we assume $\gamma > 1$, an increase in the BGP gross growth rate decreases the 'effective' discount factor, $\tilde{\beta}_{bgp}$. As we will show in detail in Section 4 and 5, a change in the policy sensitivity parameters can affect the risk-adjusted BGP, including $\hat{\mathcal{U}}_{bgp}$ and $\tilde{\beta}_{bgp}$. Importantly, a policy-induced change in $\tilde{\beta}_{bgp}$ has a much stronger impact on welfare than a change in $\hat{\mathcal{U}}_{bgp}$, which can be seen from a comparison of the elasticities of welfare with respect to the period utility and the effective discount factor, respectively

$$\mathcal{E}_{\hat{\mathcal{V}}_{bgp}, \hat{\mathcal{U}}_{bgp}} \quad \equiv \quad \left| \frac{\partial \hat{\mathcal{V}}_{bgp}}{\partial \hat{\mathcal{U}}_{bgp}} \frac{\hat{\mathcal{U}}_{bgp}}{\hat{\mathcal{V}}_{bgp}} \right| = |1| \quad < \quad \mathcal{E}_{\hat{\mathcal{V}}_{bgp}, \tilde{\beta}_{bgp}} \equiv \left| \frac{\partial \hat{\mathcal{V}}_{bgp}}{\partial \tilde{\beta}_{bgp}} \frac{\tilde{\beta}_{bgp}}{\hat{\mathcal{V}}_{bgp}} \right| = \left| - \frac{\tilde{\beta}_{bgp}}{1 - \tilde{\beta}_{bgp}} \right|, \quad \text{if } \tilde{\beta}_{bgp} > 0.5.$$

Even a mild increase in the BGP value of the gross growth rate $\Gamma_{bgp}$ could have a substantial positive impact on welfare. Intuitively, a rise in $\Gamma_{bgp}$ lowers the effective discount factor $\tilde{\beta}_{bgp}$, the stream of future period dis-utilities is discounted stronger, giving rise to a higher lifetime utility value. In fact, the role of discounting in determining welfare is so strong that even an increase in the period disutility (equivalent to a decrease in the period utility) could still be associated with an overall improvement in welfare. Indeed, in our numerical analysis, we find that the optimised macro-prudential policy is associated with a lower period utility but a higher gross growth rate (increasing discounting) which leads to significant welfare gains. These gains are not accounted for in standard models with exogenous productivity.

As is standard in the literature, we measure and compare welfare by expressing it in consumption-equivalent terms. In particular, we denote $\mathcal{C}^{equiv}$ as the percentage increase in consumption that would be required for the unregulated baseline model to reach the same level of welfare as the one with optimised macro-prudential policy ($\mathcal{V}_{bgp}^{*}$)

$$\hat{\mathcal{V}}_{bgp}^{*} \quad = \quad \left(1 - \tilde{\beta}_{bgp}\right)^{-1} \left( \frac{1}{1-\gamma} \left( (1 + \mathcal{C}^{equiv})(1-h)\hat{C}_{bgp} - \hat{\vartheta}_{bgp} \frac{L_{bgp}^{1+\varphi}}{1+\varphi} \right)^{1-\gamma} \right).$$

[15] See Appendix C for details.

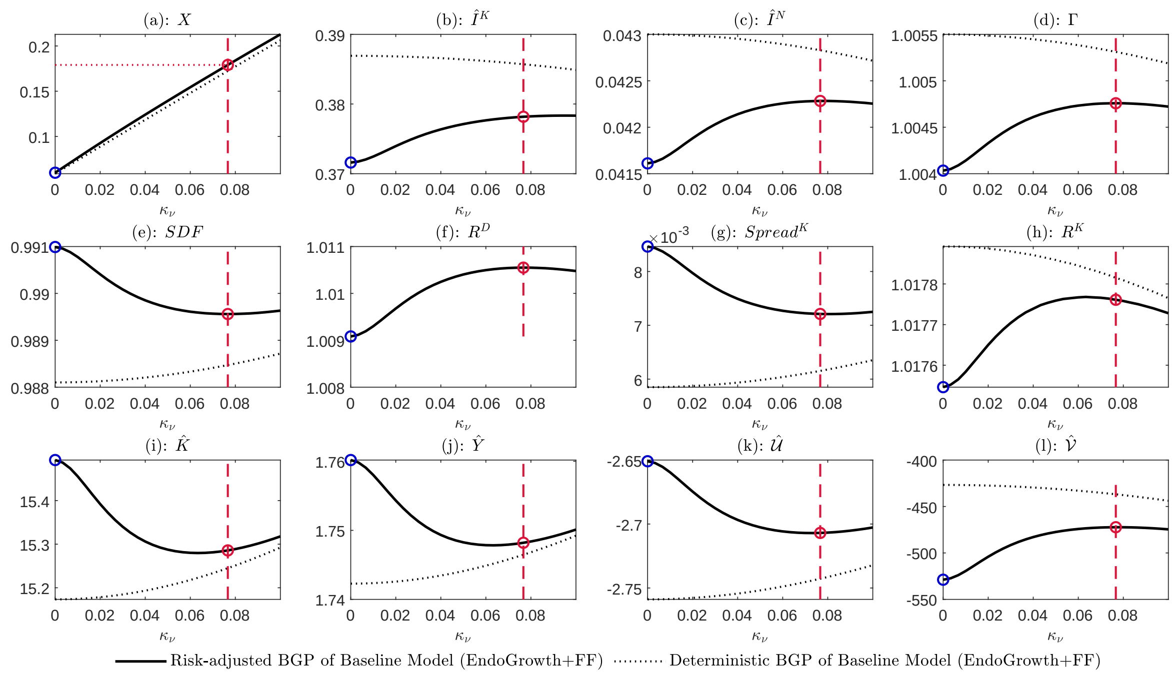
## 5.2 Optimised Macro-prudential Policy

First, in a comparative static context, we discuss the choice of the macro-prudential policy sensitivity $\kappa_\nu$ that maximises welfare. We then discuss the dynamic consequences of shocks in the presence of optimised macro-prudential policy.

**Comparative Statics** The role of macro-prudential policy is to incentivise financial intermediaries to finance a larger share of their portfolio with outside equity, rather than with short-term non-state-contingent deposits. The policy instrument of macro-prudential policy is a subsidy on outside equity $\tau_t^E$. As conjectured in equation (3.22), the subsidy on outside equity reacts to the intermediary's shadow cost of deposits with sensitivity $\kappa_\nu$. We then derive the optimised macro-prudential policy rule by picking the value $\kappa_\nu^*$ that maximises the household's lifetime utility $\hat{\mathcal{V}}_{bgp}^*$ in the risk-adjusted BGP.

In Figure 3, we illustrate the optimisation of welfare by varying the sensitivity parameter of macro-prudential policy, $\kappa_\nu$. As can be seen in Panel (l) the optimal level of $\kappa_\nu$ is around 0.075.

FIGURE 3: RISK-ADJUSTED BGP VALUES AS A FUNCTION OF $\kappa_\nu$ IN THE ENDOGENOUS GROWTH MODEL



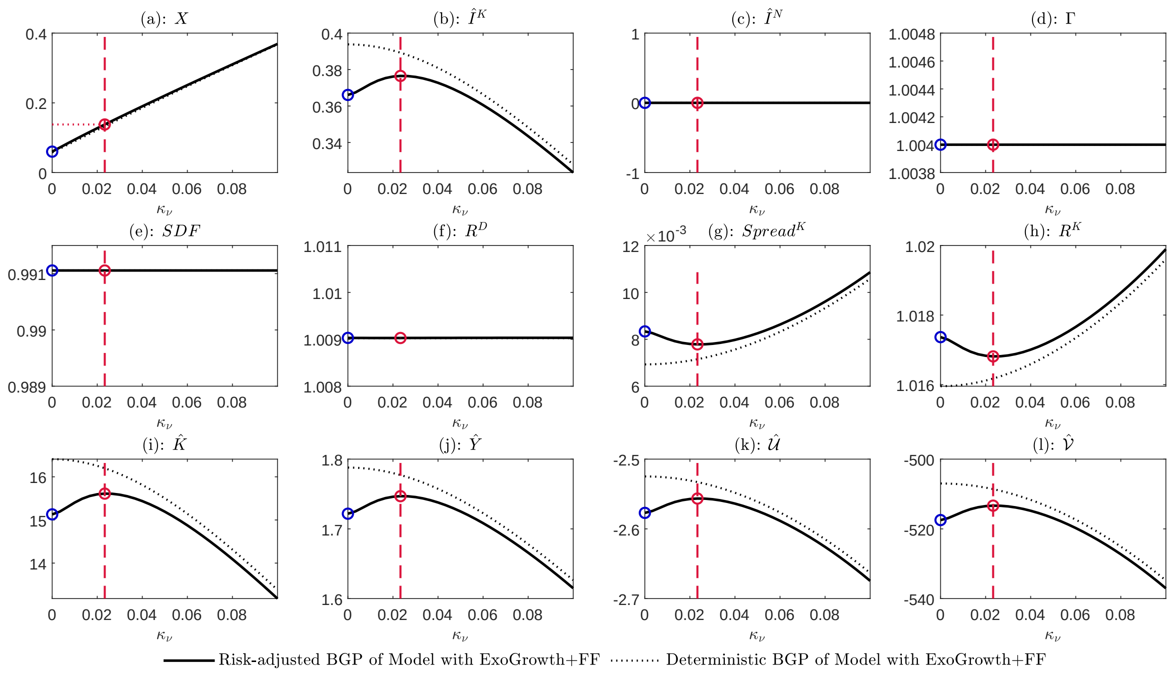

Note: The straight line depicts the values of several variables in their risk-adjusted balanced growth equilibrium as a function of the macro-prudential sensitivity parameter $\kappa_\nu$. The dashed line depicts the deterministic BGP values.

As shown in Panel (a), increasing $\kappa_\nu$ leads to an increase in the equity-to-asset ratio $X_{BGP}$. This is because a higher value of $\kappa_\nu$ corresponds to a more aggressive macro-prudential policy stance. Starting at the unregulated level of roughly $X_{BGP} = 6\%$ (blue circle), the increase in $X_{BGP}$ via the increase in $\kappa_\nu$ is initially welfare improving because the increased reliance on outside equity financing strengthens the financial system's resilience and its shock absorption capacity. The reduction in volatility mitigates the incentive compatibility problem between the household and the financial intermediary, spreads decline, the capacity of the financial system to channel funds to firms increases, which has a positive effect on investment in physical and intangible capital. However, as stated above in equation (3.9), the increase in outside equity finance also aggravates the incentive compatibility problem between the household and the financial intermediary since the absconding rate $\Theta$ increases in $X_{BGP}$. Thus, once the latter effect starts dominating the former, an increase in $\kappa_{\nu}$ will lower welfare $\mathcal{V}_{BGP}$.[16] For this reason, a tightening of the macro-prudential policy stance will always lower welfare in a deterministic BGP, since only the cost (increase in $\Theta$ and hence increase in spreads) is captured, while the benefit (reduction in volatility) is ignored.

Increasing $\kappa_{\nu}$ also leads to a reduction in spreads. The increase in the BGP value of intangible capital investment leads to an rise in the BGP value of the productivity growth rate from 1.004 to 1.0048. The increase in the growth rate is associated with a rise in interest rates, as implied by the Euler equation and the definition of the stochastic discount factor. The increase in the deposit rate causes a rise in the rate of return on capital, $R^K$, which, in turn, reduces capital, output, consumption, employment and hence, the household *period* utility. Despite the reduction in the period utility, the household's *lifetime* utility $\hat{\mathcal{V}}$ still increases (Panel (l)) due to the rise in the growth rate $\Gamma$, as explained in Section 5.1.

In Figure 4, we repeat the same exercise in the context of an exogenous growth model. As shown in Panel (l),in this case the optimal level of $\kappa_{\nu}$ is around 0.02.

FIGURE 4: RISK-ADJUSTED BGP VALUES AS A FUNCTION OF $\kappa_{\nu}$ IN THE EXOGENOUS GROWTH MODEL




Note: The straight line depicts the values of several variables in their risk-adjusted balanced growth equilibrium as a function of the macro-prudential sensitivity parameter $\kappa_{\nu}$. The dashed line depicts the deterministic BGP values.

In the model with exogenous growth, increasing $\kappa_{\nu}$ leads to a reduction in spreads, and a reduction in the return on capital $R^K$, unlike our baseline model. The reduction in $R^K$ leads to an increase in the BGP value of capital, output, consumption, and hours worked. The increase in consumption more than offsets the rise in hours worked, which improves the household's period and lifetime utility. It is worth underscoring how, under endogenous growth, the rise in lifetime utility was driven by the increase in

[16]Refer to Liu (2016) for a detailed discussion of these two opposing effects in the model by GKQ.

the growth rate. By contrast, under exogenous growth, macro-prudential policy can only increase the household's lifetime utility by affecting the period utility.

TABLE 2: RISK-ADJUSTED BGP VALUES WITHOUT AND WITH MACRO-PRUDENTIAL POLICY

| | | Endogenous Growth Model | | Exogenous Growth Model | |
|---|---|---|---|---|---|
| | | No MacroPru | With MacroPru | No MacroPru | With MacroPru |
| **Variables** | | | | | |
| $\hat{Y}$ | Output | 1.7602 | 1.7482 | 1.7217 | 1.7469 |
| $\hat{C}$ | Consumption | 1.3470 | 1.3277 | 1.3557 | 1.3704 |
| $L$ | Labor | 0.6026 | 0.6008 | 0.5888 | 0.5930 |
| $\hat{K}$ | K Capital | 15.4930 | 15.2857 | 15.1296 | 15.6104 |
| $\hat{I}^K$ | Physical Investment | 0.3716 | 0.3782 | 0.3661 | 0.3765 |
| $\hat{I}^N$ | Intangible Investment | 0.0416 | 0.0423 | 0.0000 | 0.0000 |
| $\Gamma$ | Growth Rate | 1.0040 | 1.0048 | 1.0040 | 1.0040 |
| $Spread^K$ | Spread K | 0.0085 | 0.0072 | 0.0083 | 0.0078 |
| $Spread^N$ | Spread N | 0.0133 | 0.0114 | 0.0000 | 0.0000 |
| $\widehat{NW}$ | Net Worth | 2.2189 | 1.9129 | 3.3094 | 3.2093 |
| $X$ | Outside Equity/Assets | 0.0600 | 0.1790 | 0.0600 | 0.1379 |
| $\phi$ | Leverage | 4.6579 | 5.3423 | 4.5889 | 4.8832 |
| $\Theta$ | Absconding Rate | 0.8925 | 0.9764 | 0.6503 | 0.6782 |
| $\nu$ | Cost of Deposits | 3.9394 | 4.9238 | 2.8853 | 3.1968 |
| $\mu^E$ | Excess Value Equity | 0.0006 | 0.0016 | 0.0007 | 0.0008 |
| $R^E$ | Return Equity | 1.0091 | 1.0107 | 1.0091 | 1.0091 |
| $R^K$ | Return K Capital | 1.0175 | 1.0178 | 1.0174 | 1.0168 |
| $R^D$ | Deposit Rate | 1.0091 | 1.0106 | 1.0090 | 1.0090 |
| $\Pi$ | Inflation | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| $\tau^E$ | MacroPru Subsidy | 0.0000 | 0.0156 | 0.0000 | 0.0073 |
| $\hat{U}$ | Period Utility | -2.6509 | -2.7070 | -2.5771 | -2.5565 |
| $\hat{V}$ | HH Welfare | -528.8315 | -472.3265 | -517.4795 | -513.3466 |
| C-Welf Eqv in pct, noZLB: | | | 6.70 | | 0.46 |

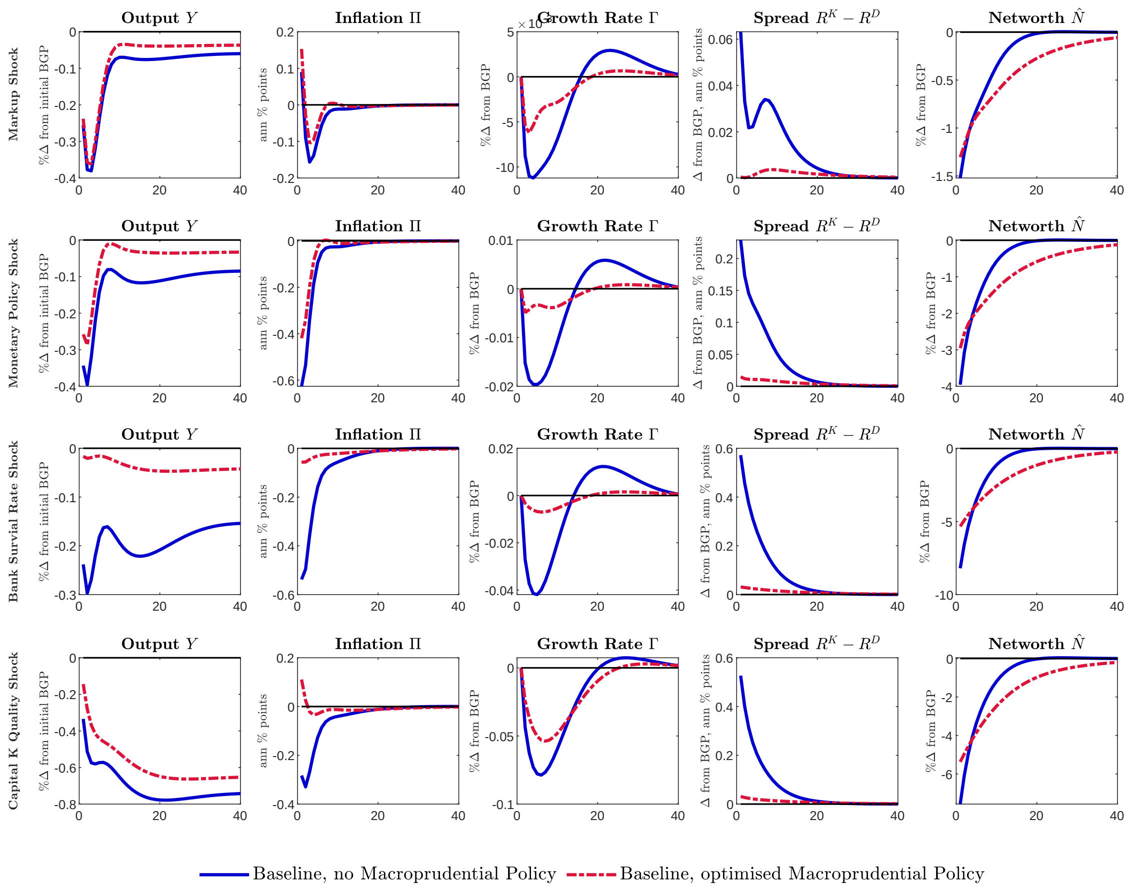
In Table 2 we report the balanced-growth path values of key variables, under endogenous and exogenous growth, without macro-prudential policy and with optimised macro-prudential policy.

In the baseline model, the optimised outside equity ratio $X^*_{bgp}$ associated with the optimised value for $\kappa_\nu$ is around 18%. In the exogenous growth model, instead, the optimised outside equity ratio is around 14%. In other words, based on this numerical lifetime optimisation approach, a stronger macro-prudential policy stance is warranted once one allows for endogenous productivity growth. Intuitively, the fact that the growth rate of productivity and the BGP are subject to shocks, which are amplified via the balance sheet constraints of financial intermediaries, justifies a stronger macro-prudential response.

The optimised macro-prudential policy regime in the endogenous growth model is associated with a consumption welfare-equivalence gain of 6.7% compared to unregulated regime. This value is 14 times larger than in the exogenous growth model, which is only 0.46%. The welfare-gains are also significantly larger than previously found in the literature. For example, GKQ find these welfare improvements to be only 0.29%.

**Dynamics** In Figure 5, we show the impulse response functions (IRFs) to a markup shock (upper row), a monetary policy shock (second row), a bank survival rate shock (third row) and a $K$ capital quality shock (fourth row) for output, inflation, productivity growth, $K$ spreads and net worth in the baseline model with endogenous growth and financial frictions. We compare the case without macro-prudential policy (blue-solid line) and the case with the optimised macro-prudential intervention (red-dashed line). Macro-prudential policy mitigates the initial drop in output for all four shocks. The permanent loss in output is also much smaller, e.g. less than half of the loss under the no-regulation case for the intermediary survival rate shock. The stabilisation gains from macro-prudential policy are smaller for the other three shocks. As described above, macro-prudential policy increases the resilience of the financial system and thus facilitates the intermediation of credit even when the economy is hit by adverse financial shocks. Since macro-prudential policy mitigates the decline in investment, also the growth rate of the economy will fall less. The milder drop in the growth rate means that the post-shock BGP will deviate less from the initial pre-shock path. Output, expressed in terms of its deviation from the initial BGP, will thus decline much less under optimised macro-prudential policy.

FIGURE 5: IMPULSE RESPONSES TO MARKUP, MONETARY POLICY AND SURVIVAL RATE SHOCK




Note: The upper row depicts the IRFs for a (+1stdev) markup shock, the middle row for a (+1stdev) monetary policy shock and the lower row for a (-1stdev) survival rate shock. The blue-straight line depicts the responses of the baseline model, with endogenous growth and financial frictions without macro-prudential policy, the red-dashed line depicts the variables in the same model but with optimised macro-prudential policy.

In Table 3, we report the standard deviations of some key variables. We compare the model standard deviations to their empirical counterparts for the US postwar sample period and the period since 1984Q1. The baseline model in the absence of macro-prudential policy is calibrated to match these empirical moments. Under the optimised macro-prudential policy, the variables are much less volatile. While the unregulated model economy with endogenous growth implies an output growth rate volatility of 0.68%, the optimised macro-prudential policy reduces it to 0.49%. The spreads in particular become much less volatile, reflecting a stabilisation gain from macro-prudential policy for shocks that transmit via the financial system.

We also report how often the central bank's policy instrument, the deposit rate, hits the ZLB. By simulating our baseline model, we find this to occur about 1.1% of the time.[17] In the model with optimised macro-prudential policy, the lower bound is never attained. This result highlights an additional benefit of macro-prudential policy: reducing the likelihood of hitting the ZLB on the central bank's policy rate.

TABLE 3: VOLATILITIES AND ZLB FREQUENCY WITHOUT AND WITH MACRO-PRUDENTIAL POLICY

| | | **Endogenous Growth Model** | | **Exogenous Growth Model** | |
|---|---|---|---|---|---|
| | | No MacroPru | With MacroPru | No MacroPru | With MacroPru |
| **Variables** | | | | | |
| **Moments** | **Data, full sample (post 1984Q1)** | | | | |
| StDev(Y) | 0.99 ( 0.58) | 0.68 | 0.49 | 0.55 | 0.43 |
| StDev(C) | 0.84 ( 0.55) | 0.47 | 0.39 | 0.35 | 0.33 |
| StDev(IK) | 2.30 ( 1.77) | 1.73 | 1.08 | 1.69 | 1.11 |
| StDev(IRnD) | 2.12 ( 1.45) | 1.61 | 0.84 | 0.00 | 0.00 |
| StDev(L) | 0.89 ( 0.66) | 0.75 | 0.55 | 0.67 | 0.60 |
| StDev(Pi) | 0.66 ( 0.37) | 0.35 | 0.16 | 0.38 | 0.26 |
| StDev(SpreadK) | 0.47 ( 0.36) | 0.34 | 0.03 | 0.38 | 0.14 |
| StDev(SpreadRnD) | 0.57 ( 0.57) | 0.57 | 0.06 | 0.00 | 0.00 |
| freq(ZLB binds), in pct | | 1.13 | 0.00 | 3.07 | 0.18 |

## 5.3 Macro-prudential Policy and the ZLB

This subsection analyses the effects of macro-prudential policy when the deposit rate hits the ZLB. In Figure 6, we show the IRFs of several key variables in response to an adverse bank survival rate shock of 8 standard deviations in period 1. The magnitude of the shock implies an increase in spreads and an initial drop in output consistent with US data for the Great Financial Crisis in late 2008. To highlight the implications of the occasionally-binding ZLB constraint and its interaction with macro-prudential policy, we compare three scenarios: one without ZLB and macro-prudential policy (blue-solid line), one with (a binding) ZLB constraint but without macro-prudential policy (green-dashed line) and lastly one scenario with (a non-binding) ZLB constraint and macro-prudential policy (red-dotted line).

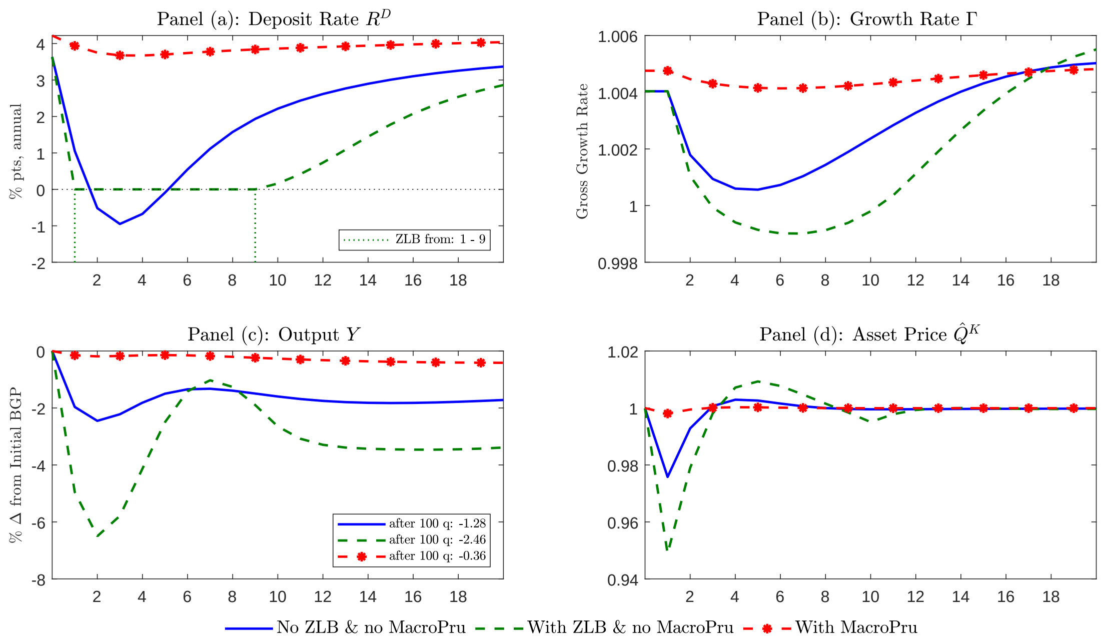
The negative financial shock causes a fall in output and inflation, which calls for a significant cut in the central bank's deposit rate. In the unconstrained case, the deposit rate is allowed to fall below zero. In the constrained case without macro-prudential policy, the deposit rate hits the ZLB in period one and starts to rise again in period 10. The liquidity trap causes a substantial fall in inflation expectations and a rise in the real rate, which amplifies the drop in output by about three times, compared to the unconstrained case. Besides the fall in output, the liquidity trap also amplifies the fall in both types of investment and the decline in the productivity growth rate. Due to the stronger fall in the growth rate, the permanent effects on the output level are significantly more severe with a binding ZLB constraint. In particular, output remains more than 3 per cent below its initial BGP, compared to less than 2 per cent in the unconstrained case.

Under the optimised macro-prudential policy (MacroPru) the financial system is more resilient, and asset prices fall less,[18] which mitigates the tightening in credit conditions. Consequently, the fall in

[17] This corresponds to an annualised frequency of roughly 4.5%. It should be noted that we do not actually impose the ZLB in the simulation. We only assess the effects of a binding lower bound in Section 5.3. Given that once the lower bound is hit and policy is constrained, the economy contracts sharply and the desired nominal rate would drop further, we therefore underestimate the duration and thus the actual frequency of quarters during which the lower bound binds.

[18] The asset price response of intangible capital claims $\hat{Q}_t^N$ would be very similar.

FIGURE 6: ZLB SCENARIO FOR A BANK SURVIVAL RATE SHOCK




Note: The Figure displays IRFs for a negative bank survival rate shock (8 stdev). The blue-straight line depicts the case in which the deposit rate is not constrained by the ZLB. The green-dashed line depicts the case in which the deposit rate is constrained by the ZLB. The red-dotted line depicts the case with optimised macro-prudential policy.

investment in physical and intangible capital, output and inflation are more muted. The milder decline in macroeconomic conditions calls for a less drastic cut in the policy interest rate, which does not reach the ZLB. The presence of MacroPru significantly mitigates the fall in the growth rate and, thus, the permanent output losses, which amount to only $-0.36\%$.

In addition to the welfare-gains in normal times shown in section 5.2, this last analysis highlights the importance of macro-prudential policy in avoiding the short-term and long-term adverse consequences of a liquidity trap.

Finally, it is important to note that our exercises assumed the ZLB to bind for only eight quarters. In reality, in the US the Federal Funds Rate stayed at ZLB for more than 20 quarters between 2009 and 2015. Keeping the ZLB binding for such an extended period would further strengthen our results and suggest even larger stabilisation gains from MacroPru.

# 6 Concluding Remarks

This paper shows that disruptions in financial intermediation are followed by slowdowns in productivity growth and lead to a permanently lower level of real economic activity. A tightening in financial conditions can be particularly harmful to investment in both physical and intangible capital, such as R&D, and cause a significant slowdown in the growth rate of productivity.

We study how macro-prudential policy can mitigate the adverse short-run and long-run consequences of financial shocks, thereby significantly improving aggregate welfare. To this end, we build a medium-scale DSGE model with financial frictions and endogenous growth.

Our baseline model implies an optimal bank capital ratio of 18 per cent, about four percentage points higher than in a specification with exogenous growth. We find that the optimal macro-prudential policy reduces the slowdown in productivity growth and the permanent losses in output by more than half.

Our main result is that, when we account for its potential long-term benefits, macro-prudential policy leads to substantial welfare improvements. In particular, compared to the unregulated scenario, macro-prudential policy increases welfare, translated into consumption terms, by 6.7 per cent against the 0.46 per cent implied by a model with exogenous growth. Our work highlights the importance of taking the long-term costs of financial crises into account when assessing the benefits of macro-prudential policy. MacroPru's surprisingly small welfare gains commonly found in the theoretical literature are a consequence of ignoring long-term effects and endogenous growth elements.

We also highlight that macro-prudential policy reduces the probability of the monetary policy rate reaching the zero lower bound. While in the unregulated economy, the quarterly probability of ending up in a liquidity trap is about 1.1 per cent, this falls to zero under macro-prudential regulation. The importance of this result is underscored by analysing how the macroeconomic variables respond to an adverse financial shock at ZLB in the unregulated economy. The binding ZLB is particularly costly, amplifying the decline in output by about three times within the first two quarters and about 1.5 times after 20 quarters. Macro-prudential regulation substantially mitigates these costs by preventing the economy from falling in a liquidity trap in the first place. Thus, the resulting output decline is about 30 times smaller on impact and seven times smaller permanently than in the unregulated case with a binding ZLB.

Our comprehensive analysis does not consider alternative macro-prudential policies such as adjustments of risk-weights or the implementation of capital requirements. Last, in our model, the corporate debt structure is relatively stylised, while other studies have considered more detailed and granular modelling approaches. Although beyond the scope of this paper, we believe these are all interesting avenues for future research.

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# A Data used in Empirical Analysis of the Effects of Banking Crises

**Total Factor Productivity** We collect data on TFP from the Total Economy Database (Conference Board, 2019), henceforth TED, and from Coe et al. (2009), henceforth CHH. The data from TED are available from 1990 until 2018, while those from CHH go back to 1970 until 2004. We splice the TED data with the second dataset for the period 1970 − 1989 by using the OLS fitted values (using the OLS parameter estimate for the overlapping period 1990 − 2004).

**Research and Development** We take data on annual real 2015 PPP USD Business Enterprise Expenditure on R&D (henceforth BERD) and Gross Domestic Expenditure on R&D (GERD) from the OECD's Main Science and Technology Indicators (OECD, 2019). To extend the time-series dimension, we also take the TFP series from Coe et al. (2009). Similarly as in CHH, for Austria, we fill gaps in the data by using the fitted value of the regression of the BERD on the (at constant prices and PPP USD). For TFP, we then extrapolate backwards the BERD data with the CHH R&D data using the OLS fitted values.

**Gross Domestic Product** We take data on annual real per capita GDP in 2018 PPP USD from the Total Economy Database (Conference Board, 2019) which is available from 1950.

**Banking Crisis and Other Recessions** Data on banking crisis dates are taken from Laeven and Valencia (2018), who define a banking crisis as events that were characterised by both heightened financial distress in the banking system as well policy interventions to respond to significant losses in the banking system. We construct the data on recession dates from quarterly real GDP data from the OECD. In particular, we label a year as a recession, if it was characterised by at least two quarters with negative GDP growth. Similarly, as in Queralto, we only take the first year of the banking crisis (or the recession). For example, if country A had a banking crisis or a recession from 1992 until 1994 (included), our dummy would have a one only in 1992 and zeros in 1993 and 1994. If a country experienced a double-dip recession in 2 consecutive years, e.g. negative GDP growth in 1992Q1-1992Q2 and 1993Q2-1993Q3, then both years (1992 and 1993), would appear with a one. We define "Other recessions" by removing the banking crisis dates from the recessions. We report the Banking Crises and other recessions in Table 4.

TABLE 4: BANKING CRISIS AND RECESSION DATES

| Country | Banking Crises | Other Recessions |
|---|---|---|
| Australia | | 1971, 1975, 1977, 1981, 1982, 1991 |
| Austria | 2008 | 1981, 1982, 1984, 1992, 2001, 2012 |
| Belgium | 2008 | 1974, 1976, 1980, 1992, 2001, 2012 |
| Canada | | 1974, 1980, 1981, 1990, 2008, 2015 |
| Denmark | 2008 | 1973, 1977, 1980, 1986, 1989, 1992, 1997, 2001, 2006 |
| Finland | 1991 | 1971, 1975, 1977, 1980, 1990, 1992, 1995, 2008, 2012 , 2013, 2014 |
| France | 2008 | 1974, 1992, 2012 |
| Germany | 2008 | 1974, 1980, 1982, 1991, 1992, 2000, 2001, 2002, 2004, 2012 |
| Greece | 2008 | 1974, 1975, 1977, 1978, 1979, 1980, 1981, 1982, 1983 |
| | | 1984, 1987, 1989, 1990, 1992, 1994, 2004, 2007, 2010 |
| Iceland | 2008 | 1974, 1982, 1988, 1991, 1994, 1999, 2000, 2009 |
| Ireland | 2008 | 1975, 1982, 1985, 2007, 2011 |
| Israel | 1977 | 2000 |
| Italy | 2008 | 1974, 1977, 1982, 1992, 2001, 2003, 2011, 2018 |
| Japan | 1997 | 1993, 1998, 2001, 2008, 2010, 2012, 2015 |
| South Korea | 1997 | 1979 |
| Netherlands | 2008 | 1973, 1974, 1980, 1981, 2011, 2012 |
| New Zealand | | 1974, 1976, 1978, 1985, 1991, 1997, 2008, 2010 |
| Norway | 1991 | 1980, 1981, 1988, 1992, 2009, 2010, 2016 |
| Portugal | 2008 | 1974, 1980, 1983, 1992, 2002, 2010 |
| Spain | 2008 | 1975, 1978, 1981, 1992, 2009, 2011 |
| Sweden | 1991, 2008 | 1971, 1976, 1990, 1992, 2012 |
| Switzerland | 2008 | 1974, 1977, 1981, 1990, 1992, 1996, 1998, 2001, 2002, 2015, 2018 |
| United Kingdom | 2007 | 1973, 1975, 1980, 1990, 2008 |
| United States | 1988, 2007 | 1974, 1980, 1981, 1990, 2008 |

# B Model Derivation

**Households**

$$\max_{C_t,L_t,D_t,E_t,B_t^{G,TIPS}} \sum_{t=0}^{\infty} \beta^t \mathcal{U}(C_t, C_{t-1}, L_t) \quad \Leftrightarrow \quad \mathcal{V}_t = \mathcal{U}_t + \beta \mathbf{E}_t[\mathcal{V}_{t+1}] \tag{B.1}$$

$$\text{where } \mathcal{U}_t = \frac{1}{1-\gamma}\left(C_t - h\Gamma_t C_{t-1} - \vartheta_t \frac{L_t^{1+\varphi}}{1+\varphi}\right)^{1-\gamma} \tag{B.2}$$

$$\text{and } \vartheta_t = \chi (N_t)^{\iota} (\vartheta_{t-1})^{1-\iota} \tag{B.3}$$

$$\text{s.t.} \quad P_t C_t + P_t \hat{Q}_t^E E_t + D_t = W_t^N L_t + \Xi_t - T_t + R_t^E P_{t-1} \hat{Q}_{t-1} E_{t-1} + R_{t-1}^D D_{t-1}$$

$$\mathcal{L}_t = \sum_{s^t} \sum_{t=0}^{\infty} \beta^t \left[\mathcal{U}(C_t, L_t) + \lambda_t \left(-P_t C_t - P_t \hat{Q}_t^E E_t - D_t + W_t^N L_t + \Xi_t - T_t + R_t^E P_{t-1} \hat{Q}_{t-1}^E E_{t-1} + R_{t-1}^D D_{t-1}\right)\right]$$

$$\frac{\partial \mathcal{L}_t}{\partial L_t} = \mathcal{U}_L + \lambda_t W_t^N = 0 \quad \Leftrightarrow \lambda_t = -\frac{\mathcal{U}_{L,t}}{W_t^N}, \text{ note } W_t \equiv \frac{W_t^N}{P_t}$$

$$\frac{\partial \mathcal{L}_t}{\partial C_t} = \mathbf{E}_t \mathcal{U}_C - \lambda_t P_t = 0 \quad \Leftrightarrow \lambda_t = \frac{\mathbf{E}_t \mathcal{U}_{C,t}}{P_t}$$

$$\frac{\partial \mathcal{L}_t}{\partial D_t} = -\lambda_t + \beta \mathbf{E}_t\left[\lambda_{t+1} R_t^D\right] = 0 \quad \Leftrightarrow 1 = \beta \mathbf{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t} R_t^D\right]$$

$$\frac{\partial \mathcal{L}_t}{\partial E_t} = -\lambda_t + \beta \mathbf{E}_t\left[\lambda_{t+1} R_{t+1}^E\right] = 0 \quad \Leftrightarrow 1 = \beta \mathbf{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t} R_{t+1}^E\right]$$

$$-\mathcal{U}_L = \mathbf{E}_t \mathcal{U}_C W_t = \mathbf{E}_t \mathcal{U}_C \frac{W_t^N}{P_t} \tag{B.4}$$

$$1 = \mathbf{E}_t\left[\Lambda_{t,t+1} \frac{R_t^D}{\Pi_{t+1}}\right] \tag{B.5}$$

$$1 = \mathbf{E}_t\left[\Lambda_{t,t+1} \frac{R_{t+1}^E}{\Pi_{t+1}}\right] \tag{B.6}$$

$$\Lambda_{t,t+1} \equiv \beta \frac{\mathcal{U}_{C,t+1}}{\mathcal{U}_{C,t}} \tag{B.7}$$

$$\mathcal{U}_t^L \equiv -\vartheta_t L_t^{\varphi} \left(C_t - h\Gamma_t C_{t-1} - \vartheta_t \frac{L_t^{1+\varphi}}{1+\varphi}\right)^{-\gamma} \tag{B.8}$$

$$\mathcal{U}_t^C \equiv \left(C_t - h\Gamma_t C_{t-1} - \vartheta_t \frac{L_t^{1+\varphi}}{1+\varphi}\right)^{-\gamma} - \beta h \Gamma_{t+1} \left(C_{t+1} - h\Gamma_{t+1} C_t - \vartheta_{t+1} \frac{L_{t+1}^{1+\varphi}}{1+\varphi}\right)^{-\gamma} \tag{B.9}$$

**Production**

$$Y_{m,t} = \left(\varepsilon_t^K U_{m,t}^K K_{m,t}\right)^{\alpha} \left(\mathcal{X}_{m,t}^{LAP} L_{m,t}\right)^{1-\alpha}. \tag{B.10}$$

$$\mathcal{X}_{m,t}^{LAP} = \left(U_{m,t}^N N_{m,t}\right)^{\eta} \left(U_t^N N_t\right)^{1-\eta}. \tag{B.11}$$

$$K_{m,t} = B_{m,t-1}^K \tag{B.12}$$

$$N_{m,t} = B_{m,t-1}^N. \tag{B.13}$$

$$\min \left\{ \tau_t^{\mathcal{M}} \left( P_t \hat{W}_t L_{m,t} + \left(P_{t-1} \hat{Q}_{t-1}^K R_t^K - P_t \hat{Q}_t^K (1-\delta_{m,t}^K)\right) \varepsilon_t^K K_{m,t} + \left(P_{t-1} \hat{Q}_{t-1}^N R_t^N - P_t \hat{Q}_t^N (1-\delta_{m,t}^N)\right) N_{m,t} \right) \right\}$$

$$s.t. \quad \left(\varepsilon_t^K U_{m,t}^K K_{m,t}\right)^{\alpha} \left(\left(U_{m,t}^N N_{m,t}\right)^{\eta} \left(U_t^N N_t\right)^{1-\eta} L_{m,t}\right)^{1-\alpha} \geq \left(\frac{P_{m,t}}{P_t}\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} Y_t.$$

$$\mathcal{L}_{m,t} = -\tau_t^{\mathcal{M}} \left( P_t \hat{W}_t L_{m,t} + \left(P_{t-1} \hat{Q}_{t-1}^K R_t^K - P_t \hat{Q}_t^K (1-\delta_{m,t}^K)\right) \varepsilon_t^K K_{m,t} + \left(P_{t-1} \hat{Q}_{t-1}^N R_t^N - P_t \hat{Q}_t^N (1-\delta_{m,t}^N)\right) N_{m,t} \right)$$

$$+ P_t \widehat{MC}_{m,t} \left( \left(\varepsilon_t^K U_{m,t}^K K_{m,t}\right)^{\alpha} \left(\left(U_{m,t}^N N_{m,t}\right)^{\eta} \left(U_t^N N_t\right)^{1-\eta} L_{m,t}\right)^{1-\alpha} - \left(\frac{P_{m,t}}{P_t}\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} Y_t \right)$$

$$\frac{\partial \mathcal{L}}{\partial L_{m,t}} = 0 \Leftrightarrow P_t \hat{W}_t = \frac{P_t \widehat{MC}_{m,t}}{\tau_t^{\mathcal{M}}}(1-\alpha)\frac{Y_{m,t}}{L_{m,t}} \tag{B.14}$$

$$\frac{\partial \mathcal{L}}{\partial K_{m,t}} = 0 \Leftrightarrow R_t^K = \frac{\frac{P_t \widehat{MC}_{m,t}}{\tau_t^{\mathcal{M}}}\alpha \frac{Y_{m,t}}{K_{m,t}} + (1-\delta_{m,t}^K) P_t \hat{Q}_t^K \varepsilon_t^K}{P_{t-1}\hat{Q}_{t-1}^K} \tag{B.15}$$

$$\frac{\partial \mathcal{L}}{\partial K_{m,t}} = 0 \Leftrightarrow R_t^N = \frac{\frac{P_t \widehat{MC}_{m,t}}{\tau_t^{\mathcal{M}}}(1-\alpha)\eta \frac{Y_{m,t}}{N_{m,t}} + (1-\delta_{m,t}^N) P_t \hat{Q}_t^N}{P_{t-1}\hat{Q}_{t-1}^N} \tag{B.16}$$

$$\hat{\mathcal{R}}_t^K \equiv \frac{\widehat{MC}_{m,t}}{\tau_t^{\mathcal{M}}}\alpha \frac{Y_{m,t}}{K_{m,t}} \tag{B.17}$$

$$\hat{\mathcal{R}}_t^N \equiv \frac{\widehat{MC}_{m,t}}{\tau_t^{\mathcal{M}}}(1-\alpha)\eta \frac{Y_{m,t}}{N_{m,t}} \tag{B.18}$$

$$\frac{\partial \mathcal{L}}{\partial U_{m,t}^K} = 0 \Leftrightarrow \frac{\widehat{MC}_{m,t}}{\tau_t^{\mathcal{M}}}\alpha \frac{Y_{m,t}}{U_{m,t}^K} = \left(\frac{\partial \delta_{m,t}^K}{\partial U_{m,t}^K}\right)\hat{Q}_t^K \tag{B.19}$$

$$\frac{\partial \mathcal{L}}{\partial U_{m,t}^N} = 0 \Leftrightarrow \frac{\widehat{MC}_{m,t}}{\tau_t^{\mathcal{M}}}(1-\alpha)\eta \frac{Y_{m,t}}{U_{m,t}^N} = \left(\frac{\partial \delta_{m,t}^N}{\partial U_{m,t}^N}\right)\hat{Q}_t^N. \tag{B.20}$$

$$\delta_{m,t}^K = \bar{\delta}^K + \frac{b^K}{1+\zeta^K}\left(U_{m,t}^K\right)^{1+\zeta^K} \tag{B.21}$$

$$\delta_{m,t}^N = \bar{\delta}^N + \frac{b^N}{1+\zeta^N}\left(U_{m,t}^N\right)^{1+\zeta^N}. \tag{B.22}$$

**Inter-temporal Pricing Problem of Final Output Producers** The objective of each final output producer is to maximise its profits $Profit_{m,t}^Y$. They are given by $Profit_{m,t} = P_{m,t}Y_{m,t} - \tau_t^{\mathcal{M}} P_t \left\{\hat{W}_t L_{m,t} + \hat{\mathcal{R}}_t^K K_{m,t} + \hat{\mathcal{R}}_t^N N_{m,t}\right\}$. Some firms may not be able to set their desired price $P_{m,t}^*$. With probability $\phi_P$ a firm cannot reset its price. In this case the firm is stuck with its previous-period price indexed to a composite of previous-period inflation and steady-state inflation so that

$$P_{m,t} = \begin{cases} P_{m,t}^* & \textit{with probability:} \quad 1-\phi_P \\ P_{m,t-1}\left(\left(\Pi_{ss}^P\right)^{1-ind_P}\left(\Pi_{t-1}^P\right)^{ind_P}\right) & \textit{with probability:} \quad \phi_P \end{cases}$$

where $ind_P \in [0,1]$ is the weight attached to previous-period inflation. Consider a firm who can reset its price in the current period $P_{m,t} = P_{m,t}^*$ and who is then stuck with its price until future period $t+s$. The price in this case would be $P_{i,t+s} = P_{m,t}^*\left[(\bar{\Pi})^{s(1-ind_P)}\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{ind_P}\right]$. The final output producing firms solve the following optimisation problem $\max_{P_{m,t}^*} E_t \sum_{s=0}^{\infty}(\phi_P\beta)^s \frac{\mathcal{U}_{t+s}^C}{\mathcal{U}_t^C}\frac{P_t}{P_{t+s}}\left[P_{i,t+s}Y_{i,t+s|t} - (1+(1-\alpha)\eta)P_t\widehat{MC}_{t+s}Y_{i,t+s|t}\right]$ subject to the above derived demand constraint and assuming that a firm $i$ always meets the demand for its good at the current price $Y_{i,t+s} = \left(\frac{P_{i,t+s}}{P_{t+s}}\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}}Y_{t+s}$. Substitute the demand schedule and the relation for $P_{i,t+s}$ into the objective function to get

$$\max_{P_{m,t}^*} E_t \sum_{s=0}^{\infty}(\beta\phi_P)^s \frac{\mathcal{U}_{t+s}^C}{\mathcal{U}_t^C}\frac{P_t}{P_{t+s}}\left[\left(P_{m,t}^*\right)^{1-\frac{\mathcal{M}}{\mathcal{M}-1}}\left((\bar{\Pi})^{s(1-ind_P)}\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{ind_P}\right)^{1-\frac{\mathcal{M}}{\mathcal{M}-1}}\left(\frac{1}{P_{t+s}}\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}}Y_{t+s}\right.$$
$$\left.-(1+(1-\alpha)\eta)P_{t+s}\widehat{MC}_{t+s}\left[\left(P_{m,t}^*\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}}\left(\frac{\left[(\bar{\Pi})^{s(1-ind_P)}\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{ind_P}\right]}{P_{t+s}}\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}}Y_{t+s}\right]\right], \quad \frac{P_{t+s-1}}{P_{t-1}} = \prod_{g=0}^{s-1}\Pi_{t+g}^P.$$

Taking the derivative with respect to $P_{m,t}^*$ and rearranging delivers $P_{m,t}^* = P_t^* = \frac{\mathcal{F}_{1,t}\mathcal{M}}{\mathcal{F}_{2,t}}$ where

$$\mathcal{F}_{1,t} \equiv (P_t)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} E_t \sum_{s=0}^{\infty}(\phi_P)^s\beta^s \frac{\mathcal{U}_{t+s}^C}{\mathcal{U}_t^C}\frac{P_t}{P_{t+s}}\left[P_{t+s}\widehat{MC}_{t+s}\left[\left(\frac{\left[(\bar{\Pi})^{s(1-ind_P)}\left(\prod_{g=0}^{s-1}\Pi_{t+g}^P\right)^{ind_P}\right]}{P_{t+s}}\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}}Y_{t+s}\right]\right]$$

$$\mathcal{F}_{2,t} \equiv (P_t)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} E_t \sum_{s=0}^{\infty} (\phi_P)^s \beta^s \frac{\mathcal{U}^C_{t+s}}{\mathcal{U}^C_t} \frac{P_t}{P_{t+s}} \left[ \left( \left[ (\bar{\Pi})^{s(1-ind_P)} \left( \prod_{g=0}^{s-1} \Pi^P_{t+g} \right)^{ind_P} \right] \right)^{1-\frac{\mathcal{M}}{\mathcal{M}-1}} \left[ \left( \frac{1}{P_{t+s}} \right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} Y_{t+s} \right] \right].$$

$$\mathcal{F}_{1,t} = (1+(1-\alpha)\eta) P_t \widehat{MC}_t Y_t + \phi_P E_t \left[ \Lambda_{t,t+1} \Pi^{-1}_{t+1} \left( \zeta^{\Pi}_{t+1} \right)^{\frac{\mathcal{M}}{\mathcal{M}-1}} \mathcal{F}_{1,t+1} \right] \tag{B.23}$$

$$\mathcal{F}_{2,t} = Y_t + \phi_P E_t \left[ \Lambda_{t,t+1} \left( \zeta^{\Pi}_{t+1} \right)^{\frac{1}{\mathcal{M}-1}} \mathcal{F}_{2,t+1} \right] \tag{B.24}$$

$$\zeta^{\Pi}_t \equiv \frac{\Pi_t}{\left[ (\bar{\Pi})^{(1-ind_P)} (\Pi_{t-1})^{ind_P} \right]}. \tag{B.25}$$

An expression for the aggregate final output price index $P_t$ can be derived from

$$P_t = \left( \int_F \left( P^*_{m,t} \right)^{\frac{1}{1-\mathcal{M}}} di + \int_{\bar{F}} \left( \left[ (\bar{\Pi})^{(1-ind_P)} \left( \Pi^P_{t-1} \right)^{ind_P} \right] P_{m,t-1} \right)^{\frac{1}{1-\mathcal{M}}} di \right)^{1-\mathcal{M}}$$

where $F$ is the set of those final output producers who can reoptimise their price. The Phillips Curve is given by

$$\frac{P^*_t}{P_t} = \left[ \frac{1-(\phi_P)\left(\zeta^{\Pi}_t\right)^{\frac{-1}{1-\mathcal{M}}}}{1-\phi_P} \right]^{1-\mathcal{M}} = \frac{\mathcal{F}_{1,t}\mathcal{M}}{\mathcal{F}_{2,t}}. \tag{B.26}$$

Capital good production is given by

$$I^K_t = K_{t+1} - \left[ 1 - \delta^K_t \right] K_t \varepsilon^K_t \tag{B.27}$$

$$Q^K_t = 1 + \frac{\psi_{I^K}}{2} \left( \frac{I^K_t}{I^K_{t-1}} - \bar{\Gamma} \right)^2 + \left( \frac{I^K_t}{I^K_{t-1}} \right) \psi_{I^K} \left( \frac{I^K_t}{I^K_{t-1}} - \bar{\Gamma} \right) - \mathbf{E}_t \left[ \Lambda_{t,t+1} \left( \frac{I^K_{t+1}}{I^K_t} \right)^2 \psi_{I^K} \left( \frac{I^K_{t+1}}{I_t} - \bar{\Gamma} \right) \right] \tag{B.28}$$

$$I^N_t = N_{t+1} - \left[ 1 - \delta^N_t \right] N_t. \tag{B.29}$$

$$Q^N_t = 1 + \frac{\psi_{I^N}}{2} \left( \frac{I^N_t}{I^N_{t-1}} - \bar{\Gamma} \right)^2 + \left( \frac{I^N_t}{I^N_{t-1}} \right) \psi_{I^N} \left( \frac{I^N_t}{I^N_{t-1}} - \bar{\Gamma} \right) - \mathbf{E}_t \left[ \Lambda_{t,t+1} \left( \frac{I^N_{t+1}}{I^N_t} \right)^2 \psi_{I^N} \left( \frac{I^N_{t+1}}{I^N_t} - \bar{\Gamma} \right) \right]. \tag{B.30}$$

**Banks** Recall the flow of funds relation

$$RE_{j,t} + (1+\tau^N_t) Q^N_t B^N_{j,t} + (1+\tau^K_t) Q^K_t B^K_{j,t} - NW_{j,t} - (1+\tau^E_t) Q^E_t E_{j,t} = D_{j,t}.$$

Consider that a policy makers wants to subsidise the issuance of outside equity via a tax on assets such that the budget is always balanced $\tau^E_t Q^E_t E_{j,t} = \tau^{S^N}_t Q^N_t B^N_{j,t} + \tau^{S^K}_t Q^K_t B^K_{j,t}$. Moreover, we assume that the risk profile associated with each type of asset affects the tax rate charged so that $\tau^K_t = \Delta^K \tau^{S^N}_t$. Since claims associated with phyiscal capital have a higher liquidation value, they are less risky, and the tax rate charged on them is thus lower than the tax rate charged on claims associated with intangible capital, so that $\tau^K_t < \tau^{S^N}_t$. Combine with this with the accumulation of retained earnings

$$\begin{aligned} NW_{j,t+1} = & \Big[ \phi^{B^N}_{j,t} \left( R^N_{t+1} - R^D_t \right) + \phi^{B^K}_{j,t} \left( R^K_{t+1} - R^D_t \right) + X_{j,t}\phi_{j,t} \left( R^D_t - R^E_{t+1} \right) \\ & + R^D_t \left( 1 - \tau^K_t \phi^{B^K}_{j,t} - \tau^N_t \phi^{B^N}_{j,t} + \tau^E_t X_{j,t}\phi_{j,t} \right) + \frac{RE_{j,t}}{NW_{j,t}} \left( R^{RE}_t - R^D_t \right) \Big] NW_{j,t}. \end{aligned}$$

$$\Theta_t = \theta \left( 1 + \omega_1 X_{j,t} + \frac{\omega_2}{2} X^2_{j,t} \right), X_{j,t} \equiv \frac{Q^E_t E_{j,t}}{\Delta^K Q^K_t B^K_{j,t} + Q^N_t B^N_{j,t}}$$

We introduced the definition of the risk-weighted equity-to-asset ratio and the leverage ratios

$$\phi_{j,t} \equiv \frac{\Delta^K Q^K_t B^K_{j,t} + Q^N_t B^N_{j,t}}{NW_{j,t}}, \phi^{B^K}_{j,t} \equiv \frac{Q^K_t B^K_{j,t}}{NW_{j,t}}, \phi^N_{j,t} \equiv \frac{Q^N_t B^N_{j,t}}{NW_{j,t}}.$$

The 'value' of the bank in period $t$, $V_{j,t}$, is the expected payout from the terminal net worth

$$V_{j,t} = \mathbf{E}_t \left[ \sum_{\tau=t+1}^{\infty} (\sigma_t)^{\tau-t-1} \Lambda_{t,\tau} \Pi_{t,\tau}^{-1} \left( (1-\sigma_t) NW_{b,\tau} \right) \right].$$

Recall the LoM derived for net worth by combining the balance sheet identity with the retained earnings expression

$$\begin{aligned}
\mathcal{G}_{j,t} \quad \equiv \quad \frac{NW_{j,t+1}}{NW_t} = & \Bigg[ \phi_{j,t}^{B^N} \left( R_{t+1}^N - R_t^D \right) + \phi_{j,t}^{B^K} \left( R_{t+1}^K - R_t^D \right) + X_{j,t}\phi_{j,t} \left( R_t^D - R_{t+1}^E \right) \\
& + R_t^D \left( 1 - \tau_t^K \phi_{j,t}^{B^K} - \tau_t^N \phi_{j,t}^{B^N} + \tau_t^E X_{j,t}\phi_{j,t} \right) + \frac{RE_{j,t}}{NW_{j,t}} \left( R_t^{RE} - R_t^D \right) \Bigg]
\end{aligned}$$

so that

$$g(NW_{j,t+1}, NW_{j,t}, X_{j,t}, \phi_{j,t}, \phi_{j,t}^{S^K}, \phi_{j,t}^{S^N}) \equiv \mathcal{G}_{j,t} NW_{j,t} - NW_{j,t+1} \quad = \quad 0$$

$$\max V_{j,t} \quad s.t. \quad g(NW_{j,t+1}, NW_{j,t}, X_{j,t}, \phi_{j,t}, \phi_{j,t}^{S^K}, \phi_{j,t}^{S^N})$$

$$\mathcal{L}_{j,t} \quad = \quad \mathbf{E}_t \left[ \sum_{\tau=t+1}^{\infty} \sigma_t^{\tau-t-1} SDF_{t,\tau} \Pi_{t,\tau}^{-1} \left\{ (1-\sigma_t) NW_{b,\tau} + \Omega_{b,\tau} g(NW_{j,t+1}, NW_{j,t}, X_{j,t}, \phi_{j,t}, \phi_{j,t}^{S^K}, \phi_{j,t}^{S^N}) \right\} \right]$$

where $\Omega_{b,\tau}$ is the Lagrange multiplier associated with the net worth accumulation budget constraint. Rewrite the Lagrangian using the double-sum

$$\begin{aligned}
\mathcal{L}_{j,t} \quad &= \quad \sum_{\mathcal{S}_t} \pi^{\mathcal{S}_t} \left[ \sum_{\tau=t+1}^{\infty} \sigma_t^{\tau-t-1} SDF_{t,\tau} \Pi_{t,\tau}^{-1} \left\{ (1-\sigma_t) NW_{b,\tau} + \Omega_{b,\tau} g(NW_{b,\tau}, NW_{b,\tau-1}, X_{b,\tau-1}, S_{b,\tau-1}^K, S_{b,\tau-1}^N) \right\} \right] \\
\frac{\partial \mathcal{L}_{j,t}}{\partial NW_{b,\tau=t+1}} \quad &= \quad \pi^{\mathcal{S}_t} \sigma_t^0 \Lambda_{t,t+1} \Pi_{t,t+1}^{-1} \left\{ (1-\sigma_t) + \Omega_{j,t+1} \frac{\partial g_{t+1}}{\partial NW_{j,t+1}} \right\} + \sum_{\mathcal{S}_{t+1}|\mathcal{S}_t} \pi^{\mathcal{S}_{t+1}} \sigma_t^1 SDF_{t,t+2} \Pi_{t,t+2}^{-1} \left\{ \Omega_{j,t+2} \frac{\partial g_{t+2}}{\partial NW_{j,t+1}} \right\} = 0 \\
0 \quad &= \quad \Lambda_{t,t+1} \Pi_{t,t+1}^{-1} \left\{ (1-\sigma_t) + \Omega_{j,t+1} (-1) \right\} + \sigma_t \mathbf{E}_{t+1} SDF_{t,t+2} \Pi_{t,t+2}^{-1} \left\{ \Omega_{j,t+2} \left( \mathcal{G}_{j,t+1} \right) \right\} \\
\Omega_{j,t+1} \quad &= \quad (1-\sigma_t) + \sigma_t \mathbf{E}_{t+1} \Lambda_{t+1,t+2} \Pi_{t+1,t+2}^{-1} \left\{ \Omega_{j,t+2} \left( \mathcal{G}_{j,t+1} \right) \right\} \\
\mathcal{G}_{j,t} \quad &\equiv \quad \frac{NW_{j,t+1}}{NW_{j,t}} = \Bigg[ \phi_{j,t}^{B^N} \left( R_{t+1}^N - R_t^D \right) + \phi_{j,t}^{B^K} \left( R_{t+1}^K - R_t^D \right) + X_{j,t}\phi_{j,t} \left( R_t^D - R_{t+1}^E \right) \\
& \qquad + R_t^D \left( 1 - \tau_t^K \phi_{j,t}^{B^K} - \tau_t^N \phi_{j,t}^{B^N} + \tau_t^E X_{j,t}\phi_{j,t} \right) + \frac{RE_{j,t}}{NW_{j,t}} \left( R_t^{RE} - R_t^D \right) \Bigg] \\
\mathcal{G}_{j,t+1} \quad &= \quad \Bigg[ \phi_{j,t+1}^{B^N} \left( R_{t+2}^N - R_{t+1}^D \right) + \phi_{j,t+1}^{B^K} \left( R_{t+2}^K - R_{t+1}^D \right) + X_{j,t+1}\phi_{j,t+1} \left( R_{t+1}^D - R_{t+2}^E \right) \\
& \qquad + R_{t+1}^D \left( 1 - \tau_{t+1}^K \phi_{j,t+1}^{B^K} - \tau_{t+1}^N \phi_{j,t+1}^{B^N} + \tau_{t+1}^E X_{j,t+1}\phi_{j,t+1} \right) + \frac{RE_{j,t+1}}{NW_{j,t+1}} \left( R_{t+1}^{RE} - R_{t+1}^D \right) \Bigg]
\end{aligned}$$

$$\begin{aligned}
\Omega_{j,t+1} \quad = \quad & (1-\sigma_t) + \sigma_t \Bigg\{ \mathbf{E}_{t+1} SDF_{t+1,t+2} \Pi_{t+1,t+2}^{-1} \Omega_{j,t+2} \phi_{j,t+1}^{B^K} (R_{t+2}^K - R_{t+1}^D) \\
& + \mathbf{E}_{t+1} SDF_{t+1,t+2} \Pi_{t+1,t+2}^{-1} \Omega_{j,t+2} \phi_{j,t+1}^{B^N} (R_{t+2}^N - R_{t+1}^D) + \mathbf{E}_{t+1} SDF_{t+1,t+2} \Pi_{t+1,t+2}^{-1} \Omega_{j,t+2} \phi_{j,t+1} X_{j,t+1} \left( R_{t+1} - R_{t+2}^E \right) \\
& + \mathbf{E}_{t+1} SDF_{t+1,t+2} \Pi_{t+1,t+2}^{-1} \Omega_{j,t+2} R_{t+1}^D \left( 1 - \tau_t^K \phi_{j,t}^{B^K} - \tau_t^N \phi_{j,t}^{B^N} + \tau_t^E X_{j,t}\phi_{j,t} \right) \\
& + \mathbf{E}_{t+1} SDF_{t+1,t+2} \Pi_{t+1,t+2}^{-1} \Omega_{j,t+2} \frac{RE_{j,t+1}}{NW_{j,t+1}} \left( R_t^{RE} - R_t^D \right) \Bigg\}
\end{aligned}$$

Using the auxiliary definitons

$$\begin{aligned}
\nu_{j,t} &\equiv \mathbf{E}_t\left[\Lambda_{t,t+1}\Omega_{j,t+1}\Pi_{t,t+1}^{-1}\left(R_t^D\right)\right]\\
\mu_{j,t}^{B^K} &\equiv \mathbf{E}_t\left[\Lambda_{t,t+1}\Omega_{j,t+1}\Pi_{t,t+1}^{-1}\left(R_{t+1}^K-R_t^D\right)\right]\\
\mu_{j,t}^{B^N} &\equiv \mathbf{E}_t\left[\Lambda_{t,t+1}\Omega_{j,t+1}\Pi_{t,t+1}^{-1}\left(R_{t+1}^N-R_t^D\right)\right]\\
\mu_{j,t}^{RE} &\equiv \mathbf{E}_t\left[\Lambda_{t,t+1}\Omega_{j,t+1}\Pi_{t,t+1}^{-1}\left(R_t^{RE}-R_t^D\right)\right]\\
\mu_{j,t}^{E} &\equiv \mathbf{E}_t\left[\Lambda_{t,t+1}\Omega_{j,t+1}\Pi_{t,t+1}^{-1}\left(R_t^D-R_{t+1}^E\right)\right]
\end{aligned}$$

**Guess the Banks Franchise Value and Verify**

$$\begin{aligned}
V_{j,t}(NW_{j,t}) &= \mathbf{E}_t\Lambda_{t,t+1}\Pi_{t+1}^{-1}\Delta_{t+1}NW_{j,t+1}\\
V_t(B_{j,t}^G,B_{j,t}^K,B_{j,t}^N,X_{j,t},NW_{j,t}) &= \mathbf{E}_t\Lambda_{t,t+1}\Pi_{t+1}^{-1}\left\{(1-\sigma_t)NW_{j,t+1}+\sigma_t\max_{B_{j,t+1},X_{j,t+1}}\mathbf{E}_tV_{t+1}(B_{j,t+1}^G,B_{j,t+1}^K,B_{j,t+1}^N,X_{j,t+1},NW_{j,t+1})\right\}\\
\mathbf{E}_t\Lambda_{t,t+1}\Pi_{t+1}^{-1}\Delta_{t+1}NW_{j,t+1} &= \mathbf{E}_t\Lambda_{t,t+1}\Pi_{t+1}^{-1}\left\{(1-\sigma_t)NW_{j,t+1}+\sigma_t\mathbf{E}_t\left(SDF_{t+1,t+2}\Pi_{t+1,t+2}^{-1}\Delta_{t+2}NW_{j,t+2}\right)\right\}\\
\mathbf{E}_t\Lambda_{t,t+1}\Pi_{t+1}^{-1}\Delta_{t+1}NW_{j,t+1} &= \mathbf{E}_t\Lambda_{t,t+1}\Pi_{t+1}^{-1}\left\{(1-\sigma_t)NW_{j,t+1}+\sigma_t\mathbf{E}_t\left(SDF_{t+1,t+2}\Pi_{t+1,t+2}^{-1}\Delta_{t+2}\mathcal{G}_{j,t+1}NW_{j,t+1}\right)\right\}\\
\Omega_{j,t+1} &= \Delta_{t+1}=\left\{(1-\sigma_t)+\sigma_t\mathbf{E}_t\left(SDF_{t+1,t+2}\Delta_{t+2}\mathcal{G}_{j,t+1}\right)\right\}\\
V_{j,t}(NW_{j,t}) &= \mathbf{E}_t\left[\Lambda_{t,t+1}\Pi_{t+1}^{-1}\Omega_{t+1}NW_{j,t+1}\right].
\end{aligned}$$

Recall

$$\Lambda_{t,t+1}\Omega_{j,t+1} = \Lambda_{t,t+1}(1-\sigma_t)+\sigma_t\mathbf{E}_{t+1}SDF_{t,t+2}\left\{\Omega_{j,t+2}\left(\mathcal{G}_{j,t+1}\right)\right\}$$

and

$$\begin{aligned}
V_{j,t}(NW_{j,t}) &= \mathbf{E}_t\Lambda_{t,t+1}\Pi_{t+1}^{-1}\Omega_{j,t+1}NW_{j,t+1}\\
V_{j,t}(NW_{j,t}) &= \mathbf{E}_t\left[\left(\Lambda_{t,t+1}\Pi_{t+1}^{-1}(1-\sigma_t)NW_{j,t+1}+\sigma_t\mathbf{E}_{t+1}SDF_{t,t+2}\Pi_{t+2}^{-1}\left\{\Omega_{j,t+2}\left(\mathcal{G}_{j,t+1}NW_{j,t+1}\right)\right\}\right)\right]\\
V_{j,t}(NW_{j,t}) &= \mathbf{E}_t\left[\left(\Lambda_{t,t+1}\Pi_{t+1}^{-1}(1-\sigma_t)NW_{j,t+1}+\sigma_t\underbrace{\mathbf{E}_{t+1}\left\{SDF_{t,t+2}\Pi_{t+2}^{-1}\Omega_{j,t+2}NW_{j,t+2}\right\}}_{V_{t+1}}\right)\right]\\
V_{j,t}(NW_{j,t}) &= \mathbf{E}_t\Omega_{j,t+1}\Lambda_{t,t+1}\Pi_{t+1}^{-1}NW_{j,t+1}\\
V_{j,t}(NW_{j,t}) &= \left[\mu_{j,t}^{RE}\frac{RE_{j,t}}{NW_{j,t}}+\mu_{j,t}^{B^N}\phi_{j,t}^{B^N}+\mu_{j,t}^{B^K}\phi_{j,t}^{B^K}+\mu_{j,t}^{E}\phi_{j,t}X_{j,t}+\nu_{j,t}\left(1-\tau_t^K\phi_{j,t}^{B^K}-\tau_t^N\phi_{j,t}^{B^N}+\tau_t^E\phi_{j,t}X_{j,t}\right)\right]NW_{j,t}.
\end{aligned}$$

Next, we maximise

$$V_{j,t}(NW_{j,t}) = \left[\mu_{j,t}^{RE}\frac{RE_{j,t}}{NW_{j,t}}+\mu_{j,t}^{B^N}\phi_{j,t}^{B^N}+\mu_{j,t}^{B^K}\phi_{j,t}^{B^K}+\mu_{j,t}^{E}\phi_{j,t}X_{j,t}+\nu_{j,t}\left(1-\tau_t^K\phi_{j,t}^{B^K}-\tau_t^N\phi_{j,t}^{B^N}+\tau_t^E\phi_{j,t}X_{j,t}\right)\right]NW_{j,t}$$

subject to the incentive compatibility constraint

$$V_t \geq \Theta(X_{j,t})\left(\Delta^KQ_t^KB_{j,t}^K+Q_t^NB_{j,t}^N\right)$$

over $X_{j,t}$, $RE_{j,t}$, $B_{j,t}^K$ and $B_{j,t}^N$, so that the Lagrangien is given by

$$\begin{aligned}
\mathcal{L}_t &= \left\{\left[\mu^{RE}_{j,t}\frac{RE_{j,t}}{NW_{j,t}} + \mu^{B^N}_{j,t}\phi^{B^N}_{j,t} + \mu^{B^K}_{j,t}\phi^{B^K}_{j,t} + \mu^{E}_{j,t}\phi_{j,t}X_{j,t} + \nu_{j,t}\left(1-\tau^K_t\phi^{B^K}_{j,t} - \tau^N_t\phi^{B^N}_{j,t} + \tau^E_t\phi_{j,t}X_{j,t}\right)\right]NW_{j,t}\right\} \\
&\quad + LM_1\left(V_t - \Theta(X_{j,t})\left(\Delta^K Q^K_t B^K_{j,t} + Q^N_t B^N_{j,t}\right)\right) \\
\frac{\partial\mathcal{L}}{\partial X_{j,t}} &= 0:\ \left[\mu^E_{j,t}\phi_{j,t} + \nu_{j,t}(\tau^E_t\phi_{j,t})\right]NW_{j,t} - LM_1\Theta'(X_{j,t})\left(\Delta^K Q^K_t B^K_{j,t} + Q^N_t B^N_{j,t}\right) =!0, \left[\mu^E_{j,t} + \nu_{j,t}\tau^E_t\right] = LM_1\Theta'(X_{j,t}) \\
\frac{\partial\mathcal{L}}{\partial RE_{j,t}} &= 0:\ \mu^{RE}_{j,t} = 0 \\
\frac{\partial\mathcal{L}}{\partial B^K_{j,t}} &= 0:\ \mu^{B^K}_{j,t}Q^K_t + \mu^E_{j,t}X_{j,t}\Delta^K Q^K_t + \nu_{j,t}\underbrace{\left(-\tau^K_t Q^K_t + \tau^E_t X_{j,t}\Delta^K Q^K_t\right)}_{=0,\text{ under tax assumption B}} - LM_1\Theta(X_{j,t})\Delta^K Q^K_t ! = 0, \frac{\mu^{B^K}_{j,t}}{\Delta^K} + \mu^E_{j,t}X_{j,t} = LM_1\Theta(X_{j,t})
\end{aligned}$$

$$\frac{\partial\mathcal{L}}{\partial B^N_{j,t}} = 0:\ \mu^{B^N}_{j,t}Q^N_t + \mu^E_{j,t}X_{j,t}Q^{S^N}_t + \nu_{j,t}\underbrace{\left(-\tau^{S^N}_t Q^N_t + \tau^E_t X_{j,t}Q^N_t\right)}_{=0,\text{ under tax assumption B}} - LM_1\Theta(X_{j,t})Q^N_t ! = 0 \Leftrightarrow \mu^{B^N}_{j,t} + \mu^E_{j,t}X_{j,t} = LM_1\Theta(X_{j,t})$$

Combine the optimality conditions to get

$$\frac{\left[\mu^E_{j,t} + \nu_{j,t}\tau^E_t\right]}{\mu^{B^N}_{j,t} + \mu^E_{j,t}X_{j,t}} = \frac{\Theta'(X_{j,t})}{\Theta(X_{j,t})}, \qquad \mu^{B^K}_{j,t} = \Delta^K\mu^{B^N}_{j,t}$$

Combine the closed-form expression of the banks franchise value $V_{j,t}$ with the incentive compatibility constraint to derive the endogenous limit on the banks leverage

$$\begin{aligned}
\Theta(X_t)\left(\Delta^K Q^K_t B^K_{j,t} + Q^N_{j,t}B^N_{j,t}\right) &= \Bigg[\overbrace{\mu^{RE}_{j,t}}^{=0}\frac{RE_{j,t}}{NW_{j,t}} + \overbrace{\mu^{B^G}_{j,t}}^{\Delta^G\mu^N_{j,t}}\phi^{B^G}_{j,t} + \mu^{B^N}_{j,t}\phi^{B^N}_{j,t} + \overbrace{\mu^{B^K}_{j,t}}^{\Delta^K\mu^N_{j,t}}\phi^{B^K}_{j,t} + \mu^E_{j,t}\phi_{j,t}X_{j,t} \\
&\quad + \nu_{j,t}\underbrace{\left(1-\tau^G_t\phi^{B^G}_{j,t} - \tau^K_t\phi^{B^K}_{j,t} - \tau^N_t\phi^{B^N}_{j,t} + \tau^E_t\phi_{j,t}X_{j,t}\right)}_{=1,\text{ under tax assumption, B}}\Bigg]NW_{j,t} \\
\Theta(X_t)\phi_{j,t} &= \mu^{B^N}_{j,t}\overbrace{\left(\Delta^K\phi^{B^K}_{j,t} + \phi^{B^N}_{j,t}\right)}^{\phi_{j,t}} + \mu^E_{j,t}\phi_{j,t}X_{j,t} + \nu_{j,t}, \quad \phi_{j,t} = \frac{\nu_{j,t}}{\Theta_t - \left(\mu^{B^N}_{j,t} + \mu^E_{j,t}X_{j,t}\right)}.
\end{aligned}$$

### Collecting Terms

$$\Theta_t = \theta\left(1 + \omega_1 X_{j,t} + \frac{\omega_2}{2}X^2_{j,t}\right) \tag{B.31}$$

$$\phi_{j,t} \equiv \frac{\Delta^K Q^K_t B^K_{j,t} + Q^N_t B^N_{j,t}}{NW_{j,t}} \tag{B.32}$$

$$\nu_{j,t} \equiv \mathbf{E}_t\left[\Lambda_{t,t+1}\Omega_{j,t+1}\Pi^{-1}_{t,t+1}\left(R^D_t\right)\right] \tag{B.33}$$

$$\Delta^K\mu^{B^N}_{j,t} \equiv \mathbf{E}_t\left[\Lambda_{t,t+1}\Omega_{j,t+1}\Pi^{-1}_{t,t+1}\left(R^K_{t+1} - R^D_t\right)\right] \tag{B.34}$$

$$\mu^{B^N}_{j,t} \equiv \mathbf{E}_t\left[\Lambda_{t,t+1}\Omega_{j,t+1}\Pi^{-1}_{t,t+1}\left(R^N_{t+1} - R^D_t\right)\right] \tag{B.35}$$

$$\mu^E_{j,t} \equiv \mathbf{E}_t\left[\Lambda_{t,t+1}\Omega_{j,t+1}\Pi^{-1}_{t,t+1}\left(R^D_t - R^E_{t+1}\right)\right] \tag{B.36}$$

$$\Omega_{j,t+1} \equiv (1-\sigma_t) + \sigma_t\left[\nu_{j,t+1} + \phi_{j,t+1}\left(\mu^{B^N}_{j,t+1} + X_{j,t+1}\mu^E_{j,t+1}\right)\right)\right] \tag{B.37}$$

$$\left(\mu^E_{j,t} + \nu_{j,t}\tau^E_t\right)/\left(\mu^{B^N}_{j,t} + \mu^E_{j,t}X_{j,t}\right) = \Theta'(X_{j,t})/\Theta(X_{j,t}) \tag{B.38}$$

$$\phi_{j,t} = \nu_{j,t}/\left(\Theta_t - \left(\mu^{B^N}_{j,t} + \mu^E_{j,t}X_{j,t}\right)\right). \tag{B.39}$$

**Return on Bank's Outside Equity** In order to find the flow return on equity via the flow return on total capital $\mathcal{K}$, the production function can be rewritten in terms of total capital as follows

$$
\begin{aligned}
Y_{m,t} &= \left(\varepsilon_t^K U_{m,t}^K K_{m,t}\right)^{\alpha} \left(\left(U_{m,t}^N N_{m,t}\right)^{\eta} \left(U_t^N N_t\right)^{1-\eta} L_{m,t}\right)^{1-\alpha} \\
Y_{m,t} &= \left(\varepsilon_t^K\right)^{\alpha} \underbrace{\left(K_{m,t}\right)^{\alpha} \left(N_{m,t}\right)^{\eta(1-\alpha)}}_{\equiv \mathcal{K}_{m,t}^{\alpha+\eta(1-\alpha)}} \underbrace{\left(U_{m,t}^K\right)^{\alpha} \left(U_{m,t}^N\right)^{\eta(1-\alpha)}}_{\equiv \left(U_{m,t}^{\mathcal{K}}\right)^{\alpha+\eta(1-\alpha)}} \left(U_t^N N_t\right)^{(1-\eta)(1-\alpha)} \left(L_{m,t}\right)^{1-\alpha} \\
Y_{m,t} &= \left(\varepsilon_t^E U_{m,t}^{\mathcal{K}} \mathcal{K}_{m,t}\right)^{\alpha+\eta(1-\alpha)} \left(U_t^N N_t\right)^{(1-\eta)(1-\alpha)} \left(L_{m,t}\right)^{1-\alpha}
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{K}_{m,t} &= \left(\left(K_{m,t}\right)^{\alpha} \left(N_{m,t}\right)^{\eta(1-\alpha)}\right)^{\frac{1}{\alpha+\eta(1-\alpha)}} \\
U_{m,t}^{\mathcal{K}} &= \left(\left(U_{m,t}^K\right)^{\alpha} \left(U_{m,t}^N\right)^{\eta(1-\alpha)}\right)^{\frac{1}{\alpha+\eta(1-\alpha)}} \\
\varepsilon_t^E &\equiv \left(\varepsilon_t^K\right)^{\frac{\alpha}{\alpha+\eta(1-\alpha)}}.
\end{aligned}
$$

We can derive the flow return $\mathcal{R}_t^{\mathcal{K}}$ by taking the derivative of output with respect to total capital

$$
\frac{\partial Y_{m,t}}{\partial \mathcal{K}_{m,t}} = \left(\alpha + \eta(1-\alpha)\right) \frac{Y_{m,t}}{\mathcal{K}_{m,t}} \equiv \mathcal{R}_{m,t}^{\mathcal{K}} = R_{m,t}^K \frac{K_{m,t}}{\mathcal{K}_{m,t}} + R_{m,t}^N \frac{N_{m,t}}{\mathcal{K}_{m,t}}.
$$

We can still detrend the production function by dividing by $N_t$

$$
\begin{aligned}
\frac{Y_t}{\mathcal{K}_t} &= \left(\varepsilon_t^K\right)^{\alpha} \left(U_t^{\mathcal{K}}\right)^{\alpha+\eta(1-\alpha)} \left(\mathcal{K}_t\right)^{\alpha-1+\eta(1-\alpha)} \left(U_t^N N_t\right)^{(1-\eta)(1-\alpha)} \left(L_t\right)^{1-\alpha} \\
\frac{Y_t}{\mathcal{K}_t} &= \left(\varepsilon_t^K\right)^{\alpha} \left(U_t^{\mathcal{K}}\right)^{\alpha+\eta(1-\alpha)} \mathcal{K}_t^{(1-\alpha)(\eta-1)} \left(N_t\right)^{(1-\eta)(1-\alpha)} \left(U_t^N\right)^{(1-\eta)(1-\alpha)} \left(L_t\right)^{1-\alpha} \\
\frac{Y_t}{\mathcal{K}_t} &= \left(\varepsilon_t^K\right)^{\alpha} \left(U_t^{\mathcal{K}}\right)^{\alpha+\eta(1-\alpha)} \left(\frac{\mathcal{K}_t}{N_t}\right)^{(\eta-1)(1-\alpha)} \left(U_t^N\right)^{(1-\eta)(1-\alpha)} \left(L_t\right)^{1-\alpha}
\end{aligned}
$$

so that the ratio $\frac{Y_t}{\mathcal{K}_t}$ is not trending. We can express total aggregate capital in detrended terms as follows

$$
\begin{aligned}
\mathcal{K}_t &= \left(\left(K_t\right)^{\alpha} \left(N_t\right)^{\eta(1-\alpha)}\right)^{\frac{1}{\alpha+\eta(1-\alpha)}} \\
\frac{\mathcal{K}_t}{N_t} &= \frac{1}{N_t^{\frac{\alpha+\eta(1-\alpha)}{\alpha+\eta(1-\alpha)}}} \left(\left(K_t\right)^{\alpha} \left(N_t\right)^{\eta(1-\alpha)}\right)^{\frac{1}{\alpha+\eta(1-\alpha)}} \\
\hat{\mathcal{K}}_t \equiv \frac{\mathcal{K}_t}{N_t} &= \left(\frac{1}{N_t^{\alpha+\eta(1-\alpha)}} \left(K_t\right)^{\alpha} \left(N_t\right)^{\eta(1-\alpha)}\right)^{\frac{1}{\alpha+\eta(1-\alpha)}} \\
\hat{\mathcal{K}}_t &\equiv \left(\hat{K}_t\right)^{\frac{\alpha}{\alpha+\eta(1-\alpha)}}.
\end{aligned}
$$

This implies that the flow return on total capital and thus equity can be written as

$$
\mathcal{R}_t^E \equiv \mathcal{R}_t^{\mathcal{K}} = R_t^K \hat{K}_t^{\frac{\eta(1-\alpha)}{\alpha+\eta(1-\alpha)}} + R_t^N \hat{K}_t^{\frac{-\alpha}{\alpha+\eta(1-\alpha)}} \tag{B.40}
$$

and

$$
R_t^E = \frac{\mathcal{R}_t^E + Q_t^E \varepsilon_t^E}{Q_{t-1}^E}. \tag{B.41}
$$

**Monetary Policy**

$$\frac{R_t^{TR}}{\bar{R}^{TR}} = \left(\frac{R_{t-1}^{TR}}{\bar{R}^{TR}}\right)^{\rho_{RTR}} \left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\kappa_\Pi}\left(\frac{MC_t}{\overline{MC}}\right)^{\kappa_Y}\right]^{1-\rho_{RTR}} \varepsilon_t^{MP} \tag{B.42}$$

$$R_t^D = \max\left(1, R_t^{TR}\right). \tag{B.43}$$

**Macro-prudential Policy** We assume that the risk profile associated with each type of capital affects the tax rate charged so that $\tau_t^K = \Delta^K \tau_t^N$. This implies

$$\begin{aligned}
\tau_t^E Q_t^E E_{j,t} &= \tau_t^K Q_t^K B_{j,t}^K + \tau_t^N Q_t^N B_{j,t}^N \\
\tau_t^E Q_t^E E_{j,t} &= \tau_t^{S^N}\left(\Delta^K Q_t^K B_{j,t}^K + Q_t^N B_{j,t}^N\right) \\
\tau_t^E X_{j,t} &= \tau_t^{S^N} \quad \Leftrightarrow \quad \Delta^K \tau_t^E X_{j,t} = \tau_t^K.
\end{aligned}$$

We assume the macro-prudential policy follows a simple rule

$$\tau_t^E = \kappa_\nu \nu_t^{-1} \tag{B.44}$$

**Aggregation**

**Final Output Goods Market Clearing and Price Dispersion** Equating the aggregate supply of final output goods $Y_t^s$ with the aggregate demand $Y_t$ we get

$$\int_0^1 Y_{m,t}^s di = \int_0^1 \left(\frac{P_{m,t}}{P_t}\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} Y_t di, \quad Y_t^s = Y_t \int_0^1 \left(\frac{P_{m,t}}{P_t}\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} di.$$

We define price dispersion $Disp_t^P$ and write it recursively, to get

$$\begin{aligned}
Disp_t^P &\equiv \int_0^1 \left(\frac{P_{m,t}}{P_t}\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} di = \left(\frac{1}{P_t}\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} \int_0^1 \left(P_{m,t}\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} di \\
Disp_t^P &= \left(\frac{1}{P_t}\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} \left[\int_{i\in F} \left(P_{m,t}^*\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} di + \int_{i\in \bar{F}} \left(P_{m,t}\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} di\right]
\end{aligned}$$

where $F$ is the set of those firms that can reoptimise their price. Recall that

$$P_{m,t} = \begin{cases} P_{m,t}^* & \textit{with probability: } 1-\phi_P \\ P_{m,t-1}\left(\left(\Pi_{ss}^P\right)^{1-ind_P}\left(\Pi_{t-1}^P\right)^{ind_P}\right) & \textit{with probability: } \phi_P \end{cases}$$

so that

$$\begin{aligned}
Disp_t^P &= \left(\frac{1}{P_t}\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} \left[\int_{i\in F} \left(P_{m,t}^*\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} di + \int_{i\in \bar{F}} \left(P_{m,t-1}\left(\left(\Pi_{ss}^P\right)^{1-ind_P}\left(\Pi_{t-1}^P\right)^{ind_P}\right)\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} di\right] \\
Disp_t^P &= (1-\phi_P)\left(\frac{P_t^*}{P_t}\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} + \left(\frac{\Pi_t^P}{\left(\left(\Pi_{ss}^P\right)^{1-ind_P}\left(\Pi_{t-1}^P\right)^{ind_P}\right)}\right)^{\frac{\mathcal{M}}{\mathcal{M}-1}} (\phi_P)\, Disp_{t-1}^P.
\end{aligned}$$

The detrended recursive expression of price dispersion is given by

$$Disp_t^P = (1-\phi_P)\left(\frac{1-\phi_P\left(\zeta_t^\Pi\right)^{\frac{1}{\mathcal{M}-1}}}{1-\phi_P}\right)^{\mathcal{M}} + \phi_P\left(\zeta_t^\Pi\right)^{\frac{\mathcal{M}}{\mathcal{M}-1}} Disp_{t-1}^P. \tag{B.45}$$

This implies

$$Y_t^s = Y_t Disp_t^P.$$

We will use the latter relation to replace $Y_t^s$.

**Aggregate Resource Constraint** By combining the household budget constraint, factor prices, capital accumulation equationst, the profits of the perfectly competitive capital goods producers, the aggregate bank balance sheet identity and the net worth accumulation equation one can obtain the aggregate market clearing relationship

$$\begin{aligned} P_t Y_t &= P_t C_t + \left[1 + \Psi_t^{I^K}\right] P_t I_t^K + \left[1 + \Psi_t^{I^N}\right] P_t I_t^N \\ \Xi_t &= \left(P_t - \frac{MC_t}{\tau_t^M}(1 + (1-\alpha)\eta) Disp_t^P\right) Y_t - \xi + (1-\sigma_t) NW_t^o \end{aligned} \tag{B.46}$$

# C Stationarisation

$$\mathcal{V}_t = \max_{C_t, C_{t-1}, L_t} \{\mathcal{U}_t + \beta \mathbf{E}_t \mathcal{V}_{t+1}\} \quad \Leftrightarrow \quad \hat{\mathcal{V}}_t \equiv \frac{\mathcal{V}_t}{N_t^{1-\gamma}} = \left\{\hat{\mathcal{U}}_t + \beta \mathbf{E}_t \Gamma_{t+1}^{1-\gamma} \hat{\mathcal{V}}_{t+1}\right\} \tag{C.1}$$

$$\mathcal{U}_t = \frac{1}{1-\gamma}\left(C_t - h\Gamma_t C_{t-1} - \vartheta_t \frac{L_t^{1+\varphi}}{1+\varphi}\right)^{1-\gamma} \quad \Leftrightarrow \quad \hat{\mathcal{U}}_t = \frac{\mathcal{U}_t}{N_t^{1-\gamma}} = \frac{1}{1-\gamma}\left(\hat{C}_t - h\hat{C}_{t-1} - \hat{\vartheta}_t \frac{L_t^{1+\varphi}}{1+\varphi}\right)^{1-\gamma} \tag{C.2}$$

$$\vartheta_t = \chi(N_t) \quad \Leftrightarrow \quad \hat{\vartheta}_t \equiv \frac{\vartheta_t}{N_t} = \chi \tag{C.3}$$

$$\mathbf{E}_t\left[\frac{\mathcal{U}_t^C}{N_t^{-\gamma}} \frac{W_t}{P_t N_t}\right] = -\frac{\mathcal{U}_t^L}{N_t{}^{1-\gamma}} \quad \Leftrightarrow \quad \mathbf{E}_t \hat{\mathcal{U}}_t^C \hat{W}_t = -\hat{\mathcal{U}}_t^L \tag{C.4}$$

$$1 = \mathbf{E}_t\left[\Lambda_{t,t+1} \frac{R_t^D}{\Pi_{t+1}}\right] \tag{C.5}$$

$$1 = \mathbf{E}_t\left[\Lambda_{t,t+1} \frac{R_{t+1}^E}{\Pi_{t+1}}\right] \tag{C.6}$$

$$1 = \mathbf{E}_t\left[\Lambda_{t,t+1} R_t^R\right] \tag{C.7}$$

$$R_t^E = \frac{P_t}{P_{t-1}} \frac{\hat{\mathcal{R}}_t^E + \hat{Q}_t^E}{\hat{Q}_{t-1}^E} \quad \Leftrightarrow \quad \frac{R_t^E}{\Pi_t} = \frac{\hat{\mathcal{R}}_t^E + \hat{Q}_t^E}{\hat{Q}_{t-1}^E} \tag{C.8}$$

$$\Lambda_{t,t+1} = \beta \frac{\mathcal{U}_{C,t+1}}{\mathcal{U}_{C,t}} = \beta \frac{\hat{\mathcal{U}}_{C,t+1}}{\hat{\mathcal{U}}_{C,t}} \frac{N_{t+1}{}^{-\gamma}}{N_t{}^{-\gamma}} \quad \Leftrightarrow \quad \Lambda_{t,t+1} = \beta \frac{\hat{\mathcal{U}}_{C,t+1}}{\hat{\mathcal{U}}_{C,t}} \frac{1}{\Gamma_{t+1}^{\gamma}} \tag{C.9}$$

$$\begin{aligned} \frac{\mathcal{U}_t^C}{N_t^{-\gamma}} &= \left(\frac{C_t}{N_t} - h\frac{C_{t-1}}{N_{t-1}} - \frac{\vartheta_t}{N_t}\frac{L_t^{1+\varphi}}{1+\varphi}\right)^{-\gamma} - \beta h \Gamma_{t+1}\left(\frac{C_{t+1}}{N_t} - h\Gamma_{t+1}\frac{C_t}{N_t} - \frac{\vartheta_{t+1}}{N_t}\frac{L_{t+1}^{1+\varphi}}{1+\varphi}\right)^{-\gamma} \\ \hat{\mathcal{U}}_t^C &= \left(\hat{C}_t - h\hat{C}_{t-1} - \hat{\vartheta}_t \frac{L_t^{1+\varphi}}{1+\varphi}\right)^{-\gamma} - \beta h \Gamma_{t+1}^{1-\gamma}\left(\hat{C}_{t+1} - h\hat{C}_t - \hat{\vartheta}_{t+1}\frac{L_{t+1}^{1+\varphi}}{1+\varphi}\right)^{-\gamma} \end{aligned} \tag{C.10}$$

$$\mathcal{U}_t^L \equiv -\vartheta_t L_t^{\varphi}\left(C_t - h\Gamma_t C_{t-1} - \vartheta_t \frac{L_t^{1+\varphi}}{1+\varphi}\right)^{-\gamma} \quad \Leftrightarrow \quad \frac{\mathcal{U}_t^L}{N_t^{1-\gamma}} = -\hat{\vartheta}_t L_t^{\varphi}\left(\hat{C}_t - h\hat{C}_{t-1} - \hat{\vartheta}_t \frac{L_t^{1+\varphi}}{1+\varphi}\right)^{-\gamma} \tag{C.11}$$

# D Summary of Baseline Model Equations

**Households**

$$\hat{\mathcal{V}}_t = \hat{\mathcal{U}}_t + \beta \mathbf{E}_t \left[\Gamma_{t+1}^{1-\gamma} \hat{\mathcal{V}}_{t+1}\right] \tag{D.1}$$

$$\hat{\mathcal{U}}_t = \frac{1}{1-\gamma}\left(\hat{C}_t - h\hat{C}_{t-1} - \hat{\vartheta}_t \frac{L_t^{1+\varphi}}{1+\varphi}\right)^{1-\gamma} \tag{D.2}$$

$$\hat{\vartheta}_t = \chi \left(\frac{\hat{\vartheta}_{t-1}}{\Gamma_t}\right)^{1-\iota} \tag{D.3}$$

$$\mathbf{E}_t \hat{\mathcal{U}}_t^C \hat{W}_t = -\hat{\mathcal{U}}_t^L \tag{D.4}$$

$$1 = \mathbf{E}_t \left[SDF_{t,t+1} \Pi_{t+1}^{-1} R_t^D\right] \tag{D.5}$$

$$1 = \mathbf{E}_t \left[SDF_{t,t+1} \Pi_{t+1}^{-1} R_{t+1}^E\right] \tag{D.6}$$

$$\hat{\mathcal{U}}_t^L \equiv -\hat{\vartheta}_t L_t^{\varphi} \left(\hat{C}_t - h\hat{C}_{t-1} - \hat{\vartheta}_t \frac{L_t^{1+\varphi}}{1+\varphi}\right)^{-\gamma} \tag{D.7}$$

$$\hat{\mathcal{U}}_t^C \equiv \left(\hat{C}_t - h\hat{C}_{t-1} - \hat{\vartheta}_t \frac{L_t^{1+\varphi}}{1+\varphi}\right)^{-\gamma} - \beta h \Gamma_{t+1}^{1-\gamma} \left(\hat{C}_{t+1} - h\hat{C}_t - \hat{\vartheta}_{t+1} \frac{L_{t+1}^{1+\varphi}}{1+\varphi}\right)^{-\gamma} \tag{D.8}$$

$$\Lambda_{t,t+1} = \beta \frac{\hat{\mathcal{U}}_{t+1}^C}{\hat{\mathcal{U}}_t^C} \frac{1}{\Gamma_{t+1}^{\gamma}} \tag{D.9}$$

**Firms**

$$Disp_t^P \hat{Y}_t = \left(\varepsilon_t^K U_t^K \hat{K}_t\right)^{\alpha} \left(\hat{\mathcal{X}}_t^{LAP} L_t\right)^{1-\alpha} \tag{D.10}$$

$$\hat{\mathcal{X}}_t^{LAP} = U_t^N \tag{D.11}$$

$$\hat{K}_t = \hat{B}_{t-1}^K \frac{1}{\Gamma_t^N} \tag{D.12}$$

$$1 = \hat{B}_{t-1}^N \frac{1}{\Gamma_t^N} \tag{D.13}$$

$$\hat{W}_t = \frac{\widehat{MC}_t}{\tau^{\mathcal{M}}} \left((1-\alpha) \frac{Disp_t^P \hat{Y}_t}{L_t}\right) \tag{D.14}$$

$$R_t^K \Pi_t^{-1} = \left[\hat{\mathcal{R}}_t^K + \hat{Q}_t^K (1-\delta_{m,t}^K) \varepsilon_t^K\right] / \hat{Q}_{t-1}^K \tag{D.15}$$

$$R_t^N \Pi_t^{-1} = \left[\hat{\mathcal{R}}_t^N + \hat{Q}_t^N (1-\delta_{m,t}^N)\right] / \hat{Q}_{t-1}^N \tag{D.16}$$

$$\hat{\mathcal{R}}_t^K = \frac{\widehat{MC}_t}{\tau^{\mathcal{M}}} \alpha \frac{Disp_t^P \hat{Y}_t}{\hat{K}_t} \tag{D.17}$$

$$\hat{\mathcal{R}}_t^N = \frac{\widehat{MC}_t}{\tau^{\mathcal{M}}} (1-\alpha) \eta Disp_t^P \hat{Y}_t \tag{D.18}$$

$$\frac{\widehat{MC}_t}{\tau^{\mathcal{M}}} \alpha \frac{Disp_t^P \hat{Y}_t}{U_t^K} = \hat{Q}_t^K \hat{K}_t b^K \left(U_t^K\right)^{\zeta^K} \tag{D.19}$$

$$\frac{\widehat{MC}_t}{\tau^{\mathcal{M}}} (1-\alpha) \eta \frac{\hat{Y}_t}{U_t^N} = \hat{Q}_t^N b^N \left(U_t^N\right)^{\zeta^N} \tag{D.20}$$

$$\delta_t^K = \delta_c^K + \frac{b^K}{1+\zeta^K} \left(U_t^K\right)^{1+\zeta^K} \tag{D.21}$$

$$\delta_t^N = \delta_c^N + \frac{b^N}{1+\zeta^N} \left(U_t^N\right)^{1+\zeta^N} \tag{D.22}$$

$$\frac{\hat{\mathcal{F}}_{1,t} \mathcal{M}}{\hat{\mathcal{F}}_{2,t}} = \left[\frac{1 - (\phi_P) \left(\zeta_t^{\Pi}\right)^{\frac{-1}{1-\mathcal{M}}}}{1-\phi_P}\right]^{1-\mathcal{M}} \tag{D.23}$$

$$\hat{\mathcal{F}}_{1,t} = \widehat{MC}_t \hat{Y}_t + \phi_P E_t \left[\Lambda_{t,t+1} \Gamma_{t+1} \left(\zeta_{t+1}^{\Pi}\right)^{\frac{\mathcal{M}}{\mathcal{M}-1}} \hat{\mathcal{F}}_{1,t+1}\right] \tag{D.24}$$

$$\hat{\mathcal{F}}_{2,t} = \hat{Y}_t + \phi_P E_t \left[\Lambda_{t,t+1} \Gamma_{t+1} \left(\zeta_{t+1}^{\Pi}\right)^{\frac{1}{\mathcal{M}-1}} \hat{\mathcal{F}}_{2,t+1}\right] \tag{D.25}$$

$$\zeta_t^{\Pi} = \frac{\Pi_t^P}{(\bar{\Pi})^{1-ind_P}\left(\Pi_{t-1}^P\right)^{ind_P}} \tag{D.26}$$

$$\hat{I}_t^K = \Gamma_{t+1}\hat{K}_{t+1} - \left[1-\delta_{m,t}^K\right]\hat{K}_t \tag{D.27}$$

$$\hat{Q}_t^K = 1 + \frac{\psi_{I^K}}{2}\left(\frac{\hat{I}_t^K}{\hat{I}_{t-1}^K}\Gamma_t - \bar{\Gamma}\right)^2 + \frac{\hat{I}_t^K}{\hat{I}_{t-1}^K}\Gamma_t\psi_{I^K}\left(\frac{\hat{I}_t^K}{\hat{I}_{t-1}^K}\Gamma_t - \bar{\Gamma}\right) - E_t\Lambda_{t,t+1}\left(\frac{\hat{I}_{t+1}^K}{\hat{I}_t^K}\Gamma_{t+1}\right)^2\psi_{I^K}\left(\frac{\hat{I}_{t+1}^K}{\hat{I}_t}\Gamma_{t+1} - \bar{\Gamma}\right) \tag{D.28}$$

$$\hat{I}_t^N = \Gamma_{t+1} - \left[1-\delta_{m,t}^N\right] \tag{D.29}$$

$$\hat{Q}_t^N = 1 + \frac{\psi_{I^N}}{2}\left(\frac{\hat{I}_t^N}{\hat{I}_{t-1}^N}\Gamma_t - \bar{\Gamma}\right)^2 + \frac{\hat{I}_t^N}{\hat{I}_{t-1}^N}\Gamma_t\psi_{I^N}\left(\frac{\hat{I}_t^N}{\hat{I}_{t-1}^N}\Gamma_t - \bar{\Gamma}\right) - E_t\Lambda_{t,t+1}\left(\frac{\hat{I}_{t+1}^N}{\hat{I}_t^N}\Gamma_{t+1}\right)^2\psi_{I^N}\left(\frac{\hat{I}_{t+1}^N}{\hat{I}_t}\Gamma_{t+1} - \bar{\Gamma}\right) \tag{D.30}$$

**Banks**

$$\Theta_t = \theta\left(1+\omega_1 X_{j,t} + \frac{\omega_2}{2}X_{j,t}^2\right) \tag{D.31}$$

$$\phi_{j,t} \equiv \frac{\Delta^K Q_t^K B_{j,t}^K + Q_t^N B_{j,t}^N}{NW_{j,t}} \tag{D.32}$$

$$\nu_{j,t} \equiv \mathbf{E}_t\left[\Lambda_{t,t+1}\Omega_{j,t+1}\Pi_{t,t+1}^{-1}\left(R_t^D\right)\right] \tag{D.33}$$

$$\Delta^K\mu_{j,t}^{B^N} \equiv \mathbf{E}_t\left[\Lambda_{t,t+1}\Omega_{j,t+1}\Pi_{t,t+1}^{-1}\left(R_{t+1}^K - R_t^D\right)\right] \tag{D.34}$$

$$\mu_{j,t}^{B^N} \equiv \mathbf{E}_t\left[\Lambda_{t,t+1}\Omega_{j,t+1}\Pi_{t,t+1}^{-1}\left(R_{t+1}^N - R_t^D\right)\right] \tag{D.35}$$

$$\mu_{j,t}^{E} \equiv \mathbf{E}_t\left[\Lambda_{t,t+1}\Omega_{j,t+1}\Pi_{t,t+1}^{-1}\left(R_t^D - R_{t+1}^E\right)\right] \tag{D.36}$$

$$\Omega_{j,t+1} \equiv (1-\sigma_t) + \sigma_t\left[\nu_{j,t+1} + \phi_{j,t+1}\left(\mu_{j,t+1}^{B^N} + X_{j,t+1}\mu_{j,t+1}^E\right)\right)\Bigg] \tag{D.37}$$

$$\frac{\left[\mu_{j,t}^E + \nu_{j,t}\tau_t^E\right]}{\mu_{j,t}^{B^N} + \mu_{j,t}^E X_{j,t}} = \frac{\Theta'(X_{j,t})}{\Theta(X_{j,t})} \tag{D.38}$$

$$\phi_{j,t} = \frac{\nu_{j,t}}{\Theta_t - \left(\mu_{j,t}^{B^N} + \mu_{j,t}^E X_{j,t}\right)} \tag{D.39}$$

$$R_t^E\Pi_t^{-1} = \frac{\hat{R}_t^E + \hat{Q}_t^E\left(\varepsilon_t^K\right)^{\alpha/(\alpha+\eta(1-\alpha))}}{\hat{Q}_{t-1}^E} \tag{D.40}$$

$$\mathcal{R}_t^E \equiv \mathcal{R}_t^{\mathcal{K}} = R_t^K\hat{K}_t^{\frac{\eta(1-\alpha)}{\alpha+\eta(1-\alpha)}} + R_t^N\hat{K}_t^{\frac{-\alpha}{\alpha+\eta(1-\alpha)}} \tag{D.41}$$

**Policy**

$$\frac{R_t^{TR}}{\bar{R}^{TR}} = \left(\frac{R_{t-1}^{TR}}{\bar{R}^{TR}}\right)^{\rho_{RTR}}\left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\kappa_\Pi}\left(\frac{MC_t}{\overline{MC}}\right)^{\kappa_Y}\right]^{1-\rho_{RTR}}\varepsilon_t^{MP} \tag{D.42}$$

$$R_t^D = \max\left(1, R_t^{TR}\right) \tag{D.43}$$

$$\tau_t^E = \kappa_\nu\nu_t^{-1} \tag{D.44}$$

**Market Clearing**

$$Disp_t^P = (1-\phi_P)\left(\frac{1-\phi_P\left(\zeta_t^{\Pi}\right)^{\frac{1}{\mathcal{M}-1}}}{1-\phi_P}\right)^{\mathcal{M}} + \phi_P\left(\zeta_t^{\Pi}\right)^{\frac{\mathcal{M}}{\mathcal{M}-1}} Disp_{t-1}^P \tag{D.45}$$

$$Y_t = C_t + \left(1+\frac{\psi_{I^K}}{2}\left(\frac{I_t^K}{I_{t-1}^K}\Gamma_t^N - \bar{\Gamma}^{I^K}\right)^2\right)I_t^K + \left(1+\frac{\psi_{I^N}}{2}\left(\frac{I_t^N}{I_{t-1}^N}\Gamma_t^N - \bar{\Gamma}^{I^N}\right)^2\right)I_t^N \tag{D.46}$$

$$\begin{aligned}\Gamma_t\Pi_t\widehat{NW}_t = \sigma_t\Big[&\hat{Q}_{t-1}^K\hat{B}_{t-1}^K\left(R_t^K - R_{t-1}^D\right) + \hat{Q}_{t-1}^N\hat{B}_{t-1}^N\left(R_t^N - R_{t-1}^D\right) \\ &+X_{t-1}\phi_{t-1}\left(R_{t-1}^D - R_{t-1}^E\right)\widehat{NW}_{t-1} + R_{t-1}^D\widehat{NW}_{t-1}\Big] + \xi\end{aligned} \tag{D.47}$$

**Exogenous Processes**

$$\log\varepsilon_t^K = \rho_K\log\varepsilon_{t-1}^K + \varsigma_K\eta_t^K \tag{D.48}$$

$$\log\varepsilon_t^{\mathcal{M}} = \rho_{\mathcal{M}}\log\varepsilon_{t-1}^{\mathcal{M}} + \varsigma_{\mathcal{M}}\eta_t^{\mathcal{M}} \tag{D.49}$$

$$\log\varepsilon_t^{\sigma} = \rho_{\sigma}\log\varepsilon_{t-1}^{\sigma} + \varsigma_{\sigma}\eta_t^{\sigma} \tag{D.50}$$

$$\log\varepsilon_t^{MP} = \rho_{MP}\log\varepsilon_{t-1}^{MP} + \varsigma_{MP}\eta_t^{MP} \tag{D.51}$$

**Auxiliary Variables**

$$Spread_t^K \equiv \left(R_t^K - R_{t-1}^D\right) \tag{D.52}$$

$$Spread_t^N \equiv \left(R_t^N - R_{t-1}^D\right) \tag{D.53}$$

$$Spread_t^E \equiv \left(R_{t-1}^D - R_t^E\right) \tag{D.54}$$

$$TETA_t \equiv \frac{NW_t + Q_t^E E_t}{Q_t^K B_t^K + Q_t^N B_t^N} \tag{D.55}$$

$$INOUTE_t \equiv \frac{NW_t}{X(\Delta^K Q_t^K B_t^K + Q^N B^N)} \tag{D.56}$$

## D.1 Exogenous Growth + FF and Endogenous Growth +noFF

In the main text above we compare the baseline model, Equations (D.1) - (D.56) to a model with exogenous growth and financial frictions, and to a model with endogenous growth but without financial frictions. In the case of exogenous growth, it holds that $\Gamma_t = \bar{\Gamma}$, which can be thought of as a limiting case in which the investment adjustment cost for research is infinitely high $\psi_{I^N} \to \infty$. All variables associated with intangible capital become constants. In the model with endogenous growth and financial frictions, we can disregard the Equations associated to the banking block and all spreads become zero.

# E The Risky Steady State

In this paper, we evaluate the dynamics of the linearized model around what Coeurdacier et al. (2011) refer to as the risky steady state. Loosely speaking, the risky steady state is the state of the economy where agents choose to stay when they expect future risk and if the realization of shocks at this period is 0. Opposite to the deterministic steady state where agents anticipate no future shocks, the risky steady state incorporates information relative to the stochastic nature of the economy. Such information can be crucial to characterize banks' optimal portfolio choice or household welfare.[19]

We provide a brief description of the method used to compute the risky steady state. We follow de Groot (2013) who uses an iterative method relying on the second-order approximation of the decision rules to compute the risky steady state.[20]

Consider the equilibrium conditions describing the behavior of our model,

$$\begin{aligned} &E_t[f(y_{t+1},y_t,x_{t+1},x_t,z_{t+1},z_t)] = 0, \\ &z_{t+1} = \Lambda z_t + \eta\sigma\varepsilon_{t+1}, \end{aligned} \tag{E.1}$$

where $y_t$ is an $n_y \times 1$ vector of non-predetermined variables, $x_t$ is an $n_x \times 1$ vector of predetermined variables, $z_t$ is an $n_z \times 1$ vector of exogenous variables, and $\varepsilon_{t+1}$ is an $n_z \times 1$ vector of i.i.d exogenous disturbances. $\Lambda$ and $\eta$ are parameters matrices of size $n_z \times n_z$ and $\sigma$ is the stochastic scale of the model so that if $\sigma = 0$ the model is deterministic.

We define functions $h$ and $g$ as the decision rules that solve the equilibrium conditions defined in (E.1) with $y_t = g(x_t,z_t,\sigma)$ and $x_{t+1} = h(x_t,z_t,\sigma)$. We can now formally define the risky steady state as the vector $x^r$ that solves,

$$x^r = h\left(x^r,0,\sigma\right).$$

Computing a Taylor approximation of the decision rules $h$ around the deterministic steady state $x^d$ yields:

$$x^i_{t+1} = x^{d,i} + h^{d,i}_x\left(x_t - x^d\right) + \frac{1}{2}\left(x_t - x^d\right)' h^{d,i}_{xx}\left(x_t - x_d\right) + \frac{1}{2} z_t' h^{d,i}_{zz} z_t + \left(x_t - x^d\right)' h^{d,i}_{xz} z_t + \frac{1}{2} h^{d,i}_{\sigma\sigma}\sigma^2, \tag{E.2}$$

for $i = 1,...,n_x$, where the vector $h^{d,i}_x$ and the matrices $h^{d,i}_{xx}$, $h^{d,i}_{xz} z_t$ and $h^{d,i}_{\sigma\sigma}$ correspond respectively to the jacobian and the hessians of the decision rules evaluated at the deterministic steady state. Because at the risky steady state, all shocks are zero, it is possible to write (E.2) as:

$$x^{r,i} = x^{d,i} + h^{d,i}_x\left(x^r - x^d\right) + \frac{1}{2}\left(x^r - x^d\right)' h^{d,i}_{xx}\left(x^r - x^d\right) + \frac{1}{2} h^{d,i}_{\sigma\sigma}\sigma^2. \tag{E.3}$$

Stacking each of the decision rules in (E.3) and defining $x^* \equiv x^r - x^d$, we obtain the following quadratic equation,

$$C + Bx^* + Avec\left(x^* x^{*\prime}\right) = 0, \tag{E.4}$$

where,

$$C \equiv h^d_{\sigma\sigma}\frac{\sigma^2}{2}, \quad B \equiv h^d_x - I_{n_x}, \quad A \equiv \frac{1}{2}\begin{bmatrix} vec(h^{d,1}_{xx})' \\ \vdots \\ vec(h^{d,n_x}_{xx})' \end{bmatrix}.$$

Finally, we obtain the risky steady state using a nonlinear solver to resolve (E.4).

[19] See for instance Devereux and Sutherland (2011) for an example of risky steady state applied to solve a portfolio choice problem.

[20] The method is also described in Juillard (2011).