BANK OF ENGLAND

Staff Working Paper No. 920

Unemployment risk, liquidity traps and monetary policy

Dario Bonciani and Joonseok Oh

May 2021

Staff Working Paper No. 920

Unemployment risk, liquidity traps and monetary policy

Dario Bonciani[(1)] and Joonseok Oh[(2)]

Abstract

When the economy is in a liquidity trap and households have a precautionary motive to save against unemployment risk, adverse demand shocks cause severe deflationary spirals and output contractions. In this context, we study the implications of optimal monetary policy, which consists of keeping the nominal rate at zero longer than implied by current macroeconomic conditions. Under such policy and incomplete markets, expected improvements in labour market conditions mitigate the rise in unemployment risk and decline in demand. As a result, market incompleteness may alleviate contractions in output and inflation during a liquidity trap. However, reducing market incompleteness mitigates the fall in demand under realistic monetary policy rules.

Key words: Unemployment risk, Liquidity trap, Zero lower bound, Monetary policy.

JEL classification: E21, E24, E32, E52, E61.

(1) Bank of England and Centre for Macroeconomics. Email: dario.bonciani@bankofengland.co.uk
(2) Freie Universität Berlin. Email: Joonseok.oh@fu-berlin.de

The views expressed in this paper are those of the authors, and not necessarily those of the Bank of England or its committees. We are grateful to Daeha Cho, Federico di Pace, ShinHyuck Kang, Mira Kim, Riccardo Masolo, Ricardo Reis and Vincent Sterk, and an anonymous referee for the Bank of England Staff Working Paper Series for insightful discussions and helpful suggestions.

The Bank's working paper series can be found at www.bankofengland.co.uk/working-paper/staff-working-papers

Bank of England, Threadneedle Street, London, EC2R 8AH
Email enquiries@bankofengland.co.uk

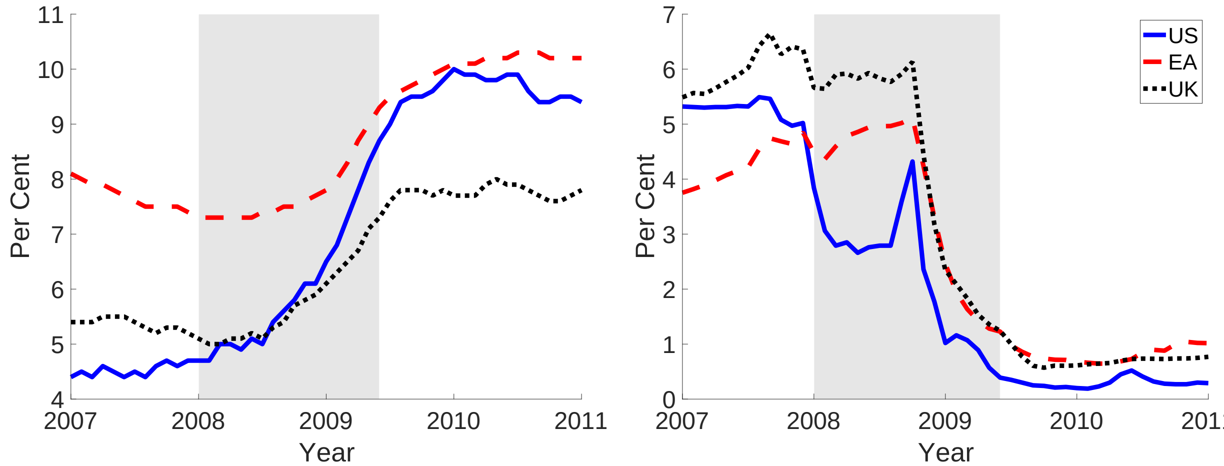
ISSN 1749-9135 (on-line)

1 Introduction

The Great Recession in 2008-2009 caused a significant and persistent increase in the unemployment rate across major advanced economies, as shown in Figure 1(a). The worsening in labour market conditions increased uncertainty about job prospects, which potentially gave rise to precautionary savings, putting further downward pressure on real economic activity and prices (see, e.g., Den Haan et al., 2018 and Challe, 2020). Moreover, in response to the severe drop in demand, central banks worldwide cut short-term nominal interest rates that rapidly approached the zero lower bound (ZLB), where they remained for a prolonged time (see Figure 1(b)).

How effective is monetary policy at responding to a contraction in demand and increase in uninsurable unemployment risk when the nominal rate is at the ZLB? In this paper, we address this question through the lenses of a Heterogeneous Agents New Keynesian (HANK) model with nominal price rigidities, labour search frictions, imperfect unemployment insurance, and an occasionally binding ZLB constraint. In particular, the model features two types of households: workers and firm owners. Workers face the risk of becoming unemployed and earning a lower income. The presence of idiosyncratic unemployment risk leads employed workers to save for precautionary reasons. Firm owners, instead, do not face any idiosyncratic risk. Households face a zero-debt limit and, as a result, end up consuming all their income. This ingredient of the model allows us to abstract from any distributional effects of monetary policy and rather concentrate on the interaction between monetary policy and countercyclical unemployment risk. On the production side of the economy, wholesale firms operate in a monopolistically competitive market and face adjustment costs when adjusting prices. These nominal rigidities allow monetary policy to affect real economic activity. The central bank responds to aggregate demand shocks by setting the nominal policy rate, subject to a ZLB constraint.

In such a context, we study the impact of monetary policy in response to a negative demand shock that leads the economy into a liquidity trap. To this end, we first analyse the economic outcomes when the central bank only responds to current inflation (strict inflation targeting), comparing the cases with complete and incomplete markets. Given this benchmark, we then study how the economy responds when the central bank follows the Ramsey optimal monetary policy. We find that, under the strict-inflation-targeting policy, an adverse demand shock has significantly stronger effects under incomplete markets. This is because the fall in demand reduces job creation and raises unemployment risk, which induces households to increase their savings for precautionary reasons. The precautionary-savings effect leads to a stronger fall in inflation and inflation expectations. Since the nominal rate is stuck at zero, the real rate rises, putting further downward



(a) Unemployment Rate

(b) Short-Term Rate

Figure 1: Unemployment Rates and Short-Term Interest Rates in the Great Recession

Note: The figure displays the unemployment rates and short-term interest rate for the United States (USA, blue-solid line), Euro Area (EA, red-dashed line), and United Kingdom (UK, black-dotted line). Grey shaded areas represent NBER recession dates. Source: OECD Main Economic Indicators, Volume 2021 Issue 1.

pressure on consumption and output. In other words, when asset markets are incomplete, and the central bank is unable to cut the interest rate, an adverse demand shock gives rise to a deflationary spiral and a severe contraction in real activity due to a worsening in expected labour market conditions.

Under the optimal policy, instead, the central bank responds to the negative demand shock by committing to keep its nominal rate at zero longer than implied by current economic conditions. This policy has the effect of increasing inflation expectations and reducing the real rate. By keeping the interest rate lower for longer, agents expect improvements in labour market conditions, which reduces their precautionary-savings behaviour in the presence of imperfect unemployment insurance. As a result, market incompleteness amplifies the rise in inflation expectations and the reduction in the real rate, thereby mitigating the decline in real activity. Specifically, when the central bank sets an optimal path for the policy rate, an adverse demand shock causes smaller contractions in real economic activity under incomplete markets than under perfect risk-sharing.

Finally, we show that a central bank can mitigate the deflationary spiral caused by the ZLB and incomplete markets by following simple policy rules that introduce history dependence in the nominal policy rate. In particular, we consider three alternative policies: (i) a Taylor rule augmented with the lagged value of the shadow policy rate, i.e., the theoretical policy rate that would prevail in the absence of a ZLB constraint;

(*ii*) a price-level-targeting rule; and (*iii*) an average-inflation-targeting policy. Following a fall in inflation due to a negative demand shock, these policy rules force the nominal rate to remain at zero longer than implied by contemporaneous macroeconomic conditions. Similarly, as under the optimal monetary policy, the presence of countercyclical uninsured unemployment risk leads to a rise in inflation expectations and a fall in the real rate. Therefore, these type of policy rules can be particularly effective under imperfect insurance. However, unlike the optimal-policy case, these simple and more realistic policy rules do not fully neutralise the deflationary spiral caused by market incompleteness. For this reason, we conclude that, in practice, unemployment insurance policies aimed at reducing market incompleteness are desirable tools, alongside monetary policy, to stabilise output at the ZLB.

Related Literature This paper builds primarily on two strands of the literature. First and foremost, by analysing the optimal monetary policy conduct in a model with uninsurable unemployment risk and frictions in the labour market, our paper is particularly related to the literature on HANK models with incomplete markets. By studying the interaction between incomplete markets and the ZLB, our work is also strictly related to the literature on monetary policy in a liquidity trap. To the best of our knowledge, we are the first to study optimal monetary policy at the ZLB in a model with uninsured unemployment risk arising endogenously from labour market frictions.

This work builds on the growing literature on unemployment risk in models with incomplete markets. McKay and Reis (2016) document that a reduction in unemployment benefits, increasing precautionary savings against uninsured unemployment risk, may raise investment and the capital stock, thereby reducing consumption volatility. Challe et al. (2017) estimate a medium-scale DSGE model with imperfect unemployment insurance and show that an adverse feedback loop between precautionary savings and aggregate demand contributes to explain the severity of the Great Recession. Ravn and Sterk (2017) build a model where households face uninsured unemployment risk, sticky prices, and search-and-matching frictions. In such a framework, a higher risk of job loss and worse job-finding prospects induce a precautionary-savings motive that causes a decline in the demand for goods. Lower demand, in turn, reduces job vacancies and the job-finding rate, producing an amplification mechanism due to endogenous countercyclical income risk. Den Haan et al. (2018) show that the combination of incomplete markets and sticky nominal wages increases business cycle volatility. Acharya et al. (2020) study optimal monetary policy in a HANK framework, where the planner's objective function includes reducing consumption inequality, besides stabilising output and inflation. When income risk is countercyclical, they find that policy curtails the fall in output in recessions to alleviate the increase in inequality. Ravn and Sterk (2020) show that in a heterogeneous agents model with labour market frictions, the precautionary-savings motive may lead the economy to get stuck in a high-unemployment steady-state. Challe (2020) analyses optimal monetary policy in a similar framework. By increasing unemployment risk, contractionary cost-push or productivity shocks lead to a rise in precautionary savings and a fall in inflation, which call for an accommodative monetary policy.[1] Our work extends the analysis in Challe (2020) to the liquidity trap case, where the deflationary spiral induced by countercyclical unemployment risk is particularly severe. Unlike McKay et al. (2016), and in line with Werning (2015) and Acharya and Dogra (2020), our results imply that incomplete markets do not attenuate the effects of forward guidance if idiosyncratic income risk is countercyclical. These two papers examine the sensitivity of aggregate demand to future monetary policy shocks using models where the cyclicality of idiosyncratic income risk can be time-varying but parameterised. Our work, instead, studies optimal policy at the ZLB in a model where labour market frictions endogenously give rise to countercyclical income risk.

This paper is also related to the strand of the macroeconomic literature studying the optimal conduct of monetary policy when nominal short-term rates are at the ZLB. Eggertsson and Woodford (2003) examines the implications of the ZLB on the ability of a central bank to contrast deflation. A credible commitment to the right sort of history-dependent policy can largely mitigate the distortions created by the ZLB. Jung et al. (2005) shows that at the ZLB, the optimal monetary policy response implies policy inertia, i.e., a zero interest rate policy should be continued for a while even after the natural rate returns to a positive level.[2] Adam and Billi (2007) study optimal monetary policy in a model where the ZLB on the nominal interest rate is an occasionally binding constraint. Rational agents anticipate the possibility of reaching the lower bound in the future, and this amplifies the effects of adverse shocks well before the bound is reached, which calls for a more aggressive response by the central bank. Bilbiie (2019) studies how long a central bank should keep interest rates at a low level after a liquidity trap ends. The paper argues that the optimal duration is approximately half the time the economy spent in a liquidity trap. Nakata et al. (2019) show that in a framework where the stimulating ability of forward guidance is relatively muted, and the economy is in a liquidity trap, the monetary policy authority should commit to keeping the policy rate at zero for a significantly long time.[3]

The remainder of the paper is structured as follows. In Section 2, we describe the model. Section 3 presents the main mechanisms at play, based on a three-period version of the model. In Section 4, we set out our

[1] Other papers dealing with monetary policy in heterogeneous agents models with incomplete markets and sticky prices are Heathcote et al. (2010), Braun and Nakajima (2012), Heathcote and Perri (2018), Kekre (2019), and Oh and Rogantini Picco (2020).

[2] Hills and Nakata (2018) and Bonciani and Oh (2020a) show that monetary policy inertia reduces the size of government spending multipliers and removes the "Paradox of flexibility" when the economy is in a liquidity trap.

[3] A non-exhaustive list of papers dealing with monetary policy at the ZLB are Nakov (2008), Christiano et al. (2011), Nakata (2017), Nakata and Schmidt (2019), Masolo and Winant (2019), and Bonciani and Oh (2020b).

numerical analysis. In Section 5, we discuss the results under alternative policy rules. Finally, in Section 6, we provide some concluding remarks.

2 The Model

Given our interest in studying the implications of uninsurable unemployment risk on optimal monetary policy at the ZLB, we consider a relatively stylised framework that mostly abstracts from distributional issues and rather focuses on the optimal stabilisation of aggregate demand. More specifically, following Challe (2020), the economy consists of two types of households, workers and firm owners. Workers can be either employed or unemployed, and their wage results from a Nash bargaining process. On the production side, the economy includes three types of firms, producing intermediate, wholesale, and final goods. In particular, intermediate-goods firms hire workers in a frictional labour market to produce their output. These firms sell the intermediate goods to wholesale firms, which operate in a monopolistically competitive market and face price adjustment costs. Last, final-goods firms produce their output using the wholesale good as input.

2.1 Working Households

Working household $i \in [0,1]$ can be employed or unemployed, and maximises its lifetime utility (1) subject to a budget constraint (2) and a zero-debt-limit constraint (3). The optimisation problem of a working household writes as follows:

$$\max_{c_{i,t},a_{i,t}} E_0 \sum_{t=0}^{\infty} \beta^t \log c_{i,t}, \tag{1}$$

subject to

$$\frac{a_{i,t}}{z_t} + c_{i,t} = e_{i,t} w_t + (1 - e_{i,t})\,\delta_t + \frac{1 + i_{t-1}}{1 + \pi_t} a_{i,t-1}, \tag{2}$$

$$a_{i,t} \geq 0, \tag{3}$$

$$\log z_t = \rho_z \log z_{t-1} + \sigma_z \varepsilon_t^z, \quad \epsilon_t^z \sim \mathcal{N}(0,1). \tag{4}$$

The parameter β is the subjective discount factor. The household derives utility from its consumption $c_{i,t}$. The dummy variable $e_{i,t}$ defines the employment status of the household. If $e_{i,t} = 1$, the household is employed, works full-time without any associated disutility, and earns a wage income $w_t > 0$. If $e_{i,t} = 0$, the household is unemployed and only gets an exogenous home-production income $\delta_t \in (0, w_t)$. The employment status of the workers is random and the associated income risk is uninsured, i.e., there is no compensation for the income loss. $a_{i,t}$ represents risk-free bonds issued by the workers. z_t is an aggregate demand shock[4] with persistence $\rho_z \in [0, 1)$ and volatility σ_z. The net nominal interest rate is represented by i_t, whereas π_t is the inflation rate. At the beginning of time, workers are assumed to hold no assets $a_{-1} = 0$.

2.2 Firm Owners

There is a unit mass of households, who own the various firms in the economy. These households choose consumption c_t^F to maximise their lifetime utility (5) subject to their budget constraint (6) and a zero-debt-limit constraint (7). Unlike workers, firm owners do not face any idiosyncratic income risk. Their optimisation problem looks as follows:

$$\max_{c_t^F, a_t^F} E_0 \sum_{t=0}^{\infty} \beta^t \log c_t^F, \tag{5}$$

subject to

$$\frac{a_t^F}{z_t} + c_t^F = \Pi_t^W + \Pi_t^I + \varpi + \tau_t + \frac{1 + i_{t-1}}{1 + \pi_t} a_{t-1}^F, \tag{6}$$

$$a_t^F \geq 0, \tag{7}$$

a_t^F represents the bonds issued by the firm owners that pay the risk-free nominal interest rate i_t. Π_t^W and Π_t^I are the dividends the firm owners receive from the ownership of wholesale and intermediate-goods firms, whereas $\varpi \geq 0$ and τ_t are respectively a home-production income and a lump-sum fiscal transfer. Similarly as for the workers, firm owners hold no assets at the beginning of time $a_{-1} = 0$.

2.3 Final Goods Firms

The final good y_t is produced by aggregating wholesale inputs $y_t(h)$ with a constant elasticity of substitution technology:

$$y_t = \left(\int_0^1 y_t(h)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \tag{8}$$

where θ is the elasticity of substitution of wholesale goods. The cost-minimisation problem for the final good firm implies that the demand for the intermediate good i is given by:

$$y_t(h) = \left(\frac{p_t(h)}{p_t} \right)^{-\theta} y_t, \tag{9}$$

[4] Smets and Wouters (2007) interprets z_t as a risk-premium shock.

where $p_t(h)$ is the price of the wholesale good. Finally, the zero-profit condition implies that the price index is expressed as:

$$p_t = \left(\int_0^1 p_t(h)^{1-\theta} dh \right)^{\frac{1}{1-\theta}}. \tag{10}$$

2.4 Wholesale Firms

There exists a continuum of wholesale firms, indexed by $h \in [0, 1]$, that produce a differentiated product using a homogeneous intermediate good as input. The production function of a wholesale good h is given by:

$$y_t\left(h\right) = x_t\left(h\right), \tag{11}$$

where $x_t\left(h\right)$ is the input of intermediate goods demanded by the wholesale firm h, purchased at price φ_t. $y_t\left(h\right)$ represents the output of firm h. These wholesale firms act in a monopolistically competitive market and set their price $p_t(h)$ facing quadratic adjustment costs à la Rotemberg (1982). Since these firms are owned by the firm owners, the stream of profits $\Pi^W_{t+j}(i)$ is discounted by pricing kernel $M^F_{t,t+j}$. The optimisation problem of these firms is given by:

$$\max_{p_t(h)} E_t \sum_{j=0}^{\infty} M^F_{t,t+j} \Pi^W_{t+j}(h), \tag{12}$$

$$\Pi^W_t(h) = \left(\frac{p_t(h)}{p_t}\right)^{1-\theta} y_t - \left(1-\tau^W\right)\varphi_t \left(\frac{p_t(h)}{p_t}\right)^{-\theta} y_t - \frac{\psi}{2}\left(\frac{p_t(h)}{p_{t-1}(h)} - 1\right)^2 y_t, \tag{13}$$

where Equations (12) and (13) represent the stream of lifetime profits, φ_t is the price of intermediate goods relative to the final good's price, and τ^W is a production subsidy. In a symmetric equilibrium, the maximisation problem delivers the following New Keynesian Phillips curve:

$$\psi\left(1+\pi_t\right)\pi_t = \psi E_t M^F_{t+1}\left(1+\pi_{t+1}\right)\pi_{t+1}\frac{y_{t+1}}{y_t} + 1 - \theta + \theta\left(1-\tau^W\right)\varphi_t. \tag{14}$$

The profits of the wholesale firm, which are returned to the firm owners in the form of dividends, are given by:

$$\Pi^W_t = \left(1 - \left(1-\tau^W\right)\varphi_t - \frac{\psi}{2}\pi_t^2\right) y_t. \tag{15}$$

2.5 The Labour Market

At the beginning of each period t, firms post v_t vacancies and u_t unemployed workers look for a job. The matching technology takes the form of a Cobb-Douglas function:

$$m_t = \mu u_t^{\gamma} v_t^{1-\gamma}, \tag{16}$$

where m_t represents the number of successful matches, $\gamma \in (0,1)$ and $\mu > 0$ scales the matching efficiency. The job-filling rate, i.e., the probability that a vacancy is matched with a worker searching a job, is defined as:

$$\lambda_t = \frac{m_t}{v_t}. \tag{17}$$

The job-finding rate, i.e., the probability that an unemployed searching for a job is matched with an open vacancy, is given by:

$$f_t = \frac{m_t}{u_t}. \tag{18}$$

At the beginning of every period, there are n_{t-1} workers and a fraction ρ are laid off. Thus, the number of workers who keep their jobs is $(1-\rho)\, n_{t-1}$. At the same time, m_t new matches are formed. Assuming that new hires start working immediately when they are hired, aggregate employment evolves according to the following law of motion:

$$n_t = (1-\rho)\, n_{t-1} + m_t, \tag{19}$$

while the number of unemployed workers seeking a job is given by:

$$u_t = 1 - (1-\rho)\, n_{t-1}. \tag{20}$$

2.6 Intermediate Goods Firms

If an intermediate-good firm can successfully hire a worker, it produces one unit ($x_t = 1$) of its good with its only employee. If a firm finds a match, it obtains a flow profit in the current period after paying the worker. In the next period, if the match survives (with probability $1-\rho$), the firm continues. If the match breaks down (with probability ρ), the firm posts a new job vacancy at a fixed cost κ with the value J_t^v. The value of a firm with a match (denoted by J_t^F) is therefore given by the Bellman equation:

$$J_t^F = \left(1-\tau^I\right)(\varphi_t - w_t + T) + E_t M_{t,t+1}^F \left((1-\rho)\, J_{t+1}^F + \rho J_{t+1}^v\right), \tag{21}$$

where $\tau^I \in [0,1]$ is a corporate tax rate and T a wage subsidy. If the firm posts a new vacancy in period t, it costs κ units of final goods. The vacancy can be filled with probability λ_t, in which case the firm obtains the value of the match. Otherwise, the vacancy remains unfilled and the firm goes into the next period with the value J_{t+1}. Thus, the value of an open vacancy is given by:

$$J_t^v = -\kappa + \lambda_t J_t^F + (1-\lambda_t)\, E_t M_{t,t+1}^F J_{t+1}^v. \tag{22}$$

Free entry implies that $J_t^v = 0$, so that:

$$\frac{\kappa}{\lambda_t} = J_t^F. \tag{23}$$

This relation describes the optimal job creation decisions. The benefit of creating a new job is the match value J_t^F. The expected cost of creating a new job is the flow cost of posting a vacancy κ multiplied by the expected duration of an unfilled vacancy $1/\lambda_t$. Finally, the aggregate period profits of intermediate-goods firms are given by:

$$\Pi_t^I = n_t \left(1 - \tau^I\right)(\varphi_t - w_t + T) - \kappa v_t. \tag{24}$$

2.7 Workers' Value Function

If a worker is employed, he obtains wage income w_t. At time $t+1$, the worker is laid off with probability ρ and may find a new job with probability f_{t+1}. A separated worker may fail to find a new match in period $t+1$, thereby entering the unemployment pool, with probability $s_{t+1} = \rho(1 - f_{t+1})$. The worker continues to be employed with probability $1 - s_{t+1}$. The value of an employed worker, V_t^e, writes as:

$$V_t^e = \log w_t + \beta E_t \left((1 - s_{t+1}) V_{t+1}^e + s_{t+1} V_{t+1}^u\right), \tag{25}$$

where V_t^u denotes the value of an unemployed worker. They obtain the home-production income δ_t and, in period $t+1$, they have the chance of finding a new job with probability f_{t+1}. Thus, the value of an unemployed worker satisfies the Bellman equation:

$$V_t^u = \log \delta_t + \beta E_t \left(f_{t+1} V_{t+1}^e + (1 - f_{t+1}) V_{t+1}^u\right). \tag{26}$$

2.8 The Nash Bargaining Wage

Firms and workers bargain over wages. If we define $S_t^W \equiv V_t^e - V_t^u$, the Nash bargaining problem writes as:

$$w_t^N = \underset{w_t}{\operatorname{argmax}} \left(S_t^W\right)^{1-\alpha} \left(J_t^F\right)^{\alpha}, \tag{27}$$

where $\alpha \in (0,1)$. The first-order condition is then given by:

$$(1 - \alpha) J_t^F = \alpha S_t^W w_t^N. \tag{28}$$

2.9 Wage Rigidity

In practice, however, the equilibrium real wage may differ significantly from the Nash bargaining solution. For this reason, to generate empirically reasonable volatilities of vacancies and unemployment, the literature assumes some form of real wage rigidity (Hall, 2005). We assume, therefore, that the actual wage is obtained by weighing the Nash wage w_t^N against the (constrained-efficient) steady-state value w:

$$w_t = w^{\phi} {w_t^N}^{1-\phi}, \tag{29}$$

where the parameter $\phi \in (0,1)$ represents the degree of wage inertia.

2.10 Government

Monetary Policy In our baseline specification, we assume that the monetary policy authority sets the nominal interest rate optimally in response to aggregate shocks. In other words, it maximises the following social welfare function subject to all equilibrium conditions and the ZLB constraint (i.e., $i_t \geq 0$):

$$W_t = E_0 \sum_{t=0}^{\infty} \beta^t U_t, \tag{30}$$

where U_t is the sum of instantaneous utilities of all households: employed, unemployed, and firm owners. In Section 2.12, we explicitly define W_t and U_t, while we set up the problem and derive the first-order conditions in Appendix B. To highlight the benefits of the optimal policy, we also consider the implications of a simple strict-inflation-targeting rule:[5]

$$\pi_t = 0 \ \text{ s.t. } \ i_t \geq 0. \tag{31}$$

Fiscal Policy In order to achieve a constrained-efficient allocation in steady state, we assume that the fiscal authority sets constant taxes and subsidies τ^w, τ^I, and T, which are rebated lump-sum to firm owners:

$$\tau_t = \tau^I n_t \left(\varphi_t - w_t\right) - \tau^W \varphi_t y_t - n_t \left(1 - \tau^I\right) T. \tag{32}$$

The first term of the expression represents a corporate tax, the second is a production subsidy, and the last is a wage subsidy. In Section 2.13, we report the values of taxes and subsidies associated with the constrained-efficient allocation.

[5] In the three-period model of Section 3 and the infinite-horizon model in Section 4, the allocation under the simple strict-inflation-targeting rule is the same to that under the optimal discretionary policy.

2.11 Market Clearing and Equilibrium

The model is closed by the following market-clearing conditions for bonds, final goods, and wholesale goods:

$$\int_{[0,1]} a_{i,t} di + a_t^F = 0, \tag{33}$$

$$\int_{[0,1]} c_{i,t} di + c_t^F + \kappa v_t = y_t + (1 - n_t)\,\delta_t - \frac{\psi}{2} {\pi_t}^2 y_t + \varpi, \tag{34}$$

$$y_t = n_t. \tag{35}$$

For the sake of conciseness, we report the full set of equilibrium conditions in Appendix A. It bears noting that, as in Ravn and Sterk (2017, 2020) and Challe (2020), the model does not give rise to a distribution of wealth across workers. The reason for this is that with a zero debt limit (Equations (3) and (7)), no one is issuing the assets that the precautionary savers would be willing to purchase for self-insurance. In other words, the precautionary-savings motive of employed workers puts downward pressure on the real interest rate. Given the low level of the real rate, unemployed workers and firm owners would prefer to borrow and face, therefore, a binding debt limit. For this reason, the equilibrium supply of assets ends up being zero, and all households just consume their current income. Thus, employed workers consume their wage, $c_{e,t} = w_t$, and their Euler equation holds with equality:

$$E_t M_{t,t+1}^e \frac{(1 + i_t)\, z_t}{1 + \pi_{t+1}} = 1, \tag{36}$$

where their stochastic discount factor writes as:

$$M_{t,t+1}^e = \beta \frac{(1 - s_{t+1})\, u'(w_{t+1}) + s_{t+1} u'(\delta_{t+1})}{u'(w_t)}. \tag{37}$$

The two conditions above determine the saving/consumption choice of the employed households. In particular, two forces drive this decision: (i) changes in w_t make agents want to save more when wages are temporarily high (aversion to intertemporal substitutions); (ii) in times of high unemployment risk, i.e., high job-loss probability s_t, employed households wish to self-insure against the possibility of becoming unemployed (precautionary savings).

Unemployed households consume their home-production income, $c_{u,t} = \delta_t$. Since $\delta_t < w_t$, they are relatively poor at time t and would like to borrow in expectation of a higher income at time $t + 1$. As a result, they face a binding debt limit, and their Euler equation holds with strict inequality:

$$E_t M^u_{t,t+1} \frac{(1+i_t)\, z_t}{1+\pi_{t+1}} < 1, \tag{38}$$

where the stochastic discount factor is given by:

$$M^u_{t,t+1} = \beta \frac{(1-f_{t+1})\, u'(\delta_{t+1}) + f_{t+1} u'(w_{t+1})}{u'(\delta_t)}. \tag{39}$$

Also firm owners do not have any precautionary-savings motive, as they do not face any unemployment risk. For this reason, they face a binding debt limit and their Euler equation holds with strict inequality:

$$E_t M^F_{t,t+1} \frac{(1+i_t)\, z_t}{1+\pi_{t+1}} < 1, \tag{40}$$

where the firm owners' stochastic discount factor is equal to:

$$M^F_{t,t+1} = \beta \frac{u'\left(c^F_{t+1}\right)}{u'\left(c^F_t\right)}. \tag{41}$$

The consumption of a firm owner can be derived by combining Equations (6), (15), (24), (32), and (34):

$$c^F_t = y_t - w_t n_t - \frac{\psi}{2}\pi_t^2 y_t - \kappa v_t + \varpi. \tag{42}$$

2.12 Social Welfare

The central bank's objective following an optimal policy is to maximise social welfare, given by the sum of value functions of all agents in the economy. In particular, assuming the same welfare weight across working households, we have that:

$$W_t = n_t V^e_t + (1-n_t)\, V^u_t + \Lambda V^F_t = U_t + \beta E_t W_{t+1}, \tag{43}$$

where $\Lambda = \frac{c^F}{w}$ is the relative welfare weight on firm owners, V^e_t and V^u_t are defined by Equations (25) and (26), and the value of firm owners V^F_t is given by:

$$V^F_t = \log c^F_t + \beta V^F_{t+1}. \tag{44}$$

U_t in Equation (43) is the sum of instantaneous utilities:

$$
\begin{aligned}
U_t &= n_t \log c_{e,t} + (1-n_t) \log c_{u,t} + \Lambda \log c_t^F \\
&= n_t \log w_t + (1-n_t) \log \delta_t + \Lambda \log \left(y_t - w_t n_t - \frac{\psi}{2} \pi_t^2 y_t - \kappa v_t + \varpi \right), \quad (45)
\end{aligned}
$$

where the last equality is a result of households consuming all their income each period.

2.13 Constrained-Efficient Steady State

The economy features three distortions: monopolistic competition in the wholesale sector, congestion externalities in the labour market, and imperfect insurance against unemployment risk. To simplify the analysis about optimal policy, we assume a constrained-efficient steady state. To this end, we consider the appropriate values of steady-state inflation (π) and the tax instruments (τ^W, τ^I, T) that eliminate the various distortions in steady state:[6]

$$
\pi = 0, \quad \tau^W = \frac{1}{\theta}, \quad T = \frac{u(w^*) - u(\delta^*)}{u'(w^*)}, \quad \tau^I = 1 - \frac{(1-\gamma)(1-\beta(1-\rho))}{1-\beta(1-\rho)(1-\gamma f^*)}, \quad (46)
$$

where f^* is given by:

$$
f^* = \left(\frac{(1-\tau^I)\, \mu^{\frac{1}{1-\gamma}}}{\kappa (1-\beta(1-\rho))} \left(1 - w^* + \frac{u(w^*) - u(\delta^*)}{u'(w^*)} \right) \right)^{\frac{1-\gamma}{\gamma}}. \quad (47)
$$

The production subsidy τ^W ensures that the price markup is 1 in steady state, thereby eliminating monopolistic competition. The hiring subsidy T corrects the lack of unemployment insurance, whereas the corporate tax τ^I corrects the congestion externalities in the labour market. Finally, in order to ensure the decentralised wage to be constrained-efficient in steady state, we also need to assume:

$$
\alpha = \left(1 + \frac{S^W w^*}{J^F} \right)^{-1}. \quad (48)
$$

2.14 Solution and Calibration

The three-period version of the model considered in Section 3 is solved with a perfect foresight algorithm using the Levenberg-Marquardt mixed complementarity problem solver (Adjemian et al., 2011). For the numerical analysis in Section 4, instead, the model is solved via a piecewise linear approximation using the approach suggested by Guerrieri and Iacoviello (2015), in order to consider the effects of the occasionally

[6]For a detailed derivation and discussion of the constrained-efficient allocation, please refer to Section 3 in Challe (2020).

Table 1: Calibration

Parameters		I.I.	P.I.	Targets/Sources		I.I.	P.I.
Sym.	Description	Value	Value	Sym.	Description	Value	Value
β	Discount factor	0.989	0.995	$4i$	Annual interest rate	2%	-
θ	Monopoly power	6.000	-	$\frac{1}{\theta-1}$	Markup rate	20%	-
ψ	Price stickiness	1088.6	1119.2	-	Calvo stickiness	0.84	-
γ	Elasticity of matching	2/3	-	-	Shimer (2005)	-	-
κ	Vacancy cost	0.044	0.040	κ/w	% of wage	4.5%	-
w	Real wage	0.979	0.888	f	Job-finding rate	80%	-
μ	Matching efficiency	0.765	-	λ	Vacancy-filling rate	70%	-
ρ	Job-destruction rate	0.250	-	s	Job-loss rate	5%	-
δ	Workers' home prod.	0.882	0.888	$1-\frac{\delta}{w}$	Cons. loss upon unemp.	10%	0%
ϖ	Firm owners' home prod.	0.484	0.351	$\frac{wn}{c^F+wn}$	Labour share	65%	-
ϕ	Wage inertia	0.900	-	P.I.: 10% output drop & 2%p inflation drop			
ρ_z	RP shock persistence	0.925	-	(Same)			
σ_z	RP shock volatility	0.017	-	(Same)			

Note: The tables presents the calibrated value of our baseline model with imperfect insurance (I.I.) and a version of the model with perfect insurance (P.I.).

binding ZLB. In our numerical exercises, we compare the baseline model with imperfect unemployment insurance (I.I.), i.e., $w_t > \delta_t$, to a version of the model with perfect-insurance (P.I.), i.e., $w_t = \delta_t$. It is important to note that in the I.I. model we assume that the home-production income δ_t varies such that δ_t/w_t is constant. This assumption implies that the income risk faced by employed households only depends on variations in the job-loss rate s_{t+1} and not on changes in δ_t/w_t.

Table 1 lists the model parameters and the empirical moments we aim to target. It is important to note that the calibration of some parameters differs between the I.I. and P.I. models to match the steady-state target values. The discount factor β is set to 0.989 (I.I.) or 0.995 (P.I.), targeting an average annualised nominal interest rate of 2%. The elasticity of substitution between intermediate goods θ is set to 6, which is standard in the literature and implies an average markup rate of 20 per cent. We set the Rotemberg price stickiness parameter to 1088.58 (1119.18), which, in a Calvo setting, would imply firms do not readjust their price with a probability of 0.84, consistent with Nakata et al. (2019). Regarding the labour market parameters, the γ parameter in the matching function is equal to 2/3, in line with Shimer (2005). Following Challe (2020), the flow cost of a vacancy κ is set to 0.044 (0.04) to match an average vacancy cost-to-wage ratio of 4.5 per cent. The steady-state real wage is 0.979 (0.888) to match an average job-finding rate of 80%. The average matching efficiency μ is 0.765, targeting a vacancy-filling rate of 70 per cent. The job-separation rate ρ is equal to 0.25, implying a 5 per-cent job-loss rate. The average home-production income δ is set to 0.882 (0.888), such that the average proportional consumption loss upon unemployment $1 - \frac{\delta}{w} = 0.1$. In the three-period model, we consider two additional counterfactual scenarios where $1 - \frac{\delta}{w}$ is set equal to 0.2 or 0.3. The steady-state level of the firm owners' home-production income is set to 0.484 (0.351) to match a 65% labour share. The real wage rigidity parameter is set to $\phi = 0.9$.

Finally, we calibrate the exogenous risk-premium shock process to $\rho_z = 0.925$ and $\sigma_z = 0.017$. This calibration induces a 10 per-cent drop in output, a 2 percentage-point fall in inflation, and the ZLB constraint to bind for 16 quarters when the central bank conducts a strict-inflation-targeting rule in the P.I. version of the model.

3 Three-Period Model

Before discussing our main numerical results, based on the infinite-horizon model, we consider first a simple three-period version of the model to highlight the key mechanism behind our results.[7] In particular, for this exercise, we assume agents have perfect foresight, and we consider the impact of a two per cent increase in the period-0 risk premium ($z_0 = 1.02$). In the following periods, the risk premium returns to its steady-state value ($z_1 = z_2 = 1.0$). The rise in the risk premium leads the nominal interest rate to hit the ZLB on impact, i.e., $i_0 = 0$. We then compare how the responses depend on the degree of unemployment insurance under strict inflation targeting and the optimal monetary policy. We consider four different possible levels of the ratio δ_t/w_t, such that a smaller value implies lower unemployment insurance.

Figure 2 displays the responses of the model variables to the rise in the risk premium when the central bank follows a strict inflation targeting rule. The increase in the risk premium causes employed workers to reduce their consumption via their Euler equation. Given that prices are sticky, firms reduce their production y_0 and labour demand n_0 to adjust to the falling demand, whereas inflation π_0 declines more sluggishly. The fall in the firm's profits causes a decline in the firm owners' consumption c_0^F. Furthermore, the fall in demand causes a tightening in labour market conditions, reducing vacancies v_0, the job-finding rate f_0, and wages, and increasing the job-loss rate s_0. Since the nominal rate is at zero, the central bank cannot reduce it to respond to the fall inflation. Hence, the real rate rises and the fall in demand is larger than away from the ZLB.

When there is perfect risk-sharing between working households ($\delta_t/w_t = 1$), a rise in the job-loss rate does not affect their saving behaviour. In the imperfect-insurance case ($\delta_t/w_t < 1$), instead, a tightening in labour-market conditions increases the stochastic discount factor of employed workers, who increase their

[7] In an infinite-horizon setting with a strict-inflation-targeting policy, the response of inflation becomes rapidly very large as we decrease the degree of unemployment insurance.

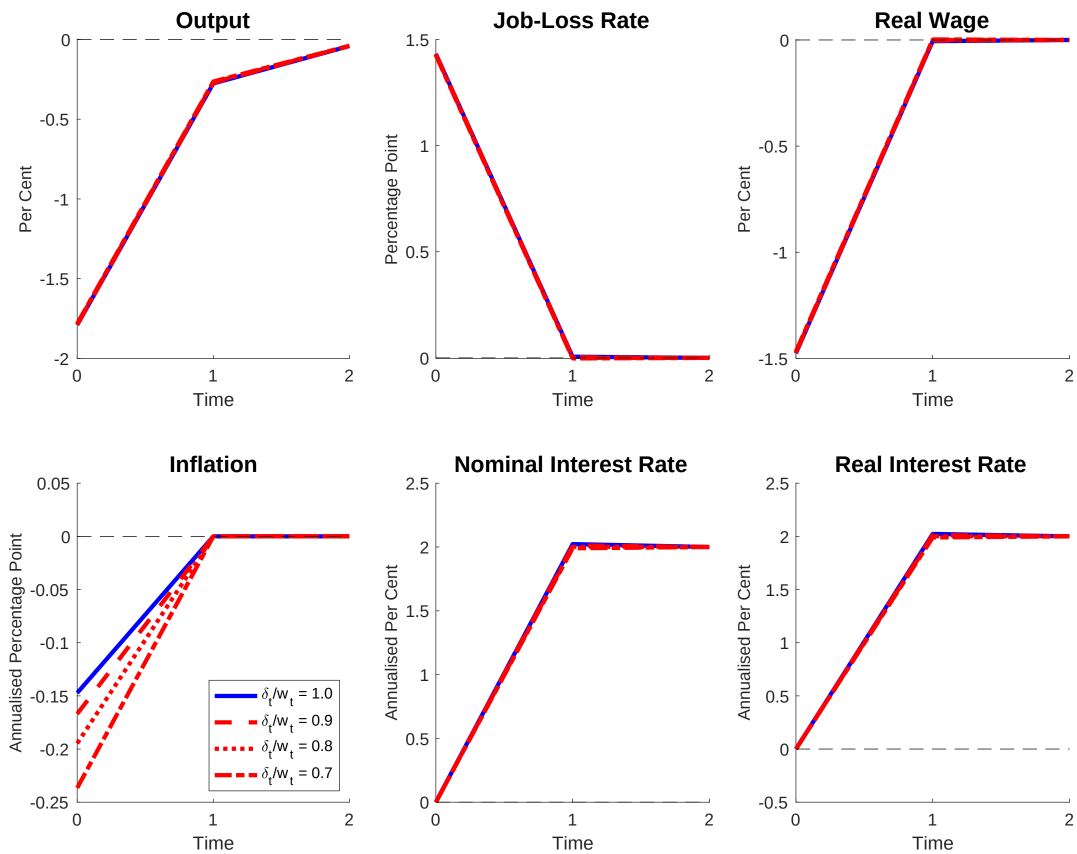


Figure 2: Strict Inflation Targeting in a Three-Period Model

Note: The figure displays the responses to an adverse risk-premium shock that leads the ZLB constraint to bind in period 0 under strict inflation targeting. Each line represents a different degree of unemployment-risk-sharing.

savings for precautionary reasons. Precautionary savings further amplify the initial decline in inflation. Since in period 1 the ZLB constraint does not bind anymore, the monetary policy authority can adjust the interest rate to bring inflation back to zero ($\pi_1 = 0$). As a result, the real interest rate in period 0 is the same both under perfect or imperfect unemployment insurance ($r_0 \approx i_0 - \pi_1 = 0$). Similarly, the decline in output, employment and real wages are unaffected by the degree of unemployment insurance.

Under the optimal monetary policy, as displayed in Figure 3, the central bank can commit to a specific path for the nominal interest rate. In particular, the central bank keeps the rate at zero for one additional period. The lower interest rate (compared to the strict-inflation-targeting policy) has a positive effect on y_1 and π_1. The increase in inflation expectations reduces the period-0 real interest rate r_0, which attenuates the decline in real activity y_0 and inflation π_0 (standard forward guidance channel). In the presence of incomplete markets, future improvements in labour market conditions further strengthen this mechanism.

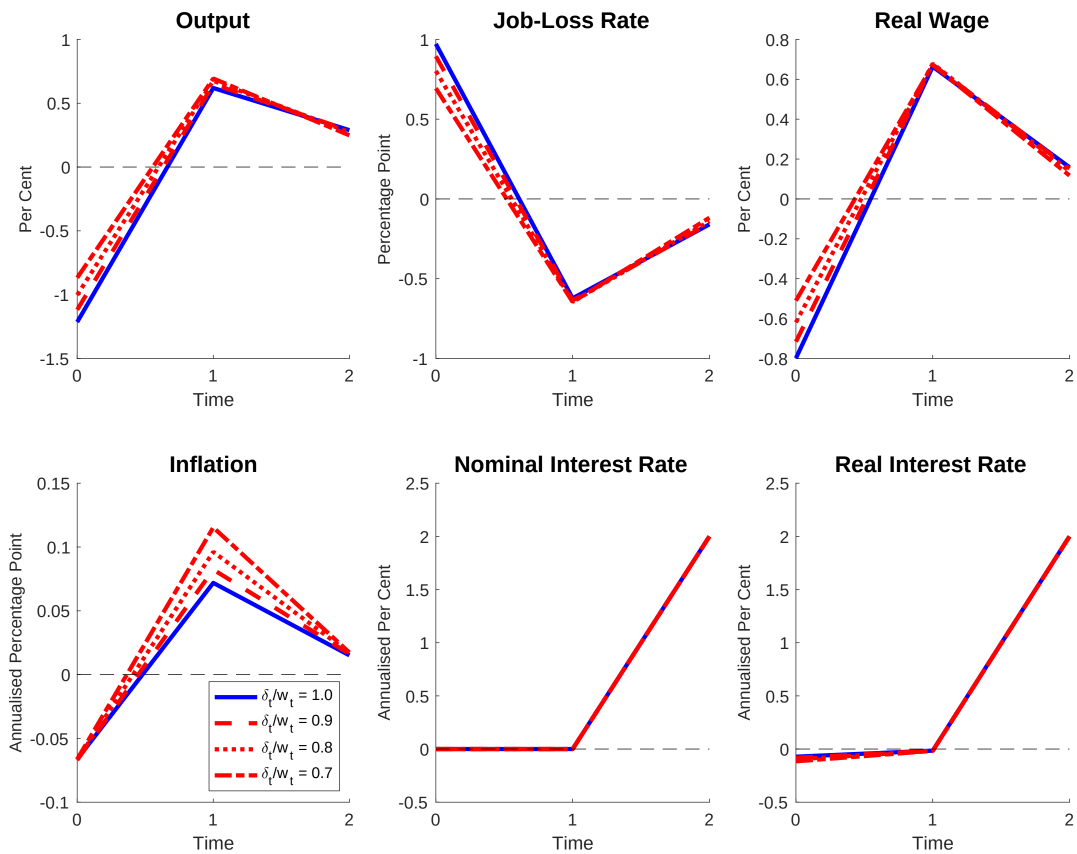


Figure 3: Optimal Monetary Policy in a Three-Period Model

Note: The figure displays the responses to an adverse risk-premium shock that leads the ZLB constraint to bind in period 0 under strict inflation targeting. Each line represents a different degree of unemployment-risk-sharing.

In other words, $i_1 = 0$ has a positive effect on the period-1 job finding rate f_1 and a negative one on the job-loss rate s_1. The latter decreases the stochastic discount factor of employed workers, hence mitigating their period-0 precautionary savings and fall in consumption $c_{e,0}$. As a result of the optimal policy, we see that the smaller the degree of unemployment-risk sharing, i.e., the smaller δ_t/w_t, the more muted is the response of output, employment, and the real wage to a negative demand shock.

4 Infinite-Horizon Model

In this section, we analyse the impact of an adverse risk-premium shock that causes the ZLB constraint to bind for 16 quarters when the central bank follows a strict inflation targeting rule. In line with the previous section, the shock causes a decline in output, employment, wages and inflation. As displayed in Figure 4, under a strict-inflation-targeting rule, the central bank cannot react to the fall in demand, which causes a significant decline in inflation expectations and an increase in the real interest rate. The latter further

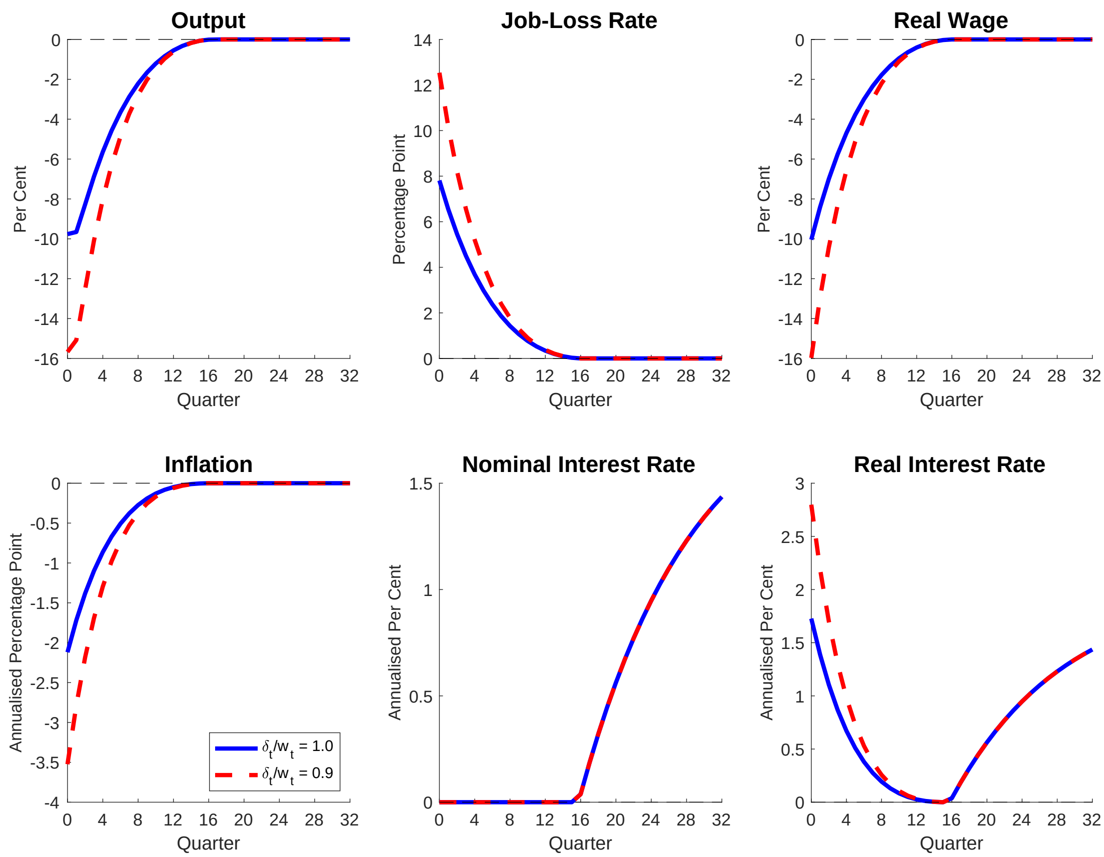


Figure 4: Strict Inflation Targeting in the Infinite-Horizon Model

Note: The figure displays the responses to an adverse risk-premium shock that leads the ZLB constraint to bind for 16 quarters under strict inflation targeting. Each line represents a different degree of unemployment-risk-sharing.

amplifies the initial drop in real activity and inflation. In the imperfect-insurance case (red-dashed line), a worsening in labour market conditions induces employed workers to increase their savings for precautionary reasons, which causes inflation to fall even more substantially on impact. Because of the binding ZLB constraint on the policy rate, inflation expectations decline more severely under imperfect insurance, causing a larger increase in the real rate. Consequently, the fall in output and employment is six percentage points larger than under perfect unemployment-risk sharing.

When the central bank is able to commit to an optimal interest rate path, as shown in Figure 5, the effects of an adverse risk-premium shock are significantly milder than with a strict inflation targeting policy rule. By keeping the interest rate at zero for nine quarters longer, the central bank boosts inflation expectations, reduces the real rate and substantially mitigates the drop in output. In the presence of imperfect insurance, the optimal path of the policy rate is nearly unchanged compared to the perfect-insurance case.[8] Because

[8] For lower δ_t/w_t, the central bank tends to lift off the interest rate earlier.

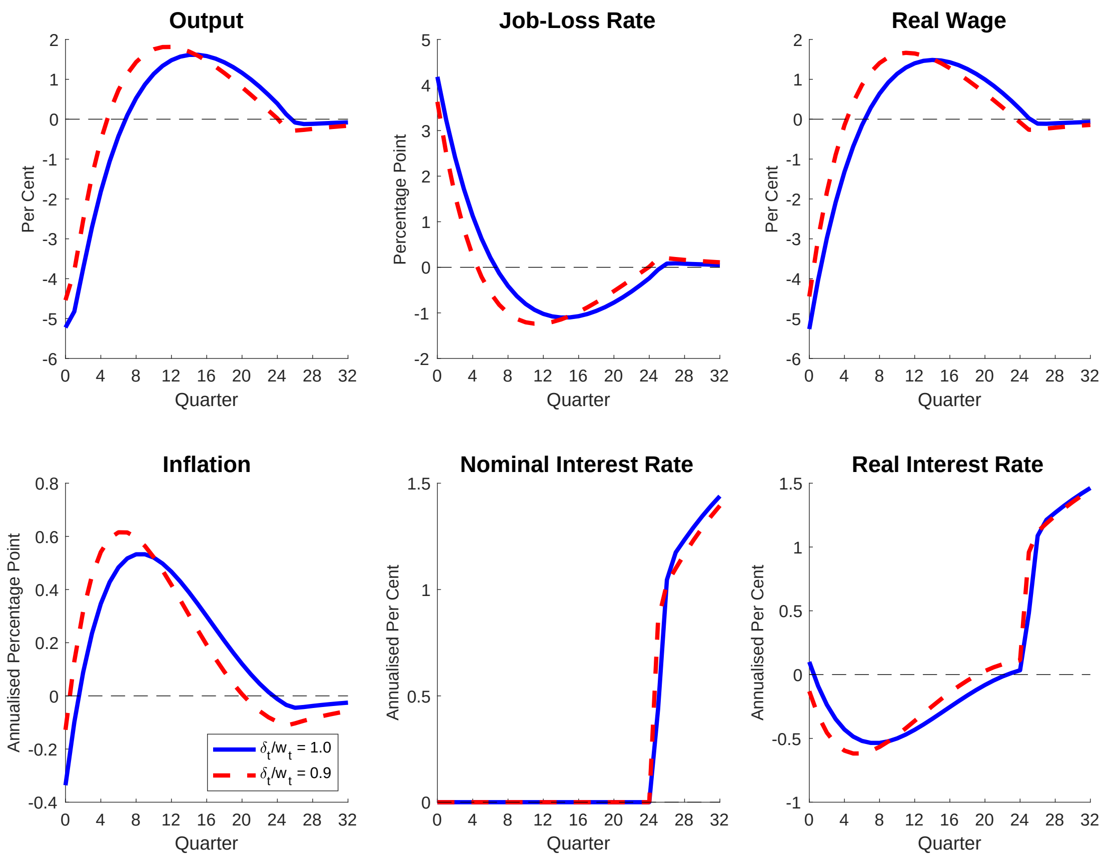


Figure 5: Optimal Monetary Policy in the Infinite-Horizon Model

Note: The figure displays the responses to an adverse risk-premium shock that leads the ZLB constraint to bind for 16 quarters under strict inflation targeting. Each line represents a different degree of unemployment-risk-sharing.

the nominal rate is kept low for an extended period, households expect labour market conditions to improve, which attenuates the employed workers' precautionary-savings motive. Inflation declines less and overshoots more than in the case of perfect unemployment-risk sharing. As a result, the decline in real activity, employment, and real wages is more muted than under perfect unemployment insurance. In other words, under the optimal policy, the central bank is able to neutralise the deflationary spiral caused by the ZLB and the precautionary-savings behaviour. Lower market incompleteness (e.g., via unemployment insurance policies), provided an optimal path of monetary policy, would not be beneficial in terms of output stabilisation.

5 Alternative Policy Rules

In this section, we consider alternative policy rules, which can significantly attenuate the negative impact of demand shocks, both under perfect and imperfect unemployment insurance, and deliver results close to those found under the optimal policy. In particular, we consider an inertial Taylor rule, a price-level-targeting

(PLT) rule, and an average-inflation-targeting (AIT) rule. Unlike the optimal policy case, under these simple monetary policy rules, market incompleteness amplifies output contractions in response to negative demand shocks and unemployment insurance policies are, therefore, useful tools to stabilise output in a liquidity trap.

5.1 Shadow Rate Smoothing

The first alternative policy we consider includes the lagged shadow policy rate into a standard truncated Taylor-type rule:

$$i_t = \max\left\{i_t^{\star}, 0\right\}, \tag{49}$$

$$i_t^{\star} = \rho_i i_{t-1}^{\star} + (1-\rho_i)\left(i + \phi_\pi \pi_t\right). \tag{50}$$

While the actual nominal rate, i_t, is bounded from below, the shadow (or notional) rate $i_t^{\star}$ is not. The shadow rate represents the theoretical rate that would prevail in the absence of a ZLB constraint. The central bank sets its shadow rate $i_t^{\star}$ in response to deviations of the inflation rate from its steady-state value. Moreover, we assume that the monetary authority has a preference for smoothing the shadow rate, which is given by the autoregressive component in Equation (50). The parameter ρ_i controls the degree of policy inertia, while ϕ_π indicates the responsiveness of the shadow rate to inflation. It bears noting that the strict-inflation-targeting rule considered above implies the parameter $\phi_\pi \to +\infty$ and $\rho_i = 0$. In this section, we assume that $\rho_i = 0.9$, which is broadly in line with the literature (see e.g., Hills and Nakata, 2018 and Billi and Galí, 2020), and set ϕ_π to a large value (10^5).

Figure 6 displays the responses of our model variables under the inertial policy. First, comparing these results with those in Figure 4, one can see how the inertial policy significantly mitigates the drop in output, employment, wages, and inflation. Second, in line with the optimal policy case, the inertial policy is more effective at reducing the decline in real activity under imperfect insurance. In particular, with perfect unemployment-risk sharing, output falls by eight per cent under an inertial policy, against a ten per-cent drop in the absence of inertia. When there is imperfect unemployment insurance and employed workers feature a precautionary-savings motive, the decline in output is about six percentage points smaller under inertial policy compared to the standard strict-inflation-targeting rule.

Intuitively, in the absence of inertia, a fall in the shadow rate does not have any implications about the future path of the actual policy rate. Therefore, as displayed in Figure 4, the policy rate lifts off after 16 quarters,

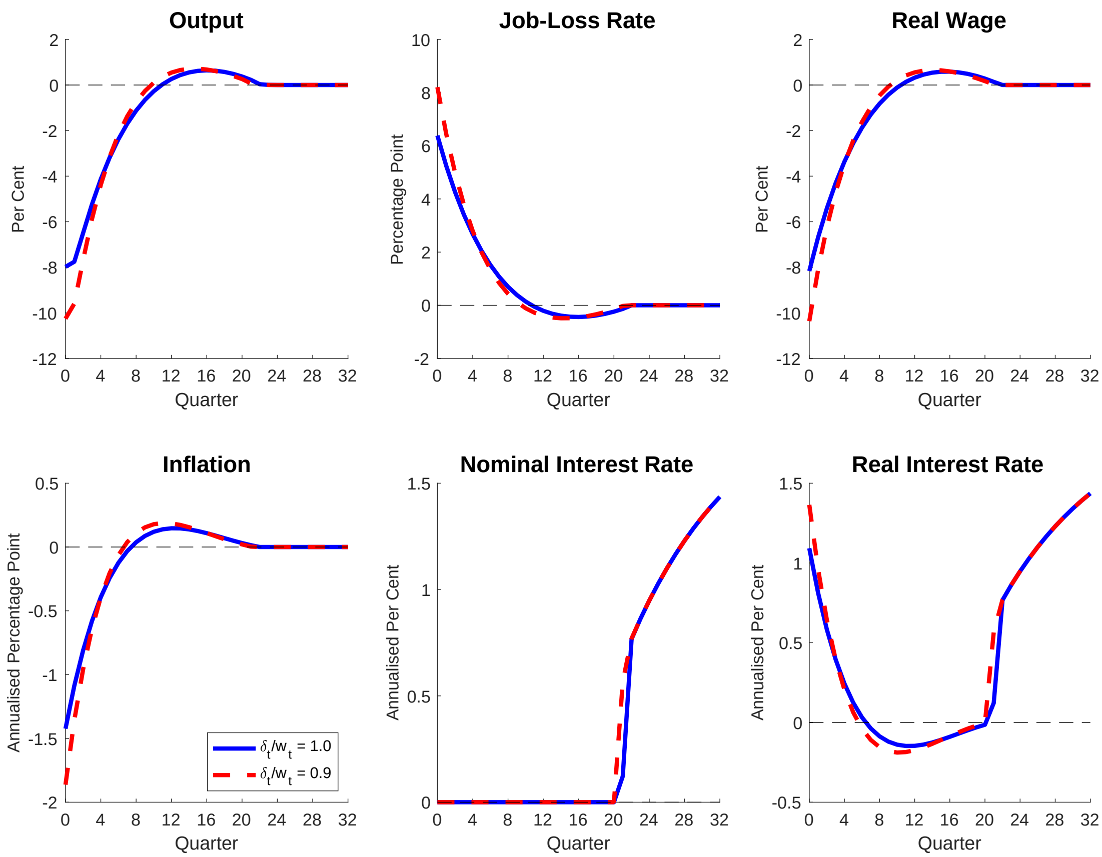


Figure 6: Inertial Policy Rule

Note: The figure displays the responses to an adverse risk-premium shock that leads the ZLB constraint to bind for 16 quarters under strict inflation targeting. Each line represents a different degree of unemployment-risk-sharing. The inertial policy rule assumes $\rho_i = 0.9$.

as soon as the ZLB constraint is not binding anymore. With the inertial policy instead, a reduction in the shadow rate implies that the actual policy rate will remain lower for longer. Indeed, as shown in Figure 6, the nominal interest rate is kept at zero for 21 quarters, as long as the shadow rate is negative. By keeping the nominal rate lower for longer, the central bank is boosting expectations about future inflation, output, and employment. The rise in inflation expectations leads to a smaller initial increase in the real rate, which undershoots after a few quarters. As a result, the declines in output, employment, and real wage are significantly more muted. However, unlike the optimal policy case, the inertial monetary policy does not fully neutralise the deflationary spiral induced by market incompleteness. Hence, incomplete markets amplify output contractions in response to negative demand shocks under this monetary policy. Finally, it bears noting how the optimal policy discussed above implies an even larger (and empirically implausible) degree of policy inertia.

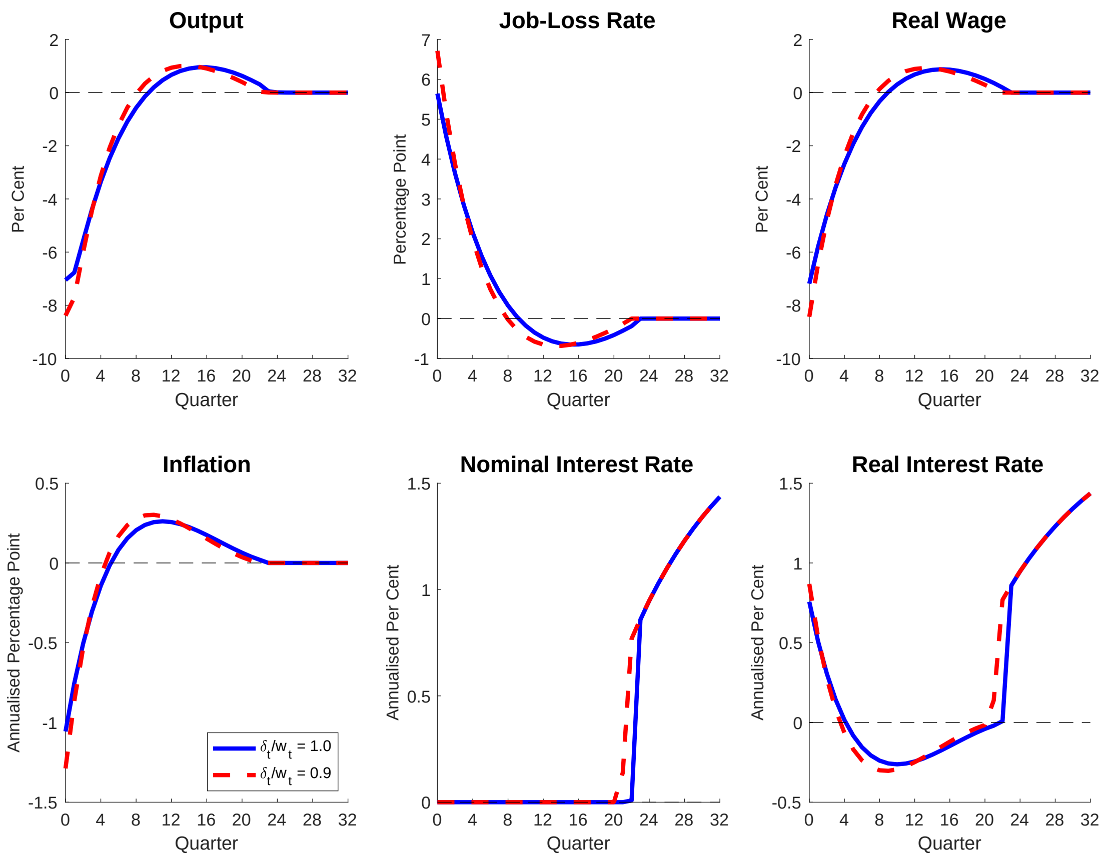


Figure 7: Price Level Targeting

Note: The figure displays the responses to an adverse risk-premium shock that leads the ZLB constraint to bind for 16 quarters under strict inflation targeting. Each line represents a different degree of unemployment-risk-sharing.

5.2 Price Level Targeting

The second alternative policy specification we study is a PLT rule, defined by:

$$\log p_t = 0, \tag{51}$$

where:

$$\frac{p_t}{p_{t-1}} = \pi_t + 1. \tag{52}$$

The steady-state price level can be normalised such that $\log p = 0$.

Figure 7 displays the results under this policy specification. It bears noting that, similarly as with the inertial policy rule, PLT implies history dependence in the policy rate. As a consequence, the responses follow a similar pattern as described above. Following a negative demand shock, the nominal policy rate is

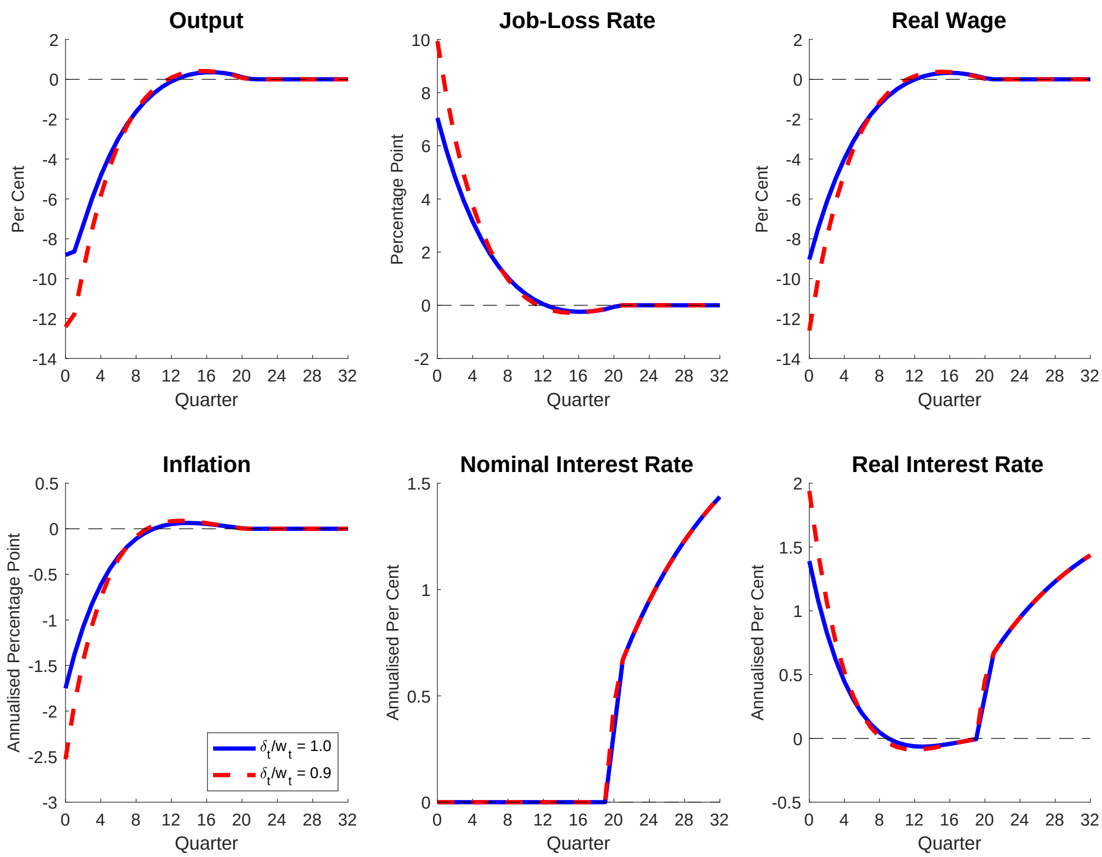


Figure 8: Average Inflation Targeting

Note: The figure displays the responses to an adverse risk-premium shock that leads the ZLB constraint to bind for 16 quarters under strict inflation targeting. Each line represents a different degree of unemployment-risk-sharing. The averaging window parameter is set to $\omega = 0.2$.

kept at zero for longer than implied by contemporaneous macroeconomic variables. As a result, inflation and output overshoot after the initial decline. Also in this case, the gap between the economies with complete and incomplete markets narrows. However, the initial fall in output and inflation remains stronger under incomplete markets.

5.3 Average Inflation Targeting

The last policy specification we consider is an AIT rule, defined by:

$$\hat{\pi}_t = 0, \tag{53}$$

where:

$$\hat{\pi}_t = \omega \pi_t + (1 - \omega)\, \hat{\pi}_{t-1}, \tag{54}$$

with $\omega \in (0, 1)$. In other words, the central bank aims to stabilise an exponential moving average inflation rate $\hat{\pi}_t$, as defined in equation (54). When $\omega \to 0$, the rule becomes a PLT rule. When $\omega \to 1$, we fall back in the SIT policy case. Following Budianto et al. (2020), we consider an inflation-averaging parameter equal to $\omega = 0.2$.

Figure 8 displays the results under the AIT policy. Following a negative demand shock, the nominal policy rate is kept at zero for longer than implied by contemporaneous macroeconomic variables. Since the policy represents an average between the PLT and SIT policies, the rate is kept at zero less than under PLT. As a result, the overshoots in inflation and output are less marked than under PLT. Also in this case, the gap between the economies with complete and incomplete markets is more narrow than under SIT.

To sum up, we find that all three alternative (and more realistic) policy specifications are effective at easing the deflationary spiral caused by market incompleteness. Nevertheless, unlike the optimal monetary policy, the deflationary spiral cannot be completely neutralised by these policies. Therefore, in practice, unemployment insurance policies are desirable to stabilise output at the ZLB.

6 Conclusion

In this paper, we study optimal monetary policy in response to adverse demand shocks when the short-term rate is at the ZLB and there is countercyclical uninsurable unemployment risk. Imperfect insurance gives rise to a precautionary-savings motive, which may significantly amplify the drop in inflation and inflation expectations, depending on the monetary policy response. Under a strict-inflation-targeting policy rule, the central bank is unable to respond to the fall in inflation, and, for this reason, the real rate rises. As a result, the decline in real activity is substantially larger than in the perfect-unemployment-insurance case.

The central bank's optimal response is to commit to keeping the interest rate at zero for an extended period after exiting the liquidity trap. The policy increases inflation expectations and reduces the real rate, sustaining current economic conditions both under complete and incomplete markets. The policy also has the additional benefit of improving the future economic outlook and expected labour market conditions, attenuating the precautionary-savings motive of households under imperfect unemployment insurance. As a result, we find that, in response to a negative demand shock, the contraction in real activity is milder under incomplete markets than under perfect risk sharing.

Finally, we consider the impact of alternative policy rules that introduce history dependence in the policy rate and could, therefore, operationalise the optimal policy prescriptions. In particular, we consider an inertial rule, including the lagged shadow policy rate, a price-level-targeting rule, and an average-inflation-targeting rule. We find that these simple (and more realistic) policies ease but not fully neutralise the deflationary spiral caused by the ZLB and the precautionary-savings behaviour. Therefore, we conclude that, in practice, unemployment insurance (UI) policies are desirable tools, alongside monetary policy, to stabilise output at the ZLB.

Our analysis has two important limitations. First, in order to concentrate on the role of countercyclical unemployment risk, the model relies on a zero-liquidity assumption, therefore abstracting from potential effects of monetary policy on the wealth distribution, which is an important transmission channel in standard HANK models. Second, correcting for the "Forward Guidance puzzle" may reduce the strength of optimal monetary policy. Despite these caveats, our results underscore that in the face of recessions in times of low interest rates, monetary policy can be an effective tool alongside UI policies in mitigating the negative consequences of heightened unemployment risk. Understanding the optimal mix of monetary and UI policies is an important open question, which should be further investigated in future research.

References

Acharya, Sushant and Keshav Dogra, "Understanding HANK: Insights from a PRANK," *Econometrica*, 2020, *88* (3), 1113–1158.

_ , Edouard Challe, and Keshav Dogra, "Optimal Monetary Policy According to HANK," 2020. Working Paper.

Adam, Klaus and Roberto M. Billi, "Discretionary Monetary Policy and the Zero Lower Bound on Nominal Interest Rates," *Journal of Monetary Economics*, 2007, *54* (3), 728–752.

Adjemian, Stéphane, Houtan Bastani, Michel Juillard, Fréderic Karamé, Junior Maih, Ferhat Mihoubi, George Perendia, Johannes Pfeifer, Marco Ratto, and Sébastien Villemot, "Dynare: Reference Manual Version 4," Dynare Working Papers 1, CEPREMAP 2011.

Bilbiie, Florin O., "Optimal Forward Guidance," *American Economic Journal: Macroeconomics*, 2019, *11* (4), 310–345.

Billi, Roberto M. and Jordi Galí, "Gains from Wage Flexibility and the Zero Lower Bound," *Oxford Bulletin of Economics and Statistics*, 2020, *Forthcoming.*

Bonciani, Dario and Joonseok Oh, "Monetary Policy Inertia and the Paradox of Flexibility," 2020. Bank of England Staff Working Paper No. 888.

_ and _ , "Revisiting the New Keynesian Policy Paradoxes under QE," 2020. Working Papar.

Braun, R. Anton and Tomoyuki Nakajima, "Uninsured Countercyclical Risk: An Aggregation Result and Application to Optimal Monetary Policy," *Journal of the European Economic Association*, 2012, *10* (6), 1450–1474.

Budianto, Flora, Taisuke Nakata, and Sebastian Schmidt, "Average inflation targeting and the interest rate lower bound," Working Paper Series 2394, European Central Bank April 2020.

Challe, Edouard, "Uninsured Unemployment Risk and Optimal Monetary Policy in a Zero-Liquidity Economy," *American Economic Journal: Macroeconomics*, 2020, *12* (2), 241–283.

_ , Julien Matheron, Xavier Ragot, and Juan F. Rubio-Ramirez, "Precautionary Saving and Aggregate Demand," *Quantitative Economics*, 2017, *8* (2), 435–478.

Christiano, Lawrence, Martin Eichenbaum, and Sergio Rebelo, "When Is the Government Spending Multiplier Large?," *Journal of Political Economy*, 2011, *119* (1), 78–121.

Den Haan, Wouter J., Pontus Rendahl, and Markus Riegler, "Unemployment (Fears) and Deflationary Spirals," *Journal of the European Economic Association*, 2018, *16* (5), 1281–1349.

Eggertsson, Gauti B. and Michael Woodford, "The Zero Bound on Interest Rates and Optimal Monetary Policy," *Brookings Papers on Economic Activity*, 2003, *2003* (1), 139–211.

Guerrieri, Luca and Matteo Iacoviello, "OccBin: A Toolkit for Solving Dynamic Models with Occasionally Binding Constraints Easily," *Journal of Monetary Economics*, 2015, *70*, 22–38.

Hall, Robert E., "Employment Fluctuations with Equilibrium Wage Stickiness," *American Economic Review*, 2005, *95* (1), 50–65.

Heathcote, Jonathan and Fabrizio Perri, "Wealth and Volatility," *Review of Economic Studies*, 2018, *85* (4), 2173–2213.

_ , _ , and Giovanni L. Violante, "Unequal we stand: An empirical analysis of economic inequality in the United States," *Review of Economic Dynamics*, 2010, *13* (1), 15–51.

Hills, Timothy S. and Taisuke Nakata, "Fiscal Multipliers at the Zero Lower Bound: The Role of Policy Inertia," *Journal of Money, Credit and Banking*, 2018, *50* (1), 155–172.

Jung, Taehun, Yuki Teranishi, and Tsutomu Watanabe, "Optimal Monetary Policy at the Zero-Interest-Rate Bound," *Journal of Money, Credit and Banking*, 2005, *37* (5), 813–835.

Kekre, Rohan, "Unemployment Insurance in Macroeconomic Stabilization," 2019. Working Papar.

Masolo, Riccardo M. and Pablo E. Winant, "The Stochastic Lower Bound," *Economics Letters*, 2019, *180*, 54–57.

McKay, Alisdair and Ricardo Reis, "The Role of Automatic Stabilizers in the U.S. Business Cycle," *Econometrica*, 2016, *84* (1), 141–194.

_ , Emi Nakamura, and Jón Steinsson, "The Power of Forward Guidance Revisited," *American Economic Review*, 2016, *106* (10), 3133–3158.

Nakata, Taisuke, "Uncertainty at the Zero Lower Bound," *American Economic Journal: Macroeconomics*, 2017, *9* (3), 186–221.

_ and Sebastian Schmidt, "Gradualism and Liquidity Traps," *Review of Economic Dynamics*, 2019, *31*, 182–199.

__ , **Ryota Ogaki, Sebastian Schmidt, and Paul Yoo**, "Attenuating the Forward Guidance Puzzle: Implications for Optimal Monetary Policy," *Journal of Economic Dynamics and Control*, 2019, *105*, 90–106.

Nakov, Anton, "Optimal and Simple Monetary Policy Rules with Zero Floor on the Nominal Interest Rate," *International Journal of Central Banking*, 2008, *4* (2), 73–127.

Oh, Joonseok and Anna Rogantini Picco, "Macro Uncertainty and Unemployment Risk," 2020. Sveriges Riksbank Working Paper No. 395.

Ravn, Morten O. and Vincent Sterk, "Job Uncertainty and Deep Recessions," *Journal of Monetary Economics*, 2017, *90*, 125–141.

__ **and** __ , "Macroeconomic Fluctuations with HANK & SAM: an Analytical Approach," *Journal of the European Economic Association*, 2020, *Forthcoming.*

Rotemberg, Julio J., "Sticky Prices in the United States," *Journal of Political Economy*, 1982, *90* (6), 1187–1211.

Schmitt-Grohé, Stephanie and Martín Uribe, "Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model," in Mark Gertler and Kenneth Rogoff, eds., *NBER Macroeconomics Annual*, Vol. 20, The University of Chicago Press, 2005, pp. 383–425.

Shimer, Robert, "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, 2005, *95* (1), 25–49.

Smets, Frank and Rafael Wouters, "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 2007, *97* (3), 586–606.

Werning, Iván, "Incomplete Markets and Aggregate Demand," 2015. Working Papar.

Woodford, Michael, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, 2003.

Appendices

A Equilibrium Conditions

- Workers
  - Home production

$$\delta_t = \frac{\delta}{w} w_t, \tag{A.1}$$

  - Euler equation

$$E_t M^e_{t,t+1} \frac{(1+i_t)\, a_t}{1+\pi_{t+1}} = 1, \tag{A.2}$$

  - IMRS of employed workers

$$M^e_{t-1,t} = \beta \frac{(1-s_t)\, {w_t}^{-1} + s_t {\delta_t}^{-1}}{{w_{t-1}}^{-1}}, \tag{A.3}$$

- Firm owners
  - Total consumption of firm owners

$$c^F_t = y_t - w_t n_t - \kappa v_t - \frac{\psi}{2} {\pi_t}^2 y_t + \varpi, \tag{A.4}$$

  - IMRS of firm owners

$$M^F_{t-1,t} = \beta \left(\frac{c^F_t}{c^F_{t-1}} \right)^{-1}, \tag{A.5}$$

- Labor market flows
  - Job finding rate

$${f_t}^{\frac{\gamma}{1-\gamma}} = \left(1-\tau^I\right) (\varphi_t - w_t + T)\, \mu^{\frac{1}{1-\gamma}} / \kappa + (1-\rho)\, M^F_{t,t+1} {f_{t+1}}^{\frac{\gamma}{1-\gamma}}, \tag{A.6}$$

  - Period-to-period job-loss rate

$$s_t = \rho\,(1-f_t)\,, \tag{A.7}$$

  - Employment rate

$$n_t = (1-s_t)\, n_{t-1} + (1-n_{t-1})\, f_t, \tag{A.8}$$

– Vacancies

$$v_t = \left(\frac{n_t - (1-\rho)\, n_{t-1}}{(1-(1-\rho)\, n_{t-1})^{\gamma}}\right)^{\frac{1}{1-\gamma}}, \tag{A.9}$$

- Wholesale firms
  – New Keynesian Phillips curve

$$\psi\,(1+\pi_t)\,\pi_t = \psi M^F_{t,t+1}\,(1+\pi_{t+1})\,\pi_{t+1}\frac{y_{t+1}}{y_t} + 1 - \theta + \theta\left(1-\tau^W\right)\varphi_t, \tag{A.10}$$

- Nash wage
  – Value of being employed ($V^e - V^u$)

$$S^W_t = \log w_t - \log \delta_t + \beta\,(1 - s_{t+1} - f_{t+1})\, S^W_{t+1}, \tag{A.11}$$

  – Job value (from free-entry condition)

$$J^F_t = \kappa \frac{f_t^{\frac{\gamma}{1-\gamma}}}{\mu^{\frac{1}{1-\gamma}}}, \tag{A.12}$$

  – Nash wage

$$\frac{S^W_t}{S^W} = \left(\frac{J^F_t}{J^F}\right)\left(\frac{w^N_t}{w}\right)^{-1}, \tag{A.13}$$

  – Nash-bargaining wage

$$w_t = w^{\phi} {w^N_t}^{1-\phi}, \tag{A.14}$$

- Market clearing

$$y_t = n_t, \tag{A.15}$$

- Zero Lower Bound

$$i_t \geq 0. \tag{A.16}$$

B Ramsey Optimal Policy Problem

Following Schmitt-Grohé and Uribe (2005), we assume that, in every period, the Ramsey planner honors commitments made in the very distant past, i.e., $t = -\infty$, in choosing optimal policy. This means that the constraints that the planner faces at date $t \geq 0$ are the same as those at date $t < 0$, implying that the predetermined Lagrange multipliers at date $t = 0$ are not necessarily assumed to be zero. This form of policy is referred to as an optimal policy from the timeless perspective (Woodford, 2003).

Let $\lambda_{1,t}$, $\lambda_{2,t}$, $\lambda_{3,t}$, $\lambda_{4,t}$, $\lambda_{5,t}$, $\lambda_{6,t}$, $\lambda_{7,t}$, $\lambda_{8,t}$, $\lambda_{9,t}$, $\lambda_{10,t}$, $\lambda_{11,t}$, $\lambda_{12,t}$, $\lambda_{13,t}$, $\lambda_{14,t}$, $\lambda_{15,t}$, and $\lambda_{16,t}$ be Lagrange multipliers on the constraints (A.1) to (A.16). Given $\{n_t, w_t, \delta_t, c_t^F, M_{t-1,t}^e, i_t, \pi_t, s_t, y_t, v_t, M_{t-1,t}^F, \varphi_t, f_t, S_t^W, J_t^F, w_t^N\}_{-\infty}^{-1}$, $\{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}, \lambda_{7,t}, \lambda_{8,t}, \lambda_{9,t}, \lambda_{10,t}, \lambda_{11,t}, \lambda_{12,t}, \lambda_{13,t}, \lambda_{14,t}, \lambda_{15,t}, \lambda_{16,t}\}_{-\infty}^{-1}$, and a stochastic process $\{z_t\}_0^\infty$, a Ramsey equilibrium consists of a set of control variables $\{n_t, w_t, \delta_t, c_t^F, M_{t-1,t}^e, i_t, \pi_t, s_t, y_t, v_t, M_{t-1,t}^F, \varphi_t, f_t, S_t^W, J_t^F, w_t^N\}_0^\infty$ and a set of co-state variables $\{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}, \lambda_{4,t}, \lambda_{5,t}, \lambda_{6,t}, \lambda_{7,t}, \lambda_{8,t}, \lambda_{9,t}, \lambda_{10,t}, \lambda_{11,t}, \lambda_{12,t}, \lambda_{13,t}, \lambda_{14,t}, \lambda_{15,t}, \lambda_{16,t}\}_0^\infty$ that solve:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(n_t \log w_t + (1-n_t)\log \delta_t + \Lambda \log c_t^F\right), \tag{B.1}$$

subject to (A.1) to (A.16). Predetermined Lagrangian multipliers are set equal to their steady state. The augmented Lagrangian for the optimal policy problem then reads as follows:

$$\begin{aligned}
L = \max E_0 \sum_{t=0}^{\infty} \beta^t \Bigg[& n_t \log w_t + (1-n_t)\log \delta_t + \Lambda \log c_t^F + \lambda_{1,t}\left(\delta_t - \frac{\delta}{w} w_t\right) \\
& + \lambda_{2,t}\left(1 - M_{t,t+1}^e \frac{(1+i_t)\, a_t}{1+\pi_{t+1}}\right) + \lambda_{3,t}\left(M_{t-1.t}^e {w_{t-1}}^{-1} - \beta\left((1-s_t)\, {w_t}^{-1} + s_t {\delta_t}^{-1}\right)\right) \\
& + \lambda_{4,t}\left(y_t - w_t n_t - \kappa v_t - \frac{\psi}{2} {\pi_t}^2 y_t + \varpi - c_t^F\right) + \lambda_{5,t}\left(\beta {c_t^F}^{-1} - M_{t-1,t}^F {c_{t-1}^F}^{-1}\right) \\
& + \lambda_{6,t}\left(\left(1-\tau^I\right)(\varphi_t - w_t + T)\frac{\mu^{\frac{1}{1-\gamma}}}{\kappa} + (1-\rho)\, M_{t,t+1}^F {f_{t+1}}^{\frac{\gamma}{1-\gamma}} - {f_t}^{\frac{\gamma}{1-\gamma}}\right) \\
& + \lambda_{7,t}\left(s_t - \rho(1-f_t)\right) + \lambda_{8,t}\left((1-s_t)\, n_{t-1} + (1-n_{t-1})\, f_t - n_t\right) \\
& + \lambda_{9,t}\left(v_t (1-(1-\rho)\, n_{t-1})^{\frac{\gamma}{1-\gamma}} - (n_t - (1-\rho)\, n_{t-1})^{\frac{1}{1-\gamma}}\right) \\
& + \lambda_{10,t}\left(\psi(1+\pi_t)\,\pi_t y_t - \psi M_{t,t+1}^F (1+\pi_{t+1})\,\pi_{t+1} y_{t+1} - (1-\theta)\, y_t - \theta\left(1-\tau^W\right)\varphi_t y_t\right) \\
& + \lambda_{11,t}\left(\log w_t - \log \delta_t + \beta(1-s_{t+1}-f_{t+1})\, S_{t+1}^W - S_t^W\right) + \lambda_{12,t}\left(J_t^F - \kappa \frac{{f_t}^{\frac{\gamma}{1-\gamma}}}{\mu^{\frac{1}{1-\gamma}}}\right) \\
& + \lambda_{13,t}\left(\frac{S_t^W}{S^W} - \left(\frac{J_t^F}{J^F}\right)\left(\frac{w_t^N}{w}\right)^{-1}\right) + \lambda_{14,t}\left(w_t - w^\phi {w_t^N}^{1-\phi}\right) + \lambda_{15,t}(n_t - y_t) + \lambda_{16,t} i_t \Bigg].
\end{aligned} \tag{B.2}$$

The first-order conditions are as follows:

$$[n_t]: \quad \log w_t - \log \delta_t - \lambda_{4,t} w_t - \lambda_{8,t} - \lambda_{9,t} \frac{1}{1-\gamma} \left(n_t - (1-\rho) n_{t-1}\right)^{\frac{1}{1-\gamma}-1} + \lambda_{15,t}$$
$$+ \beta \lambda_{8,t+1} \left(1 - s_{t+1} - f_{t+1}\right) \tag{B.3}$$
$$-\beta \lambda_{9,t+1} \left(v_{t+1} \frac{\gamma}{1-\gamma} \left(1 - (1-\rho) n_t\right)^{\frac{\gamma}{1-\gamma}-1} (1-\rho) - \frac{1}{1-\gamma} \left(n_{t+1} - (1-\rho) n_t\right)^{\frac{1}{1-\gamma}-1} (1-\rho) \right) = 0,$$

$$[w_t]: \quad \frac{n_t}{w_t} - \lambda_{1,t} \frac{\delta}{w} + \lambda_{3,t} \beta \left(1 - s_t\right) w_t^{-2} - \lambda_{4,t} n_t - \lambda_{6,t} \left(1 - \tau^I\right) \frac{\mu^{\frac{1}{1-\gamma}}}{\kappa} + \lambda_{11,t} w_t^{-1} + \lambda_{14,t}$$
$$-\beta \lambda_{3,t+1} M^e_{t,t+1} w_t^{-2} = 0, \tag{B.4}$$

$$[\delta_t]: \quad \frac{1-n_t}{\delta_t} + \lambda_{1,t} + \lambda_{3,t} \beta s_t \delta_t^{-2} - \lambda_{11,t} \delta_t^{-1} = 0, \tag{B.5}$$

$$\left[c^F_t\right]: \quad \frac{\Lambda}{c^F_t} - \lambda_{4,t} - \lambda_{5,t} \beta c^{F^{-2}}_t + \beta \lambda_{5,t+1} M^F_{t,t+1} c^{F^{-2}}_t = 0, \tag{B.6}$$

$$\left[M^e_{t-1,t}\right]: \quad \lambda_{3,t} w_{t-1}^{-1} - \frac{1}{\beta} \lambda_{2,t-1} \frac{\left(1 + i_{t-1}\right) z_{t-1}}{1 + \pi_t} = 0, \tag{B.7}$$

$$[i_t]: \quad \lambda_{2,t} M^e_{t,t+1} \frac{z_t}{1 + \pi_{t+1}} + \lambda_{16,t} = 0, \tag{B.8}$$

$$[\pi_t]: \quad -\lambda_{4,t} \psi \pi_t y_t + \lambda_{10,t} \psi \left(1 + 2\pi_t\right) y_t + \frac{1}{\beta} \lambda_{2,t-1} M^e_{t-1,t} \frac{\left(1 + i_{t-1}\right) z_{t-1}}{\left(1 + \pi_t\right)^2} - \frac{1}{\beta} \lambda_{10,t-1} \psi M^F_{t-1,t} \left(1 + 2\pi\right) y_t = 0, \tag{B.9}$$

$$[s_t]: \quad \lambda_{3,t} \beta \left(w_t^{-1} - \delta_t^{-1}\right) + \lambda_{7,t} - \lambda_{8,t} n_{t-1} - \frac{1}{\beta} \lambda_{11,t-1} \beta S^W_t = 0, \tag{B.10}$$

$$[y_t]: \quad \lambda_{4,t} \left(1 - \frac{\psi}{2} \pi_t^2\right) + \lambda_{10,t} \left(\psi \left(1 + \pi_t\right) \pi_t - 1 + \theta - \theta \left(1 - \tau^W\right) \varphi_t\right) - \lambda_{15,t}$$
$$-\frac{1}{\beta} \lambda_{10,t-1} \psi M^F_{t-1,t} \left(1 + \pi_t\right) \pi_t = 0, \tag{B.11}$$

$$[v_t]: \quad -\lambda_{4,t} \kappa + \lambda_{9,t} \left(1 - (1-\rho) n_{t-1}\right)^{\frac{\gamma}{1-\gamma}} = 0, \tag{B.12}$$

$$\left[M^F_{t-1,t}\right]: \quad -\lambda_{5,t} c^{F^{-1}}_t + \frac{1}{\beta} \lambda_{6,t-1} (1-\rho) f_t^{\frac{\gamma}{1-\gamma}} - \frac{1}{\beta} \lambda_{10,t-1} \psi \left(1 + \pi_t\right) \pi_t y_t = 0, \tag{B.13}$$

$$[\varphi_t]: \quad \lambda_{6,t} \left(1 - \tau^I\right) \frac{\mu^{\frac{1}{1-\gamma}}}{\kappa} - \lambda_{10,t} \theta \left(1 - \tau^W\right) y_t = 0, \tag{B.14}$$

$$[f_t]: \quad -\lambda_{6,t} \frac{\gamma}{1-\gamma} f_t^{\frac{\gamma}{1-\gamma}-1} + \lambda_{7,t} \rho + \lambda_{8,t} \left(1 - n_{t-1}\right) - \lambda_{12,t} \kappa \frac{\gamma}{1-\gamma} \frac{f_t^{\frac{\gamma}{1-\gamma}-1}}{\mu^{\frac{1}{1-\gamma}}}$$
$$+ \frac{1}{\beta} \lambda_{6,t-1} (1-\rho) M^F_{t-1,t} \frac{\gamma}{1-\gamma} f_t^{\frac{\gamma}{1-\gamma}-1} - \frac{1}{\beta} \lambda_{11,t-1} \beta S^W_t = 0, \tag{B.15}$$

$$\left[S^W_t\right]: \quad -\lambda_{11,t} + \lambda_{13,t} \frac{1}{S^W} + \frac{1}{\beta} \lambda_{11,t-1} \beta \left(1 - s_t - f_t\right) = 0, \tag{B.16}$$

$$\left[J_t^F\right]: \quad \lambda_{12,t} - \lambda_{13,t}\left(\frac{1}{J^F}\right)\left(\frac{w_t^N}{w}\right)^{-1} = 0, \tag{B.17}$$

$$\left[w_t^N\right]: \quad \lambda_{13,t}\left(\frac{J_t^F}{J^F}\right)\left(\frac{w_t^N}{w}\right)^{-2}\frac{1}{w} - \lambda_{14,t}\left(1-\phi\right)w^{\phi}{w_t^N}^{-\phi} = 0, \tag{B.18}$$

$$\left[\lambda_{1,t}\right]: \quad \delta_t - \frac{\delta}{w}w_t = 0, \tag{B.19}$$

$$\left[\lambda_{2,t}\right]: \quad 1 - M_{t,t+1}^e \frac{\left(1+i_t\right)z_t}{1+\pi_{t+1}} = 0, \tag{B.20}$$

$$\left[\lambda_{3,t}\right]: \quad M_{t-1,t}^e {w_{t-1}}^{-1} - \beta\left(\left(1-s_t\right){w_t}^{-1} + s_t{\delta_t}^{-1}\right) = 0, \tag{B.21}$$

$$\left[\lambda_{4,t}\right]: \quad y_t - w_t n_t - \kappa v_t - \frac{\psi}{2}{\pi_t}^2 y_t + \varpi - c_t^F = 0, \tag{B.22}$$

$$\left[\lambda_{5,t}\right]: \quad \beta {c_t^F}^{-1} - M_{t-1,t}^F {c_{t-1}^F}^{-1} = 0, \tag{B.23}$$

$$\left[\lambda_{6,t}\right]: \quad \left(1-\tau^I\right)\left(\varphi_t - w_t + T\right)\frac{\mu^{\frac{1}{1-\gamma}}}{\kappa} + \left(1-\rho\right)M_{t,t+1}^F {f_{t+1}}^{\frac{\gamma}{1-\gamma}} - {f_t}^{\frac{\gamma}{1-\gamma}} = 0, \tag{B.24}$$

$$\left[\lambda_{7,t}\right]: \quad s_t - \rho\left(1-f_t\right) = 0, \tag{B.25}$$

$$\left[\lambda_{8,t}\right]: \quad \left(1-s_t\right)n_{t-1} + \left(1-n_{t-1}\right)f_t - n_t = 0, \tag{B.26}$$

$$\left[\lambda_{9,t}\right]: \quad v_t(1-\left(1-\rho\right)n_{t-1})^{\frac{\gamma}{1-\gamma}} - \left(n_t - \left(1-\rho\right)n_{t-1}\right)^{\frac{1}{1-\gamma}} = 0, \tag{B.27}$$

$$\left[\lambda_{10,t}\right]: \quad \psi\left(1+\pi_t\right)\pi_t y_t - \psi M_{t,t+1}^F\left(1+\pi_{t+1}\right)\pi_{t+1}y_{t+1} - \left(1-\theta\right)y_t - \theta\left(1-\tau^W\right)\varphi_t y_t = 0, \tag{B.28}$$

$$\left[\lambda_{11,t}\right]: \quad \log w_t - \log\delta_t + \beta\left(1-s_{t+1}-f_{t+1}\right)S_{t+1}^W - S_t^W = 0, \tag{B.29}$$

$$\left[\lambda_{12,t}\right]: \quad J_t^F - \kappa\frac{{f_t}^{\frac{\gamma}{1-\gamma}}}{\mu^{\frac{1}{1-\gamma}}} = 0, \tag{B.30}$$

$$\left[\lambda_{13,t}\right]: \quad \frac{S_t^W}{S^W} - \left(\frac{J_t^F}{J^F}\right)\left(\frac{w_t^N}{w}\right)^{-1} = 0, \tag{B.31}$$

$$\left[\lambda_{14,t}\right]: \quad w_t - w^{\phi}\left(w_t^N\right)^{1-\phi} = 0, \tag{B.32}$$

$$\left[\lambda_{15,t}\right]: \quad n_t - y_t = 0, \tag{B.33}$$

$$\left[\lambda_{16,t}\right]: \quad i_t \geq 0. \tag{B.34}$$