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Horses for courses: measuring foreign supply chain exposure

Richard Baldwin,⁽¹⁾ Rebecca Freeman⁽²⁾ and Angelos Theodorakopoulos⁽³⁾

Abstract

Perceptions of global supply chains (GSCs) have shifted in recent years from a positive to a more cautious view. Standard GSC measures have mostly not adapted to this change as they focus on participation in, rather than exposure to, foreign supply chains. This paper presents the tools necessary to track foreign GSC exposure, and introduces a systematic shocks approach to GSC indicator design. We use this to develop indicators that gauge the impact of a variety of foreign shocks. We argue that different indicators are appropriate to different questions and show that they can provide qualitatively different answers to the same foreign exposure question.

Key words: Global supply chains, global value chains, foreign exposure, globalisation.

JEL classification: D57, F13, F14, F15, F60, R15.

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1 Introduction

Perceptions of global supply chains have shifted in recent years. When supply chains started becoming more complex and international from around 1990, they were generally perceived in a very positive light. G7 manufacturing output grew, and exports grew even faster—rising 50% from 1990 to 2008. Several low wage economies began industrialising and growing at historically unprecedented rates thanks in part to international production networks (Taglioni and Winkler 2016). Hundreds of millions of people were lifted out of dire poverty. The rules-based multilateral system thrived and expanded to embrace nations that had previously stood aside.

There are many names for this phase of globalisation, but the 'global value chains (GVCs) revolution' is perhaps the most apt. Advanced and emerging economies alike thought of participation in GVCs as a plus and so naturally sought indicators of GVC participation (Hummels et al. 1998, 2001; Johnson and Noguera 2012; Daudin et al. 2011; Koopman et al. 2014).¹ In the early 2010s, the publication of global input-output (IO) tables—e.g. the World IO Database, WIOD (Timmer et al. 2015), OECD IO tables (OECD 2015), and EORA Database (Lenzen et al. 2012, 2013)—and the popularisation of indicators that can be derived from them—e.g. the OECD Trade in Value Added, TiVA, Database (Guilhoto et al. 2022)—greatly expanded the use of GVC indicators, especially the most well-known ones like backward and forward linkages (Hummels et al. 2001). Recent important work has extended and refined GVC indicators and the types of questions they are geared to address in many directions (see Antràs et al. 2012, Antràs and Chor 2013, De Backer and Miroudot 2013, Borin and Mancini 2015, 2019, Imbs and Pauwels 2020, 2022, and Borin et al. 2021).

More recently, perceptions of international supply linkages have shifted to a less rosy view, at least in advanced economies. The 2016 US presidential elections provide a convenient landmark for the shift. In G7 nations, domestic opposition to trade with low-wage nations has intensified, and supply disruptions during the Covid-19 pandemic strengthened the shift to a more cautious view of GVCs and several calls to reshore production.

While perceptions and concerns shifted, the measures used to track exposure to foreign suppliers did not. Measures of GVC participation were not built for the purpose of gauging exposure to foreign supply chains.

For example, many GVC indicators, including the most common ones (backward and forward linkages), look only at trade flows that cross borders at least twice; the reasoning being that this test unambiguously identifies the trade as part of an internationalised production process.² But importing inputs creates a foreign exposure even if the imports

¹In particular, the seminal work of Koopman et al. (2014) integrated the literature on vertical specialisation, i.e. trade that crosses borders at least twice (Hummels et al. 2001), with that on value-added trade (Johnson and Noguera 2012; Daudin et al. 2011).

 $^{^{2}}$ The two-borders test seems to have been introduced by Hummels et al. (2001) to distinguish traditional

are not used in export-oriented production. To give a concrete example, German electricity providers are exposed to imports of all Russian gas—not just the gas that is used to produce electricity for exporting firms. Moreover, the main GVC indicators focus on value-added trade flows rather than gross trade flows. Value-added trade measures have the merit of eliminating double counting which is inherent in gross trade statistics,³ but many supply shocks concern gross trade. For instance, when Canadian truckers blocked a key Canada-US entry point for auto parts, US industry was exposed to the full value of the blocked parts—not just the Canadian value added in those parts. Likewise, when the port of Singapore forbade crew changes to reduce Covid-19 transmission during 2020, gross trade flows were disrupted, not value-added trade flows.

Another lacuna in the use of GVC measures as indicators of foreign exposure concerns the way trade is measured. Measures of bilateral GVC engagement, like the backward linkages indicator, include the direct bilateral imports of value-added as well as all the roundabout imports that arrive after having been embedded in goods made in third nations. While meritorious for gauging GVC participation, this mismeasures exposure to bilateral trade disruptions such as the Canadian truckers' blockage mentioned above. In that case, the exposure was only to direct, bilateral imports; imports from Mexico that contain Canadian value added were unaffected.

To twist a British idiom, these GVC 'horses' were not meant for the foreign exposure 'courses'. The main goal of this paper is to propose a set of indicators that measure foreign supply chain exposure. The plan of the paper is simple. Sections 2-5 exposit the tools needed to track foreign exposure, namely Inter-Country Input-Output (ICIO) tables. This task is necessary since while ICIO tools are straightforward—and well-known to specialists—they are not yet as familiar as they should be given the long-standing and widespread recognition of the importance of intermediates in trade (Batra and Casas 1973; Dixit and Grossman 1982; Krugman and Venables 1995; Feenstra and Hanson 1996; Campa and Goldberg 1997; Jones and Kierzkowski 2001; Grossman and Rossi-Hansberg 2008; Caliendo and Parro 2015; Antràs et al. 2017; Baqaee and Farhi 2022, among others). For a more technical introduction to ICIO tools, see Antràs and Chor (2022).

trade in intermediates from the internationalised production processes that arose in the 1990s. This is clearest in the working paper version Hummels et al. (1999). Not all GVC measures embrace the two-borders rules. Baldwin and Lopez-Gonzalez (2015) and Timmer et al. (2014, 2021), for example, include all intermediates that cross borders irrespective of how many border crossings are involved. Timmer et al. (2014) uses a concept of GVC income that includes intermediate inputs imported for domestic consumption. Borin and Mancini (2015) compute fully additive measures that allow to sum trade that crosses two borders with that which crosses one border. Further, Borin and Mancini (2019) focus on gauging the source of value added in bilateral trade flows. These measures are used intensively in many studies, including the 2020 World Bank World Development Report (World Bank 2020). Nagengast and Stehrer (2016) focus on the origin of the value added in a nation's consumption (sources) and the national destination of the value added in its production (sinks) regardless of the number of border crossings.

³Various studies have tackled the issue of double counting head on, as GVC measures can be biased if double counting is not addressed for the question at hand; see Los et al. (2016), and Miroudot and Ye (2018, 2022). Antràs (2020) and Borin and Mancini (2019) provide further discussions of potential biases and methodologies to correct for double counting.

Section 6 next turns to developing ways of measuring exposure to foreign suppliers. The main contribution is to introduce a systematic approach to indicator design that we call the shocks approach. We use this to develop a handful of indicators that gauge the impact of a variety of foreign shocks, ranging from shocks to foreign gross production, or to foreign value-added, or to bilateral trade flows. The approach suggests many other indicators that could be developed.

Section 7 develops the horses for courses theme by showing that different indicators provide qualitatively different answers to the same foreign exposure question. For instance, the question *which foreign nation is your nation's largest supplier of intermediate inputs?* has more than one answer. Different indicators lead to different conclusions.⁴ This is why there must be a horses for courses perspective when it comes to measuring foreign exposure. The indicator used should be matched to the foreign exposure under study. The point is that global production networks are complex and multidimensional, and different indicators illuminate different facets. Finally, Section 8 offers concluding remarks.

To be precise, we use the term global supply chains (GSCs) to refer to trade flows where inputs from one nation are used in another, and the GVC term to refer to trade that meets the two-border-crossings criteria. GSC is thus a more comprehensive concept than GVC, in our terminology.⁵

2 Input-Output tables made easy: basic concepts

The uses of IO tables like those from WIOD, the OECD, EORA, etc. can be massively complex, even though the underlying tables are simple. But there is a catch. The simplicity emerges only after we wrap our minds around key concepts that are not taught in most economic programs. To get started, we simplify to clarify by beginning with a minimalist model. Complexity/realism are added progressively. The goal of this section is to explain well-known and well-documented concepts to non-specialists, such that they are equipped with the tools to navigate the classic GVC indicators and foreign exposure GSC indicators proposed in Section $6.^{6}$

 $^{^4\}mathrm{Borin}$ and Mancini (2019) make this point very effectively in the context of supply chain indicators based on value-added trade.

⁵Definitions have varied widely over time, across authors, and across literatures. For instance, GSC is used in logistics, economic geography, and international business and management. The GVC terminology was introduced by sociologist Gary Gereffi and his colleagues at Duke University's Global Value Chains Initiative (Gereffi 1994) and the phase quickly supplanted the cacophony of terms used at the time (ranging from 'fragmentation' and 'offshoring' to 'slicing up the value chain', and 'vertical specialisation'). More recently, many scholars have embraced GSC. Antràs and Chor (2022) is a notable exception to this which eschews the two-border test. In our paper, we stick to the GSC versus GVC distinction for clarity's sake.

⁶There are many excellent expository pieces including Timmer et al. (2015); Lenzen et al. (2012, 2013); Borin and Mancini (2015); World Bank (2017); Borin and Mancini (2019). An excellent and comprehensive book-length introduction is found in Miller and Blair (2009).

2.1 Introducing the most basic concepts with a minimalist model

The simplest model is a closed, Ricardian model with intermediate inputs, so that's where we start. Specifically, we assume: (i) perfect competition and constant returns (so no profits); (ii) labour is the only productive factor; and (iii) there is a single sector that uses its own output as an intermediate input. There is no government, no international trade, and no investment. Also, prices are fixed (more on this rather irregular assumption below).

To be concrete, think of the single sector as corn (or maize as it is called on the Continent). Corn can be used as an intermediate input (seed corn) and as a final consumption good (eating corn). By assumption, corn for eating and planting are exactly the same good.

2.1.1 Critical distinction: gross versus net output

Turning to production, it is important to distinguish between two related concepts.

- Gross production of a sector is the total value of goods produced in the sector (typically measured in dollars). In textbook trade models, we would simply call this 'production' and it would equal the sales of firms. Once we have intermediate inputs, we need to be more specific. That's why we will use the term gross production or gross output to distinguish it from our next concept, net production.
- Net production is the value of gross production minus the value of the intermediate inputs (again typically in dollars).

The next linchpin concept concerns the distinction between accounting for gross production on the supply-side and the sales-side (or alternatively, on the input- and output-sides).

2.1.2 Critical distinction: use versus cost accounting

There are two approaches to measuring the value of a firm's output: (i) on the output side we track where the value goes; and (ii) on the cost side we track the cost of all inputs. Since we are working with constant returns and perfect competition, the two accounting methods produce exactly the same answer. That is, the value of the output (in dollars) exactly matches the value of the inputs (in dollars). Each approach involves an accounting identity—both of which are straightforward.

• Use accounting identity: In this minimalist economy, gross production gets used either as final consumption or intermediate inputs. This approach is the one most used in the GVC literature.

In our example, the use accounting for the gross production of corn, i.e. the total sales of corn, equals the value of the corn purchased for consumption by consumers and value purchased by firms for use as inputs. • **Cost accounting identity:** The value of gross production (i.e. sales) equals the cost of productive factors employed plus the cost of intermediate goods used.

In our corn example, the cost of gross output is the cost of the seed corn plus the wage payments. Since there is perfect competition, the cost is equal to the value of sales.

To be sure that the distinction between the approaches is crystal clear, we show the identities in a diagram. Figure 1 illustrates the one-sector corn economy. The two accounting identities are shown schematically. The blue arrows depict the use accounting identity (corn is used as a final good, or it is used as an intermediate good). The red arrows show the cost accounting identity (the cost of producing the corn equals the wage bill plus the cost of intermediate inputs).

Figure 1: One sector, one factor, closed economy with intermediate goods



Source: Authors' illustration.

Naturally, everything adds up in this economy. The total cost of inputs equals the total value of sales, which, in turn, equals the total value of usage for consumption and intermediate inputs. Note that, as one would expect, the dollar value of intermediate corn sales (in blue) are identical to the dollar cost of intermediate corn usage (in red). To see this critical distinction from yet another angle, in Figure 2 we translate the identities into word equations. Recall that both identities are anchored to gross output.

Figure 2: U	Use and cost	accounting	identities
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Use accounting identity				Cost accounting identity					
Sales of intermediates +	Sales of final goods	=	Gross output (sales basis)	=	Gross output (cost basis)	=	Costs of factor inputs	+	Sales of intermediates
Source: Authors' illus	tration.								

The use accounting identity stresses how the gross output—Gross output (sales basis)—must equal the sales of the goods to other firms as intermediate inputs and sales as final goods to final customers. The cost accounting identity highlights that the total cost of the gross output—Gross output (cost basis)—must equal the cost of factor inputs plus the cost of intermediate inputs. The equality between the two holds because of our assumption of perfect competition and constant returns (which implies zero pure profits, i.e. costs equal sales).

2.1.3 Frequently Asked Questions: Input-Output tables

To clarify concepts and avoid confusions, consider four frequently asked questions (FAQs):

FAQ 1: Are gross and net production of a single sector made up of the same good?

The answer is yes. This is a key simplifying assumption in IO work; it is called the homogeneity assumption. This is easy to think about for a sector like corn where the intermediate usage (seed corn) and final good usage (eating corn) both use corn. It also works for fertiliser that can be used by firms to produce goods, or directly by consumers in their flower garden. Of course, it is fairly rare that the goods used for consumption and intermediates are exactly the same good at any reasonable level of aggregation. One could avoid the problem by having tens of thousands of sectors, but even if we could find the data for that, it would hardly make it easier to understand real-world supply chains. To be a little philosophical about it, a map, as often noted, is not useful when it is at the one-to-one scale; that's no different than walking around the actual world. We have to simplify to clarify. We must ignore certain details to better understand the whole picture (the trick is to ignore the details that don't matter much for the issues at hand). In any case, all existing IO tables have to rely on the homogeneity assumption.

FAQ 2: Does a sector supply intermediate goods before they are produced into final goods?

The answer is no—again due to the homogeneity assumption and the fact that IO analysis is timeless, i.e. it is a static analysis where production and consumption happen simultaneously. Corn is again a good placeholder for this answer concerning the timing problem. There is no way to use the corn at an intermediate stage of processing, i.e. before the sector has produced all its gross output.

FAQ 3: Why have I not heard of the gross versus net production distinction before?

Textbook economic models typically assume away intermediate inputs, so gross and net production are the same thing. That's why the gross-net distinction is rarely necessary. In the real world, however, intermediate inputs are pervasive. Data shows that about half of all output of the manufacturing sector is used as intermediates rather than as final goods. This is why the distinction is critical, especially when thinking about international supply chain issues. After all, GVCs and GSCs are all about intermediate inputs.

FAQ 4: Are net production and final demand the same thing?

The answer is yes, almost by definition. If one sector sells to another sector, those sales do not count as net production. Thus, each sector sells all its net output to final uses (final consumption in this simple model, but we have investment, government purchases, capital formation, and direct purchases abroad in more sophisticated IO tables).

With the basics laid out, we add complexity by asking: What do these concepts look like in a more sophisticated economy?

2.2 Moving beyond the obvious: Input-Output in a two-sector economy

Figure 3 presents a basic IO table for a single economy with two sectors (two goods to be concrete). In real IO tables, many of the sectors are services sectors since services make up such a large share of real economies. But to avoid having to continually write goods and services we simplify by assuming away services in our examples. We continue to simplify by assuming perfect competition, constant returns, labour as the only productive factor, no government, no investment, and no international trade.

There are now two goods. To be concrete, think of them as corn and fertiliser. It is easy to imagine fertiliser as an input into corn production. The reverse? Not so much. Corn is not used in making fertiliser, but we could deal with this by putting a zero into the appropriate element of the matrix of intermediate inputs (these are the Ts in Figure 3 but we leave the entries as variables to stay general).

		Buying	g sector		
		Intermedia	ate use (T)	Final	Gross
		Nati	on A	demand	output
		Sector 1	Sector 2	(F)	(X)
Selling Nation	Sector 1	T_{11}	T_{12}	F_1	X_1
sector Nation A	Sector 2	T_{21}	T_{22}	F_2	X_2
Value	Added (V)	V_1	V_2		
Gross	output (\boldsymbol{X})	X_1	X_2		

Figure 3: Basic closed economy IO table

Source: Authors' illustration.

Notes: This IO table is simplified in two main ways. First, F is a single column vector whereas standard IO tables break this out into final consumption by household, government, and gross fixed capital formation, inter alia. Second, we exclude taxes less subsidies on intermediate and final products, which are also typically included in IO matrices as a row vector above V. The ordering of sector subscripts is always 'from, to', i.e. T_{21} is sector 2's sales of intermediates to sector 1.

The first row of Figure 3 resembles the use accounting identity from the one-sector case in Figure 1. For example, reading right to left across sector 1's row, some of sector 1's gross output (denoted as X_1) goes to consumption (shown as F_1). The rest of sector 1's gross output goes to intermediate usage in sectors 1 and 2. The values of these intermediate purchases are denoted as T_{11} and T_{12} (the ordering of the subscripts is always 'row, column', which is easy to remember as 'from, to', i.e. from row to column). The second row sector, sector 2, has analogous relations and notation.

Quick review.— Each sector's gross production is used for intermediate inputs and for consumption. In symbols this is $X_1 = T_{11} + T_{12} + F_1$, and $X_2 = T_{21} + T_{22} + F_2$. These

are the use accounting identities. As mentioned, they show the allocation of each row's gross output among intermediate and final uses.

For convenience, the labour input for each sector is listed under the rows. Thus, the payment to labour employed in sector 1 (i.e. its wage bill) is denoted as V_1 . Likewise, V_2 is the wage bill for sector 2. With the Vs we see the cost accounting identity in the table by reading down each sector's column. For instance, the value of sector 1's production (X_1) equals the cost of its purchased inputs $(T_{11} + T_{21})$ plus its payment to labour (V_1) . The analogous identity holds for sector 2, i.e. $X_2 = T_{12} + T_{22} + V_2$.

Aggregate GDP in this simple economy can be measured in two ways: the income- or expenditure-side (i.e. final demand/output). On the income-side, the whole economy's GDP is the sum of value-added payments, $\text{GDP} = V_1 + V_2$. On the expenditure-side it is $\text{GDP} = F_1 + F_2$. Sectoral GDP is less straightforward because $T_{11} + T_{21}$ does not equal $T_{11} + T_{12}$. Or, taking our 'corny' example from above (where now sector 1 is fertiliser and sector 2 is corn), the intermediate input of corn and fertiliser in the production of fertiliser, which equals $T_{11} + T_{21}$, does not equal the intermediate usage of corn for the production of corn and fertiliser, i.e. $T_{11} + T_{12}$. Consequently, V_1 does not equal F_1 , so value added in the fertiliser sector is not the same when measured by value-added inputs, V_1 , and when measured by final output, F_1 .

2.2.1 Matrix notation for the use-accounting identity

In preparation for more complex cases, we introduce matrix notation that allows us to compactly manipulate the use and cost accounting equations implicit in the IO table. The notation we use throughout this paper (and which is fairly standard in the literature) is:

$$\boldsymbol{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, \boldsymbol{F} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}, \boldsymbol{V} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, \boldsymbol{T} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}, \text{ and } \boldsymbol{\iota} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

where X, F, and V are vectors of gross production, final demand, and value added, and T is the matrix of intermediate usage from the origin sector (first subscript) to the destination sector (second subscript). ι is a column vector of 1s used for aggregation purposes to sum across matrix rows. The use accounting identity for the two sectors is thus:

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} + \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
(1)

The first row shows the allocation of output of sector 1 to final production of good 1 and use of good 1 as an input to the production of good 1 and good 2. The second row shows the same for sector 2.

There is another linchpin matrix to get our minds around, the A matrix. This is really the heart of IO analysis.

2.2.2 The A matrix of input-output coefficients

So far, we have worked in total values. For example, T_{12} is the value of intermediates that go from sector 1 to sector 2. Since we want to consider changes in the level of activity in various sectors, we need to decompose the total into a technical (per unit) coefficient in relation to the gross output of the relevant sector. This is simple, but crucial to fix in our mental map.

To this end, we separate each flow into the per-dollar of output requirement on the one hand, and the overall gross output value of the buying sector, on the other. For instance, the intermediates from sector 2 that are used in sector 1, T_{21} , can be separated into the per-dollar amount and sector 1's gross output:

$$a_{21} = T_{21}/X_1$$

These a_{ij} (where *i* refers to the selling sector and *j* refers to the buying sector) are called input-output coefficients, or fixed-input coefficients, or technical coefficients. To avoid confusion, note that a_{ij} are not measured in dollars; they are measured in units of input. Importantly, the a_{ij} are assumed to be fixed; they do not respond to relative price changes as we might expect. To put it differently, IO analysis assumes Leontief technology. To be explicit, we write out the matrix of the a_{ij} and their relation to the elements of Tand gross outputs for the basic Figure 3 closed economy example as:

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \equiv \begin{bmatrix} T_{11}/X_1 & T_{12}/X_2 \\ T_{21}/X_1 & T_{22}/X_2 \end{bmatrix}$$

As noted above, it is the gross output of the *buying sector* (i.e. the column sector) that is used to scale the elements of T since we want the number of units of input from the row sector per unit of output of the column sector. Recall that the rows correspond to producing/selling sectors and columns correspond to using/buying sectors. Further, observe that with some simple matrix multiplication $T\iota = AX$.

2.2.3 Why is translating $T\iota$ into AX useful or insightful?

The merit of translating $T\iota$ into AX stems from three facts.

1. First, the vector AX gives us the sum of the intermediate usage of every sector's gross output. Thus, the use accounting identity in matrix notation is:

$$\boldsymbol{X} = \boldsymbol{A}\boldsymbol{X} + \boldsymbol{F} \leftrightarrow \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$
(2)

This formula is at the core of IO analysis. It shows how gross output is allocated between intermediate use (the AX) and final use (the F).

2. Second, breaking down T into A and X lets us distinguish between intensities of

input usage and the scale of production; the intensities are given by a_{ij} and are scaled by \boldsymbol{X} . This distinction is handy in many applications considered below.

3. The a_{ij} coefficients reveal the direct linkages among every sector in every country—which provides a simple indicator of supply chain connections that is based on the observable trade flow between every pair of sectors. When we turn to constructing indicators of supply chain exposure to foreign production, we will use these observed links for some indicators

2.2.4 The Leontief inverse matrix

The decomposition yields a very tight link between gross and net output. Solving equation (2) for \boldsymbol{X} yields:

$$\boldsymbol{X} = (\boldsymbol{I} - \boldsymbol{A})^{-1} \boldsymbol{F}$$
(3)

where I is the identity matrix. This tells us how much output each sector would need to produce a given vector of consumption given by F; it is the mathematical expression for the distinction between gross output, X, and net output, F, for every sector. In words, it tells us the gross production in every sector that is needed to produce a given vector of final output in every sector, taking account of all direct and indirect linkages.⁷ For example, if a nation wanted to make more airplanes, this relationship would tell planners how much they would have to boost production in every sector in order to end up with, say, 100 more airplanes.

Note that X is, element by element, larger than F (or at least as big) since the final output of a sector is its gross output minus its output that is used as intermediate inputs. The $(I - A)^{-1}$ matrix is so famous that it has a name—the Leontief inverse (Leontief 1986).⁸ For clarity's sake, we write out the Leontief inverse for the Figure 3 example:

$$(\boldsymbol{I} - \boldsymbol{A})^{-1} \equiv \boldsymbol{L} = \begin{bmatrix} \ell_{11} & \ell_{12} \\ \ell_{21} & \ell_{22} \end{bmatrix}$$
(4)

Here we give the matrix a shorthand name of its own, L, and use the cursive lower-case ℓ to indicate its elements. L and ℓ are mnemonics for Leontief. As we show below, the ℓ s are key in understanding a nation's exposure to inputs and foreign demand. With this:

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} \ell_{11} & \ell_{12} \\ \ell_{21} & \ell_{22} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

An important difference between the A and L matrices is that A reflects only the

⁷What do we mean by indirect linkages? Note that $(I - A)^{-1} = I + A + A^2 + A^3 + \dots$ Thus, the expression $(I - A)^{-1}$ shows all of the intersectoral linkages among all sectors which are observed (the A matrix) and of higher order $(A^2$ and above).

⁸This term is named after Wassily Leontief, a Russian-born American economist, who was a pioneer in IO analysis. He won a Nobel Prize in Economics for it in 1973.

observed (direct) links among sectors while L captures these direct links plus the indirect purchases by one sector from another that are used in the making of intermediate purchases from third nations.

2.2.5 Frequently Asked Questions about the Leontief inverse

While the Leontief inverse matrix is the proverbial meat-and-potatoes for IO mavens, it is a novel thinking tool for many economists, so we present a couple of FAQs and answers.

FAQ 1: What is the plain language meaning of a typical element, ℓ_{ij} ?

The Leontief inverse matrix maps final output into gross output, so the ℓ_{ij} s tell us how much gross output is needed to produce \$1 of final output. As the foundation of Lis I - A, it is clear that the diagonal and off-diagonal elements will be systematically different.

FAQ 2: How should we think about the columns of L?

Each column of L is a complete list of what every sector must produce in order for the economy to produce \$1 of final output of the goods in the corresponding column. Say F_1 is corn and F_2 is fertiliser. Then ℓ_{11} is the amount of corn needed to make \$1 of final good corn plus the corn that is needed to make fertiliser used in making corn. Obviously, then, $\ell_{11} \ge 1$. The element ℓ_{21} is the amount of fertiliser needed to make \$1 of final corn. Obviously $\ell_{21} \ge 0$, but beyond that, it could be anything depending on the technology. This means that the column elements necessarily sum to greater than unity, i.e. $\ell_{11} + \ell_{21} \ge 1$.

FAQ 3: How should we think about the rows of L?

Each row of L provides a complete list of the ultimate destinations of the gross output of the corresponding sector. As noted, the diagonal elements all exceed unity and all other elements are non-negative, so the sum of each row of L exceeds unity. For example, $\ell_{11} + \ell_{12} \ge 1$.

Most forms of IO analysis rely upon the use accounting approach, but some employ the cost accounting identity, so we turn to that next.

2.2.6 Cost-accounting identity and the Ghosh inverse matrix

As noted, the cost accounting approach looks at the cost components of gross output. The cost components are the value added (the sector's wage bill in this simple example) and the cost of the intermediate inputs. The sum of these is the value of the output of the sector since the cost of producing output and the value of the output are identical with perfect competition and constant returns. In matrix notation:

$$\boldsymbol{X} = \boldsymbol{V} + \boldsymbol{T}'\boldsymbol{\iota} \leftrightarrow \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} T_{11} & T_{21} \\ T_{12} & T_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
(5)

The top row shows the cost of making X_1 , which consists of the sector value-added cost, V_1 , and the cost of the intermediate inputs from sectors 1 and 2, namely T_{11} and T_{21} , respectively. The second row does the same for sector 2. Note that, in order to keep the column structure of the gross output and value-added vectors, we have transposed the T matrix using the ' symbol.

As before, we will want a decomposition of the intermediate usage into scale and intensity factors, but rather than normalising the elements of T by the gross output of the sector which has purchased it, we normalise by the gross output of the selling sector, i.e. the row sector. For example, the intermediates from sector 2 that are used in sector 1, T_{21} , can be separated into the per dollar amount and sector 2's gross output:

$$b_{21} = T_{21}/X_2$$

where the b_{ij} are referred to as allocation coefficients (as opposed to technical coefficients). We then form a matrix of these allocation coefficients, and label the matrix as \boldsymbol{B} , with elements denoted as b_{ij} . Writing out the \boldsymbol{B} matrix for the Figure 3 example:

$$\boldsymbol{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \equiv \begin{bmatrix} T_{11}/X_1 & T_{12}/X_1 \\ T_{21}/X_2 & T_{22}/X_2 \end{bmatrix}$$

Plainly, there is a close, but imperfect, analogy with the \boldsymbol{A} matrix. It is imperfect since the elements of the \boldsymbol{T} matrix are divided by the gross output of the *selling sector*, not the buying sector. To be clear, note that the \boldsymbol{B} matrix is not the transpose of the \boldsymbol{A} matrix. The \boldsymbol{A} and \boldsymbol{B} matrices have the same source (the \boldsymbol{T} matrix), but \boldsymbol{A} is normalised by the column sector's gross output (so it measures inputs into the sector) while \boldsymbol{B} is normalised by the row sector's gross output (so it measures the allocation of the sector's output to other sectors).

With this, the two-sector cost accounting identity can be compactly written as:

$$\boldsymbol{X} = \boldsymbol{B}'\boldsymbol{X} + \boldsymbol{V} \leftrightarrow \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$
(6)

Solving equation (6) for \boldsymbol{X} yields:

$$\boldsymbol{X} = \left(\boldsymbol{I} - \boldsymbol{B}'\right)^{-1} \boldsymbol{V}$$
⁽⁷⁾

The matrix $(I - B')^{-1}$ is not as famous as the *L* matrix, but it is famous enough to have its own name—the Ghosh inverse matrix (Ghosh 1958).⁹ For shorthand, we refer to the Ghosh inverse matrix as *G*, and write its elements with lower-case *gs*. *G* and *g_{ij}* are

⁹Here we have simplified the notation. In standard IO usage, the Ghosh inverse matrix is $(I - B)^{-1}$ and the cost accounting identity is $X' = V' (I - B)^{-1}$. For further details see chapter 12 of Miller and Blair (2009).

mnemonics for Ghosh. This is expressed as:

$$\left(\boldsymbol{I} - \boldsymbol{B}'\right)^{-1} \equiv \boldsymbol{G} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}, \text{ thus } \boldsymbol{X} = \boldsymbol{G}\boldsymbol{V}$$
(8)

The expression X = GV gives us the full relationship between production and employment (and value added in more general examples). It tells us how much gross output we get in each sector for a given amount of labour employed in each sector. Alternatively, it tells us the source of value added that is embodied in gross production taking account of the direct value (the V in X = B'X + V) and the value added in all the intermediate inputs that the sector uses (the B'X in X = B'X + V).

Using the reasoning from the FAQs above, we know that the row sums of G are not less than unity, and the same holds for the column sums.

2.2.7 Keeping the Leontief and Ghosh approaches straight

To summarise, the use and cost accounting approaches provide two alternative ways of thinking about the gross output vector:

$$X = LF$$
, and $X = GV$

In the first approach (use accounting), we can think of net production driving gross production (i.e. $F \to X$). In the second (cost accounting), we can think of employment as driving gross production (i.e. $V \to X$). It takes no imagination at all to combine these to get a full mapping of the linkages between gross production, X, net consumption, F, and employment, V, one version of which is: $F = L^{-1}GV$.

2.2.8 There are no prices

Importantly, prices are assumed to be fixed in IO analysis. Or, more precisely, IO tables are generated from data that include no prices (only values, all of which are measured in the relevant currency—typically expressed in basic prices).¹⁰ This is why prices never enter the analysis.

Implicitly this means that prices are taken as fixed. Keep this in mind when considering shocks to the system in Section 6; IO analysis is not an approach where price adjustments, factor substitution, and consumption substitution play the roles they usually do in trade models. While the lack of prices is highly unusual in economic analyses, it is worth pointing out that the famous Rybczynski analysis takes prices as fixed, so the assumption is not entirely without precedent in trade theory.

¹⁰The OECD glossary of statistical terms defines basic prices as: the amount receivable by the producer from the purchaser for a unit of a good or service produced as output minus any tax payable, and plus any subsidy receivable, on that unit as a consequence of its production or sale; it excludes any transport charges invoiced separately by the producer (OECD 2013).

It is straightforward to expand the closed economy IO table to a multi-country setting since the world is a closed economy, and we have already seen inter-sectoral 'trade' within our closed economy.

2.3 Adding trade: Inter-Country Input-Output tables

Allowing more countries simply adds more rows and columns to the IO system without changing any of the fundamental relationships. For instance, if we had 10 countries, each with two sectors, we would have 20 buying sectors and 20 selling sectors in the T matrix. We would also have 10 F vectors (each with 20 elements corresponding to the 10x2 sectors), and 1 V vector with 20 elements (each showing the corresponding nation's employment in each sector).

Figure 4 shows an inter-country IO table with two nations, each with two sectors. To keep track of nations and sectors, we introduce a clunky but explicit notation where, for example, T_{1A1B} represents goods produced by sector 1 in nation A that are used as intermediate inputs in sector 1 in nation B. As before, it is easy to remember the order of the subscripts by remembering 'from, to', i.e. T_{1A1B} represents the intermediates from sector 1A that are going to sector 1B.

		Intermediate use (\boldsymbol{T})			Final demand (\boldsymbol{F})		Gross	
		Nati	on A	Nation B		Nation A	Nation B	output
		Sector 1	Sector 2	Sector 1	Sector 2	Nation A Nation B		(\boldsymbol{X})
Nation A	Sector 1	T_{1A1A}	T_{1A2A}	T_{1A1B}	T_{1A2B}	F_{1AA}	F_{1AB}	X_{1A}
Sector 2	Sector 2	T_{2A1A}	T_{2A2A}	T_{2A1B}	T_{2A2B}	F_{2AA}	F_{2AB}	X_{2A}
Nation B	Sector 1	T_{1B1A}	T_{1B2A}	T_{1B1B}	T_{1B2B}	F_{1BA}	F_{1BB}	X_{1B}
Ivation D	Sector 2	T_{2B1A}	T_{2B2A}	T_{2B1B}	T_{2B2B}	F_{2BA}	F_{2BB}	X_{2B}
Value Ad	ded (V)	V_{1A}	V_{2A}	V_{1B}	V_{2B}			
Gross out	$\operatorname{tput}(\boldsymbol{X})$	X_{1A}	X_{2A}	X_{1B}	X_{2B}			

Figure 4: Basic inter-country IO table

Source: Authors' illustration.

Notes: This IO table is simplified in two main ways. First, F is a single column vector whereas standard IO tables break this out into final consumption by household, government, gross fixed capital formation, inter alia. Second, we exclude taxes less subsidies on intermediate and final products, which are also typically included in IO matrices. The ordering of sector subscripts is always 'from, to', i.e. T_{21} is sector 2's sales of intermediates to sector 1 as a row vector above V. The ordering of sector subscripts is always 'from, to', i.e. T_{2B1A} sector 2's in country B sales of intermediates to sector 1 in country A.

Observe that the T matrix here includes domestic and international flows. Domestic intermediate usage is in the light blue blocks along the diagonal. International usage (i.e. trade in intermediates) is in the off-diagonal blocks in darker blue. For example, reading down the first column of T, we see that nation A's sector 1 is using inputs from domestic

sectors 1 and 2 (as before) but it is also using intermediates from nation B's sector 1 (T_{1B1A}) and sector 2 (T_{2B1A}) . These would be observed in trade data as the import by nation A of the output of nation B's sectors 1 and 2 which is destined for intermediate consumption.

In standard trade data, it can be hard to distinguish between imports used as intermediates (the T elements) and those used as final goods (the F elements), but the distinction is clear in ICIO tables. We use a related notation to indicate the source and destination of the final good sales. For example, F_{1AB} indicates final sales to nation B from sector 1 in nation A.

By way of notation, we now write the gross output, final demand, and value-added vectors, and the intermediate matrix in Figure 4 as:

$$\mathbf{X} = \begin{bmatrix} X_{1A} \\ X_{2A} \\ X_{1B} \\ X_{2B} \end{bmatrix}, \ \mathbf{F} = \begin{bmatrix} F_{1AA} + F_{1AB} \\ F_{2AA} + F_{2AB} \\ F_{1BA} + F_{1BB} \\ F_{2BA} + F_{2BB} \end{bmatrix}, \ \mathbf{V} = \begin{bmatrix} V_{1A} \\ V_{2A} \\ V_{1B} \\ V_{2B} \end{bmatrix}$$
$$\mathbf{T} = \begin{bmatrix} T_{1A1A} & T_{1A2A} & T_{1A1B} & T_{1A2B} \\ T_{2A1A} & T_{2A2A} & T_{2A1B} & T_{2A2B} \\ T_{1B1A} & T_{1B2A} & T_{1B1B} & T_{1B2B} \\ T_{2B1A} & T_{2B2A} & T_{2B1B} & T_{2B2B} \end{bmatrix}, \ \text{and} \ \boldsymbol{\iota} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

As before, $\boldsymbol{\iota}$ is a column vector of 1s, which is useful for summing matrices across rows ($\boldsymbol{\iota}$) or columns ($\boldsymbol{\iota}'$).

2.4 Adding in complexity and realism

Throughout the paper, we continue to assume that prices are fixed, competition is perfect, and returns to scale are constant. We also maintain the homogeneity assumption. To move the discussion closer to the full complexity of the ICIO literature, we relax some of the other simplifying assumptions.

Some goods and services are consumed by governments, some are used in investment, and some are exported, so we expand our view of what is in the \mathbf{F} vector to allow for these other forms of final demand. Note that, by definition, every use except intermediate use is a final use. The terms final goods, final output, and net output are all interchangeable in our discussion. Some authors refer to the sum of consumption and investment as absorption. We continue to abstract from inventories here, so net production and absorption are identical in our discussion. Likewise, we assume away taxes, transfers, and subsidies.

In the real world, however, taxes and subsidies can be a large part of the economy. In many EU nations, for example, this type of government tax revenue and spending is a considerable share of GDP. How do these enter ICIO tables? First, taxes less subsidies on intermediate goods gets added as an extra row above the value-added row in Figure 4, and taxes less subsidies are taken account of in the value-added row since, for instance, payroll taxes paid by firms must be added to the cost accounting approach. Second, taxes less subsidies on final goods gets added as a row under the final demand rows in Figure 4. We do not add these to the figure since they complicate the analysis without providing compensating insights into the measurement of foreign exposure.

With the basics pinned down, we next turn to some uses of ICIO tables that are pertinent to the measurement of foreign exposure.

3 Using ICIO tables made easy: where are 'Americanmade' autos actually made?

How do ICIO tables help us measure foreign exposure? The core issues turn around the question: *Where are things made*? There are three levels of answers to this where-is-it-made question.¹¹

1. When a Ford rolls off the assembly line in Dearborn Michigan, we can say that the car is made in Dearborn.

That is the first-level truth; but it is not the whole truth.

2. The second level works off the fact that the Dearborn plant buys inputs from other sectors which are located at home and abroad.

If sector 1 in nation A in Figure 4 represents cars made in Dearborn, we could answer the question at the second level by reading down the first column of T to get the sector's direct purchases of inputs from all sectors in all nations, including its own. That is the second level answer. But those purchased inputs also use purchased inputs, so the second-level answer is still not the whole truth.

3. The third-level answer is the whole truth, but it is more involved since it means taking account of all the purchased inputs used to make all the purchased inputs.

For instance, one of the direct inputs purchased by sector 1A is T_{2B1A} . This is produced by sector 2 in nation B, so to find the inputs used to make T_{2B1A} , we read down the fourth column of T. There we see that the American car has indirect inputs from both sectors in both nations (the elements T_{1A2B} , T_{2A2B} , T_{1B2B} , T_{2B2B}). Plainly, we are not done tracing down all the inputs since each second-tier input also has inputs.

It would be a literally never-ending job to track down all the inputs to all the inputs since it involves an infinite sequence. This is where matrix algebra comes to the rescue.

¹¹This section draws extensively on Baldwin and Freeman (2022).

3.1 Leontief (direct and roundabout) trade linkages

In a single stroke of matrix algebra, we can solve the aforementioned infinite sequence problem and find all the inputs used to make all the inputs needed to, for example, make American autos. We do this by solving the use accounting identity, AX + F = X, for X. The solution, as we saw above, involves the famous Leontief matrix:

$$X = LF$$
, where $L \equiv (I - A)^{-1}$

The elements of L show the total linkages, where total here means taking account of all direct and indirect linkages—or as we phrased it before, counting all the inputs to make all the inputs. Because of this, the Leontief inverse is also called the total requirements matrix.

3.1.1 Vaccine requirements example

To see that the ℓ s provide the full answer to the where-is-it-made question, consider the vaccine sector. Namely, suppose the top row sector in Figure 4 is the vaccine industry in the US, and we want to know how much gross output the US would need from every sector in the world to satisfy its final demand for vaccines—taking account of all the direct and indirect linkages among all sectors.

To find the answer, we post-multiply L by the F vector with a one in the top row and zeros elsewhere (this says we need only \$1 of vaccine as a final good). The result is a list of how much gross output we need from every sector in every nation to produce \$1 of vaccines in the US—taking account of the all the direct and indirect linkages among sectors. Using matrix notation:

$$\begin{bmatrix} \ell_{1A1A} \\ \ell_{2A1A} \\ \ell_{1B1A} \\ \ell_{2B1A} \end{bmatrix} = \boldsymbol{L} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where the vector on the left is the F vector evaluated at the point where only \$1 of vaccines is produced in the US. In this sense, the ℓ s are the whole-truth answer to the *where-is-it-made* question. Specifically, each column of ℓ s gives the total requirements in terms of inputs from each sector in every nation.

3.1.2 Leontief versus direct flows

To fix ideas, contrast this list of total requirements (i.e. the third-level answer) with the gross output flows that correspond only to the direct linkages (second-level answer), i.e. the observed purchases of gross output from all sectors. For this we use the A matrix instead of the L matrix:

$\begin{bmatrix} a_{1A1A} \end{bmatrix}$		1
a_{2A1A}	_ 1	0
a_{1B1A}	-A	0
$\begin{bmatrix} a_{2B1A} \end{bmatrix}$		

To recap, the *as* give us the second-level answer to where-things-are-made while the ℓ s give us the third-level answer.¹²

3.1.3 Leontief trade flows

The Leontief matrix opens the door to a way of thinking about bilateral trade relationships that is not commonly used in international economics. The standard way is to simply look at customs data to see how much one nation, say nation A, is importing from another, say nation B. In standard writing on international economics, we would simply call this imports. In the ICIO literature, it is called direct imports to distinguish it from 'roundabout trade', or 'indirect imports'. Indirect imports refer to the intermediate goods from nation A that arrive in nation B after having been used in making goods in a third nation, say nation C. Here roundabout and indirect are taken as synonyms.

For brevity's sake we refer to the full measure of bilateral imports (direct and roundabout) as 'Leontief imports'. While routine among ICIO scholars (under various monikers), the Leontief import concept may be unfamiliar to many, so it is worth illustrating with a diagram (Figure 5).

The diagram depicts a three-nation world (China, US, and Mexico), where the US and Mexico both import parts from China, and additionally, the US imports parts from Mexico which contain Chinese parts. To be concrete, we use the traditional way of valuing imports (their value when they cross the border, or gross imports in the ICIO jargon). In this example, US Leontief imports from China count the direct US imports from China (solid orange line) plus the gross value of Chinese goods that were used as intermediate inputs into the production of goods the US imports from Mexico (orange dashed line).

Why might we worry about such indirect exposure? For example, if one nation, call it nation A, imposes sanctions on another, call it nation B, then indicators of the impact could take two natural forms. If the sanctions affect only direct from-B-to-A exports, then the right indicator of exposure would be B's exports to nation A. But if nation A gets really serious about the sanctions, it could forbid imports of goods from third nations that contain nation-B inputs. To understand exposure to this type of sanction, the analyst should use nation-A's Leontief imports from nation B instead of the standard, directly measured bilateral trade.

¹²As an aside, note that we can get two relationships between net output (the F vector) and gross output (the X vector) from the use accounting identity: $X = AX + F \leftrightarrow X = LF$ and $F = L^{-1}X$. The expression X = LF tells us how much gross output we would need to produce a given final output vector. The expression $F = L^{-1}X \equiv (I - A)X$ shows the vector of final output (i.e. net output) that corresponds to a given vector of gross output.

Figure 5: US Leontief imports from China: direct and via Mexican production



Source: Authors' illustration.

Thinking this through—and realising that some of Chinese direct exports to the US may contain inputs from Mexico—we see that there are likely to be 'innocent bystanders' hit by bilateral sanctions in sectors marked by GSCs (Grossman and Helpman 2020). In essence, this point is exactly why preferential trade agreements have rules of origin (RoO). If the US has tariffs on imports from China, but not from Mexico, there is an incentive to relabel Chinese-made goods as Mexican-made goods. RoO impose limits that prevent the most obvious forms of this by requiring, for instance, that duty-free imports contain a minimum value of Mexican value added, or that all the Chinese inputs to Mexican exports to the US involve inputs that are very different from Mexican exports.

The question *where was it made?* is not the only line of inquiry in ICIO thinking. Work and production are two very distinct concepts in a world with intermediate inputs, so we next turn to answering work-related questions.

3.2 Value-added trade: 'where was it produced' versus 'where was the work done'

So far, we have been dealing with inputs measured in terms of output flows. But just because something is produced somewhere, it doesn't mean that was where the work was done. Indeed, that is exactly why we had to ask where an American-made car was made when we knew it was assembled in Dearborn. Put differently, there is an important separation between where the production takes place and where the work takes place—exactly because of the work that is embedded in intermediate goods made in other countries and sectors. The point we are focusing on here turns on the difference between a sector's gross output and its value added. For instance, sector 1A may be buying a lot of inputs from sector 2B, but sector 2B might be an outward processing sector that imports components, adds a bit of value, and then re-exports. For this reason, the value of 1A's purchases from 2B need not be an accurate reflection of how much of the work was done in nation B's sector 2. This is exactly the point of the famous iPhone example that suggested that only 10% of the value in an iPhone is added in China, even though all iPhones are 'made in China' (Dedrick et al. 2010). With this in mind, we now turn to using ICIO analysis to solve the where-was-the-work-done question. Note that we are using the word 'work' as a shorthand for all value added whether it is generated by labour, capital, natural resources, etc.

How to track down where the value was added.—A whole set of important policy questions focus on value added. Developing nations, for example, are often interested in getting a larger share of the value added done within their borders (Taglioni and Winkler 2016). As it turns out, all the elements needed to construct measures that involve value added are already on our workbench.

The Leontief matrix shows all the intersectoral flows of gross output when every sector is producing \$1 of final goods. But, as noted, the amount of value-added embedded in any given gross output flow differs across sectors and countries. Or to put it differently, sectors' value-added-to-gross-output ratios vary.¹³ For example, the crude oil sector has a very high value-added-to-gross-output ratio since the sector uses relatively few intermediate inputs. The clothing sector, by contrast, would have a relatively low value-added-to-gross-output ratio since the value of the clothes produced (the gross output) embodies quite a lot of intermediate inputs such as cloth, which itself embed inputs such as thread and dyes, which in turn embed raw cotton, chemicals, and the like.

To get the value-added flows among all sectors, we pre-multiply each element of the L matrix with the value-added-to-gross-output ratio for the selling sector, i.e. the row sector.¹⁴ Writing out the result gives us the value-added matrix, which we call VA:

$$\mathbf{VA} = \begin{bmatrix} (V_{1A}/X_{1A}) \ell_{1A1A} & \cdots & (V_{1A}/X_{1A}) \ell_{1A2B} \\ \vdots & \ddots & \vdots \\ (V_{2B}/X_{2B}) \ell_{2B1A} & \cdots & (V_{2B}/X_{2B}) \ell_{2B2B} \end{bmatrix}$$

In short, the VA matrix shows the value-added content of the intermediate flows from the row sector to the column sector (per dollar of column-sector production).¹⁵ How should

¹³The ratio V_{1A}/X_{1A} is the value-added-to-gross-output ratio for sector 1A.

¹⁴More specifically, we pre-multiply the L matrix with a square matrix that contains value-added-togross-output ratios for the selling sector along the diagonal, and zeros on the off diagonal.

¹⁵In the IO literature this matrix is often called the value-added multiplier matrix, or the value-added requirements matrix. Further, in the context of the computation of GVC indicators the term VA has sometimes been used to refer to a vector of value-added-to-gross-output ratios (see, for example, Guilhoto et al. 2022) or a value-added content matrix once what we refer to as VA is multiplied element-wise by

we think about the elements of VA? As the vaccine example showed, each column of ℓ s shows the gross production needed to produce \$1 of final output to the corresponding row sector. As the ℓ s are multiplied by the value-added-to-gross-output ratios, each row of VA shows the value-added input (from every sector in every nation) needed to produce a dollar of final output of the corresponding row sector.

Since value added here is synonymous with work (as we are using the term), the VA matrix is a full map of where all the work was done to produce final output in every sector in every nation. The VA matrix, in other words, is what we need to answer the *where-was-the-work-done?* question. To be explicit, we introduce notation for the elements of VA for the two country, two sector example:

VA =	v_{1A1A}	v_{1A2A}	v_{1A1B}	v_{1A2B}
	v_{2A1A}	v_{2A2A}	v_{2A1B}	v_{2A2B}
	v_{1B1A}	v_{1B2A}	v_{1B1B}	v_{1B2B}
	v_{2B1A}	v_{2B2A}	v_{2B1B}	v_{2B2B}

The lowercase upsilons, the vs, show the value added from the row sector to the column sector that is embedded in a dollar of output produced by the column sector.

3.3 The VA matrix as the workbench of GVC measures

VA is a matrix that is critical to understand since we can think of it as the workbench for some of the most widely used/popularised GVC measures, including backward linkages and forward linkages (Hummels et al. 2001) and the ratio of value added to gross exports (VAX) (Bems et al. 2009, 2011; Johnson and Noguera 2012).

To make the connection with the concepts already covered, note that we read across the rows of the VA matrix to find the source of the value added that is contained in the output of a particular sector in a particular country. Say row one is for the fertiliser sector in nation A, then the top row of VA tells us how much of the total value-added produced by A's fertiliser sector was added in each of the four sectors in the world (sectors 1 and 2 in nations A and B). This is akin to the exercise we did with the Leontief matrix when the question was *where are things made*? There is a close analogy with the cost accounting approach, but only thinking about the value-added part of costs.

Another way to phrase that is to say that the columns of L tell us where things are actually made (in the gross production meaning of 'made'); the rows of VA tell us where the work was actually done (in the value-added production meaning of 'work'). Looking down the columns of the VA matrix reveals a different type of information. The column elements show where the work done by a particular sector ends up being used by other sectors. There is a close analogy looking down the columns of L to see where a sector's output is used as an input. There is a parallel with use accounting approach, but again

final demand or exports.

the focus is on value added.

There are two further topics to address before turning to the standard GVC measures. The first is linking value added and final demand. The sum of a nation's value added is its GDP (in our simple example). As is well known, GDP can also be measured as the sum of final demand, namely the value of consumption, investment, government purchases, and net exports (NX). These two approaches to measuring GDP give the same answer at the aggregate level if the country is running balanced trade (NX=0). Another use of the VA matrix is to show the links between each sector's value added and its corresponding final demand:¹⁶

$$\mathbf{V} = \mathbf{V}\mathbf{A} \ \mathbf{F} \leftrightarrow \begin{bmatrix} V_{1A} \\ V_{2A} \\ V_{1B} \\ V_{2B} \end{bmatrix} = \begin{bmatrix} v_{1A1A} & v_{1A2A} & v_{1A1B} & v_{1A2B} \\ v_{2A1A} & v_{2A2A} & v_{2A1B} & v_{2A2B} \\ v_{1B1A} & v_{1B2A} & v_{1B1B} & v_{1B2B} \\ v_{2B1A} & v_{2B2A} & v_{2B1B} & v_{2B2B} \end{bmatrix} \begin{bmatrix} F_{1AA} + F_{1AB} \\ F_{2AA} + F_{2AB} \\ F_{1BA} + F_{1BB} \\ F_{2BA} + F_{2BB} \end{bmatrix}$$
(9)

The second concerns exports. Here we will need a last bit of notation, and an important distinction. For some purposes we will be interested in exports as they are normally measured, namely as the value of the goods that cross an international border. We call these gross exports to be explicit even though in most situations we would just call them exports. The precision is necessary since in other situations we shall be interested only in the value added that is embedded in gross exports. In the literature, these are called value-added exports (i.e. the value added that is embedded in gross export flows).

In the notation from Figure 4, the vector of gross exports from all sectors and nations is:¹⁷

$$\boldsymbol{E} = \begin{bmatrix} T_{1A1B} + T_{1A2B} + F_{1AB} \\ T_{2A1B} + T_{2A2B} + F_{2AB} \\ T_{1B1A} + T_{1B2A} + F_{1BA} \\ T_{2B1A} + T_{2B2A} + F_{2BA} \end{bmatrix} \equiv \begin{bmatrix} e_{1AB} \\ e_{2AB} \\ e_{1BA} \\ e_{2BA} \end{bmatrix}$$

Here E and the *es* are, respectively, mnemonics for the export matrix and its elements.

One of the most active uses of ICIO tables in recent years has been measuring nations' and sectors' involvement in GVCs. A number of indicators of GVC involvement have come to dominate, so the next section explains these measures.

¹⁶Starting with X = LF, we pre-multiply each element of X by its corresponding value-added-tooutput ratio, e.g., V_{1A}/X_{1A} . This converts X into V. Given the procedure used to calculate the VAmatrix, we see that V = VAF.

¹⁷Note that we have simply zeroed out domestic flows.

4 Traditional GVC indicators made easy: backward and forward linkages

The GVC term was coined by the Duke University sociologist Gary Gereffi (Gereffi 1994). It was originally applied to the full gamut of international production linkages. However, in much of the trade literature which emerged in the 2000s, GVC trade was narrowed to mean only goods and services that crossed borders more than once. For example, the US import of Japanese marine engines used to make boats that are exported counts as GVC trade, but the same boats sold in the US are not considered in the standard GVC measures. The narrower definition, which assumes 'vertical specialisation' with value added crossing at least two borders was inspired by the work of Hummels et al. (2001). These authors also introduced the classic indicators of GVC participation that are now so widely used, namely backward and forward linkages.¹⁸ They were further popularised by their easy access via the OECD's TiVA database (OECD 2021a).¹⁹

The classic GVC indicators focus on the two logical aspects of a firm's, sector's, or nation's participation in GVCs—the buying-side and the selling-side. The buying-side looks at inputs that firms buy from firms abroad; the selling-side looks at inputs the firms sell to firms abroad. For historical reasons, the sourcing-side links are often called backward linkages (think of supplies being delivered to a factory's receiving ramp at the back of the factory), and the sales-side links are called forward linkages (think of the intermediate goods produced as going out the factory's front door). In a nutshell, backward linkages are about the international sourcing of value added used in exports, while forward linkages are about the international selling pattern of value added that is used in other nations' exports.

More specifically:

1. Backward Linkages (BL) capture a nation's importing to export (I2E).

The BL indicator gauges the importance of imported intermediates in exports. Note that it focuses only on exports, since GVC activity includes only goods that cross borders more than once. Importing intermediates to produce locally sold goods is ignored by the BLmeasure. To normalise, the *I2E* flow is divided by the exporting nation's gross exports.

¹⁸Here we deal with the trade-related indicators. Subsequent studies have continued to adopt the vertical specialisation assumption in their computation of further GVC participation measures (see for example Koopman et al. 2014; Borin and Mancini 2019; Borin et al. 2021). In contrast, another branch of the literature presents indicators based on final demand, where imports for domestic consumption are accounted for (see for example Johnson 2018 and Antràs and Chor 2022 who focus on GVC income). In addition to developing indicators of GVC participation, Johnson (2018), Borin and Mancini (2019), and Antràs and Chor (2022) provide comprehensive reviews of the seminal work in the field and accompanying GVC measures.

¹⁹For aggregate trade with the partner region as the world, the OECD TiVA database OECD (2021a) backward linkages indicator is $EXGR_FVASH$ (foreign value added share of gross exports) and the forward linkages indicator is $EXGR_DVAFXSH$ (domestic value added embodied in foreign exports as a share of gross exports).

2. Forward linkages (FL) capture a nation's exporting to re-export (E2R).

The FL indicator looks at how a nation is *selling* into GVCs. As such, this captures the selling-side of GVC participation. To normalise, the E2R flow is divided by the exporting nation's gross exports.

Note that there is a mismatch in the concepts used in the numerators and denominators. For historical reasons, the numerators are on a value-added basis while trade flows in the denominator are on a gross basis. Before turning to the details—and details really matter when it comes to these measures—it is worth fixing ideas with an example (Figure 6). The diagram shows a simple GVC where Canada makes parts (using domestic and imported inputs), some of which are sold to an American assembly plant, which adds other parts and other value added in turning out final autos, some of which are exported to Mexico.



Figure 6: Illustration of backward and forward linkages in a simple GVC

Specifically, the gross value of Canadian parts exported to the US is \$10,000 per car, but the Canadian value added in those parts amounts to only \$7,000 (say, the other \$3,000 comprises parts that Canada imported from Germany). The US car industry uses these parts from Canada to make final autos and exports some of these to Mexico. The gross value of one final auto exported from the US to Mexico is \$20,000 in this example.

These value-added and gross export flows are shown in Figure 6 as arrows. The top arrow reflects Canadian exports of parts with the breakdown between the gross export

Source: Authors' illustration.

per car and the value-added export per car. The bottom arrow shows US exports of a car to a Mexican customer with the gross export value noted. The key point of the example is that the \$7,000 of Canadian value added that is embedded in Canadian parts to the US are: (i) the numerator for the FL for Canada; and (ii) the numerator of BL for the US. The difference between the FL and BL lies in the denominators. The FL for Canada involves Canada's gross exports (and thus 7,000/10,000), while the BL measure for the US uses US gross exports (and thus 7,000/20,000).

The BL and FL measures are mainly used to assess a nation's or sector's involvement in GVC activity on the sourcing-side (BL) or selling-side (FL). That is to say, by showing how much imported value added is embodied in exports, BL reflects the dependence of those exports on international sourcing. Likewise, FL looks at how engaged a nation's or sector's producers are involved in GVC activity on the sales-side, but focusing only on sales to downstream firms that end up using the inputs in their exports.

To be more precise, we turn to the formulas. We introduce the formula for the backward linkage measure by answering a question in the context of different example.

4.1 Backward linkages in GVCs as an indicator of sourcing-side involvement

To construct the BL measure, we have two steps. Let's take the US and China as example nations in this case. First, we work out how much American value added goes into \$1 of China's gross production in each sector, and then scale this up by each Chinese sector's gross exports. Second, we normalise this by China's total gross exports.

If we take China to be nation B and America to be nation A, we can read off from the VA matrix how much direct and indirect value added from America is going to China. The value added from America going to Chinese sector 1B is equal to $v_{1A1B} + v_{2A1B}$ per dollar of sector 1B output. Similarly, the per-dollar amount going to Chinese sector 2B is $v_{1A2B} + v_{2A2B}$. These sums are the answer to: How much American value added goes into \$1 of China's gross production in each sector?²⁰

To find the American value added that is embedded in Chinese exports, we multiply $v_{1A1B} + v_{2A1B}$ by sector 1B's gross exports (e_{1BA}) and $v_{1A2B} + v_{2A2B}$ by 2B's gross exports (e_{2BA}) . The sum of these provides the answer to the first step—how much American value added there is in Chinese exports. This forms the numerator of the *BL* measure.

The second step is to normalise this flow by China's total exports, so the denominator of the *BL* measure is China's gross exports, namely, $e_{1BA} + e_{2BA}$. In words, the *BL* measure for China is thus:

²⁰Due to the homogeneity assumption, we assume the input in Chinese production for domestic and export sales is identical, so the input into \$1 of gross exports is the same as the input into \$1 of exports.

 $BL_{\text{China}} = \frac{\text{American value added embedded in inputs used to make Chinese exports}}{\text{Chinese gross exports}}$

or more precisely:

$$BL_{\text{China}} = \left(v_{1A1B} + v_{2A1B}\right) \frac{e_{1BA}}{e_{1BA} + e_{2BA}} + \left(v_{1A2B} + v_{2A2B}\right) \frac{e_{2BA}}{e_{1BA} + e_{2BA}}$$

Put differently, this is the share of Chinese exports where the work was actually done in America (including sourcing from 1A and 2A). It equals the sum of each sector's US value added embedded in the imported intermediates needed to make \$1 of output with each sector's sum, weighted by the sector's share of gross exports. Importantly, note that the weights are the sectoral export shares of the exporting nation (in this case China).

The BL measure can also be defined at the country-pair and sector level. In this case its numerator is simply the foreign value added embedded in the bilateral, sector-specific intermediate imports:

$$BL_{1\text{China}} = \frac{(v_{1A1B} + v_{2A1B}) e_{1BA}}{\text{Sector gross exports}}$$

The denominator is the sector's gross exports by the exporting nation (China in this case).²¹ In the two country case, this simplifies but normally we work with many countries and sectors; this introduces one point to keep in mind. With more nations, the US value added in Chinese exports could have arrived there via a third nation, e.g. the US value added might have gotten to China embedded in Japanese intermediates that China uses.

A very natural question is: Why the mismatch of trade flow concepts in the numerator and denominator of BL? The main answer is historical; that's the way it has always been done. One logic justifying it is based on the idea that BL is a decomposition of exports as normally measured (i.e. in gross terms). If a nation is highly involved in buying from global supply chains, then a large share of its exports will involve value that was added abroad. The same justification also applies to the FL indicator.

4.2 Forward linkages in GVCs as an indicator of sales-side involvement

The second classic GVC indicator is FL. This uses the same ingredients as the BL measure but focuses on the value added from the supplying country that ends up its trade partners' exports. In essence, FL is a sales-side measure, rather than a buying-side measure like BL.

²¹This is how it is defined in the OECD TiVA database (OECD 2021b). In the 2021 OECD TiVA database, the indicator foreign value added as a share of gross exports ($EXGR_FVASH$) is identical to the indicator described above at the country level. At the sector level $EXGR_FVASH$ sets the denominator to sector-specific exports. When expressed this way, $EXGR_FVASH$ represents the share of a sector's gross exports which are made up of foreign value added. Other variants of this use the country's total gross exports in the denominator, so a country's sector BLs can be added to get the country's total BL.

Sticking with the US and China example, the direct and indirect value-added flows from China to the two American sectors (1A and 2A) can be read from the **VA** matrix as before. The first Chinese sector, 1B, supplies value-added (embedded in intermediate inputs) per dollar of output of sectors 1A and 2A equal to $v_{1B1A} + v_{1B2A}$. Equally, the other Chinese sector, 2B, supplies value added to sectors 1A and 2A equal to $v_{2B1A} + v_{2B2A}$ per dollar of American output.

To get the total Chinese value added that ends up in American exports (i.e. that is re-exported by America), we multiply the per-dollar amounts by the value of American exports by sector, e.g. e_{1AB} . This total is then normalised by Chinese gross exports, so the *FL* measure for China is:

 $FL_{\text{China}} = \frac{\text{Chinese value added embedded in inputs to American exports}}{\text{Chinese gross exports}}$

or more precisely:

$$FL_{\text{China}} = \left(v_{1B1A} + v_{2B1A}\right) \frac{e_{1AB}}{e_{1BA} + e_{2BA}} + \left(v_{1B2A} + v_{2B2A}\right) \frac{e_{2AB}}{e_{1BA} + e_{2BA}}$$

Note that the weights involve US exports in the numerator and Chinese exports in the denominator. There is thus no expectation that the weights sum to unity. In words, FL for China gauges how much of its exported value added ends up in other nations' exports as a share of its total gross exports.

Given that BL and FL share a common denominator when defined at the country level (namely, gross exports), they can be summed.²² This is why their sum, called the 'GVC participation index', is often taken as a comprehensive picture of a country's integration into GVCs, as noted in Koopman et al. (2010, 2014) and De Backer and Miroudot (2013).

So far, the classic GVC indicators presented were designed to gauge a very specific aspect of international supply chain linkages. They look only at linkages where valueadded flows cross borders at least twice. This is certainly an important indicator of foreign involvement, but not the only one of interest.

4.3 Measuring a sector's overall supply chain exposure

The classic GVC indicators we have looked at are focused on foreign value chain involvement. A related question is: *How exposed is a whole sector to supply chain considerations?* The answer helps put the BL and FL indicators into perspective by contrasting them with domestic plus foreign exposure to supply chain shocks.

²²This holds for the example given in this text, however, is not always the case across publicly available sources which have pre-computed indicators readily available for download. For instance, the OECD TiVA forward participation indicator, $EXGR_DVAFXSH$, cannot be summed with the OECD TiVA backward linkage indicator, $EXGR_FVASH$, especially at the bilateral-sector level. This is because there is a mismatch in the country used in the denominator across indicators (whereby the forward participation indicator is traced within third countries' exports). Similarly, the industry dimension of $EXGR_DVAFXSH$ refers to the industry of the exports of third countries and not the domestic industry. See Section 3 of Borin et al. (2021) for further considerations and discussion.

The logic of the Leontief inverse matrix provides a simple-to-calculate and intuitively obvious measure at the level of a whole sector in a particular nation. Recall that each column of the Leontief inverse matrix provides a full list of the gross production necessary to raise the net output of the corresponding sector by \$1. Consequently, the sum of all ℓ s in a column gives us the ratio of the gross output needed by a sector and the final output of the sector (equal to \$1).

To take a specific case, the sum $\ell_{1A1B} + \ell_{2A1B} + \ell_{1B1B} + \ell_{2B1B}$ is the gross output (measured in dollars) that corresponds to \$1 of final output in sector 1B. This ratio is a straightforward gauge of how important input-output links are in a particular sector in a particular nation. A sector that uses very few intermediate inputs will have a Leontief inverse column sum that is close to unity; one that uses multiple inputs will have a column sum that is well above unity. This could be called supply chain intensity.²³

5 Measuring GSC exposure versus GVC involvement: general issues

The two classic GVC measures were not designed to measure foreign exposure. BL and FL were developed to measure GVC involvement. As such, they leave out important aspects of a nation's exposure to foreign links. The BL measure, for example, is based on value-added imports that are used to produce exports. This leaves out imports of foreign intermediates that are used to produce domestically consumed goods. In words, the BL indicator does not capture the exposure to foreign inputs of domestic production that are sold for domestic uses. This can be important.

Consider the case of India's motorcycle industry (largest producer in the world). The industry relies on imported intermediates, and for the sake of argument say half the value of its gross production consists of the cost of imported inputs. As a point of fact, India exports only $1/7^{\text{th}}$ of its production (the local market is huge). By contrast, Costa Rica's aerospace industry relies heavily on imported intermediates and again say for the sake of argument that the inputs comprise half of the value of its gross output. As the local market is non-existent, Costa Rica exports all its aerospace production. In some sense both countries are equally dependent upon foreign intermediate inputs. In both nations, foreign inputs make up 50% of gross production. But the classic *BL* GVC measure would suggest that Costa Rica is far more exposed to foreign suppliers since it only considers the value-added inputs that cross borders at least twice. This illustration points out the limits of using *BL*—which was developed to measure GVC participation—as a measure of foreign production exposure.

We turn next to an enumeration of the types of foreign exposure that do not make it into the BL and FL indicators.

 $^{^{23}}$ When domestic inputs are included, it is equivalent to the downstreamness indicator of Miller and Temurshoev (2017) and Antràs and Chor (2018).

5.1 Shades of foreign exposure: the trade onion

Figure 7, which is borrowed from Baldwin and Freeman (2022), put some numbers to the various flows we might use to measure foreign exposure. The numbers are for the US in 2015 and were calculated using the 2018 version of the OECD TiVA database. The figure shows a cascade of measures of US exposure to foreign inputs.





The outermost circle—which represents the US's total imports—is the broadest and most straightforward measure of a nation's aggregate exposure to its trade partners. The total import flow, however, comprises both final and intermediate goods and services. For many policy questions, we want to distinguish between imports for final consumption versus intermediate usage. We would want to separate, say, US imports of bananas from Honduras (a final good), and US imports of Korean semiconductors (an input). To get at this, the next layer looks at imported intermediate inputs, i.e. imports that are used in US production (or importing to produce, *I2P*, as in Baldwin and Lopez-Gonzalez 2015).

The next layer down is important when we want to distinguish between where the production versus work was done on the foreign imports. For example, the US imports many industrial inputs from Mexico that contain Chinese value added. This distinction is made by looking at the origin of the value added contained in intermediates. We could call this importing value added to produce (IVA2P). This can be important in getting a full picture of the US' exposure to foreign intermediates since the gross trade flows (I2P) may misrepresent the true location of where the work was done. In the North American context, the US exports parts for processing in Mexico and then re-imports them to be used in US production. It is not clear that we would want to include US value added in US imports of intermediates as an element of its exposure to foreign production. This leads to the use of value-added measures (the third innermost circle).

Source: Authors' replication of Figure 2 in Baldwin and Freeman (2022).

Finally, the innermost circle is the trade flow corresponding to the BL measure of GVC participation which hones in on US exports of foreign value added. This trade flow could be called importing value added to export or IVA2E.

There is, indubitably, an export trade onion with the same hierarchy. We omit the discussion for the sake of space except to say that when thinking about national exposure to foreign production, imports are the natural trade flow. When thinking about national export exposure to foreign demand, exports would be the natural flow.

The point of this discussion is to highlight the plain fact that there is nothing natural or inevitable in the value-added trade flows that are used in the classic GVC indicators. The interdependence of national producing sectors is multifaceted. Different measures spotlight different aspects of interdependence. One should think hard about what one wants to measure before selecting the trade flow (and indicator).

These points are unfair in a way since BL and FL were not designed to measure foreign exposure. There are other indicators in the OECD TiVA database that use broader trade flows. For example, one indicator called $VALU_FFDDVA$ reflects the share of domestic value added in a particular sector that is absorbed by foreign final demand.

A second shortcoming of BL as a gauge of foreign exposure is the distinction between the *where-is-it-produced* question and the *where-is-the-work-done* question, i.e. the difference between the value added in a trade flow, and the usual, gross value, of the flow.

In February 2022, when Canadian protesters blocked a cross-border bridge that is critical to the US-Canada supply chain in autos, they did not just stop the Canadian value embodied in the parts going south. They blocked the whole flow. In our terms, they disrupted the gross trade, not the value-added trade. In this way, the BL indicator measures only part of the exposure to US industry on Canadian suppliers.

5.2 Which trade flows should be used?

There are four natural categories for any measure of foreign exposure. Since we are talking about international linkages, the focal point of the measure will be a trade flow. Here there are two dimensions. Trade flows can be measured on a gross basis, which is the value observed when the goods cross the border (which is why it is sometimes called observed trade). Trade can also be measured on a value-added basis using the ICIO calculations described in Section 3. The second dimension distinguishes between direct flows and flows on a Leontief basis (i.e. that take account of direct and roundabout trade). These types of flows exist for imports and for exports.

This is where the horses for courses expression enters. As with most things in economics, the answer to when to use a specific type of flow is: *it depends*. Specifically, the choice of trade flow in the numerator depends upon what aspect of the foreign exposure we are interested in and what sorts of shocks we are worried about.

5.2.1 When gross trade flows are apt

In the case of a shock to transportation, the natural concept would be gross, not valueadded, trade flows. As we discussed above, when protesters closed a bridge that was vital to the US auto industry, which traverses the Canada-US border, the protesters stopped gross trade, not just the Canadian value added embodied in the goods. Likewise, the Covid-19 pandemic-related snarls in sea cargo, or the running aground of the Evergreen ship in the Suez Canal, were a shock to gross trade flows regardless of where the work was actually done.

To take another example, During the Covid-19 pandemic, sea-based transportation was disrupted by things like countries' restrictions on foreigners entering the country to prevent the spread of the virus. This made crew changes impossible at ports. One example where this happened was in Singapore. Since Singapore is so central to cargo shipping, factories and consumers in Europe had trouble getting goods from China. The impact on, say, France did not depend upon the amount of value added in the shipping containers, it depended on their full value.

Many types of industrial disruptions could also be naturally associated with shocks to gross flows rather than net flows since industrial disruptions tend to alter industries' total output, not just their local value-added component.

5.2.2 When value-added trade flows are apt

One situation where it is clear that value-added rather than gross flows should be used arises from questions related to the vulnerability of value-added activity. The basic relationship V = VA F shows that shocks to net output, F, and value-added activities, V, will pass through value-added trade flows.

Another argument for using value-added flows requires a step beyond the simple ICIO framework. For example, if one Caribbean nation is making clothes that are exported to the US but where very little value is added to all the components the Caribbean nation imports, then it might be rather easy to move the production to an alternative producer. The idea here is that a small value-added flow would suggest much less vulnerability than the gross flow would indicate. This line of reasoning uses the value-added-to-trade ratio as an imperfect yardstick for substitutability. As the whole ICIO structure relies on extreme forms of substitutability (namely, there is none), this reasoning is outside the model.

5.2.3 Observed versus Leontief concepts for trade flows

A more straightforward choice involves the distinction between observed and Leontief imports (i.e. imports by the nation concerned both directly from a bilateral partner and via inputs embedded in goods that arrive after passing through third nations). It is more straightforward since the nature of the shock will normally provide guidance. For example, a natural disaster that hit a particular sector in a particular nation suggests using a sectoral source-country trade flow. Since the disruption would hinder the exporters' sales everywhere, we should use a trade flow that encompasses direct and roundabout trade, which means the Leontief imports concept. If, by contrast, the shock is to the bilateral transportation flow only, then roundabout flows will not be disrupted so the observed measure would provide a clearer indicator of exposure to the bilateral shock.

Since the dollar value of the foreign exposure has to be judged as big or small compared to something, indicators will scale the trade flow by some measure of domestic activity. That is the next topic.

5.3 Candidate normalisers: aggregates of concern

The normaliser choice (which will be the denominator in a typical indicator of foreign exposure) depends on the analyst's concern. The point is obvious in the classic measure of foreign exposure, the trade-to-GDP ratio. Trade is a dollar-value measure of exposure and GDP is a dollar-value measure of the thing that is exposed. GDP, however, is not the only conceivable normaliser. GDP is measured on a value-added basis while observed trade is measured on a gross value basis, so one might want to use the gross-value equivalent of GDP which is the nation's gross production (the sum of its Xs).

Another approach puts the foreign exposure in the context of exposure to all sources, both domestic and foreign. When it comes to purchasing inputs from a foreign country (imports), the normaliser in this approach is the total purchases of intermediates from all sources, which we can call total inputs. When it comes to sales to foreign customers (exports), the normaliser is the gross sales to domestic and foreign customers. One advantage of this approach is that the resulting indicator of foreign exposure will be expressed as a share. On the buying-side, it will be the foreign exposure as a share of total inputs, \boldsymbol{L} ; on the selling-side, it will be a share of total sales, \boldsymbol{X} . This is the approach we follow explicitly below, but before turning to those calculations, we note that there are alternative normalisers.

In the ICIO context, there are two additional obvious normalisers, production measured: (i) on a net basis; and (ii) on a value-added basis. These correspond to elements of \mathbf{F} and \mathbf{V} . Another set of normalisers involves components of final demand. Aggregate final demand of a nation is equal to its GDP (if trade is balanced), but final demand comprises household consumption, government purchases, and investment. Depending upon the economic activity of concern, the scaling factor for the trade flow could be household or government consumption, or investment. For example, the share of food that is imported would focus on the vulnerability of food consumption to foreign production. If the concern is, say, government purchases of equipment for the army, the suitable denominator would be total military spending on foreign inputs. If the aggregate of concern was capital formation, then the right denominator might be investment. Finally, there will be situations where a trade flow is the natural matter of concern. If a country is worried about its foreign currency inflow, then its exports would be an ordinary divisor. For Canada, one might use the total value of exports to the US. If, for instance, Canada is particularly worried about its export earnings, it might want to consider the fundamental international supply chain linkage as a share of all Canadian exports—either to the world, or just to the US.

So, which normaliser is the right one? The answer depends on one's concerns, in other words, on the fundamental international linkage to which one is worried about being exposed. Consider another example. Colombia has an auto sector that uses many imported inputs. General Motors (GM), in particular, has an annual production capacity in the country of about 100,000 units, so by the gross production concept, the size of the economic activity is the value of all the final vehicles sold by the Colombian subsidiary of GM. But many of the parts for these cars are imported, so the value added of the GM activity is far less than the sales. Roughly speaking, one could associate the value-added number with employees, and the gross production number with business activity, or turnover. When thinking about Colombia's foreign involvement in the auto sector, both value-added and gross production figures could be of interest. The choice boils down to the focus of the exposure under study. If the issue is the exposure of Colombian jobs, value-added might be the right choice. If it is business activity more broadly, then gross production might be more appropriate.

The main point is that there is no correct normaliser. The analyst will have to think hard about which aspect of the domestic economy is exposed to the foreign supply linkage.

5.4 Exposure is a two-way street

The last general point to stress is that foreign exposure is not a unilateral concept; it cuts both ways. When a firm in one nation sells an intermediate to a firm in another nation, both the seller and the buyer are exposed to a foreign risk. The main point here is that a single trade flow creates foreign exposure on the sourcing-side for the buyer, and on the selling-side for the seller. The point was illustrated graphically in Figure 4. The gross export of \$10,000 worth of parts from Canada to the US will be the numerator for an indicator of US risk, on the one hand, and Canadian risk, on the other. This is why, when thinking about the appropriate indicator, the analyst has to be clear about who the foreigner is in any foreign exposure measure.

Finally, we note that confusion can arise from the fact that there are several common phrases for the same concepts; there are many ways of referring to the two sides of the two-way street. On the selling-side, we can call this exposure to the: 'export-side'; 'demand-side'; 'selling-side'; or 'forward linkage-side'. On the buying-side, we can call it the: 'import-side'; 'supply-side'; 'sourcing-side'; or 'backward linkage-side'. Given this ambiguity of language, one ultimately must look at the formulas to understand what is really being specified by a particular indicator of foreign exposure.

6 Measuring foreign exposure with ICIO tables: indicator design

Many indicators of international supply chain relationships are available online. The OECD TiVA database (OECD 2021b), for instance, lists over 30 different indicators. The different indicators embrace various linkages and normalisers. Many of these choices seem intuitive, but intuition doesn't always provide clear guidance on which indicators are most suitable to the numerous questions a policymaker may face. This section introduces a systematic approach to the design of foreign exposure indicators. The framework might be called the 'shocks approach' as it seeks to connect the measure of exposure to various types of foreign shocks (see Miller and Blair 2009 for related analysis). Box 1 lists some caveats to keep in mind when using the shocks approach to indicator design.

Box 1: Caveats to the shocks approach to indicator design

These caveats are related to the fact that the indicators are trying to capture what might happen in reaction to shocks without allowing the normal adjustments that would work through price changes.

- The first point is that the analysis takes prices as fixed. Of course, prices will react to shocks, and the price changes would lead to substitutions in production and consumption. It is therefore best to think of the comparative statics—and thus the exposure indicators—as linear approximations of the impacts of small shocks. Finding the full impact would require a simulation model with vastly greater sophistication, but also a great deal more assumptions.
- The second point is that the variables we will be looking at, X, F, and V, are endogenous. As such, it is not really correct to shock them. We should have a model of the mechanism by which the endogenous variables are shocked, e.g. things like an explicit treatment of shipping linkages that could be disrupted, or labour forces that could go on strike. The defence for doing so is that we want to think about events outside the model—disruptions such as labour strikes (dV), flooded factories (dX), or consumer boycotts (dF).
- The third point is that the values in ICIO tables are in current prices. This is not much of an issue when comparing across nations for a single year, but it is when looking at the measures over time—especially over a longer time horizon when commodity input prices fluctuated enormously.

To be precise, in our approach X = LF is used to gauge the impact of shocks to F on X, or vice versa, V = VAF is used to gauge the impact of shocks to F on V, or
vice versa, and X = GV is used to gauge the impact of shocks to V on X, or vice versa. Combining these we could also look at the V to F linkage with $F = L^{-1}GV$.

6.1 The basics of the shocks approach

The starting point for all indicators of foreign exposure is an international trade flow. These are implicitly defined by the set of three key ICIO identities discussed above. These tell us how the three primary variables of interest are related, namely gross output, X (which specifies where overall business activity takes place), net output F (which specifies where the work is done). But which flows should be used?

The shocks approach answers the question by totally differentiating the relevant identity. To be concrete, we start with the use accounting identity, X = LF. The question of how sectors located in two different economies are linked via supply chains is fully answered by the L matrix. For instance, if one nation, say nation B, wants to increase net output of one sector, say sector 1, the L matrix tells us how much gross production has to rise in a particular foreign sector, say sector 2 in nation A.

To find the linkages between domestic and foreign sectors, we totally differentiate X = LF:

$$\begin{bmatrix} dX_{1A} \\ dX_{2A} \\ dX_{1B} \\ dX_{2B} \end{bmatrix} = \begin{bmatrix} \ell_{1A1A} & \ell_{1A2A} & \ell_{1A1B} & \ell_{1A2B} \\ \ell_{2A1A} & \ell_{2A2A} & \ell_{2A1B} & \ell_{2A2B} \\ \ell_{1B1A} & \ell_{1B2A} & \ell_{1B1B} & \ell_{1B2B} \\ \ell_{2B1A} & \ell_{2B2A} & \ell_{2B1B} & \ell_{2B2B} \end{bmatrix} \begin{bmatrix} dF_{1A} \\ dF_{2A} \\ dF_{1B} \\ dF_{2B} \end{bmatrix}$$
(10)

where d represents a given change in the variable of interest. For convenience, we introduce a shorthand for the sum of home and foreign consumption/absorption of this final output, e.g. $F_{1A} = F_{1AA} + F_{1AB}$.

Two comments are in order. First, this expression shows the necessary gross production increases (dX_{ij}) —in every sector in every nation—that are necessary to increase net output (dF_{ij}) in every sector in every nation. This establishes foreign exposure since it means that the net output in one nation is dependent upon—or to put it differently, exposed to—gross production in another economy. The second comment is that these are not behavioural equations. We cannot say that F causes X since we are working with the manipulation of the identify X = LF. As written, it looks like shocks to F are causing changes in X, but we could equally think of the shocks to X causing changes in F.²⁴

The second step in the shocks approach is to select a specific shock to consider. We may be interested in, for example, the increase in Indian adjuvant production that would be needed to boost French vaccine production. To be concrete, we consider the example of a shock to nation B's net output of sector 1 (dF_{1B}), so the specific question is: How much more output would sector 1 in importing-nation B need from sector 2 in exporting-nation

 $^{^{24}\}mathrm{See}$ Box 1 for further caveats to the shocks approach.

A to raise the net output of 1B? In symbols, the question is what happens when the shock is $dF_{1B} = \$1?^{25}$

To get the overview of all necessary inputs, we note that for the shock under consideration (dF_{1B}) , we take the third element of \mathbf{F} as equals 1 and the others as zero, so equation (10) simplifies to:

$$\begin{bmatrix} dX_{1A} \\ dX_{2A} \\ dX_{1B} \\ dX_{2B} \end{bmatrix} = \begin{bmatrix} \ell_{1A1B} \\ \ell_{2A1B} \\ \ell_{1B1B} \\ \ell_{2B1B} \end{bmatrix} dF_{1B}$$
(11)

In words, the third column of Leontief coefficients provides a list of extra gross production necessary to raise the net output of sector 1B by \$1. Two of these four sectors (1B and 2B) are domestic in the case at hand, so they will not be a concern when thinking about foreign exposure.

The third step is to choose a foreign supplying sector on which to focus. Below we expand this to include dependence on multiple foreign supplying sectors; for now, we take sector 2A. Thus, equation (11) simplifies to:

$$dX_{2A} = \ell_{2A1B} \, dF_{1B} \tag{12}$$

This defines the linchpin of foreign dependence. It says that 1B can only expand if 2A also expands. In this sense, it gauges 1B's exposure to 2A.

To be absolutely clear about the approach here, we explain it in terms of Figure 8, where the red lines show the connection between an increase in the net output of sector 1B and the requisite increase in sector 2A. The answer is ℓ_{1A1B} per dollar increase in 1B net output.

Figure 8: The shocks approach to indicator design with Leontief identity Foreign Production Exposure: Import-side: If home nation-B wants to expand final output in sector 1 $(dF_{1B} > 0)$, how much do Leontief imports from foreign nation-A's sector 2 have to rise (dX_{2A}) ?

$$dX_{2A} = \ell_{2A1B} dF_{1B} \begin{bmatrix} dX_{1A} \\ dX_{2A} \\ dX_{1B} \\ dX_{2B} \end{bmatrix} = \begin{bmatrix} \ell_{1A1A} & \ell_{1A2A} & \ell_{1A1B} & \ell_{1A2B} \\ \ell_{2A1A} & \ell_{2A2A} & \ell_{2A1B} & \ell_{2A2B} \\ \ell_{1B1A} & \ell_{1B2A} & \ell_{1B1B} & \ell_{1B2B} \\ \ell_{2B1A} & \ell_{2B2A} & \ell_{2B1B} & \ell_{2B2B} \end{bmatrix} \begin{bmatrix} dF_{1A} \\ dF_{2A} \\ dF_{1B} \\ dF_{2B} \end{bmatrix}$$

Source: Authors' illustration.

The last step is to choose a normaliser for the trade flow, i.e. a denominator for the indicator of foreign exposure. To illustrate the approach, we derive six indicators that are useful in many situations, as we shall see in the next section.

²⁵Note that a final output surge $(dF_{1B} > 0)$ and disruption $(dF_{1B} < 0)$ are the same type of shock with opposite signs, so the answer we discover applies equally to a sudden loss of final output.

6.2 Foreign exposure indicators based on Leontief gross trade

The two indicators introduced in this subsection use Leontief trade on a gross basis for the numerator. The first looks at the foreign exposure of the buying sector.

6.2.1 FPEM: import-side exposure normalised by purchases from all sources

The expression (12) shows the dollar value of international supply chain linkages between sectors 1B and 2A. To gauge how exposed the buying sector 1B is, we normalise by its total purchases from all sectors (on a gross production basis), namely:

$$TI_{1B} \equiv F_{1B} \left(\ell_{1A1B} + \ell_{2A1B} + \ell_{1B1B} + \ell_{2B1B} \right)$$

where TI_{1B} stands for total intermediate inputs. This, in words, is the total value of gross production that is purchased by sector 1B from all sources including itself.

Forming shares and proportional changes using standard hat algebra, equation (12) becomes:

$$\frac{dX_{2A}}{TI_{1B}} = 100 * \left(\frac{F_{1B}\,\ell_{2A1B}}{TI_{1B}}\right)\frac{dF_{1B}}{F_{1B}} \tag{13}$$

In words, the right-hand side defines the increase in intermediate inputs 1B would need from 2A in order to raise 1B's output by dF_{1B}/F_{1B} ; the increase is presented as a share of all the inputs that 1B is currently using.

Taking $dF_{1B}/F_{1B} = 1$, turning it into an index based on 100, and simplifying, gives us the *FPEM* indicator:

$$FPEM_{2A1B} = 100 * \left(\frac{\ell_{2A1B}}{\ell_{1A1B} + \ell_{2A1B} + \ell_{1B1B} + \ell_{2B1B}}\right)$$

which stands for Foreign Production Exposure: Import-side (FPEM). In words, the indicator can be expressed as:

$$FPEM_{2A1B} = 100 * (Sector 2A's share of 1B's intermediate inputs)$$
 (14)

This is a gauge of how exposed the importing sector 1 in nation B is to inputs from sector 2 in nation A^{26}

To clarify concepts and avoid common confusions, consider four FAQs:

FAQ 1: Does FPEM mix value added and gross concepts like the BL and trade/GDP ratio?

The answer is no. *FPEM* involves imports on a Leontief basis in the numerator using the gross production concept. The denominator of *FPEM* is on the same basis, namely Leontief and gross.

²⁶Here we maintain the 'from, to' subscript convention; the subscripts indicate the buying sector and selling nation respectively, i.e. in this case, sector 1B is buying gross production from 2A.

FAQ 2: Why does the denominator include domestically sourced inputs as well as foreign sourced inputs?

The answer is that we are interested showing how important foreign exposure is to domestic production, so we scale the foreign sourcing by all inputs.

FAQ 3: Does FPEM necessarily lie between zero and 100?

The answer is yes. *FPEM* is part of a proper and complete decomposition of the sources of the gross production from every sector and every nation necessary to make \$1 of final 1B output.

FAQ 4: Does FPEM only work for single sector exposures?

The answer is no. Below we consider versions of *FPEM* that include more foreign sectors (say, all nation A supplying sectors), and more domestic sectors (say, all nation B sectors that buy intermediate inputs from nation A).

6.2.2 FPEX: Export-side exposure normalised by sales to all buyers

When a firm in one nation sells something to a firm in another nation, both firms have a foreign exposure. *FPEM* was a measure of the buyer's exposure; here we motivate a measure of the seller's exposure.

The indicator of the seller's exposure uses the same dollar value of the linkage between 2A and 1B, $F_{1B}\ell_{2A1B}$, but we instead divide it by 2A's total sales to determine how much the linkage matters to sector 2A. For sector 2A, total sales equals total output, specifically:

$$X_{2A} = F_{1A} \ell_{2A1A} + F_{2A} \ell_{2A2A} + F_{1B} \ell_{2A1B} + F_{2B} \ell_{2A2B}$$

The resulting indicator is:

$$\frac{dX_{2A}}{X_{2A}} = 100 * \left(\frac{F_{1B}\,\ell_{2A1B}}{X_{2A}}\right) \frac{dF_{1B}}{F_{1B}}$$

We call this Foreign Production Exposure: Export-side, or FPEX for short. In words:

 $FPEX_{2A1B} = 100 * (Sector \ 1B's \text{ purchases as a share of } 2A's \text{ gross output})$

As with *FPEM*, *FPEX* is bound between zero and 100 due to X = LF. To recap, $FPEX_{2A1B}$ is a measure of how dependent sector 2A is on sales to sector 1B.²⁷

There are two caveats to keep in mind with Leontief, gross-trade indicators like *FPEM* and *FPEX*. First, they are based on gross (and not net) production concepts. As such, both the direct and indirect components of Leontief imports include accumulated gross production from third nations. Second, and relatedly, since the indicators are based on gross trade and production concepts, they will necessarily double count trade flows.

Depending upon the intended use of the measure, it might therefore be preferable to use value-added trade flows instead of gross trade flows. We now turn to considering such

 $^{^{27} {\}rm Imbs}$ and Pauwels (2020, 2022) introduce a corresponding measure called 'Higher Order Trade (HOT)' as well as other HOT variants.

indicators.

6.3 Exposure indicators based on Leontief imports on a valueadded trade basis

The next pair of indicators concentrates on linkages that connect shocks to final sales in one nation to sectoral value-added in another. Why would we change the focus? As argued above, we can loosely think of gross production as business turnover, and value added as employment (of factors). To get indicators of value-added exposure to foreign production, we perform isomorphic calculations with the value-added identity: V = VA F.²⁸

Given the simplicity of the shocks approach, we do not repeat the steps, but jump straight to the analogue of equation (12). The value-added response in nation A (the exporting nation) that is necessary to accommodate a net production shock in nation B's sector 1 is governed by the relevant elements of the VA matrix:

$$dV_{2A} = v_{2A1B} \, dF_{1B} \tag{15}$$

In words, dV_{2A} is the 2A value-added increase that is necessary to accommodate an increase of 1B final sales. To evaluate how important this is for sector 1B, i.e. how vulnerable 1B is to 2A, we normalise dV_{2A} by the sum of all the value-added responses necessary to accommodate dF_{1B} . This is the total value of intermediate inputs measured on a value-added basis (TIV): $TIV_{1B} = F_{1B} (v_{1A1B} + v_{2A1B} + v_{1B1B} + v_{2B1B})$.

The resulting indicator is:

$$FPEMV_{2A1B} = 100 * \left(\frac{v_{2A1B}}{v_{1A1B} + v_{2A1B} + v_{1B1B} + v_{2B1B}}\right)$$

We call this the FPEMV indicator, where the suffix V refers to the fact that the flows of the FPEM indicator from above are evaluated on a value-added (rather than a gross) basis. Since the value-added valuation and Leontief trade concepts are used in the numerator and denominator, it is an index that ranges from zero to 100.

In words, FPEMV gauges the importing sector's exposure to the value-added content in its purchases from a foreign sector. The term in parentheses is the value-added content of sector 1B's purchase of intermediate inputs from sector 2A as a share of the value added in all of 1B's intermediate input purchases.

As before, the importing sector's exposure to the foreign supplier of intermediates is also a foreign exposure for that supplier. That is, the exact same bilateral linkage, $F_{1B}v_{2A1B}$, is the basis of an indicator of sector 2A's exposure to 1B. The difference is that we normalise by the selling sector's total value added, V_{2A} , instead of the buying sector's total purchases of value added as in *FPEMV*. Thus:

$$FPEXV_{2A1B} = 100 * \left(\frac{F_{1B} v_{2A1B}}{V_{2A}}\right)$$

²⁸Recall that V = VAF is derived from X = LF, so we can think of it as a use accounting identity that tells us how the value added of each sector is ultimately distributed (used) by all other sectors.

where, as before, the V suffix denotes that the flows of the *FPEX* indicator are evaluated on a value-added basis. Note that the term in parenthesis is a proper fraction since:

$$V_{2A} = F_{1A} v_{2A1A} + F_{2A} v_{2A2A} + F_{1B} v_{2A1B} + F_{2B} v_{2A2B}$$

due to equation (9). Resultantly, FPEXV ranges from zero to 100.

6.4 Ghosh-based indicators: linkages between value added and gross production

The same shocks approach can be easily applied to looking at linkages between the value added of every sector in every nation and the gross production in every sector using the Ghosh identity, X = GV. There are a few differences that deserve attention.²⁹

Figure 9: The shocks approach to indicator design with Ghosh identity Foreign Value-added Exposure: Import-side: If home nation-B wants to expand value added in sector 1 $(dV_{1B} > 0)$, how much does sector 2's gross production have to expand in foreign nation A to accommodate this rise (dX_{2A}) ?

$$dX_{2A} = g_{2A1B}dV_{1B} \begin{bmatrix} dX_{1A} \\ dX_{2A} \\ dX_{1B} \\ dX_{2B} \end{bmatrix} = \begin{bmatrix} g_{1A1A} & g_{1A2A} & g_{1A1B} & g_{1A2B} \\ g_{2A1A} & g_{2A2A} & g_{2A1B} & g_{2A2B} \\ g_{1B1A} & g_{1B2A} & g_{1B1B} & g_{1B2B} \\ g_{2B1A} & g_{2B2A} & g_{2B1B} & g_{2B2B} \end{bmatrix} \begin{bmatrix} dV_{1A} \\ dV_{2A} \\ dV_{1B} \\ dV_{2B} \end{bmatrix}$$

Source: Authors' illustration.

As illustrated by Figure 9, the touchstone international supply chain linkage in this case is given by:

$$dX_{2A} = g_{2A1B} \, dV_{1B}$$

The left-hand side of the expression is the amount that gross production in 2A must rise to absorb (or, equivalently, accommodate) a slight increase in V_{1B} . A natural normaliser for this flow is the sum of the increases in gross production that are necessary to absorb that higher value added in 1B. This corresponds to the total intermediate inputs used by sector 1B, now defined on a Ghosh basis:

$$TI_{1B} = V_{1B} \left(g_{1A1B} + g_{2A1B} + g_{1B1B} + g_{2B1B} \right)$$

Our indicator is:

$$\frac{dX_{2A}}{TI_{1B}} = \left(\frac{V_{1B}\,g_{2A1B}}{TI_{1B}}\right)\frac{dV_{1B}}{V_{1B}}$$

²⁹The IO literature points out that there are difficulties in mapping the Ghosh identity into economic mechanisms. The Ghosh model, also called the supply-side version of the Leontief model, has value added determining gross output and this has been criticised for its difficult interpretation and bizarre implications (Aroche Reyes and Marquez Mendoza 2021). When shocking the value added of one sector, the Ghosh identity shows the increase in gross output in all sectors that must occur to accommodate the shock. However, gross output cannot increase in the IO setting without the value added of other sectors increasing as well. In other words, it works as a system, but is incomplete when thinking about a single value-added shock. Here we ignore the knock-on adjustments and take the sector-to-sector responses as an approximation of the full adjustment.

In words, the indicator is the relevant element of the Ghosh matrix divided by the sum of the relevant Ghosh column. It simplifies to:

$$FVEM_{2A1B} = 100 * \left(\frac{g_{2A1B}}{g_{1A1B} + g_{2A1B} + g_{1B1B} + g_{2B1B}}\right)$$

Here FVEM stands for Foreign Value-added Exposure: Import-side. The same international supply chain linkage, $dX_{2A} = g_{2A1B} dV_{1B}$, creates a foreign exposure for sector 2A. The natural normaliser here is gross output of 2A, so the indicator is:

$$\frac{dX_{2A}}{X_{2A}} = \left(\frac{V_{1B}\,g_{2A1B}}{X_{2A}}\right)\frac{dV_{1B}}{V_{1B}}$$

which simplifies to:

$$FVEX_{2A1B} = 100 * \left(\frac{V_{1B} g_{2A1B}}{X_{2A}}\right)$$

Here FVEX stands for Foreign Value-added Exposure: Export-side. This ranges from zero to 100 since $X_{2A} = V_{1A} g_{2A1A} + V_{2A} g_{2A2A} + V_{1B} g_{2A1B} + V_{2B} g_{2A2B}$.

6.5 Shocks involving sector to sector disruptions

The shocks considered have all involved sector-level changes. The approach can be applied to more specific shocks when, say, something disrupts the exports of one sector in one nation to a sector in another nation. For example, if the US cut off semiconductor sales to the car industry in Thailand. For this sort of shock, the first step in the shocks approach is to totally differentiate with respect to the elements of the Leontief matrix as well as the net and gross output vectors:

$$\begin{bmatrix} dX_{1A} \\ dX_{2A} \\ dX_{1B} \\ dX_{2B} \end{bmatrix} = \begin{bmatrix} d\ell_{1A1A} & d\ell_{1A2A} & d\ell_{1A1B} & d\ell_{1A2B} \\ d\ell_{2A1A} & d\ell_{2A2A} & d\ell_{2A1B} & d\ell_{2A2B} \\ d\ell_{1B1A} & d\ell_{1B2A} & d\ell_{1B1B} & d\ell_{1B2B} \\ d\ell_{2B1A} & d\ell_{2B2A} & d\ell_{2B1B} & d\ell_{2B2B} \end{bmatrix} \begin{bmatrix} F_{1A} \\ F_{2A} \\ F_{1B} \\ F_{2B} \end{bmatrix} + \begin{bmatrix} \ell_{1A1A} & \ell_{1A2A} & \ell_{1A1B} & \ell_{1A2B} \\ \ell_{2A1A} & \ell_{2A2A} & \ell_{2A1B} & \ell_{2A2B} \\ \ell_{1B1A} & \ell_{1B2A} & \ell_{1B1B} & \ell_{1B2B} \\ \ell_{2B1A} & \ell_{2B2A} & \ell_{2B1B} & \ell_{2B2B} \end{bmatrix} \begin{bmatrix} dF_{1A} \\ dF_{2B} \\ dF_{1B} \\ dF_{2B} \end{bmatrix}$$

The exact numerator of the indicator would depend upon the nature of the shock. One might ask, for example, what the impact would be on, say, 2A gross output, if $d\ell_{1A2B} < 0$ but there was no change in the F vector. Plainly, many other logical possibilities arise.

6.6 Aggregate foreign exposure measures: the issue of sector weights

Next, we consider a broader shock involving all nation B net production levels, not just 1B. Here the first question is: How much more output would nation B need from nation A sectors to raise its net output in all sectors by \$1? As before, we start with the touchstone linkages from X = LF, which in this case are:

$$d(X_{1A} + X_{2A}) = (\ell_{1A1B} + \ell_{2A1B}) dF_{1B} + (\ell_{1A2B} + \ell_{2A2B}) dF_{2B}$$

To see if this increase in gross production, $d(X_{1A} + X_{2A})$, is a big number for importing nation B, we normalise it by nation B's entire purchases of inputs used for its final goods production (measured on a gross production basis), namely:

$$TI_B = F_{1B} \left(\ell_{1A1B} + \ell_{2A1B} + \ell_{1B1B} + \ell_{2B1B} \right) + F_{2B} \left(\ell_{1A2B} + \ell_{2A2B} + \ell_{1B2B} + \ell_{2B2B} \right)$$

Taking $dF_{iB}/F_{iB} = 1$, for i = 1, 2, the aggregate *FPEM* is thus:

$$FPEM_B = \frac{F_{1B} \left(\ell_{1A1B} + \ell_{2A1B}\right) + F_{2B} \left(\ell_{1A2B} + \ell_{2A2B}\right)}{TI_B}$$

Having gone through several applications of the shocks approach, we leave the derivation of other indicators to readers. In particular, the ICIO identity X = GV could be used with the shocks approach to look at linkages between value-added and gross output shocks.

One whole set of indicators we did not yet discuss concern direct, or observed imports, which is to say the bilateral imports that can be directly read from trade data. These are measured in gross production terms. Although such flows miss the indirect flows, they may be relevant for certain types of shocks. For instance, sanctions and tariffs are imposed on direct bilateral trade flows, so there will be instances when analysts should use direct imports rather than Leontief imports. Such indicators are the next topic.

6.7 Bilateral issues: Leontief versus direct trade flows

Calculated bilateral trade relationships include direct and roundabout linkages (Figure 5). If an analyst wanted to calculate the impact of sanctions on a particular economy, the choice of direct or Leontief would depend upon the nature of the sanctions. For example, the Trump administration first imposed restrictions on Chinese purchases of US chips, so the sanctions were purely bilateral. In this case, the direct trade flow is relevant. Subsequently, the sanctions were extended to try to prevent third nations from selling semiconductors or semiconductor manufacturing equipment to China. Here the Leontief trade concept is appropriate.

To design indicators of direct-only bilateral foreign exposure, we must switch identities. Recall that in the use-accounting identity intermediate goods can be expressed as:

$T\iota = AX$

Observe that the AX vector lists all the trade in intermediate inputs from every sector in every economy to every sector in every economy. It is thus the A matrix that will be the ultimate source of indicators rather than the L matrix, as with *FPEM* and *FPEX*.

Having worked through the shocks approach, it is clear that in the same spirit we will substitute elements of A for elements of L when constructing analogous indicators of direct exposure. For instance, the direct-only equivalent of *FPEM* is:

$$OFPEM_{2A1B} = 100 * \left(\frac{a_{2A1B}}{a_{1A1B} + a_{2A1B} + a_{1B1B} + a_{2B1B}}\right)$$

where OFPEM stands for Observed Foreign Production Exposure: Import-side. The O stands for observed since it is based on trade flows as they are actually observed in official trade data.³⁰ There are obvious indicators for direct-only flows that correspond to those developed above. As their derivation is straightforward, we leave this to the reader.

6.8 A mental map to indicator design and naming convention

Our naming convention uses F for foreign, PE for production exposure, and VE for value-added exposure. The M and X in the indicator names reflect whether the indicators are defined on the import-side (M) or the export-side (X). The suffix V is added when the numerator is evaluated on a value-added basis rather than a gross basis and the prefix O is added to denote observed trade relationships.

Having introduced the concepts above, we now present how the different indicators fit together via a mental map. For brevity, Figure 10 focuses on the set of indicators which are derived from the use accounting identity (some of which are also featured in the subsequent section) and presents them via an indicator mental map. The second row of the map shows the linkage type (observed versus Leontief).

Subsequently, these nodes split to show whether the trade flow valuation uses gross or value-added concepts, and the related ICIO identity used in the total differentiation for each indicator's numerator. From here, we depict whether the foreign exposure is on the import- or export-side, which informs the denominator choice. As can be seen, on the import-side the options are total direct inputs (T) for the *OFPEM* measure, and total Leontief inputs (L) and total value-added inputs (VA) for the *FPEM* and *FVEM* indicators, respectively. On the export-side, the denominator choice for both the *OFPEX* and *FPEX* measures is the exporter's total gross outputs (X)³¹ while that for the *FPEXV*

³⁰Note that in *OFPEM*, all *a* terms are normalised by the same gross output $(X_{1B}$ in this example). As such, the indicator simplifies to $OFPEM_{2A1B} = 100 * \left(\frac{T_{2A1B}}{T_{1A1B}+T_{2A1B}+T_{1B1B}+T_{2B1B}}\right)$. In words, this is the ratio of imported intermediate inputs to total inputs (domestic and imported).

 $^{^{31}}$ As discussed earlier, one could also choose a different denominator. For example, the *OFPEX* measure could also be normalised using total exports.

measure is the exporter's value added (V).



Figure 10: Indicator mental map for use accounting identity

Source: Authors' illustration.

Notes: BL is conceptually linked to FPEMV but only uses imported value added that is embedded in a nation's total gross exports and is denomenated by total gross exports. FL is somewhat conceptually linked to FPEXV, but only uses the exported domestic value added that is embedded in partners' exports, as a share of the home country's total gross exports. Summing across columns of the L matrix (i.e. the denominator of FPEM) is equivalent to the downstreamness indicator of Miller and Temurshoev (2017) and Antràs and Chor (2018).

Of course, while the indicator mental map shows the relationships for the use accounting identity, an analogous mental map could also be derived for the cost accounting identity. We turn now to some straightforward applications of the indicators. In particular, we use the various indicators from Figure 10 above—in addition to the classic BL GVC participation measure—to spotlight the horses for courses point and paint a line sketch important headline facts about the world's supply chain. We focus exclusively on manufacturing as this sector is often the focus of concern when it comes to foreign exposure.

7 Horses for courses: how different indicators illuminate different facets of the GSC reality

This section takes the indicators developed above for a 'test drive'. It uses the indicators to illuminate a handful of facts that are critical to understanding the realities of today's international supply chain linkages, and through them, the realities of foreign exposure. In doing so, we elucidate some differences among the various indicators as well with traditional GVC measures. In short, this is the section where we show that analysts should match 'horses' (foreign exposure indicators) with 'courses' (the shocks and domestic variables of interest). For consistency, all the indicators in this section are computed using the 2021 release of the OECD's ICIO tables. Unless mentioned explicitly, all figures refer to the manufacturing sector as the buying or selling sector, but we include the sector's intermediate purchases from and sales to all sectors.³²

We start with a stark example of how different indicators can lead to different answers to the same question: To which foreign supplier of intermediates is a particular nation most exposed: China, the US, or Germany? Figure 11 collects the answers for all 66 nations in the ICIO tables and presents them as the share of the 66 for whom the answer is China, the US, Germany, or other. The four columns in the chart reflect the answers given by four different indicators, OFPEM, FPEM, FPEMV, and BL.



Figure 11: Share of nations with China, US, or Germany as their top supplier, 2018 The answer depends upon the indicator used

Source: Authors' calculations based on OECD 2021 ICIO Tables (OECD 2021a). Note: See Section 6 for indicator definitions.

Using the *FPEM* indicator, we conclude that over 40% of the nations in the sample have China as their top supplier, and only about 10% have the US as their top supplier. However, if we use *FPEMV* instead the facts seem quite different. Here the roles of the US and China are more symmetric. If we use the *BL* indicator, the shares are even more

 $^{^{32}}$ As it turns out, the facts are quite similar for the total economy since manufacturing trade accounts for the lion's share of international commerce (exceptions are mostly related to large commodity exporters).

symmetric (although BL was not designed to pick up foreign exposure per se). Answering the question with *OFPEM* gives the lowest share for China and a much larger share for Germany, in part since there are so many European countries in the dataset and Germany is the hub for intermediates in Europe.

The difference between FPEM and FPEMV is easy to understand. FPEM is based on Leontief trade on a gross production basis while FPEMV is based on Leontief trade on a value-added basis. The value-added content of intermediates from China is lower than its gross content when compared to the US and Germany (more on this below). The figures for the BL are not strictly comparable since the BL indicator does not look at a nation's full reliance on inputs from a partner.³³

The fact that different indicators give different answers is not an indictment of any single indicator; it is a 'buyer beware' warning and a call for horses for courses thinking. If the analyst is mainly interested in observed bilateral exposure, OFPEM is the right measure for exposure on a gross production basis. If the issue instead is how much of value added in nation's GDP depends upon value added that comes from, say, China, then FPEMV is more suitable. If the question is the foreign value-added content in gross exports, then BL is the measure to use.

7.1 World manufacturing is very concentrated in the Giant-4

The Giant-4 manufacturers—China, the US, Germany, and Japan—account for almost 60% of the world's manufacturing output (Figure 12). Manufacturing output can be measured in two ways—in value-added and in gross-output terms—but the 60% figure holds for both measures. The left panel shows the figures for size measured by value added (i.e. manufacturing GDP) while the right panel shows the share of world gross production (i.e. manufacturing GDP plus all purchased intermediate inputs).³⁴

In a way, this 60% figure tells us that the whole world is heavily exposed to the Giant-4 when it comes to manufacturing. This whole-world exposure, however, includes the Giant-4 exposure to themselves, so it is important to note that three of the four are rather closed; they are mostly producing for and buying from their own industry. China's gross exports to gross production ratio in 2018 is just 12%, so 88% of Chinese gross output is sold in China. For the US, the share is 16% and for Japan it is 22%. Germany is an exception with a share of 46%, but most of this involves sales to the EU market.

While the left and right panels of the figure are quite similar, the horses for courses point can be seen by focusing on inferences. The big change—which is obvious with either measure—is China's soaring share of world manufacturing. The timing and size of

 $^{^{33}}$ However, since *BL* examines intermediate input flows on a value-added basis, this indicator is most comparable to *FPEMV*.

 $^{^{34}}$ As Section 2 explains at length, gross production corresponds roughly to the total sales of all manufacturers in the world—and so includes some double counting. Value added measures all payments to factors of production and taxes less subsidies, but it subtracts payments for intermediate inputs and thus eliminates double counting.

China's rise, however, is different for gross and value-added measurement. Specifically, when measured by value added, China surpassed the US as the largest manufacturer in 2010 (left panel) but surpassed it already in 2008 on a gross production basis (right panel). In gross production terms, China's world share exceeds the sum of the other three (right panel), but this is not true in value-added terms (left panel).

Figure 12: The Giant-4's share of world manufacturing, 1995-2018

The Giant-4 collectively dominate world manufacturing throughout even as the share distribution among them shifted with China's rise



Source: Authors' calculations based on OECD 2021 ICIO Tables (OECD 2021a).

What is the difference between the gross and the value-added valuations in terms of economic interpretation? Roughly speaking, output measured in value-added terms indicates where the work was done because it is based on employment of primary factors of production. Output measured in gross production terms, by contrast, shows the local of total manufactured sales.

Why are the gross and value-added numbers so different for China, but not for the other three? The answer turns on three linchpin facts exemplified in Figure 13. First, Chinese industry is far more exposed to supply chains—domestic and international—than the other three Giants (left panel). The share of China's manufacturing gross output that is made up of intermediate inputs is about 75% and this figure has been fairly steady since 1995. The corresponding share for the other Giants is much lower. Second, Chinese industry is less exposed to foreign intermediates than the other giants (right

panel). Specifically, the right panel illustrates the fact with the share of gross output in the Giant-4 made up of imported intermediates. China's share is substantially lower in 2018 than the others'. The US' exposure to imported intermediates is twice and Germany's is three times that of China. Finally, China's exposure to foreign supply chains has fallen since the mid-2000s, while the other three Giants have seen their import exposure rise steadily since 1995 (right panel).



Figure 13: The Giant-4's exposure to supply chains, 1995-2018

Source: Authors' calculations based on OECD 2021 ICIO Tables (OECD 2021a). Notes: Left panel shows intermediate inputs on a gross-valuation basis (% of national gross output); right panel shows the imported intermediates on a gross-valuation basis (% of national gross output).

It is worth noting that all of the Giant-4 are quite self-reliant when it comes to intermediate inputs. The most exposed is Germany, but even then it sources about 85% of all its intermediates from itself. China's asymmetric engagement in global supply chains will play a large role in the discussions that follow. The aggregate-level facts for the Giant-4 help us understand the global pattern of bilateral exposures, which is our next topic.

7.2 Measuring bilateral foreign exposure: sourcing-side

Here we look first at bilateral exposure as measured by gross flows on the sourcing-side (FPEM, OFPEM, and FPEMV). We look at foreign exposure on the selling-side in the following subsection.

7.2.1 The global pattern of foreign exposure as measured by FPEM

The matrices, or heatmaps, in Figure 14 illustrate bilateral foreign exposure when it comes to intermediate inputs. Each cell shows the exposure of the column nation to the row nation's intermediate inputs. The diagonal of the left-hand panel is blacked out since we are focusing on foreign exposure. The left panel shows the levels of exposure in 2018 as measured by FPEM; the right panel focuses on the evolution of bilateral exposure between 1995 and 2018.

Figure 14: Percent of column nations' total intermediate usage sourced from row nations in 2018 (left panel) and ppt change in this share from 1995 to 2018 (right panel)

Global supply chains are mostly regional, but China is an exception as it has become the 'OPEC of industrial inputs' worldwide



Source: Authors' calculations based on OECD 2021 ICIO Tables (OECD 2021a).

Notes: In words, FPEM is the share of the buyer's total usage of intermediate inputs that come from the seller with the numerator and denominator evaluated on a gross production basis. See Section 6 for FPEM's formula and motivation.

Turning to the left panel, recall that the numerator of *FPEM* uses intermediates imported on a Leontief basis and evaluated on a gross production basis and then divides this by the nation's total purchases of intermediates (measured in the same way) from all sources (domestic and foreign). Thus, both the numerator and denominator are in gross production terms, and the share captures the full exposure that arises from the full set of GSC linkages. The figures in the cells are thus the share of row-nation intermediate inputs that come from the column nation. We focus on manufacturing as the buying sectors as before (but note that the shares include manufacturing's purchases from all sectors, not just from foreign manufacturing).

How should one read the heat map? Since FPEM is based on a full decomposition, the level of the numbers in the cells in the left panel can be directly interpreted. For instance, the 12.5 in the US-to-Mexico cell tells us that 12.5% of all intermediate inputs used by the

Mexican manufacturing sector are sourced from the US. The 1.6 in the Mexico-to-US cell means that US manufacturing sources 1.6% of its intermediates from Mexico. To make the numbers easier to interpret at a glance, the relative magnitude of the bilateral foreign exposure is emphasised by colouring the cells—bilateral exposures that are relatively large are shaded with darker shades of brown.

There are a number of important messages in the figure. A first key takeaway from the left panel is the marked regionalisation of world supply chain exposures into three 'factories': North America; Europe; and Asia. For example, in the North American block in the top left corner, the dark colours indicate that these nations source a lot of intermediates from each other. The middle (European) block shows a similar pattern, as does the Asian block in the lower right corner. For a shorthand, we can call these Factory North America, Factory Europe, and Factory Asia, respectively (Baldwin 2008).

In Factory Asia, we see that Japan, Korea, India, and China all source substantially from each other. Korea's total share from the other three is 10%, while its sourcing from all other nations in the matrix amounts to only 5.4%. Japan sources 5.4% within Factory Asia and only 4.3% outside Factory Asia. For India, the corresponding figures are 5.2% and 3.4%. China is the most balanced in its exposure as it sources equally from inside and outside Factory Asia (2.5% for both). The regionalisation of sourcing is even more prominent in Factory North America than it is in Factory Asia. Canada and Mexico source 15.0% and 12.5%, respectively, from the US, while the US sources 1.9% from Canada and 1.6% from Mexico.

Inside the regional factories, there is a clear hub-and-spoke pattern. The US and China are the hubs in Factory North America and Factory Asia, while Germany is the hub in Factory Europe. This is evident from the asymmetric exposures. For instance, Korea, which is a spoke in Factory Asia, depends upon China for 7% of its intermediates, but the Chinese dependence on Korean intermediates is only 1.3%. Likewise, Canada (a spoke in Factory North America) buys 15% of its intermediates from the hub (the US) while the hub buys only 1.9% of its inputs from the spoke. The hub-and-spoke pattern is somewhat less marked in Factory Europe. The exposure of the spokes (France, Italy, and the UK) to hub (German) inputs is at least twice the reverse exposure.

Shifting focus from regions to the Giant-4 we see that China and the US are quite different from Germany and Japan. A second takeaway from the left panel is the global importance of China and the US as suppliers of intermediates—especially China. As the darkness of the China row in the heatmap indicates, China could, in 2018, be called the workshop of the world. The manufacturing sectors of every nation listed source at least 2.2% of their intermediates from China on a Leontief basis. The figure rises to 7.8% for Mexico and 7.0% for Korea. Concentrating on the non-China members of the Giant-4, we see that Japan is the most exposed to Chinese intermediates with an *FPEM* of 4.2%. The US is the next most exposed to Chinese intermediates with a bilateral *FPEM* of 3.3%. Germany's corresponding figure is 2.7%.

A third takeaway is the marked asymmetry of foreign exposures among the Giant-4, with China's asymmetry being especially noteworthy. The US exposure to China, as indicated by *FPEM*, is 3.3% while China's exposure to the US is just 1.0%. Likewise, Japan's and Germany's exposure to China are 4.2% and 2.7%, respectively, while the reverse exposures are 1.0% and 0.6%.

When thinking about the implications of the asymmetric foreign exposure, we should keep in mind that our measures do not take account of substitutability of inputs. The ICIO approach assumes no change in the input-coefficients, but in reality it may be that the intermediates that China imports from the US may be harder for China to replace than the intermediates that the US imports from China. This is a fundamental limitation of our foreign exposure measures. Addressing it would require a full-blown general equilibrium trade model.

A similar, but muted pattern of asymmetry holds between Germany and Japan on the one hand, and the US, on the other. When it comes to intermediates, the US is the source of 1.8% of Germany's and 2.1% of Japan's exposure, while the reverse exposures are 0.8% and 1.0%.

Was China always as dominant as it is in 2018? The right panel answers the question by displaying the percentage point (ppt) changes between the *FPEM* matrix in 1995 (not shown) and the 2018 *FPEM* matrix shown in the left panel of the figure. To emphasis the pattern of changes, negative numbers are highlighted with various shades of blue and positive numbers with various shades of brown. As before, dark colours indicate greater magnitudes in either direction.

A few key points with respect to China's role in GSCs worldwide are apparent in the right panel. All major manufacturing nations' exposure to China on the input-side has risen significantly since 1995 as indicated by the universally brown colouring of the Chinese row. The increase in China's exposure to foreign supply chains has been quite modest as most of the cells in the Chinese column are light blue or light brown; the exceptions are sharp drops in exposure to inputs from the US (-0.3 ppt) and Japan (-1.4 ppt). And, China's rise has resulted in some de-regionalisation in Factory North America and Factory Europe.

In Factory Asia, the big shift was away from Japan as a supplier of inputs and towards China as a supplier. Likewise, many of the elements in the Factory Europe bloc are negative—indicating a downward trend in bilateral exposure. In North America, the biggest change was Canada's exposure to US inputs (-3.1 ppt) that was more than matched by Canadian exposure to Chinese inputs (+3.8 ppt).

The change in Japan's position as a world hub for intermediate inputs changed almost as dramatically as China's, but in the opposite direction. All the elements in Japan's row are negative indicating that all column nations are less exposed to Japan. In short, Japan's role in the world supply chain diminished significantly between 1995 and 2018. The drop in exposure to Japan for intermediate inputs fell from 2.2 ppt for Korea, to almost no change for Mexico. For the other Giant-4, the drop in exposure to Japan was -1.2 ppt for the US, -0.2 ppt for Germany, and -1.4 ppt for China. Note that this was not a zero sum shift from exposure to Japan to exposure to China. The rises in China's row are, element by element, larger in absolute value than those in Japan's row.

The role of the other two Giant-4 is more mixed than that of China and Japan. All nations except Canada, China, and Korea increased their reliance on US intermediates. For Germany, its importance as a supplier rose in all nations except India.

The right panel also presents an aggregate measure of how much each nation has increased its exposure to GSCs as a whole. Since the *FPEM* measure adds to 100% when taking account of the sourcing of intermediates from domestic and foreign sources, the change in exposure to domestic intermediates is a gauge of the nation's change in self-reliance, i.e. change in foreign exposure globally. The main point is that, as we saw explicitly in the right panel, all major manufacturing nations are more exposed to foreign suppliers in 2018 than they were in 1995, except China where the opposite trend is observed. Specifically, the changes in self-reliance are shown in the diagonal elements. For instance, the -4.0 in the US-US cell means that the US sourced 4 ppt less of intermediate inputs from itself between 1995 and 2018. All the diagonal elements are negative except for China; in 2018 China sourced 0.5 ppt more from itself than it did in 1995.

7.2.2 Bilateral foreign exposure: trade measured on a direct versus Leontief basis

The bilateral *FPEM* between, say, France and China, includes Chinese intermediates measured on a Leontief basis. That is to say, intermediate goods that are imported directly from China as well as indirectly (i.e. Chinese inputs embedded in intermediates that France imports from third nations). Measuring imports on a Leontief basis is really the only way to get a full picture of foreign exposure (see Section 3 for discussion of this point).

But using Leontief imports is not the only, and certainly not the most intuitive, way of measuring bilateral exposure. It is more common to measure only direct imports since these are the bilateral flows that are observed (rather than calculated). The traditional approach is to look at observed import shares, e.g. if the UK's share of imports from the EU exceeds the EU's share of imports from the UK, we would say that the UK is more exposed to the EU than vice versa.

The next horses for courses point concerns the difference between the *FPEM* indicator, which uses bilateral imports on a Leontief basis, and *OFPEM*, which is the analogue that uses bilateral trade on an observed basis. As we shall see, *FPEM* and *OFPEM* can give different answers to the same question.

The right panel in Figure 15 is identical to the left panel of Figure 14; it is reproduced to ease the comparison with the left panel which shows the observed-trade version of *FPEM*, namely *OFPEM*. Comparing the exposure of, say Canada, to intermediates from, say the

US, *OFPEM* shows that Canada sources 20.5% of its intermediates from the US. Looking at the same supply chain exposure using *FPEM*, the answer is 15%. Thus, *OFPEM* shows substantially more exposure than does *FPEM*. The reason is that the production of US intermediate goods uses a significant amount of foreign inputs from places like China, Mexico, and Canada itself. Some of Canada's imports of US intermediates measured on the observed trade basis (*OFPEM*) are actually made in China. That is why the *OFPEM* for Canadian exposure to the US is higher than its *FPEM* exposure.

Figure 15: Percent of column nations' intermediate usage sourced from row nations in 2018 on an observed trade basis (left panel) and on a Leontief basis (right panel)

Measuring foreign exposure on an observed trade basis misallocates the exposure since some of the inputs in the observed trade are actually made elsewhere



Source: Authors' calculations based on OECD 2021 ICIO Tables (OECD 2021a).

Notes: In words, FPEM is the share of the buyer's total usage of intermediate inputs that come from the seller with the numerator and denominator evaluated on a gross production basis. OFPEM is calculated in an analogous fashion with trade measured on an observed instead of a Leontief basis. See Section 6 for formulas and motivation.

A final illustration of the difference between observed and Leontief trade comes from an examination of China's row in the two tables. When looking at countries' exposure to China we see that every single *FPEM* number is higher than the corresponding *OFPEM* number; all the China row elements in the *FPEM* table are larger than those in the *OFPEM* table. Why? The reason is that *FPEM* allows Chinese inputs to accumulate as they move through GSCs while *OFPEM* does not. Since China is an important supplier of intermediates to all nations (regardless of the indicator), allowing cumulation really changes the numbers.

Some of the intermediates a country imports are actually made in third nations, so the foreign exposure based on observed trade flows can be misleading. However, if the question at hand is, for example, how big of a shock would it be to, say, the UK economy if bilateral trade with, say, Germany were cut off, then it might be more informative to look at observed imports. Alternatively, if the question is what will be the impact on the UK of a substantive reduction in German output, then the Leontief flow would be more apt since the German shock will reverberate via imports from third nation suppliers like France and Italy, who are themselves quite exposed to German intermediate production.

Foreign exposure works both ways since both the buyers and sellers of intermediates are exposed to the transaction. In recent years, when supply chain disruptions have led to a shortage of intermediate inputs, like semiconductors or chemicals used in vaccine production, policymakers have been especially worried about sourcing-side, i.e. importside, exposure. Next, we turn to sales-side, i.e. export-side, exposure that arises when there is an economic disruption to a foreign economy that hinders exports, or policies like embargoes that directly shock exports.

7.3 Measuring bilateral foreign exposure: sales-side

This subsection looks at the empirical implementation of the sales-side exposure measure FPEX. The facts are presented in Figure 16. The left panel shows the bilateral FPEX indicator and, for comparison, the right panel reproduces the FPEM heatmap from above.

Figure 16: Sales-side exposure (FPEX) versus sourcing-side exposure (FPEM), 2018

Sales-side (i.e. export-side) exposures are much smaller than FPEM exposures. The denominators are gross output and most nations buy most of their own gross production; exceptions are found in Factory North America and Factory Asia



Source: Authors' calculations based on OECD 2021 ICIO Tables (OECD 2021a). Notes: In words, *FPEX* is the share of the seller's production sold as total intermediate inputs to each buyer with the numerator and denominator evaluated on a gross production basis. *FPEM* is the share of the buyer's total usage of intermediate inputs that come from the seller with the numerator and denominator evaluated on a gross production basis. See Section 6 for formulas and motivation.

Recall that FPEX is the share of a sector's gross output (i.e. total sales) that goes to a particular partner nation. Here we focus on the manufacturing sector of the row nations but look at sales to all sectors abroad. For example, the first column of the second row in the left panel shows that Canadian manufactured exports, both direct and indirect, to all US sectors are 20.7% of Canadian manufacturing gross output. Canadian industry, in other words, is heavily exposed to the US market on the sales-side.

Two of the overarching facts we saw on the sourcing-side find a clear echo on the selling-side. First, GSC sales-side linkages are highly regionalised in Factory Asia, Factory North America, and Factory Europe. This can be seen by the marked concentration of shading within the regional blocks, especially in Factory Asia and Factory Europe.

Second, inside the regional factories, there is a clear hub-and-spoke pattern that is more marked on the sales-side (left panel) than it is on the sourcing-side (right panel). The ratio of exposure shares for hubs versus spokes is larger with *FPEX* than *FPEM*. Take the US and Canada. Canadian manufacturing industry sales to the US amount to 20.7% while in the reverse direction (US manufacturing exposure to Canada) the figure is 1.6%—a spoke-hub exposure ratio of about 13. On the sourcing-side, the spoke-hub ratio for the same pair is almost 8. The same holds in Factory Asia, but the hub-spoke lop-sidedness is less clear in Factory Europe since out-of-region markets and suppliers are more important.

8 Concluding remarks

The risks of relying on foreign suppliers has long been known. In sectors like agriculture and military procurement, many aspects of public policy are explicitly aimed at reducing exposure to foreign suppliers. Events in recent years such as Covid-19-related disruptions and shocks created by the Russian invasion of Ukraine have greatly heightened concerns about nations' vulnerability to suppliers based abroad.

This paper develops an approach to measuring such vulnerability that we call the shocks approach. A number of crucial distinctions are highlighted with our approach. The first is the distinction between indicators of foreign exposure that include only observed trade flows, and those that include direct and roundabout trade via third nations (what we call Leontief trade since one needs the Leontief inverse matrix to calculate them). The second is the valuation basis—gross versus value-added—of the trade flows used. Using value-added valuations avoids double counting but may miss exposure that involves disruption of gross trade flows rather than just the value added in the trade flow. Third, we discuss alternative ways of normalising the four types of trade flows. Finally, we spotlight the importance of indicator design by demonstrating that different indicators provide different answers to the same foreign exposure question.

The measures we explicitly introduced here always use the same valuation approach in the numerator and denominator and choose the numerator so that the foreign exposure indicator ranges between 0 and 100. This contrasts with many standard measures, like trade-to-GDP, the VAX ratio, and backward and forward linkages measures, all of which use a mixture of value-added and gross valuations in either the numerator or denominator. Our approach, however, leaves the door open to such mixing and matching. Our main point is that the analyst should think hard about what needs to be measured when deciding on the trade flow to be used in the numerator and the normaliser to be used in the denominator. Otherwise stated, 'horses' (foreign exposure indicators) should match 'courses' (the shocks and domestic variables of interest).

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