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Abstract

The global financial crisis prompted the rapid development of macro-prudential frameworks and an increased reliance on borrower-based policy tools, which influence the demand for credit. This paper studies the optimal design of one such tool, a loan-to-value (LTV) limit, and its implications for monetary policy in a model with nominal rigidities and financial frictions. The welfare-based loss function features a role for macro-prudential policy to enhance risk-sharing. Optimal LTV limits are strongly countercyclical. In a house price boom-bust episode, the active use of LTV limits alleviates debt-deleveraging dynamics and prevents the economy from falling into a liquidity trap.

Key words: Monetary and macro-prudential policy; financial crisis; zero lower bound.

JEL classification: E52, E58, G01, G28.

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1 Introduction

A persistent boom in house prices and a large increase in private indebtedness planted the seeds for the 2008 financial crisis (Figure 1). Once house prices collapsed, the turmoil in the financial sector and the ensuing deleveraging process caused the worst US recession since the Great Depression (Hall, 2011). Many other countries, including Ireland, Spain, and the UK, experienced similarly long-lived booms followed by deep recessions. To prevent the repeat of similar episodes, policy authorities around the world have introduced macro-prudential frameworks that include both 'lender-based' policy tools that primarily influence credit supply (such as capital requirements on banks) and 'borrower-based' tools that primarily affect credit demand (such as limits on household borrowing).

Motivated by the rapid increase in the use of borrower-based policy instruments in recent years, this paper focuses on one such tool—a loan-to-value (LTV) limit—and its implications for monetary policy. In our model, the key financial friction is a collateral requirement on borrowers (Kiyotaki and Moore, 1997), which limits mortgage debt to a certain fraction of the value of housing. When we allow this fraction, the 'LTV limit', to be used as a macro-prudential policy instrument, we find that the optimal LTV policy is strongly countercyclical. In our simulation experiments, the active use of LTV limits alleviates the burden of macroeconomic stabilization on monetary policy and can avoid a liquidity trap that would otherwise occur in the absence of macro-prudential policy.

Our findings contribute to a growing literature exploring the conduct of macro-prudential policy and its interaction with monetary policy. From a theoretical perspective, Farhi and Werning (2016) and Korinek and Simsek (2016) present detailed analyses of the financial market distortions that macro-prudential policy can address in the presence of aggregate demand externalities, such as nominal rigidities and the zero lower bound (ZLB) on the nominal interest rate, while Davila and Korinek (2018) emphasize the pecuniary externalities due to an endogenous collateral constraint.

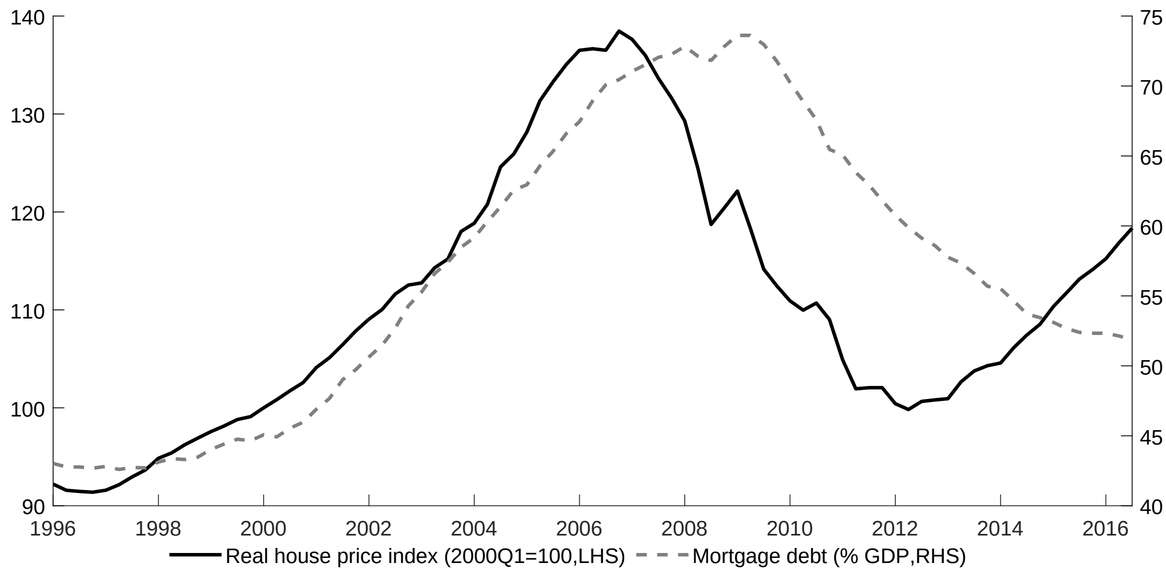
We obtain our results in a simple model that builds on Cúrdia and Woodford (2016) and combines both types of externality. As is standard in the New Keynesian literature, nominal rigidities arise because of staggered price setting (Calvo, 1983), so that monetary policy has real effects. The key financial friction is a collateral requirement on borrowers. As in Kiyotaki and Moore (1997), an underlying moral hazard problem implies that lenders are only willing to lend up to a given fraction of the value of housing collateral. We allow the macro-prudential authority to impose an LTV limit on mortgage borrowing that is no greater than this fraction.

The model also features a second financial friction. Borrowers obtain loans through perfectly competitive financial intermediaries (banks), which raise equity and deposits from savers. Banks seek to minimize equity issuance, which is costly as in Justiniano et al. (2019), but an equity requirement places an upper bound on their leverage. Exogenous changes in this requirement map into movements of the spread between borrowing and deposit rates. These financial disturbances, which we label 'credit spread shocks', are the exogenous source of fluctuations in our model.

The resulting framework is rich enough to generate meaningful policy tradeoffs, but sufficiently tractable that, up to a second-order approximation, the welfare-based loss function clearly identifies how the inefficiencies in the economy map into four policy objectives. Two of the terms in the welfare-based loss function, inflation and the output gap, stem from nominal rigidities and are standard in the New Keynesian literature (e.g., Clarida et al., 1999, and Woodford, 2003). The remaining two terms are due to imperfect risk sharing between borrowers and savers. In particular, the policymaker seeks to stabilize the differences (or 'gaps') in the marginal utility of non-durable consumption and housing between the two types of household.

The welfare-based loss function shares a number of similarities with those derived in Andres et al. (2013), Benigno et al. (2020) and Cúrdia and Woodford (2016). While those papers focus on optimal monetary policy, our contribution is to explore its interaction with the optimal setting of LTV limits.

Figure 1: Real house prices and mortgage debt in the United States.



NOTE: Real house prices correspond to the FHFA index deflated by the CPI (normalized to 100 in 2000q1). Mortgage debt corresponds to 'Households and Nonprofit Organizations, One-to-Four-Family Residential Mortgages,' expressed as a fraction of GDP in percentage points. All data are from FRED, Federal Reserve Bank of St. Louis.

When the collateral constraint is always binding and the nominal interest rate never hits the zero lower bound, the jointly optimal monetary and macro-prudential plan in a linear-quadratic approximation of the model satisfies a pair of targeting rules. The optimal targeting rule for monetary policy manages the standard tradeoff between inflation and the output gap. The optimal targeting rule for macro-prudential policy prescribes that the marginal utility gap and the output gap co-move. Thus, LTV limits are countercyclical, limiting consumption of borrowers relative to that of savers in a boom (and vice versa in a recession).

Our quantitative experiments allow for occasionally binding constraints, as in Guerrieri and Iacoviello (2017). In addition to the ZLB on the nominal interest rate, we also account for occasionally binding constraints on macro-prudential policy. If the collateral constraint is slack, the policymaker cannot force borrowers to hold more debt than the amount demanded at market prices (a 'credit demand' constraint). At the same time, the policymaker cannot force lenders to extend credit at an LTV ratio that is higher than the level required to avoid the underlying moral hazard problem, which creates an upper bound on the LTV limit (a 'credit supply' constraint).

A slow and persistent decline in credit spreads, followed by a sharp tightening, drives our quantitative experiments. The model generates a boom-bust cycle in house prices which captures the salient features of the data in the US and other advanced economies before and after the 2008 financial crisis. We compare a baseline scenario characterized by a standard interest rate rule for monetary policy and a fixed LTV limit with a regime in which the policymaker optimally sets the nominal interest rate and the LTV limit to minimize the welfare-based loss function. In the baseline scenario, the collateral constraint remains slack during the boom and becomes binding in the bust, during which the economy experiences a liquidity trap and a deep recession. Conversely, under optimal policy, the LTV limit declines markedly during the boom, such that the collateral constraint binds tightly, and rises at the time of the crisis, relaxing the collateral constraint. The countercyclical LTV policy avoids a large buildup in private debt during the expansion so that, when house prices fall, the recessionary consequences of the deleveraging process are limited. As a result, the nominal interest rate remains above the ZLB, and the economy avoids the recession. In this sense, the optimal setting of the LTV limit is indeed prudential, at least as far as macroeconomic objectives are concerned.

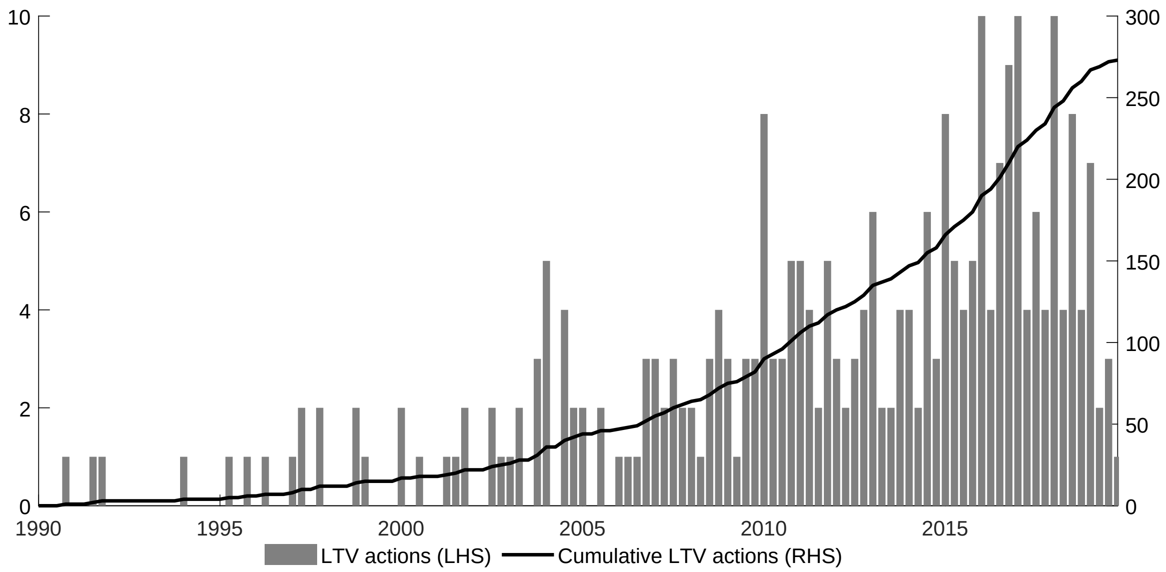
Finally, we present two experiments that demonstrate how the efficacy of macro-prudential policies may depend on prevailing macroeconomic conditions. The first shows that, in line with the existing empirical evidence, the sign of the effect of an LTV limit tightening on house prices depends on whether or not monetary policy is constrained. The second experiment shows that the benefits of setting the LTV limit optimally only once the bust occurs are much smaller than if macro-prudential policy is active also during the boom. The high level of debt built up during the boom limits the space for macro-prudential policy, so that the credit supply constraint becomes binding during the bust.

Our paper contributes to a growing literature studying borrower-based macro-prudential policy tools and their interaction with monetary policy. Rubio and Carrasco-Gallego (2014) postulate a simple rule for LTVs while Lambertini et al. (2013) study optimized simple rules for monetary policy and LTV limits in the context of boom-bust cycles generated by news shocks. Angelini et al. (2012) consider optimal monetary and macro-prudential policies (including both capital requirements and LTV limits) using an ad-hoc loss function as the policymaker's objective. Gelain et al. (2013) evaluate the effects of tighter LTV restrictions on household debt and output in a model that generates excess volatility by relaxing the assumption of rational expectations.[1] A distinguishing feature of our work is the combination of the normative analysis with a welfare-based loss function and the explicit consideration of occasionally binding constraints, including on policy instruments.

The focus on LTV limits in this paper complements a number of contributions that examine lender-based policy tools, in particular the role of capital requirements.[2] Several papers extend the analysis to the interaction between capital requirements and monetary policy. For example, Bean et al. (2010) study the optimal setting of capital requirements with ad-hoc loss functions in a simplified version of Gertler and Karadi (2011). In models with bank runs, Angeloni and Faia (2013) compare alternative properties of capital requirements that mimic the Basel I, II, and III accords, while Gertler et al. (2020a,b) study a simple capital requirement linked to the net worth of financial intermediaries. Quint and Rabanal (2014) specify rules for the leverage ratio that banks can afford as a function of credit aggregates in an estimated model of the Euro Area. Collard et al. (2017) and Van der Ghote (2021) study the jointly optimal setting of interest rates and capital requirements in environments with moral hazard frictions. Finally, Mendicino et al. (2020) evaluate the tradeoffs associated with increasing capital requirements depending on the state of the business cycle.

The rest of the paper is organized as follows. Section 2 documents the use of LTV limits as a macro-prudential tool, showing the increasing use of this type of borrower-based policy instrument in recent years. Section 3 introduces the model, focusing on the key innovations to the treatment of the household sector. Section 4 presents the linear-quadratic framework for optimal policy analysis and derives the targeting rules abstracting from occasionally binding constraints. Section 5 illustrates the optimal joint conduct of monetary and macro-prudential policy with occasionally binding constraints via numerical simulations. Section 6 concludes. The derivations and the description of the computational details are in the appendix.

Figure 2: LTV policy actions.



NOTE: The grey bars (left scale) are the number of LTV policy actions in each quarter. The black line (right scale) is the cumulative number of LTV policy actions over time at quarterly frequency. The data source is the IMF integrated macro-prudential policy database (iMaPP).

2 LTV Limits as a Policy Tool

The use of LTV limits as a policy tool has a fairly long history. For example, Hong Kong introduced LTV restrictions in 1991. Wong et al. (2011) argue that this innovation cushioned the aggregate effects of the 1997 Asian financial crisis on lenders' resilience in the face of a 40% decline in property prices. Similarly, the macro-prudential authority in South Korea set the maximum LTV to 60% in 2002, subsequently tightening the limit to as low as 40%. Crowe et al. (2013) document the notable moderation in house price appreciation that followed those policy change. Together with other measures, regulators in Canada lowered LTV limits four times between 2008 and 2012, from 100% to 80%. Krznar and Morsink (2014) estimate that a 1 percentage point reduction in the LTV limit reduced year-on-year credit growth by 0.4 percentage points.

By 2013, LTV limits, together with debt-to-income ratios, had become the most commonly deployed macro-prudential instrument, both in emerging market economies and developed countries (Claessens, 2015).[3] Their adoption has continued to grow ever since. Using data from the IMF integrated macro-prudential policy database (see Alam et al., 2019, for a description), Figure 2 reports the number of LTV policy actions in each quarter (grey bars) and their cumulative number (solid black line) globally between 1990 and 2019. The chart shows a clear acceleration in the use of LTVs since the financial crisis. In Europe alone, Arena et al. (2020) count 19 jurisdictions with LTV policies in place by 2018. Their evidence shows a decline in the extension of high LTV mortgage loans following the introduction of these

[1] Our analysis also shares some similarities with De Paoli and Paustian (2017), though their model emphasizes a different credit relationship (between entrepreneurs and households) and focuses on a different macro-prudential policy instrument (a tax/subsidy on the cost of borrowing for entrepreneurs).

[2] A non-exhaustive list includes Van den Heuvel (2008), Gertler et al. (2012), Miles et al. (2013), Admati and Hellwig (2014), Clerc et al. (2015), Christiano and Ikeda (2016) and Corbae and D'Erasmo (2021).

[3] Cerutti et al. (2017) document the relative prevalence of borrower-based instruments, including LTV limits, in advanced economies, while emerging markets often rely additionally on foreign exchange related measures. Institutional details on macro-prudential frameworks, both in terms of the tools available and the authorities in charge, vary greatly across countries (Akinci and Olmstead-Rumsey, 2018).

controls in all countries in their sample.

A plausible explanation for these developments is that the financial crisis of 2008 brought renewed attention to the terms and conditions governing borrowers' access to debt finance. The importance of housing markets and the tight links between housing net worth, household consumption and aggregate demand called for instruments that could limit the propagation of financial shocks via the indebtedness of the household sector. Indeed, in many advanced economies, mortgages are at the same time the single largest asset class on the balance sheet of banks and the single largest liability class on the balance sheet of households (Jordá et al., 2016).

As the use of LTV policies has broadened, more systematic empirical analysis has become possible. Kuttner and Shim (2016) use data on 57 countries covering the period 1980 to 2011 for a range of advanced and emerging market economies. Similarly, Araujo et al. (2020) conduct a meta analysis encompassing 58 studies of a broad set of macro-prudential policy actions. Both papers find that LTV requirements had a significant effect on household credit but not on house prices. Richter et al. (2019) reach a different conclusion regarding the effects of LTV policies on house prices by combining a narrative approach to identification and local projection methods for inference. Their estimates also give a sense of the macroeconomic consequences of LTV policies. In their sample, an exogenous 10 percentage point tightening of LTV limits generates a 1.1% decline in output and an increase in consumer prices of a broadly similar magnitude.[4]

A potential explanation for these heterogeneous estimates of the effects of LTV limits on house prices (and potentially other variables) is the role of the systematic monetary policy response to the aggregate consequences of changes in financing conditions. The VAR evidence in Bachmann and Rüth (2020) speaks directly to this point. Using quarterly US data spanning the period 1978 to 2008, expansionary LTV shocks increase GDP and business investment but lead to a decline of residential investment and house prices. Crucially, the typical response of monetary policy to these shocks is a rise in the nominal interest rate. Through a counterfactual exercise these authors show that, if the interest rate does not increase, the responses of residential investment and house prices change sign.

These findings illustrate that the interaction of LTVs with monetary policy has an important bearing on their macroeconomic effects. Our analysis studies this interaction both in normal times (i.e. when monetary policy is unconstrained) and at the zero lower bound, while also accounting for the possibility that the LTV limit may not bind at all times.

3 Model

This section describes the key building blocks of the model. The household sector contains the key frictions that give rise to a role for macro-prudential policy. We extend the approach of Cúrdia and Woodford (2016) to incorporate an occasionally binding collateral constraint on household borrowing and allow the macro-prudential policymaker to vary the LTV limit over time.

Households are ex-ante identical, but at any point in time their preferences are heterogeneous due to stochastic realizations of the coefficient of relative risk aversion. A random switch in this utility parameter is sufficient to deliver a separation between savers and borrowers, which is the focus of our paper. At the beginning of time households sign state-contingent contracts, but only have access to payoffs from those contracts in the event of a new utility parameter draw. Intermittent access to the contract payments provides a role for financial intermediation to smooth consumption over time while avoiding ever-diverging marginal utilities of income due to different individual histories.

The rest of the model is standard. Banks raise funds from savers through a mix of deposits and equity, and transfer resources to borrowers. Imperfectly competitive wholesale firms set prices on a

[4] The response in emerging markets drives the estimated effects on output. In addition, the paper presents some evidence of an asymmetric response (greater for a tightening).

staggered basis. Retailers are perfectly competitive and combine intermediate inputs to produce the final consumption good. The government conducts monetary and macro-prudential policy to address the inefficiencies arising from nominal rigidities and imperfect risk sharing.

Below, we present the problem of each economic actor. Appendix A describes the microfoundations of the household problem with intermittent access to state-contingent transfers and the full derivations of the equilibrium conditions.

3.1 Households

A continuum of measure one of ex-ante identical households populate the economy. At time t, a household can be of type b ("borrower") or s ("saver"). Households maximize the present discounted value of utility

$$\mathbb{V}_0^{\tau_t} \equiv \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\sigma^{\tau_t}} (C_t^{\tau_t})^{1-\sigma^{\tau_t}} + \frac{\chi_H}{1-\sigma_h} (H_t^{\tau_t})^{1-\sigma_h} - \frac{\chi_L}{1+\varphi} (L_t^{\tau_t})^{1+\varphi} \right] \right\}, \tag{1}$$

where $\tau_t \in \{b, s\}$ indicates the type at time t. The variable $C_t^{\tau_t}$ denotes consumption of goods, $H_t^{\tau_t}$ consumption of housing services (assumed to be proportional to the stock of housing), and $L_t^{\tau_t}$ hours worked. The parameter $\beta \in (0, 1)$ is the individual discount factor, σ^{τ_t} the coefficient of risk aversion (with $0 < \sigma^b < \sigma^s$), $\sigma_h > 0$ the inverse elasticity of housing demand, $\varphi > 0$ the inverse elasticity of labor supply, and χ_H and χ_L are positive parameters that determine steady state housing demand and hours worked, respectively.

The type τ evolves as an independent two-state Markov chain. With probability $\delta \in [0, 1]$, the type remains unchanged. With probability $1 - \delta$, a household draws a new type, independently of the previous one. The probability of drawing type τ is $\xi^\tau \in [0, 1]$. Since we assume the economy consists of a continuum of households of measure one, the probability of drawing a certain type corresponds to the share of that type in the population. To simplify the notation, we set $\xi^b = \xi$, which in turn implies $\xi^s = 1 - \xi$.

Households sign state-contingent contracts with each other at time $t_0 \leq 0$ to insure against idiosyncratic and aggregate risk. A competitive insurance agency provides the payments associated with such contracts. These payments, however, take place if and only if a household draws a new type in period t (and before knowing the new type).

The beginning-of-period financial wealth inclusive of transfers for a household of type τ is

$$A_t^\tau \equiv R_{t-1}^d \max\{D_{t-1}, 0\} + R_{t-1}^e \max\{E_{t-1}, 0\} - R_{t-1}^b \max\{B_{t-1}, 0\} + T_t^\tau,$$

where D_t denotes deposits that pay a gross nominal interest rate R_t^d, E_t equity in banks that pays a gross nominal return R_t^e, B_t debt that carries a gross nominal interest rate R_t^b, and T_t^τ state-contingent transfers.[5]

The budget constraint for a saver is

$$P_t C_t^s + Q_t H_t^s + P_t \Gamma_{ht}^s + D_t + E_t + P_t \Gamma_{et} = A_t^s + W_t L_t^s + Q_t H_{t-1} + \Omega_t^s,$$

where P_t is the consumption price index, Q_t the nominal price of housing, W_t the nominal wage, and Ω_t^τ denotes the share of profits from intermediate goods producers accruing to a household of type τ net of

[5] Households who do not draw a new type at time t do not receive a transfer and simply carry their financial wealth on from the previous period. Appendix A shows that, due to the transfers from the insurance agency, households who have just drawn a new type start the period with zero wealth. Effectively, the insurance agency redistributes resources across households by pooling together debts and assets of all those who draw a new type.

taxes. The function Γ_{et} measures equity holding costs in deviations from the target level $\overline{E}_t$

$$\Gamma_{et} \equiv \frac{\Psi_e}{2}\left(\frac{E_t}{\overline{E}_t}-1\right)^2 \overline{E}_t,$$

with $\Psi_e > 0$.[6] As in Justiniano et al. (2019), the presence of holding costs generates a premium for equity over deposits and a well-defined liability structure in the banks' balance sheet, thus capturing the idea that in practice deposits are generally more liquid and easier to adjust than equity.[7]

Similarly, the function Γ^{τ}_{ht} measures housing holding costs, expressed in deviation from a target level that we assume to be the symmetric steady state level of housing consumption H,

$$\Gamma^{\tau}_{ht} \equiv \frac{\Psi_h H}{2}\left(\frac{H^{\tau}_t}{H}-1\right)^2,$$

where $\Psi_h > 0$. As in Greenwald (2018) and Menno and Oliviero (2020), housing holding costs limit the degree of reallocation between types over the business cycle. This effect introduces a simple form of housing market segmentation, as discussed for example in Guerrieri et al. (2013) and Piazzesi and Schneider (2016).[8]

The budget constraint for a borrower is

$$P_t C^b_t + Q_t H^b_t + P_t \Gamma^b_{ht} - B_t = A^b_t + W_t L^b_t + Q_t H_{t-1} + \Omega^b_t.$$

Borrowers do not invest in equity of banks, and thus do not face the extra cost Γ_{et}, but are subject to a collateral constraint (Kiyotaki and Moore, 1997)

$$B_t \leq \gamma_d \max\{B_{t-1}, 0\} + (1-\gamma_d)\Theta_t Q_t H^b_t, \tag{2}$$

where $\gamma_d \in [0, 1)$ controls the extent of debt inertia (Justiniano et al., 2015) and $\Theta_t \in [0, \Theta]$ represents the maximum LTV ratio available at time t.[9] The standard interpretation of a collateral constraint like (2) is that lenders (banks in this model) require borrowers to have a stake in a leveraged investment to prevent moral hazard behavior.[10]

Our formulation of the collateral constraint contains two features that are important for the analysis. First, the LTV limit Θ_t varies over time reflecting the assumption that the macro-prudential authority may set the maximum LTV that banks can extend to borrowers. To respect the underlying incentive compatibility constraint encoded in the collateral requirement, the LTV limit Θ_t cannot exceed the

[6]Savers take the target level of equity as given. For analytical convenience, we set $\overline{E}_t \equiv \tilde{\kappa}\xi B_t/(1-\xi)$. As we discuss below, this assumption, coupled with the exogenous leverage restriction on banks, ensures that the cost function Γ_{et} does not have direct welfare consequences.

[7]Little of substance would change in the first-order accurate solution to the model that we examine if we specified bank equity as a state-contingent claim.

[8]Using detailed micro data, Landvoigt et al. (2015) document a high degree of segmentation for the San Diego metropolitan area. Justiniano et al. (2014) consider the case of full segmentation between borrowers and savers. Poterba (1991) discusses how the segmentation between borrowers and savers that our formulation builds into the model may be related to demographic factors.

[9]In principle, all households are subject to the collateral constraint. We abstract from its presence for savers since, in equilibrium, the constraint would never bind for this type. Moreover, since borrowers who have drawn their type at time t have their previous financial wealth reset to zero, the inertia in the collateral constraint only applies to households who were already borrowers at $t-1$ and did not draw a new type.

[10]In some contexts (e.g. Kiyotaki and Moore, 1997), the collateral constraint depends on expected future, rather than contemporaneous, asset prices. As the focus of our analysis is the real estate market, current prices are particularly suitable because the amount of mortgage loans typically depends on the current value of the house being purchased. Appendix B.1 shows that the two formulations are essentially identical in our main quantitative exercise. The reason is that, up to a first order approximation around a steady state in which the collateral constraint is slack, the house price equation is the same in the two formulations.

value $\Theta \in [0, 1]$, which is the maximum LTV consistent with the absence of a moral hazard problem.[11] Therefore, the inequality $\Theta_t \leq \Theta$ (the credit supply constraint) imposes an upper bound on the LTV limit.

Second, the collateral constraint (2) incorporates a degree of inertia governed by the parameter γ_d. The inertial formulation of the collateral constraint captures, in reduced form, the idea that only a fraction of borrowers experience a change to their borrowing limit each period, which may be associated with moving or re-mortgaging (Guerrieri and Iacoviello, 2017). This modification generates more persistent movements in debt and its marginal value. In particular, debt adjusts only gradually to changes in the value of the housing stock, which is consistent with the data in Figure 1. When $\gamma_d = 0$, the collateral constraint collapses to the familiar contemporaneous specification.

Appendix A reports the first-order conditions for the problems of savers and borrowers. The key difference between this model and one with either complete markets or incomplete markets and fixed types is that the expected future marginal utility of consumption is a weighted average of the marginal utility conditional on no type change and the average marginal utility of the two types.[12]

3.2 Banks

A continuum of perfectly competitive banks raise funds from savers in the form of deposits and equity (the banks' liabilities), and make loans (the banks' assets) to borrowers. Thus, the balance sheet of a generic bank is

$$B_t = D_t + E_t. \tag{3}$$

In addition, we assume that equity must account for at least a fraction $\tilde{\kappa}_t$ of the total amount of loans banks extend to borrowers

$$E_t \geq \tilde{\kappa}_t B_t, \tag{4}$$

where the equity requirement $\tilde{\kappa}_t$ is an exogenous shock.

The presence of the cost function Γ_{et} in the household problem breaks down the irrelevance of the capital structure (the Modigliani-Miller theorem). Savers demand a premium for holding equity, which banks pass on to borrowers in the form of a higher interest rate. From the perspective of a bank, equity is expensive, so that deposits are the preferred source of funding. In the absence of any constraint, banks would choose to operate with zero equity and leverage would be unbounded. Equation (4) ensures finite leverage for banks. In equilibrium, the capital requirement constraint is always binding because banks seek to minimize their equity requirement.[13]

Banks' profits are

$$\mathcal{P}_t \equiv R_t^b B_t - R_t^d D_t - R_t^e E_t = [R_t^b - (1 - \tilde{\kappa}_t) R_t^d - \tilde{\kappa}_t R_t^e] B_t,$$

where the second equality follows from substituting the balance sheet constraint (3) and the capital requirement (4) at equality. The zero-profit condition implies that the loan rate is a linear combination

[11] In other words, the policymaker cannot force banks to lend to households at a higher LTV than the level that ensures households will honor the debt contract.

[12] The appendix also shows that, as in Cúrdia and Woodford (2016), the marginal utility of consumption is independent of the household type history. The same result also holds for the multiplier on the collateral constraint, which is specific to this model.

[13] Since banks are identical, if the capital constraint of all banks were slack, one bank could marginally increase its leverage, charge a lower loan rate, and take the whole market. Therefore, competition drives the banking sector against the constraint.

of the return on equity and the return on deposits

$$R_t^b = \tilde{\kappa}_t R_t^e + (1 - \tilde{\kappa}_t) R_t^d,$$

where the weight on the return on equity corresponds to the time-varying capital requirement. An increase in $\tilde{\kappa}_t$ forces banks to delever and raises credit spreads—the difference between the loan rate R_t^b and the deposit rate R_t^d. Vice versa, a relaxation of the equity requirement (a lower $\tilde{\kappa}_t$) reduces spreads. Given the mapping between the equity requirement shock and credit spreads, from now on we refer to $\tilde{\kappa}_t$ as a credit spread shock, which is the key driver for the quantitative experiments that we study later.

We stress that our analysis focuses on the case in which $\tilde{\kappa}_t$ is exogenous, relying on the notion that financial institutions target a certain leverage ratio due to market forces (Adrian and Shin, 2010). An alternative interpretation would be that the macro-prudential authority sets the capital requirement on financial institutions, and thus controls $\tilde{\kappa}_t$ as a policy tool. We do not pursue this approach in this paper for two reasons. First, we believe that properly studying capital requirements, and their interaction with monetary policy, would require a more detailed specification of the financial sector. While our parsimonious description of financial intermediation does capture a connection between capital requirements and credit spreads, the model completely abstracts from a key mechanism—the accumulation of net worth—that determines banks' profitability and may be crucial to understand the effects of capital requirements. Second, as discussed in the introduction, the existing literature has extensively studied capital requirements, either in isolation or in connection with monetary policy. We aim to complement this body of work by focusing on the implications of LTV limits for macroeconomic stability and their interaction with monetary policy decisions, which have been relatively less explored.

3.3 Firms

A representative retailer combines intermediate goods according to a technology with constant elasticity of substitution $\varepsilon > 1$

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $Y_t(f)$ represents the intermediate good produced by firm $f \in [0, 1]$. Expenditure minimization implies that the demand for a generic intermediate good is

$$Y_t(f) = \left[\frac{P_t(f)}{P_t} \right]^{-\varepsilon} Y_t, \tag{5}$$

where $P_t(f)$ is the price of the variety produced by firm f and the aggregate price index is

$$P_t = \left[\int_0^1 P_t(f)^{1-\varepsilon} df \right]^{\frac{1}{1-\varepsilon}}.$$

Intermediate goods producers operate in monopolistic competition, are owned by savers and borrowers according to their shares in the population, and employ labor to produce variety f according to

$$Y_t(f) = L_t(f).$$

As in Calvo (1983), intermediate goods producing firms keep their price unchanged with probability $\alpha \in (0, 1)$. Those that can adjust choose the price of their product $\widetilde{P}_t(f)$ to maximize expected future profits conditional on no further changes, taking as given the demand for their variety and their marginal cost, which is equal to the real wage ($MC_t = W_t/P_t$) and is independent of firm-specific characteristics.

The optimal price setting decision for firms that adjust at time t solves

$$\max_{\tilde{P}_t(f)} \mathbb{E}_t \left\{ \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{\lambda_{t+j}}{\lambda_t} \left[(1+\tau^f) \frac{\tilde{P}_t(f)}{P_{t+j}} - MC_{t+j} \right] Y_{t+j}(f) \right\},$$

subject to (5), where $\tau^f > 0$ is a subsidy to make steady state production efficient. Since shares are non-tradable and the two types of households own firms in proportion to their size in the population, the discount factor for future profits corresponds to the average marginal utility of consumption

$$\lambda_t = \xi \lambda_t^b + (1-\xi)\lambda_t^s,$$

where $\lambda_t^\tau = (C_t^\tau)^{-\sigma^\tau}$ is the marginal utility of type τ.

3.4 Market Clearing and Equilibrium

The goods market equilibrium requires that total production equals the sum of consumption of the two types plus the resources spent for housing and equity holding costs

$$Y_t = \xi C_t^b + (1-\xi) C_t^s + \Gamma_t,$$

where $\Gamma_t \equiv (1-\xi)\Gamma_{et} + \xi \Gamma_{ht}^b + (1-\xi)\Gamma_{ht}^s$.[14]

We assume housing is in fixed supply, so that the housing market equilibrium requires

$$H = \xi H_t^b + (1-\xi) H_t^s,$$

where H is the total available stock of housing.

In the credit market, total bank lending must equal total household borrowing. Thus, the aggregate balance sheet of the financial sector respects

$$\xi B_t = (1-\xi)(D_t + E_t).$$

Finally, per-capita real private debt, derived by aggregating the budget constraint of new and existing borrowers over their respective measures, evolves according to

$$\begin{aligned} \frac{B_t}{P_t} = \delta \frac{R_{t-1}^b}{\Pi_t} \frac{B_{t-1}}{P_{t-1}} + \frac{Q_t}{P_t} [(H_t^b - H_{t-1}^b) + (1-\xi)(1-\delta)(H_{t-1}^b - H_{t-1}^s)] \\ + \Gamma_{ht}^b + C_t^b - Y_t - (1-\xi) \frac{W_t}{P_t} (L_t^b - L_t^s). \end{aligned}$$

The probability of not changing type drives the persistence of private debt. Differently from Cúrdia and Woodford (2016), housing demand enters the law of motion of debt. In this respect, what matters for debt is not only the change in debt of borrowers but also the difference (or 'gap') in the existing level of housing between borrowers and savers, due to the switching between types.

For a given specification of monetary and macro-prudential policy, an imperfectly competitive equilibrium for this economy is a sequence of quantities and prices such that all agents in the economy (households, banks and firms) maximize their objectives subject to the relevant constraints and all markets clear. Appendix A.2 reports the full list of variables and equilibrium conditions for the private sector.

[14] The resource constraint follows from combining the budget constraints of the two types (aggregated over their respective measures) with the banks' balance sheets.

4 Optimal Policy Problem

This section derives a linear-quadratic (LQ) characterization of the jointly optimal monetary and macro-prudential policy problem. The LQ approach allows us to derive some analytical results and is tractable enough to study numerically cases in which the lower bound on the nominal interest rate and the collateral constraint are occasionally binding. A second-order approximation to the welfare-based loss function around the efficient steady state contains components reflecting the effects of distortions generated by both nominal rigidities and imperfect risk sharing. The targeting rules demonstrate that optimal macro-prudential policy accounts for a potential tradeoff between stabilizing the effects of the two distortions.

We proceed by presenting in turn the welfare-based loss function, the log-linearized equilibrium conditions that constrain the optimal policy problem, and a pair of targeting rules for monetary and macro-prudential policy abstracting from occasionally binding constraints, which we later consider in section 5.

4.1 Loss Function

We derive the loss function for the economy by taking the average of the utility functions of borrowers and savers, weighting each type according to their share in the population. A second-order approximation of the resulting objective around an efficient zero-inflation steady state in which the marginal utility of the two types is the same gives

$$\mathcal{L}_0 \propto \frac{1}{2}\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(y_t^2 + \lambda_\pi \pi_t^2 + \lambda_h \tilde{h}_t^2 + \lambda_\omega \omega_t^2 \right) \right], \tag{6}$$

where y_t is output, π_t is the inflation rate, $\tilde{h}_t \equiv h_t^b - h_t^s$ is the housing gap between borrowers and savers, and $\omega_t \equiv \sigma^s c_t^s - \sigma^b c_t^b$ is the marginal utility gap.[15]

The loss function (6) features two sets of terms. The first includes output and inflation—the standard variables that appear in the welfare-based loss function of a large class of New Keynesian models.[16] Their presence in the loss function reflects the two distortions associated with price rigidities. First, such rigidities open up a 'labor wedge,' causing the level of output to deviate from its efficient level. Second, staggered price setting implies an inefficient dispersion in prices, which is proportional to the rate of inflation.

The second set of terms in (6), comprising the housing gap and the marginal utility gap, arise from the heterogeneity between household types. Incomplete financial markets prevent full risk sharing of goods and housing consumption. The collateral constraint further limits the amount of debt that borrowers can undertake, thus creating different marginal propensities to consume between the two types. Imperfect risk sharing therefore becomes a source of welfare losses that a benevolent policymaker will take into account when setting optimal monetary and macro-prudential policy.[17]

4.2 Log-Linearized Equilibrium Conditions

In this section, we report the set of log-linearized equilibrium conditions, expressed in terms of welfare-relevant variables, that constrain the LQ approximation to the optimal policy problem. Appendix A.7

[15] From now on, all variables should be understood as log-deviations from the steady state unless otherwise stated. The relative weights on inflation (λ_π), the housing gap (λ_h), and the marginal utility gap (λ_ω) in (6) are functions of the structural parameters (see appendix A.6 for details).

[16] Since productivity is constant, efficient output is simply equal to its steady state value, and the efficient output gap corresponds to the deviations of output from steady state.

[17] Although the choice of equity involves resource costs through the function Γ_{et}, the assumption that the leverage ratio is an exogenous—albeit time-varying—constraint implies that its fluctuations are independent of policy, and thus irrelevant for ranking alternative policies in terms of welfare.

describes the derivations in detail, including the definitions of the composite parameters.

Taking the average of the Euler equations of borrowers and savers yields an aggregate demand equation

$$y_t = -\bar{\sigma}^{-1}(i_t - \mathbb{E}_t\pi_{t+1} - r_t^*) + \mathbb{E}_t y_{t+1}, \tag{7}$$

where i_t is the net nominal interest rate (the log-deviation of the deposit rate from steady state) and r_t^* is the equilibrium real interest rate. Differently from the baseline New Keynesian model, the equilibrium real interest rate is endogenous and proportional to the expected quasi-difference of the marginal utility gap

$$r_t^* \equiv (\sigma_\omega + \delta\xi)\mathbb{E}_t\omega_{t+1} - (\sigma_\omega + \xi)\omega_t. \tag{8}$$

Our numerical experiments document how expansionary credit spread shocks generate a negative marginal utility gap, as borrowers increase their consumption more rapidly than savers, thus linking a buildup of private debt with an increase of the equilibrium real interest rate, as in Eggertsson and Krugman (2012) and Benigno et al. (2020). Conversely, a shock that forces borrowers to cut consumption in order to delever generates downward pressure on the equilibrium real interest rate.

The difference between the Euler equations of the two types gives an equation for the marginal utility gap

$$\omega_t = \kappa_t + \mu_t - \beta\gamma_d[\delta + (1-\delta)\xi]\mathbb{E}_t\mu_{t+1} + \delta\mathbb{E}_t\omega_{t+1}, \tag{9}$$

where κ_t is a scaled version of the equity requirement, which we interpret as a credit spread shock (see section 5.2 for more details), and μ_t is the Lagrange multiplier on the collateral constraint.[18]

Taking the average of the housing demand equations of borrowers and savers yields a pricing equation for housing

$$q_t = (1-\beta)[\bar{\sigma}y_t + \xi\tilde{\sigma}_h\tilde{h}_t + (\xi + \sigma_\omega)\omega_t] + \beta(\mathbb{E}_t q_{t+1} - i_t + \mathbb{E}_t\pi_{t+1}), \tag{10}$$

where q_t denotes real house prices. House prices are increasing in aggregate income and expected future house prices, and negatively related to the real interest rate, as in a standard user-cost equation. However, since borrowers are credit constrained, house prices are also increasing in both the housing gap and the marginal utility gap.

The difference between the housing demand equations of the two types gives an equation for the housing gap

$$(1-\beta)\tilde{\sigma}_h\tilde{h}_t = (1-\gamma_d)\Theta\mu_t - \omega_t + \beta\delta\mathbb{E}_t\omega_{t+1}. \tag{11}$$

The housing gap is positively related to the tightness of the collateral constraint and negatively to the marginal utility gap.

The approximation of the collateral constraint gives

$$b_t \le \ln\mathcal{M} + \frac{\delta\gamma_d}{\mathcal{M}}(b_{t-1} - \pi_t) + \frac{(1-\gamma_d)\Theta\eta_q}{\mathcal{M}}[\theta_t + q_t + (1-\xi)\tilde{h}_t], \tag{12}$$

where θ_t is the log-deviation of the LTV limit from its steady-state level (Θ) and $\mathcal{M}$ denotes the steady-state ratio of the value of the collateral constraint to the real value of debt.[19] Therefore, the complementary slackness conditions are (12), $\mu_t \ge 0$, and

$$\mu_t\left\{b_t - \ln\mathcal{M} - \frac{\delta\gamma_d}{\mathcal{M}}(b_{t-1} - \pi_t) - \frac{(1-\gamma_d)\Theta\eta_q}{\mathcal{M}}[\theta_t + q_t + (1-\xi)\tilde{h}_t]\right\} = 0. \tag{13}$$

[18]The multiplier on the collateral constraint is expressed in levels, rather than as a log-deviation, since its steady state value is zero.

[19]Since the collateral constraint is slack in steady state, $\mathcal{M} > 1$ and hence $\ln\mathcal{M} > 0$.

Real debt evolves according to

$$b_t = \frac{\delta}{\beta}(b_{t-1} + i_{t-1} + \kappa_{t-1} - \pi_t) + (1-\xi)\eta_q(\tilde{h}_t - \delta\tilde{h}_{t-1}) + \eta_d \left[\left(\frac{\varpi^b \bar{\sigma}}{\sigma^b} - 1 \right) y_t - (1-\xi)\sigma_\varphi \omega_t \right], \quad (14)$$

where we have used the equation for the nominal interest rate faced by borrowers ($i_t^b = i_t + \kappa_t$) that comes from the banks' problem.

Finally, the Phillips curve is

$$\pi_t = \gamma[(\bar{\sigma} + \varphi)y_t + \sigma_\omega \omega_t] + \beta \mathbb{E}_t \pi_{t+1}. \quad (15)$$

Because of the different labor supply behavior of borrowers and savers, financial frictions affect inflation dynamics through the marginal utility gap as in Cúrdia and Woodford (2016), thus playing the role of an endogenous cost-push shock.

Given the sequence of policy instruments $\{i_t, \theta_t\}_{t=0}^{\infty}$, exogenous shocks $\{\kappa_t\}_{t=0}^{\infty}$, and initial conditions on debt, housing gap and the nominal interest rate $\{b_{-1}, \tilde{h}_{-1}, i_{-1}\}$, an equilibrium for the log-linear version of the model is a sequence $\{y_t, \pi_t, \omega_t, \tilde{h}_t, q_t, b_t, \mu_t\}_{t=0}^{\infty}$ that satisfies (7), (9), (10), (11), (12), (14), and (15), $\forall t \geq 0$, as well as a set of inequality constraints. These inequality constraints include the contemporary slackness condition (13) and $\mu_t \geq 0$, and the constraints on the policy instruments. In particular, the zero lower bound requires $i_t \geq \ln \beta$ and the credit supply constraint requires $\theta_t \leq 0$.

4.3 Optimal Targeting Rules

This section builds intuition for the quantitative experiments, which include occasionally binding constraints, by studying two simplified examples. First, we consider the case in which the collateral constraint never binds. Second, we examine optimal policy when the collateral constraint is always binding. In both cases, we assume the nominal interest rate never violates the ZLB ($i_t > \ln \beta \ \forall t$). We focus on the discretionary solution for comparability with the numerical experiments in the next section.[20] The main result is that, in both examples, the optimal monetary policy targeting rule is identical to the one obtained in the baseline New Keynesian model. In addition, when the collateral constraint binds, optimal macro-prudential policy balances a tradeoff between the stabilization of the distortions caused by financial frictions and those caused by nominal rigidities.

We begin with the case in which the collateral constraint never binds ($\mu_t = 0 \ \forall t$). This example is particularly simple because the credit spread shock fully determines the marginal utility gap (from equation 9), which in turn pins down the housing gap (from equation 11). As a consequence, the cost-push shock component associated with the marginal utility gap in the Phillips curve becomes exogenous. Therefore, since the marginal utility gap and the housing gap become independent of policy, the loss function only depends on output and inflation, as in the standard New Keynesian model. Moreover, because house prices and debt enter neither the aggregate demand equation (7) nor the Phillips curve (15), the solution of the optimal policy problem corresponds to a standard flexible targeting rule that trades off output and inflation

$$\varepsilon \pi_t + y_t = 0. \quad (16)$$

Macro-prudential policy has no bearing on the equilibrium allocation and the optimal LTV ratio is indeterminate. Despite its simplicity, this case clarifies that in our model macro-prudential policy is effective only if the collateral constraint binds, is expected to bind at some point in the future, or if the macro-prudential authority can tighten the LTV limit enough to make the constraint bind. In this

[20] Appendix A.8 reports the Lagrangian formulation of the optimal policy problem for the second example and derives the optimal targeting rules under both discretion and commitment. The discussion in this section informally summarizes the results.

simple example, those three conditions are ruled out by assumption.

In the second case that we study analytically, we assume that the collateral constraint always binds ($\mu_t > 0\ \forall t$). Since θ_t only enters (12), we can use this equation residually to derive the value of the LTV limit that implements the optimal policy plan.[21] Following the same logic, house prices only affect (10), so that this equation is not a binding constraint for the optimal policy problem. In principle, debt, the nominal interest rate and the housing gap are state variables for this problem through equation (14). Appendix A.8 however proves that the optimal policy plan can be characterized without reference to the law of motion of debt. Thus, the optimal policy problem is purely forward looking, and equation (14) can be used to determine the equilibrium level of debt. This conclusion also implies that equation (7) determines residually the nominal interest rate.

The simplified optimal policy problem then consists of minimizing (6) subject to (9), (11), and (15). The optimal targeting rule for monetary policy follows from combining the first order conditions of the optimal policy problem for output and inflation. The central bank continues to trade off output and inflation exactly as in the baseline New Keynesian model and in equation (16). Distributional variables (the marginal utility gap and the housing gap) do not enter directly the optimal targeting rule for monetary policy.

The optimal targeting rule for macro-prudential policy follows from combining the first order conditions for the marginal utility gap, the housing gap, and the multiplier on the collateral constraint

$$\lambda_\omega \omega_t - \frac{\sigma_\omega}{\bar{\sigma} + \varphi} y_t - \frac{[1 - (1 - \gamma_d)\Theta]\lambda_h}{(1 - \beta)\tilde{\sigma}_h} \tilde{h}_t = 0. \tag{17}$$

If the collateral constraint has no inertia ($\gamma_d = 0$) and the steady state LTV ratio is 100% ($\Theta = 1$), the optimal targeting rule for macro-prudential policy only trades off the marginal utility gap and the output gap. In particular, optimal policy requires that the two variables move in the same direction because $\sigma_\omega > 0$.[22]

Optimal macro-prudential policy is therefore countercyclical. A positive output gap calls for a positive marginal utility gap (higher marginal utility of consumption of borrowers relative to savers). The macro-prudential authority achieves this result by tighter LTV limits, which reduce borrowers' consumption relative to savers' consumption. In this way, the macro-prudential authority strikes an optimal balance between aggregate and distributional variables.

More generally (i.e., without restrictions on γ_d and Θ), the housing gap matters too. Since the coefficient multiplying the housing gap is also always positive, optimal macro-prudential policy induces positive co-movement between a combination of the housing gap and the output gap on the one hand, and the marginal utility gap on the other. The targeting rule (17) also suggests that optimal policy does not involve a complete separation of objectives. The presence of output in the optimal targeting rule for macro-prudential policy implies a direct feedback effect from monetary policy.[23]

5 Quantitative Experiments

This section presents our main results, demonstrating the power of optimal LTV limits to stabilize business cycle fluctuations when the lower bound on the nominal interest rate and the collateral constraint

[21] We also assume that the resulting LTV limit respects the feasibility constraint $\theta_t < 0$.

[22] The sign of $\sigma_\omega > 0$ depends on the difference between ϖ^b/σ^b and ϖ^s/σ^s, where $\varpi^\tau \equiv C^\tau/Y$ is the steady state ratio between consumption and output for type τ. This difference is always positive because, by assumption, borrowers have a lower coefficient of relative risk aversion ($\sigma^b < \sigma^s$), which in turn implies that their steady state consumption share is higher ($\varpi^b > \varpi^s$).

[23] Optimal targeting rules are not unique. Indeed, inflation could replace the output gap in the optimal targeting rule for macro-prudential policy, as substituting (16) into (17) shows, although the interpretation would not change.

Table 1: Parameter values.

Parameter	Description	Value
β	Individual discount factor	0.995
ξ	Fraction of borrowers	0.625
δ	Probability of not changing type	0.990
σ^b	Coefficient of risk aversion (borrowers)	0.707
σ^s	Coefficient of risk aversion (savers)	3.537
χ_H	Housing utility parameter	0.024
σ_h	Inverse elasticity of substitution for housing	1.000
χ_L	Labor utility parameter	0.985
φ	Inverse Frisch elasticity	1.000
Θ	Maximum LTV	0.750
γ_d	Collateral constraint inertia	0.700
α	Probability of keeping price unchanged	0.870
ε	Elasticity of substitution among varieties	6.000
Ψ_h	Housing holding cost	0.800
ψ_π	Taylor rule feedback on inflation	1.500
ψ_y	Taylor rule feedback on output gap	0.125
b/Y	Mortgage debt relative to GDP	2.880
q/b	Value of real estate relative to mortgage debt	1.625
$\rho_{\hat{\kappa}}$	Persistence of temporary spread component	0.925
$\rho_{\bar{\kappa}}$	Persistence of quasi-permanent spread component	0.995

can be occasionally binding. A sequence of credit spread shocks generates a boom-bust scenario for house prices and private debt similar to the dynamics observed in the United States (Figure 1), including a recession in which the ZLB constrains the nominal interest rate. In the baseline scenario, monetary policy follows a simple Taylor rule and the LTV limit is constant at its steady state level. Optimal policy calls for a tightening of LTV limits during the boom phase which prevents the run-up in private debt and mitigates the effects of the bust. Indeed, under the optimal policy plan the nominal interest rate does not encounter the ZLB.

Before illustrating our results, we discuss the calibration of the model, including the process for the credit spread shocks, and the simulation methodology.

5.1 Parameter Values

Table 1 reports the parameter values used in the simulations. The individual discount factor β equals 0.995 so that the annualized steady state real interest rate is 2%. The share of borrowers ($\xi = 0.625$) corresponds to the fraction of mortgagors in the US from Cloyne et al. (2020), after adjusting for the absence of renters in our model. We set the probability that the type does not change (δ) equal to 0.99 to generate a high persistence in mortgage debt as in the data.[24] Following Cúrdia and Woodford (2016), we choose the ratio between the coefficient of risk aversion of the two types (σ^b/σ^s) equal to 0.2, and back out their levels by imposing that the inverse elasticity of output to the real interest rate $\bar{\sigma}$ (see appendix A.6 for its expression) in the aggregate demand equation (7) is equal to one, a common value in the literature (e.g. Galí, 2015). Also standard is the value for the elasticity of substitution among varieties ($\varepsilon = 6$), which we choose to deliver a steady state markup of 20%. While we assume the existence of

[24] The calibrated value of δ is slightly higher than in Cúrdia and Woodford (2016) but is consistent with the idea that being a net borrower/saver is a persistent characteristic over the life cycle.

a subsidy to eliminate the steady state monopolistic distortion, this parameter remains important in governing the optimal monetary policy tradeoff between inflation and the output gap (see equation 16).

The curvature of the utility from housing services (σ_h) and the inertia parameter in the collateral constraint (γ_d) correspond to the calibrated value and posterior mode, respectively, in Guerrieri and Iacoviello (2017). The steady state LTV ratio (Θ) equals 75%, based on the average between 1973 and 2000 in the Federal Housing Finance Agency's Monthly Interest Rate Survey (Table 17). We set the parameter of the housing holding cost function (Ψ_h) to 0.8 in order to match the relative decline in housing wealth between savers and borrowers between 2007 and 2010.[25]

We assume that the inverse elasticity of labor supply (φ) is equal to 1, within the range of the macro estimates (Peterman, 2016), though closer to the estimates from micro data (Chetty et al., 2011). The value of the probability that firms do not adjust their price in a quarter ($\alpha = 0.87$) is in line with the recent estimates in Del Negro et al. (2015).[26] The baseline specification for monetary policy (discussed in Section 5.3) is an interest rate rule with the coefficients on inflation and the output gap ($\psi_\pi = 1.5$ and $\psi_y = 0.125$) set to standard values (Taylor, 1993).

Finally, we calibrate the steady-state level of mortgage debt to 45% of (annual) GDP, in line with US data in 2000, when the housing boom began. The ratio between the value of housing and mortgage debt, which in US data between 1995 and 2016 is 2.6, gives us a target for q/b.[27]

5.2 Exogenous Shocks and Simulation Methodology

In the model, up to a first-order approximation, κ_t corresponds to the credit spread, that is, the difference between the interest rate on loans and on deposits ($\kappa_t = i_t^b - i_t$). We generate a boom-bust scenario in house prices consistent with the US data in Figure 1 through a sequence of small unanticipated negative credit spread shocks followed by one large positive shock. The credit spread declines persistently during the boom before spiking at the time of the bust.

The shocks determining the persistent fall of the credit spread seek to capture the process of financial liberalization and innovation that took place in the US starting in the second half of the 1990s (Mian and Sufi, 2009; Boz and Mendoza, 2014). The large contractionary shock approximates the tightening of credit standards at the onset of the recession (e.g. Chen et al., 2020).

Figure 3 plots the process for the credit spread that drives the simulations.[28] We assume that κ_t is the sum of two components ($\bar{\kappa}_t$ and $\hat{\kappa}_t$), each following a stationary first-order autoregressive process, with persistence $\rho_{\hat{\kappa}} < \rho_{\bar{\kappa}} \to 1$, respectively. We calibrate $\rho_{\bar{\kappa}}$ to 0.995 and refer to $\bar{\kappa}_t$ as the near-permanent component of spreads, while we set the persistence of the temporary component $\rho_{\hat{\kappa}}$ to 0.925.

Negative shocks to the near-permanent component generate the secular decline in the spread. The spike is the combination of a positive shock to the temporary component and a positive shock to the near-permanent component that partially reverses the previous decline. The boom lasts for 32 quarters.

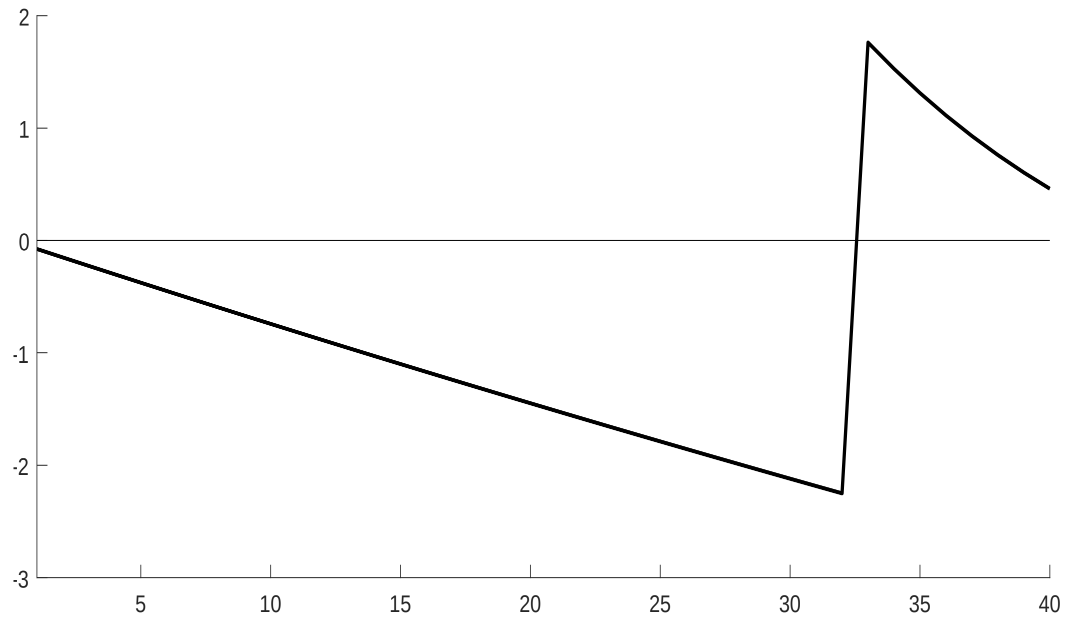
[25] Using data from the Survey of Consumer Finances, Menno and Oliviero (2020) document large differences in the decline of housing wealth held by borrowers and savers, and separately calibrate the costs (with an identical functional form to ours) for the two types to match those targets. For analytical tractability, we assume the same parameter for borrowers and savers. Setting $\Psi_h = 0.8$ delivers a relative decline of borrowers and savers' housing wealth (measured as the ratio of the log changes in housing wealth) consistent with the data between 2007 and 2010.

[26] The implied slope of the Phillips curve is $(\bar{\sigma} + \varphi)(1 - \alpha)(1 - \alpha\beta)/\alpha = 0.04$.

[27] In the model, we adjust per-capita debt b by the share of borrowers ξ to ensure proper comparability with the data. Appendix A.5 describes how we use per-capita debt/GDP and the value of housing relative to mortgage debt to back out the constants χ_L and χ_H, and hence a number of other steady-state variables.

[28] In practice, housing demand shocks are likely to have contributed to the gyrations of house prices during the first decade of the 2000s (Adelino et al., 2016). As this type of shock is more difficult to calibrate, we limit our attention to credit spread shocks inferred from spreads. In an earlier version of the paper (Ferrero et al., 2018), we show that credit spread and housing demand shocks have very similar consequences for macroeconomic variables, especially in the absence of macro-prudential policy.

Figure 3: The exogenous credit spread shock.



NOTE: The figure plots the path of $\kappa_t = \bar{\kappa}_t + \hat{\kappa}_t$ (in annualized units). Both components follow a stationary first-order autoregressive process, with parameters $\rho_{\hat{\kappa}} < \rho_{\bar{\kappa}} < 1$.

The two contractionary shocks arrive in the following period and no further shocks occur thereafter.

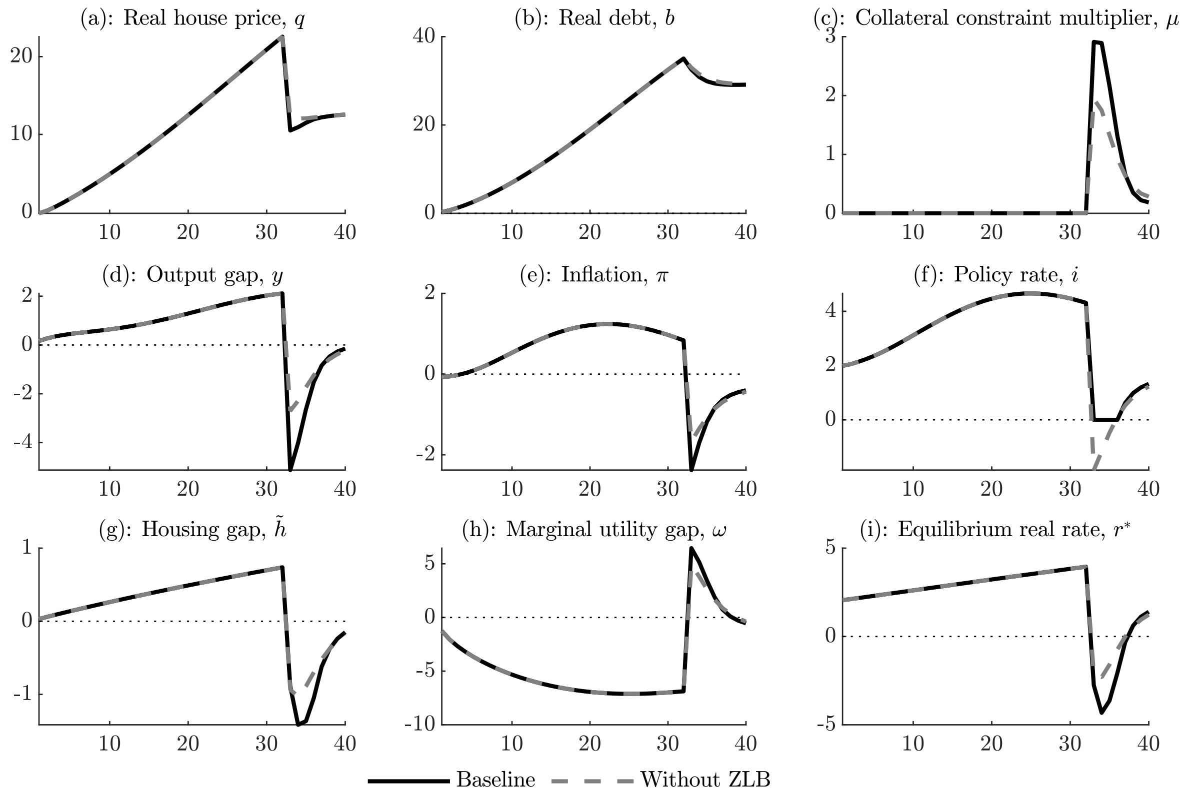
The decline of approximately 200 basis points in our simulation is consistent with the evolution of the spread between the average mortgage rate in the Private Label Securities Database (PLSD) and the 10-year Treasury yield between 2000 and 2007 (Justiniano et al., 2021). Over this period, the marginal mortgagor likely belonged to this category. Unfortunately, due to the dry-up of the private label segment of the market at the onset of the financial crisis, the data from the PLSD end in 2007. However, even conventional mortgage rates spiked at that time. Menno and Oliviero (2020) document the dramatic rise by about 450 basis points of the spread between the one-year adjustable rate mortgage and the federal funds rate in 2008-2009. Fixed-rate mortgage rates also spiked at this time. For example, during the same period, the spread between the 30-year fixed mortgage rate and the average of the 5 and 10-year Treasury yields rose by around 150 basis points (Walentin, 2014). The calibrated increase of the spreads in the simulation falls squarely within this range.[29]

We solve the model using a piecewise-linear solution method to account for the possibility that (i) the zero lower bound on the short-term nominal interest rate becomes binding and/or (ii) the borrowers' collateral constraint (12) becomes slack.[30] This approach accounts for the possibility that the occasionally binding constraints may apply in future periods, although not for the risk that future shocks may cause the constraints to bind. Thus, our approach, which is based on the methods developed by Guerrieri and Iacoviello (2015) and Harrison and Waldron (2021), abstracts from the skewness in the expected distribution of future endogenous variables arising from the possibility of being constrained in future.

[29] At the time of the crisis, spreads soared pretty much in all segments of private credit markets (Gilchrist and Zakrajšek, 2012).

[30] In our model, a binding zero lower bound on the short-term nominal interest rate also implies a zero nominal rate of return to savings. Theoretically, a zero lower bound on savings rates arises from the existence of an unmodeled zero-interest-bearing alternative saving instrument (e.g. cash). In practice, the evidence on negative interest rates (e.g. Eisenschmidt and Smets, 2019) suggests that deposit rates feature a hard floor at zero, although anecdotally some banks may have introduced new fees on deposit accounts when official interest rates became negative.

Figure 4: Baseline scenario (constant LTV limit).



NOTE: All variables are scaled by 100 and plotted as log-deviations from steady state, except for the multiplier on the collateral constraint, which is in levels. Inflation, the policy rate and equilibrium real interest rate are shown in annualized units.

Appendix C contains a full description of the approach.

5.3 Baseline Scenario: The Pre-Crisis Consensus

This section presents our baseline scenario, which we label 'the pre-crisis consensus'. In the pre-crisis consensus, the central bank controls the short-term nominal interest rate to stabilize fluctuations in inflation and output while macro-prudential policy is inactive. This policy configuration captures well the reliance on monetary policy for macroeconomic stabilization and the general absence of macro-prudential policy frameworks that prevailed in many economies before the financial crisis of 2008.

In the context of our model, inactive macro-prudential policy implies that the LTV limit remains constant at its steady-state level ($\theta_t = 0,\ \forall t$). We assume that the central bank conducts monetary policy according to a standard nominal interest rate rule (Taylor, 1993)

$$i_t = \psi_\pi \pi_t + \psi_y y_t, \tag{18}$$

where $\psi_\pi > 1$ and $\psi_y \geq 0$ (see section 5.1 for the calibration of these two parameters).

Figure 4 shows the response of the economy to the evolution of credit spreads under the baseline policy assumptions. The solid black lines correspond to the case in which both the collateral constraint and the zero bound on the policy rate can be occasionally binding. To illustrate the importance of the zero lower bound, the dashed gray lines display the results when the nominal interest rate can become negative.

The initial decline in mortgage spreads encourages borrowers to increase their leverage and purchase more housing. During the boom period, house prices (panel a) go up by slightly more than 20 percentage points while debt (panel b) rises by more than 30 percent relative to its steady state value. Borrowers remain unconstrained during the boom period (panel c), while the housing gap becomes positive (panel g), which reflects the increase of housing demand by borrowers relative to savers.

The large boom in house prices and the increase in debt coincides with a moderate expansion of real economic activity. The output gap approaches two percent at the onset of the bust (panel d). Inflation also rises above target during the boom period (panel e), though the fall of the marginal utility gap (panel h) due to the increase in credit availability mitigates the effect of the boom acting like an endogenous negative cost-push disturbance. During this phase, the nominal interest rate increases by about 200 basis points (panel f). The policy rate rises alongside (albeit not exactly one for one) the equilibrium real interest rate (panel i), which is itself inversely related to the marginal utility gap, as equation (8) shows.

The baseline scenario also captures the broad contours of the Great Recession. As house prices collapse, borrowers start to delever. The persistence in the collateral constraint slows down the adjustment, which lasts for several quarters, in line with the decoupling between house prices and mortgage debt observed in the data. As in Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2017), the deleveraging process pushes the equilibrium real interest rate into negative territory. The nominal interest rate falls all the way to zero and remains at the lower bound for four quarters.[31]

During this time, the economy experiences a severe recession, exacerbated by the inability of monetary policy to provide full accommodation. Output falls five percentage points below trend and inflation misses the central bank's target by slightly more than two percentage points on an annualized basis.[32]

When the housing bust occurs, the collateral constraint becomes binding and the shadow value of an additional unit of debt increases significantly. The tightening of the collateral constraint contributes to amplifying the impact of the shock (Guerrieri and Iacoviello, 2017). The bust entails substantial redistribution from borrowers to savers as the patterns of both the marginal utility gap and the housing gap observed during the boom reverse sharply. The welfare-based loss (6) suggests that a monetary policy response focused solely on inflation and the output gap, as the baseline rule prescribes, fails to address all the costs of the recession. Once the downturn is over, the process of monetary policy normalization is gradual, consistent with the sluggish recovery in the data.

The results from the simulation that ignores the lower bound on the interest rate shed further light on the interplay between financial frictions and monetary policy during the housing bust. As the dashed gray lines in Figure 4 show, allowing the policy rate to fall below zero substantially mitigates the effects of the crisis. As the negative shock hits, the policy rule prescribes a decline of the nominal interest rate by around 600 basis points, deep into negative territory. This response cushions the drop of output and inflation. While the housing gap still falls sharply on impact, the recovery is much swifter and the increase in the marginal utility gap is slightly less extreme. Similarly, the collateral constraint binds less tightly so that house prices fall by a few percentage points less than in the baseline case.

The presence of the ZLB therefore exacerbates the effects of the debt deleveraging process on spending and inflation (Korinek and Simsek, 2016), which reflect the relation between the equilibrium real interest rate and the marginal utility gap.[33] The ZLB limits the feasible reduction of the nominal and hence real interest rate, thus depressing in particular borrowers' spending, increasing the marginal utility gap, and tightening the collateral constraint. As a result, the equilibrium real rate falls further, which creates a negative aggregate demand loop.

[31] At the time of the financial crisis, market participants expected the Fed funds rate to remain at the ZLB for one year (Moore, 2008).

[32] Our results are consistent with a moderate decline of inflation (and little actual deflation relative to a two percent target) because the inflationary effect of the marginal utility gap in the Phillips curve partly compensates the deflationary pressures associated with the decline in aggregate demand (Gilchrist et al., 2017).

[33] A similar channel arises in Eggertsson and Krugman (2012) and Benigno et al. (2020).

5.4 Optimal LTV Policy

The last section highlighted how a tightening of the collateral constraint can generate large declines in the equilibrium real interest rate and amplify the effects on aggregate demand, especially if monetary policy is unable to adequately respond because of the zero lower bound. These results suggest that macro-prudential policies directly affecting the collateral constraint, and hence its tightness, may be able to mitigate the effects of a sharp increase in the cost of credit. In this section, we verify this conjecture by allowing the policymaker to set the macro-prudential LTV limit θ_t during the boom-bust scenario.

A natural benchmark is the joint determination of monetary and macro-prudential policy to minimize the welfare-based loss function (6). As we focus on time-consistent policies, the policymaker is unable to make promises about future actions in order to improve stabilization outcomes today.[34] One motivation for studying time-consistent policies is to limit the power of monetary policy at the zero lower bound. While optimal commitment policies can be very effective at mitigating the negative consequences of the ZLB in standard New Keynesian models (see, for example, Eggertsson and Woodford, 2003), several recent contributions have questioned their empirical relevance (e.g. Del Negro et al., 2012). Our setting rules out these commitments and maximizes the potential scope for macro-prudential policies to improve outcomes when used alongside monetary policy.[35] Qualitatively, however, a jointly optimal commitment policy delivers similar results to the time-consistent policy considered here (see appendix B.2).

Figure 5 compares the outcomes in the housing boom-bust scenario under the baseline assumptions of a Taylor rule and a constant LTV limit (solid black lines) with the case in which a single policymaker jointly sets the interest rate and the LTV limit to minimize the welfare-based loss function (6) (dashed gray lines).

The key result is that the active LTV policy markedly improves stabilization of the welfare-relevant variables during both the boom and the bust. The jointly optimal policy plan almost fully stabilizes inflation, the output gap, and the marginal utility gap, while the volatility of the housing gap visibly declines.

Focusing first on the boom, we observe that under optimal policy debt actually declines in response to the reduction of credit spreads, while its shadow value rises significantly (panels b and c, respectively). The tightness of the collateral constraint, together with an approximately constant real interest rate (panel i), push house prices higher than under the baseline policy (panel a). This result echoes the discussion of the empirical evidence on the effects of LTV limits on house prices in section 2.

Interestingly, output and inflation remain close to their target values (panels d and e) despite a stable path of the nominal interest rate. The reason is that the active LTV policy stabilizes the marginal utility gap (panel h). As a consequence, no material tradeoff between output and inflation stabilization emerges. The only welfare-relevant variable that significantly moves away from target is the housing gap (panel g), although its increase is somewhat smaller than in the baseline scenario.

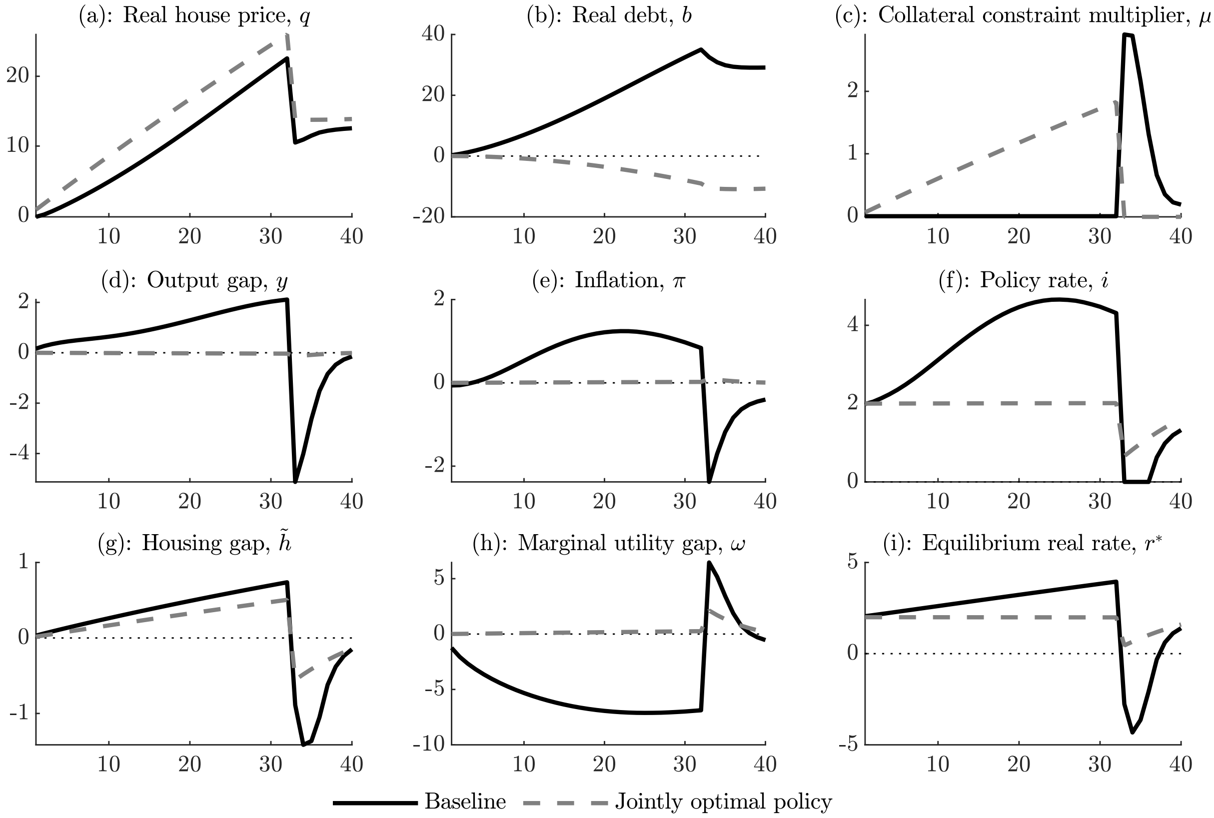
When credit spreads reverse, so does the policy stance.[36] From the macro-prudential perspective, the

[34] In this case, we can treat the multiplier on the collateral constraint μ_t as the macro-prudential instrument. When the collateral constraint binds, a one-to-one mapping links the LTV limit and the multiplier on the collateral constraint so that selecting μ_t as the policy instrument merely represents a change of variables in the policy problem. When the collateral constraint is slack, a range of values for θ_t above a certain threshold is consistent with the equilibrium. As the multiplier cannot be negative, the lower bound on μ_t corresponds to a constraint on policy. This credit demand constraint rules out cases in which the policymaker forces borrowers to hold more debt than demanded at market prices. Moreover, under this interpretation of the policy problem, the credit supply constraint $\theta_t \leq 0$ can be recast as a time-varying lower bound on μ_t. Appendix C discusses the technical details of the solution.

[35] Our analysis therefore contributes to an emerging literature studying monetary and macro-prudential policies under discretion. Bianchi and Mendoza (2018) argue that the nature of financial frictions generates an inherent time-inconsistency problem for macro-prudential policymakers. Laureys and Meeks (2018) demonstrate that discretionary policies can generate better outcomes than a class of simple macro-prudential policy rules studied in the existing literature.

[36] In practice, without the boom period, the financial crisis may have never happened and spreads may have not

Figure 5: Jointly optimal policy.



NOTE: All variables are scaled by 100 and plotted as log-deviations from steady state, except for the multiplier on the collateral constraint, which is in levels. Inflation, the policy rate and equilibrium real interest rate are shown in annualized units.

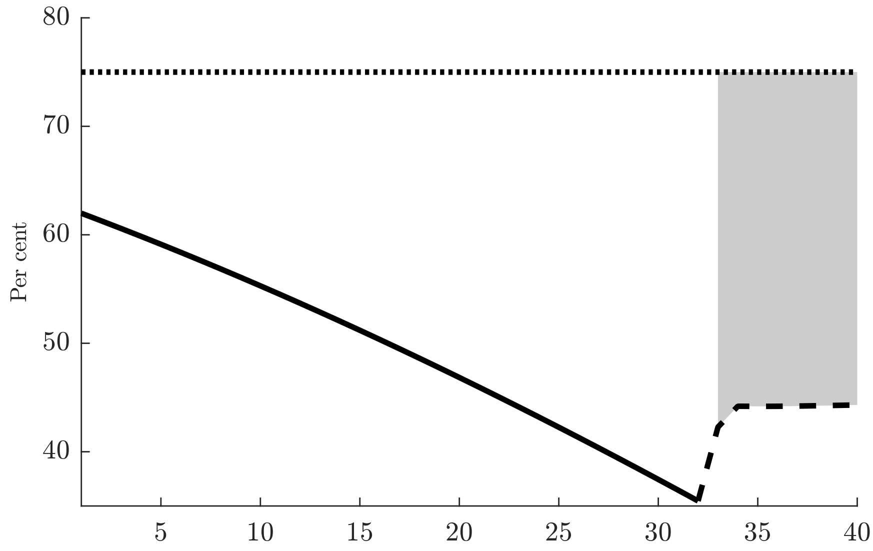
policymaker would ideally support borrowing, effectively subsidising debt by setting $\mu_t < 0$. However, the contemporary slackness conditions imply a lower bound on the multiplier ($\mu_t \geq 0$) and the credit demand constraint binds. The policymaker cuts the nominal interest rate, though without reaching the ZLB. The introduction of an active LTV policy mitigates the impact of the shock on the marginal utility gap, which in turn translates into a smaller decline of the equilibrium real interest rate. The combined monetary and macro-prudential response continues to ensure almost full stabilization of output and inflation, as during the boom. The housing gap falls, but its movement is less than half of that in the baseline scenario.

Figure 6 plots the level of the LTV limit (Θ_t) under the jointly optimal policy plan, confirming the prediction in section 4.3 that optimal macro-prudential policy is strongly countercyclical. Indeed, optimal policy requires an aggressive reduction of the LTV limit during the boom, which implies a tightening of the collateral constraint. As discussed above, this tightening increases the shadow value of debt and reduces its equilibrium level. During the bust, the credit demand constraint ($\mu_t \geq 0$) prevents the policymaker from inducing borrowers to hold more debt and further closing the marginal utility gap. The gray shaded area shows the range of LTV limits that are consistent with the equilibrium, in which the collateral constraint is slack.[37] The adjustments of the LTV limit required to deliver the jointly optimal policy are substantial, although not unprecedented. For example, as noted in section 2, South

spiked. We nevertheless find it instructive to discuss the optimal policy configuration in response to an increase in credit spreads.

[37]Figure 6 also clarifies that the LTV limit never exceeds its steady state limit of 75% (dotted line).

Figure 6: LTV limit under the jointly optimal policy plan.



NOTE: The figure plots the level of the LTV limit under the jointly optimal policy plan. The solid line corresponds to periods in which the collateral constraint binds. The dashed line corresponds to periods in which the collateral constraint is slack. In these periods, any value in the shaded grey area is consistent with the equilibrium. The dotted line corresponds to the steady-state value of the collateral constraint (75%), which the simulation treats as an upper bound.

Korea implemented similarly restrictive LTV limits shortly after their introduction as macro-prudential tools.

Our results illustrate the extent to which LTV limits may act as a substitute for monetary policy action. During the boom, active macro-prudential policy eliminates upward pressure on the equilibrium real interest rate and the need for a monetary policy tightening. When credit conditions worsen, the collateral constraint becomes slack, so that macro-prudential policy is unable to support borrowing as to fully stabilize the equilibrium real interest rate. Nevertheless, the macro-prudential policy loosening cushions the decline in the equilibrium real interest enough to avoid the ZLB and the associated feedback effects on the equilibrium real rate observed in the baseline scenario.

Finally, we note that the countercyclical adjustment of the LTV limit is the main source of improvement in macroeconomic outcomes relative to the baseline scenario. Appendix B.3 considers the case in which monetary policy continues to follow the baseline Taylor rule (18), whereas the macro-prudential policymaker sets the LTV limit to minimize the welfare-based loss function. The results for that policy configuration are almost identical to those under jointly optimal policy shown in Figure 6, with the exception of small differences in the paths of the output gap and inflation.[38]

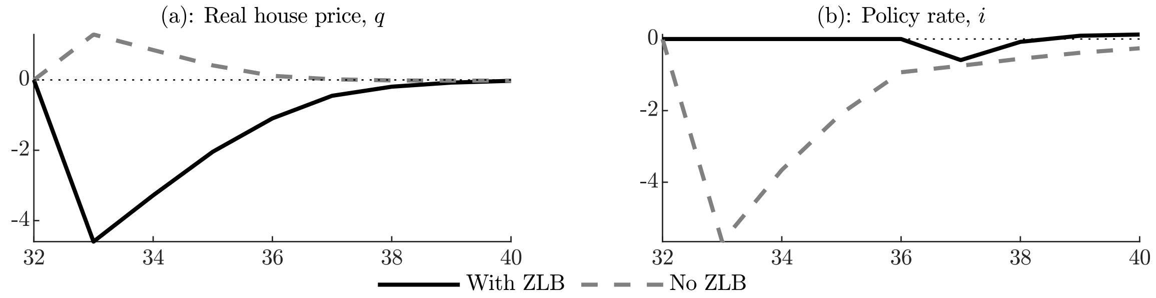
5.5 State-Contingent Effects of LTV Limits

Two striking results emerge from the simulations in section 5.4. First, house prices rise by more during the boom under the jointly optimal policy plan than in the baseline simulation, even though optimal policy involves a substantial tightening of the LTV limit. Second, the active use of LTV limits is very effective in cushioning the consequences of the bust on welfare-relevant variables, even though the credit demand constraint somewhat limits the scope of macro-prudential policy.

In this section we use two experiments, inspired by observed macro-prudential policy actions, to

[38]These differences reflect the fact that the baseline policy rule implies a slightly different tradeoff between output gap and inflation stabilization than an optimal monetary policy. See appendix B.3 for a further discussion.

Figure 7: Macro-prudential tightening in the bust.



NOTE: Both variables are plotted as log-deviations from the baseline simulation. Real house prices are scaled by 100 while the policy rate is shown in annualized percentage points.

explore the state-contingent nature of LTV limits, that is, their efficacy in relation to the broader macroeconomic environment.

5.5.1 The Monetary Policy Response

We begin by investigating how the effects of changes in the LTV limit on house prices depend on the response of monetary policy in an empirically relevant scenario. In this experiment, the macro-prudential authority implements a persistent reduction of the LTV limit by 5 percentage points at the same time as credit spreads spike. While the main objective of this experiment is simply to illustrate the importance of the monetary policy response, the Canadian experience in 2008 provides a realistic target for the scale of the shock (Allen et al., 2017).[39]

To show the importance of the monetary policy reaction to the LTV tightening, we simulate the model with and without ZLB constraint on the policy rate. In both cases, the monetary authority follows the baseline interest rate rule (18). We present the results for each case in deviations from the baseline scenario constructed in section 5.3, in which the ZLB is binding. This approach permits a straightforward comparison of the macroeconomic effects of the LTV tightening under different assumptions about the monetary policy response.[40]

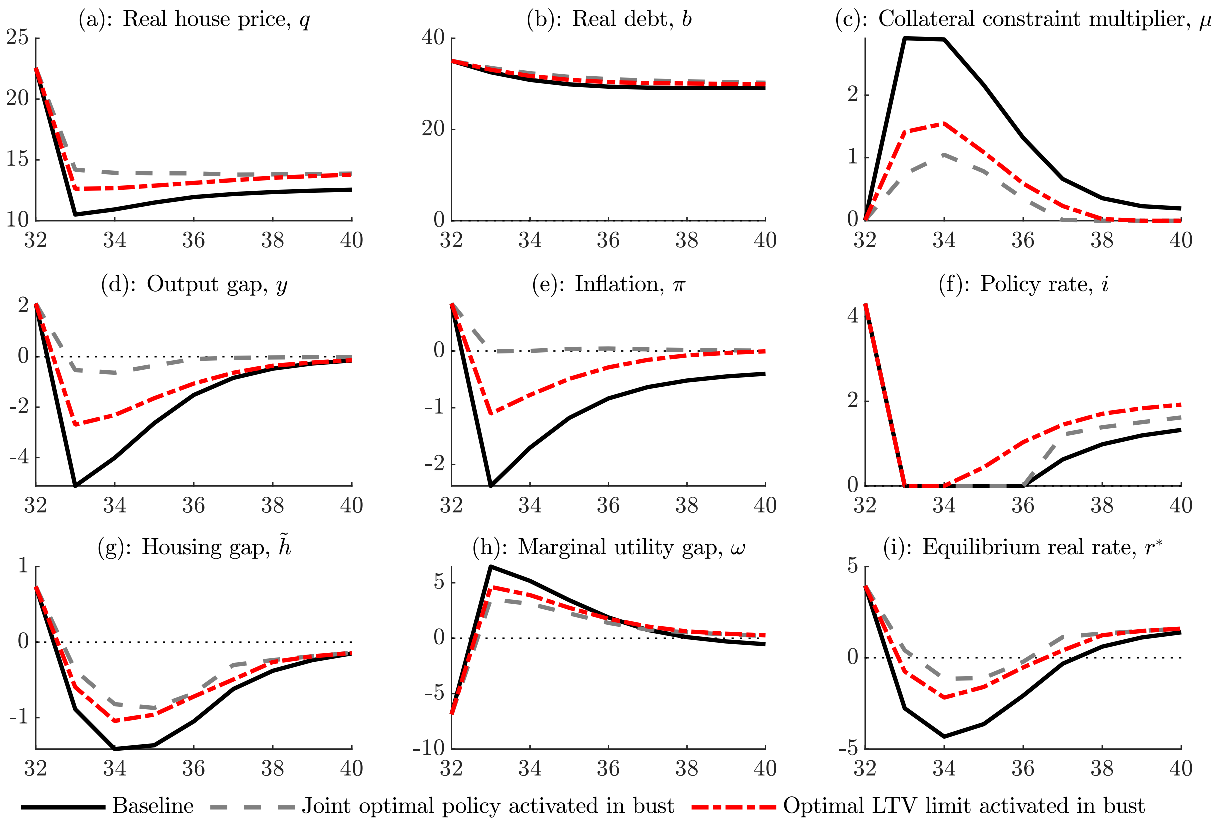
The solid black line in the left panel of Figure 7 shows the response of house prices in deviations from the baseline scenario. The dashed gray line corresponds to the case in which we ignore the ZLB constraint following the tightening of the LTV limit. The right panel plots the response of the nominal interest rate, also in deviations from the baseline scenario.

The figure demonstrates that our model, when confronted with an empirically relevant experiment, is consistent with the evidence in Bachmann and Rüth (2020) discussed in section 2. The sign of the response of house prices to a tightening of the LTV limit crucially depends on the monetary policy response. In the absence of the ZLB, house prices actually increase relative to the baseline in response to the LTV tightening because of the large contemporaneous decline of the nominal interest rate. Conversely, when the ZLB binds, house prices fall because monetary policy is relatively tight. This exercise therefore documents an important dimension of the interaction between monetary and macro-prudential policy that should be of relevance to policymakers when setting their respective instruments.

[39]The experiment assumes that the LTV limit θ_t is exogenous and follows a first-order autoregressive with high persistence ($\rho_\theta = 0.995$). A single innovation to the process at the time the bust occurs determines the size of the calibrated initial LTV limit reduction.

[40]A simple LTV tightening shock starting from the steady state would require a counterfactually large shock to make the ZLB binding. Appendix B.4 reports the full set of responses for this exercise. Unsurprisingly, the impact of the LTV tightening on both aggregate and distributional variables in less pronounced when the ZLB is not binding.

Figure 8: Optimal policy activated in the bust.



Note: All variables are scaled by 100 and plotted as log-deviations from steady state, except for the multiplier on the collateral constraint, which is in levels. Inflation, the policy rate and equilibrium real interest rate are shown in annualized units.

5.5.2 Debt Accumulation and the Credit Supply Constraint

Our second experiment illustrates the extent to which the ability of optimal policy to cushion the economy from the effects of the housing bust hinges upon contemporaneously preventing a substantial run up in debt during the boom. For this purpose, we study the introduction of the jointly optimal policy plan only at the time of the housing bust. This simulation mimics, in a stylized way, the adoption of macroprudential policy frameworks in the wake of the global financial crisis.

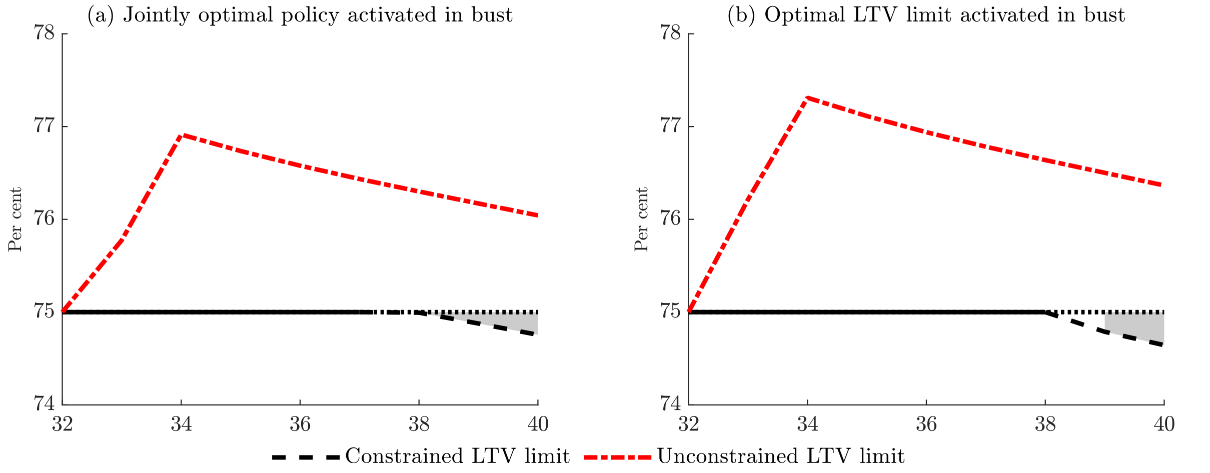
We assume that, for the duration of the boom, policy follows the baseline assumptions: the interest rate is set according to the Taylor rule (18) and the LTV limit is fixed ($\theta_t = 0$). When credit spreads spike, policy switches to the jointly optimal policy configuration: the interest rate and the LTV limit are set to minimize the welfare-based loss function (6).[41]

Figure 8 displays the results of the experiment, focusing on the responses of variables during the bust phase of the simulation. The dashed gray lines depict the case of jointly optimal policy activated in the bust. For comparison, the dashed-dotted red lines correspond to the case in which the policymaker optimally sets only the LTV limit when credit spreads spike, while continuing to follow the baseline interest rate rule for monetary policy. The solid black lines show the responses under the baseline policy configuration during both the boom and bust.

Optimal policy seeks to loosen the collateral constraint in order to cushion the effect of the spike in credit spreads on borrowers. Ideally, the policymaker would like to increase the LTV limit sufficiently to

[41]The policy 'regime change' is completely unanticipated. Appendix C.4 provides details of the solution method in this case.

Figure 9: LTV limit under optimal policy in the bust.



NOTE: The two panels plot the level of the LTV limit under the jointly optimal policy plan activated during the bust, and under the optimal LTV policy activated during the bust, respectively. The solid black lines correspond to periods in which the collateral constraint binds. The dashed black lines corresponds to periods in which the collateral constraint is slack. In these periods, any value in the shaded grey area is consistent with the equilibrium. The dotted black lines correspond to the steady-state value of the collateral constraint (75%), which the simulation treats as an upper bound. The dashed-dotted red lines show the results abstracting from the upper bound on the LTV limit (the credit supply constraint).

ensure that the collateral constraint is slack, as is the case when optimal policy is in place also during the boom. However, the baseline policy configuration followed during the boom has allowed debt to increase by around 30 percentage points above steady state when credit spreads spike. The inertia in the collateral constraint implies that such a large stock of debt accumulated during the boom hinders the ability of optimal policy to mitigate the recessionary effects of the bust. In this experiment, the collateral constraint can become slack only if the credit supply constraint is violated.

Our simulation imposes the credit supply constraint ($\theta_t \leq 0$), which forces the multiplier on the collateral constraint μ_t to remain temporarily positive. Figure 9 shows that the credit supply constraint binds when policy (either jointly or macro-prudential only) becomes optimal in the bust. In this figure, the solid black lines depict the path of the LTV limit that respects the credit supply constraint. As before, the dashed black lines and the shaded grey areas indicate the range of LTV limits consistent with an optimally slack collateral constraint. Finally, the dotted-dashed red lines show the results of a counterfactual simulation in which we ignore the credit supply constraint. In this case, the LTV limit persistently exceeds the level consistent with avoiding the moral hazard problem underpinning the collateral constraint.

When the jointly optimal policy plan becomes active in the bust, the LTV limit is at its upper bound for five quarters. If only the optimal LTV limit policy becomes active, the LTV limit remains at the upper bound for one additional quarter. While the LTV limit is constrained, the economy experiences a slightly milder variant of the debt-deflation dynamics that also characterize the baseline simulation. In particular, the binding collateral constraint reduces the equilibrium real interest rate, which becomes somewhat negative. As a result, the policy rate hits the ZLB, and the inability to track the decline of the equilibrium real interest rate generates a recession, which is significantly deeper when only the LTV policy is optimal.

The key difference between activating only the optimal LTV policy relative to activating the jointly optimal plan is the degree of monetary accommodation. Under the jointly optimal policy, the nominal interest rate exits the ZLB at the same time as in the baseline scenario. In contrast, when just the optimal LTV policy becomes active, the baseline interest rate rule prescribes an anticipated liftoff while the credit supply constraint (and hence the collateral constraint) is still binding. The early monetary tightening generates a deeper recession. This experiment therefore demonstrates that a binding credit supply constraint impairs the ability of the optimal LTV limits to deliver a similar performance to that achieved by the jointly optimal policy.

After the credit supply constraint ceases to bind, the LTV limit lies just below the steady-state level of 75%. From this point onward, the credit supply constraint never binds again. As a result, the policymaker is able to adjust the LTV limit to set the desired level of μ_t, and the equilibrium allocations for the welfare-relevant variables coincide exactly with those observed when the jointly optimal policy plan is in place also during the boom.[42] This experiment thus also demonstrates that the LTV limit by itself is not necessarily a sufficient statistic to gauge the stance of macro-prudential policy.

6 Conclusion

This paper has studied the jointly optimal monetary and LTV policy in a New Keynesian model with borrowers and savers. If the collateral constraint on borrowers either never or always binds, the monetary policy tradeoff between output and inflation is unchanged compared to the baseline New Keynesian model.

The interaction between monetary and macro-prudential policy becomes particularly important when both the lower bound on the nominal interest rate and the collateral constraint can be occasionally binding. Strongly countercyclical LTV limits can avoid a liquidity trap caused by the endogenous debt-deleveraging response to a credit spread shock. Optimal policy prevents an excessive accumulation of debt during a house price boom and the associated widening of the gap in the marginal utility of consumption between borrowers and savers. As a consequence, the equilibrium real interest rate remains roughly constant and the policymaker can stabilize output and inflation without significant changes of the nominal interest rate.

If macro-prudential policy becomes active only after a housing bust has occurred, the LTV limit remains at its maximum level for the duration of the liquidity trap. In this case, the nominal interest rate reaches its lower bound even under optimal policy. However, the use of LTV limits still greatly mitigates the effects of the recession. Conversely, an exogenous LTV tightening during a recovery, possibly for reasons other than macroeconomic stabilization, generates a deeper recession and delays the liftoff of the nominal interest rate from the ZLB.

Our results demonstrate the power of active borrower-based policies in containing a buildup of private leverage arising from the housing market. Of course, LTV limits are only one of the many tools available to macro-prudential authorities. In this paper, we have focused on their implications for monetary policy. Going forward, the extent to which the presence of multiple instruments reduces the burden of adjustment on LTVs certainly deserves further consideration. Research along these lines may uncover broad-ranging policy lessons as both borrower-based and lender-based tools are likely to play major roles in the future development of macro-prudential policy frameworks. The implications of the wide array of macro-prudential policy actions observed in practice for the monetary policy stance, in particular via their effects on the equilibrium real interest rate, are likely to become another area of active policy debate. We leave the study of these questions for future research.

[42] The level of debt and the LTV limit however differ between the two equlibria. The policymaker adjusts the LTV limit to deliver the optimal path for the multiplier on the collateral constraint. In turn, the exact level of the LTV limit required to achieve a certain path for the multiplier crucially depends on the accumulated level of debt.

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Appendix

A Derivations

This appendix presents additional derivations that complement the analysis in the paper.

A.1 Household Problem

Drawing heavily from Cúrdia and Woodford (2016) (henceforth CW), the first part describes the household problem in detail, including the assumptions and results that make endogenous variables independent of past histories and only dependent on the current household type. The problem of banks and firms is standard.

Let $\vartheta^t \equiv (..., \vartheta_{t-1}, \vartheta_t)$ denote the history of exogenous shocks (the aggregate exogenous state), where ϑ_t is the vector of exogenous shocks at time t. In addition, let $\tau^t(i)$ denote household i's type history, which can take values in $\{0, b, s\}$, where 0 represents no type change from the previous period, whereas either b (borrower) or s (saver) enters $\tau^t(i)$ if the household draws that type in any given period. As a consequence, the current type $\tau_t(i)$ is the most recent non-zero element of $\tau^t(i)$.

Types evolve as an independent two-state Markov chain. With probability $\delta \in (0, 1)$, the type remains unchanged, while with probability $1 - \delta$ a household draws a new type. The probability of drawing type $\tau = \{b, s\}$ is ξ^τ, with $\{\xi^b, \xi^s\} \in (0, 1)$. Since a continuum of household of measure one populate the economy, to simplify the notation we set $\xi^b = \xi$, which implies $\xi^s = 1 - \xi$.

Formally, the notation for a generic endogenous variables x at the individual level $i \in [0, 1]$ is $x(\vartheta^t, \tau^t(i))$, where ϑ^t captures the dependency on the aggregate history and $\tau^t(i)$ on the individual type history. However, since all households are ex-ante identical, in what follows, we drop the index i. In addition, for simplicity, we use the subscript t to denote dependency on aggregate history ϑ^t.

Transfers

At some initial date $t_0 \leq 0$, ex-ante identical households choose state-contingent transfers. In every period, households discover if their type changes. Conditional on a type change, an insurance agency pays out transfers before decisions are taken. Transfers are such that:

- A household does not receive a transfer if a new type is not drawn at t

$$T_t(\tau^{t-1}, 0) = 0.$$

- Transfers cannot be contingent upon the newly drawn type

$$T_t(\tau^{t-1}, b) = T_t(\tau^{t-1}, s).$$

- Let $T_t^\dagger(\tau^{t-1})$ denote the transfer a household with history τ^{t-1} who has access to the insurance agency in period t independently of which type the household actually draws. Transfers then must satisfy the intertemporal budget constraint

$$\mathbb{E}_{t_0}\left[\sum_{t=t_0}^{\infty}\sum_{\tau^{t-1}} p(\tau^{t-1})\mathcal{P}(\vartheta^t)T_t^\dagger(\tau^{t-1})\right] = 0,$$

  where $p(\tau^{t-1})$ is the time-t_0 probability of reaching the type history τ^{t-1} and $\mathcal{P}(\vartheta^t) > 0$ is the the time-t_0 state-contingent nominal price of the transfer.

- The transfer market clears if and only if

$$\sum_{\tau^{t-1}} p(\tau^{t-1}) \int T_t^{\dagger}(\tau^{t-1})(i)di = 0.$$

Budget Constraints

Households' preferences (equation (1) in the main text) are heterogeneous in terms of the coefficient of risk aversion. Nevertheless, the first part of this section shows that the optimal choices of each household type is independent of previous type histories.

Assumption 1: *In equilibrium, every type s is a saver ($D_t(\tau^t) + E_t(\tau^t) > 0$). Conversely, every type b is a borrower ($B_t(\tau^t) > 0$).*

Households enter the period with some financial wealth (which can be positive or negative). At the beginning of the period, before any decision is taken, households learn if their type changes or not. If the type changes, households learn their new type and receive the transfer payment $T_t(\tau^{t-1})$. Therefore, after-transfer financial wealth is

$$\begin{aligned} A_t(\tau^{t-1}) \equiv R^d_{t-1} \max\{D_{t-1}(\tau^{t-1}), 0\} + R^e_{t-1} \max\{E_{t-1}(\tau^{t-1}), 0\} \\ - R^b_{t-1} \max\{B_{t-1}(\tau^{t-1}), 0\} + T_t(\tau^{t-1}). \end{aligned}$$

Endowed with this definition, we can write the budget constraint for a saver as

$$\begin{aligned} P_t C_t(\tau^t) + Q_t H_t(\tau^t) + P_t \Gamma_h(H_t(\tau^t)) + D_t(\tau^t) + E_t(\tau^t) + P_t \Gamma_e(E_t(\tau^t)) \\ = A_t(\tau^{t-1}) + W_t L_t(\tau^t) + Q_t H_{t-1}(\tau^{t-1}) + \Omega_t(\tau^t), \end{aligned}$$

where $\Gamma_h(\cdot)$ and $\Gamma_e(\cdot)$ are the functions that measure the costs of deviating from the housing and equity target levels, respectively. The budget constraint for a borrower as

$$P_t C_t(\tau^t) + Q_t H_t(\tau^t) + P_t \Gamma_h(H_t(\tau^t)) - B_t(\tau^t) = A_t(\tau^{t-1}) + W_t L_t(\tau^t) + Q_t H_{t-1}(\tau^{t-1}) + \Omega_t(\tau^t).$$

In addition, households also face the collateral constraint

$$B_t(\tau^t) \leq \gamma_d \max\{B_{t-1}(\tau^{t-1}), 0\} + (1 - \gamma_d)\Theta_t Q_t H_t(\tau^t).$$

While in principle all households are subject to the borrowing constraint, Assumption 1 implies the collateral constraint is always slack for savers.

Since the per-period utility function is separable, the real marginal utility of consumption is

$$\lambda_t(\tau^t) = C_t(\tau^t)^{-\sigma^{\tau}}.$$

The optimal ex-ante choice of state-contingent insurance requires that between any two states (defined as a pair of aggregate and type history) the following condition holds

$$\beta^{T-t} \frac{\lambda_T^{\dagger}(\tau^{T-1})}{\lambda_t^{\dagger}(\tau^{t-1})} = \frac{\mathcal{P}_T / P_T}{\mathcal{P}_t / P_t}, \tag{19}$$

where $\lambda_t^{\dagger}(\tau^{t-1})$ is the real marginal utility of real consumption in a state $(\vartheta^t, \tau^{t-1})$ in which the household has access to the transfer before knowing the new type

$$\lambda_t^{\dagger}(\tau^{t-1}) \equiv \xi\lambda_t(\tau^{t-1}, b) + (1-\xi)\lambda_t(\tau^{t-1}, s). \tag{20}$$

Equation (19) shows that $\lambda_t^{\dagger}(\tau^{t-1})$ is independent of type history τ^{t-1}. Therefore, in what follows, we can simply write $\lambda_t^{\dagger}(\tau^{t-1}) = \lambda_t^{\dagger}$.[43]

First Order Conditions

Starting with savers, the Euler equation for the optimal choice of deposits is

$$\lambda_t(\tau^t) = \beta\mathbb{E}_t\left\{\frac{R_t^d}{\Pi_{t+1}}[\delta\lambda_{t+1}(\tau^t, 0) + (1-\delta)\lambda_{t+1}^{\dagger}]\right\}.$$

Assumption 2: *In equilibrium, at all times*

$$\frac{R_t^d}{\Pi_{t+1}} < \frac{1}{\beta\delta},$$

and

$$0 < \lambda_t(\tau^t) < \lambda^* < \infty.$$

Proposition 1: *If Assumption 1 and 2 are satisfied, the real marginal utility of consumption for a current saver is independent of prior history.*

Proof. We can rewrite the Euler equation for deposits as

$$\lambda_t(\tau^t) = \beta(1-\delta)\mathbb{E}_t\left[\frac{R_t^d}{\Pi_{t+1}}\lambda_{t+1}^{\dagger}\right] + \beta\delta\mathbb{E}_t\left[\frac{R_t^d}{\Pi_{t+1}}\lambda_{t+1}(\tau^t, 0)\right].$$

At time $t+1$, the same equation for a saver who has not changed type is

$$\lambda_{t+1}(\tau^t, 0) = \beta(1-\delta)\mathbb{E}_{t+1}\left(\frac{R_{t+1}^d}{\Pi_{t+2}}\lambda_{t+2}^{\dagger}\right) + \beta\delta\mathbb{E}_{t+1}\left[\frac{R_{t+1}^d}{\Pi_{t+2}}\lambda_{t+2}(\tau^t, 0, 0)\right].$$

Replacing into the previous expression and rearranging, we obtain

$$\lambda_t(\tau^t) = \beta(1-\delta)\left\{\mathbb{E}_t\left[\frac{R_t^d}{\Pi_{t+1}}\lambda_{t+1}^{\dagger}\right] + \beta\delta\mathbb{E}_t\left[\frac{R_t^d}{\Pi_{t+1}}\frac{R_{t+1}^d}{\Pi_{t+2}}\lambda_{t+2}^{\dagger}\right]\right\} + (\beta\delta)^2\mathbb{E}_t\left[\frac{R_t^d}{\Pi_{t+1}}\frac{R_{t+1}^d}{\Pi_{t+2}}\lambda_{t+2}(\tau^t, 0, 0)\right].$$

After repeating the iteration, we arrive at

$$\lambda_t(\tau^t) = \beta(1-\delta)\mathbb{E}_t\left[\sum_{j=0}^{T-1}(\beta\delta)^j\left(\prod_{k=0}^{j}\frac{R_{t+k}^d}{\Pi_{t+k+1}}\right)\lambda_{t+j+1}^{\dagger}\right] + (\beta\delta)^T\mathbb{E}_t\left[\left(\prod_{k=0}^{T}\frac{R_{t+k}^d}{\Pi_{t+k+1}}\right)\lambda_{t+T}(\tau^t, 0, 0, ..., 0)\right].$$

In any equilibrium consistent with Assumption 2,

$$\lim_{T\to\infty}(\beta\delta)^T\mathbb{E}_t\left[\left(\prod_{k=0}^{T}\frac{R_{t+k}^d}{\Pi_{t+k+1}}\right)\lambda_{t+T}(\tau^t, 0, 0, ..., 0)\right] = 0,$$

because the marginal utility of consumption is finite and the product inside the expectation consists of

[43] In the text, we refer to the average marginal utility of consumption simply as λ_t since the type-specific marginal utilities carry the superscript $\tau \in \{b, s\}$.

terms which are always less than $(\beta\delta)^{-1}$. Therefore, taking the limit for $T \to \infty$ yields

$$\lambda_t(\tau^t) = \beta(1-\delta)\mathbb{E}_t \left[\sum_{j=0}^{\infty} (\beta\delta)^j \left(\prod_{k=0}^{j} \frac{R^d_{t+k}}{\Pi_{t+k+1}} \right) \lambda^\dagger_{t+j+1} \right].$$

The right-hand side of last expression is a function of (forecasts of) aggregate variables only. Therefore, $\lambda_t(\tau^t)$ is the same for all type histories with current type s, and we can use λ^s_t to denote the marginal utility of savers. Moreover, given the expression for $\lambda^\dagger_t$ in (20), we can conclude that $\lambda_t(\tau^t)$ is the same also for all type histories with current type b, and use λ^b_t to denote the marginal utility of borrowers. Finally, given the separability of the utility function, the level of consumption for both types is also independent of type histories, and hence use C^b_t and C^s_t to refer to consumption of borrowers and savers, respectively. □

The Euler equation for the optimal choice of equity is

$$[1 + \Gamma'_e(E_t(\tau^t))]\lambda^s_t = \beta\mathbb{E}_t \left\{ \frac{R^e_t}{\Pi_{t+1}} [\delta\lambda^s_{t+1} + (1-\delta)\lambda^\dagger_{t+1}] \right\},$$

which implies that the optimal choice of equity is independent of type history ($E_t(\tau^t) = E_t$). The optimal choice of housing implies

$$\lambda^s_t \frac{Q_t}{P_t} = \chi_H H_t(\tau^t)^{-\sigma_h} - \lambda^s_t \Gamma'_h(H_t(\tau^t)) + \beta\mathbb{E}_t \left\{ [\delta\lambda^s_{t+1} + (1-\delta)\lambda^\dagger_{t+1}] \frac{Q_{t+1}}{P_{t+1}} \right\},$$

which implies that the optimal choice of housing for a saver (denoted with H^s_t) is independent of history. The first order condition for labor supply is

$$\lambda^s_t \frac{W_t}{P_t} = \chi_L L_t(\tau^t)^\varphi,$$

which implies that savers' labor supply (denoted with L^s_t) is independent of history. Finally, through the budget constraint, the optimal choice of deposits (denoted with D_t) is also independent of the previous type history.

Moving on to borrowers, the Euler equation for the optimal choice of debt for an existing borrower is

$$\lambda^b_t = \tilde{\mu}_t(\tau^{t-1}, 0) + \beta\mathbb{E}_t \left\{ \frac{R^b_t}{\Pi_{t+1}} [\delta\lambda^b_{t+1} + (1-\delta)\lambda^\dagger_{t+1}] - \frac{\gamma_d}{\Pi_{t+1}} [\delta\tilde{\mu}_{t+1}(\tau^t, 0) + (1-\delta)\mu^\dagger_{t+1}(\tau^t)] \right\},$$

where $\tilde{\mu}_t(\tau^t)/P_t$ is the Lagrange multiplier on the collateral constraint and, consistent with the notation for the marginal utility of consumption, $\mu^\dagger_t(\tau^{t-1})$ denotes the multiplier on the collateral constraint for a household who has access to the transfer. Since the collateral constraint never binds for savers, this multiplier is actually proportional to the multiplier for a new borrower

$$\mu^\dagger_t(\tau^{t-1}) = \xi\tilde{\mu}_t(\tau^{t-1}, b). \tag{21}$$

Similarly, the Euler equation for the optimal choice of debt for a new borrower is

$$\lambda^b_t = \tilde{\mu}_t(\tau^{t-1}, b) + \beta\mathbb{E}_t \left\{ \frac{R^b_t}{\Pi_{t+1}} [\delta\lambda^b_{t+1} + (1-\delta)\lambda^\dagger_{t+1}] - \frac{\gamma_d}{\Pi_{t+1}} [\delta\tilde{\mu}_{t+1}(\tau^t, 0) + (1-\delta)\mu^\dagger_{t+1}(\tau^t)] \right\}.$$

The last two equations imply that $\tilde{\mu}_t(\tau^{t-1}, 0) = \tilde{\mu}_t(\tau^{t-1}, b) = \tilde{\mu}_t$, that is, also the multiplier on the collateral constraint is independent of previous histories. Using the result in (21), we can rewrite the

Euler equation for debt, which holds for both new and existing borrowers, as

$$\lambda_t^b = \tilde{\mu}_t + \beta \mathbb{E}_t \left\{ \frac{R_t^b}{\Pi_{t+1}} [\delta \lambda_{t+1}^b + (1-\delta)\lambda_{t+1}^\dagger] - \gamma_d [\delta + (1-\delta)\xi] \frac{\tilde{\mu}_{t+1}}{\Pi_{t+1}} \right\}.$$

The optimal choice of housing for borrowers then is

$$[\lambda_t^b - (1-\gamma_d)\Theta_t \tilde{\mu}_t] \frac{Q_t}{P_t} = \chi_H H_t(\tau^t)^{-\sigma_h} - \lambda_t^b \Gamma_h'(H_t(\tau^t)) + \beta \mathbb{E}_t \left\{ [\delta \lambda_{t+1}^b + (1-\delta)\lambda_{t+1}^\dagger] \frac{Q_{t+1}}{P_{t+1}} \right\},$$

which shows that housing demand (denoted with H_t^b) for borrowers is independent of type history. The first order condition for labor supply is

$$\lambda_t^b \frac{W_t}{P_t} = \chi_L L_t(\tau^t)^\varphi,$$

which shows that labor supply (denoted with L_t^b) is independent of type history. Finally, the budget constraint implies that the optimal choice of debt (denoted with B_t) is also independent of prior histories.

Following CW, we further assume that households who have access to the insurance agency all start with the same after-transfer wealth, so that $A_t(\tau^{t-1}, s) = A_t(\tau^{t-1}, b) = A_t$. The insurance arrangement implies that all households who draw a new type in period t (a fraction $1-\delta$ of both types) pool their financial wealth. The insurance agency then redistributes these resources across households so that every household that draws a new type starts with the same level of wealth. Therefore, in each period, the agency resource constraint is

$$(1-\xi)(R_t^d D_t + R_t^e E_t) - \xi R_t^b B_t = A_t.$$

Note, however, that the left-hand side of the last expression is just the aggregate profit of the financial intermediation sector. Since we assume perfect competition in the banking sector, each bank makes zero profit, and thus the initial level of financial wealth for agents that have access to the insurance agency is also zero. Essentially, having access to the insurance agency means resetting to zero all prior assets and liabilities. In order to achieve this outcome, the agency transfers resources from savers to borrowers.

A.1.1 Banks

The balance sheet of a representative bank is

$$B_t = D_t + E_t.$$

In addition, banks are subject to the capital requirement

$$E_t \geq \widetilde{\kappa}_t B_t.$$

The bank's profits are

$$\mathcal{P}_t = R_t^b B_t - R_t^d D_t - R_t^e E_t.$$

We can use the balance sheet to substitute out deposits from the expression for profits. Denoting with ζ_t the Lagrange multiplier on the capital constraint, the first order condition for loans is

$$(R_t^b - R_t^d) - \zeta_t \widetilde{\kappa}_t = 0,$$

and the first order condition for equity is

$$-(R_t^e - R_t^d) + \zeta_t = 0.$$

As discussed in the text, perfect competition in the banking sector implies that, in equilibrium, the capital requirement always binds ($\zeta_t > 0$). Solving for the Lagrange multiplier from the last expression and replacing in the previous one yields

$$R_t^b = \widetilde{\kappa}_t R_t^e + (1 - \widetilde{\kappa}_t) R_t^d.$$

In these equilibria, changes in the capital requirements affect credit spreads.

A.1.2 Firms

Firms produce with a linear technology in labor

$$Y_t(f) = L_t(f),$$

taking as given the demand for their variety from retailers

$$Y_t(f) = \left[\frac{P_t(f)}{P_t}\right]^{-\varepsilon} Y_t.$$

Profits in real terms are

$$\mathcal{P}_t(f) = (1 + \tau^f)\frac{P_t(f)}{P_t} Y_t(f) - \frac{W_t}{P_t} L_t(f),$$

where $\tau^f > 0$ is a subsidy that makes steady state production efficient. Using the production function and the fact that cost minimization implies the real wage $w_t \equiv W_t/P_t$ equals the marginal cost of production MC_t, we can write the pricing problem as

$$\max_{\widetilde{P}_t(f)} \mathbb{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{\lambda_{t+j}^\dagger}{\lambda_t^\dagger} \left\{ (1 + \tau^f) \left[\frac{\widetilde{P}_t(f)}{P_{t+j}}\right]^{1-\varepsilon} Y_{t+j} - MC_{t+j} \left[\frac{\widetilde{P}_t(f)}{P_{t+j}}\right]^{-\varepsilon} Y_{t+j} \right\},$$

where $\alpha \in (0, 1)$ is the probability of not being able to change the price in any given period, and we have assumed that the relevant stochastic discount factor for firms is a function of the average marginal utility of the two types.

The first order condition for the optimal pricing problem implies that the relative price is

$$\frac{\widetilde{P}_t(f)}{P_t} = \frac{X_{1t}}{X_{2t}},$$

where X_{1t} represents the present discounted value of real costs times the markup, which we can write recursively as

$$X_{1t} = \frac{\varepsilon}{\varepsilon - 1} \lambda_t^\dagger Y_t MC_t + \alpha\beta \mathbb{E}_t(\Pi_t^\varepsilon X_{1t+1}),$$

and X_{2t} represents the present discounted value of real revenues including the subsidy, which we can write recursively as

$$X_{2t} = (1 + \tau^f) \lambda_t^\dagger Y_t + \alpha\beta \mathbb{E}_t(\Pi_t^{\varepsilon - 1} X_{2t+1}).$$

After combining the optimal relative price with the overall price index, we obtain the non-linear Phillips curve

$$\frac{X_{1t}}{X_{2t}} = \left(\frac{1-\alpha\Pi_t^{\varepsilon-1}}{1-\alpha}\right)^{\frac{1}{1-\varepsilon}}.$$

Firms hire workers in a homogeneous labor market. Market clearing requires

$$\xi L_t^b + (1-\xi)L_t^s = \int_0^1 L_t(f)df \equiv L_t.$$

Substituting the labor supply conditions into the expression above yields

$$L_t = \left(\frac{1}{\chi_L}\right)^{\frac{1}{\varphi}} [\xi(\lambda_t^b)^{\frac{1}{\varphi}} + (1-\xi)(\lambda_t^s)^{\frac{1}{\varphi}}] \left(\frac{W_t}{P_t}\right)^{\frac{1}{\varphi}}.$$

The aggregate production function is

$$\Delta_t Y_t = L_t,$$

where Δ_t measures price dispersion

$$\Delta_t \equiv \int_0^1 \left[\frac{P_t(f)}{P_t}\right]^{-\varepsilon} df,$$

and evolves according to

$$\Delta_t = \alpha\Delta_{t-1}\Pi_t^{\varepsilon} + (1-\alpha)\left(\frac{1-\alpha\Pi_t^{\varepsilon-1}}{1-\alpha}\right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

A.1.3 Aggregate Debt Dynamics

Total debt at the end of period t consists of debt of existing borrowers who did not draw a new type, and of both previous borrowers and savers who drew the borrower type. Before knowing their type and taking decisions at time t, the latter two groups receive the insurance transfer that resets their existing assets and liabilities.

Debt for a borrower who did not switch is

$$B_t = R_{t-1}^b B_{t-1} + Q_t(H_t^b - H_{t-1}^b) + P_t\Gamma_h(H_t^b) + P_tC_t^b - W_tL_t^b - \Omega_t^b,$$

and the size of this group is $\xi\delta$.

Debt for new borrowers is

$$B_t = Q_t(H_t^b - H_{t-1}^{\tau}) + P_t\Gamma_h(H_t^b) + P_tC_t^b - W_tL_t^b - \Omega_t^b,$$

where the τ on lagged housing captures the fact that these borrowers may have been of either type in the previous period. The size of the group of new borrowers who were borrowers in $t-1$ and drew the borrower type again is $\xi^2(1-\delta)$. The size of the group of new borrowers who were savers in $t-1$ is $\xi(1-\xi)(1-\delta)$.[44] The only difference in terms of debt between the latter two groups is the initial housing stock. Because new borrowers take decisions after receiving the transfer from the insurance agency, their choices are identical to those of the existing borrowers. Total debt is then the sum of debt of the three

[44] The size of the three groups of borrowers adds up to ξ, which is the fraction of borrowers in the economy.

groups. Therefore, we have

$$
\begin{aligned}
\xi B_t = & \underbrace{\xi\delta[R^b_{t-1}B_{t-1} + Q_t(H^b_t - H^b_{t-1}) + P_t\Gamma_h(H^b_t) + P_tC^b_t - W_tL^b_t - \Omega^b_t]}_{\text{borrowers at } t-1 \text{ with no access to insurance}} \\
& + \underbrace{\xi^2(1-\delta)[Q_t(H^b_t - H^b_{t-1}) + P_t\Gamma_h(H^b_t) + P_tC^b_t - W_tL^b_t - \Omega^b_t]}_{\text{new borrowers who were borrowers at } t-1 \text{ with access to insurance}} \\
& + \underbrace{\xi(1-\xi)(1-\delta)[Q_t(H^b_t - H^s_{t-1}) + P_t\Gamma_h(H^b_t) + P_tC^b_t - W_tL^b_t - \Omega^b_t]}_{\text{new borrowers who were savers at } t-1 \text{ with access to insurance}}
\end{aligned}
$$

Adding up and simplifying yields

$$B_t = \delta R^b_{t-1}B_{t-1} + Q_t[(H^b_t - H^b_{t-1}) + (1-\xi)(1-\delta)(H^b_{t-1} - H^s_{t-1})] + P_t\Gamma_h(H^b_t) + P_tC^b_t - W_tL^b_t - \Omega^b_t.$$

Profits nets of taxes are

$$\Omega_t = P_tY_t - \xi W_tL^b_t - (1-\xi)W_tL^s_t.$$

Substituting into the expression for debt above yields

$$B_t = \delta R^b_{t-1}B_{t-1} + Q_t[(H^b_t - H^b_{t-1}) + (1-\xi)(1-\delta)(H^b_{t-1} - H^s_{t-1})] + P_t\Gamma_h(H^b_t) + P_tC^b_t - P_tY_t - (1-\xi)W_t(L^b_t - L^s_t),$$

or in real terms

$$\frac{B_t}{P_t} = \delta\frac{R^b_{t-1}}{\Pi_t}\frac{B_{t-1}}{P_{t-1}} + \frac{Q_t}{P_t}[(H^b_t - H^b_{t-1}) + (1-\xi)(1-\delta)(H^b_{t-1} - H^s_{t-1})] + \Gamma_h(H^b_t) + C^b_t - Y_t - (1-\xi)\frac{W_t}{P_t}(L^b_t - L^s_t).$$

A.2 Equilibrium

Given the monetary policy and macro-prudential policy instruments (a sequence of deposit rates and LTV limits $\{R^d_t, \Theta_t\}^\infty_{t=0}$), the values of the credit shock $\{\widetilde{\kappa}_t\}^\infty_{t=0}$, and initial conditions on the state variables

$$\left\{\frac{B_{-1}}{P_{-1}}, \Delta_{-1}, H^b_{-1}, H^s_{-1}, R^b_{-1}\right\},$$

an equilibrium for this economy is a sequence

$$\left\{\lambda^b_t, \lambda^s_t, C^b_t, C^s_t, H^b_t, H^s_t, \frac{B_t}{P_t}, X_{1t}, X_{2t}, Y_t, R^e_t, R^b_t, \frac{Q_t}{P_t}, \Pi_t, MC_t, \Delta_t, \mu_t\right\}^\infty_{t=0},$$

that satisfies the following equations:

1. Savers' Euler equation for deposits:

$$1 = \beta\mathbb{E}_t\left\{\frac{(1-\delta)\xi\lambda^b_{t+1} + [\delta + (1-\delta)(1-\xi)]\,\lambda^s_{t+1}}{\lambda^s_t}\frac{R^d_t}{\Pi_{t+1}}\right\}.$$

2. Savers' Euler equation for equity:

$$1 + \Psi_e\left(\frac{\widetilde{\kappa}_t}{\widetilde{\kappa}} - 1\right) = \beta\mathbb{E}_t\left\{\frac{(1-\delta)\xi\lambda^b_{t+1} + [\delta + (1-\delta)(1-\xi)]\,\lambda^s_{t+1}}{\lambda^s_t}\frac{R^e_t}{\Pi_{t+1}}\right\},$$

where we used the aggregate balance sheet of the financial intermediation sector

$$\xi B_t = (1-\xi)(D_t + E_t),$$

and the aggregate equity requirement at equality is

$$(1-\xi)E_t = \widetilde{\kappa}_t \xi B_t.$$

These last two equations determine D_t and E_t residually.

3. Savers' housing demand:

$$\frac{Q_t}{P_t} = \frac{\chi_H (H_t^s)^{-\sigma_h}}{\lambda_t^s} - \Psi_h \left(\frac{H_t^s}{H} - 1 \right) + \beta \mathbb{E}_t \left\{ \frac{(1-\delta)\xi \lambda_{t+1}^b + [\delta + (1-\delta)(1-\xi)] \lambda_{t+1}^s}{\lambda_t^s} \frac{Q_{t+1}}{P_{t+1}} \right\}.$$

4. Savers' marginal utility:

$$\lambda_t^s = (C_t^s)^{-\sigma^s}.$$

5. Borrowers' Euler equation for debt:

$$1 - \mu_t = \beta \mathbb{E}_t \left\{ \frac{[\delta + (1-\delta)\xi] \lambda_{t+1}^b + (1-\delta)(1-\xi)\lambda_{t+1}^s}{\lambda_t^b} \frac{R_t^b}{\Pi_{t+1}} - \gamma_d [\delta + (1-\delta)\xi] \frac{\lambda_{t+1}^b}{\lambda_t^b} \frac{\mu_{t+1}}{\Pi_{t+1}} \right\},$$

   where $\mu_t \equiv \tilde{\mu}_t / \lambda_t^b$.

6. Borrowers' housing demand:

$$\begin{aligned} [1 - (1-\gamma_d)\Theta_t \mu_t] \frac{Q_t}{P_t} = & \frac{\chi_H (H_t^b)^{-\sigma_h}}{\lambda_t^b} - \Psi_h \left(\frac{H_t^b}{H} - 1 \right) \\ & + \beta \mathbb{E}_t \left\{ \frac{[\delta + (1-\delta)\xi] \lambda_{t+1}^b + (1-\delta)(1-\xi)\lambda_{t+1}^s}{\lambda_t^b} \frac{Q_{t+1}}{P_{t+1}} \right\}. \end{aligned}$$

7. Aggregate borrowing constraint:

$$\mu_t \left[\frac{B_t}{P_t} - \delta \gamma_d \frac{B_{t-1}}{P_{t-1}} \frac{1}{\Pi_t} - (1-\gamma_d)\Theta_t \frac{Q_t}{P_t} H_t^b \right] = 0,$$

   where we have rewritten debt in real terms and δ appears in front of lagged debt because a fraction $1-\delta$ of borrowers has received the state-contingent transfer payment at time t, and hence have seen their debt cancelled.

8. Borrowers' marginal utility:

$$\lambda_t^b = (C_t^b)^{-\sigma^b}.$$

9. Spreads:

$$R_t^b = \widetilde{\kappa}_t R_t^e + (1-\widetilde{\kappa}_t) R_t^d.$$

10. Phillips' curve:

$$\frac{X_{1t}}{X_{2t}} = \left(\frac{1 - \alpha \Pi_t^{\varepsilon - 1}}{1-\alpha} \right)^{\frac{1}{1-\varepsilon}}.$$

11. PDV of real costs:

$$X_{1t} = \frac{\varepsilon}{\varepsilon - 1} [\xi \lambda_t^b + (1-\xi)\lambda_t^s] Y_t MC_t + \alpha \beta \mathbb{E}_t (\Pi_t^{\varepsilon} X_{1t+1}).$$

12. PDV of real revenues:

$$X_{2t} = (1+\tau^f)[\xi \lambda_t^b + (1-\xi)\lambda_t^s] Y_t + \alpha \beta \mathbb{E}_t (\Pi_t^{\varepsilon-1} X_{2t+1}).$$

13. Labor market equilibrium:

$$(\Delta_t Y_t)^{\varphi} = \frac{\widetilde{\lambda}_t}{\chi_L} MC_t,$$

where we have used the aggregate production function $\Delta_t Y_t = L_t$, which pins down L_t residually. In addition, the solution to the intermediate goods producers cost minimization problem ($MC_t = w_t$) determines the real wage, while the definition of $\widetilde{\lambda}_t$ links this variable to the marginal utility of consumption of the two types

$$\widetilde{\lambda}_t = \left[\xi(\lambda_t^b)^{\frac{1}{\varphi}} + (1-\xi)(\lambda_t^s)^{\frac{1}{\varphi}}\right]^{\varphi}.$$

14. Price dispersion:

$$\Delta_t = \alpha \Delta_{t-1} \Pi_t^{\varepsilon} + (1-\alpha)\left(\frac{1-\alpha \Pi_t^{\varepsilon-1}}{1-\alpha}\right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

15. Debt dynamics:

$$\frac{B_t}{P_t} = \delta \frac{R_{t-1}^b}{\Pi_t}\frac{B_{t-1}}{P_{t-1}} + \frac{Q_t}{P_t}[(H_t^b - H_{t-1}^b) + (1-\xi)(1-\delta)(H_{t-1}^b - H_{t-1}^s)] + \Gamma_{ht}^b$$
$$+ C_t^b - Y_t - (1-\xi)\chi_L^{-\frac{1}{\varphi}} MC_t^{\frac{1+\varphi}{\varphi}}\left[(\lambda_t^b)^{\frac{1}{\varphi}} - (\lambda_t^s)^{\frac{1}{\varphi}}\right],$$

where

$$\Gamma_{ht}^{\tau} = \frac{\Psi_h H}{2}\left(\frac{H_t^{\tau}}{H} - 1\right)^2,$$

and we used the labor supply equation for each type

$$MC_t = \frac{\chi_L (L_t^{\tau})^{\varphi}}{\lambda_t^{\tau}},$$

which pins down hours worked by each type.

16. Housing market equilibrium:

$$H = \xi H_t^b + (1-\xi)H_t^s.$$

17. Resource constraint:

$$Y_t = \xi C_t^b + (1-\xi)C_t^s + (1-\xi)\Gamma_{et} + \xi \Gamma_{ht}^b + (1-\xi)\Gamma_{ht}^s,$$

where

$$\Gamma_{et} = \frac{\Psi_e \widetilde{\kappa}}{2}\left(\frac{\widetilde{\kappa}_t}{\widetilde{\kappa}} - 1\right)^2 \frac{\xi B_t}{P_t}.$$

A.3 Welfare Objective and Optimal Policy Problem

We assume the welfare objective for society is weighted average of utility of the two types

$$\mathbb{W}_0 \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\xi \frac{(C_t^b)^{1-\sigma^b}}{1-\sigma^b} + (1-\xi)\frac{(C_t^s)^{1-\sigma^s}}{1-\sigma^s}\right.$$
$$\left. + \xi \frac{\chi_H}{1-\sigma_h}(H_t^b)^{1-\sigma_h} + (1-\xi)\frac{\chi_H}{1-\sigma_h}(H_t^s)^{1-\sigma_h} - \xi \frac{\chi_L}{1+\varphi}(L_t^b)^{1+\varphi} - (1-\xi)\frac{\chi_L}{1+\varphi}(L_t^s)^{1+\varphi}\right].$$

We can rewrite the labor supply condition for type τ as

$$L_t^\tau = \left(\frac{\lambda_t^\tau w_t}{\chi_L}\right)^{\frac{1}{\varphi}}.$$

Using this expression, the last term of the welfare objective becomes

$$\begin{aligned}
\xi\frac{\chi_L}{1+\varphi}(L_t^b)^{1+\varphi} + (1-\xi)\frac{\chi_L}{1+\varphi}(L_t^s)^{1+\varphi} &= \frac{1}{1+\varphi}\left[\xi\chi_L\left(\frac{\lambda_t^b w_t}{\chi_L}\right)^{\frac{1+\varphi}{\varphi}} + (1-\xi)\chi_L\left(\frac{\lambda_t^s w_t}{\chi_L}\right)^{\frac{1+\varphi}{\varphi}}\right] \\
&= \frac{(\chi_L)^{-\frac{1}{\varphi}}}{1+\varphi}\left[\xi(\lambda_t^b)^{\frac{1+\varphi}{\varphi}} + (1-\xi)(\lambda_t^s)^{\frac{1+\varphi}{\varphi}}\right] w_t^{\frac{1+\varphi}{\varphi}}.
\end{aligned}$$

We can then eliminate the real wage from the labor market equilibrium condition to obtain

$$\begin{aligned}
&\xi\frac{\chi_L}{1+\varphi}(L_t^b)^{1+\varphi} + (1-\xi)\frac{\chi_L}{1+\varphi}(L_t^s)^{1+\varphi} \\
&\quad = \frac{(\chi_L)^{-\frac{1}{\varphi}}}{1+\varphi}\left[\xi(\lambda_t^b)^{\frac{1+\varphi}{\varphi}} + (1-\xi)(\lambda_t^s)^{\frac{1+\varphi}{\varphi}}\right](\chi_L)^{\frac{1+\varphi}{\varphi}}[\xi(\lambda_t^b)^{\frac{1}{\varphi}} + (1-\xi)(\lambda_t^s)^{\frac{1}{\varphi}}]^{-(1+\varphi)}L_t^{1+\varphi} \\
&\quad = \frac{\chi_L}{1+\varphi}\left[\xi(\lambda_t^b)^{\frac{1+\varphi}{\varphi}} + (1-\xi)(\lambda_t^s)^{\frac{1+\varphi}{\varphi}}\right][\xi(\lambda_t^b)^{\frac{1}{\varphi}} + (1-\xi)(\lambda_t^s)^{\frac{1}{\varphi}}]^{-(1+\varphi)}L_t^{1+\varphi}.
\end{aligned}$$

As in CW, we define

$$\widetilde{\Lambda}_t \equiv \left[\xi(\lambda_t^b)^{\frac{1+\varphi}{\varphi}} + (1-\xi)(\lambda_t^s)^{\frac{1+\varphi}{\varphi}}\right]^{\frac{\varphi}{1+\varphi}},$$

and

$$\widetilde{\lambda}_t \equiv \left[\xi(\lambda_t^b)^{\frac{1}{\varphi}} + (1-\xi)(\lambda_t^s)^{\frac{1}{\varphi}}\right]^{\varphi}.$$

We can then rewrite the last term in the welfare objective as

$$\xi\frac{\chi_L}{1+\varphi}(L_t^b)^{1+\varphi} + (1-\xi)\frac{\chi_L}{1+\varphi}(L_t^s)^{1+\varphi} = \frac{\chi_L}{1+\varphi}\left(\frac{\widetilde{\Lambda}_t}{\widetilde{\lambda}_t}\right)^{\frac{1+\varphi}{\varphi}} L_t^{1+\varphi}.$$

Finally, using the aggregate production function, the last expression becomes

$$\xi\frac{\chi_L}{1+\varphi}(L_t^b)^{1+\varphi} + (1-\xi)\frac{\chi_L}{1+\varphi}(L_t^s)^{1+\varphi} = \frac{\chi_L}{1+\varphi}\left(\frac{\widetilde{\Lambda}_t}{\widetilde{\lambda}_t}\right)^{\frac{1+\varphi}{\varphi}} (\Delta_t Y_t)^{1+\varphi}.$$

Going back to the overall welfare objective, we finally have

$$\begin{aligned}
\mathbb{W}_0 = \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Bigg[&\xi\frac{(C_t^b)^{1-\sigma^b}}{1-\sigma^b} + (1-\xi)\frac{(C_t^s)^{1-\sigma^s}}{1-\sigma^s} \\
&+\xi\frac{\chi_H}{1-\sigma_h}(H_t^b)^{1-\sigma_h} + (1-\xi)\frac{\chi_H}{1-\sigma_h}(H_t^s)^{1-\sigma_h} - \frac{\chi_L}{1+\varphi}\left(\frac{\widetilde{\Lambda}_t}{\widetilde{\lambda}_t}\right)^{\frac{1+\varphi}{\varphi}}(\Delta_t Y_t)^{1+\varphi}\Bigg]. \qquad (22)
\end{aligned}$$

The optimal policy problem consists of choosing a sequence of deposit rates and LTV limits $\{R_t^d, \Theta_t\}_{t=0}^{\infty}$ that maximize (22) subject to the set of equations that characterize the equilibrium reported in section A.2.

In what follows, we derive a linear-quadratic (LQ) approximation that allows us to derive analytically the optimal targeting rules for monetary and macro-prudential policy in one special case. We use the LQ approximation to the optimal policy problem also to study a number of numerical experiments in which the zero lower bound on the nominal interest rate and the collateral constraint occasionally bind.

A.4 Efficient Steady State

In order to obtain a LQ approximation to the optimal policy problem, we take a second order approximation of the welfare objective and a first order approximation of the equilibrium conditions about the efficient steady state, which we discuss in this section.

We focus on a zero inflation steady state ($\Pi = 1 \Rightarrow \Delta = 1$) and no spread (built into the equity cost function), in which the marginal utility of consumption is the same across the two types ($\lambda^b = \lambda^s = \lambda = \tilde{\lambda}$). As a consequence, from the labor supply relations, we also obtain $L^s = L^b = L$. For analytical convenience in deriving the quadratic approximation of the welfare objective, we ensure that steady state output is efficient by setting the subsidy $\tau^f = (\varepsilon - 1)^{-1}$, which implies $MC = 1$.

From the savers' Euler equations, we obtain $R^d = R^e = R = 1/\beta$. In turn, from the spread equation, we have $R^b = R$, and, from the borrowers' Euler equation, we obtain $\mu = 0$. The steady state borrowing constraint, therefore, must be slack.

From the housing demand equations we can then conclude that housing consumption must also be symmetric ($H^s = H^b$). If we normalize the total stock of housing to one ($H = 1$), from the resource constraint we obtain $H^s = H^b = 1$. The borrowing constraint thus requires

$$\frac{(1-\delta\gamma_d)b}{(1-\gamma_d)\Theta q} \leq 1,$$

where $q \equiv Q/P$ is the steady state real price of housing

$$q = \frac{\chi_H}{(1-\beta)\lambda}, \tag{23}$$

and steady state real debt $b \equiv B/P$ follows from the debt accumulation equation

$$b = \frac{\beta(\varpi^b - 1)}{\beta - \delta} Y, \tag{24}$$

with $\varpi^\tau \equiv C^\tau / Y$.

The steady state resource constraint is

$$\xi C^b + (1-\xi) C^s = Y. \tag{25}$$

Since in steady state $\lambda^b = \lambda^s$, we have $(C^b)^{-\sigma^b} = (C^s)^{-\sigma^s}$, which implies $C^s = (C^b)^\varrho$, where $\varrho \equiv \sigma^b/\sigma^s$. Using this relation in the resource constraint, we can solve for the level of steady state output as a function of the borrowers' consumption share

$$Y = \left[\frac{1-\xi\varpi^b}{(1-\xi)(\varpi^b)^\varrho}\right]^{\frac{1}{\varrho-1}}. \tag{26}$$

Finally, we can back up the marginal utility (and, hence, the level) of consumption from the steady state labor market equilibrium relation

$$Y^\varphi = \frac{\lambda}{\chi_L}. \tag{27}$$

A.5 Calibration of χ_L and χ_H

Table 1 in the text reports the parameter values used in the simulation exercises. Here we describe the calibration strategy to pin down the utility parameters χ_L and χ_H.

Given the calibrated steady state level of mortgage debt as a fraction of GDP, and the values of β and δ, we can back out the steady state consumption-GDP ratio for borrowers (ϖ^b) from (24) and the correspondent variable for savers from the resource constraint (25).

Given the calibrated value of σ^b/σ^s and the value of ϖ^b, we can calculate steady state output from (26). We can then back out the level of consumption for both types from ϖ^τ and the steady state marginal utility of consumption (which is equal across types) given the values of σ^τ. Steady state output and the marginal utility of consumption, together with the calibrated value of φ, allow us to obtain the value of χ_L consistent with the steady state labor market equilibrium condition (27).

Finally, we back out the value of χ_H by taking the ratio between the value of housing (23) and mortgage debt (24), both expressed as a fraction of GDP, which in US data between 1995 and 2016 is just below 2.5.

A.6 Second Order Approximation of the Welfare Objective

This appendix derives a second order approximation of the welfare objective (22). For convenience, we break up the per-period function in three terms. The first is the utility from consumption of goods of the two types

$$\mathcal{W}_t^1 \equiv \xi \frac{(C_t^b)^{1-\sigma^b}}{1-\sigma^b} + (1-\xi)\frac{(C_t^s)^{1-\sigma^s}}{1-\sigma^s}.$$

The second is the utility from consumption of housing of the two types

$$\mathcal{W}_t^2 \equiv \frac{\chi_H}{1-\sigma_h}\left[\xi (H_t^b)^{1-\sigma_h} + (1-\xi)(H_t^s)^{1-\sigma_h}\right].$$

Finally, the third is the disutility from labor of the two types

$$\mathcal{W}_t^3 \equiv -\frac{\chi_L}{1+\varphi}\left(\frac{\widetilde{\Lambda}_t}{\widetilde{\lambda}_t}\right)^{\frac{1+\varphi}{\varphi}} (\Delta_t Y_t)^{1+\varphi}.$$

In the next three subsections we proceed to take a second order approximation of each term.

A.6.1 First Term $\mathcal{W}_t^1$

Starting with the first term in $\mathcal{W}_t^1$, we can write

$$\begin{aligned}
\xi \frac{(C_t^b)^{1-\sigma^b}}{1-\sigma^b} &\approx \xi (C^b)^{-\sigma^b}(C_t^b - C^b) - \frac{1}{2}\xi\sigma^b (C^b)^{-\sigma^b-1}(C_t^b - C^b)^2 + t.i.p. + \mathcal{O}(\|\epsilon\|^3) \\
&= \xi (C^b)^{1-\sigma^b}\left[\frac{C_t^b - C^b}{C^b} - \frac{1}{2}\sigma^b\left(\frac{C_t^b - C^b}{C^b}\right)^2\right] + t.i.p. + \mathcal{O}(\|\epsilon\|^3),
\end{aligned}$$

where $t.i.p.$ stands for "terms independent of policy" (that is, terms up to the second order not involving endogenous variables) and $\mathcal{O}(\|\epsilon\|^3)$ collects terms of order three or higher. Up to the second order

$$\frac{C_t^b - C^b}{C^b} \approx c_t^b + \frac{1}{2}(c_t^b)^2 \Rightarrow \left(\frac{C_t^b - C^b}{C^b}\right)^2 \approx (c_t^b)^2,$$

where lower-case variables denote log-deviations from steady state (e.g., $c_t^b \equiv \ln(C_t^b/C^b)$ in the case of borrowers' consumption) and we dropped the notation for terms independent of policy and of order higher than two. Replacing in the expression above, we obtain

$$\xi \frac{(C_t^b)^{1-\sigma^b}}{1-\sigma^b} \approx \xi (C^b)^{1-\sigma^b}\left[c_t^b + \frac{1}{2}(1-\sigma^b)(c_t^b)^2\right].$$

Since the approximation of the second term is similar, we can write

$$\mathcal{W}_t^1 \approx \xi (C^b)^{1-\sigma^b}\left[c_t^b + \frac{1}{2}(1-\sigma^b)(c_t^b)^2\right] + (1-\xi)(C^s)^{1-\sigma^s}\left[c_t^s + \frac{1}{2}(1-\sigma^s)(c_t^s)^2\right].$$

Given the equality of marginal utilities across groups in steady state, we can rewrite

$$\mathcal{W}_t^1 \approx \lambda\left\{\xi C^b\left[c_t^b + \frac{1}{2}(1-\sigma^b)(c_t^b)^2\right] + (1-\xi)C^s\left[c_t^s + \frac{1}{2}(1-\sigma^s)(c_t^s)^2\right]\right\}.$$

A second order approximation of the resource constraint gives

$$\frac{Y_t - Y}{Y} = \xi\frac{C^b}{Y}\frac{C_t^b - C^b}{C^b} + (1-\xi)\frac{C^s}{Y}\frac{C_t^s - C^s}{C^s} + \frac{\Psi_h}{2}[\xi(H_t^b - 1)^2 + (1-\xi)(H_t^s - 1)^2] + t.i.p.,$$

where $t.i.p.$ appears because the cost of deviating from target in terms of equity in the resource constraint is quadratic in $\widetilde{\kappa}_t - \widetilde{\kappa}$, which is exogenous under our assumption. Rewriting the approximation in log-deviations from steady state, we have

$$y_t + \frac{1}{2}y_t^2 = \xi\varpi^b\left[c_t^b + \frac{1}{2}(c_t^b)^2\right] + (1-\xi)\varpi^s\left[c_t^s + \frac{1}{2}(c_t^s)^2\right] + \frac{\Psi_h}{2}[\xi(h_t^b)^2 + (1-\xi)(h_t^s)^2],$$

and, up to the first order,

$$y_t = \xi\varpi^b c_t^b + (1-\xi)\varpi^s c_t^s.$$

Going back to the expression for $\mathcal{W}_t^1$, we can then rewrite

$$\begin{aligned}\mathcal{W}_t^1 &\approx \lambda Y\left\{\xi\varpi^b\left[c_t^b + \frac{1}{2}(c_t^b)^2\right] + (1-\xi)\varpi^s\left[c_t^s + \frac{1}{2}(c_t^s)^2\right] - \frac{1}{2}\left[\xi\varpi^b\sigma^b(c_t^b)^2 + (1-\xi)\varpi^s\sigma^s(c_t^s)^2\right]\right\}\\ &= \lambda Y\left\{y_t + \frac{1}{2}y_t^2 - \frac{\Psi_h}{2}[\xi(h_t^b)^2 + (1-\xi)(h_t^s)^2] - \frac{1}{2}\left[\xi\varpi^b\sigma^b(c_t^b)^2 + (1-\xi)\varpi^s\sigma^s(c_t^s)^2\right]\right\}.\end{aligned}$$

Next, we note that for marginal utilities, we have

$$\hat{\lambda}_t^\tau = -\sigma^\tau c_t^\tau.$$

Therefore, we can alternatively rewrite the resource constraint, up to the first order, as

$$\begin{aligned}y_t &= -\frac{\xi\varpi^b}{\sigma^b}\hat{\lambda}_t^b - \frac{(1-\xi)\varpi^s}{\sigma^s}\hat{\lambda}_t^s = -\left[\frac{\xi\varpi^b}{\sigma^b} + \frac{(1-\xi)\varpi^s}{\sigma^s}\right]\hat{\lambda}_t^b + \frac{(1-\xi)\varpi^s}{\sigma^s}\omega_t\\ &= \frac{\sigma^b}{\bar{\sigma}}c_t^b + \frac{(1-\xi)\varpi^s}{\sigma^s}\omega_t,\end{aligned}$$

where $\omega_t \equiv \hat{\lambda}_t^b - \hat{\lambda}_t^s$ is the marginal utility gap relative to steady state marginal utility λ and

$$\bar{\sigma}^{-1} \equiv \frac{\xi\varpi^b}{\sigma^b} + \frac{(1-\xi)\varpi^s}{\sigma^s}.$$

Therefore, we can rewrite borrowers' consumption as

$$c_t^b = \frac{\bar{\sigma}}{\sigma^b}y_t - \frac{(1-\xi)\varpi^s\bar{\sigma}}{\sigma^b\sigma^s}\omega_t.$$

With similar steps, we can derive an expression for savers' consumption

$$c_t^s = \frac{\bar{\sigma}}{\sigma^s}y_t + \frac{\xi\varpi^b\bar{\sigma}}{\sigma^b\sigma^s}\omega_t.$$

Using the last two results, we can write

$$\begin{aligned}\xi\varpi^b\sigma^b(c_t^b)^2+(1-\xi)\varpi^s\sigma^s(c_t^s)^2 &= \frac{\xi\varpi^b}{\sigma^b}\left[\bar{\sigma}y_t-\frac{(1-\xi)\varpi^s\bar{\sigma}}{\sigma^s}\omega_t\right]^2+\frac{(1-\xi)\varpi^s}{\sigma^s}\left[\bar{\sigma}y_t+\frac{\xi\varpi^b\bar{\sigma}}{\sigma^b}\omega_t\right]^2\\ &= \bar{\sigma}\left[y_t^2+\frac{\xi\varpi^b}{\sigma^b}\frac{(1-\xi)\varpi^s}{\sigma^s}\omega_t^2\right].\end{aligned}$$

Plugging back into the expression for the first term, we get

$$\mathcal{W}_t^1 \approx \lambda Y\left[y_t+\frac{1}{2}(1-\bar{\sigma})y_t^2-\frac{\Psi_h}{2}[\xi(h_t^b)^2+(1-\xi)(h_t^s)^2]-\frac{1}{2}\frac{\xi\varpi^b}{\sigma^b}\frac{(1-\xi)\varpi^s}{\sigma^s}\bar{\sigma}\omega_t^2\right].$$

A.6.2 Second Term $\mathcal{W}_t^2$

A second order approximation of the second term gives

$$\mathcal{W}_t^2 \approx \xi\chi_H(H_t^b-1)+(1-\xi)\chi_H(H_t^s-1)-\frac{\sigma_h}{2}\xi\chi_H(H_t^b-1)^2-\frac{\sigma_h}{2}(1-\xi)\chi_H(H_t^s-1)^2.$$

We can then rewrite

$$\mathcal{W}_t^2 \approx \chi_H\left\{\xi h_t^b+(1-\xi)h_t^s+\frac{1}{2}(1-\sigma_h)[\xi(h_t^b)^2+(1-\xi)(h_t^s)^2]\right\}.$$

From the housing market clearing condition, up to a second order approximation, we have

$$0=\xi\left[h_t^b+\frac{1}{2}(h_t^b)^2\right]+(1-\xi)\left[h_t^s+\frac{1}{2}(h_t^s)^2\right],$$

and, up to the first order,

$$0=\xi h_t^b+(1-\xi)h_t^s.$$

Replacing these two results into the approximation for the second and third term, we have

$$\mathcal{W}_t^2 \approx -\frac{1}{2}\sigma_h\chi_H\left[\xi(h_t^b)^2+(1-\xi)(h_t^s)^2\right].$$

Finally, again from the housing resource constraint, we have

$$h_t^b=(1-\xi)(h_t^b-h_t^s) \qquad \text{and} \qquad h_t^s=-\xi(h_t^b-h_t^s).$$

Using this result, we conclude

$$\mathcal{W}_t^2 \approx -\frac{1}{2}\sigma_h\xi(1-\xi)\chi_H\tilde{h}_t^2,$$

where $\tilde{h}_t \equiv h_t^b-h_t^s$ is the housing gap.

A.6.3 Third Term $\mathcal{W}_t^3$

We now move to the third term. To begin, we notice that in steady state $\widetilde{\Lambda} = \widetilde{\lambda} = \lambda$ and $\Delta = 1$. In addition, as showed later (and well known) Δ_t is of order two. Therefore, we can write

$$\begin{aligned}\mathcal{W}_t^3 \approx -\chi_L Y^{1+\varphi} \Bigg[&\frac{1}{\varphi}\left(\frac{\widetilde{\Lambda}_t - \lambda}{\lambda}\right) + \frac{1}{2\varphi^2}\left(\frac{\widetilde{\Lambda}_t - \lambda}{\lambda}\right)^2 - \frac{1}{\varphi}\left(\frac{\widetilde{\lambda}_t - \lambda}{\lambda}\right) + \frac{1}{2\varphi^2}\left(\frac{\widetilde{\lambda}_t - \lambda}{\lambda}\right)^2 \\ &- \frac{1+\varphi}{\varphi^2}\left(\frac{\widetilde{\Lambda}_t - \lambda}{\lambda}\right)\left(\frac{\widetilde{\lambda}_t - \lambda}{\lambda}\right) + \frac{1+\varphi}{\varphi}\left(\frac{\widetilde{\Lambda}_t - \lambda}{\lambda}\right)\left(\frac{Y_t - Y}{Y}\right) - \frac{1+\varphi}{\varphi}\left(\frac{\widetilde{\lambda}_t - \lambda}{\lambda}\right)\left(\frac{Y_t - Y}{Y}\right) \\ &+ \left(\frac{Y_t - Y}{Y}\right) + \frac{1}{2}\varphi\left(\frac{Y_t - Y}{Y}\right)^2 + (\Delta_t - 1)\Bigg].\end{aligned}$$

Collecting terms and simplifying, we can then write

$$\mathcal{W}_t^3 \approx -\chi_L Y^{1+\varphi}\left[\frac{1}{\varphi}(\hat{\widetilde{\Lambda}}_t - \hat{\widetilde{\lambda}}_t) + \frac{1}{2\varphi}\left(\frac{1+\varphi}{\varphi}\right)(\hat{\widetilde{\Lambda}}_t - \hat{\widetilde{\lambda}}_t)^2 + \left(\frac{1+\varphi}{\varphi}\right)(\hat{\widetilde{\Lambda}}_t - \hat{\widetilde{\lambda}}_t)y_t + y_t + \frac{1}{2}(1+\varphi)y_t^2 + \hat{\Delta}_t\right],$$

where $\hat{\widetilde{\Lambda}}_t \equiv \ln(\widetilde{\Lambda}_t/\lambda)$ and $\hat{\widetilde{\lambda}}_t \equiv \ln(\widetilde{\lambda}_t/\lambda)$.

Next, we focus on the approximation of $\widetilde{\Lambda}_t$ and $\widetilde{\lambda}_t$. Starting with $\widetilde{\Lambda}_t$, we have

$$\begin{aligned}\hat{\widetilde{\Lambda}}_t + \frac{1}{2}\hat{\widetilde{\Lambda}}_t^2 \approx \xi[\hat{\lambda}_t^b + \frac{1}{2}(\hat{\lambda}_t^b)^2] + (1-\xi)[\hat{\lambda}_t^s + \frac{1}{2}(\hat{\lambda}_t^s)^2] + \frac{1}{2\varphi}\xi(1-\xi)(\hat{\lambda}_t^b)^2 + \frac{1}{2\varphi}\xi(1-\xi)(\hat{\lambda}_t^s)^2 \\ - \frac{1}{\varphi}\xi(1-\xi)\hat{\lambda}_t^b\hat{\lambda}_t^s.\end{aligned}$$

For $\widetilde{\lambda}_t$, we have

$$\begin{aligned}\hat{\widetilde{\lambda}}_t + \frac{1}{2}\hat{\widetilde{\lambda}}_t^2 \approx \xi[\hat{\lambda}_t^b + \frac{1}{2}(\hat{\lambda}_t^b)^2] + (1-\xi)[\hat{\lambda}_t^s + \frac{1}{2}(\hat{\lambda}_t^s)^2] + \frac{1-\varphi}{2\varphi}\xi(1-\xi)(\hat{\lambda}_t^b)^2 + \frac{1-\varphi}{2\varphi}\xi(1-\xi)(\hat{\lambda}_t^s)^2 \\ - \frac{1-\varphi}{\varphi}\xi(1-\xi)\hat{\lambda}_t^b\hat{\lambda}_t^s.\end{aligned}$$

Note that up to the first order, $\hat{\widetilde{\Lambda}}_t = \hat{\widetilde{\lambda}}_t$. Therefore, we also have $\hat{\widetilde{\Lambda}}_t^2 = \hat{\widetilde{\lambda}}_t^2$. Taking the difference between the two approximations and using the equality above, we can then write

$$\hat{\widetilde{\Lambda}}_t - \hat{\widetilde{\lambda}}_t \approx \frac{1}{2}\xi(1-\xi)\omega_t^2.$$

Since $\hat{\widetilde{\Lambda}}_t - \hat{\widetilde{\lambda}}_t$ is of order two, its square in $\mathcal{W}_t^3$ drops out. Therefore, we can rewrite

$$\mathcal{W}_t^3 \approx -\chi_L Y^{1+\varphi}\left[y_t + \frac{1}{2}(1+\varphi)y_t^2 + \hat{\Delta}_t + \frac{1}{2\varphi}\xi(1-\xi)\omega_t^2\right].$$

Approximation of the Price Dispersion Term

Lastly, we take a second order approximation of the price dispersion index, which yields

$$\hat{\Delta}_t = \alpha\hat{\Delta}_{t-1} + \frac{1}{2}\frac{\alpha\varepsilon}{1-\alpha}\pi_t^2.$$

Solving the previous difference equation backward, we have

$$\hat{\Delta}_t = \alpha\hat{\Delta}_{-1} + \frac{1}{2}\frac{\alpha\varepsilon}{1-\alpha}\sum_{j=0}^{t}\alpha^j\pi_{t-j}^2,$$

for some initial level of price dispersion $\hat{\Delta}_{-1}$. We are interested in the present discounted value of the previous expression, that is

$$\sum_{t=0}^{\infty}\beta^t\hat{\Delta}_t = \frac{1}{2}\frac{\alpha\varepsilon}{1-\alpha}\sum_{t=0}^{\infty}\beta^t\sum_{j=0}^{t}\alpha^j\pi_{t-j}^2,$$

where we have dropped the initial level of price dispersion as it is independent of policy. Let us now focus on the double sum on the right-hand side of the last expression, which we can expand to obtain

$$\begin{aligned}
\sum_{t=0}^{\infty}\beta^t\sum_{j=0}^{t}\alpha^j\pi_{t-j}^2 &= \pi_0^2 + \beta(\alpha\pi_0^2 + \pi_1^2) + \beta^2(\alpha^2\pi_0^2 + \alpha\pi_1^2 + \pi_2^2) + ... \\
&= \sum_{j=0}^{\infty}(\beta\alpha)^j\pi_0^2 + \beta\sum_{j=0}^{\infty}(\beta\alpha)^j\pi_1^2 + \beta^2\sum_{j=0}^{\infty}(\beta\alpha)^j\pi_2^2 + ... \\
&= \pi_0^2\sum_{j=0}^{\infty}(\beta\alpha)^j + \beta\pi_1^2\sum_{j=0}^{\infty}(\beta\alpha)^j + \beta^2\pi_2^2\sum_{j=0}^{\infty}(\beta\alpha)^j + ... \\
&= \sum_{t=0}^{\infty}\beta^t\pi_t^2\sum_{j=0}^{\infty}(\beta\alpha)^j = \frac{1}{1-\beta\alpha}\sum_{t=0}^{\infty}\beta^t\pi_t^2.
\end{aligned}$$

Therefore, we can write

$$\sum_{t=0}^{\infty}\beta^t\hat{\Delta}_t = \frac{1}{2}\frac{\alpha\varepsilon}{(1-\alpha)(1-\beta\alpha)}\sum_{t=0}^{\infty}\beta^t\pi_t^2.$$

Plugging back into the approximation of the third term, we arrive at

$$\mathcal{W}_t^3 \approx -\chi_L Y^{1+\varphi}\left[y_t + \frac{1}{2}(1+\varphi)y_t^2 + \frac{1}{2}\frac{\alpha\varepsilon}{(1-\alpha)(1-\beta\alpha)}\pi_t^2 + \frac{1}{2\varphi}\xi(1-\xi)\omega_t^2\right].$$

A.6.4 Putting the Pieces Together

We begin by summing $\mathcal{W}_t^1$ and $\mathcal{W}_t^3$

$$\begin{aligned}
\mathcal{W}_t^1 + \mathcal{W}_t^3 = \lambda Y&\left[y_t + \frac{1}{2}(1-\bar{\sigma})y_t^2 - \frac{\xi(1-\xi)\Psi_h}{2}\tilde{h}_t^2 - \frac{1}{2}\xi(1-\xi)\frac{\varpi^b}{\sigma^b}\frac{\varpi^s}{\sigma^s}\bar{\sigma}\omega_t^2\right] \\
&- \chi_L Y^{1+\varphi}\left[y_t + \frac{1}{2}(1+\varphi)y_t^2 + \frac{1}{2}\frac{\alpha\varepsilon}{(1-\alpha)(1-\beta\alpha)}\pi_t^2 + \frac{1}{2}\frac{\xi(1-\xi)}{\varphi}\omega_t^2\right],
\end{aligned}$$

where we have used the relations between h_t^τ and $\tilde{h}_t$. In the efficient steady state, $\lambda = \chi_L Y^\varphi$. Therefore, we can rewrite

$$\mathcal{W}_t^1 + \mathcal{W}_t^3 = -\frac{\lambda Y}{2}\left[(\bar{\sigma}+\varphi)y_t^2 + \xi(1-\xi)\Psi_h\tilde{h}_t^2 + \xi(1-\xi)\left(\frac{1}{\varphi} + \frac{\varpi^b}{\sigma^b}\frac{\varpi^s}{\sigma^s}\bar{\sigma}\right)\omega_t^2 + \frac{\alpha\varepsilon}{(1-\alpha)(1-\beta\alpha)}\pi_t^2\right].$$

Finally, we add the second term

$$\begin{aligned}
\mathcal{W}_t^1 + \mathcal{W}_t^2 + \mathcal{W}_t^3 = -\frac{\lambda Y}{2}&\left[(\bar{\sigma}+\varphi)y_t^2 + \xi(1-\xi)\left(\Psi_h + \frac{\sigma_h\chi_H}{\lambda Y}\right)\tilde{h}_t^2 + \xi(1-\xi)\left(\frac{1}{\varphi} + \frac{\varpi^b}{\sigma^b}\frac{\varpi^s}{\sigma^s}\bar{\sigma}\right)\omega_t^2\right. \\
&\left. + \frac{\alpha\varepsilon}{(1-\alpha)(1-\beta\alpha)}\pi_t^2\right],
\end{aligned}$$

or, more compactly,

$$\mathcal{W}_t^1 + \mathcal{W}_t^2 + \mathcal{W}_t^3 = -\frac{\Xi}{2}(y_t^2 + \lambda_h \tilde{h}_t^2 + \lambda_\omega \omega_t^2 + \lambda_\pi \pi_t^2),$$

where

$$\begin{aligned}
\Xi &\equiv \lambda Y(\bar{\sigma} + \varphi) \\
\lambda_h &\equiv \frac{\xi(1-\xi)}{\bar{\sigma} + \varphi}\left(\Psi_h + \frac{\sigma_h \chi_H}{\chi_L Y^{1+\varphi}}\right) \\
\lambda_\omega &\equiv \frac{\xi(1-\xi)\sigma_\varphi}{\bar{\sigma} + \varphi} \\
\lambda_\pi &\equiv \frac{\varepsilon}{\gamma(\bar{\sigma} + \varphi)} \\
\gamma &\equiv \frac{(1-\alpha)(1-\beta\alpha)}{\alpha} \\
\sigma_\varphi &\equiv \left(\frac{1}{\varphi} + \frac{\varpi^b}{\sigma^b}\frac{\varpi^s}{\sigma^s}\bar{\sigma}\right).
\end{aligned}$$

A.7 First-Order Approximation of the Equilibrium Conditions

In this section, we derive a first-order approximation to the set of equilibrium conditions that constitute the constraints of the linear-quadratic optimal policy problem. For all variables, we take a log-linear approximation around the efficient steady state with zero inflation and equal marginal utility of consumption between borrowers and savers. The exception is the Lagrange multiplier on the collateral constraint, which is equal to zero in steady state, for which we take a linear approximation.

Up to the first order, the savers' Euler equation becomes

$$\hat{\lambda}_t^s = i_t - \mathbb{E}_t \pi_{t+1} + (1-\delta)\xi \mathbb{E}_t \hat{\lambda}_{t+1}^b + [\delta + (1-\delta)(1-\xi)]\mathbb{E}_t \hat{\lambda}_{t+1}^s.$$

where $i_t \equiv \ln(R_t^d / R^d)$ is approximately equal to the net nominal interest rate. Similarly, the savers' Euler equation for equity gives

$$\hat{\lambda}_t^s + \Psi_e \kappa_t = i_t^e - \mathbb{E}_t \pi_{t+1} + (1-\delta)\xi \mathbb{E}_t \hat{\lambda}_{t+1}^b + [\delta + (1-\delta)(1-\xi)]\mathbb{E}_t \hat{\lambda}_{t+1}^s,$$

where, using the same notation as for the return on deposits, i_t^e is approximately equal to the nominal net return on equity. The savers' housing demand equation becomes

$$q_t = (1-\beta)(-\sigma_h h_t^s - \hat{\lambda}_t^s) - \tilde{\Psi}_h h_t^s + \beta\left\{(1-\delta)\xi \mathbb{E}_t \hat{\lambda}_{t+1}^b + [\delta + (1-\delta)(1-\xi)]\mathbb{E}_t \hat{\lambda}_{t+1}^s - \hat{\lambda}_t^s + \mathbb{E}_t q_{t+1}\right\},$$

where $\tilde{\Psi}_h \equiv \Psi_h / q$. Savers' marginal utility is

$$\hat{\lambda}_t^s = -\sigma^s c_t^s.$$

For borrowers, the Euler equation for debt is

$$\hat{\lambda}_t^b = i_t^b - \mathbb{E}_t \pi_{t+1} + [\delta + (1-\delta)\xi]\mathbb{E}_t \hat{\lambda}_{t+1}^b + (1-\delta)(1-\xi)\mathbb{E}_t \hat{\lambda}_{t+1}^s + \mu_t - \beta\gamma_d[\delta + (1-\delta)\xi]\mathbb{E}_t \mu_{t+1},$$

where i_t^b is the net nominal borrowing rate. Borrowers' housing demand gives

$$q_t = (1-\beta)(-\sigma_h h_t^b - \hat{\lambda}_t^b) - \tilde{\Psi}_h h_t^b + (1-\gamma_d)\Theta\mu_t \\ + \beta\left\{[\delta + (1-\delta)\xi]\mathbb{E}_t\hat{\lambda}_{t+1}^b + (1-\delta)(1-\xi)\mathbb{E}_t\hat{\lambda}_{t+1}^s - \hat{\lambda}_t^b + \mathbb{E}_t q_{t+1}\right\}.$$

Borrowers' marginal utility is

$$\hat{\lambda}_t^b = -\sigma^b c_t^b.$$

We rewrite the collateral constraint as

$$\frac{B_t}{P_t} \leq \mathcal{D}_t \equiv \delta\gamma_d \frac{B_t}{P_{t-1}}\frac{1}{\Pi_t} + (1-\gamma_d)\Theta_t \frac{Q_t}{P_t} H_t^b.$$

In log-deviations from steady state, the previous equation becomes

$$b_t - \hat{\mathcal{D}}_t \leq \ln\left(\frac{\mathcal{D}}{b}\right) \equiv \ln\mathcal{M},$$

where

$$\mathcal{M} \equiv \frac{\mathcal{D}}{b} = \delta\gamma_d + (1-\gamma_d)\frac{\Theta q}{b}.$$

Up to the first order, we have

$$\hat{\mathcal{D}}_t = \frac{\delta\gamma_d}{\mathcal{M}}(b_{t-1} - \pi_t) + (1-\gamma_d)\frac{\Theta q}{\mathcal{D}}(\theta_t + q_t + h_t^b).$$

The equation for the lending rate becomes

$$i_t^b = \widetilde{\kappa} i_t^e + (1-\widetilde{\kappa})i_t.$$

The Phillips curve is

$$\pi_t = \gamma mc_t + \beta\mathbb{E}_t\pi_{t+1}.$$

Up to the first order, the labor market equilibrium condition gives

$$\varphi y_t = \hat{\lambda}_t + mc_t,$$

where average marginal utility is

$$\hat{\lambda}_t = \xi\hat{\lambda}_t^b + (1-\xi)\hat{\lambda}_t^s.$$

The resource constraint gives

$$y_t = \xi\varpi^b c_t^b + (1-\xi)\varpi^s c_t^s.$$

The housing market equilibrium condition gives

$$0 = \xi h_t^b + (1-\xi)h_t^s.$$

Finally, the law of motion of debt is

$$b_t = \frac{\delta}{\beta}(b_{t-1} + i_{t-1}^b - \pi_t) + \frac{q}{b}[(h_t^b - h_{t-1}^b) + (1-\xi)(1-\delta)(h_{t-1}^b - h_{t-1}^s)] + \frac{Y}{d^b}\left(\varpi^b c_t^b - y_t - \frac{1-\xi}{\varphi}\omega_t\right).$$

A.7.1 Gap Representation

In this section, we express the equilibrium conditions in terms of welfare-relevant gaps. Combining the resource constraint and the expression for the average marginal utility we can write

$$\hat{\lambda}_t = -\bar{\sigma}\left[y_t + \xi(1-\xi)\left(\frac{\varpi^b}{\sigma^b} - \frac{\varpi^s}{\sigma^s}\right)\omega_t\right].$$

Replacing into the first order approximation of the labor market equilibrium, we obtain

$$mc_t = (\bar{\sigma} + \varphi)y_t + \sigma_\omega \omega_t,$$

where

$$\sigma_\omega \equiv \xi(1-\xi)\bar{\sigma}\left(\frac{\varpi^b}{\sigma^b} - \frac{\varpi^s}{\sigma^s}\right).$$

Therefore, we can rewrite the Phillips curve as

$$\pi_t = \gamma[(\bar{\sigma} + \varphi)y_t + \sigma_\omega \omega_t] + \beta \mathbb{E}_t \pi_{t+1}.$$

Taking the difference between the savers' Euler equations for deposits and equity we obtain

$$i_t^e = i_t + \Psi_e \kappa_t.$$

Combining this expression with the zero profit condition for banks we get

$$i_t^b = i_t + \kappa_t,$$

where we have normalized $\Psi_e \widetilde{\kappa}$ to one.[45] Next, taking a population-weighted average of the savers' Euler equation for deposits and the borrowers' Euler equation for debt, we obtain

$$\hat{\lambda}_t = i_t - \mathbb{E}_t \pi_{t+1} + \mathbb{E}_t \hat{\lambda}_{t+1} + \xi\{\kappa_t + \mu_t - \beta\gamma_d[\delta + (1-\delta)\xi]\mathbb{E}_t \mu_{t+1}\}.$$

Using the relation between the average marginal utility of consumption, output and marginal utility gap at the beginning of this section, we can rewrite the last expression as

$$\bar{\sigma} y_t + \sigma_\omega \omega_t = -(i_t - \mathbb{E}_t \pi_{t+1}) + \bar{\sigma}\mathbb{E}_t y_{t+1} + \sigma_\omega \mathbb{E}_t \omega_{t+1} - \xi\{\kappa_t + \mu_t - \beta\gamma_d[\delta + (1-\delta)\xi]\mathbb{E}_t \mu_{t+1}\}.$$

Taking the difference between the two Euler equations yields

$$\omega_t = \kappa_t + \mu_t - \beta\gamma_d[\delta + (1-\delta)\xi]\mathbb{E}_t \mu_{t+1} + \delta \mathbb{E}_t \omega_{t+1},$$

which shows that the wedge in aggregate Euler equation (the term in the curly bracket) is proportional to the quasi-difference in the marginal utility gap. We can use this expression to eliminate the wedge and rearrange to get

$$y_t = -\bar{\sigma}^{-1}(i_t - \mathbb{E}_t \pi_{t+1} - r_t^*) + \mathbb{E}_t y_{t+1},$$

where the equilibrium real interest rate is

$$r_t^* \equiv \bar{\sigma}[(\sigma_\omega + \delta\xi)\mathbb{E}_t \omega_{t+1} - (\sigma_\omega + \xi)\omega_t].$$

The equilibrium real interest rate is endogenous in this model, and in particular is a function of the

[45] The normalization is innocuous since $\Psi\widetilde{\kappa}$ always pre-multiplies κ_t in the log-linear approximation of the model.

quasi-difference of the marginal utility gap.

Next, we take a population-weighted average of the housing demand equations to obtain

$$q_t = -(1-\beta)\hat{\lambda}_t + \beta(\mathbb{E}_t q_{t+1} - i_t + \mathbb{E}_t\pi_{t+1}) + \xi[(1-\gamma_d)\Theta\mu_t + \beta(\delta\mathbb{E}_t\omega_{t+1} - \omega_t)],$$

where we have used the savers' and borrowers' Euler equation to substitute for expected future marginal utilities of consumption, and housing costs disappear using the housing resource constraint. If we take the difference between the two housing demand equations, we get

$$[(1-\beta)\sigma_h + \tilde{\Psi}_h]\tilde{h}_t = (1-\gamma_d)\Theta\mu_t - \omega_t + \beta\delta\mathbb{E}_t\omega_{t+1}.$$

We can use this second expression, together with the expression for the average marginal utility of consumption, to eliminate the last term in the previous one and rewrite

$$q_t = (1-\beta)[\bar{\sigma}y_t + \xi\tilde{\sigma}_h\tilde{h}_t + (\xi+\sigma_\omega)\omega_t] + \beta(\mathbb{E}_t q_{t+1} - i_t + \mathbb{E}_t\pi_{t+1}),$$

where $\tilde{\sigma}_h \equiv \sigma_h + \tilde{\Psi}_h/(1-\beta)$.

We can rewrite the law of motion of debt as

$$b_t = \frac{\delta}{\beta}(b_{t-1} + i_{t-1} + \kappa_{t-1} - \pi_t) + (1-\xi)\eta_q(\tilde{h}_t - \delta\tilde{h}_{t-1}) + \eta_d\left[\left(\frac{\varpi^b\bar{\sigma}}{\sigma^b} - 1\right)y_t - (1-\xi)\sigma_\varphi\omega_t\right],$$

where

$$\eta_q \equiv \frac{q}{b} = \frac{\chi_H\eta_d}{(1-\beta)\chi_L Y^{1+\varphi}},$$

and

$$\eta_d \equiv \frac{Y}{b} = \frac{\beta-\delta}{\beta(\varpi^b - 1)}.$$

Finally, the borrowing constraint is

$$b_t \leq \ln\mathcal{M} + \frac{\delta\gamma_d}{\mathcal{M}}(b_{t-1} - \pi_t) + \frac{(1-\gamma_d)\Theta\eta_q}{\mathcal{M}}[\theta_t + q_t + (1-\xi)\tilde{h}_t].$$

Given the policy instruments $\{i_t, \theta_t\}_{t=0}^{\infty}$, the exogenous shocks $\{\kappa_t\}_{t=0}^{\infty}$, and initial conditions $\{b_{-1}, \tilde{h}_{-1}, i_{-1}\}$, an approximated equilibrium for this economy is a sequence $\{y_t, \pi_t, \omega_t, \tilde{h}_t, q_t, b_t, \mu_t\}_{t=0}^{\infty}$ that satisfies:

1. Phillips curve

$$\pi_t = \gamma[(\bar{\sigma}+\varphi)y_t + \sigma_\omega\omega_t] + \beta\mathbb{E}_t\pi_{t+1}.$$

2. Aggregate demand

$$y_t = -\bar{\sigma}^{-1}(i_t - \mathbb{E}_t\pi_{t+1} - r_t^*) + \mathbb{E}_t y_{t+1},$$

where

$$r_t^* \equiv (\sigma_\omega + \delta\xi)\mathbb{E}_t\omega_{t+1} - (\sigma_\omega + \xi)\omega_t.$$

3. House prices:

$$q_t = (1-\beta)[\bar{\sigma}y_t + \xi\tilde{\sigma}_h\tilde{h}_t + (\xi+\sigma_\omega)\omega_t] + \beta(\mathbb{E}_t q_{t+1} - i_t + \mathbb{E}_t\pi_{t+1}).$$

4. Housing gap:

$$(1-\beta)\tilde{\sigma}_h\tilde{h}_t = (1-\gamma_d)\Theta\mu_t - \omega_t + \beta\delta\mathbb{E}_t\omega_{t+1}.$$

5. Debt dynamics:

$$b_t = \frac{\delta}{\beta}(b_{t-1} + i_{t-1} + \kappa_{t-1} - \pi_t) + (1-\xi)\eta_q(\tilde{h}_t - \delta\tilde{h}_{t-1}) + \eta_d \left[\left(\frac{\varpi^b \bar{\sigma}}{\sigma^b} - 1 \right) y_t - (1-\xi)\sigma_\varphi \omega_t \right],$$

6. Borrowing constraint:

$$\mu_t \left\{ b_t - \ln \mathcal{M} - \frac{\delta\gamma_d}{\mathcal{M}}(b_{t-1} - \pi_t) - \frac{(1-\gamma_d)\Theta\eta_q}{\mathcal{M}}[\theta_t + q_t + (1-\xi)\tilde{h}_t] \right\} = 0.$$

7. Lagrange multiplier:

$$\omega_t = \kappa_t + \mu_t - \beta\gamma_d[\delta + (1-\delta)\xi]\mathbb{E}_t\mu_{t+1} + \delta\mathbb{E}_t\omega_{t+1}.$$

A.8 Optimal Targeting Rules

In this section we focus on the case in which the nominal interest rate is always positive ($i_t > \ln\beta$) and the collateral constraint is always binding ($\mu_t > 0$). The optimal joint monetary and macro-prudential policy problem consists of maximizing

$$\mathbb{W}_0 = -\frac{\Xi}{2}\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t (y_t^2 + \lambda_h \tilde{h}_t^2 + \lambda_\omega \omega_t^2 + \lambda_\pi \pi_t^2) \right],$$

subject to the constraints 1. to 7. at the end of last section plus the definition of r_t^*.

A.8.1 Discretion

The state variables for the optimal policy problem are $\tilde{h}_t$, b_t and i_t. The Lagrangian under discretion is

$$\begin{aligned}
\mathcal{L}_t(\tilde{h}_{t-1}, b_{t-1}, i_{t-1}) \;&=\; y_t^2 + \lambda_h \tilde{h}_t^2 + \lambda_\omega \omega_t^2 + \lambda_\pi \pi_t^2 + \beta\mathbb{E}_t\mathcal{L}_{t+1}(\tilde{h}_t, b_t, i_t) \\
&-\; 2\phi_{1t}[\pi_t - \gamma(\bar{\sigma} + \varphi)y_t - \gamma\sigma_\omega\omega_t - \beta\mathcal{F}_{\pi t}] \\
&-\; 2\phi_{2t}\{y_t + \bar{\sigma}^{-1}[i_t - \mathcal{F}_{\pi t} - (\sigma_\omega + \delta\xi)\mathcal{F}_{\omega t} + (\sigma_\omega + \xi)\omega_t] - \mathcal{F}_{yt}\} \\
&-\; 2\phi_{3t}\{q_t - (1-\beta)[\bar{\sigma}y_t + \xi\tilde{\sigma}_h\tilde{h}_t + (\xi + \sigma_\omega)\omega_t] - \beta(\mathcal{F}_{qt} - i_t + \mathcal{F}_{\pi t})\} \\
&-\; 2\phi_{4t}[(1-\beta)\tilde{\sigma}_h\tilde{h}_t - (1-\gamma_d)\Theta\mu_t + \omega_t - \beta\delta\mathcal{F}_{\omega t}] \\
&-\; 2\phi_{5t}\left\{ b_t - \frac{\delta}{\beta}(b_{t-1} + i_{t-1} + \kappa_{t-1} - \pi_t) - (1-\xi)\eta_q(\tilde{h}_t - \delta\tilde{h}_{t-1}) \right. \\
&\qquad \left. -\eta_d \left[\left(\frac{\varpi^b\bar{\sigma}}{\sigma^b} - 1 \right) y_t - (1-\xi)\sigma_\varphi\omega_t \right] \right\} \\
&-\; 2\phi_{6t}\left\{ b_t - \ln\mathcal{M} - \frac{\delta\gamma_d}{\mathcal{M}}(b_{t-1} - \pi_t) - \frac{(1-\gamma_d)\Theta\eta_q}{\mathcal{M}}[\theta_t + q_t + (1-\xi)\tilde{h}_t] \right\} \\
&-\; 2\phi_{7t}\{\omega_t - \kappa_t - \mu_t - \delta\mathcal{F}_{\omega t} + \beta\gamma_d[\delta + (1-\delta)\xi]\mathcal{F}_{\mu t}\},
\end{aligned}$$

where ϕ_{jt}, for $j = \{1, ..., 7\}$, are Lagrange multipliers and $\mathcal{F}_{zt} = \mathbb{E}_t z_{t+1}$, with $z = \{\pi, y, q, \omega, \mu\}$, are time$-t$ expectation terms that the policymaker takes as given. Time-consistency requires the policymakers to take into account the consequences of its current policy decisions on future losses via the state variables.

The first order condition for output is

$$y_t + \gamma(\bar{\sigma} + \varphi)\phi_{1t} - \phi_{2t} + (1-\beta)\bar{\sigma}\phi_{3t} + \eta_d \left(\frac{\varpi^b \bar{\sigma}}{\sigma^b} - 1 \right) \phi_{5t} = 0.$$

The first order condition for inflation is

$$\lambda_\pi \pi_t - \phi_{1t} - \frac{\delta}{\beta}\phi_{5t} - \frac{\delta\gamma_d}{\mathcal{M}}\phi_{6t} = 0.$$

The first order condition for the marginal utility gap is

$$\lambda_\omega \omega_t + \gamma\sigma_\omega \phi_{1t} - \bar{\sigma}^{-1}(\sigma_\omega + \xi)\phi_{2t} + (1-\beta)(\sigma_\omega + \xi)\phi_{3t} - \phi_{4t} - \eta_d(1-\xi)\sigma_\varphi \phi_{5t} - \phi_{7t} = 0.$$

The first order condition for the housing gap is

$$\lambda_h \tilde{h}_t + \beta \mathbb{E}_t \frac{\partial \mathcal{L}_{t+1}}{\partial \tilde{h}_t} + (1-\beta)\xi\tilde{\sigma}_h \phi_{3t} - (1-\beta)\tilde{\sigma}_h \phi_{4t} + (1-\xi)\eta_q \phi_{5t} + \frac{(1-\gamma_d)\Theta\eta_d}{\mathcal{M}}(1-\xi)\phi_{6t} = 0,$$

where for simplicity we have omitted the arguments of the Lagrangian function at $t+1$. The first order condition for debt is

$$\beta \mathbb{E}_t \frac{\partial \mathcal{L}_{t+1}}{\partial b_t} - \phi_{5t} - \phi_{6t} = 0.$$

The first order condition for the nominal interest rate is

$$\beta \mathbb{E}_t \frac{\partial \mathcal{L}_{t+1}}{\partial i_t} - \bar{\sigma}^{-1}\phi_{2t} - \beta\phi_{3t} = 0.$$

The first order condition for the multiplier on the borrowing constraint is

$$(1-\gamma_d)\Theta\phi_{4t} + \phi_{7t} = 0.$$

The first order condition for house prices is

$$-\phi_{3t} + \frac{(1-\gamma_d)\Theta\eta_d}{\mathcal{M}}\phi_{6t} = 0.$$

Finally, the first order condition for the LTV ratio is

$$\frac{(1-\gamma_d)\Theta\eta_d}{\mathcal{M}}\phi_{6t} = 0.$$

The envelope condition with respect to the housing gap is

$$\frac{\partial \mathcal{L}_t}{\partial \tilde{h}_{t-1}} = -(1-\xi)\eta_d \delta \phi_{5t}.$$

The envelope condition with respect to debt is

$$\frac{\partial \mathcal{L}_t}{\partial b_{t-1}} = \frac{\delta}{\beta}\phi_{5t} + \frac{\delta\gamma_d}{\mathcal{M}}\phi_{6t}.$$

The envelope condition with respect to the nominal interest rate is

$$\frac{\partial \mathcal{L}_t}{\partial i_{t-1}} = \frac{\delta}{\beta}\phi_{5t}.$$

Since the first order condition for the LTV ratio gives us $\phi_{6t} = 0$, from the first order condition for house prices, we obtain $\phi_{3t} = 0$. In addition, combining the result with the envelope condition for debt and plugging in the first order condition for that variable, we can see that the only stable solution is $\phi_{5t} = 0$, which implies that also the derivatives of the continuation values with respect to the housing gap and the nominal interest rate are also zero. Furthermore, from the first order condition for the nominal interest rate, we obtain $\phi_{2t} = 0$.

The collateral constraint and the aggregate demand equations residually determine the macro-prudential policy and monetary policy instruments, respectively, that implement the optimal policy plan. The equations for house prices and debt determine the value of those two variables consistent with the equilibrium under optimal policy.

The simplified system of equations that characterize the optimal policy plan then reduces to five equations:

$$\begin{aligned} y_t + \gamma(\bar{\sigma} + \varphi)\phi_{1t} &= 0 \\ \lambda_\pi \pi_t - \phi_{1t} &= 0 \\ \lambda_\omega \omega_t + \gamma \sigma_\omega \phi_{1t} - \phi_{4t} - \phi_{7t} &= 0 \\ \lambda_h \tilde{h}_t - (1-\beta)\tilde{\sigma}_h \phi_{4t} &= 0 \\ (1-\gamma_d)\Theta\phi_{4t} + \phi_{7t} &= 0. \end{aligned}$$

Combining the first two, we obtain the standard targeting rule for monetary policy under discretion

$$\varepsilon \pi_t + y_t = 0,$$

where we also used the definition of the relative weight on inflation.

We can use the fourth condition to express ϕ_{4t} as a function of $\tilde{h}_t$

$$\phi_{4t} = \frac{\lambda_h}{(1-\beta)\tilde{\sigma}_h}\tilde{h}_t.$$

Replacing the result into the fifth equation, we can then also express ϕ_{7t} as a function of $\tilde{h}_t$

$$\phi_{7t} = -\frac{(1-\gamma_d)\Theta\lambda_h}{(1-\beta)\tilde{\sigma}_h}\tilde{h}_t.$$

Finally, from the first, we can express ϕ_{1t} as a function of y_t

$$\phi_{1t} = -\frac{1}{\gamma(\bar{\sigma} + \varphi)}y_t.$$

Substituting these three multipliers into the third condition we obtain

$$\lambda_\omega \omega_t - \frac{\sigma_\omega}{\bar{\sigma} + \varphi}y_t - \frac{[1-(1-\gamma_d)\Theta]\lambda_h}{(1-\beta)\tilde{\sigma}_h}\tilde{h}_t = 0.$$

We interpret this equation as the optimal targeting rule for macro-prudential policy. If the collateral constraint has no inertia and the steady state LTV ratio is 100%, the optimal targeting rule for macro-prudential policy only trades off the marginal utility gap and the output gap. In particular, optimal policy requires that the two variables move in the same direction since $\sigma_\omega > 0$. More generally, the housing gap matters too. Since the coefficient in front of the housing gap is always positive, macro-prudential policy moves the marginal utility gap and a combination of the output gap and the housing gap in the same direction.

A.8.2 Commitment

In order to write the optimal policy problem under commitment, we leverage the result that the collateral constraint, the house price equation, the debt equation and the aggregate demand equation are residual for the optimal policy problem as demonstrated in the previous section. For simplicity, we focus on the case $\gamma_d = 0$ (no inertia in the collateral constraint). The Lagrangian for the optimal policy problem under commitment is then

$$\begin{aligned}
\mathcal{L}_0 \quad = \quad & \mathbb{E}_0 \textstyle\sum_{t=0}^{\infty} \beta^t \Big\{ (y_t^2 + \lambda_h \tilde{h}_t^2 + \lambda_\omega \omega_t^2 + \lambda_\pi \pi_t^2) \\
& -2\varkappa_{1t}[\pi_t - \gamma(\bar{\sigma} + \varphi)y_t - \gamma\sigma_\omega\omega_t - \beta\mathbb{E}_t\pi_{t+1}] \\
& -2\varkappa_{2t}[(1-\beta)\tilde{\sigma}_h\tilde{h}_t - \Theta\mu_t + \omega_t - \beta\delta\mathbb{E}_t\omega_{t+1}] \\
& -2\varkappa_{3t}[\omega_t - \mu_t - \kappa_t - \delta\mathbb{E}_t\omega_{t+1}\} ,
\end{aligned}$$

where $\varkappa_{\ell t}$, with $\ell = \{1, 2, 3\}$, are Lagrange multipliers.

The first order condition for output is

$$y_t + \gamma(\bar{\sigma} + \varphi)\varkappa_{1t} = 0.$$

The first order condition for inflation is

$$\lambda_\pi \pi_t - \varkappa_{1t} + \varkappa_{1t-1} = 0.$$

Combining these two expressions, we obtain the standard optimal targeting rule for monetary policy under commitment

$$\varepsilon\pi_t + y_t - y_{t-1} = 0.$$

The same expression would also hold under commitment in the case in which the collateral constraint is always slack.

The first order condition for the marginal utility gap is

$$\lambda_\omega\omega_t + \gamma\sigma_\omega\varkappa_{1t} - \varkappa_{2t} + \delta\varkappa_{2t-1} - \varkappa_{3t} + \frac{\delta}{\beta}\varkappa_{3t-1} = 0.$$

The first order condition for the housing gap is

$$\lambda_h\tilde{h}_t - (1-\beta)\tilde{\sigma}_h\varkappa_{2t} = 0.$$

Finally, the first order condition for the multiplier on the collateral constraint is

$$\Theta\varkappa_{2t} + \varkappa_{3t} = 0.$$

We can solve for $\varkappa_{2t}$ from the first order condition for the housing gap

$$\varkappa_{2t} = \frac{\lambda_h}{(1-\beta)\tilde{\sigma}_t}\tilde{h}_t.$$

Using the result in the first order condition for the multiplier on the collateral constraint yields

$$\varkappa_{3t} = -\frac{\lambda_h\Theta}{(1-\beta)\tilde{\sigma}_t}\tilde{h}_t.$$

Substituting the expressions for these two Lagrange multipliers and the solution for $\varkappa_{1t}$ from the first order condition for output into the first order conditions for the marginal utility gap gives

$$\lambda_\omega \omega_t - \frac{\sigma_\omega}{\bar{\sigma} + \varphi} y_t - \frac{\lambda_h}{(1-\beta)\sigma_h}\left[(1-\Theta)\tilde{h}_t - \delta\left(1 - \frac{\Theta}{\beta}\right)\tilde{h}_{t-1}\right] = 0.$$

If the LTV ratio is 100%, the contemporaneous housing gap disappears from the optimal targeting rule as in the case of discretion. However, a lagged housing gap remains, which captures the standard state-dependency of the commitment solution (this effect disappears only in the special case $\Theta = \beta$).

B Additional Results

In this section, we report a number of additional results. First, we look at a different specification of the collateral constraint. Second, we compare optimal policy under commitment to the discretionary outcome in the boom-bust numerical example. Third, we show that most of the welfare gains associated with the jointly optimal policy plan depend on setting LTVs optimally. Fourth, we report the full set of responses to a tightening in the collateral constraint during the housing bust.

B.1 Expected Value of Housing in the Collateral Constraint

The formulation of the collateral constraint in the main text is

$$B_t \leq \gamma_d \max\{B_{t-1}, 0\} + (1-\gamma_d)\,\Theta_t Q_t H_t^b,$$

and the borrower's first order condition for housing consumption is

$$\begin{aligned}\frac{Q_t}{P_t} - (1-\gamma_d)\,\Theta_t\mu_t\frac{Q_t}{P_t} = {} & \frac{\chi_H\left(H_t^b\right)^{-\sigma_h}}{\lambda_t^b} - \Phi_h\left(\frac{H_t^b}{H} - 1\right) \\ & + \mathbb{E}_t\left\{\frac{\left[\delta + (1-\delta)\,\xi\right]\lambda_{t+1}^b + (1-\delta)(1-\xi)\,\lambda_{t+1}^s}{\lambda_t^b}\frac{Q_{t+1}}{P_{t+1}}\right\}. \end{aligned} \tag{28}$$

These two equations are the only equilibrium conditions that are influenced by the nature of the collateral constraint. In particular, the second term on the left-hand side of (28) captures the marginal effect on the collateral constraint of the choice of housing.

Suppose the collateral constraint depends instead on the expected value of housing (as in e.g. Iacoviello and Neri, 2010), that is

$$B_t \leq \gamma_d \max\{B_{t-1}, 0\} + (1-\gamma_d)\,\Theta_t \mathbb{E}_t Q_{t+1} H_t^b. \tag{29}$$

The first order condition (28) then becomes

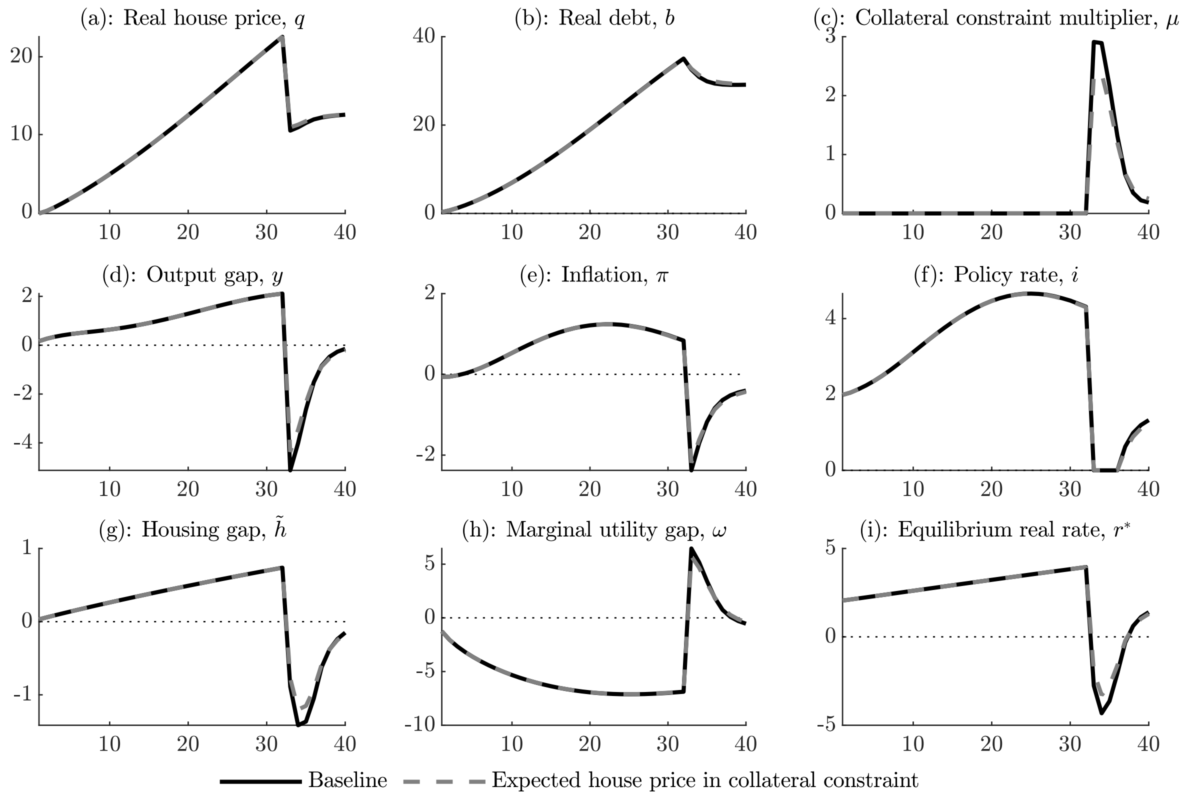
$$\begin{aligned}\frac{Q_t}{P_t} - (1-\gamma_d)\,\Theta_t\mu_t\frac{\mathbb{E}_t Q_{t+1}}{P_t} = {} & \frac{\chi_H\left(H_t^b\right)^{-\sigma_h}}{\lambda_t^b} - \Phi_h\left(\frac{H_t^b}{H} - 1\right) \\ & + \mathbb{E}_t\left\{\frac{\left[\delta + (1-\delta)\,\xi\right]\lambda_{t+1}^b + (1-\delta)(1-\xi)\,\lambda_{t+1}^s}{\lambda_t^b}\frac{Q_{t+1}}{P_{t+1}}\right\}. \end{aligned} \tag{30}$$

Since the collateral constraint is slack in steady state ($\mu = 0$), the first-order approximation to (28) and (30) are identical. Therefore, the effect of including the expected value of housing in the collateral constraint, rather than its current value, is negligible, at least when the collateral constraint is slack and expected to remain slack in the near future. When the collateral constraint binds, the alternative formulation of the constraint (29) influences the value of the multiplier μ_t.

Figure 10 demonstrates these results by plotting the baseline simulation under the two formulations of the collateral constraint, with the contemporaneous value of housing (solid black lines) and with its expected value (dashed gray lines). During the boom the collateral constraint is slack, and the two simulations coincide. The bust displays some small differences between the two cases, due to the different implications for the multiplier μ_t (panel c), but the broad contours of the two simulations remain very similar.

The result is even stronger under optimal policy. Figure 11 shows that macroeconomic outcomes are identical under the jointly optimal policy plan regardless of the specification of the collateral constraint. This finding follows from the fact that the jointly optimal policy problem can be cast as one in which

Figure 10: Baseline simulation with expected housing value in the collateral constraint.

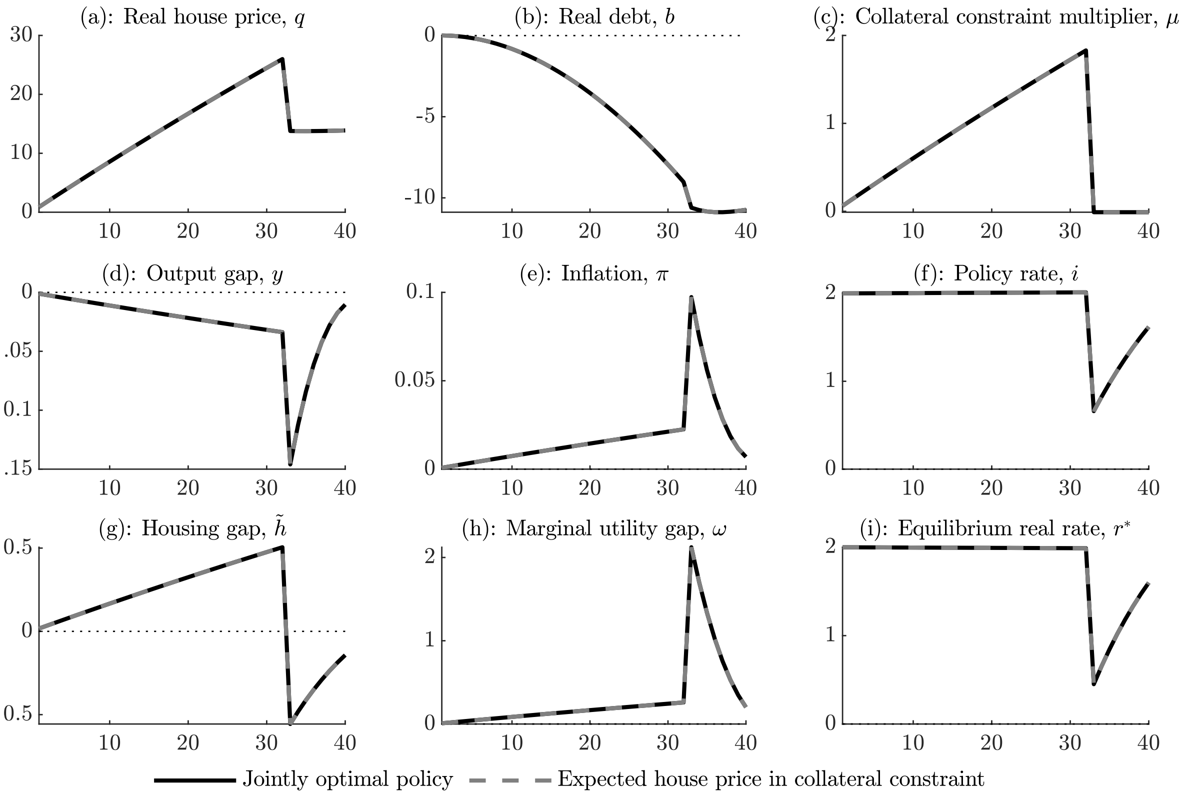


NOTE: All variables are scaled by 100 and plotted as log-deviations from steady state, except for the multiplier on the collateral constraint (μ) which is in levels. Inflation, the policy rate and equilibrium real interest rate are shown in annualized units.

μ_t is the macro-prudential policy instrument. As discussed in the main text, the collateral constraint determines the LTV ratio that supports the optimal setting of μ_t.[46]

[46] As a result, the alternative specification of the collateral constraint only manifests itself in terms of the optimal path of the LTV limit. However, the differences between the LTV limits (available on request) are quantitatively small (indistinguishable to the naked eye). This result reflects the high persistence of house prices under these policy specifications, which implies that $\mathbb{E}_t q_{t+1} \approx q_t$, particularly during the bust when jointly optimal policy implies an almost flat profile for q_t.

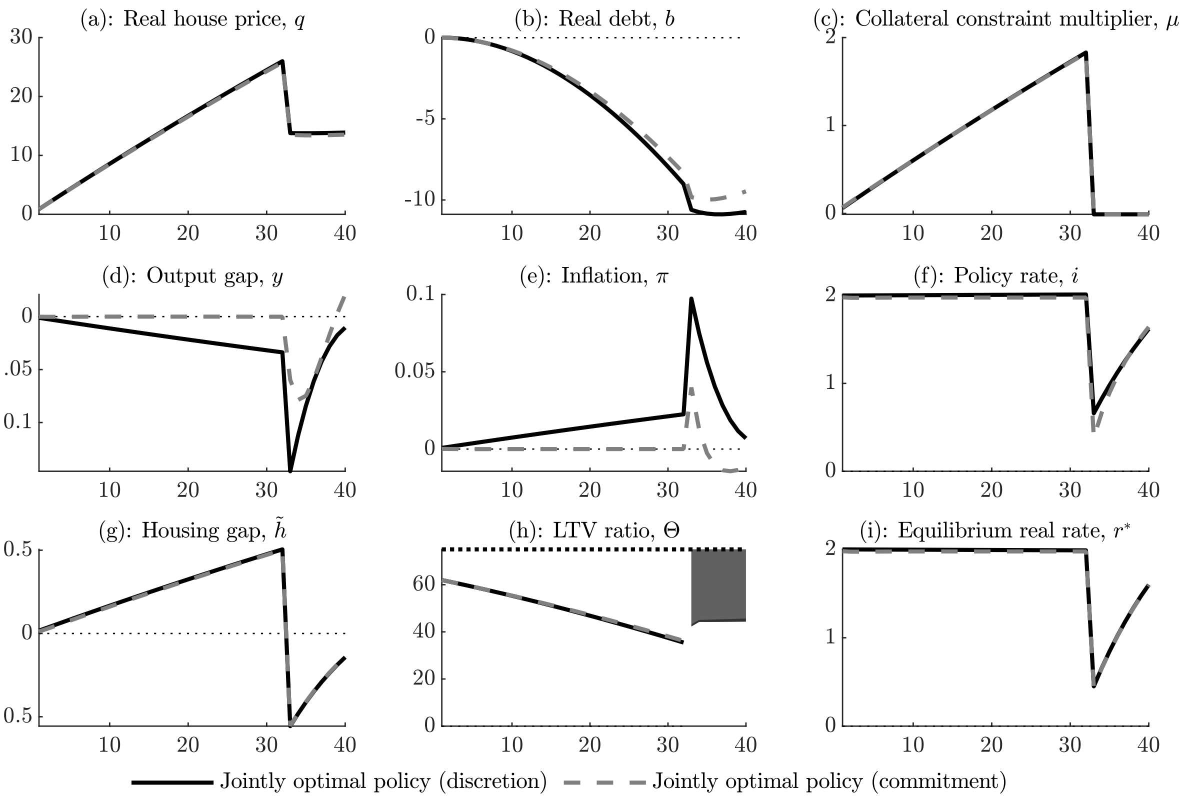
Figure 11: Jointly optimal policy with expected housing value in collateral constraint.



NOTE: All variables are scaled by 100 and plotted as log-deviations from steady state, except for the multiplier on the collateral constraint (μ) which is in levels. Inflation, the policy rate and equilibrium real interest rate are shown in annualized units.

B.2 Jointly Optimal Commitment Policy

Figure 12: Jointly optimal policy under commitment.



Note: All variables are scaled by 100 and plotted as log-deviations from steady state, except for the multiplier on the collateral constraint (μ) which is in levels. Inflation, the policy rate and equilibrium real interest rate are shown in annualized units.

Figure 12 compares the results under jointly optimal policy with commitment (dashed grey lines) with the case of discretion (solid black lines) discussed extensively in the text. We compute the commitment solution using the algorithm presented in section 3 of Harrison and Waldron (2021).

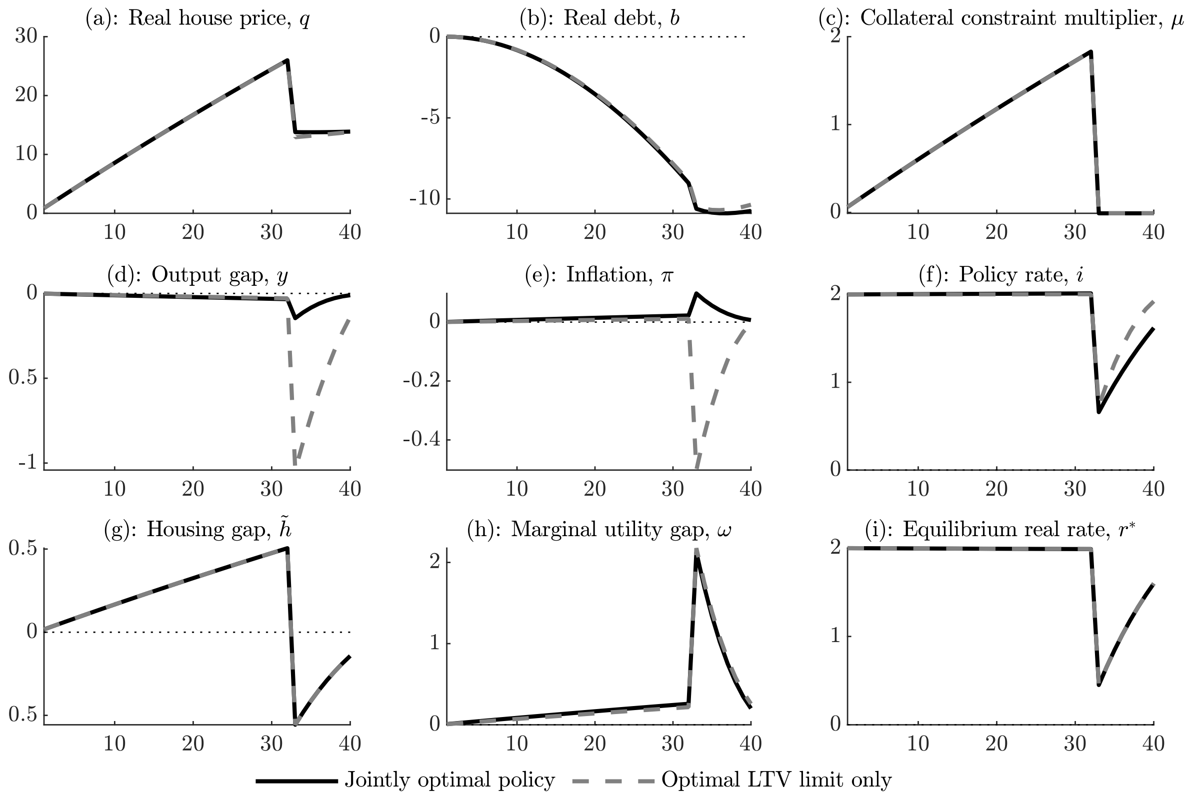
The figure shows minimal quantitative differences for most variables. Under commitment, the policymaker limits the volatility of output and inflation even more than under discretion, both during the boom and the bust, while debt is slightly higher. Commitment involves a marginally more accommodating monetary policy stance during the recession while the path of the LTV limit essentially coincides with the case of discretion.

B.3 Optimal LTV Limits vs. Jointly Optimal Policy

To explore the relative contribution of monetary and macro-prudential policy to the improved stabilization outcomes under the jointly optimal plan relative to the baseline scenario, figure 13 compares the jointly optimal policy plan (solid black lines) with the case in which the policymaker only sets the LTV limit optimally (dashed gray lines). In this configuration, the policymaker chooses the LTV limit to minimize the welfare-based loss (6) under discretion, taking as given the nominal interest rate, which follows the baseline Taylor rule (18).

As mentioned in the text, the figure demonstrates that the paths of most variables are virtually identical, with only perceptible differences in the responses of inflation and the output gap during the 'bust' phase of the scenario. Under the jointly optimal policy, the policymaker favors the stabilization of

Figure 13: Optimal LTV limits.

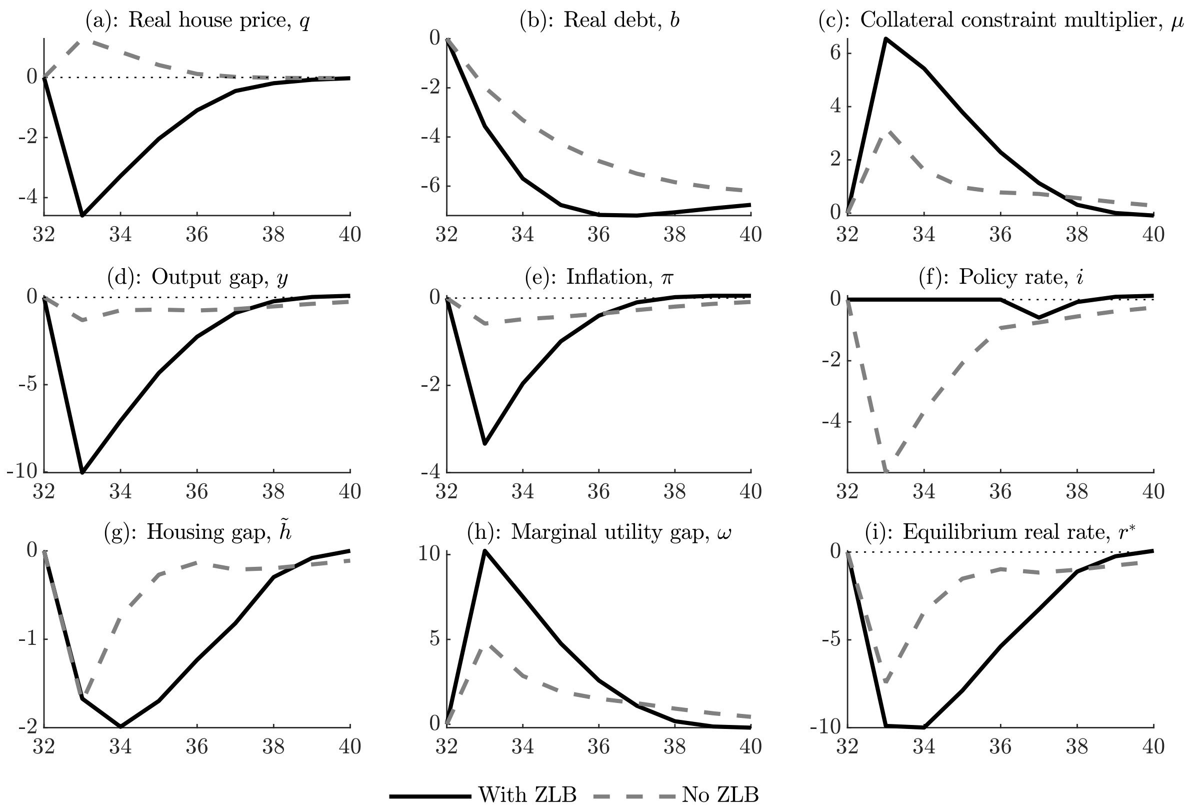


Note: All variables are scaled by 100 and plotted as log-deviations from steady state, except for the multiplier on the collateral constraint (μ) which is in levels. Inflation, the policy rate and equilibrium real interest rate are shown in annualized units.

inflation relative to the output gap. Since in our calibration (and, more generally, in models with price stickiness à la Calvo, 1983) λ_π is much greater than the relative weight on inflation in the Taylor rule (ϕ_π/π_y), the inflation-output tradeoff under the jointly optimal policy generates smaller welfare losses than those obtained under the baseline monetary policy rule. The paths for the LTV limits under the two policy configurations are nearly identical.

B.4 LTV Tightening in the Bust

Figure 14: Macro-prudential tightening in the bust.



NOTE: All variables are scaled by 100 and plotted as log-deviations from the baseline simulation, apart from the policy rate, inflation and the multiplier μ which are plotted as an absolute deviation and the LTV limit which is plotted as a level (in per cent). Inflation and the policy rate is shown in annualized units.

Figure 14 shows the responses of all variables, measured relative to the baseline simulation, for the case of a macro-prudential tightening at the time of the spike in credit spreads considered in section 5.5.1. The solid black lines account for the ZLB on the nominal interest rate while the dashed grey lines do not. Unsurprisingly, the presence of the the ZLB generates a much deeper recession.

C Computational Details

This appendix describes the implementation of optimal monetary and macro-prudential policy in the linear-quadratic approximation of the model, accounting for occasionally binding constraints. We apply the methods developed by Harrison and Waldron (2021), who present a toolkit for analyzing optimal policy problems in this type of setting.

The toolkit permits the analysis of optimal time-consistent policies ('discretion') and optimal commitment policies. The equilibrium concept under discretion is a Markov-perfect policy equilibrium. In each period, the policymaker acts as a Stackelberg leader with respect to private agents and future policymakers. The current policymaker takes the decision rules of future policymakers as given. In equilibrium, the decisions of policymakers in the current period satisfy the decision rule followed by future policymakers.

The piecewise linear solution method applies the 'perfect foresight' assumption. Agents account for the effects of occasionally binding constraints on the most likely future trajectory of the economy, but not for the risk that those constraints may bind on other possible future trajectories. As such, the piece-wise linear solution concept is identical to that studied by Guerrieri and Iacoviello (2015) in their 'OccBin' toolkit.[47]

C.1 Baseline Policy Configuration

The baseline policy configuration studied in section 5.3 is a simple Taylor rule, which has fixed coefficients. Therefore, the method in Holden and Paetz (2012) accurately computes the effects of anticipated disturbances to the model equations.

In order to impose that the zero lower bound on the monetary policy rate may be occasionally binding, a 'proxy shock' augments the policy rule to ensure that the contemporary slackness condition holds

$$(i_t - \psi_\pi \pi_t - \psi_y y_t)(i_t - \ln\beta) = 0.$$

Specifically, the Taylor rule is written as:

$$i_t = \psi_\pi \pi_t + \psi_y y_t + \delta_t^i$$

where δ^i is the 'proxy shock' used to ensure that the contemporary slackness condition is satisfied. Thus, if $\psi_\pi \pi_t + \psi_y y_t > \ln\beta$, the 'shock' takes the value of zero. Otherwise, δ_t^i is set equal to the value necessary to ensure that $i_t = \ln\beta$.

Similarly, to impose that the collateral constraint is occasionally binding, we add a 'proxy shock' $\delta_t^d = \mu_t$ that we use to enforce the contemporary slackness condition

$$\mu_t \left\{ b_t - \ln\mathcal{M} - \frac{\delta\gamma_d}{\mathcal{M}}(b_{t-1} - \pi_t) - \frac{(1-\gamma_d)\Theta\eta_q}{\mathcal{M}}[\theta_t + q_t + (1-\xi)\tilde{h}_t] \right\} = 0.$$

When the borrowing constraint is slack, $\delta_t^d = 0$, which implies that $\mu_t = 0$. When the borrowing constraint binds, an appropriate choice of δ_t^d ensures that

$$b_t - \ln\mathcal{M} - \frac{\delta\gamma_d}{\mathcal{M}}(b_{t-1} - \pi_t) - \frac{(1-\gamma_d)\Theta\eta_q}{\mathcal{M}}[\theta_t + q_t + (1-\xi)\tilde{h}_t] = 0,$$

and the borrowing constraint determines the level of debt.

As in Harrison and Waldron (2021), we use the algorithm in Holden and Paetz (2012) to find the

[47] The key innovation in Harrison and Waldron (2021) is to allow for flexible specifications of a variety of optimal policy behaviors.

required values of the ‘proxy shocks’. The approach requires writing the structural equations of the model (the equilibrium conditions describing the private sector behavior plus first order conditions of the optimal policy problem) as

$$H_F \mathbb{E}_t x_{t+1} + H_C x_t + H_B x_{t-1} = \Psi_\epsilon \epsilon_t + \Psi_\delta \delta_t. \tag{31}$$

The matrices H_F, H_C, and H_B collect the coefficients on the endogenous variables. The matrix Ψ_ϵ collects the coefficient on the exogenous shocks, which are in the vector ϵ_t. The vector δ_t contains the ‘proxy shocks’ to impose the occasionally binding constraints. These shocks enter the model equations with coefficient matrix Ψ_δ.

To solve for the values of δ_t that impose the occasionally binding constraints, we first obtain the rational expectation solution of (31) using the algorithm of Anderson and Moore (1985)

$$x_t = B x_{t-1} + \Phi_\epsilon \epsilon_t + \sum_{i=0}^{\infty} F^i \Phi_\delta \mathbb{E}_t \delta_{t+i}. \tag{32}$$

Secondly, we apply the method in Holden and Paetz (2012) to (32) in order to construct a mapping from the proxy shocks δ_{t+i} to the set of constrained variables (a subset of x_t). The algorithm uses a quadratic programming approach to find the sequence of shocks $\{\delta_t\}_{t=1}^T$ that impose the contemporary slackness conditions for periods $t = 1, \ldots, T$, where T is the last period when the constraint binds.

While the quadratic programming approach is typically efficient for solving such problems, in some cases a variant of the ‘inversion’ algorithm in the MAPS toolkit described in Burgess et al. (2013) improves the computational speed. In our case, the Holden-Paetz method turned out to be fast and reliable for the cases in which the collateral constraint alone was binding. For simulations in which both the collateral constraint and zero bound were binding, we used the MAPS inversion approach and verified that the shocks derived from that approach minimized the objective function of the Holden-Paetz quadratic programming problem.

C.2 Jointly Optimal Policy and Optimal LTV Limits

The main text shows that, when the collateral constraint is always binding, the policymakers uses LTV policy alongside monetary policy to minimize the welfare-based loss function (6). In this case, the optimal targeting rules are

$$\begin{aligned} \varepsilon \pi_t + y_t &= 0 \\ \lambda_\omega \omega_t - \frac{\sigma_\omega}{\bar{\sigma} + \varphi} y_t - \frac{[1 - (1 - \gamma_d)\Theta]\lambda_h}{(1 - \beta)\tilde{\sigma}_h} \tilde{h}_t &= 0, \end{aligned}$$

Though both of these targeting rules are static, the second one, which characterizes optimal macroprudential policy, includes an endogenous state variable (the housing gap). When we introduce occasionally binding constraints, we employ the method in Harrison and Waldron (2021), which solves the optimal policy problem using a finite-horizon value function iteration method. The policy instruments are the nominal interest rate, $i_t \geq \ln \beta$, and the multiplier on the collateral constraint, $\mu_t \geq 0$. This formulation exploits the one-to-one mapping between μ_t and the value of θ_t discussed in the text, provided the LTV constraint consistent with the equilibrium respects the upper bound $\Theta_t \leq \Theta$ ($\theta_t \leq 0$ in log-deviations). The next section offers further details on this additional requirement.

To implement the dynamic programming solution, Harrison and Waldron (2021) write the model as

$$\widetilde{H}^F_{\tilde{x}} \mathbb{E}_t \widetilde{x}_{t+1} + \widetilde{H}^C_{\tilde{x}} \widetilde{x}_t + \widetilde{H}^B_{\tilde{x}} \widetilde{x}_{t-1} + \widetilde{H}^F_r \mathbb{E}_t r_{t+1} + \widetilde{H}^C_r r_t = \widetilde{\Psi}_{\tilde{z}} \widetilde{z}_t,$$

which corresponds to equation (31) in partitioned form. Specifically, the vector of endogenous variables x_t is partitioned into a vector of non-policy variables $\widetilde{x}_t$ and a vector of policy instruments r_t. The shock vector $\widetilde{z}_t$ stacks the vectors of shocks ε_t and the vector of proxy shocks δ_t. The coefficient matrices exclude the equations for the policy instruments by deleting the relevant rows of the coefficient matrices in (31).

The policy instruments are set to minimize a loss function of the form

$$\mathcal{L}_t \equiv \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left\{ (\widetilde{x}_{t+i})' W (\widetilde{x}_{t+i}) + (r_{t+i})' Q (r_{t+i}) \right\},$$

subject to inequality constraints on the policy instruments expressed as

$$Sr_t \geq b, \tag{33}$$

where S is a conformable matrix and b is a vector that collects the bounds on the policy instruments.

Harrison and Waldron (2021) show that the optimal time consistent policy can be written as

$$\widetilde{x}_t = B_{\tilde{x}\tilde{x},t}\widetilde{x}_{t-1} + \sum_{s=0}^{T-t} \Xi_{s,\tilde{x}\tilde{z},t}\widetilde{z}_{t+s} + \gamma_{\tilde{x},t} \tag{34}$$

$$r_t = B_{r\tilde{x},t}\widetilde{x}_{t-1} + \sum_{s=0}^{T-t} \Xi_{s,r\tilde{z},t}\widetilde{z}_{t+s} + \gamma_{r,t} \tag{35}$$

$$\vartheta_t = B_{\vartheta\tilde{x},t}\widetilde{x}_{t-1} + \sum_{s=0}^{T-t} \Xi_{s,\vartheta\tilde{z},t}\widetilde{z}_{t+s} + \gamma_{\vartheta,t}, \tag{36}$$

where ϑ_t are the Lagrange multipliers on (33). Their paper also provides recursive formulas to compute the terms in (34), (35) and (36). These formulas depend on an assumption about the periods in which the constraints on the instruments in (33) are binding.[48] In particular, section 5.1 in Harrison and Waldron (2021) shows how the time-variation in the matrices that relate the variables to the state variables ($B_{\tilde{x}\tilde{x},t}$, $B_{r\tilde{x},t}$ and $B_{\vartheta\tilde{x},t}$) is determined by whether the constraints on the policy instruments are binding in period t.

C.3 Imposing the LTV Limit Feasibility Constraint

In some cases, the implicit value for the LTV limit required to implement the optimal policy for μ_t may be infeasible. To respect the underlying incentive compatibility constraint embodied in the collateral requirement, the level of the LTV limit Θ_t can be no greater than the steady-state level Θ. In log-deviation terms, the constraint is $\theta_t \leq 0$.

To appreciate the interaction of the various constraints, first recall that the value of the borrowing limit in deviations from steady state can be written as

$$\hat{\mathcal{D}}_t = \frac{\delta\gamma_d}{\mathcal{M}}(b_{t-1} - \pi_t) + (1-\gamma_d)\frac{\Theta\eta_q}{\mathcal{M}}\left[\theta_t + q_t + (1-\vartheta)\tilde{h}_t\right].$$

The distance between actual debt and the borrowing limit is then

$$bgap_t = b_t - \hat{\mathcal{D}}_t.$$

[48]The equilibrium is the result of a guess-and-verify procedure. Given a guess for the number of periods in which the instrument constraints bind, the algorithm computes the equilibrium and checks the contemporary slackness conditions. If these conditions are satisfied, the algorithm stops and the initial guess is a valid solution. Otherwise, the algorithm updates the guess and iterates the procedure until convergence.

Since the borrowing limit is occasionally binding, we require that

$$bgap_t \leq \ln \mathcal{M},$$

and that $bgap_t = \ln \mathcal{M}$ when the borrowing limit is binding.

We can define the level of the borrowing limit that would apply when the LTV limit is at its maximal level (and hence $\theta_t = 0$) as

$$\hat{\mathcal{D}}_t^{max} \equiv \frac{\delta \gamma_d}{\mathcal{M}} (b_{t-1} - \pi_t) + (1 - \gamma_d) \frac{\Theta \eta_q}{\mathcal{M}} \left[q_t + (1 - \vartheta) \tilde{h}_t \right].$$

The corresponding gap between debt and this value of the borrowing limit is

$$bgap_t^{max} = b_t - \hat{\mathcal{D}}_t^{max}.$$

The variable $bgap_t^{max}$ is the 'maximal debt gap'. The feasibility constraint $\theta_t \leq 0$ implies that $\mathcal{D}_t^{max} \geq \hat{\mathcal{D}}_t$ and hence that $bgap_t^{max} \leq bgap_t$.

Notice also that, when the constraint is binding (at a feasible value of θ_t), we have

$$\ln \mathcal{M} = b_t - \hat{\mathcal{D}}_t = b_t - \mathcal{D}_t^{max} - (1 - \gamma_d) \frac{\Theta \eta_q}{\mathcal{M}} \theta_t,$$

which implies

$$\theta_t = \frac{\mathcal{M}}{(1 - \gamma_d) \Theta \eta_q} (b_t - \mathcal{D}_t^{max} - \ln \mathcal{M}) = \frac{\mathcal{M}}{(1 - \gamma_d) \Theta \eta_q} (bgap_t^{max} - \ln \mathcal{M}).$$

The last equation means that the constraint $\theta_t \leq 0$ requires

$$bgap_t^{max} \leq \ln \mathcal{M}.$$

Several combinations of constraints may occur at any date t:

1. The collateral constraint binds with a valid LTV (i.e., lower than its steady state value)
   (a) $\mu_t > 0$
   (b) $bgap_t = \ln \mathcal{M}$
   (c) $\theta_t < 0$
2. The collateral constraint is slack
   (a) $\mu_t = 0$
   (b) $bgap_t < \ln \mathcal{M}$
   (c) A range of $\theta_t \leq 0$ are compatible with equilibrium
3. The collateral constraint binds with the maximum LTV
   (a) $\mu_t > 0$
   (b) $bgap_t = bgap_t^{max} = \ln \mathcal{M}$
   (c) $\theta_t = 0$.

To allow for case 3 and impose $\theta_t \leq 0$ as an occasionally binding constraint, we first define an 'augmented multiplier' as

$$\widehat{\mu}_t \equiv \mu_t - \delta_t^{\mu}$$

where δ_t^μ is a 'proxy shock'. This formulation allows us to treat the augmented multiplier $\widehat{\mu}_t$ as the policy instrument. Imposing a zero lower bound on the multiplier $\widehat{\mu}_t \geq 0$ corresponds to imposing that $\mu_t \geq \delta_t^\mu$. In this way, we can impose a time-varying and positive lower bound on the feasible μ_t, reflecting the upper bound on θ_t.

To implement the upper bound on θ_t requires use to choose the correct sequence of $\{\delta_t^\mu\}_{t=1}^T$ to ensure that $\theta_t \leq 0$. To do so we use the representation of the equilibrium given by equations (34), (35) and (36). Our method for imposing the upper bound on θ_t involves finding the sequence of proxy shocks that ensures that $\theta_t = 0$ in the relevant periods when the upper bound is binding. Conditional on these values, the equilibrium for non-policy variables $\widetilde{x}_t$ and instruments r_t can be computed using (34) and (35).

Based on the observations of the properties of the constraints when $\theta_t = 0$ (case 3 above), we compute the values of the proxy shocks δ_t^μ that ensure that $bgap_t^{max} = \ln \mathcal{M}$ in the relevant periods. The starting point is a simulation in which proxy shocks are set to zero in all periods ($\delta_t^\mu = 0, \ \forall t$) and $\widehat{\mu}_t \geq 0$ is the policy instrument in the optimal policy problem, imposing in addition the ZLB on the policy rate. The outcome of this first step is what we call the 'initial simulation'. We denote the solution for the non-policy variables associate with the initial simulation as $\{\widetilde{x}_t^0\}_{t=1}^T$.

We can then check the initial simulation to determine whether the equilibrium is always consistent with cases 1 and 2 detailed above. In particular, if we observe periods in which the collateral constraint is slack, but $bgap_t^{max,0} > \ln \mathcal{M}$, then the initial simulation violates the upper bound on θ_t, since the LTV limit cannot be increased enough to ensure that the collateral constraint becomes slack. To enforce this constraint, we use the proxy shocks to impose a time-varying, positive lower bound on μ_t. To find the required values of the shocks, we compute their effects on $bgap_t^{max}$, since enforcing $bgap_t^{max} \leq \mathcal{M}$ is equivalent to ensuring that the optimal policy is constrained by the upper bound on θ_t.

From (34), the effects of the shocks $\{\delta_t^\mu\}_{t=1}^T$ on the maximal debt gap in period 1 of the simulation are given by

$$\Delta bgap_1^{max} = C \sum_{s=0}^{T-1} \Xi_{s,\tilde{x}\delta,1} \delta_{s+1}^\mu, \tag{37}$$

where $\Xi_{s,\tilde{x}\delta,1}$ is the column of $\Xi_{s,\tilde{x}\tilde{z},1}$ that corresponds to the proxy shock δ_t^μ and C is a matrix that selects $bgap_t^{max}$ from the vector of non-policy variables.

We can then develop a recursive scheme for building a matrix that maps the effects of δ_t, for $t = 1, \ldots, T$, on to the maximal debt gap in each period. The first (block) row of the matrix follows from expanding (37)

$$\Delta bgap_1^{max} = C \underbrace{\begin{bmatrix} \Xi_{0,\tilde{x}\delta,1} & \ldots & \Xi_{k-1,\tilde{x}\delta,1} & \ldots & \Xi_{T-1,\tilde{x}\delta,1} \end{bmatrix}}_{\equiv \omega_1} \underbrace{\begin{bmatrix} \delta_1^\mu \\ \vdots \\ \delta_k^\mu \\ \vdots \\ \delta_T^\mu \end{bmatrix}}_{\equiv \boldsymbol{\delta}},$$

so that $\omega_t \boldsymbol{\delta}$ denotes the effects of current and future proxy shocks on the maximal debt gap at horizon

t. The weights ω_t on current and future policy shocks at horizon t are then

$$\begin{aligned}
\omega_2 &= \begin{bmatrix} 0 & \Xi_{0,\tilde{x}\delta,2} & \ldots & \Xi_{k-2,\tilde{x}\delta,2} & \ldots & \Xi_{T-2,\tilde{x}\delta,2} \end{bmatrix} \\
&\ldots \\
\omega_k &= \begin{bmatrix} 0 & 0 & \ldots & \Xi_{0,\tilde{x}\delta,k} & \ldots & \Xi_{T-k,\tilde{x}\delta,k} \end{bmatrix} \\
&\ldots \\
\omega_T &= \begin{bmatrix} 0 & 0 & \ldots & 0 & \ldots & \Xi_{0,\tilde{x}\delta,H} \end{bmatrix}.
\end{aligned}$$

Equation (34) then implies that the effects of past and future proxy shocks on all non-policy variables at horizon t are given by

$$\mathcal{X}_t = B_{\tilde{x}\tilde{x},h}\mathcal{X}_{t-1} + \omega_t,$$

for $t = 1, \ldots, T$ and with $\mathcal{X}_0 = 0$. More compactly, the effect on the maximal debt gap in period t is given by $C\mathcal{X}_t$, which we can write as a matrix mapping the proxy shocks δ_t shocks to the maximal debt gap:

$$\Delta = \mathcal{W}\boldsymbol{\delta}$$

where

$$\Delta = \begin{bmatrix} \Delta bgap_1^{max} \\ \vdots \\ \Delta bgap_k^{max} \\ \vdots \\ \widehat{\Delta} bgap_T^{max} \end{bmatrix}, \quad \mathcal{W} = \begin{bmatrix} C\mathcal{X}_1 \\ \vdots \\ C\mathcal{X}_k \\ \vdots \\ C\mathcal{X}_T \end{bmatrix}$$

Let the initial simulation for the level of the maximal debt gap be defined as

$$\mathcal{B}^0 = \begin{bmatrix} bgap_1^{max,0} \\ \vdots \\ bgap_k^{max,0} \\ \vdots \\ bgap_T^{max,0} \end{bmatrix}.$$

So far we have recorded the initial simulation for the level of the maximal debt gap ($\mathcal{B}^0$), and the marginal effects of the proxy shocks ($\boldsymbol{\delta}$) on the maximal debt gap (Δ). These computations assume that the shocks are applied over the entire simulation horizon ($t = 1, \ldots, T$) and the marginal effects of those shocks are computed for all periods in the simulation horizon. However, the upper bound on θ_t may only bind in some periods within the full simulation horizon. Let $\mathcal{P}$ be a $j \times T$ matrix that selects the j periods in which the upper bound on θ_t is violated in the initial simulation (and therefore needs to be enforced).[49] The effects of these j shocks only on the maximal debt gap are given by $\mathcal{W}\mathcal{P}'\mathcal{P}\boldsymbol{\delta}$, since the post-multiplication of $\mathcal{W}$ by $\mathcal{P}'$ selects the relevant columns on $\mathcal{W}$ and pre-multiplication of $\boldsymbol{\delta}$ by $\mathcal{P}$ selects the relevant rows of $\boldsymbol{\delta}$. Thus, $\widetilde{\boldsymbol{\delta}} \equiv \mathcal{P}\boldsymbol{\delta}$ represents the $j \times 1$ vector of proxy shocks required to enforce the upper bound on θ_t.

To compute the set of proxy shocks that will enforce the upper bound on θ_t requires finding $\widetilde{\boldsymbol{\delta}}$ such that

$$\mathcal{P}\mathcal{B}^0 + \mathcal{P}\mathcal{W}\mathcal{P}'\widetilde{\boldsymbol{\delta}} = \mathbf{1}\ln\mathcal{M}$$

[49] Each row of $\mathcal{P}$ contains a single unit entry in the relevant column corresponding to a period in which the upper bound on θ_t is violated. All other elements are zero.

where $\mathbf{1}$ is a $j \times 1$ unit vector. The solution to this equation is

$$\widetilde{\boldsymbol{\delta}} = \left(\mathcal{P}\mathcal{W}\mathcal{P}'\right)^{-1}\left(\mathbf{1}\ln\mathcal{M} - \mathcal{P}\mathcal{B}^0\right) \tag{38}$$

The solution method therefore consists of the following steps:

1. Use the algorithm in Harrison and Waldron (2021) to construct an initial simulation under jointly optimal policy assuming:
   (a) $\widehat{\mu}_t$ is the macro-prudential policy instrument, subject to a lower bound of $\widehat{\mu}_t \geq 0$;
   (b) i_t is the monetary policy instrument, subject to a lower bound of $i_t \geq \ln\beta$;
   (c) the proxy shocks are zero in all periods ($\delta_t^{\mu} = 0$ for $t = 1, \ldots, T$).
2. Record the maximal debt gap in the simulation $\mathcal{B}^0$ and compute the matrices necessary to find the values of the proxy shocks.
   (a) Form the matrices $\mathcal{W}$ and $\boldsymbol{\delta}$ using the solution outputs from the algorithm in Harrison and Waldron (2021) as described above.
   (b) Construct the matrix $\mathcal{P}$ to select the periods in which the proxy shocks should be applied to enforce the upper bound on θ_t.
3. Solve for the proxy shocks using (38).
4. Apply the proxy shocks to the initial solution and re-compute the equilibrium using the algorithm in Harrison and Waldron (2021).

The method relies on the fact, already noted above, that the solution characterized by equations (34), (35) and (36) depends on an assumption about the periods in which the instrument bounds (33) are binding (in our case, $\widehat{\mu}_t \geq 0$). The solution method described here will be valid if the addition of the proxy shocks does not change the periods in which the constraint $\widehat{\mu}_t = 0$ binds in equilibrium, which we check is indeed the case in our applications.[50]

C.4 Macro-Prudential Policy in the Bust

The experiments in section 5.5.2 assume that the policy behavior during the 'bust' (when credit spreads spike) differs from policy behavior during the housing boom, when credit spreads are gradually falling. We implement these experiments under the assumption that the change in policy behavior is completely unanticipated by agents in the economy.

Specifically, we record the outcomes from our baseline scenario (section 5.3) for periods $t = 1, \ldots, T_B$, where $T_B = 32$ denotes the final period of the housing boom. We record the outcomes for the endogenous variables in this period in the vector x_{T_B}, which we then use as the initial state vector for the simulations conducted under alternative policy assumptions (namely optimal LTV limits and jointly optimal monetary and macro-prudential policies).

[50] If that were not the case, an additional step could be added to the method to iterate between steps 3 and 4, that is, solving for the proxy shock values and the equilibrium 'constraint indicator function' (see Harrison and Waldron (2021) for further details).