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Why you should not use the LSV herding measure

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Why you should not use the LSV herding measure

Simon Jurkatis[(1)]

Abstract

Here are three reasons. (a) This paper proves that the popular investor-level herding measure is a biased estimator of herding. Monte Carlo simulations demonstrate that the measure underestimates herding by 20% to 100% of the estimation target. (b) The bias varies with the number of traders active in an asset such that regression type analyses using LSV to understand the causes and consequences of herding are likely to yield inconsistent estimates if controls are not carefully chosen. (c) The measure should be understood purely as a test on binomial overdispersion. However, alternative tests have superior size and power properties.

Key words: Herding, estimation, market microstructure, overdispersion.

JEL classification: C13, C58, G14, G40.

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1 Introduction

Given the potentially destabilizing forces of masses acting in accord on financial markets, the subject of herd behavior continues to attract attention. The theoretical literature on the subject of herding has converged to a fairly common framework, particularly within the field of market microstructure (Banerjee, 1992; Avery and Zemsky, 1998; Park and Sabourian, 2011), that defines herding as traders taking the same action as the majority before them, even though they have private information that advised them to act differently (Brunnermeier, 2001). This definition makes explicit the potential inefficiencies that can arise from herding, ranging from excess volatility to bubbles and crashes. The empirical literature, on the other hand, is more divided on the question of how to define, measure and test for herding. Approaches range from estimating structural parameters of a herd-model (Cipriani and Guarino, 2014) to more descriptive methods like measuring clustering of prices or trades (Christie and Huang, 1995; Patterson and Sharma, 2010).

A particularly popular measure when analysing herding of subgroups of investors is the one proposed by Lakonishok et al. (1992) (LSV) (see, for example, Deng et al., 2018; Cai et al., 2019; Zheng et al., 2021, for just a few recent studies). It measures the dispersion of the fraction of buyers around the average fraction across the sample of assets. In essence, it captures whether the subgroup of investors buys and sells the same assets. The measure is intuitively appealing and easily implemented.

Yet, a formal definition of what the measure is supposed to estimate has not been given. If one was able to say that 5 out 10 traders engaged in herd behavior, what number would one want to get out of LSV? Without a specific estimation target, a rigorous evaluation of the measure's performance is impossible. Moreover, it makes a comparison to other empirical approaches, most of which face the same short-coming, as well as linking it to the theoretical literature difficult. For example, another popular approach by Chiang and Zheng (2010) interprets herding as decreased dispersion of returns around the market return (during times of increased volatility). This interpretation seems to clash with LSV's understanding of herding as an increased dispersion of buy-ratios. Still, both approaches refer to the same literature on herding.

In this paper, I provide a formal estimation target based on typical interpretations of LSV: the fraction of traders that engaged in herding net of what would be expected if traders acted random and independently. Evaluating the measure against that target within a general, statistical framework that includes some specific models from the microstructure literature as special cases, I prove that LSV underestimates herding. Monte Carlo simulations show that the underestimation can be anywhere between 20% to 100% of the estimation target.

The more formal treatment of LSV's estimation performance also allows me to connect the measure to the theoretical framework on herd behavior more clearly. I show that the measure cannot be interpreted in the same way the theoretical microstructure literature interprets herding, where coordination induced by private information does not count. LSV, however, cannot differentiate between coordination induced by private or public information (such as observed behavior of others, price movements etc.). Empirical studies, therefore, try to uncover the causes of herding by regressing LSV on covariates that proxy for different sources of potential coordination (see Kremer and Nautz, 2013, and references therein). However, given that LSV's bias varies, for example, with the number of traders, and, in practice, may differ for different sources of coordination, it cannot be expected that such regressions yield consistent estimates.

Finally, I argue that LSV is best understood as a purely statistical test on binomial overdispersion. Treated as such, I show that LSV has inferior size and power properties than alternative tests less known in the field of finance.

Measuring herding is an inherently difficult task, particularly if one considers the stricter, microstructure definition of herding since the motivation to trade in a certain direction is not observable. Unfortunately, this paper does not offer a solution.

Rather, the contribution of this paper is to uncover and highlight significant short-comings of one of the most popular measures in the field. In doing so, I urge the practitioner and researcher to apply and interpret the measure with caution. Note that the bias that I demonstrate in this paper arises in a framework that is internally consistent with LSV's own reduced-form view of herding. Earlier critics of the measure have already pointed out that LSV cannot differentiate between coordination that arises from imitation vs acting on the same information, i.e. intentional vs spurious herding (Bikhchandani and Sharma, 2000). Such discussions, however, stress the potential over-estimation of herding when herding is defined in its stricter, microstructure sense. Instead, I demonstrate that LSV *under*-estimates herding even if herding is specified in the way the measure is typically interpreted.

Another paper that demonstrates a bias in the LSV measure is the one by Frey et al. (2014). Their demonstration, however, is purely numerical and for one specific statistical model of herding. Moreover, their statistical model assumes that within any given asset, traders still act independently from each other which is a narrower view on herding than the one taken in this paper. Instead, I prove the bias analytically under a very general framework that incorporates basic ideas from the theoretical herding literature and I present more extensive numerical exercises that show that the bias is non-negligible.

The remainder of this paper is organized as follows. Section 2 introduces the LSV measure. Section 3 defines the estimation target, states the bias of LSV analytically and demonstrates its magnitude via Monte Carlo simulations. In Section 4, I contrast LSV with the market microstructure literature and attempt a more conceptual interpretation. Section 5 evaluates LSV's size and power characteristics as a test of binomial over-dispersion. In Section 6, I discuss related measures of investor herding. Finally, Section 7 concludes.

2 The *LSV* herding measure

The measure of Lakonishok et al. (1992) is applied to transaction data of a subgroup of investors, such as quarterly equity holdings of mutual funds or daily stock market transactions of retail traders (see Brown et al., 2014; Dorn et al., 2008). The measure is based on the buy-ratio statistic defined as the number of buyers, B_i, over the total number of traders among the considered subgroup of investors, N_i, in stock i.[1] Specifically, the LSV measure is computed as

$$LSV_i = \left| \frac{B_i}{N_i} - p \right| - \mathbb{E}_0 \left| \frac{B}{N_i} - p \right|, \qquad (1)$$

where, under the null of no herding, it is assumed that $B \sim \text{Bino}(N_i, p)$. That is, p is the expected proportion of traders buying in case of no herding. Note that p is assumed to be constant over assets $i = 1, \ldots, I$ and is thus estimated as $\hat{p} = \sum_i B_i / \sum_i N_i$.

Let me consider a specific example to explain the basic idea of LSV. First, note that the measure looks at the aggregated behavior of investors over a fixed time-horizon, there is no inter-temporal component, even though the measure is usually applied to panel data. I, therefore, consider a single period of arbitrary length throughout the paper and do not carry a time subscript. Now, because the measure considers only a subgroup of investors, buy-ratios can deviate from 0.5. For the moment, neglect the expectation term in (1) and let me assume that the investor group buys as much as it is selling across the set of assets such that $\hat{p} = 0.5$. Therefore, in any single asset where the investor group buys as much as it sells, the measure is zero, indicating no herding. If, on the other hand, there is a majority of investors buying or selling an asset, the buy-ratio will significantly deviate from 0.5, indicating herding. That is, LSV captures the degree to which investors buy and sell the same assets at the same time, focusing on the "crowd" aspect of herding. Since buy-ratios would deviate from 0.5 purely by chance, the expectation term centres the measure over zero under the null model.

The LSV measure is typically interpreted at its average level $LSV = \sum_i LSV_i / I$. Assuming an average measure of 0.03 a typical interpretation reads:

[1] The number of buyers is usually based on the net-position of traders. Alternatively, one may count each single transaction as in Choe et al. (1999).

> “We can think of this average herding measure as meaning that if 100 funds trade a given stock-quarter, then approximately three more funds trade on the same side of the market than would be expected if each fund randomly and independently chose stocks.” (Wermers, 1999, p. 593)

3 Biased herding estimation

3.1 Analytical result

From the interpretation of the LSV measure, one can conclude that it attempts to estimate the fraction of traders that engaged in herd behavior. More specifically, let the number of buyers B be additively separable into the number of buyers B^0 that were generated under a no-herding regime and the number of buyers B^h generated under a state of herding. The estimation target can then be expressed as $\theta := |B^h - pN^h|/N$ where N^h is the number of transactions under the herding regime and p is the expected proportion of buys under no-herding. That is, the estimation target is the number of herding buys net of the buys that we would have expected in case of no-herding over the total number of trades.

I prove in the Appendix that the LSV measure is a biased estimator of this target. Specifically, let $B = B^0 + B^h$, where $B^0 \sim \text{Bino}(N - N^h, p)$ and $B^h \sim F$ with $0 < N^h < N$, $p \in (0, 1)$ and F being some arbitrary distribution different from $\text{Bino}(N^h, p)$ with existing mean absolute deviation, then

$$\mathbb{E}[LSV] \leq \mathbb{E}[\theta]. \tag{2}$$

A necessary condition for (2) to hold with equality is the trivial case of $N^h = 1$ and $pN \in \mathbb{Z}$.

3.2 Numerical result

To demonstrate the magnitude of the bias, I generate data according to the following setups.

Setup 1 $F = w\text{Bino}(N^h, p+\delta) + (1-w)\text{Bino}(N^h, p-\delta)$ with $\delta, w \in \{0.05, 0.2\} \times \{0.5, 0.7\}$.

Setup 2 $F = \text{Beta-Bino}(N^h, \alpha, \beta)$ with $(\alpha, \beta) \in \{(10, 10), (5, 15)\}$.

I fix the trader population at $N = 100$ and set $N^h = 1, \ldots, N-1$.[2] The no-herding trades are generated according to $Bino(N - N^h, p)$ with $p = 0.5$. For now I assume p to be known. The cross-section of assets is fixed at $I = 100$. For each setup I generate 10000 draws. For each draw the bias is computed as[3]

$$Bias := \frac{\sum_i LSV_i}{I} - \frac{\sum_i |B_i^h - pN^h|/N}{I}. \tag{3}$$

Note that both setups are very much in line with the theoretical literature on herding. Setup 1 closely resembles Park and Sabourian (2011) where a fraction of trader-types (δ) changes its behavior (from selling to buying in case of buy-herding, and vice versa for sell-herding) after having observed a sufficient imbalance of buys/sales. In any single asset, this leads trades to be generated under two different regimes of Bernoulli distributions. The term w is the fraction of assets in which buy-herding occurs, and $1 - w$ the fraction of assets experiencing sell-herding.

Setup 2 reflects Pólya's urn model, which can be thought of as a continuous version of the previous setup. After each draw from an urn containing balls of two colors, the same ball and an additional ball of the same color are returned to the urn. That is, with each buy/sell a marginal trader-type enters the market wanting to trade in the same direction as the previous one. The limiting distribution of the number of balls drawn of a certain color from such an urn is the beta-binomial distribution.

Figure 1 shows the average bias (left column) and the average relative bias (right column) across all draws. For each setup the bias is sizeable and fairly similar, converging to an under-estimation of around 4%, which is equivalent to 20% to 80% relative to the estimation target. The relative frequency of buy- as opposed to sell-herding (w) does not affect the bias, because p is assumed to be known and buy-herding is as strong as sell-herding.

4 Further considerations

4.1 Herding vs contrarian behavior

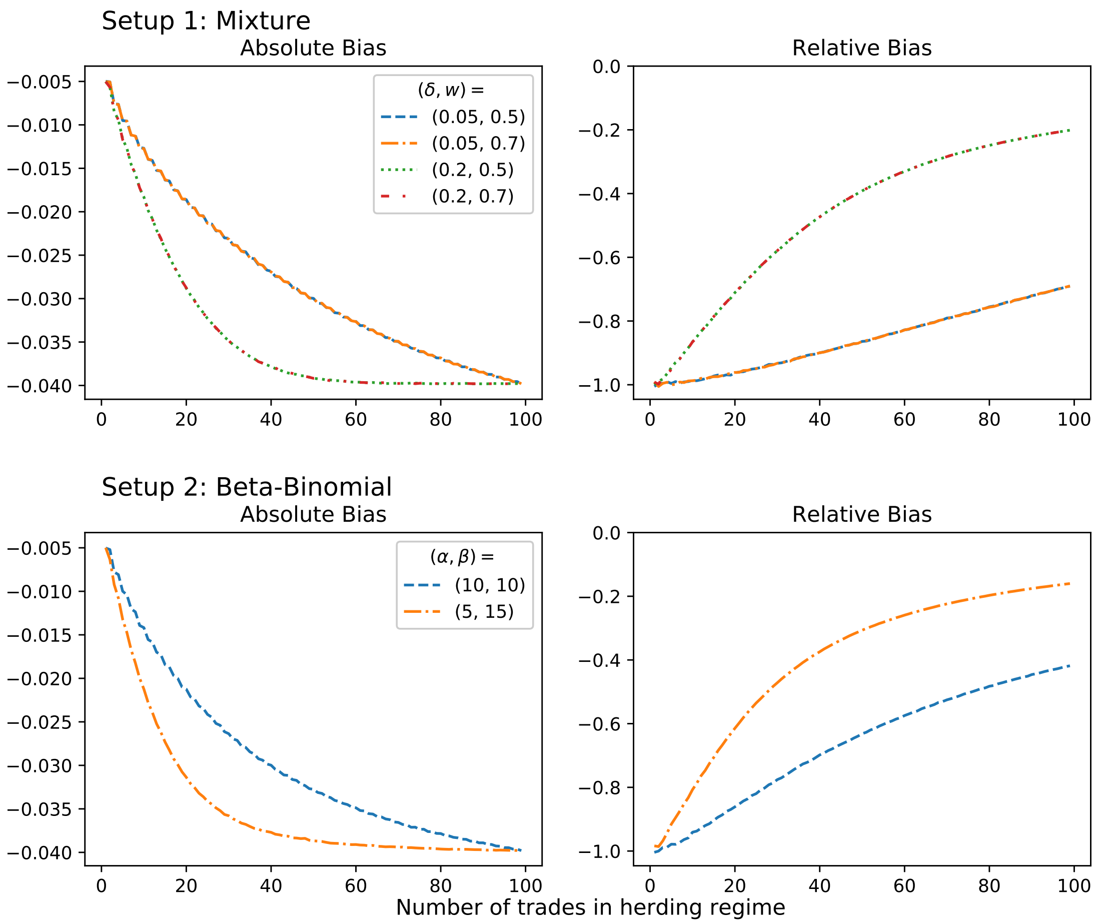
The market microstructure literature on herding distinguishes between two types of investor behavior: herding, where past trades increase the probability of observing another trade in the same direction, and contrarianism, where past trades increase the probability of observing a trade in the opposite direction.

The LSV measure distinguishes between herding and contrarianism implicitly. Statistically speaking, LSV is a measure of binomial overdispersion. So the assumption underlying LSV is that herding leads to more extreme buy-ratios than

[2]Results for a smaller trader population of $N = 20$ are very similar.

[3]For Setup 1 the estimation target can be computed analytically, as shown in the Appendix.

Figure 1: Bias evaluation - p known



Notes: This figures shows the average absolute and relative bias across 10000 simulations of two different setups of trading under herding. In Setup 1, buys during the herding regime are generated according to $B_i^h \sim w\text{Bino}(N^h, p+\delta)+(1-w)\text{Bino}(N^h, p-\delta)$ with $\delta, w \in \{0.05, 0.2\} \times \{0.5, 0.7\}$. In Setup 2, $B_i^h \sim \text{Beta-Bino}(N^h, \alpha, \beta)$ with $(\alpha, \beta) \in \{(10, 10), (5, 15)\}$. For each asset $i = 1, \ldots, I$ with $I = 100$, observed buys are given by $B_i = B_i^0 + B_i^h$ where $B_i^0 \sim \text{Bino}(N - N^h, p)$ with $N = 100$, $p = 0.5$ and $N^h = 1, \ldots, N-1$ (x-axis). For each simulation, the bias is given by $Bias := \sum_i (LSV_i - \theta_i)/I$. The relative bias is given by $Bias / \sum_i \theta_i / I$.

what would be expected if investors traded independently and with the same propensity to buy; and that the self-defeating nature of contrarianism leads to less extreme buy-ratios, clustering more strongly around the overall propensity to buy. Given that (2) is valid independent of the dispersion of buy-ratios generated under F, the LSV measure underestimates herding and over-estimates contrarianism, if we follow LSV's implicit distinction of these two types of behaviors.

4.2 Sources of investor coordination

Another important distinction in the microstructure literature on herding is the one between public and investors' private information. Herding arises when investors take the same action as the majority before them (past actions are part of the public information set by being either directly observable or implicitly contained in price movements) even if they had private information advising against that action. However, private information on its own is already a source for creating imbalances in buy-ratios.

Viewing the LSV measure through the lens of measuring binomial dispersion shows that LSV is incapable of distinguishing between the different sources of coordination. Take as an example Pólya's urn model of Setup 2 above. The beta-binomial distribution resulting from that urn experiment in the limit is over-dispersed compared to a binomial distribution. However, we can obtain the same distribution by letting traders act independently from each other and with the same propensity to buy, but drawing that propensity before the start of trading from a beta distribution. The beta distribution simply reflects that across assets information is distributed heterogeneously.

So while (2) established an under-estimation of herding, the previous example shows that in a more strict, microstructure sense of the term, the LSV measure can over-estimate herding as well. This is sometimes referred to as spurious vs intentional herding (Bikhchandani and Sharma, 2000).

Not being able to differentiate between different sources of coordination is particularly problematic if those sources have different implications for an asset. While one form of investor coordination might be beneficial for the price discovery process (informed trading), another form might be harmful to financial stability (fire-sales).

Therefore, in order to uncover the consequences of herding measured using LSV, a popular approach is to sort assets into portfolios depending on their level of LSV and check whether the return on the high-minus-low portfolio displays persistent (indication of informed trading) or transitory gains (Cai et al., 2019). To understand the consequences of herding, on the other hand, LSV is often regressed on covariates reflecting different sources of coordination as predicted by the theoretical literature (Kremer and Nautz, 2013). For example, small cap stocks are expected to be associated with higher levels of herding, because asymmetric information is assumed to be stronger in such stocks, which, in turn, gives rise to herding (Venezia et al., 2011).

These approaches to understand the causes and consequences of herding may result in spurious or inconclusive results given that the bias varies, for example, with the number of traders. Moreover, in practice we cannot expect that the bias is uncorrelated with different sources of coordination, leading to potentially inconsistent estimates of the actual effects of these sources on herding. Finally, to make predictions on the effects of variables on empirically measured herding requires that one understand how the measure relates to the model from which the predictions are derived. The LSV measure, however, does not offer such a link, nor do many of the other empirical measures.

4.3 Estimating p

The under-estimation of herding demonstrated in (2) was derived under the assumption that the buy-propensity of traders under no-herding is known. The over-estimation sketched in the previous subsection resulted because the asset specific buy-propensities were not known. If p has to be estimated, LSV may under- or over-estimate herding, depending on the distance of p to its estimator, $\hat{p}$, and the number of trades during the herding phase (see Appendix).

The dependence of the bias on these parameters is non-monotonic. This adds another complication in interpreting the LSV measure and for using it in comparative studies, such as regression analyses, where relative differences of the LSV measure across time or population may be more important than its level.

4.3.1 Constant p

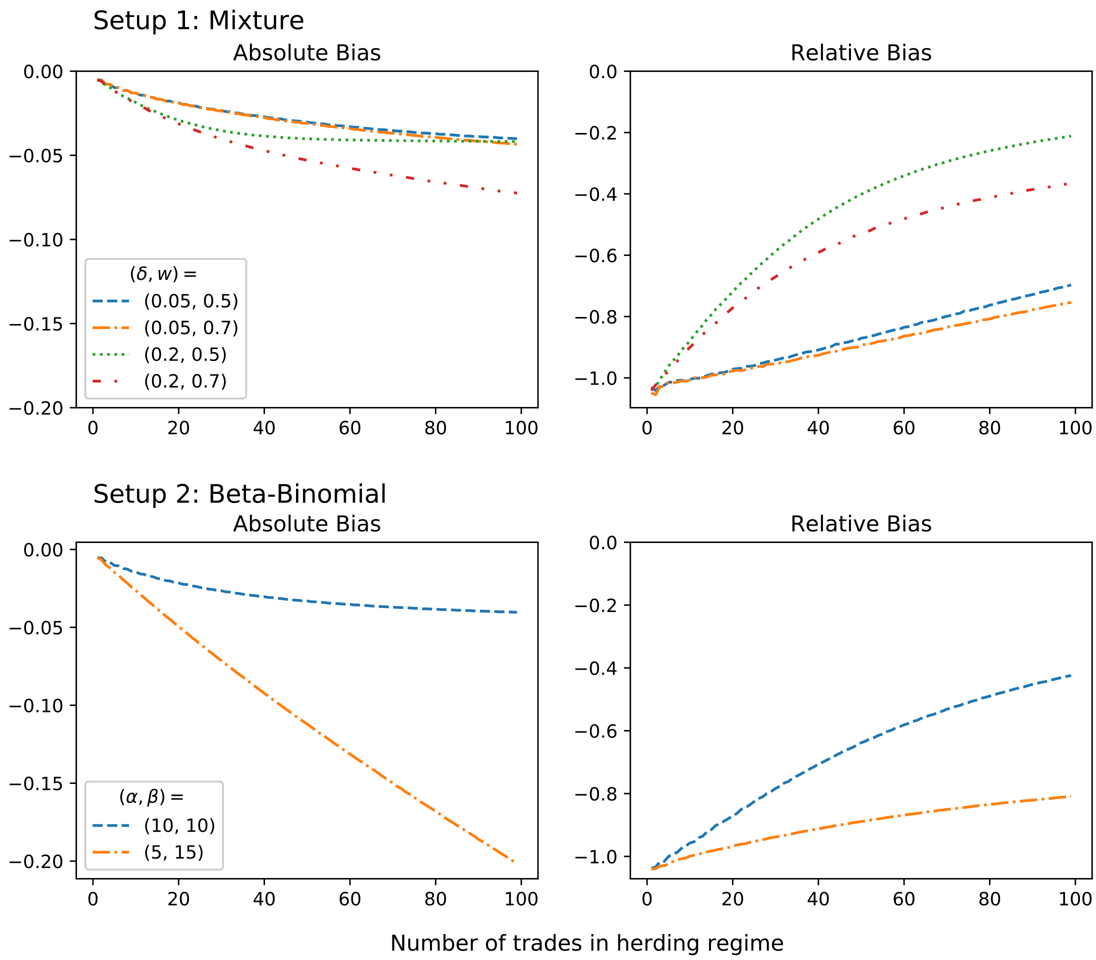
To demonstrate the effect of estimating p, I repeat Setup 1 and 2 but with p estimated via $\hat{p}$. The probability of a buy in the no-herding regime is still assumed to be the same across assets.

Figure 2 shows the results. In setups where buy and sell herding occur asymmetrically, the estimation of the true propensity to buy in the no-herding regime adds a significant proportion to the bias. Particularly for the beta-binomial setup, the asymmetric case yields a bias of 20% (or 80% when measured relative to the estimation target) when all but one trade are generated under the herding regime.

4.3.2 Idiosyncratic p

Finally, to demonstrate how over-estimation can arise from unknown, idiosyncratic buy-propensities, I consider a third setup:

Figure 2: Bias evaluation - p estimated



Notes: This figures shows the average absolute and relative bias across 10000 simulations of two different setups of trading under herding. In Setup 1, buys during the herding regime are generated according to $B_i^h \sim w\text{Bino}(N^h, p+\delta) + (1-w)\text{Bino}(N^h, p-\delta)$ with $\delta, w \in \{0.05, 0.2\} \times \{0.5, 0.7\}$. In Setup 2, $B_i^h \sim \text{Beta-Bino}(N^h, \alpha, \beta)$ with $(\alpha, \beta) \in \{(10, 10), (5, 15)\}$. For each asset $i = 1, \ldots, I$ with $I = 100$, observed buys are given by $B_i = B_i^0 + B_i^h$ where $B_i^0 \sim \text{Bino}(N - N^h, p)$ with $N = 100$, $p = 0.5$ and $N^h = 1, \ldots, N-1$ (x-axis). For each simulation, the bias is given by $Bias := \sum_i (LSV_i - \theta_i)/I$. The relative bias is given by $Bias / \sum_i \theta_i / I$. p is not known and is estimated by $\hat{p} = \sum_i B_i / IN$.

Setup 3 $B_i = B_i^0$ where $B_i^0 \sim \text{Bino}(N, p_i)$ and $p_i \sim Beta(\alpha, \beta)$ with $(\alpha, \beta) \in \{(10, 10), (5, 15)\}$.

The size of the cross-section of assets is fixed at $I = 100$. I let the number of traders active in the cross-section vary from $N = 10, \ldots, 100$. Table 1 shows the average, percentage LSV measure for this setup. Given that herding does not occur in this setup, LSV equals the bias.

For $N = 100$, not accounting for the idiosyncrasy in buy-propensities across assets leads to an over-estimation of herding of 5.61% for the symmetric case ($\alpha = \beta$) and 4.81% for the asymmetric one. The bias is smaller in the asymmetric case, because the variance of the beta distribution is smaller compared to the symmetric setup.

The monotonic increase in the LSV measure from $N = 10$ to $N = 100$ is striking and adds another complication to its interpretation. Let me take the standpoint of a proponent of the LSV measure and argue that the idiosyncrasy of buy-propensities is a valid case of investor coordination that I wish to measure (even if it simply reflects that traders act on similar information and that signals across different assets have different information strength, but otherwise traders still act independently from each other). In that case, the estimation target is equivalent to the mean absolute dispersion of buy-propensities, here given by the mean absolute deviation of the beta distribution, $\mathbb{E}|p - \mathbb{E}[p]|$.[4] However, the estimated dispersion varies with the number of investors active in the asset, even though the underlying distribution of buy-propensities did not change. Also note that convergence to the target value cannot be achieved with conventional sample sizes. This is an undesirable property of LSV that, so far, has not received much attention and must concern even the hardest advocate of the measure.[5]

5 Power and size of LSV

Given the bias and difficulty of economic interpretability of the LSV measure, one may simply treat it as a purely statistical measure of binomial over-dispersion. Therefore, I compare it to other tests of binomial excess variability better known in the natural science literature. Specifically, I will compare the LSV measure

[4]Note the subtle difference to the estimation target and its underestimation presented earlier. Previously, the model included a constant, unknown buy-propensity under no-herding such that instead of $\mathbb{E}[p]$ we would have used pN^h/N, with p being the true buy-propensity.

[5]An exception is the study by Frey et al. (2014). Using a binomial-mixture model for the cross-section of assets with two buy-propensities, one for buy-, one for sell-herding, they show that LSV underestimates the deviation of these buy-propensities from the one under no-herding, and that LSV increases with the number of traders such that the bias decreases.

Table 1: *LSV* in % under Setup 3

	N										MAD
$(\alpha,\,\beta)$	10	20	30	40	50	60	70	80	90	100	
(10,10)	2.58	3.48	4.02	4.40	4.72	4.97	5.16	5.34	5.49	5.61	8.81
(5,15)	2.11	2.96	3.43	3.77	4.05	4.25	4.43	4.59	4.70	4.81	7.59

Notes: This table shows the average LSV in % across 10000 simulations of trading without herding, but unobserved, asset specific buy-propensities. Specifically, for each asset $i = 1, \ldots, I$ with $I = 100$, $B_i = B_i^0$ where $B_i^0 \sim \text{Bino}(N, p_i)$ with $N = 10, \ldots, 100$ and $p_i \sim Beta(\alpha, \beta)$ with $(\alpha, \beta) \in \{(10, 10), (5, 15)\}$. For each simulation, LSV is given by $\sum_i LSV_i/I = \sum_i |br_i - \hat{p}|/I - \mathbb{E}_0|\frac{B}{N} - \hat{p}|$, with $\hat{p} = \sum_i B_i/IN$. MAD is the mean absolute deviation of the beta distribution, $\mathbb{E}|p - \mathbb{E}[p]|$.

against the binomial variance test (Cochran, 1954) and the $C(\alpha)$ test (Tarone, 1979) which are given by

$$X_v := \sum_{i=1}^{I} \frac{(B_i - \hat{p}N_i)^2}{N_i\hat{p}(1-\hat{p})} \quad \overset{a}{\sim} \chi^2(I-1) \quad \text{and} \tag{4}$$

$$X_c := \frac{(\sum_{i=1}^{I} \frac{(B_i - \hat{p}N_i)^2}{\hat{p}(1-\hat{p})} - \sum_{i=1}^{I} N_i)^2}{2\sum_{i=1}^{I} N_i(N_i - 1)} \quad \overset{a}{\sim} \chi^2(1), \tag{5}$$

respectively.

The significance of LSV is typically determined by t-tests based on standard asymptotic arguments.[6]

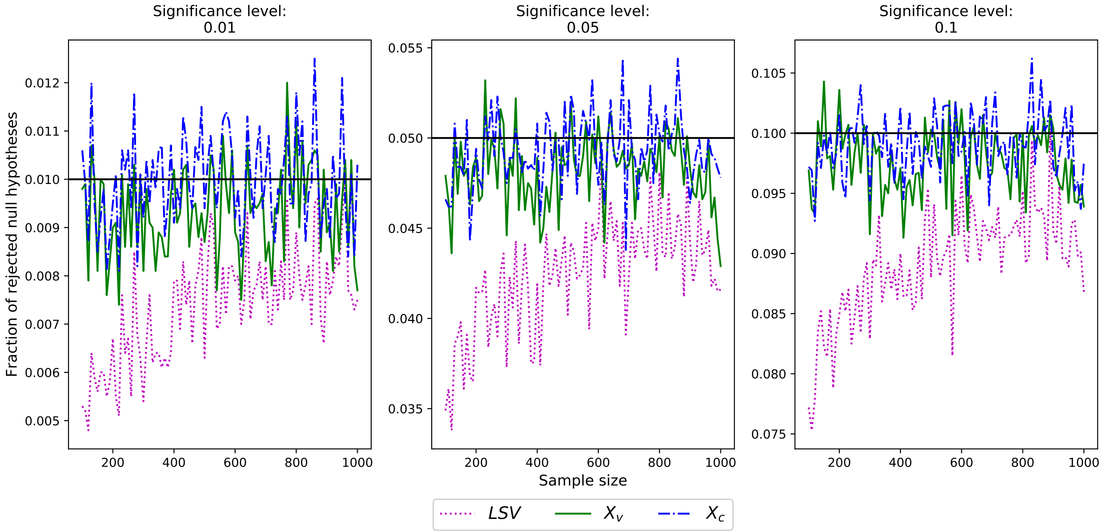
5.1 Size

For the size evaluation I draw the number of buys from a binomial distribution with $p = 0.5$ and $N_i = 5 + x_i$ with $x_i \sim Pois(\lambda)$ where $Pois$ is the Poisson distribution. That is, I let the number of traders in each asset vary as it would be the case in empirical applications. I set $\lambda = 45$. I draw 10000 samples for varying sample sizes $I = 100, \ldots, 1000$. The fraction of rejected null hypotheses is examined at the usual significance levels $\alpha = 0.1, 0.05, 0.01$. Figure 3 presents the results.

I find that the LSV measure is too conservative compared to the nominal significance level. That is, it rejects the null-hypothesis not often enough, even in

[6]Note that in empirical applications the number of traders in each asset typically varies, which means that in practice very large cross-sections may be needed to make the asymptotic arguments apply.

Figure 3: Size evaluation



Notes: This figure shows the fraction of rejected null hypotheses of no-herding (y-axis) among 10000 simulations under no herding. In each simulation buys are generated according to $B_i \sim \text{Bino}(N_i, p)$ with $N_i \sim 5 + Pois(\lambda)$ with $p = 0.5$ and $\lambda = 45$ for $i = 1, \ldots, I$ and sample size $I = 100, \ldots, 1000$ (x-axis).

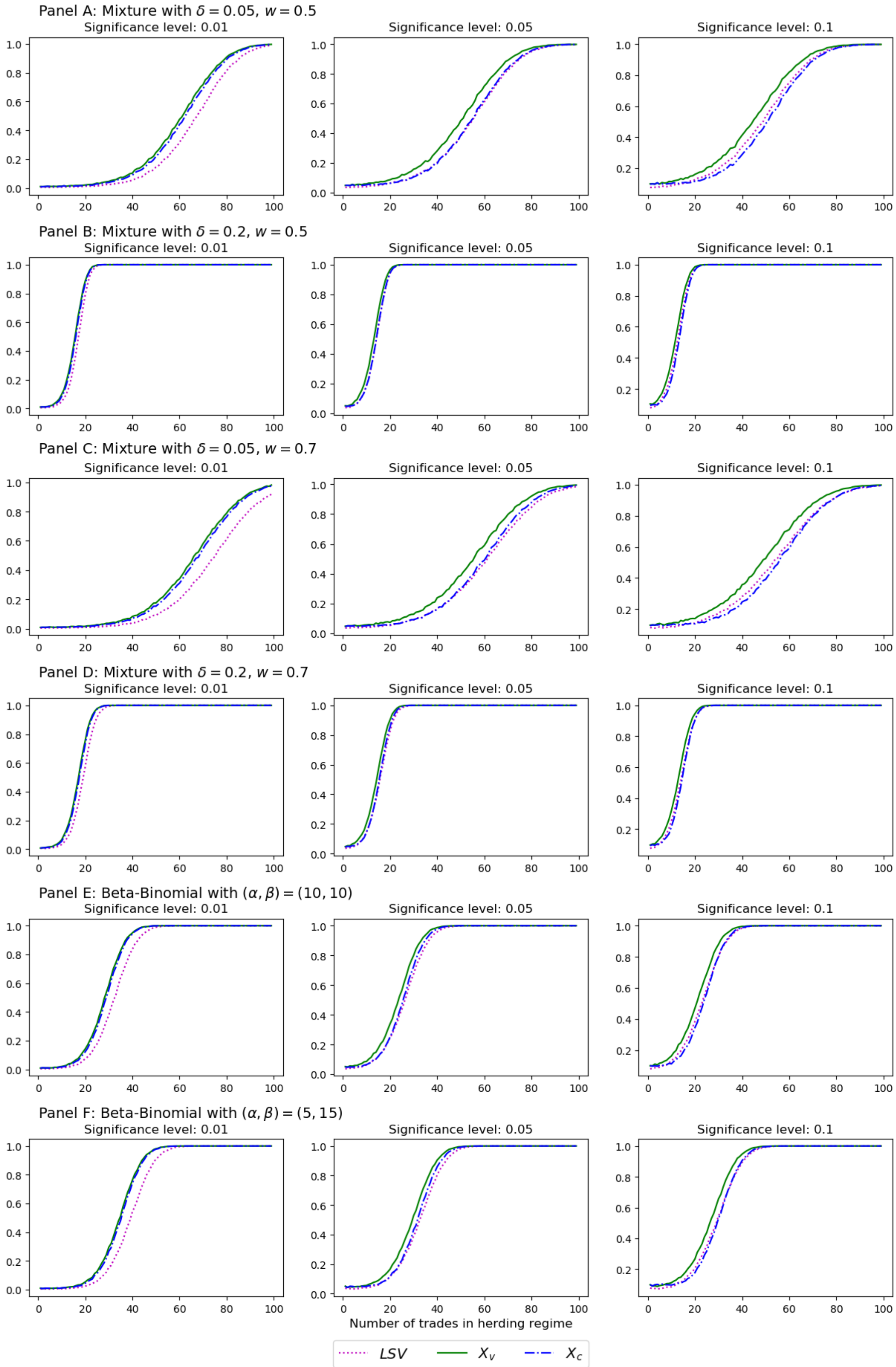
large cross-sections. The rejection rates of the other test-statistics are relatively close to their nominal levels.

5.2 Power

To evaluate the power of the tests, I use the alternative models presented in Setup 1 and 2. Contrary to the bias evaluation exercise, however, I will exclusively consider setups with p being estimated. I again fix N and I at 100. To control the degree of the deviation from the null model I vary the number of trades in the herding regime from 1 to N-1. I repeat each setup for 10000 draws.

Figure 4 presents the rejection rates. We find that across the different setups, LSV usually has the smallest power, the variance test the highest.

Figure 4: Power evaluation



Notes: This figure shows the fraction of rejected null hypotheses of no-herding (y-axis) among 10000 simulations under the alternative of herding. In each simulation the buys are generated according to $B_i = B_i^0 + B_i^h$, where $B_i^0 \sim \text{Bino}(N - N^h, p)$ with $N = 100$ and $p = 0.5$ for $i = 1, \ldots, 100$ and $N^h = 1, \ldots, N - 1$ (x-axis). B_i^h is either drawn from $F = w\text{Bino}(N^h, p + \delta) + (1 - w)\text{Bino}(N^h, p - \delta)$ with $\delta, w \in \{0.05, 0.2\} \times \{0.5, 0.7\}$ (Panel A-D), or from $F = \text{Beta-Bino}(N^h, \alpha, \beta)$ with $(\alpha, \beta) \in \{(10, 10), (5, 15)\}$ (Panel E-F).

6 Other measures of investor herding

While *LSV* is probably the most widely used empirical measure of herding based on investor level data, other measures have been proposed, most notably the one by Sias (2004). Sias (2004) measures the correlation of the buy-ratios over adjacent periods and is therefore not directly comparable. Still, as the estimation of the average buy-ratio in each period is an integral part of the measure, some of the criticism, especially around spurious vs intentional herding, applies here as well.

More directly related to the LSV measure is the one by Frey et al. (2014), which uses the squared instead of the absolute mean deviation. One can derive an equivalent bias to the one demonstrated above for their measure as well (a proof is given in the Appendix). This seems to be in contradiction with the unbiasedness shown by Frey et al. (2014).

The working assumption in Frey et al. (2014) is that under herding the number of buys are distributed according to a binomial mixture model in the cross-section of assets. In any individual instance, therefore, the number of buys are still the realization from a binomial distribution, either drawn from a binomial with higher or lower buy-propensity compared to the one under no herding. That is, even under herding each individual trader still trades independently of the others. This is at odds with the common idea of herd behavior.

In contrast, I have assumed that in the case of herding only a part of the trades in an individual asset follows a binomial distribution. The rest of the trades under the herding regime follow some other distribution.

7 Conclusion

I have shown that the LSV measure is a biased estimator and that a structural, economic interpretation of the measure is difficult at best. The measure is best understood as a purely statistical measure of binomial over-dispersion, in which case, however, other measures exist that have better size and power properties.

More broadly, this paper has shown that the empirical research of investor herding requires further attention. It lacks a clear definition of what constitutes herd behavior, a description of how the empirical treatment is linked to the theoretical literature, and a demonstration that the empirical approach taken indeed measures an economically meaningful target.

Appendix

A Proof of LSV bias

A.1 Known p

Let $B = B^0 + B^h$ where $B^0 \sim \text{Bino}(N - N^h, p)$ and $B^h \sim F$ with $0 < N^h < N$, $p \in (0, 1)$ and F some arbitrary distribution different from $\text{Bino}(N^h, p)$ with existing mean absolute deviation.

LSV is given by

$$LSV = \left| \frac{B}{N} - p \right| - \mathbb{E}_0 \left| \frac{B}{N} - p \right|, \tag{6}$$

where $\mathbb{E}_0$ indicates that expectation is taken under the assumption that $B \sim \text{Bino}(N, p)$. Substituting for B, expanding with pN^h, and rearranging terms we get

$$LSV = \frac{1}{N} \left\{ |B^h - pN^h + B^0 - p(N - N^h)| - \mathbb{E}_0 |B - pN| \right\}. \tag{7}$$

From the triangle inequality it follows that

$$LSV \leq \frac{|B^h - pN^h|}{N} + \frac{1}{N} \left\{ |B^0 - p(N - N^h)| - \mathbb{E}_0 |B - pN| \right\}. \tag{8}$$

Taking expectations, we get

$$\mathbb{E}[LSV] \leq \mathbb{E}[\theta] + \frac{1}{N} \left\{ \mathbb{E}|B^0 - p(N - N^h)| - \mathbb{E}_0 |B - pN| \right\}. \tag{9}$$

What remains to be shown is that

$$\mathbb{E}|B^0 - p(N - N^h)| \leq \mathbb{E}_0 |B - pN| \tag{10}$$

with equality if and only if $pN \in \mathbb{Z}$ and $N^h = 1$, which follows immediately from Theorem 3 in Diaconis and Zabell (1991).

A.2 Unknown p

If p has to be estimated via $\hat{p}$ we can arrive at a similar inequality to (9):

$$\mathbb{E}[LSV] \leq \mathbb{E}[\theta] + \frac{N^h |p - \hat{p}|}{N} + \frac{1}{N} \left\{ \mathbb{E}|B^0 - \hat{p}(N - N^h)| - \mathbb{E}_0 |B - pN| \right\}. \tag{11}$$

Note that $\mathbb{E}_0$ evaluates the expectation at $\hat{p}$, whereas $\mathbb{E}$ evaluates the expectation at the true parameter p. It is not possible to establish if one expectation is generally larger than the other. Additionally, the presence of $N^h |p - \hat{p}| / N$ complicates the matter, as it adds an upward bias.

A.3 Estimation target under Setup 1

Let the estimation target be $\mathbb{E}[\theta] = \mathbb{E}[\frac{\lfloor B^h - pN^h \rfloor}{N}]$ and $B^h \sim w_1\text{Bino}(N^h, p_1) + w_2\text{Bino}(N^h, p_2)$. Then

$$\mathbb{E}[\theta] = \sum_{j=1}^{2} w_j \big\{ 2v(1-p_j)g(v; N^h, p_j) + (p_j - p)N^h(1 - 2G(k-1; N^h, p_j)) \big\}, \quad (12)$$

where $v = \lfloor N^h p \rfloor$, G is the distribution function of the binomial and g its density.

The proof follows from the following corollary that I derive along the lines of the proof of Todhunter's formula stated in Lemma 1 in Diaconis and Zabell (1991).

Corollary 1. *For all integers* $0 \le a < b \le n$,

$$\sum_{k=a}^{b} (k - n\tilde{p})g(k; n, p) = \quad (13)$$
$$a(1-p)g(a; n, p) - (n-b)pg(b; n, p) + (p - \tilde{p})nG(a, b; n, p).$$

Proof.

$$\sum_{k=a}^{b} (k - \tilde{p}n)g(k; n, p) \quad (14)$$
$$= \sum_{k=a}^{b} ((p + 1 - p)k - \tilde{p}n + np - np)g(k; n, p)$$
$$= \sum_{k=a}^{b} ((1-p)k - (n-k)p)g(k; n, p) + \sum_{k=a}^{b} (p - \tilde{p})ng(k; n, p)$$
$$= a(1-p)g(a; n, p) - (n-b)pg(b; n, p) + (p - \tilde{p})nG(a, b; n, p).$$

□

Using this corollary it is straight forward to show that

$$\sum_{k=0}^{n} |k - \tilde{p}n| g(k; n, p) \quad (15)$$
$$= 2\lfloor n\tilde{p} \rfloor (1-p)g(\lfloor n\tilde{p} \rfloor; n, p) + (p - \tilde{p})n(1 - 2G(\lfloor n\tilde{p} \rfloor - 1; n, p)),$$

from which the result in (12) follows immediately.

B Bias of Frey et al.'s (2014) measure

The measure of Frey et al. (2014) is given by

$$FHW = \frac{(B - pN)^2 - Np(1-p)}{N(N-1)}. \tag{16}$$

Given that FHW work with the squared, rather than absolute deviation we define an equivalent estimation target, $\tilde{\theta} := (B^h - pN^h)^2/(N-1)N$.

Operating under the same assumptions as in A.1, substituting for B and expanding with pN^h, we obtain

$$FHW = \frac{(B^h - pN^h)^2}{N(N-1)} \tag{17}$$

$$+ \frac{2(B^h - pN^h)(B^0 - p(N - N^h))}{N(N-1)} \tag{18}$$

$$+ \frac{(B^0 - p(N - N^h))^2 - Np(1-p)}{N(N-1)}. \tag{19}$$

Taking expectations, we get

$$\mathbb{E}[FWH] = \mathbb{E}[\tilde{\theta}] + \frac{\mathrm{Cov}(B^h, B^0)}{N(N-1)} - \frac{N^h p(1-p)}{N(N-1)}. \tag{20}$$

So far, I have only specified the marginal distributions of B^h and B^0, which seems to suggest independence between the two random variables. If that would be the case, FWH underestimates herding. Of course, more realistically the herding and no-herding regimes are not independent. In that case, without further assumptions, the bias can go either direction.

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