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Beliefs- and fundamentals-driven job creation

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Abstract

This paper studies whether beliefs about future labour productivity independent of fundamentals at any horizon are important drivers of job creation. It develops a model with search frictions in the labour market that accounts for imperfectly observed permanent labour productivity changes. The estimation of the model shows that beliefs are important drivers of job creation in economies with larger search frictions. Beliefs explain 2%, 35%, and 55% of employment fluctuations for the US, the UK and France respectively. Furthermore, exogenous belief changes exert a more powerful influence on job creation during times when unemployment is low.

Key words: Labour productivity, information frictions, fundamentals and beliefs, equilibrium unemployment growth model, search and matching, business cycles.

JEL classification: E24, E32, E37.

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1   Introduction

Does an increase in job creation reflect an improvement in labour productivity or merely a change in beliefs about the future? An increase in job creation is often attributed to improving fundamentals.[1] When labour markets are frictionless this is correct. However, in any dynamic model with labour market frictions, the job creation decision becomes a forward-looking investment problem determined by current fundamentals *and* expectations about the future. Search and matching frictions make the job creation condition forward-looking since firms hire workers on the expectation of future payoffs from the job match. These expectations may turn out to be correct or they may have been purely the result of changing *beliefs* without underlying changes in fundamentals.

In this paper I examine whether beliefs or fundamentals drive job creation. I test the theory in three advanced economies: the United States, the United Kingdom, and France. I start by developing a dynamic stochastic general equilibrium model (DSGE) with a frictional labour market where labour productivity is subject to permanent and temporary shocks. The labour market has search frictions and displays an equilibrium unemployment rate along a balanced growth path. Information frictions prevent agents in the economy from disentangling permanent from temporary changes to labour productivity. Agents receive a noisy signal allowing them to forecast labour productivity. The result is that news and noise about future labour productivity drive the job creation decision. The setup then allows for decomposing drivers of job creation into economic fundamentals and pure beliefs. This paper is the first to model, estimate, and quantify the extent to which labour markets are driven by pure beliefs and fundamentals.

Fundamental shocks are defined as those where optimal agent decisions can be attributed to the dynamics of observed present or future labour productivity fundamentals. Meanwhile, pure belief shocks are the changes to expectations and the resulting decisions that are independent of labour productivity innovations at any horizon. The model identifies belief and fundamental shocks in the following way. Any observed changes in the number of vacancies posted, the unemployment rate, and the job-finding rate that cannot be explained by contemporaneous labour productivity changes reflect shifts in agents' expectations about future labour productivity. In fundamentals-driven economies, agents react quickly to these expectation shifts, since they know that information problems are minimal. In contrast, in belief-driven economies, agents display a sluggish behaviour when reacting to these expec-

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[1]For example: "... I'd say that we and a lot of private sector forecasters see strong growth and strong job creation starting right now. So really, the outlook has brightened substantially. And that's the base case. ..." Jerome Powell, 60 - minutes, CBS news, April 11, 2021

tation shifts, since they know that information problems are severe. Agents know they face a higher risk of hiring sub-optimally by reacting to a positive signal about the future and they therefore wait longer until fundamentals are realised. The model estimation fits the parameters and shocks such that the likelihood for the observed labour market behaviour and labour productivity fundamentals is maximised.

Estimating the model on labour market and labour productivity data between 1990 and 2020 shows that job creation for the two economies with less fluid labour markets, the United Kingdom and France, is driven to a larger extent by belief shocks. Concretely, I estimate the share of pure belief shock driving the employment rate to be around 35% for the United Kingdom and around 55% for France while it is merely 2% for the United States. Furthermore, belief shocks across economies are found to be stronger and more volatile drivers of the unemployment rate during times when the unemployment rate is low.

These results can be explained within the conventional search and matching framework. I show that the share of the value of a job match driven by expectation changes is higher in economies with more sclerotic labour markets. In more sclerotic labour markets with lower job-finding and job destruction rates a larger share of the surplus of a match is based on future expectations. Firms have to rely in their hiring decisions to a larger extent on productivity forecasts as it takes longer to achieve the optimal employment level in response to the expected productivity change. As a result, future expectations matter more to agents than the currently observed fundamentals in more sclerotic labour markets. This makes economies with more sclerotic labour markets potentially more susceptible to expectation-shifting belief shocks about future labour productivity.

While the literature has found that pure beliefs are important drivers of consumption and capital investment choices by focusing on the standard household consumption Euler equation, no studies have so far examined forward-looking employment choices. Chahrour and Jurado (2018) show that news shocks that are neutralised before their realisation by counteracting news or surprise shocks are observationally equivalent to noise shocks. As a result, they suggest decomposing shocks into fundamental shocks and pure belief shocks that are neutral to fundamental productivity changes at any horizon. I follow this approach for defining and identifying fundamental and pure belief shocks. Chahrour and Jurado (2018) show that with their decomposition, the share of consumption variation driven by pure belief shocks as estimated with the approaches in Barsky and Sims (2012) and Blanchard et al. (2013) increases to more than a third. Only for the model presented in Schmitt-Grohé and Uribe (2012) do they find a smaller than a third beliefs component in present aggregate consumption choices. Forni et al. (2017) use an identification for noise which is closely re-

lated to the Chahrour and Jurado (2018) concept of pure beliefs. They require signalled improvements to realise. Otherwise, they are classified as noise. I use their identification approach to show that noise shocks may be powerful drivers of job creation.

Optimal inter-temporal hiring decisions are at the heart of the Mortensen-Pissarides search and matching model (Mortensen and Pissarides (1994) and Pissarides (2000)). Following the finding by Beaudry and Portier (2006) that news shocks are important drivers of business cycles, several authors found that such anticipated shocks also are important drivers in a search and matching environment. Adjustment costs to labour input are found to be important for explaining observed business cycles with news shocks in Jaimovich and Rebelo (2009). Zanetti and Theodoridis (2016) show that accounting for news channels in a model with hiring frictions greatly improves the performance of the search and matching model. Den Haan and Kaltenbrunner (2009) and Krusell and McKay (2010) show that theoretical real business cycle models with search and matching frictions to produce Pigou cycles, with an empirically plausible co-movement of labour input, investment, and consumption, with output. Di Pace et al. (2021) show that systematically wrong expectations of aggregate wages can be explained by adaptive forecasting rules and contribute to the greater volatility of the vacancy response of the standard random search and matching model. All these papers show that a good part of aggregate job creation systematically responds to shifts in expectations about future productivity. This paper contributes to this area of research by estimating the extent to which these shifts in expectations result from changing fundamentals, which is critical for policies aimed at fostering job creation and smoothing unemployment over the business cycle.

While there is little research on the effects of information frictions in forming expectations about the aggregate component of job matches, previous research has focused on information frictions in forming expectations about the idiosyncratic productivity of a job match. Moscarini (2005) finds that labour markets with higher information frictions impacting the information extraction problem for forming expectations about the idiosyncratic productivity process of a job match are inherently more sclerotic. This paper shows a complementary relationship with regard to the information extraction problem for forming expectations about the aggregate productivity process, though the causality of the mechanism is reversed. Information frictions in forming expectations about the aggregate labour productivity process matter more in more sclerotic labour markets. This makes these markets more noise and beliefs driven.

Prior to presenting the main structural DSGE model developed in this paper, I present evidence that noise may be a substantial driver of labour market decision-making by adapting

the structural vector auto-regression approach in Forni et al. (2017) to the labour market. The model shows that the identified noise shocks may be a potential driver of the employment rate. However, the identification relies on instruments for the information agents hold about future labour productivity. As these instruments may be weak, the model has insufficient statistical power to identify noise as a certain driver of job creation. Such instruments are not required for estimating noise or beliefs as drivers of job creation in the main model.

In the main model developed in this paper, I set up the information problem about labour productivity as in Lorenzoni (2011) and Blanchard et al. (2013). I then embed this into an equilibrium unemployment growth model as presented in Pissarides (2000). However, this model is subject to the Shimer neutrality (Shimer, 2010), which states that labour productivity shocks will not affect the unemployment rate or vacancy posting when all variables entering the job creation condition are contemporaneously scaled with the labour productivity process. The reason is that the surplus created by agents being matched over being unmatched, the so-called fundamental surplus of the job match (Ljungqvist and Sargent, 2017), would not be altered. This renders a productivity shock neutral. This neutrality does not apply when the outside benefit and the vacancy posting cost are proportional to productivity. The paper contributes to these findings by extending this neutrality to future expectations about the labour productivity process. I then propose a tractable catch-up process for the cost of posting a vacancy and the unemployment benefit to overcome the neutrality result, present evidence that this catch-up process captures the response in the data well, and propose a method to calibrate it.

The remainder of the paper is structured as follows. Section two presents with a structural vector auto-regression which shows that beliefs and noise may play a substantial role in the labour market. Section three presents the equilibrium unemployment model with search and information frictions. Section four discusses the Shimer neutrality in an equilibrium unemployment model with permanent productivity innovations and the proposed catch-up process. Section five describes the data sources and presents the calibration of the model to the three target economies. Section six presents and interprets the results of the estimation. Section seven concludes.

2 A structural vector auto-regression showing noise shocks as potential drivers of job creation

In this section, I develop a simple structural vector auto-regression to identify noise-driven job creation in the labour market. The model shows that the identified noise shocks may be

important drivers of the employment rate and job creation in all three economies.

I use the identification approach suggested in Forni et al. (2017) to identify noise shocks. The approach relies on the assumption that agents ultimately learn with certainty whether any signal about future labour productivity changes was correct or merely noise. The method and the relation to the main DSGE model developed in Section 3 are explained in detail in Appendix A. I describe the method here briefly. As a first step, I estimate a vector-auto regression and identify reduced form shocks with short-run restrictions. The estimated model showing the ordering is in equation (1):

$$
\begin{bmatrix} \Delta P_t \\ z_t \\ \Delta p(\theta_t) \\ \Delta n_t \end{bmatrix} = \sum_{i=1}^{4} A_i \begin{bmatrix} \Delta P_{t-i} \\ z_{t-i} \\ \Delta p(\theta_{t-i}) \\ \Delta n_{t-i} \end{bmatrix} + \begin{bmatrix} u_t \\ s_t \\ x_{3,t} \\ x_{4,t} \end{bmatrix} . \tag{1}
$$

The ordering is kept in coherence with the labour market model presented in Section 3 and is natural to a labour market with search frictions. A shock u_t to changes in labour productivity ΔP_t is assumed to contemporaneously affect all series. Agents receive a noisy signal about the future z_t. This signal summarises the agents' entire information set at time t about future changes in future labour productivity. A shock s_t to the signal z_t may only affect the signal series, the changes in the job-finding rate $p(\theta)$ and changes in the employment rate n contemporaneously. A shock to changes in the job-finding rate will affect changes in the employment rate contemporaneously, while a shock to changes in the employment rate, won't affect changes in the job-finding rate until the next period.

Forni et al. (2017) then suggest that the shocks u_t and z_t can be decomposed into fundamental and noise shocks when the following assumptions hold. Agents learn after a set maximum number of periods with certainty whether a signal movement was due to a fundamental or noise shock. A noise shock is assumed not to affect the fundamental labour productivity series at any length within this period. These assumptions are clearly closely related to the definition of a belief shock in Chahrour and Jurado (2018). If the assumptions hold then if one can observe a series which can serve as an instrument for the true signal received by agents, then one can separate noise and fundamentals with the help of this series as a measure for z_t in the vector-auto-regression. The shocks identified with the short-run restrictions u_t and z_t are rotated until the restriction of the noise shock not affecting fundamentals within the given period is satisfied.

The identification of fundamentals and noise in this section relies on the OECD's composite

leading indicator being used as the instrument for the true information set of the agents. The series instruments for the true signal received by agents. The series selection is described in detail in Section 5 and Appendix B. I assume that agents learn if a shock was fundamental or noise after two years meaning the vector auto-regression is estimated with 24 lags.

The shocks u_t and s_t to fundamental labour productivity ΔP_t and the signal z_t are identified via short-run restrictions. Figure 1 shows these impulse responses on the left side. The reduced form regressions show the signal series for all three economies driving the employment rate and job creation. The response of the employment rate to a positive signal is visibly delayed in the case of the United Kingdom and France compared to the United States a result explained via higher information frictions in the estimation of the DSGE model in Section 6.

Implementing the identification of Forni et al. (2017) and rotating residuals to an identification where the identified noise shock has no significant impact on labour productivity shows that noise shocks could lead to large and prolonged employment rate movements. The model vector auto-regression has the statistical power to identify noise shocks as a significant driver of the employment rate for the United States. The statistical power is insufficient to identify noise shocks as certain drivers of job creation for all three economies. This may be due to the OECD composite leading indicator being a too weak instrument for the agents' true information set for the United Kingdom and France. However, the estimation shows that noise shocks could be powerful drivers of job creation. Most notably it cannot be said with certainty whether fundamental shocks have a large effect on the employment rate and job creation or whether the identified noise has a larger impact on the employment rate.

The next section develops the main DSGE model. The model identifies fundamentals and belief shocks from the behaviour and agents and therefore doesn't need to rely on a signal series instrumenting for the information set of the agents. With this model, we can answer whether noise or belief shocks are drivers of job creation and the employment rate.

6

Short-Run Restrictions **Rotation of the Residuals**

United States
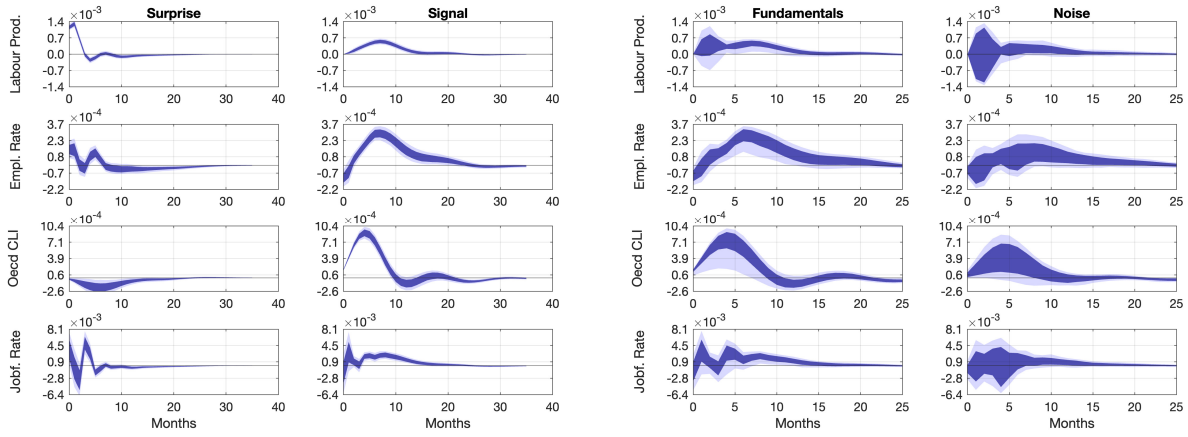
United Kingdom
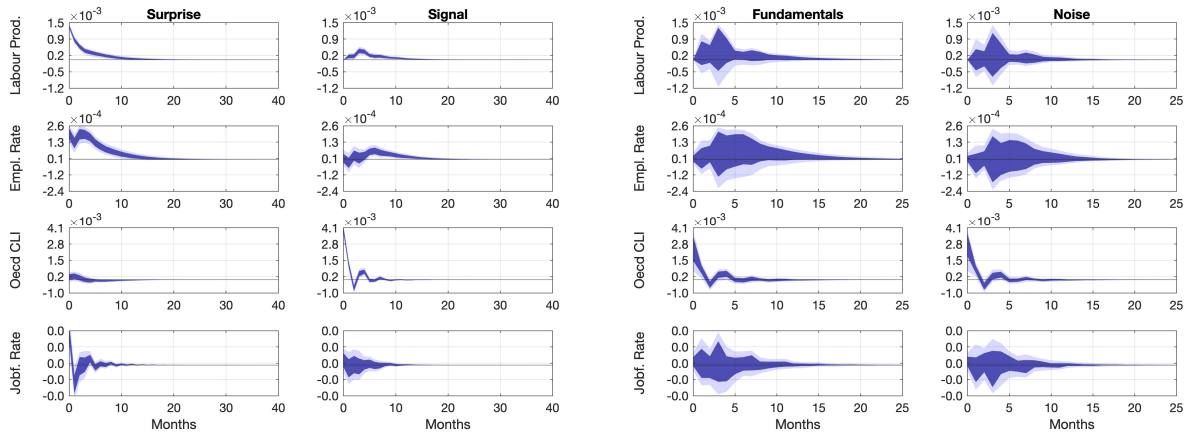
France
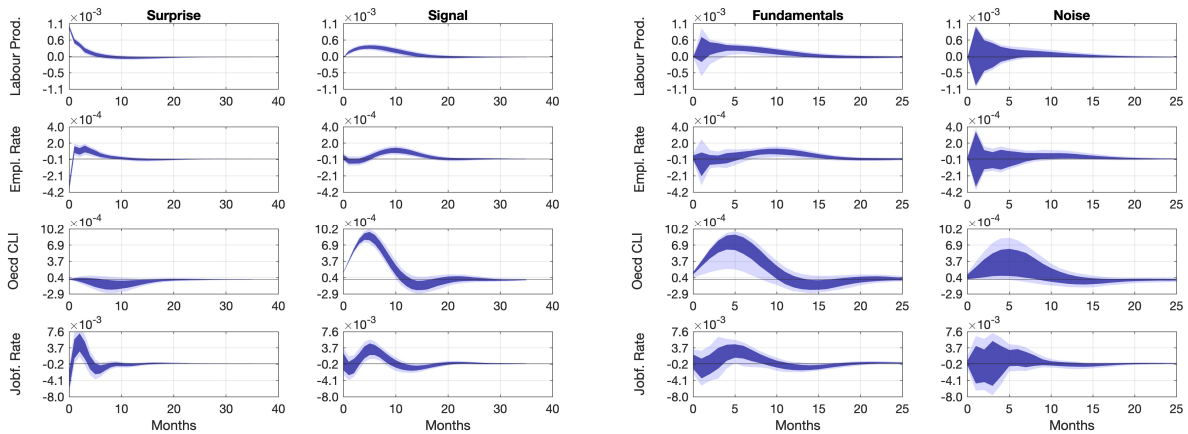
Figure 1: Impulse response functions following the identification by short-run restrictions are presented on the left for a surprise and a signal shock. These reduced-form shocks are then used to identify noise and fundamental shocks following the method suggested in Forni et al. (2014). The signal series is the OECD composite leading indicator. Light blue shows the 90% confidence interval, while dark blue shows the 67% confidence interval of the response.

3 The model

The model is composed of a representative household which supplies the economy with labour and owns firms. All agents in the economy face the same information frictions that prevent them from perfectly forecasting aggregate labour productivity. Firms optimally choose the number of vacancies to post based on these forecasts and the state of the employment rate.

3.1 Labour productivity

Section 3.1.1 describes the fundamental aggregate labour productivity process, which determines the output of all firm-worker matches in the economy. The next Section 3.1.2 specifies the information set available to agents, while the last Section 3.1.3 describes how agents solve the information problem with the agent Kalman filter. The model estimation focuses on estimating the parameters determining this agent Kalman filter.

3.1.1 Productivity fundamentals

The model takes the non-stationary aggregate labour productivity process formulated in Blanchard et al. (2013) and integrates it into a labour market with search frictions and equilibrium unemployment. The observed aggregate labour productivity process a in equation (2), determines the flow product produced by a worker-firm match and consists of an unobserved permanent component x and an unobserved temporary component z.

$$a_t = x_t + z_t \tag{2}$$

The first difference of the permanent component, and the temporary component are assumed to be stationary processes. The process x can be viewed as capturing permanent changes in production technology, while z captures short-term labour productivity deviations, for instance via temporary demand shocks. ϵ_t and η_t are independently, identically, and normally distributed exogenous shocks with mean 0 and constant and known variances σ_ϵ^2 and σ_η^2.

$$\Delta x_t = \rho^x \Delta x_{t-1} + \epsilon_t \qquad (3) \qquad\qquad z_t = \rho^z z_{t-1} + \eta_t \qquad (4)$$

I restrict parameters such that there is no information on the future path of a_t contained in its past realisations alone. Thus without any further signals the process a_t would appear as a random walk. This property is the result of choosing the parameter relations in equation (5) and (6) for the permanent and temporary component ((Lorenzoni, 2011) and (Blanchard et al., 2013)).
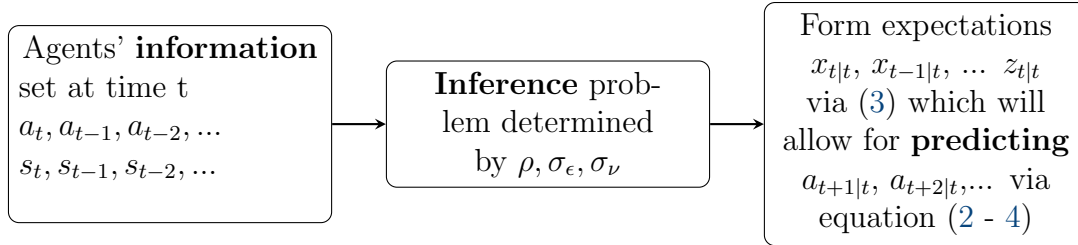
Figure 2: Information extraction problem

$$\rho^x = \rho^z = \rho \qquad (5) \qquad\qquad \rho\sigma_\epsilon^2 = (1-\rho)^2\sigma_\eta^2 \qquad (6)$$

3.1.2 Information

Only the present and past joint realizations of the permanent and temporary match productivity components $a_t, a_{t-1}, a_{t-2}, a_{t-3}, ...$ are part of the agents' information set in period t, making it impossible for agents to tell whether future match productivity will grow or decline just from observing the productivity process. All information that may help agents disentangle the permanent from the temporary process and thereby predict future match productivity growth is summarised in a noisy signal in equation (7) containing information about the permanent component.

All agents receive this noisy signal s_t. ν_t is independently, identically, and normally distributed noise shocks with mean 0 and constant and known variance σ_ν^2.

$$s_t = x_t + \nu_t \qquad (7)$$

3.1.3 Forming expectations over future match productivity

All agents have the same information set and are assumed to know the distributions of shocks and form of the productivity process. The agents information set consists of all past and present productivity realisations and past and present signals. Assuming agents observe the processes and signals over a long time this information problem is optimally resolved by rational agents with a converged Kalman filter.

The random walk assumption ensures that without a signal the agents would expect for all future j periods $E_t(a_{t+j}) = a_t$. Due to agents receiving a signal over the permanent component of productivity x, and thus over the future growth of productivity to be expected, a signal extraction problem evolves. As in Blanchard et al. (2013) this information extraction problem is solved with a typical agent Kalman filter closer described in Appendix A. The

9

Kalman gain will converge to a 3x2 matrix with 6 elements. Each element of the Kalman gain will be a function of the parameter values defining the properties of the shocks to the exogenous processes $\sigma_\epsilon, \sigma_\nu$, and ρ.

$$K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \\ K_{31} & K_{32} \end{bmatrix} = K(\sigma_\epsilon, \sigma_\nu, \rho) \tag{8}$$

The main objective of the maximum likelihood estimation of the model is to estimate the parameters defining the Kalman gain in equation (8) as these define the importance of the temporary component, the permanent component, and noise. These drivers can then be decomposed into fundamentals and pure beliefs.

Using the elements of the converged Kalman gain and the observed new observations a_t and s_t agents form their expectations over future productivity growth by updating their expectations about the values of the past and present value of the permanent productivity component $x_{t|t}$, $x_{t-1|t}$ (description in Appendix A). The upper section in the model summaries in table 5 and in table 1 shows equations for the exogenous processes and the information extraction process. Chahrour and Jurado (2018) show that these processes have an equivalent representation decomposing the permanent, and temporary, noise processes instead into pure fundamentals and pure belief shocks. Pure fundamental shocks change productivity, while pure belief shocks are neutral to productivity at any point in time. This decomposition of the exogenous processes is equivalent to the baseline model and hence the agent information extraction remains unchanged. Both the belief and noise estimations are equivalent representations of the information problem, but the decomposition of the problem into pure beliefs and pure noise addresses the criticism in Chahrour and Jurado (2018) of the news and noise identification in Blanchard et al. (2013).

3.2 Household

There exists a large representative household, whose members maximize the present discounted value of the sum of future expected utilities. The instant utility function has a constant-elasticity of substitution specification.

$$\max_{g_t, c_t} E_t \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\iota}}{1-\iota} \tag{9}$$

The household maximises utility subject to the budget constraint:

$$c_t + g_t = (w_t n_t + d_t + b_t u_t + r_{t-1} g_{t-1}). \tag{10}$$

Here β is the discount factor, ι is the elasticity of inter-temporal substitution, g_t are one-period government bonds r_{t-1} is the interest rate on government bonds of the past period determined in period $t-1$ and received in period t. The interest rate is the equilibrium rate at which household members would be willing to lend to each other. w_t is the real wage, b_t is the value of household production for an unemployed worker, and d_t is the profit created by firms and passed on via dividends to the household. The net supply of government bonds will be 0 and the aggregate constraint requires that the sum of wages and profits equals the output from production $a_t n_t$ minus the cost of hiring $\kappa_t v_t$, $w_t n_t + d_t = (a_t n_t - \kappa_t v_t)$.

As usual the first order conditions provide the expected shadow value of the period budget constraint being $c_t^{-\iota} = \mu_t$, where μ is the Lagrangian multiplier on the budget constraint. The inter-temporal Euler equation for bonds is $\mu_t = \beta r_t E_t(\mu_{t+1})$. We can rewrite this as:

$$\frac{1}{r_t} = \beta \frac{E_t(\mu_{t+1})}{\mu_t}. \tag{11}$$

3.3 Production

For ease of exposition, the model derivation focuses on production with exogenous job destruction. Production with endogenous job destruction follows a similar derivation and is driven by similar forces. While the main equations to extend the model to endogenous job destruction are briefly described at the end of the section, a detailed derivation can be found in Appendix A. All model equations used in the estimations are in table 5 and table 1.

Firms are assumed to be identical and to produce goods in period t using only labour as an input and producing output $y_t = a_t n_t$. In order to produce, firms have to hire workers in a labour market with search and matching frictions. The number of firms and workers that meet each other in every period is determined by the matching function $M(m_t, v_t, u_t) = m_t v_t^{1-\xi} u_t^\xi$. m_t is an exogenous matching efficiency parameter. In the model with exogenous job destruction, match efficiency is assumed to be subject to temporary exogenous shocks equation (12). These movements in match efficiency are endogenised in the model with endogenous job destruction following Sedláček (2014).

v_t is the number of posted vacancies in a given period, and $u_t = 1 - (1-\lambda)n_{t-1}$ is the number

of unemployed before matching occurs in a given period. Jobs are destroyed at the constant rate λ following the arguments in Shimer (2012). It costs a firm κ_t to post a one-period vacancy. m_t is subject to exogenous match efficiency shocks in the model with exogenous job destruction:

$$m_t = (1 - \rho_m)\mu_m + \rho_m m_{t-1} + \epsilon_m . \tag{12}$$

Firms are assumed to be small enough to take the probability of matching with a worker and filling a vacancy $q(\theta_t) = \frac{M(m_t, v_t, u_t)}{v_t}$ as given. $\theta_t = \frac{v_t}{u_t}$ is a measure of labour market tightness. Firms are also assumed to be hiring in markets large enough compared to the number of workers they employ that the law of motion for employees for an individual firm corresponds to the law of motion of the labour market as a whole. Finally, the law of motion for the rate of employment is $n_t = (1 - \lambda)n_{t-1} + M(m_t, v_t, u_t)$. Workers will take up production in the same period that they are hired. Further, note that workers can be separated and re-employed in the same period, which means that there are more workers searching for a job in a current period than the number of unemployed $(1 - n_{t-1})$ in the previous period.

Firms will have to decide how many workers to hire based on their expectations on present and future labour productivity. The firm's management then chooses v_t and n_t to maximize the value of current and future expected profits. These are discounted by the expected utility contribution of the production value, which in a model with a bond market is equal to the interest rate.

$$\max_{v_t, n_t} E_t \left\{ \sum_{t=0}^{\infty} \beta^t \mu_t (a_t n_t - w_t n_t - \kappa_t v_t) \right\} \tag{13}$$

The present discounted value of future profits is maximised subject to the law of motion of employment:

$$n_t = (1 - \lambda)n_{t-1} + v_t q(\theta_t) . \tag{14}$$

$\mu_{n,t}$ is the Lagrange multiplier on the constraint in equation (14). Combining the first order condition for v_t, $\mu_{n,t} q(\theta_t) - \mu_t \kappa_t = 0$, with the first order condition for n_t, $\mu_t a_t - \mu_{n,t} + (1 - \lambda)\beta E_t(\mu_{n,t+1}) = 0$ yields the job creation condition where the expected cost of hiring a new worker $\frac{\kappa_t}{q(\theta_t)}$ equals the benefit of hiring a new worker:

$$\frac{\kappa_t}{q(\theta_t)} = a_t - w_t + \beta(1-\lambda)E_t\left\{\frac{\mu_{t+1}}{\mu_t}\frac{\kappa_t}{q(\theta_{t+1})}\right\} . \tag{15}$$

We can simplify equation (15) by substituting with the interest rate r_t (11). Firms and workers split the expected surplus of a successful match according to a Nash bargaining protocol with the firm's bargaining strength being π. The negotiated wage is in equation (16):

$$w_t = \pi b_t + (1-\pi)[a_t + (1-\lambda)\frac{1}{r_t}E_t\left\{\kappa_{t+1}\theta_{t+1}\right\} . \tag{16}$$

Substituting equation (15) back into the job creation condition yields the job creation condition as a function of productivity, tightness, the outside value and the vacancy positing cost:

$$\frac{\kappa_t}{q(\theta_t)} = \pi(a_t - b_t) + (1-\lambda)E_t\left\{\frac{1}{r_t}[\frac{\kappa_t}{q(\theta_{t+1})} - (1-\pi)\kappa_{t+1}\theta_{t+1}]\right\} . \tag{17}$$

3.3.1 Production with endogenous job destruction

I describe the derivation of the model to endogenous job destruction in detail in Appendix A. All relevant equations of the model are further in Table 1. In this section, I focus on the key assumptions and present the main equations necessary to extend the model setup to include endogenous job destruction. First, I allow for the productivity of a job match to be subject to idiosyncratic productivity shocks ζ drawn from a distribution $H(\zeta)$. These idiosyncratic productivity shocks are non-persistent. Second, I allow for the shocks to be already observed at matching before job creation leading to stochastic matching for workers and firms meeting in the search process. Third, I extend the model with firing cost $\kappa_{F,t}$ paid by the firm. These costs are not paid when the match is newly formed and has not yet entered production. These kinds of costs together with stochastic matching endogenise movements of match productivity over the business cycle as shown in Sedláček (2014).

The job creation and job destruction conditions for new hires resulting from these extensions are in equation (18) and equation (19). $\hat{J}_{t+1}$ is the benefit of a filled vacancy for a continuing match, which is accompanied by an equivalent job destruction condition described in Appendix A. The job creation condition is:

$$\frac{\kappa_t}{q(\theta_t)} = \int_{\tilde{\zeta}_t^N}^{\bar{\zeta}} \left[\exp(\zeta)a_t - w_t^N + \beta(1-\lambda)E_t \left\{ \hat{J}_{t+1} - H(\tilde{\zeta}_{t+1})\kappa_{F,t+1} \right\} \right] h(\zeta)d\zeta . \qquad (18)$$

$\tilde{\zeta}_t^N$ is the idiosyncratic productivity at which a new firm worker job match is destroyed:

$$\exp(\tilde{\zeta}_t^N - \hat{\zeta}_t^N) = \frac{1}{a_t} \left[b_t - (1-\lambda)\frac{1}{r_t}\frac{1}{\pi}E_t \left\{ \hat{J}_{t+1} - (1-\pi)\kappa_{t+1}\theta_{t+1} + \tilde{\kappa}_{t+1} \right\} \right] . \qquad (19)$$

This is different to the destruction condition for continuing firm worker matches in equation (20) due to the firing cost κ_F firms have to bear when dissolving continuing matches. This introduction of stochastic matching with firing cost allows for the endogenisation of match efficiency. $\tilde{\kappa}_{t+1}$ summarises further costs of dissolving a match in the next period described in detail in Appendix A. The job destruction condition for continuing matches is:

$$\exp(\tilde{\zeta}_t - \hat{\zeta}) = \frac{1}{a_t} \left[b_t - \kappa_{F,t} - (1-\lambda)\frac{1}{r_t}\frac{1}{\pi}E_t \left\{ \hat{J}_{t+1} - (1-\pi)\kappa_{t+1}\theta_{t+1} + \tilde{\kappa}_{t+1} \right\} \right] . \qquad (20)$$

The law of motion with endogenous job destruction is given by equation (21) capturing endogenous separations of new and continuing matches:

$$n_t = (1-\lambda)(1 - H(\tilde{\zeta}_t))n_{t-1} + (1 - H(\tilde{\zeta}_t^N))u_t p(\theta_t) . \qquad (21)$$

4   Equilibrium job creation with an integrated labour productivity of a job match

Equation (17) shows that the benefit of a match is driven by the non-stationary aggregate labour productivity process a_t. An increase in the permanent component of a_t, x_t leads to a permanent increase in labour productivity. This then permanently increases consumption and output. However, without further assumptions about the cost of creating a job match and the value of the outside option, this match productivity increase also leads to a permanent decrease in unemployment and an increase in vacancies as the relative cost of hiring and the value of production outside the match decline. This would deny the empirical evidence of a long-run stable Beveridge curve (Martellini and Menzio, 2020) and render the model without an unemployment equilibrium.

Pissarides (2000) resolves this problem in the equilibrium unemployment growth model presented in the book by imposing a proportional increase in cost and the outside option on the parameters governing the job creation condition. Dividing by the labour productivity process then yields a stable job creation condition. This a method frequently used in the real business cycle and news literature, for example in Schmitt-Grohé and Uribe (2012). For the unit root to be removed from the fundamental surplus of a job match and to introduce a long-run unemployment equilibrium, it is required that the cost of vacancy creation κ_t and the outside value to a match b_t are in the long-run proportional to a_t.

However, when Proposition 1 holds, namely that when parameters κ_t and b_t depend on aggregate contemporary variables, such as output per worker, consumption, or wages, then present productivity changes or shifts in productivity expectations about the output of a worker won't affect job creation. This is an extension of the neutrality result in Shimer (2010), which states that in an equilibrium unemployment growth model in the spirit of Pissarides (2000) labour productivity changes won't affect the unemployment rate.

Proposition 1. *Cost of vacancies κ_t and unemployment benefits b_t, which are proportional to contemporary variables such as output per worker, consumption, output, or wages in a plain Diamond-Mortensen-Pissarides random search DSGE model lead to any labour productivity and expected labour productivity shifts leaving the unemployment rate unaffected.*

Proof. Without dependencies on past states, any contemporary variables are proportional to a_t. Assume $\kappa_t = \phi_1 a_t$ and $b_t = \phi_2 a_t$. Here ϕ_1 and ϕ_2 are parameters, which could stand for κ and b. For simplicity set $\iota = 1$. Inserting these in the job creation condition means the job creation condition becomes equation (22),

$$\frac{\phi_1}{q(\theta_t)} = \pi(n_t - \phi_2) + (1 - \lambda)E_t \left\{ \frac{1}{r_t}\frac{a_{t+1}}{a_t}[\frac{\phi_1}{q(\theta_{t+1})}) - (1 - \pi)\phi_1\theta_{t+1}] \right\} . \tag{22}$$

Note that $\frac{1}{r_t}\frac{a_{t+1}}{a_t} = \frac{c_t}{c_{t+1}}\frac{a_{t+1}}{a_t}$ and $c_t = a_t(n_t - \psi_1 v_t + \psi_2(1 - n_t))$. Thus all changes and expected changes to the product of labour cancel out. Equivalent results can be achieved when using output per worker where $y_t = a_t\frac{n_t + \phi_2(1-n_t)}{n_t}$ or wages $w_t = a_t[\pi\phi_2 + (1 - \pi)[1 + (1 - \lambda)\phi_1 E_t(\theta_{t+1})]$. $\qquad\square$

To achieve a positive correlation of an increase in labour productivity and an increase in employment, vacancies, consumption and output in line with empirical findings on Pigou cycles (Beaudry and Portier, 2006) and Okun's law, which continues to fit the data well (Ball et al., 2013), it is necessary for current labour productivity to temporarily increase in

proportion to vacancy cost and the unemployment benefit.

This is the case when the cost of vacancies and the unemployment benefit follow the labour productivity process with a lag. This proposed catch-up process is a tractable functional form for capturing many arguments proposed for the equilibrium unemployment model to better match the data. The process can for instance capture a delayed cyclicality of the unemployment benefit (Chodorow-Reich and Karabarbounis, 2016), or the argument that a lot of the effort spent on a vacancy is already occurring in previous periods (Shimer, 2010) making the vacancy cost dependent on past labour productivity realisations.

For tractability, I propose that both κ_t and b_t catch-up to the current level of labour productivity over time at a common rate. Concretely $\kappa_t = \kappa \prod_{s=1}^{L} a_{t-s}^{\gamma_s}$ and $b_t = b \prod_{s=1}^{L} a_{t-s}^{\gamma_s}$ are weighted products of past labour productivity realisations. The parameters γ_s control the importance of each lag. Assume $\sum_{s=1}^{L} \gamma_s = 1$, which is necessary for a stable unemployment equilibrium to exist.

Dividing the job creation condition by $\prod_{s=1}^{L} a_{t-s}^{\gamma_s}$ shows that job creation and employment rate changes will depend on relative labour productivity $P_t \equiv \frac{a_t}{\prod_{s=1}^{L} a_{t-s}^{\gamma_s}}$ and $Q_{t+1} \equiv \frac{\prod_{s=1}^{L} a_{t+1-s}^{\gamma_s}}{\prod_{s=1}^{L} a_{t-s}^{\gamma_s}}$. We can rewrite job creation condition:

$$\frac{\kappa}{q(\theta_t)} = \pi(P_t - b) + \beta(1-\lambda)E_t \left\{ Q_{t+1} \frac{c_{t+1}^{-\iota}}{c_t^{-\iota}} \left(\frac{\kappa}{q(\theta_{t+1})} - (1-\pi)\kappa\theta_{t+1} \right) \right\} . \tag{23}$$

Following a shock P_t and Q_t will converge to 1. As a result, job creation will stay at a constant level, even though consumption will be permanently changed if the shock is of a permanent nature. Section 5 discusses the data and presents a proposed method for calibrating γ_s.

4.1 Stationary Dynamic Equilibrium

The stationary dynamic equilibrium of the model with exogenous job destruction is closed with the aggregate market clearing condition in equation (24). In the economy, everything produced is either spent on recruitment or consumed:

$$c_t = a_t[n_t - \kappa_t v_t + b_t(1 - n_t)] . \tag{24}$$

Increases in the permanent component x lead to permanent increases in the level of consumption. Equation (24) shows that, while long-run unemployment has an equilibrium level in this model in contrast to the simple model presented in Blanchard et al. (2013), the long-

run outcome of consumption in the model is qualitatively the same. As long as $n + bu > \kappa v$ in steady-state, the model has a positive steady-state for its labour market variables. The inequality is fulfilled for any reasonable calibration of the search and matching model as otherwise, the matching process would cost more resources than the economy produces. Absent any permanent productivity shocks, the value long-run consumption will converge to $c_\infty = (n^* + bu^* - \kappa v^*)x_\infty = (n^* + bu^* - \kappa v^*)(\frac{x_t - \rho x_{t-1}}{1-\rho})$.

The remaining equilibrium equations for the five variables: $c_t, n_t, u_t, \theta_t, v_t$, consumption, employment rate, searching unemployed, tightness and vacancies are:

$$c_t / \prod_{s=1}^{L} a_{t-s}^{\gamma_s} = P_t n_t - \kappa v_t + b(1 - n_t) \ ,$$

$$\frac{\kappa}{q(\theta_t)} = \pi(P_t - b) + \beta(1 - \lambda)E_t \left\{ Q_{t+1} \frac{c_{t+1}^{-\iota}}{c_t^{-\iota}} , (\frac{\kappa}{q(\theta_{t+1})} - (1 - \pi)\kappa\theta_{t+1}) \right\} \ ,$$

$$n_t = (1 - \lambda)n_{t-1} + u_t p(\theta_t) \ ,$$

$$u_t = 1 - (1 - \lambda)n_{t-1} \ ,$$

$$\theta_t = \frac{v_t}{u_t} \ .$$

Agents forecast these dynamic equilibria for every future period given their expectations about the labour productivity process and choose their present optimal hiring policies accordingly. $p(\theta_t)$ is the probability with which searching workers u_t achieve a match with $\theta_t p(\theta_t) = q(\theta_t)$.

4.2 Model driving forces

The system of equations defining the dynamic equilibrium of the model with exogenous job destruction is fully determined by the exogenous parameters $\beta, \lambda, \kappa, \pi, b, \iota, gamma_1, ...\gamma_s$, the state of the employment rate and past, present, and future expected values of the productivity process. The employment rate in the previous period n_{t-1} is the only labour market state variable of the model. The influence of the labour productivity process on the equilibrium is fully captured by the L past, the current, as well as the expected future values of the product of a job $a_{t-L}, ..., a_{t-1}, a_t, E_t a_{t+1|t}, ... $.

The model in linearised form (see Appendix A for the linearisation) shows that the optimal vacancy posting policy, which produces current labour market tightness θ_t given the state of

the employment rate, is a function of the exogenous parameters (summarised by ψ_1 and ψ_2) and the path of past, current, and expected future labour productivity:

$$\hat{\theta}_t = \psi_1 E_t \left\{ \sum_{s=0}^{\infty} \psi_2^s \hat{P}_{t+s} \right\} \ . \tag{25}$$

The job creation condition of the model with exogenous job destruction is approximated by equation (25). Hence the current level of tightness is approximated by a function which includes the expectation formation process and captures the value of current and future relative labour productivity. $\hat{P}_t$ is is the linearisation of $P_t = \frac{a_t}{\prod_{s=1}^{L} a_{t-s}^{\gamma s}}$. $\psi_1 = \frac{\pi m}{\kappa \xi \theta^{\xi}}$ captures the current value of a match, while $\psi_2 = \beta(1-\lambda)(1 - \frac{(1-\pi)}{\xi} m\theta^{1-\xi})$ captures the future match values.

Note that $m\theta^{1-\xi} = p$ is the steady-state job-finding rate. This rate is crucial in determining the surplus value of a match, as a higher job-finding rate in steady-state will result in a more fluid labour market and a lesser surplus generated from matching. Imposing the Hosios efficiency condition (Hosios, 1990) further results in the bargaining power of the worker $(1 - \pi)$ being equal to the elasticity of the matching function for searchers ξ meaning $\frac{(1-\pi)}{\xi} = 1$. Rewrite the approximated job creation condition:

$$\hat{\theta}_t = \underbrace{\psi_1 P_t}_{\text{Current Match Value}} + \underbrace{\psi_1 E_t \left\{ \sum_{s=0}^{\infty} (\beta(1-\lambda)(1-p))^s \hat{P}_{t+s} \right\}}_{\text{Future Expected Match Value depending negatively on } \lambda \text{ and } p} . \tag{26}$$

equation (26) says that the structure of the labour market determines how much future realisations of relative labour productivity P matter for current hiring policy. Higher churn in the labour market due to higher separations λ and job-finding rates p will mean that future expectations of labour productivity matter less. Theoretically, it is therefore clear that job creation in more sclerotic labour markets may be more susceptible to belief shocks as expectations about the future have a higher potential to drive current hiring policies.

4.3 Summary of the models

The set of equations defining model with endogenous job destruction is summarised in Table 1. Appendix A shows the model with exogenous job destruction. The models are approximated via perturbation for the estimation.

The structure of the information process requires that the model is not only solved for

the current state via perturbation but also for the correctly or wrongly expected future paths of the state variables, which include the employment rate, the expected path of labour productivity and all relevant past realisations of labour productivity which influence the value of the catch-up process. To save space, the model summary tables only include the present-time equations, but the dynamic future equilibria are also part of the model estimation.

Both models are estimated twice estimating two different sets of exogenous processes. In the first case the model is estimated with the productivity process presented above following Blanchard et al. (2013) and including permanent and temporary productivity shocks as well as noise shocks. In the second case the model is estimated using the equivalent decomposition from Chahrour and Jurado (2018) transforming the temporary, permanent and noise processes into two processes one capturing fundamentals and the other beliefs.

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5   Data and calibration

I calibrate and estimate the model on labour market and labour productivity data for the United States, United Kingdom and France. I have chosen this set of economies to represent three different kinds of labour markets. The United States provides data for a fluid labour market with high job-finding rates and low match severance costs for the firm. The United Kingdom is a less fluid labour market with low job-finding rates and low match severance costs. France is an even less fluid labour market with low job-finding rates and high layoff costs. A summary for the series of these three economies is in Appendix B.

5.1   Series description

I estimate the models on the differences of aggregate series capturing labour productivity, job-finding rates, unemployment rates, and vacancies.

For the United States and France, data is available between 1990 and 2020, while for the United Kingdom data is available from 1992 to 2020. The period of the pandemic is excluded from the estimation as the large labour market interventions are unlikely to make it possible to identify beliefs from observed agent employment policies during this period.

5.2   Calibration

The simulation and estimation require calibration of the model. The main parameters to calibrate are the weights on the catch-up process and the labour market structure. I calibrate

Equation Description	Model with Endogenous Job Destruction and Endogenised Match Efficiency	
	Productivity process	*Equivalent productivity process for the Beliefs - and Fundamentals- shock decomposition following Chahrour and Jurado (2018)*
Productivity process	$a_t = x_t + z_t$	$a_t = -\frac{\rho}{(1-\rho)^2}(ma_t + ma_{t-2}) + \frac{(1+\rho^2)}{(1-\rho)^2}ma_{t-1}$
Temporary process	$z_t = \rho z_{t-1} + \eta$	Not part of the model
Permanent process	$\Delta x_t = \rho\Delta x_{t-1} + \epsilon_t$	Not part of the model
Moving average	Not part of the model	$ma_t = (1+2\rho)ma_{t-1} - \rho(2+\rho)ma_{t-2} + \rho^2 ma_{t-3} + (1-\rho)\epsilon_t$
Signal	$s_t = x_t + \nu_t$	$s_t = ma_{t-1} + \hat{\nu}_{t-1};$
Signal process	Not part of the model	$\hat{\nu}_t = \rho(2\hat{\nu}_{t-1} - \rho\hat{\nu}_{t-2} + \delta_2^{-0.5}\nu_t - \delta_1\delta_2^{-0.5}\nu_{t-1} + \delta_2^{0.5}\nu_{t-2})$

Information Extraction

Extraction process for the current permanent component	$x_{t\|t} = K_1(x_{t-1\|t-1}, x_{t-2\|t-1}, a_t, s_t)$
Extraction process for the past permanent component	$x_{t-1\|t} = K_2(x_{t-1\|t-1}, x_{t-2\|t-1}, a_t, s_t)$
Extraction process for the temporary component	$z_{t\|t} = K_3(x_{t-1\|t-1}, x_{t-2\|t-1}, a_t, s_t)$

Additional parameter definitions: K_1 to K_3 are the relevant parts of the converged agent Kalman filter and the component forecasting process

$\delta_1 = \delta + \text{complexconjugate}(\delta)$ and $\delta_2 = \delta\,\text{complexconjugate}(\delta)$

$\delta = \frac{1}{2\rho}(1 + \rho^2 + \rho^{0.5}\sigma_u/\sigma_\nu i - [(1 + \rho^2 + \rho^{0.5}\sigma_u/\sigma_\nu i)^2 - 4\rho^2]^{0.5})$

Labour market and the household budget constraint

Job Creation Condition	$\frac{\kappa}{q(\theta_t)} = (1 - H(\tilde{\zeta}_t^N))\left[\exp(\hat{\zeta}_t^N - \hat{\zeta})\pi(P_t - b) + E_t\left\{\frac{1}{r_t}(1-\lambda)\tilde{J}_{t+1}\frac{1}{\prod_{s=1}^L a_{t+1-s}^{\gamma_s}}\right\}\right]$
Future value of continuing a match to the firm	$\tilde{J}_{t+1}\frac{1}{\prod_{s=1}^L a_{t+1-s}^{\gamma_s}} = \hat{J}_{t+1}\frac{1}{\prod_{s=1}^L a_{t+1-s}^{\gamma_s}} - (1-\pi)\kappa\theta_{t+1} + (1-\pi-H(\tilde{\zeta}_{t+1}))\kappa_F - p_{t+1}(1-\pi)(1-H(\tilde{\zeta}_{t+1}^N))\kappa_H$
Value of a continuing match	$\hat{J}_t = (1 - H(\tilde{\zeta}_t))\left[\exp(\hat{\zeta}_t - \hat{\zeta})\pi(a_t - b_t) - (1-\pi)\kappa_{F,t} + E_t\left\{\frac{1}{r_t}(1-\lambda)\tilde{J}_{t+1}\right\}\right]$
Stochastic match destruction condition	$\exp(\tilde{\zeta}_t^N - \hat{\zeta}^N) = \frac{1}{P_t}\left[b - (1-\lambda)E_t\left\{\frac{1}{r_t}\frac{1}{\pi}\frac{1}{Q_{t+1}}[\frac{1}{\prod_{s=1}^L a_{1+t-s}^{\gamma_s}}\hat{J}_{t+1} - (1-\pi)\kappa\theta_{t+1} + \tilde{\kappa}]\right\}\right]$
Job destruction condition	$\exp(\tilde{\zeta}_t - \hat{\zeta}) = \frac{1}{P_t}\left[b - \kappa_F - (1-\lambda)\frac{1}{\pi}E_t\left\{\frac{1}{r_t}\frac{1}{Q_{t+1}}[\frac{1}{\prod_{s=1}^L a_{1+t-s}^{\gamma_s}}\hat{J}_{t+1} - (1-\pi)\kappa\theta_{t+1} + \tilde{\kappa}]\right\}\right]$
Law of Motion for Employment	$n_t = (1-\lambda)(1 - H(\tilde{\zeta}_t))n_{t-1} + (1 - H(\tilde{\zeta}_t^N))u_t p(\theta_t)$
Searchers	$u_t = 1 - (1-\lambda)n_{t-1}$
Interest rate	$\frac{1}{r_t} = E_t\left\{\beta\frac{c_{t+1}^{-\iota}}{c_t^{-\iota}}\right\}$
Tightness	$\theta_t = \frac{v_t}{u_t}$
Probability of matching with a worker	$q(\theta_t) = m\theta_t^{-\xi}$
Probability of finding a match for the worker	$p(\theta_t) = m\theta_t^{1-\xi}$
Consumption	$c_t = a_t\exp(\hat{\zeta}_t)(1-\lambda)(1 - H(\tilde{\zeta}_t))n_{t-1} + a_t\exp(\hat{\zeta}_t^N)(1 - H(\tilde{\zeta}_t^N))u_t p(\theta_t) + (1-n_t)b_t$

Additionally all past states of labour productivity, and future expectations of all state variables:
$a_{t,L}, ... a_t, ...\ E(a_{t+s}|x_{t\|t}, x_{t-1\|t}, z_{t\|t})$ and $E(n_{t+s}|n_t, x_{t\|t}, x_{t-1\|t}, z_{t\|t})$.

Table 1: Summary of the two models with endogenous job destruction estimated.

other parameters such as the instant utility function and the discount rate to standard values.

5.2.1 Calibrating the relative labour productivity process

The job creation condition ensures that changes in the employment rate will be proportional to changes in the relative product of labour as given by equation (27). Given a labour market state n_{t-1}, the response of employment is going to be in the long-run proportional to the differences of current and past productivity realisations:

$$\Delta n_t \propto \frac{a_t}{\prod_{s=1}^{L} a_{t-s}^{\gamma_s}} \ . \tag{27}$$

Taking the logs of the catch-up process allows it to be written as a sum of γ_s weighted changes:

$$\log(\frac{a_t}{\prod_{s=1}^{L} a_{t-s}^{\gamma_s}}) = (\sum_{s=1}^{L} \gamma_s)\Delta a_t + (\sum_{s=2}^{L} \gamma_s)\Delta a_{t-1}... + \gamma_s \Delta a_{t-s} \ .$$

A regression of employment rate changes Δn_t on past and present changes in a reveals the relative importance of each lag. While it would not be possible to recover the exact relation equation (27) of the employment response given the model assumptions, it is possible to recover the proportion of the weights on each lag γ_s from the estimates. The reason for this is that in the long run any temporary or noise shocks will bias the coefficients downward. However, the noise brought into the regression, will not change the relative importance of each lag. As a result, we can uncover γ_s from a regression of the form in equation (28),

$$\Delta n_t = \beta_0 \sum_{s=1}^{L} \beta_s \Delta a_{t-s} + u_t \ . \tag{28}$$

We can then recover each γ_s as $\hat{\gamma}_j = \frac{\sum_{s=j}^{L} \hat{\beta}_s}{\sum_{s=1}^{L} s\hat{\beta}_s}$. The weights for each economy are plotted in Appendix B. I only find the first 7 lags to be significant and the importance is declining over the with the first or second lag having the largest weight. Calculating the proportional importance of each weight is together with the stationarity assumption of the job creation condition sufficient for calibrating the productivity process.

Parameter	Value			Description
β	0.9967			Monthly discount factor for an annual discount factor of 0.96.
ι	1			Specifying the instant utility function of the household as $u(C) = \log(c)$.
κ	0.1			Vacancy posting cost. Close to the suggested value in Hagedorn and Manovskii (2008) 0.1115.
π	$1 - \xi$			Bargaining power of the firm. Calibrated to fulfil the Hosios (1990) efficiency condition.
σ_ζ	0.15			Volatility of the idiosyncratic shocks calibrated to the value in Sedláček (2014).
	US	UK	FR	
m	0.416	0.188	0.241	Estimated match efficiency.
ξ	0.650	0.660	0.478	Estimated elasticity of the matching function.
λ (ex)	0.0394	0.0098	0.0073	Exogenous job destruction rate for the model with exogenous job destruction.
λ (end)	0.0476	0.0098	0.0072	Exogenous job destruction rate calibrated for the model with endogenous job destruction.
b (ex)	0.8028	0.9299	0.9870	Unemployment benefit for the model with exogenous job destruction.
b (end)	0.7988	0.9299	0.9870	Unemployment benefit for the model with endogenous job destruction.
κ_F	0.177	0.232	0.327	Match severance cost for the model with endogenous job destruction.

Table 2: Model calibration

5.2.2 Calibrating the labour market structure

The model is calibrated to the labour market structure of the specific economy. The discount rate β, the parameter determining the utility function of the household ι and the equilibrium vacancy posting cost κ are assumed to be common across economies. This cost is calibrated lower than the equivalent monthly value in Shimer (2005) and close to the suggested value in Hagedorn and Manovskii (2008) $0.474/4.25 = 0.1115$. The calibration is close to the values implying a high elasticity of tightness in Pissarides (2009).

I estimate the parameters of the matching function from observed job-finding rates and labour market tightness as explained in Appendix B. The long-run unemployment rate, together with the long-run job-finding rate allow for identifying the long-run exogenous job destruction rate. In the model with endogenous job destruction, the exogenous job destruction rate has to be found simultaneously with the equilibrium endogenous job destruction rates. Finally, the parameters of the matching function and other parameters determining the labour market structure allow for non-linearly solving for the unemployment benefit that will deliver the observed labour market tightness implied by the job-finding rate.

Finally, the model with endogenous job destruction requires calibration of a match severance cost for continuous matches to endogenise movements of match efficiency following Sedláček (2014). For the United States the value is set to the value in Sedláček (2014). The relative strength of these severance costs are then scaled for the United Kingdom and France using the respective relative strength of employment protection as estimated by the OECD for these two economies compared to the United States. These scaling factors are 1.7/1.3 and 2.4/1.3 respectively. The parameter calibration is summarised in table 2.

6 Estimation of Beliefs and Fundamentals

In this section, I first show that an empirical decomposition of the employment rate responds to temporary and permanent shocks as the model would predict. I then present the results of the estimation of the model with exogenous job destruction and the model with endogenous job destruction. I estimate both models twice once estimating noise, temporary, and permanent shocks as in Blanchard et al. (2013) and then estimating beliefs and fundamental shocks using the transformation of noise, permanent and temporary shocks into fundamental and pure belief shocks suggested in Chahrour and Jurado (2018). For the model with exogenous job destruction, I allow for exogenous match productivity movements. These are identified via the vacancy, unemployment and job-finding rate series. For the model with endogenous job destruction, match productivity movements are endogenised via stochastic matching and match severance cost.

Employment rates respond both to temporary and permanent changes in labour productivity. The model predicts that the employment rate rises in response to a positive labour productivity shock as well as in response to a temporary shock. However, the increase in the employment rate following a temporary labour productivity increase will be less persistent. An estimation approach using the long-run restrictions proposed by Blanchard and Quah (1993) to identify permanent and temporary shocks for the non-stationary labour productivity series and the stationary employment rate series shows that indeed a permanent increase in labour productivity increases employment for a prolonged period, while a temporary increase only leads to a brief increase in the employment rate. The impulse responses of the employment rate for the three economies with regard to permanent and temporary labour productivity improvements are shown in figure 3. The response of the labour productivity process itself and more details on the estimation Appendix C.

While this structural vector auto-regression is appropriate for identifying the direction and persistence of labour productivity shocks, beliefs about the future and information frictions would bias any identified impulse response in an undetermined direction. For this reason, we estimate the developed DSGE models instead by maximising the likelihood with respect to the persistence and volatility of the exogenous processes. The target of the estimation is to identify the volatility of noise or belief shocks σ_ν, and volatility of temporary and permanent or fundamental shocks σ_u. In the case of the models with exogenous job destruction σ_m is estimated as well. Finally, as the response is going to depend on the persistence of shocks ρ is estimated as well. These parameters influence the information problem and the expectation formation process via the agent Kalman filter and hence determine agent job creation decisions.
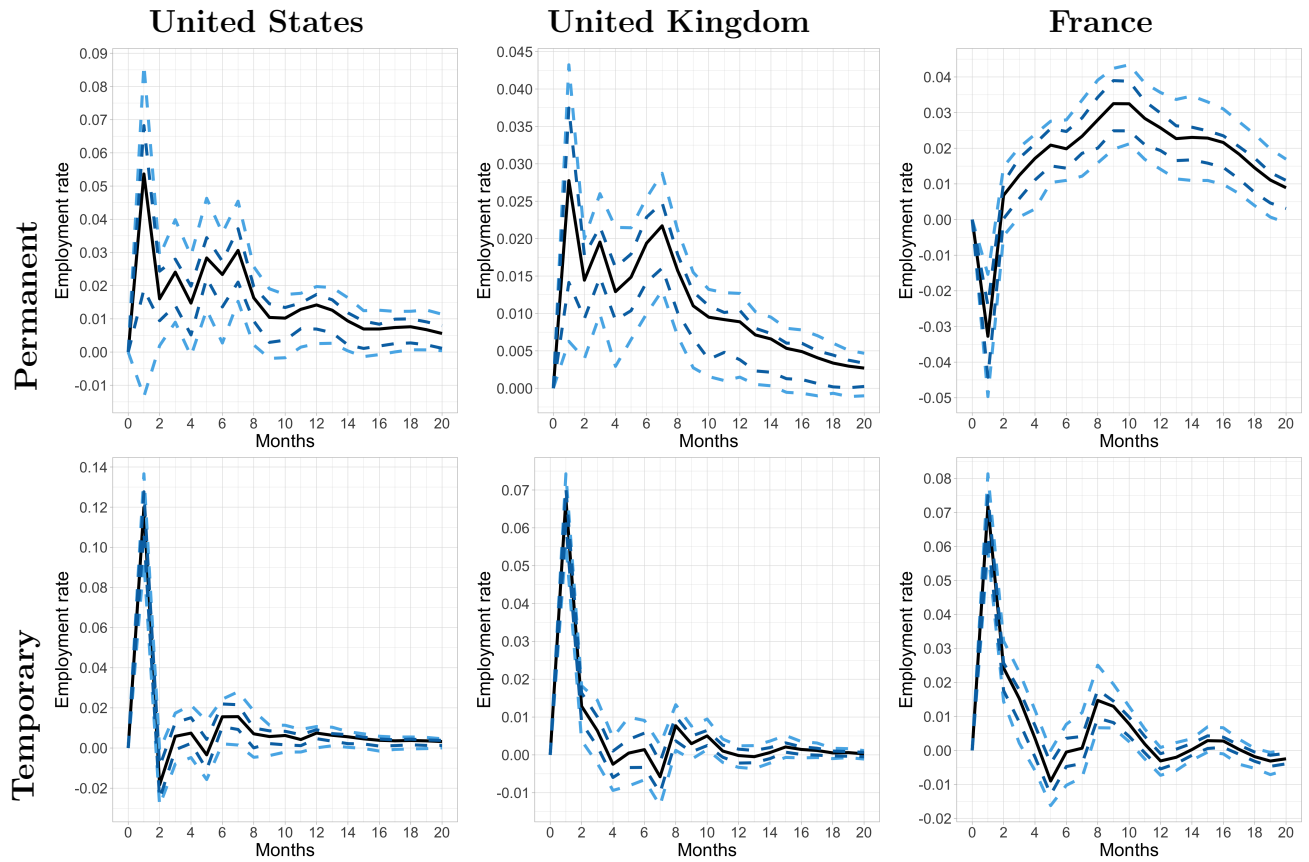
Figure 3: Response of the employment rate to a permanent and temporary labour productivity improvement following the shock identification proposed by Blanchard and Quah (1993). The light blue lines show the 90% confidence interval, while the dark blue lines show the 67% confidence interval.

Job Destruction	Country	Fundamental volatility	Belief volatility	Beliefs share	Fundamentals share
Exogenous	US	0.69	0	2.57	97.43
Exogenous	UK	0.19	0.07	20.6	79.4
Exogenous	FR	0.19	0.08	43.4	56.6
Endogenous	US	0.8	0.08	1.7	98.3
Endogenous	UK	0.39	0.01	34.54	65.46
Endogenous	FR	0.16	0.06	54.84	45.16

Table 3: The fourth and fifth columns show the estimated volatility of fundamental and belief shocks. The right two columns variance decomposition with regard to the employment rate.

6.1 Estimation results

Estimation of the models shows that beliefs play a larger role in economies with less fluid labour markets. Thus not only are these economies more susceptible to belief shocks, as explained in the above section, these economies indeed experience a larger share of belief shocks driving job creation. This result holds consistently for estimates of the model with exogenous job destruction and match efficiency movements and for estimates of the model with endogenous job destruction and endogenous match efficiency movements.

The estimates for the share of pure beliefs following the decomposition in Chahrour and Jurado (2018) are presented in Table 3[2]. Pure beliefs are movements in the employment rate which cannot be explained by current or future labour productivity movements. Noise as in Blanchard et al. (2013) forms part of these movements, but also news which is neutralised by news in an opposite direction before any observed realisation. The raw results of the estimations are found in Appendix C.

All four model specifications presented in Table 5 and Table 1 yield qualitatively similar results. Beliefs have a stronger effect in the models with endogenous job destruction as the destruction condition for new matches and the job destruction condition for continuing additionally boost the importance of the value of the future expected match output. The discussion of the impulse responses focuses on the model with endogenised destruction and match efficiency identifying beliefs as this is the fullest model encompassing all different elements. However, the impulse response function for the models with exogenous job destruction and match efficiency movements shown in Appendix C are qualitatively similar.

The higher share of beliefs in the United Kingdom and France can be seen in the estimated impulse response to belief and fundamental shocks. Figure 4 shows the response to belief

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[2]The ordering and magnitude of the estimates are robust to variations in the individual γ_s and ξ within their estimated standard deviation ranges. For the γ_s other γ_s I adjust other γ estimates such that $\sum_{s=1}^{S} \gamma_s = 1$ continues to hold.

Job Destruction	Country	Permanent volatility	Noise volatility	Noise share	Temporary share	Permanent share
Exogenous	US	0.94	0.1	0.27	41.23	58.51
Exogenous	UK	0.47	0.04	1.62	44.31	54.07
Exogenous	FR	0.11	0.09	24.03	29.57	46.41
Endogenous	US	2.04	0.01	0.01	45.29	54.7
Endogenous	UK	0.49	0.06	2.29	36.65	61.06
Endogenous	FR	0.18	0.22	37.39	32.12	30.49

Table 4: The fourth and fifth columns show the estimated volatility of noise and the volatility of permanent labour productivity. The right three columns variance decomposition with regard to the employment rate.

shocks for the model with endogenous job destruction and match efficiency. The estimated impulse response function for the other models are in Appendix C and show a similar picture. The figure shows in the first row that a fundamental shock indeed drives labour productivity, while a belief shock has no impact. The belief and fundamental decomposition combines the effect of temporary and permanent shocks. Belief shocks also drive the employment rate and job-finding rate in all three economies.

The impulse response function to a positive fundamental shock to labour productivity can be understood in the following way. The reason for the initial brief fall is that the fundamental shock combines temporary and permanent labour productivity changes to one impulse response. Appendix C shows the estimation of the impulse response function of each of these shocks separately. Improved current and expected labour productivity fundamentals cause firms to post vacancies increasing the job-finding and employment rate.

The impulse response function to a positive belief shock in contrast is neutral with regard to labour productivity. However, as the belief shock causes expectations about labour productivity fundamentals to rise firms start to hire increasing the job-finding and employment rate. Note that in economies where the belief share is larger, the response of the employment and job-finding rate is more similar for both shocks. The reason is that agents know in these economies that a positive signal about the future is close to being equally likely due to a pure belief and a fundamental shock. As a result, they adjust their hiring behaviour in both cases similarly.

Furthermore, the transmission of a fundamental shock takes longer in economies with a higher share of beliefs driving employment. It reaches its peak for the employment rate after about half a year in the United States, close to a year in the United Kingdom and one and a half years in France. The reason for this is that in more beliefs-driven economies agents face higher information frictions. This is best illustrated by the estimation results of the model identification following Blanchard et al. (2013) where the noise is estimated separately from

the permanent and temporary components table 4. Agents in the more noisy economies know that it is harder for them to disentangle fundamental labour productivity changes from pure beliefs and therefore adjust their hiring behaviour in response to positive signals more cautiously.

This means that agents in the United Kingdom and France tend to wait longer before increasing job creation. Only when agents are more confident that labour productivity indeed increased in the long run, do they increase job postings and drive the employment rate up. Less fluid labour markets amplify the effect of information frictions delaying the response to a positive labour productivity signal. The reason for this is that the expected value of a job match forms a larger part of the total match value. This total match value drives the job creation decisions and information frictions make the exact extraction of this value more difficult. This explains the delayed peaks of the employment rate and job-finding rate impulse response to fundamental and belief shocks.

The shock decomposition of drivers of the unemployment rate suggests that belief shocks are indeed correctly identified and measured in the developed DSGE model. The OECD composite leading indicators have not been used for the estimation of the model. This series is meant to capture the combined producer and consumer sentiment in each of the three economies. The series should therefore capture beliefs about the future. Figure 5 shows that beliefs and the series are significantly correlated. Thus identified belief shocks that reduce the unemployment rate are correlated with positive producer and consumer sentiments. A similar correlation cannot be found for identified fundamental shocks. This indicates that beliefs are correctly identified in the model.

6.2 Belief shocks are stronger drivers of the unemployment rate when it is low

During times of lower unemployment belief shocks driving unemployment rate changes have a higher variance and therefore a larger effect on unemployment in either direction. While the figures in Appendix C show that there is no discernible correlation between a lower unemployment rate and the direction of belief shocks, figure 6 shows that times of lower unemployment rates are accompanied by a higher variance of unemployment rate changes attributed to pure belief shocks. The figures plot the deciles of the unemployment rate on the x-axis against the corresponding variance of changes of the unemployment rate driven by belief shocks.

The correlation is stronger for the United Kingdom and France, but holds for all three
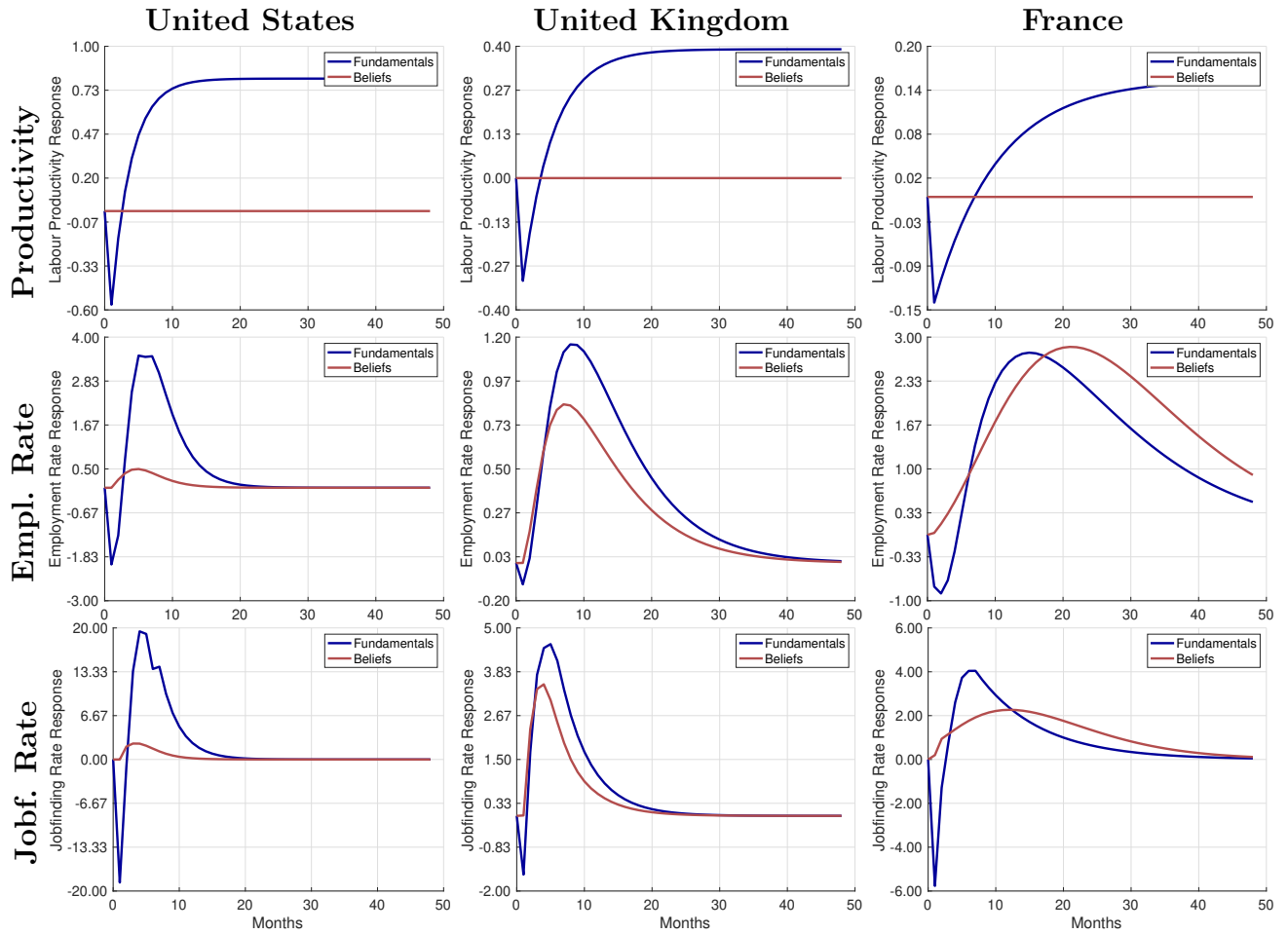
Figure 4: Impulse response to a belief and fundamental shock in the model with endogenous job destruction and endogenous match efficiency.
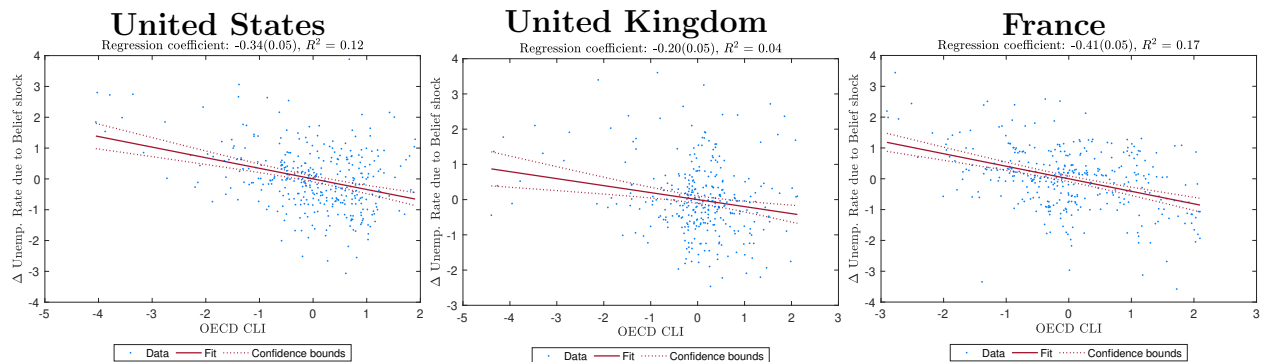

Figure 5: Decomposition of the effect of belief shocks on the unemployment rate plotted against the corresponding measure of the OECD Composite leading indicator not used for the estimation of the model. Higher sentiments are correlated with beliefs reducing the unemployment rate.
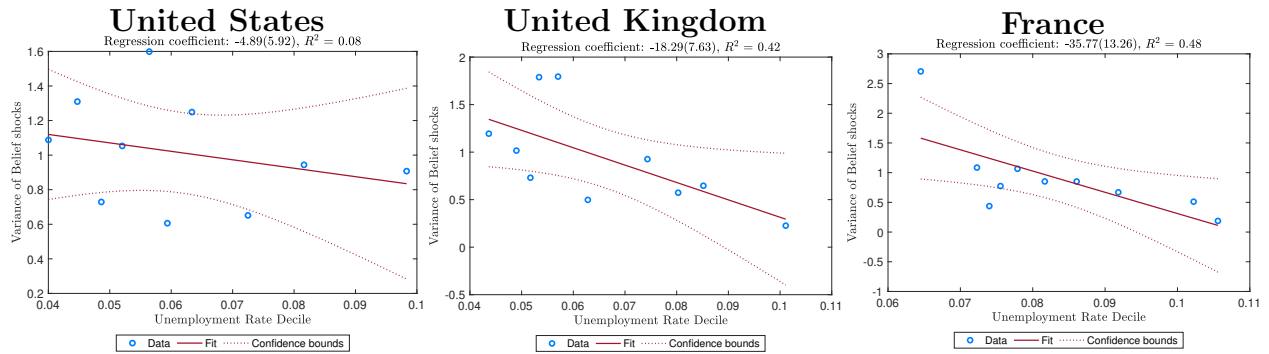
Figure 6: Variance of the effect of belief shocks on the unemployment rate plotted against deciles of the corresponding unemployment rate. Belief shocks explain larger and more dispersed movements of the labour market when the labour market is tight. The effect is stronger in labour markets with lower job-finding and separation rates.

economies. These figures show that during times of a lower unemployment rate, belief shocks have a higher potential to drive the labour market in either direction. The reason for this can be again attributed to the value of the forward-looking component.

In a tight labour market with low unemployment rates, the cost of hiring a new worker is higher. As a result, the value of achieving a match is larger and expectations of future changes to tightness contribute more to the match value. Therefore any changes to these expectations of future tightness will have large effects on job creation resulting in job creation activity rapidly changing as a result of pure belief shifts and without any changes to fundamentals.

7 Conclusion

In this paper, I show both theoretically and empirically that pure beliefs about future labour productivity matter for job creation in sclerotic labour markets with high frictions, but matter less in fluid labour markets with smaller hiring frictions. I find that pure beliefs are not an important driver of the employment rate for the United States, while they are for France and the United Kingdom.

I further find that belief shocks are more important and dispersed drivers of changes to the unemployment rate when the unemployment rate is low. Thus during these times, the labour market is more sensitive to expectations shifts that are unrelated to present or future fundamentals. Thus expectation management by policy institutions is likely to have a higher effect on employment and labour demand during times of a low unemployment rate. This effect is also found to be stronger in economies with higher labour market frictions.

The results are important for policymakers as they show that current job creation is a

reliable indicator of labour productivity fundamentals for the United States, but not a reliable indicator for the United Kingdom or France. This means that employment creation should not be given the same weight in these economies as an indicator of the effectiveness of policies or for judging the current state and future expected path of labour productivity in the economy. Further, the fact that labour markets are particularly sensitive to belief shocks during times when the labour market is tight highlights the importance of expectation management by policymakers during these periods. Not only does this mean that labour markets may become tighter beyond what is warranted by fundamentals, but the result also suggests that unemployment may rapidly rise when expectations shift beyond what labour productivity fundamentals would warrant.

A potentially promising avenue for future research is to estimate the share of beliefs driving job creation in specific sectors for specific occupations or specific types of agents. Sectors and occupations with low transition rates are more likely to be driven by beliefs as the sensitivity of job creation decisions to beliefs is strengthened by high information frictions and low job-finding and vacancy-filling probabilities. For instance, it is likely that hiring decisions for smaller firms with typically higher recruiting frictions are more sensitive to beliefs shocks as these firms are typically facing higher hiring difficulties. Similarly, industries and occupations requiring more specific skills making matching more difficult are probably driven to a higher extent by beliefs. This would make hiring in these markets more costly and less efficient requiring a re-evaluation of optimal policy in response to job creation fluctuations in these sectors or for these agents.

References

Ball, L. M., D. Leigh, and P. Loungani (2013). Okun's law: fit at fifty? Technical report, National Bureau of Economic Research.

Barsky, R. B. and E. R. Sims (2012). Information, animal spirits, and the meaning of innovations in consumer confidence. American Economic Review 102(4), 1343–77.

Beaudry, P. and F. Portier (2006). Stock prices, news, and economic fluctuations. American Economic Review 96(4), 1293–1307.

Blanchard, O. J., J.-P. L'Huillier, and G. Lorenzoni (2013). News, noise, and fluctuations: An empirical exploration. American Economic Review 103(7), 3045–70.

Blanchard, O. J. and D. Quah (1993). The dynamic effects of aggregate demand and supply disturbances: Reply. The American Economic Review 83(3), 653–658.

Chahrour, R. and K. Jurado (2018). News or noise? the missing link. American Economic Review 108(7), 1702–36.

Chodorow-Reich, G. and L. Karabarbounis (2016). The cyclicality of the opportunity cost of employment. Journal of Political Economy 124(6), 1563–1618.

Den Haan, W. J. and G. Kaltenbrunner (2009). Anticipated growth and business cycles in matching models. Journal of Monetary Economics 56(3), 309–327.

Den Haan, W. J., G. Ramey, and J. Watson (2000). Job destruction and propagation of shocks. American economic review 90(3), 482–498.

Di Pace, F., K. Mitra, and S. Zhang (2021). Adaptive learning and labor market dynamics. Journal of Money, Credit and Banking 53(2-3), 441–475.

Forni, M., L. Gambetti, M. Lippi, and L. Sala (2017). Noisy news in business cycles. American Economic Journal: Macroeconomics 9(4), 122–52.

Forni, M., L. Gambetti, and L. Sala (2014). No news in business cycles. The Economic Journal 124(581), 1168–1191.

Hagedorn, M. and I. Manovskii (2008). The cyclical behavior of equilibrium unemployment and vacancies revisited. American Economic Review 98(4), 1692–1706.

Hosios, A. J. (1990). On the efficiency of matching and related models of search and unemployment. The Review of Economic Studies 57(2), 279–298.

Jaimovich, N. and S. Rebelo (2009). Can news about the future drive the business cycle? American Economic Review 99(4), 1097–1118.

Krusell, P. and A. McKay (2010). News shocks and business cycles. FRB Richmond Economic Quarterly 96(4), 373–397.

Ljungqvist, L. and T. J. Sargent (2017). The fundamental surplus. American Economic Review 107(9), 2630–65.

Lorenzoni, G. (2011). News and aggregate demand shocks. Annu. Rev. Econ. 3(1), 537–557.

Martellini, P. and G. Menzio (2020). Declining search frictions, unemployment, and growth. Journal of Political Economy 128(12), 4387–4437.

McCracken, M. W. and S. Ng (2016). Fred-md: A monthly database for macroeconomic research. Journal of Business & Economic Statistics 34(4), 574–589.

Mortensen, D. T. and C. A. Pissarides (1994). Job creation and job destruction in the theory of unemployment. The Review of Economic Studies 61(3), 397–415.

Moscarini, G. (2005). Job matching and the wage distribution. Econometrica 73(2), 481–516.

Pissarides, C. A. (2000). Equilibrium unemployment theory. MIT press.

Pissarides, C. A. (2009). The unemployment volatility puzzle: Is wage stickiness the answer? Econometrica 77(5), 1339–1369.

Schmitt-Grohé, S. and M. Uribe (2012). What's news in business cycles. Econometrica 80(6), 2733–2764.

Sedláček, P. (2014). Match efficiency and firms' hiring standards. Journal of Monetary Economics 62, 123–133.

Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. American economic review 95(1), 25–49.

Shimer, R. (2010). Labor markets and business cycles. In Labor Markets and Business Cycles. Princeton University Press.

Shimer, R. (2012). Reassessing the ins and outs of unemployment. Review of Economic Dynamics 15(2), 127–148.

Zanetti, F. and K. Theodoridis (2016). News shocks and labor market dynamics in matching models. Canadian Journal of Economics 49(3), 906–930.

Appendix for "Beliefs- and Fundamentals-driven Job Creation"

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August 22, 2023

Contents

A Model derivations

A.1 Agents forming expectations about future labour productivity

Agents form their expectations over future labour productivity growth based on the filtered expected values of the past and present permanent productivity component $x_{t|t}$, $x_{t-1|t}$, $z_{t|t}$.

$$a_t = x_{t|t} + z_{t|t}$$

$$E_t(a_{t+1}) = (1 + \rho^x)x_{t|t} - \rho^x x_{t-1|t} + \rho^z z_{t|t}$$

$$E_t(a_{t+2}) = [(1 + \rho^x)^2 - \rho^x]x_{t|t} - (1 + \rho^x)\rho^x x_{t-1|t} + (\rho^z)^2 z_{t|t}$$

This process eventually converges in the long run to equation (29). Thus only shocks to the permanent component will permanently change aggregate labour productivity.

$$E_t(a_{t+\infty}) = \frac{x_{t|t} - \rho^x x_{t-1|t}}{1 - \rho^x} \tag{29}$$

A.2 The agent Kalman filter

The agent Kalman filter is as in Blanchard et al. (2013). Agents know that there is an unobserved process of the form in equation (30), but only observe only a_{t-1} and s_t which are however driven by the same set of shocks according to equation (31).

$$\begin{bmatrix} x_t \\ x_{t-1} \\ z_t \end{bmatrix} = \begin{bmatrix} 1 + \rho & -\rho & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \rho \end{bmatrix} \begin{bmatrix} x_{t-1} \\ x_{t-2} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} e_t \\ \eta_t \\ \nu_t \end{bmatrix} \tag{30}$$

$$\begin{bmatrix} a_t \\ s_t \end{bmatrix} = \begin{bmatrix} (1 + \rho) & -\rho & \rho \\ (1 + \rho) & -\rho & 0 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ x_{t-2} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} e_t \\ \eta_t \\ \nu_t \end{bmatrix} \tag{31}$$

Agents enter a given period with expectations about the unobserved components and receive a new observation a_{t-1} and s_t. They use this observation to update their expectation of the unobserved process using the Kalman gain. Finally, expectations on the permanent and temporary component of a_t have to add up to a_t and the signal is a combination of news and noise as shown in equation (32).

$$\begin{bmatrix} a_t \\ s_t \end{bmatrix} = D \begin{bmatrix} x_{t|t} \\ x_{t-1|t} \\ z_{t|t} \end{bmatrix} + \begin{bmatrix} 0 \\ \nu_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{t|t} \\ x_{t-1|t} \\ z_{t|t} \end{bmatrix} + \begin{bmatrix} 0 \\ \nu_t \end{bmatrix} \tag{32}$$

Given these identities, the result is the agent Kalman filter is found in equation (33).

2

$$
\begin{bmatrix} x_{t|t} \\ x_{t-1|t} \\ z_{t|t} \end{bmatrix} = \begin{bmatrix} 1+\rho & -\rho & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \rho \end{bmatrix} (I - KD) \begin{bmatrix} x_{t-1|t-1} \\ x_{t-2|t-1} \\ z_{t-1|t-1} \end{bmatrix} + K \begin{bmatrix} a_{t-1} \\ s_t \end{bmatrix} \tag{33}
$$

Where I is the identity matrix, K is the converged Kalman gain and D a 2x3 matrix. Given that the covariance matrix of the shocks is positive semi-definite, as the three shocks e, η and ν are independent with positive finite variance the Kalman gain will converge.

With equations (31), (32), and (33) the expectations about future productivity growth can be described as the result of current expectations and the fundamental shocks e, η, and ν.

A.3 Linearising the dynamic equilibrium in the exogenous job destruction case

Market clearing and the first-order conditions require that the aggregate constraint of the economy is given by the consumption equation. Define $P = \frac{a_t}{\Pi_{s=1}^{L} a_{t-s}^{\gamma_s}}$. Further, define $\hat{\Pi}_t$ as the linear approximation of the catch-up process.

$$
c_t = P_t n_t - \kappa v_t - b(1 - n_t) + \hat{\Pi}_t \tag{34}
$$

$$
\frac{\kappa}{q(\theta_t)} = \pi(P_t - b) + \beta(1-\lambda)E_t \left\{ Q_{t+1} \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \left(\frac{\kappa}{q(\theta_{t+1})} - (1-\pi)\kappa\theta_{t+1} \right) \right\} \tag{35}
$$

$$
n_t = (1-\lambda)n_{t-1} + v_t q(\theta_t) \tag{36}
$$

$$
u_t = 1 - (1-\lambda)n_{t-1} \tag{37}
$$

$$
\theta_t = \frac{v_t}{u_t} \tag{38}
$$

The steady-state value of a worker-firm relationship relative to the value of unemployment is $p = 1$. The steady-state of a is subject to change if the permanent component is shocked due to the unit root process. For the approximation here the steady-state of a is normalized at 1. Approximating the above equation at this steady-state with a first-order log-linearisation yields:

$$
\hat{c}_t = \frac{n}{c}[\hat{P}_t + \hat{n}_t] - \frac{\kappa v}{c}\hat{v}_t - \frac{bn}{c}\hat{n}_t \tag{39}
$$

$$
\hat{\theta}_t = \psi_1 \hat{P}_t + \psi_2 E_t \hat{\theta}_{t+1} + \psi_3 (E_t \hat{c}_t - E_t \hat{c}_{t+1}) \tag{40}
$$

With $\psi_1 = \frac{\pi m}{\kappa \xi \theta^\xi}$, $\psi_2 = \beta(1-\lambda)(1 - \frac{m(1-\pi)}{\xi}\theta^{1-\xi})$, and $\psi_3 = \sigma\beta(1-\lambda)\frac{(1-m(1-\pi)\theta^{1-\xi})}{\xi} = \sigma(\psi_2 + \beta(1-\lambda)\frac{1-\xi}{\xi})$

$$
\hat{n}_t = (1-\lambda)\hat{n}_{t-1} + \lambda(\hat{v}_t - \xi\hat{\theta}_t) = \frac{(1-n)(1-\lambda)}{1-(1-\lambda)n}\hat{n}_{t-1} + \lambda(1-\xi)\hat{\theta}_t \tag{41}
$$

3

$$\hat{u}_t = \frac{(1-\lambda)n}{1-(1-\lambda)n}\hat{n}_{t-1} \tag{42}$$

$$\hat{\theta}_t = \hat{v}_t - \hat{u}_t \tag{43}$$

If one assumes that $\sigma = 0$, thus that agents have a linear instant utility function it becomes straightforward to show that vacancy postings only depend on last period's employment and the expected productivity path.

$$\hat{\theta}_t = \psi_1 \hat{P}_t + \psi_2 \psi_1 E_t \hat{P}_{t+1} + \psi_2^2 E_t \hat{\theta}_{t+2} = \psi_1 E_t \left\{ \sum_{s=0}^{\infty} \psi_2^s \hat{P}_{t+s} \right\} \tag{44}$$

$$\hat{v}_t = \psi_1 E_t \left\{ \sum_{s=0}^{\infty} \psi_2^s \hat{P}_{t+s} \right\} + \frac{(1-\lambda)n}{1-(1-\lambda)n}\hat{n}_{t-1} \tag{45}$$

This result is generalized to values of $\sigma \geq 0$ by inserting equation (39) and (41), and the resulting equations into (40) substituting out for $\hat{c}_t$, $\hat{c}_{t+1}.\hat{c}_{t+2,...}$. However, the resulting equation for $\hat{\theta}_t$ as a function n_{t-1} and past, current, and future values of a_t does not have a closed form representation.

Note that even if agents knew current period productivity, thus if a_t and p_t would be part of their information set in period t, hiring in the current period will still depend on the future productivity expectations $E_t p_{t+1}, E_t p_{t+2}, ...$ and would thereby still be affected by news and noise besides observed shocks to current productivity. To emphasize the potential effect of news and noise the arguably more realistic modelling choice has been made to exclude the precise value of current aggregate productivity from the agents' information set.

A.4   Production with endogenous job destruction

In the model with endogenous job destruction, we make the same assumptions about the matching function and firm optimal hiring as in the model with exogenous job destruction. The differences consist of the matching efficiency parameter in the matching function being fixed, and pro-cyclical match efficiency movements being captured endogenously via firing cost and stochastic matching following Sedláček (2014).

To achieve this we extend the model with exogenous job destruction by allowing for the productivity of a job match to be subject to non-stochastic idiosyncratic productivity shocks ζ drawn from a distribution $H(\zeta)$. These shocks are observed by the firm and worker at the beginning of the period, before production but after job creation as in Den Haan et al. (2000). This renders the job creation condition changed to equation (46). Any remaining exogenous job destruction is assumed to take place at the end of the period following production.

$$\frac{\kappa_t}{q(\theta_t)} = \int_{\tilde{\zeta}_t^N}^{\bar{\zeta}} \left[\exp(\zeta)a_t - E_t(w_t^N) + \beta(1-\lambda)E_t \left\{ \hat{J}_{t+1} - H(\tilde{\zeta}_{t+1} - \hat{\zeta}^N)\kappa_{F,t+1} \right\} \right] h(\zeta)d\zeta \quad (46)$$

The left side of the equation is the expected cost of creating a new job, while the right is the expected benefit to a firm from creating the job. $\bar{\zeta}$ is the upper most realisation of stochastic ζ, while $\tilde{\zeta}_t$ is the ζ realisation below which the firm and worker will decide to sever the match. This will be determined via the job destruction condition in equation (equation (47)), which can be found by substituting out for the wage in equation (51). $\kappa_{H,t}$ captures hiring cost in the spirit of Pissarides (2009) and may be increased above 0 to increase unemployment volatility. Termination cost $\kappa_{F,t}$ allows for capturing changes to match efficiency endogenously over the cycle as in Sedláček (2014). Both costs are assumed to be proportional to past realisations of the match productivity similar to κ_t and b_t. Hiring or firing cost will result in the value of new hires denotes by the superscript N differing from the value of continuing matches.

Equation (47) the stationary job destruction condition for new matches.

$$\exp(\tilde{\zeta}_t^N - \hat{\zeta}^N) = E_t \left\{ \frac{\prod_{s=1}^{L} a_{t-s}^{\gamma_s}}{a_t} \left[b - (1-\lambda)\frac{1}{r_t}\frac{1}{\pi}(\frac{\prod_{s=1}^{L} a_{1+t-s}^{\gamma_s}}{\prod_{s=1}^{L} a_{t-s}^{\gamma_s}})[\frac{1}{\prod_{s=1}^{L} a_{1+t-s}^{\gamma_s}}\hat{J}_{t+1} - (1-\pi)\kappa\theta_{t+1} + \tilde{\kappa}] \right] \right\}$$
$$(47)$$

Wages for new hires are given by equation (48).

$$w_t^N = \left[\pi b_t + (1-\pi)[\exp(\zeta - \hat{\zeta}^N)a_t + (1-\lambda)E_t \left\{ \frac{1}{r_t}(\kappa_{t+1}\theta_{t+1} + p_{t+1}(1 - H(\tilde{\zeta}_{t+1}^N))\kappa_{H,t+1} - \kappa_{F,t+1}) \right\}] \right]$$
$$(48)$$

Equation (49), equation (50), and equation (51) show the job creation, job destruction, and wage equation for continuing matches.

$$\hat{J}_t = \int_{\tilde{\zeta}_t}^{\bar{\zeta}} \left[\exp(\zeta)a_t - E_t(w_t) + \beta(1-\lambda)E_t \left\{ \hat{J}_{t+1} - H(\tilde{\zeta}_{t+1})\kappa_{F,t+1} \right\} \right] h(\zeta)d\zeta \quad (49)$$

$$\exp(\tilde{\zeta}_t - \hat{\zeta}) = \frac{\prod_{s=1}^{L} a_{t-s}^{\gamma_s}}{a_t} \left[b - \kappa_F - (1-\lambda)E_t \left\{ \frac{1}{r_t}\frac{1}{\pi}(\frac{\prod_{s=1}^{L} a_{1+t-s}^{\gamma_s}}{\prod_{s=1}^{L} a_{t-s}^{\gamma_s}})[\frac{1}{\prod_{s=1}^{L} a_{1+t-s}^{\gamma_s}}\hat{J}_{t+1} - (1-\pi)\kappa\theta_{t+1} + \tilde{\kappa}] \right\} \right]$$
$$(50)$$

$$w_t = \pi b_t + (1-\pi)[\exp(\zeta)a_t + \kappa_{F,t} + (1-\lambda)\frac{1}{r_t}E_t\left\{\kappa_{t+1}\theta_{t+1} + p_{t+1}(1 - H(\tilde{\zeta}_{t+1}^N))\kappa_{H,t+1} - \kappa_{F,t+1}\right\}]$$
<div align="right">(51)</div>

$$\tilde{\kappa}_{t+1} = (1-\pi)p_{t+1}(1 - H(\tilde{\zeta}_{t+1}^N))\kappa_{H,t+1} - (1-\pi - H(\tilde{\zeta}_{t+1}))\kappa_{F,t+1}$$
<div align="right">(52)</div>

Assuming ζ is drawn from a normal distribution and $\zeta \sim N(-\sigma_\zeta^2/2, \sigma^2)$ we can compute the expected value produced by a job match. The choice of mean ensures that the unconditional distribution $\exp(\zeta)$ has an expectation of 1. Equation (53) shows the mean idiosyncratic labour productivity for new matches $\hat{\zeta}_t^N$. Equation (54) shows the mean idiosyncratic labour productivity for continuing matches $\hat{\zeta}_t$. ϕ and Φ here represent the pdf and cdf of the normal distribution.

$$\hat{\zeta}_t^N = \Phi((\sigma_\zeta^2/2 - \tilde{\zeta}_t^N)/\sigma_\zeta) - [1 - \Phi((\tilde{\zeta}_t^N + \sigma_\zeta^2/2)/\sigma_\zeta)]$$
<div align="right">(53)</div>

$$\hat{\zeta}_t = \Phi((\sigma_\zeta^2/2 - \tilde{\zeta}_t)/\sigma_\zeta) - [1 - \Phi((\tilde{\zeta}_t + \sigma_\zeta^2/2)/\sigma_\zeta)]$$
<div align="right">(54)</div>

Equations (54) and (51) let us write the job creation condition as equation (55). Finally assume that $\hat{\zeta}$ is the steady state value and that it is subtracted from $\hat{\zeta}_t$. This allows for the steady state of the expected output of a worker to be equal to 1.

$$\frac{\kappa_t}{q(\theta_t)} + (1 - H(\tilde{\zeta}_t^N))\kappa_{H,t} = (1 - H(\tilde{\zeta}_t^N))\exp(\hat{\zeta}_t - \hat{\zeta})\pi(a_t - b_t) + E_t\left\{\frac{1}{r_t}(1-\lambda)\tilde{J}_{t+1}\right\}$$
<div align="right">(55)</div>

The future value of a continuing match is given by equation (56).

$$\tilde{J}_{t+1} = [\hat{J}_{t+1} - (1-\pi)\kappa_{t+1}\theta_{t+1} + (1-\pi - H(\tilde{\zeta}_{t+1}))\kappa_{F,t+1} - p_{t+1}(1-\pi)(1 - H(\tilde{\zeta}_{t+1}^N))\kappa_{H,t+1}]$$
<div align="right">(56)</div>

The present value of a continuing match is given by equation (57).

$$\hat{J}_t = (1 - H(\tilde{\zeta}_t))\left[\exp(\hat{\zeta}_t - \hat{\zeta})\pi(a_t - b_t) - (1-\pi)\kappa_{F,t} + E_t\left\{\frac{1}{r_t}(1-\lambda)\tilde{J}_{t+1}\right\}\right]$$
<div align="right">(57)</div>

The law of motion for employment is transformed to incorporate endogenous and exogenous job destruction as shown in equation (58).

$$n_t = (1-\lambda)(1 - H(\tilde{\zeta}_t))n_{t-1} + (1 - H(\tilde{\zeta}_t^N))u_t p(\theta_t)$$
<div align="right">(58)</div>

Consumption in ant period is given by equation (59).

<div align="center">6</div>

$$c_t = a_t \exp(\hat{\zeta}_t)(1-\lambda)(1-H(\tilde{\zeta}_t))n_{t-1} + a_t \exp(\hat{\zeta}_t^N)(1-H(\tilde{\zeta}_t^N))u_t p(\theta_t) + (1-n_t)b_t \quad (59)$$

A.5 Steady state with endogenous job destruction

The steady state level tightness of the model is found via a non-linear solver defining the steady state employment rate.

Job creation condition

$$\frac{\kappa}{m}\theta^\xi(1-\Phi(\frac{\tilde{\zeta}^N+\sigma_\zeta^2/2}{\sigma_\zeta}))^{-1} + \kappa_H = \left[\pi(1-b) + \beta(1-\lambda)[\tilde{J}]\right] \quad (60)$$

$$\hat{J}\frac{1}{(1-\Phi(\frac{\tilde{\zeta}+\sigma_\zeta^2/2}{\sigma_\zeta}))} = \pi[1-b] - (1-\pi)\kappa_F + \beta(1-\lambda)[\tilde{J}] \quad (61)$$

$$\tilde{J} = [\hat{J} - (1-\pi)\kappa\theta + (1-\pi-\Phi(\frac{\tilde{\zeta}+\sigma_\zeta^2/2}{\sigma_\zeta}))\kappa_F + (1-\pi)(1-\Phi(\frac{\tilde{\zeta}^N+\sigma_\zeta^2/2}{\sigma_\zeta}))\kappa_H] \quad (62)$$

Job destruction condition

$$\exp(\tilde{\zeta}^N) = \left[b - (1-\lambda)\beta\frac{1}{\pi}[\hat{J} - (1-\pi)\kappa\theta + \tilde{\kappa}]\right] \quad (63)$$

$$\tilde{\kappa} = (1-\pi)p(1-H(\tilde{\zeta}))\kappa_H - (1-\pi-H(\tilde{\zeta}))\kappa_F \quad (64)$$

Combining the job destruction and job creation condition allows for solving for equilibrium tightness non-linearly as:

$$\frac{\frac{\kappa}{m}\theta^\xi}{(1-\Phi(\frac{\log([b-(1-\lambda)\beta[\frac{\kappa}{m}\theta^\xi-(1-\pi)\kappa\theta])+\sigma_\zeta^2/2}{\sigma}))} = \left[\pi(1-b) + \beta(1-\lambda)[\frac{\kappa}{m}\theta^\xi - (1-\pi)\kappa\theta]\right] \quad (65)$$

Equilibrium employment

$$n = (1-\Phi(\tilde{\zeta}))[(1-\lambda)n + (1-(1-\lambda)n)p(\theta_t)] \quad (66)$$

$$n = \frac{p(1-\Phi(\tilde{\zeta}))}{1-(1-\Phi(\tilde{\zeta})(1-\lambda)(1-p))} \quad (67)$$

7

A.6 Summary of the model with exogenous job destruction

Equation Description	Model with Exogenous Job Destruction and Exoegnous Match Efficiency	
	Productivity process	*Equivalent productivity process for the Beliefs - and Fundamentals- shock decomposition following Chahrour and Jurado (2018)*
Productivity process	$a_t = x_t + z_t$	$a_t = -\frac{\rho}{(1-\rho)^2}(ma_t + ma_{t-2}) + \frac{(1+\rho^2)}{(1-\rho)^2}ma_{t-1}$
Temporary process	$z_t = \rho z_{t-1} + \eta$	Not part of the model
Permanent process	$\Delta x_t = \rho \Delta x_{t-1} + \epsilon_t$	Not part of the model
Moving average	Not part of the model	$ma_t = (1+2\rho)ma_{t-1} - \rho(2+\rho)ma_{t-2} + \rho^2 ma_{t-3} + (1-\rho)\epsilon_t$
Signal	$s_t = x_t + \nu_t$	$s_t = ma_{t-1} + \hat{\nu}_{t-1};$
Signal process	Not part of the model	$\hat{\nu}_t = \rho(2\hat{\nu}_{t-1} - \rho\hat{\nu}_{t-2} + \delta_2^{-0.5}\nu_t - \delta_1\delta_2^{-0.5}\nu_{t-1} + \delta_2^{0.5}\nu_{t-2})$

	Information Extraction	
Extraction process for the current permanent component		$x_{t|t} = K_1(x_{t-1|t-1}, x_{t-2|t-1}, a_t, s_t)$
Extraction process for the past permanent component		$x_{t-1|t} = K_2(x_{t-1|t-1}, x_{t-2|t-1}, a_t, s_t)$
Extraction process for the temporary component		$z_{t|t} = K_3(x_{t-1|t-1}, x_{t-2|t-1}, a_t, s_t)$
Additional parameter definitions	K_1 to K_3 are the relevant parts of the converged agent Kalman filter and the component forecasting process	
	$\delta_1 = \delta + \text{complexconjugate}(\delta)$ and $\delta_2 = \delta\,\text{complexconjugate}(\delta)$	
	$\delta = \frac{1}{2\rho}(1 + \rho^2 + \rho^{0.5}\sigma_u/\sigma_\nu i - [(1 + \rho^2 + \rho^{0.5}\sigma_u/\sigma_\nu i)^2 - 4\rho^2]^{0.5})$	

	Labour market and the household budget constraint	
Match efficiency		$m_t = m + \epsilon_{m,t}$
Job Creation Condition		$\frac{\kappa}{q(\theta_t)} = \pi(P_t - b) + \beta(1-\lambda)E_t\left\{Q_{t+1}\frac{c_{t+1}^{-\iota}}{c_t^{-\iota}}(\frac{\kappa}{q(\theta_{t+1})} - (1-\pi)\kappa\theta_{t+1})\right\}$
Law of Motion for Employment		$n_t = (1-\lambda)n_{t-1} + v_t q(\theta_t)$
Searchers		$u_t = 1 - (1-\lambda)n_{t-1}$
Tightness		$\theta_t = \frac{v_t}{u_t}$
Probability of filling a vacancy		$q(\theta_t) = m_t\theta_t^{-\xi}$
Consumption		$c_t = a_t[n_t - \kappa_t v_t + b_t(1-n_t)]$

Additionally all states of labour productivity, and future expectations of all state variables:
$a_{t,L}, \ldots a_t, \ldots E(a_{t+s}|x_{t|t}, x_{t-1|t}, z_{t|t})$ and $E(n_{t+s}|n_t, x_{t|t}, x_{t-1|t}, z_{t|t})$.

Table 5: Summary of the two models with exogenous job destruction estimated.

A.7 Relating the Structural Vector Auto-Regression Identifying Noise to the DSGE model

Forni et al. (2017) suggest alterations to the assumptions taken in Blanchard et al. (2013) to study the reaction of consumption to noise and news in a structural vector auto-regressive model. The identification method makes assumptions similar to Chahrour and Jurado (2018)

in the sense that an identified noise shock ultimately is required not to drive labour productivity at any length, while a fundamental news shock does. If these assumptions hold and agents learn after a certain number of periods with certainty whether a signal was news or noise, then it is possible to identify noise shocks with a structural vector auto-regressive model by rotating the residuals of the vector auto-regression with regard to productivity and the signal instrument accordingly to fulfil the exclusion restriction of noise on the fundamental process.

Agents are assumed to learn with certainty after a number of periods whether a past signal was news or noise. In contrast, in the information process implemented in the DSGE model above agents only ever know with an increasing probability the true nature of past shocks. If agents can retrospectively identify noise shocks, then the econometrician is also able to identify the shocks from the data as it will reflect the choices of the agents, provided that some reliable instrument for the signal based on which agent expectations are formed is available to the econometrician. The signal used for the three economies is the OECD leading composite indicator. The reason for this choice is that it is a signal series consistently available for the time period for all three economies.

The process determining worker productivity is assumed to be a random walk with a drift τ.

$$a_t = a_{t-1} + \tau + \epsilon_{t-S} \tag{68}$$

ϵ_{t-S} is a news shock determined by a finite number of periods S in the past.

This process is related but not the same as the productivity process described in Section 3.1, where $a_t = x_t + z_t = (1+\rho)x_{t-1} - \rho x_{t-2} + \rho z_{t-1} + e_t + \eta_t = \rho a_{t-1} + x_{t-1} - \rho x_{t-2} + e_t + \eta_t$. If ρ is either 0 or 1 and in the second case the permanent component is not subject to shocks then the properties of the two processes are similar and the only difference is the timing of the signal. In the first case the result would be $a_t = x_t = x_{t-1} + \epsilon_{t-S}$, where $e_t + \eta_t = \epsilon_{t-S}$. In the second case $a_t = a_{t-1} + \eta_t = \epsilon_{t-S} = L^S \epsilon_t$.

As in Section 3.1 in each period agents are assumed to observe worker productivity a_t and to receive a noisy signal over future innovations as shown in equation (69).

$$s_t = \epsilon_t + \nu_t \tag{69}$$

There is assumed to exist a cointegrated relationship between the present discounted utility value of a filled vacancy and the value of unemployment. This renders the relative value of a filled vacancy P_t a stationary process, which is assumed to be the de-trended process a_t.

$$\Delta a_t = \Delta a_t - \tau = L^S \epsilon_t \tag{70}$$

The agents are assumed to know that both ϵ_t and ν_t are mean zero normally distributed and uncorrelated with each other and with previous and future realizations. The future

expected values of productivity are then simply projections from e_{t-S} on s_{t-S}. Thus if I_t is the information set of agents at period t then productivity changes in the future can be extracted from the signal according to equation (71).

$$E(\Delta P_{t+1}|I_t) = \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_\nu^2} s_{t-S} \tag{71}$$

It follows that the expected long-run change in the value of productivity is the sum of current productivity and the projections from the at time t available signals on future productivity shocks. This is described by equation (72).

$$E(P_{t+\infty} - P_t|I_t) = \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_\nu^2} \sum_{i=0}^{S} s_{t-i} \tag{72}$$

Given the dynamic equilibrium of the DSGE model it becomes straightforward to form equations for approximating the labour market described in it. Both labour market tightness and the job-finding rate will be a function of the equilibrium value plus deviations to the employment equilibrium in the previous period plus expected deviations of the relative value of the firm-worker relationship, plus an error term to capture exogenous shocks such as shocks to matching productivity.

$$\theta_t = \theta^* + \phi_{0,1}(n^* - n_{t-1}) + E_t\left\{\sum_{i=0}^{\infty} \phi_{0,2+i}\Delta p_{t+1+i}\right\} + e_{3,t} \tag{73}$$

Labour market tightness can be proxied for by the observed job-finding rate.

$$p(\theta_t) = p(\theta^*) + n^* + \phi_{1,1}(n^* - n_{t-1}) + E_t\sum_{i=0}^{\infty} \phi_{1,2+i}\Delta p_{t+1+i} + e_{3,t} \tag{74}$$

Finally using equation (75) in equation (74).

$$p(\theta_t) = p(\theta^*) + n^* + \phi_{1,1}(n^* - n_{t-1}) + \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_\nu^2} \sum_{i=0}^{S} \phi_{1,2+i}L^i s_t \tag{75}$$

Here θ^* and n^* are constants and expected to be the steady-state values when no productivity shocks occur. $e_{3,t}$ captures other shocks to the job-finding rate such as shocks to matching productivity. Finally, the law of motion of employment, or of unemployment can be captured by the equation (76).

$$n_t = n^* + \phi_{2,1}(n^* - n_{t-1}) + \phi_{2,2}(p(\theta^*) - p(\theta_t)) + e_{4,t} \tag{76}$$

The structural VAR system is then found in equation (77).

$$
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \frac{-\sigma_\epsilon^2}{\sigma_\epsilon^2+\sigma_\nu^2}\sum_{i=0}^{S}\phi_{1,2+i}L^i & 1 & 0 \\ 0 & 0 & \phi_{2,2} & 1 \end{bmatrix} \begin{bmatrix} \Delta P_t \\ s_t \\ p(\theta_t) \\ n_t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ p(\theta^*)+\phi_{1,1}n^* \\ (1+\phi_{2,1})n^*+\phi_{2,2}p(\theta^*) \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\phi_{1,1} \\ 0 & 0 & 0 & -\phi_{2,1} \end{bmatrix} \begin{bmatrix} \Delta P_{t-1} \\ s_{t-1} \\ p(\theta_{t-1}) \\ n_{t-1} \end{bmatrix} + \begin{bmatrix} L^S & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \epsilon_t \\ \nu_t \\ e_{3,t} \\ e_{4,t} \end{bmatrix} \quad (77)
$$

This model has a clear ordering. The job-finding rate will affect employment contemporaneously, but not vice versa. This ordering is used in the short-run restrictions imposed in equation (1) to identify surprise and signal shocks.

The reduced form shocks after estimating the vector auto-regression with short run restrictions are then separated into news and noise shocks by rotating the residuals until the identified noise shock has no significant effect on the fundamental series at the assumed length of the signal. In this case the assumed length is to be a maximum of two years, meaning $L = 24$.

B   Data and Empirics

B.1   Summary of the data

United States

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Year.Month	361	2,005.000	8.696	1,990	1,997.5	2,012.5	2,020
Delta YpL1	361	−0.0003	0.002	−0.010	−0.001	0.001	0.006
Delta YpL2	361	0.001	0.002	−0.006	−0.0001	0.002	0.006
Unemployment Rate	361	0.062	0.017	0.037	0.049	0.073	0.106
Delta Employment Rate	361	0.0001	0.002	−0.006	−0.001	0.001	0.005
Monthly Job-finding Rate	361	0.365	0.078	0.182	0.328	0.409	0.543
Vacancy Rate (stock)	230	0.031	0.008	0.015	0.025	0.037	0.049
Tightness Indicator	361	0.569	0.258	0.153	0.357	0.729	1.240
OECD Composite Confidence Indicator	361	99.801	1.246	94.739	99.189	100.713	102.181

United Kingdom

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Year.Month	333	2,006.163	8.030	1,992.167	1,999.250	2,013.083	2,020.000
Delta YpL1	332	0.0001	0.002	−0.012	−0.001	0.001	0.005
Delta YpL2	332	0.001	0.002	−0.011	0.0002	0.002	0.006
Unemployment Rate	333	0.065	0.018	0.038	0.051	0.080	0.108
Delta Employment Rate	332	0.0002	0.001	−0.003	−0.0002	0.001	0.003
Monthly Job-finding Rate	326	0.127	0.028	0.064	0.105	0.149	0.198
Vacancy Rate (stock)	333	0.018	0.005	0.006	0.014	0.021	0.025
OECD Composite Confidence Indicator	333	100.090	1.465	93.936	99.725	100.819	103.172

France

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Year.Month	361	2,005.000	8.696	1,990	1,997.5	2,012.5	2,020
Delta YpL1	361	−0.0003	0.003	−0.024	−0.001	0.001	0.003
Delta YpL2	361	0.0004	0.006	−0.023	−0.003	0.004	0.014
Unemployment Rate	361	0.083	0.013	0.057	0.074	0.090	0.106
Delta Employment Rate	361	0.00003	0.001	−0.003	−0.001	0.001	0.005
Monthly Job-finding Rate 1	361	0.077	0.018	0.044	0.064	0.089	0.118
Monthly Job-finding Rate 2	361	0.077	0.017	0.044	0.065	0.089	0.118
Vacancy Rate (flow)	361	0.009	0.002	0.004	0.008	0.011	0.012
OECD Composite Confidence Indicator	361	99.879	1.581	95.284	98.880	100.986	103.170

B.2 Data description

United States	
Series	**Description**
Delta YpL1	Labour productivity series. Result of the residuals of a regression of changes to real GDP on changes in employment. This will uncover labour productivity movements excluding movements purely due to changes in the employment rate itself. Assume a Cobb-Douglas production function $Y_t = AL^{1-\alpha}K^{\alpha}$. Then $$\Delta \log(Y_t) = \beta_0 + \beta_1 \Delta \log(L) + \epsilon_t$$ Then a regression will uncover any adjustments to total aggregate factor productivity $Cov(\Delta \log(A_t), \Delta \log(L)) > 0$ and aggregate capital utilisation $Cov(\Delta \alpha \log(K_t), \Delta \log(L)) > 0$ as well as any output changes $Cov((1 - \alpha)\Delta \log(L_t), \Delta \log(L)) > 0$ that can be purely attributed to changes in employment. The unexplained residuals ϵ_t represent cleaned labour productivity movements.
Delta YpL2	Labour productivity series using output per Worker. This is calculated as real GDP over employment $\frac{Y_t}{L_t}$.
Unemployment Rate	Downloaded from FRED and calculated from UNEMPLOY and PAYEMS series
Delta Employment Rate	$-\Delta$ Unemployment Rate
Monthly Job-finding Rate	Calculated using FRED series UEMPLT5 and UNEMPLOY following Shimer (2005)
Vacancy Rate	Calculated using FRED series JTSJOL
Tightness indicator	Taken from the updated version on the monthly FRED-MD dataset (McCracken and Ng, 2016)
OECD Composite Confidence Indicator	Downloaded from the OECD database

United Kingdom	
Series	**Description**
Delta YpL1	Labour productivity series. Result of the residuals of a regression of changes to real GDP on changes in employment. This will uncover labour productivity movements excluding movements purely due to changes in the employment rate itself. Assume a Cobb-Douglas production function $Y_t = AL^{1-\alpha}K^{\alpha}$. Then $$\Delta \log(Y_t) = \beta_0 + \beta_1 \Delta \log(L) + \epsilon_t$$ Then a regression will uncover any adjustments to total aggregate factor productivity $Cov(\Delta \log(A_t), \Delta \log(L)) > 0$ and aggregate capital utilisation $Cov(\Delta \alpha \log(K_t), \Delta \log(L)) > 0$ as well as any output changes $Cov((1 - \alpha)\Delta \log(L_t), \Delta \log(L)) > 0$ that can be purely attributed to changes in employment. The unexplained residuals ϵ_t represent cleaned labour productivity movements.
Delta YpL2	Labour productivity series using output per Worker. This is calculated as real GDP over employment $\frac{Y_t}{L_t}$.
Unemployment Rate	From ONS Labour Force Survey data
Delta Employment Rate	$-\Delta$ Unemployment Rate
Monthly Job-finding Rate	Calculated using monthly overlapping durations of unemployment from the Labour Force Survey following Shimer (2005)
Vacancy Rate	From ONS data combined with the millennium of macroeconomic data estimates of the Bank of England
OECD Composite Confidence Indicator	Downloaded from the OECD database

France	
Series	**Description**
Delta YpL1	Labour productivity series. Result of the residuals of a regression of changes to real GDP on changes in employment. This will uncover labour productivity movements excluding movements purely due to changes in the employment rate itself. Assume a Cobb-Douglas production function $Y_t = AL^{1-\alpha}K^{\alpha}$. Then $$\Delta \log(Y_t) = \beta_0 + \beta_1 \Delta \log(L) + \epsilon_t$$ Then a regression will uncover any adjustments to total aggregate factor productivity $Cov(\Delta \log(A_t), \Delta \log(L)) > 0$ and aggregate capital utilisation $Cov(\Delta \alpha \log(K_t), \Delta \log(L)) > 0$ as well as any output changes $Cov((1 - \alpha)\Delta \log(L_t), \Delta \log(L)) > 0$ that can be purely attributed to changes in employment. The unexplained residuals ϵ_t represent cleaned labour productivity movements.
Delta YpL2	Labour productivity series using output per Worker. This is calculated as real GDP over employment $\frac{Y_t}{L_t}$.
Unemployment Rate	Downloaded from FRED and from LRHUADT-TFRM156S series
Delta Employment Rate	$-\Delta$ Unemployment Rate
Monthly Job-finding Rate	Calculated using quarterly observations of the length of unemployment from the ILO following Shimer (2005). The jobfinding rate is then predicted backwards from 2003 to 1990 using observations of the unemployment rate, tightness, the OECD CLI and seasonal dummies. The primary series uses the raw estimates while the alternative only uses the predictions.
Vacancy Rate	Calculated using FRED series LMJVT-TNVFRM647S. This survey records the vacancy flow rather than the vacancy rate recorded by the other two surveys. As the model makes no distinction between flows and stocks the series are treated as identical for estimation purposes.
OECD Composite Confidence Indicator	Downloaded from the OECD database

B.3 Catch up weights

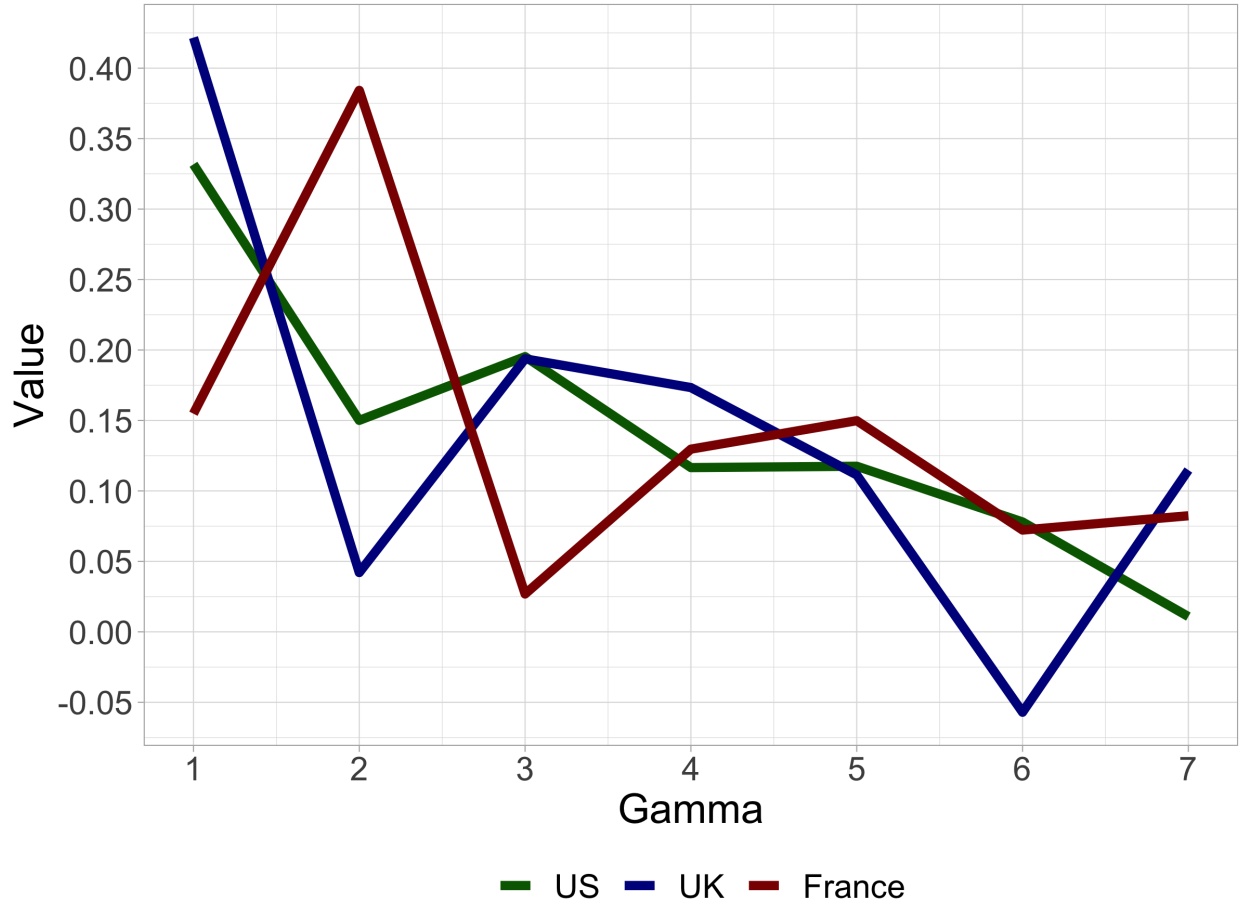The weights on the catch-up process for each economy are in figure 7.

Figure 7: Computed catch up weights

B.4 Calibration of the matching function

The matching function is estimated from observations of the job finding rate on tightness. Assuming a Cobb-Douglas matching function, the regression $log(p(\theta)) = \beta_0 + \beta_1\theta + \epsilon_t$ allows for estimating the steady state match efficiency as $m = \exp(\beta_0)$ and the elasticity of the matching function as $\xi = 1 - \beta_1$. The table below reports the results of the estimation over the country datasets.

	US	UK	FR
	(1)	(2)	(3)
ltheta	0.350	0.340	0.523
	(0.018)	(0.015)	(0.018)
Constant	−0.877	−1.673	−1.423
	(0.016)	(0.020)	(0.041)
Observations	229	325	360
Adjusted R^2	0.612	0.619	0.706
F Statistic	361.020 (df = 1; 227)	526.837 (df = 1; 323)	863.346 (df = 1; 358)

C Estimation

C.1 Structural VAR estimation with a long-run identification

The estimation uses the employment rate series for identifying shocks and the labour productivity series for identifying the effect of permanent and temporary shocks following Blanchard and Quah (1993). Estimation is based on monthly data with seven lags. The standard errors are computed at the 10 % threshold (lighter blue dashed line) and the 33% threshold (darker blue dashed line). The estimated impulse responses for the labour productivity process are in figure 8.
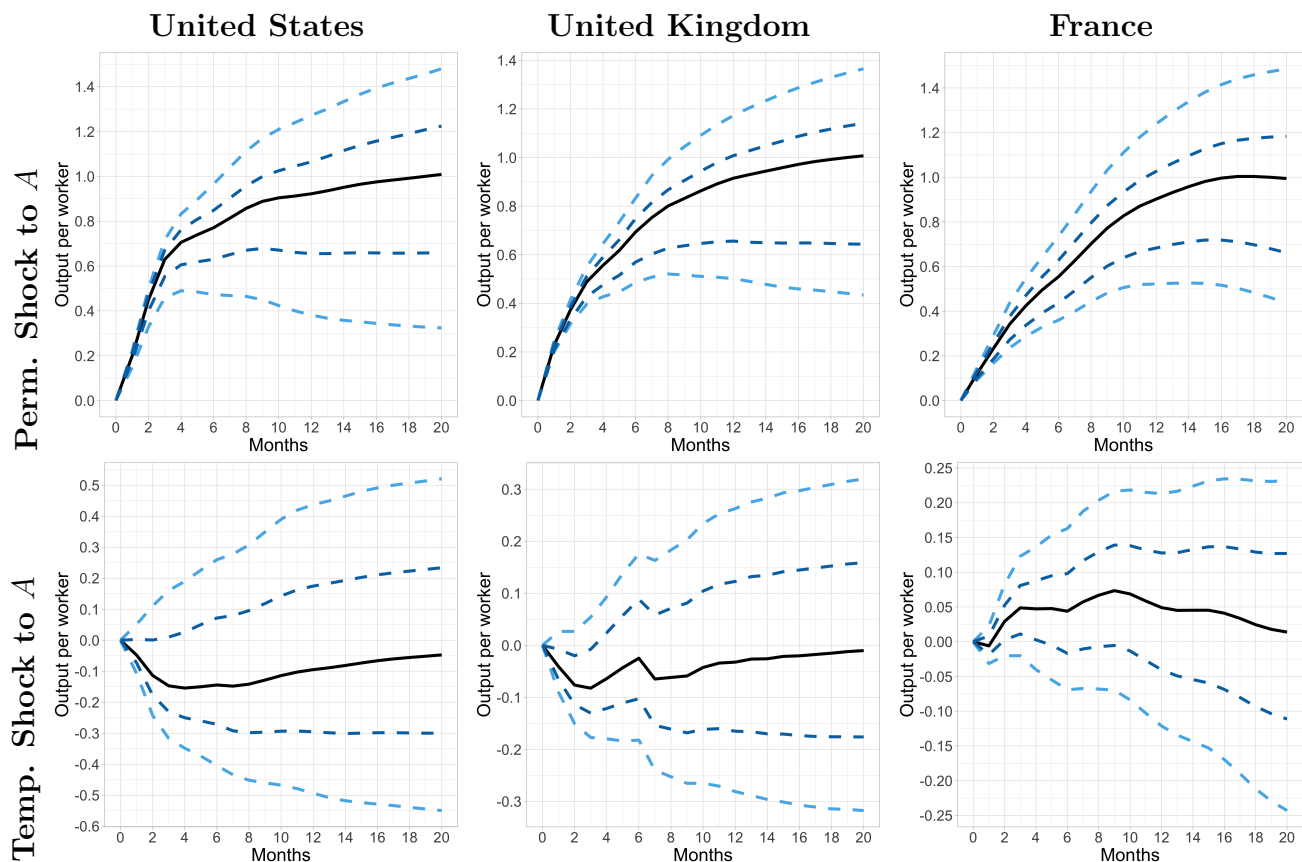
Figure 8: Response of labour productivity to permanent and temporary labour productivity improvement following the shock identification proposed by Blanchard and Quah (1993). The light blue lines show the 90% confidence interval, while the dark blue lines show the 67% confidence interval.

C.2 Details on the estimation

Estimation is based on the changes to labour productivity, the vacancy rate, unemployment rate, and the job-finding rate. To avoid stochastic singularity the models allow for an observation error of 1% of the respective variance of th selected series. The model with exogenous job destruction and exogenous shocks allows for observational error in the unemployment and job-finding rate series. In this way vacancies are assumed to identify match efficiency shocks. Meanwhile the model with endogenous job destruction and endogenous match efficiency movements allows for an observation error of 1% of the respective variance of the vacancy rate and job-finding rate series.

C.3 Raw estimated of the models

C.3.1 Noise model

Job Destruction	Country	ρ (Persistence)	σ_u (Permanent shock volatility)	σ_ν (Noise volatility)	σ_m (Shock volatility of the matching function)
Exo	US	0.78 (0.0187)	0.21 (0.011)	0.1 (0.0068)	0.026 (0.0008)
Exo	UK	0.88 (0.0101)	0.05 (0.003)	0.04 (0.0048)	0.024 (0.0011)
Exo	FR	0.8 (0.0124)	0.02 (0.0005)	0.09 (0.0018)	0.032 (0.0001)
End	US	0.84 (0.0058)	0.33 (0.0026)	0.01 (0.0009)	Endogenised
End	UK	0.81 (0.0019)	0.09 (0.0003)	0.06 (0.0002)	Endogenised
End	FR	0.88 (0.0124)	0.02 (0.0009)	0.22 (0.0085)	Endogenised

C.3.2 Beliefs model

Job Destruction	Country	ρ (Persistence)	σ_u (Fundamental shock volatility)	σ_ν (Beliefs volatility)	σ_m (Shock volatility of the matching function)
Exo	US	0.86 (0.0059)	0.1 (0.0016)	0.01 (0.0006)	0.023 (0.0008)
Exo	UK	0.73 (0.0099)	0.05 (0.0013)	0.09 (0.0045)	0.026 (0.0011)
Exo	FR	0.88 (0.0012)	0.02 (0.0001)	0.1 (0.0001)	0.037 (0.0001)
End	US	0.71 (0.0109)	0.24 (0.0047)	0.16 (0.0014)	Endogenised
End	UK	0.8 (0.0014)	0.08 (0.0002)	0.02 (0.0002)	Endogenised
End	FR	0.9 (0.0024)	0.02 (0.0004)	0.08 (0.0012)	Endogenised

C.4 Estimated impulse response

The estimated impulse responses for the different models and for noise, temporary and permanent shocks are below.

C.4.1 Noise and labour productivity impulse response in the model with exogenous job destruction and exogenous match efficiency
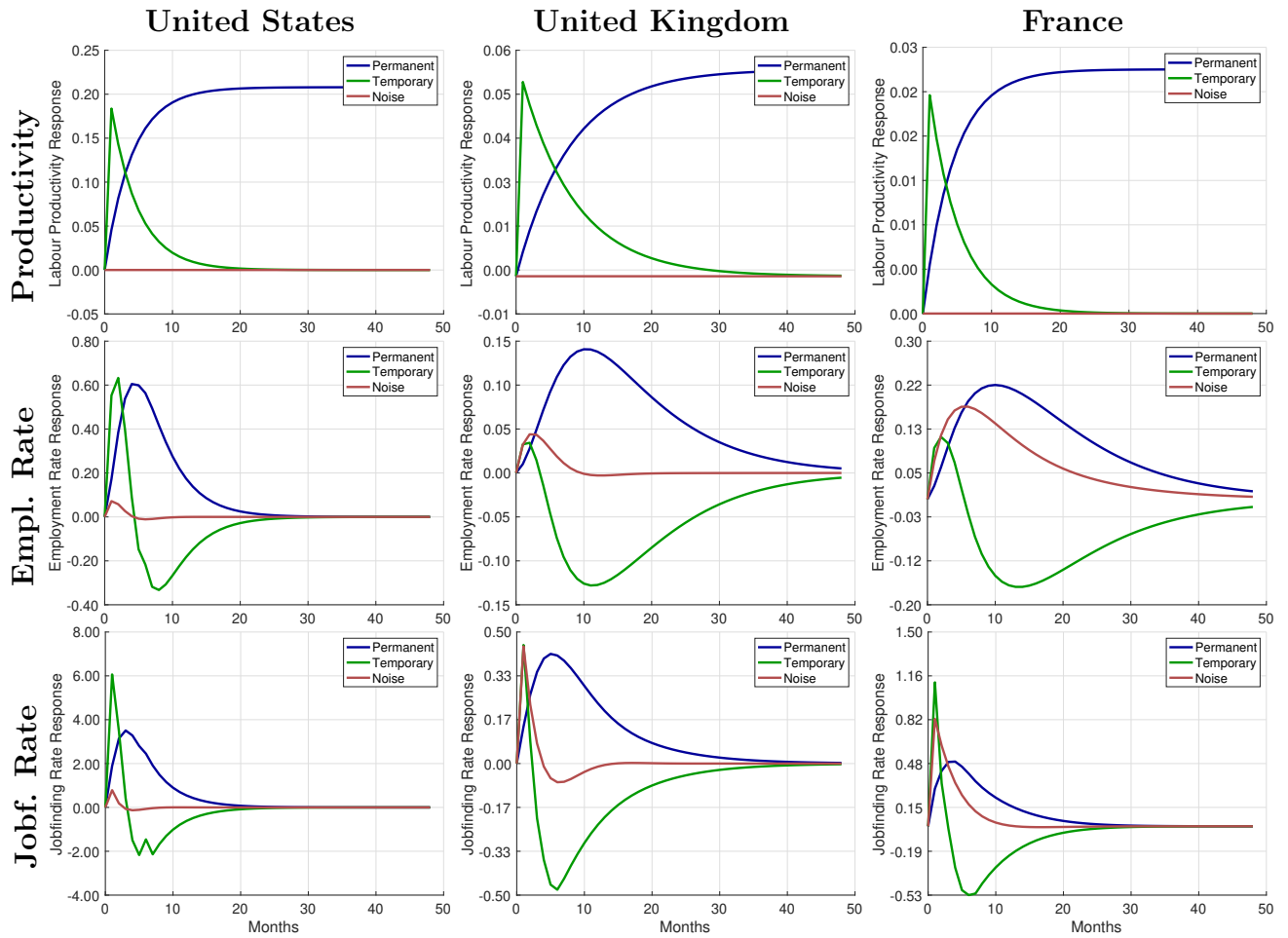

Figure 9: Noise and labour productivity impulse response in the model with exogenous job destruction and exogenous match efficiency

C.4.2 Noise and labour productivity impulse response in the model with endogenous job destruction and endogenous match efficiency
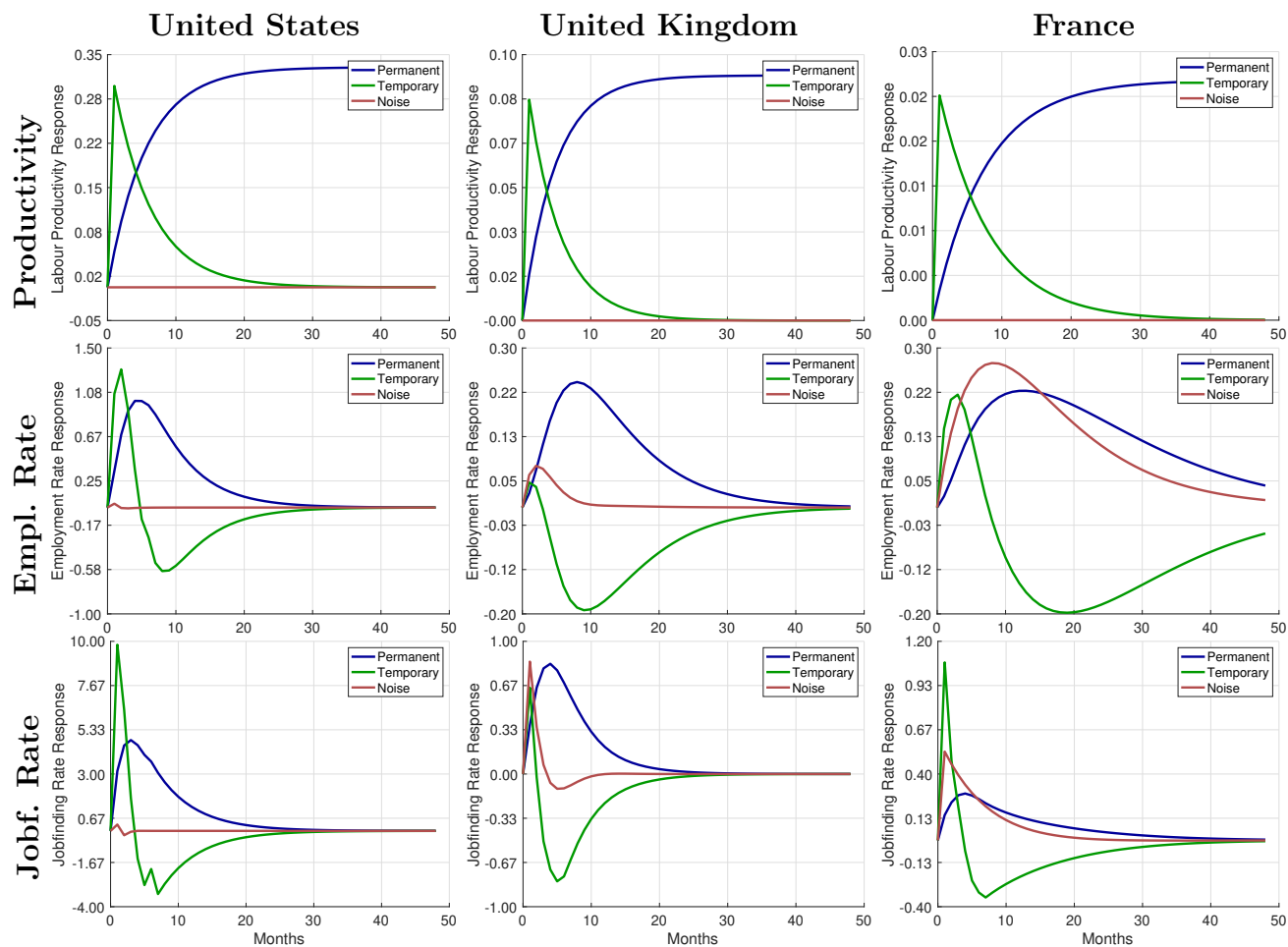

Figure 10: Noise and labour productivity impulse response in the model with endogenous job destruction and endogenous match efficiency.
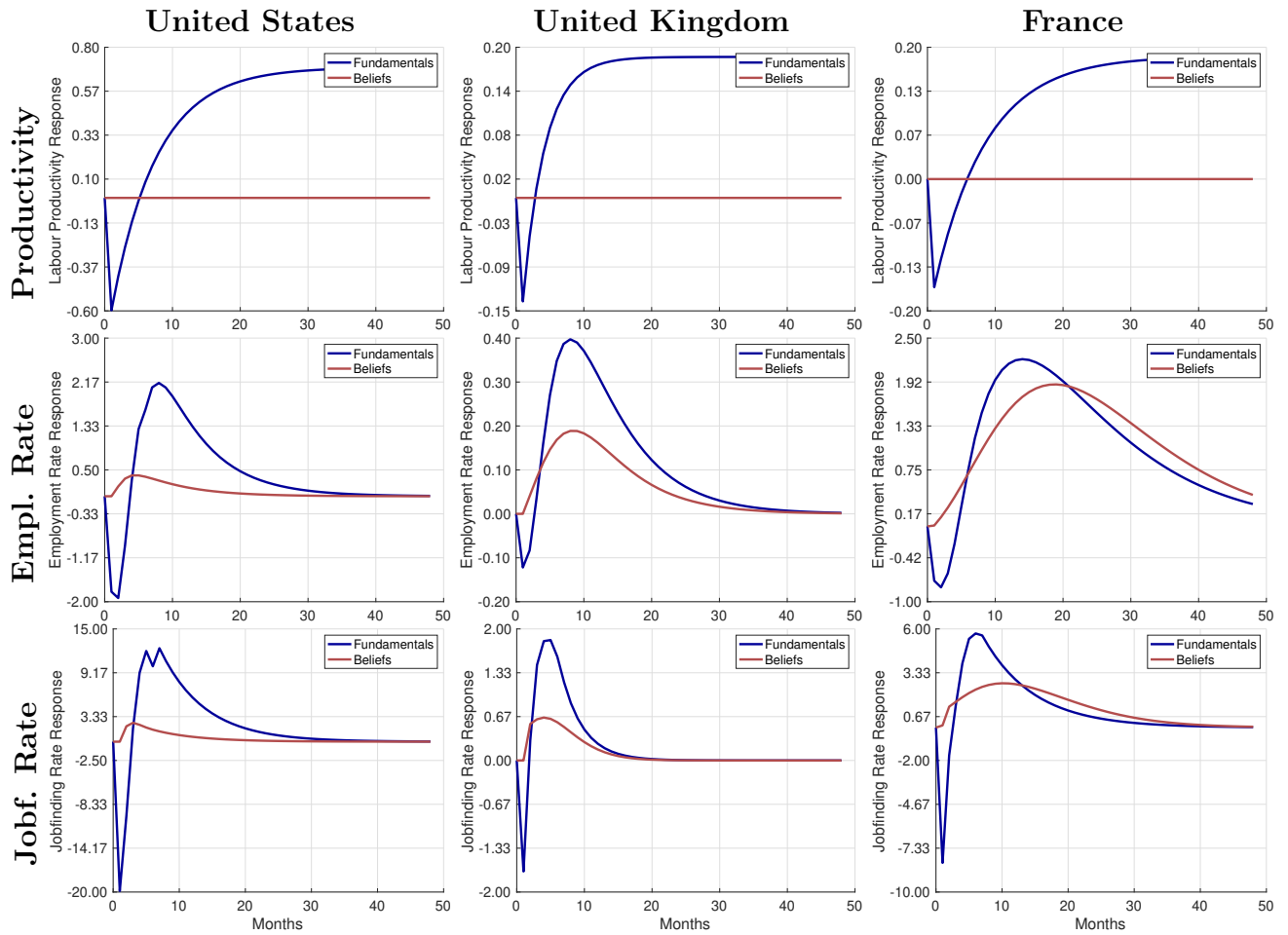
Figure 11: Beliefs and Fundamentals impulse response in the model with exogenous job destruction and exogenous match efficiency.

C.5 Shock decomposition of changes in the unemployment rate relative to the current employment rate
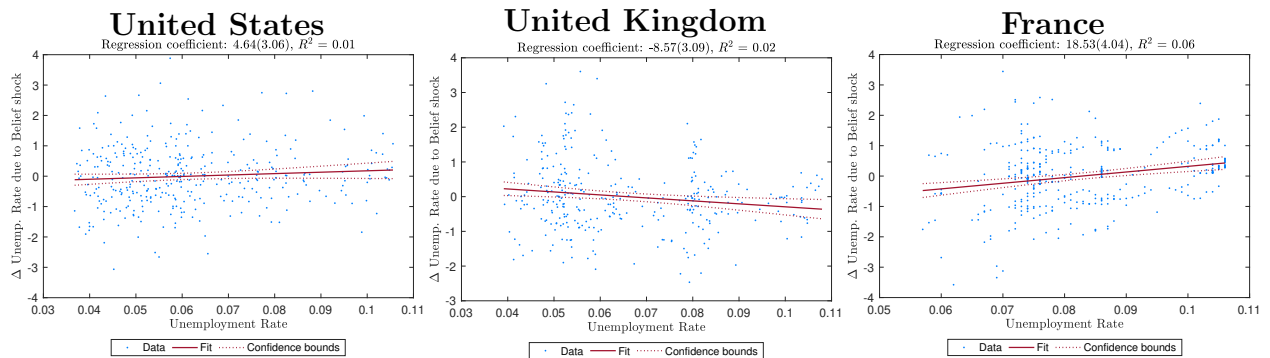

Figure 12: Decomposed belief shocks to unemployment rate movements plotted against the level of the unemployment rate.
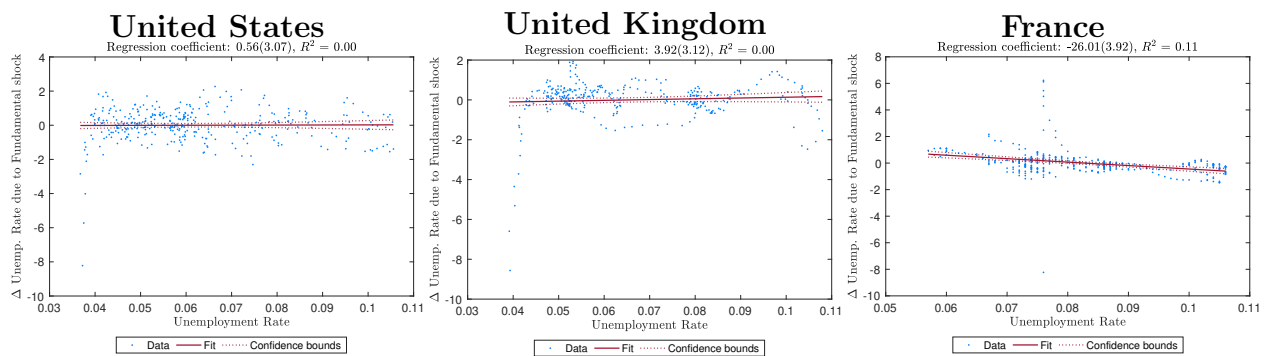

Figure 13: Decomposed fundamental shocks to unemployment rate movements plotted against the level of the unemployment rate.