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Taehyun Lee,[1] Ioannis C Moutzouris,[1] Nikos C Papapostolou[1] and Mahmoud Fatouh[2]

Abstract

We develop a four-state regime-switching model for optimal foreign exchange (FX) hedging using forward contracts. The states reflect four possible market conditions, defined by the direction and magnitude of deviation of the prevailing FX spot rate from its long-term trends. The model's performance is tested for five currencies against pound sterling for various horizons. Our analysis compares the hedging outcomes of the proposed model to those of other frequently used hedging approaches. The empirical results suggest that our model demonstrates the highest level of risk reduction for the US dollar, euro, Japanese yen and Turkish lira and the second-best performance for the Indian rupee. The risk reduction is significantly higher for lira, which suggests that the proposed model might be able to provide much more effective hedging for highly volatile currencies. The improved performance of the model can be attributed to the adjustability of the estimation horizon for the optimal hedge ratio based on the prevailing market conditions. This, in turn, allows it to better capture fat-tail properties frequently observed in FX returns. Our findings suggest that FX investors tend to use short-term memory (focus more on recent price movements) during low market conditions (relative to trend) and long-term memory in high ones. It would be also useful to build a better understanding of how investor behaviour depends on market conditions and mitigate the adverse behavioural implications of short-term memory, such as panic.

Key words: Regime switching, foreign exchange hedging, hedging effectiveness, high-volatility currencies, forward hedging.

JEL classification: G13, G15.

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1. Introduction

The expansion of international trade and finance has exposed market participants to various types of risks. One of the key issues of concern identified by market participants, regulatory bodies and researchers is foreign exchange (FX) volatility, generally referred to as FX risk. The UK is heavily exposed to FX volatility as it has the highest level of external assets and liabilities to gross domestic product (GDP) ratio among the G7.[1] In addition, pound sterling (GBP) FX turnover has increased more than tenfold in the last three decades.[2] GBP is one of the most actively traded currencies in the global FX market and the fourth-largest turnover currency after the United States dollar (USD), euro (EUR) and Japanese yen (JPY). Moreover, the UK economy is highly involved in international trade, with trade flows equivalent to 58% of the UK's GDP in 2020 (World Bank, 2020). The UK was the fourth-largest importer and twelfth-largest exporter of world merchandise and the second-largest exporting and fifth-largest importing country of commercial services globally in 2020.[3] These international trade statistics emphasise how important the management of FX risk is to the UK.

Despite several studies having examined FX hedging, a comprehensive analysis in the case of GBP remains elusive. The current paper aims to fill that gap by employing various strategies to test the hedging effectiveness of forward contracts against five GBP-related currency pairs.[4] It contributes to the FX hedging literature by comparing how hedging effectiveness differs by currency pairs, forward maturities, and hedging methods. These consist of existing methods in the FX literature: the Naïve hedge; Ordinary Least Squares (OLS); Generalised Orthogonal Generalised Autoregressive Conditional Heteroscedasticity (GO-GARCH); and Markov Regime-Switching (MRS).

In addition to these, we propose a regime-switching model that, to the best of our knowledge, had not been incorporated for FX hedging before but yields strong hedging effectiveness as it accounts for features inherent to the FX markets. More specifically, we propose a dynamic framework that employs a four-state regime-switching approach that adjusts the hedge ratio in response to changes in FX

[1] Office for National Statistics (2020); G7 refers to the United Kingdom, the United States, Japan, Italy, Germany, France and Canada.

[2] Monthly GBP turnover of $78 billion in 1989; $844 billion in 2019 (Bank for International Settlement, 2019).

[3] World Trade Organization (2021).

[4] Forward rates constitute an effective FX hedging tool as they are traded in large and liquid markets while transaction costs are low compared to other products such as futures (Briys and Solnik, 1992).

market conditions. First, the model classifies market conditions into four regimes: very low, low, high and very high. The regimes are determined by the direction and magnitude of deviation of the prevailing FX spot rate from its long-term trends. The rationale for the selected regime identification is that the optimal hedge ratio should also account for cyclicality arising from swings between market conditions. Then, the estimation method selects and applies the optimal lags, i.e., the ones that minimise the variance of the hedged portfolio, depending on the prevailing regime.

The empirical evaluation covers USD, EUR, JPY, Turkish lira (TRY), and Indian rupee (INR). The first three constitute the most traded currencies against GBP and belong to highly developed economies.[5] The last two were selected to explore the performance of the model for non-major currencies[6], yet ones which belong to fast-growing emerging economies[7] that play an increasingly important role in the world and the UK economic environment. Furthermore, INR is relatively stable while TRY has a history of extreme volatility, especially recently, which has resulted in significant policy intervention (Tarkocin, 2022). Thus, developing efficient hedging techniques for TRY would be particularly useful for risk management purposes. In conclusion, selecting these currencies allows us to investigate whether our approach would perform differently when used to determine the hedge ratios for (i) currencies of developed versus emerging economies and (ii) relatively less versus more volatile currency pairs. Finally, to examine how hedging effectiveness differs across maturities, we employ one [1M]-, three [3M]- and six-month [6M] forward contracts.

Our results indicate that the proposed four-state regime-switching (PRS) model reduces portfolio variance more effectively than other existing hedging strategies in the GBPUSD, GBPEUR, GBPJPY and GBPTRY markets. In the latter case, PRS significantly improves the hedging performance across all maturities, by more than 22% compared to the second-best performing strategy. This is an interesting

[5] The average daily FX turnover (spot) corresponded to $118 billion for GBPUSD, $24 billion for EURGBP and $13 billion for GBPJPY (Bank for International Settlement, 2019).

[6] The over-the-counter foreign exchange turnover in April 2022 was $5,811 billion for USD, $2,126 billion for EUR, and $1,108 billion for JPY while it was $122 billion and $27 billion for INR and TRY, respectively (Bank for International Settlement, 2022).

[7] Among the G20 nations, Turkey had the highest growth (11.0%) and India recorded the second-highest growth (8.3%) in the year 2021 (OECD, 2022).

finding as it suggests that the model might be able to provide much more effective hedging for highly volatile currencies.

The outperformance of the proposed model against other existing approaches suggests that it can capture asymmetry and fat-tail properties frequently detected in FX returns. In the case of GBPINR, where the spot and forward rates are close to being normally distributed, PRS shows the second-best performance, following MRS, which assumes that parameters are normally distributed. This is because our model changes the horizon used to estimate the optimal hedge ratio based on the prevailing market conditions. In other words, our results suggest that FX investors tend to use shorter-term memory (i.e., focusing mainly on the most recent events) during low market conditions, and longer-term memory (i.e., focus spans over longer time periods) in high market conditions.

Several authors have referred to the changes in investors' mode/behaviour between good and bad market conditions as an important driver of cyclicality in economic activity and asset markets (for example, De Grauwe, 2012; Williams, 2013; Adam et al., 2017; and Fatouh and Giansante, 2023). Namely, the market is more affected by more recent events during high-volatility periods than low-volatility ones. Such patterns can fuel panic and lead to runs and understanding them is crucial for policymakers. That is, policymakers could design policy interventions in low market conditions in a way that mitigate the shorter-term memory of investors, reducing panic and risk of runs. This rationale is not specific to FX markets, and can be applied in other markets. Trust and confidence are key drivers of the values of financial assets (including currencies) and can be dented more easily in troubled times. Hence, interventions that can help reinstate confidence would be more effective. More specifically, in the context of FX markets, our analysis can help policymakers build better understanding of how FX risk evolves with market conditions.

The remainder of the paper is organised as follows. Section 2 reviews the related literature. Section 3 analyses the data. Section 4 compares optimal hedge ratios of four frequently incorporated hedging methods; this will be used as a benchmark against the proposed model. Section 5 introduces the proposed four-state regime-switching model. The hedging performance assessments of the discussed methods are presented in Section 6. Section 7 concludes.

2. Literature review

FX hedging has been a significant research topic over the years, with several methods examined. The most simplistic and intuitive among these is the one-forward-contract-to-one-spot-contract trade approach (naïve hedging). More broadly, the number of forward contracts used to hedge one spot contract is called the hedge ratio; in the case of naïve hedging, this ratio is equal to 1. Hedging theories that assume stable volatility of returns were proposed by Keynes (1930), Hicks (1939) and Working (1953). Following these studies, the effectiveness of hedging has been thoroughly investigated. Specifically, hedging effectiveness is determined by the percentage reduction in the variance of returns of the hedged portfolio compared to the return variance of the unhedged one. Johnson (1960) and Stein (1961) both utilised portfolio theory for hedging. Ederington (1979) suggests that the optimal hedge ratio (OHR) (the hedge ratio that maximises hedging effectiveness) should match the slope coefficient of an OLS regression of the spot on the futures returns. This is equivalent to the covariance between spot and futures price over the variance of the futures price (Kahl, 1983) and still constitutes the most frequently used hedging approach.[8] Perold and Schulman (1988) emphasise the importance of FX hedging stating that foreign currency exposure introduces risk without sufficient reward and suggest that a long-horizon portfolio needs to be hedged against currency movements. Related to this, Campbell et al. (2010) highlight a static optimal FX hedging strategy and show its effectiveness in return volatility reduction in global equity investments.

However, the OHR static approach has been criticised for not considering market changes as it implements a fixed hedge ratio regardless of when the hedge is executed. Overall, it is not able to account for potential time-varying variances, co-integration of forward and spot prices and heteroscedasticity of residuals (e.g., Park and Bera, 1987; Bollerslev, 1990; Kroner and Sultan, 1993; and Lien et al., 2002). Instead, several studies have shown that a dynamic hedging strategy outperforms a static model, across various sectors, because it can quickly adjust to changing market conditions. Using foreign currency futures, Kroner and Sultan (1993) propose a dynamic model using bivariate error correction with an Autoregressive Conditional Heteroscedasticity (GARCH) error

[8] Relevant research includes Ederington (1979), Park and Bera (1987), Alizadeh and Nomikos (2004), Yang and Allen (2005), Kharbanda and Singh (2020) and Buyukkara et al. (2022).

structure. The study concludes that this model reduces risk more than conventional static models, and the benefit gained from the dynamic application more than offsets the transaction cost. de Roon et al. (2003) suggest that dynamic hedging materially improves the performance of USD-based stock portfolios compared to static methods. McMillan (2005), using also a GARCH model, shows that time-varying hedge ratios are significantly more effective than constant ones in the non-ferrous metal market. Schmittmann (2010) examines the merits of futures hedging and finds that currency hedging significantly reduces the volatility of FX rates, when using a quarterly investment horizon. Chang et al. (2012) emphasise the importance of incorporating conditional variances and covariances in currency hedging through a dynamic multivariate GARCH framework. Lai (2019) evaluates the hedging performances of multivariate GARCH models[9], with the empirical findings indicating that GO-GARCH is the most effective one. Kharbanda and Singh (2020) analyse hedging effectiveness in the Indian currency futures market. Their results show that the dynamic multivariate GARCH model can surpass static ones. Buyukkara et al. (2022) investigate the optimal hedge ratio and its effectiveness in the Turkish currency market, using futures contracts. They evaluate naïve, constant and time-varying approaches and find that the variance reduction from dynamic methods outperforms the ones from naïve and constant hedging ones.

Regime-switching models have been playing a significant part in the hedging literature, since Mandelbrot (1963) suggested that returns on asset tend to show regime shifts. Gray (1996) develops a generalised regime-switching (GRS) model using a conditional distribution of interest rates. The model accommodates mean reversion and conditional heteroskedasticity of short-term interest rates. Accordingly, the effectiveness is compared with the statistical fit and forecasting power of conventional approaches to hedged interest rate risk. The MRS model has been widely used in research since it can incorporate time-series features as structural changes (Kasahara and Shimotu, 2017). The MRS model considers the potential structural changes and the current state of the currency market. Engel and Hamilton (1990) find that foreign exchange rates are regime dependent.[10]

[9] These include Baba–Engle–Kraft–Kroner (BEKK-GARCH), generalised orthogonal (GO-GARCH) and dynamic conditional correlation (DCC-GARCH) models.

[10] In their study, based on the probabilities of FX rates staying in the same state, a one-directional, long-term move of USD from 0.822 to 0.928 is determined.

Consequently, their study rejects the hypothesis that FX movements follow a random walk and show that MRS has better forecasting performance than the random walk process. In line with that result, Engel (1994) examines the fitness of the MRS model for 18 exchange rates and finds that it has superior forecasting performance compared to other methods, such as random walk. To identify different regimes, the method makes use of transition probabilities. Alizadeh and Nomikos (2004) utilise the MRS approach to calculate dynamic hedge ratios for the FTSE-100 and S&P 500 stock indices. Their results suggest that MRS models may increase hedging effectiveness as well as hedgers' utility.

Lee and Yoder (2007) suggest an MRS model that extends Gray's (1996) univariate GRS to the bivariate case to estimate hedge ratios in corn and nickel markets. Alizadeh et al. (2008) implement an MRS methodology to enhance the performance of energy market hedges. Their approach links the volatility and cointegration concepts across two market states, a high and a low volatility one. By identifying more regimes, based on the detrended MSCI World Index, Zalachoris (2022) demonstrate that a four-state regime-switching model can decrease crude oil market's portfolio volatility more than naïve hedge, constant OHR, time-varying OHR and two-state MRS models.

3. Data and Descriptive Statistics

Our dataset covers the period from February 1999 to June 2022 and includes monthly spot and forward rates (1M, 3M and 6M maturities) of USD, EUR, JPY, TRY and INR against GBP. The estimation period starts in February 1999 due to the introduction of EUR in January 1999. For brevity, the currency pairs are denoted as dollar, euro, yen, lira, and rupee and in the graphs and tables as GBPUSD, GBPEUR, GBPJPY, GBPTRY and GBPINR.[11] For lira, the market data used for the period before January 2005 includes adjusted values of the New Turkish lira[12] against GBP since Turkey denominated its currency at the one-million level to accommodate high inflation.

We use forward contracts as instruments to hedge spot rate changes. We assess hedging effectiveness across different maturities, focusing on one-, three- and six-month forward contracts, as 97% of FX forwards have maturities of less than six months.[13] Each contract has 281 recorded observations. The

[11] The study utilises GBPEUR, which stands for one GBP to EUR value, even though GBP and EUR FX trade is conventionally quoted as EURGBP displaying one EUR to GBP value. Using GBPEUR helps to ensure intuitive recognition of GBP's value, and the rate aligned with the other selected FX quotation pairs.

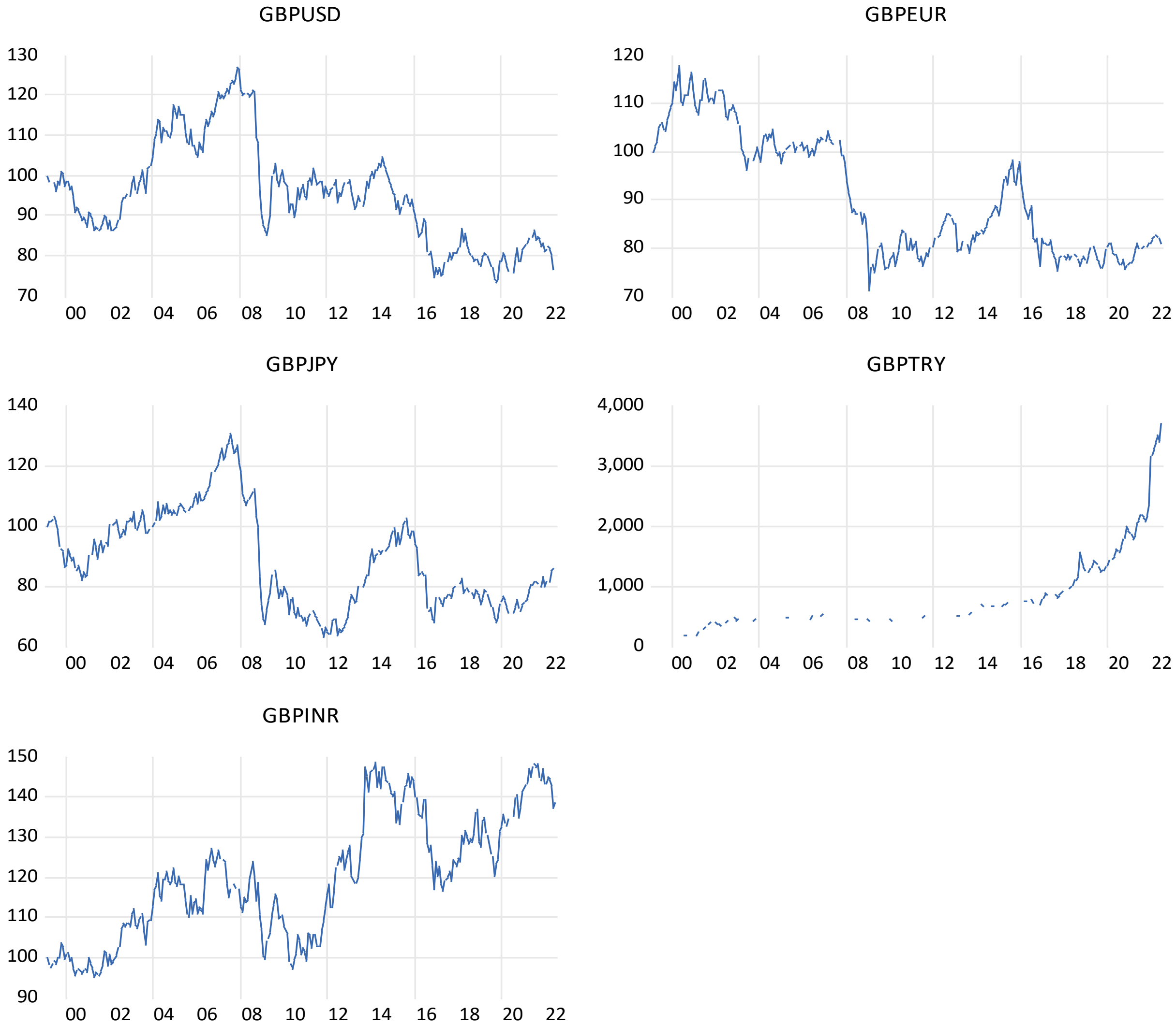
[12] Transitory term for the newly introduced Turkish lira. It had been used from January 2005 to December 2008. The term "New Turkish lira" was changed to "Turkish lira" in January 2009.

[13] Bank of England (2022).

relevant market trading datasets are gathered from the Thomson Reuters Refinitiv Eikon platform. The symbols are listed in Table A1 in Appendix II.

Figure 1 illustrates the evolution of the spot rate variables during the corresponding period. The most striking feature of the figure is the significant upward trend that lira has been following from 2013 and especially since 2020 –because of the extremely high inflation rates in Turkey.

Figure 1: Spot rate index movements[14]



Source: Refinitiv Eikon

The spot rate correlations between currency pairs are depicted in Table A2 of Appendix II. The highest level of correlation is detected between dollar and yen (0.75), which implies high similarity in the FX movements related to the two developed markets of the US and Japan. On the other hand, the correlation between dollar and lira is the lowest (-0.51), suggesting that these two currencies moved

[14] The index has a base value of 100 (as of 1 February 1999) for each respective currency pair.

moderately in the opposite way in relation to each other. Moreover, Table A2 suggests that, when the spot rates are divided into two groups, advanced market currencies (USD, EUR and JPY) and emerging ones (TRY and INR), the spot rate correlations are positive between currency pairs of the same group and negative across different groups. This could be explained by synchronisation of economic events and their effects for countries belonging to similar stages of development and lagging effects for countries across differences stages, given that macroeconomic fundamentals often account for the co-movements of exchange rates (Kühl, 2018). In line with the literature, to obtain stationary series, we calculate the log-returns of the spot and forward prices:

$$\Delta S_t = ln\frac{S_t}{S_{t-1}} \ ; \ \Delta F_t = ln\frac{F_t}{F_{t-1}} \tag{1}$$

where S_t and S_{t-1} are the spot rates in months t and $t-1$, respectively and F_t, F_{t-1} the corresponding forward rates.

Table 1: Descriptive statistics for monthly spot and forward returns.

Ccy Pair	Mean	Median	Min	Max	Std Dev	Count
USD(SPOT)	-0.0010	-0.0018	-0.1108	0.1006	0.0259	280
USD(1MF)	-0.0010	-0.0016	-0.1134	0.1005	0.0258	280
USD(3MF)	-0.0010	-0.0015	-0.1160	0.1002	0.0258	280
USD(6MF)	-0.0010	-0.0014	-0.1153	0.0999	0.0257	280
EUR(SPOT)	-0.0008	0.0009	-0.1302	0.0722	0.0217	280
EUR(1MF)	-0.0008	0.0010	-0.1303	0.0722	0.0217	280
EUR(3MF)	-0.0008	0.0009	-0.1304	0.0721	0.0216	280
EUR(6MF)	-0.0008	0.0010	-0.1298	0.0719	0.0215	280
JPY(SPOT)	-0.0005	0.0030	-0.1756	0.1223	0.0357	280
JPY(1MF)	-0.0005	0.0030	-0.1751	0.1219	0.0356	280
JPY(3MF)	-0.0005	0.0032	-0.1730	0.1219	0.0355	280
JPY(6MF)	-0.0005	0.0034	-0.1694	0.1217	0.0354	280
TRY(SPOT)	0.0129	0.0071	-0.1367	0.2961	0.0533	280
TRY(1MF)	0.0129	0.0059	-0.1478	0.8081	0.0684	280
TRY(3MF)	0.0126	0.0054	-0.1455	0.8911	0.0733	280
TRY(6MF)	0.0123	0.0039	-0.1551	1.0221	0.0822	280
INR(SPOT)	0.0012	0.0015	-0.0841	0.1188	0.0271	280
INR(1MF)	0.0012	0.0010	-0.0843	0.1190	0.0271	280
INR(3MF)	0.0012	0.0009	-0.0843	0.1187	0.0272	280
INR(6MF)	0.0011	0.0006	-0.0836	0.1161	0.0271	280

As shown in Table 1 (and Figure A1 in Appendix III), all spot and forward returns are highly volatile which motivates the need for efficient hedging in these markets. Furthermore, the longer maturity forwards tend to have the same or less volatility than the shorter ones, with the main exception being lira, for which volatility significantly increases with the maturity of the contract.[15] Also, apart from lira,

[15] In the case of rupee, volatility increases between the 1M and 3M contracts but decreases for the 6M contracts.

all spot and same-currency forward rates correlation coefficients (Corr [SP]) exceed 0.99. This suggests that even the naïve or constant OHR strategy should be able to achieve high hedging effectiveness. For lira though, the correlation is much lower, ranging from 0.52 to 0.63 (Table 1). The severe volatility of lira causes low correlation between the spot and the corresponding forward rates compared to the other currency pairs, for which volatility is much lower. As a direct consequence, there is significantly lower hedging effectiveness for lira when it comes to static hedging models (as opposed to the currency pairs where there is strong correlation between spot and forward rates). Instead, more sophisticated, dynamic techniques should be applied. Finally, the Jarque-Bera (JB) test indicates that all returns are not normally distributed, although rupee has much lower JB statistics than the others.

4. Existing Hedging Methods

The naïve, OLS, GO-GARCH and MRS models are selected for comparison and as a benchmark against the proposed framework. For each model, the optimal hedge ratio (in short, denoted by γ) and hedging effectiveness are estimated and verified.[16] The formula for the latter is:

$$Hedging\ Effectiveness = 1 - \frac{Variance\ of\ hedged\ portfolio\ return}{Variance\ of\ unhedged\ portfolio\ return} \tag{2}$$

In other words, the smaller the variance of the hedged portfolio compared to the variance of the unhedged one, the higher the risk reduction stemming from hedging and, in turn, the higher the effectiveness of the chosen method. Accordingly, the optimal hedge ratio, and thus, method is perceived as the one which results in the largest hedging effectiveness.

4.1. Naïve Method

The naïve method applies a constant hedge ratio of 1, which means that each FX spot contract corresponds to one forward contract. While it is straightforward to implement, it frequently proves to be suboptimal. This is particularly the case when the spot and forward rate changes are not identical or, from a technical point of view, when the respective correlation coefficients are not close to 1.

4.2. Ordinary Least Squares

In this case, the optimal hedge ratio is derived from the following OLS regression:

$$\Delta S_t = \gamma_0 + \gamma_1 \cdot \Delta F_t + \varepsilon_t;\ \varepsilon_t \sim \text{iid}\ (0, \sigma^2) \tag{3}$$

[16] The hedged portfolio returns are given by $R_t = \Delta S_t - \gamma \cdot \Delta F_t$.

where γ_0 is the intercept; the slope coefficient, γ_1, is a minimum variance hedge ratio; and ε_t are the residuals. In line with equation (2), R^2 of the regression measures the effectiveness of the hedge. The minimum variance hedge ratios for the currencies under consideration are presented in Table 2.

Table 2: OLS hedge ratios.

OLS (HR)	1M Fwd	3M Fwd	6M Fwd
GBPUSD	0.99673	0.99761	1.00260
GBPEUR	0.99780	1.00107	1.00801
GBPJPY	0.99771	1.00023	1.00440
GBPTRY	0.48546	0.42220	0.33720
GBPINR	0.99525	0.99135	0.99030

Evidently, while the hedge ratios for dollar, euro, yen, and rupee are close to unity, the ones for lira are below 0.5. This result can be explained by the components of the OLS hedge ratio.[17] Namely, the standard deviation of the spot rate of lira is much lower than the ones of the corresponding forward rates (whereas, for each of the other currencies, spot and forward rates are similar) and the correlation coefficients between the spot and forward rates for lira are significantly smaller than the ones for the other currency pairs, which are close to 1 (Table 1).

Furthermore, in the case of advanced market currency pairs, the hedge ratios become higher for longer maturity forward hedges. However, it is in the opposite direction for the emerging market ones and especially lira, where the hedge ratio significantly decreases with the horizon of the contract. In line with the arguments in the previous paragraph, this is because the standard deviations of the lira forward rates increase with the maturity of the contract while correlation coefficients decrease. For rupee, the decrease can be attributed to the reduction in the correlation coefficients of longer forward rates (the standard deviations of forward rates remain relatively stable in this case). In contrast, for the advanced market currency pairs, the standard deviations of forward rates decrease with the contract horizon much faster than the respective correlation coefficients (Table 1). As such, the slope coefficient of regression (3) increases in time for the advanced market currency pairs and decreases for the emerging market ones.

[17] The hedge ratio, i.e., minimum variance hedge ratio, is calculated by the product of the standard deviation of spot returns and the correlation coefficient between spot and forward returns divided by the standard deviation of forward returns.

4.3. Generalised Orthogonal GARCH (GO-GARCH)

The GO-GARCH framework is considered the most effective specification in the family of GARCH models (Appendix I.1). According to Table 3, dollar, euro and yen, have average hedge ratios larger than 1, which increase with the horizon of the contract. However, the hedge ratios for lira and rupee are less than 1 and decrease with the contract's maturity. Given that the GO-GARCH model explicitly accounts for the volatility of both the spot and forward rates, the explanation for this finding is along the same lines as for the OLS hedging results (Section 4.2). From an economic point of view, as the model is based on the conditional covariance matrix, the result suggests that each group of currency pairs possesses analogous combinations of uncorrelated economic components.

Table 3: GO-GARCH average hedge ratios.

GO-GARCH (HR)	1M Fwd	3M Fwd	6M Fwd
GBPUSD	1.00023	1.00079	1.00546
GBPEUR	1.00131	1.00469	1.01167
GBPJPY	1.00133	1.00269	1.00638
GBPTRY	0.91316	0.90489	0.82569
GBPINR	0.99875	0.99490	0.99385

4.4. Markov Regime-Switching (MRS)

In the MRS model, the spot rate at time t, S_t, can be parameterised to a first-order Markov process with transition probabilities. From an economic point of view, S_t indicates two different market states, a high and a low one. Accordingly, the first-order Markov process explains that the regime probability at time t depends on the regime process at time $t-1$ (Appendix I.2).

Table 4: MRS average hedge ratios.

MRS (HR)	1M Fwd	3M Fwd	6M Fwd
GBPUSD	0.99995	0.98352	0.99731
GBPEUR	1.00118	1.00399	1.01102
GBPJPY	1.00117	1.00257	1.00646
GBPTRY	0.88406	0.75665	0.69272
GBPINR	0.99924	0.99641	0.99614

Table 4 suggests that the average hedge ratios for the dollar, euro, yen and rupee are very close to unity across all maturities. Once again, however, the hedge ratios for lira are significantly lower than the ones for the other currencies, which can be attributed to the reasons mentioned above for the other models.

5. The Proposed Four-State Regime Switching (PRS) Model

5.1. Regime Identification

The regimes are determined by first detrending FX spot rates, accounting for the relative economic conditions in the countries whose currencies are compared. For the detrending of spot rates, we use the Hodrick–Prescott filter, as it can detect short-term volatilities caused by economic cycles (Rebelo and King, 1999; Stock and Watson, 1999; Cornea-Madeira, 2016). Observations above and below the corresponding detrended value (DV) are assumed to belong to the upper and lower regime, respectively (Figure A2 in Appendix III). The extreme states are determined through a threshold, lim:

$$lim = p \cdot \sigma \tag{4}$$

where $p \in \mathbb{R}$ denotes the number of standard deviations, σ, from the mean of the detrended series. Namely, to be on the VH (VL) state, the detrended spot rate should be $p \cdot \sigma$ above (below) the detrended mean. For the empirical analysis and after conducting a sensitivity analysis (Table A5, Appendix II), parameter p is assigned the value of 1.2. The results in Table 5 also suggest that this is a reasonable assumption, given that each of the above (H and VH) and below (L and VL) regimes on average comprises of 50% of the total observations while the extreme states (VL and VH) converge into an average of approximately 10% each.

Table 5: Regime distribution.

		VH	H	L	VL	VH+H	L+VL
GBPUSD	Count	26	120	110	25	146	135
	Ratio	9.3%	42.7%	39.1%	8.9%	52.0%	48.0%
GBPEUR	Count	33	103	119	26	136	145
	Ratio	11.7%	36.7%	42.3%	9.3%	48.4%	51.6%
GBPJPY	Count	26	115	123	17	141	140
	Ratio	9.3%	40.9%	43.8%	6.0%	50.2%	49.8%
GBPTRY	Count	11	124	132	14	135	146
	Ratio	3.9%	44.1%	47.0%	5.0%	48.0%	52.0%
GBPINR	Count	30	114	98	39	144	137
	Ratio	10.7%	40.6%	34.9%	13.9%	51.2%	48.8%
Average	**Count**	**25.2**	**115.2**	**116.4**	**24.2**	**140.4**	**140.6**
	Ratio	**9.0%**	**41.0%**	**41.4%**	**8.6%**	**50.0%**	**50.0%**

The regime distributions are depicted in Figure A3 (Appendix III). Then, we further divide each regime to two states, a normal and an extreme one (Zalachoris, 2022). Thus, the proposed model calculates the optimal hedge ratios by considering four states: very low (VL); low (L); high (H); and very high (VH). L and H are considered as normal states, and VL and VH as extreme ones. Several authors (for example, De Grauwe, 2012; Williams, 2013; Adam et al., 2017; and Fatouh and Giansante, 2020) suggest that investor's behaviour and mode can swing between high and low market conditions, and that these swings represent a key driver of cyclicality in economic activity and asset markets. We use the 4-state setup to reflect these swings in investors behaviour.

5.2. Lag Optimisation

The optimal lags are evaluated by adjusting k in Equation (5) for each regime, to obtain the best composition of the rolling periods:

$$\gamma_t^* = \frac{Cov\ (\Delta St, \Delta Ft)k}{Var\ (\Delta Ft)k} \tag{5}$$

This corresponds to a time-varying OHR calculation based on the rolling windows. The number of optimal lags is assumed to be consistent across the period under examination (Ricci, 2020). For a testing range from 3 to 24 months (assuming integer lag values), 22^4 iterations are executed in total.[18] The combinations associated with the best results allow for optimised hedging performances. Table 6 shows the average number of lags for the top-100 results, indicating that hedging in moderate regimes may be optimised when a lower number of lags is employed to compute γ_t. In contrast, hedgers may rely on a longer time horizon under extreme conditions. This result may imply that short-term memory hedge ratio determination in extreme states is not a holy grail. In other words, a moderate time window can be more effective than myopic consideration in extremely volatile market conditions.

Moreover, we should note that transactions in extreme circumstances are often vulnerable, as they can be cancelled or withdrawn, due to a liquidity squeeze in FX spot/forward markets or price spikes. Namely, during 2001 and part of 2002, the Turkish crisis led to a collapse of the Turkish lira; in 2001, the monthly excess return of the USD/TRY was above -50% (Banti et al., 2012).[19, 20] In that regard, a

[18] Portfolio return volatilities are calculated by varying lag k corresponding to the four states (VL, L, H and VL). Each state has 22 possible cases (from 3 to 24). Hence, the number of iterations is 22^4.
[19] Arnold et al. (2021).
[20] Samson et al. (2021).

short-term lag determination that mainly reflects extreme volatile periods can lead to upward or downward deviations from optimal levels. This is aligned with the results from Ricci (2020) that a short-memory (3M) hedging showed the lowest level of variance reduction, whilst a medium-memory (6M) hedging had the best performance in high volatility conditions, for commodity hedging with 3M, 6M, twelve-month (12M) and twenty-four-month (24M) futures. The lag-optimisation results also emphasise that the length of windows defining recent information is critical to pursue an optimal FX dynamic hedging.

Table 6: Lag optimisation.

Currency Pair	Term	Type	VH	H	L	VL
GBPUSD	1M Fwd	Best result	9.0	8.0	4.0	15.0
		Top 100 result avg	11.5	9.0	4.0	17.2
	3M Fwd	Best result	11.0	21.0	4.0	23.0
		Top 100 result avg	10.0	18.5	4.0	20.3
	6M Fwd	Best result	11.0	21.0	4.0	23.0
		Top 100 result avg	10.1	16.9	4.0	21.4
GBPEUR	1M Fwd	Best result	5.0	7.0	6.0	5.0
		Top 100 result avg	13.3	8.0	6.0	5.0
	3M Fwd	Best result	5.0	7.0	23.0	5.0
		Top 100 result avg	6.5	7.6	23.2	6.9
	6M Fwd	Best result	5.0	7.0	6.0	6.0
		Top 100 result avg	6.1	7.2	6.7	9.2
GBPJPY	1M Fwd	Best result	8.0	9.0	6.0	7.0
		Top 100 result avg	9.6	8.7	7.0	9.7
	3M Fwd	Best result	13.0	9.0	3.0	7.0
		Top 100 result avg	17.0	9.5	3.2	7.0
	6M Fwd	Best result	19.0	11.0	5.0	3.0
		Top 100 result avg	18.2	9.9	4.6	3.1
GBPTRY	1M Fwd	Best result	6.0	4.0	3.0	6.0
		Top 100 result avg	7.2	4.0	3.0	11.4
	3M Fwd	Best result	6.0	4.0	3.0	5.0
		Top 100 result avg	5.6	4.8	3.0	13.0
	6M Fwd	Best result	6.0	4.0	4.0	5.0
		Top 100 result avg	9.3	4.0	4.3	11.2
GBPINR	1M Fwd	Best result	3.0	6.0	4.0	8.0
		Top 100 result avg	12.1	6.1	4.0	11.2
	3M Fwd	Best result	14.0	6.0	4.0	5.0
		Top 100 result avg	12.2	6.1	4.0	10.7
	6M Fwd	Best result	5.0	6.0	4.0	5.0
		Top 100 result avg	8.2	6.5	4.0	11.3
Average	1M Fwd	Best result	6.2	6.8	4.6	8.2
		Top 100 result avg	10.8	7.1	4.8	10.9
	3M Fwd	Best result	9.8	9.4	7.4	9.0
		Top 100 result avg	10.3	9.3	7.5	11.6
	6M Fwd	Best result	9.2	9.8	4.6	8.4
		Top 100 result avg	10.4	8.9	4.7	11.2

6. Hedging Results

The hedging effectiveness results of the PRS model are superior to the ones of the other four existing methods for all FX spot and forward maturity combinations except for rupee (Table 7). For the latter, the MRS model displays the most effective hedging result, followed by PRS. MRS demonstrates the second-best hedging performance for euro and yen, while the naïve model is ranked second for dollar hedging. When it comes to lira, GO-GARCH reveals the second-best performance.

Table 7: Hedging effectiveness summary.

Ccy Pair	Strategy	1M Fwd	3M Fwd	6M Fwd
GBPUSD	Naïve	99.980790%	99.912076%	99.785932%
	OLS	99.979524%	99.910942%	99.788443%
	GO-GARCH	99.980075%	99.899887%	99.756877%
	MRS	99.980752%	99.873967%	99.776785%
	PRS	**99.982941%**	**99.918874%**	**99.801400%**
GBPEUR	Naïve	99.993001%	99.964689%	99.870321%
	OLS	99.991913%	99.965561%	99.882227%
	GO-GARCH	99.993192%	99.952710%	99.882255%
	MRS	99.993291%	99.967502%	99.887884%
	PRS	**99.995387%**	**99.977979%**	**99.921685%**
GBPJPY	Naïve	99.993966%	99.979628%	99.927330%
	OLS	99.992856%	99.979796%	99.932356%
	GO-GARCH	99.994014%	99.982363%	99.940878%
	MRS	99.994244%	99.984001%	99.944532%
	PRS	**99.995831%**	**99.987638%**	**99.955227%**
GBPTRY	Naïve	-4.228199%	-28.896689%	-77.034607%
	OLS	39.204098%	34.001937%	27.298010%
	GO-GARCH	60.782135%	56.622417%	52.998272%
	MRS	40.491097%	34.438167%	20.383186%
	PRS	**83.141162%**	**84.001011%**	**78.915534%**
GBPINR	Naïve	99.798515%	99.227975%	98.156888%
	OLS	99.797383%	99.229311%	98.159392%
	GO-GARCH	99.802370%	99.229744%	98.160363%
	MRS	**99.884125%**	**99.559434%**	**98.940634%**
	PRS	99.841340%	99.441853%	98.713022%

While for dollar, euro and yen the increase in hedging effectiveness from PRS is within the range of basis points compared to the respective second-best performing model, for lira, it is above 23% across all maturities (compared to GO-GARCH, which is the second-best performing model for that currency). For example, in the 3-month case, PRS yields 84.00% compared to 56.62%. This is an interesting finding on its own as it suggests that the proposed model might be able to provide much more effective hedging for highly volatile currencies.

Notably, the naïve method reveals a negative performance for lira. In line with Section 3 and Table 1, this might be due to the relatively large discrepancies between the spot and forward returns levels. In practice, a negative hedging effectiveness suggests that the variance of the hedged portfolio is higher than the variance of the unhedged one. As such, in the case of lira, a variance-minimising investor should be better off not hedging their position rather than using a naïve hedge strategy.

Meanwhile, MRS presents the best performing result for rupee, although, PRS follows it very closely. This could be explained by the fact that the returns distributions of the spot and forward rates of rupee are close to normal, considering excess kurtosis and skewness, as opposed to the other currency pairs (Table 1), given that MRS assumes that the incorporated variables are normally distributed.

Moreover, in all cases, the shorter maturity forward contracts are more effective in hedging compared to longer maturity ones. This can be related to the importance of market liquidity for hedging effectiveness, considering that, the shorter the maturity of the forward contract, the more liquid it is.[21] In line with that, Gupta and Singh (2009) and Gupta and Kaur (2015) suggest that liquidity significantly affects hedging effectiveness.

In conclusion, the proposed model provides higher hedging effectiveness for each advanced market currency pair while its performance in the highly volatile case of lira is clearly better than the other tested existing techniques. In line with these empirical findings, the model has advantages over other techniques when hedging against FX spot movements.

7. Conclusion

We develop a four-state regime-switching model using forward contracts to hedge foreign exchange positions. Our results indicate that the proposed model reduces portfolio variance more effectively than other existing hedging strategies for pound sterling against US dollar, euro, Japanese yen, and Turkish lira. In the pound sterling-Indian rupee market, the model shows the second-best performance. The findings suggest that, constructing the four-state regime-switching hedging with the optimised level of memory produces better results than employing a constant ratio obtained from

[21] Bank of England (2022).

the entire period. The findings are consistent with prior research that supports the use of a model that can be updated with more recent data over time (Myers and Thompson, 1989; Kroner and Sultan, 1993; Ricci, 2020). The outperformance of the proposed model compared to other dynamic approaches means that it can capture asymmetry and fat-tail properties frequently detected in FX returns. Importantly, the significantly improved hedging performance in the case of pound sterling against the Turkish lira suggests that the model might be able to provide much more effective hedging for highly volatile currencies. This is because the model automatically adjusts the horizon to estimate the optimal hedge ratio based on the prevailing market conditions. In other words, our results analysis suggests that FX investors tend to use shorter-term memory during low market conditions, and longer-term memory in high market conditions. In FX market context, the (perceived) level of FX risk might evolve with the market mode. This conclusion could have implications for policymakers. The short termism of investors would have stronger effects during high-volatility periods than low-volatility ones. Such patterns can fuel panic and lead to runs. Thus, policymakers could design policy interventions in volatile market conditions in a way that mitigates the shorter-term memory of investors, reducing panic and risk of runs. This rationale is not specific to FX markets, as it can be applied to other markets. Trust and confidence are key drivers of the values of financial assets (including currencies) and can be dented more easily in troubled times. Hence, interventions that can help reinstate confidence would be more effective.

While this paper examines the concept of hedging against sterling, it can be tested against any other currency. As an area for further research, the regime determination factor can be based on a macroeconomic indicator instead of foreign exchange spot rates. This might result in a deeper understanding of FX hedging and the dynamics with the macroeconomy, demonstrating the model's effectiveness even further.

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Appendix I

I.1. GO-GARCH Model

The model assumes that an observed process x_t follows a linear combination of uncorrelated components y_t as equation $x_t = Zy_t$, where Z, which stands for the linear map, connects the unobserved variables with observed components. The unconditional covariance matrix can be expressed as $V = ZZ^T$. Equation (A1) shows that Z is identified considering conditional information:

$$\mathrm{P}\Lambda^{\frac{1}{2}}U_0 = Z \tag{A1}$$

where U_0 is an orthogonal matrix. P and Λ are orthogonal matrices that have $[(m(m-1)/2]$ and m degrees of freedom, respectively. The orthogonal matrix U, that is an estimator of U_0, is represented with $[m(m-1)/2]$ matrices as expressed in Equation (A2):

$$U = \prod_{i<J} R_{ij}(\theta_{ij}) \qquad -\pi \leq \theta_{ij} \leq \pi \tag{A2}$$

where R_{ij} denotes the conditional covariance matrix of y_t and θ_{ij} refers to the Euler angle which defines ration points in matrices. The conditional covariance matrix V is provided in Equation (A3):

$$V = ZH_tZ^t \tag{A3}$$

where H is a diagonal matrix.

I.2. MRS Model

In the MRS model, the spot rate at time t, S_t, can be parameterised as a first-order Markov process with transition probabilities. S_t indicates two different market states. The first-order Markov process assumes that the regime probability at time t depends on the regime process at time $t-1$. The relationship between the two market states is expressed in Equation (A4):

$$\begin{gathered}\Pr[S_t = 1 \mid S_{t-1} = 1] = P_{11}\,;\Pr[S_t = 2 \mid S_{t-1} = 1\,] = P_{21}\\ \Pr[S_t = 2 \mid S_{t-1} = 2] = P_{22}\,;\Pr[S_t = 1 \mid S_{t-1} = 2] = P_{12}\end{gathered} \tag{A4}$$

where P_{21} denotes the likelihood that state 2 will occur after state 1, and P_{12} represents the likelihood that state 1 will occur after state 2 (Hamilton, 1989; Engel and Hamilton, 1990; Gray, 1996; Alizadeh and Nomikos, 2004; Lee and Yoler, 2007; Alizadeh et al., 2008; Zalachoris, 2022). The transition probabilities P_{11} and P_{22} reflect the possibility that the market's status will remain unchanged in the subsequent period. These transition probabilities, which can be estimated with the model's other parameters, are assumed to be constant across time. The obtained transition probabilities are illustrated in Table A3 in Appendix II. The figures align with other studies in the literature; the probabilities of remaining in the same regime are higher than the probabilities of regime changes for all the currency pair cases. In Equation (A5), the term γ_t^* stands for the weighted average of the minimum-variance hedge ratio.

$$\gamma_t^* = \pi_{1,t} \cdot \gamma_{1,1} + \pi_{2,t} \cdot \gamma_{1,2} \qquad \text{(A5)}$$

where $\pi_{1,t}$ and $\pi_{2,t}$ are the probabilities of being at time t in states 1 and 2, respectively; $\gamma_{1,1}$ and $\gamma_{2,2}$ indicate the minimum-variance hedge ratio of each state.

Appendix II

Table A1: List of utilised symbols.

Currency Pair	Spot	1M Forward	3M Forward	6M Forward
GBPUSD	USDOLLR	USGBP1F	USGBP3F	USGBP6F
GBPEUR	EURSTER	UKXEU1F	UKXEU3F	UKXEU6F
GBPJPY	JPAPYEN	UKJPY1F	UKJPY3F	UKJPY6F
GBPTRY	TURKLIR	UKTRY1F	UKTRY3F	UKTRY6F
GBPINR	INDRUPE	UKINR1F	UKINR3F	UKINR6F

Source: Refinitiv Eikon

Table A2: Spot rate correlations between the currency pairs.

	GBPUSD	GBPEUR	GBPJPY	GBPTRY	GBPINR
GBPUSD	1				
GBPEUR	0.43	1			
GBPJPY	0.75	0.69	1		
GBPTRY	-0.51	-0.49	-0.30	1	
GBPINR	-0.20	-0.49	-0.09	0.66	1

Table A3: Transition probabilities

		1M FWD		3M FWD		6M FWD	
GBPUSD		1	2	1	2	1	2
	1	0.909106	0.090894	0.978671	0.021329	0.972229	0.027771
	2	0.273181	0.726819	0.439840	0.560160	0.241063	0.758937
GBPEUR		1	2	1	2	1	2
	1	0.890393	0.109607	0.920095	0.079905	0.932547	0.067453
	2	0.132242	0.867758	0.044220	0.955780	0.066038	0.933962
GBPJPY		1	2	1	2	1	2
	1	0.951609	0.048391	0.835402	0.164598	0.792551	0.207449
	2	0.093414	0.906586	0.041987	0.958013	0.076944	0.923056
GBPTRY		1	2	1	2	1	2
	1	0.970362	0.029638	0.957813	0.042187	0.958308	0.041692
	2	0.238135	0.761865	0.133836	0.866164	0.098395	0.901605
GBPINR		1	2	1	2	1	2
	1	0.971243	0.028757	0.941784	0.058216	0.951992	0.048008
	2	0.173545	0.826455	0.171393	0.828607	0.075253	0.924747

Table A4: Minimum variance hedge ratio corresponding to each regime

	1M FWD		3M FWD		6M FWD		Avg	
	$\gamma_{1,1}$	$\gamma_{1,2}$	$\gamma_{1,1}$	$\gamma_{1,2}$	$\gamma_{1,1}$	$\gamma_{1,2}$	$\gamma_{1,1}$	$\gamma_{1,2}$
GBPUSD	1.0005	0.9998	1.0034	0.9826	1.0085	0.9960	1.0041	0.9928
GBPEUR	1.0020	1.0002	1.0072	1.0024	1.0175	1.0053	1.0089	1.0026
GBPJPY	1.0019	0.9998	1.0111	1.0005	1.0208	1.0012	1.0113	1.0005
GBPTRY	0.9847	0.0089	0.9616	0.0051	0.9281	0.0459	0.9581	0.0200
GBPINR	1.0004	0.9925	1.0007	0.9842	1.0032	0.9857	1.0014	0.9875

Table A5: Limit (Lim) p value sensitivity analysis

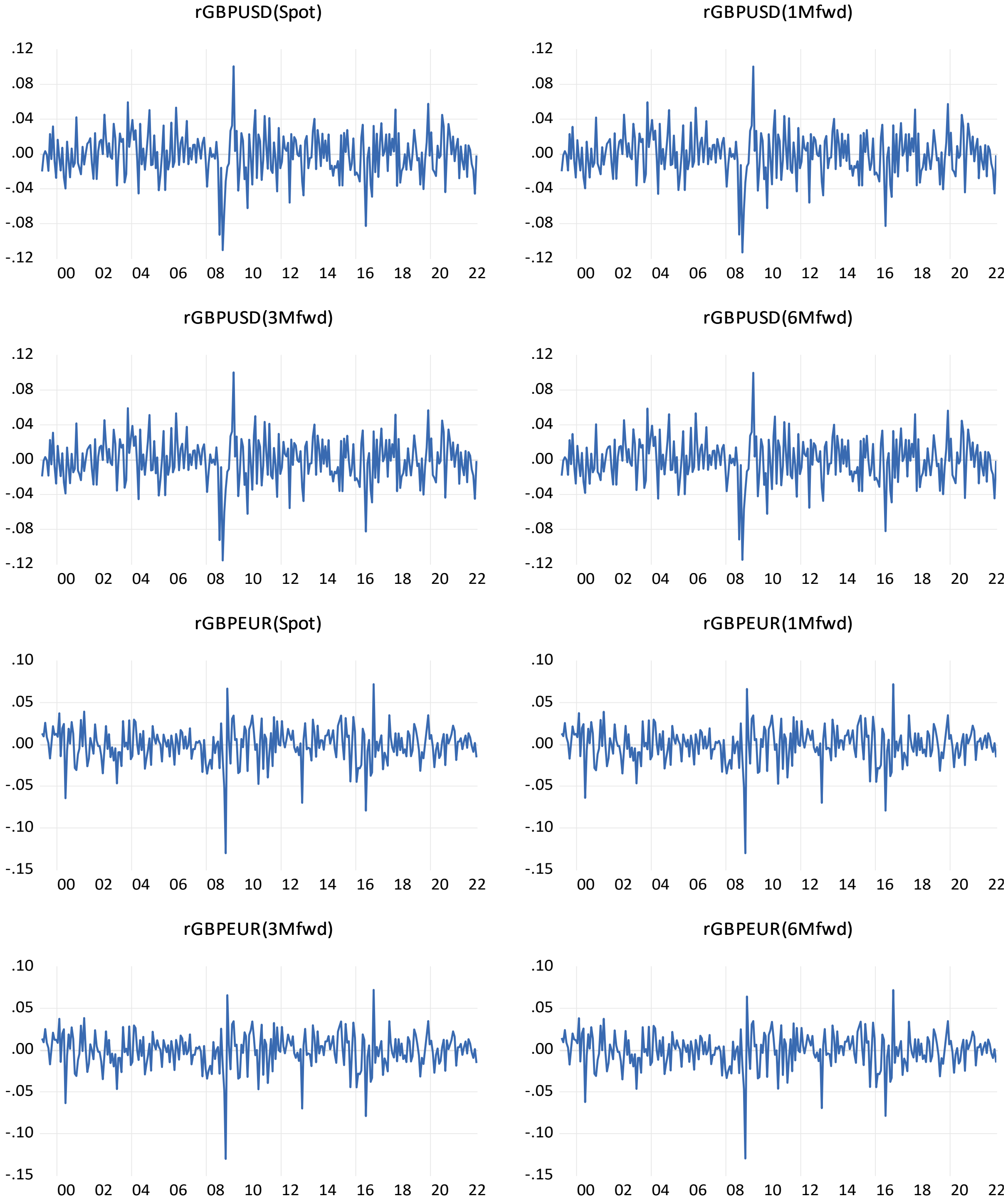
p=1.0		VH	H	L	VL	VH+H	L+VL
GBPUSD	Count	35	111	103	32	146	135
	Ratio	12.5%	39.5%	36.7%	11.4%	52.0%	48.0%
GBPEUR	Count	45	91	106	39	136	145
	Ratio	16.0%	32.4%	37.7%	13.9%	48.4%	51.6%
GBPJPY	Count	37	104	114	26	141	140
	Ratio	13.2%	37.0%	40.6%	9.3%	50.2%	49.8%
GBPTRY	Count	12	123	126	20	135	146
	Ratio	4.3%	43.8%	44.8%	7.1%	48.0%	52.0%
GBPINR	Count	45	99	90	47	144	137
	Ratio	16.0%	35.2%	32.0%	16.7%	51.2%	48.8%
Average	**Count**	**34.8**	**105.6**	**107.8**	**32.8**	**140.4**	**140.6**
	Ratio	**12.4%**	**37.6%**	**38.4%**	**11.7%**	**50.0%**	**50.0%**

p=1.2		VH	H	L	VL	VH+H	L+VL
GBPUSD	Count	26	120	110	25	146	135
	Ratio	9.3%	42.7%	39.1%	8.9%	52.0%	48.0%
GBPEUR	Count	33	103	119	26	136	145
	Ratio	11.7%	36.7%	42.3%	9.3%	48.4%	51.6%
GBPJPY	Count	26	115	123	17	141	140
	Ratio	9.3%	40.9%	43.8%	6.0%	50.2%	49.8%
GBPTRY	Count	11	124	132	14	135	146
	Ratio	3.9%	44.1%	47.0%	5.0%	48.0%	52.0%
GBPINR	Count	30	114	98	39	144	137
	Ratio	10.7%	40.6%	34.9%	13.9%	51.2%	48.8%
Average	**Count**	**25.2**	**115.2**	**116.4**	**24.2**	**140.4**	**140.6**
	Ratio	**9.0%**	**41.0%**	**41.4%**	**8.6%**	**50.0%**	**50.0%**

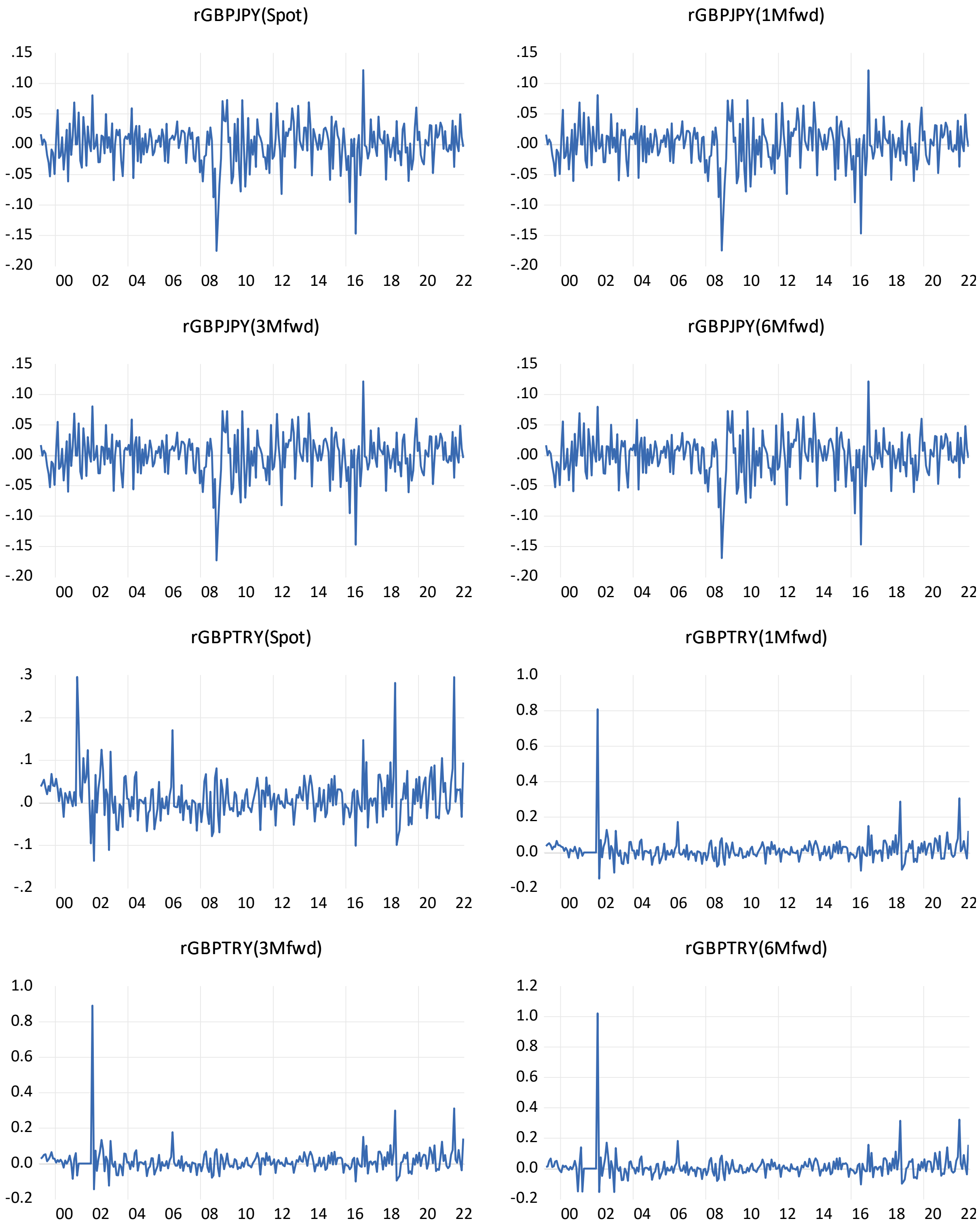
p=1.4		VH	H	L	VL	VH+H	L+VL
GBPUSD	Count	20	126	119	16	146	135
	Ratio	7.1%	44.8%	42.3%	5.7%	52.0%	48.0%
GBPEUR	Count	25	111	124	21	136	145
	Ratio	8.9%	39.5%	44.1%	7.5%	48.4%	51.6%
GBPJPY	Count	21	120	126	14	141	140
	Ratio	7.5%	42.7%	44.8%	5.0%	50.2%	49.8%
GBPTRY	Count	11	124	135	11	135	146
	Ratio	3.9%	44.1%	48.0%	3.9%	48.0%	52.0%
GBPINR	Count	21	123	105	32	144	137
	Ratio	7.5%	43.8%	37.4%	11.4%	51.2%	48.8%
Average	**Count**	**19.6**	**120.8**	**121.8**	**18.8**	**140.4**	**140.6**
	Ratio	**7.0%**	**43.0%**	**43.3%**	**6.7%**	**50.0%**	**50.0%**

Appendix III

Figure A1: Monthly FX spot and forward rate returns for GBPUSD, GBPEUR, GBPJPY, GBPTRY and GBPINR (February 1999-June 2022)



(Figure continues on next page)



(Figure continues on next page)

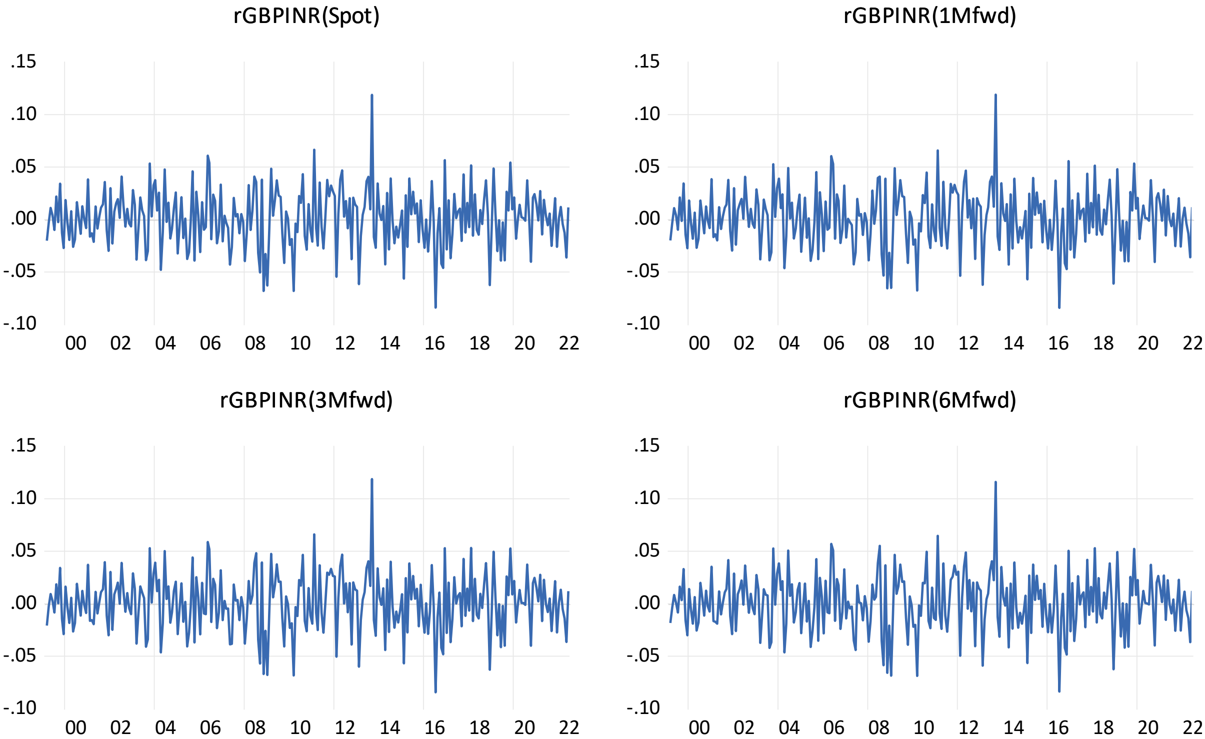
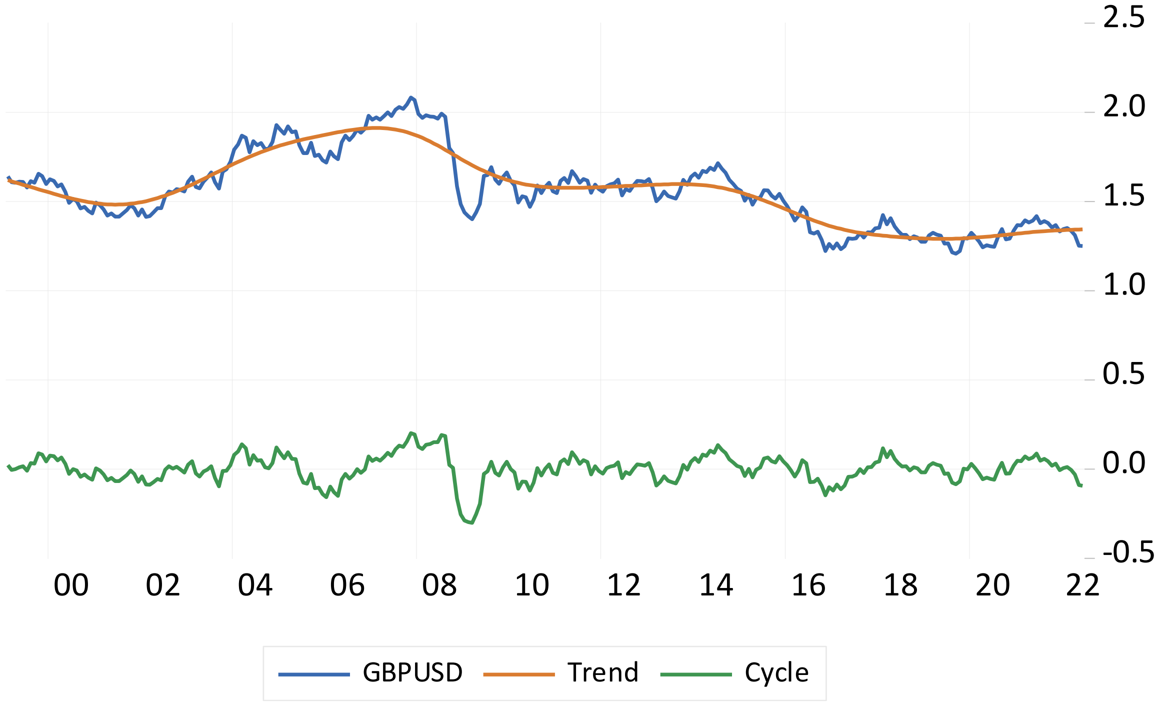
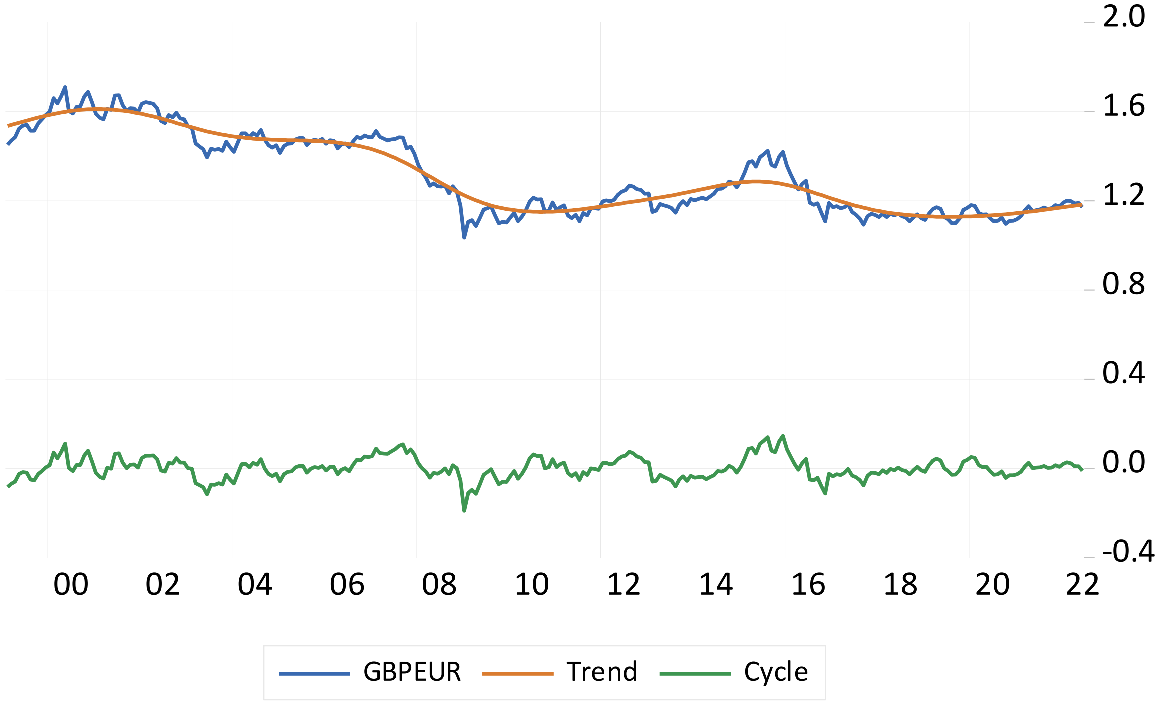

rGBPINR(Spot)
rGBPINR(1Mfwd)
rGBPINR(3Mfwd)
rGBPINR(6Mfwd)
.15
.10
.05
.00
-.05
-.10
00 02 04 06 08 10 12 14 16 18 20 22

Figure A2: Detrended FX rates. Each panel (GBPUSD, GBPEUR, GBPJPY, GPBTRY and GBPINR) includes FX spot rate, trend and cycle lines using the Hodrick–Prescott filter. Cycle means short-term fluctuations, which are decomposed from the detrended FX spot rates





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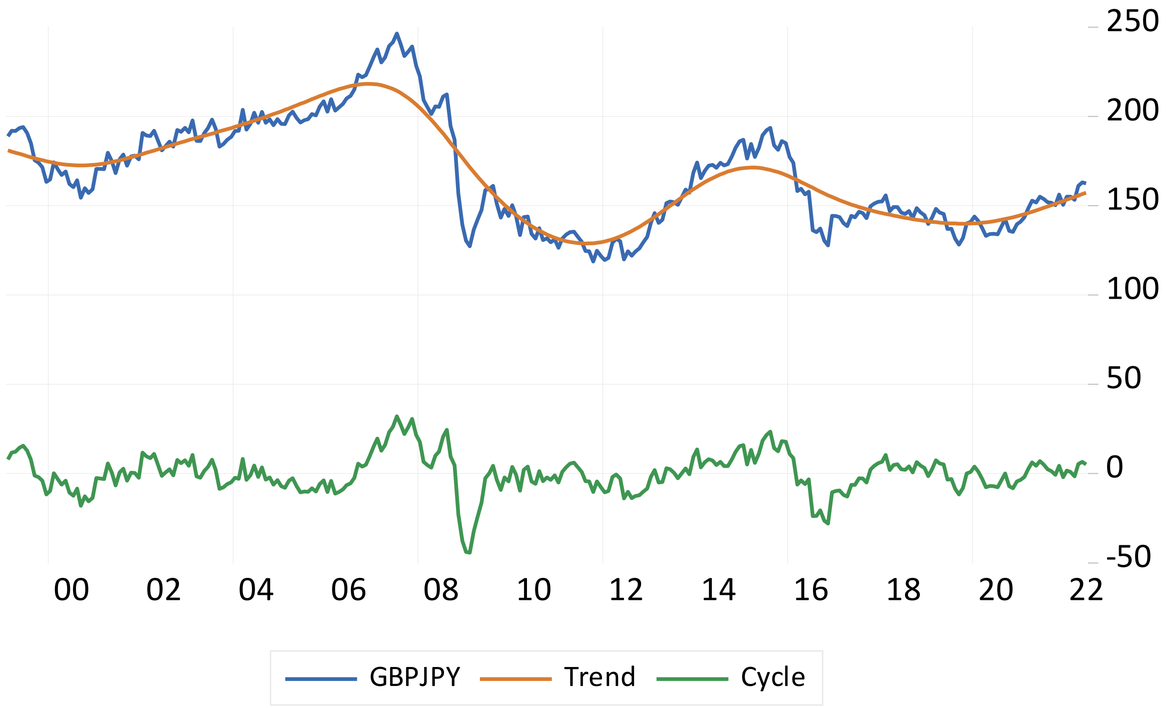
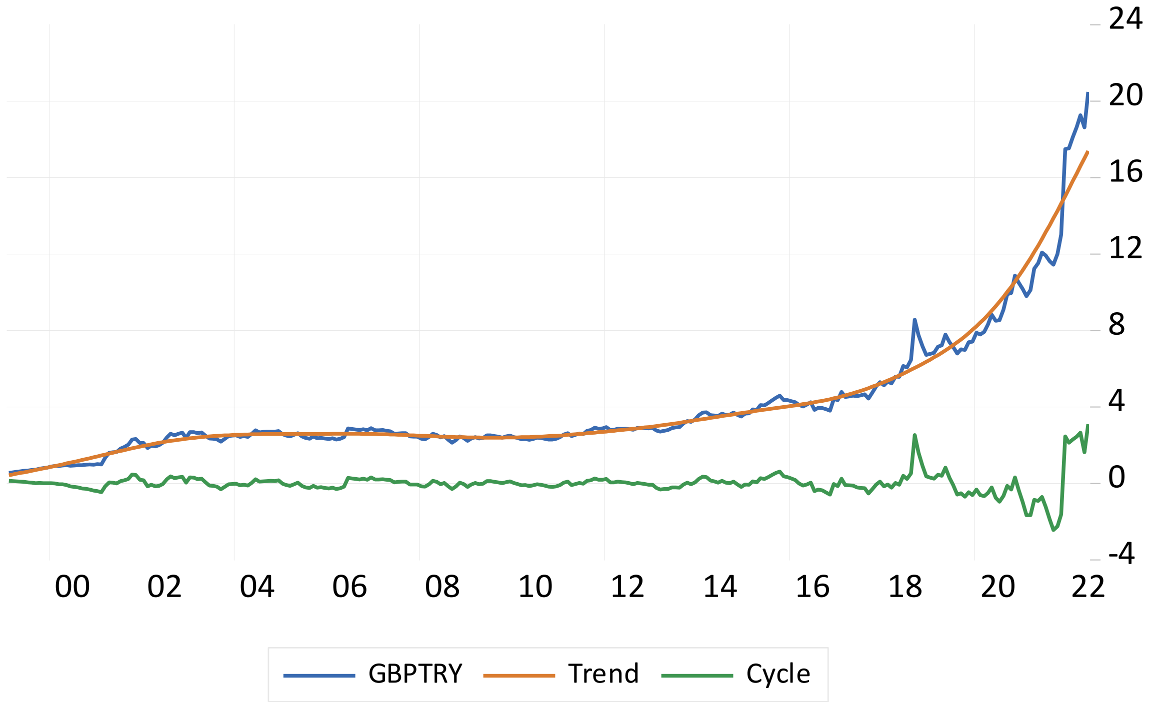




(Figure continues on next page)

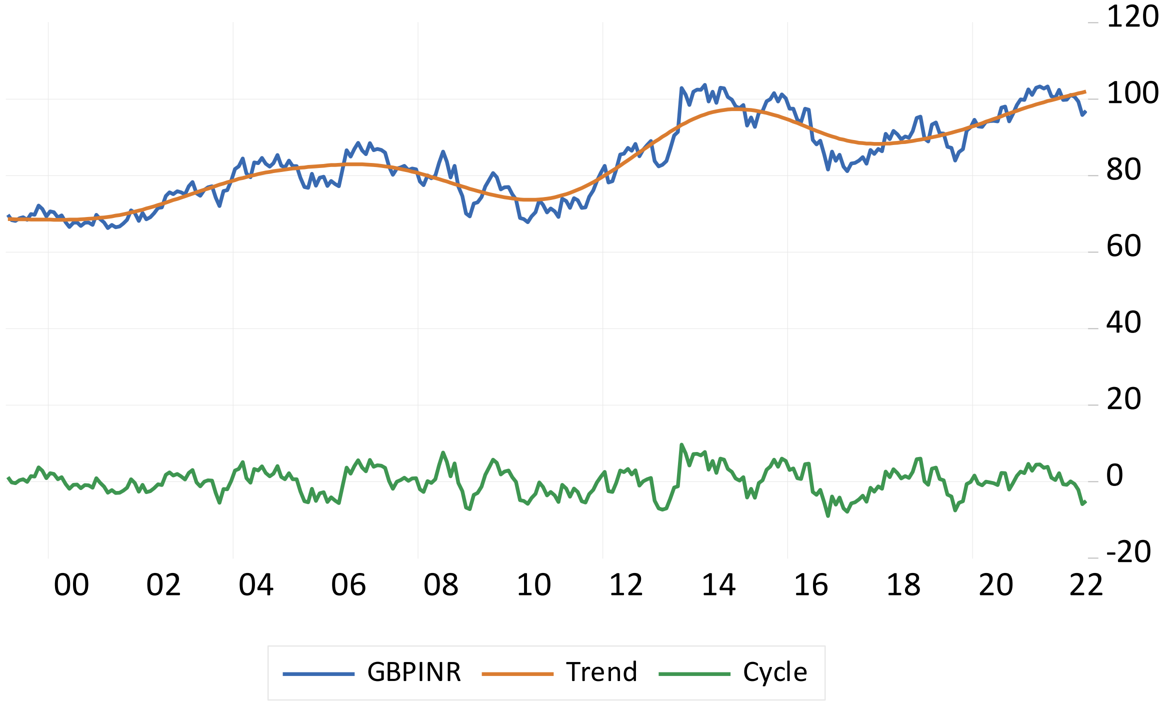
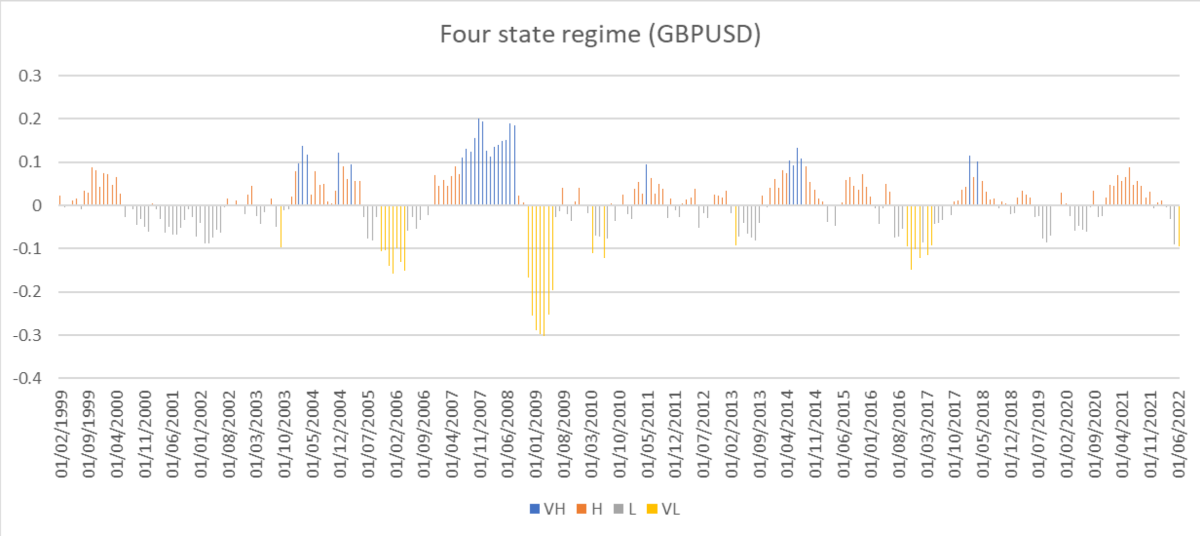
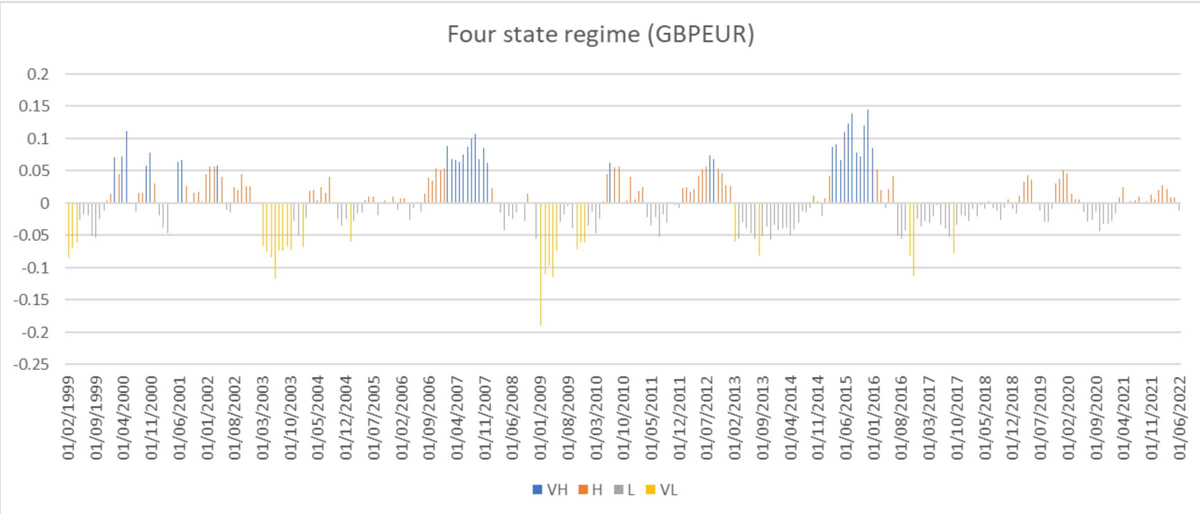
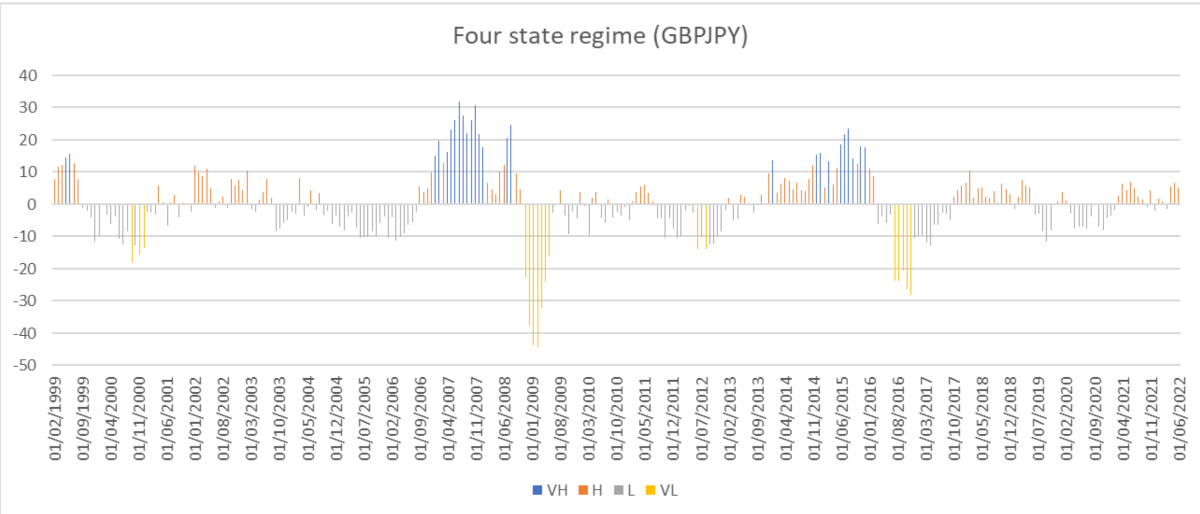

120
100
80
60
40
20
0
-20
00
02
04
06
08
10
12
14
16
18
20
22
GBPINR
Trend
Cycle

Figure A3: Four-state regimes. The following graphs show four-state distributions identified by detrending FX spot rates for GBPUSD, GBPEUR, GBPJPY, GBPTRY and GBPINR.







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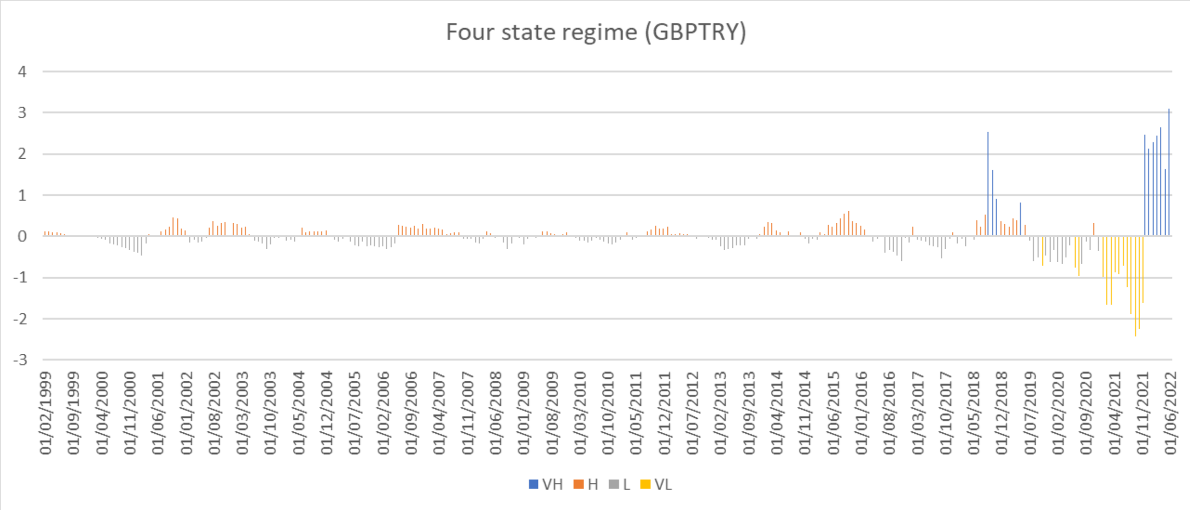
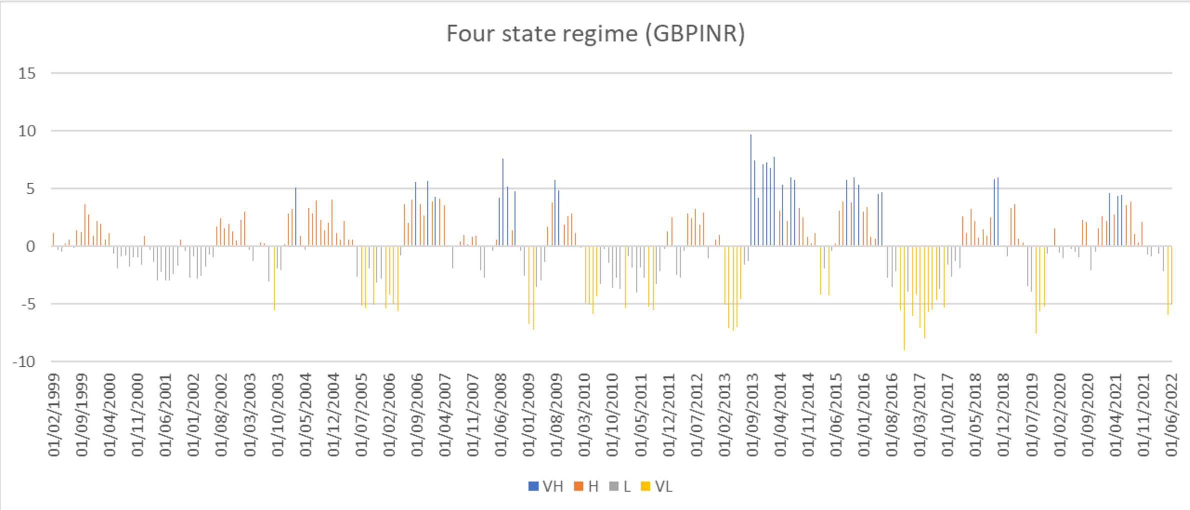

Four state regime (GBPTRY)
4
3
2
1
0
-1
-2
-3
01/02/1999
01/06/2022
VH
H
L
VL


Four state regime (GBPINR)
15
10
5
0
-5
-10
01/02/1999
01/06/2022
VH
H
L
VL