Bank of England

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## Self-fulfilling fire sales and market backstops

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### Abstract

Motivated by the March 2020 'dash for cash', we build a model in which a potential liquidity shock to bond funds can prompt a self-fulfilling fire sale in the presence of a dealer with limited trading capacity. Following the global games literature, we derive the probability of a self-fulfilling fire sale. Introducing a central bank market backstop policy, we show that if the central bank can credibly commit to (i) set the size of its potential asset purchases high enough and (ii) its price discount low enough, then it can eliminate self-fulfilling fire sales without having to purchase any bonds. If the central bank acts less aggressively, it can still reduce the probability of a self-fulfilling fire sale. However, in response to the policy, funds choose to hold more bonds, which increases the probability of a self-fulfilling fire sale and reduces the effectiveness of the market backstop.


**Key words:** Market backstops, fire sales, bond purchases.

**JEL classification:** G12.

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# 1 Introduction

Bond markets came under severe stress at the start of the COVID-19 crisis in March 2020, the so-called “dash for cash”. Investors sought to sell bonds in such volumes that dealers ran into capacity constraints and could not accommodate all desired trades. Concerned about the systemic risk posed by fire sales, central banks stepped in when dealers were overwhelmed and acted as backstop buyers of bonds to stabilise markets. This has led to a debate about whether last-resort tools for intervening in capital markets in pursuit of financial stability objectives should be part of central banks’ permanent policy toolkits. That debate has intensified following the Bank of England’s gilt market operations in September-October 2022, when it purchased long-dated securities at a spread to mid-prices to help restore market functioning (Cunliffe 2022).

One concern with such a policy is that if the central bank is expected to provide an effective market backstop, then problems akin to “moral hazard” can arise. The presence of a central bank backstop ensures that investors do not face the negative consequences of fire sales, so they optimally take more aggressive positions in risky assets. However, more aggressive positions would be more difficult to unwind in a crisis. Central bank intervention could therefore make the financial system more fragile, thus risking the incurrence of welfare costs associated with financial instability.

During the “dash for cash” in March 2020, funds were trying to sell off large amounts of bonds, while dealers were unable or unwilling to further increase their bond holdings. This imbalance became a worry for central banks, as they led to a sharp rise in gilt yields, bid-ask spreads, and premiums on repo borrowing. Markets only started to recover when the Bank of England and other central banks announced their willingness to provide a market backstop by purchasing significant amounts of bonds.[1]

This paper develops a theoretical model to examine the potential impact of central banks offering market backstops on the functioning of bond markets and the potential for moral hazard. Motivated by the empirical literature studying the “dash for cash” episode of March 2020. Haddad et al. (2021) argue that a key source of the selling pressure was driven by bond mutual funds facing immediate liquidity shocks and fearing future shocks. This selling pressure drove down bond prices because dealers struggled to intermediate given the magnitude of the selling pressure. However, Haddad et al. (2021) show how announcements by the Federal Reserve alone stabilised corporate bond markets. Kargar et al. (2021) also argue that corporate bond markets stabilised primarily because the announcements immediately reduced funds’ demand for cash.[2] This is consistent with central bank announcements reducing fears about potential consequences of future shocks.

[1] See for example Czech et al. (2021)

[2] Other papers that empirically examine various aspects of the “dash for cash” episode include Boyarchenko et al. (2022), Gilchrist et al. (2021), O’Hara & Zhou (2021), and Vissing-Jorgensen (2021). Interestingly, Vissing-Jorgensen (2021) provide evidence that Federal Reserve intervention in Treasury markets actually operated more through purchases than announcements due to the immediate liquidity needs of Treasury sellers. However, prior literature has found a role for announcement effects of Federal Reserve purchases of Treasuries (e.g., Krishnamurthy & Vissing-Jorgensen (2011)) showing that the need for purchases is not in general a special feature of the US Treasury market.

Our model therefore focuses on the behaviour of bond funds in a liquidity crisis. In the model, there is a continuum of risk-neutral funds initially endowed with a portfolio of cash and bonds. Similar to Bernardo & Welch (2004), funds then learn about the possibility of a future liquidity shock which would force them to sell their bond holdings to a dealer at a discount to the bond's expected gross return. The discount is increasing in the dealer's inventory of bonds, either because of risk-aversion (Bernardo & Welch 2004, Morris & Shin 2004) or institutional constraints.

The discount can create an incentive for funds to pre-emptively sell their bonds before they learn whether or not the liquidity shock has materialised. If a fund thinks that other funds will sell today, it will worry about being hit by the liquidity shock tomorrow and having to sell to a dealer with a large bond inventory charging a high discount. This makes it more attractive for the fund to sell today. If all funds reason in a similar manner, the optimality of the decision to sell today becomes self-fulfilling. But this is not the only possibility. If a fund thinks that all other funds will continue to hold bonds today, it will think that the dealer will have a low bond inventory tomorrow and therefore it will face the low discount if it is hit by the liquidity shock. This makes it more attractive for the fund to hold today and, if all funds reason similarly, the optimality of the decision to hold today becomes self-fulfilling. Therefore, the model features multiple equilibria: an equilibrium where funds hold and an equilibrium where funds panic and cause a self-fulfilling fire sale.

A problem with a model featuring multiple equilibria is that sunspots rather than economic fundamentals determine which equilibrium is realised. To solve this problem, we employ a global games approach (Carlsson & van Damme 1993, Morris & Shin 1998) to eliminate the multiplicity of equilibria and link the probability of a self-fulfilling fire sale to economic fundamentals. In particular, we show that there will be a self-fulfilling fire sale if and only if the capacity of the dealer to absorb bonds sales is sufficiently low. We also show that the probability of a self-fulfilling fire sale is increasing in the discount charged by the dealer in stressed market conditions and increasing in the probability of the liquidity shock occurring. The probability of a fire sale is also greater if the funds' initial endowment of bonds is higher because the dealer would have to absorb more bonds if the funds chose to unwind their positions.

Our next step is to introduce a portfolio choice decision for each fund. If there is small cost of holding bonds linked to the variance of their returns, we can show that funds optimally hold fewer bonds and more cash as the probability of a fire sale increases. The result is natural: a fire sale prevents the fund from holding the bonds to maturity and instead forces it to sell at a discount to the dealer, so a higher probability of fire sales makes holding bonds less attractive. As a result funds choose to hold more cash, which could be seen as a form of 'self insurance'. Since bond holdings of the funds itself influences the probability of a fire sale, these two variables are jointly determined in equilibrium. We show that an equilibrium exists and it is unique.

Our final step is to extend the model to introduce a central bank acting as a backstop buyer of assets. We allow the central bank to provide additional capacity to absorb bond sales so that the total capacity consists of both central bank and dealer capacity. The central bank also sets a discount (or spread) to the market price at which it is willing to purchase bonds. In practice, central banks might prefer to provide support through lending facilities ahead of a market backstop.[3] As our focus is on market backstops, however, we assume the funds in our model are non-levered, so they cannot borrow.[4]

We show that by committing to act aggressively, the central bank in our model can completely eliminate the possibility of a self-fulfilling fire sale. Moreover, it does not actually have to purchase any bonds to eliminate the self-fulfilling fire sale. The aggressive policy works via market expectations. In particular, it makes the pessimistic beliefs that drive the self-fulfilling fire sale impossible for funds to rationalise because they know that the central bank stands ready to act as a market backstop. If the central bank does not act aggressively enough, however, we show that funds will hold more bonds in response to its policy ('moral hazard'). This acts to increase the probability of a fire sale and partially offsets the effect of the policy.

It thus appears clear that the central bank should choose to act aggressively. However, we put forward two arguments as to why a central bank may not want to pursue an aggressive policy. First, whilst a commitment to act aggressively rules out self-fulfilling fire sales without the need for asset purchases, the central bank would be forced to act if there was a fire sale due to the subsequent crystallisation of the liquidity shock rather than self-fulfilling beliefs. The central bank may therefore wish to act less aggressively to trade-off the benefit of reducing self-fulfilling fire sales with the cost of intervening in fire sale events caused by fundamental liquidity shortfalls rather than expectations. Second, if market participants expect an aggressive policy, they will choose to hold more bonds. If the central bank then reneges on an aggressive policy or market participants lose faith in the ability of the central bank to act aggressively, we show that the probability of self-fulfilling fire sales could increase relative to doing nothing. This highlights the importance of a consistent and credible central bank policy.

Thus, this paper focuses on the effects of central bank market backstops when used to prevent self-fulfilling fire sales. It does not conduct a full cost-benefit analysis of such backstops or other polices that might reduce the probability of fire sales, such as reforms to enhance the resilience of market participants to liquidity shocks or to strengthen market-wide infrastructure, although we discuss the effects that such reforms may have on self-fulfilling fire sales within the context of our model in Section 4.3.[5] It also does not discuss how these policies might overlay, though our presumption is that these other policies would be considered for ongoing usage while market backstops would be only be called upon in exceptional circumstances.

[3] Potential costs of central bank market backstops include moral hazard, losses on asset holdings which taxpayers would have to bear and conflict with monetary policy if quantitative tightening were required.

[4] Non-levered funds invest shareholders' funds alone in financial assets. They do not borrow in order to increase their exposure to financial assets. This is approximately true of many funds in practice. In Europe, for example, funds regulated as Undertakings for Collective Investment in Transferable Securities (UCITS) may borrow up to a limit of 10% of their net assets, and only on a temporary basis, for example for liquidity management purposes.

[5] Hauser (2021) sets out how these policies may all play a role in enhancing the resilience of financial markets and, thus, reducing the likelihood of fire sales.

**Related literature.** Our paper builds upon the the financial market run literature initiated by Bernardo & Welch (2004) and Morris & Shin (2004). We have a similar setup to Bernardo & Welch (2004) in that funds have an incentive to pre-emptively sell because they are worried about future liquidity shocks and having to sell in depressed market conditions. Bernardo & Welch (2004) generate a downward sloping demand curve for risky assets via a risk-averse market maker. We do not micro-found the dealer's demand curve so that we can extend the model more easily to consider policy. The main methodological difference with Bernardo & Welch (2004) is that their equilibrium concept is Nash equilibrium. Their equilibrium features a mixed strategy Nash equilibrium where investors choose just the right probability of selling to equate the expected payoff from selling today and holding. We view the ability of investors to choose just the right probabilities somewhat at odds with crisis periods which are infrequent and chaotic. Instead, our equilibrium features the possibility that funds panic and all sell when fundamentals are weak but otherwise all hold. We see this as a better fit to a crisis episode.

In Morris & Shin (2004), there is also a risk-neutral trading sector and a risk-averse market making sector implying a downward sloping demand curve for risky assets. Traders may choose to pre-emptively sell their risky assets because they are worried about prices falling such that they hit a loss limit and they lose their job. Similar to our model, they devise a global game and show that there exists a threshold equilibrium where traders hold if fundamentals are good enough and sell if they are bad enough. The motivation for pre-emptively selling in our paper is a potential future liquidity shock, which is more tailored to the dash-for-cash episode. We also extend our model to allow us to investigate the effects of the central bank providing a market backstop.

More generally, this paper is related to the literature on runs in the financial system. Diamond & Dybvig (1983) pioneered this literature by showing how the liquidity mismatch inherent in banking make banks vulnerable a self-fulfilling run. The model in Diamond & Dybvig (1983) features another equilibrium with no bank run and it is unclear which equilibrium will be realised. Therefore, Goldstein & Pauzner (2005) examine a global game variant of a bank run which eliminates the multiplicity and links the probability of bank runs to economic fundamentals. Allen et al. (2018) add a government to the model of Goldstein & Pauzner (2005) to study how government guarantees affect the probability of runs and welfare. The structure of our paper follows the development of the literature for bank runs: we first consider a model with multiple equilibria, then we employ global games techniques to eliminate the multiplicity of equilibria, and finally we extend the model to study the effects of policy.

The structure of open-ended mutual funds can also make them vulnerable to runs. As argued by Chen et al. (2010), open-ended funds allow investors to redeem their investments on a daily basis whilst investing in illiquid assets. Crucially, investors are paid based on the most recent net asset value whilst the trades are conducted in the following days. A large number of withdrawals, which forces the fund to quickly sell some of its assets, can thus depress the net asset value for those remaining in the fund which creates an incentive for them to also withdraw. Goldstein et al. (2017) show that this force is stronger in funds investing in more-illiquid corporate bonds. Morris et al. (2017) provide evidence to show that asset managers hoard cash in response to redemptions which can exacerbate fire sales initiated by the redemptions themselves. These papers all examine the interaction between the fund and its investors whereas our paper examines the interaction between the funds.

Our paper is also related to two recent papers studying central bank policy in response to the dash-for-cash episode. Choi & Yorulmazer (2022) examine the role of a central bank as a market maker of last resort. They employ a cash-in-the-market framework (Allen & Gale 1994, 1998) and model the central bank policy as a promise to inject future cash into the market. They show that this promise encourages private-sector dealers to make markets today. Interestingly, by committing to act more aggressively in the future, the central bank can reduce its asset purchases and eliminate self-fulfilling pessimistic equilibria. The results of Choi & Yorulmazer (2022) therefore mirror our own results regarding the aggressiveness of central bank action within a different framework. Eisenbach & Phelan (2022) also study the dash-for-cash episode as a market run. However, they focus on showing that a flight to safety can trigger a dash for cash in times of stress. Our focus is on how the probability of fire sales, the portfolio decision of funds, and central bank policy all interact.

# 2 Self-fulfilling fire sales

In this section, we fix the initial portfolio of the bond funds. The funds are faced with the risk of a future liquidity shock and are able to pre-emptively sell their bond holdings today. We first show how multiple equilibria can arise due to self-fulfilling beliefs. If funds believe that future market conditions will be poor, they choose to pre-emptively sell their bonds which makes market conditions poor. If funds believe market conditions will be good, they choose to hold their bonds which makes the future market conditions good. We then consider a global game variation on the model by introducing some uncertainty in the capacity of dealers to absorb bonds. We show that the global game has a unique equilibrium where self-fulfilling fire sales only occur when dealer capacity is sufficiently low.

## 2.1 Multiple equilibria

There are three dates $t = 0, 1, 2$ and a continuum of risk-neutral bond funds of measure 1. The funds are initially endowed with $x$ units of bonds and $1 - x$ units of cash. Cash returns 1 in each period. Bonds have an expected return $R > 1$ at $t = 2$. Before $t = 2$, bonds can be sold to dealers at a discounted price $p = R - \delta$ where $\delta > 0$. Dealers face difficulties holding large quantities of bonds so $\delta$ increases with the number of bonds held. This can be micro-founded by assuming a risk-averse dealer sector as in Bernardo & Welch (2004) and Morris & Shin (2004) or by appealing to balance-sheet constraints.[6] In particular, assume that $\delta = \delta_L$ when the quantity of bonds held is less than $K \in [0,1]$ and $\delta = \delta_H$ when the quantity of bonds held is greater than $K$ where $\delta_L < \delta_H$.

At $t = 0$, dealers first set the price they are willing to pay for bonds $p_0$. Since their bond holdings are initially zero, they set $p_0 = R - \delta_L$. At $t = 0$, funds learn of the possibility of being hit by a liquidity shock at $t = 1$ with probability $q$ independent of the bond return. They can choose to sell their bond holdings at $t = 0$ at price $p_0$ to prepare for the liquidity shock or hold the asset. If they face the liquidity shock, they are forced to sell their bonds at $t = 1$ at price $p_1$. If they do not face the liquidity shock, they can hold the bonds through to maturity at $t = 2$ and earn the return $R$.

A key assumption in our analysis is that prices do not fully adjust within the period due to selling pressure within the same period. In particular, all selling at $t = 0$ takes place at $p_0 = R - \delta_L$, after which the price adjusts for $t = 1$. We motivate this by assuming that the fund may submit their sell order at $p_0 = R - \delta_L$ but not take into account that prices may have adjusted by the time that their sell order is executed.[7] Indeed, Haddad et al. (2021) report that during the dash-for-cash period there was evidence of “liquidity inversion” where assets that are normally more liquid experienced price discounts greater than assets that are typically more illiquid. They argue that funds did not take into account real time price adjustments when making their trading decisions due to the speed at which the crisis took place. This assumption can create an incentive to pre-emptively sell bonds: funds believe that selling at $t = 0$ guarantees a high price whereas waiting until $t = 1$ carries the risk of being forced to sell at a low price if the liquidity shock hits and other funds have already sold to the dealer.[8]

Given this assumption and since the funds are atomistic, they take $p_0$ as given and choose to sell at $t = 0$ if

$$p_0 > qp_1(\tilde{\delta}) + (1-q)R \tag{1}$$

where $\tilde{\delta} \in \{\delta_L, \delta_H\}$ is a fund’s belief about the discount applied by the dealer for any trades at $t = 1$.

[6] For example, during the dash for cash, the leverage ratio that dealers must satisfy seemingly became a binding constraint (Duffie 2020). Dealers also face a risk-weighted capital constraint that limits their ability to hold bonds on their balance sheets.

[7] Alternatively, the existence of a small friction in dealer price setting so that dealers do not instantaneously adjust the price in response to selling pressure would justify this assumption. This might also be reasonable to assume if the dealers have had some success in anticipating a period of selling pressure and have already lowered $p_0$ (reflected in the discount $delta_L$).

[8] Our assumption is similar to the assumption from the market-run literature (Bernardo & Welch 2004, Morris & Shin 2004) that execution order is not perfectly sequential. In these models, when investors submit a sell order at $t = 0$, they join a queue of investors wishing to sell at $t = 0$. Because investors do not know their exact position in the queue, the models assume that investors believe they will be in the middle of the queue. Waiting to sell until $t = 1$ ensures a position at the rear of the queue. Since investors further down the queue receive a lower price, this gives an incentive to pre-emptively sell. In our model, our assumption that funds do not take into account within-period price adjustment is equivalent to assuming that all funds believe they will be at the front of the queue rather than the middle. Therefore, the underlying reason for pre-emptive selling in our model is identical to the market run literature.

Using $p = R - \delta$, we can write (1) as

$$q\tilde{\delta} > \delta_L. \tag{2}$$

Thus, the choice of an individual fund to sell depends on whether it expects the dealer to be under strain or not. We look for an equilibrium where $\tilde{\delta} = \delta$, that is, beliefs coincide with outcomes.

**Proposition 1.** *An equilibrium with $\tilde{\delta} = \delta_L$ always exists. If (i) $q\delta_H > \delta_L$ and (ii) $x > K$ both hold, then a second equilibrium exists with $\tilde{\delta} = \delta_H$.*

*Proof.* If $\tilde{\delta} = \delta_L$, then from (2) the condition for selling is $q > 1$. Therefore, an individual fund has no incentive to sell. Since all funds are identical, no fund sells. This means that the bond holdings of the dealer going into $t = 1$ are zero. This implies $\delta = \delta_L$ so we have an equilibrium.

Suppose instead that $\tilde{\delta} = \delta_H$. If $q\delta_H \leq \delta_L$, then from (2) we see that no fund chooses to sell. This means that the bond holdings of the dealer remain zero which implies $\delta = \delta_L$. Therefore, this cannot be an equilibrium. If $q\delta_H > \delta_L$, then an individual fund wishes to sell. Therefore, all funds choose to sell and the dealer absorbs $x$ assets. If $x \leq K$, then we still have $\delta = \delta_L$ so this is not an equilibrium. If $x > K$, then the dealer will set $\delta = \delta_H$ for period 1. This gives the other equilibrium. □

From Proposition 1, we see that an equilibrium with no fire selling always exists. If funds expect future market conditions to be good, there is no incentive to pre-emptively sell bonds. Because funds do not pre-emptively sell their bonds, market conditions indeed are good.

However, we also see that there is potential for an equilibrium with self-fulfilling fire sales to exist. In this equilibrium, funds sell their bonds at $t = 0$ leading to fire sale prices at $t = 1$ simply because funds expect fire sale prices at $t = 1$. The first condition for the fire selling equilibrium to exist is that it must be sufficiently likely that funds are hit by a liquidity shock and then have to sell their bonds at depressed prices. This condition gives the incentive for funds to sell at $t = 0$ and potentially put the dealer under strain. But even if this condition is satisfied, we still require that the total quantity of bonds the dealer absorbs be large enough so that they are indeed put under strain when funds choose to sell.

### 2.2 Unique equilibrium via global games

An undesirable feature of the multiple equilibria result in Proposition 1 is that the model is silent about how likely the fire sale equilibrium is to arise. Following the global games literature (Carlsson & van Damme 1993, Morris & Shin 1998), we now introduce a small amount of private noise to the model which eliminates the multiplicity. The result is that we can link the probability of a self-fulfilling fire sale to economic fundamentals.

Now assume that funds face uncertainty about the dealer capacity $K$. Let the prior distribution of $K$ be uniform on $[0,1]$ and denote the density function by $f(\cdot)$. $K$ is also independent of the liquidity shock and the bond return. At $t=0$, each fund $i$ observes a private signal

$$z_i = K + e_i \tag{3}$$

where $e_i$ is a noise term which is uniformly distributed on $[-\varepsilon, \varepsilon]$ where $\varepsilon > 0$ is small.[9] This assumption reflects the fact that funds face some uncertainty related to the true capacity of the dealer bank because they do not know the exact details of the dealer's balance sheet position.

After observing their private signal, funds once again have a choice whether to sell their bond holdings at $p_0$ or hold. Let $a \in \{\text{sell}, \text{hold}\}$ denote the action. Denote the proportion of funds selling by $s$. Each fund's payoff $u(a, s, K)$ is a function of the action taken, the proportion of funds selling, and the dealer capacity. We can therefore write the payoffs as

$$u(\text{sell}, s, K) = p_0 = R - \delta_L \tag{4}$$

$$u(\text{hold}, s, K) = qp_1(\delta) + (1-q)R = \begin{cases} q(R-\delta_L) + (1-q)R & \text{if } sx \leq K \\ q(R-\delta_H) + (1-q)R & \text{if } sx > K. \end{cases} \tag{5}$$

Each fund holds $x$ bonds so, if a proportion $s$ sell, the dealer will have to absorb $sx$ bonds. If $sx$ exceeds $K$, the dealer sets $p_1 = R - \delta_H$. Defining $\pi(s,K) \equiv u(\text{sell}, s, K) - u(\text{hold}, s, K)$ as the payoff gain from selling, we have

$$\pi(s,K) = \begin{cases} -(1-q)\delta_L & \text{if } sx \leq K \\ q\delta_H - \delta_L & \text{if } sx > K. \end{cases} \tag{6}$$

Notice that if $q\delta_H \leq \delta_L$ or $x \leq K$, the dominant strategy is to hold. If both of these conditions fail, then the decision whether to sell or hold depends on a fund's belief about $s$. We therefore need to derive this belief.

To make progress, we look for an equilibrium where funds follow a switching strategy of the form:

$$a = \begin{cases} \text{hold} & \text{if } z_i > z^* \\ \text{sell} & \text{if } z_i \leq z^* \end{cases} \tag{7}$$

where $z^*$ is a threshold value to be determined. We are then able to derive the distribution of $s$ for a fund whose private signal is exactly the threshold $z^*$.

**Lemma 1.** *Suppose that funds follow the switching strategy around $z^*$ as in (7). Then the density of $s$ conditional on $z^*$ is uniform over $[0,1]$.*

[9] The results go through for a general density function for the noise whenever the density becomes concentrated around zero (see Morris & Shin (2003)).

*Proof.* The proof follows Morris et al. (2017). When the true dealer capacity is $K$, the signals $\{z_i\}$ are distributed uniformly over $[K-\varepsilon, K+\varepsilon]$. Funds with signals $z_i < z^*$ choose to sell. Hence,

$$s = \frac{z^* - (K-\varepsilon)}{2\varepsilon}.$$

To derive the distribution of $s$ conditional on $z^*$, we derive the cumulative distribution function. In particular, we compute the probability that $s < b$ conditional on $z^*$. Define $K_0$ as

$$b = \frac{z^* - (K_0 - \varepsilon)}{2\varepsilon} \implies K_0 = z^* + \varepsilon - 2\varepsilon b.$$

Thus, $s < b$ if and only if $K > K_0$. We therefore need the probability of $K > K_0$ conditional on $z^*$. Fund $i$'s posterior density over $K$ conditional on $z^*$ is $[z^* - \varepsilon, z^* + \varepsilon]$. Therefore,

$$\Pr(K > K_0 | z^*) = \frac{z^* + \varepsilon - K_0}{2\varepsilon} = \frac{z^* + \varepsilon - (z^* + \varepsilon - 2b\varepsilon)}{2\varepsilon} = b.$$

This is the cumulative distribution function of the uniform distribution. □

Given Lemma 1, we can now determine the equilibrium of the game. We focus on the case when $\varepsilon \to 0$, that is, the private noise vanishes. This implies that the only role of the private noise is to break down the perfect coordination that generates multiple equilibria and allows direct comparison with Proposition 1.

**Proposition 2.** *Let $\varepsilon \to 0$. If $q\delta_H \leq \delta_L$ or $x < K$, all funds have a dominant strategy to hold. If $q\delta_H > \delta_L$ and $x > K$, then there is an equilibrium where all funds hold if $K > K^*$ and all funds sell if $K < K^*$ where*

$$K^* = \left[\frac{q\delta_H - \delta_L}{q(\delta_H - \delta_L)}\right] x \equiv \alpha x < x. \tag{8}$$

*$K^*$ is the ex-ante probability of a self-fulfilling fire sale occurring and $\alpha$ is the ex-ante probability of a self-fulfilling fire sale occurring conditional on $x > K$.*

*Proof.* Consider a fund observing a signal $z_i$. Fund $i$'s expectation of $K$ is then $z_i$. If $z_i \geq x$, from (6) we see that it is always optimal for fund $i$ to hold regardless of the value of $s$. Therefore, since as $\varepsilon \to 0$ we have $z_i \to K$, we have that the funds have a dominant strategy to hold whenever $x < K$. It is also evident from (6) that funds have a dominant strategy to hold when $q\delta_H \leq \delta_L$.

If $z_i < x$, then fund $i$'s optimal decision to sell or hold depends on the value of $s$. To make progress, consider a fund observing signal $z^*$. From Lemma 1, we know that the fund believes the density of $s$ is uniform on $[0, 1]$ if all funds play the threshold strategy according to (7). Therefore, for $z^* < x$ we

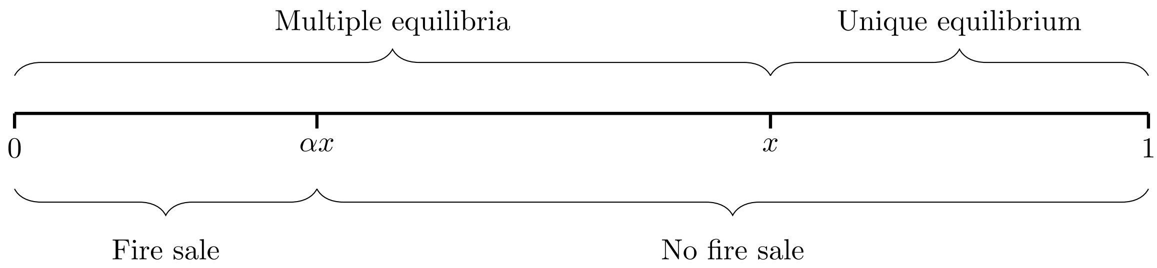


Figure 1: Equilibrium outcomes for different values of $K$

have

$$\int_0^1 \pi(s,z^*)ds = -(1-q)\delta_L \frac{z^*}{x} + \left(1 - \frac{z^*}{x}\right)(q\delta_H - \delta_L).$$

If $s < z^*/x$, then the fund believes that the dealer faces no stress. Since $s$ is uniform on $[0,1]$ conditional on $z^*$, the fund believes that the probability of no stress is $z^*/x$. This gives the first term. The second term follows because the fund believes that the probability of the dealer being under stress is $1 - z^*/x$.

The threshold value $z^*$ must satisfy $\int_0^1 \pi(s,z^*)ds = 0$. This yields

$$z^* = \left[\frac{q\delta_H - \delta_L}{q(\delta_H - \delta_L)}\right] x.$$

To complete the argument, we need to show that if $z_i < z^*$, fund $i$ prefers to sell, and if $z_i > z^*$, fund $i$ prefers to hold. The details are given in the appendix.

Letting $\varepsilon \to 0$ implies that the threshold value of $z^*$ corresponds to a threshold value of $K^*$. Therefore, funds will hold if $K > K^*$ and we will get a self-fulfilling fire sale when $K < K^*$. Since the prior distribution of $K$ is uniform on $[0,1]$, $K^*$ is also the *ex-ante* probability of a self-fulfilling fire sale. Finally, $\Pr(\text{fire sale}|x > K) = \alpha x/x = \alpha$. □

It is instructive to contrast Propositions 1 and 2. In both models, if $q\delta_H \leq \delta_L$ or $x < K$, then there is only one equilibrium where all funds hold.[10] The value of the global games approach arises when $q\delta_H > \delta_L$ and $x > K$, in which case there are multiple equilibria in the framework without private noise. The global games approach tells us which one of these equilibria will arise as a function of economic fundamentals. Figure 1 summarises the results of Propositions 1 and 2 by depicting the equilibrium outcomes as a function of $K$, with Proposition 1 above the bold line and Proposition 2 below it.

We can also see how the *ex-ante* probability of the fire sale equilibrium changes as economic fundamentals change.

**Corollary 1.** *The following are true:*

[10] The ex-ante probability of $x = K$ is zero so we ignore this case.

1. *$\partial K^*/\partial x > 0$. A higher value of $x$ means that funds hold more bonds at the outset. This means that there are more bonds that could potentially be sold to the dealer, which increases the probability that the dealer will be under stress. The incentive to sell today increases, so a fire sale is ex-ante more likely.*

2. *$\partial K^*/\partial q > 0$. A higher value of $q$ means it is more likely that a fund will get hit by a liquidity shock. The incentive to sell today increases, so a fire sale is ex-ante more likely.*

3. *$\partial K^*/\partial \delta_H > 0$. As the severity of the stressed state increases, the incentive to sell today increases. The ex-ante probability of a fire sale occurring increases.*

## 3 Optimal portfolio choice

In the analysis thus far, we have taken the choice of the quantity of bonds held as given. In this section, we endogenise this choice and examine the interaction between the probability of fire sales $K^*$ and the optimal portfolio choice.

Suppose that before funds receive their private signals, they make their choice of $x$. In the current setup, funds are risk-neutral. Therefore, they will simply choose to hold either all cash or all bonds depending on which has a higher expected payoff. In reality, however, funds are unlikely to be risk neutral and may hold some cash for self-insurance purposes, which would reduce the variance of their portfolio. To introduce this in our model, we assume that there is a penalty for variance in the portfolio $c(x) = \gamma x^2$ where $\gamma > 0$.[11] Note that the binary action choice at $t = 0$ combined with no time discounting implies that the introduction of this cost does not affect the optimal choice of a fund to sell or hold in section 2. If they sell their entire bond holdings at $t = 0$, they gain some benefit $c > 0$ for no longer holding bonds. If they hold their bonds into $t = 1$, they then either sell their bonds or they mature giving the same benefit $c$ in either case. Therefore, it is optimal to sell if $p_0 x + c > q(p_1 x + c) + (1 - q)(Rx + c)$ which simplifies to equation (1).

We focus on an equilibrium where the *ex-ante* probability of a fire sale is strictly positive. Following Proposition 2, we therefore must have that (i) $q\delta_H > \delta_L$ and (ii) $x > K$. The former condition we can simply assume. However, since we are now endogenising $x$, we cannot simply assume that $x > K$. Instead, we guess that $x > K$ and show that we can find an equilibrium where our guess is indeed true.

An individual fund thus takes the aggregate bond holding $x$ (and therefore $K^*$) as given. Bonds are available in perfectly elastic supply with the price normalised to 1. The fund therefore solves the

[11] Note that the variance of the portfolio is $\text{Var}[(1 - x) + xR_B]$ where $R_B$ is the overall return on holding bonds. The overall bond return has three sources of stochasticity: the dealer capacity $K$ which determines the occurrence of a fire sale, the liquidity shock, and the bond return itself. Using the rules for the variance operator, the portfolio variance is $x^2\text{Var}[R_B]$ which motivates the quadratic penalty term. For simplicity, we assume that $\text{Var}[R_B]$ is a constant $\gamma$, although strictly speaking $\gamma$ depends on other model parameters.

problem

$$\max_{x\in[0,1]} \quad (1-x)+x\left[\int_0^{K^*}(R-\delta_L)f(K)dK+\int_{K^*}^1(q(R-\delta_L)+(1-q)R))f(K)dK\right]-\gamma x^2. \qquad (9)$$

The first term in the square brackets gives the payoff when a self-fulfilling fire sale occurs and funds all sell their entire bond holdings at $p_0 = R - \delta_L$. This outcome occurs for values of $K$ between 0 and $K^*$. The second term gives the payoff when everyone holds. With probability $q$, the funds face a liquidity shock at $t = 1$ and are forced to sell their bond holdings. Since the stock of bonds held on the dealer's balance sheet going into $t = 1$ is zero, the price $p_1$ is $R - \delta_L$. With probability $1 - q$, the funds are able to hold the funds through to maturity and obtain the expected return $R$. These outcomes occur for values of $K$ between $K^*$ and 1.

The solution to (9) gives the optimal $x$ as a function of $K^*$. Proposition 2 derived $K^*$ as a function of $x$. An equilibrium is thus an $(x_e, K_e^*)$ pair where $x_e$ solves (9) conditional on $K_e^*$ and $x_e$ implies $K^* = K_e^*$ according to (8).

In what follows, we focus on the interesting case where a self-fulfilling fire sale may occur and hence $x_e > K$. For any realisation of $K$, we can choose $R$ large enough to ensure a self-fulfilling fire sale is possible and the unique equilibrium can then be guessed.

**Proposition 3.** *Conditional on $K^*$, aggregate fund bond holdings are*

$$x=\frac{R-1-\delta_L(q+K^*(1-q))}{2\gamma}. \qquad (10)$$

*There exists a unique equilibrium pair $(x_e, K_e^*)$ which simultaneously satisfies (8) and (10):*

$$x_e=\frac{R-\delta_L q}{2\gamma+\delta_L\alpha(1-q)}, \qquad (11)$$

$$K_e^*=\alpha x_e. \qquad (12)$$

*Proof.* First guess that $x > K$ so that it is possible for a self-fulfilling fire sale to occur. We can then solve the fund's problem in (9). Using the fact that the prior distribution of $K$ is uniform over $[0, 1]$, we can write the term in the square brackets as

$$K^*(R-\delta_L)+(1-K^*)(q(R-\delta_L)+(1-q)R).$$

This simplifies to

$$R-\delta_L(K^*(1-q)+q).$$

The fund problem can thus be written as

$$\max_{x\in[0,1]} \quad (1-x)+x(R-\delta_L(q+K^*(1-q)))-\gamma x^2.$$

Assuming the parameters are such that an interior solution exists, there is a unique maximum given by (10). Since all funds are *ex-ante* identical, this is the optimal $x$ for all funds and therefore the aggregate fund bond holdings. Substituting $K^*=\alpha x$ from (8) into (10) and solving for $x$ gives $x_e$ which then immediately implies $K^*_e=\alpha x_e$. Finally, we need to verify our guess that $x>K$. We can always ensure this guess is correct by choosing $R$ to be suitably large. □

The first result in Proposition 3 is that aggregate fund bond holdings are decreasing in $K^*$. This is because a fire sale prevents the fund from holding the bond to maturity and instead forces it to sell at a discount to the dealer. Therefore, a higher probability of fire sales makes holding bonds less attractive. Holding $K^*$ fixed, the effect of $K^*$ on $x$ is attenuated by (i) a reduction in $\delta_L$ and (ii) an increase in $q$. A reduction in $\delta_L$ makes fire sales less costly so changes in the probability of fire sales have less of an effect on bond holdings. An increase in $q$ makes it less likely that funds will be able to hold their bonds to maturity due to the $t=1$ liquidity shock so the fire sale at $t=0$ becomes less relevant.

We can build graphical intuition for the second result in Proposition 3 by drawing equations (8) and (10), which characterise equilibrium, in $(K^*,x)$ space. In particular, notice that (8) is an upward sloping line passing through the origin with slope $1/\alpha$. Equation (10), is a downward sloping line with vertical intercept $(R-1-\delta_L q)/2\gamma$ and slope $-\delta_L(1-q)/2\gamma$. The equilibrium is depicted by the black lines in Figure 2 with the intersection giving the unique mutually consistent $(x,K^*)$ pair.

We can also perform comparative statics exercises with the aid of our diagram. Suppose that there a reduction in the discount the dealer applies to the bond in stressed market conditions: a reduction in $\delta_H$. From equations (8) and (10), we see that this increases the slope of the $K^*(x)$ curve and does not affect the $x(K^*)$ curve. The new equilibrium is shown in red in Figure 2. The new equilibrium is associated with a lower *ex-ante* probability of fire sales and funds holding a higher quantity of bonds. Notice that the fall in $K^*$ would have been greater if funds did not have the ability to adjust their portfolio in response to the change in $\delta_H$. When $\delta_H$ falls, funds are *ex-ante* less likely to face a fire sale. Bonds are therefore more attractive so funds choose to hold more bonds. But if funds hold more bonds, they increase the probability of facing a fire sale. This partially undoes the effect the fall in $\delta_H$ has on reducing the probabilty of fire sales.

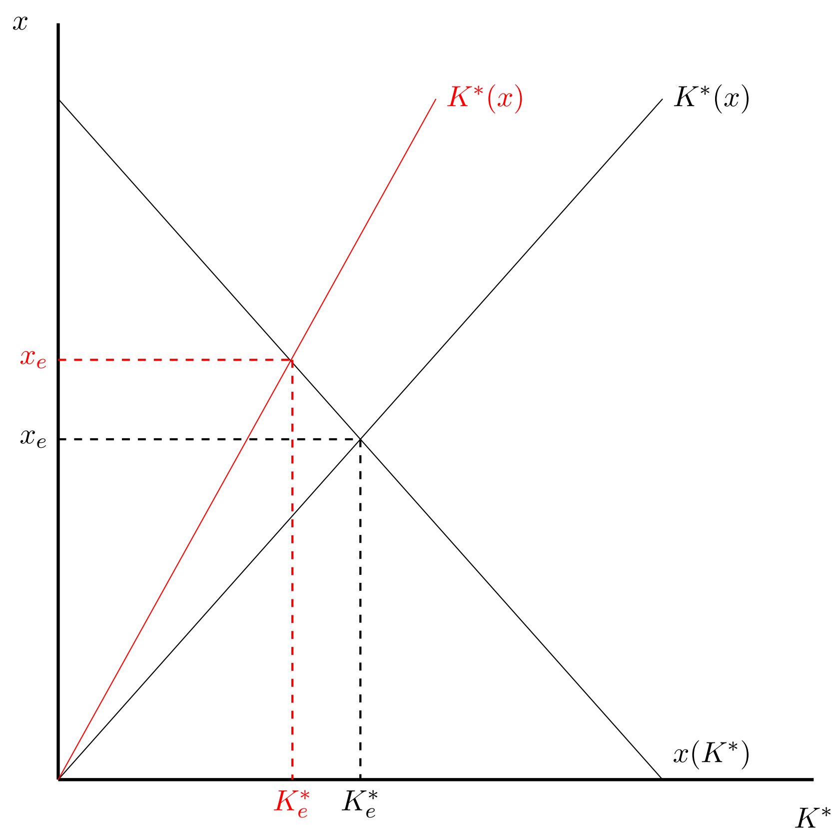


Figure 2: Joint determination of $x$ and $K^*$ in equilibrium. A reduction in $\delta_H$ reduces the *ex-ante* probability of a fire sale.

# 4 Central bank provision of a market backstop

We now extend our global games framework to study the central bank acting as a backstop buyer of assets.[12] We show that, if the central bank commits to act aggressively enough, then it is able to completely eliminate the possibility of self-fulfilling fire sales. If the central bank does not act with sufficient force, perhaps due to lack of credibility or political constraints, then it can reduce but not entirely eliminate the probability of a self-fulfilling fire sale. The effectiveness of an insufficiently forceful policy is further reduced when funds re-optimise their portfolios in response to central bank policy.

It thus appears that the central bank should simply commit to act aggressively to completely eliminate the possibility of self-fulfilling fire sales. However, using our model, we highlight some reasons why this aggressive policy stance may be undesirable. We complete this section with a discussion of alternative policies that could reduce the probability of self-fulfilling fire sales within the context of our model.

## 4.1 Using market backstops to reduce the probability of fire sales

We introduce a market backstop policy by allowing the central bank to provide additional capacity for absorbing bond sales through an asset purchasing facility. Denote this capacity by $K_{CB}$. Consistent with the central bank acting as a backstop buyer, it offers to purchase bonds at a discount $\delta_{CB} \in (\delta_L, \delta_H]$. Thus, the central bank policy is a price-quantity pair $(K_{CB}, \delta_{CB})$, where $K_{CB}$ is the maximum quantity of bonds that it is prepared to buy at any time and $\delta_{CB}$ is the discount (to the market mid-price) at which it would buy.[13] In most circumstances, market participants would not sell to the central bank because the dealer in the model would not face balance sheet constraints and so would offer to purchase bonds at a better price (i.e. smaller discount). The penal central bank discount mirrors the principle set out by Bagehot (1873) for central banks acting as a lender of last resort. Notice that this specification of policy includes the case of the central bank setting an asset price floor at $R - \delta_{CB}$. This would be achieved by setting $K_{CB}$ such that $K + K_{CB} = 1$. The market backstop policy is therefore fully described by the pair $(K_{CB}, \delta_{CB})$.

[12] As our model is highly stylised, the authority need not in fact be a central bank. Instead, it could be a fiscal authority. Indeed, that could have some advantages, such as avoiding potential conflict with monetary policy objectives and internalising potential costs to taxpayers.

[13] So $\delta_{CB}$ is like the 'reserve spread' in the September-October 2022 Bank of England gilt market interventions.

With a central bank, the payoffs become

$$u(\text{sell}, s, K) = R - \delta_L \tag{13}$$

$$u(\text{hold}, s, K) = \begin{cases} q(R-\delta_L) + (1-q)R & \text{if } sx \in [0, K] \\ q(R-\delta_{CB}) + (1-q)R & \text{if } sx \in (K, K+K_{CB}] \\ q(R-\delta_H) + (1-q)R & \text{if } sx \in (K+K_{CB}, 1]. \end{cases} \tag{14}$$

The payoff gain from selling is

$$\pi(s, K) = \begin{cases} -(1-q)\delta_L & \text{if } sx \in [0, K] \\ q\delta_{CB} - \delta_L & \text{if } sx \in (K, K+K_{CB}] \\ q\delta_H - \delta_L & \text{if } sx \in (K+K_{CB}, 1]. \end{cases} \tag{15}$$

Following similar steps to Proposition 2, we can prove the following result.

**Proposition 4.** *Let $\varepsilon \to 0$. Suppose that $q\delta_H > \delta_L$ and $x > K$. If $K + K_{CB} > x$ and $q\delta_{CB} \leq \delta_L$, then funds have a dominant strategy to hold. If $q\delta_{CB} > \delta_L$, then there is an equilibrium where all funds hold if $K > \tilde{K}^*$ and all funds sell if $K < \tilde{K}^*$ where*

$$\tilde{K}^* = \begin{cases} \left[\frac{q\delta_{CB}-\delta_L}{q(\delta_{CB}-\delta_L)}\right] x & \text{if } K + K_{CB} > x \\ \left[\frac{q\delta_H-\delta_L}{q(\delta_H-\delta_L)}\right] x - \left[\frac{\delta_H-\delta_{CB}}{\delta_H-\delta_L}\right] K_{CB} & \text{if } K + K_{CB} < x. \end{cases} \tag{16}$$

*$\tilde{K}^*$ is decreasing in $K_{CB}$ and increasing in $\delta_{CB}$.*

*Proof.* See Appendix. □

The first point to note about Proposition 4 is that by assuming that $q\delta_H > \delta_L$ and $x > K$ we provide the conditions for a possible fire sale in the model without a central bank. If a central bank dislikes fire sales, we therefore potentially have scope for policy action to reduce the probability of a self-fulfilling fire sale occurring.

The central bank can eliminate the possibility of fire sales entirely by acting aggressively. For example, if it sets $K_{CB} = 1 - K$ and $\delta_{CB} \leq \delta_L/q$, funds have a dominant strategy to hold for all values of $x$. This result corresponds with the theoretical result in Choi & Yorulmazer (2022) where the central bank may wish to act aggressively to rule out all equilibria except the good one with no fire sales. Notice that the central bank facility needs to be sufficiently large *and* the discount needs to be small enough to eliminate the fire sale outcome entirely: they are not substitutable. The reason is straightforward. If the capacity is large but the discount $\delta_{CB}$ is also large, then selling to the central bank is still a bad outcome so the central bank does not remove the incentive to pre-emptively sell. If the discount $\delta_{CB}$ is small but the capacity is small, then the central bank's purchasing facility is close to irrelevant and funds will still put positive weight on the possibility of having to sell to the dealer at discount $\delta_H$. Also notice that the discount $\delta_{CB}$ does not need to equal $\delta_L$ to eliminate the possibility of fire sales.

Importantly, the central bank only needs to credibly announce a policy for it to be effective. Moreover, the facility is not used at $t = 0$ because all funds choose to hold. This matches the empirical evidence from the dash for cash where the announcement of the Federal Reserve's corporate bond purchase programme alone calmed corporate bond markets (Haddad et al. 2021). It also fits with the experience of Euro area sovereign debt markets after Mario Draghi's "whatever it takes" speech, when Draghi's promise calmed markets without the ECB having to buy any bonds. Indeed, this relates to the general idea that asset purchases for financial stability purposes should be "catalytic" in the sense that they restore the functioning of private markets rather than involving considerable direct intervention (Cecchetti & Tucker 2021).

Now consider the case when $q\delta_{CB} > \delta_L$ so the fire sale outcome is not ruled out. If central bank capacity is sufficiently large, $K + K_{CB} > x$, the central bank is essentially setting an asset price floor at $R - \delta_{CB}$. The expression for $\tilde{K}^*$ is identical to $K^*$ in Proposition 2 with $\delta_H$ replaced by $\delta_{CB}$. A reduction in $\delta_{CB}$ therefore reduces $\tilde{K}^*$. Further changes in $K_{CB}$ have no effect.

If the central bank capacity is smaller such that $K + K_{CB} < x$, then the central bank does not rule out the possibility of a fund having to sell to a stressed dealer. In this case, both an increase in $K_{CB}$ and a reduction in $\delta_{CB}$ reduce the probability of a self-fulfilling fire sale. Thus, reducing the discount $\delta_{CB}$ and increasing the size of the facility $K_{CB}$ can substitute for each other. Moreover, the policies are complementary in the sense that $|\partial K^*/\partial K_{CB}|$ is larger if $\delta_{CB}$ is lower and $|\partial K^*/\partial \delta_{CB}|$ is larger if $K_{CB}$ is larger.

**Portfolio re-optimisation.** Now suppose that the central bank announces the policy $(K_{CB}, \delta_{CB})$ ahead of time such that funds are able to optimally choose their value of $x$ in response to the policy.

Note that the optimal portfolio choice problem is not directly affected by the central bank providing a market backstop. The only channel through which central bank policy affects the portfolio choice problem is through influencing the probability of a fire sale $\tilde{K}^*$, which the atomistic funds take as given when choosing $x$.

Consider first a reduction in $\delta_{CB}$ when $K + K_{CB} = 1$, that is, the central bank sets an asset price floor at $R - \delta_{CB}$. The equilibrium outcome is identical to Figure 2 showing a reduction in $\delta_H$. The endogenous response of funds to the policy, which is to hold more bonds and less cash, partially offsets the reduction in the probability of a self-fulfilling fire from lowering $\delta_{CB}$. The response of private-sector agents to take more risk illustrates the 'moral hazard' that is often present in the provision of public-sector insurance.

Next consider the case when $K + K_{CB} < x$. A reduction in $\delta_{CB}$ or an increase in $K_{CB}$ will reduce

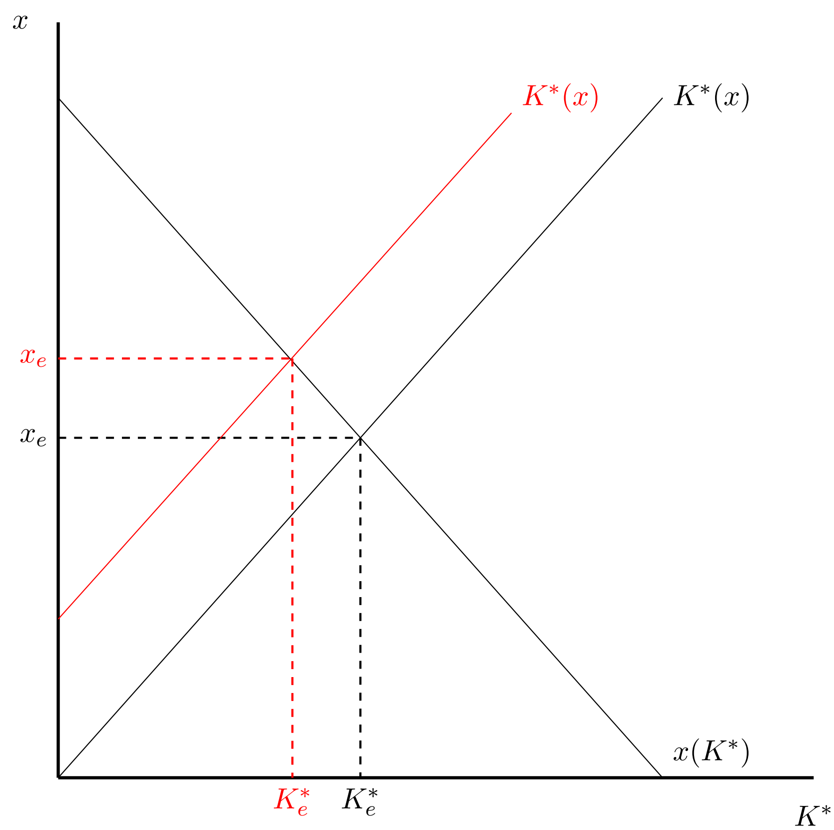


Figure 3: An increase in $K_{CB}$ and/or a reduction in $\delta_{CB}$ when $K + K_{CB} < x$

$\tilde{K}^*$ and the effect is independent of the value of $x$. This is shown graphically in Figure 3 as a leftward shift in the $K^*(x)$ curve. We again have a reduction in the equilibrium probability of a fire sale which is partially offset by the endogenous increase in funds' bond holdings $x$.

To summarise the discussion, we have seen that if the central bank commits to act with sufficient force, it can completely eliminate self-fulfilling fire sales as a potential equilibrium outcome. Moreover, the central bank will not actually have to purchase any bonds at $t = 0$ for the policy to be effective. If it acts less aggressively, it can still reduce the probability of a self-fulfilling fire sale. However, if the central bank does not take into account the effect of its policy on funds' portfolio choice, it will think its policy is more effective at reducing the probability of fire sales than it actually is. It therefore appears as if central banks have a simple decision: act aggressively to rule out the fire sale outcome. In the next section, however, we discuss some potential pitfalls with such a policy.

### 4.2 Potential problems with providing a market backstop

**Costs of central banks holding assets.** Thus far, we have implicitly assumed that the central bank only cares about reducing the probability of a self-fulfilling fire sale. If enacting a policy $(K_{CB}, \delta_{CB})$ is costless, then the central bank will optimally act aggressively to reduce the probability of a self-fulfilling fire sale to zero. However, in practice, a central bank may not wish to take certain assets onto its balance sheet. Reasons may include political economy considerations, with any losses on purchased assets ultimately falling to the taxpayer; or conflicts with its monetary policy objectives, as central bank asset purchases all else equal may loosen monetary conditions, which may not be appropriate depending on the economic outlook. We can represent this cost with a central bank cost function $\mathcal{C}(Q_{CB}, \delta_{CB})$ where $Q_{CB}$ denotes the quantity of bonds purchased by the central bank. The cost function is increasing in $Q_{CB}$ and decreasing in $\delta_{CB}$.

This cost, combined with the potential liquidity shock at $t = 1$, implies that acting aggressively to eliminate the self-fulfilling fire sale in $t = 0$ may no longer be optimal. To see this, note that when a central bank is choosing the policy $(K_{CB}, \delta_{CB})$ before the funds make any decisions, its ex-ante expected cost is

$$\mathcal{C}(Q_{CB}^{fs}, \delta_{CB})\tilde{K}^*(K_{CB}, \delta_{CB}) + \mathcal{C}(Q_{CB}^{ls}, \delta_{CB})(1 - \tilde{K}^*(K_{CB}, \delta_{CB}))q \tag{17}$$

where the dependence of $\tilde{K}^*$ on policy is made explicit. With probability $\tilde{K}^*$, there is a self-fulfilling fire sale at $t = 0$ and the central bank purchases $Q_{CB}^{fs}$ bonds at discount $\delta_{CB}$. With probability $(1 - \tilde{K}^*)q$, there is no self-fulfilling fire sale at $t = 0$ followed by a liquidity shock at $t = 1$. In this case, a central bank committed to providing a market backstop would purchase $Q_{CB}^{ls}$ bonds at discount $\delta_{CB}$. Evidently, $Q_{CB}^{fs}$ and $Q_{CB}^{ls}$ are weakly increasing in $K_{CB}$, though they may be less than $K_{CB}$ if bond sales do not exhaust the central bank's capacity to absorb bonds.

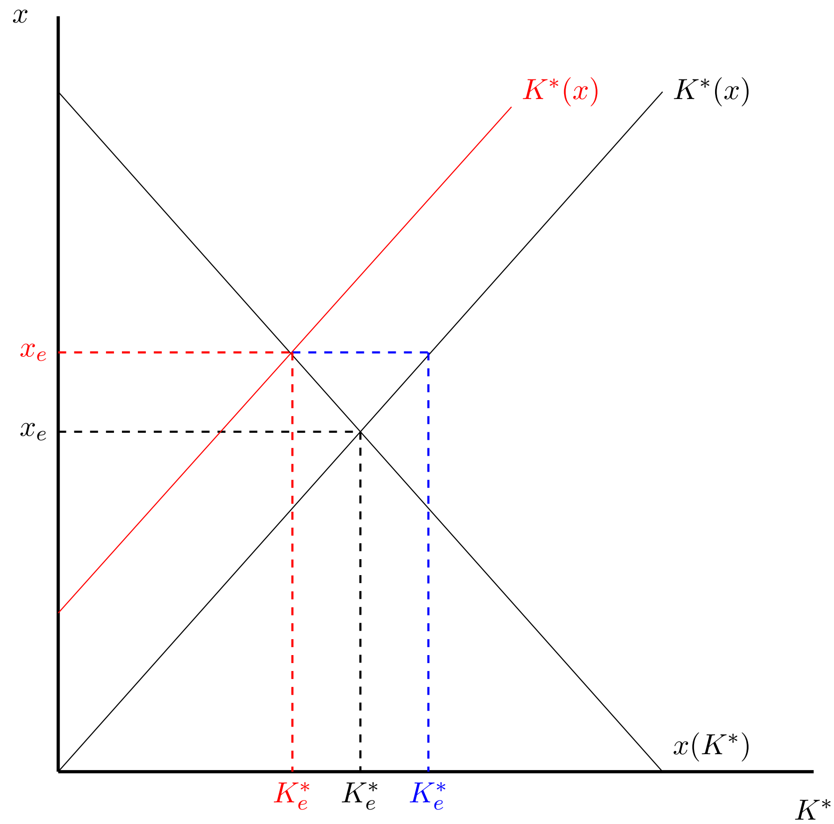


Figure 4: Shifts in market expectations can increase the probability of self-fulfilling fire sales

An aggressive backstop policy (high $K_{CB}$ and low $\delta_{CB}$) ensures that $\tilde{K}^* = 0$. With no cost of purchasing bonds and a benefit to reducing the probability of self-fulfilling fire sales, it would clearly be optimal to pursue an aggressive policy. However, from equation (17), we now see that there is a positive ex-ante expected cost from an aggressive policy, $\mathcal{C}(Q_{CB}^{ls}, \delta_{CB})q$, due to the presence of the liquidity shock. Depending on the cost function $\mathcal{C}(\cdot, \cdot)$ and the benefit of reducing the probability of self-fulfilling fire sales, it may now be optimal for a central bank to pursue a less aggressive policy. Such a policy would tradeoff an increased probability of self-fulfilling fire sales at $t = 0$ against the reduced cost of intervening aggressively when there are liquidity needs at $t = 1$. Put differently, an aggressive backstop policy is no longer a "free lunch" when there are (i) costs to intervening in markets and (ii) fire sales driven by fundamental liquidity needs rather than self-fulfilling beliefs.

**Expectations of market participants.** If the backstop policy is not clear, consistent, and credible, central bank action may increase the probability of self-fulfilling fire sales.

Suppose, for example, that market participants expect the central bank to enact a policy $(K_{CB}, \delta_{CB})$ consistent with the equilibrium given in red in Figure 4. Next suppose at the start of the crisis period at $t = 0$, expectations shift so that the relevant $K^*(x)$ curve is the black line (in this case representing no backstop policy). The market run game played between the funds would therefore take place with the higher red $x_e$ as the bond portfolio share. Following the blue line across in the example in Figure 4, we see that the probability of a self-fulfilling fire sale actually increases relative to the scenario where expectations remained anchored on the equilibrium in black.

Why might market expectations shift in this manner? One possible reason is if the central bank initially promises the policy $(K_{CB}, \delta_{CB})$ and market participants believe the promise is credible. Expectations could then shift if the central bank explicitly reneges on its policy commitment or market participants suddenly believe that the central bank will no longer follow through on its promise. Thus, any central bank committed to providing a market backstop must ensure that announced policy is consistent and can be credibly implemented in a crisis.

In the absence of an explicit policy commitment, markets may instead infer the policy $(K_{CB}, \delta_{CB})$ from central bank actions or statements. If this inference turns out to be incorrect, market expectations could shift at the onset of the crisis. Our observation thus cautions against the principle of "constructive ambiguity" advocated by some in the context of lender of last resort policies (see, for example, George (1994)). The idea of constructive ambiguity is that the central bank can avoid moral hazard by making access to emergency support facilities uncertain. However, such ambiguity creates the possibility that market participants' expectations will become overly optimistic about the extent of central bank support and we see from Figure 4 that this can increase the risk of financial instability. This point is therefore similar to Hanson et al. (2020), who show how mis-perceptions about central bank interventions in asset markets can lead to sudden price declines if investors overestimate the aggressiveness of central bank intervention.

Incorrect expectations of market participants could even force the central bank to provide a market backstop when it would prefer not to. Suppose that the costs of providing a backstop are deemed too high such that the central bank would not want to provide one: the central bank would prefer the black equilibrium in Figure 4. However, if funds expect the policy $(K_{CB}, \delta_{CB})$, they will act in a manner consistent with the red equilibrium in Figure 4. At the onset of the crisis, the central bank therefore has a choice. It could not intervene and face an increased probability of self-fulfilling fire sales relative to the black equilibrium. Alternatively, it could choose to intervene consistent with market participants' expectations and lower the probability of a self-fulfilling fire sale at the cost of providing a market backstop. Depending on the objective function of the central bank, it may choose to provide a backstop in line with the expectations of market participants.

### 4.3 Alternative policies

Given the difficulties with providing a market backstop, the central bank may wish to consider using alternative policies to reduce the probability of fire sales, with a market backstop potentially available as a last resort. Indeed, Hauser (2021) identifies three steps to strengthen market functioning: (1) reforms to improve the resilience of financial institutions to liquidity shocks, (2) strengthened market-wide infrastructure, and (3) central bank backstops. We now provide a brief discussion on steps (1) and (2) within the context of our model.

**Improving resilience to liquidity shocks.** The first fundamental driver of the self-fulfilling fire sale is the fear of being hit by a future liquidity shock. It follows that policies that reduce the severity of the liquidity shock should reduce the probability of a self-fulfilling fire sale.

In our current setup, the liquidity shock requires funds to shift to a 100% cash portfolio which does not allow us to consider a reduction in the severity of a liquidity shock. Therefore, suppose instead that a fund hit by a liquidity shock only has to sell a fraction $f$ of their bond holdings. The liquidity shock requiring 100% cash is the case when $f = 1$. One can easily show that equation (1), which gives the condition for funds to optimally sell at $t = 0$, becomes

$$p_0 > qfp_1(\tilde{\delta}) + (1 - qf)R. \tag{18}$$

A reduction in $f$, which we can interpret as a reduction in the severity of the liquidity shock to funds, therefore has the same effect as a reduction in $q$. We thus see from Corollary 1 that a reduction in $f$ will reduce the probability of a self-fulfilling fire sale (other factors equal) because it reduces the incentive to pre-emptively sell bonds. In our model, $f$ would have some baseline value consistent with the atomistic funds choosing liquidity buffers that were privately optimal. Such behaviour would not take into account any benefits to other funds of raising their liquidity buffers further and thus reducing the probability of a self-fulfilling fire sale below the baseline level.

However, enhancing liquidity management tools through regulation could reduce the severity of any given liquidity shock to funds and therefore reduce the probability of a self-fulfilling fire sale. For instance, requiring funds to hold larger liquidity buffers in normal times that could be drawn upon in the event of a shock would reduce potential sales and thus reduce the probability of a self-fulfilling fire sale. Another option could be the use of redemption gates that limit redemptions for a short period of time. By design, this reduces the fraction of bonds the fund would need to sell when hit by a liquidity shock. Another possible tool could be swing pricing (Jin et al. 2022). This tool reduces run-like incentives on open-ended funds by forcing redeeming investors to internalise the trading costs associated with selling instead of passing it onto those still invested in the fund. By reducing the incentive to run on the fund, swing pricing should reduce the number of bonds the fund needs to sell for any given liquidity shock.

**Strengthening market-wide infrastructure.** The second fundamental driver of the self-fulfilling fire sale is the presence of a constrained dealer which funds have to sell to. Policies to improve the ability of the dealer to intermediate would therefore increase the expected price of bonds at $t = 1$ and reduce the incentive to pre-emptively sell at $t = 0$. In the aftermath of the dash-for-cash episode, post-GFC financial regulations have been highlighted as a contributor to dealer capital constraints (Bessembinder et al. 2018, Dick-Nielsen & Rossi 2019). In particular, the leverage ratio, which is typically a non-binding backstop, may have become binding during the dash for cash and prevented dealers from taking more bonds onto their balance sheets (Breckenfelder & Ivashina 2021). Policymakers could therefore consider ways to relax binding capital constraints at times of stress. Alternatively, they could reduce the capital-ratio impact for dealers of new trades by mandating central clearing.[14] This means that dealers could net buy and sell trades regardless of the trading counterparties and only record net buys on the balance sheet. Both of these policies could improve the ability of dealers to intermediate and therefore reduce the fear of market participants that they may have to sell to a dealer under stress.

Alternatively, policymakers could seek to break the fund-dealer relationship by promoting all-to-all trading platforms. This would allow market participants to trade with dealers or each other. If the liquidity shock at $t = 1$ is an idiosyncratic rather than an aggregate shock, there will be $q$ funds that are sellers of bonds and $1 - q$ funds that are natural buyers of bonds. In situations where dealers were constrained and could not intermediate between these buyers and sellers, such a policy would increase the expected price of bonds at $t = 1$ and reduce the incentive to pre-emptively sell.

## 5 Conclusion

Following the "dash for cash" in March 2020, central banks stepped into secondary markets and purchased bonds to meet their financial stability objectives. However, there are only a few papers theoretically examining the use of central bank asset purchases as a financial stability tool. This paper helps to fill this gap by introducing a central bank which provides a market backstop in a model of self-fulfilling fire sales. We show that by committing to act aggressively, the central bank can eliminate the possibility of a self-fulfilling fire sale. However, a central bank may wish to act less aggressively to trade-off the benefit of reducing self-fulfilling fire sales with the cost of intervening in other fire sale events. Furthermore, if market participants expect the central bank to act aggressively and the central bank does not act in accordance with expectations, then we show that the probability of self-fulfilling fire sales can increase relative to the baseline scenario of the central bank credibly ruling out the provision of a market backstop.

Our model is highly stylised and intended as a conceptual overview of what may happen when a central bank introduces a type of market backstop. It does not provide a full cost-benefit analysis of market backstops. Such a study would quantify the benefits of market backstops beyond reducing the risk of self-fulfilling fire sales and weigh those benefits against costs such as those of encouraging greater risk taking ('moral hazard') and the central bank having to manage a portfolio of risky assets. In addition, there remain many open questions on the details of designing a market backstop. Such

[14] See, for example, Duffie (2020) who proposes central counterparty clearing for the US Treasury market.

issues include which assets to buy, how to best unwind any purchases and how to deal with market dysfunction when financial stability objectives may conflict with monetary policy. There is also a separate question about the relative merits of central bank market backstops or lending backstops, where the latter may generate less moral hazard and pose less risk to the public finances.[15] We leave such questions for future research.

# Appendix

## Proof of Proposition 2 continued

In this appendix, we complete the proof of Proposition 2 by showing that if $z_i < z^*$, fund $i$ chooses to sell, and if $z_i > z^*$, fund $i$ chooses to hold. The proof is similar to that presented in Morris & Shin (2004).

We need to compute the probability density function of $s$ conditional on fund $i$ observing private signal $z_i$ and all funds using the threshold strategy $z^*$. When the true market maker capacity is $K$, the signals $\{z_i\}$ are distributed uniformly over $[K-\varepsilon, K+\varepsilon]$. Funds with signals $z_i < z^*$ choose to sell. Hence,

$$s = \frac{z^* - (K-\varepsilon)}{2\varepsilon}.$$

To derive the conditional distribution of $s$, we derive the cumulative distribution function. In particular, we compute the conditional probability that $s < b$: $G(\cdot|z_i, z^*)$. Define $K_0$ as

$$b = \frac{z_i - (K_0 - \varepsilon)}{2\varepsilon} \implies K_0 = z_i + \varepsilon - 2\varepsilon b.$$

Thus, $s < b$ if and only if $K > K_0$. We therefore need the probability of $K > K_0$ conditional on $z_i$. Fund $i$'s posterior density over $K$ conditional on $z_i$ is $[z_i - \varepsilon, z_i + \varepsilon]$. Therefore,

$$\Pr(K > K_0|z_i) = \frac{z_i + \varepsilon - K_0}{2\varepsilon} = \frac{z_i + \varepsilon - (z^* + \varepsilon - 2b\varepsilon)}{2\varepsilon} = \frac{z_i - z^*}{2\varepsilon} + b.$$

Thus,

$$G(b|z_i, z^*) = \begin{cases} 0 & \text{if } \frac{z_i - z^*}{2\varepsilon} + b < 0 \\ 1 & \text{if } \frac{z_i - z^*}{2\varepsilon} + b > 1 \\ \frac{z_i - z^*}{2\varepsilon} + b & \text{otherwise.} \end{cases}$$

[15] Markets Committee (2022) discusses in further detail design considerations for alternative market backstop policies as well as their potential benefits and costs.

Consider the case where $z_i < z^*$ (the case where $z_i > z^*$ follows an analagous argument). We need to show that fund $i$ prefers to sell. The conditional density over the half-open interval $s \in [0, 1)$ is given by

$$g(s|z_i, z^*) = \begin{cases} 0 & \text{if } s < \frac{z^*-z_i}{2\varepsilon} \\ 1 & \text{if } s \geq \frac{z^*-z_i}{2\varepsilon} \end{cases}$$

with an atom at $s = 1$ with mass $\frac{z^*-z_i}{2\varepsilon}$.

Now recall that the payoff gain from selling compared to holding for fund $i$ after observing $z_i$ is

$$\pi(s, z_i) = \begin{cases} -(1-q)\delta_L & \text{if } sx \leq z_i \\ q\delta_H - \delta_L & \text{if } sx > z_i. \end{cases}$$

The function $\pi(s, z_i)$ is increasing in $s$ and decreasing in $z_i$. Moreover, if we impose the parameter restriction $q\delta_H > \delta_L$, the payoff gain is negative for $sx \leq z_i$ and positive for $sx > z_i$. Note that the density $g(s|z_i, z^*)$ can be obtained from the uniform density by transferring weight from the interval $\left[0, \frac{z^*-z_i}{2\varepsilon}\right]$ to $s = 1$. Therefore,

$$\begin{aligned} 0 &= \int_0^1 \pi(s, z^*) ds \\ &< \int_0^1 \pi(s, z^*) g(l|z_i, z^*) ds \\ &< \int_0^1 \pi(s, z_i) g(l|z_i, z^*) ds. \end{aligned}$$

Thus, fund $i$ strictly prefers to sell when $z_i < z^*$.

## Proof of Proposition 4

The proof is similar to Proposition 2 so we only provide a sketch of the argument.

Consider a fund observing a signal $z_i$ and thus its expectation of $K$ is $z_i$. We cannot have $z_i > x$ because as $\varepsilon \to 0$ this would imply $K > x$. Therefore, $z_i < x$.

The first case to consider is when $z_i + K_{CB} \geq x$ and $q\delta_{CB} \leq \delta_L$. Then $\pi(s, z_i) \leq 0$ for all $s$: funds have a dominant straetgy to hold. As $\varepsilon \to 0$, $z_i \to K$ so that funds have a dominant strategy to hold whenever $K + K_{CB} > x$.

The second case is $z_i + K_{CB} \geq x$ and $q\delta_{CB} > \delta_L$. Now the sign of $\pi(s, z_i)$ depends on the value of $s$. To proceed, consider the fund observing the threshold signal $z^*$. From Lemma 1, we know that $s|z^* \sim U[0, 1]$. If $s < z^*/x$, the fund believes the dealer will be under no stress. Otherwise the fund believes it will sell to the central bank at discount $\delta_{CB}$. Since $z^* + K_{CB} \geq x$, the fund does not believe it will ever sell to the dealer at discount $\delta_H$ even if $s = 1$. Therefore, the analysis is identical to Proposition 2 with $\delta_H = \delta_{CB}$.

The final case is $z_i + K_{CB} < x$. The sign of $\pi(s, z_i)$ depends on the value of $s$ so we consider the indifference condition of a fund observing signal $z^*$. There are three possibilities: (i) sell to the dealer under no stress with probability $z^*/x$; (ii) sell to the central bank with probability $K_{CB}/x$; (iii) sell to the dealer under stress with probability $1 - (z^* + K_{CB})/x$. Therefore,

$$\int_0^1 \pi(s, z^*)ds = -(1-q)\delta_L \frac{z^*}{x} + (q\delta_{CB} - \delta_L)\frac{K_{CB}}{x} + (q\delta_H - \delta_L)\left(1 - \frac{z^* + K_{CB}}{x}\right).$$

The threshold value $z^*$ (or equivalently $K^*$ as $\varepsilon \to 0$) sets this expression to zero. This gives the result in Proposition 4. A similar argument to Proposition 2 establishes the optimality of selling (holding) if $z_i < z^*$ ($z_i \geq z^*$).

By inspection and Corollary 1, we can see that $\tilde{K}^*$ is increasing in $\delta_{CB}$. For $K + K_{CB} < x$, it is evident that $\tilde{K}^*$ is decreasing in $K_{CB}$. Letting $K_{CB} \to x - K$ and performing some simple algebra shows that the expression for $\tilde{K}^*$ when $K + K_{CB}$ converges to the expression for $\tilde{K}^*$ when $K + K_{CB} > x$.

# References


Allen, F., Carletti, E., Goldstein, I. & Leonello, A. (2018), 'Government guarantees and financial stability', *Journal of Economic Theory* **177**, 518–557.

Allen, F. & Gale, D. (1994), 'Limited market participation and volatility of asset prices', *American Economic Review* **84**(4), 933–955.

Allen, F. & Gale, D. (1998), 'Optimal financial crises', *The Journal of Finance* **53**(4), 1245–1284.

Bagehot, W. (1873), *Lombard Street: A Description of the Money Market*, Henry S. King and Co.

Bernardo, A. E. & Welch, I. (2004), 'Liquidity and financial market runs', *The Quarterly Journal of Economics* **119**(1), 135–158.

Bessembinder, H., Jacobsen, S., Maxwell, W. & Venkataraman, K. (2018), 'Capital commitment and illiquidity in corporate bonds', *The Journal of Finance* **73**(4), 1615–1661.

Boyarchenko, N., Kovner, A. & Shachar, O. (2022), 'It's what you say and what you buy: A holistic evaluation of the corporate credit facilities', *Journal of Financial Economics* **144**, 695–731.

Breckenfelder, J. & Ivashina, V. (2021), Bank leverage constraints and bond market illiquidity during the COVID-19 crisis. ECB Research Bulletin No. 89.

Carlsson, H. & van Damme, E. (1993), 'Global games and equilibrium selection', *Econometrica* **61**(5), 989–1018.

Cecchetti, S. G. & Tucker, P. M. W. (2021), Understsanding how central banks use their balance sheets: A critical categorisation. VoxEU.org.

Chen, Q., Goldstein, I. & Jiang, W. (2010), 'Payoff complementarities and financial fragility: Evidence from mutual fund outflows', *Journal of Financial Economics* **97**, 239–262.

Choi, D. B. & Yorulmazer, T. (2022), Whatever it takes: Market maker of last resort and its fragility. Working Paper.

Cunliffe, J. (2022), Letter to Rt Hon Mel Stride MP. Available at: https://committees.parliament.uk/publications/30136/documents/174584/default (Accessed: 19 October 2022).

Czech, R., Gual-Ricart, B., Lillies, J. & Worlidge, J. (2021), The role of non-bank financial intermediaries in the 'dash for cash' in sterling markets. Bank of England Financial Stability Paper 47.

Diamond, D. W. & Dybvig, P. H. (1983), 'Bank runs, deposit insurance, and liquidity', *The Journal of Political Economy* **91**(3), 401–419.

Dick-Nielsen, J. & Rossi, M. (2019), 'The cost of immediacy for corporate bonds', *The Review of Financial Studies* **32**(1), 1–41.

Duffie, D. (2020), Still the world's safe haven? Redesigning the U.S Treasury market after the COVID-19 crisis. Hutchins Center Working Paper No. 62.

Eisenbach, T. M. & Phelan, G. (2022), Fragility of safe asset markets. Federal Reserve Bank of New York Staff Reports, no. 1026.

George, E. (1994), 'The pursuit of financial stability', *Bank of England Quarterly Bulletin* **34**(1), 60–66.

Gilchrist, S., Wei, B., Yue, V. Z. & Zakrajšek, E. (2021), The Fed takes on corporate credit risk: An analysis of the efficacy of the SMCCF. BIS Working Paper No. 963.

Goldstein, I., Jiang, H. & Ng, D. T. (2017), 'Investor flows and fragility in corporate bond funds', *Journal of Financial Economics* **126**, 592–613.

Goldstein, I. & Pauzner, A. (2005), 'Demand-deposit contracts and the probability of bank runs', *The Journal of Finance* **60**(3), 1293–1327.

Haddad, V., Moreira, A. & Muir, T. (2021), 'When selling becomes viral: Disruptions in debt markets in the COVID-19 crisis and the Fed's response', *The Review of Financial Studies* **34**, 5309–5351.

Hanson, S. G., Stein, J. C., Sunderman, A. & Zwick, E. (2020), Business credit programs in the pandemic era. Brookings Papers on Economic Activity.

Hauser, A. (2021), From Lender of Last Resort to Market Maker of Last Resort via the dash for cash: why central banks need new tools for dealing with market dysfunction. Speech at Reuters, London, 7 January 2021.

Jin, D., Kacperczyk, M., Kahraman, B. & Suntheim, F. (2022), 'Swing pricing and fragility in open-end mutual funds', *Review of Financial Studies* **35**(1), 1–50.

Kargar, M., Lester, B., Lindsay, D., Weill, S. L. P.-O. & Zúñiga, D. (2021), 'Corporate bond liquidity during the COVID-19 crisis', *The Review of Financial Studies* **34**, 5352–5401.

Krishnamurthy, A. & Vissing-Jorgensen, A. (2011), 'The effects of Quantitative Easing on interest rates: Channels and implications for policy', *Brookings Papers on Economic Activity* **42**(2), 215–287.

Markets Committee (2022), Market dysfunction and central bank tools. Available at: https://www.bis.org/publ/mc_insights.pdf (Accessed: 19 October 2022).

Morris, S., Shim, I. & Shin, H. S. (2017), 'Redemption risk and cash hoarding by asset managers', *Journal of Monetary Economics* **89**, 71–87.

Morris, S. & Shin, H. S. (1998), 'Unique equilibrium in a model of self-fulfilling currency attacks', *The American Economic Review* **88**(3), 587–597.

Morris, S. & Shin, H. S. (2003), Global games: Theory and applications, *in* 'Advances in Economics and Econometrics: Theory and Applications, Eighth World Congress, Volume 1', Cambridge University Press, pp. 56–114.

Morris, S. & Shin, H. S. (2004), 'Liquidity black holes', *Review of Finance* **8**, 1–18.

O'Hara, M. & Zhou, X. (2021), 'Anatomy of a liquidity crisis: Corporate bonds in the COVID-19 crisis', *Journal of Financial Economics* **142**, 46–68.

Vissing-Jorgensen, A. (2021), 'The Treasury market in Spring 2020 and the response of the Federal Reserve', *Journal of Monetary Economics* **124**, 19–47.