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Solvency and systemic risk of European life insurers

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Abstract

The paper presents two risk-based capital frameworks for systemically important European life insurers by drawing a distinction between solvency risk and systemic risk. Solvency risk arises when the value of a life insurer's assets falls below some threshold proportion of its liabilities. To assess solvency risk we implement the Merton-Vasicek portfolio credit risk model and determine capital adequacy of life insurers that correspond to a value-at-risk measure. We measure systemic risk as the expected capital shortfall of an insurer conditional on the overall European life insurance sector being in distress. Our results show that European life insurers have been growing in systemic risk exposure since 2007 and suggest that regulatory capital requirements should account for this. We also find evidence of interconnectedness between systemically important banks and insurance companies, as measured by the transmission of volatility shocks, which increased during periods of financial stress.

Key words: Solvency risk, systemic risk, insurers, banks, capital shortfall.

JEL classification: C61, C63, G01, G21, G28.

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1 Introduction

Life insurance companies' business models are exposed to a variety of risks on both sides of the balance sheet. The value of assets of a life insurer could decrease following a deterioration in financial market conditions. The value of liabilities could increase if a prolonged period of low interest rates increases their present value. Solvency risk arises when a life insurer's assets become less valuable than its liabilities. Given the long contractual horizon of life insurers policies, they are not liquid in the same way as bank deposits. To ensure that life insurers can meet future payments as they occur, they are required to keep certain assets as reserves, typically in the form of fixed-income securities such as government or corporate bonds. However, life insurers liabilities typically have a longer maturity profile than that of the fixed income assets held to meet those obligations, implying a negative duration gap that fluctuates with movements in long-term interest rates.

Insurers are traditionally considered as long-term investors as they typically hold assets until maturity. It has been argued that insurance companies have much longer-term assets and less liquid liabilities which make them less susceptible to runs on their liabilities of the type that affected banks, during the global financial crisis (GFC) of 2008-2009, or open-ended mutual funds during the shock from the COVID pandemic in March 2020. The argument rests on the view that traditional life insurance can eliminate risks through diversification and asset-liability matching (ALM), and therefore, not generate risks of a systemic nature. However, this observation would not hold for more complex hedging arrangements, and the broadening scale and scope of credit investments that life insurers have undertaken in recent years. In such a situation, insurance regulation should incorporate a systemic element in the assessment of capital requirements.

The definition of a systemically important financial institution (SIFI) related to its size, interconnectedness and lack of substitutability has been applied to insurers. In 2013, the International Association of Insurance Supervisors (IAIS (2011)) published a methodology for identifying global systemically important insurers (G-SIIs), on the basis of which, the Financial Stability Board (FSB) brought out a list of G-IIs (FSB (2013)). Systemic risk may rise from insurers' collective activities and exposures sectorwide, as well as the distress or failure of individual insurers. This was recognised in the Holistic Framework for Systemic Risk in the Insurance Sector ((IAIS (2019)).

Against this background, we aim to contribute to a more informed assessment of risk-based capital by an empirical assessment of the systemic importance of 6 European life insurers (Aviva, Legal & General, Prudential, AXA, Allianz SE and Assicurazioni Generali), which include both UK and EU insurers classified as G-IIs. To the best of our knowledge we are the first paper to provide an empirical analysis of both solvency and systemic risk in the European life insurance sector. To determine capital adequacy for addressing solvency risk, we implement an augmented version of the Merton-Vasicek portfolio credit risk model (Vasicek (2002)). The model postulates that an insurer defaults when the value of its assets falls below some threshold. Standard solvency regulation is centred around the tail risk of an individual institution's asset returns captured by their value-at-risk (VaR). Systemic risk is more concerned with the correlation that exists between the tails of asset returns of individual institutions within the financial system.

The empirical finance literature contains a number of market-based systemic risk measures which differ in terms of their scope and application. In this section we implement the SRISK methodology, developed by Acharya et al. (2012) and Brownlees and Engle (2017), for our sample of European life insurers. The objective of the SRISK methodology is to measure the capital shortfall a financial institution is expected to experience conditional on a systemic event. Different definitions of a systemic event can be adopted. In this study we define it as the occurrence of losses in the tail of the STOXX Euro 600 Insurance index above a particular threshold.

Our results show that European life insurers have been growing in systemic risk exposure since 2007. Systemic risk in the European life insurance sector peaked during the GFC in 2008-2009; the Eurozone sovereign debt crisis in 2010-2012; and the asset price shock during the onset of the COVID pandemic in 2020. During the global financial crisis in 2008-2009 and the COVID pandemic in 2020, both the Bank of England (BoE) and the European Central Bank (ECB) lowered policy rates effectively to the zero lower bound (ZLB) and conducted quantitative easing programmes that involved the purchase of large volumes of government and corporate bonds. These unconventional monetary policies compressed interest rates even further, and for a long time, giving rise to a period now referred to as the "low-for-long" era. In the face of ultra low policy rates, of less than 1 percent, life insurers' ventured into risky and less liquid asset classes in a search for yield. The financial turmoil in March 2020 resulting in the extreme 'dash for cash', led non-bank investors to sell off safe assets, including government bonds, to raise cash. Open-ended corporate bond and equity funds recorded significant outflows. The UK life insurers, also experienced a rise in systemic risk in late September and early October 2022, when highly leveraged liability-driven investment (LDI) strategies of pension funds caused severe repricing of UK financial assets, particularly affecting long-dated UK government bonds. From mid-2023 systemic risk in the European life insurance sector, as a whole, has trended downwards. But it remains non-trivial and there are signficant variations in levels of systemic risk across institutions.

Life insurers have traditionally managed interest rate risk, through efficient ALM strategies. This has typically involved matching the duration of their assets, which used to be predominantly bonds, and liabilities to immunise their financial reserves and equity from interest rate fluctuations. Since 2016 there has been a diversification out of bonds into alternative investments largely driven by a narrowing of spreads of investment grade bonds over risk-free rates.

Notwithstanding the different business models of banks and insurers, we find common variation in their systemic risk, which rises during periods of financial stress. For further evidence of co-movement, we estimate using principal components analysis (PCA), that a single factor explains 77 percent of the common variation in bank and insurance sector systemic risk. The method suggested by Diebold-Yilmaz (Diebold and Yilmaz (2012)) is used to quantify interactions between the European bank and life insurance sectors, also termed "spillovers". The empirical results show the existence of spillovers, with time-varying intensity that peak during periods of financial stress.

The rest of the paper is organised as follows: Section 2 provides a brief survey of the literature. Section 3 describes the Merton-Vasicek portfolio credit risk model. Section 4 describes the data used for the estimation. Section 5 shows how the empirical results from the model are used to determine solvency capital levels. Section 6 examines the asset allocation of European life insurers. Section 7 estimates systemic risk capital levels using the SRISK measure. Section 8 analyses the co-movement of systemic risk in European bank and life insurance sectors. Section 9 measures volatility spillovers between the bank and insurance sectors. Section 10 concludes.

2 Related Literature

Structural modelling of the risk of ruin has a strong tradition in actuarial science, dating back to Lundberg in 1903 (Braunsteins and Mandjes (2023)). The Vasicek model (Vasicek (2002)), which is based on the Merton (1974) structural model of firm default has been used to model insurance failure and to value the liabilities of insurance guarantee schemes. Shaked (1985) used the Merton model to assess the probability of default of US life insurers. He treated liabilities as growing deterministically, with life insurers more exposed to asset risks than non-life insurers. Cummins (1988) also applied the Merton model to value insurance guarantee scheme liabilities.

Douglas et al. (2017) use a structural model of insurer's balance sheets to estimate how Solvency II regulations might affect UK life insurers' incentives to hold different types of financial assets. They find that while Solvency II may partly protect insurers' solvency positions from falls in risky asset prices, the new regulations might encourage certain types of UK Life insurers to de-risk - that is, move to hold safe assets instead of risky - following falls in risk-free interest rates witnessed at the time.

Research has used data on actual failures of insurance companies, examining the factors that increase the likelihood of them. de Bandt and Overton (2022) examined the causes of failure of life insurance and non-life insurance firms in France, Japan, UK and US, over the period 1986-2006. They concluded that for life insurers a key cause of failure was default of bond instruments. Non-life insurers were more likely to fail because of low profitability or inefficiency. Duan and Liang (2022) estimated the probability of failure for insurance companies from the US, Canada and France, over the period 2008-2021, using common macro variables, and firm specific variables (liquidity, profitability, debt and size). They found that higher debt levels increase the likelihood of failure.

Research has estimated the sectoral losses, or sectoral capital shortfall, that might be experienced in the event of one or more insurance company failures to assess the value of liabilities for insurance guarantee schemes. The Joint Research Council of the European Commission (2021) used the Vasicek (2002) model of credit risk to calculate the expected cost of an insurance guarantee, and its VaR, for European life and non-life insurance sectors, from 2016-2018. Their modelling treated insurance sectors as a portfolio of investments, from the perspective of the insurance guarantee scheme. Our study follows a variant of this approach by viewing the European life insurance sector as portfolio of individual life insurers. We also augment the Vasicek (2002) model by incorporating the capital asset pricing model (CAPM) that enables greater market-sensitivity when assessing the default risk of individual insurers.

Research has applied the methodology for measuring systemic risks, to the insurance sector, developed after the GFC. Acharya and Richardson (2014) have argued that given the degree to which some insurers have shifted away from their traditional business model, the insurance sector does pose a systemic risk. Cummins and Weiss (2014) observed that insurers' noncore business activities interconnect insurers more closely with financial markets. The authors identify reinsurance as one of the primary factors driving systemic risk in the insurance industry. This is particularly relevant for the UK where rapid growth of the primary bulk purchase annuity business has led to increased demand for reinsurance.

Jobst (2014) provides a review of the regulatory efforts in defining systemic risk in the insurance sector and the designation of systemically important insurers. Bobtcheff et al. (2016) describe how insurance companies create systemic risk through their nontraditional activities and provide an indepth comparison between the insurance and banking industry. As in our study, Bobtcheff et al. (2016) stress the importance of differentiating traditional solvency risk from systemic risk. Gómez and Ponce (2018) provide a framework for the macroprudential regulation of insurance companies, where capital requirements increase in their contribution to systemic risk. They show that insurers exhibited a flight to quality during the European sovereign debt crisis and also engaged in procyclical investment behaviour through the sale of southern European assets.

Recent work has analysed how the era of exceptionally low interest rates, from the aftermath of the GFC until 2021, posed significant challenges to the traditional business model of life insurers. By examining the maturity profile of government holdings of the insurance sector in Germany, Domanski et al. (2017) show how portfolio adjustments by long-term investors aimed at containing duration mismatches may have acted as an amplificaton mechanism in the process. Brinkhoff and Sole (2022) estimate that the search for yield accounted for about one-third of the total deterioration in credit quality of European life insurers' portfolios from 2005 to 2021. In the context of the UK, Giese et al. (2021) find that institutional investors like life insurers, with large UK government bond holdings (gilts), are less sensitive to price movements than other investor groups and exhibit preferred habitat behaviour. Fay and Ghiselli (2023) investigate European SII insurers' response to the asset price shock of the Covid-19 pandemic and assess cyclical investment behaviour. The authors' demonstrate that the insurers are active and not buy-and-hold investors, given that they rebalance their portfolios during the shock in the first quarter of 2020. Farkas et al. (2023a) show that the underperformance of life insurance companies' stock prices continued even after interest rates soared in 2022 and 2023. They attribute this to the level and shape of the yield curve pulling in opposite directions following by the inversion of the yield curve in late 2022. Garavito et al. (2024) show how the migration of risks from life insurers' balance sheets and the increasing involvement of private equity firms have sustained the sector's growth and relieved capital constraints.

With insurance companies having grown in size and interconnectedness with banks, there is an expanding literature on assessing the degree of interconnectedness. Using Granger causality analysis, Nyholm (2012) observed that equity-return tail losses of insurers and banks are of similar magnitudes and that financial equity markets of Europe do not differentiate their trading of banks and insurance companies in periods of stress. Malik and Xu (2017) find evidence of interconnectedness between the banking and insurance sectors based on a global sample of systemically important banks and insurance companies. Gehrig and Iannino (2018) find a similar pattern of interconnectness in a large sample of European banks and insurance companies. Both these papers analyse systemic risk using the SRISK measure on which our analysis is also based. Our assessment of the comovement of bank and insurance sector systemic risk are consistent with their findings. Kaserer and Klein (2019) use CDS spreads to analyse systemic risk in insurance relative to banking within the context of a global financial system comprising 147 banks and 54 insurers. Their results affirm that some insurers are systemic and that overall, multi-line and life insurers tend to show the highest levels of systemic risk.

3 Modelling framework for solvency risk

This section describes the modelling framework used to assess solvency risk in the UK life insurance sector. We analyse solvency risk in terms of losses to insurers' creditors, policy holders and investors. The insurance sector is set within the framework of a structural model for portfolio credit risk. The insurance sector comprises a portfolio of individual insurers $i = 1,2,...n$.

We follow Vasicek (2002) and apply to a firm's asset values the basic structural credit risk model, attributed to Merton (1974), where the market value of a firm's assets evolve according to a Geometric Brownian Motion (GBM). This is expressed as a stochastic differential equation (1),

$$dA = \mu A dt + \sigma A dW \tag{1}$$

where A is the value of the firm's assets, μ is the expected growth rate of the firm's asset value, σ is the asset volatility and W stands for a standard Brownian motion. Structural credit risk models view a firm's liabilities as "contingent claims" on the firm's underlying assets. The basic premise underlying structural models is that default occurs if the value of the assets (A) falls below a critical value associated with the firm's liabilities (D) that is due at some point in the future (T). In this study, we assume a time horizon T of 1 year. Solving the stochastic differential equation (1), we can obtain the value of the firm's assets at time t as:

$$A_t = A_0 \exp\left\{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W\right\} \tag{2}$$

where A_0 is the asset value at time zero. It follows from the GBM assumption that the annual log asset value is normally distributed.

$$X_t = \log\frac{A_t}{A_0} = \left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W \tag{3}$$

and X_t is a normally distributed random variable.

$$X_t \sim N(\mu, \sigma^2). \tag{4}$$

To estimate the default probability we are interested in the likelihood that the firm's asset value A falls below its default point D, in the next year, i.e. $P(\log A < \log D)$. Denoting $\log A = X$ and $\log D = L$, default for the firm i occurs when

$$X_i \leq L_i = N^{-1}(\text{pd}) \tag{5}$$

where pd is the probability of default, L_i is the log of the default point. $N(.)$ is the cumulative normal distribution function and N^{-1} is the inverse of N.

The main feature of the Vasicek version of the Merton model is the introduction of dependence on a common factor in driving process of value when considering a portfolio of institutions which, in this case, are life insurers. The simplest set-up for this model is obtained by considering a single normally distributed common factor and a single correlation coefficient ρ for all insurers in the portfolio. Under this assumption the value of assets of any insurer can be seen as being driven by a combination of a common factor S_t and an idiosyncratic factor Z_{it}. The asset value of the i-th insurer at time t is given by:

$$X_{it} = S_t\sqrt{\rho} + Z_{it}\sqrt{1-\rho} \tag{6}$$

where $S_t \sim N(0,1)$ and $Z_t \sim N(0,1)$. Here X_{it} and Z_{it} are mutually independent standard normal variables.

The Vasicek model can be interpreted in the context of a trigger mechanism that is useful for modelling credit risk. A simple threshold condition determines whether insurer i defaults or not.

Integrating over S in equation (6) we denote the unconditional probability of default by pd :

$$P(X_i < L) = N^{-1}(L) = \text{pd} \tag{7}$$

The probability of default conditional on S_t can be written as:

$$P(X_i < L|S) = P\left(S\sqrt{\rho} + Z_i\sqrt{1-\rho} < L|S\right) = P\left(Z_i < \frac{L - S\sqrt{\rho}}{\sqrt{1-\rho}}|S\right) \tag{8}$$

It follows that the probability of default conditional on S, denoted as $PD(S) = P(X_i < L|S)$ can be rewritten as:

$$PD(S) = N\left(\frac{N^{-1}(\text{pd}) - S\sqrt{\rho}}{\sqrt{1-\rho}}\right) \tag{9}$$

In order to obtain the loss distribution function for the portfolio, we consider:

$$PD(S) = P(F_i = 1|S) \tag{10}$$

where F_i is a random variable equal to 1 if the insurer defaults and 0 otherwise. The total loss on the portfolio, expressed as a share between 0 and 1 (i.e. 0% to 100%), can be obtained as:

$$F = \sum_{i=1}^{n} {}^{F_i}/_{n} \tag{11}$$

Conditional on the value of S, the random variables F_i are independent equally distributed variables with finite variance. Applying the law of large numbers, the loss of the whole portfolio conditional on S converges to its expectation $PD(S)$ as n goes to infinity. This can be expressed as:

$$P(F \leq x) = P\left(\sum_{i=1}^{n} F_i/n \leq x\right) = P(PD(S) \leq x)$$

The probability that a loss smaller than x% will be incurred in a large portfolio can be written as:

$$P(F \leq x) = N\left(\frac{\sqrt{1-\rho}N^{-1}(x)\,N^{-1}(\rho)}{\sqrt{\rho}}\right) \tag{12}$$

Inverting this formula provides, for each probability α, the corresponding VaR x loss which is not going to be exceeded with probability α:

$$\alpha = P(F \leq VaR_\alpha) \Leftrightarrow VaR_\alpha = N\left(\frac{\sqrt{\rho}N^{-1}(\alpha) + N^{-1}(\rho)}{\sqrt{1-\rho}}\right) \tag{13}$$

This gives the loss in percentage terms. The expected loss can be obtained by multiplying this share by the Exposure-at-default (EAD) and the asset shortfall (LGD). The specification of the VaR in equation (13) is based on the assumption that the portfolio is equally distributed and that the law of large numbers can be applied.

Theoretically, insurer-invariant risk weights can be used to limit the probability that losses exceed total capital provided two assumptions are met. First, insurers must be 'asymptotically fine-grained,' which means that each insurer must be of negligible size. Second, one must make an 'asymptotic single risk factor' (ASRF) assumption (Gordy (2003)). The ASRF assumption means that while every insurer is exposed to idiosyncratic risk, there is only one source of common shocks. One may think of the ASRF as representing aggregate macrofinancial conditions. Each insurer may have a different correlation with the ASRF, but correlations between insurers are only driven by their link to that single factor.

The asymptotically fine-grained assumption would treat the insurance sector as being made of an infinite number of negligible players. In practice, however, the insurance sector may be characterised as being lumpy and made up a few dominant companies. This lumpiness effectively decreases the diversification of the portfolio of insurers, thereby increasing the variance of the loss distribution. So to keep the same probability of containing losses, more capital is needed for a sector with a few dominant companies than for an asymptotically fine-grained one.

Vasicek (2002) proposes an adjustment to take into consideration the market granularity of insurance companies. He proposes replacing ρ by $\rho + \delta(1-\rho)$, where δ is the quadratic sum of the weights and the weights are defined as the ratio of the size of each insurance company to the total market size.

The Vasicek model may be characterised as an Asymptotic Single Risk Factor (ASRF) model with a granularity adjustment.

$$VaR_\alpha = EAD * LGD * N\left(\frac{N^{-1}(PD) - \sqrt{\rho + \delta(1-\rho)}N^{-1}(1-\alpha)}{\sqrt{(1-\rho-\delta(1-\rho))}}\right) \tag{14}$$

The value-at-risk (VaR) estimated by equation (14) represents the basis for the computation of insurer capital requirements. Standard solvency regulation is centred around the tail risk of individual institutions' asset returns as captured by their VaR. However, regulation of SIFIs would need to consider the correlation between the tails in the distribution of asset returns of different institutions. These correlations are time-varying and increase in magnitude during periods of heightened financial stress. The criticisms of the VaR measure are based on its perceived inability to curtail the risk-taking behaviour of financial institutions and, further that it could not ensure adequate capital buffers to prevent the propagation of systemic risk.

A single factor model cannot, by definition, capture any clustering of institution-specific default risk due to common sensitivity of these components to global factors. Accounting for systemic risk enables the recognition of time-varying conditional correlations between the financial system's returns and individual institution's returns.

4 Data

The portfolio credit-risk model is calibrated using market information and balance sheet data of six global systemically important European life insurers - Allianz SE, Aviva, the AXA Group, Assicurazioni Generali SpA, Legal & General and Prudential plc. These 6 institutions are taken to be a representation of the European life insurance sector. In 2019, the 6 insurers had total assets of £3.4 trillion. Their assets amounted to 28% of those of the largest 25 life insurers in the world, according to AM Best,[1] and 70% of those of the European companies within the world top 25.

For these 6 institutions, daily market equity data and annual balance sheet data on total liabilities and total assets have been sourced from LSEG Workspace. The combined panel dataset comprises observations for each insurer from January 2005 to December 2024. [2]Our measure of the risk-free rate of interest is the spot yield on 1-year UK government bonds.[3] Daily 1-year spot yields are sourced from the estimated yield curves for the UK published by the Bank of England.

Based on the time series of the insurers' observed equity prices, the risk-free rate and debt levels, the diffusion process given by equation (1) is used to back out a corresponding series of each insurer's implied asset values as shown in equation (2). In structural credit risk models, default risk is inherently dependent on the time horizon. These models define default as a firm's asset value falling below a certain threshold level of liabilities, and the timing of this event depends on the chosen time frame. For a 1-year horizon the default point, the asset value at which the insurer will default, will lie somewhere between the value of total liabilities and the value of short-term debt. For insurers, debt holders are not the only claim holders. There are also policy holders. At a 1-year time horizon, the insurance company will need to meet both payments of claims, i.e. the benefits to policy holders falling due and the maturing debt. The mean and median value of claims and benefit payments to UK policy holders relative to total liabilities is 9 percent over the period from 2006 to 2020. As an approximation, the 1-year cash flows are consistent with a duration of liabilities of 11 years. The EU insurers write more non-life business than the three UK insurers. As a rough approximation this reduces their duration by 1 year and adds a further 1 percent of liabilities falling due at the 1-year horizon, i.e. 10 percent of liabilities falling due to policyholders instead of 9 percent.

We do not have time series data on the maturity profile of debt liabilities. For estimating the Merton model, we take debt at a 1-year horizon as 50 percent of total debt. Debt liabilities are, on average, 2 percent of total liabilities over the same period. which we then halve for our 1-year estimate. Taking 50% gives 1 percent. Based on the current year debt profile, taking 50 percent may overstate the amount of debt falling due at a 1-year horizon, i.e. the proportion of debt falling due in 1 year is less than 50% according to the most recent data, but given that claims tend to be a much larger amount relative to debt, we do not think applying this materially affects our estimates. In view of these considerations, for payments to policyholders and debt holders, we set the default barrier, consisting of both insurance claims and debt, at 10 percent of total liabilities, i.e 9 percent plus 1 percent.

[1]A.M.Best is a credit rating agency that focuses on the insurance industry

[2]Our sample of European life insurers was driven, in part, by the availability of data. We could not include Phoenix and the NN Group owing to the non-availability of consistent time series data over the period of observation.

[3]We opted for spot yields on 1-year UK gilts rather than German bunds as our measure of the risk-free rate of interest rate. This is because yields on the German bunds were negative from 2015 and 2021. 1-year government yields peform the role of a discount factor within the framework of a structural credit risk model. The model will not provide as solution with negative discount discount rates.

5 Empirical results from the portfolio credit-risk model

The model is calibrated using market information and balance sheet data of these six European insurers. First, we show the estimation of default probabilities followed by capital requirements.

5.1 Estimating the implied asset value and default probability

We implement the Merton model to back out the asset values A_t of the six insurers consistent with equation (2). Stochastic assets evolve through time relative to a distress barrier. The probability that the assets will be below the distress barrier is the probability of default. We estimate the expected change in the asset values of each individual insurer R_i with the Capital Asset Pricing Model (CAPM) as described in Löeffler and Posch (2011).

As shown in equation (15), we obtain the beta of the assets with respect to a market index, and then apply the CAPM formula for the return on the asset of insurer i:

$$E[R_i] - R = \beta_i (E[R_M] - R) \tag{15}$$

with R denoting the risk-free rate of interest given by the spot yield on 1-year UK government bonds. The STOXX Europe 600 index[4] return is taken to be a proxy for R_M, the return on the market portfolio. We regress excess returns of the insurer R_i on the excess returns of the market R_M where 'excess' is relative to the risk-free rate R. The regression output provides an estimate of the insurer's beta β. Assuming a value of 6 percent for the market risk premium, applicable to the UK, Germany and France, we obtain the drift rate μ for the asset value returns for each insurer as:[5]

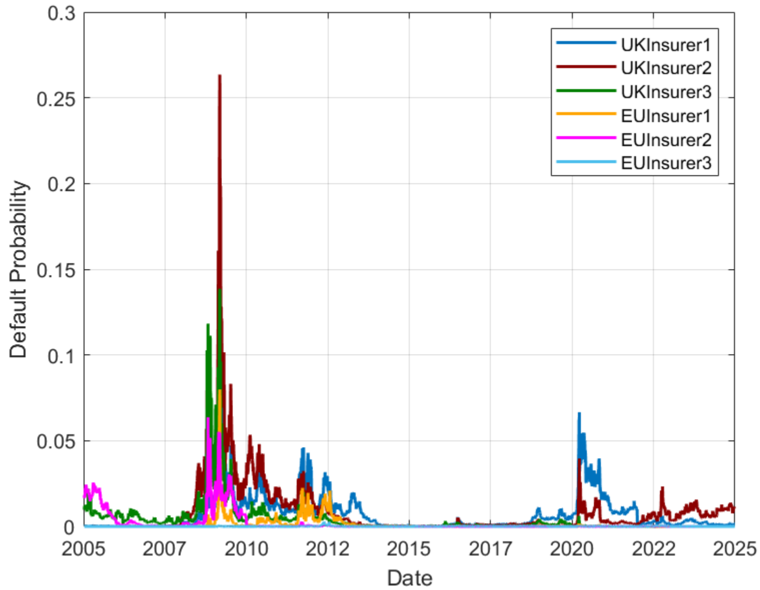
$$\mu = R + \beta * (E[R_M] - R) \tag{16}$$

We re-estimate the default probability of each insurer with the updated estimate for the drift rate μ. Including the CAPM model in the estimation of the drift of the asset value returns augments the market-sensitivity of the default risk measure. The time series evolution of default probabilities for individual institutions are shown in Figure 1.

[4] STOXX Europe 600 is a broad measure of the European equity market.

[5] Market risk premiums (MRP) measure the expected return on investment an investor looks to make. The perfect scenario for a risk-based investment would be a high rate of return with as small a risk as possible. Based on surveys, this number was about 6% for both the UK and Germany in 2025. It is currently estimated to range between 5.25% and 6.25% in France.

Figure 1: Default probability of insurers



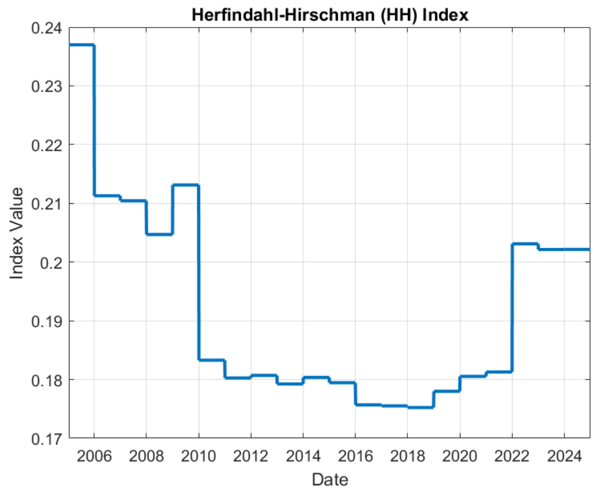
Notes: The figure shows the time series evolution in the default probability of individual insurers. Default probabilities peaked during the height of the GFC (2008-2009) and remained significant during the European sovereign debt crisis (2010 2011). UK insurers witnessed another rise in default risk during March 2020 market turmoil resulting in the extreme 'dash for cash'. The UK insurers again witnessed an uptick in default risk in late September and early October 2022, when the UK gilt market experienced a collapse in prices.
Source: Model outputs

The granularity adjustments for estimating δ, as given in equation(14) are based on the Herfindal-Hirschmann Index (HHI) of market concentration. The formula for the HH Index is:

$$HH_t = \sum_{i=1}^{N_t} w_i^2 \tag{17}$$

where w_i is given by the total liabilities of insurer i. The obtained time series are plotted in Figure 2, indicating substantial variation in δ over time. The HH Index takes the value between $1/N$ (full diversification) and 1(full concentration).

Figure 2: European Life Insurance sector: Herfindal-Hirschmann Index (HHI) of market concentration



Source: Model outputs
Notes: The HH Index is calculated by squaring the total liabilities of each insurer and summing the results.

Credit concentration as given by δ has increased significantly after Prudential sold off assets in 2021 following its decision to demerge its UK and Asia operations. The granularity effect is sensitive to the number of insurers in the portfolio. When this number decreases, the sector accounts for a larger proportion of idiosyncratic risk.

With estimates of the parameters ρ, δ and the time series of default probabilities, we compute the VaR and capital requirements for each insurer's asset returns based on the ASRF model described in equation (6). EAD is given by the total liabilities of each insurer. LGD represents the fraction of exposure that is lost when a counterparty defaults. The LGD is set at 0.25, which implies a 75% recovery rate in the event of a default.[6]

5.2 Estimating VaR and solvency capital adequacy

VaR measures risk in terms of returns at a given probability. The VaR of the random variable $R_{i,t}$, representing the log return of the estimated asset values $A_{i,t}$ of the individual insurers, is defined as the α-quantile of the return distribution and thus can be formulated in terms of returns in the following way.

$$P_r \left(R_{i,t} \leq VaR^i_{\alpha,t} \right) = \alpha \tag{18}$$

where $VaR^i_{\alpha,t}$ is the $\alpha-$quantile of the returns $R_{i,t}$ at time t. Solvency II regulation in the UK and Europe use a 1-year VaR of 99.5%. Adhering to that guideline we set the VaR level at 99.5% such that α =0.005.

The capital adequacy of individual insurers for solvency risk mirrors their respective VaR estimated

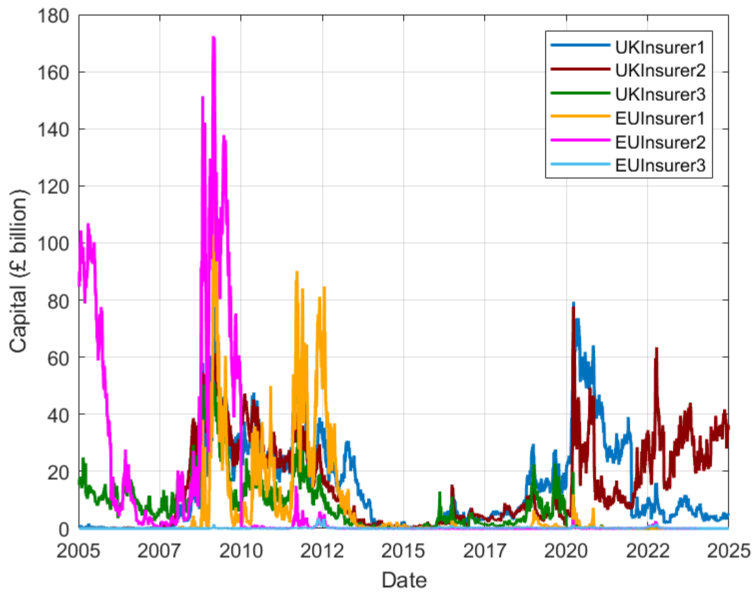
[6]The Joint Research Council of the European Commission (2021) apply a recovery rate of 85% for insurers, while other estimates of recovery rates are lower. Hull and White (2017) quotes an average recovery rate of 52% on senior, secured bonds, based on Moody's data.

in equation (14). As the EAD is expressed in GBP billions, capital adequacy is specified in the same units of GBP. Individual institution capital requirements are computed using a VaR confidence level of 99.5%, a one-year default probability and the correlation between each insurer's returns and index returns. In this paper, the capital adequacy consistent with VaR is referred to as solvency capital adequacy. We provide two interpretations of solvency capital adequacy: for individual insurers, and for the sector as a whole, expressed as a share of total assets.

Figure 3 plots the solvency capital adequacy of the individual insurers. Capital requirements for European life insurers peaked during the GFC. EU insurers, in particular, witnessed a rise in capital requirements during the Eurozone sovereign debt crisis. Life insurance companies' valuations came under pressure during this prolonged phase of exceptionally low interest rates, known as the low for long era. As life insurers' liabilities have a longer maturity profile than fixed-income assets held to meet those obligations, a decline in bond yields makes the discounted present value of those liabilities rise by more than that of assets, giving rise to a negative duration gap. To the extent that it remains unhedged, the negative duration gap would be a factor impinging upon the profit-margins of life insurers up to end-2020. Closing the duration gap would entail adding longer-dated bonds to the portfolio of assets so that the duration of assets catches up with the higher duration of liabilities. Domanski et al. (2017) have provided evidence of this search for duration amongst German life insurers. As discussed in Garavito et al. (2024), reduced profit margins induced life insurers' to develop risk-sharing strategies to cut costs and economise on capital. Brinkhoff and Sole (2022) estimate that the search for yield accounted for about one-third of the total deterioration in credit quality of European life insurers' portfolios from 2005 to 2021. Low interest rates underpinned the growth of funded reinsurance as a source of capital. Funded reinsurance is a means to gain indirect exposure to asset classes beyond the origination capacity of the traditional life insurance sector.

In March 2020, the COVID-19 shock exposed vulnerabilities in the financial system. From a solvency perspective, UK insurers were particularly affected. The primary risk to insurers was via their assets, with the potential for falling income from asset management type businesses, and the scope for default on their credit assets and loans backing annuity businesses. In the UK, a 'flight to safety', in which prices of risky assets fell and prices of government bonds increased, was followed by an abrupt and extreme dash for cash – where even safe assets were sold to obtain cash. Insurance companies and pension funds became net sellers of UK government bonds (gilts) during the dash for cash. Net sales of gilts by insurance companies, pension funds and asset managers was very large by historical standards - totalling GBP 6 billion over 9 days from 9-19 March (Czech et al. (2021)). This led to several consecutive days of very high VaR implying increased solvency capital levels in UK insurers. Balance sheet data shows that UK insurers, which are more oriented towards life business than EU insurers, have higher derivatives exposures for asset-liability management. UK insurers also have more exposure to unit-linked businesses as compared to their European counterparts. Unit-linked businesses being fee based are more vulnerable to market downturns. The EU insurers are also more diversified than the UK insurers, writing more non-life business, as well as life, and across a wider range of countries. This diversification may have helped them reduce asset-side risks to a greater degree than UK insurers.

Figure 3: Solvency capital adequacy of insurers



Notes: The figure shows the time series evolution in the solvency capital requirements of individual insurers. Solvency capital requirements are computed using a VaR confidence of 99.5%, a 1-year default probability and the correlation between each insurer's asset value returns and the index returns, where the index is the STOXX Europe 600.
Source: Model outputs

In late September and early October 2022, the UK gilt market experienced an almost unprecedented collapse in prices following the mini-budget delivered on 23 September 2022 against a background of high inflation and low economic growth. The highly leveraged LDI strategies followed by certain pension funds, compelled them to sell gilts, amplifying pressure in certain segments of the gilt market. Pension funds are a material player in the gilt market particularly at the long-end. Yields on index linked gilts and on long-dated nominal gilts began to spiral upwards, with every rise in yields triggering further asset sales by LDIs in an increasingly illiquid market. On September 28th, the Bank of England launched a novel-asset purchase programme to restore market functioning (Breeden (2022)). [7] Life insurers, as providers of annuities and defined benefit (DB) pension schemes, provide an income for the rest of the beneficiary's life. Therefore, these providers need to ensure that they hold sufficient assets to meet their future obligations to beneficiaries, after accounting for movements in interest rates and inflation. The contractual obligations of pension funds are in real terms which explains why they specialised in buying inflation-indexed bonds better known as linkers. For example, if a pension fund had to deliver a £100 in real terms in 30 years, with an expected inflation rate of 2 percent and a discount rate of 3 percent, this would be recorded as a liability of £73.60 today. The pension fund can then invest this reduced amount in an inflation-indexed bond such that it would grow to become £100 in 30 years. If the discount rate rises to 4 percent the present day liability falls to £50.50. Therefore, lower gilt yields would raise their obligations and higher gilt yields would lower their obligations. With the rise in gilt yields in September-October, life insurers and pension funds were able to discount their liabilities at a higher rate than before, apparently strengthening their funding positions. However, the

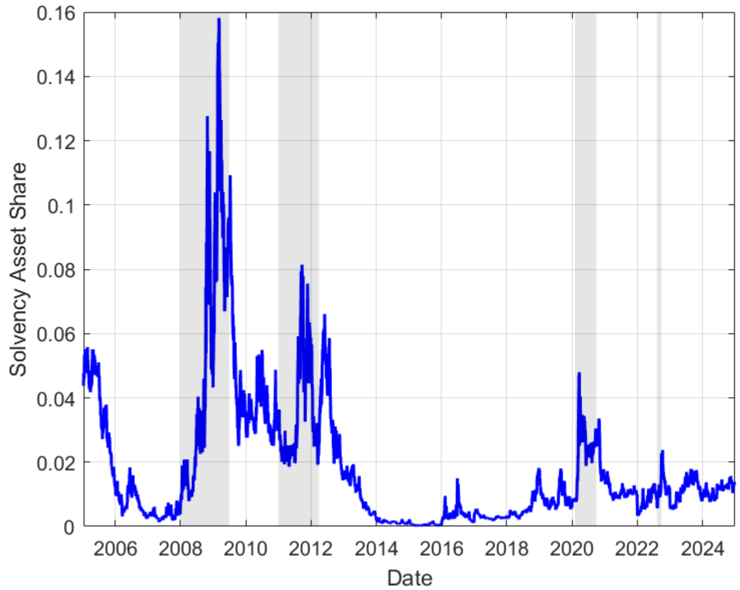
[7] Pinter (2023) and Bandera and Stevens (2024) address the gilt market crisis in September-October 2022.

problem arose from the LDI strategy followed by these DB pension schemes to hedge their exposure to long-term interest rates and inflation. A steep rise in gilt yields after the mini-Budget on 23 September forced LDI managers to post huge amounts of margin to their swap counterparties as the value of their positions declined. This led to a vicious cycle of collateral calls and forced gilt sales that led to further market dysfunction. This was a problem of liquidity that would have turned into a more severe solvency crisis had it not been for intervention by the Bank of England that involved a temporary and targeted programme of purchases of long-dated gilts.

Although life insurers have the same type of long-term obligations as DB pension schemes, and hedge their liabilities against the same risks, they operate under a much stricter prudential regulatory governance and risk management framework based on the principles of Solvency II. Individual insurers hedging strategies are specific to the institution rather than having to rely on pooled funds that underpinned the DB pension schemes. The financial stress reflected concerns about the effect of the LDI crisis on the pensions business of insurers, and the effect of market liquidity more broadly on the assets held by insurers.

Solvency capital adequacy has been estimated across insurers with varying balance sheet size, some of which have shed or acquired assets over time. To assess solvency capital at a sectoral level, we divide the insurance sector's solvency capital level by the total assets of the six insurers in the sector. This generates the asset share of solvency capital. Figure 4 shows the evolution of the asset share of solvency capital in the European life insurance sector. The level of solvency capital exhibits considerable time series variation that captures the GFC, the Eurozone sovereign debt crisis, the Covid-19 pandemic and gilt market crisis. The shaded vertical bars highlight the periods of financial stress associated with these events.

Figure 4: Asset share of solvency capital levels



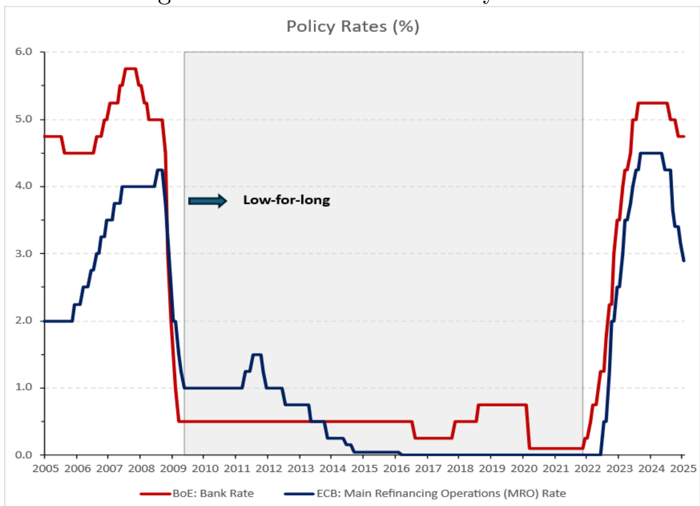
Source: Model outputs
Notes: The figure shows the time series variation in the asset share of solvency capital for the European life insurance sector over the period 2005 to 2024. Solvency capital asset share is obtained by dividing the insurance sector solvency capital level by the total value of assets of the six insurers. The shaded vertical areas refer to periods of financial stress : the global financial crisis, the Eurozone debt crisis, the shock from the COVID pandemic and the UK gilt market crisis.

Life insurers' asset durations are typically shorter than liability durations, implying a negative duration gap. A negative duration gap means assets decrease less in value than liabilities when interest rates rise, which explains why most European life insurers saw a positive impact on their solvency positions when interest rates rose in 2022 and 2023. Figure 4 shows that solvency capital levels for the European life insurance sector stabilised between 2023 and 2024.

6 Asset allocation of European Life Insurers

As insurance is a liability driven business, the duration of the commitments and the guarantees offered shape the asset allocation of insurers and their sensitivity to changes in interest rates. In the aftermath of the GFC in 2008 and the COVID pandemic in early 2020, both the Bank of England and the ECB lowered policy rates effectively to the ZLB. They also conducted unconventional monetary policy operations involving acquisitions of sovereign and corporate bonds. These purchase programmes compressed interests even further and for a long time, giving rise to the period now known as the "low-for-long" era (shown in Figure 5). In an environment of low interest rates, insurers would have difficulty generating sufficient returns to meet obligations from higher guaranteed returns promised to their policyholders. The era of low interest rates incentivised life insurers to exit core lines of business and divest assets. Private equity firms entered the life insurance sector, acquiring legacy book assets via reinsurance, to fund and expand their private credit operations.

Figure 5: BoE and ECB Policy Rates



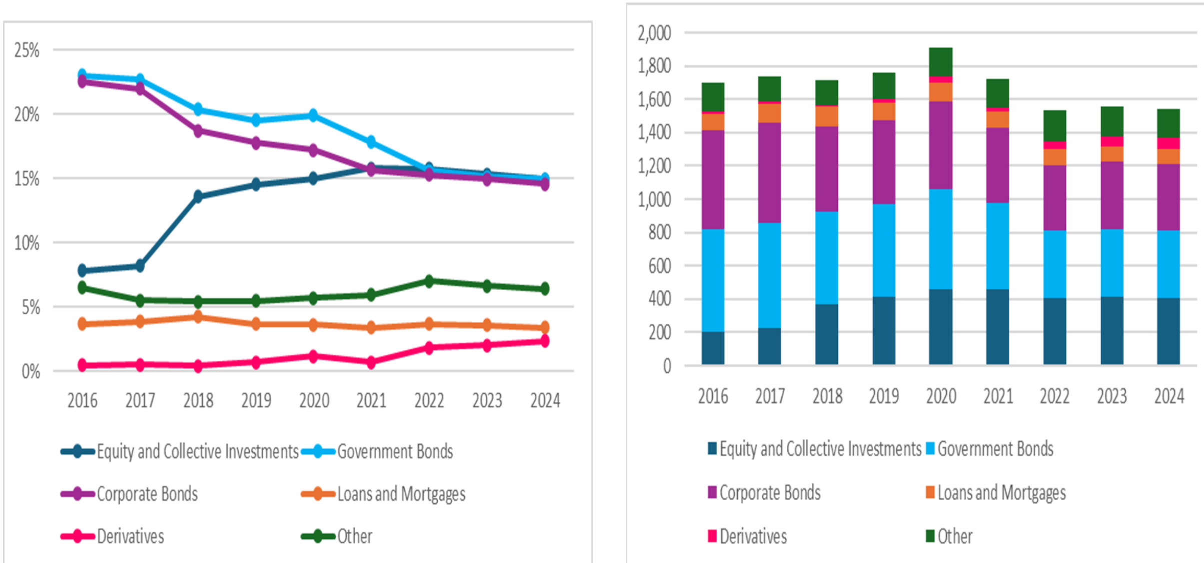
Source: Bank of England and European Central Bank

Figure 6 shows the portfolio composition of European life insurers valued at market prices. Corporate and government bonds made up around 46 per cent of assets in 2016. In 2024, this stood at 30 per cent. There has been a diversification out of bonds largely driven by a narrowing of spreads of investment grade bonds over risk-free rates. As a share of total assets of the European life insurance sector, government bonds accounted for more than 23 percent in 2016. By 2023 this share had fallen to 15 per cent and remained at that level in 2024. The share of equity and collective investments increased from 8 percent to 15 percent over the same period. In the UK, life insurers have increased their exposures to illiquid credit by engaging in direct lending to long-term financing markets such as infrastructure, equity release mortgages, social housing and commercial real estate loans. EU insurers have greater relative exposure to non-life liabilities and government bonds, as part of their asset-liability management.

In the UK, most of the longevity risk on new "bulk purchase annuity" transactions is currently being reinsured, often outside the UK. The rapid growth of the primary bulk purchase annuity business in the UK has led to increased demand for funded reinsurance, as primary insurers seek to increase their underwriting capacity without having to raise more capital. Reinsurance companies, where private equity companies have full or partial ownership, or a strategic partnership – often invest more in illiquid assets, potentially increasing risks (BOE (2024)). Reinsurance counterparty relationships would add to interconnectedness in the life insurance industry.

Figure 6: Asset allocation of 6 European Life Insurers: 2016 to 2024

Portfolio composition by asset class (percentage) Portfolio composition by asset class (GBP billions)



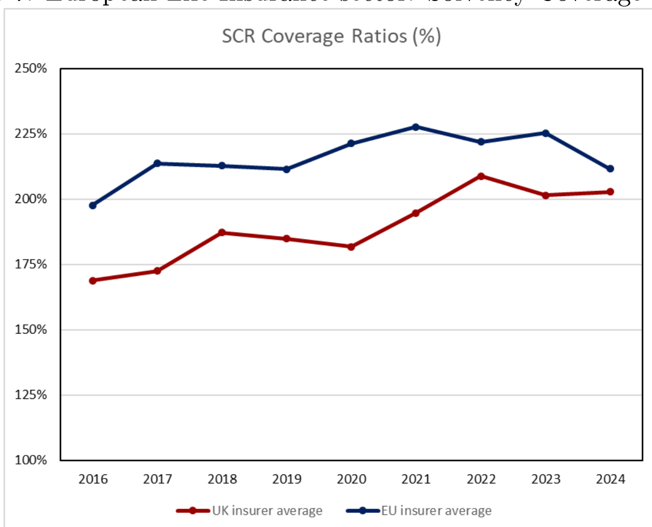
Source: Companies' Solvency and Financial Condition reports and Authors' calculations

The European financial system experienced a notable shift in its interest rate environment in 2022 and 2023, marked by the re-emergence of higher interest rates, after a prolonged period of historically low rates. Both the UK Bank Rate and ECB policy rates reverted to their pre-GFC levels, but did so more rapidly than expected. A rise in interest rates should ease pressures over the long run and improve solvency ratios. However, a very sharp rise in interest rates would entail liquidity risks for life insurers, in the short term, as the market value of assets could drop to an extent that could incentivise policy holders to lapse their policies and reinvest the proceeds in new higher-yielding products. With interest rates rising, life insurers' balance sheets should have strengthened as the present value of their assets typically declines by less than that of their liabilities. In the UK, higher interest rates have improved funding levels in DB pension schemes. There is an expectation that higher interest rates will lead to annuity-driven growth in the European life insurance sector.

The sharp rise in interest rates in 2022, also led to a significant increase in the gross market value of interest rate derivatives held by life insurers' and the requirement to pay variation margin calls. The value of derivatives liabilities, for the European life insurance sector, doubled from GBP 37bn in 2021 to GBP 74bn in 2022. At the end of 2024, derivatives liabilities remained high at GBP 87bn. About half of these derivatives exposures are interest rate derivatives. Derivatives exposures doubled from 1% to 2% of total assets between 2021 and 2022 (as shown in the left-hand-side panel of Figure 6).

Most risk-based insurance regulation, emphasises the requirement to hold enough capital to cover a larger range of risks than traditional insurance risks (GA (2016)). Solvency II, the regulatory framework that is in force since 2016, has had a significant influence on insurers' investment strategies by introducing a capital framework that directly links capital charges to asset risk. As a result, insurers face higher capital charges for assets deemed riskier, such as equities, investments in mutual funds, or lower-rated corporate bonds. The solvency coverage ratio (SCR) is the ratio of an insurance company's eligible capital to its regulatory capital requirement. This ratio is used as an indication of an insurance company's financial strength and its ability to withstand the risks they are exposed to such as falling asset prices or increased liabilities. It is usually expressed as a percentage and must exceed 100%. At the same time, insurers must fulfil the solvency capital requirement to withstand shocks with a 99.5% probability over the next year.

Figure 7: European Life Insurance sector: Solvency Coverage Ratios



Source: Companies' Solvency and Financial Condition reports and Authors' calculations

European life insurers' solvency capital positions remained stable at the end of the observation period in 2024 and were not significantly affected by the expected reduction in policy interest rates (Figure 7). Both the BoE and ECB policy rates fell in 2024. Although these declines in interest rates put some downward pressure on solvency coverage ratios, most insurers' capital positions were able to withstand such rate movements.

7 Systemic risk in the insurance sector

Systemic risk, by its nature, is characterised by both a cross-sectional and a time series dimension. The cross sectional dimension emerges from the correlation between risks of financial institutions at a given point in time, due to spillovers and common exposure effects. The time series dimension focuses on the evolution of systemic risk over time due to changes in financial market conditions and the build up of vulnerabilities in the financial system.

In this section we implement the SRISK methodology, developed by Acharya et al. (2012) and Brownlees and Engle (2017), for European life insurers. The objective of the SRISK methodology is to measure the capital shortfall a financial institution is expected to experience conditional on a systemic event. In this study, a systemic event is defined as the occurrence of losses in the tail of the STOXX Euro 600 Insurance index above a particular threshold.[8]

[8]Allianz, AXA, Generali, Prudential and Aviva are amongst the top 10 components of the index

SRISK takes into account an institution's market capitalisation, its prudential capital ratio, and the level of debt given by its total liabilities. In the context of this study, the estimation of SRISK involves the following variables: the returns of the particular insurer i at date t denoted as $R_{i,t}$, the return of the chosen equity market index s at date t denoted as $R_{s,t}$, the value of total liabilities $L_{i,t}$, the market value of equity (market capitalisation) $ME_{i,t}$ and the prudential capital requirement k. These variables are first used to calculate each insurer's long-run marginal expected shortfall (LRMES). Conditional volatilities of the returns are estimated with an assymetric GJR GARCH process (Glosten et al. (1993)) and correlations with a DCC correlation model (Engle (2002)).

For the estimation of SRISK we use daily market equity data and annual balance sheet data on total liabilities for the six European life insurers. Later in the analysis we compute SRISK asset shares and for this we require annual balance sheet data on total assets of the 6 institutions.

The Marginal Expected Shortfall (MES), proposed by Acharya et al. (2017), is defined here as the expected loss on an insurer's equity conditional on the occurrence of losses in the tail of our chosen index, the STOXX Euro 600 Insurance. Based on this definition, we take the MES of an insurer to be its short-run equity loss conditional on the insurance sector taking a loss greater than its VaR. While VaR is a quantile (α%) of the loss distribution over a prescribed holding period, expected shortfall (ES) is the expected loss knowing that the loss is above VaR.[9]By definition, the expected shortfall (ES) at the α% level is the expected return in the worst α% of the cases, but it can be extended to the general case, in which returns exceed a given threshold C. We consider a threshold C equal to the conditional VaR of the insurance sector's return, which is defined as $Pr[R_{s,t} < VaR_{s,t}(\alpha)] = \alpha$ where $\alpha = 0.005$ consistent with a VaR level at 99.5%

SRISK will decrease as the prudential capital ratio k decreases. We use a prudential capital ratio of 5.5 percent applicable for financial institutions in Europe (see Acharya and Steffen (2014)).

We define the ES of the insurance sector as the expected loss in the insurance sector s conditional on this loss being greater than C.

$$ES_{s,t}(C) = E_{t-1}(R_{s,t} \mid R_{s,t} < C) \tag{19}$$

We evaluate measures of daily performance in the event of an extreme aggregate shock, as the insurance index falls more than its 99.5% VaR. The expected daily loss of the individual insuerer returns in this case is the MES.

$$MES_{i,t}(C) = E_{t-1}(R_{i,t} \mid R_{s,t} < C) \tag{20}$$

MES measures how exposed an individual institution is to tail shocks within the system. Benoit et al. (2013) have provided a theoretical proof of the proposition that the MES of a given financial institution i is proportional to its systematic risk as measured by it's beta. As shown in equation (21), MES corresponds to the product of the insurance sector's expected shortfall (sector tail risk) and the insurer beta (insurer systematic risk). The proportionality coefficient is the expected shortfall of the

[9]The Expected Shortfall (ES) of a given system is also known as the expected tail loss, or average value-at-risk. It is the expected conditional loss, given that the VaR is exceeded (a worst-case scenario), and will be larger (or more extreme) the more fat-tailed the distribution of returns. The MES is then the contribution of an individual institution to the total estimated shortfall in the system.

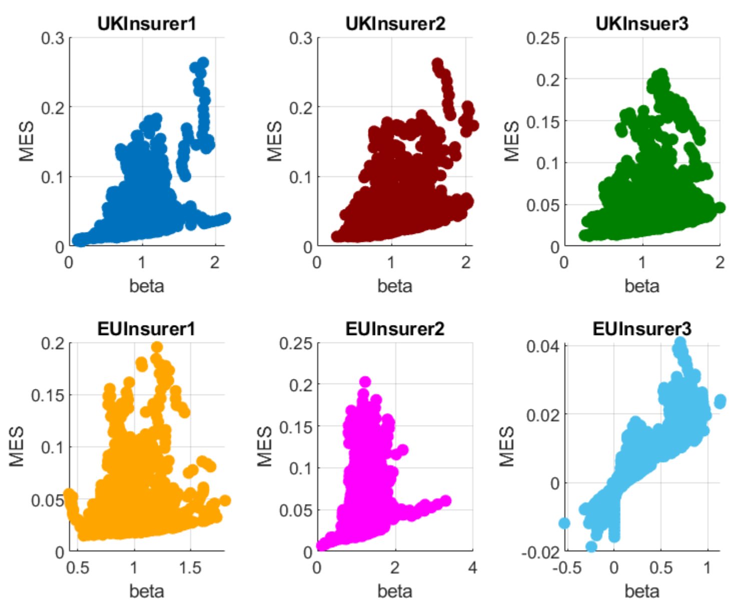
market:

$$MES_{i,t}(\alpha) = \beta_{i,t} ES_{s,t}(\alpha) \tag{21}$$

where $\beta_{i,t} = \frac{cov(R_{i,t}, R_{s,t})}{var(R_{s,t})} = \frac{\rho_{i,t}\sigma_{i,t}}{\sigma_{s,t}}$ denotes the time-varying beta of insurer i and $ES_{s,t}(\alpha)$ is the expected shortfall of the market. Since the ES of the insurance sector is not institution-specific, the greater sensitivity of an individual insurer's equity return to the sector's equity return, the more systemically risky the insurer is. Since the insurance sector ES may not be constant over time, forecasting the systematic risk of an insurer may not be sufficient to forecast the future evolution of its contribution to systemic risk. For a given insurer, the variation of its systemic risk (measured by it's MES) may be different from the variation of its systematic risk (measured by it's conditional beta).

We examined this proposition in the context of the European insurance sector. Figure 8 plots the relationship between insurer betas and MES for each of the six insurers. The scatter plot shows a positive relationship between MES and insurer beta, which implies that systemic risk rankings of insurers based on their MES should broadly conform to the rankings obtained by sorting insurers on their betas. However, there is noise in the relationship between beta and MES which could be partially explained by the time-varying nature of the ES.

Figure 8: Relationship between insurer beta and MES



Notes: The figure shows the scatter plot of conditional beta and MES for each of the six insurers. MES is the expected loss on an insurer's equity conditional on the occurrence of losses in the tail of our chosen index (STOXX Euro 600 Insurance) greater than its VaR at 99.5%. Expected shortfall (ES) is the expected loss knowing that the loss is above VaR. ES is an overall measure of sysemic risk, being the expected impact on the insurance sector of a tail shock. The contribution of each institution to the total shortfall, defines the MES. The MES of an institution is proportional to its systematic risk as measured by its beta. The proportionality coefficient is the ES of the insurance sector
Source: Model outputs

The time-series average of the MES for each insurer and its beta suggests a cross-sectional relationship between the two measures. This is shown in Figure 9. Each point corresponds to one of the six insurers listed in Table 1. The beta corresponds to the average of the time-varying beta, $\beta_{i,t}$, in equation 21. Figure 9 plots the output from regressing the MES of each institution as a linear function of its beta. The straight line in Figure 9 is the ordinary least squares (OLS) regression line with no constant term.

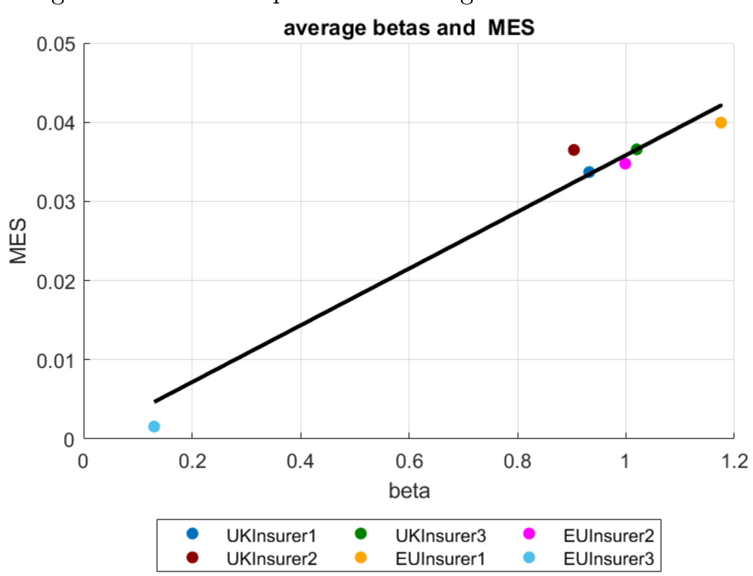
Table 1 reports the average of the time-varying beta, $\beta_{i,t}$ and MES for each insurer, as described in equation (21).

Table 1: Insurers' beta and MES

Insurer	beta β	MES
UKInsurer1	0.93	0.0337
UKInsurer2	0.90	0.0365
UKInsurer3	1.02	0.0366
EUInsurer1	1.18	0.0399
EUInsurer2	1.00	0.0348
EUInsurer3	0.13	0.0016

Table 1 shows the varying levels of systematic risk in the insurance sector. Two inferences can be made. First, the systematic risk of an insurer with a conditional beta of 1.0 will, on average, move in the same direction and magnitude as its systemic risk. Second, the systematic risk of an insurer with a conditional beta greater than 1.0 will, on average, be more volatile than it's systemic risk.

Figure 9: Relationship between average insurer betas and MES



Notes: The figure plots the output from regressing the MES of each institution as a linear function of its beta. The straight line is the OLS regression line.
Source: Model outputs

The LRMES is a measure of an institution's expected cumulative loss of equity over a prolonged period conditional upon a large shock in the financial system. There are two general approaches to calculating the LRMES, both of which require a first step of fitting a DCC-GARCH model. The first is by simulating the trajectories of insurance sector returns in future periods. We have adopted the second approach based on the following approximation proposed by Acharya et.al. (2012):

$$LRMES_{i,t} = E_{t-1}\left(R_{i,t+T} | R_{s,t+T} < C\right) \approx 1 - \exp\left(-18 * MES_{i,t}\right) \tag{22}$$

where $R_{i,t+T}$ is the cumulative return to a insurer's equity, and $R_{s,t+T}$ is the cumulative return on the insurance sector. Thereafter, SRISK is calculated as:

$$SRISK_{i,t} = \max\left(0, k_t * L_{i,t} - (1 - k_t) * ME_{i,t} * (1 - LRMES_{i,t})\right)$$

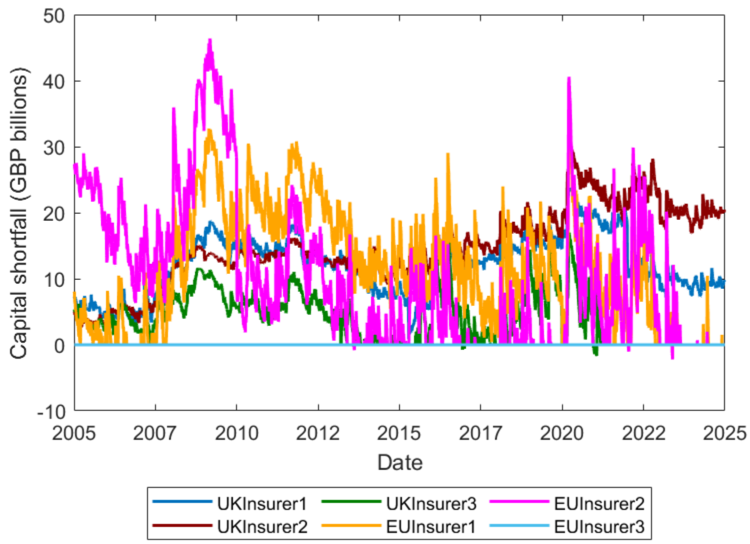
$$SRISK_{i,t} = \max\left(0, ME_{i,t}\left[k_t * LEV_{i,t} - (1 - k_t) * (1 - LRMES_{i,t} - 1)\right]\right) \tag{23}$$

where $LEV_{i,t} = \frac{L_{i,t} + ME_{i,t}}{ME_{i,t}}$ is the quasi-leverage ratio. SRISK can vary depending on the value of the parameter k, the prudential capital ratio. This implies that an insurer with a capital surplus has an SRISK value of zero.

Therefore, SRISK adjusts for the amount of regulatory capital that the insurer is mandated to hold, and can be decomposed into three main components: (i) the market capitalisation of the insurer; (ii) how leveraged the insurer is; and (iii) how risky the insurer is conditional on a distressed financial system, as proxied by its LRMES. A positive SRISK value thus suggests that the insurer, in the event of a systemic event, would experience financial distress, due to excessive debt, insufficient equity, and/or a deterioration in market-based risk-related measures of the insurer. By definition, SRISK can be negative when an insurer is expected to have excess capital in a crisis.

Figure 10 presents the evolution of the conditional capital shortfall measure SRISK for individual insurers between 2005 and 2024. Capital shortfall of individual insurers fluctuated widely over the period of observation. Capital shortfall for two EU insurers, peaked during the GFC. Both these institutions experienced heightened capital shortfall during the Eurozone sovereign debt crisis. In March 2020, the shock from COVID pandemic caused the capital shortfall to surge across insurers.

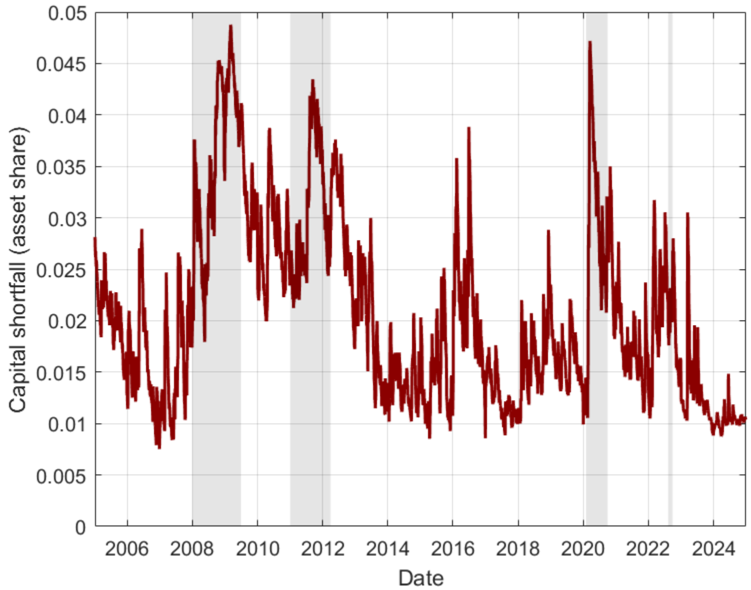
Figure 10: Conditional capital shortfall of institutions



Notes: The figure shows the evolution of the conditional capital shortfall of individual insurers.
Source: Model outputs

To get sector conditional capital shortfall, we sum the SRISK measure across the six insurers, assigning zero when the value is negative. As the insurers within the sector have different balance sheet sizes which have changed over time, we divide the sector SRISK by the total assets of insurers in the sector. The table reports period averages for the entire period of observation and during periods of stress. Figure 11 shows the evolution of capital shortfalls, expressed as asset shares, for the European insurance sector. The figure shows that capital shortfall peaked during crisis periods with the market turmoil following the shock from the COVID pandemic having the most significant impact.

Figure 11: Asset share of capital shortfall



Source: Model outputs
Notes: The chart plots the evolution of capital shortfall, measured as asset shares, for the European life insurance sector over the period 2005 to 2024. The shaded areas refer to periods of financial stress : the global financial crisis, the Eurozone debt crisis, the Covid-19 shock and the UK gilt market crisis. Capital shortfall asset shares ranged from a maximum of 0.05 to a minimum of 0.01.

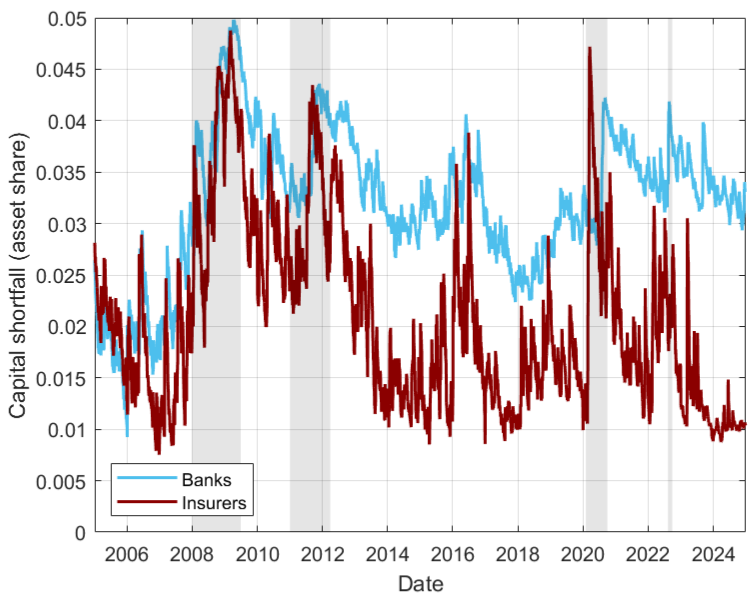
SRISK asset shares have also shown considerable time variation throughout the period of observation. A comparison of Figure 4 and Figure 11 shows that systemic risk capital adequacy peaked earlier than solvency capital during the run up to periods of financial stress. This points to the efficacy of the SRISK measure, as an ex-ante indicator that can quantify the build-up of systemic risk.

The SRISK measure, like other measures of systemic risk, does have features that would limit its scope as a supervisory instrument. Tavolaro and Visnovsky (2014) raise concerns about the information content of SRISK, as it mirrors market participants expectations, which may differ significantly from economic fundamentals. Given its reliance on market factors, SRISK may not effectively reflect the fundamentals of financial institutions. In the context of life insurers, fundamental risk could be associated with risks originating from non-performing assets or from idiosyncratic risks.

8 Comovement of bank and insurance sector systemic risk

Over the past decade insurance companies have grown in size and interconnectedness with banks. Systemic risk within the life insurance and banking sector may move together if they tap similar funding markets and have a common exposure to the macroeconomy. During the phase of low interest rates life insurers shifted to alternative assets, which included direct lending, while banks moved away from some traditional elements of direct lending. The return on the underlying assets of the insurer could be correlated to the return on assets held by other financial intermediaries, including banks, and to economic growth. To investigate this comovement we compare the SRISK asset share of the European insurance sector with a representation of the EU banking sector comprising 10 global systemically important banks (G-SIBs) - Barclays, BNP Paribus, Credit Agricole, Deutsche Bank, HSBC, ING, Santander, Societe General, Standard Chartered and UBS over the period from 2005 to 2024. In order to estimate SRISK for the banks we obtain daily market equity data and annual balance sheet data on total liabilities and total assets from LSEG. To be consistent with the insurance sector, we use a prudential capital ratio of 5.5 percent for each of the ten banks. The SRISK asset share is computed by dividing aggregate SRISK in each sector by the total assets of institutions within that sector. Figure 12 plots the SRISK asset shares of the bank and life insurance sector, along with periods of financial stress. Systemic risk in banks and life insurers follow a similar trajectory, rising during the GFC and the COVID pandemic.

Figure 12: Comovement of bank and insurance sector capital shortfall



Source: Model outputs
Notes: The chart plots the assets shares of SRISK for banks and life insurers over the period 2005 to 2024. The shaded areas refer to periods of financial stress : the GFC, the European sovereign debt crisis, the Covid-19 shock and the UK gilt market crisis.

The GFC is marked by distinct peaks in the SRISK asset shares of banks and insurers. In the early phase of the crisis in late 2007 and early 2008 the banking sector had higher levels of systemic risk.

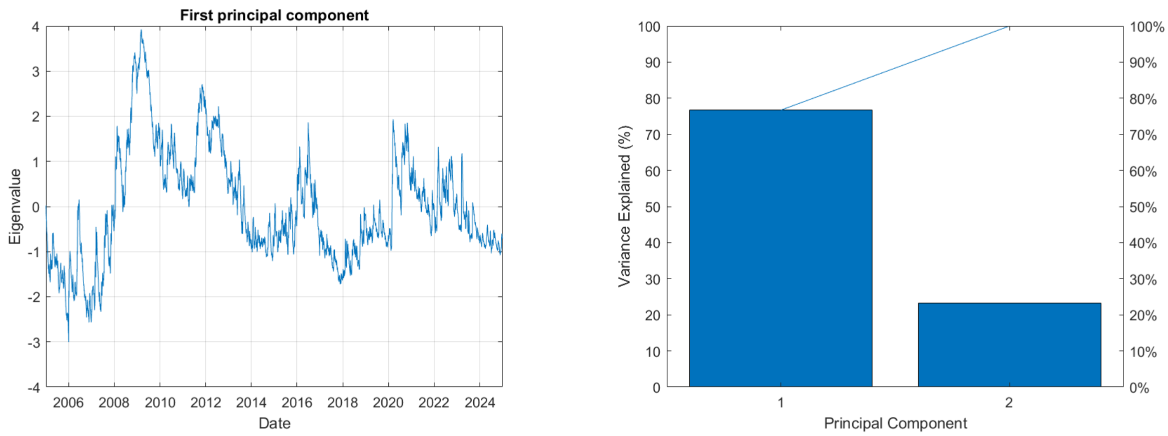
In late 2008 and early 2009 policy measures taken to stabilise the banking system, which included bank recapitalisation, resulted in a significant drop in SRISK asset shares of the banking sector. Asset shares of the European banking and insurance sectors peaked again during the Eurozone sovereign debt crisis. The March 2020 turmoil resulting in the 'dash for cash', impacted insurers more significantly than banks. The dash for cash was reflective of margin requirement for derivatives which affected insurers and other NBFIs rather than banks. On March 19, the Bank of England cut the Bank Rate (UK's official interest rate) to 0.1% at a special Monetary Policy Committee meeting. Interest rates in the Eurozone were already negative. Both the Bank of the England and the ECB were also operating unconventional monetary policies that target longer-term interest rates which would likely impact the systemic risk of life insurers more than banks. Through quantitative easing (QE), the central bank purchases assets and credits the reserves accounts of banks, which can use this liquidity to invest in relatively more illiquid assets, such as loans to households and businesses. However, QE has a more direct impact on life insurers as the lowering of long-term interest rates increases the present value of their future liabilities and also makes it harder to meet guaranteed returns on policies. There was another uptick in SRISK asset shares coinciding with the gilt market crisis in September 2022, given the weight of UK banks and insurers within the European financial system.

The unexpected surge in interest rates, that began in 2022, resulted in significant valuation losses and increases in liquidity risks, as was observed in some banks. A number of bank deposit runs took place over this period, which represented the most serious disruption to the banking sector in more than a decade. The runs were the proximate cause of the collapse of Credit Suisse, a global systemically important bank, and the failure of three US banks (SVB, Signature and First Republic) in quick succession. SRISK asset shares for the European banking sector increased in 2023 and remained elevated in 2024. For the European life insurance sector, SRISK asset shares declined during this period of high interest rates which may be attributed to the longer duration of their liabilities relative to their assets.

To analyse the degree of comovement, we apply principal components analysis (PCA) to the time series of SRISK asset shares of the two sectors. PCA involves a series of steps, including transforming the asset shares to have a mean of zero and a standard deviation of one, calculating the covariance matrix to measure the extent to which the asset shares change together, and determining the eigenvectors and eigenvalues of the covariance matrix. The eigenvectors represent the principal components. Figure 13 shows that the first principal component, or a single factor, explains 77 percent of the common variation in European bank and life insurance sector systemic risk and this factor is correlated to the macroeconomic conditions that characterise periods of financial stress. This outcome is consistent with Aradillas Fernandez et al. (2024) who found that a single factor explains 88 percent of common variation in the comovement system of systemic risk in US banks and the US NBFI sector more broadly.

Figure 13: Factors driving common variation in SRISK asset shares for Banks and Insurers

First Principal Component — Variance Explained



Source: Model outputs
Notes: The figure in the left-hand-side panel shows that the first principal component spikes during financial stress episodes. The figure on the right-hand-side panel shows that this single factor accounts for almost 80 per cent of the cumulative variance of the SRISK asset shares of banks and insurers.

It is well known that owing to the maturity mismatch between assets and liabilities, banks are intrinsically risky and highly leveraged. Bank liabilities are also readily callable rendering them highly liquid. Traditional insurance activity has been characterised by an inverted production cycle where the policy holder first pays for a service that could be delivered conditional on a predefined event that can occur in a relatively distant future. So an insurance company can become insolvent and remain liquid at the same time (Bobtcheff et al. (2016)). Insurers have much more longer time liabilities which are much less liquid than bank liabilities and whose callability can be triggered by predefined events that are not under the control of the policy holders.

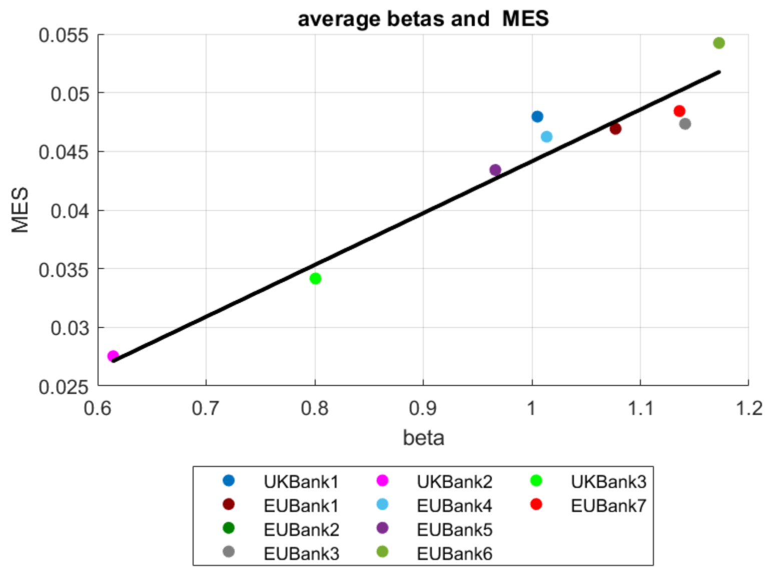
For the European banking sector, we also examined the time profile of its systemic risk, measured by its MES, and the evolution of its systematic risk measured by its conditional beta. This shown in the scatter plot in Figure 14. Each point corresponds to one of the six insurers listed in Table 2.

Table 2: Banks' beta and MES

Insurer	beta β	MES
UKBank1	1.0052	0.0482
EUBank1	1.0771	0.0469
EUBank2	1.1360	0.0484
EUBank3	1.1412	0.0473
UKBank2	0.6144	0.0275
EUBank4	1.0137	0.0076
EUBank5	0.9664	0.0434
EUBank6	1.1724	0.0542
UKBank3	0.8007	0.0342
EUBank7	1.1360	0.0484

A comparison of Table 2 with Table 1 shows that the banks' conditional betas are much lower than insurers' betas. This implies that banks have much lower levels of systematic risk. However, bank MES levels are greater than that of insurers suggesting higher systemic risk. Banks are less exposed to risks from exposure to a common factor than insurers. However, risks emanating from banks can cause larger financial losses, or even a collapse of the financial system, than insurers. The higher betas for insurers could also be attributed to their equity investments, and unit-linked business, where they earn a fee from assets under management.

Figure 14: Relationship between average bank betas and MES



Notes: The figure plots the output from regressing the MES of each institution as a linear function of its beta. The straight line is the OLS regression line.
Source: Model outputs

9 Spillovers between European bank and insurance sectors

This section measures the degree to which European bank and insurance sectors have interacted during the period from January 2005 to December 2024. The method suggested by Diebold-Yilmaz (Diebold and Yilmaz (2012)) is used to quantify such interactions, also termed "spillovers". The spillover analysis first estimates a generalised VAR model with equity market data of the sample of banks and insurers. The connectedness measure is then derived from the forecast error variance decomposition of the underlying VAR for the volatility in equity returns. Daily conditional volatility is measured by applying a GARCH(1,1) model to the return series of banks' and insurers' market value of equity. A moving data window is used such that a time series of spillover observations is generated. The procedure starts with a covariance stationary N-variable VAR(p) process:

$$x_t = \sum_{i=0}^{p} \phi_i x_{t-1} + \varepsilon_t \tag{24}$$

where x_t is an $N \times 1$ vector of endogenous variables and $\varepsilon \sim (0, \sum)$ stands for a vector of i.i.d disturbances.

The moving average representation of the above process is:

$$x_t = \sum_{i=0}^{\infty} A_i \varepsilon_{t-i} \tag{25}$$

where the coefficient matrices A_i follows:

$$A_i = \Phi_1 A_{i-1} + \Phi_2 A_{i-2} + ... + \Phi_p A_{i-p} \tag{26}$$

with A_0 an $N \times N$ identity matrix and $A_i = 0$ for $i < 0$.

Forecast error variance decompositions are transformations of the moving-average coefficients, which attributes the H-step ahead forecast error variances of each variable i, to other variables in the system.

Following Diebold and Yilmaz (2012), we consider the generalised variance decomposition of the underlying VAR. In contrast to the Cholesky decomposition proposed by Sims (1980) and related identification strategies, the generalised variance decomposition is invariant to the ordering of variables, which offers more flexibility in the modelling strategy without making any a priori assumption on the sequence of responses. The generalised variance decomposition is particularly applicable to the European bank and insurance sector with 16 financial institutions because of the infeasibility of imposing a meaningful ordering among so many entities.

Variable j's contribution to variable i's H-step-ahead generalised forecast error variance is given by:

$$\theta_{ij}^{g}(H) = \frac{\sigma_{jj}^{-1} \sum_{h=0}^{H-1} \left(e_i' A_h \sum e_j \right)^2}{\sum_{h=0}^{H-1} \left(e_i' A_h \sum A_h' e_i \right)} \tag{27}$$

For H=1,2,..., where $\sum$ depicts the variance matrix corresponding to ther error vector ε. σ_{jj} is the standard deviation of ε_j, e_i is the selection vector with the i-th element unity and zeros elsewhere. The row sums of the variance decomposition matrix are not necessarily unity, due to non-zero correlations across shocks.

$$\sum_{j=1}^{N} \theta_{ij}^{g}(H) \neq 1 \tag{28}$$

Therefore, each forecast error variance decomposition is normalised by the row sum:

$$\tilde{\theta}_{ij}^{g}(H) = \frac{\theta_{ij}^{g}(H)}{\sum_{j=1}^{N} \theta_{ij}^{g}(H)} \tag{29}$$

The construction proposed by Diebold and Yilmaz (2009) postulates that the following two relations must hold:

$$\sum_{j=1}^{N} \tilde{\theta}_{ij}^{g}(H) = 1 \tag{30}$$

$$\sum_{i,j=1}^{N} \tilde{\theta}_{ij}^{g}(H) = N \tag{31}$$

By construction, $\tilde{\theta}_{ij}^{g}(H)$ measures the pairwise directional connectedness from financial institution j to i at horizon H. In other words, $\tilde{\theta}_{ij}^{g}(H)$ captures the extent to which variations in i's asset returns or volatility can be explained by financial institution j, based on the generalised forecast error variance decomposition.

Considering the specifications on various decomposition, Diebold and Yilmaz (2012) introduce the total volatility spillover index as:

$$S^{g}(H) = \frac{\sum_{i,j=1,i\neq i}^{N} \tilde{\theta}_{ij}(H)}{\sum_{i,j=1}^{N} \tilde{\theta}_{ij}^{g}(H)} \times 100 = \frac{\sum_{i,j,i\neq j}^{N} \tilde{\theta}_{ij}^{g}(H)}{N} \times 100 \tag{32}$$

Diebold and Yilmaz (2012) also provide a specification for directional spillovers. The directional volatility spillover received by market i from all markets j is:

$$S_{i.}^{g}(H) = \frac{\sum_{j=1,j\neq i}^{N} \tilde{\theta}_{ij}^{g}(H)}{\sum_{j=1}^{N} \tilde{\theta}_{ji}^{g}(H)} \times 100 \tag{33}$$

The directional volatility spillovers propagated by market i to all other markets j is:

$$S_{.i}^{g}(H) = \frac{\sum_{j=1,j\neq i}^{N} \tilde{\theta}_{ji}^{g}(H)}{\sum_{j=1}^{N} \tilde{\theta}_{ji}^{g}(H)} \times 100 \tag{34}$$

On the basis of the two equations presented above, we can determine the net volatility spillover coming from market i to all other markets j by simple a subtraction:

$$S_{i}^{g}(H) = S_{.i}^{g}(H) - S_{.i}^{g}(H) - S_{i.}^{g}(H) \tag{35}$$

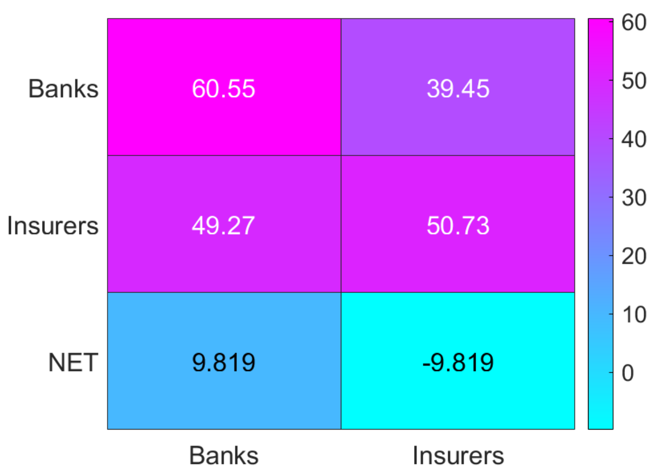
The net pairwise volatility spillover is:

$$S_{ij}^{g}(H) = \frac{\tilde{\Phi}_{ji}^{g}(H) - \tilde{\phi}_{ij}^{g}(H)}{N}.100 \quad (36)$$

In this case, the vector x_t comprises 2 endogenous variables - the conditional volatilies of banks' and insurers' equity returns - and the VAR model is estimated with 1 lag.Following Diebold and Yilmaz (2012), we calculate the overall connectedness of the European bank and insurance sector by implementing the model in a 120-week moving window with an 8-week forecast horizon. This is done to obtain to, from, and net results for the spillover index for the two sectors. The empirical results document that spillovers are present in the data, with a time-varying intensity.

Figure 15, shows the spillovers between the bank and insurance sectors. It's ijth entry is the estimated contribution to the forecast error variance of sector i coming from innovations to market j.

Figure 15: Volatility Spillovers



Notes: The figure shows from, to and net specifications of spillovers from the European bank and insurance sectors. The spillovers are first estimated using a VAR model with two endogenous variables - the conditional volatilities of banks' and insurers' equity. returns. Spillovers are then derived from the forecast error variance decomposition of the underlying VAR.
Source: Model outputs

From the figure we can see that gross directional volatility spillovers from banks to insurers is 49.27% and, from insurers to banks, it is 39.45%. This implies that volatility spillovers from banks to insurers is of greater intensity than vice versa. The net directional volatility spillovers from the banks to insurers is 9.82% and from insurers to banks it is - 9.82%. This would imply that banks are transmitters of volatility shocks whereas insurers are receivers.

10 Conclusion

This paper provides an empirical investigation into the evolution of solvency and systemic risk measures for the European life insurance sector. The European G-II life insurers, considered in this analysis, have maintained solvency capital ratios well in excess of the minimum regulatory requirements. There is some variation across insurers because of differing exposures to the cause of the crisis, such as the European sovereign debt crisis or the UK gilts market crisis. When engaged in traditional insurance activities, that can be managed through standard diversification principles, life insurance companies may not be generating systemic risk. In the absence of systemic risk, with insurers facing only idiosyncratic and diversifiable risk, capital levels derived by assessing solvency risk alone may be viewed as optimum from a regulatory perspective.

In practice, we find that life insurers, engage in activities that can no longer be characterised as traditional and that the sector is exposed to systemic risks. A prolonged phase of exceptionally low interest rates, from the GFC in 2008-09 until after 2021, changed life insurance asset management in Europe. Life insurers reduced their government bond holdings in favour of higher yielding securities and ventured into increasingly riskier and less liquid asset classes. They engaged in more direct lending and made greater use of derivatives to match asset and liability cash flows. It also spurred the transfer of risks to non-affiliate insurers, located in offshore centres. The need to economise on capital has been a driver for some life insurers turning to funded reinsurance. This exposes life insurers to reinsurance counterparty credit risk which tends to be highly concentrated. These non-core activities have altered the risk profile of the life insurance sector and may be beyond the purview of traditional solvency regulation.

As a consequence of these non-core activities, taken in response to market movements, life insurers' share more characteristics with banks. Our findings provide insights into the dynamics of systemic risk in the European bank and insurance sectors. We have shown that bank and insurance sector systemic risk co-move because they have a common exposure to financial markets and the real economy. However, the nature of the crisis determines the degree to which banking or insurance sectors are impacted. The unconventional monetary policies adopted by the BoE and ECB targeted longer term interest rates which had a greater relative impact on the systemic risk of life insurers given the longer-term duration of annuities on the liability side and longer-term corporate bonds on the asset side. On the other hand, the surge in interest rates in 2022-23 led to valuation losses and heightened funding costs in the banking sector.

The main conclusion of this paper is that there is a case for prudential regulation to consider adding a systemic component to capital adequacy that is linked to the life insurer's contribution to systemic risk.

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A Estimation method for Marginal Expected Shortfall

Following Brownlees and Engle (2017) we specify a bivariate GARCH-DCC process for returns:

$$R_t = H_t^{1/2} v_t \tag{37}$$

where $R_t = (R_t^s, R_t^i)'$ is the (2 x 1) vector of system and individual bank returns, and the random vector of the disturbance terms $v_t = \left(\varepsilon_t^s, \xi_t^i\right)'$. In this model, the disturbances $(\varepsilon_{s,t}, \xi_{i,t})$ are assumed to be independently and identically distributed over time and have zero mean and unit variance. But they are not considered to be independent of each other. Episodes of financial stress have a systemic element and tend to affect most financial institutions. H_t is the (2 x 2) variance-covariance matrix. $\sigma_{s,t}$ and $\sigma_{i,t}$ are the volatilities of the financial system and the individual institution at time t, and $\rho_{i,t}$ is the correlation at time t between R_t^s and R_t^i. The H_t matrix denotes the conditional variance-covariance matrix[10]:

$$H_t = \begin{pmatrix} \sigma_{s,t}^2 & \rho_{si}\sigma_{s,t}\sigma_{i,t} \\ \rho_{is,t}\sigma_{i,t}\sigma_{s,t} & \sigma_{i,t}^2 \end{pmatrix}, \tag{38}$$

We consider the Cholesky decomposition of the variance-covariance matrix H_t:

$$H_t^{1/2} = \begin{pmatrix} \sigma_{s,t}^2 & 0 \\ \sigma_{i,t}\rho_{i,t} & \sigma_{i,t}\sqrt{1-\rho_{it}^2} \end{pmatrix} \tag{39}$$

To estimate MES, we first model the bivariate process of firm and market returns. Given equation 37, this can be expressed as:

$$R_{s,t} = \sigma_{s,t}\varepsilon_{s,t} \tag{40}$$

$$R_{i,t} = \sigma_{i,t}\varepsilon_{i,t} \tag{41}$$

$$R_{i,t} = \sigma_{i,t}\rho_{i,t}\varepsilon_{s,t} + \sigma_{i,t}\sqrt{1-\rho_{i,t}^2\xi_{i,t}} \tag{42}$$

The MES can, therefore, be expressed more explicitly as a function of correlation and some tail expectations of the standardised innovations distribution:

$$MES_{i,t-1} = E_{t-1}(R_{i,t} \mid R_{s,t} < C) \tag{43}$$

$$= \sigma_{i,t}E_{t-1}(\varepsilon_{i,t} \mid \varepsilon_{s,t} < \frac{C}{\sigma_{s,t}}) \tag{44}$$

$$= \sigma_{i,t}\rho_{i,t}E_{t-1}(\varepsilon_{s,t} \mid \varepsilon_{s,t} < \frac{C}{\sigma_{s,t}}) + \sigma_{i,t}\sqrt{1-\rho_{i,t}^2}E_{t-1}(\xi_{i,t} \mid \varepsilon_{s,t} < \frac{C}{\sigma_{s,t}}) \tag{45}$$

[10]Engle (2002) provides a detailed description of the DCC approach

For the estimation of time-varying correlations, stochastic volatilities and tail expectations we use the model defined in equations (37) and (38.).

A.1 Volatilities

The conditional volatilities are modelled with an asymmetric GARCH specification described in Scaillet (2005).

$$\sigma_{s,t}^2 = \omega_s + \alpha_s R_{i,t-1}^2 + \gamma_s R_{s,t-1}^2 I_{s,t-1} + \beta_s \sigma_{s.t-1}^2 \tag{46}$$

$$\sigma_{i,t}^2 = \omega_i + \alpha_i R_{i,t-1}^2 + \gamma_i R_{i,t-1}^2 I_{i,t-1} + \beta_i \sigma_{i,t-1}^2 \tag{47}$$

where $I_{i,t} = 1_{R_{i,t}<0}$and $I_{s,t} = 1_{R_{s,t}<0}$which can capture the leverage effect. It has been demonstrated empircally that volatility in equity returns tends to increase more with negative shocks than positive ones.

A. 2 Correlation

The time-varying conditional correlations are modelled using the DCC approach introduced by Engle (2002). The variance covariance matrix is written as follows:

$$H_t = D_t R_t D_t \tag{48}$$

where $R_t = \begin{bmatrix} 1 & \rho_{i,t} \\ \rho_{i,t} & 1 \end{bmatrix}$ is the time-varying correlation matrix of the system and firm returns and $D_t = \begin{bmatrix} \sigma_{i,t} & 0 \\ 0 & \sigma_{s,t} \end{bmatrix}$ is a diagonal matrix for the conditional standard deviations.

The DCC framework introduces what is referred to as a pseudo-correlation Q_t, which is a positive definite matrix such that:

$$R_t = diag(Q_t)^{-1/2} Q_t diag(Q_t)^{-1/2} \tag{49}$$

where $diag(Q_t)$ is such that $diag(Q_t)_{i,j} = (Q_t)_{i,j} 1_{i=j}$.

In the DCC framework, $Q_t$is defined as

$$Q_t = (1 - a - b)S + a\eta_{t-1}\eta_{t-1}^{'} + bQ_{t-1} \tag{50}$$

where $\eta_t = (\varepsilon_{i,t}\varepsilon_{s,t})^{'}$is the vector of standardised residuals, a and b are scalars. $S = E[\epsilon_t \varepsilon_t^{'}]$ is the unconditional correlation of the standardised residuals and is referred to as the intercept matrix. Q_t is a positive definite matrix under certain conditions which are $a > 0$, $b > 0$, $a + b < 0$ and the positive definiteness of S. The matrix S is estimated by

$$\hat{S} = \frac{1}{T} \sum_{t=1}^{T} \eta_t \eta_t^{'} \tag{51}$$

The DCC model is estimated via QML. The steps involved in estimating the dynamic correlation are described in Engle and Sheppard (2008).

A. 3 Tail Expectations

If we recall equation (45), then the remaining terms to be estimated in order to obtain the MES are the two conditional tail expectations: $E_{t-1}(\varepsilon_{s,t} \mid \varepsilon_{s,t} < \frac{C}{\sigma_{s,t})})$ and $E_{t-1}(\xi_{i,t} \mid \varepsilon_{s,t} < \frac{C}{\sigma_{s,t}})$. The term $E_{t-1}(\xi_{i,t} \mid \varepsilon_{s,t} < \frac{C}{\sigma_{s,t}})$ captures the tail-spillover effects from the financial system to the bank that is not captured by the correlation. Furthermore, if both marginal distributions of the standardised returns are unknown, then the conditional expectatiion $E_{t-1}(\varepsilon_{s,t} \mid \varepsilon_{s,t} < \frac{C}{\sigma_{s,t})})$ is also unknown. As a consequence, both tail expectations must be estimated. In doing so, we use a non-parametric kernel estimation which is described in ....In this analysis we consider a threshold C equal to the VaR of the financial system i.e., $C = VaR_{i,t}(\alpha)$. Then , if the standardised innovations, $\varepsilon_{s,t}$ and $\xi_{i,t}$are i.i.d , the nonparametric estimates of the tail expectations are given by:

$$E_{t-1}(\varepsilon_{s,t} \mid \varepsilon_{s,t} < k) = \frac{\sum_{t=1}^{T} K(\frac{k-\varepsilon_{s,t}}{h})\varepsilon_{s,t}}{\sum_{t=1}^{T} K\left(\frac{k-\varepsilon_{m,t}}{h}\right)} \tag{52}$$

$$E_{t-1}\left(\xi_{i,t} \mid \varepsilon_{s,t} < k\right) = \frac{\sum_{t=1}^{T} K\left(\frac{k-\varepsilon_{s,t}}{h}\right)\xi_{i,t}}{\sum_{t=1}^{T} K(\frac{k-\varepsilon_{s,t}}{h})} \tag{53}$$

where $k = VaR_{i,t}(\alpha)/\sigma_{s,t}$, and

$$K_t(h) = \int_{\infty}^{\frac{t}{h}} k(u)du \tag{54}$$

where $k(u)$ is a kernel function and h a positive bandwith. Following ..., we fix the bandwith at $T^{-1/5}$ and choose the standard normal probability function as a kernel function, i.e., $k(u) = \phi\left(u\right)$ .Using the GARCH-DCC model desribed above to determine the conditional variance and correlation we can arrive at a an expression for the MES as follows:

$$MES_{i,t} = \sigma_{i,t}\rho_{i,t}E_{t-1}\left(\varepsilon_{s,t} \mid \varepsilon_{s,t} < k\right) + \sigma_{it}\sqrt{1-\rho^2}E_{t-1}(\xi_{it} \mid \varepsilon_{s,t} < k) \tag{55}$$

B Estimating SRISK using the multivariate GARCH-DCC model and dynamic conditional betas

SRISK is estimated without simulation in 4 steps. The first of these models the bivariate distribution of individual institution and market returns as for the MES systemic risk measure. In particular, a GJR-GARCH-DCC(1,1) (asymmetric) model is specified, where individual returns are assumed to have a skewed standard t-distribution, and the joint distribution of returns is assumed to be a multivariate t-distribution[11].

[11]That is, a simple parametric approach, as opposed to a copula or extreme-value theory approach, is employed in this paper.

The variance-covariance matrix that results from the first step is used to estimate a time-varying beta (dynamic conditional betas). A nested DCB model is then estimated (56), whose consistency is guaranteed by the consistency of the initial step's estimation procedure, allowing the data to choose between a constant and time-varying beta coefficient.

$$R_{i,t} = (\theta_1 + \theta_2 \beta_{i,t}) R_{m,t} + \sqrt{h_{i,t}} \varepsilon_{i,t} \tag{56}$$

This estimated beta is then used to calculate the LRMES by the formula presented on the New York University Stern School of Business's Volatility Institute website, $LRMES_{i,t} = 1 - exp(log(1 - P(crisis)) * \tilde{\beta_{i,t}})$. This formula reflects the 6-month joint distribution of an individual institution's returns and that of the market, conditional on a severe negative shock to the market ($crisis$). The final step calculates SRISK according to equation (23).